

A Guided Matter-Wave Sagnac Interferometer

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Abstract

This thesis presents the experimental scheme for a guided cold atom Sagnac interferometer. Although atom interferometers have become an established method for precision measurements, a feasible clock-scheme based on fully guided atoms has yet to be realised. Radio frequency potentials, generated by a chip, provide a means to create a compact, fully controllable guided atom interferometer. The experimental infrastructure for a rubidium cold atom system is built and characterised. A chip-based atomic clock scheme is presented, utilising state-dependent guiding around a ring shaped trap. The chip design to form the trap and operate the interferometer is presented, and a vacuum compatible PCB has been designed and commercially manufactured to provide atomic transport and compression towards an atom chip. This project is part of the larger MatterWave project¹. In collaboration with Birmingham and Crete, the Matterwave project aims to develop ultra-sensitive matter-wave interferometry for compact devices.

¹<http://matterwave.eu/>

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with your impressive judo throws!

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that you can't indoors wear roller-skates!

Nathan Cooper - There is nothing as exciting,
as a puzzle put in writing!

Jon Nute - A lab is for equipment keep,
not for polyphasic sleep!

Elisa da Ros - The English words you keep on tweaking,
immensibly improved my English speaking!

Jorge Ferreras - With the caffeine from your coffee making,
we fix the labs that keep on breaking!

Thomas Fernholz - All of physics you can understand,
with a complex gesture of the hand!

*The light dove cleaving in flight the thin air,
wind bearing forever on her wings,
might imagine she could fly more easily,
were it not for the resistance it brings.*

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Chapter 1

Introduction

This thesis describes the design and building of a cold atoms setup, with the ultimate aim of realising a guided matter-wave Sagnac interferometer. This project exists within a wider area of experimental physics and technological aims, and it is worth beginning the discussion in this context. As of 2013, the UK government invested £270 million to launch the national quantum technologies programme, a nation-wide effort involving the collaboration of academia and industry partners to bring quantum technology out of the lab and into operational environments¹.

As the experimental field of ultra-cold atoms has matured, applications in quantum simulation [1] [2], computation [3] and quantum metrology [4] [5] have been identified. Cold atom interferometers could potentially provide the next generation of inertial sensors [6] [7] [8] and gravimeters [9] [10] [11] [12], although major technological hurdles must still be overcome before they become attractive to an end user.

The goal of this chapter is to present the broader context of which this work is a part - from the physics of ultra-cold atoms and atom interferometry, to the applications and operational requirements of inertial sensing.

¹<http://uknqt.epsrc.ac.uk/>

1.1 Ultra-Cold atoms

Absolute zero is unreachable, but we can get arbitrarily close. Combining the techniques of laser cooling, magnetic trapping and RF evaporation, temperatures below one nanokelvin have been reached in dilute atomic vapours [13]. Below a critical temperature, a gas of bosonic atoms condense into a single quantum ground state - a Bose-Einstein condensate (BEC). Dubbed the coldest place in the universe, it was predicted in 1925 [14] and achieved 70 years later [15] [16]. A new state of matter, though hardly a state of nature, as it is hard to envision how such a state could ever occur naturally. Orders of magnitude colder than outer space, a condensate is a man-made phenomenon.

At ultra-low temperatures, quantum gasses exhibit macroscopic coherence across the cloud in a beautiful demonstration of wave-particle duality. The relationship between waves and particles was first expressed in Planck's Seminal formula $E_{\text{photon}} = hf$, relating the energy of a photon to its frequency by the Planck constant h [17]. Louis de-Broglie in 1924 [18] extended this analogy to matter, postulating that discrete particles of matter were related to an equivalent wavelength $\lambda_{dB} = h/P$ where P is the momentum of the particle. In terms of temperature, we can think of the atoms of mass m in the cloud as having a thermal de-Broglie wavelength $\lambda_{dB} = \sqrt{2\pi\hbar^2/mk_BT}$, where k_B and $\hbar$ are the Boltzmann and reduced Planck's constants. As this wavelength becomes comparable to the inter-atomic distance, the cloud begins to display a wave-like behaviour. In this sense λ_{dB} parametrizes the degree of spatial coherence of the atomic cloud.

The field of ultra-cold atoms is relatively recent. The invention of tunable dye lasers in the 1970s made it possible for lasers to be used for cooling, and stimulated an explosion of progress in the field of magnetically trapping, cooling and manipulating atoms [19]. In the 1980s, magnetic and optical trapping and cooling techniques were combined, resulting in the magneto-optical trap [20]. With this key technique, researchers were able to cool atomic clouds down to micro-Kelvin temperatures. Evaporative cooling in a

purely magnetic trap then allowed temperatures below the limits of optical cooling to be reached [21]. For the first time since a condensate was theoretically predicted, researchers were now in reach of the critical temperature, and clamoured to find the expected phase transition. Two separate groups raced to it, and both demonstrated a BEC in 1995 for which they shared the Nobel prize [15] [16] [22].

After the successful development of atom cooling techniques, the focus has shifted towards applications. The high degree of coherence and control offered by cold atom experiments makes them suitable for fundamental tests of physics [23], quantum simulations [2] [24] [25], and atomic clocks [26] [5] and interferometry [27] [10].

1.2 Atomic Clocks

From the pendulum to the quartz crystal, accurate time-keeping has relied on a precise and stable oscillator. Atomic clocks provide the most accurate time-keeping in the world, and work by locking a microwave oscillator to an atomic transition. Atomic clocks utilize the Ramsey method of separated oscillatory-fields [28], which was a development upon Rabi resonance (single oscillating-field method). The Ramsey method has since become a key technique for precision measurements in atomic physics, including atom interferometry.

Atoms, prepared initially in the ground state $|1\rangle$, are transferred to the excited state $|2\rangle$ by pulses of an on-resonance oscillatory field. Detection of the final state population provides a feedback signal to stabilise and lock the oscillator. The atoms enter an interaction region, where they receive a microwave $\pi/2$ excitation pulse. This first pulse places the atoms into a super-position of states $|0\rangle$ and $|1\rangle$. The atoms then move in a field free region for some time. A dynamical phase can be associated with the super-position, which evolves at a frequency proportional to the energy splitting between the states $|0\rangle$ and $|1\rangle$. This phase dictates the effect of a second $\pi/2$ pulse. Applying a second $\pi/2$ pulse after some evolution time, the atom is either transferred to the $|1\rangle$ state or back down to the $|0\rangle$ state.

The phase ϕ of the final $\pi/2$ pulse can be shifted, affecting the final population in the excited state. The final state population then varies with the pulse phase as $\cos^2(\phi)$. Scanning the phase and measuring the state population results in Ramsey fringes. Provided the energy splitting of the two states remain constant, and there is nothing to perturb the path length of the atoms, the Ramsey fringes should provide an absolute frequency reference, by which the microwave oscillator is locked.

Cold atoms provide an advantage on account of the reduced linewidth of the atomic transition. At the National Physics Laboratory, a clock based on a laser cooled Caesium fountain operates with a short time sensitivity of $10^{-14}s$ [5].

1.3 Atom Interferometry

Since de-Broglie first postulated the wave-like equivalent of matter [18], the Young's double-slit [29] experiment has been used to interfere electrons [30] [31], neutrons [32], and finally neutral atoms in 1991 [33] [34] [35] [36]. Atom interferometry is a diverse field with a range of types and techniques. For a thorough review see reference [10].

It is helpful to distinguish two types of interference effect. An interference pattern can be spread out in space, as in the original double slit experiment. This may be called a spatial fringe pattern. Although it is useful for demonstrating the wavelike nature of light and atoms, it is not typically desirable for metrology. In contrast, the beams can be recombined to produce a single intensity measurement at a detector, as in figure 1.1. As the phase difference of the beams is changed, fringes of intensity can be recorded at the detector. In the following discussion, we will call these amplitude fringes.

Atom interferometry can produce both types of pattern. Spatial fringes can be observed in Bragg interferometers, or simply by releasing and overlapping two condensates. Bragg schemes place the atoms in a super-position of momentum states. As Bragg schemes keep the atoms in the same internal state, they are insensitive to external electromagnetic perturbations. The

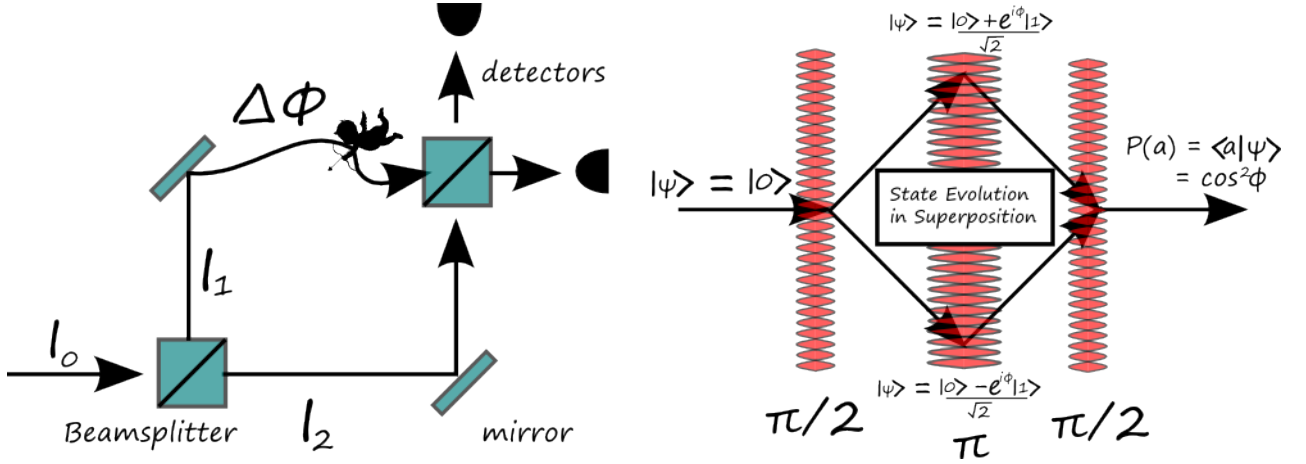


Figure 1.1: Left: Mach-Zehnder Interferometer. Intensity fringes at the detectors indicate a change in relative path length. Right: Raman beams transfer momentum to split and recombine the atoms. They also serve as interaction regions for a Ramsey clock scheme.

result is a spatial interference pattern, which must be resolved with imaging. Recently, a Bragg based gravimeter has been demonstrated [37].

Amplitude fringes can be achieved with Ramsey-Borde interferometers. A Ramsey-Borde interferometer will also split and recombine the beams, but additionally transfers the atoms into a superposition of internal states. This is then very similar to an atomic clock. A set of microwave Ramsey pulses is applied (typically $\pi/2 - \pi - \pi/2$), and the final state population is measured after the last $\pi/2$ pulse. This thesis is concerned with the Ramsey-Borde type of interferometers.

It is common for interferometry to take the form of a Mach-Zehnder configuration. This requires, in analogy with optical interferometers, that the beams can be split, reflected and recombined (see figure 1.1). Early experiments used diffraction gratings to split and recombine the atoms [33]. A spatial fringe pattern could be seen by scanning the detector laterally.

Diffraction of atoms can instead be achieved with laser-gratings, consisting of standing-wave laser beams [36]. This has significant advantages over the mechanical gratings, which have to be precisely micro-fabricated and are difficult to adjust [34] [33]. Tuning the beam width allows it to act as either a beam-splitter/re-combiner, or as a reflector. In addition to transferring momentum to the atoms, the beams can also perform the $\pi/2$ and π pulses

necessary to operate a Ramsey-Borde interferometer.

Atom interferometers are extremely sensitive, and have found applications in precision measurements of the fundamental constants [38], gravity [39] [40] [41] [37], and are sensitive to inertial forces such as linear acceleration and rotation [27] [8].

The sensitivity of atom interferometers scales with the free-evolution time of the atoms. This time is typically limited to about 0.1s in free-fall interferometers. Due to gravity, it is difficult to extend this time without making the setup very tall. This requirement imposes serious limitations on the possibility of miniaturizing the devices.

1.4 Guided Atom Interferometry

On account of the limited evolution times of schemes based on free-falling atoms, interferometry using confined atoms is very desirable. Confining the atoms against gravity could allow long evolution times while keeping the device small. Unfortunately, the confining potential typically de-coheres the phase evolution of the atoms, creating a new limitation on the evolution time. The challenge of guided atom interferometry is therefore to design a confining scheme that minimally perturbs the phase of the atoms.

Although a number of schemes have demonstrated interferometry of matter-waves in a guided setting [42] [43] [44] [45] [46] [47] [48], a truly robust scheme has not yet been demonstrated. A fundamental challenge to maintaining phase coherence is atom-atom interactions and corrugations in the confining potentials [47].

The challenges present in guided schemes can be illustrated by some examples, beginning with the case of a large magnetic storage ring for condensates [49]. A BEC is injected at the top, which splits and propagates around the ring toward the bottom where it recombines and produces an interference fringe pattern [44]. Fringes were visible, but they occurred differently on each experimental run. Although a condensate is itself coherent, in order to perform interferometry for the purposes of metrology, it is important that

the relative phase of the condensates does not change in each experimental run. Coherent splitting and recombining is required to preserve the relative phase of the atomic ensembles.

One scheme of coherent splitting involves deforming a single trap into a double well. A condensate contained in the trap should then coherently split into two, provided the deformation is slow. The recombination is then achieved by releasing the trap and letting the condensates overlap. This was first achieved in 2004 using an optical potential [42].

Schemes utilising two-dimensional confinement are possible. One group used two-dimensional confinement, with a nearly free longitudinal axis. The atoms can be guided along the path by a moving off-resonant standing-wave laser beam [47]. A $70ms$ coherence time was achieved. In this case, the free dimension is not totally free, and is effectively an elongated harmonic trap. This residual potential results in phase gradients and de-coherence along the propagation path, imposing a limitation on the path length and coherence time. Straight paths were used, as curved guides induce further de-phasing of atoms, making closed loop interferometers particularly difficult.

Atom chips allow precise, tight, and miniature confinement of atoms, and are therefore a hopeful candidate for realizing guided interferometry. Radio frequency dressed traps have been used to coherently split BECs on an atom chip [45]. Creating a condensate in two separated traps resulted in a fringe pattern with a random phase. Creating a single condensate in an RF dressed trap, splitting the trap, and releasing; resulted in a consistent fringe pattern. This demonstrated that the method of splitting was indeed coherent. A coherence time of up to $200ms$ has been achieved RF double wells [48]. The recombination was achieved by releasing the cloud and observing spatial fringes.

Although these demonstrations are promising, they have limitations. The experiments typically used condensates, which exhibit strong atom-atom interactions and give rise to an effect known as phase diffusion [47] - limiting the coherence time. Fortunately, a condensate is not necessarily required to

observe interference. Although colder atoms have advantages, the first atom interferometry was performed with a highly collimated beam of supersonic helium atoms [33]. Interferometry with cold atoms above the critical temperature ('cold thermal atoms') has been demonstrated, although the effect of temperature has been shown to give rise to a reduction in fringe visibility [50] [51].

An additional problem is that, although coherent splitting has been demonstrated, coherent recombination was not obtained in the guided sense. In the above examples, recombination was achieved by simply releasing the cloud and observing the spatial fringe pattern.

No guided scheme has yet been demonstrated that can operate as an atomic clock, producing useful amplitude fringes for the purposes of metrology. In this thesis, a fully guided scheme utilising radio frequency manipulation of a cold thermal cloud is presented. A Ramsey clock sequence is to be performed on atoms transported around a ring in a state-dependent potential. This is presented in more detail in chapter 3. The advantages of this design are manifold; robust against noise, fully controllable, and suitable for implementation on an atom chip. The most natural application for a ring geometry such as this, is for rotation sensing via Sagnac interferometry.

1.5 The Sagnac Effect

A ring interferometer experiences a phase shift proportional to the rotational rate of the inertial frame. This is known as the Sagnac effect [52]. Although it was originally demonstrated for light waves propagating around a closed loop, it is also observable in matter-waves. In fact, in the case of matter-waves a considerable advantage of sensitivity may be gained. In the case of an atom interferometer enclosing an area $\mathbf{A}$ and rotating at angular speed $\mathbf{\Omega}$, we can write the phase shift $\Delta\phi_S$ for a particle of energy E , as: [53]:

$$\Delta\phi_S = \frac{4\pi E}{hc^2} \mathbf{A} \cdot \mathbf{\Omega} \quad (1.1)$$

The sensitivity of this phase shift is proportional to the area enclosed, and the energy of the photon or atom. To demonstrate the potential sensitivity gain of using atoms, we may compare the phase shift that would result from a rotation. Comparing the rest mass energy of an atom to the energy of a photon, this phase shift is roughly a factor of 10^{10} greater for atoms than for light, for the same enclosed area (See chapter 3). It should be noted however, that atom interferometers typically have a much smaller enclosed area than their optical counterparts, which have the advantage of looping an optical fibre many times to increase the area. Nonetheless, atomic Sagnac Interferometers are already comparable to their optical counterparts [54]. The large Wettzell ring laser gyroscope achieves a sensitivity of $10^{-11} rad/\sqrt{s}$ [55]. Atom interferometers have been demonstrated with sensitivities below $10^{-9} rad/\sqrt{s}$ [56], orders of magnitude greater than commercial devices.

Precision measurements of the Sagnac effect have been applied to geodesy [57], and tests of general relativity [58]. The most common application however is an inertial sensor, particularly as part of an inertial navigation system [8].

1.6 Inertial Sensors

Inertial sensors are used to measure rotation, linear acceleration and gravity. Cold atom inertial sensors may offer many orders of magnitude greater sensitivity over classical devices, reducing the integration time of gravimeters and the bias drift of inertial navigation systems (INS) [7] [9].

INS are able to track the position of a platform on which they are mounted with respect to an initial reference point, by measuring the inertial forces as the platform rotates and accelerates [59]. The core element of an INS is the inertial measurement unit (IMU), which typically consists of three orthogonal gyroscopes and three linear accelerometers. As the determination of position requires integration over all previous measurements, the error rapidly accumulates, and consequently INS suffer from the problem of bias-drift. Even a strategic grade INS can accumulate $100m$ of error over $1000s$ of integration [7]. Cold atom interferometers, with high precision and low systematic drift, may improve the performance and autonomy of inertial navigation.

Linear accelerometers can also function as gravimeters, which measure the local gravitational acceleration. Portable gravimeters can produce gravity maps, allowing the discovery of sink-holes, underground pipes and oil reserves. Current classical gravimeters - typically based on a mechanical spring or a falling corner-cube - are slow, as they require long integration times to achieve the desired sensitivity. Portable cold atom gravimeters are being developed by a number of labs [11] [9] and companies [12]². Recently, the French aerospace Lab ONERA demonstrated absolute gravimetry on a marine platform using matter-wave interferometry [12].

²<https://www.muquans.com/>

1.7 Operational Requirements and Challenges

For application outside of a lab environment, precision is not the only important feature of a sensor. Low size, weight and power ('low SWAP'), reliability and robustness against noise are often essential for use outside of a lab environment and for commercial viability. An inertial sensor, by its nature, must be able to cope with a range of accelerations, and possibly a certain amount of shock. Additional desirable features include; measurement along multiple axes, operation in multiple orientations and high sample rate.

Typically, cold atom interferometers (CAIs) are unable to meet all of these requirements, and require integration with classical sensors as part of a hybrid device [60]. These hybrid approaches have been demonstrated [7], with the CAI making high precision measurements, while the classical sensors improve the dynamic range and make up for the various short-falls of the device. These challenges and solutions will now be discussed.

Axes and Orientation

As current CAIs rely on vertical freefall of atoms, they are typically not able to measure the acceleration along horizontal axes, nor can they cope with multiple orientations. Gimball mounted platforms are therefore required to keep the interferometer correctly oriented [12].

A six-axis linear and rotational inertial sensor was demonstrated at SYRTE in 2006 [6]. This utilised two counter-propagating sources of cold atoms in a 'butterfly' geometry. Hybrid approaches are also feasible, where classical accelerometers are used for the horizontal axes, and a CAI used for the vertical axis [7].

Sample Rate

A fast sample rate is particularly important in inertial navigation, where it is required to prevent accumulation of error. In a CAI, this amounts to elimination of the dead time between measurements. These can often be substantial in a cold atom setup, as the atomic sample must first be cooled

and prepared. This would be even more difficult in an atom chip scheme, where the atomic cloud must be first compressed into a microscopic trap. Elimination of dead-time is also a desired feature for precision measurements, as it has been identified as a major limitation to the sensitivity; on account of having the effect of aliasing high frequency noise [53] [61].

A high data-rate linear accelerometer was demonstrated at Sandia National Laboratories in New Mexico [62]. In addition, continuous operation of a cold atom Sagnac Interferometer has been demonstrated at the SYRTE research labs in Paris [8]. Operating without dead-time, they achieved a short term sensitivity of $100\text{nrad/s}/\sqrt{\text{Hz}}$. Continuous operation did not adversely affect the long term rotation stability, which was 1nrad/s after 1000s of integration time. A hybrid scheme can again be employed, with classical sensors providing the high sample rate, and the CAI providing a more accurate measurement to correct for the bias error in the classical systems [7].

Dynamic Range

In addition to a very high sensitivity, the inertial sensor must operate over the range of accelerations expected on the platform. Sensitivity comes at the cost of dynamic range, as the output phase takes the form of a periodic fringe pattern. The output phase therefore repeats itself after one half of the fringe spacing, and the effects that cause phase shifts beyond this cannot be uniquely determined.

A hybrid scheme can again be employed. A classical accelerometer can give a coarse reading of the acceleration, and inform the CAI of which fringe location it is operating on. In the case of a linear accelerometer, A MEMS accelerometer can be placed behind the mirror of a Raman beam [7]. In the case of a cold atom gyroscope, it could potentially be mounted to a rotation table, which keeps the rotational phase shift at the maximal slope of the fringe (where the sensitivity is greatest).

Schemes relying on freely-propagating atoms will nonetheless be inherently limited in dynamic range. Due to the inertia of a falling atom, the atoms will

not follow the motion of the device as it accelerates. Supposing that, during an experimental sequence the atoms must remain within a particular region, we can estimate the maximal acceleration the device can take. Accelerating at rate a , the platform and device will move a distance $d = at^2$ in time t . In the case where atoms are interrogated using Raman beams, the interaction region is the beam width, $d \approx 2\text{cm}$. The atoms should remain in this region for the duration of an experimental cycle, say $t \approx 0.1\text{s}$. The acceleration, then, must be less than $d/t^2 \approx 2g$. Therefore, a free-fall based CAI cannot sustain an acceleration of more than $\approx 2g$, as the atoms will leave the interaction region.

It is hard to envisage how this could be overcome without the confinement offered by guided schemes. By tightly confining the atoms, guided schemes could overcome many of these limitations, allowing operation over a range of accelerations, axes and orientations.

In summary, it is unlikely that a cold atom interferometer could meet the needs of a robust inertial sensor without the aid of classical devices. These hybrid schemes compensate for the limitations of sample rate, dead-time, dynamic range, and multiple axes. Atom interferometers may still play a part however, contributing to the sensitivity and long term stability of the system by correcting for the accumulated drift inherent in classical devices.

The wider research area has been discussed, and the challenges of creating such a device, and deploying outside of a lab have been outlined. With these in mind, it is not realistic to expect any single scheme to be a complete solution to all these challenges. Nonetheless, a demonstration of a guided interferometer scheme would be a great leap forward. Guided schemes could provide a way of realizing truly compact, robust and portable cold atom interferometers. However, a guided scheme which can be feasibly used to operate a Ramsey-Borde interferometer has not yet been demonstrated.

1.8 This Thesis

This thesis describes the building and designing of a cold atoms set-up for a guided matter-wave Sagnac interferometer. This involved the creation of a cold atoms lab from scratch. The electronics control, vacuum, laser and optics systems were established. Chips and a transport PCB were designed with the purpose of transporting and compressing the atomic cloud, providing an RF-dressed ring trap, and operating microwave Ramsey pulses.

Preliminary experimental results were obtained. A magneto optical trap and purely magnetic trap of rubidium-87 was demonstrated. This was implemented in the case of a macroscopic ring trap, and later in a trap formed by the transport PCB. The ultimate goal of this research project is to measure the Sagnac effect using a guided scheme. The structure of this thesis is as follows:

- Chapter 2 - Cold Atoms. An overview of the field of cold atoms is given. The common trapping and cooling techniques are discussed, including magnetic trapping, laser cooling and evaporative cooling, the Magneto Optical Trap, and Atom chips.
- Chapter 3 - A Matter-Wave Sagnac Interferometer. The Sagnac effect is presented in more detail. The optical, atomic and guided schemes are compared. The scheme for a guided matter-wave Sagnac interferometer is presented.
- Chapter 4 - Chip Design. The chip designs are presented. These include a PCB for lateral transport and compression of the atomic cloud, a microwave chip for providing Ramsey pulses, a ring chip and an RF chip for creating the controllable RF-dressed ring trap.
- Chapter 5 - Experimental Setup - Magnet Ring. The experimental set-up is described, and experimental results regarding the loading of a macroscopic ring-trap are presented.

- Chapter 6 - Experimental Setup - Transport PCB. The transport PCB set-up is described - with some preliminary results.

The work presented in this thesis has involved the publication of two research papers:

- R. Stevenson, M. R. Hush, T. Bishop, I. Lesanovsky, and T. Fernholz. Sagnac interferometry with a single atomic clock. *Phys. Rev. Lett.*, *115:163001*, Oct 2015.
- M. R. Hush, I. V. Lesanovsky, T. Bishop, F. Gentile, S. Jammi, T. Pyragius, M. G. Bason, H. Mas, S. Pandey, G. V. Konstantinos, P. Wolf von Klitzing, T. Fernholz, R. Stevenson. Towards rotation sensing with a single atomic clock. *Proc.SPIE*, *9900:14*, 2016.

Chapter 2

Cold Atoms

Magnetic trapping and laser cooling are the hallmark techniques of ultra-cold atom experiments. A magneto-optical trap (MOT) combines these techniques, and is typically the first stage of cooling. The presence of laser light limits the temperature, and transfer to a purely magnetic trap is required. Here the cloud can be evaporatively cooled. Provided that sufficient atoms can be contained, there is no clear limit to this technique, and it is regularly used to create Bose-Einstein condensates.

The ability to perform these techniques relies on an understanding of the interaction of an atom with a magnetic and optical field. The electronic energy level structure of rubidium, and its interaction with magnetic, RF and optical fields is presented; with applications in magnetic trapping and cooling, RF-dressed traps, laser cooling and MOTs. Finally, the role of atom chips for creating miniaturized traps is discussed.

2.1 Rubidium Energy Structure

Alkali atoms are widely used in cold atom experiments. A large vapour pressure at a temperature of a few hundred degrees makes them suitable for dispensing inside a vacuum chamber using a heating element. A convenient energy level structure - with transitions in the optical and microwave domain - make them suitable for laser cooling and microwave Ramsey interferometry. Rubidium, as a composite Boson with net integer spin, can also undergo Bose-Einstein condensation.

Alkali atoms have the simplest energetic structure in the periodic table. Occupying the first group of the table, they are characterised by having only a single electron in the outer shell. Although hydrogen also occupies the first group, it is not generally considered to be an alkali. Nonetheless, the energetic structure of hydrogen can be used to form the basis of a discussion on alkali energy levels.

The energetic structure can be decomposed into coarse, fine and hyperfine structure. The coarse structure is dependent on the principle quantum number n and the orbital electron spin L . The fine and hyperfine structure arise from interaction between the electron intrinsic spin $\mathbf{S}$, the electron orbital spin $\mathbf{L}$, and the nuclear spin $\mathbf{I}$ [63].

2.1.1 Course Structure

Beginning with the coarse structure, it can be largely described by the Bohr model of hydrogen. Hydrogen is a single electron atom, with a spherically symmetric Coulomb potential. The Schrödinger equation then yields a set of solutions designated by the four quantum numbers n , L , m_L and m_S . We can designate such a state then as $|nLm_Lm_S\rangle$. Due to the original Bohr model of the atom, these states are often interpreted in terms of shells. The principle quantum number n describes the electron shell and takes positive integer values.

Within this shell, so-called sub-shells may be defined by the electron orbital angular momentum quantum number L . This is related to the orbital

angular momentum operator by:

$$\hat{\mathbf{L}}^2 |nLm_L\rangle = \hbar\sqrt{L(L+1)} |nLm_L\rangle \quad (2.1)$$

L can take integer values from 0 to $n - 1$. In spectroscopic notation, the values $L = 0, 1, 2$ are labelled the 's', 'p' and 'd' orbitals. Interpreting the orbital angular momentum as a vector, we can say that $\hbar\sqrt{L(L+1)}$ is the magnitude of the angular momentum vector $\mathbf{L}$.

The magnetic quantum number m_L can be understood as the projection of $\mathbf{L}$ onto a specified axis. This is often chosen to be the z-axis by convention. m_L can take values between L and $-L$ in integer steps.

$$\hat{L}_z |nLm_L\rangle = \hbar m_L |nLm_L\rangle \quad (2.2)$$

The spin projection number m_S gives the projection of the intrinsic electron spin $\hat{\mathbf{S}}$ onto a specified axis. m_S can take values between S and $-S$ in integer steps. As an electron is a spin-1/2 particle, m_S can be $+1/2$ or $-1/2$.

In the case of hydrogen, the energy is dependent on the principal quantum number n only, and the various L sub-shells are degenerate. The energy levels in this case are given by the Rydberg formula $E_n = -R/n^2$ where R is the Rydberg constant.

We can extend some of these properties to describe the alkali atoms, which have a single valence electron outside of a number of closed shells. For an alkali atom of atomic number Z , the valence electron experiences a symmetric potential from the nucleus, which is screened by the $(Z - 1)$ electrons of the closed inner shells. The potential is therefore very similar to a hydrogen atom.

Due to the screening, the potential is however no longer Coulombic; and as such the energy becomes dependent on the angular momentum L . This can be interpreted by imagining the classical orbits of the electrons. Those orbits which are more elliptical, that is, of lower L , penetrate into the lower electron shells and experience less screening. The binding energy of these electrons thus becomes affected by L . The result is that, unlike the hydrogen

atom, the energy levels are no longer degenerate in L .

The dependence of the energy on L can be accounted for by the empirically determined quantum defect $\delta(L)$, which modifies the Rydberg formula:

$$E_{n,L} = -\frac{R}{(n - \delta(L))^2} \quad (2.3)$$

This equation is very similar to the Rydberg formula for hydrogen, except for the presence of the quantum defect term, $\delta(L)$. This describes the course structure of the alkali atoms. $\delta(L)$ is largely independent of n . It is largest for the ‘s’ orbital, and decreases for higher L states. The energy level structure of rubidium-87 is shown in figure 2.1. With 4 closed shells, and a single valence electron in the 5th shell, the ground state occurs at $L = 0$ and can be written in spectroscopic notation as $5^2S_{1/2}$. As the energy is not degenerate in L , the first excited state occurs at $L = 1$. It can be seen from figure 2.1 that the energies split into finer manifolds, each requiring the additional quantum numbers J and F to label them.

2.1.2 Fine Structure

The $L = 1$ excited state splits into two energy levels. This is the fine structure, and arises as a result of spin-orbit coupling. The two lines of the fine structure doublet are called the D_1 and the D_2 lines. It is the D_2 line which is typically of interest. The spin-orbit coupling is the interaction between the orbital electron spin $\hat{\mathbf{L}}$, and the intrinsic electron spin $\hat{\mathbf{S}}$. The energy splitting is described by the equation:

$$\Delta E_{fs} \propto \hat{\mathbf{L}} \cdot \hat{\mathbf{S}} \quad (2.4)$$

With the introduction of spin orbit coupling, $\hat{L}_z$ and $\hat{S}_z$ no longer commute with the Hamiltonian; as such their eigenvalues change over time and they no longer describe a conserved quantity. In this sense they are no longer good quantum numbers, and are not suitable for labelling the levels within the fine structure manifold. However, the total electron spin $\hat{\mathbf{J}}$ does commute, and is

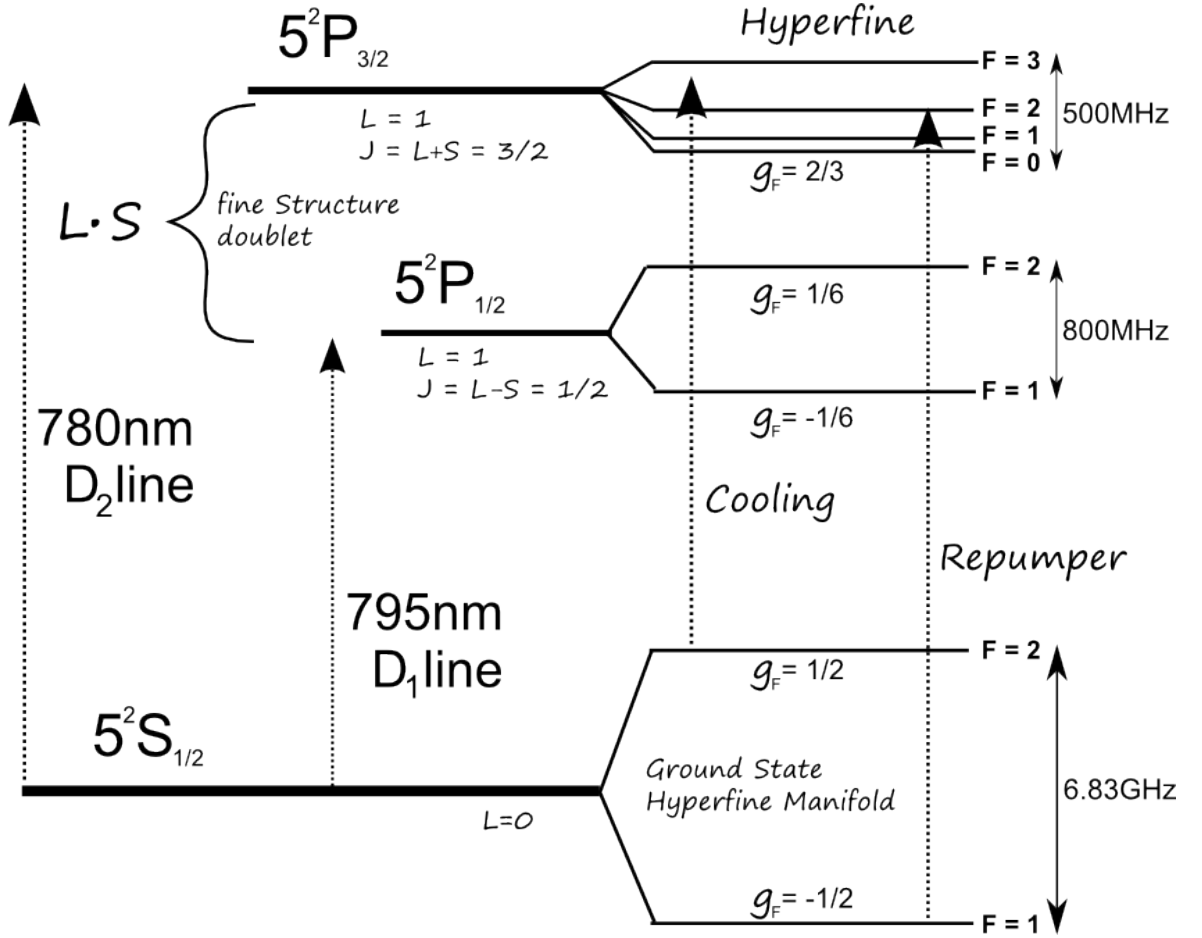


Figure 2.1: Rubidium Level Scheme. The $L = 1$ excited state splits into two lines due to spin-orbit coupling. The ground and excited states split further due to nuclear spin coupling. The hyperfine D_2 transitions are of interest. Laser cooling and re-pumper frequencies are shown. The cooling laser is detuned from the $|F = 2 \rightarrow F' = 3\rangle$ transition.

thus a conserved quantity.

$$\hat{\mathbf{J}} = \hat{\mathbf{L}} + \hat{\mathbf{S}} \quad (2.5)$$

The eigenvalues of $\hat{\mathbf{J}}$ is given by the corresponding quantum number J as $\hbar\sqrt{J(J+1)}$. The coupling term can then be written:

$$\hat{\mathbf{L}} \cdot \hat{\mathbf{S}} = \frac{1}{2}(\hat{\mathbf{J}}^2 - \hat{\mathbf{L}}^2 - \hat{\mathbf{S}}^2) \quad (2.6)$$

The energy eigenvalues of the fine structure splitting E_{hf} can now be expressed in terms of the J , L and S quantum numbers:

$$\Delta E_{fs} \propto \frac{\hbar^2}{2}(J(J+1) - L(L+1) - S(S+1)) \quad (2.7)$$

2.1.3 Hyperfine Structure

Interaction between $\hat{\mathbf{J}}$ and the nuclear spin $\hat{\mathbf{I}}$ induces further splitting, known as the hyperfine splitting. We can again find a conserved quantity whose operator commutes with the Hamiltonian, which is the total spin $\hat{\mathbf{F}}$:

$$\hat{\mathbf{F}} = \hat{\mathbf{J}} + \hat{\mathbf{I}} \quad (2.8)$$

which has eigenvalues $\hbar\sqrt{F(F+1)}$. Alternatively, we can envisage the total spin as a vector $\mathbf{F}$ with magnitude $\hbar\sqrt{F(F+1)}$. The coupling between the total electron and nuclear spin can now be written:

$$\hat{\mathbf{I}} \cdot \hat{\mathbf{J}} = \frac{1}{2}(\hat{\mathbf{F}}^2 - \hat{\mathbf{J}}^2 - \hat{\mathbf{I}}^2) = \frac{\hbar^2}{2}(F(F+1) - J(J+1) - I(I+1)) \quad (2.9)$$

The Hamiltonian for the hyperfine splitting is given as [64]:

$$\hat{H}_{hfs} = A_{hfs} \hat{\mathbf{I}} \cdot \hat{\mathbf{J}} + B_{hfs} \frac{3(\hat{\mathbf{I}} \cdot \hat{\mathbf{J}})^2 + \frac{3}{2}(\hat{\mathbf{I}} \cdot \hat{\mathbf{J}}) - I(I+1)J(J+1)}{2I(2I-1)J(2J-1)} \quad (2.10)$$

Here A_{hfs} is the magnetic dipole constant. B_{hfs} is the electric quadrupole constant, and only applies to the excited manifold of the D_2 transition. We are interested in the hyperfine structure of the ground state and $J = 3/2$ excited state. We can write the energy eigenvalues of the Hamiltonian in terms of the quantum numbers using equation 2.9. To simplify things, we make the substitution $K = F(F+1) - J(J+1) - I(I+1)$.

$$\Delta E_{hfs} = \frac{1}{2}A_{hfs}K + B_{hfs} \frac{\frac{3}{2}K(K+1) - 2I(I+1)J(J+1)}{2I(2I-1)2J(2J-1)} \quad (2.11)$$

Laser cooling utilises the transitions of the D_2 line. For Ramsey clock schemes, the hyperfine manifold of the ground state is used. Here the separation is $6.8GHz$, which is addressable using microwave fields or stimulated two-photon Raman transitions.

2.2 Interaction With a Static Magnetic Field

A neutral atom interacts with a magnetic field by virtue of its spin, with which we can associate a magnetic moment $\boldsymbol{\mu}$. The interaction between a magnetic moment and a magnetic field $\mathbf{B}$ is governed by the magnetic dipole Hamiltonian:

$$\hat{H}_B = \boldsymbol{\mu} \cdot \mathbf{B} = \frac{\mu_B}{\hbar}(g_S \hat{S}_z + g_L \hat{L}_z + g_I \hat{I}_z)B_z \quad (2.12)$$

The field has been chosen to lie along the quantization axis, which is taken to be the z-axis by convention. This choice is possible without loss of generality, as one can always define a space dependent coordinate transformation to make magnetic field vectors define the z-axis; as is demonstrated in section 2.3.

The magnetic moment has been decomposed into individual moments, each associated with their respective spin operator. Each moment is described in terms of g-factors (gyromagnetic ratios g_S , g_L and g_I); and the Bohr magneton μ_B - the magnetic moment of an electron. The g-factors describe the sensitivity of the spin to a magnetic field, and can be of different sizes and signs for different states. This different sensitivity to the field results in a splitting of the energy levels, known as the Zeeman effect. In fact, the fine and hyperfine structure can be thought of as a kind of internal Zeeman effect. If the applied field causes an energy shift that is small compared to the fine structure splitting, then J is a good quantum number and we can write the above equation in terms of J and a corresponding g-factor. For small fields, the energy levels shift linearly as:

$$\hat{H}_B = \frac{\mu_B}{\hbar}(g_J \hat{J}_z + g_I \hat{I}_z)B_z \quad (2.13)$$

Here g_J is an effective g-factor, that is defined from the other quantum numbers. Similarly, if the energy shift is small compared to the hyperfine splitting, then F is suitable for labelling the states, and we can write:

$$\hat{H}_B = \frac{\mu_B}{\hbar} g_F \hat{F}_z B_z \quad (2.14)$$

where g_F is the hyperfine Lande g-factor and is given by:

$$g_F = g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} + g_I \frac{F(F+1) + I(I+1) - J(J+1)}{2F(F+1)} \quad (2.15)$$

This is almost exactly 1/2 for the ground state hyperfine manifold. If we choose to use the commuting spin operator eigenstates of $\hat{\mathbf{F}}^2$ and $\hat{F}_z$ to label the states, then the Hamiltonian 2.14 is already diagonalised by definition. We can then read off the eigenvalues:

$$\Delta E = \mu_B g_F m_F B_z \quad (2.16)$$

The eigenvalues of the $\hat{F}_z$ operator are labelled by the m_F magnetic quantum numbers, which take $(2F+1)$ integer values between $+F$ and $-F$. In the absence of an external magnetic field, these sub-levels are degenerate. The application of a magnetic field lifts the degeneracy of these states, giving rise to the anomalous Zeeman effect. For low fields, where F is a good quantum number, the energy shifts linearly according to equation 2.16. For intermediate fields the energy eigenstates must be found numerically by diagonalizing the Hamiltonian. The Hamiltonian of rubidium when interacting with a magnetic field can be considered as the sum of the internal atomic Hamiltonian and an interaction Hamiltonian.

$$\hat{H} = \hat{H}_{hfs} + \hat{H}_B \quad (2.17)$$

We are often interested in the ground state manifold. Then $B_{hfs} = 0$ and we can write the combined Hamiltonian as:

$$\hat{H} = A_{hfs}(\hat{\mathbf{I}} \cdot \hat{\mathbf{J}}) + \frac{\mu_B}{\hbar} (g_S \hat{S}_z + g_L \hat{L}_z + g_I \hat{I}_z) B_z \quad (2.18)$$

We can numerically diagonalize the Hamiltonian to produce the energy eigenvalues as a function of field. These energies are shown in figure 2.2.

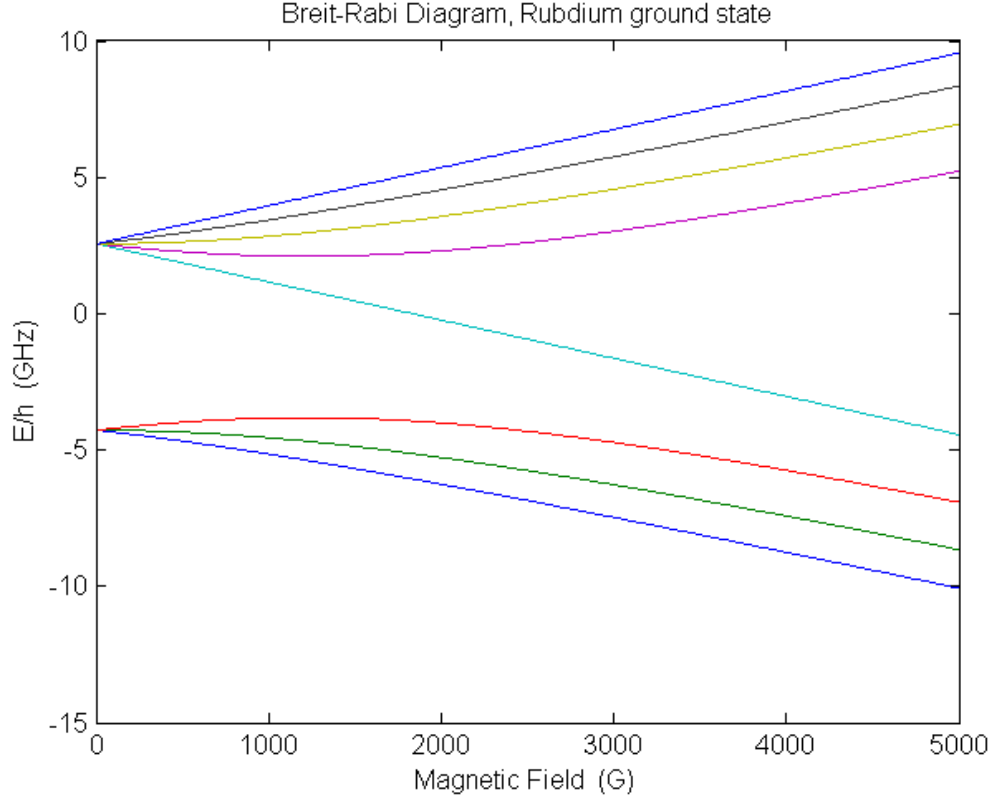


Figure 2.2: Breit-Rabi diagram of rubidium ground state manifold. The hyperfine levels split linearly for small fields, and for large fields. Intermediate fields have a non-linear response.

2.3 Atoms in an Inhomogeneous Magnetic Field

Under the dipole interaction, an atomic spin will tend to align with the field. However, in the case of an atom moving through an inhomogeneous field, the field seen by the atom is changing. The question arises over what field strengths and atomic speeds do atoms remain aligned with the field? This is important as not all spin states are magnetically trappable. We now consider a space-dependent magnetic field $\mathbf{B}(\mathbf{r})$. The Hamiltonian, including the kinetic energy of the atom, is:

$$\hat{H} = \frac{\hat{\mathbf{P}}^2}{2m} + g_F \mu_B \hat{\mathbf{F}} \cdot \mathbf{B}(\mathbf{r}) \quad (2.19)$$

As the momentum operator and the position operator do not commute, this situation will involve spin transitions between hyperfine states. We would like to find the conditions under which the spin is aligned with the field. Stated

more formally, we would like to show that the field does indeed define the quantization axis. By rotating the coordinate system such that the magnetic field defines the z-axis, the Hamiltonian can be shown to be diagonal in $\hat{F}_z$. To do this, we define a unitary transformation which performs the space-dependent rotation:

$$\hat{U}_s(\mathbf{r}) = e^{-i\hat{F}_z\alpha(\mathbf{r})}e^{-i\hat{F}_y\beta(\mathbf{r})} \quad (2.20)$$

where the angles are dependent on the magnetic field components at that point in space:

$$\alpha(\mathbf{r}) = \arctan \left[\frac{B_y(\mathbf{r})}{B_x(\mathbf{r})} \right] \text{ and } \beta(\mathbf{r}) = \arctan \left[\frac{\sqrt{B_x^2(\mathbf{r}) + B_y^2(\mathbf{r})}}{B_z(\mathbf{r})} \right] \quad (2.21)$$

Now applying the rotation to the Hamiltonian, we find:

$$\begin{aligned} \hat{U}_s^\dagger \hat{H} \hat{U}_s &= \hat{U}_s^\dagger \frac{\hat{\mathbf{P}}^2}{2m} \hat{U}_s + g_F \mu_B \hat{U}_s^\dagger \hat{\mathbf{F}} \cdot \mathbf{B}(\mathbf{r}) \hat{U}_s \\ &= \frac{1}{2m} [\hat{\mathbf{P}} + \mathbf{A}(\mathbf{r})]^2 + g_F \mu_B \hat{F}_z |\mathbf{B}(\mathbf{r})| \end{aligned} \quad (2.22)$$

The field vector now defines the z-axis and the interaction term is now dependent only on the magnitude of the field at each point. With the interaction term forced into a diagonal representation, the off-diagonal elements representing the hyperfine couplings have been transferred into a gauge potential $\mathbf{A}(\mathbf{r}) = \hat{U}_s^\dagger \hat{\mathbf{P}} \hat{U}_s$. The gauge potential represents the kinetic energy of an atom as a result of moving through an inhomogeneous magnetic field, and is responsible for inducing transitions between hyperfine states. If the gauge term can be neglected, the Hamiltonian is diagonal in $\hat{F}_z$, and the atom can be said to be aligned with the field. This is known as the adiabatic approximation, the validity of it determines whether an atom is trapped or lost, and the condition under which it is valid will now be discussed.

2.3.1 Adiabaticity

The atom remains in an instantaneous eigenstate, so long as the perturbation of its energy is slow compared to the energy gap between the states. This is the adiabatic theorem, and it amounts to neglecting the gauge potential in equation 2.22. In the case of an atom whose energy shifts linearly with magnetic field in accordance with equation 2.16, we can write the adiabatic condition as:

$$\frac{\dot{\mathbf{B}}}{|\mathbf{B}|} \ll \frac{g_F \mu_B}{\hbar} |\mathbf{B}| = \omega_L \quad (2.23)$$

This equation expresses the fact that the field must change slowly with respect to the Zeeman splitting of the states. This Zeeman splitting has been expressed in terms of the Larmor frequency $\omega_L = (\mu_B g_F / \hbar) |\mathbf{B}|$. This reflects the fact that the atomic spin can be pictured as a magnetic spinning top, precessing around the field vector at the Larmor frequency.

If the rate of change of the magnetic field is small compared to the Larmor frequency, the spin adiabatically follows. The field still defines the quantization axis, and the atom remains in its current spin state. Should the field change at a rate comparable or greater than the Larmor frequency, the spin may become unaligned. This amounts to a transition between the hyperfine states.

2.4 Magnetic Trapping

The ability to trap neutral particles in a magnetic field relies on the adiabatic following of the atomic spin with the field. Applying the adiabatic condition to the Hamiltonian in equation 2.22 by neglecting the gauge potential, the energy of the interaction potential can then be simply written as:

$$V_{mag} = -\boldsymbol{\mu} \cdot \mathbf{B} = g_F \mu_B m_F |\mathbf{B}| \quad (2.24)$$

For atoms which have $g_F m_F < 0$, the atoms are attracted to minima in the field, and are called low-field seekers. For the case of $g_F m_F > 0$, the atoms are high-field seekers. As a consequence of Earnshaw's theorem [65], no static magnetic field maximum can exist in a source-free region. High-field seekers are then untrappable, and quickly expelled from a magnetic trap. The g -factors for the rubidium ground state manifold are $g_F = \pm 1/2$.

The simplest trapping configuration to consider is the linear quadrupole trap, typically formed by a pair of coils in anti-Helmholtz configuration. The field from such a trap is:

$$\mathbf{B}(x, y, z) = B'_{xy} (x\hat{\mathbf{e}}_x + y\hat{\mathbf{e}}_y - 2z\hat{\mathbf{e}}_z) \quad (2.25)$$

The field is zero in the trap centre, and increases as a function of distance away from the centre. The parameter B'_{xy} represents the field gradient along the x and y axes. The trap formed by a set of coils in anti-Helmholtz configuration will have axial symmetry in the x - y plane. Along the z -axis the gradient is twice that of the x and y gradients. This is a consequence of Gauss's Law, which states that the divergence of the field must be zero in a source-free region $\nabla \cdot \mathbf{B} = 0$.

We can now look at the gradient required to hold the atoms against gravity. Writing the gradient along the z -axis as B'_z , the energy from equation 2.24 can be written as a function of position as $V_{mag} = g_F \mu_B m_F B'_z z$. As the force is the spatial derivative of the energy, we can equate the force of the field to

gravity, and rearrange for the gradient:

$$B'_z = \frac{m_{rb}g}{\mu_B g_F m_F} \quad (2.26)$$

Here, g is the gravitational acceleration, and $m_{rb} = 1.44 \times 10^{-25} \text{Kg}$ is the mass of a rubidium-87 atom. For atoms in the ground state manifold with spin $F = 2$, the g-factor is positive ($g_F = +1/2$), and only the $m_F = 1, 2$ states are trappable. It is instructive to look at the gradient required to do this. We find that for the $m_F = 1$ state, a gradient of 30G/cm is needed, and 15G/cm for $m_F = 2$.

We can now look at what is required to generate such a field in a typical setup. The coils of radius R in anti-Helmholtz configuration, produce a field gradient along the vertical axis of:

$$B'_z = -\frac{48\mu_0}{25\sqrt{5}R^2}NI \quad (2.27)$$

Here $\mu_0 = 4\pi \times 10^{-7} \text{N/A}^2$ is the magnetic permeability. I is the current through each coil, which is scaled by the number of turns N . The coils are typically mounted outside of the chamber. For a gradient of 30G/cm , a coil radius of 10cm , and 100 turns for each coil, a current of 30A is needed. This is a lot of current to pass, and it results in significant heating. More turns can reduce the current needed, with the drawback that the inductance scales as N^2 , which can make the coil current difficult to switch quickly.

At regions of low or vanishing field, the rate of change of the field can be comparable or greater than the Larmor frequency; the adiabatic condition no longer holds, and atoms diabatically transition to untrappable spin states. These spin-flips are known as Majorana losses [66], and are a specific case of Landau-Zener tunnelling [67]. For the case of a linear quadrupole trap, the Majorana loss rate is given by [68]:

$$\Gamma_{\text{Majorana}} = \frac{\hbar}{m_{rb}} \frac{24^{1/3}}{32} \left(\frac{g_F m_F \mu_B B'}{k_B T} \right)^2 \quad (2.28)$$

The loss rate is dependent on the trap gradient and the cloud temperature T .

Tighter traps have higher loss rates. Cooler clouds spend more time near the vanishing field and escape the trap faster. This will limit the temperature reachable in such a trap. A field-zero is in this sense a hole out of which atoms can leak. Various ways of plugging this hole exist. Most commonly, a Ioffe-Pritchard (IP) type configuration is used. Here the field-zero is replaced by a bias field, atoms adiabatically follow the field as they pass through the centre of the trap. Another alternative is a time-orbiting potential, or TOP trap. In this case the field-zero is rotated around the trap centre. A TOP trap was used to magnetically trap and evaporatively cool in the first BEC experiment [15].

In cases where the potential is harmonic (such as IP traps), the motion of the atom can be understood in terms of a trap frequency. We can write the trap frequency for the axis i , where $i = x, y, z$, as:

$$\omega_{trap,i} = 2\pi \sqrt{\frac{\mu_B g_F m_F}{m_{rb}} |\mathbf{B}|_i''} \quad (2.29)$$

This gives an idea of the field time-derivative seen by the atoms, and is useful for checking adiabaticity and for many other calculations. The trap frequency must be much less than the Larmor frequency $\omega_{trap} \ll \omega_L$ for the adiabatic approximation to hold. We can estimate the cloud temperature that is trappable by equating the thermal energy to the depth of the trap:

$$T_{max} = \frac{\mu_B g_F m_F (B_{max} - B_{min})}{K_B} \quad (2.30)$$

where K_B is the Boltzmann constant. For a trap depth of $1G$, and $m_F = 2$, we find a temperature of $70\mu K$. Thus the cloud must be substantially cooled before it can be trapped in this manner. Furthermore, the atoms must be first prepared in the magnetically trappable states, preferable $m_F = 2$. Typically, a cloud is first laser cooled in a magneto-optical trap, optically pumped to $|F = 2, m_F = 2\rangle$, and then transferred to a purely magnetic trap for evaporative cooling.

2.5 Radio Frequency Dressed Traps

Radio frequency potentials offer a way to create exotic trapping geometries from conventional magnetic traps. The dressed atom picture was originally developed in the context of the optical dipole force [69], but it was first proposed in conjunction with a conventional magnetic trap in 2001 by Zobay and Garraway [70], and first experimentally realised by Perrin in 2004 [71]. It is also a promising tool for coherently splitting clouds in atom interferometry experiments [72]. An RF field is applied with ω_{RF} close to the Larmor frequency, coupling the hyperfine spin states. A thorough treatment of the full Hamiltonian is given in [73], and a further treatment considering the geometric phases with applications to atoms interferometry is given in [74]. A simplified derivation is given here. The situation is solved by transforming the Hamiltonian into a time-independent form. The interaction is given by the Hamiltonian:

$$\hat{H} = \frac{\hat{\mathbf{P}}^2}{2m} + g_F \mu_B \hat{\mathbf{F}} \cdot (\mathbf{B}_s(\mathbf{r}) + \mathbf{B}_{RF}(\mathbf{r}, t)) \quad (2.31)$$

where $\mathbf{B}_s(\mathbf{r})$ is the static field component and $\mathbf{B}_{RF}(\mathbf{r}, t)$ is the applied RF field. As before, we can apply a rotation to align the field with the quantization axis using the unitary operator $\hat{U}_s(\mathbf{r})$ defined in equation 2.20. After applying the first rotation, we have the Hamiltonian:

$$\hat{H} = \frac{(\hat{\mathbf{P}} + \mathbf{A}(\mathbf{r}))^2}{2m} + g_F \mu_B \hat{F}_z |\mathbf{B}_s(\mathbf{r})| + g_F \mu_B \mathbf{B}'_{RF}(\mathbf{r}, t) \cdot \hat{\mathbf{F}} \quad (2.32)$$

where $\mathbf{B}'_{RF}(\mathbf{r}, t) = \hat{U}_s^\dagger \mathbf{B}_{RF}(\mathbf{r}, t) \hat{U}_s$ is the rotated radio frequency field. The first term is the kinetic energy. The gauge field has absorbed the hyperfine couplings previously present in the static field. Off-diagonals may still be present in the radio frequency term however. Any $\hat{e}_x$ and $\hat{e}_y$ components in $\mathbf{B}'_{RF}(\mathbf{r}, t)$ will induce transitions between the hyperfine states. This can be exposed by writing the field in the spherical polarisation basis. A polarised

RF field can be described in the polarisation basis of right-handed circular $\hat{\mathbf{e}}_+$, left-handed circular $\hat{\mathbf{e}}_-$, and linear π polarisation $\hat{\mathbf{e}}_0$.

$$\mathbf{B}'_{RF}(\mathbf{r}, t) = \text{Re}[(\beta_0 \hat{\mathbf{e}}_0 + \beta_+ \hat{\mathbf{e}}_+ + \beta_- \hat{\mathbf{e}}_-) e^{-i\omega_{RF}t}] \quad (2.33)$$

with the polarisation vectors:

$$\hat{\mathbf{e}}_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}, \hat{\mathbf{e}}_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}, \hat{\mathbf{e}}_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (2.34)$$

All vectors here are in the local coordinate system. That is, $\hat{\mathbf{e}}_+$ refers to a circular unit vector about the static field axis (z-axis). The new Hamiltonian is:

$$\begin{aligned} \hat{\mathbf{H}} = & \frac{(\hat{\mathbf{P}} + \mathbf{A}(\mathbf{r}))^2}{2m} + g_F \mu_B \hat{\mathbf{F}}_z |\mathbf{B}_s(\mathbf{r})| \\ & + \frac{g_F \mu_B}{\sqrt{2}} \left[(\beta_+ \hat{\mathbf{F}}_+ + \beta_- \hat{\mathbf{F}}_-) e^{-i\omega_{RF}t} + h.c. \right] \end{aligned} \quad (2.35)$$

The π polarisation component is often neglected, as it does not couple the m_F magnetic hyperfine states. We then apply an additional transformation, which moves to a frame rotating around the quantization axis at the frequency of the applied RF ω_{RF} . The $\hat{\mathbf{e}}_+$ polarised light perfectly co-rotates with the coordinate system, and thus becomes time-independent in the rotating frame. The $\hat{\mathbf{e}}_-$ component now counter-rotates at frequency $2\omega_{RF}$. In the rotating wave approximation (RWA), the $2\omega_{RF}$ components are neglected, leaving only a time-independent field. Formalizing this, we write the unitary transform as:

$$\hat{\mathbf{U}}_{RF} = e^{i\hat{\mathbf{F}}_z \omega_{RF}t} \quad (2.36)$$

This rotates the coordinate system around the z-axis at a frequency ω_{RF} . Before we apply it to the Hamiltonian, we will see how the Hamiltonian transforms under a time-dependent transformation. Applying the chain rule

of differentiation, the Hamiltonian transforms as:

$$\hat{H}' = \hat{U}_{RF}^\dagger \hat{H} \hat{U}_{RF} + i\hbar \hat{U}_{RF}^\dagger (\partial_t \hat{U}_{RF}) \quad (2.37)$$

Evaluating the second term in $\hat{H}'$, we find:

$$i\hbar \hat{U}_{RF}^\dagger (\partial_t \hat{U}_{RF}) = \hbar\omega_{RF} \hat{F}_z \quad (2.38)$$

We can now apply $\hat{U}_{RF}$ to the Hamiltonian. The kinetic energy term is neglected in the adiabatic approximation. The static field term $|\mathbf{B}_s(\mathbf{r})|$ is unaffected as it aligned with the z-axis. The RWA is applied, removing the rapidly oscillating terms and resulting in a time-independent Hamiltonian:

$$\begin{aligned} \hat{H}' &= \hbar\omega_{RF} \hat{F}_z + g_F\mu_B \hat{F}_z |\mathbf{B}_s(\mathbf{r})| + \frac{g_F\mu_B}{\sqrt{2}} \left(\beta_+ \hat{F}_+ + h.c \right) \\ \hat{H}' &= g_F\mu_B \left(|\mathbf{B}_s(\mathbf{r})| - \frac{\hbar\omega_{RF}}{g_F\mu_B} \right) \hat{F}_z + \frac{g_F\mu_B}{\sqrt{2}} |\beta_+| \hat{F}_x \\ \hat{H}' &= \hbar\delta \hat{F}_z + \hbar\Omega_+ \hat{F}_x \\ \hat{H}' &= \hbar\sqrt{\delta^2 + \Omega_+^2} \hat{\mathbf{F}} \cdot \mathbf{u} \end{aligned} \quad (2.39)$$

Here we have introduced the detuning $\delta = \omega_L - \omega_{RF}$, and the Rabi coupling term $\Omega_+ = g_F\mu_B |\beta_+| / \sqrt{2}\hbar$. We see from the final equation that we can define a new quantization axis $\mathbf{u}$:

$$\mathbf{u} = \frac{\delta}{\sqrt{\delta^2 + \Omega_+^2}} \hat{\mathbf{e}}_z + \frac{\Omega_+}{\sqrt{\delta^2 + \Omega_+^2}} \hat{\mathbf{e}}_x = \cos\theta \hat{\mathbf{e}}_z + \sin\theta \hat{\mathbf{e}}_x \quad (2.40)$$

This quantization axis is rotated around the y-axis by an angle θ . The eigenstates $|m'_F\rangle$ of the Hamiltonian are then the eigenstates of the projection of the atomic spin operator $\hat{\mathbf{F}}$ onto to the new quantization axis $\hat{F}_u = \hat{\mathbf{F}} \cdot \mathbf{u}$, with eigenvalues:

$$E = \hbar m'_F \sqrt{\delta^2 + \Omega_+^2} \quad (2.41)$$

We had omitted the kinetic energy in the above equations. Again, neglecting the gauge term is the adiabatic approximation, and the condition under which

it is valid is similar to before:

$$|\langle n'_F | \frac{\partial}{\partial t} | m'_F \rangle| = |\dot{\theta}| \ll \sqrt{\delta^2 + \Omega_+^2} \quad (2.42)$$

The neglected gauge potential contains the geometric phase information of the atoms, and was analysed in [74]. As for the atom in a magnetic trap, we can now write the final interaction potential in terms of the magnitude of an effective field:

$$E_{m'_F} = m'_F \mu_B g_F \sqrt{\left(|\mathbf{B}_s(\mathbf{r})| - \frac{\hbar \omega_{RF}}{g_F \mu_B} \right)^2 + \frac{|\beta_+|^2}{2}} = m'_F \mu_B g_F |\mathbf{B}_{eff}(\mathbf{r})| \quad (2.43)$$

Thus we have arrived at an equation analogous to equation 2.24 for a static magnetic field. The field has been replaced by an effective field, which depends on two terms. The first term is the detuning δ between the Larmor precession ω_L associated with the static field $|\mathbf{B}_s|$, and the applied radio frequency ω_{RF} . When the applied RF and the Larmor frequency are equal, the first term vanishes and an energy minima occurs along the surface of resonance. This energy minima is given by the polarisation term $|\beta_+|$, which describes the strength of the right-handed circular component of the RF in the local frame of the atom. Provided this is non-zero, the Majorana losses can be avoided.

The above derivation assumes a positive g-factor, leading to the appearance of β_+ in equation 2.43. In the case of a negative g-factor, the RWA removes the β_+ term and the β_- term becomes time-independent. For this reason, atoms in states of different g-factors may experience different potentials in a polarised RF field. This fact is used in chapter 3 to form the basis of a state-dependent guide.

2.6 Laser Cooling

That light can transfer momentum and exert a radiation pressure has long been known. But the invention of tunable dye lasers made laser use in research feasible and invigorated an interest in the use of lasers for cooling and trapping atoms. Laser cooling was proposed in 1975 by Hänsch and Schalow [75]. Atoms in the cloud have a Maxwell Boltzmann distribution of speeds, resulting in a Doppler broadened absorption line.

By red-detuning a laser, atoms experience a Doppler-shift closer towards resonance when they are moving opposite to the direction of beam propagation. An atom that absorbs a photon receives a momentum kick. Should the atom spontaneously decay, the photon is re-emitted in a random direction.

The direction of spontaneous emission is random, so the spontaneous emission force averages to zero. When the atomic momenta is much larger than the recoil kicks, this absorption cycle can be seen as a continuous molasses force. The scattering force is therefore just the momentum transfer per photon $\hbar\mathbf{k}$, multiplied by the scattering rate γ_{scatt} :

$$\mathbf{F}_{scatt} = \hbar\mathbf{k}\gamma_{scatt} \quad (2.44)$$

The goal, then, is to find γ_{scatt} . This depends upon the frequency and intensity of the light, as well as the population of atoms that are in the cooling state. The faster the atoms can decay by spontaneous emission, the faster they can absorb more cooling light. This scattering rate γ_{scatt} can be estimated theoretically using the optical Bloch equations and the density matrix approach for a two level atom with spontaneous emission. The results of this approach are summarized below, and it is described in more detail in [76].

A two-level atom is considered. A coherent laser beam drives transitions between the ground and excited state. Spontaneous emission causes the atoms to decay back to the ground state with a random phase. This results in a de-coherence of the Rabi flopping, and a steady state population ρ_{ee} in

the excited state. This is given as:

$$\rho_{ee} = \frac{S/2}{1 + S + (2\delta/\gamma)^2} \quad (2.45)$$

Here γ is the decay rate, and δ is the detuning from resonance. For the case of a moving atom, δ must be altered to account for the Doppler shift. $S = I/I_S$ is the saturation parameter, which is the ratio of the light intensity I to the saturation intensity I_S , which is given as:

$$I_S = \pi \hbar c / 3 \lambda^3 \tau \quad (2.46)$$

where τ is the lifetime of the state ($\gamma = 1/\tau$), and λ is the wavelength of the light. The saturation parameter is therefore a ratio of the absorption strength to the relaxation rate. When saturation is large, the ground and excited state populations approach equality. In this scenario, stimulated emission is equally as likely as absorption, and the medium becomes transparent. For rubidium, with σ^+ light, the saturation intensity is 1.7 mW/cm^2 , easily achievable with a diode laser. The spontaneous emission rate of the ensemble of atoms is given by taking the product of gamma with the excited state population. In steady state, this is equal to the absorption rate, thus we have for the absorption rate γ_{scatt} :

$$\gamma_{scatt} = \gamma \rho_{ee} = \frac{\gamma}{2} \frac{I/I_{sat}}{1 + (2\delta/\gamma)^2 + I/I_{sat}} \quad (2.47)$$

This is in fact a Lorentzian with a linewidth:

$$\Delta\omega = \gamma' = \gamma \sqrt{1 + I/I_{sat}} \quad (2.48)$$

This is known as the power broadened linewidth. When the saturation is low ($I \ll I_S$), the linewidth reduces to the natural linewidth γ . At an intensity equal to the saturation intensity, the natural linewidth is broadened by a factor of $\sqrt{2}$. From the equation from the scattering rate 2.47, we can also define the scattering cross section σ as the ratio of the power radiated to the

incident power, giving:

$$\sigma = \frac{\sigma_0}{1 + (2\delta/\gamma)^2 + I/I_{sat}} \quad (2.49)$$

where σ_0 is the on-resonance cross-section and is defined as:

$$\sigma_0 = \frac{\hbar\omega\gamma}{2I_{sat}} \quad (2.50)$$

In reality, the atoms are not a two-level system, and light polarisations affect the saturation intensity and absorption cross-sections.

For an atom placed into two counter-propagating beams, the resultant force is $\mathbf{F}_{molasses} = \mathbf{F}_+ + \mathbf{F}_-$, where the $+$ and $-$ denote the different directions of the beams. The force is given by equation 2.44. Substituting for the scattering rate of equation 2.47, the force from either beam is:

$$\mathbf{F}_{\pm} = \pm \frac{\hbar\mathbf{k}\gamma}{2} \frac{I_s/I}{1 + I_s/I + [2(\delta \mp |\omega_D|)/\gamma]^2} \quad (2.51)$$

To account for a moving atom, the detuning δ has been shifted by the Doppler frequency ω_D . This can then be expressed in terms of the velocity v of the atom, resulting in a total force:

$$\mathbf{F}_{molasses} \cong \frac{8\hbar k^2 \delta (I/I_{sat}) \mathbf{v}}{\gamma (1 + (2\delta/\gamma)^2 + I/I_{sat})^2} = -\beta \mathbf{v} \quad (2.52)$$

This is the optical molasses force, and provides a damping effect to the atoms in the direction of their motion. 3D laser cooling requires three such counter-propagating laser beams. Knowing the form of the damping coefficient β , the optimum detuning is found to be $\delta = \gamma/2$. Were the force truly continuous, this molasses effect would slow the atoms to a stop. However, the photon absorptions occur as discrete momentum kicks, and at low velocities the continuous forces no longer correctly describe the situation. Each spontaneous emission gives the atoms a momentum kick in a random direction. In analogy with a random walk, the average momentum of an atom will increase as $\sqrt{N}$, where N is the number of momentum kicks.

There are therefore two conflicting effects, the continuous damping force, and the heating due to the stochastic nature of the spontaneous emission. From the competition between the frictional cooling and the stochastic emission (heating), a theoretical equilibrium temperature T_D can be inferred [77].

$$K_B T_D = \hbar \frac{\gamma}{2} \quad (2.53)$$

For rubidium, this evaluates to $146\mu K$ (for a lifetime of $26ns$). This is referred to as the Doppler limit, as it was erroneously believed to be the limit achievable for such a cooling technique. Sub-Doppler temperatures of $40\mu K$ were unexpectedly observed in 1988 [78] in a gas of laser-cooled sodium atoms, well below the predicted limit of $240\mu K$. Within a year following these experimental results, previously overlooked cooling mechanisms were outlined [79]. Polarisation gradients in the beam mean that the atoms are nearly always climbing potential hills. Sub-Doppler cooling is often likened to the myth of Sisyphus, who was condemned to forever push a boulder up a hill. Another temperature milestone is the recoil limit T_R , based on the momentum imparted onto an atom when it scatters one photon:

$$K_B T_R = \frac{\hbar^2 k^2}{2m_{rb}} \quad (2.54)$$

In 1992 [80], Kasevich demonstrated optical cooling below the recoil limit. However normally sub-recoil temperatures are achieved using magnetic evaporation.

2.7 Magneto Optical Trap

The magneto-optical trap (MOT) is normally the first cooling stage in a cold atom experiment. Since it was first implemented by Steven Chu, Pritchard and others at Bell Labs in 1987 [20], it has become a very common part of cold atom experiments. As illustrated in Figure 2.3, three pairs of red detuned circularly polarised beams provide molasses cooling and trapping in three dimensions. The magnetic field does not supply the trapping force. Instead it shifts the Zeeman sub-levels closer to resonance with the oncoming beam. An atom displaced from the origin experiences an unbalanced absorption pushing it back towards the centre. In this sense the magnetic field causes the absorption force to become position dependent, resulting in a trapping force.

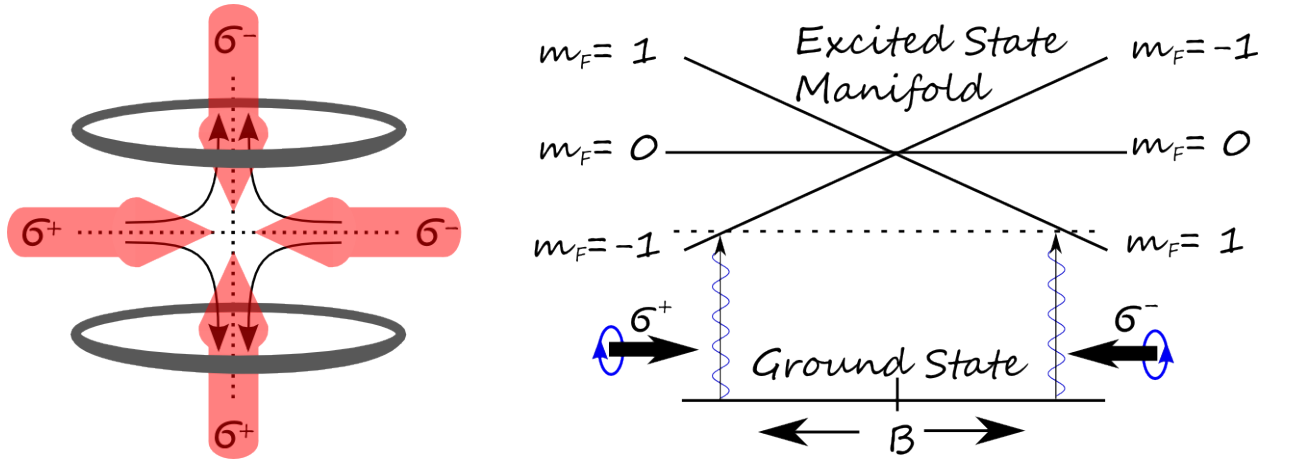


Figure 2.3: A Magneto Optical Trap. Left: Circularly polarised light illuminates an atomic cloud inside a vacuum chamber. The beams are red-detuned from resonance to provide molasses cooling. A set of Anti-Helmholtz coils produce a quadrupole magnetic field. The beams are aligned along the axis of the quadrupole. Right: The magnetic field shifts increases linearly from the centre. This shifts the Zeeman sub-levels closer to resonance.

Typically, this field is a linear quadrupole trap formed by a set of coils in anti-Helmholtz configuration, as described earlier in section 2.4. An axis can be associated with the quadrupole, along which the field vectors are aligned. These field vectors define the quantization axis of the atom, so it is important that the beams are aligned along it. The beams must be circularly polarised in order to address the magnetically sensitive states.

The polarisation in relation to the beam propagation is called the hand-

edness. From the picture, the beam propagating to the right is right-handed and vice versa. The polarisation that matters however, is in relation to the atom's quantization axis which points along the direction of the field. An atom moving to the left is more likely to absorb the right-handed, right-propagating beam. This is σ^- from the atom's perspective. An atom moving towards the right absorbs more left-handed, left-propagating light.

This is typically achieved by back-reflecting a beam and using quarter-waveplates to adjust the polarisation and flip the handedness. Normally, optical access is required along three dimensions to form a MOT, but there are some variations. In atom chip experiments, the chip normally obstructs the z-axis, and a reflection MOT must be used. Other variants such as the the diffraction grating or pyramid MOT allow only a single input beam to be used. Each has its own benefits and pitfalls. In our experiments, we use a standard MOT, and later a vertical reflection MOT.

Temperatures of about $100\mu K$ are common in a MOT. To get to lower temperatures, atoms are normally transferred to a purely magnetic trap. Note that atoms are in states with both positive and negative g-factors, and so many are not able to be purely magnetic trapped. Only those atoms with positive g-factors can be trapped, and in general it is desirable that the atoms are in the maximally stretched m_F state. For this reason optical pumping is applied before loading into the magnetic trap. Here the MOT field is switched off momentarily and replaced with a homogeneous bias field. This field defines the quantization axis, along which a pulse of circular σ^+ light optically pumps the atoms into the $m_F = 2$ state. The magnetic trap is then switched on and the atoms are caught. This process must be performed quickly, as the atoms are in free fall during this period. Typically, optical pumping pulses of $\approx 1ms$ are used. This speed is generally limited by the current switching of the power supplies. The coils are highly inductive loads which resist fast changes in current. For this reason, many labs have dedicated current switches.

2.8 Evaporative Cooling

After the MOT stage, the cloud is optically pumped, transferred into a purely magnetic trap and evaporatively cooled. The atoms with the highest thermal energy escape, reducing the average temperature of the system. As no optical absorptions are required, the recoil limit can be surpassed. It was the evaporation of sodium atoms which yielded the first BEC in 1995. Temperatures as low as $< 1nK$ have been reached [13]. Fortunately though, clouds do not need to be a BEC in order to display some degree of wave coherence. Evaporative cooling of hydrogen was proposed in 1986 [21] and implemented two years later by Harold Hess [81]. In 1995, evaporative cooling was used to produce a BEC of sodium atoms [16]. This was followed the same year by Wolfgang Petrich [15] who used a TOP trap to evaporatively cool rubidium atoms to $200nK$.

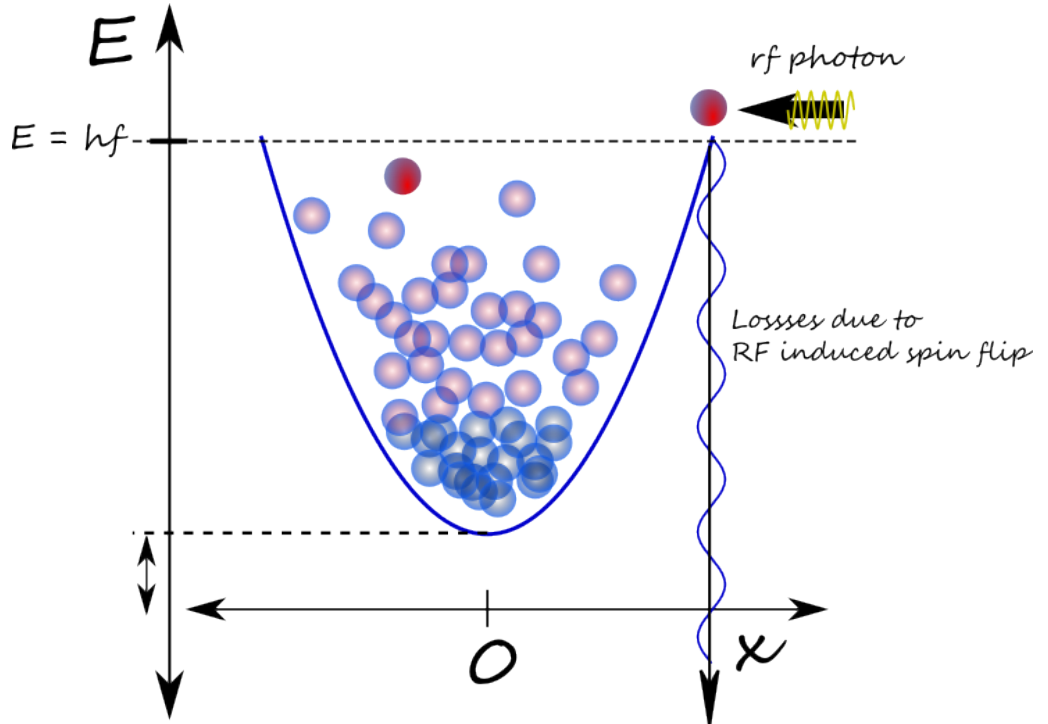


Figure 2.4: The atoms are trapped in a harmonic potential. Radio frequency absorption results in spin-flips to untrappable states. The radio frequency cuts away the high energy tail from the atomic speed distribution. As re-thermalisation of the cloud occurs, the frequency is lowered. The cooling is efficient for slow RF ramps, but a balance must be struck between losses due to RF, and losses due to background collisions.

The basic principle is as follows. The atoms are trapped in a harmonic

potential. To reach high fields, the atoms must climb a potential hill. Only the hottest atoms reach high fields. A radio frequency photon of frequency ω_{RF} is absorbed by atoms with a Larmor precession $\omega_L = \mu_B g_F B / \hbar$ equal to the radio frequency ω_{RF} . Those atoms that have an energy $E > \hbar \omega_{RF} |m_F|$ undergo RF-induced spin-flips and escape from the trap. After removal of hot atoms, the cloud re-thermalises through collisions. The speed of the re-thermalisation process is related to the density of the cloud.

The slower the frequency ramp is, the more time the atoms have to re-thermalise. Unfortunately, we don't have arbitrarily long as atoms are lost due to background collisions. A compromise must be found which optimizes the final phase space density (PSD) $\rho_{PSD} = n \lambda_{db}^3$ where n is the atom number density and $\lambda_{db} = \sqrt{2\pi\hbar^2/mk_B T}$ is the thermal de-Broglie wavelength.

Such low temperature atoms would quickly escape through a field-zero by Majorana spin-flips. Ioffe-Pritchard traps and TOP traps are ways to solve this problem (as discussed earlier in section 2.4).

2.9 Micro-fabricated Wires

Micro-fabricated wires allow for tighter control and manipulation of atoms. Integrated into substrates, the so-called atom chips are a promising strategy for miniaturization of cold atom experiments. In general, microscopic wires offer much larger field gradients, but approximately the same field strength as large current carrying structures. This can be seen by looking at the scaling laws of current and field [82]. Magnetic fields generated from current-carrying wires typically scale as I/S , where I is the wire current and S is the characteristic size of the system. Similarly, magnetic-field gradients and curvatures scale as I/S^2 and I/S^3 respectively. The result is that tighter magnetic traps can be achieved with lower power dissipation. Furthermore, the maximal current density of wires increases as the wire gets smaller. This is due to faster heat transfer into the substrate.

The simplest form of wire guide is a single current carrying wire and a bias field. This is known as a Kepler guide [83]. Adding an orthogonal bias field

B_{bias} results in a field-zero at a distance $r = (\mu_0/2\pi)(I/B_{bias})$. As before, atoms can escape from the field-zero. Adding an additional bias field in the direction of the wire raises the field-zero and plugs the hole.

A basic straight wire guide is a 2-dimensional trap, as it has no end-caps. To create a 3D trap, the wire can be bent at both ends. Bending the wire at each end creates either a "U" or a "Z" shaped wire. These so called U and Z traps are common in atom chip designs. Bending the wire into a U-shape gives a quadrupole field-zero. This is because the contributions from the bent wires cancel. When bent into a Z shape, the field from the bent wires add, resulting in a central field minimum similar to a Ioffe trap. These configurations form the basic building blocks for more complex wire configurations [84]. Atom chips should be reflective to allow for reflection imaging. Atom chips and the surrounding mounting structures obstruct the vertical axis. This means that reflection MOTs have to be used, and the atoms have to be magnetically transported to the location of the chip. This led to the use of QUIC coils [85], and transport chips utilizing a conveyor-belt design [86].

Chapter 3

A Matter-Wave Sagnac Interferometer

Sagnac interferometry has found a wide range of applications, including geodesy, seismology, and inertial navigation systems [87]. Cold atom interferometers can offer orders of magnitude greater sensitivity than their optical counterparts. In this chapter, a scheme for a guided Sagnac interferometer is presented.

3.1 Optical Sagnac Interferometer

The rotational rate of an inertial frame can be measured using the Sagnac effect, observable in both light and matter-waves. It was originally demonstrated by Georges Sagnac in 1913 [52] [88] who observed that a phase shift would arise for a closed path optical interferometer set into rotation.

Sagnac had originally intended to demonstrate the existence of the luminiferous aether and refute the theory of relativity. He believed the phase shift to arise from an optical whirling effect, caused by the movement of the system in relation to the aether. The effect was subsequently shown to be completely compatible with relativity however, and has in fact become a valuable test of general relativity.

The Sagnac effect is most naturally expressed as arising from a time-difference of arrival between counter-propagating beams on a rotating platform. This situation is shown, for a half rotation, in figure 3.1. A Beam

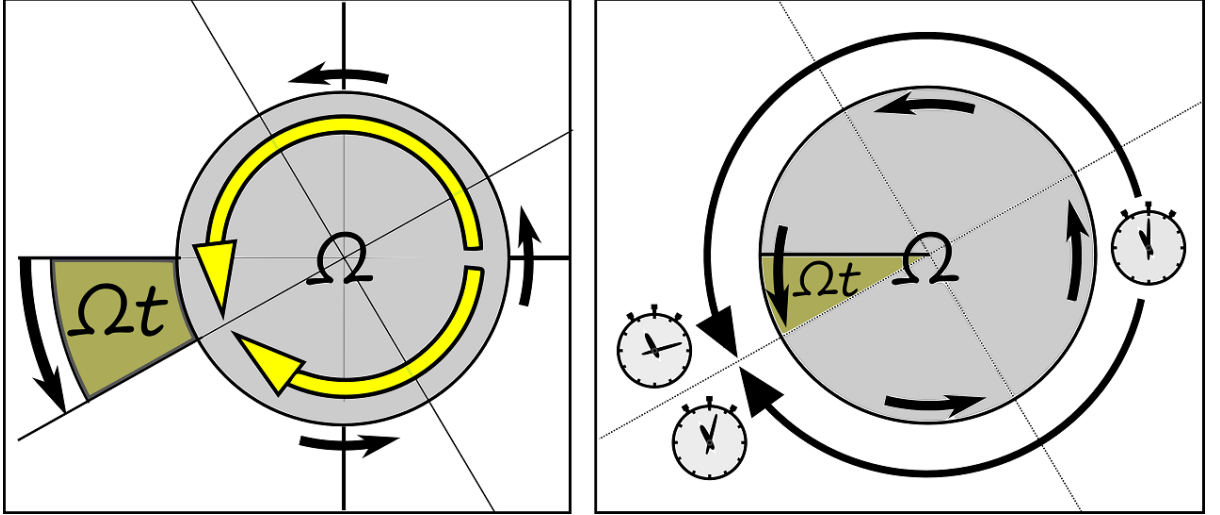


Figure 3.1: The Sagnac Effect. Left: A Beam of light travels around a loop on a rotating platform. The co-rotating light travels a longer path than the counter-rotating light. This gives rise to the Sagnac phase shift. Right: The Sagnac effect applies to any entity which travels a closed path around the ring. Two clocks are transported around the ring, and their times compared. The clocks could arrive at different times, in which case the Sagnac effect appears due to a time-difference-of-arrival, as in equation 3.3. If the clocks arrive at the same time, they must have been travelling different speeds. Then, due to time-dilation, the Sagnac effect arises as a proper-time difference, as in equation 3.5. This resembles the Hafele-Keating experiment [89].

of light travels around a loop on a rotating platform. The co-rotating light travels a longer path than the counter-rotating light. This gives rise to a path length difference between the two beams and a phase shift in the interference fringes.

Sagnac interferometers are often implemented in a common path configuration, where both light beams travel a full and overlapping path around the interferometer. This makes the interference fringes very stable against unwanted variations in path length.

To derive the Sagnac phase shift for a closed path, we consider a case where the light enters a ring of radius R , is split, counter-propagates around a fully closed loop, and is recombined in time t . We assume the source and re-combiner are at the same location on the ring. The set-up rotates in a clockwise fashion at angular velocity Ω . As the set-up rotates, the source/re-combiner moves a distance $\Delta L = R\Omega t$. Denoting the clockwise and counter-clockwise rotation as $+$ and $-$ respectively, we can write for the time taken for the light paths to travel a full cycle as:

$$t_{\pm} = \frac{2\pi R \pm \Delta L}{c} \quad (3.1)$$

where c is the speed of light, and R is the radius of the ring. This results in a time difference:

$$\Delta t = \frac{4\pi R^2 \Omega}{c^2 - R^2 \Omega^2} \quad (3.2)$$

For ordinary rotation speeds, $R\Omega \ll c$, and the time difference can be given by a simple equation, linear with the area and rotation speed:

$$\Delta t = \frac{4A\Omega}{c^2} \quad (3.3)$$

The phase can then be found by multiplying the time delay by the angular frequency ω of the light as $\Delta\phi = \omega \Delta t$. This leads us to the equation for the Sagnac phase shift:

$$\Delta\phi_S = \frac{4\omega A\Omega}{c^2} \quad (3.4)$$

The Sagnac phase shift scales with enclosed area A and the frequency of the light ω . This is the equation produced by Sagnac in his original paper in 1913, which he attributed to the effect of 'optical whirling' of the aether. In 1921, Paul Langevin described the Sagnac effect as compatible with relativity in a rotating coordinate system. Writing the Minkowski metric in cylindrical coordinates, and transforming to a rotating frame, the proper-time difference between beams counter and co-rotating with a platform is [90]:

$$\Delta\tau = \frac{4\pi R^2 \Omega}{c^2} \left(1 - \frac{\Omega^2 R^2}{c^2}\right)^{-1/2} \quad (3.5)$$

To first order this is equivalent to the time difference obtained in equation 3.3. Although the Sagnac effect was originally expressed in terms of a phase shift, the time-delay represents a more general expression of the Sagnac effect.

Equation 3.5 expresses the universality of the Sagnac effect. It holds for any observer or entity which takes a complete round-trip in a rotating frame. Provided the velocity of the two beams or entities around the ring are opposite and equal, equation 3.5 depends only on the rotation of the inertial frame, and not on the properties of the co-moving medium or the speed or nature of the beams. A slower object will of course take longer, but the time difference between the two paths is unaffected.

This has been observed in atomic clocks flown around the world. In 1971, Hafele and Keating flew atomic clocks on commercial airliners. One travelled from east to west, and the other from west to east. The time intervals were compared with geostationary clocks at the US naval laboratory. The proper-time difference arising from the different average speeds of the two clocks can then be used to infer the rotational speed of the earth [89].

3.2 Atomic Phase

Atom interferometry can be used to produce a Sagnac phase shift. In the following, we consider Ramsey-Borde interferometers, interpreting the phase shift in terms of an atomic clock experiment. Atom interferometry utilises the Ramsey clock scheme, where a microwave oscillator is stabilized to the Ramsey fringes of the nuclear hyperfine transitions. The phase measured is the relative phase between the microwave oscillator, and the dynamical phase associated with the superposition of the hyperfine states. The phase evolution of a quantum state is governed by the time-dependent Schrödinger equation. Writing the state $|\psi\rangle$ as an arbitrary superposition of the energy eigenstates $|E_n\rangle$, we see that the phase evolution of each eigenstate is determined by the energy eigenvalues E_n :

$$|\psi\rangle = \sum_n c_n e^{iE_n t/\hbar} |E_n\rangle \quad (3.6)$$

where c_n are arbitrary coefficients. For a two-level system, we can write:

$$|\psi\rangle = c_1 e^{iE_1 t/\hbar} |1\rangle + c_2 e^{iE_2 t/\hbar} |2\rangle = c_1 |1\rangle + c_2 e^{i(E_2 - E_1)t/\hbar} |2\rangle \quad (3.7)$$

The global phase is unphysical; hence we can consider only the relative phase between the two states. The relative phase of a superposition thus evolves at a frequency given by the energy difference of the two eigenstates $f = (E_1 - E_2)/h$. In the case of Rubidium, we consider the hyperfine states: $|F, m_F\rangle = |1, -1\rangle$ and $|2, 1\rangle$. These states are ideal as they are both magnetically trappable $g_F m_F > 1$, and have the same first-order Zeeman shift. For these states the frequency is 6.8GHz , and the accumulated phase is $\Delta\phi = 2\pi f t$. Operating as an atomic clock, a series of Ramsey pulses is used to interrogate the phase of the atoms. Scanning the phase of the microwave oscillator, the resultant Ramsey fringes give the relative phase of the atoms. The Sagnac effect then, is observed as a shift in the phase of the Ramsey fringes.

Discussion

We may then be inclined to ask some curious questions about atom interferometry. It is often stated that atom interferometers are advantageous on account of the tiny wavelength of the atoms. However, in the analysis presented above, the interferometer works in the microwave regime. A greater sensitivity to path-length changes could be obtained by working at optical frequencies with standard optical interferometers. So the first question is, why are atom interferometers advantageous?

Another question arises over the need to split the beams. The need for this is clear in optical interferometers, as the second arm of an optical Mach-Zehnder serves as a reference beam, resulting in stable interference fringes that are sensitive to changes in path length. However for atomic beams, where a clock scheme is operated, a single atomic beam with a microwave oscillator is enough to observe stable Ramsey fringes, fringes that would also be sensitive to changes in path length. It may then also be asked: why do we need to split the beams at all?

As it turns out, answering the first question will also answer the second. We must remember that the sensitivity of an interferometer is the amount of observed phase shift with respect to the phenomena of interest. The phase shift can arise in two ways; from a time-difference-of-arrival, or from a change in frequency (or both). If a beam of frequency f takes time t to propagate through an interferometer, we can then write the phase shift as:

$$\Delta\phi \approx 2\pi (\Delta f t + \Delta t f) \quad (3.8)$$

In the case of optical interferometers, the photons do not interact significantly with external fields, nor do they experience large changes in energy due to inertial effects. Phase shifts in optical interferometers therefore arise from changes in path length (or time-difference-of-arrival), with little change in frequency.

In the case of atomic interferometry, the opposite is true. The observed phase shift is dominated by changes in atomic frequency. In fact, in most

calculations, the path length difference is neglected. This is the perturbative approximation, which assumes that the phase shift can be found by integrating over the energy perturbation of the unperturbed path. That is, it is assumed that the phenomena of interest does not affect the path length of the atoms. Clearly, this approximation will be more justified in the microwave regime, where path length differences have much less effect on the output phase. Thus, only the energetic interaction is important. The advantage then, of atom interferometers, is the atomic energy shift in the presence of fields and inertial effects. This makes atoms suitable for measurements of gravity, where the mass of an atom leads to a shift in potential energy. In the case of the Sagnac effect, we will see in the next section that the rest-mass energy of an atom leads to a substantial gain in sensitivity towards rotation.

We can now easily answer the second question, as to why it is necessary to split the beams. A single atomic beam could indeed be operated with microwave pulses to detect changes in path length, but it would only be operating as a microwave interferometer and so would not be very sensitive. To exploit the sensitivity of an atom interferometer, the energy splitting of the clock states must change. Applying an effect to a single atomic beam may shift the clock states equally, which would not result in a change of the atomic frequency. To change the splitting, the $|1\rangle$ and $|2\rangle$ clock states must experience a different effect. This can be achieved by splitting the beams, placing the atoms into a superposition of both energy and path states. In the following, we derive the phase shift of an atomic Sagnac interferometer by considering the energy difference between the co and counter rotating paths in a rotating frame.

3.2.1 Path Integral Approach

In the case of unguided atom interferometers, the phase shift can be calculated using path integrals along the classical path of the atoms. In the case of quadratic Lagrangians within the perturbative limit, the phase shift in a wave-function (to first order) can be obtained by integrating the perturbing Lagrangian $\Delta\mathcal{L}$ over the classical unperturbed path [91].

$$\delta\phi = \frac{1}{\hbar} \oint \Delta\mathcal{L} dt \quad (3.9)$$

This formula can be easily applied to find the phase shift due to rotations. We start with the Lagrangian for a free particle with velocity $\mathbf{v}$, position $\mathbf{r}$, and mass m , as:

$$\mathcal{L} = \frac{1}{2}m\mathbf{v}^2 \quad (3.10)$$

To derive the Sagnac phase shift, we will transform the Lagrangian of a particle into a rotating frame. A vector time derivative transforms to a rotating frame as:

$$\mathbf{v}' = \mathbf{v} + \boldsymbol{\Omega} \times \mathbf{r} \quad (3.11)$$

The extra velocity term can be interpreted as the Coriolis effect. We can now write the Lagrangian in the rotating frame as:

$$\mathcal{L}' = \frac{1}{2}m(\mathbf{v} + \boldsymbol{\Omega} \times \mathbf{r})^2 = \frac{1}{2}m\mathbf{v}^2 + m\boldsymbol{\Omega} \cdot (\mathbf{r} \times \mathbf{v}) + \frac{1}{2}m(\boldsymbol{\Omega} \times \mathbf{r})^2 \quad (3.12)$$

In the first order approximation, we neglect the last term, and take the perturbation to be:

$$\Delta\mathcal{L} = m\boldsymbol{\Omega} \cdot (\mathbf{r} \times \mathbf{v}) \quad (3.13)$$

Now we use it to find the phase shift for a Sagnac interferometer. The phase is found to be:

$$\Delta\phi_S = \frac{m\mathbf{\Omega}}{\hbar} \cdot \oint_{\Gamma_0} (\mathbf{r} \times d\mathbf{r}) = \frac{2m}{\hbar} \mathbf{\Omega} \cdot \mathbf{A} \quad (3.14)$$

where A is the area within the Loop. And here we have recovered the Sagnac phase. We can now compare the sensitivity to an optical interferometer.

$$\frac{\delta\phi_{atom}}{\delta\phi_{photon}} = \frac{mc^2}{\hbar\omega_0} \approx 10^{10} \quad (3.15)$$

Taking the mass of a rubidium atom, $m = 1.4 \times 10^{-25} Kg$, and light at $500nm$, we can find the phase shift for a given rotation speed varies by 10 orders of magnitude. This demonstrates that atom interferometry is potentially much more sensitive than optical methods, although it should be remembered that this comparison assumes the area to be the same in both cases. In reality, this is very far from the truth.

3.2.2 Origin of Phase Shifts - The Compton Debate

There has been considerable debate over the origin of phase shifts in an atom interferometer. In this section, an alternative interpretation is given. We have so far understood the phase as arising from the superposition of the hyperfine states. In the following, the atomic phase shift is interpreted as arising from interference of the de-Broglie waves.

In the de-Broglie picture, a matter-wave oscillates at the Compton frequency. Combining the mass energy equation $E = mc^2$, with the Planck relation of energy and frequency $E = hf$; one finds the Compton frequency associated with a massive particle $f_C = \frac{mc^2}{h}$. From this perspective, the phase shift in an interferometer may arise from the integration of the Compton frequency over the proper-time difference-of-arrival between the two arms.

This interpretation can seem compelling, as it allows for a very straight forward derivation of the atomic Sagnac effect. Multiplying the Compton frequency by the proper-time difference of equation 3.5 yields directly the equation for the Sagnac phase shift $\phi_S = \frac{4mA\mathbf{\Omega}}{\hbar}$, which is consistent with

other derivations based on the energy difference.

However, atom interferometers do not observe fringes at the Compton frequency, but instead observe shifts in the Ramsey fringes. For comparison, the nuclear hyperfine transition corresponds to an oscillation of 6.8GHz . Taking the mass of rubidium, we find its Compton frequency to be $\approx 10^{25}\text{Hz}$. This is many orders of magnitude greater than the best state-of-the-art clocks. It is strange then, that both interpretations give the same answer for the phase shift.

This can be understood as a relativistic phenomena. From the rest frame of the atoms, the phase shift arises from a difference in proper-time between the two arms of the interferometer. From the lab frame, the phase shift arises from a difference in the Compton frequency between the two arms. This difference is due to the kinetic energy, which leads to a relativistic correction to the mass of the atoms.

This can be shown explicitly by transforming between the two frames. This is done by writing the proper-time in terms of the coordinate time. Consider the case of an atom moving with velocity v between the input and output ports of an interferometer. From the rest frame of the atoms, it takes a proper-time τ to traverse the distance. From the inertial frame of a stationary observer, it takes coordinate time t to traverse the distance. In the de-Broglie wave formalism, the free-wave phase factor in the rest frame of the atoms is:

$$\phi = \omega_C \tau = \frac{\omega_C t}{\gamma} \quad (3.16)$$

where ω_C is the angular Compton frequency and $\gamma = 1/(\sqrt{1 - v^2/c^2})$ is the Lorentz factor for an atom moving at speed v . In the low-velocity limit, we can expand the Lorentz factor to first order, $\gamma^{-1} \approx 1 - (v^2/2c^2)$. We now see that:

$$\phi = \omega_C \tau = \frac{\omega_C t}{\gamma} = \omega_C t \left(1 - \frac{v^2}{2c^2}\right) = \left(\omega_C - \frac{mv^2}{2\hbar}\right) t \quad (3.17)$$

Thus, in the case of two arms of an interferometer, the Compton frequency will drop out as a common factor, and the kinetic energy contribution determines the phase shift [92]. However, this interpretation has received criticism, especially in regard to the claims that measurements of the Compton frequency may be used to make measurements of gravitational redshift [93] [94]. Sophisticated cases have been put forward for the correct descriptions of atomic phase shifts [95] [96]. It is fair to say however, that in the case of an interferometer using atoms fully confined in a guiding potential, the atoms are not free, and the phase must be found by considering the Hamiltonian of the system.

3.3 A Guided Sagnac Interferometer

3.3.1 Sagnac Effect for Guided Atoms

The path integral approach is not suitable for guided settings, where the actual effect of the guided potential on the phase must be considered. Nonetheless, the same expression for the Sagnac phase shift can be derived by a simple argument concerning the kinetic energy of the atoms.

We consider the case of atoms transported a half-revolution around the ring in time T . Seen from a stationary observer, an atom transported in a co-rotating fashion must travel faster than the counter-rotating atom to arrive at the other end of the ring at the same time. This situation is analogous to the clock experiment of Hafele and Keating, where two atomic clocks are flown around the Earth, as shown in figure 3.1. Denoting the co-rotating and counter-rotating motions as $+$ and $-$, we can write the kinetic energy for the two paths as:

$$\frac{mv_{\pm}^2}{2\hbar}T = \frac{mR^2(\pi^2 \pm \Omega T)^2}{2\hbar T} = \frac{mR^2(\pi^2 + \Omega^2 T^2 \pm 2\pi\Omega T)}{2\hbar T} \quad (3.18)$$

Provided that this difference in kinetic energy gives rise to a difference in the frequency of phase oscillation, we can write the accumulated phase difference as:

$$\phi_{\pm} = \pm \frac{m\pi R^2 \Omega}{\hbar} = \pm \frac{mA\Omega}{\hbar} \quad (3.19)$$

This phase is the Sagnac phase accumulated over a half revolution. The total phase difference for a full revolution is then 4 times this:

$$\phi_S = \frac{4mA\Omega}{\hbar} \quad (3.20)$$

This analysis shows that the kinetic energy can change the rate of phase

evolution, giving rise to a Sagnac phase shift. This approach considers there to be no time-delay between the arriving atoms, and the phase shift arises only from the difference in the frequency of phase evolution. This can be achieved with a state-dependent guiding potential.

3.3.2 Guided Protocol

The protocol for a guided atom interferometer is shown in figure 3.2. The atoms are loaded into one side of the ring in state $|\downarrow\rangle$. A $\pi/2$ pulse puts the atoms in a superposition of states $|\downarrow\rangle$ and $|\uparrow\rangle$. A state-dependent potential guides the atoms to the other side of the ring. Atoms in the $|\uparrow\rangle$ would go clockwise for example. As each atom is in a superposition, each atom takes both paths.

During the guiding, the atoms accumulate a Sagnac phase ϕ_S , as a result of the rotational angular speed Ω_S of the lab frame. The atoms are guided around the trap in the lab frame at a rate $\omega_{\pm}$. A π pulse swaps the states. This is often known as a spin-echo pulse. It makes the interferometer insensitive to slowly varying fields/perturbations. Following the π -pulse, the atoms are again state-dependently guided back to the starting position. After this, each state has made a full revolution. This is then a common path interferometer and should exhibit stable fringes. A final $\pi/2$ pulse is applied, with a controllable phase ϕ_{ref} . The state population difference $\hat{\sigma}_z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|$ is measured as a function of ϕ_{ref} .

In the ideal case, the atoms would be fully confined, and could be moved around the trap without affecting the spin state at all, and atoms would remain adiabatically in a single spin state. We neglect any interactions or mixing of the two spin states and describe the dynamics of the interferometer by a Hamiltonian of the form:

$$\hat{H} = \hbar\omega \left(\hat{H}_{\uparrow} |\uparrow\rangle\langle\uparrow| + \hat{H}_{\downarrow} |\downarrow\rangle\langle\downarrow| \right) \quad (3.21)$$

where ω is the trapping frequency. The operation of the interferometer scheme can be described using the evolution operators for the Ramsey pulses,

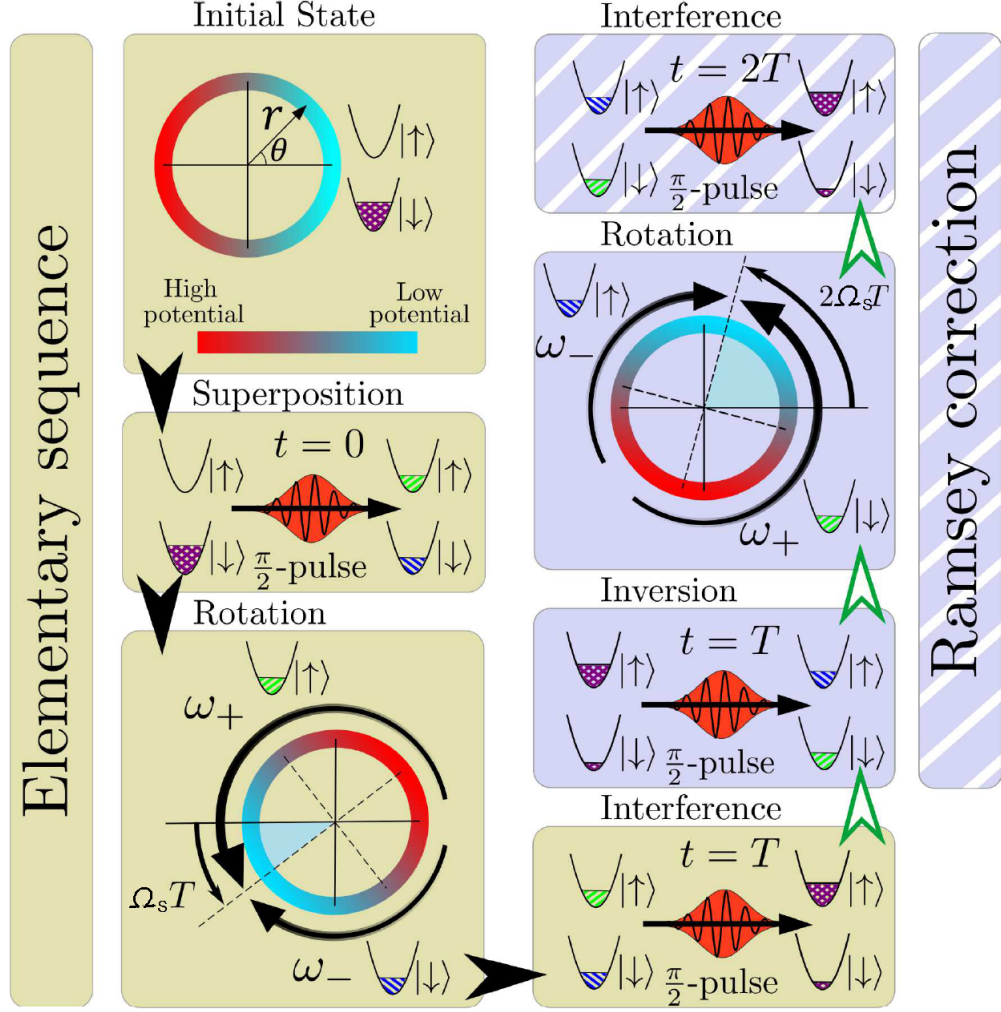


Figure 3.2: The atoms are prepared in the pure state $|\downarrow\rangle$ on one side of the ring. A $\pi/2$ pulse puts them into a superposition of $|\uparrow\rangle$ and $|\downarrow\rangle$. The atoms are state dependently transported to the opposite side, during which they evolve a dynamical and Sagnac Phase. A π pulse swaps the states of the atoms. The atoms are state-dependently guided back around to the start. A final $\pi/2$ pulse is applied, and the state population is measured. This figure is taken from [51]

and the free evolution period T . For simplicity, we treat only the elementary sequence shown in figure 3.2, ignoring the Ramsey correction (the π pulse). For the elementary sequence shown in figure 3.2 and atoms starting in motional ground state $|g\rangle$, the final atomic state can be expressed using unitary evolution operators:

$$|\Psi(T)\rangle = \hat{U}_{\pi/2}(\phi_{ref}) \hat{U}(T) \hat{U}_{\pi/2}(0) |g\rangle \otimes |\downarrow\rangle \quad (3.22)$$

where

$$\hat{U}(T) = \hat{U}_{\uparrow}(T) \otimes \hat{U}_{\downarrow}(T) \quad (3.23)$$

is the evolution imposed by the Hamiltonian and $\hat{U}_{\pi/2}(\phi)$ describes a $\pi/2$ pulse with a phase ϕ . The measured signal is the population difference $\langle \hat{\sigma}_z \rangle$, where $\hat{\sigma}_z = |\uparrow\rangle \langle \uparrow| - |\downarrow\rangle \langle \downarrow|$. This expression simplifies to:

$$\langle \hat{\sigma}_z \rangle = \frac{1}{2} \langle g | \hat{U}_{\downarrow}^{\dagger}(T) \hat{U}_{\uparrow}(T) | g \rangle e^{i\phi_{ref}} + h.c \quad (3.24)$$

Control of ϕ_{ref} enables interferometer operation near maximal dependence on laboratory rotation. Accordingly, we define the (dimensionless) sensitivity or scale factor Σ as:

$$\Sigma = \max_{\phi_{ref}} \left| \frac{d \langle \hat{\sigma}_z \rangle}{d\Omega_S} \right| \omega \quad (3.25)$$

In reality, the confinement will not be perfect, and motional excitation during transport may affect the final measurement. In general, an appropriate Hamiltonian must be found, and the final state expressed, as in equation 3.3.2, in terms of the the initial state via the evolution operators.

3.3.3 Guided Interferometer Models

We consider an idealised situation where the atoms are tightly confined to a ring of radius r , thus restricting the motional degrees of freedom to the azimuthal coordinate θ . Within this ring we assume that two harmonic potentials with trapping frequency ω are displaced in opposite directions at angular speed $\omega_P(t)$. In the laboratory frame, both paths end on the opposite side of the ring at $t = T$, imposing the condition:

$$\int_0^T \omega_P(t) dt = \pi \quad (3.26)$$

The Hamiltonian of a harmonic oscillator in 1 dimension is:

$$\hat{H} = \frac{-\hbar^2}{2mr^2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{2} \left(\theta + \sum_i \phi_i(t) |i\rangle \langle i| \right)^2 mr^2 \omega^2 \quad (3.27)$$

Here θ is the angular position around the ring, $i = 1, 2$ refers to the internal states of the atom, and ϕ_i is the trap center position for state $|i\rangle$. A state dependent transformation:

$$\hat{U} = \sum_i (\phi_i(t) |i\rangle \langle i|) \frac{\partial}{\partial \theta} \quad (3.28)$$

results in a Hamiltonian where each trap is stationary. The Hamiltonian is then transformed into second quantized notation, giving:

$$\hat{H}_{eff} = \hbar\omega \hat{a}^\dagger \hat{a} + i\hbar \sqrt{\frac{m\omega r^2}{2\hbar}} (\alpha + \Omega(t) \hat{\sigma}_z) (\hat{a} - \hat{a}^\dagger) \quad (3.29)$$

where $\Omega(t)$ is the trap speed as a function of time, and α is the global angular speed. This is a forced harmonic oscillator. The time evolution is given by the displacement operators [97]:

$$\hat{U}_{\uparrow(\downarrow)} = \exp(\alpha_{\uparrow(\downarrow)}^* \hat{a} - \alpha_{\uparrow(\downarrow)} \hat{a}^\dagger) \exp(i\phi_{\uparrow(\downarrow)}) \quad (3.30)$$

Here $\alpha_{\uparrow(\downarrow)}$ is the displacement. Substituting the evolution operators the final population difference is:

$$\langle \hat{\sigma}_z \rangle = C_{1D} \cos(\phi_S + \phi_{ref}) \quad (3.31)$$

The population difference depends simply on the Sagnac phase ϕ_S and the reference phase of the final pulse ϕ_{ref} . C_{1D} is the contrast, and depends only on the relative displacement of the atomic clouds:

$$C_{1D} = \exp(-|\Delta\alpha|^2/2) \quad (3.32)$$

The final relative displacement of the two spin operators is:

$$\Delta\alpha = \alpha_{\uparrow} - \alpha_{\downarrow} = \sqrt{2m\omega/\hbar r} \int_0^T \omega_P(t) e^{-i\omega t} dt \quad (3.33)$$

The second factor of equation 3.31 is the oscillatory part of the signal which indeed depends on the Sagnac phase $\phi_S = 2\pi m r^2 \Omega_S / \hbar$. This result shows that the Sagnac phase difference accumulated by the atoms remains independent of the temporal profile $\omega_P(\tau)$ of the path taken. However, the interferometer contrast is reduced if the final states of the two components are no longer in the ground state of the trap but (symmetrically) displaced. Any choice of temporal path that does not contain Fourier components at the trapping frequency will achieve maximal contrast by ensuring that the wavepackets overlap and appear stationary $\Delta\alpha = 0$. This has also been solved for thermal atoms of temperature Θ . In this case, we get:

$$\langle \hat{\sigma}_z \rangle_{Thermal} = \langle \hat{\sigma}_z \rangle \exp\left(-\frac{k_B \Theta}{\hbar \omega} |\Delta\alpha|^2\right) = C_{\Theta} \cos(\phi_S + \phi_{ref}) \quad (3.34)$$

The contrast is unaffected if the overlap is perfect. The overlap should be within the thermal wavelength (coherence length). This is analogous to the overlap condition for white light interferometers [10].

3.4 Dynamic Toroidal Ring Trap

The guided scheme requires a state-dependent guiding potential. This can be achieved with a radio frequency dressed trap. Here is discussed the scheme proposed by Thomas Fernholz (2007), for which the chip designs in the next chapter are based [98].

3.4.1 Ring Quadrupole

We start with a magnetic ring trap. This can be achieved, as in figure 3.3, using two concentric rings of permanently magnetized material, or current carrying wires. Taking the ring radius to be large, the two concentric rings of width w and spacing d form a ring quadrupole at a height z_0 above the surface:

$$z_0 = \frac{1}{2}\sqrt{d(2w + d)} \quad (3.35)$$

the field in this region can be written as:

$$\mathbf{B}_s \approx qr \begin{pmatrix} -\cos \theta \cos \varphi \\ -\sin \theta \cos \varphi \\ \sin \varphi \end{pmatrix} \quad (3.36)$$

B_s describes the field in the vicinity of the quadrupole. The field gradient is represented by the parameter q , and r is the distance from the quadrupole centre.

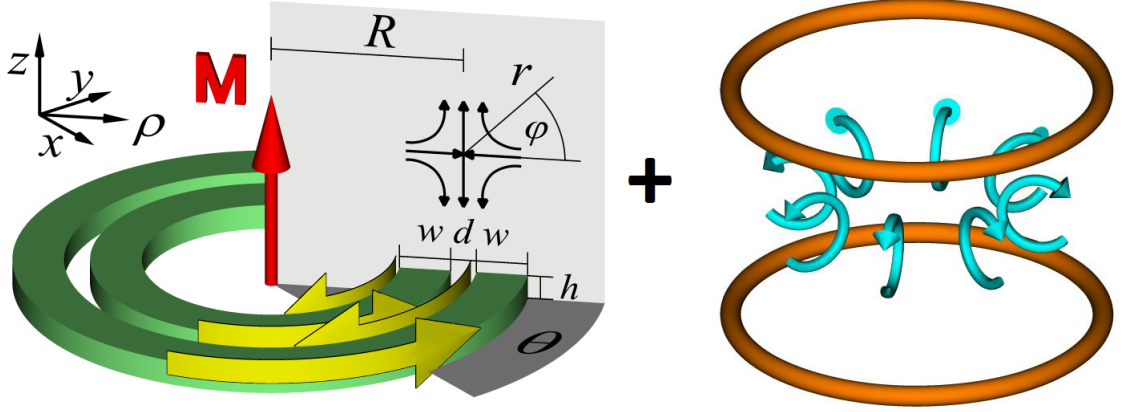


Figure 3.3: Magnetic rings form a ring-quadrupole above the plane of the chip. A pair of Radio frequency coils, operated with a 90° phase difference, can generate a toroidally polarised field. Figure taken from [99].

3.4.2 Toroidal Potential

Atoms confined in a field zero would escape through Landau-Zener transitions. The field is dressed by applying an RF field of circular or elliptical polarisation of the form shown in figure 3.3. We call this the smoke-ring field, and it can be described in global Cartesian coordinates as:

$$\mathbf{B}_{RF} = \frac{ae^{i\alpha}}{\sqrt{2}} \begin{pmatrix} \cos \theta \\ \sin \theta \\ i \end{pmatrix} + \frac{be^{i\beta}}{\sqrt{2}} \begin{pmatrix} \cos \theta \\ \sin \theta \\ -i \end{pmatrix} \quad (3.37)$$

Here θ is the angle around the ring. The polarisation has been decomposed into right and left circular components of amplitude a and b , with their respective phases α and β . The relative phase between them gives the angle of the polarisation ellipse. The polarisation relative to the atom's quantization axis determines the radio frequency potential. This axis is assumed to be defined by the magnetic field. As before, a new coordinate system is required, which matches the field distribution of the static field. In global Cartesian coordinates, this new coordinate basis is:

$$\hat{\mathbf{e}}_0 = \begin{pmatrix} -\cos \theta \cos \varphi \\ -\sin \theta \cos \varphi \\ \sin \varphi \end{pmatrix}, \hat{\mathbf{e}}_1 = \begin{pmatrix} -\sin \theta \\ \cos \theta \\ 0 \end{pmatrix}, \hat{\mathbf{e}}_2 = \hat{\mathbf{e}}_0 \times \hat{\mathbf{e}}_1 \quad (3.38)$$

$\hat{\mathbf{e}}_0$ is aligned with the static field vector in the vicinity of the ring quadrupole. $\hat{\mathbf{e}}_1$ is the tangential vector around the ring. $\hat{\mathbf{e}}_2$ is the third required vector in order to form an orthonormal triplet, and is found simply by taking the cross product between $\hat{\mathbf{e}}_0$ and $\hat{\mathbf{e}}_1$. In this basis, the left and right circular vectors can be written as:

$$\hat{\mathbf{e}}_+ = \frac{1}{\sqrt{2}}(\hat{\mathbf{e}}_1 + i\hat{\mathbf{e}}_2) \quad (3.39)$$

$$\hat{\mathbf{e}}_- = \frac{1}{\sqrt{2}}(\hat{\mathbf{e}}_1 - i\hat{\mathbf{e}}_2) \quad (3.40)$$

Remembering that $\hat{\mathbf{e}}_+$ and $\hat{\mathbf{e}}_-$ are the circular components about the static field vector, we can now find the polarisation coupling component of the RF field to the static field, β_+ . This component is found by the dot product of the RF field with the right-handed circular vector $\hat{\mathbf{e}}_+^* \cdot \mathbf{B}_{RF}$. This evaluates to:

$$\beta_+ = \frac{1}{2}(-ae^{i(\alpha-\varphi)} + be^{i(\alpha+\varphi)}) \quad (3.41)$$

The equation for the energy of an RF dressed state is:

$$E_{m_F} = m_F \mu_B g_F \sqrt{\left(B_s - \frac{\hbar \omega_{RF}}{g_F \mu_B}\right)^2 - \frac{|\beta_+|^2}{2}} \quad (3.42)$$

On the surface of resonance, $(B_s - \hbar \omega_{RF}/g_F \mu_B)^2 = 0$, so by substituting in β_+ , we get:

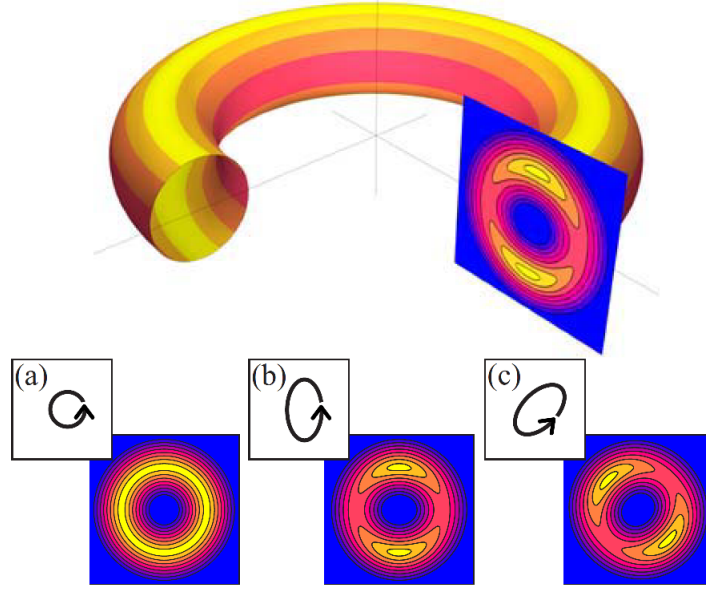


Figure 3.4: The surface of resonance of the RF dressed trap. In the case of circular polarisation, the energy minimum is uniform over the toroid, and the atoms would likely collect at the bottom. In the elliptical case, the toroid separates into two rings. Each ring can potentially be used to operate an interferometer. Figure taken from [99].

$$E_{m_F} = \frac{m_F \mu_B g_F}{2\sqrt{2}} \sqrt{a^2 + b^2 - 2ab \cos(\alpha - \beta - 2\varphi)} \quad (3.43)$$

Remember that a and b are right and left circular components, and α and β their respective phases. When either a or $b = 0$, the field is circularly polarised, the cosine term vanishes, and the potential becomes independent of φ . In general, however, the angle φ at which the potential minima occur depend upon the phases α and β of the circular components. When $a = b$, the field is completely linear, and the energy reduces to zero for certain phases, potentially resulting in unwanted Landau-Zener spin flips. When $a > b$ or vice versa, the RF field is elliptically polarised, and the potential is dependent on φ , reducing to a minima of:

$$E_{m_F} = \frac{m_F \mu_B g_F}{2\sqrt{2}} \sqrt{a^2 + b^2 - 2ab} \quad (3.44)$$

3.4.3 State-Dependent Guiding

Now, in order to stir atoms around the ring, we would like to create a toroidal minima. This can be achieved by adding an in-plane linearly polarised RF field.

$$\mathbf{B}_{\parallel} = \frac{ue^{i\zeta}}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} + \frac{ve^{i\gamma}}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} \quad (3.45)$$

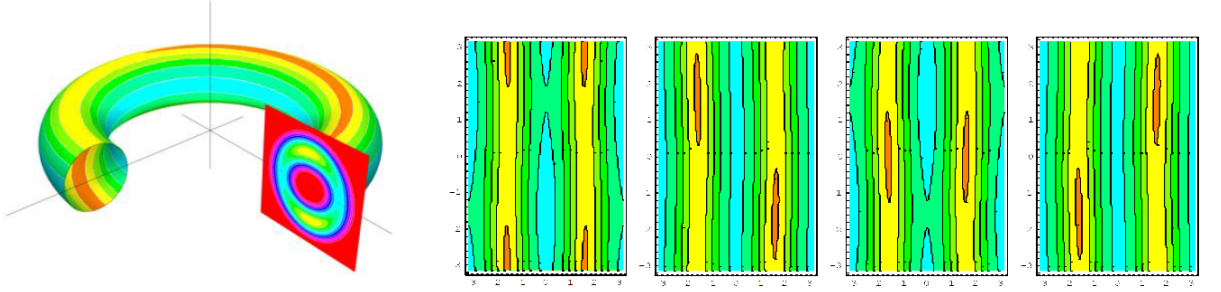


Figure 3.5: An additional in-plane RF field results in energy variation around the torus. Adjusting the phase of the in-plane RF causes the energy minima to move around the ring. The potential is state-dependent, and atoms with opposite g-factor signs experience an energy minima moving in the opposite direction. State dependent guiding for both atoms can be achieved in both rings. Figure taken from [99].

As before, we can find the coupling component β_+ of this in-plane field with the static field using $\beta_+ = \hat{\mathbf{e}}_+^* \cdot \mathbf{B}_{\parallel}$, This evaluates to:

$$\beta_+ = \hat{\mathbf{e}}_+^* \cdot \mathbf{B}_{\parallel} = \frac{ue^{i(\zeta+\theta)}}{2}(1 + \sin \varphi) - \frac{ve^{i(\gamma-\theta)}}{2}(1 - \sin \varphi) \quad (3.46)$$

Now we consider the coupling of the combined RF field with the static field.

$$\beta_+ = \hat{\mathbf{e}}_+^* \cdot (\mathbf{B}_{RF} + \mathbf{B}_{\parallel}) \quad (3.47)$$

This evaluates to:

$$\beta_+ = \frac{1}{2}(-ae^{i(\alpha-\varphi)} + be^{i(\alpha+\varphi)}) + \frac{ue^{i(\zeta+\theta)}}{2}(1 + \sin \varphi) - \frac{ve^{i(\gamma-\theta)}}{2}(1 - \sin \varphi)$$

Here the symmetry is broken, and the coupling components now depend on the toroidal angle θ . The result is an energy modulation around the ring. Adjusting the phase of the linear field moves the minima around the ring. The energy variation across the torus is plotted in figure 3.5 for varying phases of the linear field.

The two clock states of rubidium $|F, m_F\rangle = |1, -1\rangle$ and $|2, 1\rangle$ have g-factors of $g_F = -1/2$ and $+1/2$ respectively. For states of positive g-factor, it is the β_+ component that contributes to the dressed energy. In the case of a negative g-factor, the β_- component contributes instead. The above derivation can be repeated for β_- , yielding the response of a negative g-factor. It is then seen that, as the phase of the linear RF field is changed, the positive and negative g-factors experience different potentials which move in opposite directions around the ring. It is therefore possible to place the atoms in a superposition of clock states and create the counter propagating motion required to operate a Sagnac interferometer. We have thus achieved state-dependent, counter-propagating guiding of atoms. In the next chapter, the chip designs for realising this scheme will be discussed.

Chapter 4

Chip Design

We require a magnetic quadrupole and an RF dressing field to create the toroidal trap. This can be provided by magnetic and current carrying microstructures integrated onto a chip. Micro-fabricated ‘atom chips’ have the advantage of being able to produce higher field gradients and trapping frequencies than can be achieved with macroscopic wires. This is a result of the smaller distance of the trap from the wires, and higher current densities. The achievable current densities of integrated micron scale wires can reach as high as $10^7 A/cm^2$ [100], provided the heat can be dissipated effectively from the chip substrate. This should be taken to be an upper limit for short current pulses and ideal heat dissipation.

The overall experimental scheme is depicted in figure 4.1. The RF chip contains the RF coil used for the RF dressing. The ring chip creates the ring quadrupole. We created two chip designs for the ring chip, one based on permanently magnetized material, and the other based on current carrying wires (Electromagnet Chip). Magnetic materials have been used in place of conventional current carrying wires to trap and form a BEC [98]. As they carry no current, they do not suffer from current noise and Johnson noise. Additionally, they require no lead-wires, making them ideal for highly symmetric ring traps. The downside is that they cannot be switched off. Temperature measurements must then be developed to measure temperature in-situ, rather than relying on ballistic expansion. In the end we went for the electromagnet chip, due to difficulties with the magnetic material.

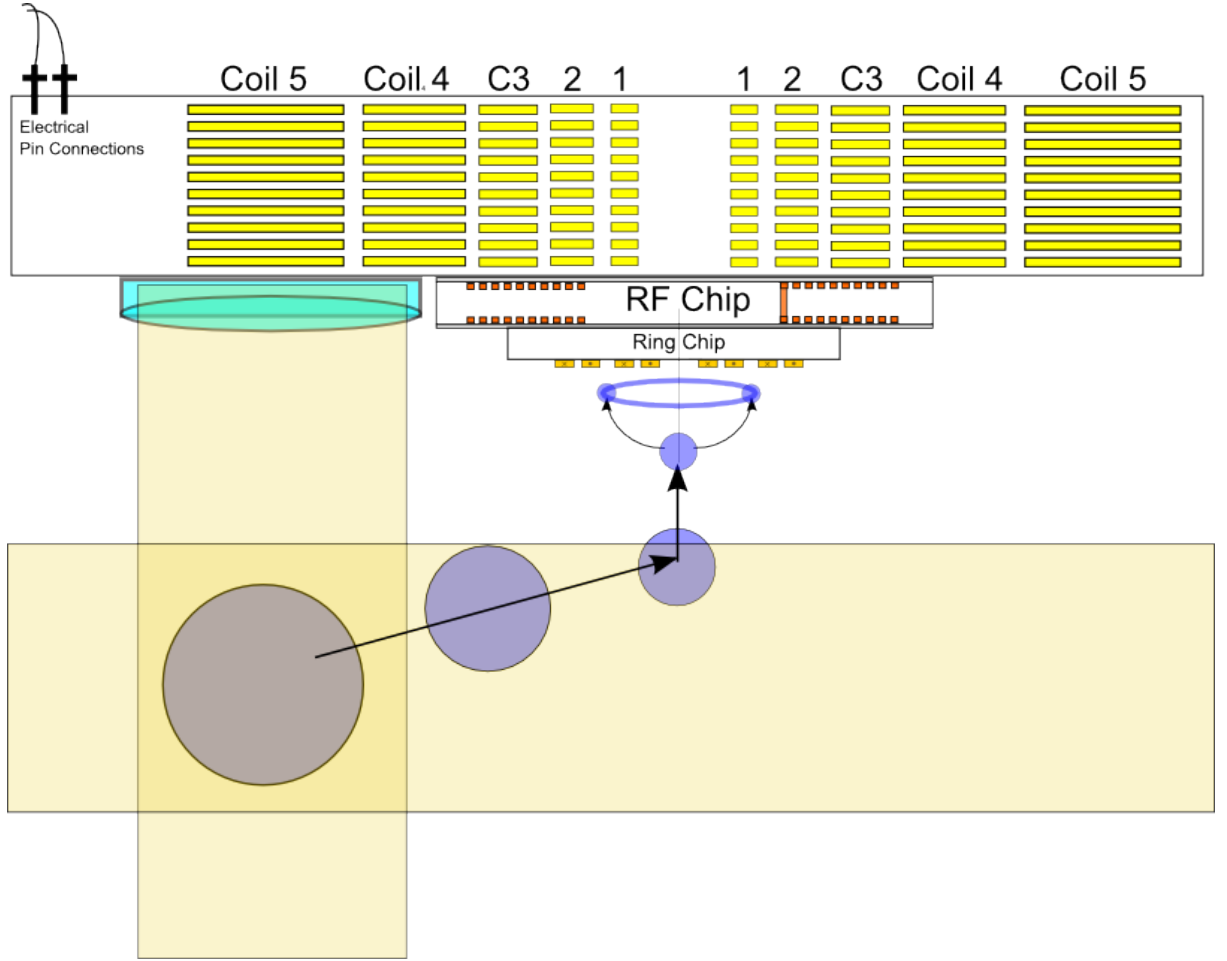


Figure 4.1: The Transport PCB, with mounted RF and Ring chips. The PCB is mounted upside down in the chamber. A MOT is formed using a reflective quarter-waveplate. The atoms are then transported laterally by 2cm using the PCB coils and assisted by the MOT coils. The atoms are compressed closer to the ring, and a loading procedure is applied.

The RF dressing must be toroidally polarised. The RF coil chip is designed only to produce a radial RF field. The vertical component is easier to produce and can be done using macroscopic coils. The RF coil is designed to be placed underneath the ring chip. All chips are mounted to a vacuum compatible Printed Circuit Board (Transport PCB). This is used for transporting the atoms from the region of the MOT to the chip. This transport is assisted by the MOT coils. In this chapter, the chip designs, PCB design and transport simulations will be presented. As operating an interferometer requires microwave pulses, a chip-based microwave resonator is also considered. A summary of the chip and PCB arrangement is shown in figure 4.1.

4.1 Electromagnet Chip

The Electromagnet (EM) chip provides the ring quadrupole using rings of counter-propagating current. First a simulation of 4 flat wires is used to choose the basic parameters. Then by simulating the current distribution in COMSOL, the realistic field is found. A set of 4 wires carrying counter-propagating current results in a quadrupole trap a distance z_0 above the surface.

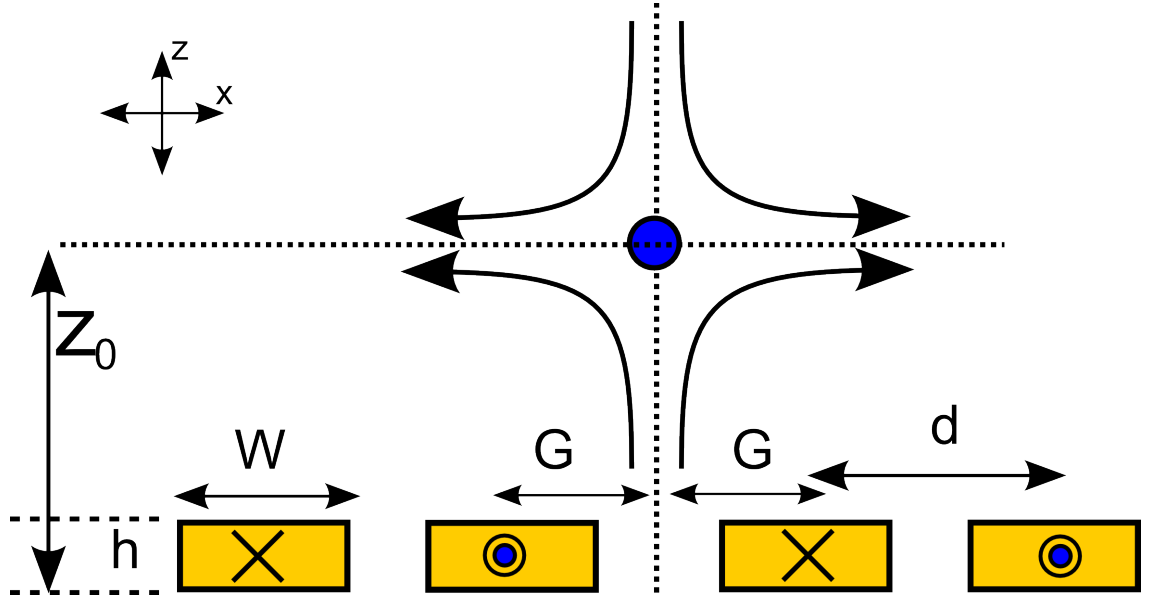


Figure 4.2: The parameters describing the Electromagnet chip. w is the width of the wire, d is the centre spacing of the inner and outer wire. $2G$ is the centre spacing between the inner wires. With current counter-propagating, a quadrupole trap at z_0 is formed.

The Equation for the field from an infinitely long straight wire carrying a current I is given as:

$$B_x = -\frac{\mu_0 I}{2\pi\sqrt{x^2 + z^2}} \frac{z}{\sqrt{x^2 + z^2}} \quad (4.1)$$

$$B_z = \frac{\mu_0 I}{2\pi\sqrt{x^2 + z^2}} \frac{x}{\sqrt{x^2 + z^2}} \quad (4.2)$$

Integrating over some width w , we get the equation for a wide flat wire of

infinite length.

$$B_{x,flat} = \frac{1}{w} \int_{-w/2}^{w/2} B_x(x - x_0, z) dx_0 = -\frac{I\mu_0 \left(\operatorname{arccot} \left[\frac{2z}{w-2x} \right] + \operatorname{arccot} \left[\frac{2z}{w+2x} \right] \right)}{2\pi w} \quad (4.3)$$

$$\begin{aligned} B_{z,flat} &= \frac{1}{w} \int_{-w/2}^{w/2} B_z(x - x_0, z) dx_0 \\ &= -\frac{I\mu_0 \left(\log \left[\frac{1}{4}(w - 2x)^2 + z^2 \right] + \log \left[(w + 2x)^2 + z^2 \right] \right)}{4\pi w} \end{aligned} \quad (4.4)$$

Now we have the equation for a flat wire. We consider the case of 4 wires carrying counter propagating currents. The wires are oriented flat on a substrate, with positions given by the parameters G and d as shown in figure 4.2. The wire height h is considered small. Should the currents be counter-propagating, the result is a magnetic field-zero a distance z_0 from the surface:

$$z_0 = \frac{1}{2} \sqrt{4dG + 4G^2 - w^2} \quad (4.5)$$

The height of the trap is then only dependent on the geometric factors if all wires carry the same current. The gradient at z_0 can be found to be:

$$\frac{\partial B}{\partial z} = \frac{2\mu_0 I}{\pi} \frac{z_0 d}{G(d + G)(d + 2G - w)(d + 2G + w)} \quad (4.6)$$

The electromagnet chip serves as an alternative to the magnetic ring chip using current carrying wires to create a ring quadrupole. A disadvantage of this design is that the lead-wires break the axial symmetry. The design therefore contains two layers. Alternating the current in the two layers as the atoms propagate around the ring should provide a continuous trap. A possible issue here is the smoothness of the trap as the current is alternated, and the effect of a double-layer on the planarisation of the wires. The wire parameters were chosen to produce a trap $80\mu m$ above the surface and a ring

radius of $500\mu m$. The design consists of a lower and upper ring separated by a planarisation layer of $3\mu m$. Satisfying both of these constraints greatly limited the choice in wire parameters. Ideally, the wires should be made as wide as possible to allow higher currents. However, to satisfy both constraints, the upper rings have a slightly smaller width than the lower rings. The width of the wires is $80\mu m$ with a height of $2\mu m$, and would carry a current of $0.5A$ to produce a trap with $3G$ depth. This amounts to a maximum current density of $10mA/\mu m^2$, which should be achievable with adequate heat drainage.

Bending these wires around in a loop should give a similar result. The Electromagnet chip design is shown in 4.3. To find the field numerically, we can either use a Biot-Savart simulation, or a finite-element method, or some combination of the two.

Biot-Savart

The Biot-Savart law describes the field from a line-element of current. The field at a position r is given by the total contribution from all current line-elements $I dl$.

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_C \frac{I d\mathbf{l} \times \mathbf{r}'}{|\mathbf{r}'|^3} \quad (4.7)$$

As any current distribution can be decomposed into a series of sufficiently small line-elements, the field at a point can be found by summing the contributions from every current element. This is a very versatile method of simulating magnetic fields.

Finite-Element Method

The 3D (or 2D) space is decomposed into a volume mesh (or area mesh). There is only one solution to the laws of physics that will satisfy the boundary conditions at every position in space. The boundary conditions are defined, and the appropriate laws are numerically solved. Axially symmetric and 2D systems can be modelled using FEMLab, which is free. We used COMSOL in our simulations for current and microwave fields.

4.1.1 Realistic Field

These calculations assumed infinitely long straight wires. In reality, the lead-wires and curvature of the wires should be taken into account. A Solidworks design of the EM chip was made using the above parameters. This is shown in figure 4.3.

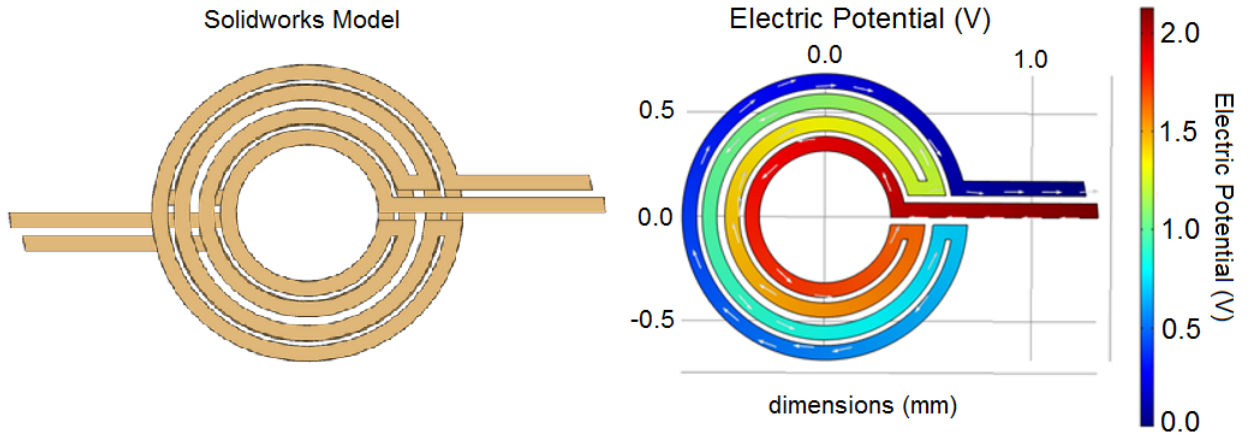


Figure 4.3: Left: The CAD (Solidworks) design of the electromagnet chip. The chip is double-layered to cancel the effect of the lead-wires. The CAD design is imported into COMSOL Right: COMSOL simulation of the EM chip. A voltage is applied to the lead-wires. Shown is the electric potential throughout the chip. This results in a current distribution which can be imported into MATLAB for a biot-savart simulation.

The Solidworks design was imported into COMSOL. A voltage was applied at the lead-wires, and the current density was found through the wires. This current was exported as a text file. A MATLAB program imported the current files and performed a biot-savart simulation.

Biot-Savart simulations are very convenient for finding magnetic fields. Although FEM methods can be used, they require that the calculation is performed in a 3D mesh that is very memory and computation intensive. For this reason COMSOL was used only for finding the current distribution through the wire. The area of interest is the cross-section of the ring field-zero. The field was plotted there for the two layers, as in figure 4.4.

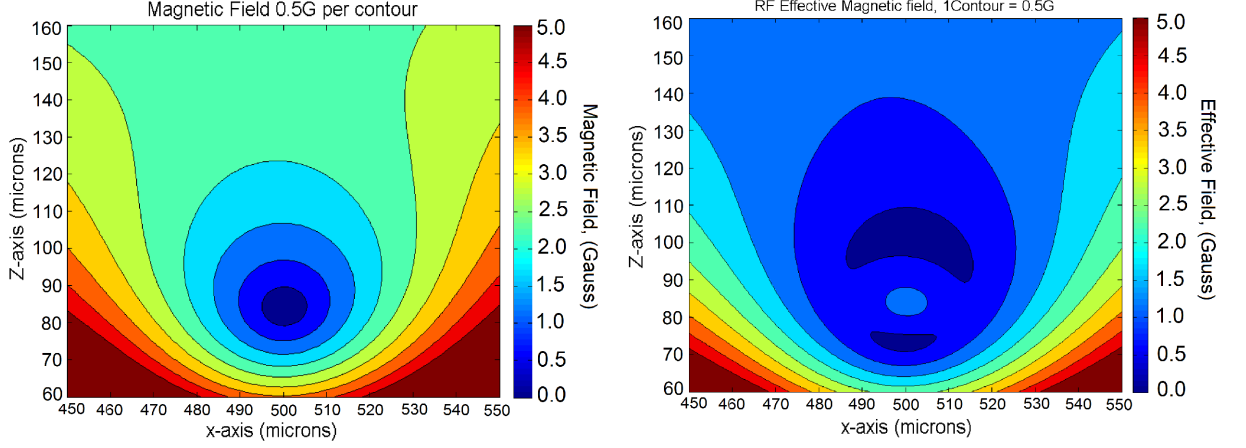


Figure 4.4: Left: The magnetic quadrupole trap above the ring trap. This is the static field only. The trap is formed $80\mu\text{m}$ above the chip surface. Right: The RF dressed effective field for an elliptically polarised 1G RF field. The trap splits into two lobes. As this is a cross section of a ring trap, these two lobes are in fact two toroidal rings.

4.1.2 Numerical RF Dressing

Once the static field $\mathbf{B}_s(\mathbf{r})$ is found, the dressed state energies can be computed. The equation for the dressed energy (see chapter 3) is:

$$E_{m_F} = m'_F \mu_B g_F \sqrt{\left(|\mathbf{B}_s(\mathbf{r})| - \frac{\hbar \omega_{RF}}{g_F \mu_B} \right)^2 + \frac{|\beta_+|^2}{2}} \quad (4.8)$$

This is sensitive to the right handed circular component β_+ in the local frame of the atom. To find β_+ , we must find the dot product of the circular vector with the polarised RF field $\mathbf{B}_{RF}$ expressed in the local frame. We must then transform to the local frame. This is a frame whose z-axis is aligned always with the local static field. This requires a space-dependent transformation, which was described in chapter 2.

Suppose we have a vector $\mathbf{v} = [v_x, v_y, v_z]$. We would like a transformation which expresses a second vector, $\mathbf{q}$, in a new coordinate system whose z-axis is defined by $\mathbf{v}$. The transformation takes the x, y and z components of $\mathbf{v}$ as arguments, and acts on $\mathbf{q}$ as: $M(v_x, v_y, v_z)\mathbf{q} = \mathbf{q}'$, where $\mathbf{q}'$ is the original vector $\mathbf{q}$ expressed in the new coordinate system. By definition, $M(v_x, v_y, v_z)\mathbf{v} = [0, 0, \sqrt{v_x^2 + v_y^2 + v_z^2}]$. This transformation matrix $M(a, b, c)$, where a, b and c represent the x, y and z components of an arbitrary vector, is:

$$M(a, b, c) = \begin{pmatrix} -\frac{b}{\sqrt{a^2+b^2}} & \frac{a}{\sqrt{a^2+b^2}} & 0 \\ -\frac{ac}{\sqrt{a^2+b^2}\sqrt{a^2+b^2+c^2}} & -\frac{bc}{\sqrt{a^2+b^2}\sqrt{a^2+b^2+c^2}} & \frac{\sqrt{a^2+c^2}}{\sqrt{a^2+b^2+c^2}} \\ \frac{a}{\sqrt{a^2+b^2+c^2}} & \frac{b}{\sqrt{a^2+b^2+c^2}} & \frac{c}{\sqrt{a^2+b^2+c^2}} \end{pmatrix} \quad (4.9)$$

At each point, the x, y and z components of the static magnetic field $\mathbf{B}_s = [B_x, B_y, B_z]$ can be used as arguments, and $M(B_x, B_y, B_z)$ can be used to transform the RF field into the local coordinate system defined by $\mathbf{B}_s$. The β_+ component is then $\beta_+ = \hat{\mathbf{e}}_+^\dagger \cdot M(B_x, B_y, B_z)\mathbf{B}_{RF}$. Taking the static field of the wires, the RF dressed energies are found. This is plotted in figure 4.4.

Field Overlap

The EM chip is double-layered to account for the lead-wires. As atoms are stirred around the ring, they are transferred to an identical trap made by the other layer. The lower ring and upper ring must create a matching trap. As the two layers are separated by $5\mu m$, the wires on each layer must have slightly different geometrical parameters to form the overlapping traps. The overlap is plotted in figure 4.5.

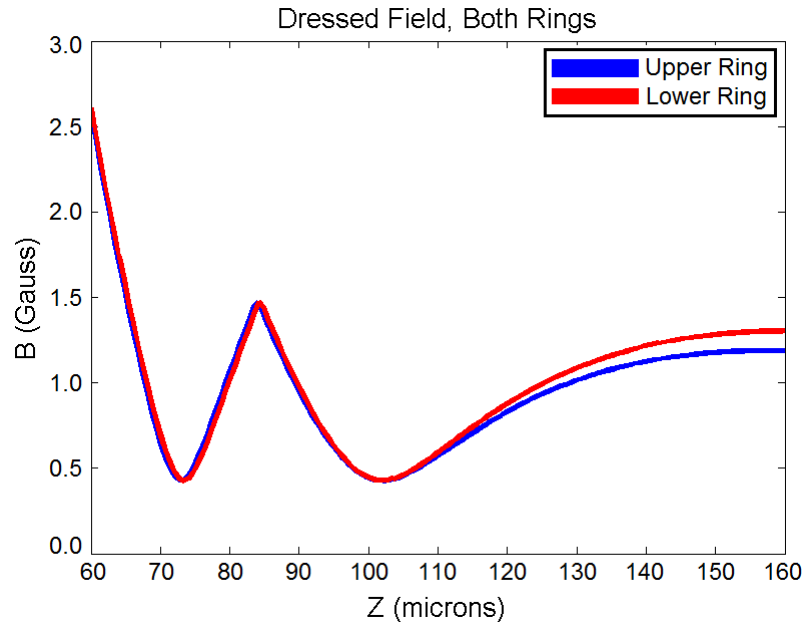


Figure 4.5: The field overlap of the two layers. The static is field is formed from the ring wires of the two layers of the chip. The two layers are separated by $5\mu m$. Shown in the figure is a vertical cut of the RF dressed potential of figure 4.4.

4.1.3 Fabrication

The electromagnet chip was fabricated in Ben-Gurion University using electroplating. The chip is shown in figure 4.6. It consists of a silicon substrate, $280\mu m$ thick, with silicon oxide insulating layers. Contact pads at the edge of the chip bond to pads on the RF chip. Alignment marks at the edge allow the chip to be correctly and precisely aligned atop the RF chip. The surface is coated with a gold mirror for reflection imaging.

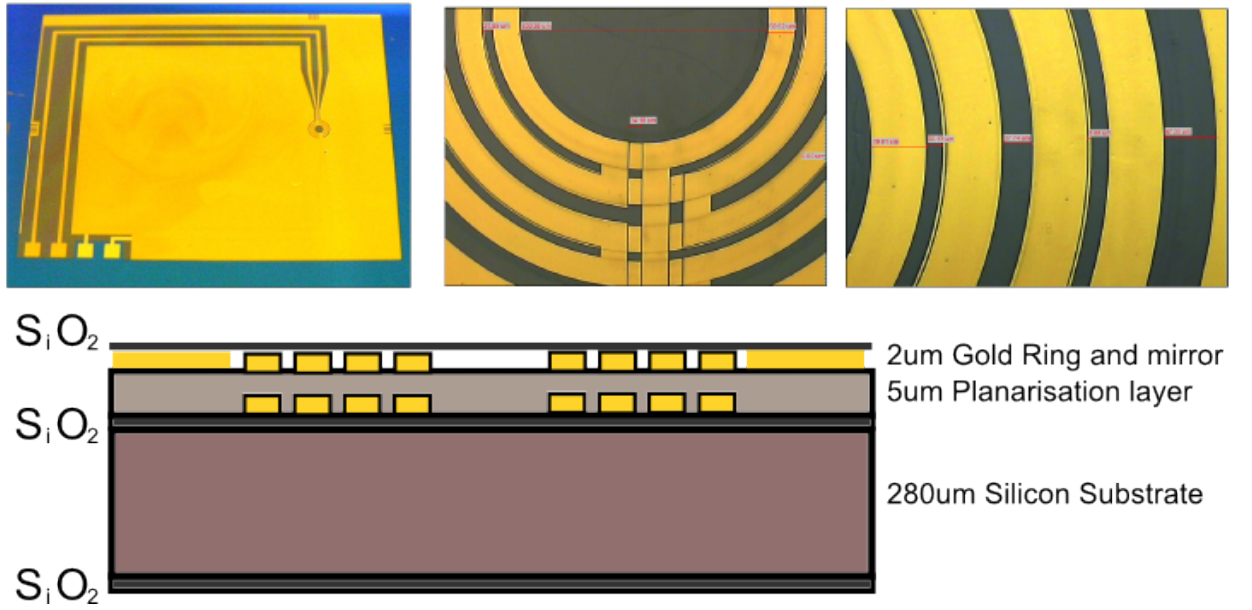


Figure 4.6: The fabrication of the EM chip. The chip consists of a silicon substrate with silicon-oxide insulation layers. Two ring layers are separated by a $5\mu m$ planarisation layer. A gold mirror coating allows reflection imaging of atoms close to the chip.

4.2 Magnetic Chip design

The magnetic chip is a permanent magnetic alternative to the EM chip. The magnetic chip consists of concentric rings, permanently magnetized out of plane. These create a magnetic ring quadrupole trap above the chip surface, at a height on the order of the ring separation. Magnetic films were grown using sputtering, following a paper [101]. The magnetisation can be modelled as edge currents flowing around the edge of the material. For a material of magnetisation M and height h magnetised out-of-plane, the field is the same as that generated by current flowing around the edge of the sample of $I_{edge} = Mh$. To model the field from two concentric rings of magnetised material, we can take four counter-propagating edge currents. To simplify things further, we take the edge currents as being infinite straight wires. This should produce a similar result providing that the radius of the ring is much larger than the ring separation. From this we find that the rings produce a field-zero a distance z_0 from the surface of the chip:

$$z_0 = \frac{1}{2} \sqrt{d^2 + h^2 - w^2} \quad (4.10)$$

Here d is the gap between the rings, w is the width, and h is the height of the film, which is negligible compared to the other parameters and can be ignored. The height of the trap is then only dependent on the geometric factors, and not on the magnetisation strength. The gradient at z_0 can be found to be:

$$\frac{\partial B}{\partial z} = Mh\mu_0 \frac{8wz_0}{\pi(d^2 + h^2)(w^2 - d^2)} \quad (4.11)$$

This equation can be further simplified, by recognising that h is small compared to the other geometric parameters. Notice that Mh here is the edge current I_{edge} . Therefore, the gradient is proportional to the edge current as would be expected. Choosing some reasonable parameters, we can see what we need to aim for when growing the magnetic film. For a ring radius $\approx 500\mu m$, and a ring width $\approx 80\mu m$, we find that an edge current of about

0.2A will produce a gradient of $2000G/cm$. This can be achieved with a material of magnetisation $1000kA/m$ (emu/cc) and a film height of $200nm$.

4.2.1 Fabrication

After looking at the basic parameters for the magnetic chip, we tried to grow a magnetic film. Fabrication involves two things - growing the film, and etching the pattern. The magnetic films were grown by myself and Andrew Rushforth at Nottingham. The fabricated films were then sent to Ben-Gurion university for etching. A high coercive field is desirable, as the chip should be able to maintain its magnetisation in the presence of the magnetic fields that will typically be used during cold atom experiments. A high Saturation magnetisation is desirable. Permanent magnetic films have been used before to form magnetic traps [98]. They used a FePt film to create a trap for a BEC. The film was patterned using electron beam lithography. Others have also investigated the use of these materials to create magnetic traps. Following from the work done by others [102] [101] [103], we decided to use FePt layers on a magnesium oxide substrate.

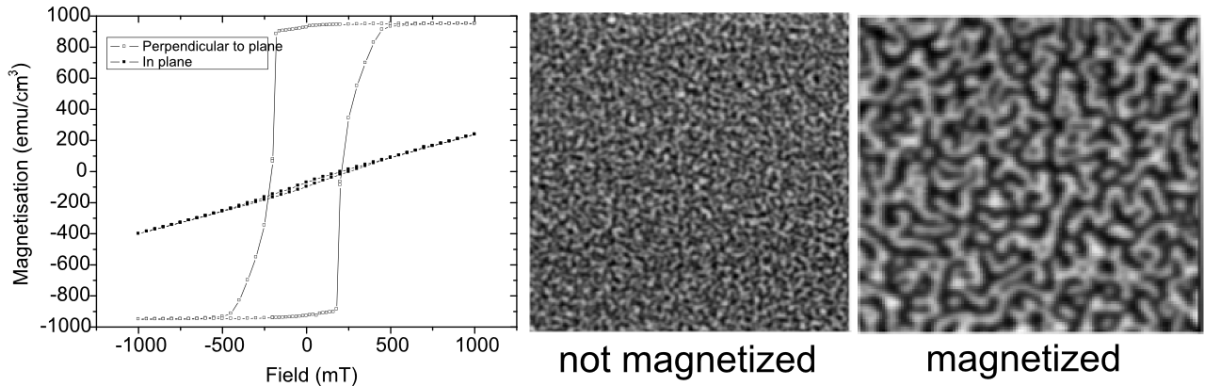


Figure 4.7: Left: The hysteresis graph of the magnetic film. A coercive field of $200mT$ and a magnetisation of $950kA/m$ was observed in a thin $20nm$ sample. These properties were not observed in thicker samples however. Right: MFM measurement of a thicker sample, before and after magnetisation. The magnetic domains are larger after magnetisation. However, the domains are too small, and will result in corrugation of the field.

The films consist of iron platinum layers on a magnesium oxide substrate, with a $2nm$ platinum cap layer to prevent oxidation. They were grown at $700^{\circ}C$ and $7.5mTorr$ by sputtering. The first films grown were $20nm$

thick. The samples were placed in a SQUID to measure the magnetic properties. The hysteresis curve is shown in figure 4.7, and shows a coercive field of $200mT$ and a magnetisation of $950kA/m$. This coercive field is suitably large to prevent demagnetisation during an experimental sequence. The magnetisation is large enough to reach an effective edge current of $0.2A$ with a $200nm$ film thickness, which should be large enough to create tight traps with a few gauss depth. Further films of thickness $150nm$ were grown and sent to Ben-Gurion university for etching. These have been completed as shown in figure 4.8. The total size of the chip is $1cm \times 1cm$. The magnetic rings occupy about $0.5mm$ in the centre of the chip. The surface of the chip has a gold mirror coating everywhere except the etched ring structure, and a diagonal cut to reduce unwanted RF induced eddy currents. Unfortunately,

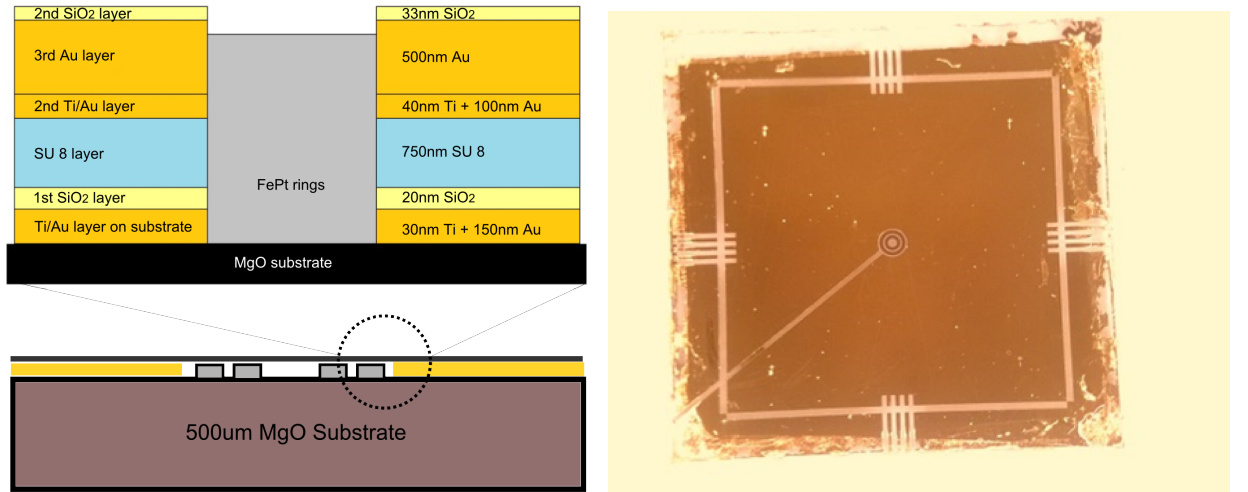


Figure 4.8: The finished magnetic Chip. As photographed by Ben-Gurion University. The total size of the chip is $1cm \times 1cm$. The magnetic rings occupy about $0.5mm$ in the centre of the chip. The surface of the chip has a gold mirror coating everywhere except the etched ring structure, and a diagonal cut to reduce unwanted RF induced eddy currents.

SQUID measurements on the thicker films have revealed substantially less magnetization. MFM measurements have confirmed the existence of very small magnetic domains on the order of microns. Further work needs to be done to retain the promising magnetic properties found on the earlier $20nm$ films. It would seem then that the thicker films do not maintain their magnetisation properties. A seed layer may improve this, or perhaps different sputtering parameters. More work needs to be done on this.

4.3 Radio Frequency Chip

The RF dressing requires elliptical polarisation. The purpose of the radio frequency chip is to provide the ‘smoke-ring’ RF dressing field. That is, an RF field that is of circular or elliptical polarisation everywhere around the magnetic ring trap. The RF field consists of radial and vertical components that oscillate with a 90° phase difference. The radial component requires a coil that has a size on the order of the ring trap. The RF coil must provide the radial component of the RF dressing. The vertical component is homogeneous everywhere around the ring, and can be created using a much larger coil. The chip is designed to sit underneath the ring chip, and on top of the transport PCB as shown in figure 4.1. The coil design is shown in figure 4.9.

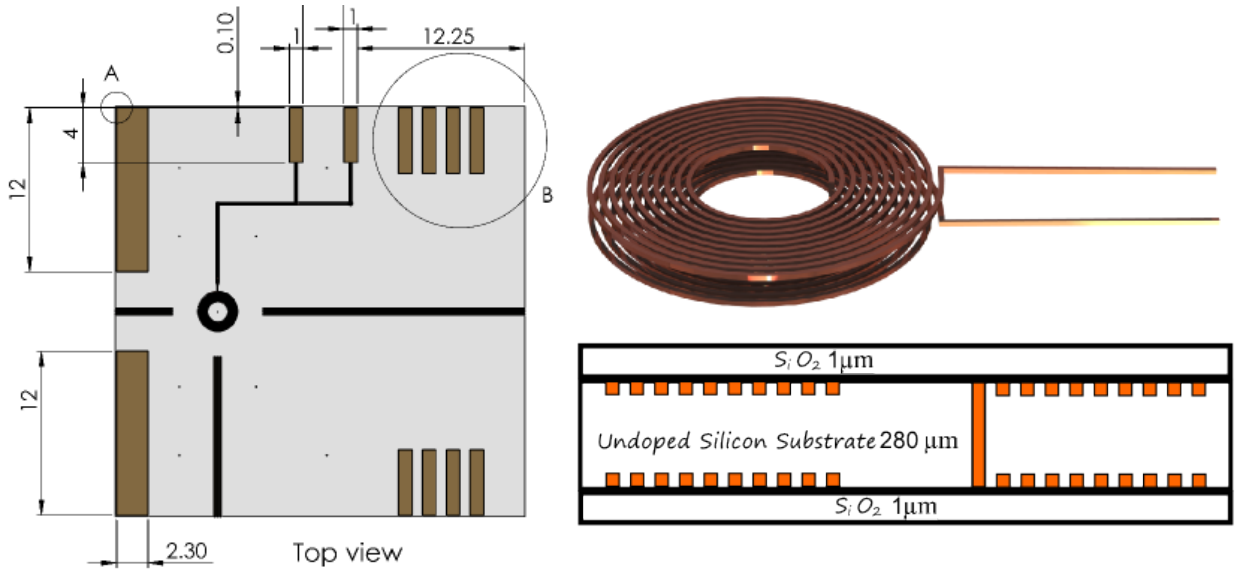


Figure 4.9: RF Chip Layout. The Ring chip is smaller than the RF chip, and sits on top. Alignment marks allow for alignment of the Ring chip onto the RF chip, and of the RF chip onto the PCB. The chip is $3\text{cm} \times 3\text{cm}$.

It consists of a double-layer spiral design, with 10 loops on each layer. The wires are 40 by 40 microns, and the outer diameter is approximately 2.5mm . It is designed to produce a radial field of $3G$ with a current of $1A$. It is currently being fabricated in Ben-Gurion University using deep reactive ion etching on a chip wafer of thickness $300\mu\text{m}$.

4.3.1 Field Design

A radial RF field polarisation is difficult to generate, as it requires an RF coil on the size order of the ring chip. The field simulation is shown in figure 4.10.

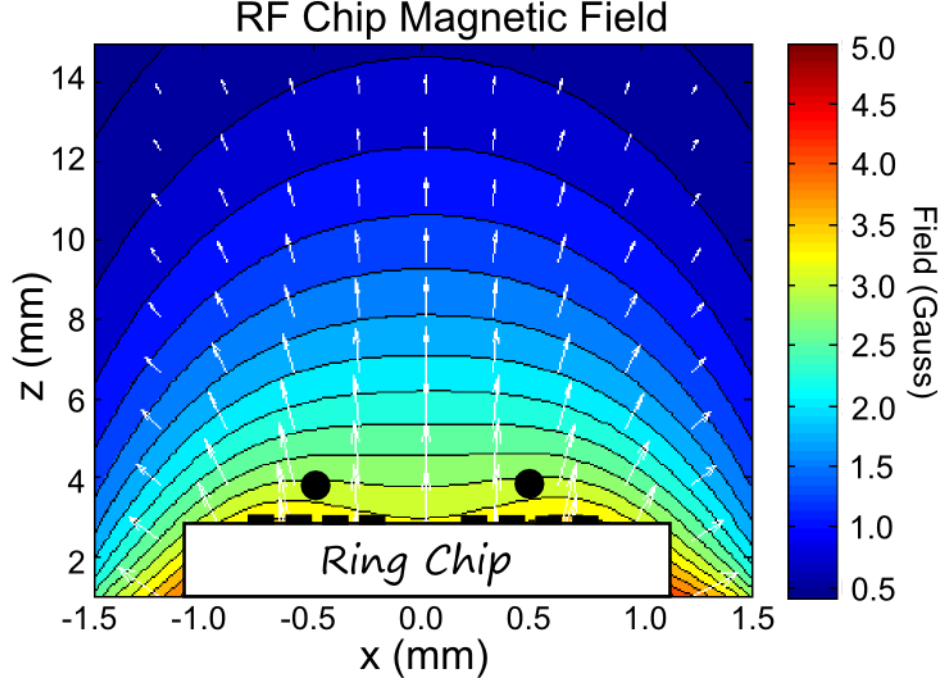


Figure 4.10: The Field from the RF chip. The current $I = 37mA$. The point of interest is the position where the ring quadrupole will be. This is depicted by the black circles. Here the radial field is $1G$, and the vertical field is $3G$.

The z -component of the field from a single coil is:

$$B_z = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}} \quad (4.12)$$

This has a gradient:

$$\frac{dB_z}{dz} = -\frac{3\mu_0 I R^2 z}{2(R^2 + z^2)^{5/2}} \quad (4.13)$$

The total divergence of the field must be zero, and the field has circular symmetry in the x - y plane. Therefore the radial gradient is half the z gradient:

$$\frac{dB_z}{dr} = -\frac{3\mu_0 I R^2 z}{4(R^2 + z^2)^{5/2}} \quad (4.14)$$

The field gradient is maximal at $z = R/2$, at this point the radial gradient is:

$$\frac{dB_z}{dr} = -\frac{12\mu_0 I}{25\sqrt{5}R^2} \quad (4.15)$$

We would like a few gauss radial field at the position of the ring. The ring trap is at the location where the radial gradient is maximal. In this region, the radial field is $1G$ and the vertical field is $3G$.

4.3.2 Fabrication

The chip was fabricated using deep reactive Ion Etching (DRIE) at Ben-Gurion university. Two chips were made. Resistance measurements were performed. The result is shown in the figure. It was found that one of the chips had substantially higher resistance. Both chips had slightly lower resistance than was theoretically estimated. The resistance for each coil was measured, and is seen to scale appropriately with coil circumference.

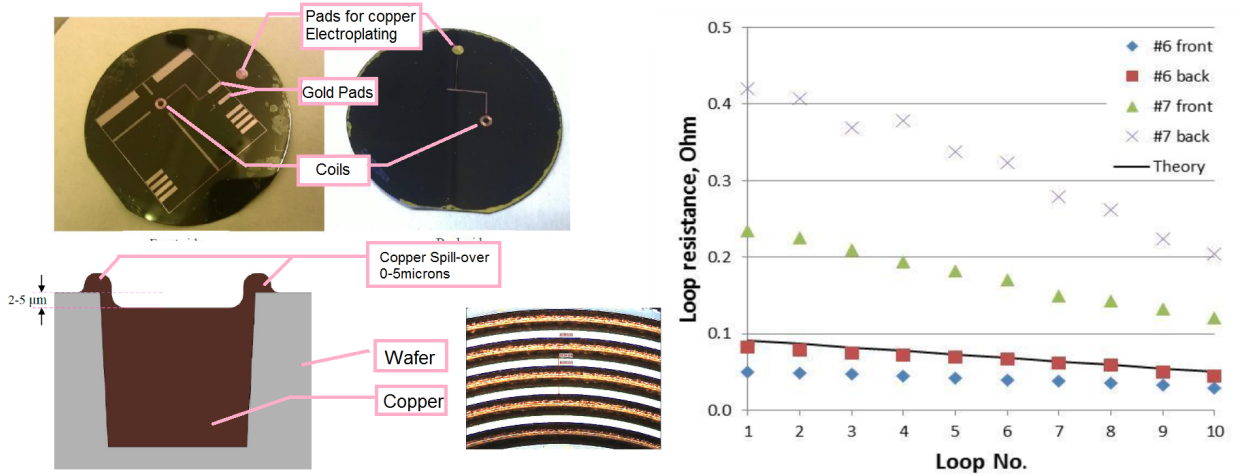


Figure 4.11: Left: RF Chip Fabrication. A photograph of the RF Chip, a picture of the copper trenches, and an optical image of the rings. Right: Resistance measurements of the chip.

4.4 Loading a Ring Trap

We consider the case of loading a magnetic ring trap. The rings also create a central quadrupole in addition to the ring quadrupole. A further field-zero exists at $z = \infty$. Applying a vertical bias field causes this zero to come down from infinite and merge with the central quadrupole. It may then be further split into a ring by applying a loading field as shown in figure 4.12. The loading field is found by finding the field along the desired trajectory. The opposite of this field is then applied. The radial component of the loading field can be provided by the RF chip. The vertical component can be provided by the larger PCB coils.

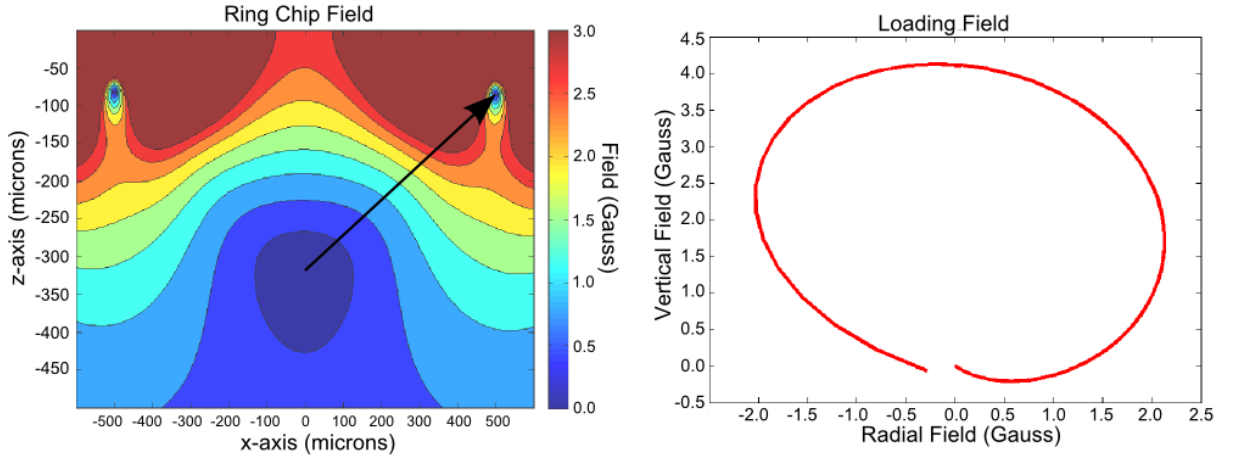


Figure 4.12: Left: The field generated by the ring chip. $1\text{Contour} = 0.33\text{G}$. In addition to the ring quadrupole, there is also a central quadrupole, and another field-zero at $z = \infty$. Applying a vertical bias field causes the two minima to merge. The atoms can then be loaded to the ring using a combination of vertical and radial fields. This loading field is found simply by taking the field along the desired trajectory, and applying the opposite. Right: The loading field expressed as an ellipse. It begins and starts at zero field as expected.

4.5 Transport PCB

In order to load the chip, we must have a way of transporting and compressing the atom cloud. Macroscopic coils [85] and dipole traps have been used by others to move atomic clouds [104]. Reichel describes a transport chip with a conveyor belt design to move atoms [105]. We provide transport using off-centre coils embedded into a PCB.

A UHV compatible, multi-layer transport PCB has been designed and manufactured¹. The PCB design is shown in figure 4.13. A reflective quarter-waveplate is glued to the PCB to provide the reflected beam for the MOT. The waveplate region is shown as a white circle in the figure. Concentric multi-layer coils, centred 2cm from the MOT region, provide transport and compression to the chips. The chips are placed 2cm to the right of the MOT centre, centred on the coils.

Layer Structure

The layer structure is shown in the figure. It consists of alternating core and prepreg layers. The core layers hold the PCB tracks and coils, and the prepreg separates and insulates the layers. The PCB has 10 coil layers. The coils on each layer are connected in series, thus each coil has 10 turns. The PCB contains 5 concentric multi-layer coils, and two auxiliary top layer coils.

The top layer is shown in red in the figure, and contains the lead-wires to the coils, and to the bonding contact pads for the chips. Bonding pads and alignment marks surround the chip area. Half-circle areas of copper provide an even surface so that the chip sits flat on the surface.

Mid-layers are shown in the figure. These consist of the coils and additional copper. This copper covers the entire unused area. This provides an even surface to each layer which is helpful to avoid inter-layer shorts through the prepreg material.

The back side of the PCB is shown in dark blue. It consists of a large flat copper area for good thermal contact to a heat sink. It also contains the

¹Manufactured by Hughes Circuits, San Diego

pin contact-pads at the edges, and SMP contact-pads for the RF coil. The current connections to the board are made by pins that extrude out the back of the PCB.

PCB Connections

Connections to the PCB are made using plated vias around the edge of the PCB. Press-fit pins are inserted into the plated vias. The head of the pins should be flush to the surface of the top layer. The tail of the pins must extrude out of the back (out from the bottom layer side).

RF connections are made to the board by an SMP connector that has been surface mount soldered. An SMP connector was chosen because it is smaller than SMA. This connector must be soldered using vacuum compatible solder, and preferably using a flame based soldering iron to avoid chemical contamination in the vacuum.

It is important that PCB connections do not obstruct optical access to the atoms. For this reason, the connections are made at the back of the PCB and the vias have been located as to avoid the optical path.

Connections between Layers

Connections are made between the layers using filled copper vias. Blind vias would be ideal for connecting the coil layers. However, blind vias are very expensive, so all vias in the design are 'through-all'. Connecting the coil layers in series using through-all vias is tricky, as each through-all via must only connect adjacent coil layers. This was achieved using a tabular design. Around each coil is a circle of vias. Each coil has a connecting tab on the edge in a different location to connect to the vias. This can be seen in the mid-layer diagrams in figure 4.13.

PCB Dimensions

The copper tracks in the mid-layers are $35\mu\text{m}$ tall. This is a standard copper thickness for PCB tracks, which are often stated in units of ounces per square

foot. A thickness of $35\mu m$ is sometimes written as a thickness of $1oz$. The top layer thickness is $70\mu m$ (or $2oz$). The total thickness of the PCB is $2mm$, and is $77mm$ across. This is just small enough to fit through a DN100CF flange.

PCB Materials

The PCB must be compatible with ultra high vacuum, lest it should out-gas and spoil the vacuum. This means that all materials must be vacuum compatible. There is no easy way to know if a material is suitable, short of testing in a test chamber. A UHV compatible PCB had been made previously by another group at Nottingham and did not present any problems, so we chose to use the same materials. These are as follows. The PCB core is Rogers 4350. The prepreg is Rogers 4450. There is no solder mask, silkscreen or nickel layer.

4.5.1 Transport Simulation

The atoms must be transported $2cm$ horizontally to the location of the chips. The transport is done with the aid of the external MOT coils. Once the transport is complete, the MOT coils are switched off and the atoms are transferred to a trap created only with the PCB. There are a number of stages in the transport procedure. Firstly a MOT is made using the reflective quarter-waveplate. Then the MOT is switched off, and the atoms are caught in a magnetic trap. The magnetic trap is transported laterally by $2cm$. This is done with the PCB coils and with the aid of the MOT coils, which provide the required magnetic trap gradient of roughly $50G/cm$. The upper MOT coil must be operated with a bipolar supply. The currents are shown in figure 4.14. The clouds are then transported to a trap made entirely from the PCB coils. The atoms are compressed and raised closer to the chip surface. The loading cycle described in figure 4.12 is then performed to transfer the atoms into the ring.

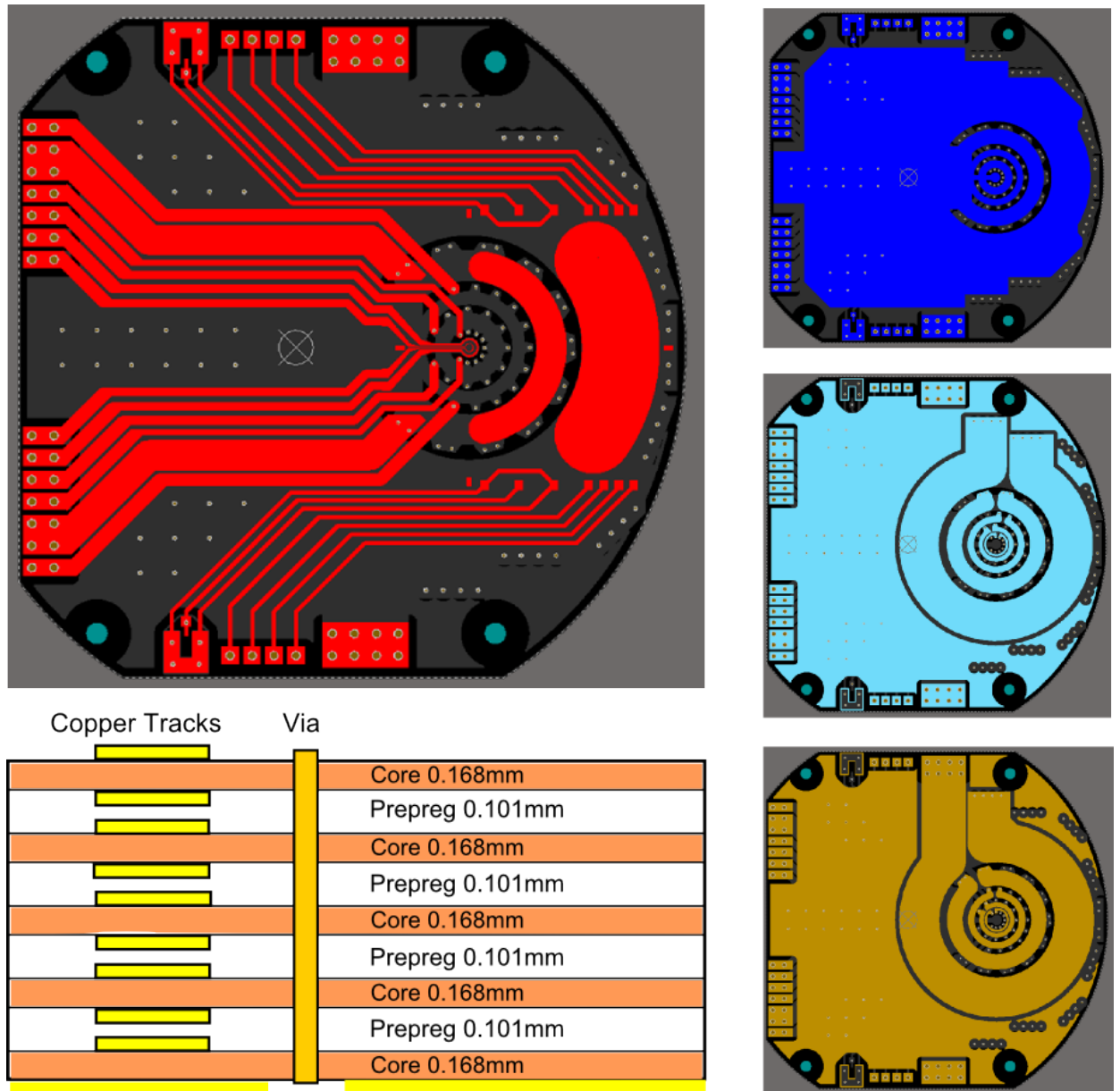


Figure 4.13: The design of the Transport PCB. **Bottom left:** The layer structure is shown. The PCB has 10 coil layers - prepreg and core. Vias connect the coils between layers. **Top left:** The top layer of the PCB, which contains the lead wires to the coils, and the contact pads for the chips. The white cross marks the point where the waveplate is glued for the MOT. Other layers are in grey. **Top right:** The Bottom layer - here the connections to the PCB are made around the edges of the board. A flat sheet of copper covers the surface of the bottom layer to provide smooth thermal contact with a heat-sink. **Mid and bottom right:** Shown are some of the mid layers which contain the coils.

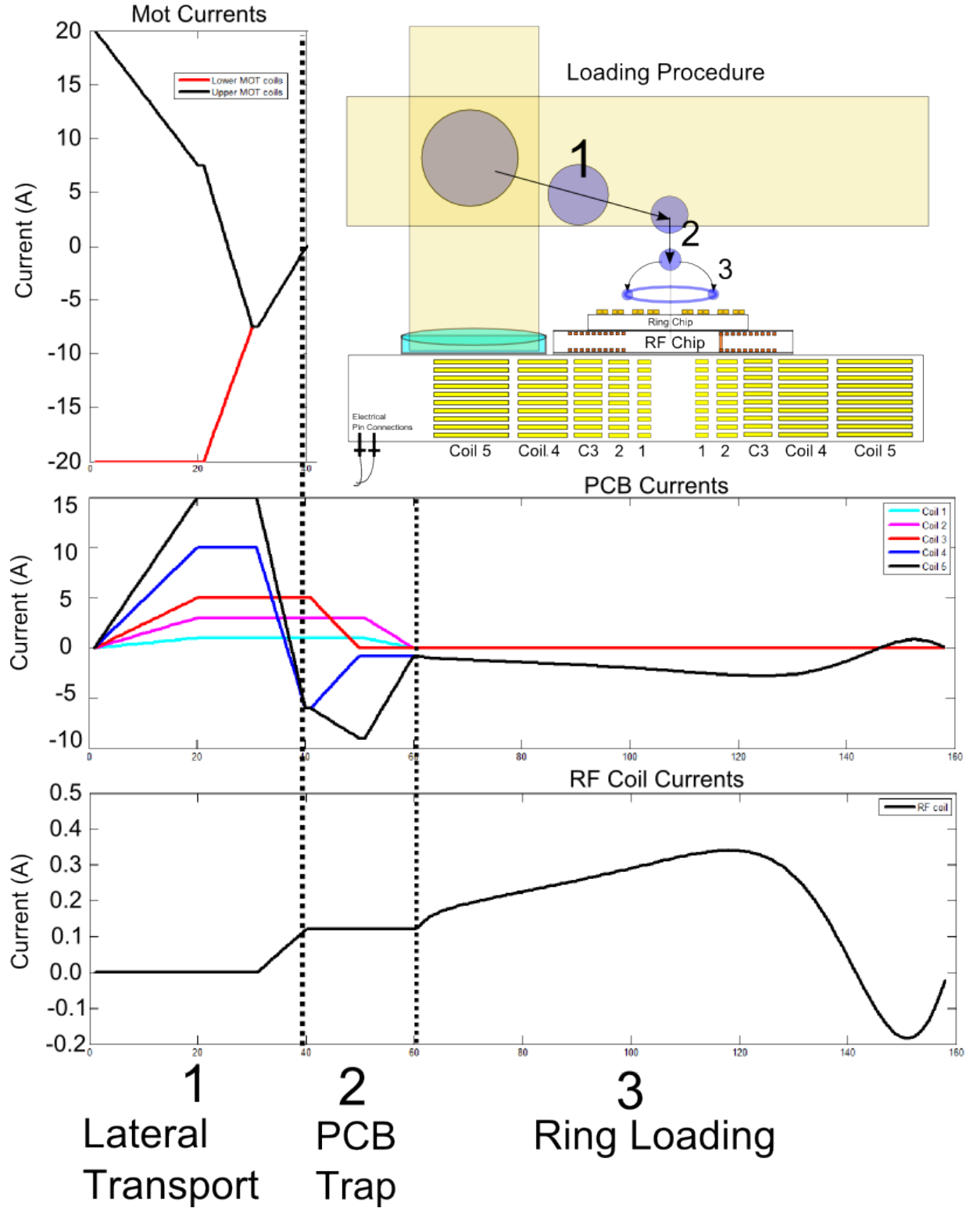


Figure 4.14: The currents needed to transport and load the atoms. The transport consists of 3 stages, depicted in the picture. The first is the transport. Assisted by the MOT coils to provide a sufficient gradient, the PCB coil currents are ramped up. Towards the end of the lateral transport, the MOT coils are ramped down and the atom is transferred into a trap made completely by the PCB coils. This requires some bipolar current supplies. The atoms are compressed and taken closer to the chip. In the final stage, the ring is loaded using the procedure described earlier.

4.5.2 Potential Losses

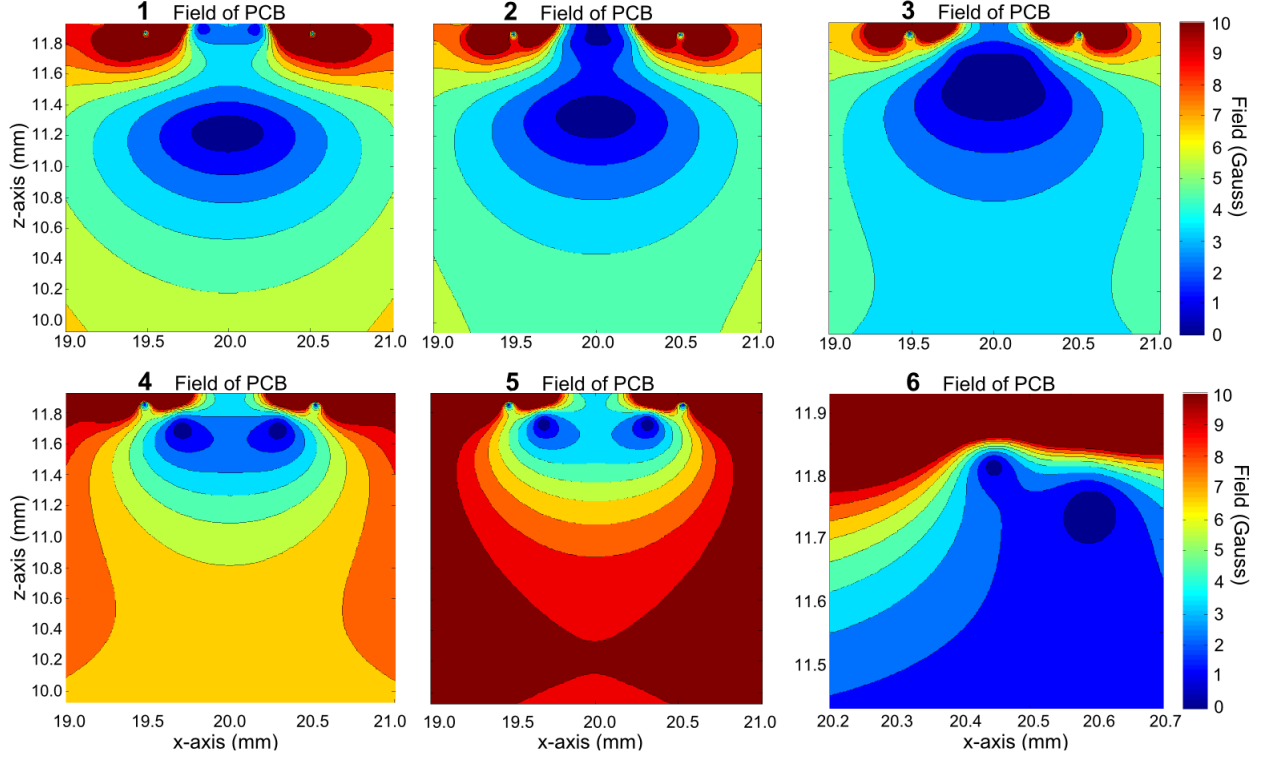


Figure 4.15: The field of the PCB and ring chip in the final ring loading stage. Each contour corresponds to 1 Gauss field strength. The effect of gravity is included. A loading field similar to figure 4.12 using the currents in stage 3 of figure 4.14 is applied. The RF coil provides the radial component.

The Field of the PCB in the final ring loading stage is shown in figure 4.15. The effect of gravity is included. A loading field is applied similar to that shown in figure 4.12 using the currents in stage 3 of figure 4.14. The RF coil provides the radial component. The Larger PCB coil provides the vertical component. Possible loss mechanisms exist. In particular, in step 2, the atoms are at risk of colliding with the trap surface. This occurs when a field-zero emerges from the chip and combines with the cloud trap before the ring split occurs. At this point, there is a 1G barrier between the cloud and the chip. This may be improved using additional gradients. Smaller single-layer coils were included on the PCB for this purpose. Gravity helps in this regard, as the chip is mounted upside down and the field barrier is lifted slightly. In step 3, the two field-zeros have merged. The field is fairly low in this region, and Majorana spin flips become likely. This loss can be

avoided by performing this step quickly. In step 6, the field is plotted in the vicinity of the ring quadrupole. The loading causes the atom cloud to take the place of the previous field-zero, which moves away to infinite. Being of larger volume, this field-zero can potentially act to leak atoms away from the trap. Shown in step 6 is the point at which the field barrier is weakest, at $1G$. In general, a larger field barrier between these minima can be achieved using a different loading trajectory. In general, choosing a trajectory which requires large loading fields results in larger barriers between the two field-zeros in step 6, and therefore allows less leakage of atoms from the trap at this stage.

4.6 Microwave Resonator

We would like to apply microwave pulses of a linear polarisation to the atoms. Although this can be achieved with a microwave horn, it is desirable to produce microwaves from a chip. We consider here a design for a chip-based microwave resonator, which creates a uniform microwave field of linear polarisation over the region of the ring trap. This is based on a split-ring resonator design. As in [106], it has been used to provide $4GHz$ uniform microwave fields to nitrogen vacancy sites in diamond. We would like to design for $6.834GHz$. The basic design is shown in figure 4.16.

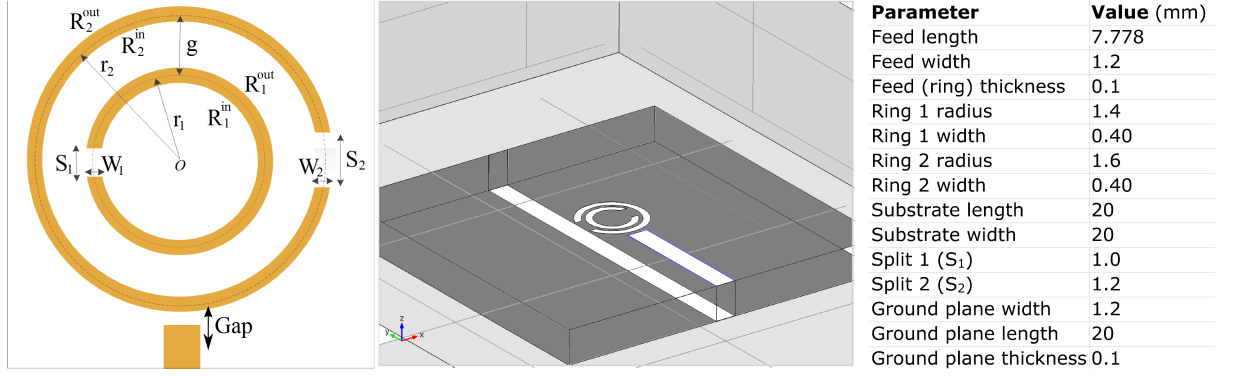


Figure 4.16: Left: The parameters of the split-ring resonator. Middle: The COMSOL model of the SRR. The substrate is shown in dark grey. Shown as white areas, a feed-line with a ground strip couples to the SRR. Right: Table of parameter values.

The resonance behaviour of the split-ring resonator (SRR) can be understood as the resonance frequency of an LC circuit $\omega_0 = 1/\sqrt{LC}$. The capacitance and inductance of SRRs have been investigated in [106]. We chose a ring of similar dimensions, and then performed parametric sweeps of the geometrical parameters to find a resonance at $6.8GHz$. The field was simulated in COMSOL and is shown in figure 4.17. It consists of a substrate with an SRR, an edge coupled feed-line and a ground strip.

An important parameter is the gap between the feed-line and the SRR, which determines the input impedance of the feed. Impedance mismatch between the feed-line and the SRR results in reflections at the gap, reducing the efficiency and disturbing the field profile. Tuning the gap correctly results in efficient transfer to the SRR at the frequency of choice.

Shown in figure 4.17 are plots for the field strengths at 3 different frequencies. The circular plots give the field strength $1mm$ directly above the resonator. Below these are plots showing the field on the surface of the substrate, of both the feed-line and the resonator.

At $6.8GHz$, the field strength at the feed-line is minimal, and is almost entirely transferred to the SRR. Above the SRR, $1mm$ above the substrate, the field strength shows a symmetric circular profile, centred on the centre of the SRR. It is clear from this that the field is most uniform along any ring centred on the SRR.

Supposing that this ring resonator was placed directly underneath the ring chip, the atoms would be contained in a ring trap about $1mm$ above the SRR, and would therefore experience a very uniform field strength.

In the case of frequencies above and below $6.8GHz$, the impedance mismatch of the feed-line results in significant fields above the feed-line, indicating an inefficient transfer of microwave energy to the resonator. In addition to this, a very non-uniform field pattern is seen above the SRR. The frequency dependence of the energy transfer is expressed in the figure by a plot of the S_{11} parameter.

Unfortunately, we have deemed that it is not currently possible to place the resonator underneath the chip, as the gold mirror of the EM chip would block the microwave field. This design is therefore not currently feasible, but may form the basis of a future design, where both the ring chip and microwave resonator are integrated onto the same chip. In the absence of a microwave chip solution, a horn placed just outside of the chamber may suffice.

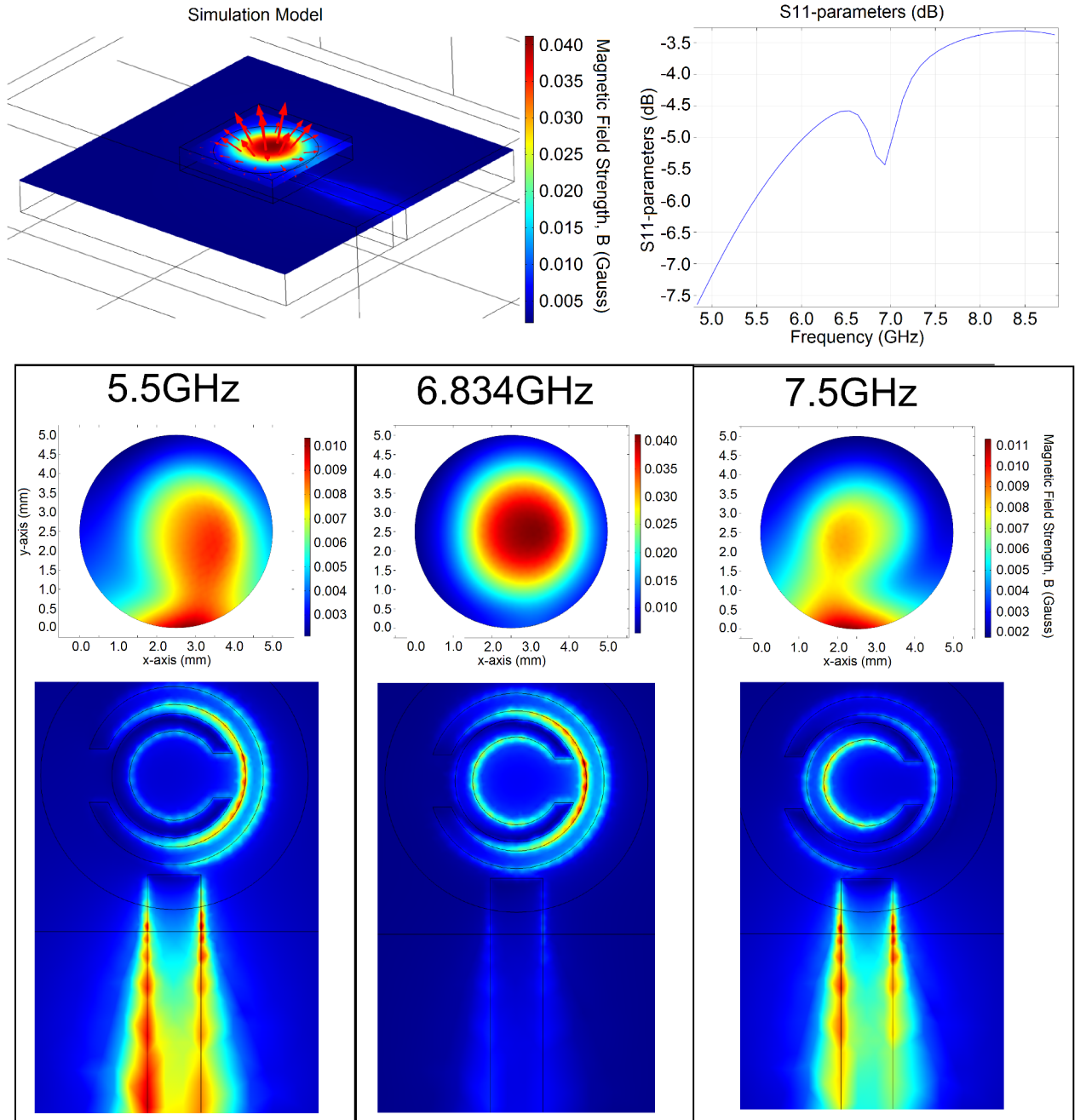


Figure 4.17: The field from the resonator. Simulated in COMSOL. Top Left. The field 1mm from the substrate surface. The field is vertically polarised. Top Right: The S_{11} parameter of the resonator. A resonance is seen at 6.8GHz. Bottom: The distribution for three different frequencies. The circular plots give the field strength 1mm directly above the resonator. Below these are plots showing the field on the surface of the substrate, of both the feed-line and the resonator. Away from resonance, the energy is lost at the feed-line, and the field is highly non-uniform. On resonance the energy is transmitted to the resonator and is uniform.

Chapter 5

Experimental Setup - Magnet Ring

This chapter describes the experimental infrastructure. Cold atoms experiments broadly consist of a laser system, a vacuum chamber, and control electronics. While we were waiting for the chips and PCB to be manufactured, we tested our set-up by loading a macroscopic ring-trap.

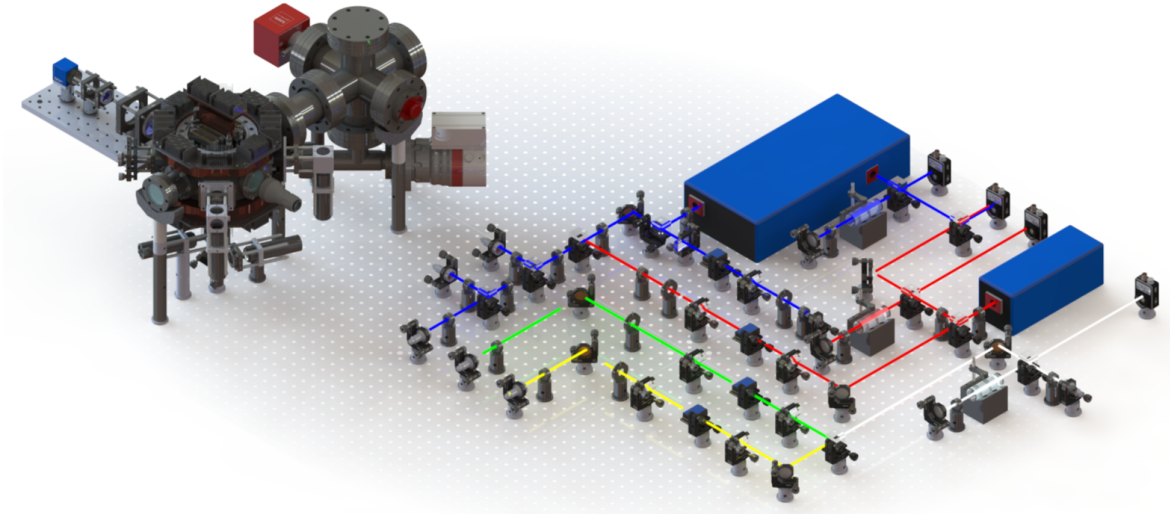


Figure 5.1: A simplified CAD image of the optics and vacuum setup. The optical elements are fixed down onto an optical bench. The large blue laser is a Topica TA-Pro which provides the cooling light, depicted in blue. The small blue laser is the Topica DL-Pro which provides the re-pumper light, depicted in red. The re-pumper and cooling light are overlapped and coupled into three fibre in-couplers. The output of a fibre laser (white beam) is split into an image beam and an optical pumping beam, depicted in yellow and green. For an optical schematic, see figure 5.5

5.1 Vacuum Setup

The Vacuum chamber is shown in figure 5.2. A pair of externally mounted Helmholtz coils provide the quadrupole field for the MOT, and any other bias fields required for trap loading. Side viewports on the main chamber allow optical access to the atoms for laser cooling. Also mounted to the vacuum chamber is a rubidium dispenser, turbo-pump (Pfeiffer HiPace 80 turbo-Pump), a combined NEG and ion pump (NEX Torr D200-5), and a pressure gauge (Agilent Technologies 9715015, Gauge, Ion, UHV-24P).

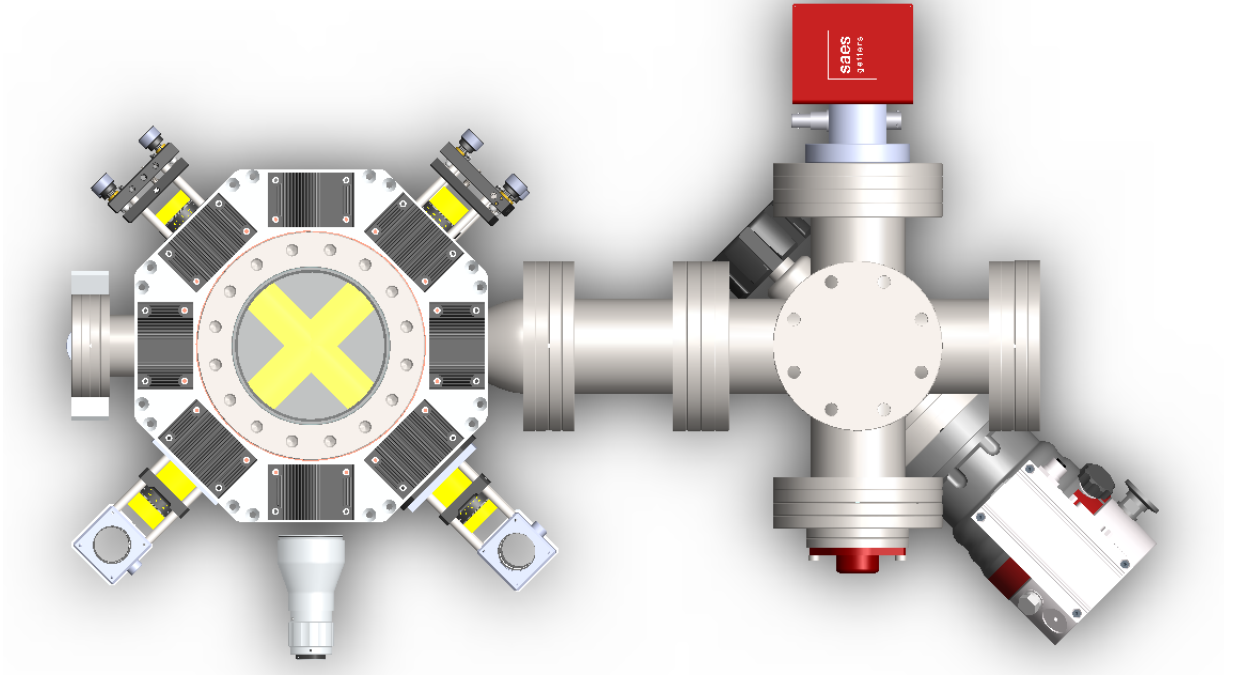


Figure 5.2: Side viewports on the main chamber allow optical access to the atoms for laser cooling. Also mounted to the vacuum chamber is a rubidium dispenser, turbo-pump (Pfeiffer HiPace 80 turbo-pump), a combined NEG and ion pump (NEX Torr D200-5), and a pressure gauge (Agilent Technologies 9715015, Gauge, Ion, UHV-24P).

There are two pumps connected to the chamber, a NEXTORR ion pump and getter, and a turbo-pump. Neither of these pumps can be operated at atmospheric pressure. Before these pumps can be operated, the pressure must be low enough. There is no single pump that can operate from atmospheric pressure to ultra-high vacuum (UHV). Reaching and sustaining UHV requires a number of pump technologies, and a bake-out to remove water vapour.

Cold atom experiments require pressures below $10^{-9}mbar$. There are a number of challenges to overcome to reach this pressure. Firstly there must be no leaks. Leaks can be avoided using a number of precautions. All flanges connected to the chamber must be rated for UHV pressures. Knife-edges bite into copper gaskets to seal the flanges. The knife-edge of the flanges must not be touched or damaged. The flange is then tightened to the port using a torque wrench. The bolts are tightened evenly in a specified order to avoid damage and to ensure a good seal.

Virtual leaks can also occur. As the air pressure decreases, the gas enters the ballistic regime. Here the air no longer acts according to fluid mechanics and pressure gradients, as there is not enough collisions between atoms. In this regime, pockets of gas can remain trapped in small areas in the chamber such as underneath screws. When this gas escapes, it can be seen as a sudden rise in pressure and is called a virtual leak. Good design of internal vacuum parts can prevent virtual leaks from occurring. For example, the use of vented screws to prevent trapped gas in blind holes.

As UHV is approached, a major contribution to the pressure comes from water vapour. To solve this, a bake-out is performed during or before pumping to remove any water vapour and other chemicals that can be removed by heating.

5.1.1 Leak Testing

The first step was to check for leaks and begin the first pumping procedure. A leak tester consists of a roughing-pump (diaphragm based pump) and a helium spectrometer. The leak tester pumps out the air. Then with a probe, helium is sprayed around the joints of the chamber. Should there be any leaks into the chamber, the helium spectrometer picks up the helium and a leak is detected. We did this and did not find any leaks.

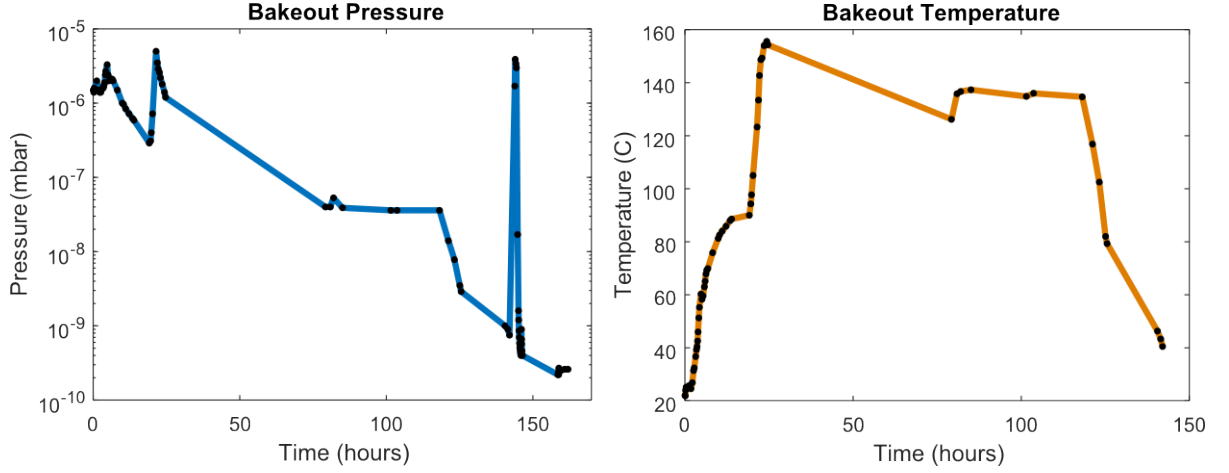


Figure 5.3: The pressure and temperature during bake-out. A spike in the pressure is seen when the ion pump is first activated.

5.1.2 Preparing for Bake-out

Heating belts were wrapped around the chamber. Large temperature gradients across the chamber should be avoided. The belts should not be overlapped anywhere, and instead evenly distributed around the chamber. Each belt was connected to an individual power transformer, so that they can be individually controlled. Temperature sensors were placed in many locations around the chamber. Thermal insulation from the environment is required. This can be done by building an oven around the chamber, but this is generally hard to do with surrounding optics. Instead, for thermal insulation we scrunched up aluminium foil and placed it all over the chamber. Many layers of foil were placed until it was completely surrounded.

5.1.3 Pumping Procedure and Bake-out

A roughing-pump (diaphragm pump) is connected to the exhaust of the turbo-pump, and is used in the initial pumping stage. Once a pressure of 10^{-3} mbar is reached, the turbo-pump is switched on. The roughing-pump remains on to provide a low exhaust pressure which is required for the turbo-pump to function. The vacuum chamber was heated to 150° during the pumping, until a low and stable pressure of 10^{-8} mbar was reached. The chamber was then cooled, and the pressure dropped to 10^{-10} mbar. Sustain-

ing UHV requires further pumps due to out-gassing of materials within the chamber. A combined ion pump and NEG (non-evaporable getter) is used for this purpose¹. The pressure and temperatures during bake-out are shown in figure 5.3.

5.2 Optical Setup

5.2.1 Transition Scheme

To create a MOT with rubidium, at least two transitions must be addressed. The first is the cooling transition. For rubidium, this is red-detuned from the $|F = 2\rangle \rightarrow |F' = 3\rangle$ transition by about 50MHz . A proportion of the atoms will subsequently decay to the $|F = 1\rangle$ ground state where they are no longer in a cool-able state. The atoms must therefore be re-pumped into the $|F = 2\rangle$ ground state. This is achieved by a second laser tuned on the $|F = 1\rangle \rightarrow |F' = 2\rangle$ transition. Atoms excited by this laser then decay to the $|F = 2\rangle$ ground state and are ready to be cooled again. This is then a closed cycle. Imaging is achieved using an absorption image on the $|F = 2\rangle \rightarrow |F' = 3\rangle$ transition. Transfer to a magnetic trap requires an additional optical pumping beam. Magnetic trapping requires a strong magnetic trapping force, which is proportional to the magnetic hyperfine quantum number m_F . Typically, the atoms will be distributed more-or-less equally over the m_F states, and many will be lost due to an inadequate trapping force during the transfer to a magnetic trap. The solution is to pump the atoms into the 'maximally stretched' state $m_F = 2$. This is achieved with circularly polarised light on the $|F = 2\rangle \rightarrow |F' = 2\rangle$ transition, as in figure 5.11. These transitions are shown in figure 5.4.

¹NEXTorr D200-5 Combined Ion and Getter

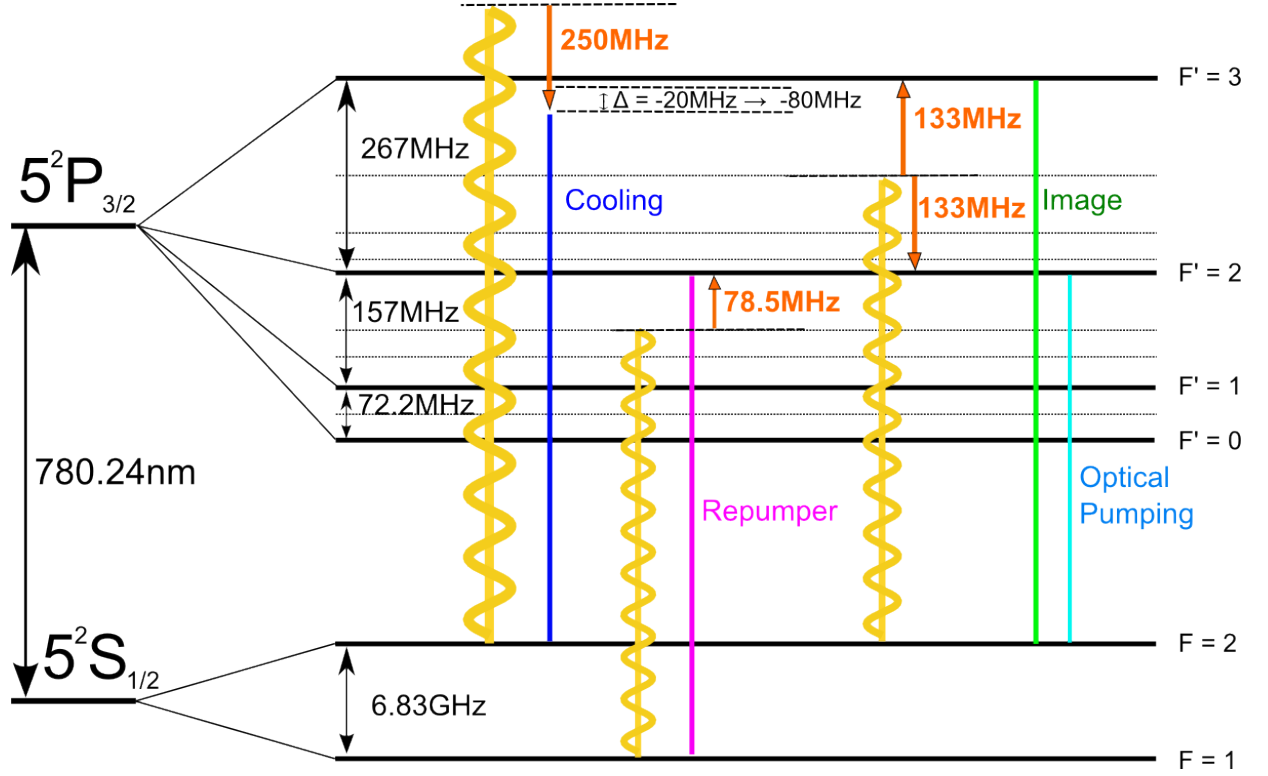


Figure 5.4: The transitions in the rubidium-87 level scheme. The transitions are represented by colour coded lines. The orange arrows represent AOM shifts. The yellow lines indicate the frequency the laser is locked at. The TA Pro provides the cooling beam, and is locked 150MHz above the $|F=2\rangle \rightarrow |F'=3\rangle$ transition. A double-pass AOM shifts the cooling beam down by 250MHz to be detuned from the cooling transition. The DL-PRO provides the re-pumper beam and is tuned to a cross-over peak. A single-pass AOM detunes the beam to the $|F=1\rangle \rightarrow |F'=2\rangle$ transition. The Fibre Laser provides the optical pumping and imaging. It is locked to a cross-over peak, and single-pass AOMs shift it up and down by 133MHz .

5.2.2 Laser Setup

Addressing the four transitions is achieved by locking the laser frequencies to spectroscopy features, and then further shifting the frequency using acousto-optic modulators (AOMs). By splitting a beam, and using AOMs, multiple transitions can be addressed with a single laser.

Two methods of laser locking are used: frequency modulation locking (FM-locking) and beat-locking. FM-locking requires a frequency reference provided by a spectroscopy feature, whereas beat-locking uses another laser as a frequency reference. Laser locking is described in more detail in section 5.2.3.

The features are provided by saturation spectroscopy. This form of spec-

troscopy has two advantages. Firstly, the Doppler broadening of the linewidth is removed, allowing the hyperfine transitions to be sharply resolved. Secondly, additional features (cross-over peaks) arise between each pair of energy levels. These are useful as they provide additional features to lock the laser to. Each laser has its own saturation spectroscopy setup for the purpose of monitoring the laser frequency and for providing a frequency reference for FM-locking.

Three lasers provide the cooling, re-pumper, imaging and optical pump beams. The transition scheme is shown in figure 5.4, and the optical schematic is shown in figure 5.5. The lasers and optical elements are fixed to an optical breadboard.

Re-pumper Laser

The re-pumper is provided by a DL-Pro laser from Toptica. Roughly $10mW$ is split off to be used for saturation spectroscopy and for the TA-Pro beat-locking. A further $45mW$ of power is sent to be used for re-pumping. The laser is FM-locked to the $|F = 1\rangle \rightarrow |F' = 1, 2\rangle$ cross-over provided by the saturation spectroscopy. A single-pass AOM detunes the beam to the $|F = 1\rangle \rightarrow |F' = 2\rangle$ transition. The light is finally overlapped with the cooler beam and coupled into three optical fibres.

Cooling Laser

The cooling light is provided by a Toptica TA-pro (tapered amplifier). It consists of two parts - a master laser and a tapered-amplifier (TA). The light from the master laser is fed into the TA, which amplifies it and outputs up to $3W$ of light to be used for the cooling. A small amount of light is also output from the side of the laser before amplification. This beam is used for saturation spectroscopy (for monitoring the frequency only), and then overlapped with the re-pumper light for beat-locking.

The laser is locked $150MHz$ above the $|F = 2\rangle \rightarrow |F' = 3\rangle$ transition. In this case, the laser is not locked directly to a spectroscopy feature, but rather

it is locked at $6.8GHz$ relative to the re-pumper laser using a beat-locking method. A double-pass AOM shifts the cooling beam down by $250MHz$ to be red-detuned from the cooling transition. The re-pumper and cooling light are overlapped and coupled into three fibres.

Optical and Imaging Laser

The optical pumping and imaging is provided by a compact fibre laser that was assembled in Birmingham. This laser is designed to be mounted to a 19-inch rack, and has an optical fibre output. It consists of a seed laser, an EOM and a fibre amplifier. The temperature and output power can be controlled by a computer through a USB port. EOM sidebands can be used to provide both re-pumper and cooling MOT light, but we used the EOM for providing the modulation for FM-locking. It can output up to $400mW$ of power.

The fibre outputs onto the optical bench that contains the other lasers. The laser is FM-locked to the $|F = 2\rangle \rightarrow |F' = 2, 3\rangle$ cross-over peak. The beam is split into two, and single-pass AOMs shift the frequencies up and down by $133MHz$. The image and optical pumping beams are then coupled into optical fibres.

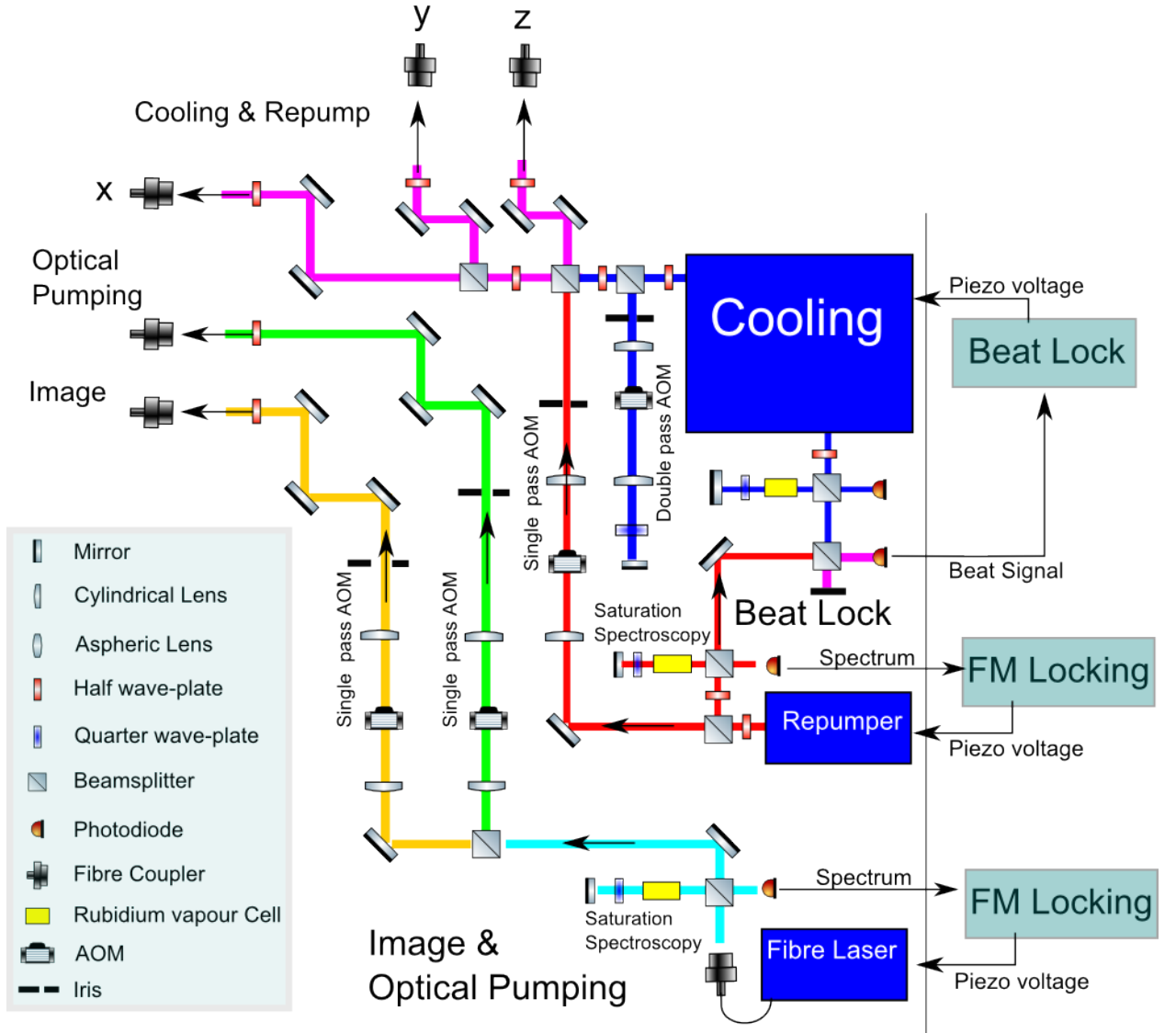


Figure 5.5: Optical Schematic. Three lasers provide the cooling, optical pumping, re-pumper and imaging. Saturation spectroscopy is used to determine the frequency of the lasers. Spectroscopy photodiode signals are sent to control electronics, which provide the feedback for frequency stabilisation. The Fibre and Re-pumper lasers are locked using FM-locking. The Cooler is locked relative to the Re-pumper using a beat-lock method. AOMs in single and double-pass configuration shift the frequency of the beams. In addition, they provide fast on/off switching of the beams. Additional shutters, not shown in the figure, were placed in the AOM setups as the AOMs did not switch off fast enough. The cooler beam and re-pumper are overlapped before they are coupled into the optical fibres. The optical fibres carry the laser light to the main chamber.

5.2.3 Laser Locking

The frequency of the lasers are affected primarily by current, temperature and piezo voltage, each of which is controlled by a laser controller. Changing the current is the fastest way to change the frequency of the laser, followed by piezo voltage and temperature. The Toptica laser controller stabilises the temperature, and provides a DC current and piezo voltage. A free running laser without feedback will drift away from the required frequency. This is the role of laser locking. We implement two forms of laser locking – frequency modulation (FM) locking for the re-pumper, optical pumping and imaging, and beat-locking for the cooling beam. The electronics flow diagram for the locking is shown in figure 5.6.

FM-locking is a fast way of producing a derivative error signal that describes the deviation of the laser frequency from a feature in the spectroscopy. The correction of the error signal is tasked to a PID controller. The principle of FM-locking is shown in figure 5.6. The laser controller scans the laser frequency using the piezo voltage input. Saturation spectroscopy determines the frequency of the laser. Applying a small MHz modulation to the laser current modulates the laser frequency, resulting in an amplitude modulation of the spectroscopy signal. The amplitude is proportional to the slope of the feature.

This modulation is generated by a function generator. The modulated spectroscopy is combined with the modulation signal in the mixer. The result is a number of frequency harmonics, the lowest of which is proportional to the slope and sign of the spectroscopy feature. The signal is zero at the extremum of the feature (at frequency f_0). A low pass filter removes higher order harmonics, leaving only the derivative error signal. This error signal is sent to a PID controller whose task is to zero it. The output of the PID is sent to the laser controller which applies the correction to the piezo voltage.

Beat-locking allows one laser to be locked relative to another by means of overlapping their beams and locking their beat frequency. The beams are overlapped at a 50:50 non-polarising beam-splitter and sent to an ultra-fast

photodiode. The beat frequency is divided down and compared to a reference frequency. The beat-lock comparator is based on a design from a paper [107]. We use a reference frequency of $10MHz$ provided by our control computer.

The cooler laser (Toptica TA PRO) consists of a master laser and a tapered amplifier. A small part of the master laser beam ($10mW$ out of $200mW$) is output at the side of the laser. This beam is useful for locking and spectroscopy. It is overlapped with the re-pumper at a non-polarising beam-splitter. The combined beam beats at a frequency given by the difference frequency of the two beams. A lens and two beam-steering mirrors direct the beam and focus it into an ultra-fast photodiode. This diode must be capable of GHz frequency differences. The signal is amplified (MiniCircuits Amplifiers are used), and sent to the 'BeatLock Box'. This consists firstly of a pre-scaler, which divides down the signal by a factor of 100. A binary counter counts the number of beats within a set time interval, and determines the frequency difference. An error signal proportional to the frequency difference is derived and sent to a PID controller for laser feedback and stabilisation.

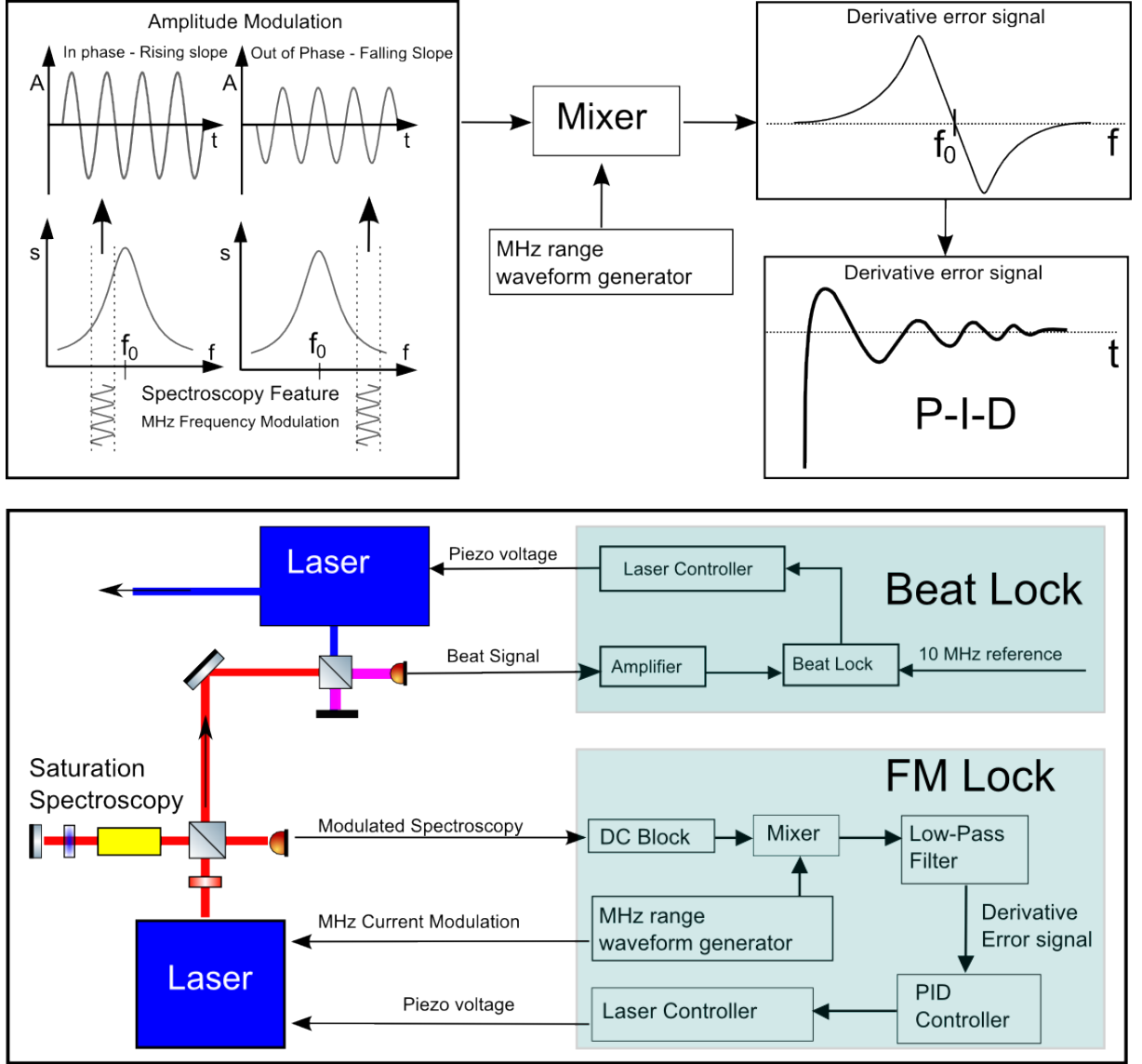


Figure 5.6: Laser locking. FM-locking and Beat-Locking is employed. Modulating the frequency gives rise to an amplitude modulation of the spectroscopy signal. The amplitude and phase give the magnitude and sign of the frequency deviation from the spectroscopy feature. Beat-locking allows one laser to be locked relative to another by means of overlapping their beams and locking their beat frequency. The beams are overlapped at a 50:50 non-polarising beam-splitter. The beams are sent to an ultra-fast photodiode. The beat frequency is divided down and compared to a reference frequency. We use a reference frequency of 10MHz provided by our control computer.

5.2.4 AOMs

Small and controllable shifts in light frequency on the order of 200MHz can be achieved using acousto-optic modulators (AOMs) placed in the beam path. The AOMs are controlled using high power RF current drivers ($\approx 1\text{W}$). A piezo-electric transducer transmits phonons through a crystal. This changes the refractive index of the crystal and results in Bragg scattering of photons, increasing the light frequency by the driven RF frequency [108].

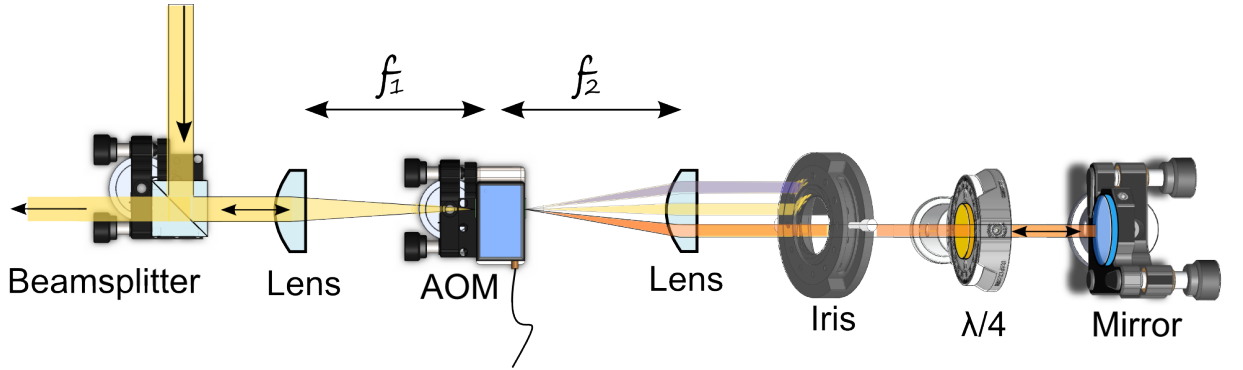


Figure 5.7: Double-pass AOM. The light is focused into the AOM crystal. An iris blocks the unwanted orders. The beam is back reflected towards the beam-splitter. A quarter-waveplate ensures the beams are transmitted at the beam-splitter.

The shift in frequency is associated with the deflection of the beam. Adjusting the frequency can cause misalignment of the beam. To make the alignment stable, that is, to make the frequency shift independent of angle, a double-pass set-up is used. Here the beam is reflected back through the AOM, and the deflection is undone while shifting the frequency by twice the amount. This has the advantage that the alignment after the double-pass is independent of the frequency shift, and greater frequency shifts can be achieved.

The AOM is mounted to an adapted Thorlabs kinematic mount and a heat sink. An SMA cable connects it to the current drivers. The beam is split off using a polarising beam-splitter, and focused into the AOM crystal. The beam is deflected into multiple orders differing by a frequency $n f_{AOM}$, where n is the diffraction order and f_{AOM} is the frequency of the RF AOM driver. The zeroth order is unshifted. An Iris blocks the unwanted orders. In the case of a single-pass AOM, the beam would then continue to the rest of the

experiment. In the case of double-pass, the beam is reflected and returns back along the same path. The net frequency shift is twice the single-pass shift and has no frequency dependent deflection. A $\lambda/4$ ensures that the beam is of the right polarisation to be transmitted or reflected at the beam-splitter.

The AOM position was adjusted to obtain maximal light in the desired deflection order. Typically we could attain 95% efficiency in single-pass and 60% in double-pass configurations.

AOMs also function as fast switches. It is for this reason that the repumper is locked to the cross-over peak and shifted towards the transition, rather than locking directly on the desired transition.

5.2.5 Fibre Coupling

Optical fibres carry the light from the optical setup to the vacuum chamber. Fibre couplers are used to couple the light into the fibre and to output and collimate the light. These are shown in figure 5.8. Polarisation maintaining fibres are used. The fibres are mounted to an FC/APC connector. This is mounted by a Thorlabs translation mount with a fibre in-coupler. The fibre in-coupler is a lens that focuses the light into the fibre. When used as an out-coupler, it collimates the output light.

Two beam-steering mirrors and a half-waveplate are placed before the fibre in-coupler. They provide enough degrees of freedom to couple the light into the fibre. This is known as beam-walking. The maximal coupling efficiency is affected by the mode-matching of the beam into the fibre. Light emerging from the fibre has a Gaussian profile. For 100% coupling efficiency, the input beam profile would have to match perfectly with the mode of the fibre. There are some basic relations which can help when choosing an output coupler to match the beam size. Typically, we achieve a maximum coupling efficiency of 50%. The situation is made more difficult as the beam profile of the TA is not good. As we are coupling two beams into the same fibre, we had to choose the in-couplers to be a compromise between the two beam sizes.

The face of the fibre is very small. A lens focuses the light into the fibre.

The fibre face is placed in the focal plane of the lens. However, the choice of focal length is not arbitrary. The focused beam should match the output divergence of the beam.

Although a Gaussian beam has no clear edge, we can define the edge as the point at which the intensity drops off by $1/e^2$ of the peak intensity. This diameter we call the mode field diameter D_{MFD} . The divergence of a Gaussian beam away from a spot of size w_0 is:

$$\theta \simeq \frac{2\lambda}{\pi w_0} \quad (5.1)$$

This is the half angle, and is equatable to the numerical aperture of the optical fibre. The spot size is the mode field diameter of the fibre and is $4.9\mu m$ in this case. For a fibre placed in the focal plane of a lens of focal length f , the beam will expand to a beam width of:

$$D = 2\theta f = \frac{4f\lambda}{\pi w_0} \quad (5.2)$$

We would like this to match the mode-field diameter of the incoming beam. Taking $D = D_{MFD}$, we can use the above equation to choose the focal length of the lens.

The fibres are polarisation maintaining. However, they only maintain polarisation that is aligned with a particular axis of the cable. For this reason, half-waveplates were placed before the fibre in-couplers to rotate the polarisation. The procedure for finding the correct polarisation angle is here described. The output light is directed at a polarising beam-splitter through a half-waveplate. The half-waveplate is adjusted to split the beams in roughly equal intensities after the beam-splitter. The beams are incident upon detectors. Variations in polarisation result in intensity fluctuations at the beam-splitter. Changing the path length of the fibre will change the polarisation if the fibre is not polarisation maintaining. Heating and cooling the fibre results in a change in path length. If the light is not correctly aligned in the fibre, the

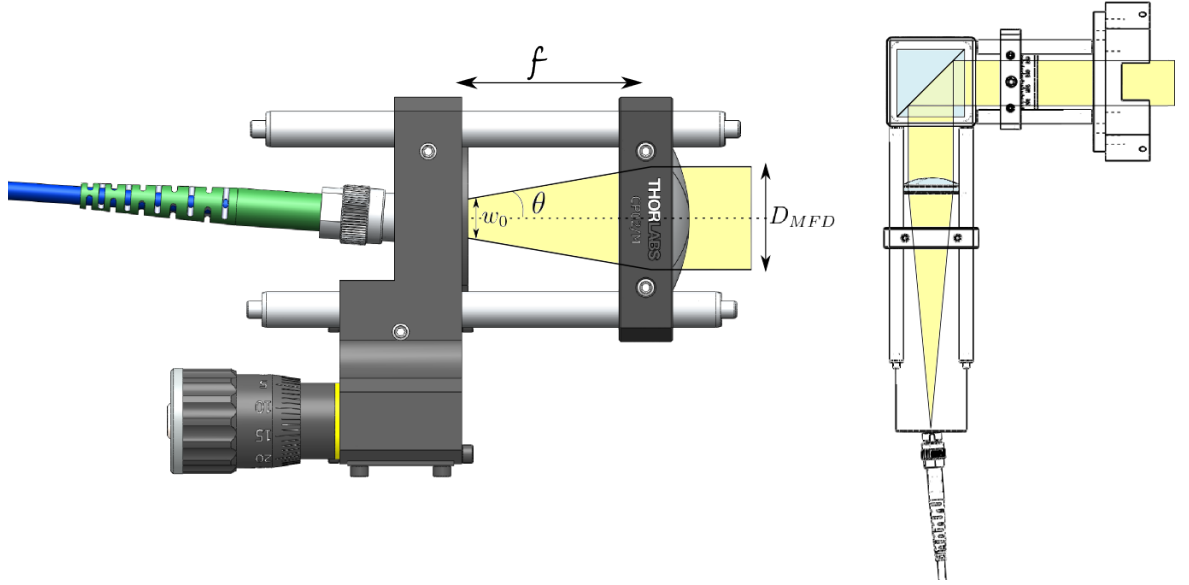


Figure 5.8: Left: In-coupler. The focusing lens must be chosen according to equation 5.2. Right: The out-coupler is mounted in a lens tube. The beam expands and is collimated by a 75mm lens, resulting in a 1-inch beam. Right: A beam-splitter reflects the beam and cleans the polarisation, and a quarter-waveplate makes the beam circularly polarised for the MOT. Mounting rods connect the parts, and are screwed to a cage plate. This is then mounted to the chamber viewport.

change in path length results in a change in output polarisation. This leads to an intensity fluctuation at the beam-splitter. The waveplate at the input is adjusted to minimise the intensity fluctuations during the heating/cooling of the fibre.

5.2.6 Chamber Optics

The optics surrounding the chamber are shown in figure 5.9. Imaging can be performed on the vertical axis or along the horizontal axis. The out-couplers are mounted to the chamber via screw-fixed cage plates. Quarter-waveplates provide circular polarised light for the MOT light and optical pumping. An imaging telescope is mounted above the chamber, on a vertically mounted breadboard. The imaging beam is split off from the MOT beam and directed towards a CCD camera using a beam-splitter. We found that some MOT light leaked through the beam-splitter and would saturate the camera. To avoid this we placed a shutter before the camera.

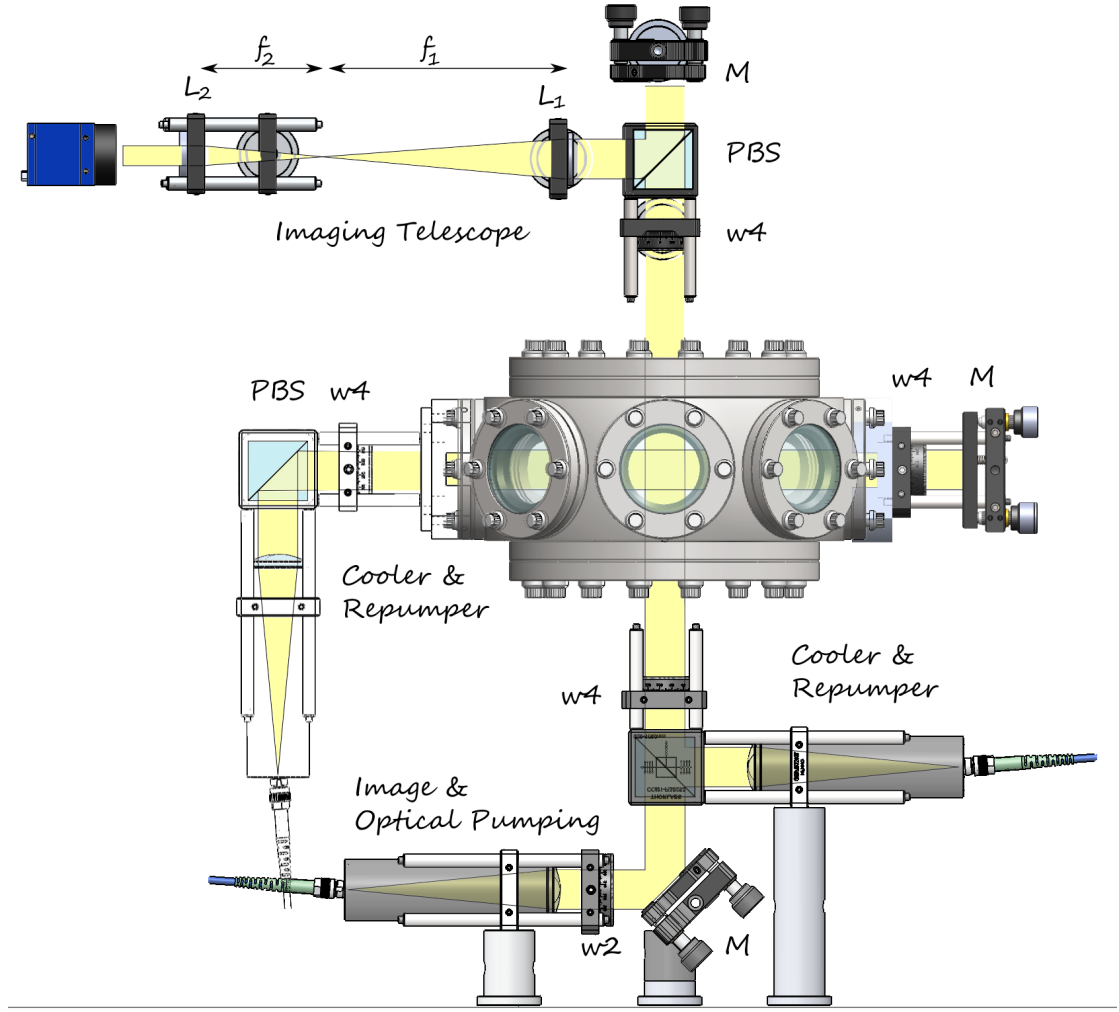


Figure 5.9: The fibres take the light to the chamber. The out-couplers are mounted to the chamber, with a mirror and a half-waveplate. This provides the MOT and re-pumper light of the correct polarisation. The z-axis is needed for MOT light, and also optical pumping and imaging. This gets crowded, and sometimes beam-splitters are needed to split off the light at the other end.

5.2.7 Imaging

We use a large output collimator², an imaging telescope and a CCD camera³. The optics are mounted to a small breadboard raised from the main optical bench at the side of the chamber. The telescope set-up is shown in figure 5.10. A telescope is used to shrink the beam before it hits the camera lens. The telescope consists of two lenses. The first lens of focal length $f_1 = 100mm$ is placed $100mm$ from the atoms. A second lens of a much smaller focal length

²60FC fibre collimator from Shafter and Kirchoff

³ImagingSource DMK 21BU618.H

($f_2 = 25mm$) is placed a distance $f_1 + f_2$ from the first lens. The result is that the beam is shrunk by a magnification factor: $M = f_1/f_2$. This must be taken into account when considering the virtual pixel size.

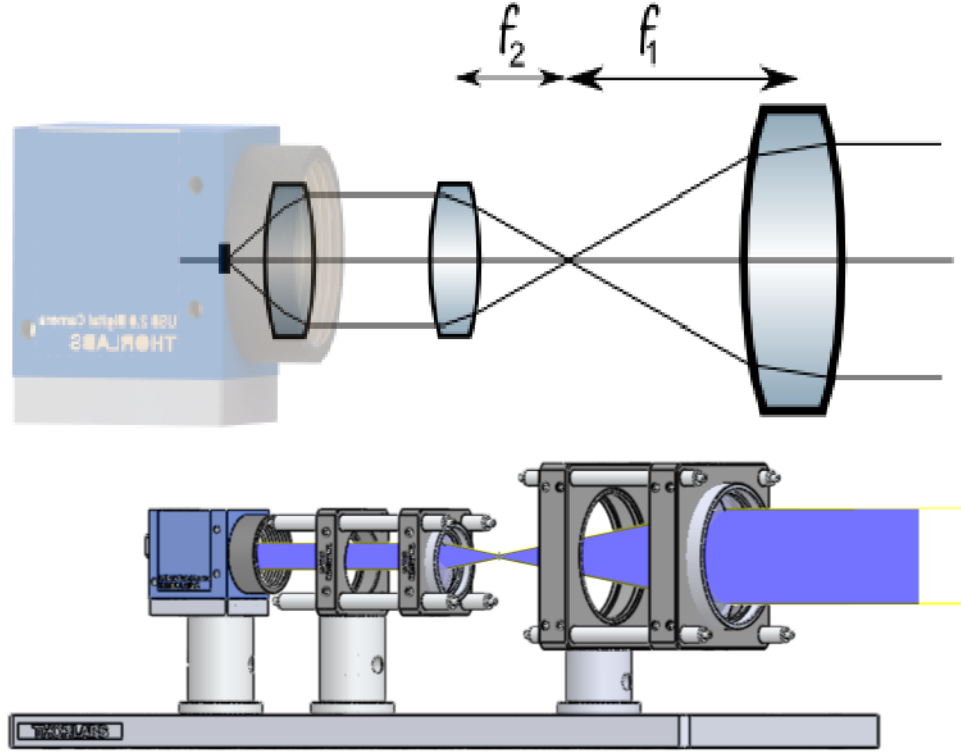


Figure 5.10: Image telescope. The optics are mounted to a breadboard at the side of the chamber. The lenses were mounted in cage mounts to allow for easy adjustment along the imaging axis.

Absorption imaging is used to determine the atom number. Three images are taken. One with the atoms, one without the atoms, and a dark image (without imaging light or atoms). These three images are combined to produce an absorption image that is corrected for the image light intensity and ambient light. Absorption imaging involves illuminating a cloud of atoms with a short pulse of light, on resonant with the cooling transition. The light is captured by a CCD camera. This camera must be triggered and exposed for a certain period of time to avoid over-exposure. A separate axis and view-port is dedicated to allowing optical axis for the cooling beam. Space around the chamber must be allocated for shrinking the beam into the small camera area.

We can understand the relationship between the atom number and light

absorption using the Beer-Lambert Law. Supposing that the light propagates in the z -axis through an atomic cloud, the absorption of the imaging beam intensity is described by the optical density:

$$OD = \ln \left(\frac{I_{probe} - I_{bg}}{I_{atoms} - I_{bg}} \right) \quad (5.3)$$

The terms I_{atoms} , I_{probe} and I_{bg} represent the received intensity of the imaging light; with atoms present, with imaging light only (no atoms), and with no light or atoms (background ambient intensity). The measured optical density can be related to the atom number using the Beer-Lambert Law. This states that the attenuation of the beam as it moves through the cloud is of the form:

$$\frac{dI}{dz} = -\sigma_0 n I \rightarrow I = I_0 e^{-\sigma_0 n z} \quad (5.4)$$

where σ_0 is the absorption cross-section of the atoms, and n is the atom number density. The absorption cross-section is polarisation dependent. For a two level atom, illuminated with light of isotropic polarisation and an intensity far below saturation, the on-resonance absorption cross-section is $\sigma_0 = 3\lambda^2/2\pi$.

The optical density is then $OD = n(x, y)\sigma_0$. In the case where the atom number density $n(x, y)$ is Gaussian, the total atom number N can be found from a Gaussian fit. In the case of a non-Gaussian atom distribution, the atom number can be found by summing the individual pixels. Each pixel represents a physical area of the cloud. The area represented by a pixel is called the virtual pixel size and is found from the magnification M of the image beam. The area represented by a pixel is then $A_{cloud} = A_{pixel} \times M^2$. The number of atoms per pixel N_{pixel} is the number density multiplied by the area $N_{pixel} = n_{pixel} \times A_{cloud}$. Summing over all the pixels gives the total atom number.

5.2.8 Optical Pumping

After the MOT stage the atoms are transferred into a magnetic trap. Only the states $m_F = 1, 2$ are trappable however, and atoms in the other m_F states will be lost from the trap. Atoms in the state $m_F = 2$ are maximally sensitive to the magnetic field and are easiest to trap. Prior to optical pumping, the atoms are uniformly distributed amongst the 5 Zeeman sub-levels. Applying σ_+ circular polarised light transfers the atoms to the $|F = 2\rangle \rightarrow |F' = 2, m_F = 2\rangle$ state. We expect then an increase in the number of atoms in the magnetic trap of up to 5 times with good optical pumping. The optical pumping is performed after the MOT stage, and before transfer into the magnetic trap. After the MOT stage, the magnetic trap is switched off and replaced with a small bias field (typically in the z-direction) in the direction of propagation of the optical pumping beam. This provides a quantization axis for the optical pumping light, and ensures all atoms see σ_+ .

Without the magnetic trap, atoms will accelerate and fall, so the optical pumping must be performed quickly. The major limitation to the speed is the switching speed of the coils. Typically, we were able to switch off the magnetic trap and apply a bias field in $2ms$. A $1ms$ pulse of optical pumping light was applied, and then a magnetic trap was switched on in approximately $1ms$. Current switches can be employed to switch the current faster. As the atoms have spent some milliseconds in free-fall, the magnetic trap must catch the atoms in such a way as to prevent sloshing. Furthermore, the magnetic trap must be correctly matched to the cloud, so as to prevent 'breathing modes'.

The optical pumping beam is derived from the fibre laser. The fibre laser is locked to the $|F = 2\rangle \rightarrow |F' = 2, 3\rangle$ cross-over peak. After being split off from the image beam, the light is shifted by a single-pass AOM by $133MHz$ down to the $|F = 2\rangle \rightarrow |F' = 2\rangle$ transition. Generally, the optimum optical pumping efficiency occurs at a detuning of approximately $20MHz$ from the transition. A scan of the optical pumping frequency is shown in figure 5.11 One explanation for this is that the beam penetrates further into the cloud

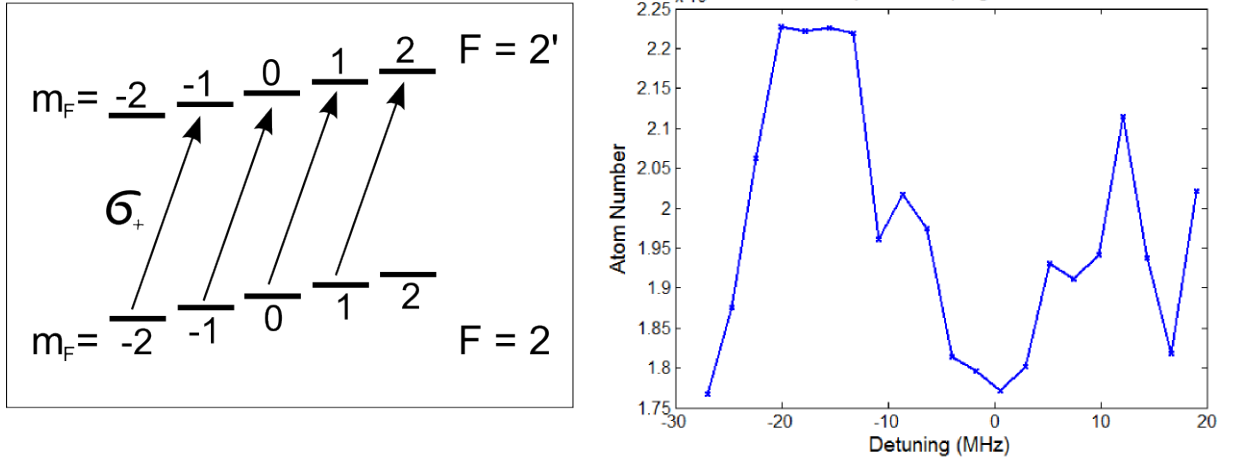


Figure 5.11: Optical Pumping. Left: The optical pumping beam is locked to the $|F = 2\rangle \rightarrow |F' = 2\rangle$ transition, and drives the atoms into the maximally stretched states $m_F = 2$. Right: The optical pumping frequency is scanned (using the AOM), and the atom number plotted. Typically, we found the optimal atom number at $10 - 20 \text{ MHz}$ detuning from the transition.

when slightly off-resonant, and can therefore address more atoms.

The optical pumping was performed on the vertical axis. This involves combining it with the cooling beam, and is shown in 5.9. In our case however, the permanent field from the magnet (see magnet ring section) complicated the optical pumping.

5.3 Labview

The experiment must be controlled by computer, and precise timing is needed to run the sequence. For this purpose, a dedicated PXI computer from National Instruments is used. In Labview an experimental sequence is defined. This sequence is then sent to an FPGA, which generates a series of well-timed voltage control signals to the experiment. These voltages control the AOMs (to scan the frequencies), shutters (to block and open the beams), power supplies (to control the magnetic coils), and camera triggers (for imaging the atoms.) The data obtained from the experiment is in the form of absorption images, and is sent to Labview to be saved and processed.

A brief overview of it is given here. The experiment is controlled through a set of user interfaces. The most important being the script editor. Here the experimental sequence is defined. Then this sequence is converted into a polynomial and sent to the FPGA (Field Programmable gate array). There are two types of FPGA - analogue and digital output types. The digital outputs consist of 0V and 5V signals, as is standard of TTL logic. These are used to switch shutters, and to trigger cameras. Analogue outputs can range from -10V to +10V. These are used to control the power supplies and the AOM frequencies (via the AOM RF current driver).

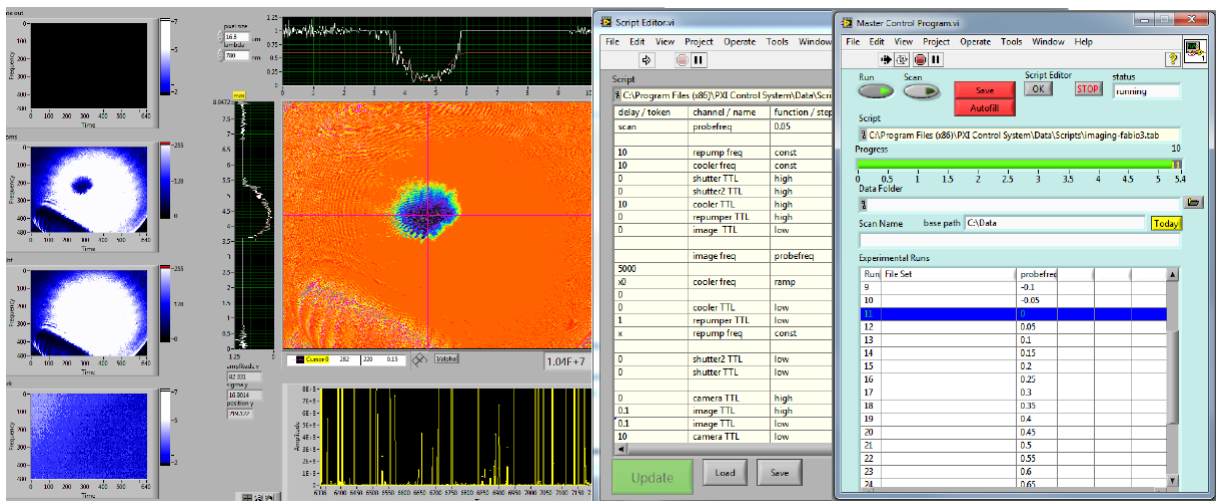


Figure 5.12: The Labview interface used to control the experiment. From right to left: The control window runs scripts and scans variables, with options to save the data. The script window defines the control voltage timings. Three images are taken, and a Gaussian curve is fit to the optical density to find the atom number.

5.4 Magnet Ring-Trap

We wanted to test our setup and test loading into a ring-shaped trap, while waiting for the atom-chips to be made. A large Ring-Magnet was placed outside the chamber, as close to the DN100 (top or bottom) viewports as possible. The principle of the ring-trap is shown in the figure 5.13.

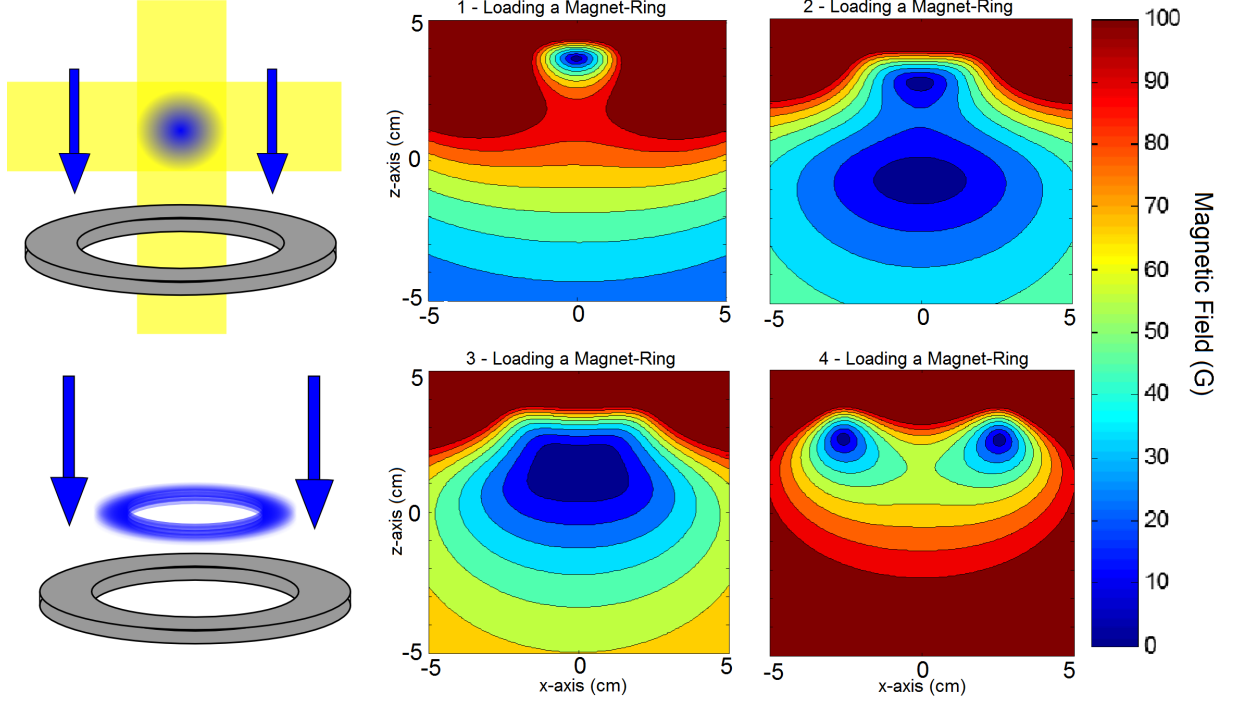


Figure 5.13: The principle of the ring. Left: A MOT is created using a bias field and a magnetic ring. Increasing the bias field compresses the trap towards the ring, where it splits into a ring shaped trap. Right: Simulations of the field for different values of the bias field: $0G$, $70G$, $95G$ and $150G$. The field is in Gauss and each contour represents $10G$.

The ring-magnet has an inner and outer radius of $30mm$ to $50mm$. It is $20mm$ thick, and magnetized along the axial direction. The field was simulated by considering an equivalent edge current $I_{edge} = Mh$ where M is the magnetization and h is the height or thickness of the ring. A 2D Biot-Savart simulation was performed to find the field. The real field from the magnet was measured along the axis of the ring only, and was then used to scale the 2D simulation.

The magnet ring has a central quadrupole field-zero at a distance $z_c \approx R/2$ where R is the approximate ring radius. Another field-zero is at $z \rightarrow \infty$. Applying a vertical downward bias field brings the field-zero from infinite

downwards towards the central quadrupole. At an applied bias field of $70G$, the gradient is $15G/cm$ and the field barrier between the two field-zeros is $25G$. This is sufficient to form a MOT. Increasing the bias field causes the field-zeros to merge and compress into a ring.

At the point of the merging of the two field-zeros (the 'coalescence point'), the field is small and the gradient is weak. This presents an opportunity for atoms to escape from the trap. This can be avoided by ramping quickly at the coalescence point.

5.4.1 Magnetic Trap Loading

Typically, atoms are lost when transferring to a magnetic trap. Good transfer involves a good overlap between the magnetic trap and the MOT centre. This can be fine-tuned with compensation coils. Bad matching can result in large atom loss and oscillations within the trap (known as breathing modes).

In addition, transfer from the MOT to a magnetic trap requires an optical pumping stage, as was described earlier. Typically, the fields would be switched off and replaced by a weak and uniform bias field. An optical pumping pulse is applied ($\approx 1ms$), and the magnetic trap field is switched on. The permanent field from the magnet complicates this, as the atoms will always experience an inhomogeneous field.

An additional problem is that inductive loads, such as the MOT coils tend to resist fast changes in current. The switching times for our power supplies, when switching the MOT coils by $20A$, was $5ms$. The atoms are in free fall during this stage, and $5ms$ is too long. Fast switching requires a dedicated current switch, which we currently do not have.

Instead of switching off the fields and replacing with a bias field, we moved the trap centre suddenly. The atoms, slow to follow the trap, see a vertical bias field. Optical pumping is performed along this axis, and the magnetic trap is ramped up to catch the atoms. Moving the trap centre, rather than switching off the fields, has the advantage that it can be done much more quickly. This should make the atoms easier to catch again in the magnetic

trap and reduce sloshing. This field is not uniform however, resulting in a spread of shifted m_F states. This is expected to lower the maximal optical pumping efficiency and broaden the effective line-width.

5.4.2 Ring Loading

The magnet was placed as close to the DN100 viewport as possible. We initially mounted the magnet underneath the chamber, and attempted to load a ring-trap. It was found that the atoms were lost during the splitting process. This is most likely due to the low gradient at this point in the sequence, and the effect of gravity causing atoms to leak to the lower field-zero where they begin to slosh. We had success when mounting the magnet from above. The low gradient at the coalescence point was still a problem however, and we used the upper MOT coil to provide an additional field gradient during the loading procedure.

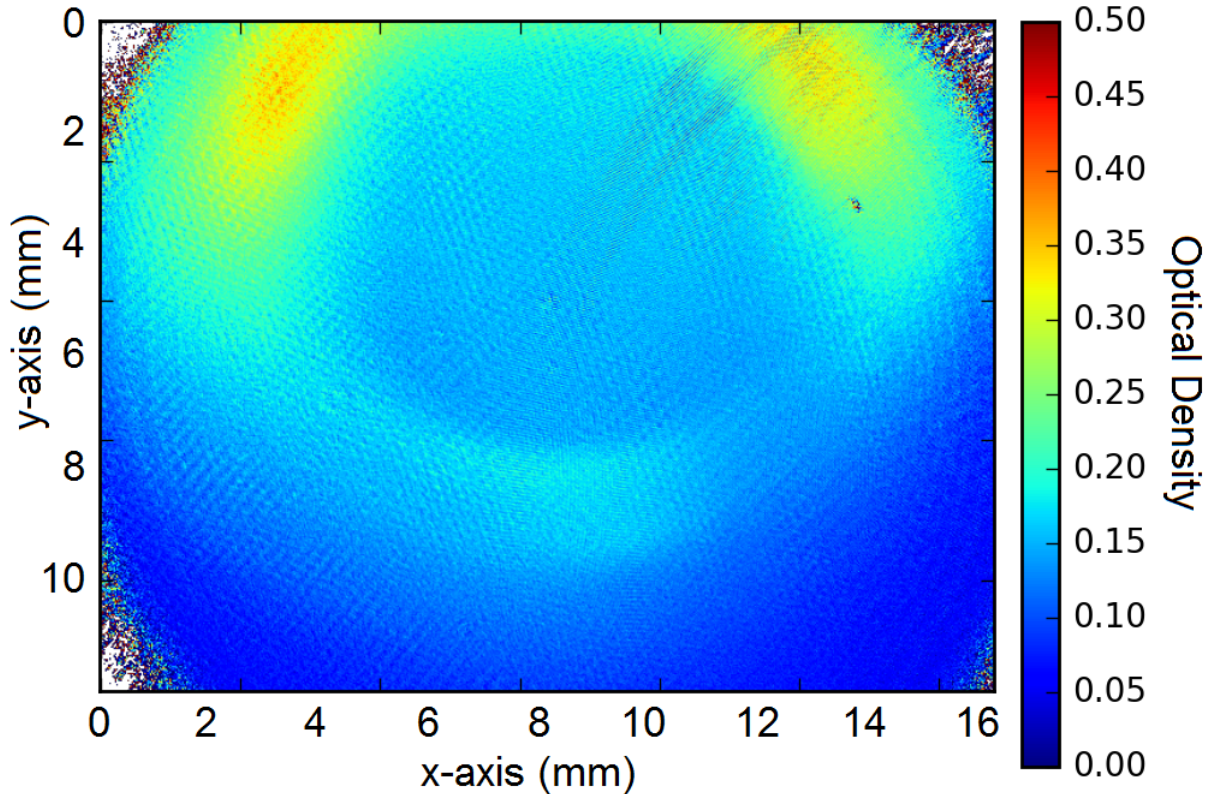


Figure 5.14: An absorption image of the ring. The atom number was found by summing the optical densities of the pixels. The atom number is 5×10^6

The atom number in a normal trap can be deduced by a Gaussian fit.

However, when the atoms are in the ring, this is no longer possible. Instead, the individual pixels are summed. An absorption image of the ring, averaged over 10 pictures, is shown in figure 5.14. Imaging the ring-trap required a large image beam on the vertical axis. The vertical imaging setup is shown earlier in figure 5.9.

5.4.3 Ring Inhomogeneity

We found in most of our absorption images that the atoms collected in nodes around the ring-trap. An inhomogeneous distribution of atoms is understandable from sloshing of the clouds as they are loaded. However, we found the nodes to be in the same place each time. We attempted to determine if it is the inhomogeneities in the ring magnetisation which cause the lobes. We mounted the magnet to a wheel. Above the magnet was a gauss meter. Turning the wheel allowed us to steadily probe and record the magnetic field. A variation of a few gauss was found around the azimuthal direction. We rotated the magnet and performed more ring loading experiments, finding that the lobes had rotated by the same amount. We can conclude then that field inhomogeneities of a few gauss are present and prevent a uniformly populated ring-trap.

5.5 RF Setup

Radio frequencies are useful for evaporative cooling, RF dressing and temperature measurement. We placed an RF coil above the chamber, as close as possible to the upper viewport. The RF coil has 5 turns of radius 3cm . We connected part of the coil to a 50Ω termination to reduce the impedance mismatch and to make it more broadband. The radio frequency is generated using a function generator⁴. The output of the generator is connected to a switch, and then to an RF amplifier⁵. The additional switch is required because the function generator did not have a feature that allows you to remotely trigger the output on and off. We can estimate the inductance using the inductance equation for a loop:

$$L_{loop} \approx N^2 \mu_0 \frac{D}{2} \left(\ln \left(\frac{8D}{d} \right) - 2 \right) \quad (5.5)$$

where N is the number of turns, D is the diameter of the loop, and d is the wire thickness. This gives us a rough value for the inductance as $4\mu\text{H}$. The total impedance of the inductor consists of an Ohmic resistance and a reactance $Z_L = R_L + jX_L$. The reactance is related to the inductance by: $X_L = \omega L$. At 1MHz , the reactance is 25Ω . As the Ohmic resistance will be on the order of 1Ω , the impedance is highly frequency dependent, and may vary by an order of magnitude over the desired frequency range ($1-10\text{MHz}$). For this reason we attach a 50Ω termination.

5.5.1 RF Temperature Measurement

Normally, the temperature of the atoms is inferred from the free ballistic expansion of the cloud, in a so-called time-of-flight measurement. However, this is not possible in this case due to the permanent magnetic field of the magnet. Instead we use a technique that was described in [98], where RF spectroscopy was used to estimate the temperature of a cloud in the presence

⁴Rigol Function generator

⁵Amplifier from Minicircuits

of a permanent magnetic trap.

As in the case of evaporative cooling, atoms will absorb RF that is resonant with the Larmor frequency, resulting in spin-flip losses from the trap. Measuring the loss rate as a function of RF frequency, the energetic distribution of the atoms can be inferred. Provided the potential energy distribution reflects the kinetic energy distribution, it should be possible to infer the temperature. A theoretical model for this was proposed and applied to measure the temperature of a cloud trapped by a permanent magnetic chip. This will be briefly described here.

The atoms have an energy $E = g_F \mu_B m_F B = \hbar \omega_{RF}$ on the resonant surface. We assume that the atoms pass over the resonant surface with a constant velocity perpendicular to the surface. The Landau-Zener equation can be used to find the spin flip probability of an atom passing through this region. This gives the loss rate:

$$\frac{dN}{dt} \approx \left(\frac{1}{2\pi\Omega^2} + \frac{\sqrt{m_F}}{4\bar{\omega}\sqrt{\pi\omega_{th}\omega_{rf}}} \right)^{-1} n_{\omega_{RF}} \quad (5.6)$$

$\bar{\omega}$ is the geometrical mean of the trap frequency, defined as $\bar{\omega} = (\omega_x \omega_y^2)^{1/3}$ [109]. The temperature is expressed in terms of a thermal frequency $\omega_{th} = K_B T / \hbar$. Provided that the Rabi frequency is sufficiently small $\Omega^2 \ll \bar{\omega} \omega_{th} / \sqrt{m_F}$, the second term can be neglected. Then the loss rate is independent of the trap parameters. $n_{\omega_{RF}} = dN/d\omega_{RF}$ is the atomic spectral density, and for a thermal cloud obeying Boltzmann statistics, it is given as:

$$n_{\omega_{RF}} = \frac{2N}{\sqrt{\pi}} \sqrt{\frac{m_F^3 \omega_{RF}}{\omega_{th}^3}} e^{-m_F \omega_{RF} / \omega_{th}} \quad (5.7)$$

The cloud is probed using a radio frequency pulse of duration $0.5ms$. We introduced oscillations in the trap and measured the centre of mass oscillations. From this we estimate the axial trap frequency to be $18Hz$, and we expect the radial component to be half of this. The Rabi frequency is difficult

to estimate, as we do not know exactly the strength of the RF seen by the atoms. We then use the Rabi frequency as a free parameter in the fitting. We use a $175mV$ signal from the function generator, and evaporate for $250ms$. Comparing the final atom number to the case of no evaporation, we estimate the atom loss. This is plotted in figure 5.15.

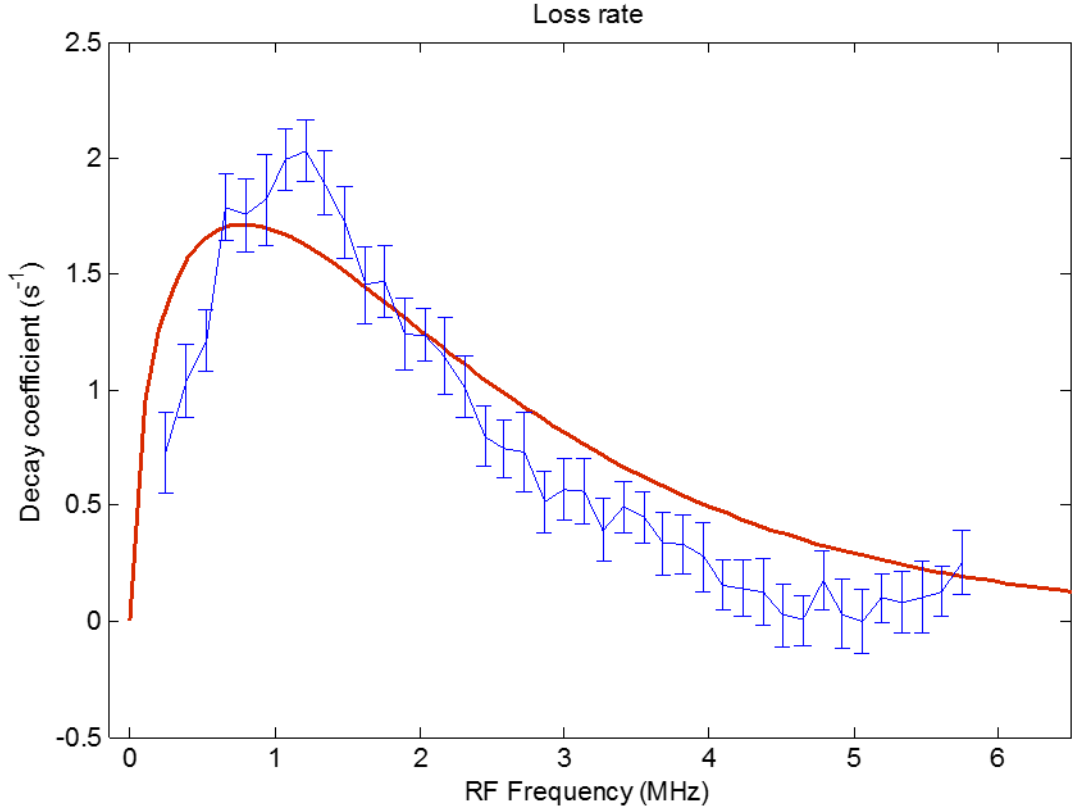


Figure 5.15: The atom loss rate as a function of radio frequency. The Rabi frequency was taken as $30Hz$. A temperature of $150\mu K$ is found. The fit is poor, possibly due to trap anisotropy and deviations from the model.

Fitting equation 5.6 to the data, we find a temperature $\approx 150\mu K$. It was difficult to match the tail and peak of the fit. This may be due to differences between our field geometry, and the quadratic form assumed by the model. Nonetheless, the temperature is of the right order, and provides a method for roughly estimating the temperature when the field parameters are not known completely, and the free ballistic expansion is not possible.

Chapter 6

Experimental Setup - Transport PCB

After the magnet-ring experiment, we changed our setup and mounted the PCB into the chamber. This chapter presents the mounting structure, the PCB preparation and tests, and finally the results obtained so far. The PCB is mounted to the upper flange and placed into the vacuum chamber. A reflective quarter-waveplate on the PCB is used to create a reflection MOT. The setup is shown in figure 6.1.

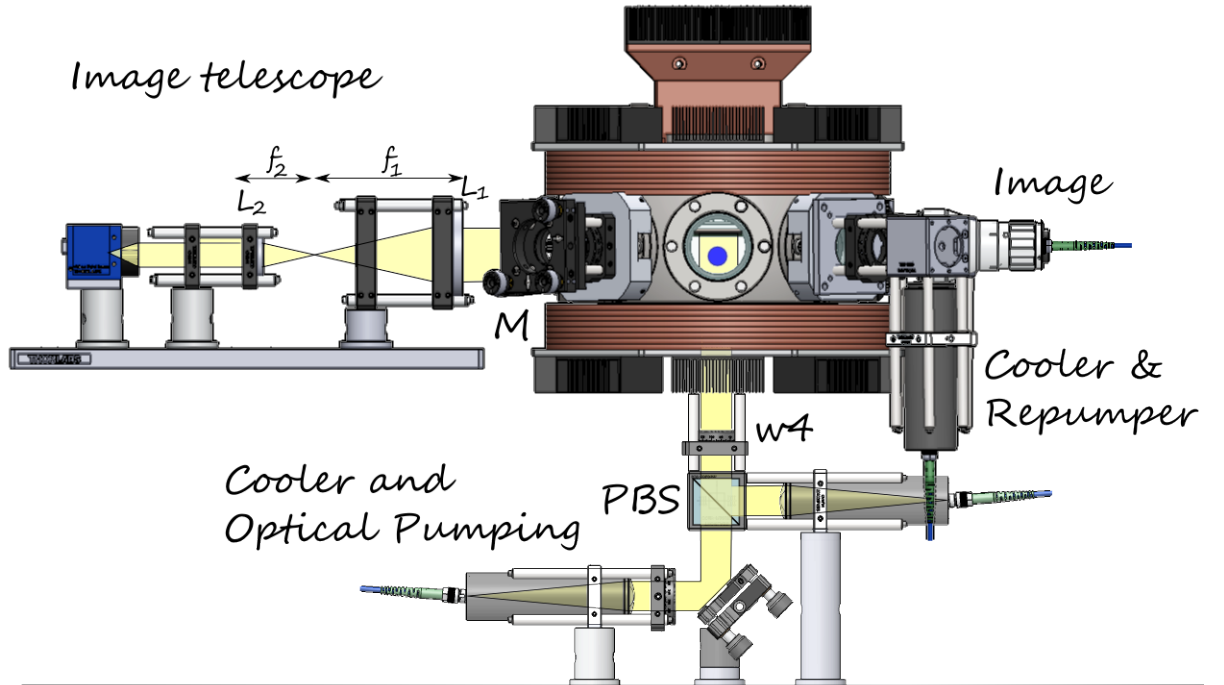


Figure 6.1: Chamber Optics. Cooling, re-pumping, optical pumping and imaging is shone into the chamber. The imaging is performed from the side. The PCB is mounted to a custom-made flange, containing feed-throughs for the PCB and chips, and a copper stem for heat dissipation.

6.1 Understructure Design

A custom flange was designed to provide current, RF signals and a heat sink. A PCB understructure was designed to mount the PCB to the flange and provide heat dissipation. The components used are vacuum compatible. The screws are vented to prevent trapped air. The flange, understructure and PCB are shown in figure 6.2.

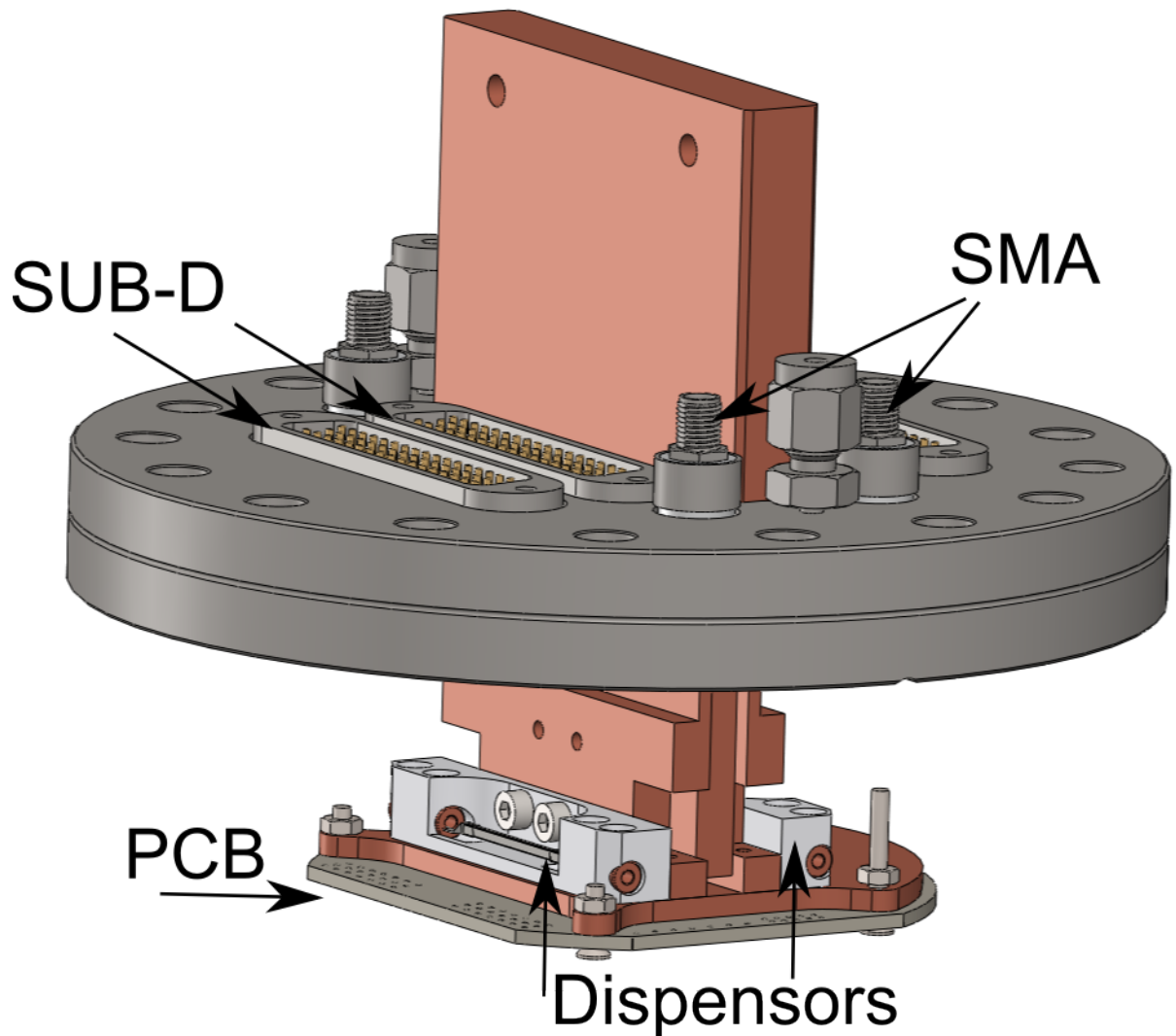


Figure 6.2: The mounted PCB. A custom flange provides 4 50-pin SUB-D connectors for current feed-through. 4 SMA connectors allow radio frequency connections, necessary for the RF chip. A copper heat sink stem allows mounting of an additional structure within the vacuum chamber. An additional dummy flange is required such that the PCB surface sits just above the centre of the chamber. A set of copper pieces mount the PCB to the copper stem. Two dispensers are mounted to the copper heat-sink.

6.2 PCB Setup

Press-fit pins¹ are used for the electrical board connections. These require no solder, and were pressed into the board using a computer controlled drill with an adapted drill bit. Female crimp connectors connect the wire to the pressfit pins². The 22AWG wires are vacuum compatible with an insulating layer of Kapton to prevent shorts³. A reflective quarter-waveplate was glued to the PCB using vacuum compatible, electrically insulating, UV cured glue. The glue used was DYMAX⁴.

The PCB and all connectors and wire parts intended for vacuum were cleaned. Firstly by washing with detergent in warm water. Then by rinsing with ultra pure water. Then finally by immersion in an ultrasound bath with ethanol for 30 minutes.

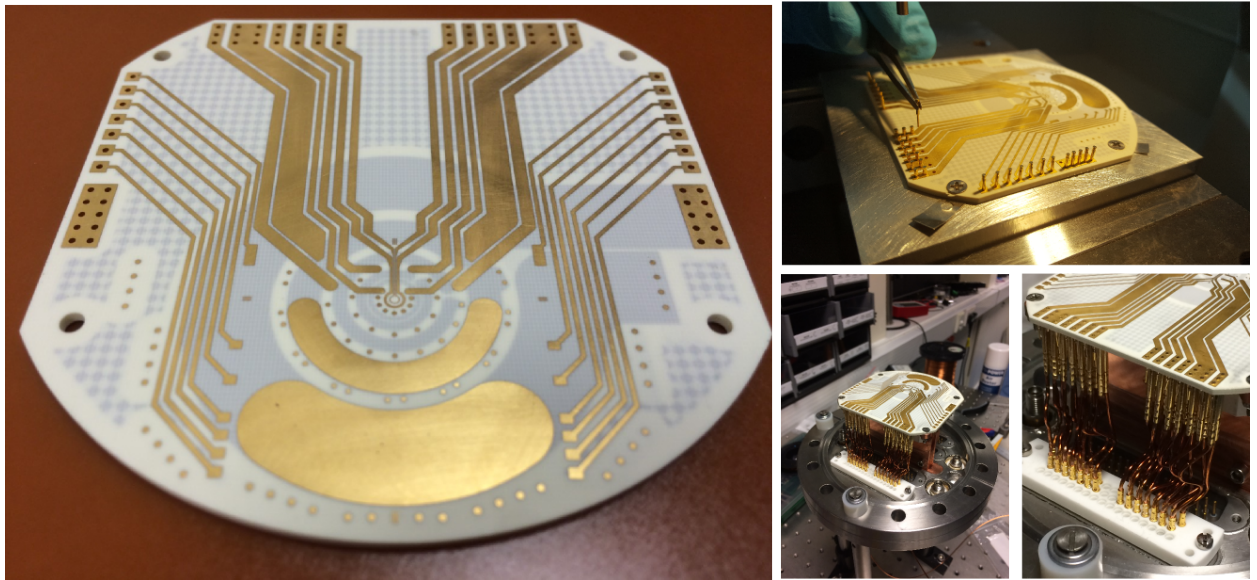


Figure 6.3: Left: A photograph of the PCB. Top Right: Placing the press-fit pins into the PCB. The PCB is mounted to a dye. This is a flat piece of aluminium with holes that match the PCB. The pins are placed by hand, and then a drill bit pushes the pin down. Bottom Right: The PCB is mounted to the understructure and flange. Wire connections are made from the pins to the SUB-D connectors.

¹MillMax 4366 Pressfit Pins for 35mil plated holes, www.mill-max.com/products/pin/4366

²MillMax 4366 Wire crimp terminal for 22AWG wire, www.mill-max.com/products/rec/1707/1707-0-19-15-30-27-10-0

³Allectra Kapton Wire 311-KAP-060-10M

⁴DYMAX, OP-67-LS

6.2.1 I-V measurements

We wanted to test how much current the PCB could handle. We looked at the resistance increase of the PCB at different times for different coils. We also used an infra-red camera to observe the temperature.

We measured the current and heat performance of the PCB. As current is passed through the coils, the coils heat up and the resistance increases. A rule-of-thumb is not to exceed a 20% increase in resistance. Starting at room temperature, we put current through the coils for 30 seconds, and measured the heat and resistance increase.

The resistance was measured by recording the current and voltage at the power supply. This then includes the resistance of the wire, and can lead to an underestimation of the resistance increase, especially for small coils. The pin connections did not heat significantly, so all resistance increase is assumed to arise from the heated PCB coils. In addition to resistance measurements, we took infra-red pictures of the PCB during the current cycles. These results are shown in figure 6.4.

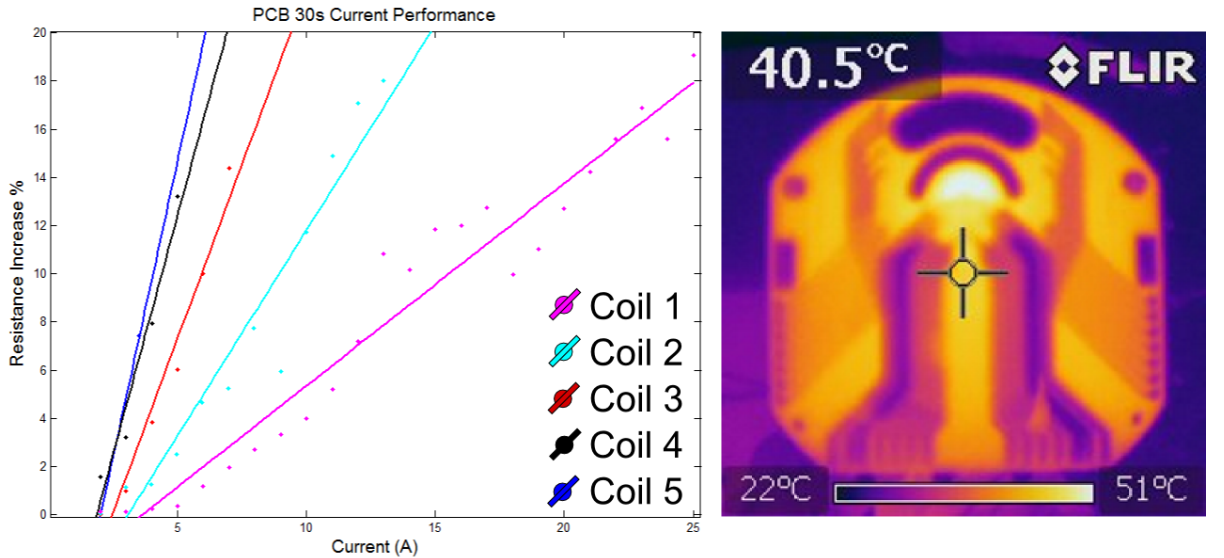


Figure 6.4: Current and heat performance of the PCB. Left: The resistance increase of the PCB coils for a 30s current cycle. Right: Infra-red image of the PCB heat distribution.

The measurements were made with the PCB mounted to the heat-sink, as would be the case in the chamber. The performance is sufficient to run the current sequence described earlier in figure 4.14. It should be noted that

these measurements were performed in air. It is expected to perform worse in vacuum due to a lower thermal conductivity.

6.3 Experimental Data

The experimental setup was characterised in a number of ways. We investigated the variation of atom number with the AOM frequencies for the cooler, imaging and optical pumping. We created a MOT, a magnetic trap, and investigated the MOT loading rate, temperatures, and losses. Unfortunately, a mistake resulted in the PCB being burnt and destroyed. A new PCB is currently being made, and protection circuits are being made to prevent over-current mistakes. In the following, the results so far are presented.

6.3.1 AOM optimisation

The AOM frequencies were scanned to optimize the atom number in the MOT. The scans are shown in figure 6.5. The optical pumping scan was shown earlier in figure 5.11. The imaging frequency is optimised for an AOM frequency of 135MHz , which is expected from the transition diagram in figure 5.4. It was found that the cooler frequency required frequent re-calibration. This was found to be a systemic error in the beat-lock box. The beat-locking attempts to lock the cooler laser 6.8GHz away from the re-pumper. The beat signal is found using a counter. Degradation of the 6.8GHz beat-signal as it travels through SMA cables and connectors gives rise to a drift in the counted frequency. We had to re-optimize the cooler frequency at the start of each day. The imaging displayed the same characteristics when it was derived from the cooler beam, and is the reason we instead derive it from the optical pumping beam. A new beat-lock system is being developed to rectify this.

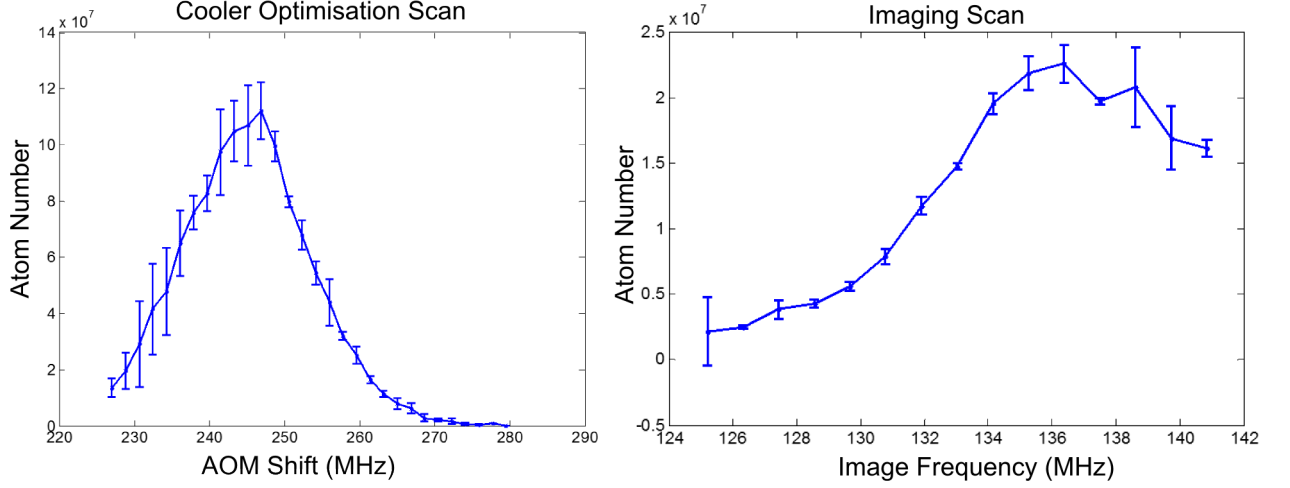


Figure 6.5: AOM Optimisation. The AOM frequencies are scanned, and optimised for atom number. Left: The cooler double-pass AOM frequency is scanned, and the atom number is measured. Right: The imaging AOM is scanned.

6.3.2 MOT

We created a MOT using the reflective quarter-waveplate on the PCB. We used 6A of current in our MOT coils, which corresponds to a gradient of $16G/cm$. A dispenser current of 3.5A was used. An absorption image of the MOT is shown below in figure 6.6. The MOT size and shape was better than in the case of the magnet-ring, where the field tended to squash the cloud shape.

We investigated the loading rate of the MOT. We wish to write the atom number N as a function of time. The loading rate of the MOT dN/dt is given by the sum of the loss rates and the capture rates [110]. Atoms are lost from the MOT by collisions with background gas atoms and two-body collisions within the trap. In the case of low densities, two-body collisions between atoms are insignificant. We then write the loading rate as:

$$\frac{dN}{dt} = R - \frac{N}{\tau} \quad (6.1)$$

where τ is the time an atom remains in the trap, and is governed mainly by collisions with the background gas. The result is then a loading rate of the form:

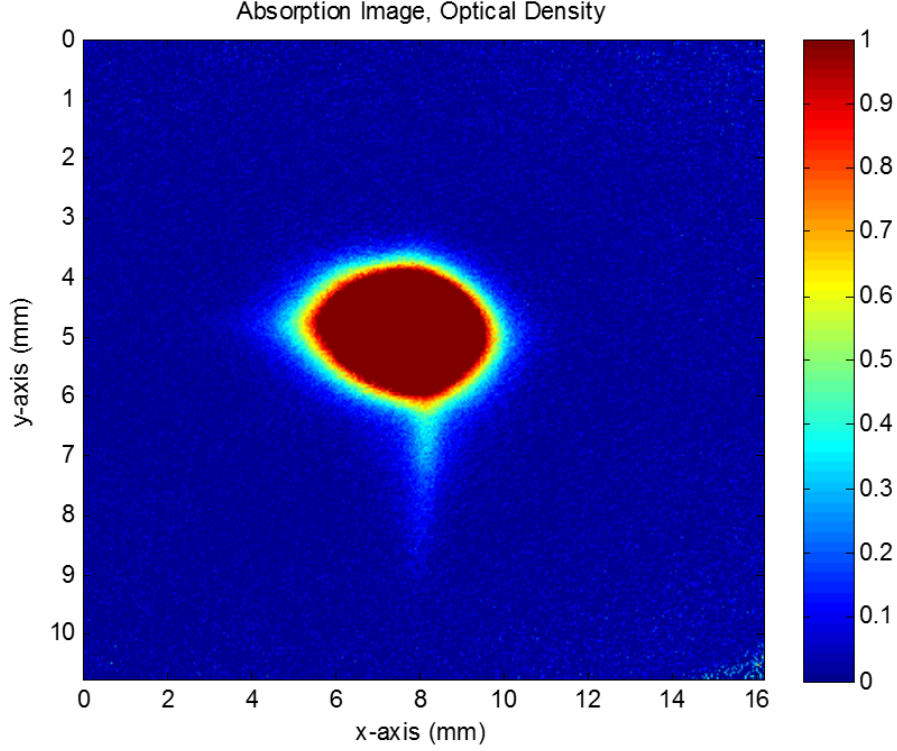


Figure 6.6: An absorption image of the MOT. The atom number is 2×10^8 . This may be an underestimate as the optical density is saturated.

$$N(t) = (N(0) - R\tau)e^{-t/\tau} + R\tau \quad (6.2)$$

The MOT duration was varied, and the atom number recorded. This is shown in figure 6.7. At small times, $N(t) \approx 0$. At large times, $N(t) = R\tau$. We find a lifetime of $2s$, and a loading rate of $2.85 \times 10^7 s^{-1}$. At large times, the steady state atom number is $R\tau = 5.88 \times 10^7$. The background pressure is dependent upon the pumping and quality of the vacuum and largely on the dispensers. At high dispenser currents, the MOT loading is faster, but the lifetime due to background collisions is significantly reduced, presenting a problem during the magnetic trap stage.

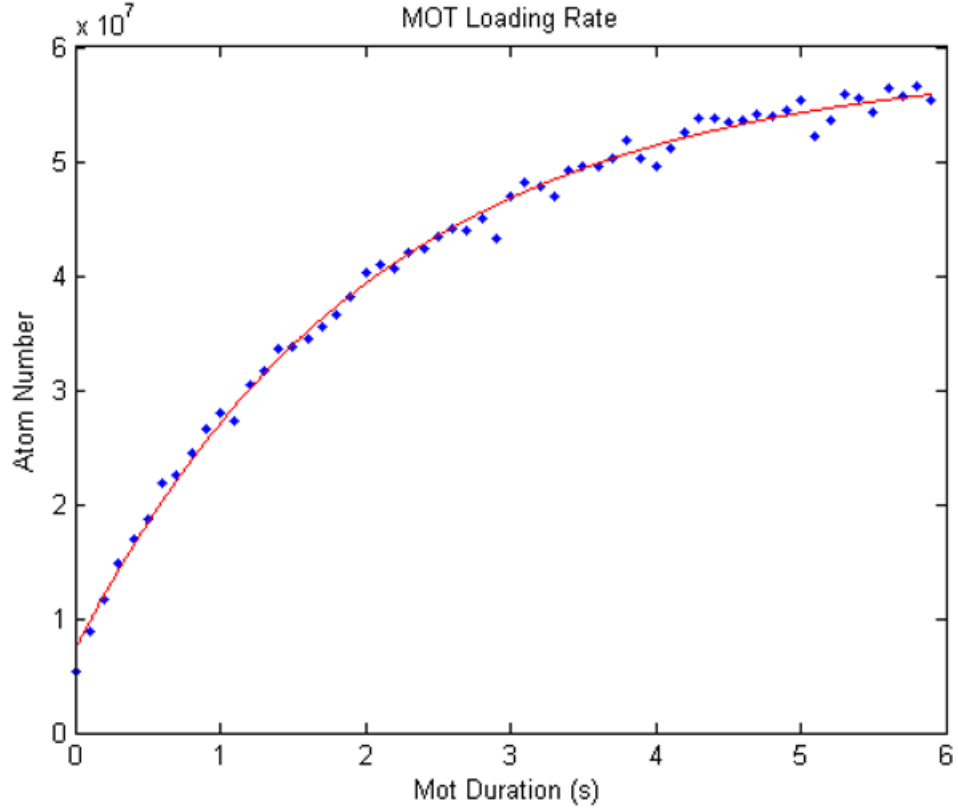


Figure 6.7: The MOT loading rate. The data is fitted by equation 6.2. The background collision time constant is found to be $2s$.

6.3.3 Temperature - Time-of-Flight

We measure temperature by looking at how the cloud expands after release from the magnetic trap or MOT. After release, the cloud falls and expands. Taking many pictures of the cloud during this phase is known as a time-of-flight measurement. Each picture requires an experimental run. The cloud expands as:

$$\sigma^2(\tau) = \sigma^2(0) + \langle v \rangle^2 t^2 \quad (6.3)$$

Here σ is the size of the cloud, and t is the time-of-flight. v is the velocity and is related to the temperature by $\langle v \rangle = \sqrt{K_B T / m_{rb}}$. This gives us the equation:

$$\sigma(t) = \sigma(0) \sqrt{1 + \frac{k_B T}{m_{rb}} t^2} \quad (6.4)$$

We performed time-of-flight measurements on the MOT. These are shown in figure 6.8 below.

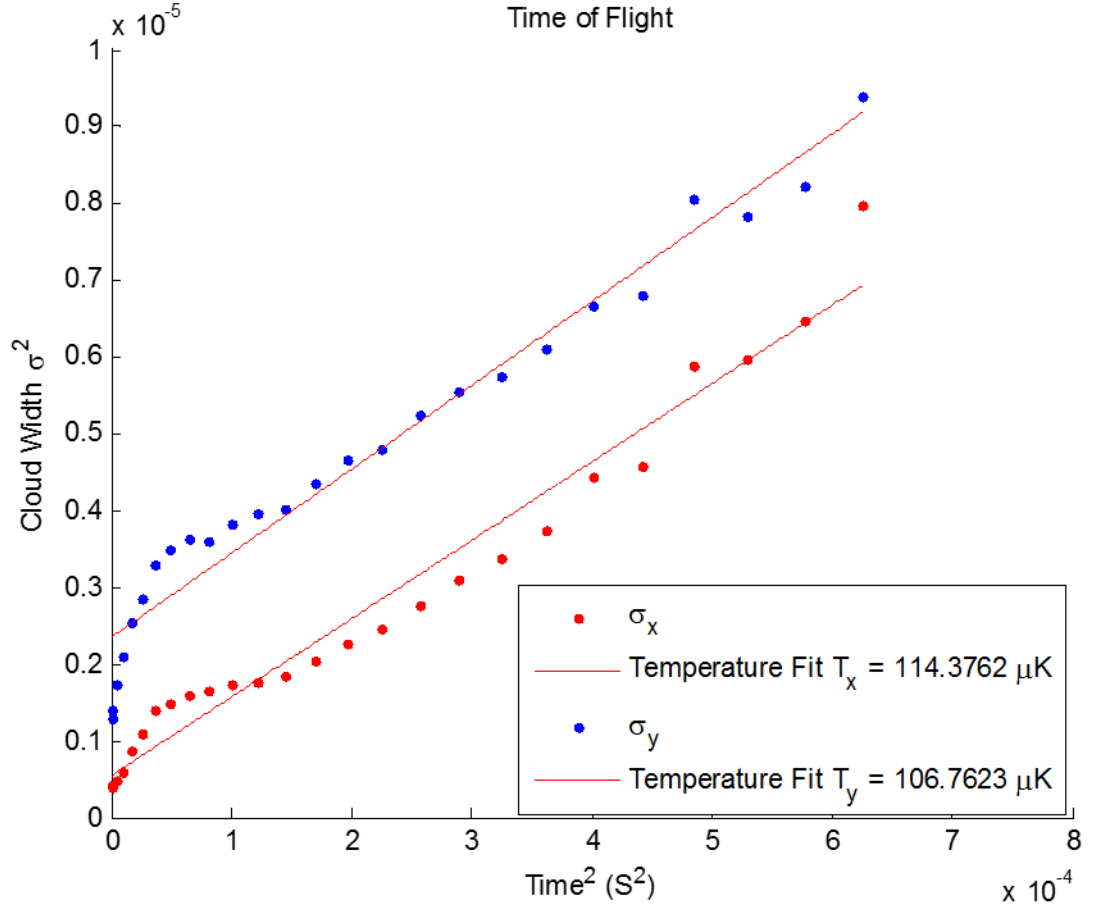


Figure 6.8: A time-of-flight measurement on the PCB MOT. The width of the cloud is recorded as a function of time. The temperature can then be inferred from equation 6.4

6.3.4 Optical Molasses

Prior to loading into a magnetic trap, the cooler detuning is ramped. We call this a compressed MOT, or CMOT. This has the effect of cooling the cloud further before transfer [111]. Time-of-flight measurements were performed for different cooler ramps. The results are shown below in figure 6.9.

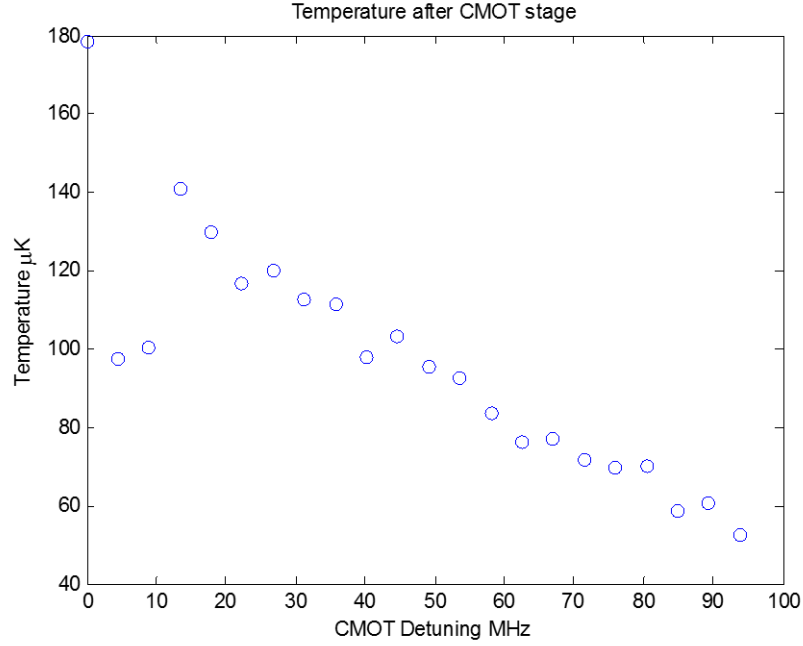


Figure 6.9: The temperature of the atomic cloud after release from the CMOT is measured using the time-of-flight method. The temperature is plotted against the detuning of the CMOT stage. A larger detuning resulted in colder clouds.

The x-axis shows the detuning of the AOM frequency used for the MOT light. The absolute value of the frequency relative to the cooling transition is unknown due to a systematic error in the beat-locking scheme. We find that ramping the cooler frequency to larger detunings results in colder clouds.

6.3.5 Lifetime

We transferred the atoms to a magnetic trap. Atoms are lost from the trap by background collisions and Majorana losses. As the atoms are reasonably hot, we expect that Majorana losses will be small and the loss rate governed by the background collision losses. From the analysis of the MOT loading procedure (section 6.3.2), we found the loss time constant to be $2s$. In the absence of any loading rate, the atom number as a function of time in the magnetic trap is then:

$$N(t) = N(0)e^{-t/\tau} \quad (6.5)$$

The lifetime is plotted in figure 6.10 below. A time constant of $3s$ is found, which is slightly larger than what we inferred from the MOT loading rate. A possible explanation is that the MOT loading experiments had a higher background pressure, which varies a lot depending on the recent dispenser usage.

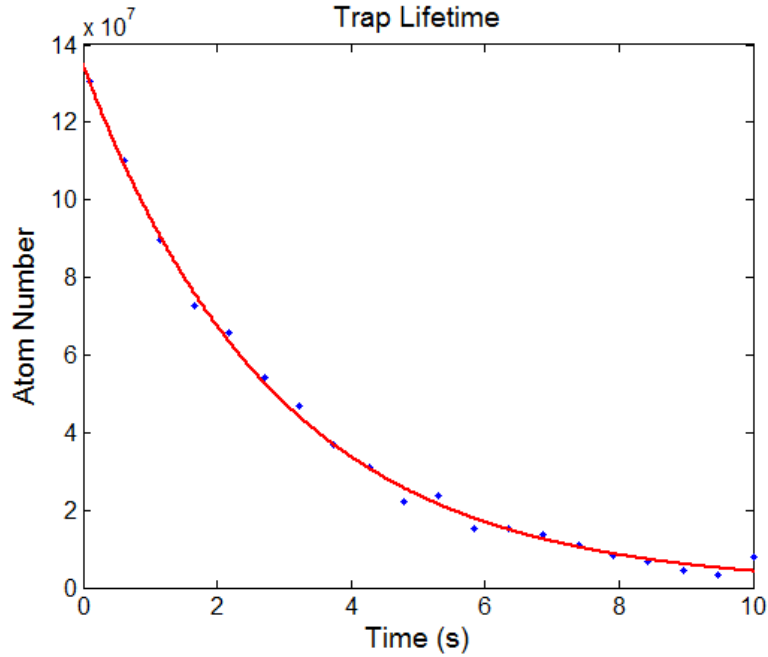


Figure 6.10: A lifetime measurement of the magnetic trap. We have a time constant of $3s$. This is similar to the MOT loading loss rate, suggesting that background collisions dominate the loss rate.

6.4 Summary

The performance of the setup has been characterised, although not as thoroughly as we would have liked. Many of the plots are presented as is, with little to no data averaging. This is due to an unfortunate incident at the time of writing, where too much current was passed through the PCB. The PCB cracked and spoiled the vacuum. As a result, neat and ideal data could not be presented here, nor any results of the PCB transport procedure. A protection circuit for the PCB is being designed to prevent this in future. On the bright side however, the plots shown are examples of the genuine data that was collected and used to optimize the setup on a weekly basis.

Regular re-optimisation was required as a result of instabilities in the laser frequency. In a beat laser-locking system, GHz frequencies are transmitted over cables, through amplifiers and processed by digital electronics. Losses and improper impedance matching result in distortions and degradation of the signal, which can vary slowly over a number of days. Consequently, the digital electronics miscount the beats, and the laser frequency drifts slowly over time. Proper impedance matching, and a redesign of the digital electronics is in process, and is expected to result in a greater stability of the overall system.

Chapter 7

Conclusions and Outlook

The vacuum chamber, optics, and control electronics have been built, but there are many areas for improvement. Significant progress has been made towards realising a guided Sagnac interferometer. A set of chips have been designed and fabricated - an electromagnet ring chip, a magnetic ring chip and an RF coil chip. Growth of magnetic thin films were investigated for the purpose of fabricating a permanent magnetic ring chip. Promising results were obtained for thin films, but these magnetic properties did not scale for thicker films. More work can be done to investigate the growth of thicker films. The next step will be to glue and bond the RF chip and EM chip to the transport PCB. Lateral transport of the atom cloud using the PCB must be tested. The atoms must be then compressed to the ring chips using the loading sequences discussed. Imaging of an atomic cloud close to a chip must be set up.

Operation of an atomic interferometer requires state-dependent imaging, and microwave pulses. A chip based microwave resonator was designed. However, it would appear that it is not suitable for use with the current ring chip design, as the gold mirror surface will block the microwaves. A microwave horn must instead be mounted externally to the chamber. Alternative designs utilising two on-chip microwave fields are being considered for operating an interferometer [72].

The transport PCB was broken when too much current was passed. A bank of fuses will protect against this occurring in future. In addition, a

protection circuit is being designed which measures the resistance increase of the coils and switches off the current above a certain threshold.

The beat-locking system presented problems with frequency drift and atom number fluctuations. The frequency drift arises from the degradation of the beat signal through the cables, resulting in a systemic counting error. The atom number fluctuations arise because the beat-lock error signal is a step function, rather than a proportional derivative signal. A new beat-lock system is being designed which rectifies both these issues.

A rotation sensor requires vibration isolation and a highly precise rotation table. The vacuum chamber will be mounted to a circular breadboard atop a rate-table. A pressurised air system allows frictionless and smooth rotation. It will be exciting to see the results of the transport and chips in the coming months.

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