Diagrams and fractions – lesson 3.1 – Adding fractions with any denominators

Summary

This lesson will introduce addition of fractions with denominators that are not multiples of each other. The central idea of the lesson is to show that the strategy the students were using up to this point (change the largest pieces into the smallest) is not sufficient and, therefore, it is necessary to expand the strategy.

Material: worksheets and sets of pieces representing the fractions (1/6, 1/8, 1/10, 1/12, 1/15 and 1/18, with the fraction written in every piece).

Outline of the lesson

Starter

The goal is to warm up recalling what they have learned in the past lessons and reinforce the strategy used in the past lessons.

The four sums explore all the basic questions solved so far in an increasing order of difficulty.
It is important to discuss this questions with the students reinforcing the strategy of changing one of the fractions to get equal denominators.
It is recommended to write the full sequence of sums on the board (as shown below), as they will be expected to do it in the next tasks.

$$\frac{1}{2}+\frac{1}{4} \; = \; \frac{2}{4}+\frac{1}{4} \; = \; \frac{3}{4}$$

Introduction

This question will evidence the limitation of the current approach: it is not possible to transform 1/2 into thirds neither 1/3 into halves.

Introduction: Use the diagrams below and the pieces to work out $\frac{1}{2}+\frac{1}{3}$.
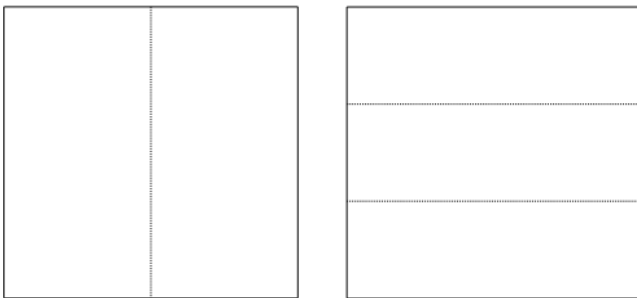
$$\frac{1}{2}+\frac{1}{3} = \underline{\quad} + \underline{\quad} = \underline{\quad}$$

Before letting the students have a go at this question, show it under the visualizer and point out the limitation of the strategy used so far. You may use terms like multiples and times tables. Then, explain that they will solve questions like this today using pieces. The idea is that students have to find pieces (fractions) that fit both given fractions, write down the equivalent fractions and find the answer.
Give them some time to solve and then discuss the solution. When doing so, reinforce that 6 (the new denominator) is multiple of 2 and 3.
Next, ask if somebody found a different solution (with 12 or 18 as denominator). If yes, show and discuss it, reinforcing that it is possible to find different solutions but they are all equivalent. If not, give some time to them to try to find at least one different denominator, and then discuss it.

Task 1 to 4

Similar to the previous task, but now they should have an opportunity to solve all the questions in a row.

The students should solve these questions using the pieces. When helping them, try to show how to choose good pieces by checking multiples instead of only going by trial and error.
As these questions are not particularly special, it is OK if you do not go through them with the whole class.
Note that is it not necessary to draw on the diagrams to solve the questions (students may only fit the pieces and remove them afterwards).

Task 5

This question is similar to the previous, but they do not have pieces representing 1/24, which would be the natural answer (note that task 1 to 4 can be solved with the product of the denominators).

Before the end of the lesson, make sure that all students spend some time trying to solve this question.
After giving them some time, discuss the answer. This question is important to reinforce multiples instead of the multiplication of the denominators.

Extension

What denominators would you need to solve the following sums?

$$\frac{1}{4}+\frac{1}{10} \qquad \frac{1}{9}+\frac{1}{6} \qquad \frac{1}{3}+\frac{1}{8} \qquad \frac{1}{10}+\frac{1}{15}$$