

DEVELOPMENT OF AN IMPROVED SCOTT-
RUSSELL COMPLIANT SPATIAL NANO-
POSITIONING STAGE BASED ON WORK-ENERGY
METHOD

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Abstract

Recently, there are fast advances in certain applications, such as optical alignment systems, stereotaxic instruments in microsurgery and nano-manufacturing, which require three degree-of-freedom spatial positioning stages to perform out-of-horizontal-plane motions. However, the research in this area is limited due to the structural complexity of the spatial compliant positioning stages, which leads to difficulty of modelling and controller design. By contrast, most of the research works focus on the traditional planar stages, which are characterized by their comparatively simple design and modelling features. Hence, the design of compact-sized spatial compliant positioning systems with high performance and accuracy are greatly required.

Therefore, this research work aims to design a spatial nano-positioning stage with high flexibility and performance. To achieve this aim, the research focuses on two issues. The first one is the structural design to improve the flexibility of the stage in terms of totally isolating the motions of the actuating units. The high flexibility is reached by adopting a monolithic structure based on a compliant Scott-Russell amplification mechanism for the actuating unit, which is driven by a piezoelectric actuator. Using three monolithic actuating units in a symmetrical mechanical arrangement results in reducing the stage size and the assembly errors and improving the structural and thermal stability of the stage. The second issue is the modelling method to obtain an accurate model, which leads to high performance. The accurate static modelling method and compliance factors are selected. In addition, an accurate method for the dynamic modelling based on the work-energy principle is developed.

Using the derived analytical models, an optimization process is conducted to obtain higher performance. Finite Element Analysis (FEA) is used to validate the performance of the proposed stage and the developed modelling method. The proposed stage has a natural frequency of 588.24Hz with 5% deviation and translational and rotational motion ranges of $299.97\text{ }\mu\text{m}$ and $3977\text{ }\mu\text{rad}$, respectively. Based on the FEA results, the proposed modelling method has the least deviation compared to the other methods, which indicates that it is more accurate than the other methods.

DEDICATION

*To my great parents
my beloved husband
my sweet children
my dear sisters and brother
and my lovely family*

AND MY SUCCESS CAN ONLY COME FROM ALLAH

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List of Symbols

Latin Symbols

		SI Units
A	Area	m^2
Amp	Displacement Amplification Ratio	-
b_{RL1}	Depth of Rigid-Link 1	m
$c_{d_a-F_a}$	Axial Compliance Factor	m/N
$c_{d_{sh}-M_{sh}}$	Shear Compliance Factor	m/N
$c_{d_{sh}-M_b}$	Shear Bending Compliance Factor	rad/N
$c_{\theta_b-M_b}$	Rotational Compliance Factor	$rad/N.m$
C	Compliance	m/N
C_{input}	Input Compliance	m/N
C_{out}	Output Compliance	m/N
$[C]$	Compliance Matrix.	-
d	Translational displacement	m
d_{in}	Input Displacement	μm
d_{out}	Output Displacement (Travel Range)	μm
d_{PZT}	Displacement of PZT actuator	μm
$\dot{d}$	Linear Velocity	m/s
$\ddot{d}$	Acceleration	m/s^2
$\bar{D}$	Displacement Vector	m
E	Modulus of longitudinal elasticity (Young's modulus).	N/m^2
f	Natural Frequency	Hz
F	Force	N
F'	Internal Force	N
F_{PZT}	Push Force of PZT actuator	N
G	Modulus of transverse elasticity (Shear Modulus)	N/m^2
GPE	Gravitational Potential Energy	J
h_{RL1}	Height of Rigid-Link 1	m
h_{LS}	Height of LS flexure hinge	m

I	Mass of Inertia	$kg.m^2$
k_{tb}	Stress Concentration Factor	-
k	Stiffness factor	N/m
K	Stiffness	N/m
K_{input}	Input stiffness	N/m
K_{PZT}	Stiffness of PZT actuator	N/m
K_{out}	Output stiffness	N/m
$[K]$	Stiffness Matrix.	-
l	Length	m
l_{LS}	Length of LS flexure hinge	m
L	Length of Scott-Russell mechanism	m
L_G	Length of Centre of Gravity	m
L_O	Distance measured from one MARC to line connecting the other two MARCs.	m
m	Mass	kg
M	Bending moment	$N.m$
M'	Internal bending Moment	$N.m$
O_{fh}	Frame of flexure hinge	-
O_{FAM}	Frame of FAM	-
O_{input}	Frame of Input Point	-
O_{out}	Frame of Output Point	-
$\bar{P}$	Internal Loading Vector	N
q	Independent Coordinate	-
Q	Non-conservative Force	N
r_O	Radius of the moving platform	m
r_{MARC}	Radius of MARC flexure hinge	m
r_{SARC}	Radius of SARC flexure hinge	m
R	Reaction	N
$[R]$	Rotation Matrix	-
S_f	Safety Factor	-
t_{base}	Thickness of the base	m
t_{LS}	Thickness of LS flexure hinge	m

t_{MARC}	Thickness of MARC flexure hinge	m
t_O	Thickness of the moving platform	m
t_{SARC}	Thickness of SARC flexure hinge	m
T	Kinetic Energy	J
$[T]$	Transformation Matrix	-
$[Trans]$	Translation Matrix	-
$[T]^{-1}$	Inverse of Transformation Matrix	-
V	Potential Energy	J
w_{RL1}	Width of Rigid-Link 1	m
w_{FAM}	Width of the FAM	m
W_{int}	Internal Work	$N.m$
W_{ext}	External Work	$N.m$

Greek Symbols

		<i>SI Units</i>
α	Angle of Scott-Russell Mechanism	rad
α_{fh}	Angel of Rotation	rad
β_a	Internal Axial Force Factor	-
β_{sh}	Internal Shear Force Factor	-
β_b	Internal Bending Moment Factor	m
σ_{max}	Maximum Stress	N/m^2
σ_y	Yield Strength	N/m^2
Φ	Mode Shape	-
β, β_{LS}	Dimensionless Geometrical Factor	-
θ	Rotational Displacement	rad
ρ	Mass Density	kg/m^3
ω	Angular Velocity	rad/s
Ψ	Stiffness Matrix	-
ζ	Mass Matrix	kg

List of Abbreviations

2-D	Two-Dimensional
3-D	Three-Dimensional
CAD	Computer-Aided Design
DC	Direct Current
DOF	Degree-of-Freedom
FAM	Flexure Amplification Mechanisms
FE	Finite Element
FEA	Finite Element Analysis
IHP	In-the-Horizontal-Plane
EBA	Energy-Based Approach
LS	Leaf-Spring
Maglev	Magnetic levitation
MARC	Multi-Axis Right-Circular
OHP	Out-of-the-Horizontal-Plane
PRBM	Pseudo-Rigid-Body-Model Method
PZT	Piezoelectric
SARC	Single-Axis Right-Circular
SMM	Stiffness Matrix Method
SRAM	Scott-Russell Amplifying Mechanism
RL	Rigid Link
WEDM	Wire-Electric-Discharge Machining

Chapter 1 Introduction

1.1 Background and Motivation

During the last two and a half decades, there was a rapid demand for high precision compliance-based positioning systems due to the extreme growth of nanoscience and nanotechnology [1]. Compliance-based nano-positioning systems address many applications, such as lithography processes, nanomaterial testing, semiconductor industry, and molecular biology. The desired characteristics of these systems are pure axial motion, high structural and thermal stability, extremely high resolution, accuracy, and fast response [1].

A compliance-based nano-positioning system can be defined as a precision mechatronic device used to move an object to a nanometer distance. The mobility of the compliant system is generated by transferring input energy to an output motion [2]. The input energy can be in a form of mechanical, electric, magnetic, etc.

Generally, compliance-based precision positioning systems can be classified into two types, serial and parallel. Serial nano-positioning stages are easy to implement and build, and the control design is relatively simple. However, the serial mechanism possesses large inertias and relatively low dynamic stiffness which lead to non-uniform performance across the workspace [3]. On the other hand, the parallel mechanism provides a structure with several advantages including shorter kinematic chains, excellent performance on stability, stiffness, and thermal induced error rejection [4].

There is high demand for three degree-of-freedom (3-DOF) spatial positioning stages to perform out-of- horizontal-plane (OHP) motions in some applications, such as optical alignment systems, stereotaxic instruments in micro-surgery and nano-manufacturing [5], which require. Unfortunately, most research works focus on the traditional planar stages (in-horizontal-plane motions). These kinds of stages characterized by their comparatively easy to design and build features. Only limited researches in OHP stages can be found [5-7]. Hence, the design of compact-sized OHP compliant positioning systems with high performance and accuracy are necessary.

The challenge arises in modelling and structural and controller design of OHP mechanisms due to their inherent structural complexity. In addition, the parasitic motion is one of the major constraints in these mechanisms, as it has direct effects on the actuating unit, which consists of a flexure amplification mechanisms (FAM) and an actuator. Consequently, this will affect the accuracy of the system. Moreover, to achieve vertical motion, the actuators must be placed in vertical arrangements, which leads to increase the size of the spatial stage. Therefore, a compact sized structure is another challenge of the stage design.

Motivated by these challenges, the goal of this work is to propose an improved structure while addressing these challenges. The research introduces some improvements during the design. These improvements are in the accuracy and flexibility of the stage. The improvement to increase accuracy and flexibility can be achieved by designing a new parallel flexure actuation system capable of moving in three different directions.

It is important to stress here that this research focuses on the structural design and the modelling of the proposed stage. The research does not address the control system and sensors, and the fabrication of the stage.

1.2 Research Objectives

The aim of this research work is to investigate the development of an OHP compact decoupled compliance-based positioning stage with high accuracy and structural and thermal stability. With the challenges stated above the objectives of the research are:

- Objective I.** Evaluate the design parameters that influence the travel range of the stage considering the overall size of the stage and the stiffness of the stage. And assess the mechanical arrangement of the stage to obtain higher natural frequency and reduce the complication of the control system design and stability.
- Objective II.** Investigate ways to reduce the parasitic motions to increase the structural stability while keeping the flexibility of the stage as high as possible by decoupling the motions of the actuating units from each other.

Objective III. Examine the dynamic models to achieve higher natural frequency to improve the dynamic response of the system and reduce vibrations.

Objective IV. Develop an accurate and simple method for the dynamic modelling.

1.3 Research Methodology

This section intends to illustrate how the research objectives will be achieved. Table 1.1 summarizes the research objectives and presents the proposed solutions to achieve these objectives.

Table 1.1 *Objectives and proposed solutions.*

OBJECTIVE	PROPOSED SOLUTION
Enlarging Travel Range & Compact-Size	➤ Implement a suitable flexure amplification mechanism which allows integrating a long stroke actuator horizontally. This can be done by using Scott-Russell amplifying mechanism (SRAM), which also leads to decrease in the height of the stage.
Simplifying Mech. Arrangement	➤ Design a monolithic Structure for the three FAMs to reduce the assembly errors and the manufacturing cost by fabricating each FAM from a single piece using wire-electrical-discharge machining (WEDM). Also, arrange the three FAMS in a triangular parallel configuration, which results in a symmetrical kinematic arrangement to increase the thermal stability of the stage.
Parasitic Motion Elimination	➤ Remove physical constraints between the actuating units to increase the structural stability of the by using 3-DOF flexure hinges to connect the actuating units with the moving platform.
Increasing Natural Frequency	➤ Reduce the size of the stage by decreasing the height of the stage and optimizing the stage components.
Developing a dynamic Modelling Method	➤ Develop an accurate and simple method for modelling the potential energy of the Lagrangian equation by introducing the work-energy method in the context of compliant mechanisms.

The objectives mentioned above address two issues: (1) the performance (2) the flexibility of the stage. Figure 1.1 is arranging the objectives according to the two categories. Each category will be answered in a separate phase. The objectives related to the flexibility of the stage will be addressed in the conceptual design phase. The performance objectives will be achieved in the modelling and optimization phase. It is important to mention that the flexibility and the performance are still interrelated. For example, increasing the travel range is achieved first in the conceptual design than in the modelling phase. The design process for achieving these objectives is explained in the following subsection.

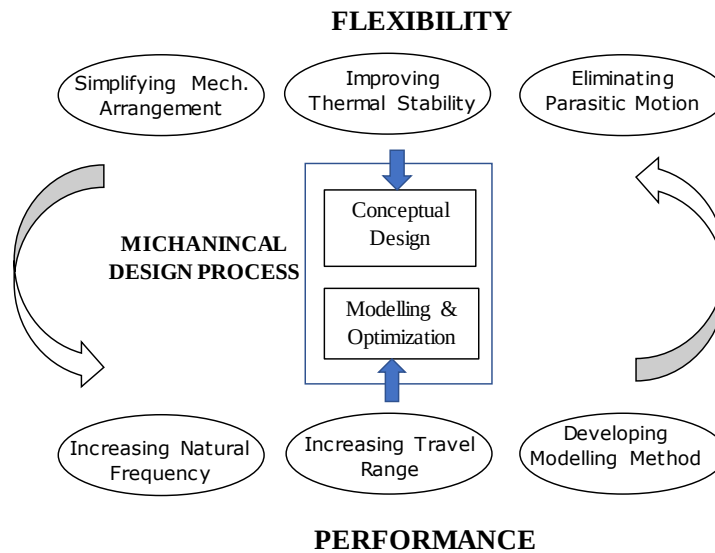


Figure 1.1 Research objectives.

The design process, which is summarized in Figure 1.2, can be broken into several steps. Each step is defined in terms of the required task to accomplish this step. The tools required to complete each task are presented. A detailed description of the design process and analysis methods will be discussed in the following sections:

- **Scope of work**

The research work is conducted based on the problem and the challenges, which are clearly defined to set the research objectives. It covers an investigation of the most accurate ways to obtain a compact-sized spatial nano-positioning stage with high performance. The investigation considers

the kinematic, static, and dynamic models of the proposed stage, which are used to develop the optimum structural design. The design requirements are determined according to the literature review. The work does not consider the control system of the proposed stage.

- **Conceptual design**

The step of the conceptual design is done according to the design requirements. Here, the objectives related to the flexibility of the stage will be answered in this step, shown in Figure 1.2. The final proposed structure of the stage will be validated to achieve the objective of the research.

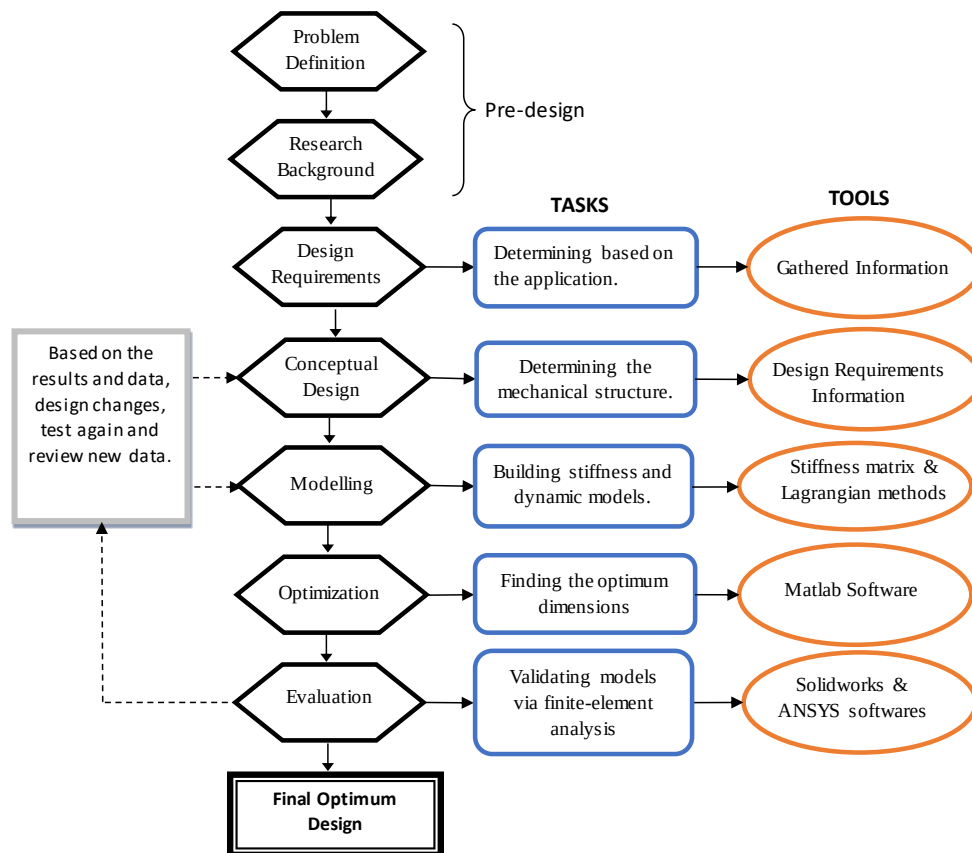


Figure 1.2 Layout of the research methodology.

- **Mechanism Modelling**

Once the proposed structure is defined, the following steps will be done:

- **Kinematic modelling**

Kinematic models of the stage will be conducted in terms of:

- defining the displacement amplification ratio of the stage, which is a factor needed throughout the design process of the stage.

- determining the linear and angular velocities of the flexure hinges, which are needed in the dynamic analysis.
- modelling the displacements of the rigid links connecting the flexure hinges, which are required for the dynamic modelling.
- testing the mobility of the stage.

▪ **Static modelling**

The static modelling is used to assess the input stiffness of the stage. The input model will be built via SMM. The stage stiffness influences the driving capacity of the actuators. In addition, it can affect the static and dynamic characteristics of the stage in terms of the output displacement, and the natural frequency.

▪ **Dynamic modelling**

A developed modelling method will be derived to achieve more simple and accurate dynamic models. The aim of the dynamic modelling is to obtain the natural frequencies of the proposed stage. The dynamic model of the stage will be obtained based on the developed method. This model will be used in the optimization process.

▪ **Optimization**

The optimum dimensions of the stage can be achieved based on the dynamic model using the MATLAB software. The objective of the optimization process is to maximize the natural frequency to achieve better dynamic performance. There is an inverse relationship between the natural frequency and the travel range of the stage. Therefore, the natural frequency will be set as the objective function of the optimization process and the travel range will be constrained.

▪ **Validation**

In this step, the performance of the whole optimized stage will be tested using Finite Element Analysis (FEA) tool. First, a Computer-Aided Design (CAD) model will be built by Solidworks software. Then, the performance of the model will be simulated in ANSYS Workbench software package. The validation will evaluate the stage performance, which involves the static, the dynamic, the deformation performance

analyses. In this step, the validation of the proposed stage will also lead to validate the developed method.

1.4 Research Contributions

The following contributions are achieved during the design, modelling, optimization and validation of the proposed stage:

- Contribution I.** Increase the affordability of the stage by reducing the height of the stage and using a monolithic structure of the actuating units made of Aluminium alloy ($Al - 7075$), which is a low-cost material.
- Contribution II.** Derivation of a simple and accurate method for dynamic modelling based on the work-energy method in the context of compliant mechanisms.
- Contribution III.** Application of a modelling method for multi-axis right circular flexure hinges in the OHP mechanisms, which is absent from the current literature.
- Contribution IV.** Demonstration of a clear framework for design and modelling OHP compliance-based mechanism, which is not available in textbooks.
- Contribution V.** A structural Design of an OHP nano-positioning stage with a natural frequency of 588.24 Hz and a travel range of $299.97\text{ }\mu\text{m}$ and $3977\text{ }\mu\text{rad}$.
- Contribution VI.** This research has resulted in the following expected publication, which is underwriting:
 - *An analytical approach for modelling input stiffness of a compliant mechanism based on work-energy method.*
 - *Design and modelling of an improved compliant spatial nano-positioning stage based on Scott-Russell mechanism.*

1.5 Thesis Outline

The research report is structured as follows:

- **Chapter 2: Literature Review**

An investigation in the current OHP compliance-based positioning stages, and their advantages and disadvantages are presented in this chapter. A conceptual design of the OHP stage is proposed to rectify the limitations found in the literature. Moreover, the current modelling methods are evaluated in this chapter and a new modelling framework is presented. The output of this chapter is the base for **Contribution I**.

- **Chapter 3: Kinematic and Static Modelling**

The kinematic quantities, which involve the displacement amplification ratio, the displacements of the flexure hinges and velocities of the rigid links, are derived. In addition, the static analysis in terms of the input stiffness of the stage using the Stiffness Matrix Method (SMM) is presented in this Chapter.

- **Chapter 4: Dynamic Modelling**

An accurate method based on the work-energy method for potential energy modelling, which includes the derivation, a full framework, and validation of the proposed method, is developed. A natural frequency model using the developed method is built. This chapter introduces **Contribution II and Contribution III**.

- **Chapter 5: Design Optimization**

An optimization process for optimal geometrical parameters using the natural frequency model as the objective function is conducted. **Contribution IV** is completed in this chapter.

- **Chapter 6: Validation**

Validation of the optimum design via FEA is discussed in this chapter. A comparison between the results achieved from the analytical models and the FEA simulation is shown. This chapter presents **Contribution V**.

- **Chapter 7: Conclusions and Future work**

The main findings of the research work are summarized in this chapter. In addition, the contributions of the research work are concluded. Some suggestions for future work to enhance and complete the design of the proposed nano-positioning stage are presented.

Chapter 2 Literature Review

The aims and the importance of this chapter are to:

- Review briefly the three common types of positioning systems (Rigid-Body, Magnetic Levitation, and Compliance-based), including their advantages and limitations.
- Assess the different types of actuation devices used for compliance-based positioning systems.
- Review the existing spatial compliance-based positioning systems. The assessment covers two issues; the design of the mechanism and the modelling methods.
- Present the conceptual structure of the stage while rectifying the limitations found in the mechanism design of the existing literature.
- Evaluate the existing modelling methods, to select the most accurate methods which suit the proposed stage. This evaluation includes the modelling methods of the flexure hinges and the compliant mechanisms. The assessment of the compliant mechanisms consists of the static and dynamic modelling methods.
- Propose a modelling method, which suits the proposed stage. The proposal for another method is based on the results achieved from the evaluation of the existing modelling methods.

These aims are discussed in three main sections. **Section 2.1** contains a literature review of the different types of the nano-positioning systems. The commonly used actuators are introduced in **Section 2.2**, where the actuating type for the proposed stage is selected. **Section 2.3** discusses the structural design of the existing compliant mechanisms while outlining their limitations. Based on these limitations, an out-of-horizontal-plane (OHP) stage is proposed in the same section. The analytical modelling methods of the flexure hinges and the compliant mechanisms are studied and evaluated in **Section 2.4**. Based on these evaluations, the most accurate modelling methods are selected, and a dynamic modelling method is suggested. Literature review of the commonly used material is conducted in Section 2.5.

2.1 Nano-positioning Stages

From a wide survey of the existing literature concerning mechanisms of stages, nano-positioning stages can be organized into three groups: (1) Rigid-body, (2) Magnetic levitation (Maglev), and (3) Compliance-based. A general description of each type and their advantages and limitations are presented in the following subsections.

2.1.1 Rigid-Body Nano-positioning Systems

Rigid-body positioning systems are considered here as systems use energy imported from mechanical or electrical actuators to transfer motions or forces through rigid links and movable joints to generate motions. The motions of the positioning systems are produced by actuation systems with high bandwidth to achieve multiple DOF [8]. These motions are designed to move with respect to a number of kinematic constraints, using ball screws [9], rolling [10-12] or sliding guideways [13, 14], to ensure the desired precision [17].

Nevertheless, these traditional manipulators with conventional joints have inherent limitations, such as errors due to a backlash, hysteresis, and manufacturing errors in the joints. In addition, these joints produce friction, which causes steady-state tracking errors [9]. Also, friction can lead to a slow motion and a limited cycle. Moreover, it is relatively difficult to consider friction compensation in controller design [9].

Therefore, achieving ultra-high precision in nano-positioning, using conventional joints, is a major challenge. This leads to a limited number of DOF and a poor resolution of the nano-positioning stage [15]. Thus, the best existing literature based on Stewart platform succeeded to reach micro-scale positioning [16-18].

There are some nano-positioning systems in the market, manufactured by PI, with high performance [19]. They are known as hexapod nano-positioning systems. These systems are six axes parallel kinematics, as shown in Figure 2.1. However, they can only be characterized as robots with several manoeuvring joints, which make them too expensive and unaffordable [20].



Figure 2.1 *Hexapod Parallel Kinematic Positioning Platform [19].*

2.1.2 Magnetic Levitation Nano-positioning Systems

Maglev technology has been proposed to overcome the drawbacks found in the rigid-body positioning systems with the conventional joints. This technology has the advantage of the absence of direct mechanical contact during operation, as shown in Figure 2.2. As a result, there are no friction and no backlash, which reduce the need for lubrication and precision bearings.

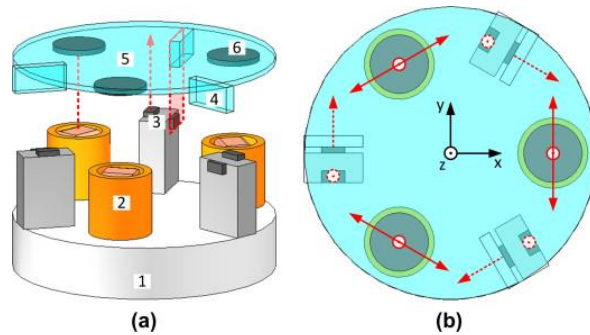


Figure 2.2 *Structure of the maglev positioner [21].*

One of earliest stages reported using this technology was fabricated by Kim and Trumper [22]. The stage was designed as a high precision Maglev stage for photolithography in semiconductor manufacturing, provided a fine 6-DOF motion. The stage had a travel of $50\text{mm} \times 50\text{mm} \times 400\mu\text{m}$. It was driven by four permanent magnetic linear suspension motors. However, the stage suffered from high-power consumption.

To overcome the previous drawbacks, a long-range scanning stage for scanned probe microscopy was built by Holmes and Hochen [21]. A 12kg moving part (platen) was floated in oil to support its weight and to provide mechanical damping. The stage had a travel range of $25\text{mm} \times 25\text{mm} \times 0.1\text{mm}$. The stage was actuated by four levitation linear motors. However, this stage suffered from the heavy weight of the moving parts as it was made of iron.

Considering this issue, a 6-DOF Maglev stage that levitates a moving part of 0.212 kg, which consisted only of magnets was designed by Verm *et al* [15]. The system had a high natural frequency due to the high structural stiff and the light weight of the moving parts. The Maglev stage had a motion range of 300 μ m in the three axes. Three vertical linear actuators were used to levitate the moving parts.

Trying to achieve sufficient robustness, low cost, and low power consumption, another 6-DOF Maglev stage using a fluid bearing was fabricated by Chen *et al* [23]. The range motion was 3 mm $\times$ 3 mm $\times$ 4 mm. In comparison with the former stages, the motion ranges were smaller in this system.

Although the Maglev stages have several advantages over the conventional positioning stages, they suffer from major limitations [24, 25]. The following points summarise these limitations:

- The motions of the axes are generated by a single levitated moving part. This leads to inherent coupling between the axes, which limits the real travel range,
- The Maglev stage is an unstable open loop,
- Because of the actuator nonlinearities and fabrication and assembly defects, the analytical model fails to capture the effects of uncertainties and unmodeled dynamics, and
- As there is no damping or restricting force on the moving part, the overshoots to the commanded steps are great.

2.1.3 Compliant Nano-Positioning Systems

Recently, the research and development work on compliant nano-positioning systems have been growing fast to achieve ultra-high precision. In comparison to other stages mentioned in the previous sections, compliant nano-positioning stages have a lower number of parts; this will result in [25-27]:

- Wear reduction and no need for lubrication.
- Increase the precision of the mechanism with no backlash.
- Significant reduction in weight.
- Reduction in the vibration and noise.

A wide variety of different DOF nano-positioning stages has been developed. In general, these stages can be classified into two groups: in-horizontal-plane (IHP) stages (planar stages), and OHP stages (spatial stages).

The planar stages are designed to provide two translational motions, such as the stages presented by Tuma *et al*, Tang and Li, and Wu and Xu [27-29], or one rotational and two translational motions, such as the stages fabricated by Hyo-Yong *et al* and Cai *et al* [30, 31]. Also, few positioning stages provide a pure rotational motion are reported in [32, 33]. All these kinds of stages are comparatively easy to design and build [33]. On the other hand, the spatial stages can offer three translations as reported in [34, 35], or three translations and three rotations as found in [36, 37].

These nano-positioning systems should be more physical compact size and flexible to travel in all directions, and capable of accurate positioning and orienting the object at the required direction [1]. The required performance can be achieved through high accuracy, large travel range, and high control bandwidth [1]. One of the vital issues that can directly affect the stage performance is the type of the actuator. The following section reviews the actuators commonly used in nano-positioning systems.

2.2 Actuation Devices

Selecting actuators for a specific nano-positioning system depends on several parameters, such as force, size, weight, power consumption, displacement range, and frequency range [38]. The following sections review the actuators commonly used in nano-positioning.

2.2.1 Conventional Devices

The following are the conventional devices that are commonly used:

- Linear motors are usually used to drive the Maglev nano-positioning stages. A three translational positioning stage using linear motors has been reported in [34]. Linear motors are usually used to increase the work-space of the stage as they have the advantage of large motion range. However, they have a bulk size, which makes them not a good candidate for the applications requiring a small size of actuation.

- Voice Coil motors, which were used to actuate a 4-DOF nano-positioning stage with flexure guides, have been reported in [39]. The stage has achieved a long motion range. Nevertheless, the control loop of these actuators used in 6-DOF is too complicated.
- Magnet–coil actuators have been used in [40-42]. They have the advantage of long travel ranges. However, these types of actuators have a low force density.
- Magnetostrictive actuators are used when large forces are needed over small distances [21]. Few positioning stages using Magnetostrictive actuators have been reported, such as in [43]. The dynamic strain at resonance of the magnetostrictive actuators is greater than piezoelectric (PZT) actuators. However, the magneto actuators require higher power consumption than PZT actuators. Other drawbacks of magneto actuators, in comparison to PZT actuators, are their low output energy per mass unit and their large size [44], which is considered a problem in the compact design of nano-positioning stages.

Generally, all these previous actuators share the same drawbacks, poor resolutions, bulky mechanical structure and not efficient when applied on spatial stages [39].

2.2.2 Piezoelectric Devices

PZT actuators are widely used in nano-positioning technology, due to their inherent advantages in terms of the following aspects [36, 45]:

- Excellent operating bandwidth,
- Fine resolution,
- Smooth and continuous movement,
- Quick response-time,
- High stiffness,
- Generation of large mechanical forces with low energy consumption, and
- Small volume.

Using a closed-loop system with integrated sensors and appropriate control scheme in nano-positioning systems can be a solution to overcome the non-linear and hysteretic behaviour, which characterized the PZT actuators [45]. Many of

previous literature worked in solving this issue, using different control methodologies, as in [46, 47].

Another aspect facing the use of PZT actuators is the limited traveling range. The PZT ceramics can generate strains of up to 0.1% enabling motions of 100 μ m with an actuator 100mm long [45]. However, this issue has been solved by using different types of amplification mechanisms. Most of the nanopositioning systems using serial or parallel flexure mechanisms, as shown in Figure 2.3, are driven by PZT actuators.

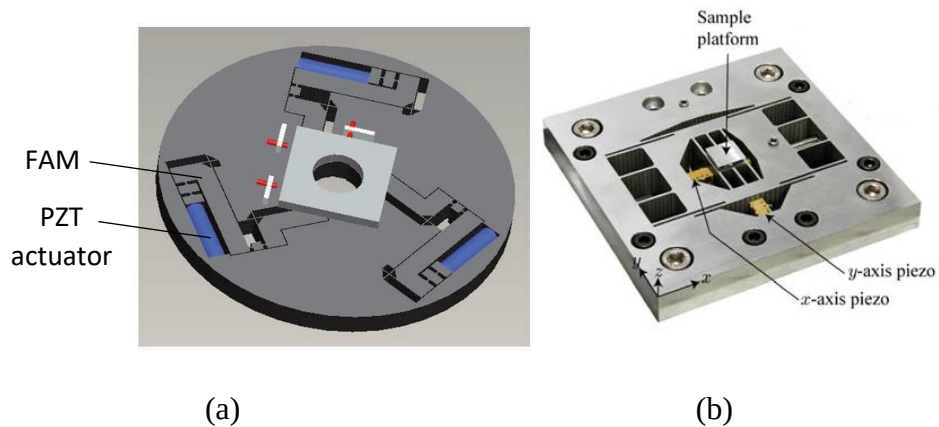


Figure 2.3 *parallel compliant nanopositioning: (a) Planar [30] and (b) Serial [48].*

2.3 Design of Compliant Mechanisms

The main components of a positioning system are a compliant mechanism, actuators, sensors, and control system. Compliant mechanisms are often driven by PZT actuators. The PZT actuators are usually integrated with flexure amplification mechanisms (FAMs). The FAMs are used to maximize the efficiency of the stage by giving the greatest range of motion from the smallest possible size of the PZT actuator.

FAMs are also used to eliminate any parasitic motion. There are several FAMs that have been used in the past, such as lever mechanism, bridge-type flexure hinge, honeycomb link, pantograph mechanism, and Scott-Russell amplification mechanism (SRAM) [4].

The FAM consists of rigid-links connected by flexure hinges, as shown in Figure 2.3. It is fabricated by removing material from a blank piece producing a monolithic mechanism. Therefore, the flexure hinges do not need repair and the mechanism will work until it fails [49].

The flexure hinge is compliant in rotation about a certain axis, which is called compliant or sensitive axis. Generally, there are three categories of flexure hinges according to their sensitive axes; (1) Single-axis flexure hinges for two-dimensional (2-D) applications (planar), and (2) Multi-axis flexure hinges for three-dimensional (3-D) applications (spatial). The flexure hinge in each category is defined according to its shape. For example, the three commonly used flexure hinges are called single-axis constant-rectangular cross section flexure hinge also known as leaf-spring flexure hinge (LS), single-axis right-circular (SARC) flexure hinge, and multi-axis right-circular flexure hinge (MARC), as shown in Figure 2.4.

The 3-DOF compliant precision positioning stages can be defined according to their motions into: (1) Planar stages (IHP), generate two translation motions and one rotation, (2) Spatial stages (OHP), produce two rotation motions and one translation, or three translation motions.

The following review will focus on OHP compliant positioning stages in terms of the design of their mechanisms and the modelling methods.

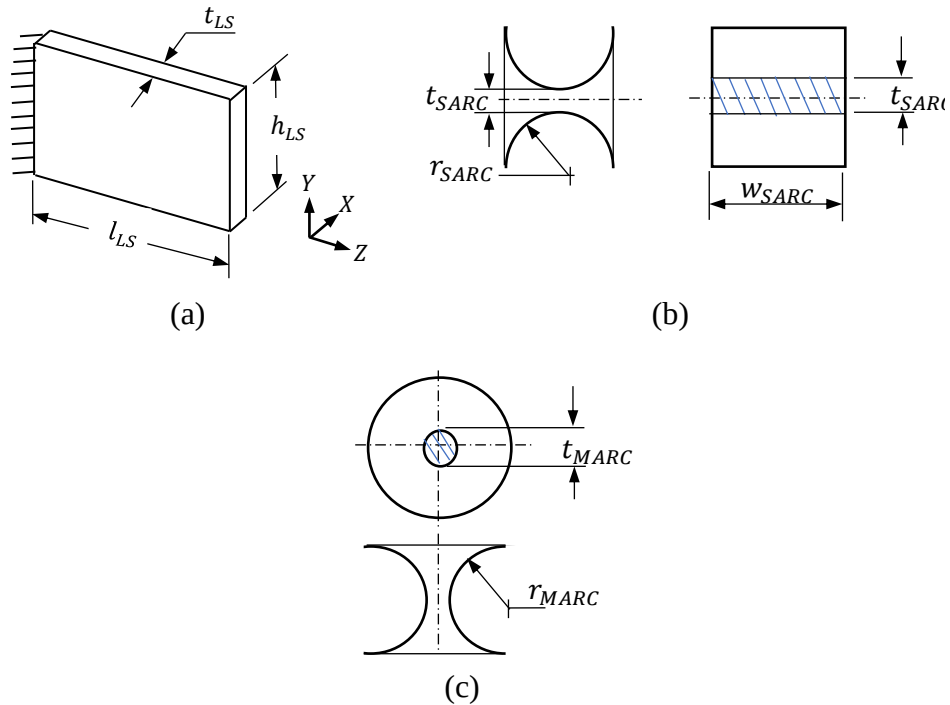


Figure 2.4 The geometrical parameters of flexure hinges: (a) LS, (b) SARC, and (c) MARC.

2.3.1 Design of Existing Mechanisms

Only a few 3-DOF spatial compliant systems have been reported in the literature. They can be found as a separate stage or integrated with a planar stage to perform 6-DOF motions.

A 6-DOF serial-parallel compliant nano-positioning stage connected by two parallel compliant stages was presented by Chao and Liu [4]. It consisted of two parallel stages, which were connected in series, as shown in Figure 2.5 . They were partially decoupled to ease the control task. The structure of the stage consisted of an upper stage, which was a spatial 3-DOF stage, and a moving plate with a 3-DOF planar stage. The OHP displacements of the system were generated by PZT actuator placed vertically with large kinematics chains. This results in increasing the stage height. Moreover, no FAMs were used, which reduce the travel range of the PZT actuator. Several 6-DOF nano-positioning stages are found, such as [42, 50, 51]. However, they are not considered here due to their complex structural design.



Figure 2.5 *Prototype of a 6-DOF micro-stage [4].*

One of the earliest spatial 3-DOF compliant stages reported was fabricated by Goo Bong *et al* [52], as shown in Figure 2.6 (a). The stage was utilized for the tele-micro-operation. It consisted of three compliant chains, and every chain had a MARC flexure hinge and a SARC flexure hinge. This system has a low natural frequency of 174 Hz. The same design of the spatial stage was improved by Yoon *et al* [53], as shown in Figure 2.6 (b). The modified stage had a higher natural frequency of 500 Hz and a travel range of 128.6 μm .

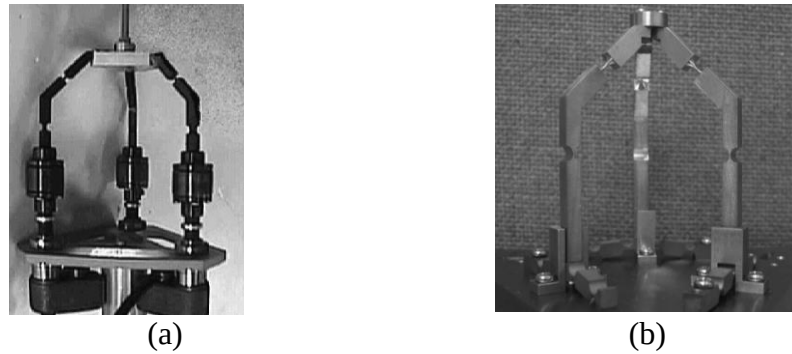


Figure 2.6 Prototype of 3-DOF tele-micro stage:(a) fabricated by [52], and (b) modified by [53].

The same limitations above mentioned are found in [54]. A 3-DOF micro-positioning table for surface grinding of 584 Hz, as shown in Figure 2.7. The stage actuated by three vertical PZT actuators. However, the PZT actuators were preloaded directly to notch flexure hinges. The notch flexure hinges were also used as a guide for the moving platform. Accordingly, when the moving platform rotates, lateral forces will be generated. These forces can damage the PZT actuators due to their vertical positions.

Another type of spatial stage is a 3-DOF translation stage, which was fabricated by Xu and Li [1]. The stage consisted of three identical kinematic structures, as shown in Figure 2.8. The structural design allowed generating motions in the X, Y, and Z axes. The actuators were placed vertically to achieve a spatial motion. In addition, the design of the actuation unit was simple and not complicated. However, the stage suffers from the same limitations mentioned above. The vertical placement of the PZT actuators increases the height of the stage and reduces the horizontal stiffness of the stage. Moreover, the stage had a small travel range because no FAM is used to amplify the PZT input displacement.

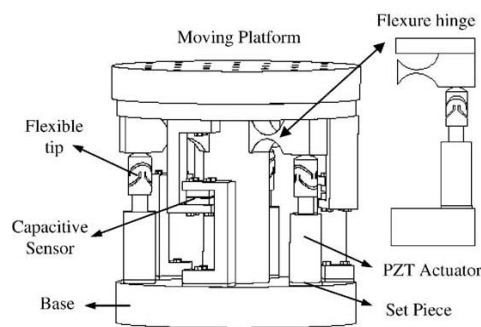


Figure 2.7 Schematic of 3-DOF micro-positioning table [54].

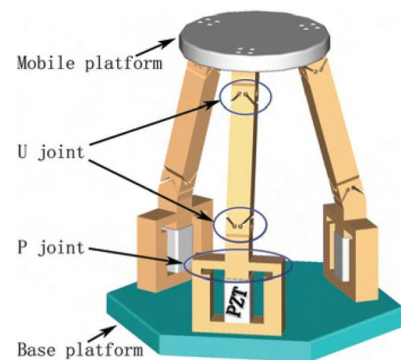


Figure 2.8 Solid model of 3-DOF stage [1].

To overcome the previous drawbacks, based on tripod parallel configuration, Kim and Cho [55] built a 3-DOF spatial stage, as shown in Figure 2.9 (a). The stage had a natural frequency of 357Hz and accuracy of $0.025\mu\text{m}$. The PZT actuator was placed horizontally to reduce the stage height, resulting in increasing the horizontal stiffness of the stage. However, the stage had two major limitations. First, the FAM adopted an assembly design of a PZT actuator, a lever, flexure hinges and rigid links, instead of monolithic design for the complexity of the structure, as shown in Figure 2.9 (b). In addition, this structural design decreased the accuracy due to the assembly errors and increased the manufacturing cost. The structural design of FAM made the PZT actuator part of the movable elements. Consequently, the lateral forces generated due to rotation can damage the PZT actuator.

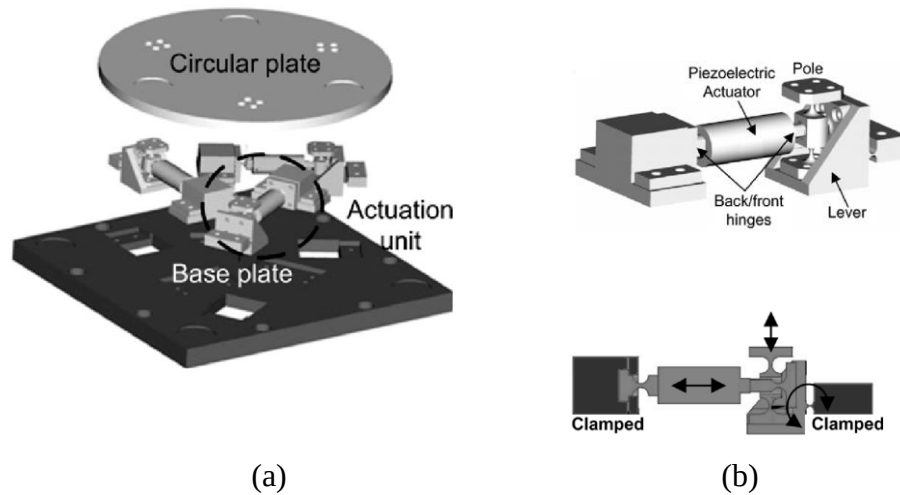


Figure 2.9 Sematic diagrams of a 3-DOF tripod micro stage [55]:(a) Configuration, and (b) Actuation unit.

To solve the former two limitations, Kim *et al.* [56] fabricated a 3-DOF vertical stage. The stage was designed for special purposes where vertical positioning was required, as shown in Figure 2.10 (a). It had a natural frequency of 156.8 Hz and an overall size of $280 \times 320 \times 270\text{ mm}^3$. It used a bridge type amplifier for the monolithic structure. The bridge-type amplifier was used as FAM, which generates upward direction motion when the PZT actuator expands, as shown in Figure 2.10 (b). However, the bridge type amplifier suffers from low output stiffness and relatively large height. The configuration of the FAMs is not symmetric, which leads to low structural and thermal stability.

Trying to achieve sufficient robustness, while keeping the advantages appeared in the former work, a compact stage was presented by Lee *et al.* [7], as shown in Figure 2.11 (a). The stage had a travel range of $190\mu\text{m}$ in the vertical direction, and $\pm 2\mu\text{rad}$ in rotational directions. The natural frequency of the stage in the vertical direction was 151.7Hz . The size of the stage was $150 \times 150 \times 30\text{ mm}^3$. The stage consisted of four monolithic actuation units. Double-bridge type amplifier was used as a FAM, as shown in Figure 2.11 (b), which generates upward direction motion when the PZT actuator expands. This stage had a compact size but low natural frequency. Moreover, the number of the actuating units was increased, which in turn will increase the cost of the stage.

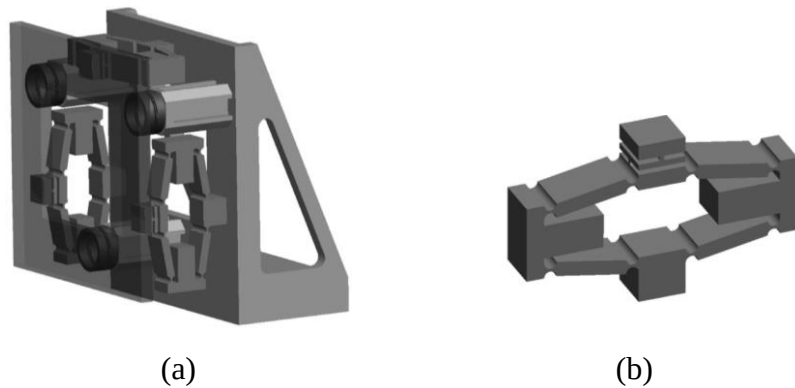


Figure 2.10 Conceptual design of 3-DOF vertical stage [56]: (a) Configuration, and (b) Actuation unit.

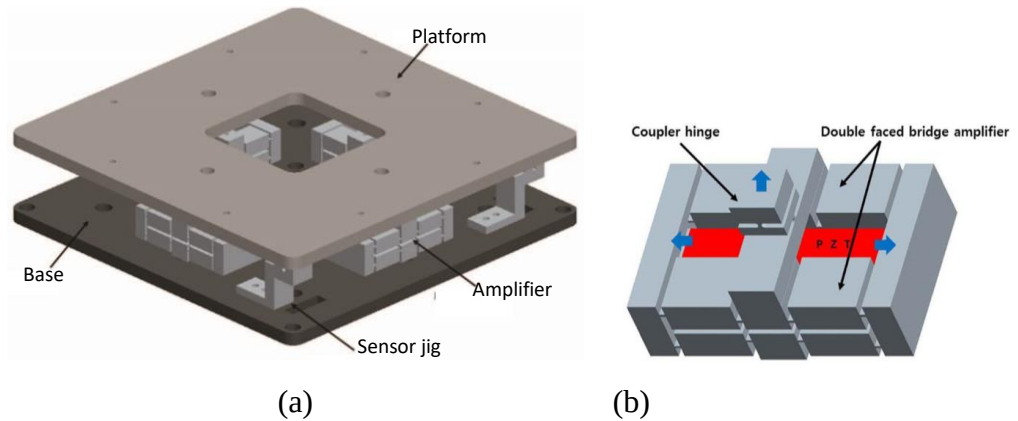


Figure 2.11 Schematic representation of 3-DOF double-bridge type amplifier stage [7]: (a) Configuration, and (b) Actuation unit.

A spatial hybrid motion mechanism with 3-DOFs was developed by Ouyang [57], as shown in Figure 2.12. It integrated two types of motion generations; a micro motion driven by direct current (DC) servomotors for large workspace and a micro motion driven by PZT actuators for high precision. The stage was

designed to achieve a natural frequency more than 400Hz . However, one of the drawbacks of the DC servomotors is that an overload can damage the motor. Also, the heat generated by the motor has a negative effect on the flexure hinges.

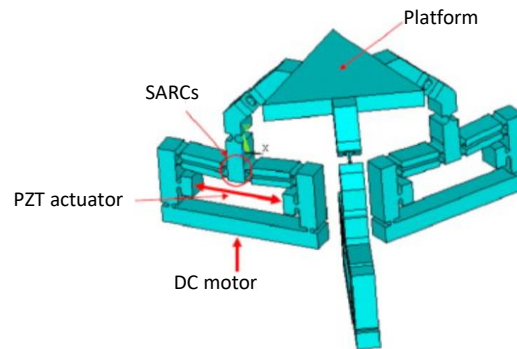


Figure 2.12 3-D model for hybrid-motion 3-DOF stage [57].

A 3-DOF parallel compliant platform was fabricated by Yun and Li [58] for micro-positioning applications with hybrid joints, as shown in Figure 2.13. Some conventional mechanical joints were adopted to increase the workspace. However, by using linear guide rails, all the disadvantages of the mechanical joints are imported to stage. In addition, the vertical positions of the PZT actuators increase the height of the stage.

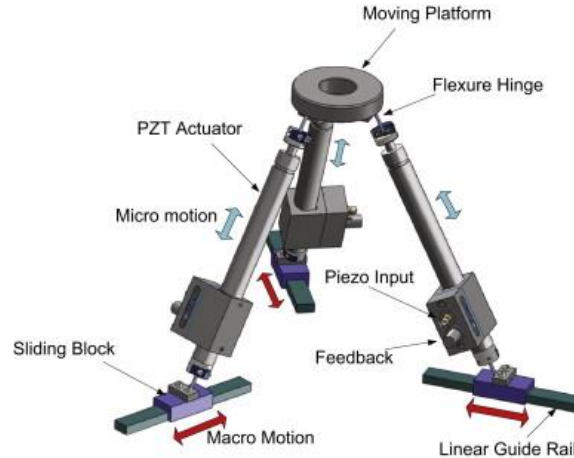


Figure 2.13 A description of hybrid joint micromanipulation [58].

A mechanical design of a novel 3-DOF parallel compliant nano-positioning stage with double amplifiers was proposed by Zhang and Xu [59]. A Computer-Aided Design (CAD) model of the stage is shown in Figure 2.14. Six MARCs were used to isolate the motions of the FAMs. The FAMs was using the principle of bridge-type amplifiers and lever amplifiers. The purpose of using two types of amplifiers was to increase the travel range. Nevertheless, utilizing bridge-type

and lever amplifiers, and six MARCs resulted in reducing the output stiffness of the stage. In addition, the stage had a low natural frequency of 41.4Hz.

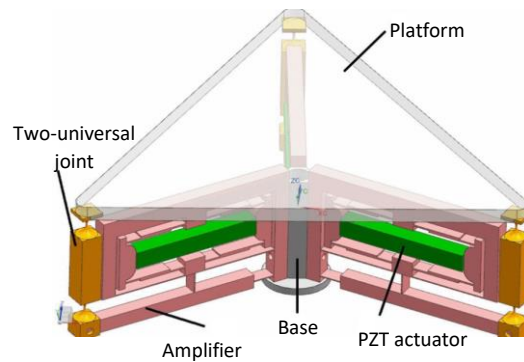


Figure 2.14 A CAD model for 3-DOF nano-positioning stage [59].

Comparing the existing 3-DOF spatial nano-positioning stages, micro-positioning table [54] shows the highest natural frequency as shown in Table 2.1. A tripod micro stage [55], vertical stage [56], and double-bridge type stage [7] have the most compact-size, although they have low natural frequencies. The design of a stage satisfying all the requirements is a challenging work, as some of the requirements are in conflict. For example, one of the methods to increase the natural frequency of the stage is to reduce the weight of the stage. This can be achieved by selecting flexure hinges and FAMs with smaller sizes. However, the small size of flexure hinges can lead to reducing the travel range.

Table 2.1 Comparison of 3-DOF positioning stages.

	Natural Frequency	Type of FAM	Main Drawbacks
Tele-micro stage [53]	500 Hz	None	Vertical Actuator position
Micro-positioning table [54]	584 Hz	None	Vertical Actuator position
Tripod micro stage [55]	375 Hz	Lever	Assembly design
Vertical stage [56]	156.8 Hz	Bridge-type	None symmetric arrangements
Double-bridge stage [7]	151.7 Hz	Modified Bridge-type	Utilization of four PZT actuators
Hybrid-motion stage [57]	400 Hz	Modified Bridge-type	DC motor utilization & long kinematic chains
Hybrid-joint stage[58]	<i>Not reported</i>	None	Vertical Actuator position
Double-amplifier stage [59]	41.4 Hz	Bridge-type & Lever	Integration of two amplifiers in vertical position

From Table 2.1, the FAMs were adopted where the increasing of the travel range was the aim of the design. Two types of FAMs were used including lever mechanism and Bridge-type mechanism. The SRAM is another type of FAMs that can provide vertical motion. It was used to develop planar micro-positioning stages [60, 61], as shown in Figure 2.15 and 2.16, or compliant grippers [62], as shown in Figure 2.17.

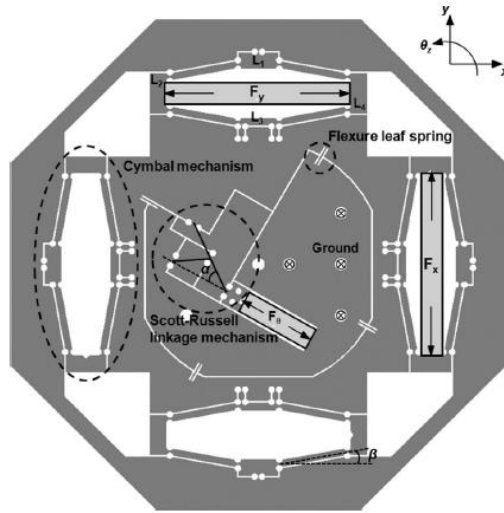


Figure 2.15 An in-plane micro-positioning stage [60].

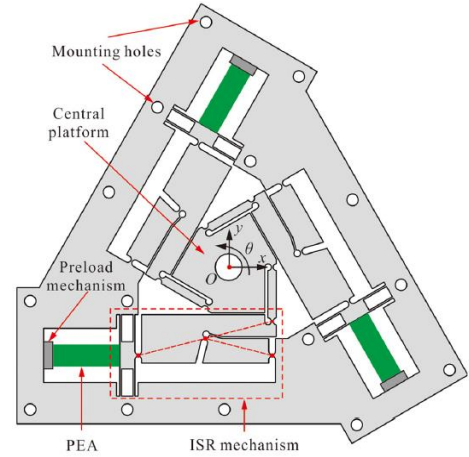


Figure 2.16 A 3-DOF planar monolithic stage [61].

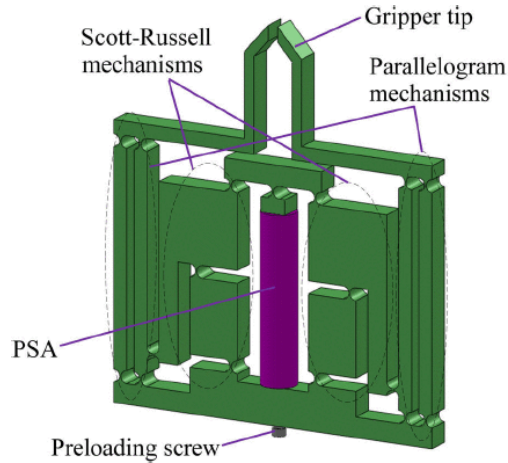


Figure 2.17 A compliant gripper mechanism [62].

Table 2.2 shows a comparison between the three types of FAMs. Considering the output stiffness being the most important factor, it is found that the Lever mechanism and the SRAM have higher output stiffness [63]. Then, setting the compact size as evaluation factor, the SRAM is selected as the amplifier in this research. However, the SRAM has a smaller amplification ratio.

Selecting a PZT actuator with large travel range is always a problem because it leads to increase the length of the PZT actuator. This results in increasing the height of the stage or increasing the size of the Lever mechanism and Bridge-type mechanism. Thus, another FAM is required allowing to integrate a long PZT actuator without affecting the size of the FAM. This can be done by integrating the PZT actuator out-side the FAM. The SR mechanism allows this kind of integration and is selected as the actuating unit of the proposed stage.

Table 2.2 *Comparison of flexure amplification mechanisms.*

Type of FAM	Advantages	Disadvantages
Lever mechanism	▪ High output stiffness	▪ Large size, applicable for planar application. ▪ Unavoidable parasitic rotations at the input and output points [61]
Bridge-type mechanism	▪ Large displacement, amplification ratio.	▪ Low output and lateral stiffness, [61] ▪ FAM size is governed by the actuator size.
Scott-Russell mechanism	▪ High output stiffness, ▪ FAM size is not governed by the actuator size.	▪ Small displacement amplification ratio.

Considering all the issues mentioned above, the following three main objectives can be extracted:

➤ High natural frequency

This can be achieved by reducing the mass of the whole stage components, which include the moving platform, the FAMs, and the flexure hinges.

➤ Large travel range

A suitable FAM and a PZT actuator with large stroke can be selected.

➤ Compact-size

This can be achieved by reducing the stage height. The solutions to achieve the first and second objectives are also resulting in a compact-sized stage.

There are some other sub-objectives that can be highlighted here, such as:

- High thermal stability can be solved by arranging the FAMs in symmetric configuration.
- High structural stability can be achieved by isolating each FAM from any motion generated in the other two FAMs.

- Simple mechanical arrangement, low-cost and low assembly errors can be achieved by selecting a monolithic FAM.

The following subsection is presenting a proposed structural design while addressing the previous solutions.

2.3.2 Proposed Mechanism Design

The conceptual design of a 3-DOF spatial parallel compliance-based nano-positioning stage is proposed while addressing some of the objectives mentioned above. The term *OHP stage* will be used throughout the work to denote the proposed stage. The design of the OHP stage is schematically shown in Figure 2.18 (a).

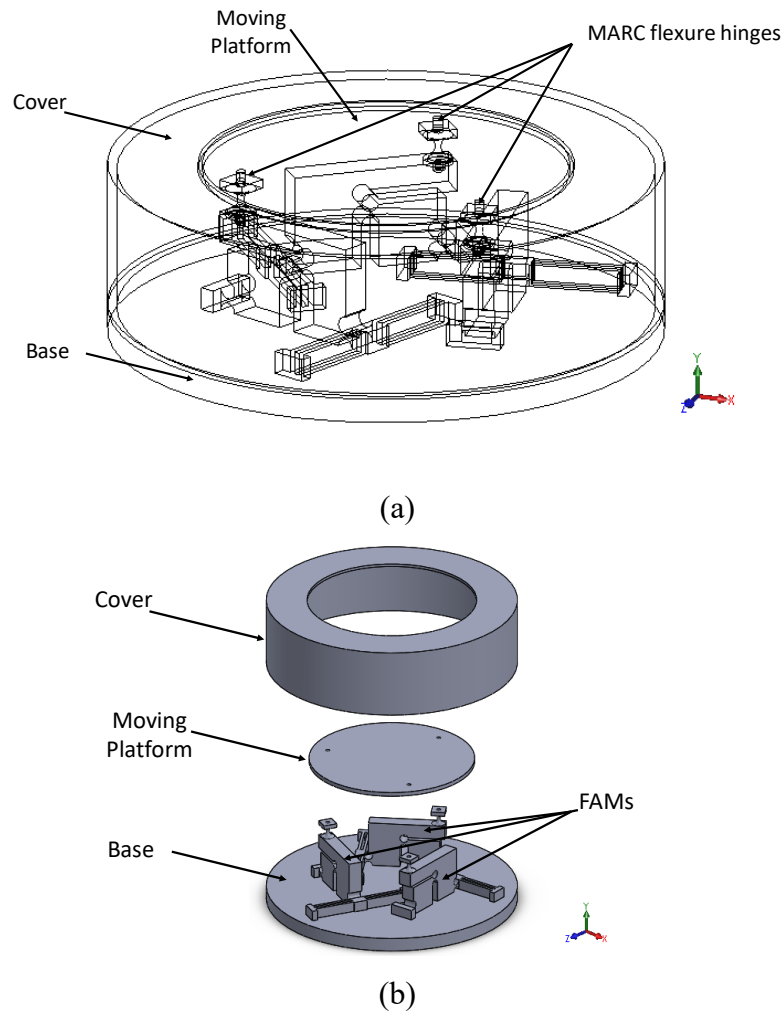


Figure 2.18 A CAD model of OHP stage: (a) Wireframe view and (b) Exploded view.

The OHP stage consists of a mobile platform connected to a fixed base by three identical parallel kinematic chains (FAMs), as presented in Figure 2.18 (b). It is

fully parallel because the number of chains is equal to the number of DOF of the moving platform and each chain having a single PZT actuator. The PZT actuator is integrated directly to the FAM.

Each FAM is fixed at the base and connected to the moving platform via a MARC, as presented in Figure 2.19. The function of the MARC is to isolate the FAM from any motions generated in the other FAMs and allow the platform to move freely in different directions. Accordingly, the PZT actuator is protected from any potential damage due to the lateral forces. As only one PZT actuator generates one output motion to move the mobile platform, the OHP stage is considered as a totally decoupled stage [64].

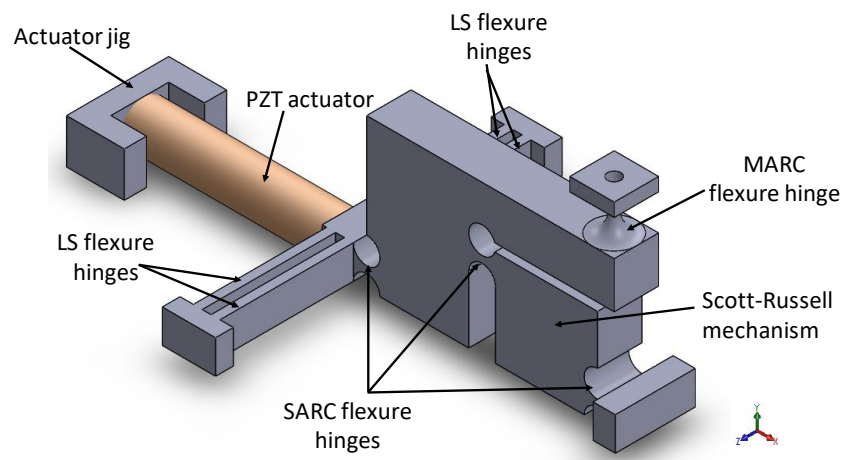


Figure 2.19 A CAD model of flexure-amplifying mechanism (FAM).

The three identical FAMs are in symmetrical configuration to ensure high thermal and structural stability. The symmetrical configuration reduces the mechanical vibration by creating a dynamic balanced environment. Accordingly, the heat imposed by the excessive stresses is reduced. A compliant SRAM is integrated to achieve three performance objectives of the FAM as follows:

- The first objective is to achieve a straight path for the output point of the stage. SRAM is capable to produce an exact straight line using the concept of the slider-crank mechanisms to reach the maximum mechanical displacement gain and the minimum lateral displacement.
- The second one is to magnify the travel range of the PZT actuator to the desired range.
- The third objective is to have a compact stage by placing the PZT actuator horizontally. SRAMs have the capability of changing the

direction of the input translation motion by 90° . This allows selecting any length of PZT actuator to increase the travel range.

Two types of flexure hinges are used in the FAM: (a) three identical SARC flexure hinges, and (b) four identical LS flexure hinges connected in parallel to ensure linear motion for the actuator, as illustrated in Figure 2.19.

The stage can provide three motions: two rotations (one or two actuators are active) and one translation (three actuators are active), as shown in Figure 2.20. The three motions of the stage are achieved by the application of an arrangement of independent force components acting through the three flexure-guided-amplified PZT actuators.

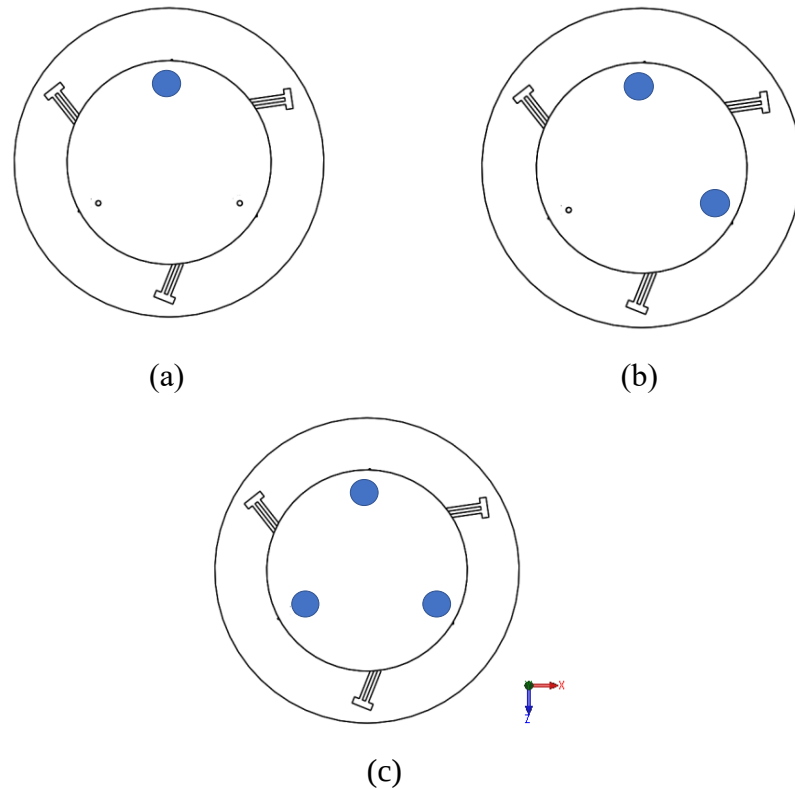


Figure 2.20 Displacements of the OHP stage: (a) One active actuator, (b) Two active actuators, and (c) Three active actuators.

2.4 Analytical Modelling Methods

The review of the existing analytical modelling methods covers the modelling of the flexure hinge and the modelling of the compliant mechanism. It is followed by an assessment of these modelling methods. The mechanism modelling includes the existing static and dynamic modelling methods.

2.4.1 Flexure Hinge Modelling

2.4.1.1 Existing Methods

As the compliant mechanisms gain the mobility from the deformations of the flexure hinges, the compliance/stiffness of the flexure hinges is considered as a fundamental factor in compliant mechanism design.

When applying an external force at a compliant mechanism which consists of several types and numbers of flexure hinges, each flexure hinge will be subjected to different types of internal loadings. The flexure hinge is held in equilibrium by three forces (an axial force, F'_Z , and two shearing forces, F'_X , and, F'_Y), and three moments (two bending moments, M'_X , and, M'_Y , and a torsional moment, M'_Z), as shown in Figure 2.21. These loadings will produce elongations associated with the internal loadings. In this analysis of the compliant mechanisms, the expression displacements will be used instead of elongations. The relationship between internal loading $\bar{P}$ and their associated displacements $\bar{D}$ can be expressed as follows

$$\bar{P} = [K]\bar{D} \quad (2.1)$$

$$\bar{D} = [C]\bar{P} \quad (2.2)$$

where K and C are the stiffness matrix and the compliance matrix, respectively.

They are inverse matrices of each other ($[K] = [C]^{-1}$).

The elements of the stiffness matrix and compliance matrix are called the stiffness factors and compliance factors, respectively.

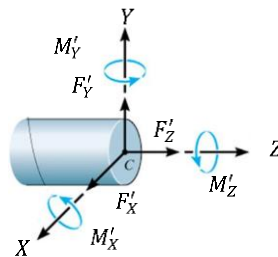


Figure 2.21 Internal loadings in a 3-D element.

The compliance models will be built first, then the inverse relationship of the compliance and stiffness will be used to obtain the stiffness models. Therefore, here only the compliance factors of the flexure hinges will be discussed.

In the 3-D analysis, any flexure hinge can be modelled as a 6-DOF spring element. From Equation (2.2), the compliance of a flexure hinge can be defined as the ratio of a resulting displacement and a load. Therefore, the compliance factors are simply obtained from the following equation

$$C_{D_i-F_i} = \frac{d_i}{F'_i} \quad (2.3)$$

or

$$C_{\theta_i-M_i} = \frac{\theta_i}{M'_i} \quad (2.4)$$

where i refers to the X-axis, Y-axis, or Z-axis.

Several approaches were adopted to derive the compliance equations of flexure hinges. The compliances of SARC flexure hinges have been studied extensively due to their large applications.

The first research work on flexure hinges using compliance-based method was presented in 1965 by Paros and Weisbord [65]. They introduced design equations and approximations for calculating the compliances of the single-axis and multiple-axis flexure hinges. The equations were achieved by the integration of linear differential equations of a rectangular cross-section beam.

A similar approach based on Paros and Weisbord, was followed by Smith [66]. This work proposed the closed-form compliance equations for elliptical flexure hinges and compared their performance to that of right circular flexure hinges. The mathematical formulae of LS, SARC and elliptic flexure hinges, and other types of flexure hinges, such as cartwheel, and circular toroidal, were formulated by Smith [67].

Adopting the Castigliano's displacement theorem, closed-form compliance equations of commonly used types of flexure hinges, such as LS flexure hinges, SARC flexure hinges, and MARC flexure hinges, were derived by Lobontiu [49]. The research also presented the equations of several new flexures, such as corner-filletted, parabolic, hyperbolic and elliptical flexure hinges for 2 and 3-D applications.

In addition, the in-plane and out-of-plane compliance factors of the stiffness matrix of several flexure hinge configurations were also provided by Lobontiu

[49]. These factors allow analysing the capacity and precision of rotation, and the stresses under fatigue conditions. Factors were derived based on the strain energy stored in the flexure hinge due to its elastic deformations. Therefore, in 3-DOF applications, the flexure hinge can be modelled as 6-DOF member.

In a comparison research presented by Yong [68], the accuracies of the compliance equations for the SARC flexure hinge, which were derived by Paros [65], Smith [66], Lobontiu [49], and others, were reviewed. It was found that the rotational compliance of Smith [66] was more accurate for thickness to radius ratio between 0.2 and 0.65. Also, the evaluation work showed that the translational compliance of Lobontiu [49] was more accurate for thickness to radius ratio between 0.05 and 0.1.

In addition, an approximation of the translational compliance at the shear axis of the single-axis right circular flexure hinge, which formulated by Paros [65], were corrected by Yoshimura [69]. This correction was adopted by Koseki [70]. In this work, another translational compliance factor at the axial axis of the SARC flexure hinges was modified.

2.4.1.2 Assessment of Existing Methods

From the previous review of the flexure hinge modelling, it is found that two main types of compliance equations of flexure hinges were derived. The first compliance equations of flexure hinges were derived by Paros and Weisbord [65]. Then, other researchers corrected or modified these equations for new flexure hinges by adopting the same equations. The second type of compliance equations was formulated based on Castigliano's displacement theorem (CDT) by Lobontiu [49].

The compliance equations of LS flexure hinges were well defined and provided in several research works, such as [48, 70]. The equations, which will be used in this research, are presented in Table 2.3. The most accurate compliance equations of SARC flexure hinges will be used. Table 2.3 lists the SARC compliance equations while considering the corrections provided by Yoshimura [69] and the accurate equations derived by Yong *et al.* [68].

Table 2.3 *LS and SARC compliance equations.*

Compliance Factors	LS	SARC
$c_{d_X-F_X}$	$\frac{4l_{LS}^3}{Eh_{LS}t_{LS}^3} + \frac{\alpha l_{LS}}{Gh_{LS}t_{LS}}$	$\frac{9\pi r_{SARC}^{\frac{5}{2}}}{2Ew_{SARC}t_{SARC}^{\frac{5}{2}}} + \frac{3\pi r_{SARC}^{\frac{3}{2}}}{2Ew_{SARC}t_{SARC}^{\frac{3}{2}}}$
$c_{d_X-M_Y}$	$\frac{6l_{LS}^2}{Eh_{LS}t_{LS}^3}$	$\frac{9\pi r_{SARC}^{\frac{3}{2}}}{2Ew_{SARC}t_{SARC}^{\frac{5}{2}}}$
$c_{d_Y-F_Y}$	$\frac{4l_{LS}^3}{Eh_{LS}^3t_{LS}} + \frac{\alpha l_{LS}}{Gh_{LS}t_{LS}}$	$\frac{12\pi r_{SARC}^2}{Ew_{SARC}^3} \left(\left(\frac{r_{SARC}}{t_{SARC}} \right)^{\frac{1}{2}} - \frac{1}{4} \right)$
$c_{d_Y-M_X}$	$-\frac{6l_{LS}^2}{Eh_{LS}^3t_{LS}}$	$-\frac{12r}{Ew_{SARC}^3} \left(\pi \left(\frac{r_{SARC}}{t_{SARC}} \right)^{\frac{1}{2}} - \frac{2+\pi}{2} \right)$
$c_{d_Z-F_Z}$	$\frac{l_{LS}}{Eh_{LS}t_{LS}}$	$\frac{1}{Ew_{SARC}} \left(\pi \left(\frac{r_{SARC}}{t_{SARC}} \right)^{\frac{1}{2}} - \frac{\pi}{2} \right)$
$c_{\theta_X-F_Y}$	$-\frac{6l_{LS}^2}{Eh_{LS}^3t_{LS}}$	$-\frac{12r_{SARC}}{Ew_{SARC}^3} \left(\pi \left(\frac{r_{SARC}}{t_{SARC}} \right)^{\frac{1}{2}} - \frac{2+\pi}{2} \right)$
$c_{\theta_X-M_X}$	$\frac{12l_{LS}}{Eh_{LS}^3t_{LS}}$	$\frac{12}{Ew_{SARC}^3} \left(\pi \left(\frac{r_{SARC}}{t_{SARC}} \right)^{\frac{1}{2}} - \frac{2+\pi}{2} \right)$
$c_{\theta_Y-F_X}$	$\frac{6l_{LS}^2}{Eh_{LS}t_{LS}^3}$	$\frac{9\pi r_{SARC}^{\frac{3}{2}}}{2Ew_{SARC}t_{SARC}^{\frac{5}{2}}}$
$c_{\theta_Y-M_Y}$	$\frac{12l_{LS}}{Eh_{LS}t_{LS}^3}$	$\frac{9\pi r_{SARC}^{\frac{1}{2}}}{2Ew_{SARC}t_{SARC}^{\frac{5}{2}}}$
$c_{\theta_Z-M_Z}$	$\frac{l_{LS}}{G\beta_{LS}h_{LS}t_{LS}^3}$	$\frac{9\pi r_{SARC}^{\frac{1}{2}}}{4Gw_{SARC}t_{SARC}^{\frac{5}{2}}}$

where $c_{d_X-F_X}$, $c_{d_X-M_Y}$, $c_{d_Y-F_Y}$, $c_{d_Y-M_X}$, $c_{d_Z-F_Z}$, $c_{\theta_X-F_Y}$, $c_{\theta_X-M_X}$, $c_{\theta_Y-F_X}$, $c_{\theta_Y-M_Y}$, and $c_{\theta_Z-M_Z}$ are the compliance factors related to the internal loadings, which can be explained more in the following displacement equation:

$$\begin{bmatrix} d_X \\ d_Y \\ d_Z \\ \theta_X \\ \theta_Y \\ \theta_Z \end{bmatrix} = \begin{bmatrix} c_{d_X-F_X} & 0 & 0 & 0 & c_{d_X-M_Y} & 0 \\ 0 & c_{d_Y-F_Y} & 0 & c_{d_Y-M_X} & 0 & 0 \\ 0 & 0 & c_{d_Z-F_Z} & 0 & 0 & 0 \\ 0 & c_{\theta_X-F_Y} & 0 & c_{\theta_X-M_X} & 0 & 0 \\ c_{\theta_Y-F_X} & 0 & 0 & 0 & c_{\theta_Y-M_Y} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{\theta_Z-M_Z} \end{bmatrix} \begin{bmatrix} F'_X \\ F'_Y \\ F'_Z \\ M'_X \\ M'_Y \\ M'_Z \end{bmatrix} \quad (2.5)$$

where d_X , d_Y , d_Z , θ_X , θ_Y , and θ_Z are the displacements associated with the internal loadings F'_X and F'_Y , F'_Z , M'_X , M'_Y , and M'_Z , respectively. E and G are Modulus of longitudinal elasticity and Modulus of transverse elasticity, respectively. α is a shape factor for the cross section used in the shear equation (for rectangular cross-section $\alpha=6/5$). β_{LS} is a geometrical factor determined by h_{LS}/t_{LS} , where, $h_{LS} > t_{LS}$. β_{LS} can be found from Table.2.4.

Table 2.4 *Geometrical factor.*

h_{LS}/t_{LS}	1.0	1.5	2.0	2.5	3.0	4.0	5.0	6.0	10.0	∞
β_{LS}	0.141	0.196	0.229	0.249	0.263	0.281	0.291	0.299	0.312	0.333

The compliance factors of the MARC flexure hinges derived by Lobontiu [3] are:

$$c_{d_Z-F_Z} = \frac{16r_{MARC}}{\pi E t_{MARC}^2} \left[\frac{2r_{MARC} + t_{MARC}}{4r_{MARC} + t_{MARC}} + 2r \sqrt{\frac{t_{MARC}}{(4r_{MARC} + t_{MARC})^3}} \arctan \sqrt{1 + \frac{4r_{MARC}}{t_{MARC}}} \right] \quad (2.6)$$

$$c_{d_X-F_X} = \left[\frac{256r_{MARC}^3}{3\pi E t_{MARC}^4 (2r_{MARC} + t_{MARC})^2 \sqrt{(4r_{MARC} + t_{MARC})^7}} \right] * \left[\sqrt{4r_{MARC} + t_{MARC}} (1664r_{MARC}^5 + 3340r_{MARC}^4 t_{MARC} + 2620r_{MARC}^3 t_{MARC}^2 + 987r_{MARC}^2 t_{MARC}^3 + 188r_{MARC} t_{MARC}^4 + 16t_{MARC}^5) + 3r(2r_{MARC} + t_{MARC})^2 \sqrt{t_{MARC}} (100r_{MARC}^2 + 84r_{MARC} t_{MARC} + 21t_{MARC}^2) \arctan \sqrt{1 + \frac{4r_{MARC}}{t_{MARC}}} \right] \quad (2.7)$$

$$c_{d_X-M_Y} = \frac{256 r_{MARC}^2}{3\pi E t_{MARC}^4} \left[3 + \frac{t_{MARC} (120r_{MARC}^4 + 176r_{MARC}^3 t_{MARC} + 92r_{MARC}^2 + 24r_{MARC} t_{MARC}^3 + 3t_{MARC}^4)}{(2r_{MARC} + t_{MARC})^2 (4r_{MARC} + t_{MARC})^3} + 24r_{MARC} (5r_{MARC}^2 + 4r_{MARC} t_{MARC} + t_{MARC}^2) \sqrt{\frac{t_{MARC}}{(4r_{MARC} + t_{MARC})^7}} \arctan \sqrt{1 + \frac{4r_{MARC}}{t_{MARC}}} \right] \quad (2.8)$$

$$c_{\theta_Z-M_Z} = \frac{E}{4Gr_{MARC}} c_{d_{sh}-M_b} \quad (2.9)$$

$$c_{\theta_X-M_X} = \frac{2G}{E} c_{\theta_a-M_a} \quad (2.10)$$

Due to the circular symmetrical structure of the cross-section, the following compliance factors are identical

$$c_{d_Y-F_Y} = c_{d_X-F_X} \quad (2.11)$$

$$c_{d_b-M_X} = -c_{d_X-M_b} = c_{\theta_X-F_Y} = -c_{\theta_Y-F_X} \quad (2.12)$$

$$c_{\theta_Y-M_Y} = c_{\theta_Y-M_Y} \quad (2.13)$$

According to the assessment carried out by Yoshimura [69], the compliance equations of the LS and SARC flexure hinges based on Paros and Weisbord's formulations will be used. The compliance equations of the MARC flexure hinge, which are only derived by Lobontiu [49], are adopted in the static analysis of the proposed stage.

2.4.2 Modelling Methods of Compliant Mechanisms

The review consists of the existing static and dynamic modelling methods followed by an evaluation to decide if these methods are applicable for analysing the proposed stage.

2.4.2.1 Static Modelling

Existing Static Modelling Methods

The static models are built to assess the stiffness of the compliant mechanisms. A stiffness modelling is defined as a modelling of a stiffness for a compliant structure under static conditions without taking actuators into consideration. The stiffness models help to examine the static behaviour of the compliant mechanisms, in terms of the output displacement, the load-carrying capacity, and the driving capacity of the PZT actuators.

The research efforts on obtaining the stiffness equations of the flexure hinges have encouraged further study works on modelling the stiffness of the compliant-based precision positioning manipulations with 2-DOF, 3-DOF, and 6-DOF. Most of these applications were utilizing LS flexure hinges or/and SARC flexure hinges.

Several methods have been used to develop the stiffness model of compliant mechanisms, such as finite-element-analysis (FEA) based technique [71-74], analytical modelling approach using the concept of the Pseudo-Rigid-Body-Model method (PRBM) [75], and the stiffness matrix method (SMM) [70].

The FEA modelling is considered the most accurate computational method to investigate the stiffness of compliant-based mechanisms. The stiffness model is achieved by modelling links and flexure hinges with their true dimensions and shapes [76].

However, this method does not establish the analytical relationship between stiffness and dimensions and the free shape of the mechanism. Consequently, the FEM modelling is appropriate for the design only after getting the specific dimensions and the free shape of the mechanism to verify the performance before fabrication and assembly [77].

The two well-defined methods introduced to analyse the stiffness of compliant mechanisms are PRBM and the SMM.

The PRBM approach has been defined and then developed by [5] and [78]. Due to its simplicity, extensive research works implemented the PRBM approach in their applications, such as [79-81]. It models the flexure hinges as revolute joints with torsion springs and the links connecting the flexure hinges as rigid bodies. The PRBM approach is widely used in stiffness modelling especially for IHP stages.

An early work of obtaining the stiffness model of an IHP compliant stage by using the SMM was established by Ryu [82]. He used the stiffness matrix to model the micro-motion of a 3-DOF spatial micro-positioning stage and to achieve an optimal design. The stage employed SARC flexure hinges using Paros and Weisbord's stiffness equations [65].

Koseki *et al.* [70] employed SMM for a kinematic analysis of a translational 3-DOF micro parallel mechanism. They listed in their work all the compliance factors of the cylinder, LS, and SARC flexure hinges. The compliance factors of the SARC flexure hinge were based on Paros and Weisbord's equations [65] and the corrections made by Koseki *et al* [70] and Yoshimura [69].

Then, SMM of multi-body dynamics of a compliant mechanism suggested by Ryu [82] and the compliance factors of the SARC flexure hinge offered by Koseki *et al.* [70] were adopted in several other research works to develop the stiffness model, such as reported in [7, 56, 64, 77, 83-85].

Based on the SMM, a stiffness analysis of a 2-DOF micro-manipulator was presented in [64]. The system displacements are governed by two paralleled mechanisms. Each mechanism composed of compound parallelograms of SARC flexure hinges. The stiffness modelling of the system is based on Paros and Weisbord [65], while considering the suggestions proposed by Yong [68]. An optimal design of a 3-DOF in-plane nano-positioning stage was carried based on maximizing the system bandwidth by Lee [83]. The objective function of the optimization process was developed with the assistance of 6×6 matrices of compliance leaf springs and SARC flexure hinges founded by [70].

Pham [77] developed a stiffness model of a 3-DOF out-of-plane precision manipulator using a double linear spring constructed of SARC flexure hinges. The stiffness of the linear spring was derived in another research work by Qu [86]. Also, Kim [56] applied the method in a 3-DOF out-of-plane precision

positioner. Nevertheless, he used the 6×6 compliance matrix of the SARC flexure hinge created by Koseki [70]. compliance matrices of 6×6 for LS and SARC flexure hinges was utilized in analysing a 3-DOF OHP nanopositioning stage [7]. This matrix's elements were formulated according to Koseki's research work [70].

Recently, this method was applied in two different works by Lobontiu [84, 85]. In [84], Lobontiu generalized this method on a quasi-static modelling of a serial in-plane compliant mechanism employing a 3×3 compliance matrix of single-axis right circular corner-filet flexure hinges. In an in-plane compliant gripper with serial and parallel combined structure, Lobontiu [85] developed an analytical model to predict the performance of the right circular corner-filet flexure hinge gripper.

In what follows, the selection of the most accurate method is presented after presenting the advantages and disadvantages of the two methods.

Evaluation of Static Modelling Methods

PRBM approach can be considered as the most currently used approach for modelling the compliant mechanisms due to its simplicity. This method assumes that flexure hinges behave as revolute joints with torsional springs in terms of its flexible behaviour and the thicker sections of the mechanism act as rigid links [78]. The model is built under the assumption that the deflection at the flexible part is small compared to the length of the rigid part of the flexure hinge.

Considering a 2-D analysis, if an external force is applied on a compliant mechanism, which consists of several flexure hinges, the flexure hinge will be subjected to three types of forces (Axial, and shear forces, and bending moment). Using the PRBM method to model this flexure hinge, a corresponding torsional spring model is obtained as shown in Figure 2.22. Comparing the two models, there are two forces not considered in the PRBM [49]. The method models only rotational stiffness, while ignoring the stiffness in the axial and the shear direction.

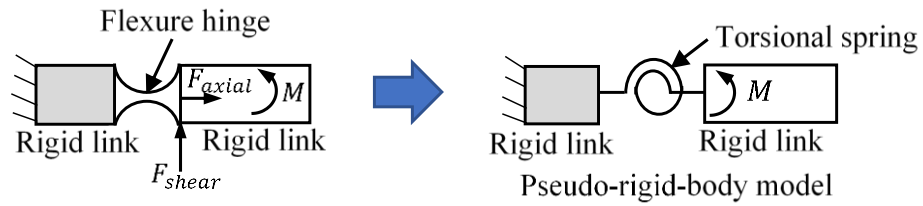


Figure 2.22 PRBM of 2-D flexure hinge.

The SMM approach takes a different method for modelling the flexure hinge. The flexure hinge is modelled as a 3-DOF member or a 6-DOF member in 2-DOF applications or 3-DOF applications, respectively. Then, the stiffness matrix of each flexure hinge is expressed in the required point by using a transformation matrix. Direct summation of the stiffness matrices of all flexure hinges to obtain the total stiffness is not accurate, because the flexure hinges have different locations, or they may have been amplified. The summation depends on the way of connections between the flexure hinges.

The advantage of this method is that it can manage all the 3×3 or 6×6 deformations of the flexure hinges of the structure. Also, it requires fewer nodes of the matrix compared with the FEA which results in less computer power [2]. The difficulty of this method occurs only when trying to obtain a value of an input or output stiffness of a compliant mechanism while having a big number of parallel connections between flexure hinges in the mechanism. The existence of several parallel connections ends the modelling with very complicated inverse matrices that can only be solved by using commercial software such as MATLAB or Mathematica.

As the accuracy is the most important evaluation factor, the SMM will be used for building the stiffness models of the proposed stage.

2.4.2.2 Dynamic Modelling

A review of the existing modelling methods used for dynamic analysis followed by an evaluation is presented in the following section.

Existing Dynamic Modelling Methods

The Lagrangian method is the almost-exclusive tool that is currently utilized. A wide range of research works has adopted Lagrange's equation to model the dynamic response. In multi-DOF mechanisms with distributed masses, it is very

difficult to use Newtonian mechanics. The Lagrange's equation, which is derived from Hamilton's principle, states that “ the variation of the kinetic energy and potential energy plus the line integral of the virtual work done by non-conservative forces during any time interval must be equal to zero” [87]. It has the advantage of requiring only the kinetic and potential energies of the system to be computed. Then the mass matrix and the stiffness matrix can be extracted from the Lagrange's equation to calculate the natural frequency. Complete details of this approach can be found in the next chapter.

Several methods have been derived for the dynamic analysis while adopting the Lagrangian approach. An early approach presented by Yang and Schweizer [88] is used to model the dynamic behaviour of a vertical motion micro-positioning stage. The single-axis flexure hinges were modelled as 1-DOF elements. The same approach has been formulated by Smith [67]. Thus, only the direct-loading stiffness is considered. This method has been reported in some literature, for example [89-93]. Moreover, this method, which will be called in this research the 1-DOFM, considers the flexure hinge only in its motion direction.

The PRBM method has been used to convert the compliant mechanism into a rigid-body mechanism with lumped compliance. It is important to mention here that the force in the flexure hinge has one component when calculating the potential energy and kinetic energy of the mechanism. Then, Lagrange's equation is used to obtain the dynamic model. This method has been used by several researchers, such as presented in [43, 94-96]. It does not consider the other loading components.

For planar compliant mechanisms, Smith [67] developed a general model for single-axis flexure hinges. This time the flexure hinge was modelled as a 3-DOF member. The stiffness matrix of a single-axis flexure hinge was expressed as follows

$$K = \begin{bmatrix} k_{d_x-F_x} & 0 & 0 \\ 0 & k_{d_y-F_y} & 0 \\ 0 & 0 & k_{\theta_z-M_z} \end{bmatrix} \quad (2.14)$$

where $K_{d_x-F_x}$, $K_{\theta_y-M_y}$, and $K_{d_z-F_z}$ are the stiffness relating the internal loadings F_x , F_z , and M_y , and the subsequent displacements d_x , d_z , and θ_y , in the X , Z , and Y axes respectively.

On the other hand, Lobontiu [49] formulated the stiffness matrix of a single-axis flexure hinge, which is modelled as a 3-DOF element, as

$$K = \begin{bmatrix} k_{d_x-F_x} & 0 & 0 \\ 0 & k_{d_y-F_y} & k_{d_y-M_z} \\ 0 & k_{d_y-M_z} & k_{\theta_z-M_z} \end{bmatrix} \quad (2.15)$$

where $k_{d_y-M_z}$ is the cross-bending stiffness, which relates the deflection d_y to the bending moment M_z at the tip of the flexure hinge.

Comparing the stiffness matrix derived by Lobontiu [49] and Smith [67], the equation formulated by Smith [67] ignores the cross-bending stiffness $k_{d_y-M_z}$.

There are other methods derived for special applications. In [97], Timoshenko beam theory was adopted to obtain the kinetic energy and potential energy. A general dynamic model of a class of parallel platforms based on Kane's method is presented in [98].

Evaluation of Dynamic Modelling Methods

Lagrange's equation contains two energy terms: the kinetic energy and the potential energy. The accuracy of modelling the flexure hinge stiffness has a direct influence on the potential energy. Comparing the four approaches, it can be concluded that the modelling method based on the energy method derived by Lobontiu [49] is the most accurate approach. However, in the existing literature, this approach has not been applied to spatial compliant mechanisms. A modelling method for calculating the potential energy considering all deformation directions of the flexure hinge is needed.

Consequently, another modelling method for computing the potential energy is proposed. The method is applicable for both planar or spatial mechanisms. Energy methods, such as work-energy method, and virtual work method, has been widely used for calculating the deflections of beams, truss and frames. The work-energy method is one of the methods used for displacement calculations of structures based on the concepts of the conservation of energy principle.

Here in this research work, this method will be developed to model the potential energy of compliance-based mechanisms and used in a different manner. The proposed method, which is an energy-based approach (EBA), allows considering all the deformation of the flexure hinges. The method is presented in detail in

Chapter 4.

2.5 Material Selection

There are many types of materials that can be used for compliant applications. The selection of material for a certain application depends on the material criteria. The ratio of yield strength to Young's modulus is the main material selection criteria for compliant mechanisms. The material of the flexure hinge with the largest deformation has the highest strength-modulus ratio [99]. While the strength-modulus ratio of materials is important, there are some other important criteria that should be considered such as the cost of the material and the machining process capability of the materials.

In nano-positioning applications where the predictable material properties are needed, metals such as steel [100, 101], aluminium [102-104], and titanium alloys [99, 105] are common metal used. They have also the advantage of having a better performance in the high-temperature environment compared with plastic materials. From the existent research work, it is found the aluminium alloy is most commonly used material.

Table 2.5 shows the common metal alloys for applications where predictable material properties are needed. The difference between the values of the strength-modulus ratios of the three materials is not large. So, other material criteria can be considered, such as the cost, machinability, and coefficient of thermal expansion.

Table 2.5 Comparison of commonly used materials.

Material	Aluminium (Al-7075-T6)	Steel (4140 Q&T@400)	Titanium (Ti-6Al-4V)
Yield strength, S_y (MPa)	503	238	880
Young's modulus, E (GPa)	71.7	30	113.8
S_y/E	7.0	7.9	7.7
CTE ($\mu\text{m}/\text{m}\cdot^\circ\text{C}$)	23.6 (20 -100 $^\circ\text{C}$)	12.2 (0-100 $^\circ\text{C}$)	8.8 (20-100 $^\circ\text{C}$)
Density, ρ (g/cm^3)	2.81	7.85	4.42
Thermal Conductivity ($\text{W}/\text{m} - \text{K}$)	130	42.6	7.1
Price (Kg)	\$1.6 [106]	\$1.8-2.5 [107]	\$24-40 [108]

In nano level of positioning applications, the thermal expansion has a direct effect on the dimensional changes in the stage components. The coefficients of temperature expansion (CTE) of the three common materials are presented in Table 2.5. Comparing the values of the CTEs, it is clear that careful consideration must be taken to maintain high positioning accuracy. While aluminium alloys exhibit a poor CTE, they have a high thermal conductivity, which aids the system to reaches thermal equilibrium quickly. Measuring the temperature to provide a correction, which is one of the control methods, can be used to handle thermal expansion [109].

If increasing the affordability of nano-positioning applications is required, the cost of the material becomes another important factor. Comparing the cost of the three material at Table 2.5, Al-7075 has the lowest cost.

The monolithic compliant mechanisms are fabricated using wire-electric-discharge machining (WEDM) process owning the ability to fabricate complicated shapes. The machining cost is not significant because most of the compliant mechanisms are in small size [110]. Therefore, the machinability of the material is another selecting criteria for material selection, which depends on the material properties. The high machinability of Al-7075 represents a significant advance over the other two alloys. Al-7075 offers additional advantages of high stiffness and low density [104]. In conclusion, the material of the stage is selected as Al-7075.

2.6 Chapter Summary

This chapter can be summarised in the following points

- A brief review of the three types of nano-positioning systems was provided. The literature showed that the characteristics of compliance-based nano-positioning systems can overcome the limitations of the other systems.
- The main types of actuation devices used to drive compliance-based nano-positioning systems were studied. The PZT actuator was selected because it offers a fine resolution, quick response, and large force generation.

- A review of the existing spatial compliant nano-positioning stages was presented. The literature focused on the mechanism structure highlighting the limitations of the existing spatial stages.
- Based on the limitations found in the literature, the research objectives were established. The structure of the proposed stage (OHP stage) was presented with the solutions for these limitations.
- An assessment of the modelling method used to formulate the compliance factors of the flexure hinges was provided. The most accurate equations of the compliance factors for the LS and SARC flexure hinges were selected based on the previous research efforts.
- The modelling methods for compliant mechanisms were reviewed. The SMM was selected because it can manage all the (3×3) or (6×6) deformations of the flexure hinges of the structure. Moreover, it was found that all the efforts applying the SMM are directed toward building stiffness models of compliant mechanisms utilizing the 3×3 or 6×6 compliance matrices of the LS and/or SARC flexure hinges. There is no focus on modelling mechanisms utilizing MARC flexure hinge types.
- The modelling methods used in analysing the dynamic behaviour of the compliant mechanisms were studied. The Lagrangian method is the almost-exclusive tool that is currently utilized because it is based on energy terms only, and therefore it is easier to use. However, in the case of the compliant mechanism, the literature considers only the rotational displacement of the flexure hinge and ignores the other two translational displacements, which leads to inaccurate modelling of the mechanism stiffness.
- A developed modelling method (EBA) was proposed for modelling the potential energy regarding all the displacements of the flexure hinges. The proposed method allows modelling the flexure hinge as 6-DOF based on the on the concepts of the energy conservation. The method is applicable for both planar and spatial applications. The explanation, validation, and application of EBA are presented in Chapter 4.

These findings will be adopted in the analytical modelling, which will be presented in the next chapters.

Chapter 3 Kinematic and Static Modelling

The aims of and importance of this chapter are to:

- Model the kinematic quantities which are needed throughout the analytical modelling process of the stage. The modelling of the kinematic quantities includes the displacement amplification ratio, the displacements of the flexure hinges and velocities of the rigid links. The kinematic modelling also contains an evaluation of the mobility of the out-of-horizontal-plane (OHP) stage.
- Carry a static analysis by assessing the stiffness of OHP stage. The Stiffness plays an important role in the precise performance of flexure-based ultra-precision manipulation systems. The stage stiffness influences the driving capacity of the piezoelectric (PZT) actuators. In addition, it can affect the static and dynamic characteristics of the stage in terms of the output displacement, load-carrying capacity, and the natural frequency. The stiffness model is derived using the stiffness matrix method (SMM). The stiffness modelling includes the input stiffness of the OHP stage when three actuators are active. The input stiffness model will be used to determine the stage performance and validating the developed modelling method.
- Determine and evaluate the design parameters of the stage, which have direct influences on the performance. These design parameters are one of the essential requirements in the design optimization process.

This chapter has two main sections to obtain these aims. **Section 3.1** offers the kinematic modelling, which includes an analysis of the displacement amplification ratio, the displacements and velocities of the flexure hinges and the rigid links connecting these flexure hinges, and the mobility of the proposed stage. **Section 3.2** presents the static modelling, which contains an introduction for SMM and modelling of the input stiffness.

3.1 Kinematic Modelling

The proposed stage consists of three flexure amplification mechanisms (FAMs), which represent the actuating units when attached to the PZT actuators. The structure of each FAM is basically made up of a compliant Scott-Russell amplification mechanism (SRAM), as shown in Figure 3.1 (a). The main function of the SRAM is to amplify the travel range of the PZT actuator. The actuator is connected to SRAM via a block, RL_1 , from one side and to the base of the stage on the other side. The block is connected to double parallel leaf-spring flexure hinge (LS) flexure hinges to prevent the PZT actuator from bending. When the PZT actuator expands, a horizontal displacement is generated at the input point, as illustrated in Figure 3.1 (b). With the aid of the structure of SRAM, this displacement is transferred to a vertical displacement pushing up the moving platform. The moving platform has a translational displacement when the three PZT actuators are operating; whereas, it has two rotational displacements when one or two PZT actuators are active, as shown in Figure 2.20.

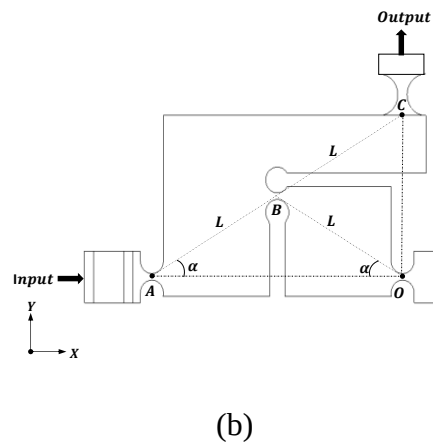
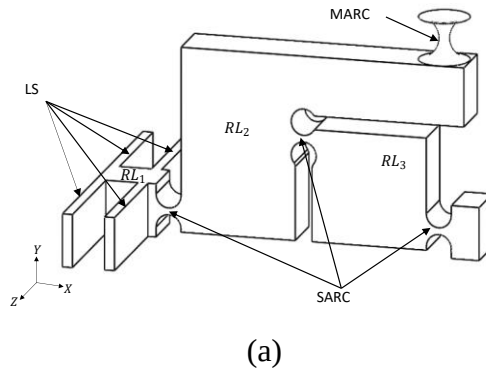


Figure 3.1 FAM: (a) CAD model and (b) Input and output points.

When the PZT actuator retracts from a large travel range, an external force supports the input point to follow the actuator and keeps the input point close to the actuator [91]. This external force is generated by the two single-axis right-circular (SARC) flexure hinges at the rigid link RL_3 , as shown in Figure 3.1 (a).

3.2.1 Displacement Amplification

The displacement amplification produced by the SRAM can be modelled by analyzing two positions of the mechanism, as depicted in Figure 3.2. The Cartesian Coordinate system is set at point O . The angle α indicated as the SRAM angle is the angle between the links OA and OB and the points A and C are the input and output points, respectively. As $AB = BO = BC = L$, where L represents the SRAM length, then the following geometric relationship can be obtained

$$y_{OC} = \sqrt{(2L)^2 - x_{AO}^2} \quad (3.1)$$

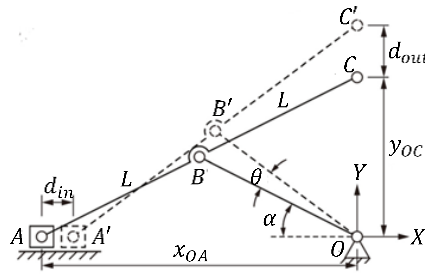


Figure 3.2 Kinematic model of Scott-Russel mechanism.

When the input point A moves along the X -axis with an input displacement of d_{in} , a vertical translation of d_{out} is generated at the output point C . From Figure 3.2, when d_{in} approaches zero, the relationship between the two displacements can be written as follows

$$\frac{d_{out}}{d_{in}} \cong \frac{dy_{OC}}{dx_{AO}} \quad (3.2)$$

By substituting (3.1) in (3.2), the relation can be expressed as

$$\frac{d(\sqrt{(2L)^2 - x_{AO}^2})}{dx_{AO}} = -\frac{x_{AO}}{\sqrt{(2L)^2 - x_{AO}^2}} = -\frac{x_{AO}}{y_{OC}} = -\cot \alpha \quad (3.3)$$

The negative sign indicates that a decrease in d_{in} will result in an increase in d_{out} .

Therefore, in small motion applications, the displacement amplification ratio, Amp , of SRAM can be considered as a fixed ratio, and can be given by

$$Amp = \cot \alpha \quad (3.4)$$

The output displacement of the SRAM can be obtained as follows

$$d_{out} = Amp d_{in} \quad (3.5)$$

From (3.4), the displacement amplification ratio of the stage is mainly affected by the SRAM angle and must be considered as an optimization variable. From Figure 3.3, it can be found that the amplification ratio will decrease with the increment of the SRAM angle. The suitable range of the SRAM angle is between $20^\circ - 30^\circ$, which provide an amplification ratio between $2.75 - 1.73$. This range will be used as a design variable limits in the design optimization.

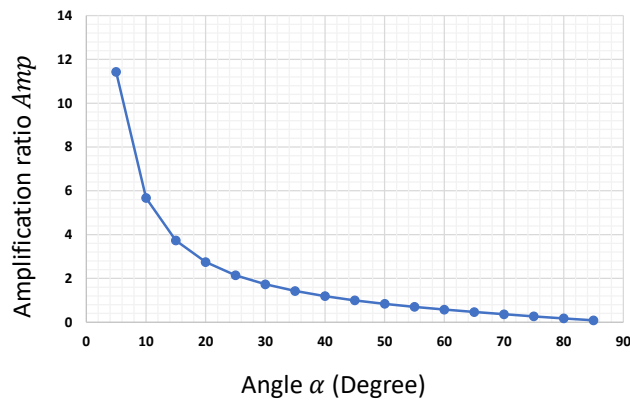


Figure 3.3 Plot of the amplification ratio and SRAM angle.

3.2.2 Displacement Analysis

From the kinematic structure of the SRAM, as presented in Figure 3.4, the distance $B'D'$ is always in the vertical direction during the operation of the mechanism, which can be described as follows

$$B'D' = L \sin(\alpha + \theta_O) = L \sin(\alpha + \theta_A) \quad (3.6)$$

where θ_O and θ_A the rotational displacement of point A' and B' .

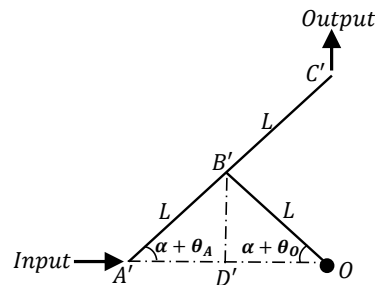


Figure 3.4 Schematic of SRAM position.

Therefore, the displacements, d_a , d_{sh} , and θ_b , of the SARC flexure hinges at point O and A, which are relevant to the axial and shear forces and the bending moment, respectively, as illustrated in Figure 3.5, can be expressed as follows

$$d_{aO} = d_{aA} \quad (3.7)$$

$$d_{shO} = d_{shA} \quad (3.8)$$

$$\theta_{bO} = \theta_{bA} \quad (3.9)$$

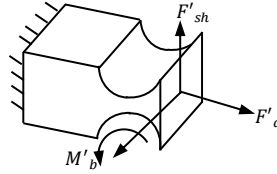


Figure 3.5 Schematic of SARC flexure hinge.

When one or two PZT actuators are active, the FAM produces an output force or forces causing the moving platform to rotate about one or two of the multi-axis right-circular (MARC) flexure hinges, as shown in Figure 3.6. The rotational displacement of MARC flexure hinge, $\theta_{b_{2,3}}$, can be approximately expressed in terms of the input displacement, d_{in} , and L_o indicated in Figure 3.6 (a), as follows

$$\sin \theta_{b_{2,3}} \approx \theta_{b_{2,3}} = \frac{d_{out}}{L_o} = \frac{Amp d_{in}}{L_o} \quad (3.10)$$

From Figure 3.6 (b), if r_0 is the radius of the moving platform, then

$$L_o = r_0 + r_0 \cos 60 = 1.5 r_0 \quad (3.11)$$

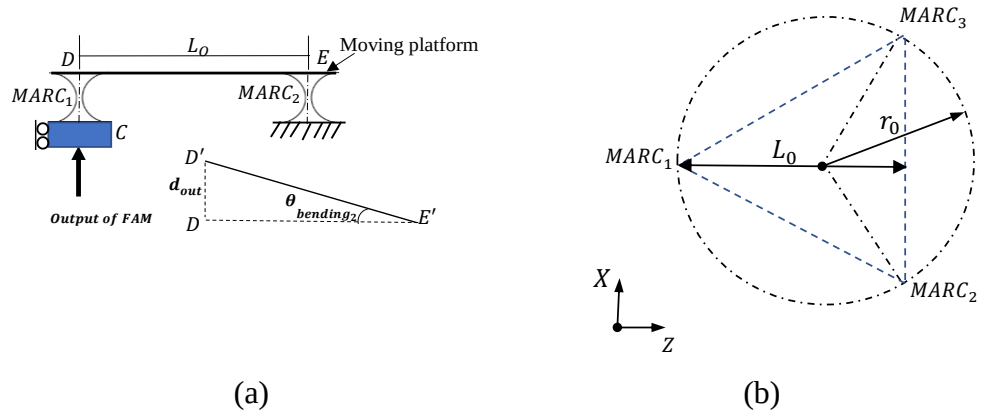


Figure 3.6 Schematic of the moving platform (a) Rotational displacement and (b) Arrangement of MARCs.

3.2.3 Velocity Analysis

The rigid-link RL_1 connecting the LS flexure hinges has a linear translation equal to the input displacement, d_{in} , as it is connected directly to the actuator, as depicted in Figure 3.2. The two links RL_2 and RL_3 of the SRAM rotate with angular displacement, θ . Therefore, the linear and angular velocities of the rigid-links as a function of the input linear velocity, $\dot{d}_{in}$, need to be developed first. From Figure 3.7, O' is an instantaneous velocity centre. The instantaneous angular velocity of link AC , which is equal to angular velocity of RL_2 , is

$$\omega_{AC} = \omega_{RL_2} = \frac{\dot{d}_{in}}{O'A'} = \frac{\dot{d}_{in}}{2L \sin(\alpha + \theta)} \cong \frac{\dot{d}_{in}}{2L \sin \alpha} \quad (3.12)$$

where ω_{RL_2} is the angular velocity of link RL_2 and $\theta \ll \alpha$.

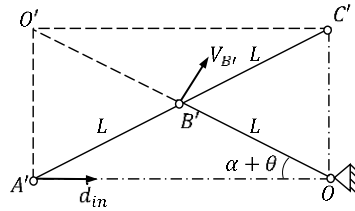


Figure 3.7 Instantaneous velocity of RL_2 .

As point B' is located at rigid link RL_2 and RL_3 , the linear velocity of point B can be achieved from the following

$$\dot{d}_{B'} = \dot{d}_{RL_3} = \omega_{AC} O'B = \omega_{AC} \cdot L \quad (3.13)$$

where $O'B' = OB' = L$.

The angular velocity of RL_3 is

$$\omega_{RL_3} = \omega_{OB} = -\omega_{AC}$$

From Figure 3.7 and (3.12), ω_{RL_3} as a function of $\dot{d}_{in}$ can be written as follows

$$\omega_{RL_3} = -\frac{\dot{d}_{in}}{2L \sin(\alpha + \theta)} \cong -\frac{\dot{d}_{in}}{2L \sin \alpha} \quad (3.14)$$

where the minus sign represents the opposite rotational direction.

According to (3.9), the angular displacement of the rigid links RL_2 and RL_3 at point O and A , which is indicated by θ_O and θ_A , can be approximately expressed as a function of d_{in} as follows

$$\theta_O = \theta_A = \theta_{SARC} = \frac{d_{in}}{2L \sin(\alpha + \theta)} \cong \frac{d_{in}}{2L \sin \alpha} \quad (3.15)$$

From Figure 3.7 and (3.12), the linear velocity of RL_2 at point C , which is equal to the linear velocity of the output point, $\dot{d}_{out}$, can be obtained by using (3.12) as follows

$$\dot{d}_{RL_2C} = \dot{d}_{out} = \omega_{RL_2} * 2L \cos(\alpha + \theta) \quad (3.16)$$

$$= \frac{2L \cos(\alpha + \theta)}{2L \sin(\alpha + \theta)} \dot{d}_{in} = \frac{1}{\tan(\alpha + \theta)} \dot{d}_{in} \cong \text{Amp } \dot{d}_{in} \quad (3.17)$$

The linear velocity of the centroid of RL_2 can be formulated as follows

$$\dot{d}_{RL_2} = L_{G_{RL_2}} \omega_{RL_2} \quad (3.18)$$

where $L_{G_{RL_2}}$ is the distance between the centre of gravity of RL_2 and point O , which can be found by using the first moment of area method and the architectural parameters shown in Figure 3.8. Table 3.1 organises all the required data to find $L_{G_{RL_2}}$ after dividing the link into two areas. From Figure 3.8 and Table 3.1, $L_{G_{RL_2}}$ can be calculated from the following

$$L_{G_{RL_2}} = \sqrt{\left((l_{3RL} + r_{SARC}) - l_{GX}\right)^2 + (l_{2RL} - l_{GY})^2} + L \quad (3.19)$$

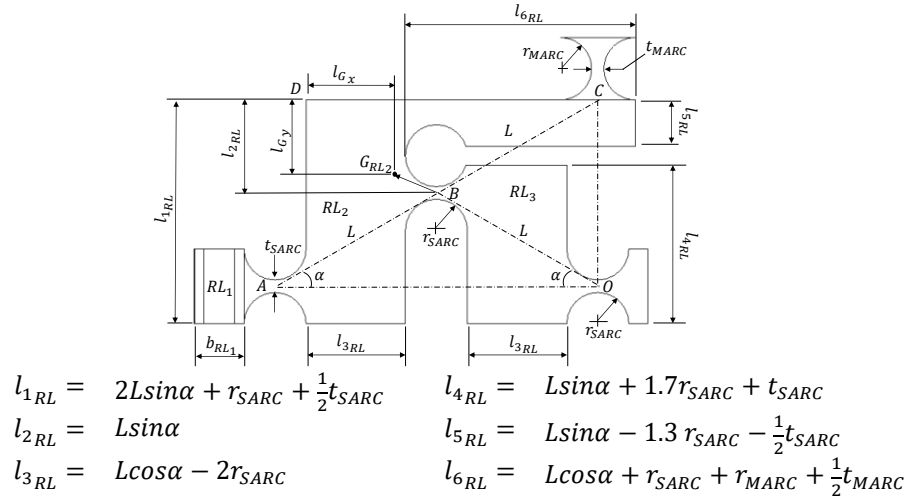
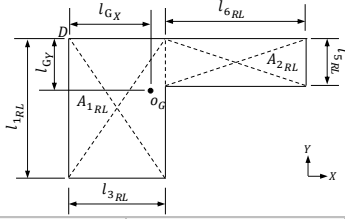


Figure 3.8 Architectural parameters of the rigid-links.

Table 3.1 The centre of gravity of RL_2 .

							
Area		Mass		Distance to centre		Area moment	
				X-direction	Y-direction	X-axis I_{AX}	Y-axis I_{AY}
A_{1RL}	$l_{1RL} l_{3RL}$	m_{1RL}	$l_{1RL} l_{3RL} w_{FAM} \rho$	$\frac{1}{2} l_{3RL}$	$\frac{1}{2} l_{1RL}$	$\frac{1}{2} l_{1RL} l_{3RL}^2$	$\frac{1}{2} l_{3RL} l_{1RL}^2$
A_{2RL}	$l_{6RL} l_{5RL}$	m_{2RL}	$l_{6RL} l_{5RL} w_{FAM} \rho$	$l_{3RL} + \frac{1}{2} l_{6RL}$	$\frac{1}{2} l_{5RL}$	$l_{6RL} l_{5RL} (l_{3RL} + \frac{1}{2} l_{6RL})$	$\frac{1}{2} l_{6RL} l_{5RL}^2$
A_{RL2}	$l_{1RL} l_{3RL} + l_{6RL} l_{5RL}$	m_{RL2}	$(l_{1RL} l_{3RL} + l_{6RL} l_{5RL}) w_{FAM} \rho$			$\frac{1}{2} l_{1RL} l_{3RL}^2 + l_{6RL} l_{5RL} (l_{3RL} + \frac{1}{2} l_{6RL})$	$\frac{1}{2} l_{3RL} l_{1RL}^2 + \frac{1}{2} l_{6RL} l_{5RL}^2$
Centre of gravity							
X-direction: $l_{GX} = I_{AX} / A_{RL2}$				Y-direction: $l_{GY} = I_{AY} / A_{RL2}$			

3.2.4 Mobility Analysis

The mobility of the stage can be determined by using Kutzbach-Grubler formula [111]:

$$DOF = 6(l - j - 1) + \sum_{i=1}^j dof_i \quad (3.20)$$

where l is the number of rigid bodies, j is the number of joints, and dof_i is the degree-of-freedom (DOF) of i^{th} joint.

For the proposed stage:

$$l = 3 * 3 + 1 + 1 = 11$$

(3 rigid bodies at each of the 3 FAMs, the fixed base, and the moving platform).

$$j = 2 * 3 + 3 * 3 + 1 * 3 = 18$$

(considering the double parallel LS as 2 joints, 3 SARCs, and 1 MARC at each of the 3 FAMs).

$$dof = (1 * 2 + 3 * 3 + 6) * 3 = 51$$

(1-DOF of LS, 3-DOFs of the SARCs, and 6 DOFs of the MARCs).

Substituting these values into (3.20), the mobility of the stage is

$$DOF = 6(11 - 18 - 1) + 51 = 3$$

This result agrees with the design requirement of the proposed stage, which is 3-DOF.

3.2 Static Modelling

The static modelling of the stage is carried out to assess the input stiffness. The stiffness of the OHP stage can be defined as the stiffness of its structure under statics conditions without taking the actuators into consideration. The input model is built via the SMM.

The flexure hinges comprising the stage are typically located in different directions. To obtain the stiffness model, the stiffness matrices of the flexure hinges at the local coordinate systems need first to be expressed in the global coordinate system by using the transformation matrices. Then, the matrices of all flexure hinges can be assembled according to the way of connections between the flexure hinges. The global coordinate system used in the analysis is a right-handed XYZ coordinate system.

The procedure of obtaining the stiffness models is shown in Figure 3.9, which is summarized as follows

- Setting the local frame of each flexure hinge, and locating the global frame,
- Determining the compliance matrix of each flexure hinge at the local frame,
- Transferring the compliance matrices of all the flexure hinges to the global frame, and
- Adding the compliances connected in serial and stiffness connected in parallel to generate the model of the mechanism.

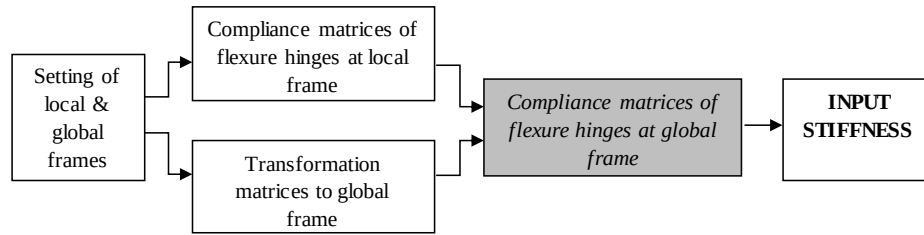


Figure 3.9 Procedure of Stiffness Matrix Method.

The modelling of the stage is under the assumption that the linear relationship between the applied force, F , and the related deformation, D , is applied (Hooke's law). This relationship can be expressed as follows:

$$F = KD \quad (3.21)$$

$$D = CF \quad (3.22)$$

where K is Stiffness matrix and C is Compliance matrix, which are inverse matrices of each other, regular and symmetric ($K = C^{-1}$).

3.2.1 Transformation of Compliance Matrices

Consider a flexure hinge part of a compliant mechanism, when an external load is applied at a point at the flexure hinge, the point will be subjected to three forces (an axial force F'_z and two shearing forces F'_x and F'_y), plus three moments (two bending moments M'_x and M'_y and a torsional moment M'_z), as shown in Figure 3.10. Under the action of these loadings, $P_{fh_i} = [F'_x \ F'_y \ F'_z \ M'_x \ M'_y \ M'_z]^T$, 3-dimensional (3-D) displacements, $D_{fh_i} = [d_x \ d_y \ d_z \ \theta_x \ \theta_y \ \theta_z]^T$, are generated.

A local coordinate system, O_{fh_i} , of flexure hinge i is defined with an origin located at the point representing the geometric centre of the area perpendicular to the longitudinal axis of the flexure hinge, as illustrated in Figure 3.10. The

displacements of the point with respect to its local coordinate frame, O_{fh_i} , can be related to the internal loadings as follows

$$D_{fh_i} = C_{fh_i} P_{fh_i} \quad (3.23)$$

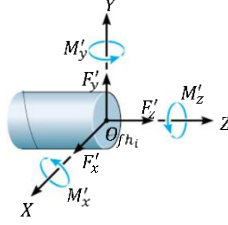


Figure 3.10 Internal loadings in a flexure hinge.

The displacements or the loadings can be oriented in another global frame, O_j , in the compliant mechanism by using a transformation matrix, $[T_{fh_i}^j]$. This can be done as follows

$$D_{fh_i}^j = [T_{fh_i}^j] D_{fh_i} \quad (3.24)$$

$$P_{fh_i}^j = [T_{fh_i}^j] P_{fh_i} \quad (3.25)$$

Solving (3.24) and (3.25) for D_{fh_i} and P_{fh_i} , respectively, and applying the results in (3.23), the following expression can be obtained

$$D_{fh_i} = \left\{ [T_{fh_i}^j]^{-1} C_{fh_i} [T_{fh_i}^j] \right\} P_{fh_i}^j = C_{fh_i}^j P_{fh_i}^j \quad (3.26)$$

Hence from the previous equation, the term $C_{fh_i}^j$ can be referred to the compliance of a flexure hinge defined in a new frame (global) and can be written as follows

$$C_{fh_i}^j = [T_{fh_i}^j]^{-1} C_{fh_i} [T_{fh_i}^j] \quad (3.27)$$

The spatial transformation matrix, $T_{fh_i}^j$, which consisted of a translation matrix, $Trans_{fh_i}^j$, and rotation matrix, $R_{fh_i}^j$, can be written as follows

$$T_{fh_i}^j = [Trans_{fh_i}^j] [R_{fh_i}^j] \quad (3.28)$$

A general form of a translation matrix of a local frame of fh_i flexure hinge in a 3-D relative to a global frame is

$$Trans_{fh_i}^j = Trans_{fh_i}^j(r_{fh_iX}, r_{fh_iY}, r_{fh_iZ}) = \begin{bmatrix} I & (\hat{r}_{fh_i}^j)^T \\ 0 & I \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & r_{fh_iZ}^j & -r_{fh_iY}^j \\ 0 & 1 & 0 & -r_{fh_iZ}^j & 0 & r_{fh_iX}^j \\ 0 & 0 & 1 & r_{fh_iY}^j & -r_{fh_iX}^j & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.29)$$

A general form of a rotation matrix of a local frame of fh_i flexure hinge in a 3-D about one of the axes of the coordinate system relative to a global frame is

$$R_{fh_i}^j(X, Y, Z, \alpha_{fh_i}) = \begin{bmatrix} R_{fh_i}^j(\alpha_{fh_{iX,Y,Z}}) & 0 \\ 0 & R_{fh_i}^j(\alpha_{fh_{iX,Y,Z}}) \end{bmatrix} \quad (3.30)$$

where α_{fh_i} is the angle of rotation about X , Y , or Z axes.

The direction of rotation which codifies the alternating signs is defined according to the right-hand rule. The positive rotation is counter-clockwise about the axis of rotation.

In modelling the input stiffness, the analysis will deal first with compliances of the flexure hinges and then later the inverse relationship between the stiffness and the compliance will be used to obtain the expression for the stiffness of the stage. The three stiffness values of the three FAMs (FAM_1 , FAM_2 , and FAM_3) are identical because of the symmetrical structural design of the stage. The displacements due to the shear, bending, and axial loadings are set in the X , Y , and Z axes, respectively, as offered in Figure 3.10.

The derivation of the input stiffness model is presented in the following subsections after defining the local compliance matrices of the flexure hinges.

3.2.2 Local Compliance Matrices

According to the elastic beam theory, when displacements $\bar{D} = [d_x \ d_y \ d_z \ \theta_x \ \theta_y \ \theta_z]^T$ occur at different directions of a flexure hinge due to the internal loading $\bar{P} = [F'_x \ F'_y \ F'_z \ M'_x \ M'_y \ M'_z]^T$ applied on its tip, as shown in Figure 2.20. The linear relationship between the displacements and the internal loadings can be written in a general matrix form as follows

$$\bar{D} = [C_{fh}] \bar{P}$$

$$\begin{bmatrix} d_x \\ d_y \\ d_z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} = \begin{bmatrix} c_{d_x-F_x fh} & 0 & 0 & 0 & c_{d_x-M_y fh} & 0 \\ 0 & c_{d_y-F_y fh} & 0 & c_{d_y-M_x fh} & 0 & 0 \\ 0 & 0 & c_{d_z-F_z fh} & 0 & 0 & 0 \\ 0 & c_{\theta_x-F_y fh} & 0 & c_{\theta_x-M_x fh} & 0 & 0 \\ c_{\theta_y-F_x fh} & 0 & 0 & 0 & c_{\theta_y-M_y fh} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{\theta_z-M_z fh} \end{bmatrix} \begin{bmatrix} F'_x \\ F'_y \\ F'_z \\ M'_x \\ M'_y \\ M'_z \end{bmatrix} \quad (3.31)$$

where, C_{fh} is the compliance matrix of a flexure hinge expressed at the local frame, and subscript fh donates the type of the flexure hinge (LS , $SARC$, and $MARC$).

The local compliant matrix of each type of the flexure hinges used in the stage is defined according to the loadings directions, as shown in Figure 3.11.

Leaf-spring flexure hinge

According to the coordinate system indicated in Figure 3.11 (a), the local compliance matrix of the LS hinge can be written as follows

$$C_{O_{LS}} = \begin{bmatrix} c_{d_x-F_{xLS}} & 0 & 0 & 0 & c_{d_x-M_{yLS}} & 0 \\ 0 & c_{d_y-F_{yLS}} & 0 & c_{d_y-M_{xLS}} & 0 & 0 \\ 0 & 0 & c_{d_z-F_{zLS}} & 0 & 0 & 0 \\ 0 & c_{\theta_x-F_{yLS}} & 0 & c_{\theta_x-M_{xLS}} & 0 & 0 \\ c_{\theta_y-F_{xLS}} & 0 & 0 & 0 & c_{\theta_y-M_{yLS}} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{\theta_z-M_{zLS}} \end{bmatrix} \quad (3.32)$$

The value of the compliance factors of the LS flexure hinge can be obtained from Table 2.3.

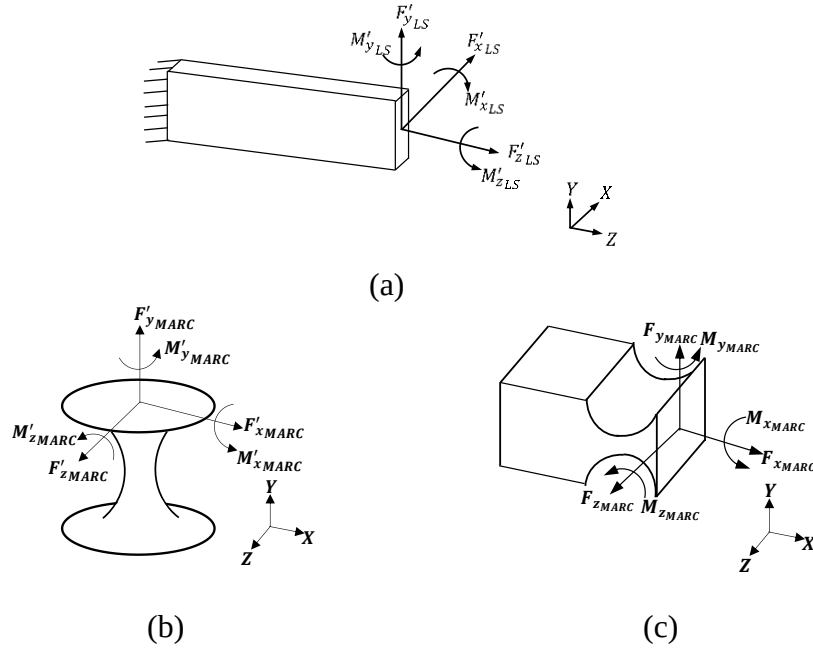


Figure 3.11 Schematic of the flexure hinges with internal loadings: (a) LS, (b) MARC, and (c) SARC.

Multi-axis right circular flexure hinge

The MARC flexure hinge is considered as multiple sensitive axes subjected to six-dimensional (6-D) loadings. From Figure 3.11 (b), the local compliance of the MARC flexure hinge can be written in a matrix form as

$$\begin{aligned}
& C_{O_{MARC}} \\
& = \begin{bmatrix} c_{d_x-F_x}_{MARC} & 0 & 0 & 0 & c_{d_x-M_y}_{MARC} & 0 \\ 0 & c_{d_y-F_y}_{MARC} & 0 & c_{d_y-M_x}_{MARC} & 0 & 0 \\ 0 & 0 & c_{d_z-F_z}_{MARC} & 0 & 0 & 0 \\ 0 & c_{\theta_x-F_y}_{MARC} & 0 & c_{\theta_x-M_x}_{MARC} & 0 & 0 \\ c_{\theta_y-F_x}_{MARC} & 0 & 0 & 0 & c_{\theta_y-M_y}_{MARC} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{\theta_z-M_z}_{MARC} \end{bmatrix} \quad (3.33)
\end{aligned}$$

The value of the compliance factors can be found in (2.6) - (2.13).

SARC flexure hinge

From Figure 3.11 (c), the local compliance matrix for SARC flexure hinge can be written as

$$\begin{aligned}
& C_{O_{SARC}} \\
& = \begin{bmatrix} c_{d_x-F_x}_{SARC} & 0 & 0 & 0 & c_{d_x-M_y}_{SARC} & 0 \\ 0 & c_{d_y-F_y}_{SARC} & 0 & -c_{d_y-M_x}_{SARC} & 0 & 0 \\ 0 & 0 & c_{d_z-F_z}_{SARC} & 0 & 0 & 0 \\ 0 & -c_{\theta_x-F_y}_{SARC} & 0 & c_{\theta_x-M_x}_{SARC} & 0 & 0 \\ c_{\theta_y-F_x}_{SARC} & 0 & 0 & 0 & c_{\theta_y-M_y}_{SARC} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{\theta_z-M_z}_{SARC} \end{bmatrix} \quad (3.34)
\end{aligned}$$

The compliance matrix elements in the matrix can be obtained from Table 2.3.

3.2.3 Input Stiffness Analysis

The input stiffness can be defined as the stiffness of the stage at the input point where the actuation force is applied, and it should not exceed the stiffness of the actuator. It can be expressed as

$$d_{input} = C_{input} F_{in} \quad (3.35)$$

where C_{input} is the input compliance of the stage.

The input compliance can be achieved by moving the local compliance matrices at the local coordinates of the flexure hinges relative to the input point at the global coordinate, as presented in Figure 3.12. Then using the inverse relation to achieve the input stiffness. The local coordinates of the flexure hinges are determined according to the direction of the displacements, which are produced due to the internal loadings. X-axis, Y-axis, Z-axis are set in the shear, bending, and axial loading directions, respectively.

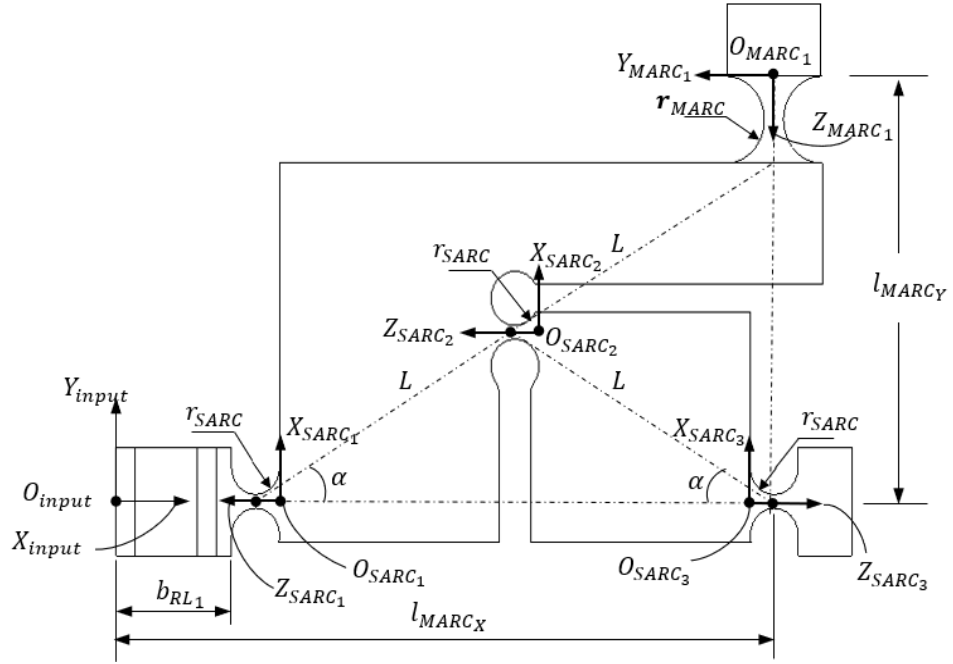


Figure 3.12 Input global and local coordinates of SARC and MARC.

3.2.3.1 Compliance Transformation Matrices

The translation matrices of the local frames of the three types of flexure hinges relative to the input global frame are derived in detailed in **Appendix A.1** and summarized as follows:

MARC Transformation Matrices

The transformation of frame O_{MARC} is made relative to the input global frame, O_{input} , as shown in Figure 3.12, by

- Rotation of $(\pi/2)$ about Y -axis,
- Rotation of $(-\pi/2)$ about Z -axis, and
- Translation of $[l_{MARC_X}, l_{MARC_Y}, 0]$.

From Figure 3.12

$$l_{MARC_X} = b_{RL1} + r_{SARC} + 2L \cos \alpha \quad (3.36)$$

$$l_{MARC_Y} = 2r_{MARC} + 2L \sin \alpha \quad (3.37)$$

The transformation matrix is

$$\begin{aligned} T_{MARC1}^{O_{input}} &= R_{O_{MARC1}}^{O_{input}}(Y, \alpha_{MARC1}) * R_{O_{MARC1}}^{O_{input}}(Z, \alpha_{MARC1}) \\ &\quad * Trans_{O_{MARC1}}^{O_{input}}(l_{X_{MARC1}}, l_{Y_{MARC1}}, l_{Z_{MARC1}}) \\ &= R_{O_{MARC1}}^{O_{input}}(Y, \pi/2) * R_{O_{MARC1}}^{O_{input}}(Z, -\pi/2) \\ &\quad * Trans_{O_{MARC1}}^{O_{input}}(l_{MARC_X}, l_{MARC_Y}, 0) \end{aligned}$$

$$= \begin{bmatrix} 0 & 0 & 1 & l_{MARC_Y} & -l_{MARC_X} & 0 \\ -1 & 0 & 0 & 0 & 0 & l_{MARC_Y} \\ 0 & -1 & 0 & 0 & 0 & -l_{MARC_X} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \quad (3.38)$$

SARC Transformation Matrices

The transformation of O_{SARC1} is made relative to the global frame, O_{input} , as presented in Figure 3.12, by the following

- Rotation of $(-\pi/2)$ about the $Z - axis$, followed by
- Rotation of $(\pi/2)$ about the $Y - axis$, and
- Translation of $[l_{SARC1_X}, 0, 0]$.

From Figure 3.12

$$l_{SARC1_X} = b_{RL1} + 2 r_{SARC} \quad (3.39)$$

The transformation matrix is

$$\begin{aligned} T_{SARC1}^{O_{input}} &= R_{O_{SARC1}}^{O_{input}}(Z, \alpha_{SARC1}) * R_{O_{SARC1}}^{O_{input}}(Y, \alpha_{SARC1}) \\ &\quad * Trans_{SARC1}^{O_{input}}(l_{x_{SARC1}}, l_{y_{SARC1}}, l_{z_{SARC1}}) \\ &= R_{O_{SARC1}}^{O_{input}}(Z, -\pi/2) * R_{O_{SARC1}}^{O_{input}}(Y, \pi/2) * Trans_{SARC1}^{O_{input}}(l_{SARC1_X}, 0, 0) \\ &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & l_{SARC1_X} \\ 0 & 0 & -1 & 0 & l_{SARC1_X} & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \end{aligned} \quad (3.40)$$

The transformation of the frame O_{SARC2} is made relative to the global frame O_{input} , as shown in Figure 3.12, by the following

- Rotation of $(-\pi/2)$ about the $Z - axis$, followed by
- Rotation of $(\pi/2)$ about the $Y - axis$, and
- Translation of $[l_{SARC2_X}, l_{SARC2_Y}, 0]$.

From Figure 3.12

$$l_{SARC2_X} = b_{RL1} + 2r_{SARC} + L \cos \alpha \quad (3.41)$$

$$l_{SARC2_Y} = L \sin \alpha \quad (3.42)$$

The transformation matrix is

$$\begin{aligned} T_{SARC2}^{O_{input}} &= R_{O_{SARC2}}^{O_{input}}(Z, \alpha_{SARC2}) * R_{O_{SARC2}}^{O_{input}}(Y, \alpha_{SARC2}) \\ &\quad * Trans_{SARC2}^{O_{input}}(l_{x_{SARC2}}, l_{y_{SARC2}}, l_{z_{SARC2}}) \end{aligned}$$

$$\begin{aligned}
&= R_{O_{SARC2}}^{O_{input}}(Z, -\pi/2) * R_{O_{SARC2}}^{O_{input}}(Y, \pi/2) \\
&\quad * Trans_{SARC2}^{O_{input}}(l_{SARC2x}, l_{SARC2y}, 0) \\
&= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & l_{SARC2x} \\ 0 & 0 & -1 & -l_{SARC2y} & l_{SARC2x} & 0 \\ -1 & 0 & 0 & 0 & 0 & -l_{SARC2y} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \quad (3.43)
\end{aligned}$$

The frame O_{SARC3} relative to the global frame O_{input} is achieved, as illustrated in Figure 3.12, by the following transformations

- Rotation of $(-\pi/2)$ about the Y – axis, followed by
- Rotation of $(-\pi/2)$ about the X – axis, followed by
- Translation of $[l_{SARC3x}, 0, 0]$.

From Figure 3.12

$$l_{SARC3x} = b_{RL1} + 2L \cos \alpha \quad (3.44)$$

The transformation matrix is

$$\begin{aligned}
T_{SARC3}^{O_{input}} &= R_{O_{SARC3}}^{O_{input}}(Y, \alpha_{SARC3}) * R_{O_{SARC3}}^{O_{input}}(X, \alpha_{SARC3}) \\
&\quad * Trans_{SARC3}^{O_{input}}(l_{xSARC3}, l_{ySARC3}, l_{zSARC3}) \\
&= R_{O_{SARC3}}^{O_{input}}(Y, -\pi/2) * R_{SARC3}^{O_{input}}(X, -\pi/2) \\
&\quad * Trans_{SARC3}^{O_{input}}(l_{SARC3x}, 0, 0) \\
&= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & l_{SARC3x} \\ 0 & 0 & 1 & 0 & -l_{SARC3x} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (3.45)
\end{aligned}$$

LS Transformation matrices

From Figure 3.13, to express the frames O_{SL1} , O_{SL2} , O_{SL3} and O_{SL4} relative to the reference frame, the following successive transformations are required:

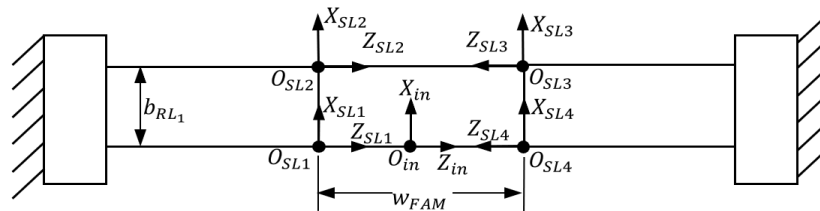


Figure 3.13 Input global and local coordinates of LS.

Frame O_{SL_1}

- Translation of $[0, 0, -\frac{w_{FAM}}{2}]$.

The transformation matrix is

$$T_{SL_1}^{O_{in}} = Trans_{O_{SL_1}}^{O_{in}} \left(0, 0, -\frac{w_{FAM}}{2} \right)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{w_{FAM}}{2} & 0 \\ 0 & 1 & 0 & \frac{w_{FAM}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.46)$$

Frame O_{SL_2}

- Translation of $[b_{RL_1}, 0, -\frac{w_{FAM}}{2}]$

The transformation matrix is

$$T_{SL_2}^{O_{in}} = Trans_{O_{SL_2}}^{O_{in}} \left(b_{RL_1}, 0, -\frac{w_{FAM}}{2} \right)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{w_{FAM}}{2} & 0 \\ 0 & 1 & 0 & \frac{w_{FAM}}{2} & 0 & b_{RL_1} \\ 0 & 0 & 1 & 0 & -b_{RL_1} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.47)$$

Frame O_{SL_3}

- Rotation of (π) about the $X - axis$, followed by
- Translation of $[b_{RL_1}, 0, \frac{w_{FAM}}{2}]$.

The transformation matrix is

$$T_{SL_3}^{O_{input}} = R_{O_{SL_3}}^{O_{input}}(X, \pi) * Trans_{O_{SL_3}}^{O_{input}} \left(b_{RL_1}, 0, \frac{w_{FAM}}{2} \right)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{w_{FAM}}{2} & 0 \\ 0 & -1 & 0 & \frac{w_{FAM}}{2} & 0 & -b_{RL_1} \\ 0 & 0 & -1 & 0 & b_{RL_1} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad (3.48)$$

Frame O_{SL_4} Rotation of (π) about the $X - axis$, followed by translation of $[0, 0, \frac{w_{FAM}}{2}]$.

The transformation matrix is

$$T_{SL_4}^{O_{in}} = R_{O_{SL_4}}^{O_{input}}(X, \pi) * Trans_{O_{SL_4}}^{O_{input}} \left(0, 0, \frac{w_{FAM}}{2} \right)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{W_{FAM}}{2} & 0 \\ 0 & -1 & 0 & \frac{W_{FAM}}{2} & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad (3.49)$$

3.2.3.2 Global Compliance Matrices

To express the local compliance matrices of the flexure hinges at the input point, the local frames, $C_{O_{fh}}^{O_{input}}$, are transferred to the input global frame, O_{input} , by using the generic following equation

$$C_{O_{fh}}^{O_{input}} = [T_{O_{fh}}^{O_{input}}]^T [C_{O_{fh}}] [T_{O_{fh}}^{O_{input}}] \quad (3.50)$$

where $C_{O_{fh}}$ is the local compliance matrix and $T_{O_{fh}}^{O_{input}}$ is transformation matrix of the flexure hinge.

The local compliance matrix of MARC flexure hinge with respect to the input global frame, O_{input} , can be expressed, by applying (3.33), (3.38), into (3.50), as follows:

$$C_{O_{MARC_1}}^{O_{input}} = [T_{O_{MARC}}^{O_{input}}]^T [C_{O_{MARC}}] [T_{O_{MARC}}^{O_{input}}] \quad (3.51)$$

By combining (3.34) and (3.40), (3.43), or (3.45), with (3.50), the compliance matrices of the three SARC flexure hinges are, respectively,

$$C_{O_{SARC_1}}^{O_{input}} = [T_{O_{SARC_1}}^{O_{input}}]^T [C_{O_{SARC}}] [T_{O_{SARC_1}}^{O_{input}}] \quad (3.52)$$

$$C_{O_{SARC_2}}^{O_{input}} = [T_{O_{SARC_2}}^{O_{input}}]^T [C_{O_{SARC}}] [T_{O_{SARC_2}}^{O_{input}}] \quad (3.53)$$

$$C_{O_{SARC_3}}^{O_{input}} = [T_{O_{SARC_3}}^{O_{input}}]^T [C_{O_{SARC}}] [T_{O_{SARC_3}}^{O_{input}}] \quad (3.54)$$

The Compliance matrix of the four LS flexure hinges can be obtained by applying (3.32), and (3.46) – (3.49), respectively, into (3.50),

$$C_{O_{LS_1}}^{O_{input}} = [T_{O_{LS_1}}^{O_{input}}]^T [C_{O_{SL}}] [T_{O_{LS_1}}^{O_{input}}] \quad (3.55)$$

$$C_{O_{LS_2}}^{O_{input}} = [T_{O_{LS_2}}^{O_{input}}]^T [C_{O_{SL}}] [T_{O_{LS_2}}^{O_{input}}] \quad (3.56)$$

$$C_{O_{LS_3}}^{O_{input}} = [T_{O_{LS_3}}^{O_{input}}]^T [C_{O_{SL}}] [T_{O_{LS_3}}^{O_{input}}] \quad (3.57)$$

$$C_{O_{LS_4}}^{O_{input}} = [T_{O_{LS_4}}^{O_{input}}]^T [C_{O_{SL}}] [T_{O_{LS_4}}^{O_{input}}] \quad (3.58)$$

3.2.3.3 Input Stiffness Model

The matrices of the flexure hinges, which are expressed with respect to the input global frame, can be assembled according to the type of connection relative to input point, as shown in Figure 3.14 (a). According to the serial connection

between $SARC_2$ and $SARC_3$, as shown in Figure 3.14 (b), using (3.53) and (3.54), the equivalent compliance matrix is

$$C_{O_{SARC_{2,3}}}^{O_{input}} = C_{O_{SARC_2}}^{O_{input}} + C_{O_{SARC_3}}^{O_{input}} \quad (3.59)$$

Due to the parallel connection, and from (3.51) and (3.59), the compliance matrix of $SARC_{2,3}$ and $MARC$ is

$$C_{O_{SARC_{2,3}MARC_2}}^{O_{input}} = \left[\left[C_{O_{SARC_{2,3}}}^{O_{input}} \right]^{-1} + \left[C_{O_{MARC_2}}^{O_{input}} \right]^{-1} \right]^{-1} \quad (3.60)$$

The four LS flexure hinges are connected in parallel. Using (3.55)-(3.58), the stiffness matrix is

$$C_{O_{SL_{1,2,3,4}}}^{O_{input}} = \left[\left[C_{O_{SL_1}}^{O_{input}} \right]^{-1} + \left[C_{O_{SL_2}}^{O_{input}} \right]^{-1} + \left[C_{O_{SL_3}}^{O_{input}} \right]^{-1} + \left[C_{O_{SL_4}}^{O_{input}} \right]^{-1} \right]^{-1} \quad (3.61)$$

In the view of the serial connection between $C_{O_{SL_{1,2,3,4}}}^{O_{input}}$, $C_{O_{SARC_1}}^{O_{input}}$, and $C_{O_{SARC_{2,3}MARC_2}}^{O_{input}}$, the input compliance C_{input} of the stage, when three actuators are active, can be assembled by using (3.52), (3.60), and (3.61), as follows

$$C_{input} = C_{O_{SL_{1,2,3,4}}}^{O_{input}} + C_{O_{SARC_1}}^{O_{input}} + C_{O_{SARC_{2,3}MARC_2}}^{O_{input}} \quad (3.62)$$

Using the inverse relationship, the stiffness and the compliance, the input stiffness of the stage is

$$K_{input} = [C_{input}]^{-1} = \left[C_{O_{SL_{1,2,3,4}}}^{O_{input}} + C_{O_{SARC_1}}^{O_{input}} + C_{O_{SARC_{2,3}MARC_2}}^{O_{input}} \right]^{-1} \quad (3.63)$$

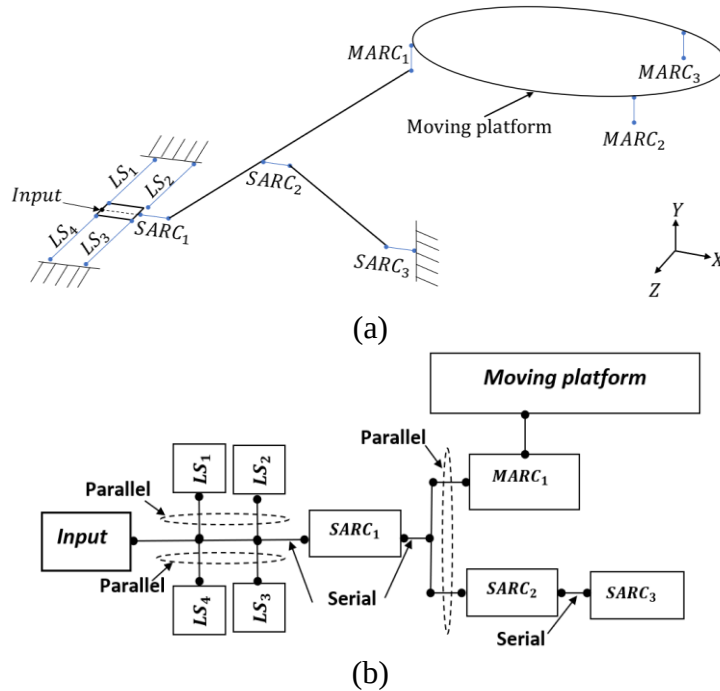


Figure 3.14 Connections between the flexure hinges relative to the input point: (a) 3-D-model and (b) 2D-model.

3.2.4 Architectural Parameter Evaluation

The effect of architectural parameters of the flexure hinges on the input stiffness of the OHP stage is studied. The architectural parameters of the stage are described in Table 3.2. Suitable dimensions are chosen after returning to a previous research [91]. The material used in the model is Aluminium alloy (*Al* – 7075) with parameters assigned in Table 3.3. All the flexure hinges have the same material parameters. MATLAB software is used to calculate the value of the input stiffness and the code used is documented in **Appendix B**.

Table 3.2 Architectural Parameters.

Architectural Parameters (m)				
t_{MARC}	r_{MARC}	t_{SARC}	r_{SARC}	b_{SARC}
0.002	0.004	0.00125	0.0025	0.01
l_{LS}	t_{LS}	b_{LS}	r_0	
0.025	0.002	0.01	0.11	

Table 3.3 Material Parameters.

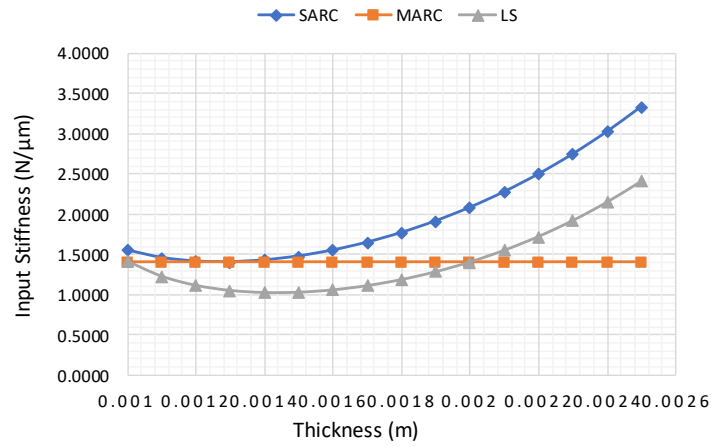
Aluminium alloy (<i>Al</i> – 7075)		
Young's Modulus (E)	Shear Modulus (G)	Yield strength (σ_y)
71.7GPa	26.692GPa	500MPa

The plots of the input stiffness versus the parameters of the flexure hinges are illustrated in Figure 3.15. When one parameter is varied, the other parameters are assigned to be constants. The minimum and maximum value of the architectural parameters in the graphs are selected according to the machining feasibility and structural compactness of the stage. The thickness-to-radius ratio of the SARC and MARC flexure hinges is chosen 0.5, which is the maximum value allowed [112].

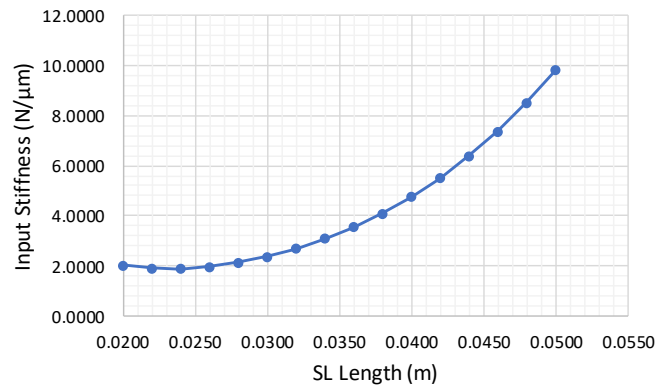
The following points can be concluded from analysing the plots of Figure 3.15:

- The thickness of the SARC flexure hinge influences the input stiffness of the stage in the specified range and the value of the input stiffness is changed, as shown in Figure 3.15 (a). Accordingly, the input stiffness is sensitive to variation in the thicknesses and radii of the SARC flexure hinges.
- In the specified range, the thickness of the MARC flexure hinge has no effect on the input stiffness of the stage and the value of the input stiffness remains constant, as shown in Figure 3.15 (a). Accordingly, the input stiffness is

insensitive to variation in the thicknesses and radii of the MARC flexure hinges.



(a)



(b)

Figure 3.15 Plots of input stiffness in terms of flexure hinge geometries: (a) in terms of LS, SARC, and MARC thickness and (b) in terms of LS length.

- From Figure 3.15 (a), the input stiffness of the stage increases along with increasing of the thickness of LS flexure hinge, which indicates that it is affected by the variation in the thickness and radius of the LS flexure hinge.
- As the length of LS flexure hinge increases, the input stiffness increases as well, as shown in Figure 3.15 (b). Consequently, the input stiffness is sensitive to the changes in the length of the LS flexure hinge.

It appears from the above analysis that the thickness and length of LS flexure hinge have a major influence on the input stiffness of the stage. A thicker and longer LS flexure hinge will produce higher input stiffness than a thinner shorter LS flexure hinge of equal material parameters. Therefore, a careful consideration

is needed for the design process. The results of this evaluation will help in selecting the constraints for the optimum design.

3.3 Chapter Summary

Kinematic and static models were built in this chapter. The following points can be extracted from the previous analysis:

Kinematic Modelling

- The displacement amplification ratio and the SRAM angle have an inverse relationship.
- Mathematical models of the rigid-links were established to be used in the dynamic modelling of the proposed stage.
- According to the proposed structural design of the OHP stage, it was proved that the stage has 3-DOF and capable to provide two rotations and one translation motions.

Static Modelling

- The input stiffness model depends basically on the flexure hinges parameters and the material properties of the stage components.
- The advantage of the SMM is that it can manage all the deformations of the flexure hinges of the structure. However, the difficulty of this method occurs when trying to obtain a value of an input stiffness of a compliant mechanism, while having a big number of parallel connections between flexure hinges in the mechanism. The existence of several parallel connections ends the modelling with very complicated inverse matrices that can only be solved by using one of commercial software such as MATLAB or Mathematica.
- The SRAM angle and the geometric parameters of the LS and SARC flexure hinges were the main design variables needed to describe the stage for the design optimization.

Using the models provided in this chapter, a dynamic model of the OHP stage will be built in **Chapter 4**, which is considered one of the important characteristics of the stage and the objective function of the design optimization process.

Chapter 4 Dynamic Modelling

The aims and the importance of this chapter are to:

- Build a dynamic model for the out-of-horizontal-plane (OHP) stage. The importance of this dynamic analysis is to determine the vibration characteristics of the stage structure such as the natural frequencies. Defining the natural frequencies of the stage helps to avoid resonance, which could lead to system failure. Lagrange's method is adopted for modelling the natural frequency of the stage as it has been justified in **Chapter 2**.
- Develop a method for modelling the potential energy of the stage. The necessity for developing another method is based on the evaluation of the existing modelling methods carried in **Chapter 2**, where it is found that the commonly used method considers the flexure hinge as 1-DOF element.
The developed method for modelling the potential energy of compliance-based mechanisms is based on the concepts of the conservation of energy principle. The work-energy method, which has been widely used for computing the deflections of beams and frames, allows considering all the deformations of the flexure hinges.
- Draw a general framework for the developed method, which is called the energy-based approach (EBA).
- Validate EBA by applying it to an existing research work and compare the results.
- Use EBA to model the potential energy for each flexure hinge and then obtain the total potential energy of the stage.
- Select the suitable natural frequency model, which is built based on the EBA, as an objective function for the design optimization.

These aims are addressed in two main sections. **Section 4.1** introduces the concept of the EBA and the derivation of the potential energy expression. It also includes the validation of the EBA. The modelling of the natural frequency of the stage is presented in **Section 4.2**. This section also contains a selection of the

suitable dynamic model for the design optimization and an evaluation of the geometric parameters.

4.1 Energy-Based Approach

The method is essentially based on the conservation of energy principle, which states that: *“The work done by gradually applied external loads acting on an elastic isolated system in equilibrium is equal to the work performed by the internal loads, or the change in its internal energy”* [113].

That means the internal work done by the internal loads is equal to the change in the system potential energy. By applying this concept to compliant mechanisms, the formulation of the method incorporates all the internal works generated in the individual flexure hinges. When an external force is applied statically (gradually) to a structure of flexure hinges, the following assumptions can be considered

- The kinetic energy of the mass of the structure and the heat energy are neglected,
- The force-displacement curve remains the same at loading and unloading, and
- The stresses are less than the elastic limit of the material.

Accordingly, the external work, W_{ext} , due to the external force, which is equal to the internal work, W_{int} , generated by the internal forces, will be completely stored in a form of potential (strain) energy, V . The term potential energy of a flexure hinge is the energy due to its deformation. When unloading the structure gradually, the flexure hinge returns to its original shape due to the recovering of the internal potential energy. That means the potential energy will be governed by the internal loads developed by the external load. The concept of conservation of energy can be mathematically expressed as

$$W_{ext} = W_{int} = V \quad (4.1)$$

When a flexure hinge deforms due to the action of an applied force, work is produced and stored in a form of internal energy. If the force is removed, the energy converts back into a work forcing the flexure hinge to unbend. That means the internal energy is a stored work. The internal work can be expressed as

$$W_{int} = \frac{1}{2} P' \delta \quad (4.2)$$

where P' is the internal loading and δ is the flexure hinge elongation.

In the following section, the work-energy method will be introduced in the context of compliant mechanisms. Equation (4.1) will be used to derive the total potential energy of the compliant mechanism. Equation (4.2) can be used to establish modelling of the total internal work of each flexure hinge.

4.1.1 Derivation of EBA

Considering a compliance-based mechanism of flexure hinges connected by rigid links, when the mechanism is subjected to an external force, deformations only occur in the flexure hinges. Thus, the input force will be transferred throughout the flexure hinges and the rigid links. Consequently, internal loadings will be developed in each flexure hinge. In a 3-dimensional (3-D) flexure hinge, a point is held in equilibrium by three forces (an axial force, F'_z , and two shearing forces, F'_x and F'_y), plus three moments (two bending moments, M'_x and M'_y , and a torsional moment, M'_z), as illustrated in Figure 4.1. The work generated by the internal loadings is stored in the flexure hinge as internal energy. Therefore, the total internal energy of the mechanism can be considered as the sum of the energies stored in the flexure hinges.

The flow of the process to obtain the potential energy of the mechanism using the EBA starts by defining the equations of the internal loadings. Then, these equations can be used in formulating the internal work, which is equal to the potential energy.

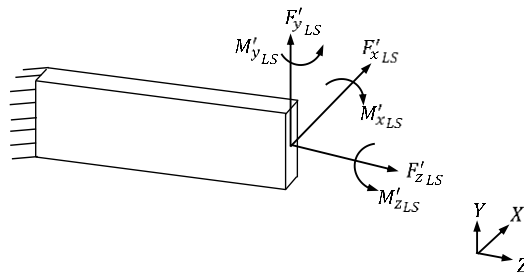


Figure 4.1 Internal loadings in LS flexure hinge.

Assuming a linear relationship between the applied force, F , and the translational displacement, d , or the bending moment, M , and the rotational displacement, θ , the following two equations hold

$$d = C F = \frac{F}{K} \quad (4.3)$$

$$\theta = C M = \frac{M}{K} \quad (4.4)$$

where C and K are the compliance and stiffness of the flexure hinge, respectively.

The external work in (4.1) can be written as a function of the stiffness or the compliance as follows

$$W_{ext} = \frac{1}{2} K d^2 \quad (4.5)$$

$$= \frac{1}{2} C F^2 \quad (4.6)$$

Using i as a general index for a respective flexure hinge, which is a part of a compliant mechanism consists of n flexure hinges, an external work, W_{ext} , is produced when the mechanism is subjected to an input force, F_{in} . By applying work-energy principle in (4.1), the total potential energy of the whole mechanism, V_{mech} , can be obtained as follows

$$W_{ext} = V_{mech} = \sum_i^n V_i \quad (4.7)$$

where V_i is the potential energy of the i^{th} flexure hinge.

Using the relationship in (4.5) and (4.6), the total potential energy of the mechanism in (4.7) can be expressed as

$$V_{mech} = \frac{1}{2} C_{in} F_{in}^2 \quad (4.8)$$

$$= \frac{1}{2} K_{in} d_{in}^2 \quad (4.9)$$

where C_{in} and K_{in} are the input compliance and input stiffness of the mechanism, respectively.

To obtain an expression for the potential energy, V_i , of the i^{th} flexure hinge, it is required first to determine the internal work, W_{int_i} , generated in each flexure hinge. Assuming that the flexure hinge is thick enough to prevent any out of plane displacements, the internal work generated in each flexure hinge is the total internal work associated with the internal loadings (Axial, F'_a , and shear, F'_{sh} , forces, and bending moment, M'_b) of each flexure hinge [113], as shown in Figure 4.2. This can be obtained from the following expression

$$W_{int_i} = W_{a_i} + W_{sh_i} + W_{b_i} \quad (4.10)$$

$$= \frac{1}{2} F'_{a_i} d_{a_i} + \frac{1}{2} F'_{sh_i} d_{sh_i} + \frac{1}{2} M'_{b_i} \theta_{b_i} \quad (4.11)$$

where W_{a_i} , W_{sh_i} , and W_{b_i} are the internal works and d_{a_i} , d_{sh_i} , and θ_{b_i} are the displacements related to the axial force, F'_{a_i} , shear force, F'_{sh_i} , and bending moment, M'_{b_i} , of the i^{th} flexure hinge, respectively.

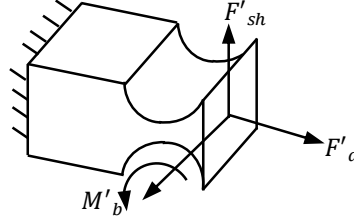


Figure 4.2 Internal loadings of SARC flexure hinge.

The internal loadings of the i^{th} flexure hinge can be defined as a function of an input force as follows

$$F'_{a_i} = \beta_{a_i} F_{in} \quad (4.12)$$

$$F'_{sh_i} = \beta_{sh_i} F_{in} \quad (4.13)$$

$$M'_{b_i} = \beta_{b_i} F_{in} \quad (4.14)$$

where β_{a_i} , β_{sh_i} , and β_{b_i} are internal loading factors, which depend on the support reactions acting on the i flexure hinge.

The generic displacement equations due to the internal loadings of each flexure hinge can be written as follows [91]:

The displacement corresponding to the internal axial force is

$$d_a = C_{d_a - F_{a_i}} F'_{a_i} \quad (4.15)$$

The displacement associated with shear internal force is

$$d_{sh_i} = C_{d_{sh} - F_{sh_i}} F'_{sh_i} + C_{d_{sh} - M_{b_i}} M'_{b_i} \quad (4.16)$$

The rotational displacement resulted from the internal bending moment is

$$\theta_b = C_{d_{sh} - M_{b_i}} F'_{sh_i} + C_{\theta_b - M_{b_i}} M'_{b_i} \quad (4.17)$$

where $C_{d_a - F_{a_i}}$, $C_{d_{sh} - F_{sh_i}}$, $C_{d_{sh} - M_{b_i}}$, and $C_{\theta_b - M_{b_i}}$ are the compliance factors and their values completely depend on the dimensions and the material of the i flexure hinge.

Replacing (4.15), (4.16), and (4.17) into (4.11), the total internal work done can be expressed as

$$W_{int_i} = \frac{1}{2} C_{d_a - F_{a_i}} F_{a_i}'^2 + \frac{1}{2} C_{d_{sh} - F_{sh_i}} F_{sh_i}'^2 + C_{d_{sh} - M_{b_i}} F_{sh_i}' M_{b_i}' + \frac{1}{2} C_{\theta_b - M_{b_i}} M_{b_i}'^2 \quad (4.18)$$

By substituting (4.12), (4.13), and (4.14) into (4.18) and applying the concept of conservation of energy in (4.1), the potential energy of the i flexure hinge as a function of the input force, F_{in} , can be formulated as follows

$$\begin{aligned}
 V_i &= W_{int_i} \\
 &= \frac{1}{2} \left[\beta_{a_i}^2 C_{d_a - F_{a_i}} + \beta_{sh_i}^2 C_{d_{sh} - F_{sh_i}} + 2\beta_{sh_i}\beta_{b_i} C_{d_{sh} - M_{b_i}} \right. \\
 &\quad \left. + \beta_{b_i}^2 C_{\theta_b - M_{b_i}} \right] F_{in}^2 \\
 &= \frac{1}{2} C_i F_{in}^2
 \end{aligned} \tag{4.19}$$

where C_i is the total compliance of the i flexure hinge, which can be written as follows

$$C_i = \beta_{a_i}^2 C_{d_a - F_{a_i}} + \beta_{sh_i}^2 C_{d_{sh} - F_{sh_i}} + 2\beta_{sh_i}\beta_{b_i} C_{d_{sh} - M_{b_i}} + \beta_{b_i}^2 C_{\theta_b - M_{b_i}} \tag{4.20}$$

Using the inverse relationship between the stiffness and the compliance, the total stiffness K_i of the i^{th} flexure hinge can be obtained from the following

$$K_i = 1/C_i \tag{4.21}$$

Using the relationship in (4.21), the potential energy of each flexure hinge can be rewritten as follows

$$V_i = \frac{1}{2} K_i d_{in}^2 \tag{4.22}$$

Substituting (4.22) into (4.7) the total potential energy of the whole mechanism can be computed from the following

$$V_{mechanism} = \frac{1}{2} \sum_i^n K_i d_{in}^2 = \frac{1}{2} K_{in} d_{in}^2 \tag{4.23}$$

The expression in (4.19) is considered as the generic potential energy equation of the EBA. Furthermore, it appears that most of the analysis in this method is only expressing all the internal loadings of the flexure hinges as a function of the input force, F_{in} .

4.1.2 Framework of EBA

By applying an external force on the compliance-based mechanism, internal loadings and support reactions are generated in the flexure hinges and the connection points to base, respectively. To derive an expression for the total potential energy of the mechanism to be used in Lagrange's equation using the EBA, it is required first to express the potential energy as a function of the input force.

From (4.18), the potential energy is a function of the internal loadings. Furthermore, in the force analysis of the mechanism, these internal loadings can be expressed in terms of the support reactions, which are functions of the input forces. Accordingly, the process of obtaining the potential energy can be started in opposite way expressing all the support reactions as a function of the input force.

The procedure for modelling the potential energy of a compliance-based mechanism by the EBA involves the following steps:

1. Free-Body Diagram

Draw a free-body diagram, define a system of coordinates keeping the external loads acting on the mechanism, and determine all the support reactions in the diagram.

2. Equations of Internal Loadings

For each flexure hinge, draw a free-body diagram. Use the section method to define the internal loadings at the ends of the flexure hinge by passing a section perpendicular to the centroidal axis of the flexure hinge at the point of deflection. Then, determine all the internal loadings by applying the equations of equilibrium to each flexure hinge. The expressions of the internal loadings will be expressed as a function of all the support reactions acting on the flexure hinge.

3. Unknown Support Reactions

All the unknown support reactions must be expressed as a function of a known term which is the input force. Then, substitute the support reactions, which are a function of the input force, into the internal loading expressions achieved in step 2 to obtain the internal loading factors. Accordingly, the internal loadings will be stated as a function of the input force. If the mechanism is statically indeterminate or has a big number of unknown support reactions, find only the unknown support reactions needed for computing the internal loads and their corresponding displacements. This can be done by applying the equations of equilibrium and conditions of the mechanism. The number of the conditions required depends on the number of degree-of-indeterminacy.

4. Equations of potential energy

Determine the potential energy equation of each flexure hinge by using the generic potential energy equation. The equations are defined in terms of the internal loading factors, the compliances and the input force. Replace the value of the internal loading factors, which is derived in step 3, in the potential equations. Then the same equations can be changed in terms of the total stiffness and the input displacement by using the inverse relationship between the compliance and the stiffness. The total potential energy for the mechanism can be obtained by joining all the potential energies of the flexure hinges.

4.1.3 Validation of EBA

A compound bridge-type flexure amplification mechanisms (FAM) is used as an application to validate the EBA. The compound bridge-type FAM is investigated in a previous research conducted by Qingsong and Yangmin [93]. The investigation included an analytical modelling, optimization, and testing of the compound bridge-type FAM, which is utilized to develop a nano-positioning stage, as shown in Figure 4.3.

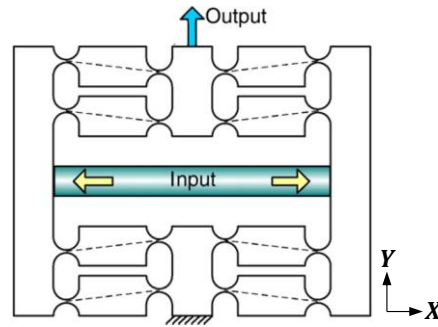


Figure 4.3 Schematic model of the compound bridge-type FAM [93].

An analytical model of the natural frequency was built based on the Lagrangian approach while assuming the flexure hinge as a 1-DOF rotational compliance. Then, it was used to optimize the parameters of the structure to obtain a large natural frequency. The parameters of the structure are illustrated in Figure 4.4. The optimized parameters and the material properties are presented in Table 4.1. The value of the first natural frequency using the analytical model was 86.76Hz.

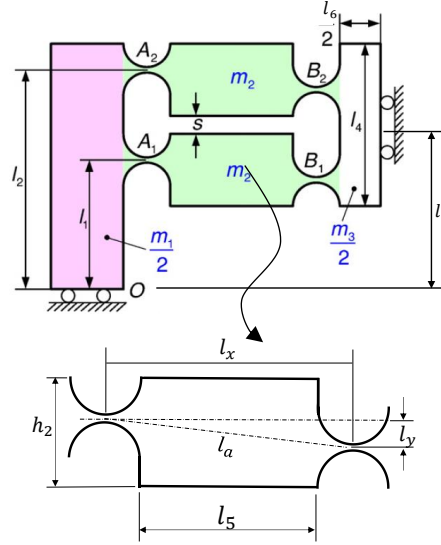


Figure 4.4 One quarter-model of the FAM [93].

Table 4.1 Geometric parameters and Material properties of bridge-type FAM.

Geometric parameters			
Parameter	Value (m)	Parameter	Value (m)
t_{SARC}	0.0008	l_1	0.03466
r_{SARC}	0.004	l_2	0.046
b_{SARC}	0.0127	l_3	0.02466
h_1	0.006	l_4	0.02466
h_2	0.01158	l_5	0.05
l_a	0.06206	l_6	0.006
l_y	0.00278	Amp	12.44
l_x	0.054		
Material properties of Al7075-T651			
Young's modulus E		Yield strength σ	Density ρ
71.7GPa		503MPa	2810Kg/m ³

The analytical model of the potential energy regarding 1-DOF derived by Qingsong and Yangmin is given as follows

$$V_{FAM}^{Q\&Y} = \frac{2 \text{ Amp}^2}{l_a^2 C_{\theta_z - M_z SARC}} \cdot d_{in}^2 = K_{FAM}^{Q\&Y} \cdot d_{in}^2 \quad (4.24)$$

where Amp is amplification of the FAM, l_a is a geometric parameter shown in Figure 4.4, $C_{\theta_z - M_z SARC}$ is the rotational compliance of single-axis right-circular (SARC) flexure hinge, and $K_{FAM}^{Q\&Y}$ is the stiffness of the flexure amplification mechanism (FAM) formulated by Qingsong and Yangmin.

It is clear from the above equation that the potential energy of the previous research [93] considering only the rotational displacement of the SARC flexure hinge (1-DOF). However, using the EBA, the potential energy obtained here is considering the three in-plane displacements in the flexure hinge (3-DOF). The three displacements are associated with the axial and shear forces and the bending moment generated in each flexure hinge of the FAM. The FAM has a double symmetrical structure, only one-quarter of the mechanism can be modelled, as shown in Figure 4.4.

4.1.3.1 Potential Energy Modelling via EBA

The procedure for modelling the potential energy of the FAM by the EBA involves the following steps:

1. Free-Body Diagram

The free-body diagram of the model is illustrated in Figure 4.5. In the diagram, the coordinate system, the input force, and the support reactions are defined. Based on the XYZ coordinate system, the axial force, the shear force, and bending moment are acting on the X, Y, and Z axes, respectively.

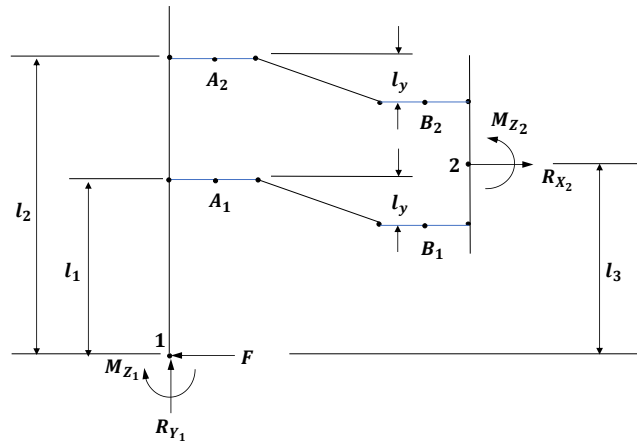


Figure 4.5 Free-body diagram of a quarter-model of the FAM.

From Figure 4.5, the mechanism has 4 unknowns and 3 equilibrium equations. Therefore, it is statically indeterminate with one degree of indeterminacy. However, some of the support reactions can be obtained by applying the equations of equilibrium, as follows:

$$\begin{aligned}\Sigma F_X &= 0 \\ R_{X_2} &= F\end{aligned}\tag{4.25}$$

$$\begin{aligned}\Sigma F_Y &= 0 \\ R_{Y_1} &= 0\end{aligned}\tag{4.26}$$

Taking the moment about point 1

$$\begin{aligned}\Sigma M_{Z_1} &= 0 \\ -M_{Z_1} + M_{Z_2} - R_{X_2} \cdot l_3 &= 0\end{aligned}\tag{4.27}$$

Substituting (4.25) into (4.27) and solving for M_{Z_2}

$$M_{Z_2} = M_{Z_1} + l_3 F\tag{4.28}$$

2. Equations of Internal Loadings

From Figure 4.5, the two kinematic chains, A_1B_1 and A_2B_2 , have the same output displacements because they are in parallel. Therefore, the displacements of flexure hinge A_1 and A_2 or B_1 and B_2 must be equal. Accordingly, the internal loadings on flexure hinge A_1 and A_2 or B_1 and B_2 must also be equal. Using the section method at the point of deflection, a free-body diagram is presented to reveal the internal loadings, as shown in Figure 4.6.

The internal loadings as a function of the applied force, F , of the four flexure hinges can be determined as follows:

Flexure hinge A_1 and A_2

From Figure 4.6 (a), the internal loadings can be obtained by applying the equations of equilibrium to flexure hinge A_1, A_2 .

The internal axial force

$$\begin{aligned}\Sigma F_X &= 0 \\ F'_{X_{A_1}} + F'_{X_{A_2}} - F &= 0\end{aligned}$$

As $F'_{X_{A_1}} = F'_{X_{A_2}}$, the internal axial force can be written as follows

$$\therefore F'_{X_{A_1}} = F'_{X_{A_2}} = F'_{X_A} = \frac{1}{2}F\tag{4.29}$$

The internal shear force

$$\begin{aligned}\Sigma F_Y &= 0 \\ F'_{Y_{A_1}} + F'_{Y_{A_2}} - R_{Y_1} &= 0\end{aligned}\tag{4.30}$$

As $F'_{Y_{A_1}} = F'_{Y_{A_2}}$ and from (4.26), the internal shear force can be written as

$$F'_{Y_{A_1}} = F'_{Y_{A_2}} = F'_{Y_A} = R_{Y_1} = 0\tag{4.31}$$

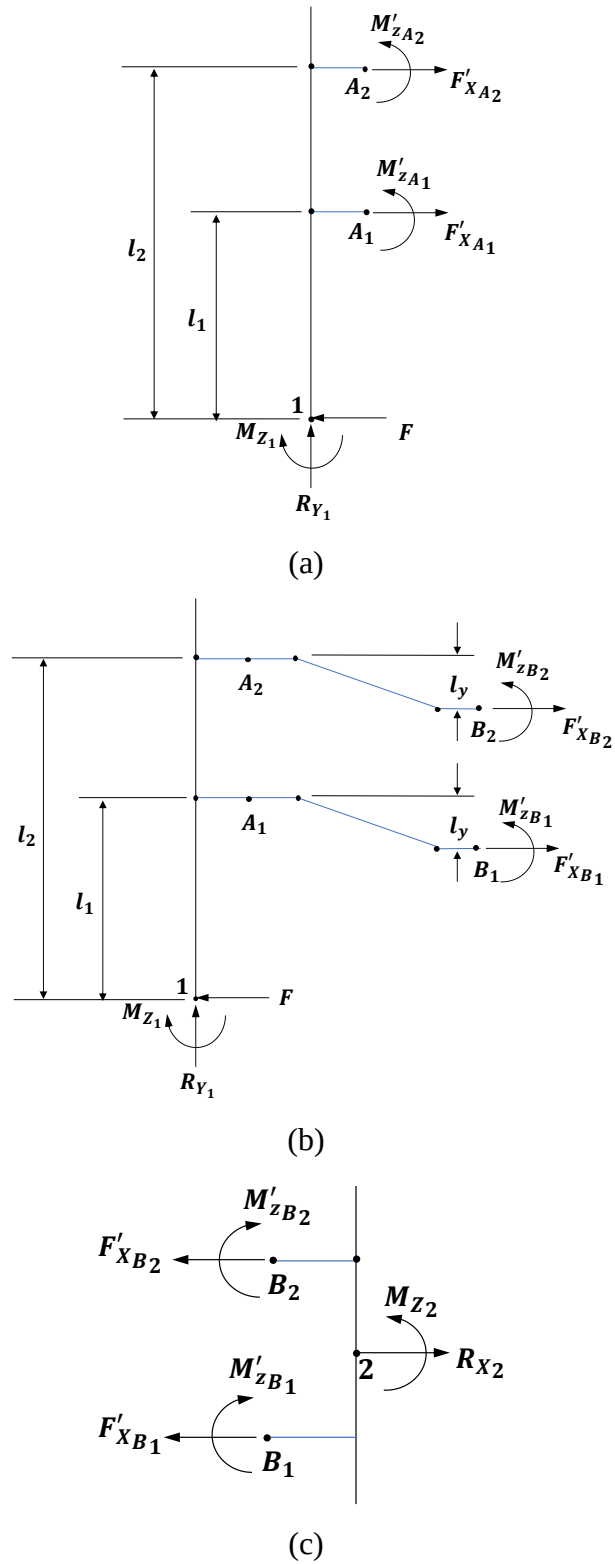


Figure 4.6 Free-body diagram using section method for: (a) Flexure hinge A_1 and A_2 , (b) Flexure hinge B_1 and B_2 regarding to point 1, and (c) Flexure hinge B_1 and B_2 regarding to point 2.

Taking the moment about 1

$$\begin{aligned}
 \sum M_Z &= 0 \\
 -M_{Z_1} + M'_{Z_{A_1}} + M'_{Z_{A_2}} - F'_{X_{A_1}} \cdot l_1 - F'_{X_{A_2}} \cdot l_2 &= 0 \\
 -M_{Z_1} + 2M'_{Z_A} - F'_{X_A}(l_1 + l_2) &= 0
 \end{aligned} \tag{4.32}$$

where $M'_{Z_{A_1}} = M'_{Z_{A_2}} = M'_{Z_A}$.

Substituting (4.29) into (4.32) and solving for M'_{Z_A}

$$M'_{Z_A} = \frac{1}{2} \left(\frac{1}{2} F(l_1 + l_2) + M_{Z_1} \right) \tag{4.33}$$

Flexure hinge B_1 and B_2

From Figure 4.6 (b), the internal loadings can be obtained by applying the equations of equilibrium to flexure hinge B_1 and B_2 .

The internal axial force

$$\begin{aligned}
 \sum F_X &= 0 \\
 F'_{X_{B_1}} = F'_{X_{B_2}} = F'_{X_B} &= \frac{1}{2} F
 \end{aligned} \tag{4.34}$$

As $F'_{Y_{B_1}} = F'_{Y_{B_2}}$, the internal shear force

$$\begin{aligned}
 \sum F_Y &= 0 \\
 F'_{Y_{B_1}} = F'_{Y_{B_2}} = F'_{Y_B} = R_{Y_1} &= 0
 \end{aligned} \tag{4.35}$$

Taking the moment at 1, the internal bending moment

$$\begin{aligned}
 \sum M_{Z_1} &= 0 \\
 -M_{Z_1} + M'_{Z_{B_1}} + M'_{Z_{B_2}} - F'_{X_{B_1}} \cdot (l_1 - l_y) - F'_{X_{B_2}} \cdot (l_2 - l_y) &= 0 \\
 -M_{Z_1} + 2M'_{Z_B} - F'_{X_B}(l_1 + l_2 - 2l_y) &= 0
 \end{aligned} \tag{4.36}$$

Replacing (4.34) into (4.36) and solving for M'_{Z_B}

$$M'_{Z_B} = \frac{1}{2} \left(\frac{1}{2} F(l_1 + l_2 - 2l_y) + M_{Z_1} \right) \tag{4.37}$$

3. Unknown Support Reactions

As the mechanism has one degree of indeterminacy, one more equation is required to obtain the unknown reaction. The unknown reaction M_{Z_1} , which appears in (4.33), and (4.37), must be expressed as a function of the input force. From Figure 4.6 (c), the bending moment reaction, M_{Z_2} , at point 2 can be expressed as follows

$$\begin{aligned}
 \sum M_Z &= 0 \\
 M_{Z_2} - 2M'_{Z_B} &= 0 \\
 M_{Z_2} &= 2M'_{Z_B}
 \end{aligned} \tag{4.38}$$

Replacing (4.28) into (4.38) and solving for M'_{Z_B}

$$M'_{Z_B} = \frac{1}{2}(M_{Z_1} + l_3 F) \quad (4.39)$$

At equilibrium state, (4.37) is equal to (4.39) and the following relation can be written

$$\left(\frac{1}{2}F(l_1 + l_2 - 2l_y) + M_{Z_1}\right) = -(M_{Z_1} + l_3 F)$$

Solving for the unknown bending support M_{Z_1}

$$M_{Z_1} = -\frac{1}{4}(l_1 + l_2 - 2l_y + 2l_3)F \quad (4.40)$$

The negative sign indicates that the assumed directional sense is in the opposite direction.

Now, the internal loadings can be expressed as a function of the applied force to obtain internal loading factors, as follows:

The internal axial forces for flexure hinge A and B can be written as follows, respectively,

$$F'_{X_A} = F'_{X_{A_1}} = F'_{X_{A_2}} = \frac{1}{2}F = \beta_{x_A}F \quad (4.41)$$

$$F'_{X_B} = F'_{X_{B_1}} = F'_{X_{B_2}} = \frac{1}{2}F = \beta_{x_B}F \quad (4.42)$$

where β_{x_A} and β_{x_B} internal axial load factors of the internal axial forces.

By applying (4.40) into (4.33) and (4.37), the internal bending moment of flexure hinge A and B can be computed from the following equations, respectively,

$$M'_{Z_A} = \frac{1}{4}\left(\frac{1}{2}(l_1 + l_2) + l_y - l_3\right)F = \beta_{Z_A}F \quad (4.43)$$

$$M'_{Z_B} = \frac{1}{4}\left(\frac{1}{2}(l_1 + l_2) - l_y - l_3\right)F = \beta_{Z_B}F \quad (4.44)$$

where β_{Z_A} and β_{Z_B} internal loading factors of the internal bending moments.

4. Equations of potential energy

The potential energy equation for each flexure hinge can be achieved by using the expression in (4.19). The equation can be defined as a function of the input force by replacing the internal loading factors derived in (4.41) - (4.44), while considering that the axial force, the shear force, and bending moment are acting on the X, Y, and Z axes, respectively. And, it can be derived as follows:

The potential energy of flexure A₁ and A₂

The potential energy of flexure A₁ and A₂ can be obtained by substituting (4.41) and (4.43) into (4.19), as follows

$$\begin{aligned} V_A &= \frac{1}{2} \left[\beta_{x_A}^2 C_{d_X - F_{X_{SARC}}} + \beta_{Z_A}^2 C_{\theta_Z - M_{Z_{SARC}}} \right] F^2 \\ &= \frac{1}{2} \left[\frac{1}{4} \left(C_{d_X - F_{X_{SARC}}} + \frac{1}{4} \left(\frac{1}{2}(l_1 + l_2) + l_y - l_3 \right)^2 C_{\theta_Z - M_{Z_{SARC}}} \right) \right] F^2 \end{aligned} \quad (4.45)$$

$$= \frac{1}{2} C_A F^2$$

where C_A is the total compliance of flexure hinge A_1 and A_2 , which can be written as follows

$$C_A = \frac{1}{4} \left(C_{d_X - F_{X_{SARC}}} + \frac{1}{4} \left(\frac{1}{2} (l_1 + l_2) + l_y - l_3 \right)^2 C_{\theta_Z - M_{Z_{SARC}}} \right) \quad (4.46)$$

The potential energy of flexure B_1 and B_2

The potential energy of flexure B_1 and B_2 can be obtained by replacing (4.42) and (4.44) into (4.19), as follows

$$\begin{aligned} V_B &= \frac{1}{2} \left[\beta_{X_B}^2 C_{d_X - F_{X_{SARC}}} + \beta_{Z_B}^2 C_{\theta_Z - M_{Z_{SARC}}} \right] F^2 \\ &= \frac{1}{2} \left[\frac{1}{4} \left(C_{d_X - F_{X_{SARC}}} + \frac{1}{4} \left(\frac{1}{2} (l_1 + l_2) - l_y - l_6 \right)^2 C_{\theta_Z - M_{Z_{SARC}}} \right) \right] F^2 \\ &= \frac{1}{2} C_B F^2 \end{aligned} \quad (4.47)$$

where C_B is the total compliance of flexure hinge B_1 and B_2 , which can be expressed as follows

$$C_B = \frac{1}{4} \left(C_{d_X - F_{X_{SARC}}} + \frac{1}{4} \left(\frac{1}{2} (l_1 + l_2) - l_y - l_6 \right)^2 C_{\theta_Z - M_{Z_{SARC}}} \right) \quad (4.48)$$

The compound bridge-type FAM consists of eight parallel kinematic chains. Each chain contains two flexure hinges A and B , which are in serial connection. Therefore, using the inverse relationship between the compliance and the stiffness, the total potential energy of the FAM can be expressed in terms of the input displacement by combining (4.46) and (4.48) as follows

$$V_{FAM} = \frac{1}{2} [8(C_A + C_B)] F^2 = \frac{1}{2} \left[\frac{1}{2} (C_A + C_B) \right] F_{in}^2 = \frac{1}{2} K_{FAM} d_{in}^2 \quad (4.49)$$

where, K_{FAM} is the input stiffness of the FAM and F_{in} is the input force. From Figure 4.3, the input force of the actuator is

$$F_{in} = 4 F \quad (4.50)$$

The value of K_{FAM} totally depends on the geometric parameters and the material properties, which are provided in Table 4.1. It can be computed as follows:

- According to the coordinate system, the axial, shear, and bending movement occur at the Z, X, and Y axes, respectively. Hence, the equations of compliance factors of SARC flexure hinge can be obtained from Table 2.3. Using the parameters in Table 4.1, the values of the compliance factors are shown in Table 4.2.

Table 4.2 Values of compliance factors.

Compliance factor	Equation	Value
$C_{d_x-F_x}$	$\frac{1}{Ew_{SARC}} \left(\pi \left(\frac{r_{SARC}}{t_{SARC}} \right)^{\frac{1}{2}} - \frac{\pi}{2} \right)$	9.89809E-07(m/N)
$C_{\theta_z-M_z}$	$\frac{9\pi r_{SARC}^{\frac{1}{2}}}{2Ew_{SARC} t_{SARC}^{\frac{5}{2}}}$	0.05422 (rad/N.m)

- From (4.46) and (4.48), the total compliances of flexure A and B are calculated by using the parameters in Table 4.1 and the compliance factors in Table 4.2. The values of the total compliances are given in Table 4.3.

Table 4.3 Values of total compliance of flexure hinge A and B.

Flexure hinge	Total compliance (m/N)
A	3.77785E-07
B	2.48726E-07

- From (4.49), the value of the input stiffness of the FAM is

$$K_{FAM} = 3.19228 \mu N/m \quad (4.51)$$

4.1.3.2 Dynamic Modelling

The natural frequency of the FAM, using the EBA, can be calculated from the following expression

$$f = \frac{1}{2\pi} \sqrt{\frac{K_{FAM}}{m_{FAM}}} \text{ (Hz)} \quad (4.52)$$

The value of m_{FAM} depends on the geometrical parameters and the material density ρ presented in Table 4.1. From the equivalent mass model built by Qingsong and Yangmin [93], is calculated from the kinetic energy equation, and the geometrical parameters in Table 4.1, the value of m_{FAM} is

$$m_{FAM} = \left(\frac{1}{4}m_1(1 + Amp^2) + \frac{1}{4}m_1\left(1 + \frac{31}{3}Amp^2\right) + \frac{1}{2}m_1Amp^2\right) = 8.59kg \quad (4.53)$$

The value of the natural frequency obtained by EBA is computed by assigning the value of (4.51) and (4.53) into (4.52), as follows

$$f^{EBA} = \frac{1}{2\pi} \sqrt{\frac{3192279.946}{8.59}} = 97.02 \text{ (Hz)} \quad (4.54)$$

4.1.3.3 Finite Element Analysis Simulation

The analytical natural frequency model of the FAM, which is built by Qingsong and Yangmin [93], was confirmed by the Finite Element Analysis (FEA)

simulation. However, the FEA simulation done by Qingsong and Yangmin [93] cannot be used to verify the two analytical (Qingsong and Yangmin and the EBA models). This is because the simulation was conducted with the consideration of the mass of the piezoelectric (PZT) actuator, whereas the two analytical models are built without regarding the actuator mass. Therefore, another the FEA simulation is conducted to evaluate the two models.

Using the dimensional parameters shown in Table 4.1, a Computer-Aided Design (CAD) model is built by Solidworks software, as shown in Figure 4.7 (a). Then, the meshed model, shown in Figure 4.7 (b), is simulated in ANSYS Workbench TM software package to obtain the natural frequency. The first natural frequency obtained is 104.64Hz as shown in Figure 4.8.

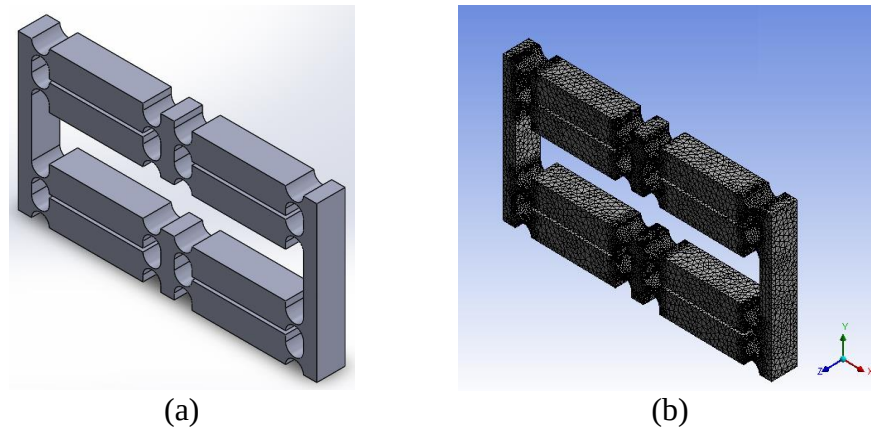


Figure 4.7 Compound bridge-type FAM (a) CAD model, and (b) Meshed model.

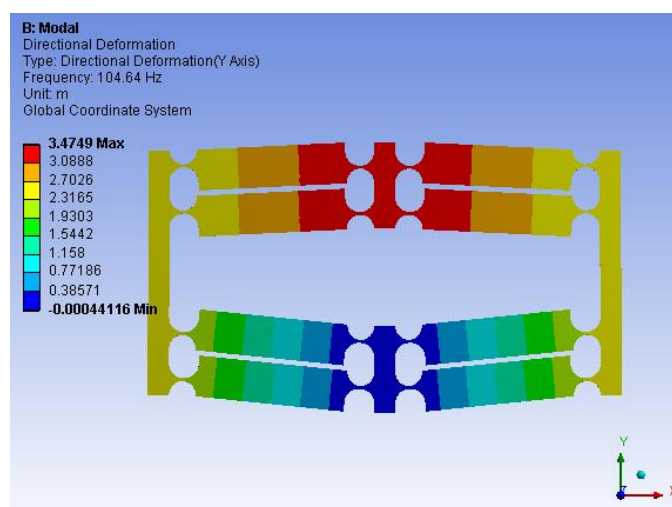


Figure 4.8 The first mode shape of the compound bridge-type FAM.

The results of the FEA simulation and the analytical equations are presented in Table 4.4.

Table 4.4 *Analytical model and FEA simulation results.*

	Qingsong and Yangmin	EBA	FEA
	Analytical model	Analytical model	
Natural Frequency (Hz)	86.76	97.02	104.64
Error (%)	17.1%	7.3%	-

Taking the FEA results as the benchmark, it is noted that the analytical results obtained by Qingsong and Yangmin and the EBA are within 17.1% error and 7.3% compared with the FEA result, respectively, as shown in Table 4.4. The following points can be concluded:

- The result of the analytical model considering only the rotational direction derived by Qingsong and Yangmin is far away from FEA result, which is attributed to neglect the effects of the internal loading in the axial directions.
- On the other hand, the EBA result is closest to FEA because all the internal loadings are considered. The deviation of 7.3% is mainly from the accuracy of compliance equations of the SARC flexure hinge used in the model.

4.1.3.4 Conclusions

It can be concluded that the analytical model of the natural frequency based on the EBA is more accurate than the one built by Qingsong and Yangmin. Although the EBA has longer steps compared to the traditional method, it is still simpler and more accurate. Hence, the EBA can be considered as an efficient approach for modelling the total potential energy of compliance-based mechanisms to be used in the Lagrange's equation.

4.2 Free Vibration Modelling

The dynamic model of the stage is obtained based on the Lagrangian approach, under the following assumptions:

- The flexure hinges and the links are modelled as springs and rigid bodies, respectively.
- The potential energy is generated from the elastic deformations of the flexure hinges. And, the kinetic energy is produced by the moving platform and the rigid links connecting the flexure hinges.
- The masses of the flexure hinges are not considered.

The natural frequency is a function of the total mass and the input stiffness. The purpose of using the Lagrangian approach is to extract the total mass and input stiffness from the developed equation. Figure 4.9 summarizes the main steps to obtain the natural frequencies of the stage. The generated kinetic and potential energies are expressed relative to the input-displacement variables after defining the generalized coordinates. Then, the values of the mass and stiffness of the stage are achieved from the Lagrange's equation to compute the natural frequencies.

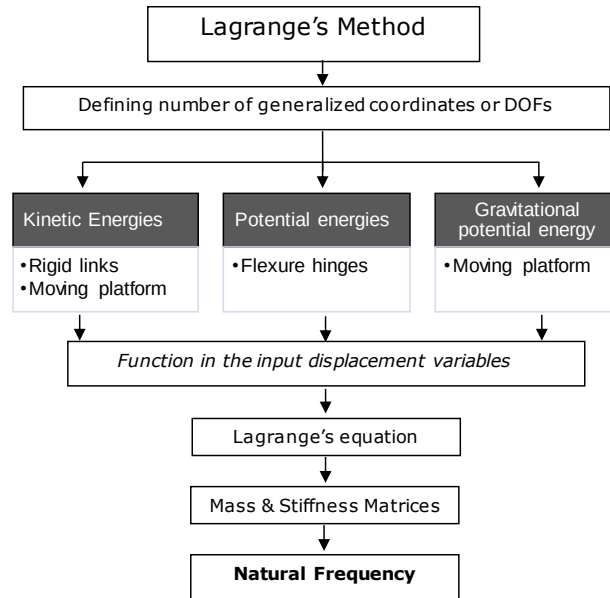


Figure 4.9 *Lagrange's Method.*

As the stage is a spatial mechanism, the gravity potential energy of the moving platform is considered. Since the structure of the FAMs is symmetric, only one FAM is modelled, as shown in Figure 4.10. In the case of rotational

displacement, only one or two actuators are operating. When the stage is in the translational motion, the three FAMs are active.

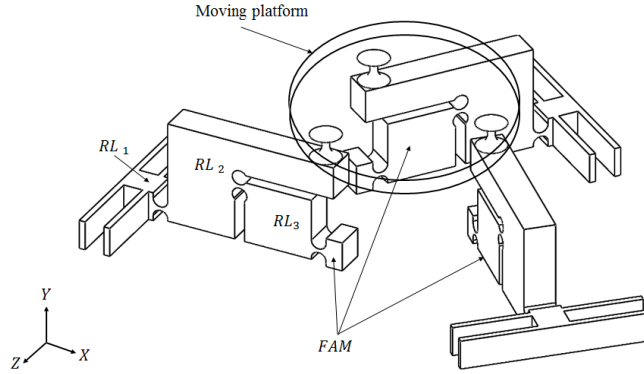


Figure 4.10 Schematic of the Configuration of FAMs.

4.2.1 Natural Frequency Equation

As the compliant flexure hinges are used in the structure of the stage, the system has no friction. Therefore, the damping is ignored in the dynamic model. Hence, the general dynamic equation in matrix form for undamped system is

$$[m] \ddot{d} + [K] d + GPE = Q \quad (4.55)$$

where $[m]$ and $[K]$ are the mass matrix and the stiffness matrix, respectively. $\ddot{d}$ and GPE are the acceleration and the gravitational potential energy, respectively. The dynamics equation describing undamped free vibration of the stage can be reduced to

$$[m] \ddot{d} + [K] d = 0 \quad (4.56)$$

From the theory of vibration, the modal equation representing the undamped free vibration of the system can be written as follows

$$(K - \omega_i^2 m) \Phi_i = 0 \quad (4.57)$$

where ω_i^2 is the circular natural frequency (for $i = 1, 2, \text{ and } 3$), Φ_i is the mode shape.

For a nonzero solution of Φ_i ,

$$\det(K - \omega_i^2 m) = 0 \quad (4.58)$$

The natural cyclic frequency can be obtained from the characteristic equation

$$\omega_i = \sqrt{K/m} \quad (4.59)$$

Hence, the natural frequency can be calculated as

$$f_i = \left(\frac{1}{2\pi} \right) \omega_i = \frac{1}{2\pi} \sqrt{\frac{K}{m}} \quad (4.60)$$

The number of the equations is equal to the number of the active actuators.

4.2.2 Concept of Lagrangian Method

Lagrange's equations, derived from Hamilton's principle, states that the variation of the kinetic energy and potential energy plus the line integral of the virtual work done by non-conservative forces during any time interval must be zero. The general form of the Lagrange's equation for undamped dynamic system is

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial \dot{q}_i} = Q_i \quad i = 1, 2, \dots, n \quad (4.61)$$

where

T is the total kinetic energy,

V is total potential energy, which includes both the strain energy and the potential of any conservative external forces,

q_i is the independent generalized coordinates or the DOF that describe the position of the system under dynamic excitation,

Q_i generalized non-conservative force, or the i th input force actuated by the PZT, (Q_i is zero in the case of the free response /modal analysis), and

n is the number of generalized coordinates. It is also equal to the DOF of a system (three coordinates for this stage).

This equation is applicable to holonomic systems. That means the number of DOFs is equal to the number of coordinates which are needed to describe the motion. The input-displacement variables, $d_{in\ i}$, of the three actuators are selected to be as the generalized coordinates, q_i . They can be expressed as follows

$$D_{in} = [d_{in1\ act} \ d_{in2\ act} \ d_{in3\ act}]^T \quad (4.62)$$

where $d_{in1\ act}$, $d_{in2\ act}$, and $d_{in3\ act}$ donates the input displacements of the three cases of motions (when one actuator is active, two actuators are active, or three actuators are active). As the stage has three DOFs, which are equal to the number of coordinates, the analysed system is holonomic.

The kinetic energy, T , and the potential energy, V , of the stage are expressed in terms of the generalized coordinates and their derivatives. Therefore, the Lagrange's equations of motion are formulated in terms of the selected coordinates.

The generic expression of the kinetic energy, T_i , of the stage can be written as follows

$$T_i = \frac{1}{2}(\zeta_i) \dot{d}_{in_i}^2 \quad (4.63)$$

where i represents 1 *act*, 2 *act*, or 3 *act*, and ζ represents the mass matrix.

The expression of the potential energy, V_i , of the stage can be written in a general form as follows

$$V_i = \frac{1}{2}\Psi d_{in_i}^2 + m_o g d_{in_i} \quad (4.64)$$

where Ψ is the input stiffness matrix, m_o is the mass of the moving platform, and g is the acceleration of gravity.

To calculate the terms of the Lagrangian equation, two derivatives are taken for (4.63), as follows:

The first term of (4.61)

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{d}_{in_i}} = \frac{d}{dt} \frac{\partial (\frac{1}{2}\zeta \dot{d}_{in_i}^2)}{\partial \dot{d}_{in_i}} = \frac{d(\zeta \dot{d}_{in_i})}{dt} = \zeta \ddot{d}_{in_i} \quad (4.65)$$

The second term of (4.61)

$$\frac{\partial T}{\partial d_{in_i}} = 0 \quad (4.66)$$

From (4.64), taking the partial derivative to obtain the stiffness term of Lagrangian equation

$$\frac{\partial V_i}{\partial d_{in_i}} = \frac{\partial (\frac{1}{2}\Psi d_{in_i}^2 + m_o g d_{in_i})}{\partial d_{in_i}} = \Psi d_{in_i} + GPE \quad (4.67)$$

By the substitution of (4.65), (4.66), and (4.67) into (4.61), we can obtain the following

$$\zeta \ddot{d}_{in_i} + \Psi d_{in_i} + GPE = Q_i \quad (4.68)$$

For undamped free vibration (4.68) can be rewritten as follows

$$\zeta \ddot{d}_{in_i} + \Psi d_{in_i} = 0 \quad (4.69)$$

Comparing (4.56) with (4.69), it is found that ζ represents the mass matrix $[m]$ while Ψ represents the input stiffness matrix $[K]$. Therefore, the task in the following analysis is formulating expressions for m and K to apply them in (4.60) to achieve the first three natural frequencies of the stage.

4.2.3 Kinetic Energy

The kinetic energy in the stage is generated by the moving platform, and the three rigid-links (RL_1 , RL_2 , and RL_3) connecting the flexure hinges at each FAM, as shown in Figure 4.10. The effects of the masses of the flexure hinges are neglected. The kinetic energy of the stage can be computed in the following subsections.

4.2.3.1 Kinetic Energy of Rigid-Links

The architectural parameters of the three rigid-links are described in Figure 4.11.

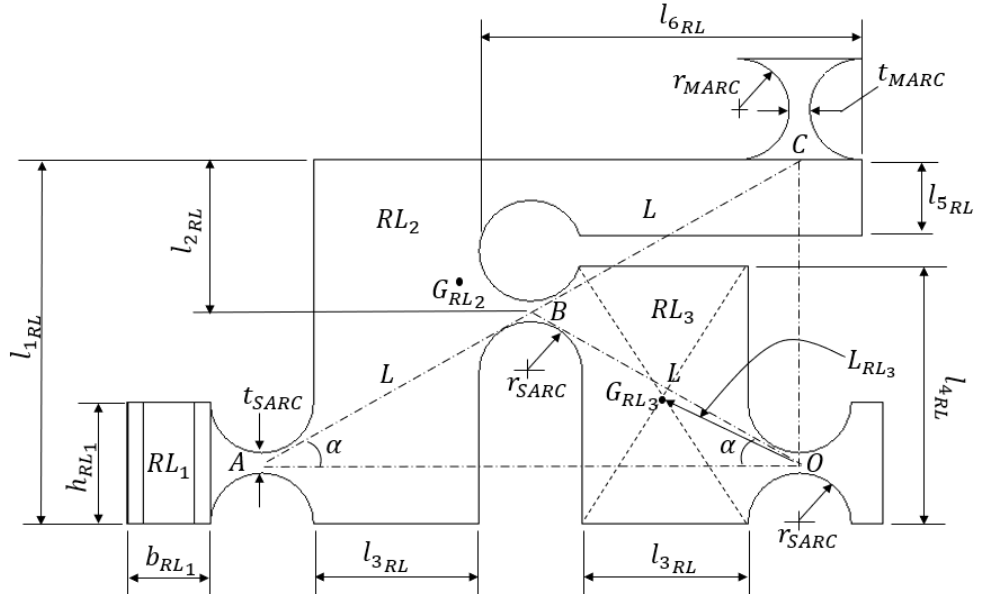


Figure 4.11 Architectural parameters of the rigid-links.

The kinetic energy can be formulated as follows:

Rigid-link RL_1

The motion of link RL_1 is a translational motion. Therefore, the kinetic energy is

$$T_{RL_1} = \frac{1}{2} m_{RL_1} \dot{d}_{in}^2 \quad (4.70)$$

where m_{RL_1} is the mass of RL_1 and $\dot{d}_{in}$ is the input linear velocity.

From Figure 4.11, the mass of link RL_1 can be computed from the following

$$m_{RL_1} = b_{RL_1} h_{RL_1} w_{FAM} \rho \quad (4.71)$$

Where w_{FAM} is the width of the FAM and ρ is the material density.

Rigid-link RL_2

Link RL_2 has both translation and rotational motions. Thus, the kinetic energy is

$$T_{RL_2} = \frac{1}{2} m_{RL_2} (\dot{d}_{RL_2})^2 + \frac{1}{2} I_{RL_2} (\omega_{RL_2})^2 \quad (4.72)$$

where m_{RL_2} is the mass, I_{RL_2} is the mass moment of inertia about point O , and $\dot{d}_{RL_2}$ and ω_{RL_2} the linear and angular velocities of RL_2 , respectively.

From Figure 4.11, the mass of RL_2 can be calculated approximately from the following expression

$$m_{RL_2} = (l_{1RL} l_{3RL} + l_{5RL} l_{6RL}) w_{FAM} \rho \quad (4.73)$$

where l_{1RL} , l_{3RL} , l_{5RL} , and l_{6RL} are architectural parameters defined in Figure 4.11.

The mass moment of inertia, I_{RL_2} , of RL_2 can be found by using the data given in Table 3.1 and expressed as follows

$$\begin{aligned} I_{RL_2} &= I_{RL_2C} + m_{RL_2} L_{G_{RL_2}}^2 \\ &= \frac{1}{12} m_{1RL_2} (l_{1RL}^2 + l_{3RL}^2) + m_{1RL_2} \left(\left(l_{GX} - \frac{1}{2} l_{3RL} \right)^2 + \left(l_{GY} - \frac{1}{2} l_{1RL} \right)^2 \right) \\ &\quad + \frac{1}{12} m_{2RL_2} (l_{5RL}^2 + l_{6RL}^2) + m_{2RL_2} \left(\left(l_{3RL} + \frac{1}{2} l_{6RL} - l_{GX} \right)^2 + \left(l_{GY} - \frac{1}{2} l_{5RL} \right)^2 \right) \\ &\quad + m_{RL_2} L_{G_{RL_2}}^2 \\ &= \left(\begin{aligned} &l_{1RL} l_{3RL} \left(\frac{1}{12} (l_{1RL}^2 + l_{3RL}^2) + \left(\left(l_{GX} - \frac{1}{2} l_{3RL} \right)^2 + \left(l_{GY} - \frac{1}{2} l_{1RL} \right)^2 \right) \right) \\ &+ l_{5RL} l_{6RL} \left(\frac{1}{12} (l_{5RL}^2 + l_{6RL}^2) + \left(\left(l_{3RL} + \frac{1}{2} l_{6RL} - l_{GX} \right)^2 + \left(l_{GY} - \frac{1}{2} l_{5RL} \right)^2 \right) \right) \end{aligned} \right) w_{FAM} \rho \\ &\quad + m_{RL_2} L_{G_{RL_2}}^2 \end{aligned} \quad (4.74)$$

where I_{RL_2C} is the mass moment of inertia about the centroid G_{RL_2} , $L_{G_{RL_2}}$ is the radius of rotation of RL_2 . l_{GX} and l_{GY} are the distances of $L_{G_{RL_2}}$ from point D as defined in Table 3.1.

Substituting the angular and linear velocities in (3.12) and (3.18) into (4.72), respectively, the kinetic energy of link RL_2 can be expressed as function of $\dot{d}_{in}$ as follows

$$\begin{aligned} T_{RL_2} &= \frac{1}{2} m_{RL_2} (L_{G_{RL_2}} \omega_{RL_2})^2 + \frac{1}{2} I_{RL_2} (\omega_{RL_2})^2 \\ &= \frac{1}{2} (m_{RL_2} L_{G_{RL_2}}^2 + I_{RL_2}) \left(\frac{1}{2L \sin \alpha} \dot{d}_{in} \right)^2 \\ &= \frac{1}{2} \left(\frac{(m_{RL_2} L_{G_{RL_2}}^2 + I_{RL_2})}{4L^2 \sin^2 \alpha} \right) \dot{d}_{in}^2 \end{aligned} \quad (4.75)$$

Rigid-link RL_3

Link RL_3 goes only through rotational motion about point O , as shown in Figure 4.11. Therefore, the kinetic energy

$$T_{RL_3} = \frac{1}{2} I_{RL_3} (\omega_{RL_3})^2 \quad (4.76)$$

where, I_{RL_3} is the mass of inertia of RL_3 about point O and ω_{RL_3} is angular velocities of RL_3 . Using Figure 4.11, it can be expressed as follows

$$\begin{aligned} I_{RL_3} &= I_{RL_3C} + m_{RL_3} L_{G_{RL_3}}^2 = \frac{1}{12} m_{RL_3} (l_{3RL}^2 + l_{4RL}^2) + m_{RL_3} L_{G_{RL_3}}^2 \\ &= m_{RL_3} \left(\frac{1}{12} (l_{3RL}^2 + l_{4RL}^2) + \left(\frac{1}{2} l_{3RL} + r_{SARC} \right)^2 + \left(\frac{1}{2} l_{4RL} - r_{SARC} - \frac{1}{2} t_{SARC} \right)^2 \right) \end{aligned} \quad (4.77)$$

where I_{RL_3C} is the mass moment of inertia about the centroid G_{RL_3} , $L_{G_{RL_3}}$ is the radius of rotation of RL_3 , l_{4RL} is architectural parameters defined in Figure 4.11, and m_{RL_3} is the mass of link RL_3 , which can be calculated from the following equation

$$m_{RL_3} = l_{3RL} l_{4RL} w_{FAM} \rho \quad (4.78)$$

Substituting (3.14) into (4.76), the kinetic energy of link RL_3 can be written as

$$T_{RL_3} = \frac{1}{2} I_{RL_3} \left(\frac{1}{2L \sin \alpha} \dot{d}_{in} \right)^2 = \frac{1}{2} \left(\frac{I_{RL_3}}{4L^2 \sin^2 \alpha} \right) \dot{d}_{in}^2 \quad (4.79)$$

The total kinetic energies of the rigid-links

The total kinetic energy is the summation of the kinetic energies of the rigid links, as follows

$$T_{RLS} = T_{RL_1} + T_{RL_2} + T_{RL_3} \quad (4.80)$$

By combining (4.70), (4.75), and (4.79) in (4.80), the total kinetic energies of the FAM can be achieved from the following

$$\begin{aligned} T_{FAM} &= \frac{1}{2} \left(m_{RL_1} + \frac{(m_{RL_2} L_{G_{RL_2}}^2 + I_{RL_2})}{4L^2 \sin^2 \alpha} + \left(\frac{I_{RL_3}}{4L^2 \sin^2 \alpha} \right) \right) \dot{d}_{in}^2 \\ &= \frac{1}{2} \left(m_{RL_1} + \frac{1}{4L^2 \sin^2 \alpha} (m_{RL_2} L_{G_{RL_2}}^2 + I_{RL_2} + I_{RL_3}) \right) \dot{d}_{in}^2 \\ &= \frac{1}{2} m_{RLS} \dot{d}_{in}^2 \end{aligned} \quad (4.81)$$

where m_{RLS} is the mass of the rigid links, which can be written as

$$m_{RLS} = m_{RL_1} + \frac{1}{4L^2 \sin^2 \alpha} (m_{RL_2} L_{G_{RL_2}}^2 + I_{RL_2} + I_{RL_3}) \quad (4.82)$$

4.2.3.2 Kinetic Energy of Moving Platform

The kinetic energy of the moving platform due to the rotational motion can be written as

$$T_{O_{rot}} = \frac{1}{2} I_O (\omega_O)^2 \quad (4.83)$$

where I_O and ω_O are the mass moment of inertia and the angular velocity of the moving platform, respectively.

The moving platform rotates about its edge. When the rotational motion of the platform is generated from one or two active actuators, the moments of inertias of the two cases are approximately equal because of the symmetrical structure of the stage. Considering the moving platform rotating about one edge, the mass of inertia of the moving platform can be obtained approximately as follows

$$I_O = I_c + m_O (r_O)^2 = \frac{1}{2} m_O r_O^2 + m_O (r_O)^2 = \frac{3}{2} m_O r_O^2 \quad (4.84)$$

where m_O and r_O are the radius and the mass of the moving platform, respectively, and I_c is the mass of inertia about the centroid.

The mass of the moving platform can be computed in terms of the material density, ρ , the radius, r_O , and the thickness, t_O , as follows

$$m_O = \pi r_O^2 t_O \rho \quad (4.85)$$

The kinetic energy of the moving platform due to the rotational displacement can be expressed as a function of the input displacement by applying (3.10), (3.11), and (4.84) into (4.83), as follows

$$\begin{aligned} T_{O_{1\ act,2\ act}} &= \frac{1}{2} \left(\frac{3}{2} m_O r_O^2 \right) \left(\frac{\dot{d}_{out\ i}}{L_O} \right)^2 = \frac{1}{2} \left(\frac{3}{2} m_O \left(\frac{r_O}{1.5r_O} \right)^2 Amp^2 \right) \dot{d}_{in\ i}^2 \\ &= \frac{1}{2} \left(\frac{2}{3} m_O Amp^2 \right) \dot{d}_{in\ i}^2 = \frac{1}{2} m_{O_{1\ act,2\ act}} \dot{d}_{in\ i}^2 \end{aligned} \quad (4.86)$$

where $m_{O_{1\ act,2\ act}}$ is the mass of the moving platform when one or two actuators are active, which can be written as

$$m_{O_{1\ act,2\ act}} = \frac{2}{3} m_O Amp^2 \quad (4.87)$$

Using (3.5), the kinetic energy occurring due to the translation motion of the centre of mass of the moving platform, is defined by

$$T_{O_{3\ act}} = \frac{1}{2} m_O \dot{d}_{out\ i}^2 = \frac{1}{2} (m_O Amp^2) \dot{d}_{in\ i}^2 = \frac{1}{2} m_{O_{3\ act}} \dot{d}_{in\ i}^2 \quad (4.88)$$

where $m_{O_{3\ act}}$ is the mass of the moving platform when three actuators are active, which can be written as

$$m_{O_{3\ act}} = m_O Amp^2 \quad (4.89)$$

4.2.3.3 Total Kinetic Energy

The kinetic energy of the whole stage is the sum of (4.81) with (4.86) or (4.88) as follows

$$T = T_O + \sum_{i=1}^n T_{FAM_i} \quad (4.90)$$

where T_{FAM_i} is the mass of the i^{th} FAM and n donates for the number of the FAMs.

From (4.65), when the input displacements of the FAMs are equal, a general expression for an equivalent mass, m_{stage} , for a parallel stage is

$$m_{stage} = m_O + \sum_{i=1}^n m_{FAM_i} \quad (4.91)$$

where m_{FAM_i} is the mass of the i^{th} FAM.

As the mass of three FAMs are identical, the total mass of the stage depending on the number of the active actuators, which can be derived as follows:

One active actuator

For rotational motion of the moving platform while one actuator is active, the total mass can be obtained by applying (4.82) and (4.86) into (4.91) and written as follows

$$m_{1\ act} = \frac{2}{3} m_O Amp^2 + m_{RL_1} + \frac{1}{4L^2 \sin^2 \alpha} (m_{RL_2} L_{G_{RL_2}}^2 + I_{RL_2} + I_{RL_3}) \quad (4.92)$$

Two active actuators

For rotational motion of the moving platform while two actuators are active, the total mass achieved by substituting (4.82) and (4.86) into (4.91) is

$$m_{2\ act} = \frac{2}{3} m_O Amp^2 + 2 \left(m_{RL_1} + \frac{1}{4L^2 \sin^2 \alpha} (m_{RL_2} L_{G_{RL_2}}^2 + I_{RL_2} + I_{RL_3}) \right) \quad (4.93)$$

Three active actuators

For translation motion of the moving platform where three actuators are active and the three FAMs are identical, the total mass of the stage, which can be found by plugging (4.82) and (4.89) into (4.91), is as follows

$$m_{3\ act} = m_O Amp^2 + 3 \left(m_{RL_1} + \frac{1}{4L^2 \sin^2 \alpha} (m_{RL_2} L_{G_{RL_2}}^2 + I_{RL_2} + I_{RL_3}) \right) \quad (4.94)$$

4.2.4 Potential Energy

The potential energy of the stage includes the strain energy stored in the flexure hinges due to their elastic deformations and the inertia of the moving platform.

As the vertical stage has out of plane motions, the gravitational potential energy is taken into consideration. When the stage is in the vertical motion, the potential energy derived considering the flexure hinge as a 1-DOF element (1-DOFM) can be found in **Appendix C**. In the following sections, the outlines of formulating the potential energy for the three cases of motions via EBA is presented, and the detailed derivation is documented in **Appendix D**.

Potential Energy Modelling via EBA

From the conceptual design of the stage, the mechanism is totally decoupled by utilizing three multi-axis right-circular (MARC) flexure hinges during the operation of the actuators. Moreover, the three stiffness values of the three FAMs are identical, as the structure design is symmetric. Consequently, the potential energy of each FAM has the same value. Figure 4.12, which is equivalent to Figure 4.10, presents the numbering of the flexure hinges.

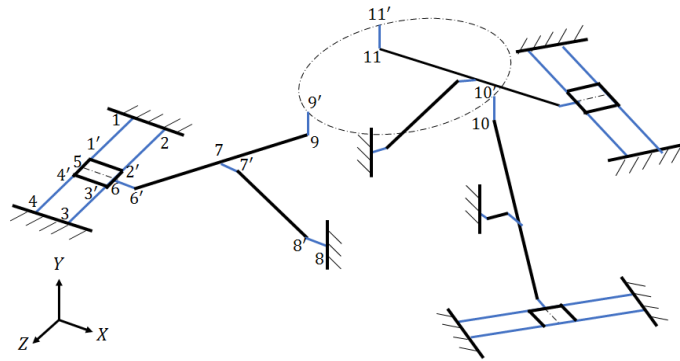


Figure 4.12 An equivalent schematic of FAMs representing the flexure hinges.

The potential energy modelling is divided into a rotational modelling and a translational modelling. The rotational modelling consists of two cases: (1) one active actuator and (2) two active actuators. In the translational modelling, three actuators are active.

The flexure hinges are thick enough to resist any out of the plane motion; therefore, only the internal axial, shear force and bending moment acting on the flexure hinges are considered. The internal loadings acting on the three types of flexure hinges and their coordinates are shown in Figure 4.13.

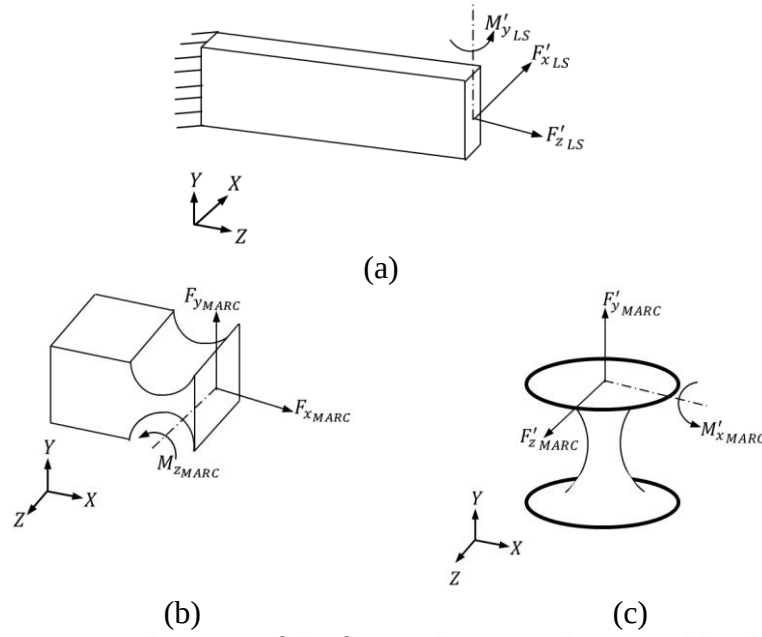


Figure 4.13 Schematic of the flexure hinges with internal loadings acting on: (a) LS, (b) SARC, and (c) MARC.

One Active Actuator

The total potential energy of the stage can be formulated as follows

$$V_{1act} = \sum V_i = \frac{1}{2} \left[\sum C_i \right] F_{in}^2 = \frac{1}{2} C_{1act} F_{in}^2 = \frac{1}{2} \frac{1}{K_{1act}} F_{in}^2 \quad (4.95)$$

where, i is LS, 6,7,8, and MARC flexure hinges shown in Figure 4.12, C_i is the total compliance and stiffness of the i^{th} flexure hinge, and C_{1act} and K_{1act} are the total compliance and the total stiffness of stage, respectively.

From (4.23), the total potential energy of the stage as a function of the input displacement is

$$V_{1act} = \frac{1}{2} K_{1act} d_{in}^2 \quad (4.96)$$

Based on the way of connections shown in Figure 4.12, the total stiffness of the flexure hinges in the stage

$$K_{1act} = \left[C_{LS_{eq}} + C_6 + C_7 + \left[C_8^{-1} + \left[C_9 + \frac{C_{10,11}}{2} \right]^{-1} \right]^{-1} \right]^{-1} \quad (4.97)$$

where $C_{LS_{eq}}$, C_6 , C_7 , C_8 , C_9 , and $C_{10,11}$ can be obtained from (D.91), (D.93), (D.95), (D.97), (D.99), (D.101), respectively.

Two Active Actuators

The total potential energy of the flexure hinges in one FAM as a function of the input displacement is

$$V_{FAM2act} = \frac{1}{2} K_{FAM2act} d_{in}^2 \quad (4.98)$$

where $K_{FAM2\ act}$ is the input stiffness of one FAM.

Based on the way of connections shown in Figure 4.12, the total stiffness of the flexure hinges in the FAM

$$K_{FAM2\ act} = \left[C_{LSeq} + C_6 + C_7 + \left[C_8^{-1} + [C_{11} + C_{92\ act}]^{-1} \right]^{-1} \right]^{-1} \quad (4.99)$$

where $C_{92\ act}$ is the compliance factor of flexure hinge 9, which can be obtained from (D.112) and C_{11} is half of the compliance of flexure hinge 11, which can be defined as

$$C_{11} = \frac{1}{2} C_{10,11} \quad (4.100)$$

When two FAMs are operating, the stiffness of the MARC flexure hinge of one inactive FAM must be considered. Accordingly, the total potential energy of the stage when two actuators are active is

$$V_{2\ act} = \frac{1}{2} (2K_{FAM2\ act}) d_{in}^2 = \frac{1}{2} K_{2\ act} d_{in}^2 \quad (4.101)$$

where $K_{2\ act}$ is the input stiffness of the stage when two actuators are active, which can be written as

$$K_{2\ act} = 2K_{FAM2\ act} \quad (4.102)$$

Three Active Actuators

The total potential energy of the flexure hinges in one FAM as a function of the input displacement is

$$V_{FAM3\ act} = \frac{1}{2} K_{FAM3\ act} d_{in}^2 \quad (4.103)$$

Based on the way of connections shown in Figure 4.12, the total stiffness of the flexure hinges in the FAM

$$K_{FAM3\ act} = \left[C_{LSeq} + C_6 + C_7 + C_8 \right]^{-1} \quad (4.104)$$

The potential energy of the moving platform only comes from its gravity. It must be considered once the translation motion along the vertical direction is activated. Therefore, the gravitational potential energy of the moving platform can be computed as

$$GPE_O = m_O g d_{out} = m_O g Amp d_{in} \quad (4.105)$$

where g is the acceleration of gravity.

The total potential energy of the entire stage, which is the sum of (4.103) and (4.105) can be expressed as follows

$$V_{3\ act} = \frac{1}{2} (3K_{FAM3\ act}) d_{in}^2 + m_O g Amp d_{in} \quad (4.106)$$

where $K_{3\ act}$ represents the input stiffness of the stage when three actuators are active expressed as follows

$$K_{3 \text{ act}} = 3K_{FAM3 \text{ act}} \quad (4.107)$$

4.2.5 Natural Frequency Equations

The number of the natural frequency equations depends on the DOF of the stage. In addition, the three FAMs have the same structure.

One active actuator

By substituting (4.92) and (4.97) into Lagrange equation (4.69), the dynamic model can be achieved as follows

$$m_{1 \text{ act}} \ddot{d}_{in} + K_{1 \text{ act}} d_{in} = 0 \quad (4.108)$$

From (4.60) and (4.108), the natural frequency of the stage can be achieved from the following

$$f_{1 \text{ act}} = \frac{1}{2\pi} \sqrt{\frac{K_{1 \text{ act}}}{m_{1 \text{ act}}}} \quad (4.109)$$

Two active actuators

Substituting (4.93), and (4.102) into (4.69), the Lagrange equation of motion can be expressed as

$$m_{2 \text{ act}} \ddot{d}_{in} + K_{2 \text{ act}} d_{in} = 0 \quad (4.110)$$

Using (4.60) and (4.110), the natural frequency of the stage when two actuators are operating is

$$f_{2 \text{ act}} = \frac{1}{2\pi} \sqrt{\frac{K_{2 \text{ act}}}{m_{2 \text{ act}}}} \quad (4.111)$$

Three active actuators

Substituting (4.94), and (4.107) into (4.69), the equation of motion can be expressed as

$$m_{3 \text{ act}} \ddot{d}_{in} + K_{3 \text{ act}} d_{in} + GPE = 0 \quad (4.112)$$

For undamped free vibration, (4.112) can be rewritten as follows

$$m_{3 \text{ act}} \ddot{d}_{in} + K_{3 \text{ act}} d_{in} = 0 \quad (4.113)$$

From (4.60) and (4.113), the natural frequency can be achieved from the following

$$f_{3 \text{ act}} = \frac{1}{2\pi} \sqrt{\frac{K_{3 \text{ act}}}{m_{3 \text{ act}}}} \quad (4.114)$$

4.2.6 Evaluation of Dynamic Models

Three dynamic models are built according to the three cases of displacements (two rotations and one translation). The suitable architectural parameters of the stage described in Table 3.2 are used to examine the input stiffness of three models. The material used in the model is Aluminium alloy (Al – 7075) with parameters assigned in Table 3.3.

The support reaction in the shear direction of leaf-spring flexure hinge (LS) flexure hinge has a direct effect on the internal loadings of the flexure hinges. Therefore, the thickness of the LS flexure hinge is taken to assess the three stiffness models in (4.97), (4.99), and (4.104). The values of the stiffness of the three models are illustrated in Table 4.5 and plotted in Figure 4.14.

Table 4.5 *Values of the stiffness of the three models.*

Thickness (mm)	Stiffness (N/ μ m)		
	1-active	2-active	3-active
0.001	0.175069	0.153218	0.153218
0.0015	0.42974	0.407925	0.407925
0.002	0.920211	0.898391	0.898391
0.0025	1.715688	1.693864	1.693863
0.003	2.878355	2.856529	2.856529
0.0035	4.462155	4.440327	4.440327
0.004	6.512017	6.490188	6.490188
0.0045	9.063493	9.041664	9.041663
0.005	12.14274	12.12092	12.12092

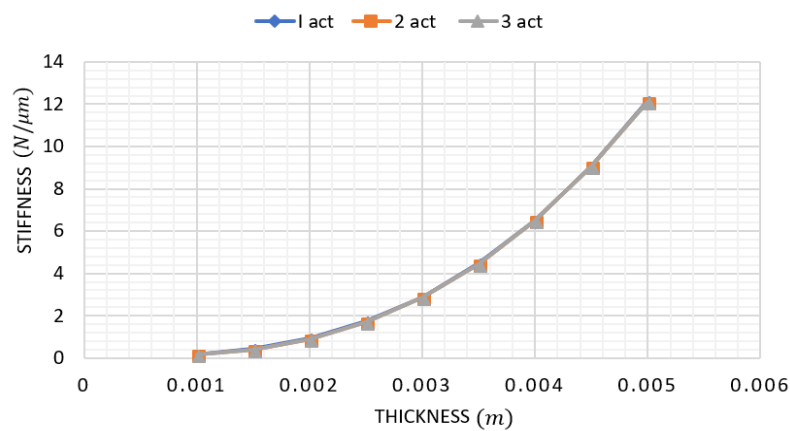


Figure 4.14 *Plots of the stiffness in terms of the thickness of LS flexure hinge.*

- Changing the thickness of the LS flexure hinge has a great effect on the stiffness. It appears that increasing the thickness aids the stiffness, but it also reduces the input displacement, as shown in (D.52), which rewritten as follows

$$d_{in\ LS} = \frac{1}{4} C_{LS} 4R_{xLS} = \frac{1}{4} (C_{dx-FxLS} - \frac{1}{2} L_{LS} C_{dx-MyLS}) 4R_{xLS} \quad (4.115)$$

where C_{LS} is the compliance factor of LS flexure hinge.

Accordingly, a careful selection of the minimum and maximum values for the thickness dimension in the design optimization must be considered.

As there is no significant difference between the three stiffness models, it can be assumed that the three stiffness values are equal. From (4.60), the natural frequency decreases with the increase of the mass. Comparing (4.92), (4.93), and (4.94), the mass model of the three-active actuators has the largest value, and accordingly it has the lowest natural frequency. As one of the objectives is to enhance the natural frequency of the stage, the natural frequency model of the three-active actuators in (4.114) will be chosen as an objective function for the optimization process.

4.3 Chapter Summary

The following points can be extracted from the previous analysis

- A developed method based on the conservation of energy (EBA) was proposed for modelling the potential energy of compliance-based mechanisms used in Lagrange's equation while considering the three types of the displacements.
- A previous research was used to validate the EBA. Taking the FEA results as the benchmark, the model of the previous research had a deviation of 17.1%, while the EBA model had a deviation of 7.3%. Therefore, it was approved that the EBA is as an effective approach.
- The analysis of parallel configurations using this approach will be very simple and easy. The calculations can be done manually or using just a spread sheet software. The simplicity of this method will assist several researchers to analyse the dynamic behaviour and the optimum design of

complicated structural compliant-based systems. It is applicable to serial or parallel systems.

- The only difficulty occurs when the system is statically indeterminate. However, this can be solved by establishing more equations using the displacement or force conditions in the mechanism. The number of these equations is equal to the number of the degree of determinacy.
- Although the EBA may take longer steps than other techniques, it is still simpler to use. It represents a straightforward and systematic approach for determination of potential energy for dynamic equations of motions. Therefore, it can be considered a cost-effective approach.
- Here in this research work, this method was adopted to model the natural frequencies of the stage. The stage had different dynamic characteristics due to the numbers of the DOFs. Three cases were considered: (1) One FAM is active, (2) Two FAMs are active, and (3) Three FAMs are active.
- As there was no significant difference between the stiffness values of the three cases and the mass model of the three active actuators, which had the largest value, the natural frequency model with three active actuators was selected.

This model will be used to optimize the parameter dimensions in **Chapter 5** and to examine the dynamic performance of the stage in **Chapter 6**.

Chapter 5 Design Optimization

The aims and the importance of this chapter are to:

- Obtain the optimal design parameters of the stage, which fulfil the objectives mentioned in **Chapter 1**, since the performance of the stage is mainly influenced by the design parameters. From the static and dynamic modelling, the design parameters are the geometrical dimensions and the material properties. Therefore, an optimization process is needed to optimize the design parameters considering the performance of the system. The objective of the optimization process is to maximize the natural frequency, in order to achieve better dynamic performance, which leads to robustness against several dynamic disturbances [56]. One of the reasons why the first natural frequency must be in a high value is that the system will not be stable during the operation if the control frequency is equal to the principle mode of vibration [56]. However, maximizing the natural frequency by reducing the size of the stage parts will decrease the travel range because of the reduction in the flexure hinge dimensions. Therefore, the objective of achieving a large travel range will be constrained, and the natural frequency will be set as the objective function of the optimization process.

Performing this process manually is very difficult due to the correlations between the design parameters and the performance of the stage. Therefore, MATLAB-Optimization Toolbox is used to maximize the objective function while satisfying the constraints.

- Select the material and the manufacturing process, which increases the affordability of the stage, and enable to fabricate the flexure amplification mechanism (FAM) as one piece.
- Select the piezoelectric (PZT) actuator with specifications allowing to achieve the required travel range of the stage.

These aims are satisfied throughout this chapter. The formulation of the design optimization problem, which includes the problem description, the data and the information collection, the design parameters definition, and the constraint

derivation, is introduced in **Section 5.1**. The processing of the optimum design and the final design results are presented in **Section 5.2**.

5.1 Design Problem Formulation

The static and dynamic models, derived in the previous chapters, provide the basis for understanding the influence of the design parameters on the performance of the system. Therefore, these models can be used to obtain the optimal design parameters of the stage that fulfil some of the objectives mentioned in Chapter 1 in terms of improving the natural frequency and the travel range of the stage.

Improving the natural frequency cannot be achieved without determining first the dimension limits of the stage components. Therefore, the optimum design process must start first by setting the dimension ranges for the geometric parameters of the stage, which includes the moving platform, the Scott-Russell amplification mechanism (SRAM), and the flexure hinges. Then, the natural frequency can be optimized using these dimension ranges. It is important to mention that the thickness ranges of all flexure hinges are governed by the material allowable stress.

Formulating the optimization problem is conducted using the following steps:

- Step 1: Problem Description
- Step 2: Information and Data Collection
- Step 3: Definition of Design Parameters
- Step 4: Formulation of Constraints

The four-step formulation procedure is detailed in the following sub-sections.

5.1.1 Problem Description

The criterion of the optimum design procedure is to maximize the first natural frequency of the system to improve the dynamic characteristics. From the natural frequency model achieved in Chapter 4, the objective function of the problem is a non-linear function. In addition, some restrictions are placed on the design problem to meet the performance requirements. The design variables, which describe the system, can have any value within their allowable range.

From the previous paragraph, the problem is classified as *constrained, continuous, nonlinear programming optimization problem*. The general idea of

the optimization process is to converge to a local minimum point starting from any point.

The selected design requirements are based on the literature review. These requirements are stated as follows:

- Travel range:
 - Translational motion range $> 190 \mu m$,
 - Rotational motion range $> 2 \mu rad$.
- Natural frequency $> 357 Hz$,
- Total height $< 60 mm$.

5.1.2 Information and Data Collection

To formulate the design optimization problem, some information and data are required. The information needed for the design problem includes expressions for the maximum stresses of the flexure hinges. Some data about the material properties, the machining capability, and the actuator specifications are also needed to formulate the design problem.

5.1.2.1 Maximum Stress

The stress of the flexure hinge must be examined to guarantee the safety and durability of the system. The maximum stress, σ_{max} , is induced by the maximum output of the PZT actuator. It occurs at the root of the four leaf-spring (LS) flexure hinges, the thinnest part of the single-axis right-circular (SARC) and the multi-axis right-circular (MARC) flexure hinges. The maximum stresses of the flexure hinges must be less than the allowable stress of the material, governed by

$$\sigma_{max} \leq \frac{\sigma_y}{S_f} \quad (5.1)$$

where σ_y is the yield strength of the material, and $S_f > 0$ is a safety factor (a safety factor of 2 is selected).

Maximum stress of leaf-spring flexure hinge

As the LS flexure hinges are connected directly to the PZT actuators, the maximum stress occurs at the roots of the hinge when the actuator reaches its maximum stroke. The maximum stress of one LS flexure hinge can be expressed as follows [114]

$$\sigma_{max_{LS}} = \frac{3Et_{LS}}{l_{LS}^2} d_{in_{max}} \quad (5.2)$$

where, $d_{in_{max}}$ is the maximum stroke of the PZT actuator, E is the Elastic modulus, and t_{LS} and l_{LS} are the thickness and length of the LS flexure hinge, respectively.

Maximum stress of single-axis right-circular flexure hinges

The maximum bending stress occurs at the thinnest part of the flexure hinge. The relationship between the maximum bending stress and the maximum rotational deformation of the SARC flexure hinge can be obtained from the following [67]:

$$\sigma_{max_{SARC}} = \frac{E(1 + \beta)^{9/20}}{\beta^2 \varepsilon_\beta} \theta_{SARC_{max}} \quad (5.3)$$

where $\beta = \frac{t}{2r}$ is a dimensionless geometry factor with a valid range of $0 < \beta < 2.3$, $\theta_{SARC_{max}} = \frac{d_{in_{max}}}{2L \sin \alpha}$ is the maximum rotational angle of the hinge found from (4.160), and ε_β is a dimensionless compliance factor defined by

$$\varepsilon_\beta = \frac{1}{2\beta + \beta^2} \left[\frac{3 + 4\beta + 2\beta^2}{(1 + \beta)(2\beta + \beta^2)} \right] + \left[\frac{6(1 + \beta)}{(2\beta + \beta^2)^{3/2}} \right] \tan^{-1} \left[\sqrt{\frac{2 + \beta}{\beta}} \right] \quad (5.4)$$

Maximum stress of multi-axis right-circular flexure hinge

The MARC flexure hinge is subjected to compressive and bending stresses. The compressive stress acts perpendicular to the sensitive axis of the flexure hinge, while the bending stress occurs at the thinnest part of the flexure hinge, which has a critical effect. Therefore, only the maximum bending stress is considered, which is given by [67]

$$\sigma_{max_{MARC}} = \frac{8k_{tb}Et_{MARC}^{1/2}}{5\pi r_{MARC}^{1/2}} \theta_{MARC_{max}} \quad (5.5)$$

where, k_{tb} is the stress concentration factor for bending, which can be selected as 1.2, $\theta_{max_{MARC}}$ is the maximum rotational displacement of the hinge. The stress concentration factors at the necks of the hinges are less than 1.2 when the hinge thickness-to-radius ratio does not exceed 0.5 [112].

The maximum rotational displacement of the MARC flexure hinge can be expressed by using (3.10) as follows

$$\theta_{MARC_{max}} = \frac{Amp}{L_o} d_{in_{max}} \quad (5.6)$$

where $d_{in_{max}}$ is the maximum displacement.

5.1.2.2 Manufacturing Process Selection

Wire-electric-discharge machining (WEDM) is the suitable manufacturing process because it is capable to fabricate the proposed FAM from a single monolithic piece. It is also used to manufacture the moving platform and the base. Turning machining process can be used to fabricate MARC flexure hinges. The machining tolerance of WEDM is given as 1.5mm .

5.1.2.3 Material Selection

Based on the literature review carried on Section 2.5, Alloy T7075 will be selected. The material properties of Aluminium Alloy T7075 are shown in Table 5.1 and 3.2.

Table 5.1 Material properties of Aluminium Alloy T7075.

Aluminium Alloy T7075	
Shear modulus	26.9 GPa
Density	2810 Kg/m ³
Poisson's Ratio	0.33
CTE, linear 250°C	25.2 $\mu\text{m}/\text{m} - ^\circ\text{C}$, Average over the range 20 – 300°C

5.1.2.4 Actuator Selection

The PZT actuator is selected based on the target travel range of the stage, which is more than $190\text{ }\mu\text{m}$. Although the travel range of the PZT actuator will be amplified by FAM, the maximum travel range must be large enough to overcome the displacement losses due to the stiffnesses of the flexure hinges. The PZT actuator produced by Physik Instrumente (PI) is selected with a maximum travel range of $180\text{ }\mu\text{m}$ and pushing force capacity of 4500N , which is the maximum force can be generated by the actuator. The specifications of the selected PZT actuator are shown in Table 5.2.

Table 5.2 Specifications of the PZT actuator.

	Travel range μm	Operating temperature range $^\circ\text{C}$	Resonant frequency (no load) kHz	Push force capacity N	Stiffness in motion direction $\text{N}/\mu\text{m}$
P-216.90	180	-40 - 80	32	4500	32

5.1.3 Definition of Design Parameters

The design parameters are divided into two groups: (a) Constant design parameters and (b) Continuous design parameters or design variables. The selection criteria for the values and the design limits of the constant design parameters and the design variables, respectively, are based on the two following considerations:

- The smallest thicknesses of flexure hinges are selected to increase the travel range,
- The masses of the rigid parts are reduced by selecting the smallest allowable size to increase the natural frequency.

Numerical values are set for the parameters that are chosen as constant design parameters. The definition of the design variables is based on the following considerations [115]:

- The design variables should be independent of each other as far as possible.
- The number of the independent design variables is reduced to give the design problem more DOF.
- A numerical limit is given for each design variable.

All the geometric parameters of the stage can be put into three groups: (a) the flexure hinge parameters, (b) the moving platform parameters, and (c) the SRAM parameters. The design ranges of the variable parameters are imposed by the travel range, the compact physical size, or the manufacturing tolerance. It is also important to ensure no out-of-the-plane displacements will occur in the flexure hinges when the actuators are operating.

5.1.3.1 Flexure hinges

The design ranges of the variable parameters are imposed by the minimum travel range, the compact physical size and the manufacturing tolerance [116]. The minimum thicknesses of the flexure hinges are set to enable the manufacturing process and the material treatment [61]. The maximum values of the design limits are determined according to the required size of the stage.

An estimated calculation is conducted to find a suitable dimension ranges for the design parameters. The estimation depends on the required travel range and the

whole size of the stage. The design limits of the three types of flexure hinges are set as follows:

LS flexure hinge

The LS flexure hinge has three geometric parameters, the height, h_{LS} , thickness, t_{LS} , and length, L_{LS} . The height, h_{LS} , is equal to the height of the base, h_{stage} , and the height of the rigid-link 1, h_{RL1} . To reduce the number of the independent design variables, the height, h_{LS} , is set as a constant parameter with a suitable value of 12 mm.

It has been approved in Section 4.2.6 that the increase of the thickness improves the stiffness of the stage. However, this increase will reduce the travel range of the stage, as shown in the following

$$d_{in\ max} = \frac{1}{4} \frac{l_{LS}^3}{E h_{LS} t_{LS}^3} F_{in\ max} \quad (5.7)$$

Therefore, a suitable value of 2 mm is selected for the thickness according to the minimum capability of machining. An approximated minimum value of 20 mm can be assigned for the design limit of the length, l_{LS} . The maximum value is chosen as 40 mm to avoid any twisting occur in the four LS flexure hinges.

SARC flexure hinge

The geometric parameters of the SARC flexure hinge are the width, w_{SARC} , thickness, t_{SARC} , and radius, r_{SARC} . The width, w_{SARC} , which is equal to the width of the SRAM, w_{SRM} , and the thickness of rigid-link 1, w_{RL1} , is chosen as a constant parameter with a value of 12 mm. The maximum allowable ratio between the thickness and radius is 0.5. To avoid any out-of-the-plane deformations and increase in the stiffness of the flexure hinge, a value of 0.4 for the thickness-to-radius ratio is chosen. The minimum value of the design limit, which is chosen as 2 mm, is set based on the minimum machining tolerance. The compliance of rotation expression, shown in Table 2.3, reflects the great effect of the thickness in the stiffness of the flexure hinge, which is given by

$$c_{\theta_b - M_b} = \frac{9\pi r_{SARC}^{\frac{1}{2}}}{2E w_{SARC} t_{SARC}^{\frac{5}{2}}} = \frac{9 * \pi * \sqrt{2.5}}{2 * E * (0.01) * (t_{SARC}^2)} \quad (5.8)$$

From (5.10), increasing the thickness will reduce the compliance, and accordingly, increase the stiffness of the flexure hinge, which has been approved

in Section 3.2.4. The maximum design limit of SARC flexure hinge is taken as 3 mm .

MARC flexure hinge

The geometric parameters of MARC flexure hinge are the thickness, t_{MARC} , and radius, r_{MARC} . The value of the thickness-to-radius ratio is chosen as 0.4. The thickness, t_{MARC} , does not have a large effect on the stiffness of the stage, as has been approved in Section 3.2.4. However, it has a direct effect on the stiffness of the flexure hinge. When the increase of the rotational displacement is the aim, the smaller the thickness the better. Therefore, the thickness is set as a constant parameter with a value of 2 mm to avoid any parasitic displacements in the moving platform.

5.1.3.2 Moving Platform

The moving platform has two geometric parameters: the thickness, t_0 , and the radius, r_0 . Increasing the values of the geometric parameters will increase the mass of the moving platform and consequently will reduce the natural frequency of the stage. Both the geometric parameters are chosen as constant parameters to reduce the number of the design variables. A suitable value of 5 mm is taken for the radius to reduce the mass. The thickness is selected with a value of 50 mm to avoid buckling.

The moving platform is subjected to bending moment when one or two FAMs are active. The Finite Element Analysis (FEA) test is established to evaluate the selected dimensions using ANSYS software. A 3-dimensional (3-D) model of the moving platform with the three MARC flexure hinges is built via Solidworks software, as presented in Figure 5.1. From Figure 5.2, no deformation occurred in the moving platform when it is subjected to the maximum force of PZT actuator.

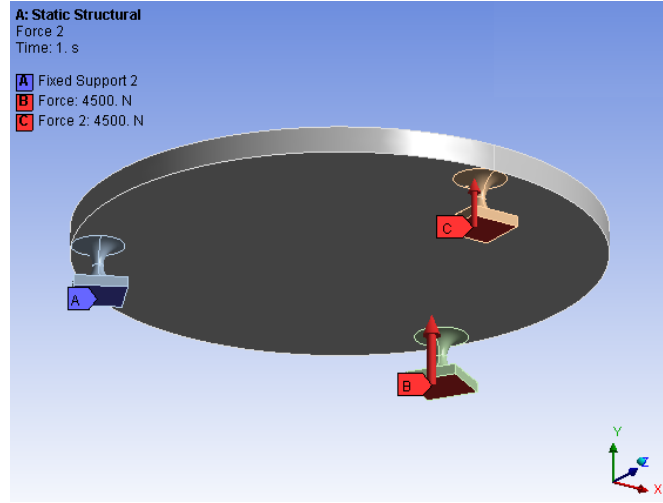


Figure 5.1 3-D Model of the moving platform.

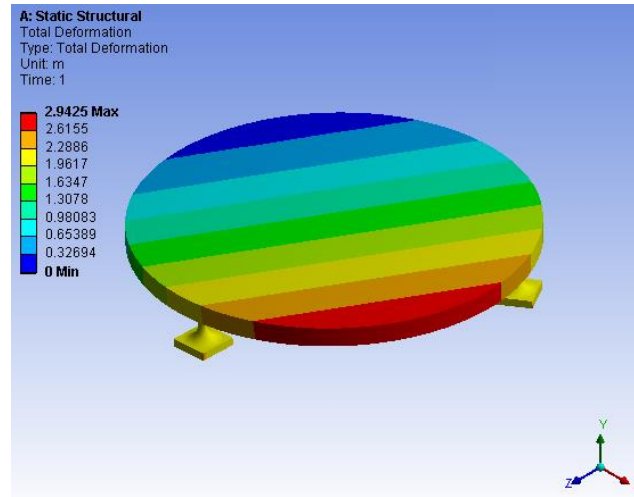


Figure 5.2 FEA Assessment of the selected dimensions of the moving platform.

5.1.3.3 Scott-Russell Mechanism

The geometric parameters of the SRAM are the angle, α , the length, L , and the width, w_{SRM} , which is equal to the width of the FAM, as shown in Figure 3.1. The design limits of the angle defined in Section 3.1.1 are within the following limits

$$20^\circ \leq \alpha \leq 30^\circ \quad (5.9)$$

From Figure 5.3, the stage height, h_{stage} , can be written as follows

$$h_{stage} = t_0 + D_{MARC} + 2L \sin \alpha + \frac{1}{2} h_{base} \quad (5.10)$$

The SRAM length, L , has a large influence on the stage height, and consequently, the weight of the stage. Therefore, the smaller the length the lighter the weight. To reduce the number of the independent design variables, the base height is

given definite value. Solving (5.10) for L while taking the minimum and maximum value of α is 20° and 30° and the maximum value of h_{stage} is 50 mm , the design limit of L is set as 30 mm and 50 mm .

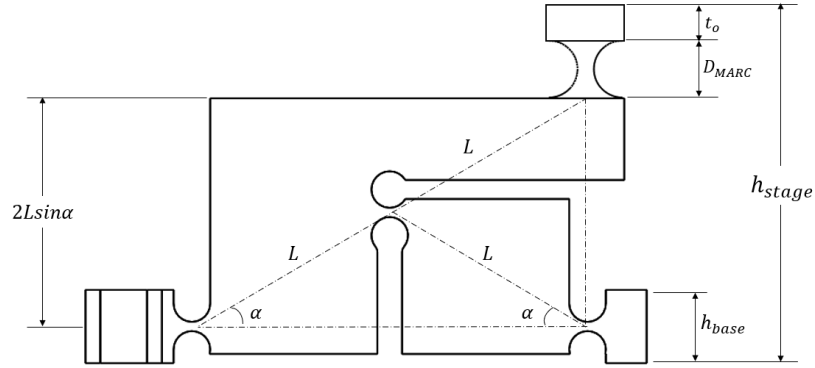


Figure 5.3 Geometric parameters of SAM.

5.1.3.4 Final Design Parameters

The constant design parameters, which include the material properties and the performance requirements, are given in Table 5.1 and 5.4, respectively. The continuous design variables are shown in Table 5.3.

Table 5.3 The constant design parameters.

Constant Parameters	Symbol	Value
Maximum travel range	d_{out}	$250 * 10^{-6}\text{m}$
Stiffness of the PZT	K_{PZA}	
Maximum displacement of PZT	$d_{PZA_{max}}$	$180 * 10^{-6}\text{m}$
Maximum force of PZT	$F_{PZA_{max}}$	4500N
Safety factor	S_f	2
Stress concentration factor	k_{tb}	1.2
Radius of the moving platform	r_0	50 mm
Thickness of the moving platform	t_0	5 mm
Thickness of LS flexure hinge	t_{LS}	2 mm
Width of rigid-link 1	w_{RL_1}	12 mm
Depth of rigid-link 1	b_{RL_1}	8 mm
Height of rigid-link 1	h_{RL_1}	12 mm
FAM width	w_{FAM}	12 mm
Thickness of MARC	t_{MARC}	2 mm
Radius of MARC	r_{MARC}	5 mm

Table 5.4 The continuous design variables.

Design variables		Symbol	Limits
LS flexure hinge	Length	l_{LS}	$10mm \leq l_{LS} \leq 30mm$
SARC flexure hinge	Thickness	t_{SARC}	$2mm \leq t_{SARC} \leq 3mm$
	Radius	r_{SARC}	$5mm \leq r_{SARC} \leq 7.5mm$
SRAM	Angle	α	$20^\circ \leq \alpha \leq 30^\circ$
	Length	L	$30\text{ mm} \leq L \leq 50mm$

5.1.4 Formulation of Constraints

The constraints of the design problem are inequality constraints. The constraint conditions of the optimization to be considered are:

a. Travel range

From the design requirements, the constraint of the travel range can be expressed as

$$d_{out} \leq \text{Amp } d_{PZT} \left(\frac{K_{PZA} + K_{FAM}}{K_{FAM}} \right) \quad (5.11)$$

b. Maximum stress

The yield stress of the stage must be considered. As the maximum stresses occur at the flexure hinges, the stresses of the flexure hinges must be less than the yield stress (allowable stresses of the hinges). The stress constraint can be written as follows:

$$\sigma_{max} \leq \frac{\sigma_y}{S_f} \quad (5.12)$$

or it can be rewritten as

$$S_f \sigma_{max} - \sigma_y \leq 0 \quad (5.13)$$

where, σ_{max} is calculated stress, σ_y is the yield stress, and S_f is a safety factor.

From (5.2), the stress constraint for LS is

$$S_f d_{in\ max} \leq \frac{l_{LSP}^2}{3Et_{LSP}} \sigma_y \quad (5.14)$$

From (5.3), the stress constraint for SARC is

$$S_f \theta_{SARC\ max} \leq \frac{\beta^2 \varepsilon_\beta}{E(1 + \beta)^{9/20}} \sigma_y \quad (5.15)$$

From (5.5), the stress constraint for MARC is

$$S_f \theta_{MARC\ max} \leq \frac{5\pi r_{MARC}^{1/2}}{8k_{tb} E t_{MARC}^{1/2}} \sigma_y \quad (5.16)$$

c. Input stiffness

The input stiffness must be less than the stiffness of the PZT actuator. A safety factor of 20% is used as follows

$$K_{in} \leq 20\%K_{PZA} \quad (5.17)$$

d. Geometric of the design variables

Considering the structural compactness and machining feasibility of the stage the constraint bounds of the variables are chosen as follows

$$0.03 \leq L \leq 0.05 \quad (5.18)$$

$$20 \leq \alpha \leq 30 \quad (5.19)$$

$$0.002 \leq t_{SARC} \leq 0.003 \quad (5.20)$$

$$0.02 \leq l_{LS} \leq 0.03 \quad (5.21)$$

5.2 Optimization Process

The objective of the process is to optimize the natural frequency for the optimum design. Due to the complicity between the design parameters, Optimization Toolbox™ with fmincon solver in MATLAB is used. This solver allows minimizing a function subject to linear and nonlinear constraints and bounds. As the aim of the optimization is to maximize the natural frequency and not to minimize, the objective function is set as the inverse of the natural frequency model built in (4.114).

By using (5.13)- (5.21) and (4.114), the design optimization model can be stated as follows:

Minimize

$$\mathbf{f}(l_{LS}, t_{SARC}, t_{MARC}, L) = \frac{1}{f}$$

Subject to

$$g_1 = d_{out} - Amp d_{PZA} \left(\frac{K_{PZA} + K_{FAM}}{K_{FAM}} \right) \leq 0$$

$$g_2 = S_f d_{in max} - \frac{l_{LSP}^2}{3Et_{LSP}} \sigma_y \leq 0$$

$$g_3 = S_f \theta_{SARC max} - \frac{\beta^2 \varepsilon_\beta}{E(1 + \beta)^{9/20}} \sigma_y \leq 0$$

$$g_4 = S_f \theta_{MARC max} - \frac{5\pi r_{MARC}^{1/2}}{8k_{tb} E t_{MARC}^{1/2}} \sigma_y \leq 0$$

$$g_5 = K_{in} - 0.2K_{PZA} \leq 0$$

and

$$0.03 \leq L \leq 0.05$$

$$20 \leq \alpha \leq 30$$

$$0.002 \leq t_{ARC} \leq 0.003$$

$$0.02 \leq l_{LS} \leq 0.03$$

The following steps summarize how the `fmincon` solver is used to minimize the objective function while satisfying the constraints [117]:

- Step 1: A file `ObjctvFreq` for the objective function is written, as documented in Appendix E.1.
- Step 2: A file `ConstrnFreq` for the constraints is written, as documented in Appendix E.2.
- Step 3: the optimization problem is set up and run with Optimization App.

The procedure of running the App is presented in the following points:

- `fmincon` is selected from the selection of solvers and the Algorithm is changed to *SQP* (Sequential Quadratic Programming).
- `@ObjctvFreq` is entered in the Objective function field to call the `ObjctvFreq.m` file.
- The minimum design limits [0.03 20 0.002 0.02] are entered in the Start point field to be used as the first start points.
- At the lower and Upper fields of Bounds, the minimum and maximum design limits of [0.03 20 0.002 0.02] and [0.05 30 0.003 0.03] are entered, respectively.
- The constraints are set by entering `@ConstrnFreq` in the Nonlinear constraint function field to call the `ConstrnFreq.m` file.

The solver starts running when clicking the start button. The optimal design results of the optimization process are given in Table 5.5. Several different starting points are used to guarantee the yield of the global minimum. Only the final start points used to achieve the optimum results are presented.

After several iterations, the optimization solver stopped when the objective function value reached its minimum value while satisfying the constraints, as shown in Figure 5.4. The final design values are achieved by rounding the

optimum values with the first natural frequency of 588.24 Hz, which is more than the design requirement, which is 357 Hz.

Table 5.5 Optimal design results.

Design variables		Symbol	Final start points	Optimum values	Design values
SRAM	Length	$L(m)$	0.05	0.030012	0.03
SRAM	Angle	α^0	20	29.986319	30
SARC flexure hinge	Thickness	$t_{SARC}(m)$	0.003	0.002005	0.002
LS flexure hinge	Length	$l_{LS}(m)$	0.01	0.02068	0.021
Dependent variables					
SARC flexure hinge	Radius	$r_{SARC}(m)$			0.005
Optimum natural frequency value (design values) = 588.24 Hz					

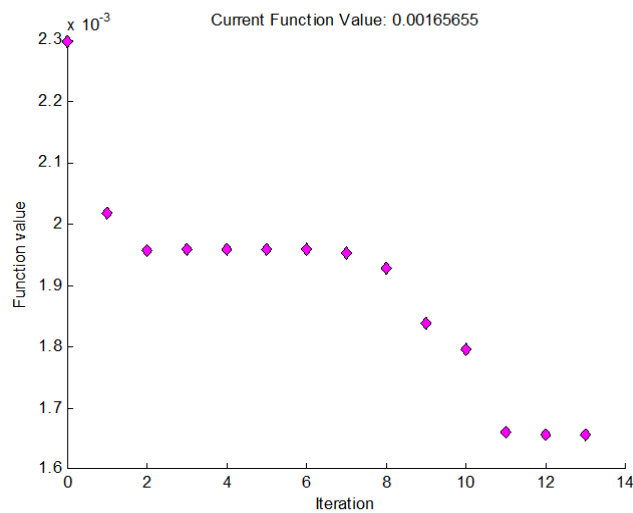


Figure 5.4 Optimization plot of the objective function.

5.3 Chapter Summary

An optimization process to obtain optimal design parameters was conducted. The following points summarise this process:

- The first natural frequency of the stage was set as the objective function of the optimization process since it must be large enough to improve the dynamic characteristics, and therefore to increase the bandwidth of the whole stage.
- Some information and data, which were needed in formulating the process, were identified and gathered. They were related to the manufacturing process, the material, and the actuator specifications:

- Wire electrical discharge machining was selected as the manufacturing process because it can fabricate the FAM from a single piece.
- Aluminium Alloy T7075 was selected because it had the lowest cost in comparison with other two materials.
- Three PZT actuators (P-025.150 from IP) were selected to actuate the stage. The selection of the actuators was based on their maximum travel range.
- The design parameters were determined as constant design parameters or continuous design parameters. The values for the former were assigned. The maximum and the minimum allowable dimensions of the later were defined.
- Expressions for the constraints, which depend on the design variables, were formulated. All the constraints were inequality constraints.
- Because of the complicated relations between the design parameters, which have direct effects on the stage performance, MATLAB software was used to implement the optimization process.
- The optimal design results were achieved with a first natural frequency of 588.24 Hz , which satisfied the design requirement.

The final dimensions will be used to build a 3-D model, which will be utilized to test the performance of the stage in **Chapter 6**.

Chapter 6 Finite Element Analysis Validation

The aims and the importance of this chapter are to:

- Validate the performance of the proposed stage by means of the Finite Element Analysis (FEA) simulation using ANSYS software. The reason for using FEA software is that it is more flexible and cheaper than the experimental work. FEA software allows to change the dimensions of the models easily and no need to build several models such as in the experimental work. The validation involves the analytical models established in **Chapter 3** and **4**. The analytical models consist of the input stiffness, the displacement amplification ratio, and the natural frequency. The characteristics of the stage include the deformation behaviour, the stress distribution, and the travel ranges of the proposed stage.
- Ensure that the design requirements are satisfied and the other performance parameters such as the travel ranges and the maximum load capacity are determined.
- Validate the developed method (EBA) by evaluating the analytical model of the natural frequency, which is basically built by using this method.
- Present a 3-dimensional (3-D) model of the out-of-horizontal-plane (OHP) stage with the final specifications.
- Fulfil some of the research objectives such as reducing the parasitic motions.

These aims are addressed throughout the chapter. First, the 3-D model is built using Solidworks software in **Section 6.1**. Then, the FEA is conducted in **Section 6.2**. It is divided into three subsections: deformation performance analysis in **Section 6.2.1**, static performance analysis in **Section 6.2.2**, and dynamic performance analysis in **Section 6.2.3**. After the validation, the final specifications of the OHP nano-positioning stage are listed in **Section 6.3**.

6.1 Three-Dimensional Modelling

Solidworks software is used to establish the 3-D model of the OHP stage because it is more flexible than using the design modeler of ANSYS-Workbench software, which is used in FEA simulations. This 3-D model is adopted throughout the FEA simulations. It is built using the optimized design parameters described in Table 6.1.

Table 6.1 Architectural parameters of OHP stage.

Design Parameters		Symbol	Dimensions
Moving platform	<i>Radius</i>	r_o	50 mm
	<i>Thickness</i>	t_o	5 mm
SRAM	<i>Angle</i>	α	30°
	<i>Length</i>	L	30 mm
Flexure hinges			
LS	<i>Length</i>	l_{LS}	21 mm
	<i>Thickness</i>	t_{LS}	2 mm
	<i>Height</i>	h_{LS}	12 mm
SARC	<i>Thickness</i>	t_{SARC}	2 mm
	<i>Radius</i>	r_{SARC}	5 mm
MARC	<i>Thickness</i>	t_{MARC}	2 mm
	<i>Radius</i>	r_{MARC}	5 mm
Rigid-Link 1	<i>Width</i>	w_{RL_1}	8 mm
	<i>Depth</i>	b_{RL_1}	12 mm
	<i>Height</i>	h_{RL_1}	12 mm
FAM	<i>Width</i>	w_{FAM}	12 mm
Base	<i>Thickness</i>	t_{base}	12 mm

Figure 6.1 (a) represents a CAD model of the OHP stage. The wireframe view of the stage, which is shown in Figure 6.1 (b), displays the components of the stage, which are the flexure amplification mechanisms (FAMs) and the moving platform. The base of the stage is disposed to expose the leaf-spring flexure hinge (LS) flexure hinges.

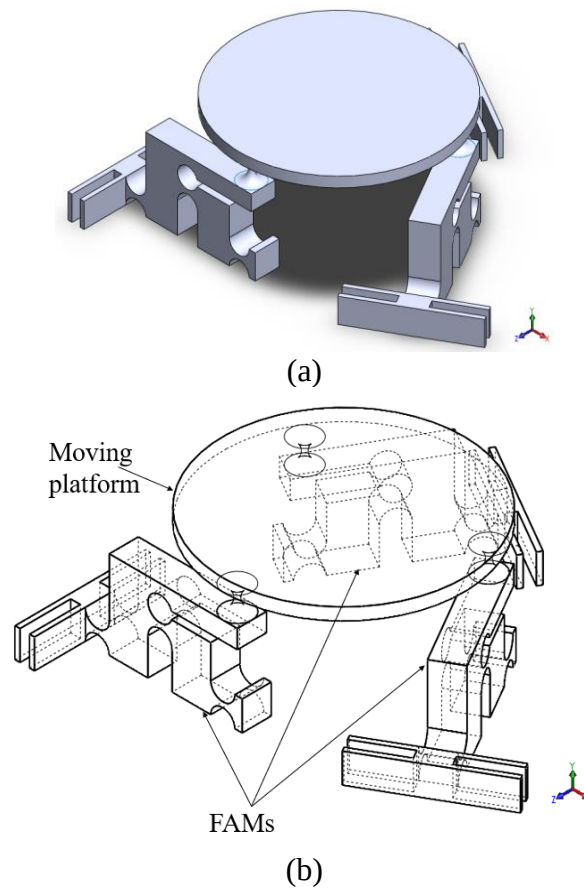


Figure 6.1 A CAD model of OHP stage: (a) Solid view and (b) Wireframe view.

6.2 Finite Element Analysis

The analysis process of the stage performance is done by using the FEA. The performance analysis of the stage is divided into three phases: (1) Static, (2) Dynamic, and (3) Deformation. Each phase starts first by verifying the FEA model to provide an accurate FEA results, then it is followed by a validation process to check the FEA results against the analytical one. The FEA simulation is conducted via ANSYS-Workbench software.

A 3-D finite element (FE) model is established in ANSYS-Workbench by importing the CAD model of the stage from Solidworks software. The material used in the model was Aluminium alloy (A17075) with the properties listed in Tables 5.1 and 5.2, and all the stage parts have the same material parameters.

6.2.1 Static Performance Analysis

The static analysis is performed to validate the input stiffness and the displacement amplification ratio models of the OHP stage. The 3-D model, which is built by Solidworks software is imported to ANSYS, as appeared in Figure 6.2.

A more refined mesh is added by inserting *Refinements* at each flexure hinge surface to obtain a finer mesh; consequently, more accurate results are achieved. The auto meshing with fine *Sizing* is used in the 3-D model after increasing the *Relevance* option to +100 for a finer meshing. The meshed model is shown in Figure 6.3. As the base of the stage blocks the view when the simulation is conducted, fixed supports are inserted at the faces of the LS hinges, which are connected to the base of the stage, as presented in Figure 6.4. These boundary conditions prevent these faces from moving or deforming.

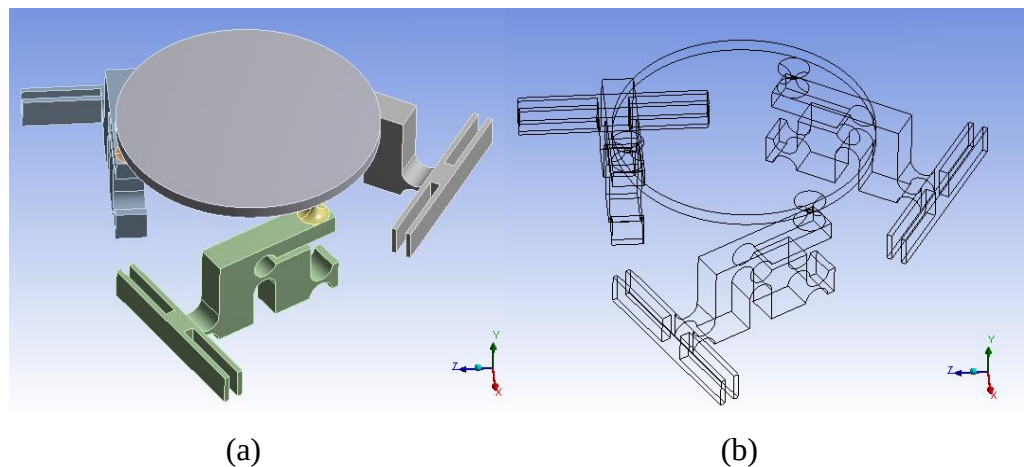


Figure 6.2 FE model in ANSYS-Workbench: (a) Shaded view and (b) Wireframe view.

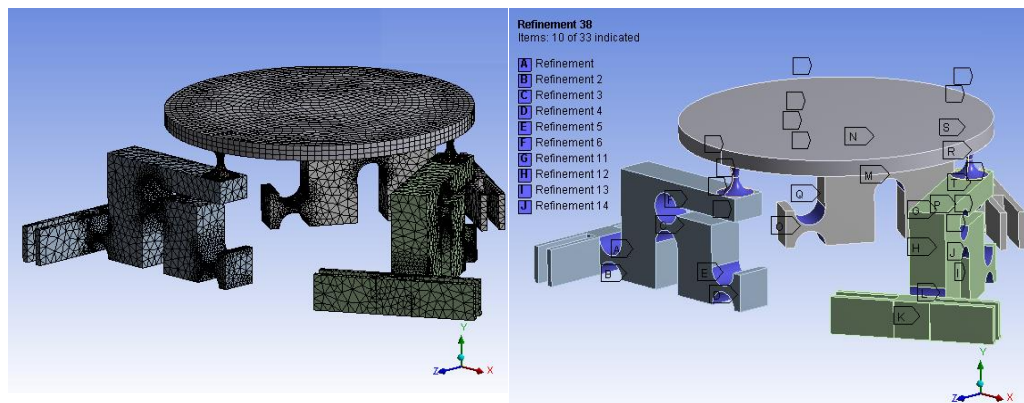


Figure 6.3 Meshing of FE model.

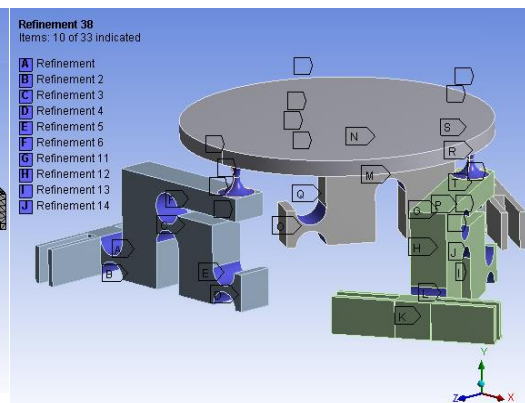
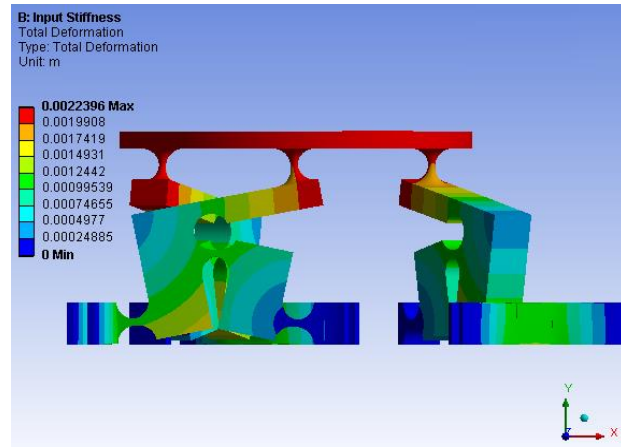


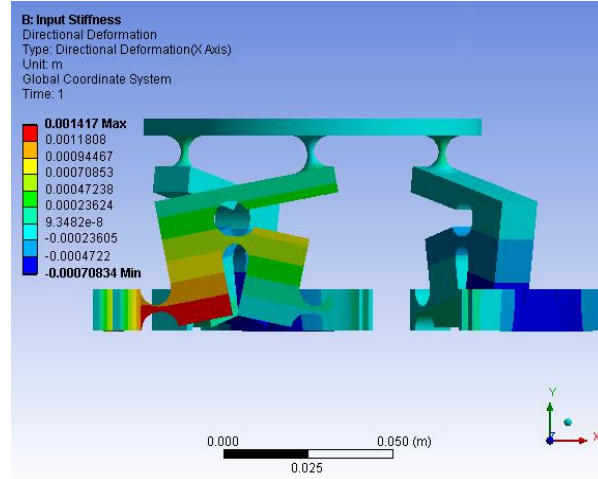
Figure 6.4 Fixed supports.

6.2.1.1 Input Stiffness

The three input forces, F_{in} , which are being applied on the input points of the FAMs, generate a vertical displacement, as shown in Figure 6.5 (a). This elastic deformation of the flexure hinges can be used to simulate the input stiffness of the stage when the moving platform is moving in the Y-direction. The directional deformation in the X-axis of the input point of the FAM, which represents the input displacement, d_{in} , is illustrated in Figure 6.5 (b).



(a)



(b)

Figure 6.5 FE model of the vertical motion: (a) Total deformation and (b) Directional deformation in X-axis orientation.

Different forces ranging between 0-1600 N are applied on the FEA model and the associated input displacements are recorded. The results of this test are plotted in Figure 6.6. The slope of the plot represents the input stiffness of the stage, which can be calculated from the following expression

$$K_{in} = F_{in}/d_{in} \quad (6.1)$$

From (6.1), the value of the simulated input stiffness of the stage using FEA is $3.1758 \text{ N}/\mu\text{m}$ by using an input force of 1100 N and an associated input displacement of $346.37 \mu\text{m}$.

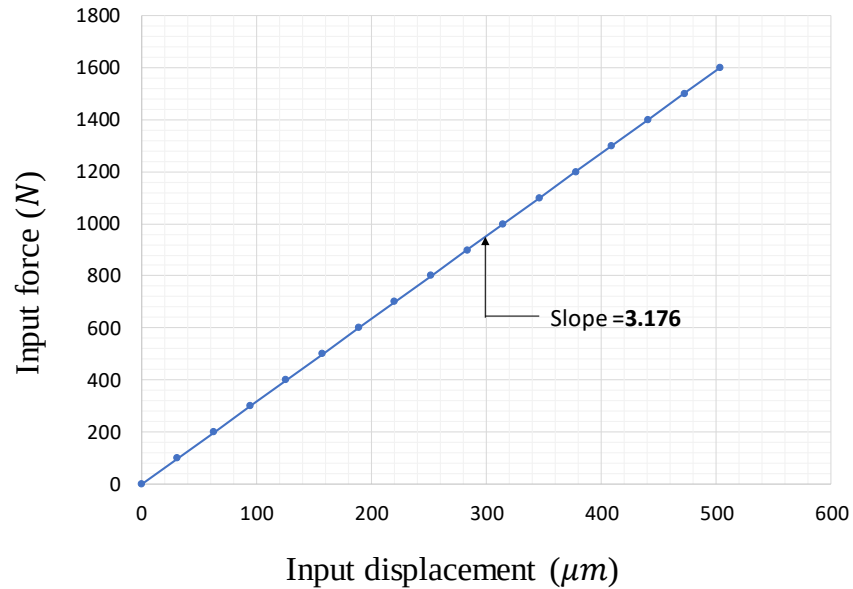


Figure 6.6 Input force versus input displacement of FEA simulation.

To compare the analytical model with the FEA result for validation, the design parameters are applied to the analytical model of the stiffness matrix method (SMM) input stiffness (3.63). The existence of several parallel connections ends the model with very complicated inverse matrices, which can only be solved by using one of the mathematical software. MATLAB is used to calculate the value of the input stiffness of the SMM model. The results of the analytical and FEA models are presented in Table.6.2.

Table 6.2 Results of the input stiffness of SMM and FEA models.

	FEA	SMM
Input stiffness ($\text{N}/\mu\text{m}$)	3.176	2.844
Deviation (%)	-	10.5

The result of the input stiffness using the SMM is compared to the FEA result to validate its accuracy. The results of the SMM model and the FEA simulation agree with each other with an error of 10.5%. This deviation is mainly from the accuracy of compliance factors of the flexure hinges used in the model.

6.2.1.2 Displacement Amplification Ratio

The directional deformation test of the established FEA model in Section 6.2.1.1 is used to assess the displacement amplification ratio, Amp . The assessment is carried out by changing the orientation of the deformation test from X-axis to Y-axis, as shown in Figure 6.7.

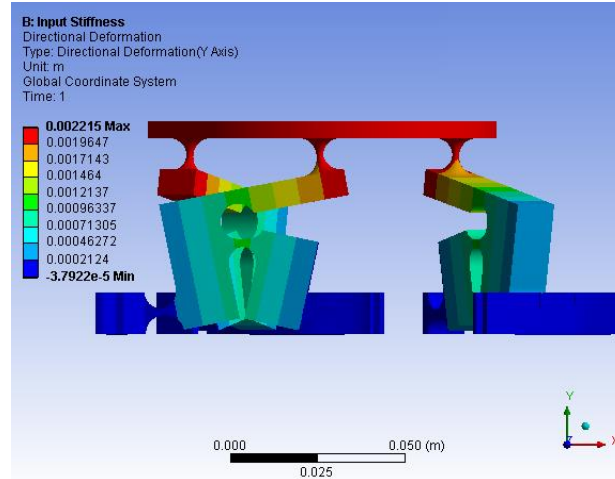


Figure 6.7 Directional deformation test of FE model in Y-axis orientation.

Different values of forces are applied at the input point of the stage and the correspondent input and out displacements of the X and Y orientations are recorded. These results are plotted in Figure 6.8 and the slop of the plot represents the amplification ratio. The value of the slop can be calculated from the following equation

$$Amp = \frac{d_{out}}{d_{in}} \quad (6.2)$$

Using the value of $220.42 \mu m$ and $343.85 \mu m$ for the input displacement, d_{in} , and the output displacement, d_{out} , respectively, the value of the displacement amplification ratio is calculated via (6.2) and presented in Table 6.3.

Table 6.3 Results of the displacement amplification ratio.

	FEA	Kinematic
Amplification ratio	1.56	1.73
Deviation (%)	-	11

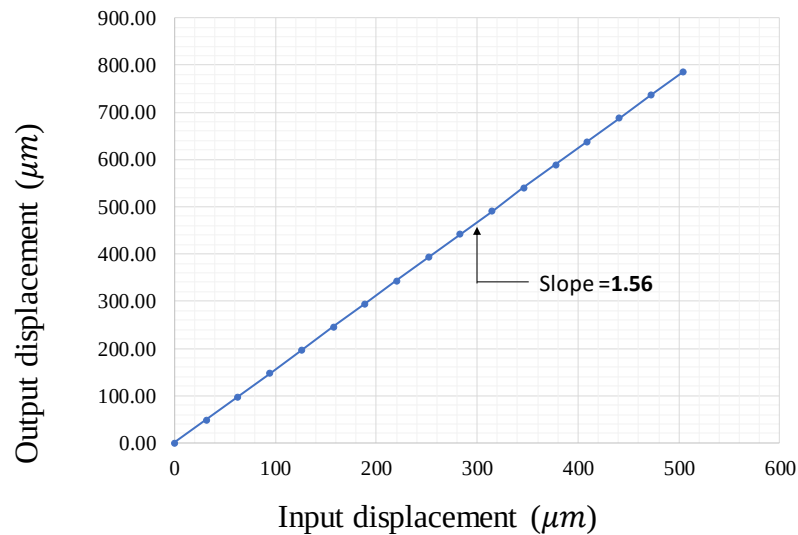


Figure 6.8 Output versus input displacements of FEA simulation.

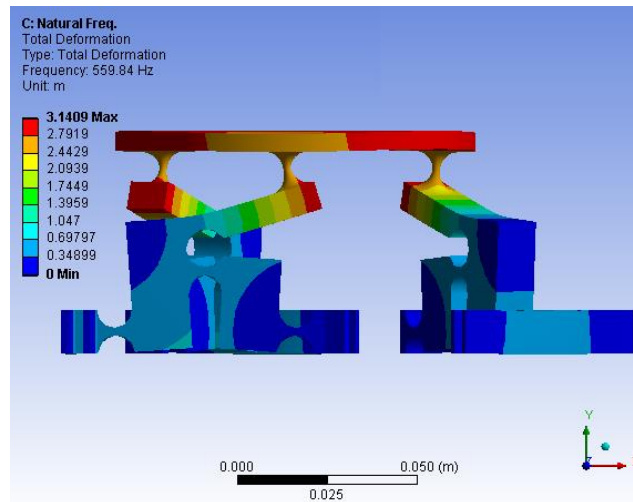
The analytical value of the amplification ratio can be achieved from the kinematic model in equation (3.4). Using the optimized value of the Scott-Russell amplification mechanism (SRAM) angle, α , which is 30° , the analytical value is calculated and given in Table 6.2. Based on the result of the FEA model, the deviation of the analytical model is 11%. The offset mainly comes from modelling the flexure hinges as ideal joints in the kinematic modelling. The result indicates that there are displacement losses.

6.2.2 Dynamic Performance Analysis

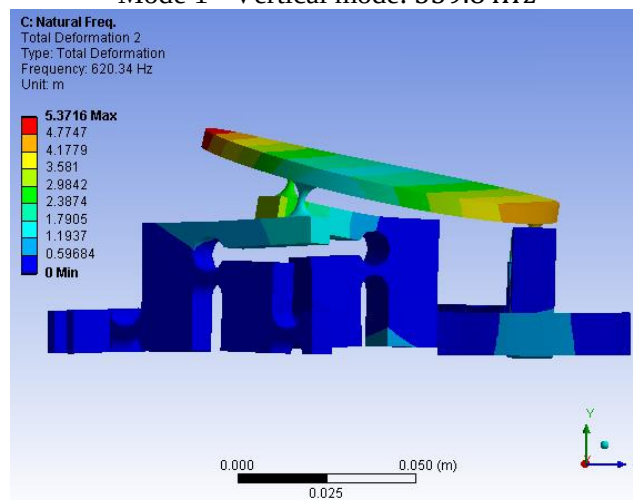
A dynamic analysis is carried out on the 3-D meshed model to simulate the natural frequencies of the proposed stage. Using ANSYS-Workbench, the *Model* tool is selected from the *Analysis Systems* Toolbox. The first six natural frequencies are listed in Table 6.4. The first three natural frequency modes are shown in Figure 6.9.

Table 6.4 FEA natural frequencies of OHP stage.

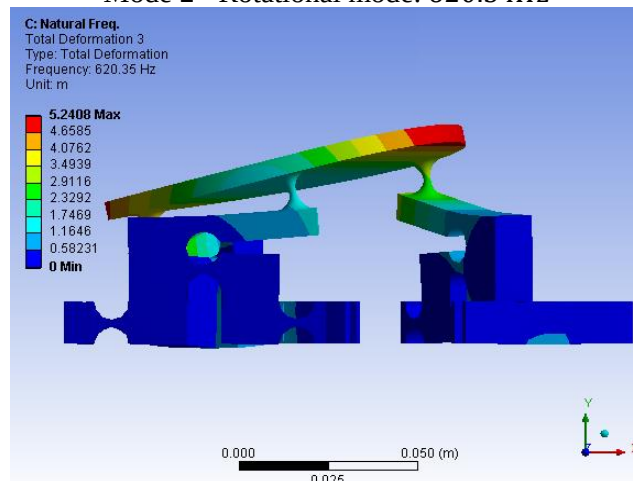
Modal shape No.	Natural frequency (Hz)
1	559.84
2	620.34
3	620.35
4	1151.5
5	1151.7
6	1628



Mode 1 - Vertical mode: 559.84Hz



Mode 2 - Rotational mode: 620.34Hz



Mode 3 - Rotational mode: 620.35Hz

Figure 6.9 Mode shapes of OHP stage from 1st to 3rd.

The following points can be taken from the modal analysis:

- The first mode of 559.84Hz corresponds to the desired vertical displacement, while the two subsequent modes of 620.34Hz , and 620.35Hz are related to the rotational motions.
- The difference between the first and the other two frequencies is not large, which indicates that the stage has closed dynamic characteristics in both the vertical and rotational displacements.
- All the first three natural frequencies are more than the design requirement, which is set as 357 Hz .

The design parameters of the proposed stage are applied to the two analytical models (EBA and 1-DOFM). All the results of the analytical and the FEA models, which are related to the vertical mode, are listed in Table 6.5.

Table 6.5 Results of 1st natural frequency of OHP stage.

	FEA	EBA	1-DOFM
Natural frequency (Hz)	559.84Hz	588.24Hz	693.68Hz
Deviation (%)	-	5	23.9

The following outcomes can be extracted from these results:

- The deviations between the EBA and 1-DOFM models and the simulated first natural frequency of FEA test are about 5% and 24%, respectively.
- The EBA model predictions match quite closely the FEA result with a deviation of only 5%.
- 1-DOFM has larger error than the EBA when compared with the FEA result.

It can be observed that the EBA is more accurate than the 1-DOFM; and accordingly, the results validate the natural frequency model. The deviation of 5% of the EBA is due to the assumptions adopted in building the analytical model. For example, the weights of the flexure hinges are totally ignored when the Lagrangian equation of motion has been derived. In addition, the deviation of the analytical model of the displacement amplification ratio is imported to this model.

As the mass is fixed in the EBA and 1-DOFM models, the natural frequencies of the two analytical models are affected by the change in the value of the input

stiffness. Therefore, the input stiffness models of the three methods (SMM, EBA and 1-DOFM) can be verified via the FEA to determine the most accurate method for modelling the input stiffness.

The three analytical models of the input stiffness of the stage, which are built using three different methods, are studied to validate their accuracies. The values of the SMM model of equation (3.63), the EBA model of equation (4.114), and 1-DOFM model equation (C.10) are individually obtained by means of the optimized design parameters and the selected material properties. The simplicity of the EBA and 1-DOFM models allow the use of Excel Spreadsheet for calculating their values. All the results of the analytical models are compared with the FEA result to validate their accuracies and elaborated in Table.6.6.

Table 6.6 Results of the input stiffness of the analytical models compared to FEA.

	FEA	SMM	EBA	1-DOFM
Input stiffness ($N/\mu m$)	3.176	2.844	2.967	3.877
Deviation (%)	-	10.5	6.6	22.1

From the values of the input stiffnesses of the analytical models and the FE model, the relations between the input displacements and the input forces are plotted in Figure 6.10.

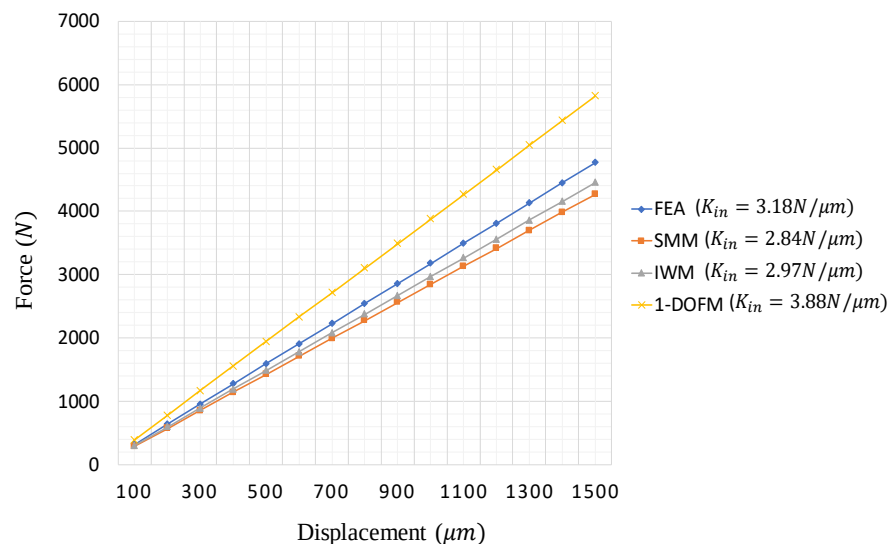


Figure 6.10 The input stiffness results of FEA and analytical models.

The following conclusions can be formulated by analysing the results in Table 6.6 and the plots in Figure 6.10:

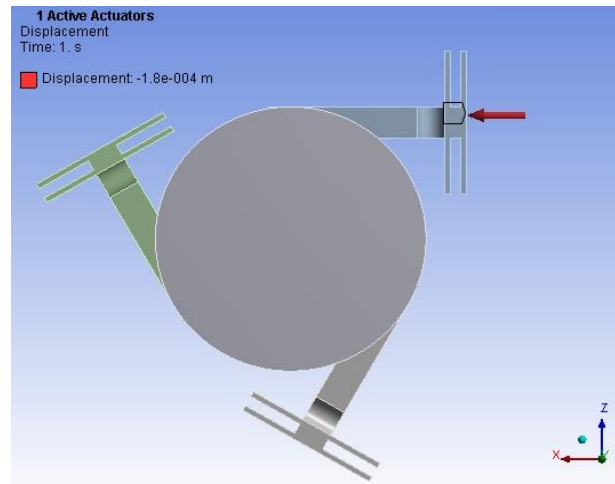
- Based on the result of the FEA simulation, the input stiffness models of the SMM and FEA agree with each other with an error of 10.5%. Although the SMM is one of the implementations of the FEA, it models the system using a smaller number of nodes connecting the elements of the system compared with the FEA, which is one of the reasons for the difference between the SMM and the FEA results. As the compliance equations used in modelling the flexure hinges cannot be calculated accurately, there is an accumulative amount of error also responsible for the deviation of 10.5%.
- The difference between the EBA result and FEA simulation is less than 7%, whereas an error of 22% is the deviation of 1-DOFM model compared with FE model.
- From the four plots of the three analytical models and the FEA simulation in Figure 6.10, the input displacement increases along with the increasing of the input force, and the four plots are linear and coincide with the mathematical results.

In conclusion, taking the FEA results as the benchmark, it is found that the developed method EBA is the most accurate method compared with the other two methods, which consequently supports the validity of the performed dynamic modelling of the OHP stage.

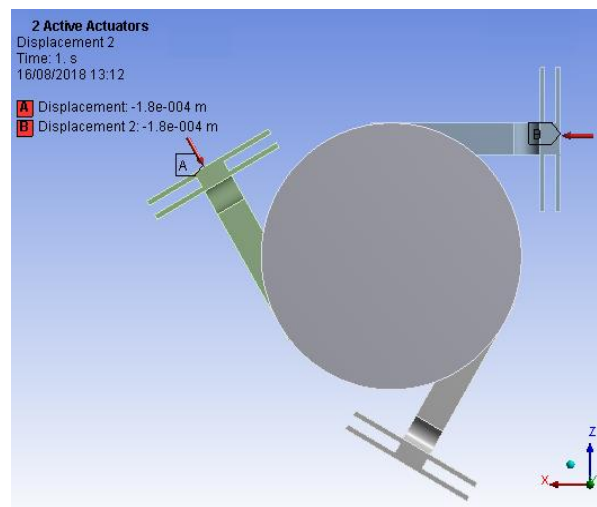
6.2.3 Deformation Analysis

A deformation test is conducted to examine the deformation behaviour and the stress distribution and to determine the travel ranges of the proposed stage. As the stiffness of the piezoelectric (PZT) actuator is not considered in the FEA model, the actuator travel range of $180\mu m$ is used in the analysis. The reason is that using the maximum input force of the PZT actuator will generate an input displacement more than travel range of the actuator.

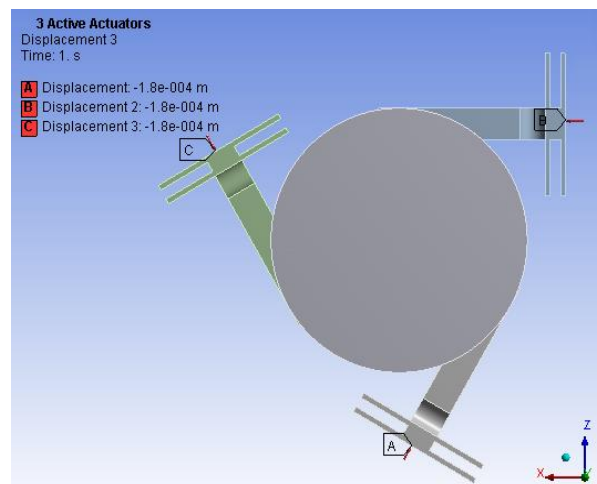
Three different tests, which represents the cases of motions, are carried out by applying the maximum travel range of the PZT actuator on each input point, as shown in Figure 6.11. These three tests are simulating the three cases of motions when one, two, or three actuators are active, which produce two types of rotational displacements or one translational displacement. These simulations can be used to examine the following performance parameters.



(a)



(b)



(c)

Figure 6.11 FE model under maximum input force: (a) One active actuator, (b) Two active actuators, and (c) Three active actuators.

6.2.3.1 Deformation Behaviour

The stability of the stage structure is mainly affected by parasitic motions that can arise at the moving platform or the FAMs during the operation. Parasitic motions can occur when the moving platform is rotating, and one of the FAMs is inactive or the three FAMs are subjected to unequal forces. To avoid these motions, the movements of the FAMs must be decoupled from each other. Therefore, it is important to carry out deformation behaviour tests to ensure the absence of these motions. The three displacement simulations shown in Figure 6.11 are used for examining the deformation of the moving platform. Figure 6.12 shows the motions of the moving platform in the three cases.

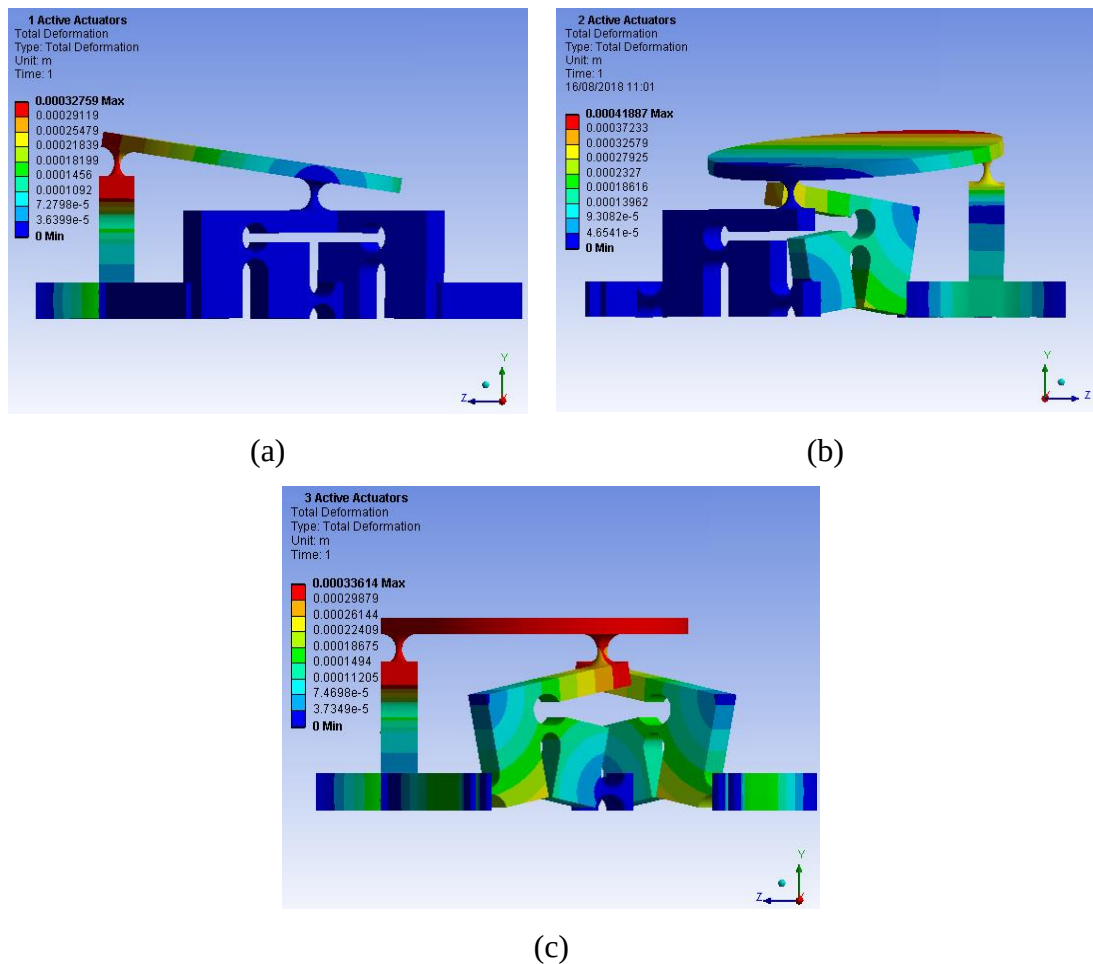


Figure 6.12 FEA deformation behaviour simulation: (a) One active actuator, (b) Two active actuators, and (c) Three active actuators.

The following points can be observed from the three tests:

- From Figure 6.12 (a) and (b), the two rotational displacements of the moving platform are achieved without any deformations occur at the

inactive FAM. This indicates that the three FAMs are totally isolated from each other and the concept of decoupling has been achieved by using the multi-axis right-circular (MARC) flexure hinges.

- From Figure 6.12 (c), a vertical displacement is achieved, while the moving platform is totally stable.
- No parasitic deformations appear in the FAMs, as presented in Figure 6.13 (the moving platform is hidden to give a clear view).

These results demonstrate that the proposed stage has an excellent decoupling capability, and the parasitic motions are eliminated; consequently, the structure is stable while keeping the flexibility of the stage high. These findings meet the conceptual design requirements of OHP stage and fulfil one of the research objectives.

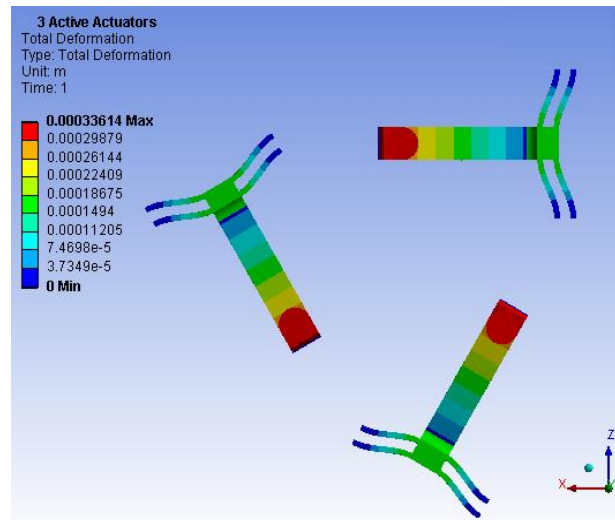


Figure 6.13 FEA total deformation behaviour simulation.

6.2.3.2 Stress Distribution

The stress analysis test is done to evaluate the impact of the maximum deformation generated by the PZT actuator. The evaluation of the maximum stress test is based on the equivalent (von-Mises) stress, which is often used in the FEA because it allows any arbitrary 3-D stress state to be represented as a single positive stress value. When equivalent/von Mises stress exceeds the uniaxial material yield strength, yielding will occur [118]. Therefore, the equivalent stresses that occur in the stage parts are assessed based on the yield strength of the stage material.

To determine the value of stress related to the travel range of the actuator, different values of forces are applied to the 3-D model in the vertical motion test, and the associated input displacements and equivalent stresses are recorded. The values of the forces are increased gradually until the equivalent stress of the stage reaches the value of the yield strength. The results of the simulation are listed in Table 6.7. For clear presentation for the FEA results, a plot of the stresses in terms of the input displacements is given in Figures 6.14.

Table 6.7 Results of FEA simulation for determining maximum input force.

Force	Equivalent	Input
(N)	Stress	Displacement
(MPa)	(μm)	
100	33.70	31.49
200	67.40	62.98
300	101.10	94.46
400	134.81	125.95
500	168.51	157.44
600	202.21	188.99
700	235.91	220.42
800	269.61	251.91
900	303.31	283.39
1000	337.01	314.88
1100	370.72	346.37
1200	404.42	377.86
1300	438.12	409.35
1400	471.80	440.80
1500	505.52	472.32
1600	539.22	503.80

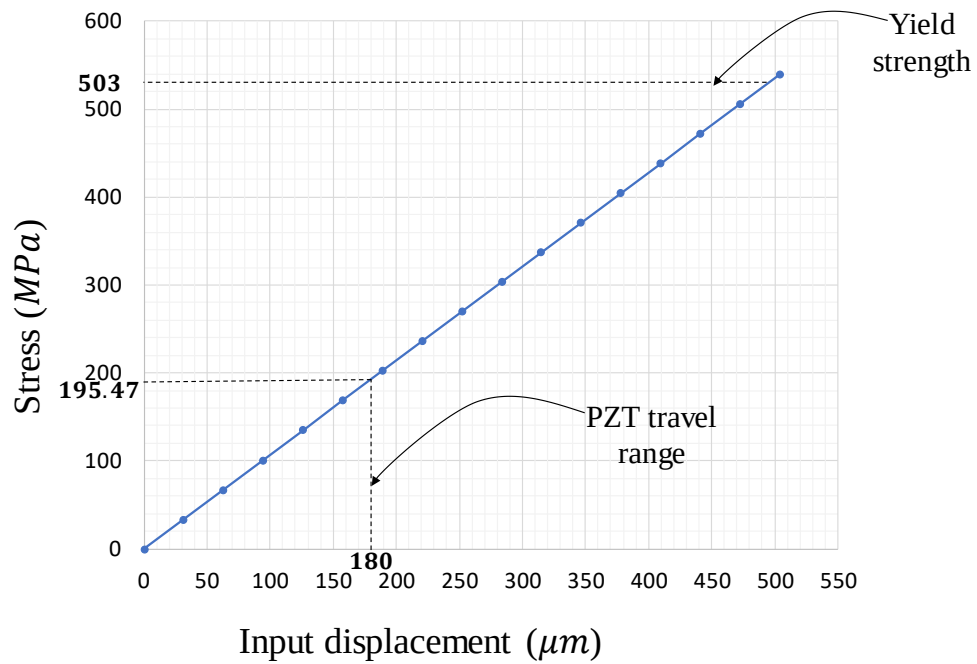


Figure 6.14 Stress plot in terms of input displacement of FEA simulation.

From Table 6.7, it is found the displacement of $180\mu\text{m}$ is generated by a force of 580N , and the associated stress is 195.47MPa , which is less than the yield strength. To examine the stress distribution of the stage, the input force of 580N is applied on the 3-D model. From the three static structural tests, it is found that the maximum equivalent stresses occurred in the flexure hinges, as shown in Figure 6.15. The following points can be extracted from the FEA tests:

- In the 1-active actuator simulation, the maximum equivalent stress of 194.37MPa occurred in the single-axis right-circular (SARC) flexure hinge, as shown in Figure 6.15 (a).
- In the 2-active actuator simulation, the maximum equivalent stress of 196.01MPa occurred in SARC and MARC flexure hinges, as shown in Figure 6.15 (b).
- In the 3-active actuator simulation, which simulates the translational motion, the maximum equivalent stress of 195.47MPa occurred in SARC flexure hinges, as shown in Figure 6.15 (c).

It can be concluded that the OHP stage has capability to achieve the maximum travel range of $180\mu\text{m}$ and the maximum stress is far below the yield strength of the material, which is 503MPa .

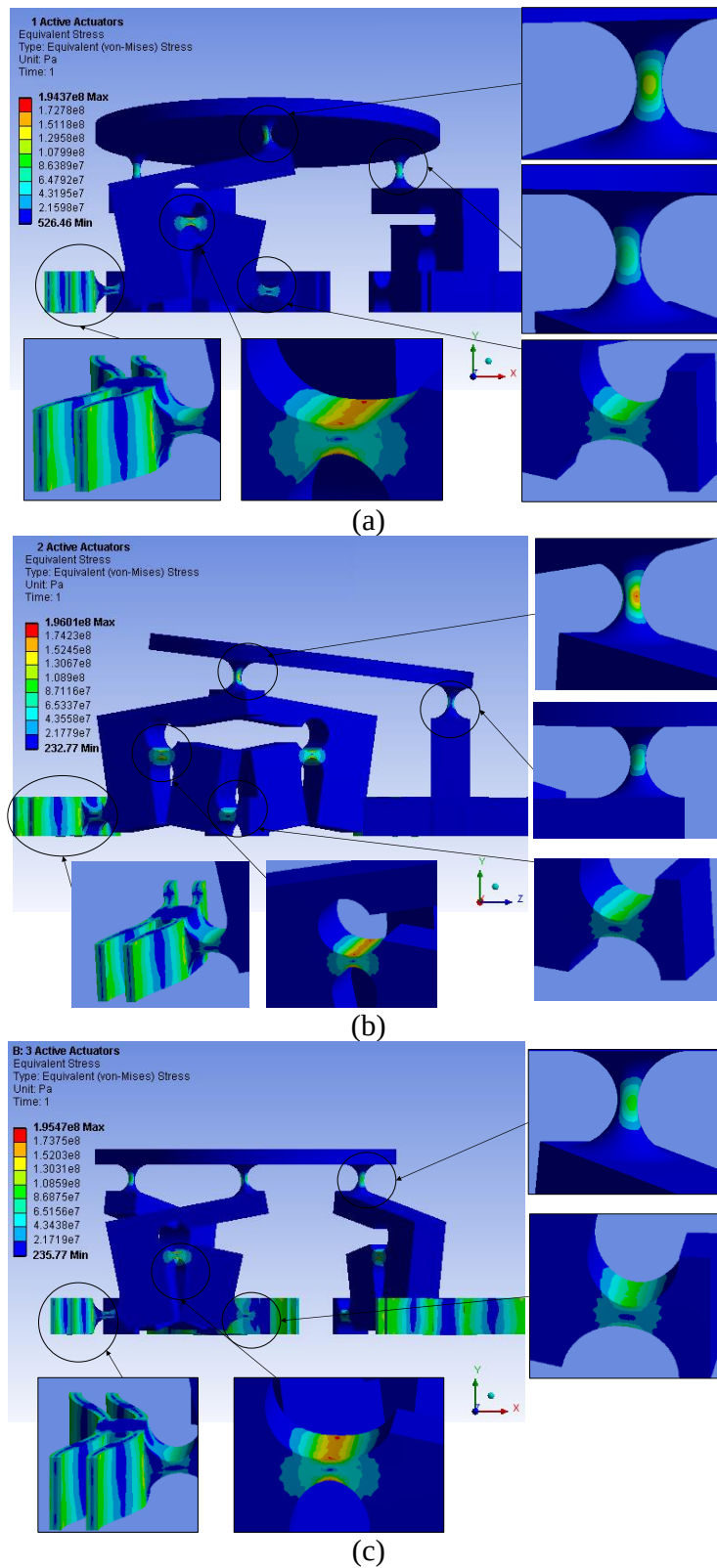


Figure 6.15 FEA Stress analysis under maximum input force:
 (a) One active actuator, (b) Two active actuators, and (c) Three active actuators.

Using the vertical motion test, the analytical models (5.2), (5.3), and (5.5) of LS, SARC, and MARC flexure hinges are individually used to examine the influence of the input force on the bending stresses of the flexure hinges, respectively. The models are a function of the maximum input displacement, hence, the results of the input displacements in Table 6.7 are used with the architectural parameters of the flexure hinges presented in Table 6.1. The analytical results of the bending stresses of the flexure hinges are shown in Table 6.8 and plotted in Figure 6.16.

Table 6.8 *Analytical results of the bending stresses of the flexure hinges.*

Force	Input	Stress	Stress	Stress
	Displacement	LS	SARC	MARC
(N)	(μm)	(MPa)	(MPa)	(MPa)
100	31.49	30.419	26.417	22.313
200	62.98	60.838	52.835	44.627
300	94.46	91.247	79.244	66.933
400	125.95	121.67	105.66	89.247
500	157.44	152.08	132.08	111.56
600	188.99	182.56	158.55	133.92
700	220.42	212.92	184.91	156.19
800	251.91	243.34	211.33	178.5
900	283.39	273.75	237.74	200.81
1000	314.88	304.17	264.16	223.12
1100	346.37	334.59	290.57	245.43
1200	377.86	365.01	316.99	267.75
1300	409.35	395.43	343.41	290.06
1400	440.80	425.81	369.79	312.35
1500	472.32	456.25	396.24	334.68
1600	503.80	486.66	422.64	356.99

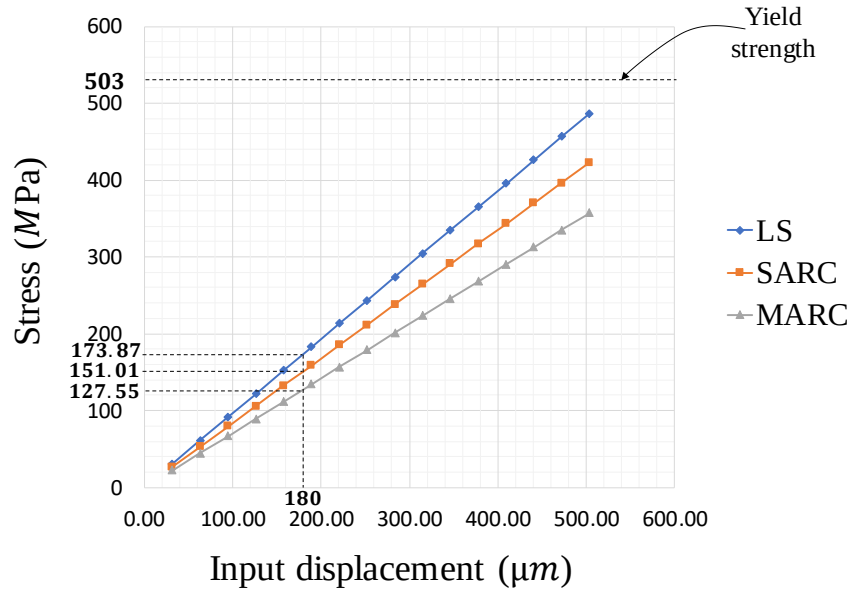


Figure 6.16 Stress plot in terms of input displacement of the flexure hinges.

It is found that the maximum stresses of 173.87 MPa, 151.01 MPa, and 127.55 MPa of LS, SARC, and MARC flexure hinges, respectively, are far below the yield stress of 503 MPa, as presented in Figure 6.16. The difference between the analytical and simulation results of the maximum stresses reflect the importance of using the equivalent stresses.

6.2.3.3 Travel Range

The travel ranges of the OHP stage can be determined by applying the input displacement of $180\mu m$, which is the maximum displacement can be generated by the PZT actuator. For more accurate results, a coordinate system is assigned at the out point and the exact output displacement is recorded by using deformation probe.

The FEA results of the directional deformation simulations in the Y-direction of the three cases of motions are shown in Table 6.9. Using these results, the maximum rotational travel range, $\theta_{stage_{max}}$, of the centre of the stage can be calculated as follows

$$\begin{aligned}\theta_{stage_{max}} &= 0.01745 \tan^{-1}(d_{Y_{FEA}} / (L_o)) \\ &= 0.01745 \tan^{-1}(d_{Y_{FEA}} / (1.5 r_o))\end{aligned}\quad (6.3)$$

where r_o is the radius of the moving platform of $0.05m$, and $d_{Y_{FEA}}$ is the maximum directional deformation of FEA in the Y-orientation.

Table 6.9 FEA directional deformation results in Y orientation.

	Output Displacement (μm)	Rotational Displacement (μrad)
1 active actuator	293.86	3917
2 active actuators	298.47	3977
3 active actuators	299.97	-

From FEA results, the travel ranges and the design requirements of the stage are listed in Table 6.10. Travel ranges of $299.97 \mu m$ and $3977 \mu rad$ are obtained, which are more than the values of the design requirements. The reason of achieving a large rotational displacement is the using of the MARC flexure hinges, which provide large rotational deformations.

Table 6.10 FEA simulation results of travel ranges of OHP stage.

	FEA	Design Requirement
Rotational motion range (μrad)	1-Act. 3917	> 2
	2-Act. 3977	> 2
Translational motion range (μm)	3-Act. 299.97	> 190

The FEA results indicate that the OHP stage succeeded to achieve the maximum travel ranges of $299.97 \mu m$ and $3977 \mu rad$, and accordingly, the design requirements are met.

6.2.3.4 Load Capacity

The load capacity is the load that can be applied at the top of the moving platform without reducing the travel range of the stage. To determine the value of the load capacity, different values of forces are applied on the top of the moving platform in the opposite direction of the output displacement. The values of the forces are increased gradually until a loss in the travel range of stage appears. A force of $10N$ is achieved, which has no negative effects on the stage performance, as shown in Figure 6.17. The stresses of the stage due to the load capacity in the three cases of motions are examined. Stresses of $268.7MPa$, $230MPa$, and

228MPa are found, which are below the yield strength. It can be concluded that the OHP stage is capable to support a load of 10N without failure.

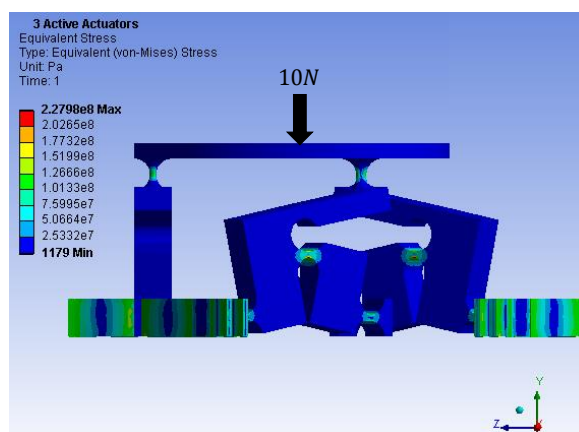
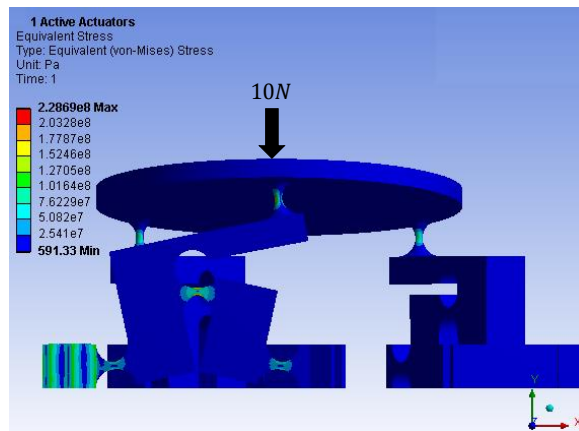


Figure 6.17 FEA simulation of the load capacity: (a) One active actuator, (b) Two active actuators, and (c) Three active actuators

6.3 OHP Specifications

The specifications of the OHP nano-positioning stage are listed in Table 6.11. Figure 6.18 presents the dimensional drawing of the stage. It is observed that the height of the stage is 51 mm, which is less than the design requirement of 60 mm.

Table 6.11 Specifications of OHP stage.

Name	Out-of-Horizontal-Plane Nano-positioning Stage.
Application	Nano-manufacturing.
Travel Range	Vertical motion 299.97 μm and Rotational motion 3977 μrad
Platform Diameter	100 mm.
Height	51 mm.
Actuator	MTP105 from Queensgate
Material	Aluminium alloy (A17075)
Operation Temperature	< 20°C
Maximum Centre Loading	1 kg
Natural Frequency	588.24 Hz

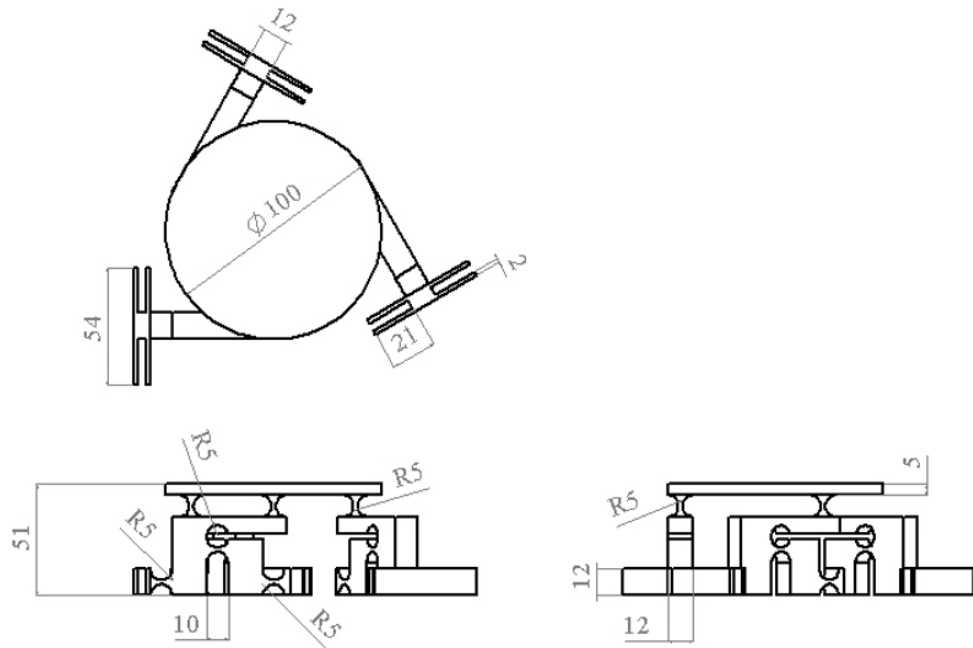


Figure 6.18 Dimensional drawing of OHP stage.

6.4 Chapter Summary

The FEA was performed to validate the performance of the OHP stage. First, the CAD model was built by Solidworks software using the optimized architectural parameters of the stage. The model was imported from Solidworks to ANSYS-Workbench to carry out the FEA simulations. Based on the FEA results, the performance of the stage was validated in terms of the following:

- The offset of the displacement amplification ratio mainly came from neglecting the flexure hinges in the analytical model. The result indicated that there were displacement losses.
- Based on FEA result, it was found that the developed method (EBA) had the highest accuracy with a deviation of 6.6%, which consequently validate the dynamic modelling of the OHP stage.
- The results of the deformation analysis demonstrated that the proposed stage had an excellent decoupling capability and the parasitic motions were eliminated, and consequently, the structure was stable while keeping the flexibility of the stage high.
- It was determined that the OHP stage was capable to support a load of 10 *N* without failure.
- The OHP stage succeeded to achieve maximum travel ranges of 299.97 μm and 3977 μrad , and therefore, the design requirements were met.

In conclusion, the established analytical models were validated by ANSYS software and the simulation results agreed very well with the analytical results. And the OHP stage performance achieved the design requirements and the research objectives, which will be discussed in detail in **Chapter 7**.

Chapter 7 Conclusion and Future Work

The demands of spatial compliant nano-positioning stages in applications, such as optical alignment systems, micro-surgery instruments and nano-manufacturing, increase rapidly. However, the complexity of the structural design and modelling methods of the spatial compliant nano-positioning stages is the main reason for the limited researches in this area. Therefore, research works in the design and modelling of spatial stages with high flexibility and accuracy is very valuable.

An out-of-horizontal-plane (OHP) nano-positioning stage was proposed in the research. This research handled two aspects: (a) the structural design and (b) the modelling methods. A complete structural design for the stage was presented. Scott-Russell mechanism is applied to the actuating unit. And three piezoelectric (PZT) actuators were utilized to generate one vertical and two rotational motions. Furthermore, a developed method for modelling the potential energy was formulated based on work-energy method.

7.1 Contributions

The two main contributions of the research were the design of an OHP stage with higher performance and flexibility and the derivation of a simple and the development of an accurate method for the dynamic modelling. A desire to achieve high structural stability and a compact size by reducing the size of the stage and using a monolithic structure of the actuating units led to increase of the affordability, which was one of the contributions. The fourth contribution of the research was presenting an application of a modelling method for multi-axis right circular flexure hinges in the spatial mechanisms, which is absent from the existing literature. The last contribution of the work was the demonstration of a clear framework for design and modelling the OHP compliance-based mechanism, which is not available in textbooks. These contributions towards achieving the research objectives are discussed hereafter, along with key conclusions and the *assumptions* and the *limitations* of the research.

7.1.1 A large travel range and a simple mechanical arrangement

Based on the limitations found in the literature, the research objectives were established, and the structure of the proposed stage was presented with the solutions for these limitations.

A compliant Scott-Russell amplification mechanism (SRAM) was adopted in the flexure amplification mechanism (FAM) because it allows integrating the actuator outside the FAM in a horizontal position. This horizontal position permits to select a PZT actuator with a long stroke to increase the travel range. Also, it reduces the height of the stage, which led to reduce the size of the stage.

The structure of the stage consisted of the base, moving platform, and three FAMs. The design of the FAM was a monolithic structure. WEDM was selected as the manufacturing process because the FAM can be fabricated from a single piece. This kind of structures decreases the assembly errors and the manufacturing cost. The three FAMs were symmetric in a triangular parallel configuration, which results in a simple kinematic arrangement with high thermal stability.

Each FAM was fixed at the base and connected to the moving platform via a multi-axis right-circular (MARC) flexure hinge. The MARC flexure hinge allows isolating the FAM from any motions generated in the other FAMs and permitting the platform to move freely in different directions, and each PZT actuator to generate one output motion. This integration of MARC flexure hinges assisted to attain a totally decoupled stage and enhance the structural stability of the stage. This unique structural design was one of the research contributions.

7.1.2 Accurate Modelling Methods

The modelling of the kinematic quantities included the displacement amplification ratio, the displacements of the flexure hinges and velocities of the rigid links. It was proved that the stage had three DOF and a capability to provide two rotations and one translation motions. All the kinematic models of the complaint are built without considering the stiffness of the flexure hinges, which reflect a **limitation** of this kind of modelling.

The input stiffness model was derived using the stiffness matrix method (SMM). The modelling of the stage was under the **assumption** that the relationship

between input force and correspondent deformation is linear. The advantage of the SMM is that it can manage all the deformations of the flexure hinges of the structure. However, the existence of several parallel connections ends the modelling with very complicated inverse matrices that can only be solved by using one of commercial software such as MATLAB, which is considered a **limitation** of this method.

Lagrange's equation was used to obtain the dynamic model because it is very difficult to use Newtonian mechanics in multi-DOF mechanisms with distributed masses. Lagrange's equation has the advantage of computing only the kinetic and potential energies of the system. However, from the literature review, it was found that most of the existing researches model the flexure hinges as a 1-DOF element, which leads to an inaccurate stiffness model because only the direct-loading stiffness is considered.

A modelling method for calculating the potential energy considering all deformation directions of the flexure hinge was developed. This method, which was called the energy-based approach (EBA), was developed based on the Work-Energy Method. The EBA was validated first via a previous research work (Qingsong and Yangmin [93]) using the Finite Element Analysis (FEA) before it was used to model the potential energy of the OHP stage. The analytical models and FEA simulation results are shown in Table 7.1.

Table 7.1 Validation of EBA based on natural frequency.

	Qingsong and Yangmin	EBA	FEA
	Analytical model	Analytical model	
Natural Frequency (Hz)	86.76	97.02	104.64
Error (%)	17.1%	7.3%	-

Based on the FEA results, the result of the analytical model considering only the rotational direction derived by Qingsong and Yangmin was far away from the FEA result with a derivation of 17.1%, which was attributed to neglect the effects of the internal loading in the axial directions. On the other hand, the EBA result was closest to the FEA results because all the internal loadings were considered. The deviation of 7.3% was mainly from the accuracy of compliance equations of the single-axis right-circular (SARC) flexure hinge used in the

model. It was concluded that the analytical model of the natural frequency built by using the EBA was more accurate than the one of Qingsong and Yangmin.

Although the EBA had longer steps compared to the traditional method, it is still simple and accurate. Hence, the EBA was considered an efficient approach for modelling the potential energy of Lagrange's equation of compliance-based mechanisms. The EBA was under the **assumptions** that the flexure hinges were connected by totally rigid links and the deformations only occur in the flexure hinges, and finally, Hook's law was still valid. The kinetic energy model of Lagrange's equation was built under the **assumption** that the weight of the flexure hinges can be neglected.

The analysis of parallel configurations using this approach will be very easy and simple because the analysis based only on finding the forces acting on the flexure hinges. The calculations can be done manually or using just a spreadsheet software. The simplicity of this method can assist several researchers to study the dynamic behaviour of complicated structures of serial or parallel compliant-based systems. The EBA has only one **limitation** when the mechanism is statically undetermined. It may take longer steps than other techniques, however, it is still simpler to use and more accurate. EBA represents one of the main contributions of this research.

Using the EBA, three natural frequency models of the three cases of motions were built and evaluated to select the model with lower input stiffness for the optimization process. The natural frequency model of the translational motion was selected because it had the largest value of a mass and there was no significant difference between the stiffness values of the three cases.

The employment and modelling of the MARC flexure hinges in parallel OHP applications, such as in this research work, presented one of the research contributions.

7.1.3 High Natural Frequency

An optimization process to obtain optimal geometrical parameters was conducted. The first natural frequency of the stage was set as the objective function of the optimization process since it must be large enough to improve the dynamic characteristics. The maximum values of the large travel range, the

stresses of the flexure hinges, and the input stiffness were set as the constraints of the design optimization. Aluminium Alloy T7075 was selected because it has the lowest cost in comparison with Steel (4140 Q&T@400) and Titanium (Ti-6Al-4V), which are the commonly used materials in this kind of applications. MATLAB-Optimization Toolbox was used to maximize the objective function while satisfying the constraints. The optimal design results were achieved with a first natural frequency of 588.24 *Hz*, which satisfied the design requirement as shown in Table 7.2.

Table 7.2 *Fulfilment of design requirements.*

	Design Requirement	Research Achievement
Natural frequency	$> 357 \text{ Hz}$	588.24 Hz
Travel range:		
Translational motion	$> 190 \mu\text{m}$	299.97 μm
Rotational motion	$> 2 \mu\text{rad}$	3977 μrad
Total height	$< 60 \text{ mm.}$	51 mm

7.1.4 Design of an OHP nano-positioning stage with higher flexibility and performance

From the 3-dimensional (3-D) model built using the optimum dimensions, it was observed that the height of the stage was 51 *mm*, which was less than the design requirement presented in Table 7.2. Using the 3-D model, the proposed design was validated via the FEA.

ANSYS-Workbench software was used to analyse the performance of the proposed stage in terms of the static, dynamic, and deformation performance tests. The static analysis was carried out to validate the input stiffness and the displacement amplification ratio models of the OHP stage. It was observed that the accuracy of the input stiffness model derived by the SMM was with a deviation of 10.5% due to the accuracy of the selected compliance factors. Furthermore, it was found that the deviation of the displacement amplification ratio analytical model mainly came from neglecting the stiffness of the flexure hinges in the analytical model.

In the dynamic analysis, the design parameters of the proposed stage were applied to the two analytical models (EBA and 1-DOFM). The results of the

analytical and the FEA models, which are related to the vertical mode, are listed in Table 7.3.

Table 7.3 Validation of the dynamic model based on the 1st natural frequency.

	FEA	EBA	1-DOFM
Natural frequency (Hz)	559.84Hz	588.24Hz	693.68Hz
Deviation (%)	-	5	23.9

The EBA model predictions match quite closely the FEA result with an error of only 5%. This deviation due to the assumptions adopted in building the analytical model. For example, the weights of the flexure hinges were totally ignored when the Lagrangian equation of motion was derived. In addition, the deviation of the analytical model of the displacement amplification ratio was imported to this model.

The analytical input stiffness models of the three methods (SMM, EBA and 1-DOFM) were verified via the FEA to determine the most accurate method for modelling the input stiffness. The results of the analytical models and the FEA simulation are given in Table 7.4. Based on the FEA result, it was found that the developed method (EBA) had the highest accuracy with a deviation of 6.6% compared with SMM and 1-DOFM; and consequently, validates the dynamic modelling of the OHP stage.

Table 7.4 Validation of EBA based on input stiffness.

	FEA	SMM	EBA	1-DOFM
Input stiffness (N/μm)	3.176	2.844	2.967	3.877
Deviation (%)	-	10.5	6.6	22.1

The results of the deformation analysis demonstrated that the proposed stage had an excellent decoupling capability and the parasitic motions were prevented; and consequently, the structure was stable while keeping the flexibility of the stage high. In addition, it was found that the OHP stage was capable to support a load up to 10N without failure. Furthermore, the OHP stage succeeded to achieve maximum travel ranges of 299.97 μ m and 3977 μ rad; and therefore, the design requirements were met, as presented in Table 7.2.

From the static, dynamic, and deformation performance analysis, which were validated by the FAE, it was found that the design requirements and the research objectives were achieved. In comparison with the other positioning stages, which were found in the literature, the OHP nano-positioning stage had shown a larger travel range, higher natural frequency, and smaller height. Furthermore, the whole research work provided clear steps for the design of compliant parallel mechanisms.

The contributions highlighted in this conclusion successfully fulfilled the research objectives mentioned in **Section 1.2**.

7.2 Future Work

Some suggestions for future work are presented to enhance and complete the design of the OHP nano-positioning stage. The work is directed toward the following points:

- Examine the stress relaxation for the flexure hinges.
- Perform a complete kinematic in terms of inverse and forward kinematics for the motion control system.
- Build a model for the displacement amplification ratio while considering the stiffness of the flexure hinges.
- Establish an analytical model for calculating the workspace of the stage, which is the volume of space that the moving space can reach.
- Manufacture a prototype of the proposed stage and conduct an experimental investigation of the stage performance, such as the resolution.
- Apply the EBA method on a planar stage and compare it with the spatial stage to evaluate the accuracy of the method in the two types of stages.

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Appendix A

Transformation Matrices of the flexure hinges

The translation matrices of the local frames of the three types of flexure hinges relative to the input global frame are derived as follows:

A.1 The MARC Transformation Matrices

The transformation of frame O_{MARC1} is made relative to the input global frame, O_{input} , as shown in Figure A.1, by

- Rotation of $(\frac{\pi}{2})_Y$ about Y -axis,
- Rotation of $(-\frac{\pi}{2})_Z$ about Z -axis, and
- Translation of $[l_{MARCX}, l_{MARCY}, 0]$.

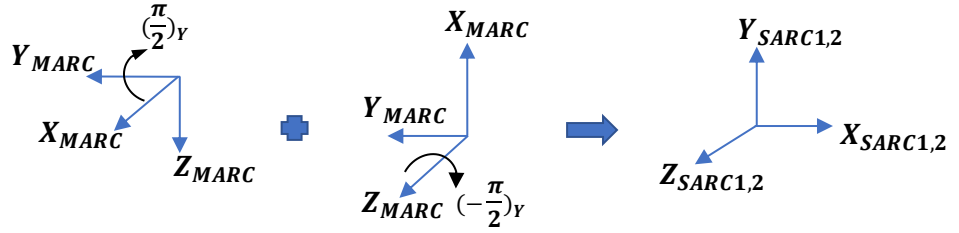


Figure A.1 Rotation of MARC local frame.

The transformation matrix is

$$\begin{aligned}
 T_{MARC1}^{O_{input}} &= R_{O_{MARC1}}^{O_{input}}(Y, \alpha_{MARC1}) * R_{O_{MARC1}}^{O_{input}}(Z, \alpha_{MARC1}) \\
 &\quad * Trans_{O_{MARC1}}^{O_{input}}(l_{X_{MARC1}}, l_{Y_{MARC1}}, l_{Z_{MARC1}}) \\
 &= R_{O_{MARC1}}^{O_{input}}(Y, \pi/2) * R_{O_{MARC1}}^{O_{input}}(Z, -\pi/2) \\
 &\quad * Trans_{O_{MARC1}}^{O_{input}}(l_{MARCX}, l_{MARCY}, 0) \\
 &= \begin{bmatrix} R_{O_{MARC1}}^{O_{input}}(\pi/2)_Y & 0 \\ 0 & R_{O_{MARC1}}^{O_{input}}(\pi/2)_Y \end{bmatrix} \\
 &\quad * \begin{bmatrix} R_{O_{MARC1}}^{O_{input}}(-\pi/2)_Z & 0 \\ 0 & R_{O_{MARC1}}^{O_{input}}(-\pi/2)_Y \end{bmatrix} * \begin{bmatrix} I & (\hat{r}_{MARC1}^{O_{input}})^T \\ 0 & I \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
&= \begin{bmatrix} \cos(\pi/2) & 0 & \sin(\pi/2) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin(\pi/2) & 0 & \cos(\pi/2) & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\pi/2) & 0 & \sin(\pi/2) \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sin(\pi/2) & 0 & \cos(\pi/2) \end{bmatrix} \\
&\quad * \begin{bmatrix} \cos(\pi/2) & -\sin(\pi/2) & 0 & 0 & 0 & 0 \\ \sin(\pi/2) & \cos(\pi/2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\pi/2) & -\sin(\pi/2) & 0 \\ 0 & 0 & 0 & \sin(\pi/2) & \cos(\pi/2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
&\quad * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -l_{MARC1Y}^{O_{input}} \\ 0 & 1 & 0 & 0 & 0 & l_{MARC1X}^{O_{input}} \\ 0 & 0 & 1 & l_{MARC1Y}^{O_{input}} & -l_{MARC1X}^{O_{input}} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
&\quad * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -l_{MARC_Y} \\ 0 & 1 & 0 & 0 & 0 & l_{MARC_X} \\ 0 & 0 & 1 & l_{MARC_Y} & -l_{MARC_X} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -l_{MARC_Y} \\ 0 & 1 & 0 & 0 & 0 & l_{MARC_X} \\ 0 & 0 & 1 & l_{MARC_Y} & -l_{MARC_X} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 1 & l_{MARC_Y} & -l_{MARC_X} & 0 \\ -1 & 0 & 0 & 0 & 0 & l_{MARC_Y} \\ 0 & -1 & 0 & 0 & 0 & -l_{MARC_X} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \tag{A.1}
\end{aligned}$$

where

$$l_{MARC_X} = b_{RL_1} + 2r_{SARC} + 2L \cos \alpha \tag{A.2}$$

$$l_{MARC_Y} = 2r_{MARC} + 2L \sin \alpha \tag{A.3}$$

A.2 The SARC Transformation Matrices

The transformation of O_{SARC1} is made relative to the global frame O_{input} , as presented in Figure 3.12, by the following

- Rotation of $(-\frac{\pi}{2})$ about the $Z - axis$, followed by
- Rotation of $(\frac{\pi}{2})$ about the $Y - axis$, and

- Translation of $[l_{SARC1X}, 0, 0]$.

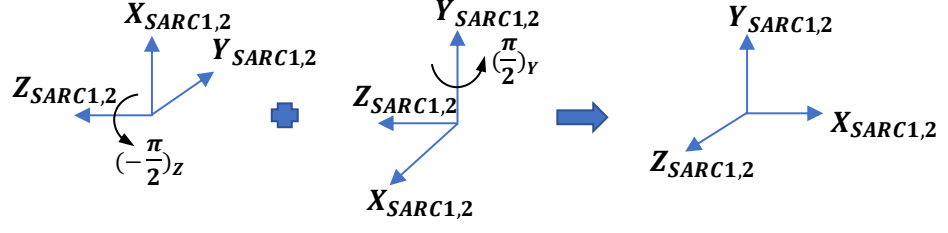


Figure A.2 Rotation of MARC local frame.

where

$$l_{SARC1X} = b_{RL1} + 2 r_{SARC} \quad (\text{A.4})$$

The transformation matrix is

$$\begin{aligned}
 T_{SARC1}^{Oinput} &= R_{O_{SARC1}}^{Oinput}(Z, \alpha_{SARC1}) * R_{O_{SARC1}}^{Oinput}(Y, \alpha_{SARC1}) \\
 &\quad * Trans_{SARC1}^{Oinput}(l_{x_{SARC1}}, l_{y_{SARC1}}, l_{z_{SARC1}}) \\
 &= R_{O_{SARC1}}^{Oinput}(Z, -\pi/2) * R_{O_{SARC1}}^{Oinput}(Y, \pi/2) * Trans_{SARC1}^{Oinput}(l_{SARC1X}, 0, 0) \\
 &= \begin{bmatrix} R_{O_{SARC1}}^{Oinput}(-\pi/2)_Z & 0 \\ 0 & R_{O_{SARC1}}^{Oinput}(-\pi/2)_Z \end{bmatrix} * \begin{bmatrix} R_{O_{SARC1}}^{Oinput}(\pi/2)_Y & 0 \\ 0 & R_{O_{SARC1}}^{Oinput}(\pi/2)_Y \end{bmatrix} \\
 &\quad * \begin{bmatrix} I & (\hat{r}_{SARC1}^{Oinput})^T \\ 0 & I \end{bmatrix} \\
 &= \begin{bmatrix} \cos(-\pi/2) & -\sin(-\pi/2) & 0 & 0 & 0 & 0 \\ \sin(-\pi/2) & \cos(-\pi/2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(-\pi/2) & -\sin(-\pi/2) & 0 \\ 0 & 0 & 0 & \sin(-\pi/2) & \cos(-\pi/2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
 &\quad * \begin{bmatrix} \cos(\pi/2) & 0 & \sin(\pi/2) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin(\pi/2) & 0 & \cos(\pi/2) & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\pi/2) & 0 & \sin(\pi/2) \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sin(\pi/2) & 0 & \cos(\pi/2) \end{bmatrix} \\
 &\quad * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & l_{SARC1X}^{Oinput} \\ 0 & 0 & 1 & 0 & -l_{SARC1X}^{Oinput} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
&= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \\
&\quad * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & l_{SARC1_X}^{O_{input}} \\ 0 & 0 & 1 & 0 & -l_{SARC1_X}^{O_{input}} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & l_{SARC1_X} \\ 0 & 0 & -1 & 0 & l_{SARC1_X} & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \tag{A.5}
\end{aligned}$$

The transformation of the frame O_{SARC2} is made relative to the reference frame O_{input} , as shown in Figure A.2, by the following

- Rotation of $(-\frac{\pi}{2})$ about the $Z - axis$, followed by
- Rotation of $(\frac{\pi}{2})$ about the $Y - axis$, and
- Translation of $[l_{SARC2_X}, l_{SARC2_Y}, 0]$.

From Figure 3.12

$$l_{SARC2_X} = b_{RL_1} + 2r_{SARC} + L \cos \alpha \tag{A.6}$$

$$l_{SARC2_Y} = L \sin \alpha \tag{A.7}$$

The transformation matrix is

$$\begin{aligned}
T_{SARC2}^{O_{input}} &= R_{O_{SARC2}}^{O_{input}}(Z, \alpha_{SARC2}) * R_{O_{SARC2}}^{O_{input}}(Y, \alpha_{SARC2}) \\
&\quad * Trans_{SARC2}^{O_{input}}(l_{x_{SARC2}}, l_{y_{SARC2}}, l_{z_{SARC2}}) \\
&= R_{O_{SARC2}}^{O_{input}}(Z, -\pi/2) * R_{O_{SARC2}}^{O_{input}}(Y, \pi/2) \\
&\quad * Trans_{SARC2}^{O_{input}}(l_{SARC2_X}, l_{SARC2_Y}, 0) \\
&= \begin{bmatrix} R_{O_{SARC2}}^{O_{input}}(-\pi/2)_Z & 0 \\ 0 & R_{O_{SARC2}}^{O_{input}}(-\pi/2)_Z \end{bmatrix} \\
&\quad * \begin{bmatrix} R_{O_{SARC2}}^{O_{input}}(\pi/2)_Y & 0 \\ 0 & R_{O_{SARC2}}^{O_{input}}(\pi/2)_Y \end{bmatrix} * \begin{bmatrix} I & (\hat{r}_{SARC2}^{O_{input}})^T \\ 0 & I \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
&= \begin{bmatrix} \cos(-\pi/2) & -\sin(-\pi/2) & 0 & 0 & 0 & 0 \\ \sin(-\pi/2) & \cos(-\pi/2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(-\pi/2) & -\sin(-\pi/2) & 0 \\ 0 & 0 & 0 & \sin(-\pi/2) & \cos(-\pi/2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
&\quad * \begin{bmatrix} \cos(\pi/2) & 0 & \sin(\pi/2) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin(\pi/2) & 0 & \cos(\pi/2) & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\pi/2) & 0 & \sin(\pi/2) \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sin(\pi/2) & 0 & \cos(\pi/2) \end{bmatrix} \\
&\quad * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -l_{SARC2Y}^{O_{input}} \\ 0 & 1 & 0 & 0 & 0 & l_{SARC2X}^{O_{input}} \\ 0 & 0 & 1 & l_{SARC2Y}^{O_{input}} & -l_{SARC2X}^{O_{input}} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & l_{SARC2Y} \\ 0 & 1 & 0 & 0 & 0 & l_{SARC2X} \\ 0 & 0 & 1 & l_{SARC2Y} & -l_{SARC2X} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & l_{SARC2X} \\ 0 & 0 & -1 & -l_{SARC2Y} & l_{SARC2X} & 0 \\ -1 & 0 & 0 & 0 & 0 & -l_{SARC2Y} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \tag{A.8}
\end{aligned}$$

The frame O_{SARC3} relative to the reference frame O_{input} is achieved, as illustrated in Figure A.3, by the following transformations

- Rotation of $(-\pi/2)$ about the Y – axis, followed by
- Rotation of $(-\pi/2)$ about the X – axis, followed by
- Translation of $[l_{SARC3X}, 0, 0]$.

From Figure 3.12

$$l_{SARC3X} = b_{RL1} + 2L \cos \alpha \tag{A.9}$$

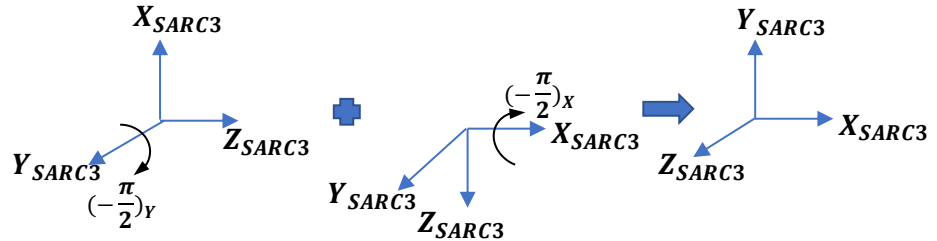


Figure A.3 Rotation of MARC local frame.

The transformation matrix is

$$\begin{aligned}
T_{SARC3}^{Oinput} &= R_{O_{SARC3}}^{Oinput}(Y, \alpha_{SARC3}) * R_{O_{SARC3}}^{Oinput}(X, \alpha_{SARC3}) \\
&\quad * Trans_{O_{SARC3}}^{Oinput}(l_{xSARC3}, l_{ySARC3}, l_{zSARC3}) \\
&= R_{O_{SARC3}}^{Oinput}(Y, -\pi/2) * R_{SARC3}^{Oinput}(X, -\pi/2) \\
&\quad * Trans_{SARC3}^{Oinput}(l_{SARC3x}, 0, 0) \\
&= \begin{bmatrix} R_{O_{SARC3}}^{Oinput}(-\pi/2)_Y & 0 \\ 0 & R_{O_{SARC3}}^{Oinput}(-\pi/2)_Y \end{bmatrix} \\
&\quad * \begin{bmatrix} R_{O_{SARC3}}^{Oinput}(-\pi/2)_X & 0 \\ 0 & R_{O_{SARC3}}^{Oinput}(-\pi/2)_X \end{bmatrix} * \begin{bmatrix} I & (\hat{r}_{SARC3}^{Oinput})^T \\ 0 & I \end{bmatrix} \\
&= \begin{bmatrix} \cos(-\pi/2) & 0 & \sin(-\pi/2) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin(-\pi/2) & 0 & \cos(-\pi/2) & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(-\pi/2) & 0 & \sin(-\pi/2) \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sin(-\pi/2) & 0 & \cos(-\pi/2) \end{bmatrix} \\
&\quad * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos(-\pi/2) & -\sin(-\pi/2) & 0 & 0 & 0 \\ 0 & \sin(-\pi/2) & \cos(-\pi/2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos(-\pi/2) & -\sin(-\pi/2) \\ 0 & 0 & 0 & 0 & \sin(-\pi/2) & \cos(-\pi/2) \end{bmatrix} \\
&\quad * \begin{bmatrix} 1 & 0 & 0 & 0 & l_{SARC3Z}^{Oinput} & -l_{SARC3Y}^{Oinput} \\ 0 & 1 & 0 & -l_{SARC3Z}^{Oinput} & 0 & l_{SARC3X}^{Oinput} \\ 0 & 0 & 1 & l_{SARC3Y}^{Oinput} & -l_{SARC3X}^{Oinput} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \\
&\quad * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & l_{SARC3X}^{Oinput} \\ 0 & 0 & 1 & 0 & -l_{SARC3X}^{Oinput} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & l_{SARC3X} \\ 0 & 0 & 1 & 0 & -l_{SARC3X} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}
\end{aligned} \tag{A.10}$$

A.3 The LS Transformation matrices

To express the frames O_{SL_1} , O_{SL_2} , O_{SL_3} and O_{SL_4} relative to the reference frame, the following successive transformations are required:

Frame O_{SL_1}

- Translation of $[0, 0, -\frac{w_{FAM}}{2}]$.

The transformation matrix is

$$\begin{aligned}
 T_{SL_1}^{O_{in}} &= Trans_{O_{SL_1}}^{O_{in}}(l_{x_{SL_1}}, l_{y_{SL_1}}, l_{z_{SL_1}}) = Trans_{O_{SL_1}}^{O_{in}}\left(0, 0, -\frac{w_{FAM}}{2}\right) \\
 &= \begin{bmatrix} I & (\hat{r}_{SL_1}^{O_{in}})^T \\ 0 & I \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 & 0 & l_{SL_1 z}^{O_{in}} & 0 \\ 0 & 1 & 0 & -l_{SL_1 z}^{O_{in}} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{w_{FAM}}{2} & 0 \\ 0 & 1 & 0 & \frac{w_{FAM}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (A.11)
 \end{aligned}$$

Frame O_{SL_2}

- Translation of $[b_{RL_1}, 0, -\frac{w_{FAM}}{2}]$

The transformation matrix is

$$\begin{aligned}
 T_{SL_2}^{O_{in}} &= Trans_{O_{SL_2}}^{O_{in}}(l_{x_{SL_2}}, l_{y_{SL_2}}, l_{z_{SL_2}}) = Trans_{O_{SL_2}}^{O_{in}}\left(a_a, 0, -\frac{w_{FAM}}{2}\right) \\
 &= \begin{bmatrix} I & (\hat{r}_{SL_2}^{O_{in}})^T \\ 0 & I \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 & 0 & l_{SL_2 z}^{O_{input}} & 0 \\ 0 & 1 & 0 & -l_{SL_2 z}^{O_{input}} & 0 & l_{SL_2 x}^{O_{input}} \\ 0 & 0 & 1 & 0 & -l_{SL_2 x}^{O_{input}} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{w_{FAM}}{2} & 0 \\ 0 & 1 & 0 & \frac{w_{FAM}}{2} & 0 & b_{RL_1} \\ 0 & 0 & 1 & 0 & -b_{RL_1} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (A.12)
 \end{aligned}$$

Frame O_{SL_3}

- Rotation of (π) about the $X - axis$, followed by
- Translation of $[b_{RL_1}, 0, \frac{w_{FAM}}{2}]$.

The transformation matrix is

$$\begin{aligned}
T_{SL3}^{O_{input}} &= R_{O_{SL3}}^{O_{input}}(X, \alpha_{SL3}) * Trans_{O_{SL3}}^{O_{input}}(l_{x_{SL3}}, l_{y_{SL3}}, l_{z_{SL3}}) \\
&= R_{O_{SL3}}^{O_{input}}(X, \pi) * Trans_{O_{SL3}}^{O_{input}}\left(b_{RL1}, 0, \frac{w_{FAM}}{2}\right) \\
&= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos(\pi) & -\sin(\pi) & 0 & 0 & 0 \\ 0 & \sin(\pi) & \cos(\pi) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos(\pi) & -\sin(\pi) \\ 0 & 0 & 0 & 0 & \sin(\pi) & \cos(\pi) \end{bmatrix} \\
&\quad * \begin{bmatrix} 1 & 0 & 0 & 0 & l_{SL3}^{O_{input}}{}_z & 0 \\ 0 & 1 & 0 & -l_{SL3}^{O_{input}}{}_z & 0 & l_{SL2}^{O_{input}}{}_x \\ 0 & 0 & 1 & 0 & -l_{SL3}^{O_{input}}{}_x & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{w_{FAM}}{2} & 0 \\ 0 & -1 & 0 & \frac{w_{FAM}}{2} & 0 & -b_{RL1} \\ 0 & 0 & -1 & 0 & b_{RL1} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{A.13}
\end{aligned}$$

Frame O_{SL4} Rotation of (π) about the $X - axis$, followed by translation of $[0,0, \frac{w_{FAM}}{2}]$.

The transformation matrix is

$$\begin{aligned}
T_{SL4}^{O_{in}} &= R_{O_{SL4}}^{O_{input}}(X, \alpha_{SL4}) * Trans_{O_{SL4}}^{O_{input}}(l_{x_{SL4}}, l_{y_{SL4}}, l_{z_{SL4}}) \\
&= R_{O_{SL4}}^{O_{input}}(X, \pi) * Trans_{O_{SL4}}^{O_{input}}\left(0, 0, \frac{w_{FAM}}{2}\right) \\
&= \begin{bmatrix} R_{O_{SL4}}^{O_{input}}(\pi)_X & 0 \\ 0 & R_{O_{SL4}}^{O_{input}}(\pi)_X \end{bmatrix} * \begin{bmatrix} I & (\hat{r}_{SL4}^{O_{input}})^T \\ 0 & I \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos(\pi) & -\sin(\pi) & 0 & 0 & 0 \\ 0 & \sin(\pi) & \cos(\pi) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos(\pi) & -\sin(\pi) \\ 0 & 0 & 0 & 0 & \sin(\pi) & \cos(\pi) \end{bmatrix} \\
&\quad * \begin{bmatrix} 1 & 0 & 0 & 0 & r_{SL4}^{O_{input}}{}_z & 0 \\ 0 & 1 & 0 & -r_{SL4}^{O_{input}}{}_z & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{w_{FAM}}{2} & 0 \\ 0 & -1 & 0 & \frac{w_{FAM}}{2} & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{A.14}
\end{aligned}$$

Appendix B

MATLAB Codes of Input Stiffness Model

```

function [ ] = INPUTSTIFFNESS02( x )
%Constants
pi=3.14159; E=71000000000; G =26692000000;k2=0.312;
%Alfa=30;
Wfam=0.012;
bRL1=0.008;
hls=0.012;

%Variables:
L = x (1);
Tmarc = x (2);
Tsarc = x (3);
Lls = x (4);
Tls = x (5);
Alpha = x (6);

%Intermediates:
Rmarc = 2.5*Tmarc;
Rsarc = 2.5*Tsarc;
SinAlpha = sind (Alpha);
CosAlpha = cosd (Alpha);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Input stiffness of LS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
DxFxLS = ((4*Lls^3)/(E*hls*Tls^3)) + (1.2*Lls/(G*Bls*Tls));
DxMyLS = (6*Lls^2)/(E*hls*Tls^3);
DyFyLS = ((4*Lls^3)/(E*hls^3*Tls)) + ((1.2*Lls/(G*Bls*Tls)));
DyMxLS = -(6*Lls^2)/(E*hls^3*Tls);
DzFzLS = Lls/(E*hls*Tls);
ThxFyLS = -(6*Lls^2)/(E*hls^3*Tls);
ThxMxLS = (12*Lls)/(E*hls^3*Tls);
ThyFyLS = (6*Lls^2)/(E*hls*Tls^3);
ThyMyLS = (12*Lls)/(E*hls*Tls^3);
ThzMzLS = Lls/(G*k2*hls*Tls^3);

%Define the Compliant Matrix of LS at local reference
syms CSLdxFx CSLdxMy CSLdyFy CSLdyMx CSLdzFz CSLthxFy CSLthxMx CSLthyFx
CSLthyMy CSLthzMz
CSL = [CSLdxFx 0 0 0 CSLdxMy 0; 0 CSLdyFy 0 CSLdyMx 0 0; 0 0 CSLdzFz 0 0 0; 0
CSLthxFy 0 CSLthxMx 0 0; CSLthyFx 0 0 0 CSLthyMy 0; 0 0 0 0 CSLthzMz];
%set values to the matrix elements
ComMatSL =
subs(CSL,{CSLdxFx,CSLdxMy,CSLdyFy,CSLdyMx,CSLdzFz,CSLthxFy,CSLthxMx,CSLthyFx,
CSLthyMy,CSLthzMz},{DxFxLS,DxMyLS,DyFyLS,DyMxLS,DzFzLS,ThxFyLS,ThxMxLS,ThyFyLS,
ThyMyLS,ThzMzLS});

%Transfers Matrices of LS to global reference( wlinka/2=0.005, b= 0.01 )
TransS11 = [1 0 0 0 -0.006 0; 0 1 0 0 0.006 0 0; 0 0 1 0 0 0; 0 0 0 1 0 0; 0 0
0 0 1 0; 0 0 0 0 0 1];
TransS12 = [1 0 0 0 -0.006 0; 0 1 0 0 0.006 0 0.008; 0 0 1 0 -0.008 0; 0 0 0 1
0 0; 0 0 0 0 1 0; 0 0 0 0 0 1];
TransS13 = [1 0 0 0 0.006 0; 0 -1 0 0 0.006 0 -0.008; 0 0 -1 0 0.008 0; 0 0 0 1
0 0; 0 0 0 0 -1 0; 0 0 0 0 0 -1];
TransS14 = [1 0 0 0 0.006 0; 0 -1 0 0 0.006 0 0; 0 0 -1 0 0 0; 0 0 0 1 0 0; 0 0
0 0 -1 0; 0 0 0 0 0 -1];

%Compliance Matrices at global reference
CompInSL1= double (transpose(TransS11)* ComMatSL *TransS11 );
CompInSL2= double ( transpose(TransS12)* ComMatSL * TransS12);
CompInSL3= double (transpose(TransS13)* ComMatSL * TransS13);
CompInSL4= double (transpose(TransS14)* ComMatSL * TransS14);

%Stiffness Matrices at global reference
StiffInSL1= inv(CompInSL1) ;
StiffInSL2= inv(CompInSL2) ;
StiffInSL3= inv(CompInSL3) ;
StiffInSL4= inv(CompInSL4) ;

```

```

EqCompInLS = inv(StiffInSL1 + StiffInSL2 + StiffInSL3 + StiffInSL4);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Input stiffness of MARC
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Intermediates:
DyFyMARC
=((256*Rmarc^3)/(3*pi*E*(Tmarc^4)*((2*Rmarc+Tmarc)^2)*sqrt((4*Rmarc+Tmarc)^7))
)* (sqrt(4*Rmarc+Tmarc)*(1664*(Rmarc^5)+3340*(Rmarc^4)*Tmarc+2620*(Rmarc^3)*
(Tmarc^2)+987*(Rmarc^2)*(Tmarc^3)+188*Rmarc*(Tmarc^4)+16*(Tmarc^5)))+(3*Rmarc*
(2*Rmarc+Tmarc)^2)*
sqrt(Tmarc)*(100*(Rmarc^2)+84*Rmarc*Tmarc+21*(Tmarc^2)))*atan(sqrt(1+(4*Rmarc
/Tmarc)))));
DyMxMARC=((256*(Rmarc^2))/(3*pi*E*(Tmarc^4)))*
(3+(Tmarc*(120*(Rmarc^4)+176*(Rmarc^3)*Tmarc+92*(Rmarc^2)+24*Rmarc*(Tmarc^3)+
3*(Tmarc^4))/((2*Rmarc+Tmarc)^2*(4*Rmarc+Tmarc)^3))+
24*Rmarc*(5*(Rmarc^2)+4*Rmarc*Tmarc+(Tmarc^2))*sqrt(Tmarc/((4*Rmarc+Tmarc)^7))
)* atan (sqrt(1+4*Rmarc/Tmarc)));
DxFxMARC = DyFyMARC;
DxMyMARC = DyMxMARC;
DzFzMARC =
(16*Rmarc/(pi*E*Tmarc^2))*(((2*Rmarc+Tmarc)/(4*Rmarc+Tmarc)))+(2*Rmarc*
sqrt(Tmarc/(4*Rmarc+Tmarc)^3))*atan(sqrt(1+4*Rmarc/Tmarc)));
ThxFyMARC = DyMxMARC;
ThyFxMARC = DxMyMARC;
ThzMzMARC = (E/(4*Rmarc*G))*DyMxMARC;
ThyMyMARC= (2*G/E)*ThzMzMARC;
ThxMxMARC= ThyMyMARC;

LyMARC_Val= Rmarc + L; %(2*L*sin (Alpha30)=0.04
LxMARC_Val= Wlinka + 2*Rsarc + 1.7321* L; %(2*L*cos (Alpha30)0.06996
%Compliant Matrix of MARC
syms CMARCDxFx CMARCDxMy CMARCDyFy CMARCDyMx CMARCDzFz CMARCThxFy CMARCThxMx
CMARCThyFx CMARCThyMy CMARCThzMz
CMARC = [CMARCDxFx 0 0 0 CMARCDxMy 0; 0 CMARCDyFy 0 CMARCDyMx 0 0; 0 0
CMARCDzFz 0 0 0; 0 CMARCThxFy 0 CMARCThxMx 0 0; CMARCThyFx 0 0 0 CMARCThyMy
0; 0 0 0 0 0 CMARCThzMz];
CompMatxMARC =
subs(CMARC,{CMARCDxFx,CMARCDxMy,CMARCDyFy,CMARCDyMx,CMARCDzFz,CMARCThxFy,CMAR
CThxMx,CMARCThyFx,CMARCThyMy,CMARCThzMz},{DxFxMARC,DxMyMARC,DyFyMARC,DyMxMARC
,DzFzMARC,ThxFyMARC,ThxMxMARC,ThyFxMARC,ThyMyMARC,ThzMzMARC});

%Transfers Matrices of MARC
%syms LyMARC LxMARC

%TransMARC1 = [0 0 1 LyMARC -LxMARC 0; -1 0 0 0 0 LyMARC; 0 -1 0 0 0 -LxMARC;
0 0 0 0 0 1; 0 0 0 -1 0 0; 0 0 0 0 -1 0];

%TransMatxMARC1 = subs(TransMARC1,{LyMARC,LxMARC},{LyMARC_Val,LxMARC_Val});
TransMatxMARC1 = [0 0 1 0.04 -0.06996 0; -1 0 0 0 0 0.04; 0 -1 0 0 0 -
0.06996; 0 0 0 0 0 1; 0 0 0 -1 0 0; 0 0 0 0 -1 0];

CompofMARC1 = transpose (TransMatxMARC1)* (CompMatxMARC) * TransMatxMARC1 ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Input stiffness of SARC
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Intermediates:
DxFxSARC =
(9*pi*(Rsarc^2.5))/(2*E*Wfam*(Tsarc^2.5))+(3*pi*(Rsarc^1.5))/(2*E*Wfam*(Tsarc
^1.5));
DxMySARC = (9*pi*(Rsarc^1.5))/(2*E*Wfam*(Tsarc^2.5));
DyFySARC = ((12*pi*Rsarc^2)/(E*Wfam^3))*(((Rsarc/Tsarc)^0.5)-(2.57));
DyMxSARC = -(12*Rsarc/(E*Wfam^3))*(pi*((Rsarc/Tsarc)^0.5)-(2.57));
DzFzSARC = (1/(E* Wfam))*(pi*((Rsarc/Tsarc)^0.5)-(pi/2));
ThxFxSARC = -(12*Rsarc/(E*Wfam^3))*(pi*((Rsarc/Tsarc)^0.5)-(2.57));
ThxMxSARC = (12/(E*Wfam^3))*(pi*((Rsarc/Tsarc)^0.5)-(2.57));
ThyFxSARC = (9*pi*(Rsarc^1.5))/(2*E*Wfam*(Tsarc^2.5));
ThyMySARC = (9*pi*(Rsarc^0.5))/(2*E*Wfam*(Tsarc^2.5));
ThzMzSARC = (9*pi*Rsarc^0.5)/(4*G*Wfam*Tsarc^2.5);

LxSARC1_Val = bRL1 + 2*Rsarc;
LySARC2_Val = SinAlpha*L;
LxSARC2_Val = bRL1 + 2* Rsarc+ CosAlpha* L;
LxSARC3_Val = bRL1 + 2*CosAlpha*L;

%Compliant Matrix of SARC

```

```

syms CSARCDxFx CSARCDxMy CSARCDyFy CSARCDyMx CSARCDzFz CSARCDzMy
CSARCDzFx CSARCDzMy CSARCDzFy CSARCDzMx CSARCDzFz CSARCDzMy
CSARCDzFx CSARCDzMy CSARCDzFy CSARCDzMx
CSARC = [CSARCDxFx 0 0 0 CSARCDxMy 0; 0 CSARCDyFy 0 CSARCDyMx 0 0 ; 0 0
CSARCDzFz 0 0 0; 0 CSARCDzFx 0 CSARCDzMy 0 0; CSARCDzFy 0 0 0 CSARCDzMy
0; 0 0 0 0 0 CSARCDzFz];
ComMatSARC=
subs (CSARC, {CSARCDxFx,CSARCDxMy,CSARCDyFy,CSARCDyMx,CSARCDzFz,CSARCDzMy,CSARCDzFx,CSARCDzMy,CSARCDzFy,CSARCDzMx}, {DxFxSARC,DxMySARC,
DyFySARC,DyMxSARC,DzFzSARC,ThxFxSARC,ThxMxSARC,ThyFySARC,ThyMySARC,ThzMzSARC}
);

%Transfers Matrices of SARC
syms LxSARC1 LxSARC2 LySARC2 LxSARC3
TransSARC1 = [0 1 0 0 0 LxSARC1; 0 0 -1 0 LxSARC1 0; -1 0 0 0 0 0; 0 0 0 0 1
0; 0 0 0 0 0 -1; 0 0 0 -1 0 0];
TransSARC2 = [0 1 0 0 0 LxSARC2; 0 0 -1 -LySARC2 LxSARC2 0 ; -1 0 0 0 0 -
LySARC2; 0 0 0 0 1 0; 0 0 0 0 0 -1; 0 0 0 -1 0 0];
TransSARC3 = [0 1 0 0 0 LxSARC3; 0 0 1 0 -LxSARC3 0; 1 0 0 0 0 0; 0 0 0 0 1
0; 0 0 0 0 0 1; 0 0 0 1 0 0];

%Assing values to LxSARC1 LySARC2 LxSARC2 LxSARC3
TransMatxSARC1 = double (subs (TransSARC1,{LxSARC1},{LxSARC1_Val}));
TransMatxSARC2 = double
(subs (TransSARC2,{LxSARC2,LySARC2},{LxSARC2_Val,LySARC2_Val}));
TransMatxSARC3 = double (subs (TransSARC3,{LxSARC3},{LxSARC3_Val}));

CompofSARC1 = transpose (TransMatxSARC1) * ComMatSARC *TransMatxSARC1 ;
CompofSARC2 = transpose (TransMatxSARC2) * ComMatSARC *TransMatxSARC2 ;
CompofSARC3 = transpose (TransMatxSARC3) * ComMatSARC *TransMatxSARC3 ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Input Stiffness of the spatial stage
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
InputStiff= double( inv(EqCompInLS +
CompofSARC1+(inv(inv(CompofSARC2+CompofSARC3)+ inv(CompofMARC1)))));
InputStiff

```

Appendix C

Potential Energy Modelling Via 1-DOFM

C.1 Potential energy of flexure hinges

The potential energy is generated by the strain energy stored as elastic deformations of the flexure hinges and the gravity of the moving platform. As the stage has out of plane motions, the gravity potential energy is taken into consideration. The potential energies of the entire stage can be obtained from the following:

Potential energy of LS flexure hinges

There are four similar leaf-spring flexure hinges connected to a rigid link and all have the same linear displacements. The rigid link, $RL1$, is directly connected to the PZT actuator. Therefore, linear translation of the rigid link is equal to the input displacement, d_{in} , of the actuator. The potential energy of the four LS flexure hinges can be achieved from the following

$$V_{LS} = 4 \left(\frac{1}{2} k_{LS} d_{in}^2 \right) = \frac{1}{2} (4k_{LS}) d_{in}^2 \quad (C.1)$$

where k_{LS} is the stiffness of LS flexure hinge expressed by

$$k_{LS} = \frac{E h_{LS} t_{LS}^3}{l_{LS}^3} \quad (C.2)$$

Potential energy of SARC flexure hinges

There are three similar SARC flexure hinges employed at each FAM. From Figure 3.2, when the SARC flexure hinges at point O and A rotate with an angular displacement of θ_{SARC} , the angular displacement of SARC flexure hinge at point B will be $2\theta_{SARC}$. Therefore, the potential energy of SARC flexure hinges is can be achieved from the following

$$V_{SARC} = 2 \left(\frac{1}{2} k_{SARC} \theta_{SARC}^2 \right) + \frac{1}{2} k_{SARC} (2 \theta_{SARC})^2 = \frac{1}{2} (6 k_{SARC}) \theta_{SARC}^2 \quad (C.3)$$

where k_{SARC} is the rotational stiffness of SARC flexure hinge and equal to the inverse of the bending compliance listed in Table 2.3, which is expressed as

$$k_{SARC} = \frac{2Ew_{SARC}t_{SARC}^{2.5}}{9\pi r_{SARC}^{0.5}} \quad (C.4)$$

with the relationships of $t_{SARC} \ll r_{SARC}$ and $t_{SARC} \ll w_{SARC}$. θ_{SARC} is the deformation angle of SARC flexure hinge, which is formulated in (3.18) as follows

$$\theta_{SARC} = \frac{d_{in}}{2L \sin \alpha} \quad (C.5)$$

From (A.3) and (A.5), the Potential energy of SARC flexure hinges, V_{SARC} , can be expressed as a function of d_{in} as follows

$$V_{SARC} = \frac{1}{2} (6 k_{SARC}) \left(\frac{d_{in}}{2L \sin \alpha} \right)^2 = \frac{1}{2} \left(\frac{3 k_{SARC}}{2 L^2 \sin^2 \alpha} \right) d_{in}^2 \quad (C.6)$$

C.2 Gravitational potential energy

Potential energy of the moving platform, GPE_O , only comes from its gravity. Therefore, the gravitational potential energy computed in (4.113) is

$$GPE_O = m_O g \text{ Amp } d_{in} \quad (C.7)$$

C.3 Total potential energy

The total potential energy of the entire stage is the sum of the equations (C.1), (A.6), and (A.7) as follows

$$\begin{aligned} V &= V_{LS} + V_{SARC} + GPE \\ &= \frac{1}{2} (4k_{LS}) d_{in}^2 + \frac{1}{2} \left(\frac{3 k_{SARC}}{2 L^2 \sin^2 \alpha} \right) d_{in}^2 + m_O g \text{ Amp } d_{in} \\ &= \frac{1}{2} \left(4k_{LS} + \frac{3 k_{SARC}}{2 L^2 \sin^2 \alpha} \right) d_{in}^2 + m_O g \text{ Amp } d_{in} \\ &= \frac{1}{2} (K_{FAM1-DOFM}) d_{in}^2 + m_O g \text{ Amp } d_{in} \end{aligned} \quad (C.8)$$

where $K_{FAM1-DOFM}$ is the input stiffness of the FAM, can be written as follows

$$K_{FAM1-DOFM} = 4k_{LS} + \frac{3 k_{SARC}}{2 L^2 \sin^2 \alpha} \quad (C.9)$$

From (C.9) and (4.94), the natural frequency can be achieved from the following

$$f_{1-DOFM} = \frac{1}{2\pi} \sqrt{\frac{K_{1-DOFM}}{m_{3 \text{ act}}}} \quad (C.10)$$

Appendix D

Potential Energy Modelling Via EBA

D.1. One Active Actuator

A schematic model of one FAM connected to the moving platform is shown in Figure D.1. In addition, only the two MARC flexure hinges of the other FAM are presented. The potential energy model of the stage in the case of operating one actuator is presented.

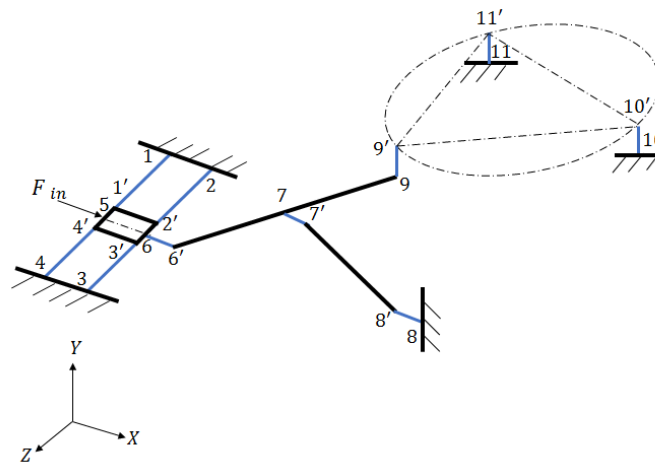


Figure D.1 Schematic model of one FAM.

To obtain the potential energy, the application of EBA requires following the steps of the approach offered in Section 4.1.2.

1. Free-Body Diagram

Using X, Y, and Z coordinate system, it is necessary first to determine the support reactions that acting on the mechanism and their locations in the free-body diagram, as shown in Figure D.2. Keeping the external force acts at the input point, imaginary sections are passed through the mechanism at the points where the internal loadings are to be determined.

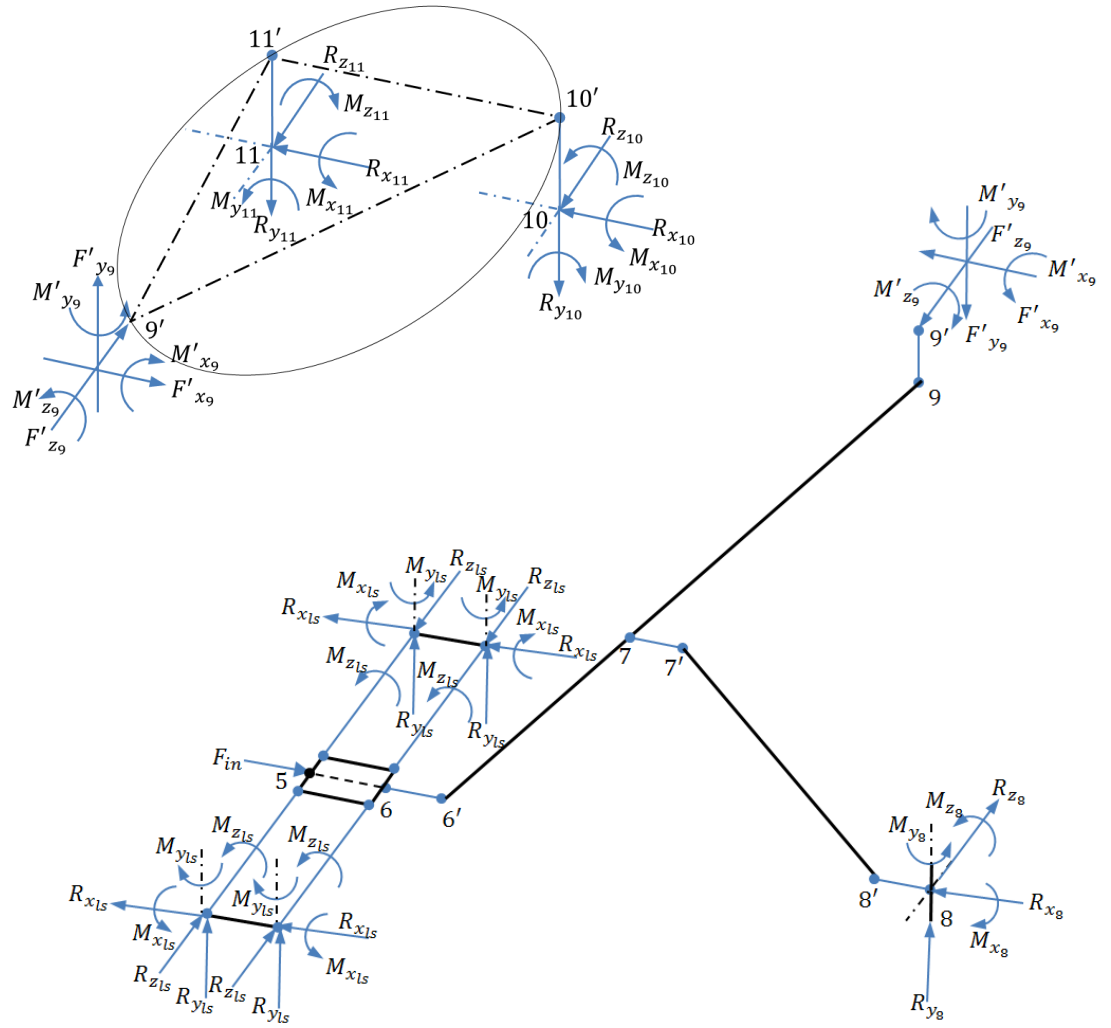


Figure D.2 Free-body diagram for support reactions and external loadings for FAM and the moving platform.

2. Equations of Internal Loadings

To define the internal loadings acting on the flexure hinges, it is required first to draw the free-body diagram of one cut segment for each flexure hinge and indicate the unknown internal loadings at the section. These loadings are usually placed at the geometric centre of the sectioned area.

The internal loadings of each flexure hinge in the mechanism are defined in terms of the support reactions by applying the equations of equilibrium. The sign convention is those counter-clockwise bending moments are positive and clockwise bending moments are negative. If an internal loading yields a negative value that means the assumed directional sense is in the opposite direction.

If the flexure hinge is subjected to a coplanar system of loads and rigid enough to prevent any out-of-plane loads, then only axial-force, shear-force, and bending moment components will be considered in the analysis. Therefore, the structural design of each flexure hinge is considered rigid enough to prevent any out-of-plane displacements. For each flexure hinge, only the unknown internal loadings produce displacements are found.

LS Flexure hinge 1,2,3, and 4

The in-plane loadings of LS flexure hinges are the axial force, shear force, and bending moment at Z-axis, X-axis, and Y-axis directions, respectively, as shown in Figure D.3 (a). From Figure D.3 (b), LS flexure hinge can be modelled as a cantilever beam, fixed at one end and free but guided at the other end. Therefore, the flexure hinge has 1-DOF in the X-axis direction. Consequently, only the internal shear force is considered.

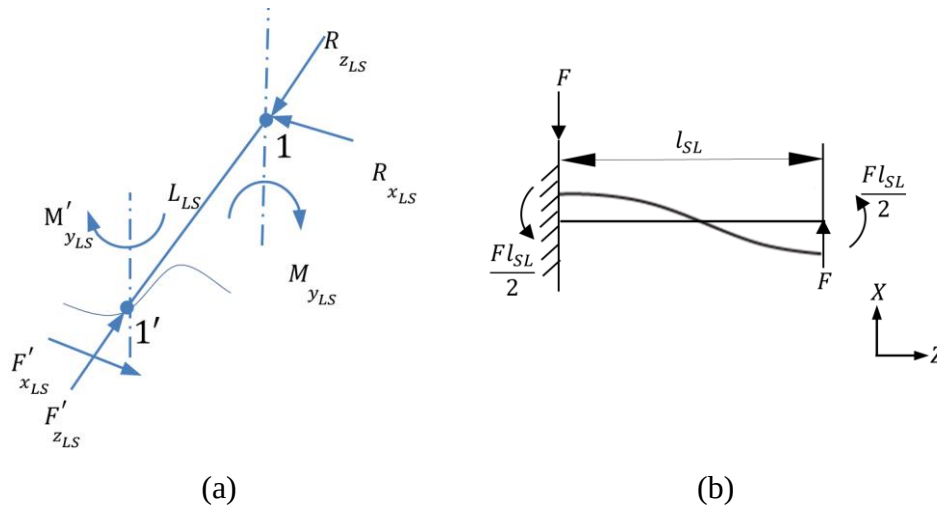


Figure D.3 Free-body diagram of a cut segment LS flexure hinge shows:

(a) Internal loadings, (b) Bending moment diagram.

The four LS flexure hinges are symmetric. Therefore, only one flexure is modelled. From Figure D.3 (a), by applying the equilibrium equations, the internal shear force is

$$\Sigma F_x = 0$$

$$F'_{x1} = F'_{x2} = F'_{x3} = F'_{x4} = F'_{xLS} = R_{xLS} \quad (D.1)$$

The internal bending moment

$$\Sigma M_{y1'} = 0$$

$$-M'_{yLS} + R_{xLS} L_{LS} - M_{yLS} = 0$$

$$M'_{yLS} = R_{xLS} L_{LS} - M_{yLS} \quad (D.2)$$

Considering the Leaf-spring flexure hinge as fixed-guided cantilever beam, and from Figure D.3 (b), the bending moment reaction can be written as

$$M_{y_{LS}} = \frac{1}{2} R_{x_{LS}} L_{LS} \quad (D.3)$$

Therefore, substituting (D.3) into (D.2), the internal bending moment can be expressed as a function of one unknown as follows

$$M'_{y_{LS}} = R_{x_{LS}} L_{LS} - \frac{1}{2} R_{x_{LS}} L_{LS} = \frac{1}{2} R_{x_{LS}} L_{LS} \quad (D.4)$$

SARC Flexure hinge 6

The internal loadings causing displacements are the axial force, shear force, and bending moment at X-axis, Y-axis, and Z-axis directions, respectively. Using Figure D.4 and applying the equations of equilibrium, the internal loadings are as follows:

The internal axial force

$$\begin{aligned} \Sigma F_x &= 0 \\ -F'_{x_6} - 4R_{x_{LS}} + F_{in} &= 0 \\ \therefore F'_{x_6} &= F_{in} - 4R_{x_{LS}} \end{aligned} \quad (D.5)$$

The internal shear force

$$\begin{aligned} \Sigma F_y &= 0 \\ -F'_{y_6} + 4R_{y_{LS}} &= 0 \\ \therefore F'_{y_6} &= 4R_{y_{LS}} \end{aligned} \quad (D.6)$$

The internal bending moment

$$\begin{aligned} \Sigma M_{z_{6'}} &= 0 \\ M'_{z_6} + 4M_{z_{LS}} - 2R_{y_{LS}} L_{5-6'} - 2R_{y_{LS}} D_{SARC} &= 0 \\ \therefore M'_{z_6} &= 4R_{y_{LS}} \left(\frac{1}{2} L_{5-6} + D_{SARC} \right) - 4M_{z_{LS}} \end{aligned} \quad (D.7)$$

where L_{5-6} is the distance from point 5 to 6 and D_{SARC} is the diameter of the SARC .

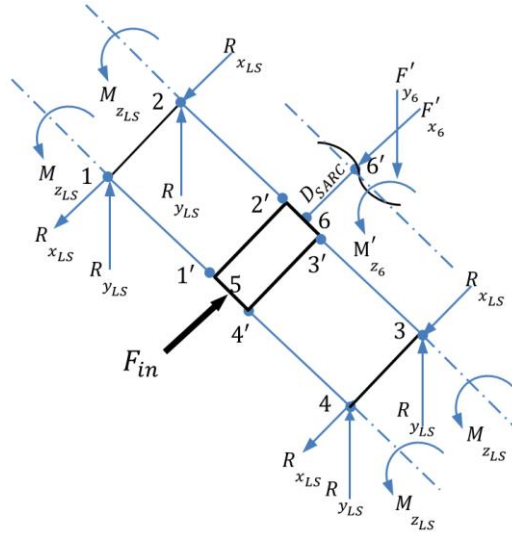


Figure D.4 Cut segment of the SARC flexure hinge 6.

SARC Flexure hinge 7

Applying the equations of equilibrium to the segment in Figure D.5 (a), the internal loadings are:

The internal axial force

$$\begin{aligned}\Sigma F_x &= 0 \\ F'_{x_7} &= R_{x_8}\end{aligned}\tag{D.8}$$

The internal shear force

$$\begin{aligned}\Sigma F_y &= 0 \\ F'_{y_7} &= R_{y_8}\end{aligned}\tag{D.9}$$

The internal bending moment

$$\begin{aligned}\Sigma M_{z_7} &= 0 \\ M'_{z_7} + M_{z_8} - R_{x_8} L \sin \alpha + R_{y_8} (D_{SARC} + L \cos \alpha) &= 0 \\ \therefore M'_{z_7} &= R_{x_8} L \sin \alpha - R_{y_8} (D_{SARC} + L \cos \alpha) - M_{z_8}\end{aligned}\tag{D.10}$$

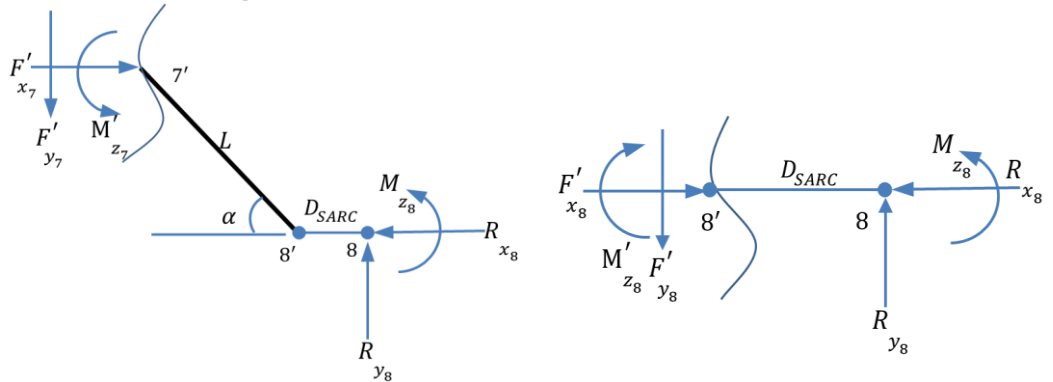


Figure D.5 Cut segment of: (a) SARC flexure hinge 7, and (b) SARC flexure hinge 8.

SARC Flexure hinge 8

From Figure D.5 (b), applying the three equations of equilibrium:

The internal axial force

$$\begin{aligned}\sum F_x &= 0 \\ F'_{x_8} &= R_{x_8}\end{aligned}\tag{D.11}$$

The internal shear force

$$\begin{aligned}\sum F_y &= 0 \\ F'_{y_8} &= R_{y_8}\end{aligned}\tag{D.12}$$

The internal bending moment

$$\begin{aligned}\sum M_{z_8'} &= 0 \\ -M'_{z_8} + M_{z_8} + R_{y_8} D_{SARC} &= 0 \\ \therefore M'_{z_8} &= M_{z_8} + R_{y_8} D_{SARC}\end{aligned}\tag{D.13}$$

MARC Flexure hinge 9

The internal axial and shear forces and bending moment are acting on the Y-axis and Z-axis and the rotation about X- axis, respectively, as shown in Figure D.2.

The internal loadings can be expressed as follows:

The internal axial force is

$$\begin{aligned}\sum F_y &= 0 \\ F'_{y_9} - 2R_{y_{10,11}} &= 0 \\ F'_{y_9} &= 2R_{y_{10,11}} = R_{y_8} + 4R_{y_{LS}}\end{aligned}\tag{D.14}$$

The internal shear force is

$$\begin{aligned}\sum F_z &= 0 \\ F'_{z_9} - R_{z_8} &= 0 \\ F'_{z_9} &= R_{z_8}\end{aligned}\tag{D.15}$$

The internal bending moment is

$$\begin{aligned}\sum M_{x_9'} &= 0 \\ M'_{x_9} - M_{x_8} + R_{z_8} L_{8-9'} &= 0 \\ \therefore M'_{x_9} &= M_{x_8} - R_{z_8} L_{8-9'}\end{aligned}\tag{D.16}$$

MARC Flexure hinge 10 & 11

The MARC flexure hinge 10 and 11 are under the same internal loadings, therefore

$$F'_{y_{10}} = F'_{y_{11}} = F'_{y_{10,11}}\tag{D.17}$$

$$F'_{z_{10}} = F'_{z_{11}} = F'_{z_{10,11}}\tag{D.18}$$

$$M'_{x_{10}} = M'_{x_{11}} = M'_{x_{10,11}}\tag{D.19}$$

The internal loadings at flexure hinge 10 and 11, as shown in Figure D.6, can be written as follows:

The internal axial force

$$\begin{aligned}\Sigma F_y &= 0 \\ F'_{y_{10,11}} - R_{y_{10,11}} &= 0 \\ F'_{y_{10,11}} &= R_{y_{10,11}}\end{aligned}\quad (D.20)$$

The internal shear force

$$\begin{aligned}\Sigma F_z &= 0 \\ F'_{z_{10,11}} - R_{z_{10,11}} &= 0 \\ F'_{z_{10,11}} &= R_{z_{10,11}}\end{aligned}\quad (D.21)$$

The internal bending moment

$$\begin{aligned}\Sigma M_{x_{10',11'}} &= 0 \\ -M'_{x_{10,11}} + M_{x_{10,11}} - R_{z_{10,11}} D_{MARC} &= 0 \\ M'_{x_{10,11}} &= M_{x_{10,11}} - R_{z_{10,11}} D_{MARC}\end{aligned}\quad (D.22)$$

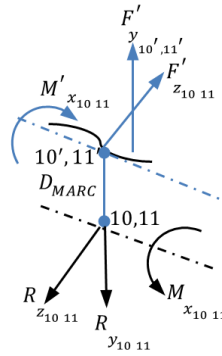


Figure D.6 Cut segment of the SARC flexure hinge 10 and 11.

3. Unknown Support Reactions

To fully describe the internal loadings of the flexure hinges, only the support reactions, which appear in the expressions of loadings, must be expressed as a function of F_{in} .

From Figure D.2, the equations of equilibrium can be applied as follows:

$$\begin{aligned}\Sigma F_x &= 0 \\ F_{in} - R_{x_8} - 4R_{x_{LS}} + R_{x_{10}} - R_{x_{11}} &= 0 \\ F_{in} - R_{x_8} - 4R_{x_{LS}} &= 0\end{aligned}\quad (D.23)$$

$$\begin{aligned}\Sigma F_y &= 0 \\ 4R_{y_{LS}} + R_{y_8} - 2R_{y_{10,11}} &= 0\end{aligned}\quad (D.24)$$

where $R_{y_{10}} = R_{y_{11}} = R_{y_{10,11}}$

$$\begin{aligned}
\Sigma F_z &= 0 \\
2R_{z_{LS}} - 2R_{z_{LS}} + R_{z_8} - R_{z_{10}} - R_{z_{11}} &= 0 \\
\therefore R_{z_8} &= 2R_{z_{10,11}}
\end{aligned} \tag{D.25}$$

where $R_{z_{10}} = R_{z_{11}} = R_{z_{10,11}}$ and L_{6-8} and L_{8-9} are the distances between point 6 and 8 and point 8 and 9, respectively.

$$\begin{aligned}
\Sigma M_{x_8} &= 0 \\
M_{x_{10}} + M_{x_{11}} - M_{x_8} - 2R_{y_{10,11}}L_0 + 2R_{z_{10,11}}L_{8-9} &= 0 \\
2M_{x_{10,11}} - M_{x_8} - 2R_{y_{10,11}}L_0 + 2R_{z_{10,11}}L_{8-9} &= 0
\end{aligned} \tag{D.26}$$

$$\begin{aligned}
\Sigma M_{y_8} &= 0 \\
M_{y_8} &= 0
\end{aligned} \tag{D.27}$$

$$\begin{aligned}
\Sigma M_{z_8} &= 0 \\
4M_{z_{LS}} + M_{z_8} - 2R_{y_{LS}}L_{6-8} - 2R_{y_{LS}}L_{5-8} &= 0 \\
4M_{z_{LS}} + M_{z_8} - 4R_{y_{LS}}\left(L_{6-8} + \frac{1}{2}L_{5-6}\right) &= 0
\end{aligned} \tag{D.28}$$

There are 12 internal loadings in terms of 12 unknown support reactions and only 6 equilibrium equations. Therefore, 6 more equations are required in order to express all the support reactions as a function of the input force. These additional equations can be achieved by using the displacement conditions of the flexure hinges.

From the kinematic analysis in Section 3.1.3, by using (3.7), (3.8), and (3.9) the displacement conditions of SARC flexure hinges 6 and 8 can be rewritten as follows

$$d_{x_6} = d_{x_8} \tag{D.29}$$

$$d_{y_6} = d_{y_8} \tag{D.30}$$

$$\theta_{z_6} = \theta_{z_8} \tag{D.31}$$

Using (4.15) to obtain the axial displacements of flexure 6 and 8 and replacing the results into (D.29), the following condition can be achieved

$$\begin{aligned}
C_{dx-Fx_{SARC}}F'_{x_6} &= C_{dx-Fx_{SARC}}F'_{x_8} \\
F'_{x_6} &= F'_{x_8}
\end{aligned} \tag{D.32}$$

Substituting (D.5) and (D.11) into (D.32)

$$\begin{aligned}
(F_{in} - 4R_{x_{LS}}) &= R_{x_8} \\
F_{in} - 4R_{x_{LS}} - R_{x_8} &= 0
\end{aligned} \tag{D.33}$$

Therefore, the assumption that $R_{x_{10}}$ and $R_{x_{11}}$ have opposite directions is correct.

The support reactions $R_{x_{LS}}$ and R_{x_8} have direct relationship with the input force. Therefore, to express all the other support reactions as a function of F_{in} , they need first to be related to $R_{x_{LS}}$ or R_{x_8} . This is done in the following points by using the relationships and conditions of the support reactions.

R_{y_8} , $R_{y_{LS}}$, and $R_{y_{10,11}}$ as a function of R_{x_8}

From (D.30) and (D.31), the following conditions can be concluded

$$F'_{y_6} = F'_{y_8} \quad (D.34)$$

and

$$M'_{z_6} = M'_{z_8} \quad (D.35)$$

Replacing (D.6) and (D.12) into (D.34)

$$4R_{y_{LS}} = R_{y_8} \quad (D.36)$$

Substituting (D.36) into (D.24), $R_{y_{10,11}}$ is

$$R_{y_{10,11}} = 4R_{y_{LS}} = R_{y_8} \quad (D.37)$$

The mechanism, which consists of Link 6-7 and 7-8, can be treated as crank-slide mechanism, as shown Figure D.7. Therefore, by using Lami's formula from the force diagram of point 6 shown in Figure D.8, the following relationship can be obtained

$$\frac{F_{in} - 4R_{x_{LS}}}{\sin(90 - \alpha)} = \frac{4R_{y_{LS}}}{\sin \alpha} = \frac{F_{67}}{\sin 90} \quad (D.38)$$

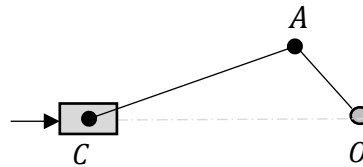


Figure D.7 A Crank-slide mechanism.



Figure D.8 (a) Free-body diagram of flexure hinge 6, (b) force polygon.

From (D.36) and (D.38), the support reactions $4R_{y_{LS}}$ and R_{y_8} as a function of R_{x_8} can be expressed as

$$4R_{y_{LS}} = R_{y_8} = \frac{\sin \alpha}{\sin(90 - \alpha)} (F_{in} - 4R_{x_{LS}}) = R_{x_8} \tan \alpha \quad (D.39)$$

M_{z_8} and, $4M_{z_{LS}}$ as a function of R_{x_8}

Replacing (D.7) and (D.13) into (D.35), the relationship between the bending moment $4M_{z_{LS}}$ and M_{z_8} , while considering the opposite direction, can be written as follows

$$\begin{aligned} 4R_{y_{LS}} \left(\frac{1}{2}L_{5-6} + D_{SARC} \right) - 4M_{z_{LS}} &= -(M_{z_8} + R_{y_8} D_{SARC}) \\ 4M_{z_{LS}} &= R_{y_8} D_{SARC} + 4R_{y_{LS}} \left(\frac{1}{2}L_{5-6} + D_{SARC} \right) + M_{z_8} \end{aligned} \quad (D.40)$$

From (D.39), (D.40) can be rewritten as follows

$$4M_{z_{LS}} = R_{y_8} \left(\frac{1}{2}L_{5-6} + 2D_{SARC} \right) + M_{z_8} \quad (D.41)$$

Substituting (D.35) and (D.41) into (D.28), and solving for M_{z_8}

$$M_{z_8} = R_{y_8} \left(\frac{1}{2}L_{6-8} - D_{SARC} \right) \quad (D.42)$$

Rewriting M_{z_8} as a function of R_{x_8} , by replacing (D.39) with (D.42) as follows

$$M_{z_8} = R_{x_8} \left(\frac{1}{2}L_{6-8} - D_{SARC} \right) \tan \alpha \quad (D.43)$$

By substituting (4.143) into (4.142), $4M_{z_{LS}}$ is

$$4M_{z_{LS}} = R_{y_8} \left(\frac{1}{2}L_{5-8} + D_{SARC} \right) \quad (D.44)$$

Substituting (D.36) into (D.44), $4M_{z_{LS}}$ can be expressed as

$$4M_{z_{LS}} = R_{x_8} \left(\frac{1}{2}L_{5-8} + D_{SARC} \right) \tan \alpha \quad (D.45)$$

$R_{z_{10,11}}$, $M_{x_{10,11}}$ and M_{x_8} as a function of R_{x_8}

Figure D. 9 (a) represents an equivalent 2-D plane structure of the moving platform shown in Figure D.2. The structure possesses five unknown reactions and three equilibrium equations, as shown in Figure D.9 (b). Therefore, the moving platform is statically indeterminate with two degrees of indeterminacy. Accordingly, two more equations are needed to obtain all the unknown loadings as a function of R_{x_8} . These two equations can be achieved by applying the displacement conditions of the moving platform, as follows:

As flexure hinge 10 and 11 are symmetric and in parallel connection, the equivalent compliance factors can be achieved from the following

$$\begin{aligned} C_{d_z - M_{x_{MARC}} eq} &= \frac{1}{2} C_{d_z - M_{x_{MARC}}} \\ C_{\theta_x - M_{x_{MARC}} eq} &= \frac{1}{2} C_{\theta_x - M_{x_{MARC}}} \end{aligned} \quad (D.46)$$

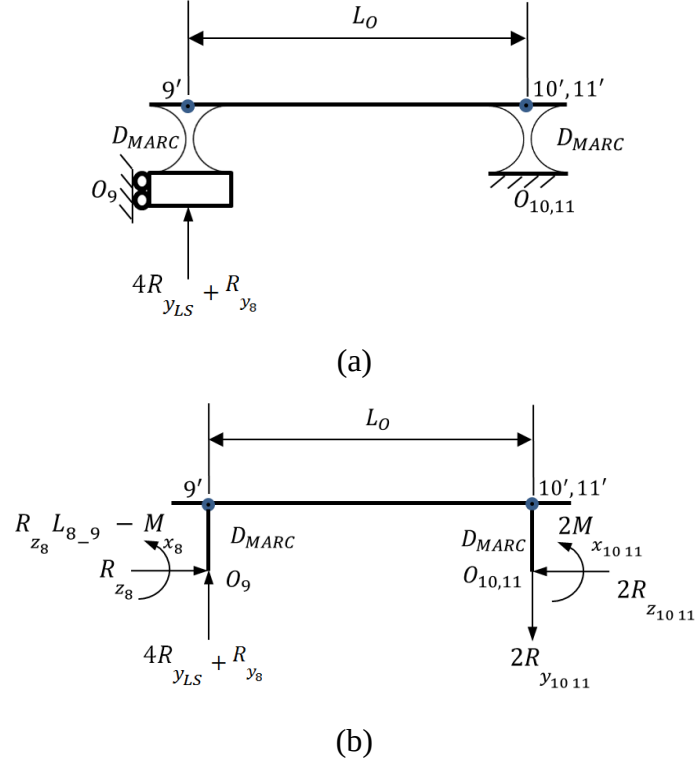


Figure D.9 The moving platform (a) an equivalent scheme and (b) Free-body-diagram.

From (4.17) and (D.46), the rotational displacement of flexure hinge at 10,11 can be written as

$$\begin{aligned}\theta_{x_{10,11}} &= C_{d_z-M_{x_{MARCeq}}} * 2F'_{z_{10,11}} - C_{\theta_x-M_{x_{MARCeq}}} * 2M'_{x_{10,11}} \\ &= C_{d_z-M_{x_{MARC}}} F'_{z_{10,11}} - C_{\theta_x-M_{x_{MARC}}} M'_{x_{10,11}}\end{aligned}\quad (D.47)$$

Replacing (D.21) and (D.22) into (D.47), the rotational displacement is

$$\theta_{x_{10,11}} = C_{d_z-M_{x_{MARC}}} R_{z_{10,11}} - C_{\theta_x-M_{x_{MARC}}} (M_{x_{10,11}} - R_{z_{10,11}} D_{MARC}) \quad (D.48)$$

From (D.9), the rotational displacement $\theta_{x_{10,11}}$ can be expressed in terms of the input displacement d_{in} and L_O , as follows

$$\sin \theta_{x_{10,11}} \approx \theta_{x_{10,11}} = \frac{Amp}{L_O} d_{in} \quad (D.49)$$

The translation displacement of point 6 is equal to the axial displacement of the four LS flexure hinges. The displacement of one LS flexure hinge can be found from the following

$$\begin{aligned}d_{LS} &= C_{d_x-F_{x_{LS}}} F'_{x_{LS}} - C_{d_x-M_{y_{LS}}} M'_{y_{LS}} \\ &= \left(C_{d_x-F_{x_{LS}}} - \frac{1}{2} L_{LS} C_{d_x-M_{y_{LS}}} \right) R_{x_{LS}} = C_{LS} R_{x_{LS}}\end{aligned}\quad (D.50)$$

where C_{LS} is the compliance of each LS flexure hinge, which can be written as

$$C_{LS} = C_{d_x-F_{x_{LS}}} - \frac{1}{2} L_{LS} C_{d_x-M_{y_{LS}}} \quad (D.51)$$

The displacement of the four LS flexure hinges $d_{in\ LS}$ known as a double-parallelgram flexure hinge can be written as follows

$$d_{in\ LS} = d_{in} = \frac{1}{4} C_{LS} 4R_{xLS} = \frac{1}{4} (C_{d_x-F_{xLS}} - \frac{1}{2} L_{LS} C_{d_x-M_{yLS}}) 4R_{xLS} \quad (D.52)$$

Substituting (D.52) into (D.49) and replacing the results with (D.48), the following relationship can be achieved

$$\begin{aligned} \frac{Amp\ C_{LS}}{2L_o} 4R_{xLS} &= 2C_{d_z-M_{xMARC}} R_{z_{10,11}} \\ &\quad - C_{\theta_x-M_{xMARC}} (2M_{x_{10,11}} - 2R_{z_{10,11}} D_{MARC}) \end{aligned} \quad (D.53)$$

From Figure D. 9 (b), the rotational displacement at point $O_{10,11}$ is zero, which can be expressed as

$$\theta_{x_{O_{10,11}}} = -2C_{d_z-M_{xMARC}} R_{z_{10,11}} + 2C_{\theta_x-M_{xMARC}} M_{x_{10,11}} = 0 \quad (D.54)$$

Solving for $2M_{x_{10,11}}$

$$2M_{x_{10,11}} = 2 \left(C_{d_z-M_{xMARC}} / C_{\theta_x-M_{xMARC}} \right) R_{z_{10,11}} \quad (D.55)$$

By substituting (D.55) into (D.53), $2R_{z_{10,11}}$ and R_{z_8} can be expressed as

$$2R_{z_{10,11}} = R_{z_8} = \frac{Amp\ C_{LS}}{2L_o D_{MARC} C_{\theta_x-M_{xMARC}}} 4R_{xLS} = \zeta (F_{in} - R_{x_8}) \quad (D.56)$$

Replacing (D.56) into (D.55)

$$2M_{x_{10,11}} = \zeta \left(C_{d_z-M_{xMARC}} / C_{\theta_x-M_{xMARC}} \right) (F_{in} - R_{x_8}) \quad (D.57)$$

With the substitution of (D.58) and (D.57) into (D.26), M_{x_8} can be written as follows

$$M_{x_8} = \zeta \left(C_{d_z-M_{xMARC}} / C_{\theta_x-M_{xMARC}} + L_{8-9} \right) (F_{in} - R_{x_8}) - 2 R_{y_{10,11}} L_o \quad (D.58)$$

R_{xLS} and R_{x_8} as a function of F_{in}

From (D.33), it is necessary to find another relation between R_{xLS} and R_{x_8} to define them in terms of F_{in} . According to (3.14) and [2], the rotational displacement of flexure hinge 6 can be approximately expressed in terms of the input displacement as follows

$$\theta_{z_6} \approx \frac{d_{in}}{2L \sin \alpha} \quad (D.59)$$

Replacing (D.6) and (D.7) into (4.17), the rotational displacement is

$$\theta_{z_6} = -4R_{yLS} C_{d_y-M_{zSARC}} + \left(4R_{yLS} \left(\frac{1}{2} L_{5-6} + D_{SARC} \right) - 4M_{zLS} \right) C_{\theta_z-M_{zSARC}} \quad (D.60)$$

Substituting (D.39) and (D.45) into (D.60) and taking the absolute value of the angular displacement

$$\theta_{z_6} = \left(C_{d_y-M_{zSARC}} + \frac{1}{2} L_{6-8} C_{\theta_z-M_{zSARC}} \right) \tan \alpha R_{x_8} \quad (D.61)$$

By substituting (D.52) and (D.61) into (D.59), the relationship between $R_{x_{LS}}$ and R_{x_8} can be derived as follows

$$\begin{aligned} & (C_{d_y-M_{z_{SARC}}} + \frac{1}{2}L_{6-8}C_{\theta_z-M_{z_{SARC}}}) \tan \alpha R_{x_8} \\ &= \frac{1}{4} \left(C_{d_x-F_{x_{LS}}} - \frac{1}{2}L_{LS} C_{d_x-M_{y_{LS}}} \right) 4R_{x_{LS}}/2L\sin\alpha \end{aligned} \quad (D.62)$$

Solving for $R_{x_{LS}}$

$$R_{x_{LS}} = 2L\sin\alpha \tan \alpha \left(\frac{C_{d_y-M_{z_{SARC}}} + \frac{1}{2}L_{6-8}C_{\theta_z-M_{z_{SARC}}}}{C_{d_x-F_{x_{LS}}} - \frac{1}{2}L_{LS} C_{d_x-M_{y_{LS}}}} \right) R_{x_8} = \varepsilon R_{x_8} \quad (D.63)$$

where

$$\varepsilon = 2L\sin\alpha \tan \alpha \left(\frac{C_{d_y-M_{z_{SARC}}} + \frac{1}{2}L_{6-8}C_{\theta_z-M_{z_{SARC}}}}{C_{d_x-F_{x_{LS}}} - \frac{1}{2}L_{LS} C_{d_x-M_{y_{LS}}}} \right) \quad (D.64)$$

Replacing (D.63) with (D.23), the support reaction R_{x_8} can be expressed as a function of F_{in} as follows

$$R_{x_8} = \frac{1}{4\varepsilon + 1} F_{in} \quad (D.65)$$

Replacing (D.65) with (D.23)

$$4R_{x_{LS}} = \frac{4\varepsilon}{4\varepsilon + 1} F_{in} \quad (D.66)$$

Writing $4R_{y_{LS}}$ and R_{y_8} in terms of F_{in} by joining (D.65) with (D.39), as follows

$$4R_{y_{LS}} = R_{y_8} = \frac{\tan \alpha}{4\varepsilon + 1} F_{in} \quad (D.67)$$

Writing M_{z_8} as a function of F_{in} by replacing (D.67) with (D.43) as follows

$$M_{z_8} = \frac{\tan \alpha}{4\varepsilon + 1} \left(\frac{1}{2}L_{6-8} - D_{SARC} \right) F_{in} \quad (D.68)$$

Substituting (4.168) into (4.146), $4M_{z_{LS}}$ can be written as a function of F_{in} as follows

$$4M_{z_{LS}} = \frac{\tan \alpha}{4\varepsilon + 1} \left(\frac{1}{2}L_{5-8} + D_{SARC} \right) F_{in} \quad (D.69)$$

By the substitution of (D.66) into (D.45), R_{z_8} can be expressed as follows

$$2R_{z_{10,11}} = R_{z_8} = \zeta \left(\frac{4\varepsilon}{4\varepsilon + 1} \right) F_{in} \quad (D.70)$$

Replacing (D.66) into (D.57)

$$2M_{x_{10,11}} = \zeta \left(\frac{4\varepsilon}{4\varepsilon + 1} \right) \left(C_{d_z-M_{x_{MARC}}} / C_{\theta_x-M_{x_{MARC}}} \right) F_{in} \quad (D.71)$$

Applying (D.37) and (D.66) in (D.58), M_{x_8} can be written as follows

$$M_{x_8} = \frac{1}{4\varepsilon + 1} \left(4\varepsilon \zeta \left(C_{d_z-M_{x_{MARC}}} / C_{\theta_x-M_{x_{MARC}}} + L_{8-9} \right) - 2L_0 \tan \alpha \right) F_{in} \quad (D.72)$$

Rewriting the internal loadings of the flexure hinges as a function of F_{in} as follows:

LS Flexure hinge 1,2,3, and 4

As the four LS flexure hinges has 1-DOF, only the displacement in the shear direction is considered. From (D.3) and (D.66), the shear force acting on the double parallel LS flexure hinges is

$$4R_{xLS} = \left(\frac{4\varepsilon}{4\varepsilon + 1} \right) F_{in} \quad (D.73)$$

SARC Flexure hinge 6

Combining (D.5) and (D.66)

$$F'_{x_6} = \frac{1}{4\varepsilon + 1} F_{in} = \beta'_{x_6} F_{in} \quad (D.74)$$

Joining (D.6) with (D.67)

$$F'_{y_6} = \frac{\tan \alpha}{4\varepsilon + 1} F_{in} = \beta'_{y_6} F_{in} \quad (D.75)$$

Replacing (D.67) and (D.69) into (D.7)

$$M'_{z_6} = -\frac{1}{2}L_{6-8} \left(\frac{\tan \alpha}{4\varepsilon + 1} \right) F_{in} = \beta'_{z_6} F_{in} \quad (D.76)$$

SARC Flexure hinge7

Substituting (D.65) into (D.8)

$$F'_{x_7} = -\frac{1}{4\varepsilon + 1} F_{in} = \beta'_{x_7} F_{in} \quad (D.77)$$

Combining (D.67) with (D.9)

$$F'_{y_7} = \frac{\tan \alpha}{4\varepsilon + 1} F_{in} = \beta'_{y_7} F_{in} \quad (D.78)$$

Replacing (D.65), (D.67), and (D.68) with (4.11)

$$\begin{aligned} M'_{z_7} &= \frac{1}{4\varepsilon + 1} \left(L \sin \alpha - \tan \alpha \left(L \cos \alpha + \frac{1}{2} L_{6-8} \right) \right) F_{in} \\ &= \beta'_{z_7} F_{in} \end{aligned} \quad (D.79)$$

SARC Flexure hinge 8

From (D.11) and (D.65)

$$F'_{x_8} = \frac{1}{4\varepsilon + 1} F_{in} = \beta'_{x_8} F_{in} \quad (D.80)$$

Joining (D.67) with (D.12)

$$F'_{y_8} = \frac{\tan \alpha}{4\varepsilon + 1} F_{in} = \beta'_{y_8} F_{in} \quad (D.81)$$

Replacing (D.67) and (D.68) into (D.13)

$$M'_{z_8} = \frac{1}{2} L_{6-8} \left(\frac{\tan \alpha}{4\varepsilon + 1} \right) F_{in} = \beta'_{z_8} F_{in} \quad (D.82)$$

MARC Flexure hinge 9

From (D.14) and (D.67)

$$F'_{y_9} = 2 \frac{\tan \alpha}{4\varepsilon + 1} F_{in} = \beta'_{y_9} F_{in} \quad (D.83)$$

From (D.15), and (D.70)

$$F'_{z_9} = \zeta \left(\frac{4\varepsilon}{4\varepsilon + 1} \right) F_{in} = \beta'_{z_9} F_{in} \quad (D.84)$$

With the substitution of (D.70), and (D.72) into (D.16)

$$\begin{aligned} M'_{x_9} &= \frac{1}{4\varepsilon + 1} \left(4\varepsilon \zeta \left(C_{d_z - M_{x_{MARC}}} / C_{\theta_x - M_{x_{MARC}}} - D_{MARC} \right) - 2L_0 \tan \alpha \right) F_{in} \\ &= \beta'_{x_9} F_{in} \end{aligned} \quad (D.85)$$

MARC Flexure hinge 10 and 11

From (D.20) and (D.67)

$$F'_{y_{10,11}} = \frac{\tan \alpha}{4\varepsilon + 1} F_{in} = \beta'_{y_{10,11}} F_{in} \quad (D.86)$$

And from (D.21) and (D.70)

$$F'_{z_{10,11}} = -\frac{1}{2} \zeta \left(\frac{4\varepsilon}{4\varepsilon + 1} \right) F_{in} = \beta'_{z_{10,11}} F_{in} \quad (D.87)$$

With the substitution of (D.70) and (D.71) into (D.22)

$$\begin{aligned} M'_{x_{10,11}} &= -\frac{1}{2} \zeta \left(\frac{4\varepsilon}{4\varepsilon + 1} \right) \left(C_{d_z - M_{x_{MARC}}} / C_{\theta_x - M_{x_{MARC}}} - D_{MARC} \right) F_{in} \\ &= \beta'_{x_{10,11}} F_{in} \end{aligned} \quad (D.88)$$

4. Equation of potential energy

From the generic potential energy equation (4.19), the potential energy of each flexure hinge can be computed as follows:

LS Flexure hinge 1,2,3, and 4

The four LS flexure hinges are symmetric and connected in parallel. Therefore, they have the same potential energy, which can be achieved as follows

$$V_{LS} = \frac{1}{2} d_{in\ LS} 4R_{x_{LS}} \quad (D.89)$$

From (D.52) and (D.66), the potential energy of the double parallel LS flexure hinges is

$$V_{LS} = \frac{1}{2} \left(\frac{1}{4} C_{LS} 4R_{x_{LS}} \right) 4R_{x_{LS}} = \frac{1}{2} \left[\frac{1}{4} \left(\frac{4\varepsilon}{4\varepsilon + 1} \right)^2 C_{LS} \right] F_{in}^2 = \frac{1}{2} C_{LS_{eq}} F_{in}^2 \quad (D.90)$$

where $C_{LS_{eq}}$ is the compliance of the double parallel LS flexure hinges.

From (D.51), $C_{LS_{eq}}$ can be written as follows

$$C_{LS_{eq}} = \frac{1}{4} \left(\frac{4\varepsilon}{4\varepsilon + 1} \right)^2 \left(C_{d_x - F_{x_{LS}}} - \frac{1}{2} L_{LS} C_{d_x - M_{y_{LS}}} \right) \quad (D.91)$$

SARC Flexure hinge 6

The potential energy of flexure hinge 6 is

$$V_6 = \frac{1}{2} \left[\beta_{x_6}'^2 C_{d_x - F_{xSARC}} + \beta_{y_6}'^2 C_{d_y - F_{ySARC}} + 2\beta_{y_6}'\beta_{z_6}' C_{d_y - M_{zSARC}} + \beta_{z_6}'^2 C_{\theta_z - M_{zSARC}} \right] F_{in}^2 = \frac{1}{2} C_6 F_{in}^2 \quad (D.92)$$

where C_6 is the compliance of 6 flexure hinge, which can be expressed, by using (D.74), (D.75), and (D.76), as follows

$$C_6 = \left(\frac{1}{4\varepsilon + 1} \right)^2 \left(\begin{array}{c} C_{d_x - F_{xSARC}} + \tan^2 \alpha \\ C_{d_y - F_{ySARC}} + L_{6-8} C_{d_y - M_{zSARC}} \\ + \frac{1}{4} L_{6-8}^2 C_{\theta_z - M_{zSARC}} \end{array} \right) \quad (D.93)$$

SARC Flexure hinge 7

The potential energy of flexure hinge 7 is

$$V_7 = \frac{1}{2} \left[\beta_{x_7}'^2 C_{d_x - F_{xSARC}} + \beta_{y_7}'^2 C_{d_y - F_{ySARC}} + 2\beta_{y_7}'\beta_{z_7}' C_{d_y - M_{zSARC}} + \beta_{z_7}'^2 C_{\theta_z - M_{zSARC}} \right] F_{in}^2 = \frac{1}{2} C_7 F_{in}^2 \quad (D.94)$$

where C_7 is the compliance of 7 flexure hinge, which can be written as follows by substituting (D.77), (D.78), and (D.79) into (D.94)

$$C_7 = \left(\frac{1}{4\varepsilon + 1} \right)^2 \left(\begin{array}{c} C_{d_x - F_{xSARC}} + \tan^2 \alpha C_{d_y - F_{ySARC}} + 2 \tan \alpha \\ \left(L \sin \alpha - \tan \alpha \left(L \cos \alpha + \frac{1}{2} L_{6-8} \right) \right) C_{d_y - M_{zSARC}} \\ + \left(L \sin \alpha - \tan \alpha \left(L \cos \alpha + \frac{1}{2} L_{6-8} \right) \right)^2 C_{\theta_z - M_{zSARC}} \end{array} \right) \quad (D.95)$$

SARC Flexure hinge 8

The potential energy of flexure hinge 8 is

$$V_8 = \frac{1}{2} \left[\beta_{x_8}'^2 C_{d_x - F_{xSARC}} + \beta_{y_8}'^2 C_{d_y - F_{ySARC}} + 2\beta_{y_8}'\beta_{z_8}' C_{d_y - M_{zSARC}} + \beta_{z_8}'^2 C_{\theta_z - M_{zSARC}} \right] F_{in}^2 = \frac{1}{2} C_8 F_{in}^2 \quad (D.96)$$

where C_8 is the compliance of 8 flexure hinge.

From (D.80), (D.81), and (D.82), the compliance of 8 flexure hinge can be written as follows

$$C_8 = \left(\frac{1}{4\varepsilon + 1} \right)^2 \left(\begin{array}{c} C_{d_x - F_{xSARC}} + \tan^2 \alpha \\ C_{d_y - F_{ySARC}} + L_{6-8} C_{d_y - M_{zSARC}} \\ + \frac{1}{4} L_{6-8}^2 C_{\theta_z - M_{zSARC}} \end{array} \right) \quad (D.97)$$

MARC Flexure hinge 9

The potential energy of flexure hinge 9 is

$$V_9 = \frac{1}{2} \left[\beta_{y_9}'^2 C_{d_y - F_{yMARC}} + \beta_{z_9}'^2 C_{d_z - F_{zMARC}} + 2\beta_{z_9}'\beta_{x_9}' C_{d_z - M_{xMARC}} + \beta_{x_9}'^2 C_{\theta_x - M_{xMARC}} \right] F_{in}^2 = \frac{1}{2} C_9 F_{in}^2 \quad (D.98)$$

where C_9 is the compliance of flexure hinge 9.

By substituting (D.83), (D.84), and (D.85) into (D.98), C_9 can be expressed as follows

$$C_9 = \left(\frac{1}{4\varepsilon + 1} \right)^2 \left(\begin{array}{c} 4 \tan^2 \alpha C_{d_y - F_{y_{MARC}}} + 16(\zeta\varepsilon)^2 \\ C_{d_z - F_{z_{MARC}}} + 2 \left(\left(C_{d_z - M_{x_{MARC}}} / C_{\theta_z - M_{z_{MARC}}} - D_{MARC} \right) - 2L_0 \tan \alpha \right) \\ C_{d_z - M_{x_{MARC}}} + \\ \left(\left(C_{d_z - M_{x_{MARC}}} / C_{\theta_z - M_{z_{MARC}}} - D_{MARC} \right) - 2L_0 \tan \alpha \right)^2 C_{\theta_x - M_{x_{MARC}}} \end{array} \right) \quad (D.99)$$

MARC Flexure hinge 10 and 11

The potential energy of flexure hinge MARC is

$$V_{10,11} = \frac{1}{2} \left[\beta'_{y_{10,11}}{}^2 C_{d_y - F_{y_{MARC}}} + \beta'_{z_{10,11}}{}^2 C_{d_z - F_{z_{MARC}}} + 2\beta'_{z_{10,11}}\beta'_{z_{10,11}} C_{d_z - M_{x_{MARC}}} + \beta'_{x_{10,11}}{}^2 C_{\theta_x - M_{x_{MARC}}} \right] F_{in}^2 = \frac{1}{2} C_{10,11} F_{in}^2 \quad (D.100)$$

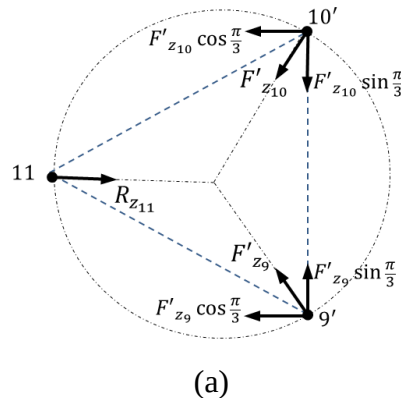
where $C_{10,11}$ is the compliance of flexure hinge 8.

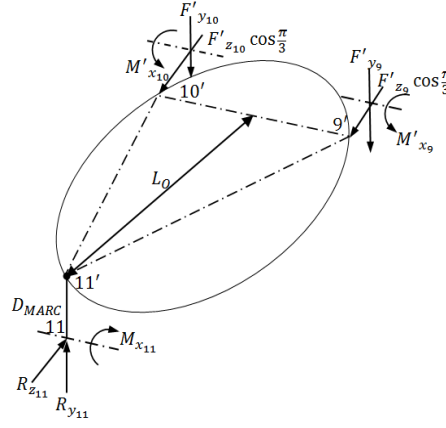
From (D.86), (D.87), and (D.88), $C_{10,11}$ can be written as follows

$$C_{10,11} = \left(\frac{1}{4\varepsilon + 1} \right)^2 \left(\begin{array}{c} (\tan \alpha)^2 C_{d_y - F_{y_{MARC}}} + 4(\zeta\varepsilon)^2 \\ \left(C_{d_z - F_{z_{MARC}}} + 2 \left(C_{d_z - M_{x_{MARC}}} / C_{\theta_x - M_{x_{MARC}}} - D_{MARC} \right) C_{d_z - M_{x_{MARC}}} \right) \\ + \left(C_{d_z - M_{x_{MARC}}} / C_{\theta_x - M_{x_{MARC}}} - D_{MARC} \right)^2 C_{\theta_x - M_{x_{MARC}}} \end{array} \right) \quad (D.101)$$

D.2. Two Active Actuators

When the two FAMs attached to flexure hinge 9 and 10 are operating, the axes of the rotation of the two flexures are in the same alignment, as shown in Figure D. 10 (a). Accordingly, the internal shear loading F'_{z_9} of flexure 9 has two components.





(b)

Figure D.10 Schematic of the moving platform: (a) Top view and (b) Free-body diagram.

From Figure D. 10 (b), by applying the equilibrium equation

$$\sum F_z = 0$$

$$F'_{z9} + F'_{z10} - R_{z11} = 0 \quad (D.102)$$

As $F'_{z9} = F'_{z10}$, (D.102) can be written as

$$F'_{z9} = \frac{1}{2 \cos \frac{\pi}{3}} R_{z11} = \frac{1}{4} R_{z11} \quad (D.103)$$

$$\sum F_y = 0$$

$$F'_{y9} + F'_{y10} - R_{y11} = 0 \quad (D.104)$$

As $F'_{y9} = F'_{y10}$, (D.104) can be written as

$$R_{y11} = 2F'_{y9} \quad (D.105)$$

$$\sum M_{x11'} = 0$$

$$2M'_{x92 act} - M_{x11} - 2F'_{y9}L_0 + R_{z11}D_{MARC} = 0 \quad (D.106)$$

Solving for M'_{x9}

$$M'_{x92 act} = \frac{1}{2}(M_{x11} + 2F'_{y9}L_0 - R_{z11}D_{MARC}) \quad (D.107)$$

Comparing Figure D.2 and D.10, it can be found that

$$R_{z11} = 2R_{z10,11} \quad (D.108)$$

and

$$M_{x11} = 2M_{x10,11} \quad (D.109)$$

Therefore, (D.56) and (D.57) can still be used in this analysis. Thus, the internal loadings of flexure hinge 9 can be expressed as follows:

From (D.70), (D.103) and (D.108), the internal shear force is

$$F'_{z9} = \frac{1}{4} R_{z11} = \frac{1}{4} \zeta \left(\frac{4\varepsilon}{4\varepsilon + 1} \right) F_{in} = \beta'_{z9} F_{in} \quad (D.110)$$

Substituting (D.70), (D.71), and (D.83) into (D.107), the internal bending moment of flexure hinge 9 can be written as

$$M'_{x_{92act}} = \left(\frac{2}{4\varepsilon + 1} \right) \left(\zeta\varepsilon \left(C_{d_z-M_{xMARC}} / C_{\theta_x-M_{xMARC}} - \frac{1}{4} D_{MARC} \right) + \tan \alpha L_0 \right) F_{in} = \beta'_{x_{92act}} F_{in} \quad (D.111)$$

The potential energy of flexure hinge 9 can be obtained by substituting (D.83), (D.110) and (D.111) into (D.98), as follows

$$V_{92act} = \frac{1}{2} \left[\left(\frac{4}{4\varepsilon + 1} \right)^2 \left(\tan^2 \alpha C_{d_y-F_{yMARC}} + (\zeta\varepsilon)^2 C_{d_z-F_{zMARC}} - \zeta\varepsilon \left(\left(\zeta\varepsilon \left(C_{d_z-M_{xMARC}} / C_{\theta_x-M_{xMARC}} - \frac{1}{4} D_{MARC} \right) + \tan \alpha L_0 \right) C_{d_z-M_{xMARC}} + \left(\left(\zeta\varepsilon \left(C_{d_z-M_{xMARC}} / C_{\theta_x-M_{xMARC}} - \frac{1}{4} D_{MARC} \right) + \tan \alpha L_0 \right) \right)^2 C_{\theta_x-M_{xMARC}} \right) \right] F_{in}^2 = \frac{1}{2} C_{92act} F_{in}^2 \quad (D.112)$$

where C_9 is the compliance of flexure hinge 9.

D.3. Three Active Actuators

When the stage is moving in the vertical direction, the three FAMs are active. As the three FAMs are identical, only one FAM is modelled. In each FAM, the deformations occur in LS and SARC flexure hinges. There is small rotational deformation in MARC flexure hinge. This is because the output displacement is produced in the vertical direction only. Therefore, it is assumed that no rotational displacement occurs in MARC flexure hinge. In this model, the weight of the moving platform F_0 is considered, as shown Figure D.11 (a).

From Figure D.11 (b), by applying the equilibrium equation in Y direction the weight of the moving platform can be expressed as

$$\Sigma F_y = 0$$

$$4R_{y_{LS}} + R_{y_8} - F_0 = 0 \quad (D.113)$$

By comparing (D.24) and (D.113), it is found that

$$2R_{y_{10,11}} = F_0 \quad (D.114)$$

With the absence of the bending moment reactions of MARC flexure hinge, there is no difference between the models of one or three active actuators in X and Z directions. Accordingly, the internal loadings acting on the LS and SARC flexure hinges derived for one active actuator can still be used in this model.

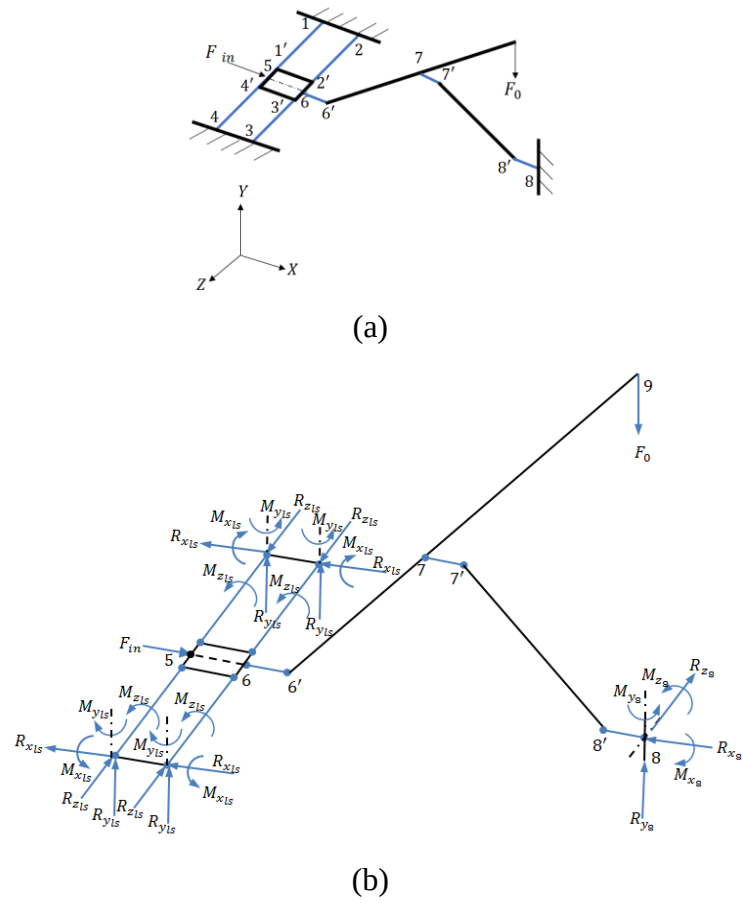


Figure D.11 Model of one FAM: (a) with the external forces and (b) free-body diagram.

Appendix E

MATLAB® Codes of Design Optimization

E.1 Objective Function Code

```
function f = ObjctvFreq( x )
pi    = 3.14159;
E     = 71000000000;
G     = 26692000000;
Dnsty = 2810;
Wsarc = 0.012;% 2*Rmarc + Tmarc
Tls   = 0.002;
hls   = 0.012;
R0    = 0.05;
T0    = 0.005;
Tmarc = 0.002;
Rmarc = 2.5*Tmarc;
bRL1  = 0.008;
hRL1  = 0.012;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Variables:
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L      = x(1);
Alpha  = x(2);
Tsarc  = x(3);
Lls    = x(4);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Intermediates:
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
SinAlpha = sind (Alpha);
CosAlpha = cosd (Alpha);
TanAlpha = tand (Alpha);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Rsarc = 2.5*Tsarc;
L6     = 3*2*Rsarc+2*L*CosAlpha;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Mass
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Parameters:
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L1RL= 2*L*SinAlpha + Rsarc+ 0.5*Tsarc;
L2RL= L*SinAlpha;
L3RL= L*CosAlpha - 2*Rsarc;
L4RL= L*SinAlpha + 1.7*Rsarc + Tsarc;
L5RL= L*SinAlpha - 1.3*Rsarc-0.5*Tsarc;
L6RL= L*CosAlpha + Rsarc + Rmarc + 0.5*Tmarc;
Lgx=(0.5*L1RL*L3RL^2+L6RL*L5RL*(L3RL+0.5*L6RL))/(L1RL*L3RL+L5RL*L6RL);
Lgy= (0.5*L3RL*L1RL^2+0.5*L6RL*L5RL^2)/(L1RL*L3RL+L5RL*L6RL);
Lg= sqrt (Lgx^2+Lgy^2);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
LG = sqrt (((L3RL+Rsarc)-Lgx)^2+(L2RL-Lgy)^2) + L;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
M0   = pi*R0^2*T0*Dnsty;
MRL1 = bRL1*hRL1*Wsarc*Dnsty;
MRL2 =(L1RL*L3RL+L5RL*L6RL)*Wsarc*Dnsty;
IRL2 =MRL2*LG^2+(L1RL*L3RL*((1/12)*(L1RL^2+L3RL^2)+(Lgx-0.5*L3RL)^2+(Lgy-0.5*L1RL)^2)+L6RL*L5RL*((1/12)*(L5RL^2+L6RL^2)+(Lgx-L3RL-0.5*L6RL)^2+(Lgy-0.5*L2RL)^2))*Wsarc*Dnsty;
IRL3 =L3RL*L4RL*Wsarc*Dnsty*((1/12)*L3RL^2+L4RL^2)+(0.5*L3RL+Rsarc)^2+(0.5*L4RL-Rsarc-0.5*Tsarc)^2);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Mass of The Stage
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Mstage = M0*(TanAlpha)^-2 + 3* (MRL1+(1/(2*L*SinAlpha)^2)*(MRL2* LG^2+IRL2+IRL3));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Compliances of Flexure hinges
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Compliances SARC
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
CdyFySARC = (9*pi*(Rsarc^2.5))/(2*E*Wsarc*(Tsarc^2.5))+(3*pi*(Rsarc^1.5))/(2*E*Wsarc*(Tsarc^1.5));
CdyMzSARC = (9*pi*(Rsarc^1.5))/(2*E*Wsarc*(Tsarc^2.5));
CdxFxSARC = (1/(E* Wsarc))*(pi*((Rsarc/Tsarc)^0.5)-(pi/2));
CThzMzSARC = (9*pi*(Rsarc^0.5))/(2*E*Wsarc*(Tsarc^2.5));
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Compliances of LS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
CdxFxLS = ((4*Lls^3)/(E*hls*Tls^3))+(1.2*Lls/(G*hls*Tls));
CdxMyLS = (6*Lls^2)/(E*hls*Tls^3);
CLS = CdxFxLS-0.5*Lls*CdxMyLS;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Parameters:
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Eps = 2*L*SinAlpha*TanAlpha*((CdyMzSARC + 0.5*L6*CThzMzSARC)/CLS);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Total Compliances:
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
FHLS = 0.25*CLS*(4*Eps/(4*Eps+1))^2;
FH6 = (1/(4*Eps+1))^2*(CdxFxSARC+(TanAlpha)^2*(CdyFySARC -
L6*CdyMzSARC+0.25*(L6)^2*CThzMzSARC));
FH7 = (1/(4*Eps+1))^2*(CdxFxSARC+(TanAlpha)^2*CdyFySARC + 2*TanAlpha*(L*SinAlpha-
TanAlpha*(L*cosAlpha+0.5*L6))*CdyMzSARC+(L*SinAlpha-
TanAlpha*(L*cosAlpha+0.5*L6))^2*CThzMzSARC);
FH8 = (1/(4*Eps+1))^2*(CdxFxSARC+(TanAlpha)^2*(CdyFySARC -
L6*CdyMzSARC+0.25*(L6)^2*CThzMzSARC));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Input Stiffness of the Stage
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Kfam = (inv(FHLS + FH6 + FH7 + FH8));
Kstage = 3*Kfam;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Natural Frequency
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Frq = (1/(2*pi))*sqrt(Kstage/Mstage);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% The Objective Function
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
f= 1/Frq;

end

```

E.2 Nonlinear Constrains Code

```

function [ c , ceq ] = ConstrnFreq( x )
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Constants
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
pi = 3.14159;
E = 71700000000;
G = 26900000000;
Sf = 2;
Segma = 503000000;
ktb = 1.2;
Wsarc = 0.01;
Tls = 0.002;
hls = 0.01;
R0 = 0.06;
Tmarc = 0.002;
Rmarc = 2*Tmarc;
Dout = 0.0002;
Dinmax = 0.00025;
Kpza = 32000000;
Dpzamax = 0.00018; %Max. travle range of PZT actuator
Finmax = 4500;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Variables:
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L = x(1);
Alpha = x(2);
Tsarc = x(3);
Lls = x(4);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Intermediates:
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Rsarc = 2 * Tsarc;
SinAlpha = sind(Alpha);
CosAlpha = cosd(Alpha);
TanAlpha = tand(Alpha);
Amp = 1/TanAlpha;
I0 = 1.5*R0;
L6 = 3*2*Rsarc+2*L*cosAlpha;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Compliance of Flexure hinge
%Compliances SARC
CdyFySARC =
(9*pi*(Rsarc^2.5))/(2*E*Wsarc*(Tsarc^2.5))+(3*pi*(Rsarc^1.5))/(2*E*Wsarc*(Tsarc^1.5));

```

```

CdyMzSARC = (9*pi*(Rsarc^1.5))/(2*E*Wsarc*(Tsarc^2.5));
CdxFxSARC = (1/(E*Wsarc))*(pi*(Rsarc/Tsarc)^0.5)-(pi/2));
CThzMzSARC = (9*pi*(Rsarc^0.5))/(2*E*Wsarc*(Tsarc^2.5));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Compliances of LS
CdxFxLS = ((4*Lls^3)/(E*hls*Tls^3))+(1.2*Lls/(G*hls*Tls));
CdxMyLS = (6*Lls^2)/(E*hls*Tls^3);
CLS = CdxFxLS-0.5*Lls*CdxMyLS;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Compliances of MARC
CDyMxMARC
= ((256*(Rmarc^2))/(3*pi*E*(Tmarc^4)))*(3+(Tmarc*(120*(Rmarc^4)+176*(Rmarc^3)*Tmarc+92*(Rmarc^2)+24*Rmarc*(Tmarc^3)+3*(Tmarc^4))/(2*Rmarc+Tmarc)^2*(4*Rmarc+Tmarc)^3)+
24*Rmarc*(5*(Rmarc^2)+4*Rmarc*Tmarc+(Tmarc^2))*sqrt(Tmarc/((4*Rmarc+Tmarc)^7))*atan
(sqrt(1+4*Rmarc/Tmarc));
CThxMxMARC = (1/(2*Rmarc))*CDyMxMARC;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ThitaSARCmax = Dinmax/(2*L*SinAlpha);
ThitaMARCmax = 2 * Amp * Dinmax /L0 - CThxMxMARC * L0* Finmax;
Bita = Tsarc/(2*Rsarc);
fBita =
(1/(2*Bita+Bita^2))*(((3+4*Bita+2*Bita^2)/((1+Bita)*(2*Bita+Bita^2))))+((6*(1+Bita))/((
2*Bita+Bita^2)^1.5))*atan(sqrt((2+Bita)/Bita));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Eps = 2*L*SinAlpha*TanAlpha*((CdyMzSARC + 0.5*L6*CThzMzSARC)/CLS);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Total Compliances:
FHLS = 0.25*CLS*(4*Eps/(4*Eps+1))^2;
FH6 = (1/(4*Eps+1))^2*(CdxFxSARC+(TanAlpha)^2*(CdyFySARC +
L6*CdyMzSARC+0.25*(L6)^2*CThzMzSARC));
FH7 = (1/(4*Eps+1))^2*(CdxFxSARC+(TanAlpha)^2*CdyFySARC - 2*TanAlpha*(L*SinAlpha-
TanAlpha*(L*CosAlpha+0.5*L6))*CdyMzSARC+(L*SinAlpha-
TanAlpha*(L*CosAlpha+0.5*L6))^2*CThzMzSARC);
FH8 = (1/(4*Eps+1))^2*(CdxFxSARC+(TanAlpha)^2*(CdyFySARC +
L6*CdyMzSARC+0.25*(L6)^2*CThzMzSARC));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Input Stiffness of FAM
Kfam = (inv(FHLS + FH6 + FH7 + FH8));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Constraints
Strsls = (Sf* Dinmax) - ((Lls^2)/(3*E*Tls))*Segma;
Strssarc = (Sf* ThitaSARCmax) - (((Bita^2*fBita)/(E*(1+Bita)^0.45))*Segma);
Strsmarc = (Sf* ThitaMARCmax) - (((5*pi*Rmarc^0.5)/(8*ktb*E*Tmarc^0.5))*Segma);
InputStif = Kfam - (0.2*Kpza);
Trvlrange = Dout- Amp*Dpzamax * ((Kpza+Kfam)/Kfam);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Nonlinear inequality constraints:
c=[Strsls-0.; Strssarc-0.; Strsmarc-0.; InputStif-0.;Trvlrange-0.];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Nonlinear equality constraints:
ceq= [];
end

```