

Non-Equilibrium Feshbach Association of Fermionic ${}^6\text{Li}$ Atoms



Guy Simmonds BSc

**Supervisor: Dr Lucia
Hackermueller**

A Thesis presented for the degree of
MSc (by Research) Physics

September 2019

School of Physics & Astronomy
University of Nottingham

Acknowledgements

Firstly, I would like to extend thanks to my supervisor Dr. Lucia Hackermueller for giving me the opportunity to work in this fantastic group. The advice and support throughout my time on this experiment has been invaluable. I am also grateful for the opportunity to attend two major conferences and present this work from this thesis at QuAMP.

I would also like to thank Dr. Weibin Li and his group for their discussion on the theory of Feshbach molecule formation.

Thankyou to my partner on the Lithium experiment Vineetha, for patiently passing on her knowledge of the experiment and ultracold atoms as a whole. I have thoroughly enjoyed working with you this year and wish you all the best for the future.

I would like to extend thanks to the other members of our research group. Nathan, for his numerous solutions to problems in the lab and providing insight to my writing. Bradley, for sharing with me the ebbs and flows of beginning in research and for many interesting conversations. Elisa for always knowing where to go and who to see when you are in need of something and Somaya for her encouragement when presenting at QuAMP.

Also, I would like to thank the wider Cold Atoms group for making my time in Nottingham extremely enjoyable. Thankyou for the welcoming atmosphere and the engaging lunchtime discussions. You cant have a bad day surrounded by such a lovely group of people.

Thank you to my family for their support not just at my time in Nottingham, but for the whole of my time at University both as an undergraduate and a postgraduate. I could not be in this position without their encouragement.

Finally, to my partner Katie. Thank you for all the support you have given me throughout this adventure and your patience with the goals that I have set myself during my time at University. I cannot emphasize how much I appreciate it.

Abstract

In this thesis, an experiment capable of producing 10^5 ${}^6\text{Li}_2$ molecules at temperatures between $0.1\text{-}1\ \mu\text{K}$ will be presented. By firstly loading 10^8 ${}^6\text{Li}$ atoms into a Magneto-Optical Trap (MOT) and transferring them to a far-red detuned optical dipole trap, 10^6 atoms are loaded at a temperature of $400\ \mu\text{K}$. Evaporative cooling techniques are used to further cool the cloud to the region of quantum degeneracy. Feshbach coils capable of producing fields up to 1200G are also implemented to exploit the broad Feshbach resonance between the $|F = 1/2, m_F = \pm 1/2\rangle$ states of ${}^6\text{Li}$.

Through this Feshbach resonance the scattering length of the atoms can be tuned to allow atoms to associate, forming so called “Feshbach Molecules”. In the case of ${}^6\text{Li}$, this allows for Fermionic atoms to form bound Bosonic states. In experiments that utilize Feshbach molecules, adiabaticity in their production is assumed, but is not always the case. By performing linear magnetic field sweeps from the BCS to the BEC side of the broad Feshbach resonance in ${}^6\text{Li}$, the conversion efficiency of atoms to bound molecules will be presented as a function of both the sweep rate and cloud temperature. Statistical tests comparing Landau-Zener and power law models of the BCS-BEC transition will be presented and comparison to experimental results discussed. The effect of the cut-off field of the sweep on molecule conversion efficiency will be presented and compared to numerical simulations.

Contents

1	Introduction	5
2	Atomic Theory	8
2.1	Atomic Structure in ${}^6\text{Li}$	8
2.2	Interaction With Magnetic Fields	12
2.3	Interaction with Light	15
2.4	Scattering Theory	18
2.5	Feshbach Resonances	21
3	Laser Cooling and Trapping	24
3.1	Zeeman Slower	24
3.2	Magneto-Optical Trap	26
3.2.1	Temperature Limit	27
3.3	Optical Dipole Trap	29
3.3.1	Focused Beam Trap	30
3.3.2	Fermi Energy and Temperature	32
3.4	Imaging and Temperature Measurements	34
3.4.1	Measuring the Temperature of an Atomic Cloud	35
4	Experimental Sequence	37
4.1	Atom Source	37
4.2	Laser Systems	38
4.3	Sequence Software	41
4.4	Magneto-Optical Trap	43
4.4.1	MOT Compression	43
4.4.2	State Preparation	45
4.5	Transfer to Dipole Trap	46
4.5.1	Evaporative Cooling Stage	46
4.5.2	Lifetime of Dipole Trap	49
4.5.3	Trapping Frequency Measurement	50

4.6	Feshbach Coils	52
4.6.1	High-Field Imaging of Atomic Cloud	52
4.6.2	Magnetic Field Calibration	53
5	Feshbach Molecule Formation	56
5.1	Models of Production Efficiency	56
5.1.1	Landau-Zener Approach	57
5.1.2	Power-Law Approach	60
5.1.3	Comparison to Experiment	62
5.2	Temperature Dependence of Production	65
5.3	Effect of Final Field on Molecule Formation	67
5.4	Initial Theory Work for Molecule Formation	71
6	Conclusions and Outlook	74
	Appendices	76
	Bibliography	79

List of Figures

2	Chapter 2	8
2.1	Fine and Hyperfine structure of ${}^6\text{Li}$	11
2.2	Breit-Rabi Diagram of ${}^2\text{S}_{1/2}$ state in ${}^6\text{Li}$	13
2.3	Breit-Rabi Diagram of ${}^2\text{P}_{3/2}$ state in ${}^6\text{Li}$	14
2.4	Zoomed Breit-Rabi Diagram of ${}^2\text{P}_{3/2}$ state in ${}^6\text{Li}$	14
2.5	Scattering rate as a Function of Saturation Parameter	17
2.6	Scattering rate as a function of Laser detuning	17
2.7	Visualization of “spherical box” model of channels.	21
2.8	Feshbach Resonance diagram for ${}^6\text{Li}$	23
3	Chapter 3	24
3.1	Schematic of Zeeman Slower Profile	25
3.2	$J = 0$ to $J' = 1$ transition in a MOT and diagram of MOT including molasses beams.	27
3.3	Focused beam dipole trap and trap potential vs position.	30
3.4	Schematic diagram of crossed beam optical dipole trap	32
4	Chapter 4	37
4.1	Experiment Laser Transitions and Energy Levels for ${}^6\text{Li}$, with re- spective detuning	40
4.2	Diagram of the optics used in this experiment	41
4.3	Image of the control sequence used for this experiment	42
4.4	MOT Loading versus time	44
4.5	Atom number in Dipole trap vs. Final Detuning in MOT compres- sion stage.	44
4.6	Top down diagram of MOT chamber	45
4.7	Atom number and Temperature at different plain evaporation times	49
4.8	Atom number versus hold time after a period of forced evaporation	50

4.9	Cloud Size versus Time after a Breathing Mode kick at 1W of Dipole Power	51
4.10	Atom number versus High Field VCO Voltage at 860G	53
4.11	Primary method of Feshbach coil calibration	55
4.12	Secondary method of Feshbach coil calibration	55
5	Chapter 5	56
5.1	Visual schematic of model comparison measurements	57
5.2	All data sets used for model comparison with model fitting	63
5.3	Reduced χ_r^2 values for Landau-Zener and Power-law models of molecular production versus Temperature	64
5.4	Molecule Conversion efficiency at different BCS-BEC sweep rates at various initial temperatures	66
5.5	Model Comparison to Adiabatic limit of Landau-Zener model to fitted data versus temperature.	66
5.6	Visual schematic of the Final Field molecule conversion measurements	67
5.7	Molecular Fraction vs. Final magnetic field of a BCS-BEC magnetic field sweep starting at 860G.	70
5.8	Molecule dynamics in a BCS-BEC sweep in small-coupling limit . .	72
5.9	Molecule dynamics in a BCS-BEC sweep with varied ramp rate . .	73
	Appendices	76
1	Measurement of Dipole Trap Temperature after Forced Evaporation	76

Chapter 1

Introduction

The field of ultracold atoms has developed into a mature and versatile tool for the study of a variety of physical phenomena. Since 1911, when the resistance of Mercury was found to drop to zero below a critical temperature [1], there has been a fascination with low-temperature physics. In 1985, the principle of laser cooling was demonstrated in Sodium atoms [2] and the experimental realisation of the Doppler cooling limit of $T_D = 240 \mu K$ in Sodium was achieved, opening up an entirely new field of research. Rapid developments were then made in the field, with the first implementation of a magneto-optical trap in 1987 confining Sodium atoms for up to 2 minutes [3], the principle of which allowed for the studying of ultra-cold atom systems.

The ultra-cold atom field blossomed with the implementation of cooling techniques that reached temperatures below the Doppler limit, for example evaporative cooling of bosonic gases [4], allowing a wide variety of physical phenomena to be probed. Alkali atoms in particular have been widely used in such experiments due to large vapour pressures at easily reachable temperatures in a lab, and have received interest due to their large and easily tunable scattering lengths, for example about a Feshbach Resonance [5]. These cooling techniques led to the experimental realization of BEC phenomena in bosonic ^{87}Rb [6], ^{23}Na [7] and ^7Li [8] in 1995. The first realization of a molecular BEC comprised of ultra-cold bosonic $^6\text{Li}_2$ was not until 2003, when a group from MIT trapped ^6Li and used high magnetic fields to tune the scattering length of the atoms to a bound state region to achieve condensation of $^6\text{Li}_2$ molecules [9].

Recently, utilizing the phenomena of the Feshbach resonance has gained interest in creating a quantum heat engine by modulating the scattering length of a BEC to form an Otto cycle [10] as well as molecular BEC's (mBEC) being used to detect gravitational waves by measuring phonon excitations caused by space-time distortions [11]. However, the optimal production of so called “Feshbach Molecules”, the kind used to produce a BEC comprised of fermionic atoms, is a matter which is not fully understood. Through this thesis, the basics of the Feshbach Resonance will be covered, as well as magneto-association of fermionic atoms. Competing models will be presented of molecule conversion efficiency as a function of magnetic field sweep rate through linear BCS-BEC crossings of the Feshbach Resonance with fixed start and end points. The models will be compared to experimental results, at different initial cloud temperatures, in a non-equilibrium ^6Li system. The effect of the temperature of an atomic cloud on the optimum conversion efficiency will also be presented and compared to a simple statistical model. Following this, experimental results considering molecule formation in linear BCS-BEC magnetic field ramps with varying final fields will be probed at different sweep rates and contextualized with similar studies on narrow resonances. Finally, initial work in the group on providing a simulation of fermion association in the BCS-BEC crossover will be presented and used to interpret the results in the previous section.

For the contents of this thesis, the work presented will be surrounding ^6Li . Previously, the experiment had been running as part of a dual-species setup with ^{133}Cs , but when I had joined it had been converted to two separate setups [12]. Work from previous students has been done on investigating different models of the density distribution of a BEC of $^6\text{Li}_2$ [13]. Through optimization of experimental parameters, as well as fine tuning of optical components, efficient loading of a Magneto Optical Trap (MOT) and Optical Dipole Trap (ODT) will be presented. The experiment routinely loads up to 10^6 atoms at magnetic fields up to 1200G as well as using evaporative techniques to cool as many as 3×10^5 atoms to temperatures of around 500nK. The magnetic field and consequently the scattering length of the atoms in the ultra-cold cloud can then be manipulated freely in order to probe the Feshbach resonance of the lowest energy states of ^6Li .

Chapter 1 – Introduction

An outline of the structure of the thesis is as follows:

Chapter 2 - Atomic Structure in ${}^6\text{Li}$

In this chapter I will discuss the fundamentals of atomic theory and their application to ${}^6\text{Li}$. This will include interaction with electric and magnetic fields, scattering theory, as well as a description of the Feshbach Resonance phenomena. These topics will form the basis of future chapters.

Chapter 3 - Laser Cooling and Trapping

In this chapter, a description of the theory behind the technical apparatus used in this experiment will be given. It is intended to lead directly to Chapter 4, where a description of how these are implemented and optimized in the experimental set up will be laid out.

Chapter 4 - Experimental Sequence

This chapter will contain the details of the sequence that has been used starting with ejection from an atomic source. Focus will be placed on how optimization and calibration of the apparatus is done in order to produce high numbers of trapped ultra-cold Feshbach molecules of ${}^6\text{Li}$.

Chapter 5 - Feshbach Molecule Formation

Here the main results of this thesis will be presented. Factors effecting molecule conversion efficiency such as temperature, sweep rate and final magnetic field in BCS-BEC magnetic field sweeps will be presented and compared to experimental data. This will be contextualized with similar studies carried out on narrow resonances. Lastly, I will discuss the initial work on simulating the molecule dynamics and its outlook.

Chapter 2

Atomic Theory

In order to perform and understand the results of this experiment, it is important to have an understanding of the fundamentals of atomic theory, as well as their application to ${}^6\text{Li}$. I will begin by discussing atomic structure in alkali atoms, as well as their interaction with Magnetic and Electric fields. The physics of Feshbach Resonances will be shown, as well as how this relates to so called ‘Feshbach Molecules’ and how their properties can be described. I will also present parameters specific to the broad Feshbach resonance of the lowest energy states of ${}^6\text{Li}$. This will be the foundation of later chapters.

2.1 Atomic Structure in ${}^6\text{Li}$

Alkali atoms such as ${}^6\text{Li}$ have a single valence electron in their outer shell. Laser cooling techniques for such atoms were developed in 1982 with Na atoms [14]. The rest of the electrons form closed shells, which have a net angular momentum of zero in a spherically symmetric distribution. This results in large excitation energies in comparison to the valence electron, so only the valence electron’s energy levels are considered in the treatment of atomic structure.

For this reason, to resolve the gross structure of the energy levels of the atom, a hydrogen-like model, known as the **Central Field Approximation**, can be applied. The Hamiltonian for such a system is given by: [15]

$$\hat{H}_o = \frac{\hat{\mathbf{P}}^2}{2m_e} - \mathbf{V}_{eff}(r) \quad (2.1)$$

The first term in the Hamiltonian represents the kinetic energy of the electron where m_e is the mass of the electron and $\hat{\mathbf{P}}$ is the momentum operator. The second term, the effective potential, arises due to coulomb interaction of the valence electron and the effective nuclear charge. The effective charge arises due to shielding of the total charge of the protons in the nucleus from the closed shell electrons. The solution is similar to that of the hydrogen atom, with the probability distribution of the electron being the product of radial and spherical harmonic functions. The solution is expressed in the basis [16]:

$$|n, l, m_l\rangle$$

Here n is the principle quantum number, l the azimuthal quantum number and m_l the magnetic quantum number. The energy levels of this gross structure will differ from Hydrogen, being dependent on the principle and azimuthal quantum numbers, due to the fact that this is not a true central coulomb potential. At lower l this effect is more significant, and at large values of n the solution tends towards the Hydrogen solution. This treatment however is insufficient to resolve the full structure of alkali metals, as we must take into account magnetic effects from the electron's and nuclear spin, as well as relativistic effects.

The corrections to this model are very small, of the order of the square of the fine structure constant α^2 , so we can treat them as a perturbation. To do this, the Dirac equation can be solved for an electron inside a potential created by a proton in the weakly relativistic limit, the details of which are outside the scope of this thesis. The resulting Hamiltonian takes the form [15]:

$$\hat{H} = \hat{H}_o + m_e c^2 - \frac{\hat{\mathbf{P}}^4}{8m_e^3 c^2} + \frac{1}{2m_e^2 c^2} \frac{1}{r} \frac{dV_{eff}(r)}{dr} \mathbf{L} \cdot \mathbf{S} + \frac{\hbar^2}{8m_e^2 c^2} \Delta V_{eff}(r) + \dots \quad (2.2)$$

Second order terms in the power expansion of the perturbation here have been neglected. Here, $\mathbf{L}$ and $\mathbf{S}$ refer to the operators for orbital and spin angular momentum respectively. The term $\mathbf{L} \cdot \mathbf{S}$, known as the spin-orbit coupling term, is responsible for the effects of fine structure. We can write the operator for the electron's total angular momentum as $\mathbf{J} = \mathbf{L} + \mathbf{S}$, which has eigenstates in the basis $|n, l, s, j, m_j\rangle$ and can take values $|\mathbf{L} - \mathbf{S}| \leq |\mathbf{L} + \mathbf{S}|$ [16]. The fine structure operator is written with the spin-orbit term as [16]:

$$\begin{aligned}\frac{A_{fs}}{2\hbar^2} \mathbf{L} \cdot \mathbf{S} |n, l, s, j, m_j\rangle &= \frac{A_{fs}}{2\hbar^2} (\mathbf{J}^2 + \mathbf{L}^2 + \mathbf{S}^2) |n, l, s, j, m_j\rangle \\ &= \frac{A_{fs}}{2} (j(j+1) - l(l+1) - s(s+1)) |n, l, s, j, m_j\rangle\end{aligned}\quad (2.3)$$

From this, it is shown that the energy shift caused by the spin orbit coupling is proportional to $(j(j+1) - l(l+1) - s(s+1))$. It will later be shown in Fig. 2.1 that this produces two sub-levels in the $l = 0$ state of ${}^6\text{Li}$ as s can take values $\pm\frac{1}{2}$, to which the transitions from the ${}^2S_{1/2}$ state are known as the D_1 and D_2 lines respectively.

This however does not address the interaction between the magnetic moments of the nucleus and the electron. This leads to what is known as hyperfine structure, which is an effect ≈ 2000 times smaller than fine structure splitting [16]. By defining the nuclear spin as $\mathbf{I}$, the Hamiltonian for the hyperfine structure is given by:

$$H_{hfs} = \frac{A_{hfs}}{2\hbar^2} \mathbf{I} \cdot \mathbf{J} \quad (2.4)$$

By defining the total angular momentum of the atom as $\mathbf{F} = \mathbf{I} + \mathbf{J}$, the eigenstates of this Hamiltonian can be written in the basis $|n, l, s, j, f, m_f\rangle$, and the energy splitting due to the hyperfine structure can be expressed as:

$$\Delta E_{HFS} = \frac{A_{hfs}}{2} (F(F+1) - I(I+1) - J(J+1)) \quad (2.5)$$

This splitting will depend on all indices with the exception of m_f . A magnetic field is required to break the degeneracy in m_f . For ${}^6\text{Li}$, the effects of hyperfine structure cause a doublet in the ${}^2S_{1/2}$ ground state (Fig 2.1). Another important point for ${}^6\text{Li}$ is that the natural linewidth of the D_2 transition is larger than the hyperfine splitting of the excited state, meaning that the hyperfine structure of the D_2 line is unresolved. A consequence of this is that sub-Doppler cooling is impossible on the D_2 line, and the minimum achievable temperature in a MOT is doppler-limited (See Section 3.2).

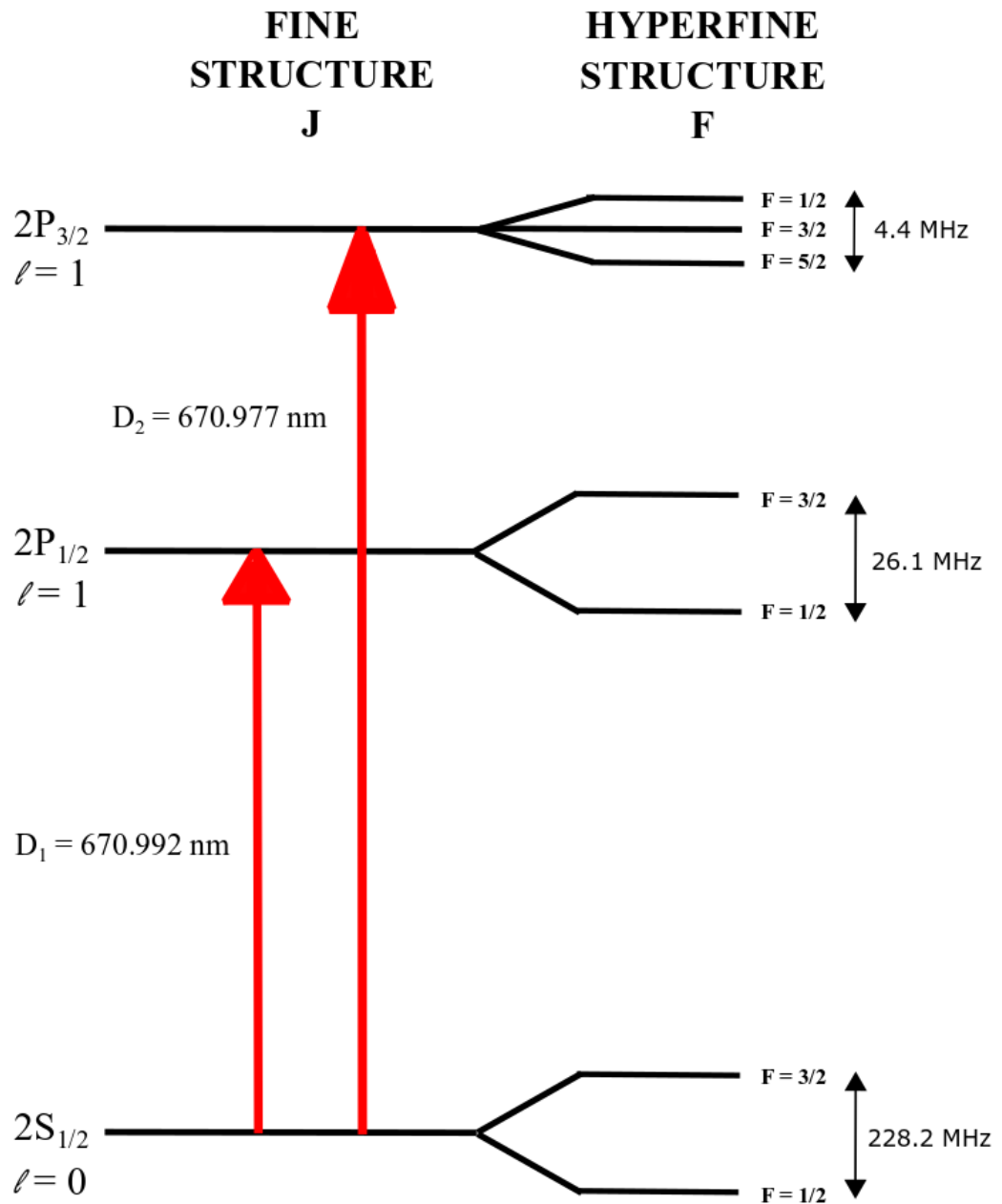


Figure 2.1: Energy level diagram for the D_1 and D_2 lines of ${}^6\text{Li}$. The levels are denoted with their spectroscopic notation. The energy levels and their splitting are not to scale. Data taken from [17]

2.2 Interaction With Magnetic Fields

The phenomena of the Feshbach Resonance require holding the ultracold atomic cloud in large magnetic fields. For this reason, it is important to understand how neutral atoms interact with static magnetic fields. One consequence of the interaction is that the m_f sub-levels will be split with increasing magnetic field, known as Zeeman Splitting. This has experimental consequences, for example it will need to be addressed when probing an atomic transition for imaging.

We can treat the interaction between an external magnetic field $\mathbf{B}$ and the nuclear, orbital and magnetic moments as an addition to the Hyperfine structure Hamiltonian. The Zeeman Hamiltonian, along the z -axis, is written as [18]:

$$\begin{aligned}\hat{H}_{ZM} &= -\mathbf{B} \cdot (\boldsymbol{\mu}_L + \boldsymbol{\mu}_S + \boldsymbol{\mu}_J) \\ &= \mu_B B_Z (g_L L_Z + g_S S_Z + g_J J_Z)\end{aligned}\tag{2.6}$$

Where g_L , g_S and g_J are the respective gyromagnetic ratios of the atom. μ_B is simply the Bohr magneton. With the addition of the Hyperfine structure Hamiltonian (Equation 2.4), the full Hamiltonian for the Hyperfine structure of an atom in an external magnetic field becomes [18]:

$$\hat{H}_{HFS} = A_{hfs} \mathbf{I} \cdot \mathbf{J} - \mu_B B_Z (g_L L_Z + g_S S_Z + g_J J_Z)\tag{2.7}$$

From this we can define three regions. Firstly, the region where $\mu_B B \gg A_{hfs}$, the high field or Paschen-Back regime. Secondly, the low field regime where $\mu_B B \ll A_{hfs}$. Finally, the intermediate regime where the energy splitting has no analytical solution, but can be numerically solved. Diagrams of the full numerical solutions across all three regimes can be found in the Breit-Rabi diagrams Figure 2.2 and Figure 2.3 for the $^2S_{1/2}$ and $^2P_{3/2}$ states in ^6Li . The details of this can be found in [19].

In the low field regime, F and m_F remain good quantum numbers. We can treat the Zeeman terms as a perturbation to the Hyperfine structure Hamiltonian, and the energy shift with respect to the magnetic field is given by [18]:

$$\Delta E = g_F \mu_B m_F B\tag{2.8}$$

Where g_F is the Landé g-factor for a given transition, given by:

$$g_F = g_J \frac{F(F+1) + J(J+1) + I(I+1)}{2F(F+1)} \quad (2.9)$$

In this low-field regime, the energy splitting varies linearly with magnetic field. For the high-field regime, F and m_F are no longer good quantum numbers. The electron's total angular momentum and the nuclear angular momentum couple to the external field directly and the Hyperfine structure Hamiltonian is treated as the perturbation. The energy splitting for this regime is given by [16]:

$$\Delta E = g_J \mu_B m_J B + A_{hfs} m_I m_J \quad (2.10)$$

This is important as the results that will be presented in this thesis concern a Feshbach Resonance at 832G. From the Breit-Rabi diagrams below, it is clear that this magnetic field is in the high-field regime for both the $^2S_{1/2}$ and $^2P_{3/2}$ states of ^6Li .

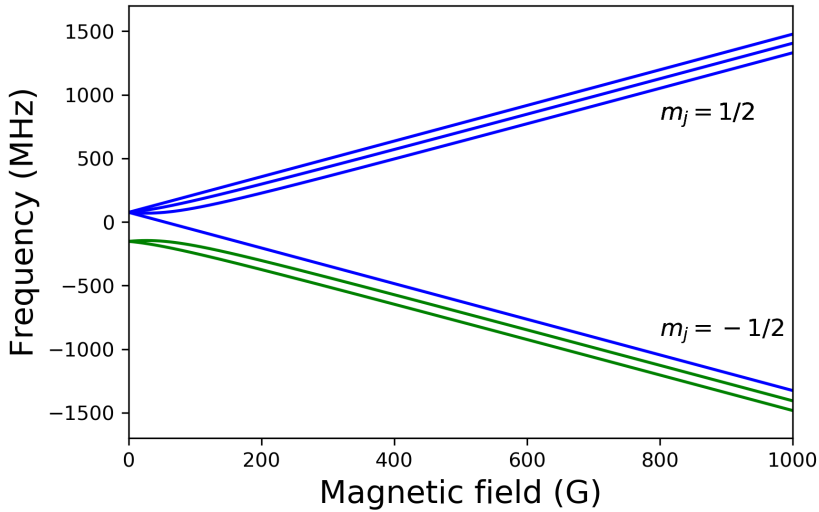


Figure 2.2: Breit Rabi diagram of the $^2S_{1/2}$ state in ^6Li . The levels are grouped according to F in the low field and m_J in the high field regime.

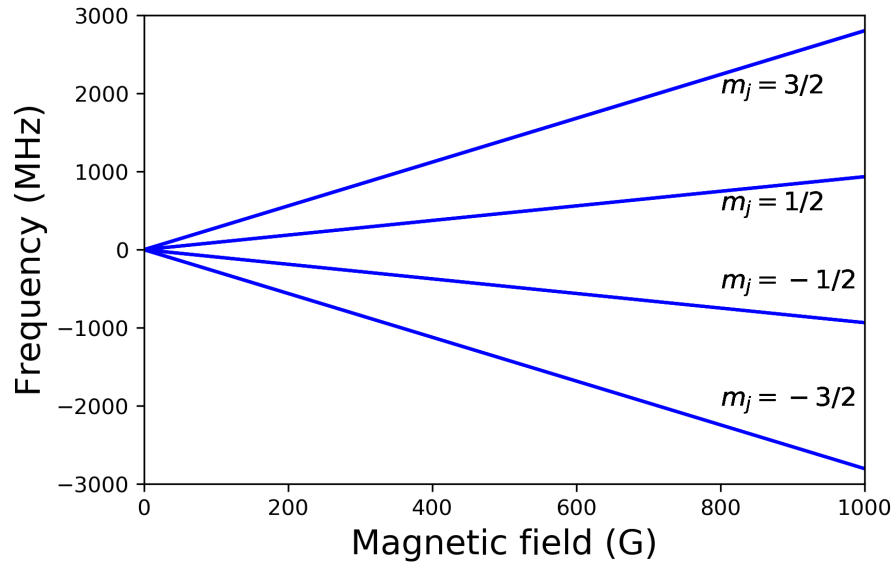


Figure 2.3: Breit Rabi diagram of the $^2P_{3/2}$ state in ^6Li . The levels are grouped according m_J in the high field regime. It should be noted that the high field regime is reached after a few G.

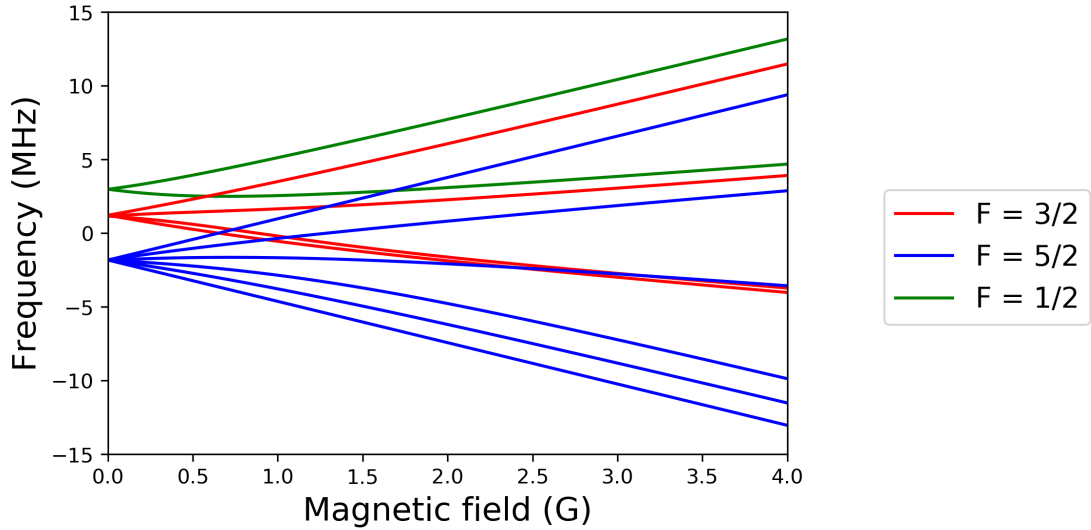


Figure 2.4: Zoomed Breit Rabi diagram of the $^2P_{3/2}$ state in ^6Li in the low-field regime. The energy levels are grouped by F in the low field regime. This is used to illustrate that the high field regime is already reached after a few G.

2.3 Interaction with Light

In ultra-cold atom experiments such as the one described in this thesis, the interaction of atoms and light is an important concept. In this section, the atom will be modelled as a two level system in the presence of a classical oscillating electric field in order to derive expressions for the photon scattering force F_{scat} and the dipole force F_{dip} .

We model the interaction as a perturbation $\hat{\mathbf{H}} = \hat{\mathbf{H}}_o + \hat{\mathbf{H}}'$ due to an oscillating field $\mathbf{E} = E_o \cos(\omega t)$ that induces a dipole moment $\mathbf{p} = \alpha \mathbf{E}$ where α is the complex polarizability. The interaction Hamiltonian is given by [16]:

$$\hat{\mathbf{H}}' = -\alpha E_o \cos(\omega t) \quad (2.11)$$

The solutions to the Schrödinger equation for this Hamiltonian will be of the form $|\Psi(t)\rangle = C_1(t) |1\rangle e^{-iE_1 t/\hbar} + C_2(t) |2\rangle e^{-iE_2 t/\hbar}$ where $C_1(t)$ and $C_2(t)$ are the respective probability amplitudes of the eigenstates $|1\rangle$ and $|2\rangle$. The values E_1 and E_2 correspond to the eigenvalues of the Hamiltonian. The wavefunction is normalized such that $|C_1|^2 + |C_2|^2 = 1$. The density matrix ρ can be introduced, given by [16]:

$$\rho = |\Psi\rangle \langle \Psi| = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \quad (2.12)$$

Where $\rho_{11} = |C_1|^2$ is the population in the ground state and $\rho_{22} = |C_2|^2$ is the population in the excited state. These terms can be found using steady state solutions to the optical Bloch equations, and the excited state population is given by [16]:

$$\rho_{22} = \frac{\Omega^2/4}{\delta^2 + \Omega^2/2 + \Gamma^2/4} \quad (2.13)$$

Where Ω is the Rabi frequency of the transition, δ is the laser detuning from resonance and Γ is the line-width of the transition. The Rabi frequency is related to the laser intensity I as $\Omega^2 = \frac{\Gamma^2 I}{2I_{sat}}$, where the saturation intensity I_{sat} is defined as $I_{sat} = \frac{\pi \hbar c \Gamma}{3\lambda^3}$ where λ is the wavelength of the transition. The saturation parameter is defined as $s_o = I/I_{sat}$, such that the excited state population is given by:

$$\rho_{22} = \frac{s_o/2}{1 + s_o + 4\delta^2/\Gamma^2} \quad (2.14)$$

It can be seen that for an on resonance beam with $I \gg I_{sat}$, the excited state population will tend to a maximum value of $1/2$. Upon absorption of a photon, the atom will experience a momentum change based on the momentum of the photon $\mathbf{p} = \hbar\mathbf{k}$. The following spontaneous emission of the photon is equally likely to happen in every direction, and thus the net force of the emission process will average to zero. The average force due to the scattering of photons is given by [16]:

$$\langle F_{scat} \rangle = \hbar\mathbf{k}\Gamma_{scat} \quad (2.15)$$

Where Γ_{scat} is the photon scattering rate, given by:

$$\Gamma_{scat} = \Gamma\rho_{22} \quad (2.16)$$

If the laser propagates in the opposite direction of the atom, this will cause a dissipative effect, slowing the atom down. This is exploited for example in an optical molasses, with 3 pairs of counter propagating beams, in a magneto optical trap (see Section 3.2). Diagrams of how the scattering rate varies as a function of the saturation parameter and the detuning are displayed in Figures 2.5 and 2.6. The maximum possible deceleration rate will be given by [16]:

$$\mathbf{a}_{max} = \frac{\langle F_{scat} \rangle_{max}}{m} = \frac{\hbar k\Gamma}{2m} \quad (2.17)$$

The dipole force, which is a conservative force and allows for trapping of atoms, arises from the real component of the complex polarizability interacting with the electric field. The dipole force can be expressed as $F_{dip} = -\nabla U_{dip}$ and is given by [16]:

$$F_{dip} = -\frac{\hbar\delta}{2} \frac{\Omega}{\delta^2 + \Omega^2/2 + \Gamma^2/4} \nabla\Omega \quad (2.18)$$

For zero detuning, it can be seen that the dipole force vanishes. If far detuned light is incident on an atom, the dipole potential varies as $U_{dip} \propto 1/\delta$, compared to the scattering rate in Equation 2.16 that varies as $\Gamma_{scat} \propto 1/\delta^2$. This shows that heating of the cloud due to scattering of photons can be suppressed by using a far-detuned laser.

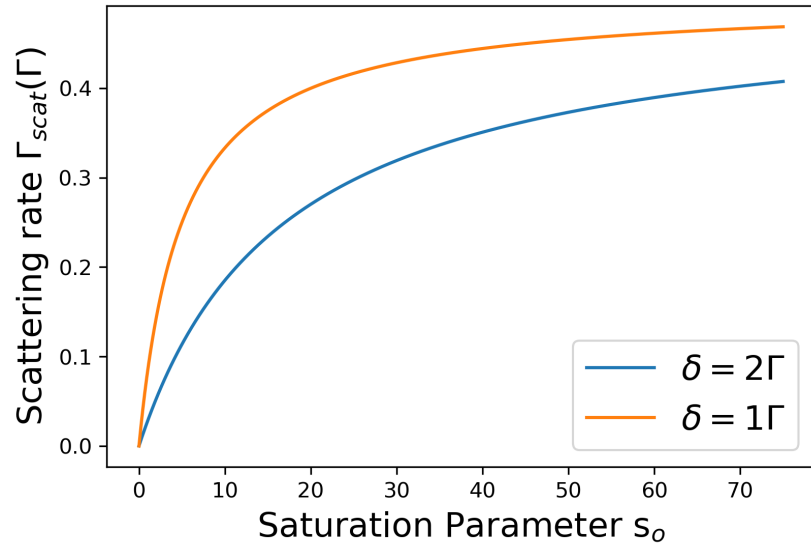


Figure 2.5: Scattering rate as a function of the saturation parameter for constant values of the laser detuning.

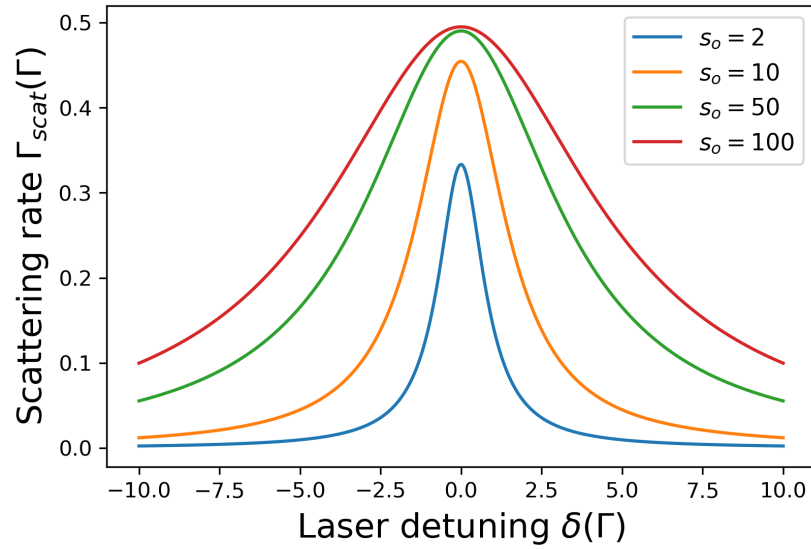


Figure 2.6: Scattering rate as a function of the laser detuning for different values of the saturation parameter. As the saturation parameter increases, the distribution is broadened.

2.4 Scattering Theory

Inter-particle scattering plays an important role in ultracold gases. For example, one factor that contributes to the trap lifetime is inelastic collisions, where the energy of colliding atoms is transferred between internal and external degrees of freedom, leading to heating and therefore ejection from the trap. Elastic collisions, where the total energy of the particles is the same after the collision, are a requirement for evaporative cooling to occur. This will be seen in later chapters. In this chapter I will discuss elastic collisions in quantum mechanics, and how this relates to s -wave scattering in fermions and bosons [20].

Consider the scattering event between two particles of mass m_1 and m_2 with momentum $\mathbf{p}_1$ and $\mathbf{p}_2$ in a centrally symmetric potential $\mathbf{V}(\mathbf{r})$, with $\mathbf{r}$ the relative distance between the two particles $\mathbf{r}_1 - \mathbf{r}_2$, such that the Hamiltonian in a centre of mass frame is given by:

$$\hat{H} = \frac{\mathbf{P}_G^2}{2M} + \frac{\mathbf{p}^2}{2m_r} + \mathbf{V}(\mathbf{r}) \quad (2.19)$$

Where M is the sum of the two masses, m_r is the reduced mass of the system $m_r = m_1 m_2 / (m_1 + m_2)$, $\mathbf{p} = \mathbf{p}_1 - \mathbf{p}_2$ and $\mathbf{P}_G = \mathbf{p}_1 + \mathbf{p}_2$. By doing this, we have a Hamiltonian that describes a free particle and a scattered particle under a central potential $\mathbf{V}(\mathbf{r})$. The goal is to not describe the specific dynamics, but to describe the probability distribution of scattering angles θ . By considering the particles at a distance far away from the scattering centre, the wavefunction of the system can be described by a superposition of a plane wave describing the free particle and a spherical wave describing the scattered particle. Thus, the approximate solution at large distances from the scattering centre is given by [20]:

$$\Psi_k(r) \approx e^{ik \cdot z} + f(\theta) \cdot \frac{e^{ikr}}{r} \quad (2.20)$$

Here $f(\theta)$ is the scattering amplitude and k is the wave-vector of the scattered wave. The z -axis is defined as the axis in which the plane wave propagates. The ratio of the number of particles per unit time incident on an infinitesimal area $d\sigma$ scattered into an infinitesimal solid angle $d\Omega$ is given by the differential cross section. This is related to the scattering amplitude as [20]:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \quad (2.21)$$

Any solution of the Schrödinger equation in a central field can be written as a sum of Spherical Harmonic, $Y_l^m(\theta, \phi)$ and Radial Functions $R_l(r)$, where the Radial Functions satisfy the radial form of the Schrödinger equation given by [20]:

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr} R_l(r) \right) + \left(k^2 - \frac{l(l+1)}{r^2} - \frac{2m_r}{\hbar^2} V(r) \right) R_l(r) = 0 \quad (2.22)$$

The form of the equation $R_l(r)$ in the asymptotic limit $r \rightarrow \infty$ is given by [20]:

$$R_l(r) \approx a_l \frac{\sin(kr - \pi l/2 + d_l)}{r} \quad (2.23)$$

Where a_l is the partial wave amplitude and d_l is the phase of the Radial Function. It will be shown how this relates to the total cross section σ . The full solution to the Schrödinger equation, obtained by combining the Radial and Spherical Harmonic terms, is [20]:

$$\begin{aligned} \Psi &\approx \sum_{l=0}^{\infty} (2l+1) A_l P_l \cos(\theta) \frac{\sin(kr - \pi l/2 + d_l)}{kr} \\ &= \sum_{l=0}^{\infty} (2l+1) A_l P_l \cos(\theta) \frac{i}{2kr} (\exp[-i(kr - l\pi/2 + d_l)] \\ &\quad - \exp[i(kr - l\pi/2 + d_l)]) \end{aligned} \quad (2.24)$$

Here P_l are Legendre Polynomials of order l . We can compare this to Equation 2.20 to find the exact form of the coefficients A_l . This can be done by expanding the plane wave solution in terms of a combination of spherical waves. This leads to defining the coefficient $A_l = i^l \exp[id_l]$ such that an approximate solution to the Schrödinger equation is given by:

$$\Psi \approx \frac{i}{2kr} \sum_{l=0}^{\infty} (2l+1) P_l \cos(\theta) [(-1)^l e^{-ikr} - e^{2id_l} e^{ikr}] \quad (2.25)$$

In this form, a direct comparison can be made to Equation 2.20 and the scattering amplitude can be read off as:

$$f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1) [e^{2id_l} - 1] P_l \cos(\theta) \quad (2.26)$$

To find the total cross section σ we integrate over all solid angles, using the form of the differential cross section given in Equation 2.21, noting that $d\Omega = 2\pi \sin(\theta) d\theta$:

$$\sigma = 2\pi \int_0^\pi \sin(\theta) |f(\theta)|^2 d\theta \quad (2.27)$$

By noting that all Legendre Polynomials of different order are orthogonal, the total effective cross-section can be evaluated as [20]:

$$\sigma = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2(d_l) \quad (2.28)$$

We must now consider the fact that Bosons and Fermions have different symmetry relations. The previous derivation considers only purely distinguishable particles. For Bosons, Ψ must be symmetric, ie $\Psi(r_1, r_2) = \Psi(r_2, r_1)$ and for Fermions it must be anti-symmetric, ie $\Psi(r_1, r_2) = -\Psi(r_2, r_1)$. This gives rise to the following two expressions for either case [21]:

$$\sigma_{BOSONS}(k) = \frac{8\pi}{k^2} \sum_{\text{even } l} (2l+1) \sin^2(d_l) \quad (2.29)$$

$$\sigma_{FERMIONS}(k) = \frac{8\pi}{k^2} \sum_{\text{odd } l} (2l+1) \sin^2(d_l) \quad (2.30)$$

We can note from the radial form of the Schrödinger equation (Equation 2.22) that there is a centrifugal barrier term for $l \geq 1$. For Lithium, the height of this is 8mK [22]. At low energies, this has the consequence that p -wave scattering is suppressed for ultracold bosons; only the $l = 0$ term contributes to the scattering cross section. By defining the scattering length a as:

$$a = -\lim_{k \rightarrow 0} \frac{\tan(d_l)}{k} \quad (2.31)$$

The scattering cross section in the limit of low energy is given by:

$$\sigma \rightarrow 8\pi a^2 \quad (2.32)$$

So far this has given the details of elastic scattering identical particles. For a composite of identical fermions, s -wave scattering cannot exist, therefore evaporative cooling is much less efficient. However, in a cloud of distinguishable fermions such as the two hyperfine states of ${}^6\text{Li}$ used in this experiment elastic collisions can occur as there is no constraint of antisymmetry under particle exchange, allowing for evaporative cooling to take place.

2.5 Feshbach Resonances

Feshbach Resonances provide a useful tool in ultracold atom experiments for the manipulation of the scattering length of atomic species. The results in this thesis are based on phenomena associated with the Feshbach Resonance, so a simple description using a two-channel box potential model will be laid out here.

In this description, the resonance is modelled as a system of two identical atoms of mass m undergoing a scattering event described by a spherical box potential $\hat{v}$. The system supports an open channel $|o\rangle$, corresponding to the scattering continuum and a closed channel $|c\rangle$ corresponding to a bound molecular state of two atoms. The wave function of such a system can be expressed as a function of the atomic separation r [23]:

$$|\Psi\rangle = \psi_o(r) |o\rangle + \psi_c(r) |c\rangle \quad (2.33)$$

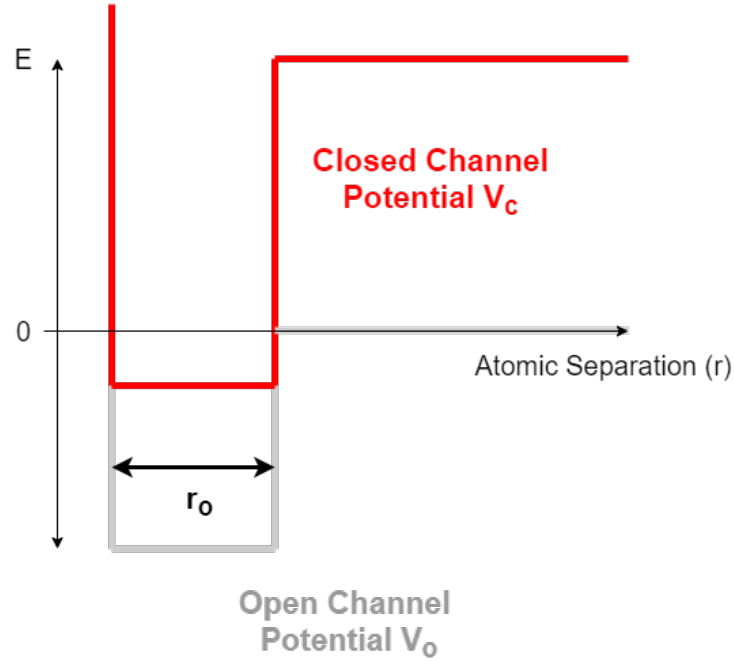


Figure 2.7: Visualization of “spherical box” model of channels. The energy of the open and closed channels differs based on their relative magnetic moment. When the energy of the closed channel approaches the energy of the open channel, a Feshbach Resonance occurs.

The interaction is presented as a “spherical box” potential (Figure 2.7) with interaction range r_o . For $r > r_o$, the potential energy of the closed channel will be

∞ and the open channel 0. For $r < r_o$ the potential between the open and closed channels will be attractive in nature. Introducing the potential $\hat{v}$ and a coupling Ω between the channels, the Schrödinger equation for this system is [23]:

$$E |\Psi\rangle = -\frac{\hbar^2}{2m} (\nabla^2 - \hat{v}) |\Psi\rangle \quad (2.34)$$

$$\hat{v} = \begin{cases} \begin{bmatrix} V_o & \Omega \\ \Omega & V_c(B) \end{bmatrix} & r < r_o \\ \begin{bmatrix} 0 & 0 \\ 0 & \infty \end{bmatrix} & r > r_o \end{cases} \quad (2.35)$$

Here V_o and V_c are the respective minima of the box potentials of the open and closed channels. We can tune V_c through varying the magnetic field applied to the atoms. It should be noted that a coupling $\Omega > 0$ is required for the model to exhibit a Feshbach Resonance. The solution to this is determined by imposing boundary conditions $\psi_c(r_o) = 0$ and $\frac{\psi_c(r_o)}{\psi'_c(r_o)} = r_o(\frac{r_o}{a} - 1)$, where a is the scattering length between the two channels. For ultracold atoms, where Feshbach resonances are typically induced by spin-spin interactions, we can also take the limit $\Omega \ll V_o, V_c$, such that the solution to the scattering length as a function of magnetic field is given by [23]:

$$a(B) = a_{bg} \left(1 - \frac{\Delta B}{B - B_o} \right) \quad (2.36)$$

Here ΔB is the width and B_o the position of the Feshbach Resonance. a_{bg} is the background scattering length of the open channel. In this thesis, the resonance considered is between the $|F = 1/2, m_F = \pm 1/2\rangle$ states of ${}^6\text{Li}$, which exhibit a broad resonance. This allows for precise tuning of the scattering length. Values for the parameters of this resonance are given below:

Parameter	Value
a_{bg}	$-1582a_o$
ΔB	-262.3G
B_o	832.18G

Table 2.1: Feshbach Resonance parameters of ${}^6\text{Li}$ for the broad resonance between the $|F = 1/2, m_F = \pm 1/2\rangle$ states. Data taken from [24].

In the region where $a > 0$, also known as the BEC region, the two atoms can form a weakly bound state, known as so called “Feshbach Molecules”. For ${}^6\text{Li}$, this allows for two fermionic components to form a bosonic molecule. The binding energy of the molecules is given by [23]:

$$E_b = -\frac{\hbar^2}{2m_r a^2} \quad (2.37)$$

Where m_r is the reduced mass of the formed molecule. In ultracold atom experiments there are two methods of producing Feshbach Molecules, either by holding the atomic cloud in a constant magnetic field for some time to allow association to happen via low energy collisions, or by adiabatic magnetic field sweeps of the negative scattering BCS side of the resonance to the BEC side. In this thesis I will investigate molecule formation when the resonance is crossed in a non-adiabatic manner. A diagram of the scattering length and binding energy as a function of magnetic field is given in Figure 2.8:

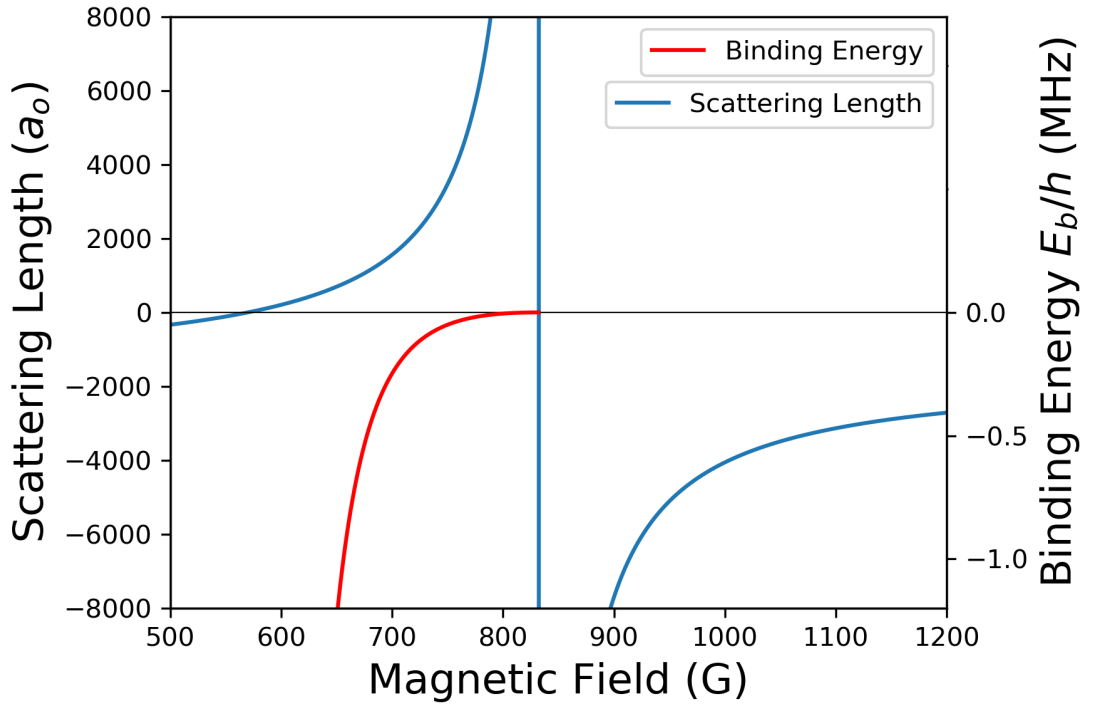


Figure 2.8: Scattering length and binding energy versus magnetic field for ${}^6\text{Li}$ across the broad resonance between the $|F = 1/2, m_F = \pm 1/2\rangle$ states.

Chapter 3

Laser Cooling and Trapping

Throughout this chapter, I will use the tools developed in the Chapter 2 to describe the principles of the techniques applied in this experiment to trap, cool and image ultra-cold clouds of ^6Li . I will start with the Zeeman Slower, used to slow atoms before the MOT. I will also describe the principle of the optical dipole trap used in the experiment as well as the physics behind using the thermal expansion of an atomic cloud to measure its temperature.

3.1 Zeeman Slower

In this experiment, the ^6Li atoms are produced from an oven that operates at temperatures of 400°C . The most probable velocity of the atoms at a temperature T is defined as $v_p = \sqrt{2k_B T/m}$ and for this experiment will be roughly 1390 m/s . This is too large in order to be trapped by the Magneto-Optical Trap described in Section 3.2 and therefore a Zeeman slower is used to slow the atoms before entering the MOT. It has been described in the previous chapter how the scattering force of an opposing laser beam can decrease the atoms velocity, and is maximized when the detuning $\delta = 0$. This can be utilized to slow the atomic beam before entering the MOT.

The treatment of light-matter interactions in the previous chapter neglects the fact that the doppler shift relative to the transition will change over time as the atomic beam decelerates. This will increase the total detuning from the atomic transition, therefore decreasing the force on the atom, as the atom will “see” a different frequency. In general there are two methods to combat this. Firstly, the actual frequency of the laser can be modulated as the atom decelerates to stay on

resonance. This is known as “chirp cooling” and was first carried out in 1985 by W. Ertmer et al [25]. The other method is to utilize the Zeeman effect to “match” the doppler shift with the Zeeman shift such that the laser is always resonant with the atom and does not require modulating. This was first implemented in 1982 by W. Phillips and H. Metcalf for Sodium atoms [26] and is the method utilized in this experiment.

The total detuning δ is given by summing the laser detuning from resonance δ_o , the Doppler shift of velocity v and the Zeeman shift:

$$\delta = \delta_o + kv + \frac{\mu B}{\hbar} \quad (3.1)$$

Where μ is the magnetic moment of the atom. For an atom that is uniformly decelerating, the velocity profile as a function of its position in one dimension is given by $v(z) = \sqrt{v_o^2 - 2az}$ where a is its deceleration [16]. Substitution into Equation 3.1, setting the total detuning $\delta = 0$ and re-arranging for B leads to:

$$B(z) = \frac{\hbar}{\mu} \left(\delta_o + k\sqrt{v_o^2 - 2az} \right) \quad (3.2)$$

The deceleration of the beam is simply the maximum acceleration of the scattering force given in Equation 2.17. A diagram of the magnetic field profile is shown in Figure 3.1:

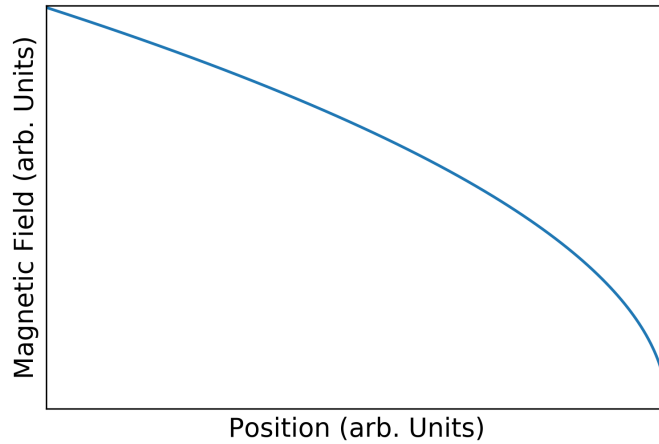


Figure 3.1: Visual schematic of the profile of the Magnetic Field of a Zeeman slower. This serves as a visual guide and the units are not representative of the experiment. The details of the Zeeman slower implemented in this experiment can be found in [27].

3.2 Magneto-Optical Trap

A Magneto-Optical Trap (MOT) is used after the Zeeman slower to trap, cool and compress the atomic cloud to prepare it for evaporative cooling. In a MOT a cloud of atoms is placed in a set of 3 orthogonal counter-propagating red-detuned, near resonant laser beams. An atom moving in one direction will have an increased rate of absorption from the opposing beam, which slows the atom down, due to the dissipative nature of the force (see Section 2.3). The total force on an atom moving with velocity v is expressed as [16]:

$$F_{MOL} = F_{SCAT}(\omega - \omega_o - kv) - F_{SCAT}(\omega - \omega_o + kv) \quad (3.3)$$

Which, for velocities $|kv| \ll \Gamma$, approximates to:

$$\begin{aligned} F_{MOL} &\approx F_{SCAT}(\omega - \omega_o) - kv \frac{\partial F_{SCAT}}{\partial \omega} - \left[F_{SCAT}(\omega - \omega_o) + kv \frac{\partial F_{SCAT}}{\partial \omega} \right] \\ &\approx -2kv \frac{\partial F_{SCAT}}{\partial \omega} \end{aligned} \quad (3.4)$$

Extrapolating this to three orthogonal beams, it is evident that in such a setup, the atom would be slowed in all directions. This is known as an “Optical Molasses” due to the dissipative nature of the force. However, this does not confine the atoms as it is possible for them to diffuse from the beams.

The MOT combines an optical molasses with the Zeeman effect to confine and cool atoms. This is most simply illustrated by a $J = 0$ to $J' = 1$ transition as shown in Figure 3.2. The atoms are loaded using a set of coils in anti-Helmholtz configuration, creating a quadrupole field that has a zero-field point in the centre of the trap. Close to this zero-field point, there is a linear gradient field that perturbs the energy levels of the atoms through the Zeeman effect (see Section 2.2). The molasses beams will have opposite circular polarization and will be red detuned from the transition. Referring to Figure 3.2, for an atom lying in the $z < 0$ region, the $m_J = -1$ transition will lie closer to resonance, and selection rules favour the absorption of the σ_- beam, which will provide a force towards $z = 0$. The opposite will be true for an atom in the $z > 0$ region. This confines the atom in this region as the molasses cools it. A modification of Equation 3.4 to

include the Zeeman effect is required [16]:

$$\begin{aligned}
 F_{MOT} &= F_{SCAT}(\omega - \omega_o - kv - \beta z) - F_{SCAT}(\omega - \omega_o + kv + \beta z) \\
 &\approx -2kv \frac{\partial F_{SCAT}}{\partial \omega} + 2\beta z \frac{\partial F_{SCAT}}{\partial \omega_o} \\
 &\approx -\alpha v - \frac{\alpha\beta}{k} z
 \end{aligned} \tag{3.5}$$

Where $\alpha = 2k \frac{dF_{SCAT}}{d\omega}$. Here βz accounts for the frequency shift for the $m_J = \pm 1$ transitions. This is the same as the optical molasses, with a term that mirrors a restoring force with spring constant $\alpha\beta/z$. Practically, the MOT uses 3 pairs of counter-propagating laser beams to confine atoms in all spatial directions. A diagram of this is given in Figure 3.2 below:

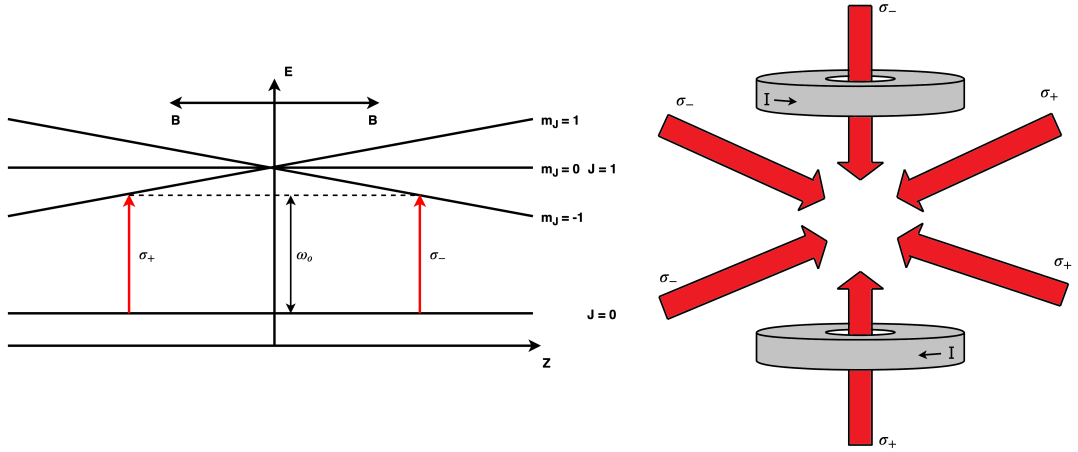


Figure 3.2: **Left:** Diagram of an example $J = 0$ to $J' = 1$ transition in a MOT. The Zeeman effect of the MOT coils shifts the energy of the m_J sublevels closer to resonance with the circularly polarized beams. **Right:** Diagram of the arrangement of MOT coils and molasses beams.

3.2.1 Temperature Limit

In an absorption-emission cycle, atoms that absorb photons will experience an input of momentum $\hbar k$ and have a recoil energy $E_r = \frac{\hbar^2 k^2}{2m}$. As mentioned in Section 2.3, the average force due to spontaneous emission of the photon will be zero. What is not considered is the fluctuations of the two processes, as an atom will not absorb the same amount of photons at any given time. This leads to heating effects, and the kinetic energy input can be related to the temperature

by the equipartition theorem. The result of this gives an equation for the cloud temperature T as a function of the laser detuning [16]:

$$k_B T = -\frac{\hbar\Gamma}{4} \frac{1 + (2\delta/\Gamma)^2}{2\delta/\Gamma} \quad (3.6)$$

Where k_B is the Boltzmann constant. This equation is minimized when $\delta = -\Gamma/2$, and leads to the Doppler temperature T_D :

$$T_D = \frac{\hbar\Gamma}{2k_B} \quad (3.7)$$

The Doppler Temperature represents the minimum temperature achievable via laser cooling in a two level system. For cooling on the D_2 line of ${}^6\text{Li}$, the Doppler Temperature is $T_D = 140 \mu\text{K}$. In reality, most alkali atoms have well resolved hyperfine energy levels that allow for sub-doppler cooling, but for ${}^6\text{Li}$ the ${}^2P_{3/2}$ excited states are unresolved and therefore sub-doppler cooling in a MOT is impossible. The excited state overlap also creates a broader effective linewidth of the excited state which further increases the temperatures achievable in the MOT to $\approx 600 \mu\text{K}$.

3.3 Optical Dipole Trap

In order to cool the atomic cloud beyond the Doppler temperature an optical dipole trap, which utilizes the effect of the dipole force between atoms and incident radiation, is implemented along with evaporative cooling techniques. With this, temperatures of $T < 1 \mu K$ can be reached. Referring to Equation 2.18, the dipole force on an atom with incident radiation was defined as a conservative force $F_{dip} = -\nabla U_{dip}$. For a two level atom with large detuning from resonance and ignoring saturation effects, the dipole potential and scattering rate become [28]:

$$U_{dip} = -\frac{3\pi c^2 \Gamma}{2\omega_o^3} \left(\frac{1}{\omega_o - \omega} + \frac{1}{\omega_o + \omega} \right) I(\mathbf{r}) \quad (3.8)$$

$$\Gamma_{SC} = \frac{3\pi c^2 \Gamma^2}{2\hbar\omega_o^3} \left(\frac{\omega}{\omega_o} \right)^3 \left(\frac{1}{\omega_o - \omega} + \frac{1}{\omega_o + \omega} \right)^2 I(\mathbf{r}) \quad (3.9)$$

Here, $I(\mathbf{r})$ denotes the intensity profile of the incident laser. These expressions show two resonant conditions, both the usual resonance at $\omega = \omega_o$ and the counter-rotating resonance at $\omega = -\omega_o$. If the magnitude of the detuning is much smaller than the resonant frequency, then the counter rotating term can be neglected. This is known as the rotating wave approximation and leads to the following expressions:

$$U_{dip} = -\frac{3\pi c^2 \Gamma}{2\omega_o^3} \left(\frac{1}{\delta} \right) I(\mathbf{r}) \quad (3.10)$$

$$\Gamma_{SC} = \frac{3\pi c^2 \Gamma^2}{2\hbar\omega_o^3} \left(\frac{1}{\delta} \right)^2 I(\mathbf{r}) \quad (3.11)$$

Here, the expressions have been simplified by labelling the detuning $\delta = \omega - \omega_o$ as in Section 2.3. It should also be noted that by combining Equations 3.10 and 3.11 there is a simple relation between the two quantities:

$$U_{dip} = \frac{\hbar\delta\Gamma_{SC}}{\Gamma} \quad (3.12)$$

From this it is clear that for ‘red’ detunings $\delta < 0$, the dipole potential will be negative and the light-matter interaction will attract atoms to the field. Consequently, potential minima are found at positions where $I(\mathbf{r})$ is maximized. Also, as mentioned in Section 2.3, the scattering rate in a dipole field increases with $I(\mathbf{r})/\delta$

whereas the magnitude of the potential increases as $I(\mathbf{r})/\delta^2$. For this reason, an ideal optical dipole trap uses a large laser detuning with high intensities to minimize the scattering rate for a given potential depth. Whilst this is an appropriate treatment for a two-level atom, in the case of ${}^6\text{Li}$ the contribution of the D_1 line (see Section 2.1) must be included in the expression for the dipole potential, such that Equation 3.10 becomes [28]:

$$U_{dip} = -\frac{3\pi c^2}{2} \left[\frac{\Gamma_1}{3\omega_1^3 \delta_1} + \frac{2\Gamma_2}{3\omega_2^3 \delta_2} \right] I(\mathbf{r}) \quad (3.13)$$

Where Γ_1/Γ_2 , ω_1/ω_2 and δ_1/δ_2 correspond to the line-widths, transition frequencies and detuning of the D_1 and D_2 lines of ${}^6\text{Li}$. For this experiment, the dipole trap is created at the centre of a focused, red-detuned beam that will now be described.

3.3.1 Focused Beam Trap

The dipole trap in this experiment uses a far-red detuned focused beam to trap the atomic cloud after the MOT stage. In practice a variant known as a crossed beam trap, where the incident beam is reflected back towards the cloud and intersects at the centre of the MOT is used. A diagram of a collimated Gaussian beam that is focused and then re-collimated via a pair of lenses is given below in Figure 3.3:

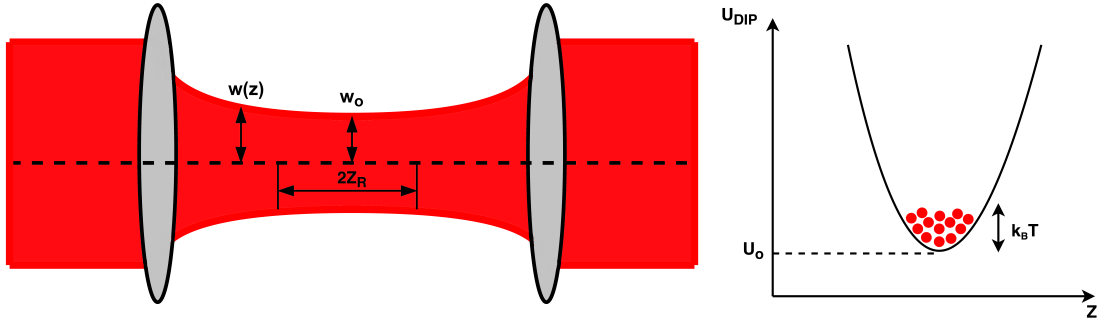


Figure 3.3: **Left:** Diagram of the focused beam trap. A collimated beam is focused at one lens and re-collimated at the other. The Rayleigh length is denoted by Z_R , the beam waist by w_0 and the $1/e^2$ radius by $w(z)$. **Right:** Dipole trap potential U_{DIP} vs position. U_0 is denoted as the point of minimum potential at the centre of the trap.

For a far-red detuned focused beam of power P , its intensity distribution $I(r, z)$ along the z axis is given by [28]:

$$I(r, z) = \frac{2P}{\pi w(z)^2} e^{-2\frac{r^2}{w(z)^2}} \quad (3.14)$$

Where $w(z)$ denotes the $1/e^2$ intensity radius of the collimated beam as a function of position, given by:

$$w(z) = w_o \sqrt{1 + \left(\frac{z}{Z_R}\right)^2} \quad (3.15)$$

Here w_o corresponds to the beam radius at the focus of the trap and Z_R the Rayleigh length is defined as $Z_R = \pi w_o^2 / \lambda$. Given that $Z_R \gg w_o$, the potential in the radial direction is steeper than in the axial direction. This also ensures that the gravitational force does not exceed the trapping force, providing the trap is aligned along the horizontal axis.

If the thermal energy $k_B T$ of a trapped atomic cloud is smaller than the potential depth, the potential in the dipole trap can be well approximated by a symmetric harmonic oscillator potential [28]:

$$U_{dip}(r, z) \approx -U_o \left[1 - 2 \left(\frac{r}{w_o}\right)^2 - \left(\frac{z}{Z_R}\right)^2 \right] \quad (3.16)$$

Where U_o is defined as the potential at the centre of the trap $|U(0, 0)| = U_o$. This can be derived explicitly by substitution of Equation 3.14 into Equation 3.10, such that U_o can be read off as:

$$U_o = \frac{3c^2 P \Gamma}{\omega_o^3 w_o^2 \delta} \quad (3.17)$$

The oscillation frequencies of a trapped atom in the radial and axial direction are given by [28]:

$$\omega_r = \sqrt{\frac{4U_o}{m w_o^2}} \quad \omega_z = \sqrt{\frac{2U_o}{m Z_R^2}} \quad (3.18)$$

As mentioned previously, the Rayleigh length is significantly larger than the beam waist $Z_R \gg w_o$, therefore the trapping frequency in the axial direction would be

much smaller than in the radial direction. This has the consequence of reducing the collision rate between the atoms and hence reducing the evaporation rate (see Section 4.5.1). Therefore, in this experiment a crossed beam trap is used in which the focal point of two red-detuned focused beams intersect at an angle ϕ to the z -axis to trap the atoms. A schematic of this can be found in Figure 3.4. In this case, the trapping frequencies in the x, y and z directions are given by [28]:

$$\begin{aligned}\omega_x &= \sqrt{\frac{4U_o}{m} \left(\frac{2 \cos^2(\phi)}{w_o^2} + \frac{\sin^2(\phi)}{Z_R^2} \right)} \\ \omega_y &= \sqrt{\frac{4U_o}{m} \left(\frac{2 \sin^2(\phi)}{w_o^2} + \frac{\cos^2(\phi)}{Z_R^2} \right)} \\ \omega_z &= \sqrt{\frac{8U_o}{mw_o^2}}\end{aligned}\tag{3.19}$$

The angle between the beams for this experiment is $\phi = 6.8$ degrees. The measurement of these quantities for the setup in this experiment will be discussed later. These quantities can also be used to define a temperature scale for measurements, the Fermi temperature T_F .

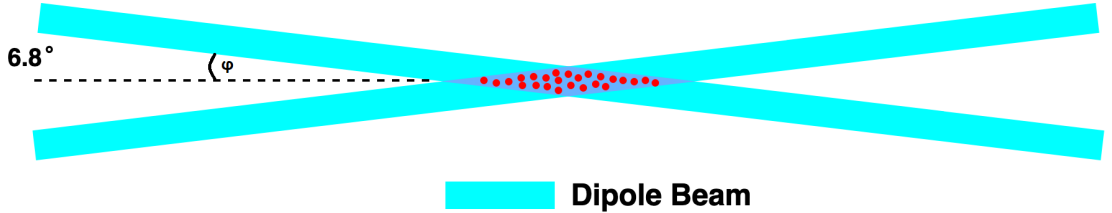


Figure 3.4: Schematic diagram of a crossed beam optical dipole trap. The angle ϕ denotes the intersection angle of the beams to the z -axis.

3.3.2 Fermi Energy and Temperature

The purpose of this thesis is to explore phenomena in the BCS-BEC crossover at varying initial cloud temperatures. Therefore, it is useful to define an energy and temperature scale to contextualize the temperatures reported.

The single particle Hamiltonian for a system of N spin polarized fermions of mass m in an azimuthally symmetric harmonic trap is given by [29]:

$$\hat{H}(r, p) = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + \frac{m\omega_r^2}{2}(x^2 + y^2 + \lambda^2 z^2) \quad (3.20)$$

Here ω_r is the trapping frequency described in the previous section. The factor λ relates the ratio of the trapping frequencies as ω_z/ω_r . The single particle energy levels $E(n_x, n_y, n_z)$ of this Hamiltonian are given by:

$$E(n_x, n_y, n_z) = \hbar\omega_r(n_x + n_y + \lambda n_z) \quad (3.21)$$

Where n_i are non-negative integers describing the energy level in each direction. For thermal energies $k_B T$ that are significantly larger than the level spacing, which is a good approximation for this experiment, it is a reasonable approximation to replace this discrete energy spacing with a continuum of energy states described by a density of states given by [29]:

$$g(E) = \frac{E^2}{2\lambda(\hbar\omega_r)^3} \quad (3.22)$$

The number of atoms in the cloud is given by $N = \int g(E)f(E)dE$ where $f(E)$ is the Fermi distribution. For a gas of Fermions with density of states given by Equation 3.22, this can be written explicitly as:

$$N = \frac{1}{2\lambda(\hbar\omega_r)^3} \int \frac{E^2}{e^{(E-\mu)/kT} + 1} dE \quad (3.23)$$

The Fermi Energy E_F is defined as the energy difference between the lowest energy state and the highest occupied energy state at zero temperature. At $T = 0$, this integral can be evaluated to give the result:

$$E_F = \hbar\omega_r(6\lambda N)^{1/3} \quad (3.24)$$

The Fermi temperature T_F is defined as E_F/k_B and is therefore explicitly given as:

$$T_F = \frac{\hbar\omega_r}{k_B}(6\lambda N)^{1/3} \quad (3.25)$$

3.4 Imaging and Temperature Measurements

Probing an ultra-cold atomic cloud is complicated, as it is impossible to take direct measurements with a physical probe. In this experiment, state-selective absorption imaging is utilized to image the density profile of the cloud in order to determine its atom number and temperature. This however will destroy the atomic cloud. Fluorescence imaging, based on detecting the scattering of light off the atoms from the trapping beams is also used for diagnostic purposes.

To produce an absorption image, resonant light is incident on an atomic cloud and the resultant shadow is measured on a CCD camera. The measured intensity I after passing through the cloud as a function of position z is given by Beers Law [31]:

$$\frac{dI}{dz} = -n\sigma(I)I \quad (3.26)$$

Where n is the atomic density of the cloud and $\sigma(I)$ is the absorption cross section, given by:

$$\sigma(I) = \frac{\sigma_o}{1 + 4\left(\frac{\delta}{\Gamma}\right)^2 + \frac{I}{I_{SAT}}} \quad (3.27)$$

Here, δ is the laser detuning from resonance, Γ is the natural linewidth of the transition and σ_o is the on-resonance cross section given by $\sigma_o = \hbar\omega_o\Gamma/2I_{SAT}$. Equation 3.26 can be solved to give:

$$\frac{I}{I_o} = e^{-n_z\sigma} \quad (3.28)$$

Where n_z is the column density of the cloud given by $n_z(x, y) = \int n(x, y, z)dz$ and I_o is the intensity of the light incident on the cloud. We can re-write Equation 3.28 for the column density as:

$$n_z = -\frac{1}{\sigma} \ln\left(\frac{I}{I_o}\right) \quad (3.29)$$

In the limit that the incident beam is on resonance $\delta = 0$ and $I_o \ll I_{SAT}$, the absorption cross section $\sigma \rightarrow \sigma_o$. The total atom number in the cloud can be found by integrating over the column density:

$$N = \int n_z(x, y) dx dy \quad (3.30)$$

Hence, 3 images are taken to produce an absorption image. A first image is taken of the transmitted light through the cloud, followed by an image of the incident light with no atoms present. The ratio of these two images is simply the quantity I/I_o . Finally, a background image with no light is taken and the intensity subtracted from the resulting value of I/I_o to eliminate any artefacts caused by stray light.

3.4.1 Measuring the Temperature of an Atomic Cloud

The results presented in this thesis will be based substantially on thermal effects of the atomic cloud on Feshbach association, therefore precise determination of the temperature of the atomic cloud is important. Upon release from a trap, a thermal cloud of atoms will expand in all directions, the rate of which can be used to determine the temperature of the cloud at the point of release, under the assumption of negligible interactions. To gain an accurate measurement this can be repeated for the same initial conditions. This is known as Time-Of-Flight (TOF) imaging.

For a cloud of atoms of mass m in a harmonic trap, their energy can be expressed as:

$$E(\mathbf{r}, \mathbf{p}) = \sum_j \frac{m\omega_j^2 \mathbf{r}_j^2}{2} + \frac{\mathbf{p}_j^2}{2m} \quad (3.31)$$

Where ω_j is the frequency of the harmonic trap in the x, y and z directions, $\mathbf{r}_j$ and $\mathbf{p}_j$ are the relative position and momentum of the atoms in the trap in the three spatial directions. If the gas is assumed to be essentially classical, the probability to find an atom at a position $\mathbf{r}, \mathbf{p}$ at a time $t = 0$ immediately after release from a trap, can be described by a Boltzmann distribution function [32]:

$$f_o(\mathbf{r}, \mathbf{p}) \propto e^{-\frac{E(\mathbf{r}, \mathbf{p})}{k_B T}} \quad (3.32)$$

Here, k_B is the Boltzmann constant and T is the temperature of the gas. This equation must be normalized such that $\int_0^\infty f_o(\mathbf{r}, \mathbf{p}) dp dx dy dz = 1$. Using Equation 3.31, the normalization condition leads to the following expression:

$$f_o(\mathbf{r}, \mathbf{p}) = \left(\frac{m\omega^2}{2\pi k_B^2 T^2} \right)^{3/2} e^{-\frac{\mathbf{r}^2 m \omega^2}{2k_B^2 T^2}} \cdot \left(\frac{1}{2\pi k_B^2 T^2} \right) e^{-\frac{\mathbf{p}^2}{2k_B^2 T^2}} \quad (3.33)$$

In order to measure the temperature of the cloud, after releasing from the trap the cloud is left to expand for a time t . In this time, it will have expanded a distance $\mathbf{p}t/m$, so defining a new position co-ordinate $\mathbf{r}' = \mathbf{r} + \mathbf{p}t/m$, Equation 3.33 becomes:

$$f_o(\mathbf{r}', \mathbf{p}) = \left(\frac{m\omega^2}{2\pi k_B^2 T^2} \right)^{3/2} e^{-\frac{(\mathbf{r}' - \mathbf{p}t/m)^2 m \omega^2}{2k_B^2 T^2}} \cdot \left(\frac{1}{2\pi k_B^2 T^2} \right) e^{-\frac{\mathbf{p}^2}{2k_B^2 T^2}} \quad (3.34)$$

By integrating over all momentum dp , the expression f_o has its momentum dependence removed. This equation then becomes:

$$f_o(\mathbf{r}') = \left(\frac{1}{2\pi\sigma^2} \right)^{3/2} e^{-\frac{\mathbf{r}'^2}{2\sigma^2}} \quad (3.35)$$

Where $\sigma^2 = k_B T t^2 / m + k_B T / m \omega^2$. This corresponds to a normalized Gaussian with standard deviation σ . Given the initial size of the cloud $\sigma_o = k_B T / m \omega^2$, we can write the width of the cloud as a function of time t as:

$$\sigma(t) = \sqrt{\frac{k_B T}{m} t^2 + \sigma_o^2} \quad (3.36)$$

In this experiment, to measure the temperature, a series of measurements on the size of the cloud are taken for different expansion times. It should be noted that the turning off of the trap must be done in a rapid, non-adiabatic fashion, or else a cooling effect will influence the measurement. However, as absorption imaging is a destructive process, the experiment must be run for the same initial conditions many times. Cloud size is plotted against time and fitted to Equation 3.36 to give a best fit of the temperature T . Examples of this will be found in later chapters when a description of the ODT optimizations is presented.

Chapter 4

Experimental Sequence

In this chapter the experimental sequence that allows for Feshbach molecules to be produced will be outlined, with a focus on how the implementation of the techniques in the previous chapter are calibrated and optimized to produce high atom number ultra-cold ^6Li clouds. I will also present the details of the laser systems that have been utilized.

4.1 Atom Source

At room temperature, the vapour pressure of ^6Li is too low to produce an atom flux that would be required for good loading into a MOT. The vapour pressure P of a gas of pure components at a temperature T is given by the Antoine Equation [33]:

$$\log_{10}(P) = A - \frac{B}{C + T} \quad (4.1)$$

Here A , B and C are constants specific to the atomic component. In this experiment, a solid ^6Li source is heated to 410°C . Using constants found in Yaw's handbook [34], this corresponds to a vapour pressure of 10^{-4} mbar. The atomic flux from the oven is given by [35]:

$$I = \frac{nA\bar{v}}{4} \quad (4.2)$$

Where n is the atomic density in the oven, A is the area of the aperture of the oven and $\bar{v}$ is the mean velocity of the atoms given by $2v_p/\pi$. For an oven aperture of 3mm^2 , combining Equations 4.1 and 4.2 leads to an atomic flux of around 10^9

atoms/s.

4.2 Laser Systems

In this experiment, a total of four ECDL's (External Cavity Diode Lasers) as well as a 100W High Power Fibre laser produced by IPG are used. The ECDL's can be frequency shifted using piezo-electric crystals on the diode mounts and can be modulated in-sequence via AOM (Acousto-Optic modulator) systems to provide the detuning needed to cool and trap the atoms. The uses for these, as well as their power output measured after passing through an optical isolator, are outlined in Table 4.1.

Laser	Usage	Power Output (mW)
Home Built Reference	Use Doppler-Free Spectroscopy to offset lock other lasers to the $^2S_{1/2} \rightarrow ^2P_{3/2}$ transition.	10
Home Built Imaging	Used to image atomic cloud at low and high field using absorption imaging.	10
Toptica TA Pro	Used for repumper light for the MOT and Zeeman Slower as well as providing seed power for the home built TA.	265
Home Built TA	Used for cooling light for MOT and Zeeman Slower.	320

Table 4.1: Usage and Power Output as measured after passing through an optical isolator for the lasers used in this experiment.

Referring to the level diagram of ^6Li (Fig. 2.1), the reference laser is locked to the D_2 line such that other lasers can be locked with respect to this transition. The details of how the offset locks are implemented can be found in previous theses [27][36].

In this experiment, a homebuilt Tapered Amplifier (TA) is used to provide the cooling light for the atoms in the MOT. Referring to Section 2.1 where a description of atomic structure was outlined and Figure 2.1, denoting the excited states of the $^2S_{1/2} \rightarrow ^2P_{3/2}$ transition as $F' = \frac{1}{2}, \frac{3}{2}$ and $\frac{5}{2}$ and ground states as $F = \frac{1}{2}$ and $\frac{3}{2}$,

the cooling in the MOT is done via the $F = \frac{3}{2}$ to $F' = \frac{5}{2}$ transition. As mentioned in Section 2.1, the excited states in ${}^6\text{Li}$ are unresolved and therefore an emission cycle from the excited state can transition into the $F = \frac{1}{2}$ ground state, which is a dark state to the laser locked to the cooling transition. For this reason the $F = \frac{1}{2}$ to $F' = \frac{3}{2}$ must be driven with similar power to keep cooling the cloud, which is known as repumping, although in this context also provides a cooling element. On average in this experiment, the cooling transition is driven with 1.5 times the repumping transition power.

The Toptica TA Pro is locked to the Reference transition $F = \frac{1}{2}$ to $F' = \frac{3}{2}$ and the majority of the light from it is used as repumping light. A beam splitter is placed on its output to provide seed light for the homebuilt TA, which is then used for cooling light. Both TA's pass through AOM's that shift their frequencies by ± 114 MHz to cover the width of the hyperfine splitting of the $l = 0$ state, giving the cooler and repumping beams the same detuning from the excited state. A diagram of the laser transitions as well as the beam paths used in this experiment can be found in Figures 4.1 and 4.2.

The homebuilt imaging laser needs to deliver much less power than the two TA's. Via a voltage controlled offset (VCO), this laser can have large detuning up to -1200 MHz from the reference transition, which is needed for imaging in the high field regime. The details of which are contained in Section 4.6.1.

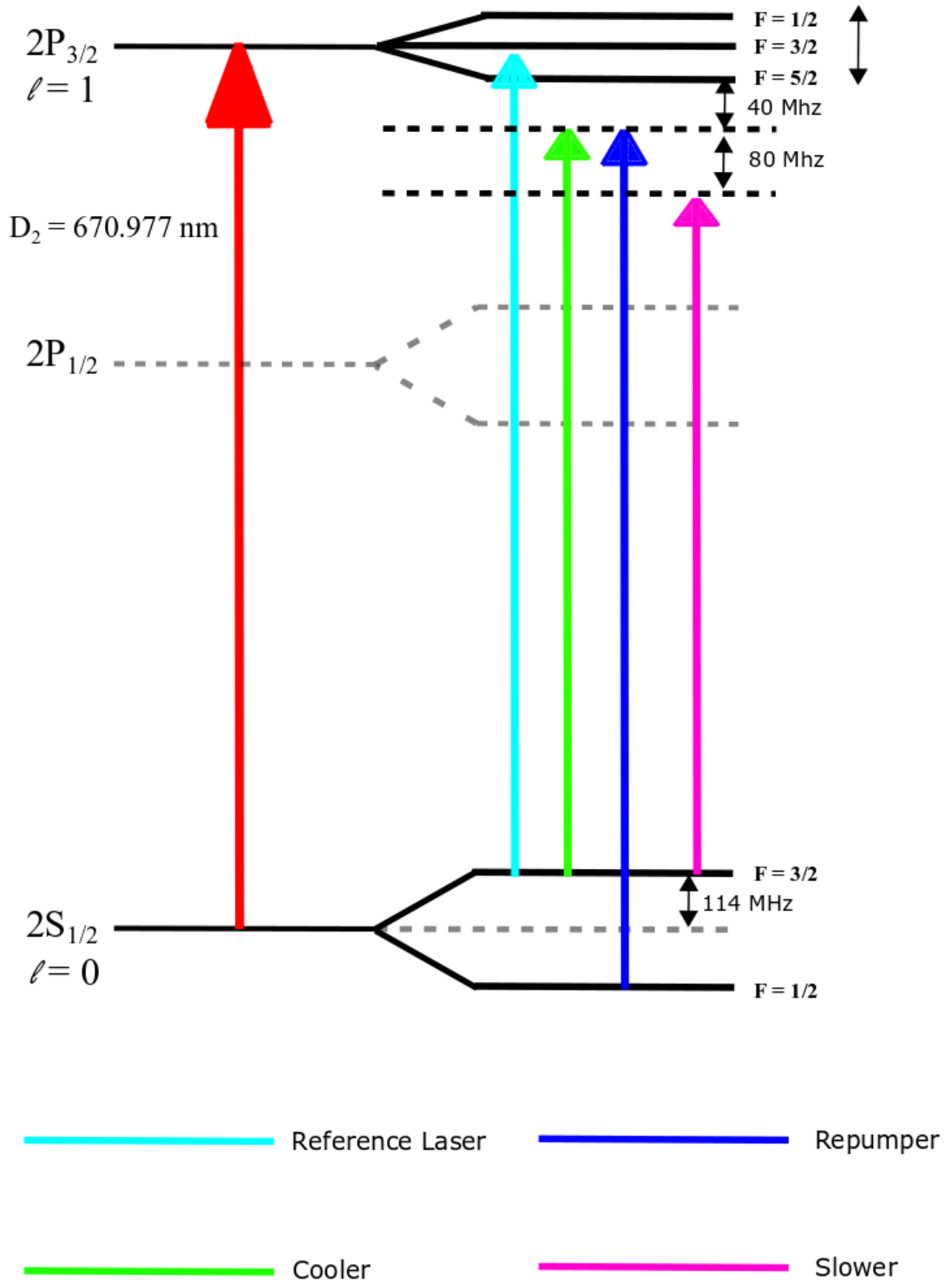


Figure 4.1: Schematic of the lasers described in Table 4.1 with respect to atomic transitions in ${}^6\text{Li}$. Shown also is the final detuning for the cooler, repumper and Zeeman slower lasers. The red arrow denotes the D_2 line which the reference laser is locked to.

Lithium Optics

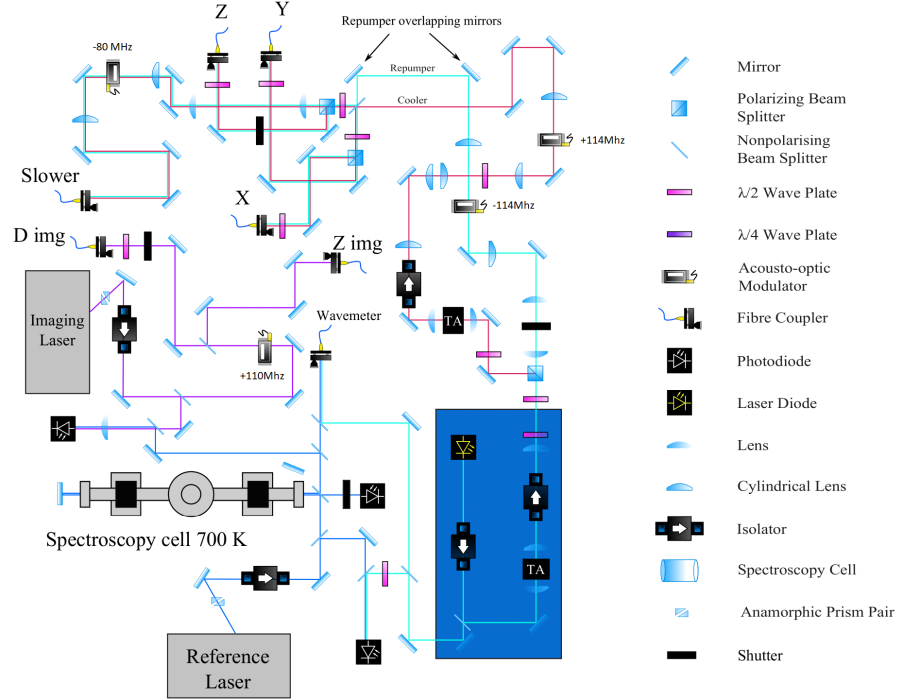


Figure 4.2: Schematic diagram of the lasers and beam path of this experiment. Original schematic was produced in [12] and has been modified for the current setup.

4.3 Sequence Software

To produce a cloud of ultracold ${}^6\text{Li}$, atoms ejected from the oven are passed through a Zeeman slower into the MOT and are then placed in a Dipole trap to begin evaporative cooling. A set of Feshbach coils that produce fields up to 1200 G (See Section 4.6) are then switched on to control the scattering length around a Feshbach resonance.

The control of timings for VCO's, shutters and AOM's are handled remotely via software developed in the PyPlayer python framework. The details of the software and how it is written is outside the scope of this thesis, but can be found in a previous PhD thesis [37]. An image of the control sequence used to run this experiment can be found in Figure 4.3.

Such a method of operating this experiment allows for easy tuning and optimization of the apparatus utilized in this experiment, the details of which will be presented in future sections within this chapter.

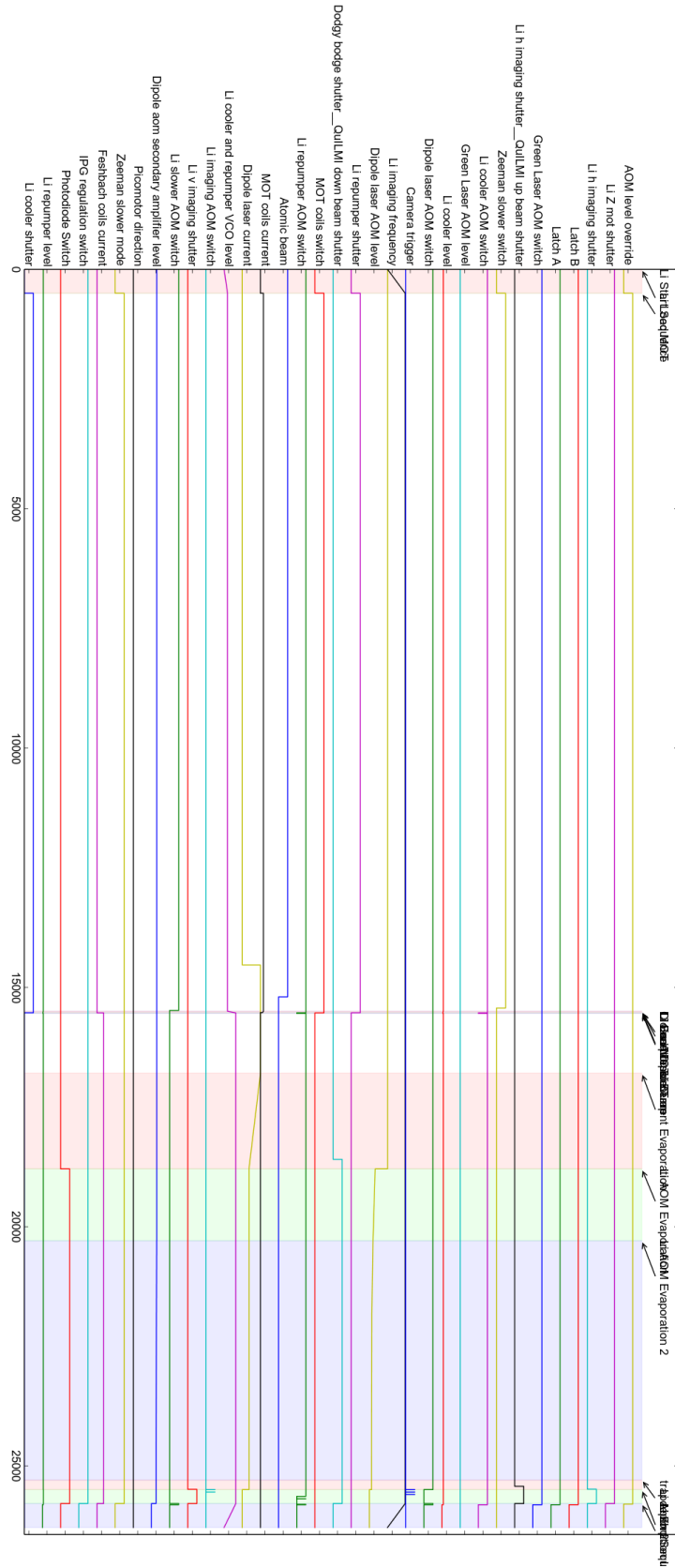


Figure 4.3: Image of the control sequence in PyPlayer software used to create Feshbach molecules in this experiment.

4.4 Magneto-Optical Trap

Light is coupled from the two TA's described in Section 4.2 through polarization maintaining optical fibres and collimated via outcouplers before the chamber with 10 mm lenses. The beams in the MOT have a typical $1/e^2$ radius of 12 mm and are aligned in the centre of the MOT chamber. A single beam is used for each of the three spatial directions and they are retro-reflected on the other side of the MOT chamber to make 6 beams in the molasses. A separate beam carries the light for the Zeeman slower. A diagram of the MOT chamber is shown in Figure 4.6. The beam power going into the outcouplers in the x , y and z direction as well as for the Zeeman slower can be found in Table 4.2.

The MOT is optimized for maximal atom number whilst maintaining the atomic cloud at millikelvin temperatures, to ensure good loading into the dipole trap. One way in which this is done is to optimize the time in which the MOT is loaded from the output of the Zeeman slower. The MOT loading in a constant density limit is given by [38]:

$$N(t) = \frac{R}{L} (1 - e^{-Lt}) \quad (4.3)$$

Here, R corresponds to the loading rate from the Zeeman slower and the lifetime of the MOT is given by $\tau = 1/L$. Whilst this equation has no optimum value, there will be a trade-off between atom number in the MOT and time taken for the loading sequence. For the MOT in this experiment, the graph of atom number in the MOT against loading time is displayed in Figure 4.4 and fitting equation Equation 4.3 gives a lifetime of 2.6s and a loading rate of $4.5 \times 10^7 s^{-1}$. From this, a typical loading time was chosen as 15s and the MOT is loaded with around 10^8 atoms.

4.4.1 MOT Compression

After the MOT loading period, the temperature in the MOT is typically on the order of 1 mK . At this point the detuning of the MOT beams from the $^2P_{3/2}$ excited state is 40 MHz (6.8Γ) to allow for a large capture velocity. It should be noted from Equation 3.7 that the Doppler Temperature, the minimum temperature achievable in a MOT, is reached for a detuning $\delta = -\Gamma/2$. Therefore, to cool the atomic cloud further, the detuning is ramped to this optimal value within 10-15

ms. As the final detuning is modulated by an externally controlled AOM, this optimization is done in practice by measuring the atom number after dipole trap loading and choosing the value that produces the largest atom number, explained in Section 4.5. A graph of atom number in the dipole trap as a function of the final compression detuning is given in Figure 4.5.

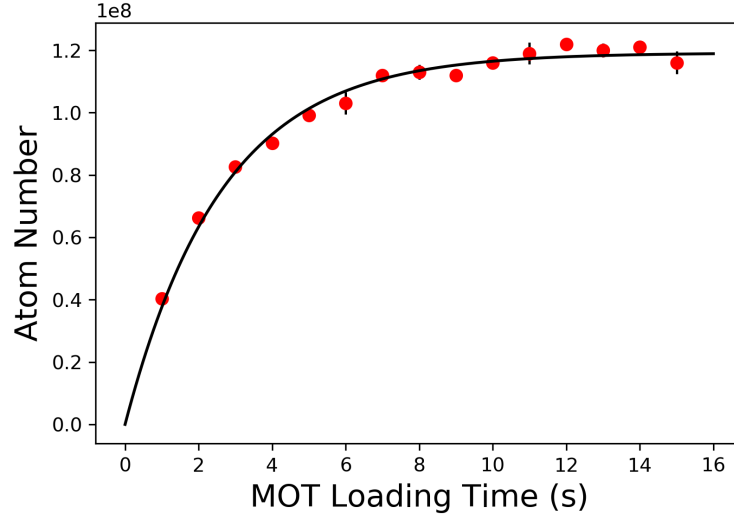


Figure 4.4: Atom number in the MOT against time for a loading sequence. The atom number saturates between 12-15s. The error in the points represents the standard error from 4 repetitions.

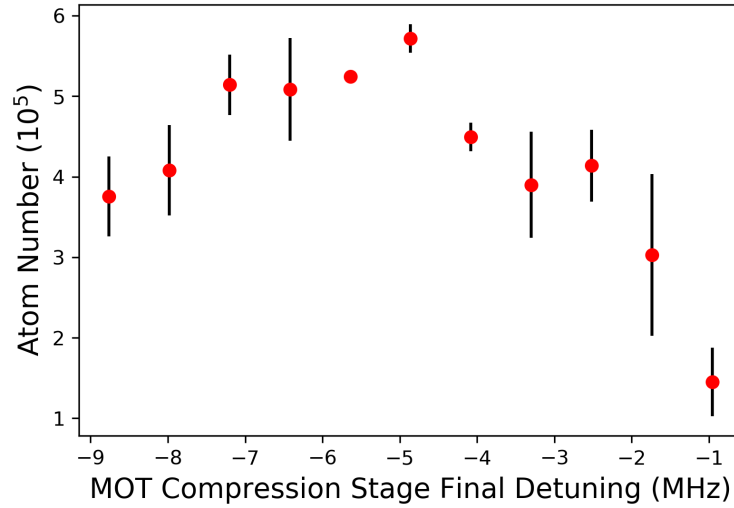


Figure 4.5: Atom number in Dipole trap against Final Detuning in the MOT compression stage. The peak value is chosen for the MOT compression stage. The error in the points represents the standard error from three repetitions. The peak value differs slightly from the optimal theoretical value of $-\Gamma/2$.

4.4.2 State Preparation

As mentioned in Section 2.4, low temperature collisions of fermions in the same state are suppressed and therefore evaporative cooling cannot occur. However, for an equal mixture of ^6Li atoms in the $|F = 1/2, m_F = \pm 1/2\rangle$ states, this is no longer an issue. At the end of MOT compression, the atoms in the MOT will be in a mixture of $F = 1/2$ and $F = 3/2$ states. To pump the atoms trapped in the MOT into the $F = 1/2$ state, the repumper beam is shut off 500 ms before the cooler beam before transfer into the dipole trap, known as the F pumping time. It is also important to ensure the atoms in the cloud are in the $F = 1/2$ state as that ensures only $m_F = \pm 1/2$ states are available, making it easy to tune interactions through a Feshbach resonance. It is the broad Feshbach resonance between these two states that forms the basis of this thesis.

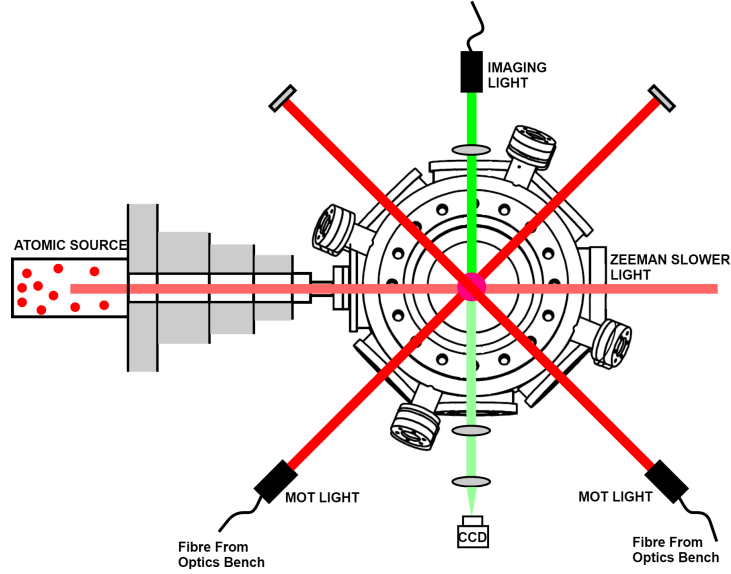


Figure 4.6: Top down diagram of MOT chamber, complete with x and y MOT beams and imaging beam. The MOT z beam is not shown. The beam waists, Zeeman coils and atomic oven are not to scale.

	Cooler Power (mW)	Repumper Power (mW)
x	35	28
y	31	22
z	41	26
Zeeman Slower	21	12

Table 4.2: Cooler and Repumper MOT beam powers in each direction as measured before the outcouplers.

4.5 Transfer to Dipole Trap

After the MOT compression stage, a cloud of atoms in an equal spin mixture of $|F = 1/2, m_F = \pm 1/2\rangle$ is produced at a temperature of roughly $600 \mu K$. The cloud is then loaded into a focused beam dipole trap (See Section 3.3). The dipole trap is created by a far-red detuned 1064nm Yb IPG fibre laser with a maximum power output of 100W collimated to a beam radius of 5 mm. The beam is focused to a waist size of $80 \mu m$ in the centre of the chamber and reflected back to create a crossed-beam trap.

4.5.1 Evaporative Cooling Stage

The atom number after dipole trap loading drops from around 10^8 atoms to roughly 10^6 . It is at this point that the evaporative cooling process begins. The principle of evaporative cooling is to remove hot atoms from the trap and allow the remaining atoms to re-thermalize via elastic collisions. Ideally, the rate at which energy is removed from the trap should not lead to large atom loss from the trap. Both a plain evaporative cooling stage, where the dipole trap depth remains constant, and a forced evaporative cooling stage where the trap depth is ramped to a progressively smaller values are utilized to cool the atomic cloud to temperatures in the nano-kelvin range.

Referring to the dipole trap centre potential U_o in Section 3.3, an atom must have an energy $E = U_o + f_r k_B T$ to be removed from the trap, where f_r corresponds to the fraction of thermal energy removed by the outgoing atom. This can be thought of as atoms in the tail of the Boltzmann distribution with energies above U_o being preferentially removed from the trap. The truncation parameter can be written as $\nu = U_o/k_B T$ and the atom loss rate given by [39]:

$$\frac{dN}{dt} = -N n_o \sigma \bar{v} \nu e^{-\nu} \quad (4.4)$$

Where n_o is the peak atomic density in the trap, σ is the s-wave scattering cross section (See Section 2.4) and $\bar{v}$ is the average relative velocity between atoms in the trap. This equation can be combined with the energy lost per atom to give an equation for the rate of energy loss in an evaporative process:

$$\frac{dE}{dt} = (U_o + f_r k_B T) \frac{dN}{dt} \quad (4.5)$$

It should be noted that if the scattering cross section is zero then the atom loss rate, and consequently the energy loss rate in a plain evaporative process should also be zero. Thus, there should be no temperature decrease in the atomic cloud. As the atom loss rate $dN/dt \propto e^{-\nu}$, with a decrease in temperature, the truncation parameter will increase, leading to a reduction in the atom loss rate. Hence, the temperature decrease due to a plain evaporative process will eventually stagnate.

The solution to this problem is a technique known as “Forced Evaporation”. In this case, the dipole trap depth is modulated with decreasing cloud temperature to keep the truncation parameter constant. A time dependant trap potential $U(t, x)$ can be considered, and assumed to be a separable function [40]:

$$U(t, x) = -U(t)g(x) \quad (4.6)$$

Where $g(x)$ describes the positional dependence of the trap shape. In the case of a time dependant potential, the previous equation for energy loss is incorrect due to a change in trap depth causing a change in energy. Assuming that evaporation is taking place at temperatures $k_B T \ll U$, the atoms oscillate at the potential minima of the trap where the average potential energy can be expressed as [40]:

$$\langle U \rangle = \frac{E}{2} \quad (4.7)$$

A modification to Equation 4.5 to account for the change in potential energy is required. If the relative change in the potential over time is $\dot{U}/U$, the total energy will evolve in time as [40]:

$$\frac{dE}{dt} = (U(t) + f_r k_B T) \frac{dN}{dt} + \frac{E}{2U(t)} \frac{dU(t)}{dt} \quad (4.8)$$

This equation can be solved in the limit of constant truncation parameter ν to give equations for the atom number N and phase space density ρ , defined as $\rho = N(h\bar{\omega}/k_B T)^3$ where $\bar{\omega}$ is the mean frequency of the trap, as they evolve over

time. These relationships are given by [40]:

$$\begin{aligned} \frac{N}{N_o} &= \left(\frac{U}{U_o} \right)^{\frac{3}{2(\nu'-3)}} \\ \frac{\rho}{\rho_o} &= \left(\frac{U_o}{U} \right)^{\frac{3(\nu'-4)}{2(\nu'-3)}} = \left(\frac{N_o}{N} \right)^{\nu'-4} \end{aligned} \tag{4.9}$$

Here $\nu' = \nu + f_r$. Provided $\nu' > 4$ during a forced evaporation period, as the atom number in the trap decreases, the phase space density will increase and the temperature of the atomic cloud will decrease.

In practice this experiment starts with a period of plain evaporation, where the atomic cloud is held at a magnetic field in the high-field Paschen-Back regime close to the Feshbach Resonance such that the scattering cross section is large. The effect of plain evaporation on atom number and temperature can be measured by holding the atomic cloud for different times and measuring the atom number and temperature by time of flight absorption imaging. An example of this for the magnetic field at the zero crossing of the Feshbach and a highly interacting region is shown in Figure 4.7.

As was mentioned at the start of this section and as seen in Figure 4.7, at a scattering length and consequently a scattering cross section of zero, atom loss during plain evaporation does not lead to a temperature decrease. In the measurements presented here there is a slight decrease in atom number and temperature due to small inaccuracies in the magnetic field. In the measurement at 698G, the temperature in the trap decreases rapidly from around $400 \mu K$ to $220 \mu K$. To optimize this stage, it is important to stop the plain evaporation when the temperature decrease stagnates, so that there is no unnecessary atom loss from the trap before the forced evaporation stage begins. We therefore stop the plain evaporation period after 1000ms.

After this period of plain evaporation, a period of forced evaporation takes place. The trap depth is lowered by decreasing the power in the dipole beams. This will decrease the frequency of the trap, therefore reducing the collision rate of the atoms, but this is counteracted by holding the cloud in the vicinity of the Feshbach Resonance where the scattering cross section is large. The forced evaporation stage is a two stage process. Firstly, the dipole beam power is reduced from the plain

evaporation value of 100W to 30W in 2000ms. From this point, the dipole beam is further reduced via an AOM feedback system to as low as 70mW, details of the feedback system can be found in previous theses [37][12]. The power value as well as ramp timings can be controlled externally to produce cloud temperatures between 0.1-2 μK . An example of a measurement of the temperature of the dipole trap, using the TOF method detailed in Section 3.4.1, can be found in Appendix A.

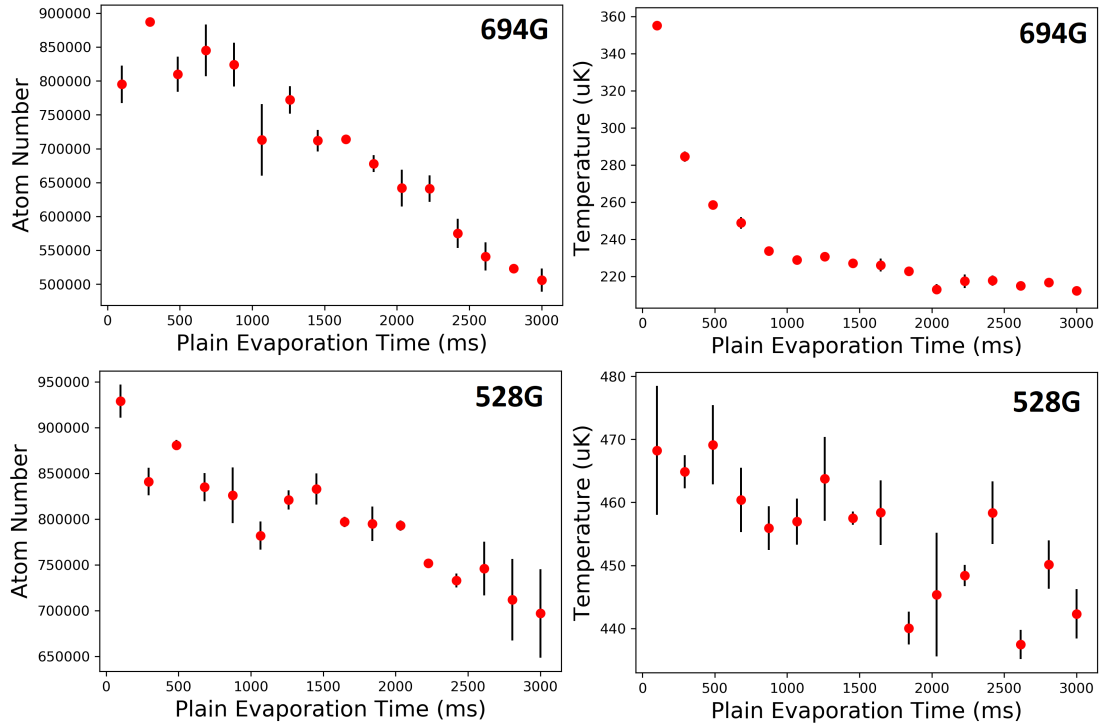


Figure 4.7: **Top:** Atom Number and Temperature for different values of Plain Evaporation time at 698G, corresponding to a scattering length of $1500a_o$. **Bottom:** Atom Number and Temperature for different values of Plain Evaporation time at 528G, the zero crossing of the Feshbach Resonance. The error bars correspond to the standard error of three measurements.

4.5.2 Lifetime of Dipole Trap

After the period of forced evaporation concludes, it is important to ensure that the atoms in the trap are long-lived and do not diffuse from the trap quickly. Specifically, it was important to ensure the lifetime in the dipole trap is much longer than the ramp time of the BCS-BEC crossover that constitutes the results of this thesis. To measure this, after forced evaporation at a trap depth of 25 μK , the atomic cloud was held in the dipole trap for different times up to 20s at a

field of 860G. The atom number for each hold time was measured via absorption imaging. The graph of atom number versus hold time was fitted to an exponential decay equation $N = N_o \exp(-t/\tau)$ in order to determine the mean lifetime τ , the time at which there is $1/e$ of the initial atom population left in the trap. The results of this can be seen below in Figure 4.8:

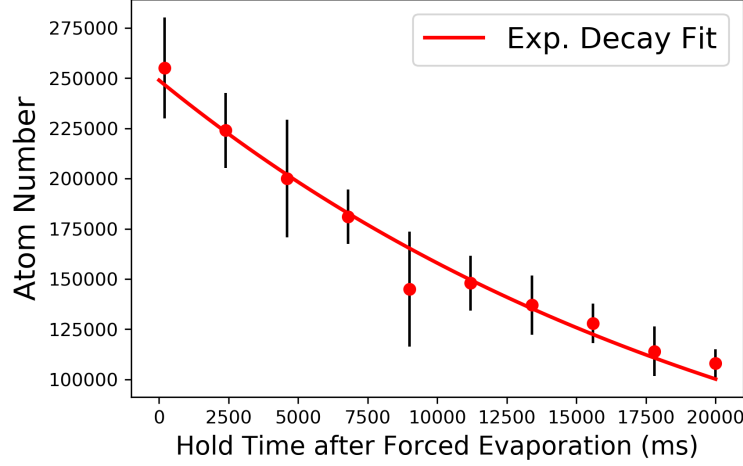


Figure 4.8: Atom number versus hold time after a period of forced evaporation. An exponential fit is given to determine the mean lifetime in the dipole trap. The error bar refers to the standard error of three repetitions.

The fitting was done via the Scipy package in Python. The mean lifetime τ was fitted as 21s, which is much longer than the duration of any of the BCS-BEC field ramps performed in this experiment (See Sections 5.1 and 5.3). This is important as it means that when measuring the atom number after the ramp, the remnant atomic fraction is not being over-counted due to atom loss from the trap.

4.5.3 Trapping Frequency Measurement

Characterization of the geometric parameters of the dipole trap is important as, for example, the mean frequency of the trap is a parameter in defining the Fermi temperature T_F (See Section 4.6). The trapping frequency ν and trap depth U_o as a function of the beam power P are defined as $\nu = A\sqrt{P}$ and $U_o = cP$.

In general there are two methods of measuring the frequency of a trap. First the method known as parametric heating, where the atomic cloud is held at a given beam power and the intensity of the dipole beam is modulated sinusoidally around this point. Once the modulation reaches twice the trapping frequency, there will

be enhanced atom loss from the trap due to parametric heating [41]. The second method is to induce breathing modes of the atomic cloud. This is done by giving the cloud a “kick” by suddenly increasing the trap power for a short duration. After this kick, the cloud will oscillate at twice the trap frequency. Absorption images can be taken of the cloud for different times after the initial kick and the radius of the cloud measured as a function of time. A decaying oscillator function can be fitted to a graph of cloud size versus time to determine the trap frequency in the radial and longitudinal directions for a given beam power. The calibration for the dipole trap in this experiment has been carried out previously [36][42], and an example of the breathing mode method is presented below for a cloud held at 1W of laser power:

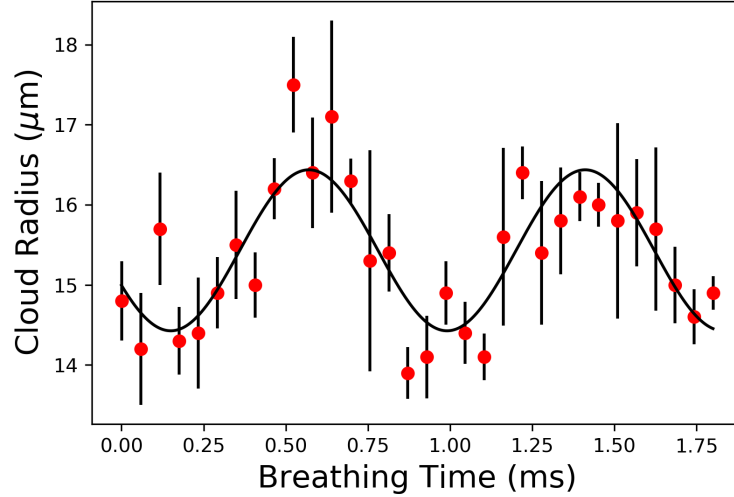


Figure 4.9: Cloud Radius versus Time after a breathing Mode kick at 1W of dipole power. Measurement taken from [36].

These measurements have been taken for many trap powers to calibrate the proportionality constants for the trap depth and trap frequency. The proportionality constants for the trap frequency are $A = 525 \text{ HzW}^{-1/2}$ (Radial) and $62.7 \text{ HzW}^{-1/2}$ (Longitudonal). For the trap depth, $c = 12.1 \text{ } \mu\text{K/W}$.

4.6 Feshbach Coils

A set of Feshbach coils in a Helmholtz configuration is used to produce the fields required to probe the broad Feshbach resonance of the two lowest hyperfine states of ^6Li . At a supplied current of 400A, the coils are capable of producing fields of up to 1200G. The coils are arranged such that there is a homogenous field in the centre of the trap. In this section, I will describe how the large magnetic fields produced by the Feshbach coils are accounted for when probing imaging transitions as well as how the field is calibrated with respect to supplied current.

4.6.1 High-Field Imaging of Atomic Cloud

As mentioned in Section 4.5, the dipole trap is loaded with an equal atom mixture in the $|F = 1/2, m_F = \pm 1/2\rangle$ states. For these two states, as can be seen in the Breit-Rabi diagram in Section 2.2, the high-field Paschen-Back region is reached at around 200G, which is a much lower field than the broad Feshbach resonance position of these two hyperfine states. In this situation, the hyperfine states are written as $|m_J = -1/2, m_I = 0\rangle$ and $|m_J = -1/2, m_I = 1\rangle$. The Zeeman splitting of these states must be accounted for in probing the imaging transition.

For imaging purposes, a laser is locked to the $F = 1/2$ to $F' = 3/2$ transition in ^6Li and for zero external field this is sufficient. When imaging in the high field regime, the laser frequency is shifted via a high field voltage control offset (VCO) to compensate for the additional effect of the Zeeman splitting. The VCO can tune the frequency of the imaging laser between -600 and -1200 MHz from resonance, such that magnetic fields between 450G and 950G can be imaged. Practically, the voltage supplied to the VCO can be scanned via the PyPlayer software discussed in Section 4.3, rather than the frequency directly, and absorption images taken of the cloud at different voltage offsets to ensure that the exact offset is used. This is done by choosing the control voltage that produces an absorption image with the largest visible atom number, corresponding to the best absorption, in the dipole trap before the plain evaporation stage. An example graph for this process is displayed in Figure 4.10.

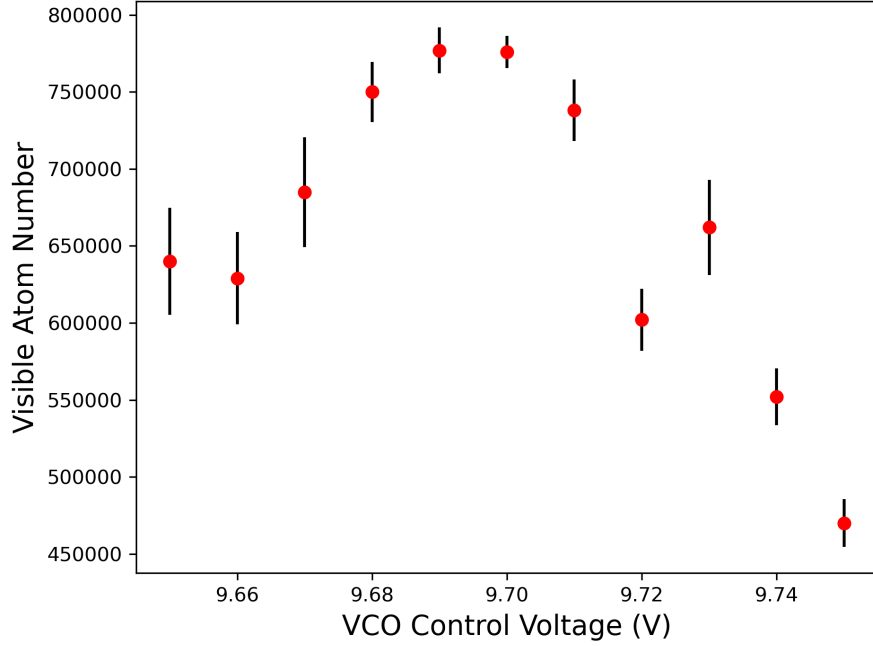


Figure 4.10: Measured atom number via absorption imaging at different values of the high field VCO at a magnetic field of 860G. The VCO values correspond to an imaging laser offset range of -1110 to -1125 MHz. The error displayed is the standard error of four scan repetitions.

4.6.2 Magnetic Field Calibration

The measurements taken in this experiment require a precise characterization of the magnetic field produced by the Feshbach coils, as it is important to know exactly the starting and end field of the BCS-BEC sweep. It is impossible for a physical probe to be placed inside the chamber to determine the magnetic field, however physical properties of the atomic cloud can be used to determine the field that it is placed in.

There are two methods employed in this experiment to determine the scaling relationship between the current supplied to the Feshbach coils and the magnetic field produced at the centre of the trap. The first method employed is based on the fact that non-interacting atoms will not thermalise during plain evaporation (see Section 2.5). The atomic cloud, prior to evaporation, is held at full power for five seconds at different values of coil load current. The atom number is measured at each current via absorption imaging. A peak in atom number occurs during this measurement at the coil current corresponding to the zero crossing of the

Feshbach Resonance at 528G. The Feshbach coils can be scaled assuming a linear dependence on current and zero offset based off of this point. Using this method, the calibration for the Feshbach coils in this experiment was measured in [27] and the result presented in Figure 4.11. As this is not a rigorous method of calibration, this was cross-checked with a secondary measurement using a separate method.

The second method for measuring the magnetic field is based on high field imaging. For a given value of input current to the Feshbach coils, the voltage applied to the imaging VCO is scanned as described in Section 4.6.1. The offset that the VCO provides to the imaging beam can be measured using a frequency counter and compared to the calculated frequency shift in the imaged state due to the Zeeman effect at high magnetic fields (see Section 2.2) in order to determine the magnetic field that the atomic cloud is held in. This was done for many values of input current to produce a calibration graph to compare to the previous measurement, and is presented in Figure 4.12. The two methods give good quantitative agreement. The magnetic field in the centre of the trap varies linearly with input current and a fit is given to the data in Figure 4.12. The proportionality constant was fitted as 2.91 G/A.

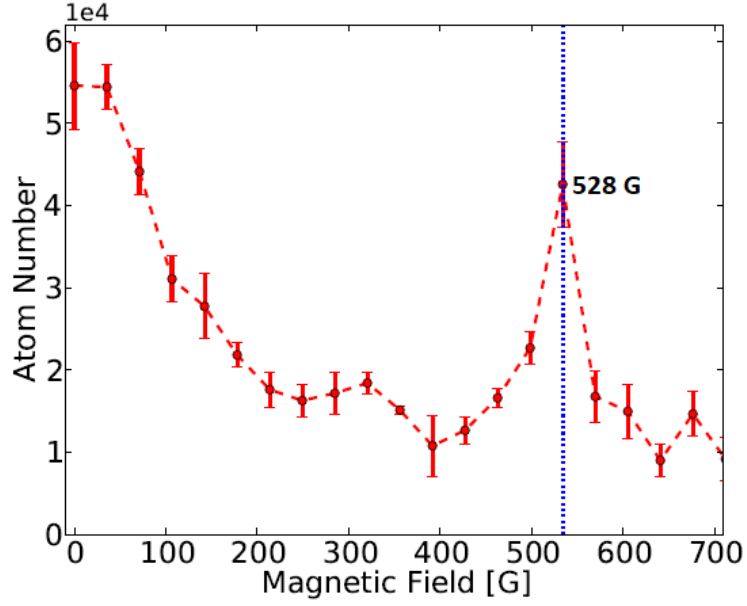


Figure 4.11: Primary measurement of Feshbach coil calibration. There is a peak in atom number after 5s of plain evaporation at the zero crossing of the Feshbach resonance. The x axis is scaled based off of this peak. Measurement taken by a previous PhD student [27].

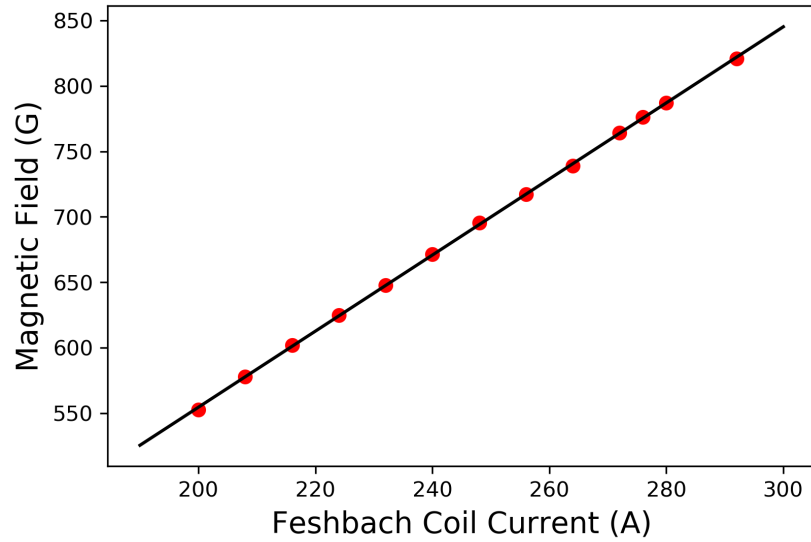


Figure 4.12: Secondary measurement of Feshbach coil calibration. The offset provided for the imaging beam by the VCO was measured and compared to the corresponding magnetic field of the Zeeman splitting of the lowest hyperfine state of ^6Li . This was done for many input currents. A linear fit is provided, with proportionality constant 2.91 G/A .

Chapter 5

Feshbach Molecule Formation

This chapter contains the primary results of this experiment, based on linear magnetic field sweeps from the BCS to the BEC side of the broad 832G resonance between the $|F = 1/2, m_F = \pm 1/2\rangle$ states in ^6Li . I will start by presenting the Landau-Zener and Power-Law approximations to molecular conversion as a function of the magnetic field sweep rate and compare them to experimental results. From that, the effect of the initial temperature on maximum conversion will be presented. Next the effect on molecular conversion of the final magnetic field of the sweep, whilst keeping the ramp rate constant, will be shown and results discussed in the context of similar studies on narrow resonances. Finally, initial theoretical studies using a zero temperature single bosonic mode model will be presented and used to interpret the results in the previous sub-section.

5.1 Models of Production Efficiency

After the evaporative cooling stages in the dipole trap are finished, around 10^5 atoms remain in the trap at temperatures between $0.5\text{-}1.5\ \mu\text{K}$. For these measurements, the atomic cloud was initially placed in a homogenous magnetic field of 860.6G, such that it lies on the BCS side of the Feshbach resonance. From there, the magnetic field was ramped linearly to a value of 707.1G on the BEC side of the resonance. This was done for different values of sweep rate, on the order of magnitude of $\dot{B} = 1\ \text{G/ms}$, in order to plot graphs of remnant atomic fraction Γ_A against inverse sweep rate. A visual schematic of these measurements, overlayed on the Feshbach resonance graph seen in Section 2.5, is presented in Figure 5.1. The remnant atomic fraction was measured by taking the difference between the atoms measured before and after the magnetic field ramp by absorption imaging.

It should be noted that this is a reasonable method of molecule detection, as the process is reversible such that the atom number is recovered after reversing the ramp back to the BCS side of the resonance. Therefore, losses do not contribute to over-estimating the remnant atomic fraction. How molecule formation in a BCS-BEC sweep depends on sweep rate is a matter that has not been fully resolved, as so far any experimental comparison to theory has only considered a single data set [43]. In this section, the current models of molecule formation will be presented and a χ_r^2 statistical test (See Appendix. A) will be performed on fits to several experimental data sets, taken at different initial cloud temperatures, in order to provide insight into which model is the better approach.

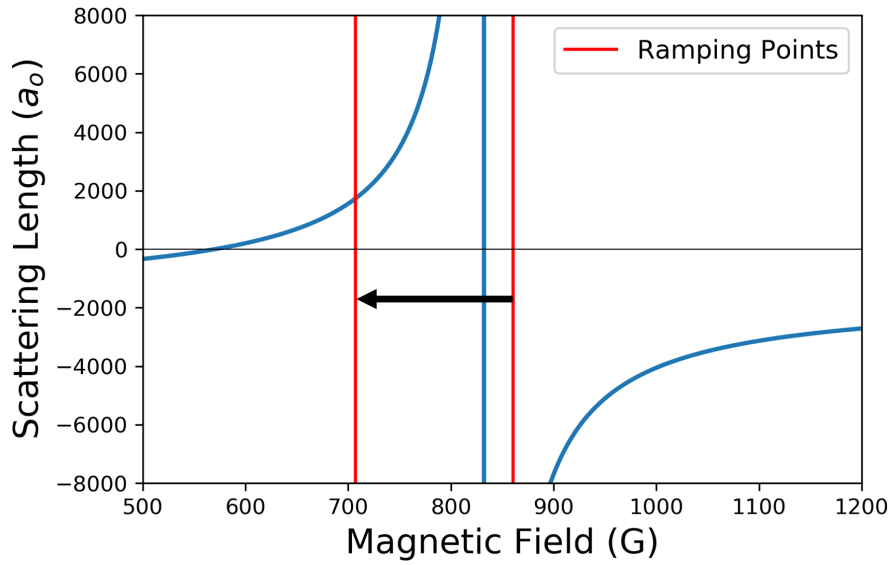


Figure 5.1: Visual schematic of the measurements taken for the molecule conversion model comparison tests. The red lines correspond to the start and end ramp points at 860.6G and 707.1G respectively.

5.1.1 Landau-Zener Approach

To derive the Landau-Zener formula for a two-level energy crossing, I will follow the original approach of C. Zener [44] and describe how it applies to magnetic field tunable interactions via a Feshbach resonance [45].

The Landau-Zener formula considers the probability of transition between a lower energy eigenstate to a higher energy eigenstate of a two-level quantum system. For this experiment it corresponds to an unbound free energy state of two atoms $|\psi_f\rangle$ and a bound molecular state of two atoms $|\psi_b\rangle$ during a contact interaction.

The wave-function for such a system can be expressed as a linear combination of the two states:

$$|\Psi\rangle = |\psi_b\rangle + |\psi_f\rangle \quad (5.1)$$

The Hamiltonian for a two-level quantum system can be accurately described by the general form of a two-channel matrix of the relative kinetic energy of the atoms given by [45]:

$$\hat{H}_{2B} |\Psi\rangle = \begin{pmatrix} -\frac{\hbar^2}{m}\nabla^2 + V_b & g \\ g & -\frac{\hbar^2}{m}\nabla^2 + V_f \end{pmatrix} \begin{pmatrix} \psi_b \\ \psi_f \end{pmatrix} \quad (5.2)$$

Here, g denotes the coupling between the bound molecular state and the unbound free atom state and V_b and V_f correspond to the potentials of the bound and free states respectively. We can therefore write the coupled system of differential equations for the two states:

$$\begin{aligned} \hat{H}_{2B} |\psi_b\rangle &= g |\psi_f\rangle + E_f |\psi_f\rangle \\ \hat{H}_{2B} |\psi_f\rangle &= g |\psi_b\rangle + E_b |\psi_b\rangle \end{aligned} \quad (5.3)$$

Where E_f and E_b are the energy eigenvalues of the free and bound molecule states. In order to derive the transition probability when crossing a Feshbach resonance, some assumptions have to be made. Firstly, the inter-channel coupling g is independent of time. Secondly, the energy separation between the eigenenergies $E_b - E_f$ varies linearly in time. This can be represented as [44]:

$$\begin{aligned} \frac{d}{dt}(E_b - E_f) &= \alpha \\ \frac{d}{dt}g &= 0 \end{aligned} \quad (5.4)$$

With α a constant. C. Zener's approach to solving this problem is to start from the form of the wave equation given by [44]:

$$\begin{aligned} \left(\hat{H}_{2B} - \frac{\hbar}{i} \frac{d}{dt} \right) [c_f(t) \exp\left(-\frac{4\pi^2 i}{\hbar} \int E_f dt\right) |\psi_f(x)\rangle \\ + c_b(t) \exp\left(-\frac{4\pi^2 i}{\hbar} \int E_b dt\right) |\psi_b(x)\rangle] = 0 \end{aligned} \quad (5.5)$$

Where $c_b(t)$ and $c_f(t)$ correspond to the probability amplitudes for the bound and free states respectively. The coupled system in Equation 5.3 can be used to reduce this equation to a system of two simultaneous equations for the free and bound

amplitudes [44]:

$$\begin{aligned}\frac{\hbar}{i} \frac{d}{dt} c_b(t) &= g \exp \left(-\frac{4\pi^2 i}{\hbar} \int (E_b - E_f) dt \right) c_f(t) \\ \frac{\hbar}{i} \frac{d}{dt} c_f(t) &= g \exp \left(\frac{4\pi^2 i}{\hbar} \int (E_b - E_f) dt \right) c_b(t)\end{aligned}\tag{5.6}$$

These equations can be solved analytically, by reducing them to a single differential equation for the bound molecule amplitude $c_b(t)$ [44]:

$$\frac{d^2}{dt^2} c_b(t) + \left[\frac{4\pi^2 i}{\hbar} (E_b - E_f) - \frac{1}{g} \frac{d}{dt} g \right] \frac{d}{dt} c_b(t) + \left(\frac{4\pi^2 g^2}{\hbar} \right) c_b(t) = 0\tag{5.7}$$

By substitution of the relationships given in the Landau-Zener approximations (Equation 5.4), this equation reduces to a form of the Weber equation, given by [44]:

$$\frac{d^2}{dt^2} U + \left(f^2 - \frac{i\alpha}{2} + \frac{\alpha^2 t^2}{4} \right) U = 0\tag{5.8}$$

Where the variable f and the function U are given by:

$$\begin{aligned}c_b(t) &= \exp \left(-\frac{2\pi^2 i}{\hbar} \int (E_b - E_f) dt \right) U \\ f &= \frac{4\pi^2 g}{\hbar}\end{aligned}\tag{5.9}$$

For an atom pair initially in the unbound state at $t = 0$, this can be explicitly solved to give the probability of the atoms to be in a bound state P_b and remaining in the free state P_f at a time $t = \infty$ as a function of the rate at which energy difference between the two levels changes and is given by [44]:

$$\begin{aligned}P_b &= |c_b(t)|^2 = \exp(-2\pi\delta_{LZ}) \\ P_f &= 1 - P_b = 1 - \exp(-2\pi\delta_{LZ})\end{aligned}\tag{5.10}$$

Where δ_{LZ} is known as the Landau-Zener parameter, given by [44]:

$$\delta_{LZ} = \frac{4\pi^2 g}{\hbar \frac{d}{dt} (E_b - E_f)}\tag{5.11}$$

In cold atom experiments in which the atom's cross a Feshbach resonance, this energy difference varies in time due to the Zeeman splitting of the hyperfine states

of the considered resonance changing with respect to the modulation of the external magnetic field (see Section 2.2). As such, the Landau-Zener Parameter is given by [45]:

$$\delta_{LZ} = \frac{4\pi n\hbar|a_{bg}||\Delta B|}{m\dot{B}} \quad (5.12)$$

Where m is the mass of the atom in the pair, n is the peak atomic density and $|a_{bg}|$ and $|\Delta B|$ have been defined in Section 2.5. Whilst this is only an exact solution for a single two-level crossing, the assumption is made that the same behaviour will be observed collectively for a group of atoms, replacing the transition probability with a remnant atomic fraction Γ_A that has an overall efficiency E_M in the limit $\dot{B} \rightarrow 0$. The remnant fraction as a function of inverse ramp rate in this approximation is given by:

$$\Gamma_A(\dot{B}^{-1}) = 1 - E_M \exp(-2\pi\delta_{LZ}) \quad (5.13)$$

In practice, the fitting for the presented data considers the Landau-Zener parameter as a free parameter. This is similar to the results presented by the group of K. Strecker [46], who performed a similar study on the narrow Feshbach resonance of Li^6 , also keeping the Landau-Zener parameter and the infinitely slow-ramp efficiency as free parameters.

5.1.2 Power-Law Approach

A competing model for molecular conversion proposed by E. Pazy et al [43] proposes that instead of the single pair approximation in the Landau-Zener model, the dynamics can be explained by a power law in the limit of large particle numbers N . The exact form of the power law depends on the initial conditions of the atom/molecule mixture undergoing the magnetic field sweep.

The derivation of this result considers a collision-less, interaction based single boson mode Hamiltonian, given by [43]:

$$\hat{H} = \sum_{k,\sigma} \epsilon_k c_{k,\sigma}^\dagger c_{k,\sigma} + E(t) b_o^\dagger b_o + g \left(\sum_k c_{k,\uparrow} c_{-k,\downarrow} b_o^\dagger + c_{k,\uparrow}^\dagger c_{-k,\downarrow}^\dagger b_o \right) \quad (5.14)$$

Here, ϵ_k corresponds to the kinetic energy of an atom with wave number k , the

$c_k/c_k^\dagger$ operators correspond to creation and annihilation operators for atoms and $b_o/b_o^\dagger$ for molecules. The subscript $\uparrow / \downarrow$ corresponds to the atom operators in each spin state. The atom-molecule coupling is given as g , and $E(t)$ refers to the time dependent molecular energy, which in the case of linear magnetic field sweeps is given by $E(t) = \frac{\Delta\mu}{\hbar}(B(t) - B_o)$.

The assumption is made that a degenerative model can be used, as the exact values of ϵ_k do not affect the conversion efficiency. The following operators are defined [43]:

$$J_- = \frac{b_o^\dagger \sum_k c_{k,\uparrow} c_{-k,\downarrow}}{(N/2)^{3/2}}, J_+ = \frac{\sum_k c_{-k,\downarrow}^\dagger c_{k,\uparrow}^\dagger b_o}{(N/2)^{3/2}}, J_z = \frac{\sum_{k,\sigma} c_{k,\sigma}^\dagger c_{k,\sigma} - 2b_o^\dagger b_o}{N} \quad (5.15)$$

Where N corresponds to the total atom number. The J_x and J_y operators are also defined as $J_x = J_+ + J_-$ and $J_y = -i(J_+ - J_-)$. The Hamiltonian in Equation 5.14 in this basis is given as [43]:

$$\hat{H} = \frac{N}{2} \left(\Delta(t) J_z + g \sqrt{\frac{N}{2}} J_x \right) \quad (5.16)$$

The group used these operators in the Heisenberg picture $-i\dot{A} = [A, \hat{H}]$ to produce equations of motion for J_x, J_y and J_z . These operators are then replaced by their expectation values, corresponding to the mean field limit of the equations of motion. In this limit, the set of Equations 5.15 are analogous to Bloch equations on a two-dimensional surface. The details of how these are solved are beyond the scope of this thesis, therefore the limit in the two extremities of initial conditions of the gas before the magnetic field sweep will be presented.

If the initial molecular population is lower than the quantum noise level, fluctuations dominate the conversion process and the remnant atomic fraction Γ_A is given by [43]:

$$\Gamma_A \propto \dot{B} \quad (5.17)$$

If quantum fluctuations can be neglected, it is expected that the remnant atomic fraction will follow a cubic in the field sweep rate. This can be written as [43]:

$$\Gamma_A \propto \dot{B}^{1/3} \quad (5.18)$$

It is expected for the molecule formation to follow a power law beyond a critical value of the sweep rate. For ${}^6\text{Li}$ this corresponds to values slower than $\dot{B}^{-1} = 0.5 \text{ ms/G}$ [43]. As a result of the effect of pre-formed pairs on the BCS side of a Feshbach resonance in fermionic systems being an open question [47], the fitting to the data presented keeps the exponent of the Power-Law as a free parameter.

5.1.3 Comparison to Experiment

As mentioned previously, to make a quantitative comparison of the two models presented here, around 3×10^5 atoms are loaded into an optical dipole trap at 860.61G. The magnetic field is then ramped to 707.1G at different ramp rates. The remnant atomic fraction is measured by comparing an absorption image of the cloud before and after ramping. This has been done for five initial cloud temperatures in the range $0.5 - 2 T/T_F$, in order to produce graphs of remnant atomic fraction versus inverse magnetic field sweep rate. The five data sets, as well as a comparison to the data from the study on the narrow resonance of ${}^6\text{Li}$ from the group of K. Strecker [46] is presented in Figure 5.2, with accompanying Landau-Zener and power-law best fits. The data for the calculated exponents of the Power-law best fits can be found in Table 5.1.3. The mean value of the exponents of the separate data sets are consistent with the lower-bound of the Power-law model predictions, but many were significantly lower, suggesting a breakdown of the model for molecule formation in this experiment.

The best-fit curves in this plot are produced via a least-squared method. As mentioned in Sections 5.1.1 and 5.1.2, both the overall efficiency and Landau-Zener parameter as well as the exponent of the power law are taken as free parameters. The significance of the derived values is discussed in later sections. Similar graphs have been produced for five different initial temperatures. A reduced χ_r^2 test was performed on the two fittings in each case (See Appendix B) and the results of this are presented in Figure 5.3. In general, the Landau-Zener model provided a better or equally good fit to the Power-law model. Whilst this does not conclusively rule out the Power-law model, it adds to the findings of K. Strecker that could not differentiate between the two models, due to the consideration of only a single data set [46][43]. It is also evident from observing the graphs that at the tail of the fit, corresponding to the slow ramp regime, that the Power-law model looks to break down as the molecule conversion appears to saturate. Future work could include taking data for slower BCS-BEC ramps, with a view to more clearly resolve

the saturation of molecule production and rule out the Power-Law conclusively.

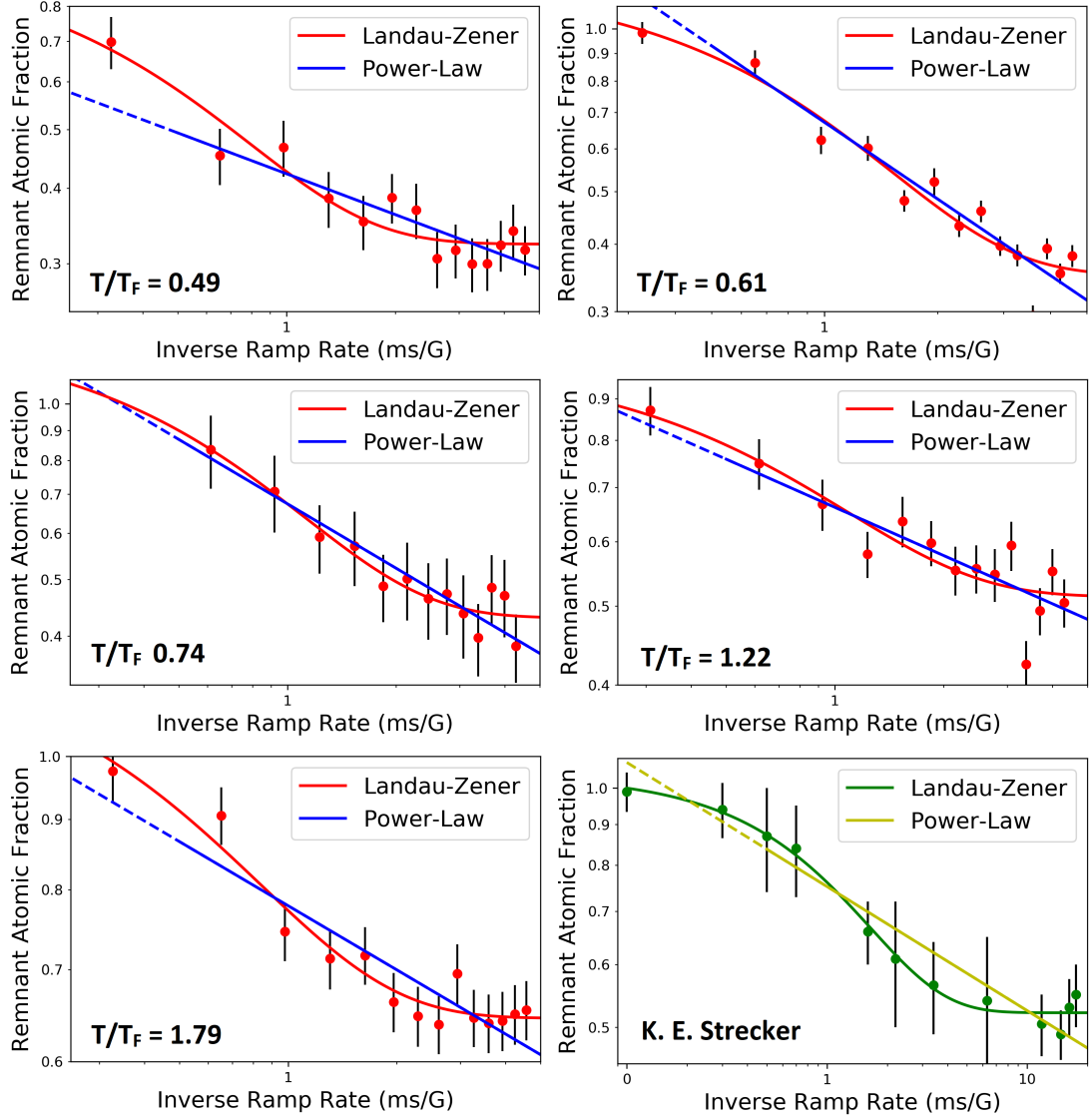


Figure 5.2: The five data sets of Remnant Atomic Fraction versus Inverse Ramp rate for a linear BCS-BEC sweep in the broad 832G resonance in ^6Li used for the molecule formation model comparison, including the relevant model fits. The initial cloud temperature is noted at the bottom left inset of the graphs. The dotted line in the Power-law model corresponds to extrapolating the model beyond 0.5 ms/G. The data set from the group of K. Strecker [46] from the narrow resonance in ^6Li at 543G can be found in the bottom right figure.

T/T_F	Power-Law Exponent
0.49	0.224
0.61	0.471
0.74	0.366
1.22	0.197
1.79	0.154
Average	0.282 ± 0.053

Table 5.1: Calculated exponents for the Power-law fitting presented in Figure 5.2. The average value between the data sets is consistent with the lower estimate of the scaling relationship. The error in the average value corresponds to the standard error between the five results.

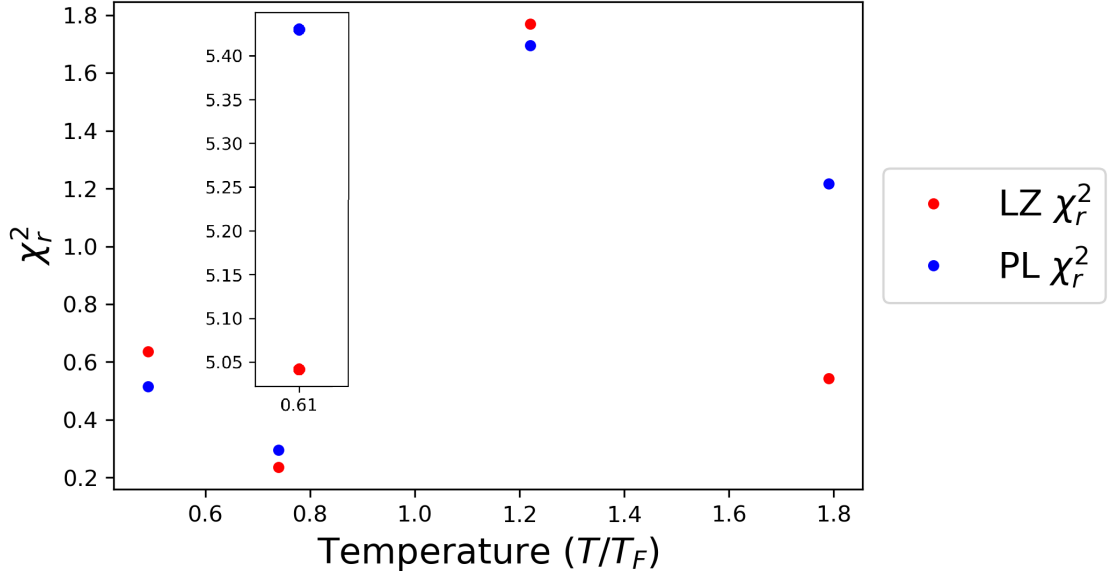


Figure 5.3: Reduced χ_r^2 values for the Landau-Zener and Power-law models of Molecule production in a Fermi gas for different initial cloud temperatures. In each case, the Landau-Zener approximation returned a better or equally as good fit.

5.2 Temperature Dependence of Production

During molecule formation, thermal effects are expected to play a significant role. To test this for the five data sets that were used for the molecule formation model comparison, the molecular fraction in the quasi-adiabatic limit of the Landau-Zener model was taken. This corresponds to large values of $\dot{B}^{-1}$, or the limit of an infinitely slow magnetic field sweep. Referring to Equation 5.13, the value of E_M was fit using origin software for all data sets in the region of $0.5 - 2 T/T_F$. A graph of the data sets with corresponding fits, as well as the corresponding values of molecular fraction as a function of T/T_F is shown in Figure 5.4.

The fitted values for molecular fraction have been compared to a semi-classical model. It treats the ultra-cold mixture of atoms and molecules as a classical canonical ensemble in chemical equilibrium at finite temperature T , corresponding to the quasi-adiabatic limit of a BCS-BEC sweep, with partition function [48]:

$$Z = \frac{Z_s^{2N_a} Z_s^{N_m} \exp(N_m E_b / k_B T)}{N_a! N_a! N_m!} \quad (5.19)$$

Where Z_s corresponds to a single particle partition function, N_a and N_m are the respective atom and molecule numbers in the gas and E_b is the binding energy of the molecules in the ensemble. Whilst outside the scope of this thesis, by minimizing the Helmholtz free energy $F = k_B T \ln(Z)$ an implicit expression can be derived for the molecular fraction M_F , given by [48]:

$$\frac{(1 - M_F)^2}{M_F} = 6 \left(\frac{T}{T_F} \right)^3 \exp \left(\frac{E_b}{k_B T} \right) \quad (5.20)$$

Here T_F corresponds to the Fermi temperature in a harmonic trap (See Section 3.3.2). As the data sets correspond to different initial atom numbers prior to the sweep, an exact curve fit cannot be applied to this data, but a point by point comparison can be made. As can be seen in Figure 5.5, the trend of this classical model has good agreement but quantitatively differs at colder temperatures due to the assumption of a classical canonical ensemble breaking down.

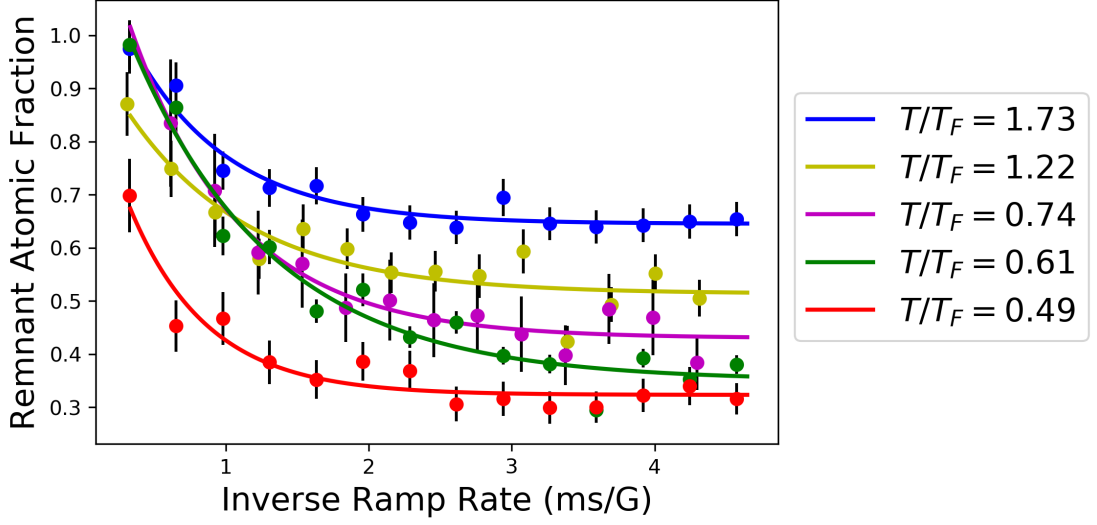


Figure 5.4: Collated graph of Remnant atomic fraction versus Inverse Ramp Rate for BCS-BEC magnetic field sweeps for different values of initial cloud temperature. A fit of the Landau-Zener model has been applied to each data set.

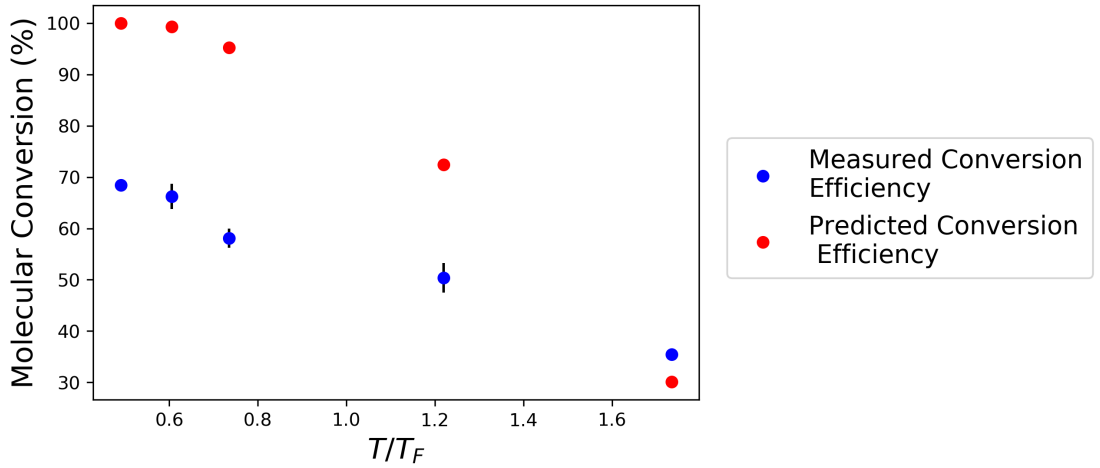


Figure 5.5: Extrapolated value of Molecule Conversion percentage of the quasi-adiabatic limit of the Landau-Zener model versus T/T_F . For each point, the value for the classical model presented in Section 5.2 has been plotted.

5.3 Effect of Final Field on Molecule Formation

In this section, the effect of the final field of a linear BCS-BEC sweep on molecule formation will be explored. This goes beyond Landau-Zener like predictions, as such a model detailed in Section 5.1.1 does not depend on the initial or final magnetic field, rather just the rate of change of energy between the two channels. Various attempts have been proposed to address this issue, both classical and fully quantum models [49][50][51]. These approaches have been fitted to data for narrow resonances in Fermi gases, including the narrow resonance between the two hyperfine states of ^6Li that are presented in this thesis [46]. Comparisons of such models to BCS-BEC sweeps in broad resonances have so far not been done.

Similar to the previous section, an atomic cloud of ^6Li in an equal mixture of the two hyperfine states $|F = 1/2, m_F = \pm 1/2\rangle$ was produced at an initial magnetic field of 860.6G. For these measurements, the initial temperature of the gas was kept constant at $1.5 \mu\text{K}$. The magnetic field was swept to different values on the BEC side of the broad Feshbach resonance at 832G, for a constant ramp rate, and the molecular fraction was measured by taking absorption images of the gas before and after the magnetic field sweep. A graphic illustrating these measurements can be found in Figure 5.6.

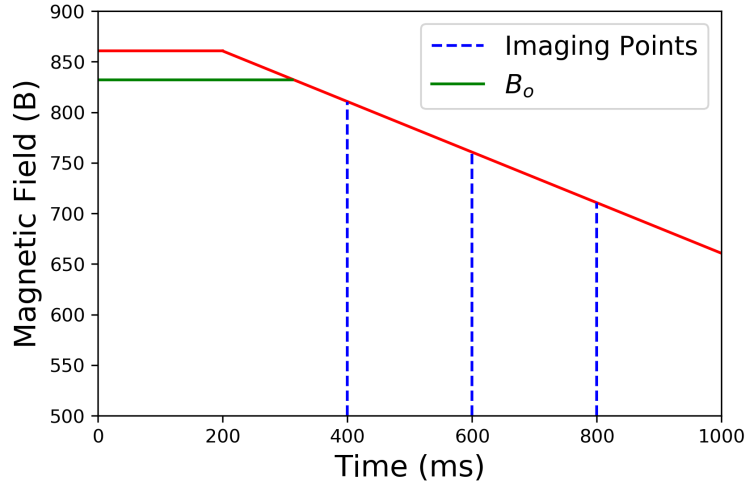


Figure 5.6: Visual schematic of the “Final Field” measurements of molecule conversion in BCS-BEC sweeps. The imaging points displayed do not represent the imaging points used in the measurements displayed in Figure 5.7. Here, B_o refers to the Feshbach resonance position of the broad resonance utilized in this experiment.

I will initially focus on the thermodynamical approach proposed by J. E. Williams

et al [49], as this is the most commonly used analytical approach to fitting such measurements [46][52][53][54]. In contrast to Landau-Zener like models where the association dynamics are assumed to be governed by a single collision between nearest-neighbour atoms, an assumption in their model is that on average an atom will collide with other atoms multiple times during a BCS-BEC sweep. The average time between collisions τ_{col} can be approximated by $\tau_{col}^{-1} \approx n\sigma\bar{v}_{rel}$ where $\bar{v}_{rel} = 4\sqrt{k_B T_F / m\pi}$ is the average relative velocity between atoms in the gas. In the unitary limit the collisional cross section $\sigma = 4\pi a^2 / (1 + k^2 a^2)$, which at the Feshbach resonance position where a diverges, can be approximated by $\sigma = 4\pi / k_F^2$. Approximating the atomic density as $n = (2mk_B T_F / \hbar^2)^{3/2} / 6\pi^2$, the average collision time becomes $\tau_{col} = \hbar / k_B T_F$. For these measurements, with a typical Fermi temperature of $T_F = 1.5 \mu K$, the average collisional time will be on the order of microseconds, which is significantly lower than the typical time of the field sweep over the broad resonance. It is therefore a good approximation that an individual atom will have multiple collisions during molecule formation.

A simple model for the rate of molecule formation is chosen, based on a formation rate γ_f and a dissociation rate γ_d :

$$\frac{dN_M}{dt} = \gamma_f N_A(t) - \gamma_d N_M(t) \quad (5.21)$$

Here, N_M and N_A are the molecule and atom populations respectively. The formation and dissociation rates follow from collisional Boltzmann equations for an atom-molecule mixture and are given by [49]:

$$\begin{aligned} \gamma_d = \frac{4\pi^2 g^2}{\hbar^7 n_M} \int dp_1 \int dp_2 \int dp_3 \delta(p_1 + p_2 - p_3) \\ \times \delta(E_A(p_1) + E_A(p_2) - E_M(p_3)) f_M(p_3) \end{aligned} \quad (5.22)$$

$$\begin{aligned} \gamma_f = \frac{4\pi^2 g^2}{\hbar^7 n_A} \int dp_1 \int dp_2 \int dp_3 \delta(p_1 + p_2 - p_3) \\ \times \delta(E_A(p_1) + E_A(p_2) - E_M(p_3)) f_A(p_1) f_A(p_2) \end{aligned} \quad (5.23)$$

Here, E_A and E_B are the relative energies of the atom and molecule states, given by $E_A = p^2/2m + U$ and $E_M = p^2/4m + 2U + E_{res}$ where E_{res} is the closed channel energy of the Feshbach resonance and U is the external trapping potential. The n_M and n_A terms correspond to the molecular and atomic densities respectively. In general, these equations cannot be solved without knowing the specific form of the distribution functions. In the case of the mixture being in chemical equilibrium,

where the distribution functions are defined as $f_j = \exp(-(E_j - \mu_j)/k_B T)$, the equations can be solved explicitly, and are given as [49]:

$$\begin{aligned}\gamma_d &= \frac{2m^{1/2}}{\hbar^2} a_{bg} \Delta\mu \Delta B \sqrt{E_{\text{res}}} \Theta(E_{\text{res}}) \\ \gamma_f &= \frac{2^{3/2} n \hbar^3}{(2\pi m k_B T)^{3/2}} \exp(-E_{\text{res}}/k_B T) \gamma_d\end{aligned}\tag{5.24}$$

Where $\Delta\mu$ is the difference in magnetic moment between the unbound and bound states. The Θ term corresponds to a unit step function and ensures that when E_{res} is tuned below zero, the molecules become stable and cannot dissociate. The form of E_{res} depends on how far from the resonance position the atomic cloud is held. Far from the resonance, E_{res} is given as $E_{\text{res}}(B) = \frac{\Delta\mu}{\hbar}(B - B_o)$ and close to the resonance this can be approximated as $E_{\text{res}} \approx \hbar^2/ma^2$. Other parameters are defined in Section 2.5. The dissociation rate is analogous to the form of molecule decay derived via Fermi's golden rule (See Appendix C).

How the molecular fraction of the gas evolves over time can be solved analytically, starting with a purely atomic sample, but is well approximated by an error function [49]. Data that have been taken have been fitted to the equation:

$$M_F(B) = M_o(1 - \text{erf}[\sqrt{2}(B - B_{\text{cen}})/\delta B]) + y_o\tag{5.25}$$

Which has also been fitted well to previous studies [52][53]. An offset y_o was added to account for any non-equilibrium effects at the resonance position. It should be noted that the centre of the error function B_{cen} in this model does not coincide with the resonance position B_o , rather is given by [49]:

$$B_{\text{cen}} = B_o + \Delta B a_{bg} \sqrt{m k_B T_F} / \hbar\tag{5.26}$$

As mentioned previously, starting with a ^6Li cloud in an equal mixture of $|F = 1/2, m_F = \pm 1/2\rangle$ states at a temperature of $1.5 \mu\text{K}$, the magnetic field was ramped from 860G on the BCS side of the broad Feshbach resonance to different values on the BEC side of the resonance. The molecule conversion percentage was measured by taking an absorption image before and after the ramp. It is assumed that due to the bound molecules binding energy, the imaging beam described in Section 4.2 no longer is able to excite the imaging transition, therefore the atoms imaged are simply the ones that did not form molecules. This has been verified by reversing the sweep to the BCS side, where the original atom number is recovered

therefore no losses occur. This measurement was carried out for three different ramp rates corresponding to $\dot{B}^{-1} = 0.6, 1$ and 4 ms/G respectively. The results of this measurement are displayed in Figure 5.7.

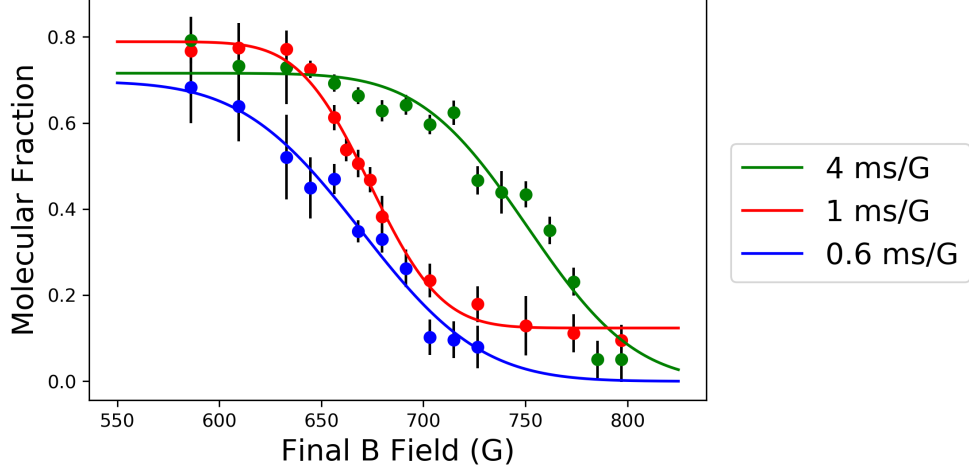


Figure 5.7: Molecular Fraction versus Final Magnetic Field of a BCS-BEC sweep starting from 860G. The data is fitted with the error function in Equation 5.25. For all data points the temperature was kept constant at $1.5 \mu\text{K}$. The error in the data corresponds to the standard error of ten repetitions.

As expected, due to the initial cloud temperature remaining constant, the three data sets converge at a similar maximum molecular fraction. This is slightly larger than the molecular fractions reported at similar temperatures in the previous section, due to the fact that Landau-Zener like models do not consider the end points of the magnetic field ramp. It is observed that for even the quasi-adiabatic case ($\dot{B}^{-1} = 4$ ms/G) that the converted molecular fraction saturated at lower fields than was considered in Section 5.1.3.

A key result of this measurement is that the magnetic field at the central point of the fitted error function appears to decrease with increased ramp rate. From Equation 5.26, variations in Fermi temperature between the three measurements can only account for a 7G difference in B_{cen} , far less than the observed difference. This is in contrast to studies on narrow Feshbach resonances in Lithium and Potassium that have been fit with the same model, where both the width and central position of the error function coincides with the values for the Feshbach resonance [46][52][53][54]. However, it could be that the dynamics presented here are unresolvable on the order of fractions of gauss, corresponding to the width of such narrow resonances, and the shifting of the central position of the curve required a measurement over a broad resonance allowing for longer sweep durations.

5.4 Initial Theory Work for Molecule Formation

In the previous section, some anomalous results on molecule formation that cannot be explained with current semi-classical approaches was presented. In this section, I will show the initial work on providing a simulation of the atom-molecule dynamics in a BCS-BEC sweep and, when using dummy variables, what insight can be had on how the rate at which the magnetic field is swept affects molecule formation. Starting from a formalism of the collision-less, single boson mode BCS Hamiltonian, given by:

$$\hat{H}_{BCS} = \delta(t)b^\dagger b + \sum_j \epsilon_j (a_{j,\downarrow}^\dagger a_{j,\downarrow} + a_{j,\uparrow}^\dagger a_{j,\uparrow}) + g \sum_j b^\dagger a_{j,\downarrow} a_{j,\uparrow} + b a_{j,\downarrow}^\dagger a_{j,\uparrow}^\dagger \quad (5.27)$$

Where $\delta(t)$ is the atom-molecule energy difference given by $\delta(t) = \Delta\mu(B(t) - B_o)/\hbar$. The ϵ_j term corresponds to the energy of an oscillator of level j , ie $\epsilon_j = (j + 1/2)\bar{\omega}$ where $\bar{\omega}$ is the mean geometric frequency of the dipole trap. The atom-molecule coupling is given by g and assumed to not vary in time. The creation and annihilation operators for molecules and atoms are given by $b/b^\dagger$ and $a/a^\dagger$ respectively. For the atom operators, the $\uparrow / \downarrow$ terms correspond to the operators for the $m_F = \pm 1/2$ states in ^6Li . To simplify the problem, the Anderson Pseudo Spin operators are defined as [55]:

$$\begin{aligned} S_j^+ &= a_{j,\downarrow}^\dagger a_{j,\uparrow}^\dagger \\ S_j^- &= a_{j,\downarrow} a_{j,\uparrow} \\ S_j^z &= 1/2(a_{j,\downarrow}^\dagger a_{j,\downarrow} + a_{j,\uparrow}^\dagger a_{j,\uparrow} - 1) \end{aligned} \quad (5.28)$$

Which have commutation relations $[S_j^+, S_j^-] = 2S_j^z$, $[S_j^+, S_j^z] = -S_j^-$ and $[S_j^-, S_j^z] = S_j^-$. Substitution of Equations 5.28 into Equation 5.27 leads to:

$$\hat{H}_{BCS} = \delta(t)b^\dagger b + 2 \sum_j \epsilon_j S_j^z + g \sum_j b S_j^+ + b^\dagger S_j^- \quad (5.29)$$

In this formalism, the total particle number is conserved as $N = 2b^\dagger b + \sum_j S_j^z + 1$. The pseudo spin operators in Equation 5.28 can be substituted into the Heisenberg relation $i\dot{S}_j^n = [S_j^n, \hat{H}]$ to give a set of coupled differential equations for the time evolution of the spin and molecule creation operators:

$$\begin{aligned}
 i\dot{b} &= \delta(t)b + g \sum_j S_j^- \\
 i\dot{S}_j^- &= 2\epsilon_j S_j^- - 2gb S_j^z \\
 i\dot{S}_j^+ &= -2\epsilon_j S_j^+ + 2gb^\dagger S_j^z \\
 i\dot{S}_j^z &= g(b S_j^+ - b^\dagger S_j^-)
 \end{aligned} \tag{5.30}$$

To solve these equations for a system of BCS pairs, each pair is placed initially in the same state, $S^+ = S^- = 0, S^z = 1/2$, corresponding to one atom in each of the two hyperfine states. The atomic ensemble is treated as a zero temperature gas, such that the atoms fill the Fermi sea with two atoms in each oscillator state up to the Fermi energy. The differential equations are then solved via a fourth order runge-kutta method for each BCS pair. These simulations are numerically intensive and as such will only be presented for a few atoms. A way to interpret this time evolution is that when the energy difference $\delta(t)$ passes the energy of a filled oscillator state, the atoms have an increased probability of forming a Feshbach molecule. This can be seen in the limiting case of small coupling g and slow magnetic field sweep rates, as there will be a ‘stepping’ in molecule formation. A simulation of the differential equations in Equation 5.30 with unphysical variables in this limit for 5 atom pairs is presented in Figure 5.8:

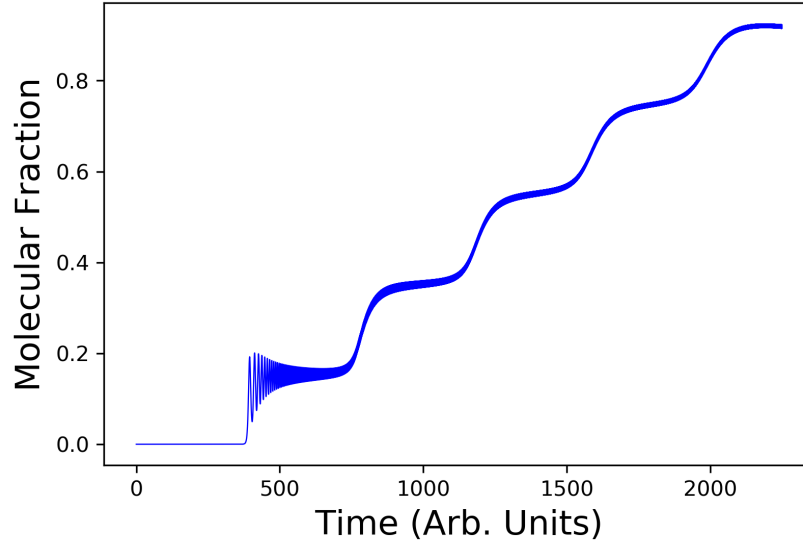


Figure 5.8: Molecular fraction versus time for a simulation of Equations 5.30. Here $g = 0.2$, $\bar{\omega} = 2$, $\gamma = 0.01$ and $\delta(0) = -1$. A stepping in molecule formation when the atom-molecule energy difference passes a filled oscillator state is observed.

Here, the atom molecule energy difference is represented as a linear sweep $\delta(t) =$

$\delta(0) + \gamma t$. The molecule fraction stepping happens at regular intervals, corresponding to the linear sweep of filled oscillator states.

As a result of the limitations of this model, the fermionic ensemble being at zero temperature and the difficulty in running a simulation for large numbers of atoms, an exact simulation of the molecule formation dynamics presented in Section 5.3 is impossible. However, by performing simulations on small atom numbers N , the hypothesis of sweep rate being a determining factor in the centre of the formation curve can be tested for unphysical parameters.

The simulation was ran for varied sweep rates in a parameter regime where the stepping in molecule formation does not occur, the results of which can be seen in Figure 5.9. It is evident from these simulations that even a small change in sweep rate causes a shift in the molecule production as function of time. Whilst this formalism cannot replicate an exact fit of Figure 5.7, these simulations provide motivation for future work to extend this to finite temperature, for example in the spirit of J. Javanainen et al, where finite temperature enters via the fermi distribution of the atoms [50].

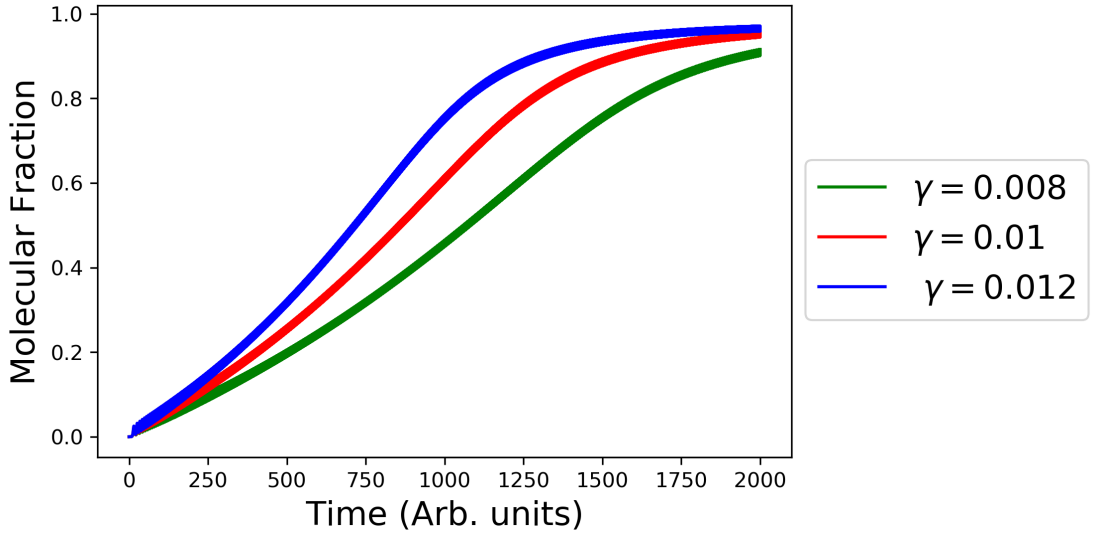


Figure 5.9: Molecular Fraction versus time for a simulation of Equations 5.30, with varied sweep rate. The other parameters were kept constant as $N = 12$, $g = 0.5$, $\omega = 0.5$, $\delta(0) = 1$. A shift in the molecule formation is observed for small variations in sweep rate.

Chapter 6

Conclusions and Outlook

The BCS-BEC crossover in fermionic atoms, and the production of Feshbach molecules, provides an interesting and useful resource in ultracold experiments. In this thesis, it has been shown how the production of Feshbach molecules in an ultracold fermionic ^6Li system during a linear BCS-BEC magnetic field ramp is affected by factors such as sweep rate, final magnetic field and temperature. An extensive test, on data sets at varying initial cloud temperatures, of the competing models of the molecule production efficiency dependence on field sweep rate has been presented and the quasi-adiabatic limit of the Landau-Zener model investigated as a function of initial cloud temperature. The effect of the final magnetic field of linear sweeps has been probed as a function of different constant sweep rates and the anomalous result that the rate of the BCS-BEC magnetic field sweep affects the position of formation have been presented in comparison to similar studies on narrow resonances. Finally, the initial work in the group on providing a simulation of the atom-molecule dynamics, in an attempt to explain the previous measurement, has been presented.

The experimental sequence of the Lithium experiment at Nottingham has also been presented. The experiment is now routinely capable of loading 10^8 atoms into a Magneto-Optical trap and transferring 10^6 atoms into a far-red detuned Optical Dipole trap. Through evaporative cooling techniques, the atomic cloud can be reduced to temperatures of hundreds of nano-kelvin whilst retaining 10^5 atoms in the trap. Whilst not shown in this thesis, as the interest was in a regime where thermal effects dominate, in principle this experiment has been shown in previous work to be capable of achieving Bose-Einstein condensation of the produced Feshbach molecules.

For the next steps of this experiment, it would be useful to implement a direct detection scheme for the molecules as for now none such exists. This has been implemented in other groups in a variety of ways, for example via Stern-Gerlach methods of separating the atomic and molecular clouds or by transferring the unpaired atoms to an excited state before taking an absorption image of the molecular cloud. Another future implementation to the experimental setup will be to replace the current AV Manta G-235B camera with an EMCCD camera from ANDOR, which will allow for more sensitive imaging of the atomic cloud.

Prospects for future study with this system include realization of the so called “Feshbach Engine”, a quantum engine wherein the modulation of the interaction strength of a BEC, controlled by a Feshbach resonance, is used to form an Otto cycle [10]. Another possibility is to measure the Dynamical Casimir Effect (DCE) in a BEC, as it has been hypothesized that modulation of trapping potential of a uniform BEC will induce phononic excitations, also having implications for possibility of measuring gravitational waves [56][11]. Both of which require the experiment to operate in the temperature region of mBEC formation. In the short term, an extension of the initial theory work presented in Section 5.4 to finite temperature will be the final step before publication of the results.

Appendix A - Temperature Measurement of Dipole Trap after Forced Evaporation

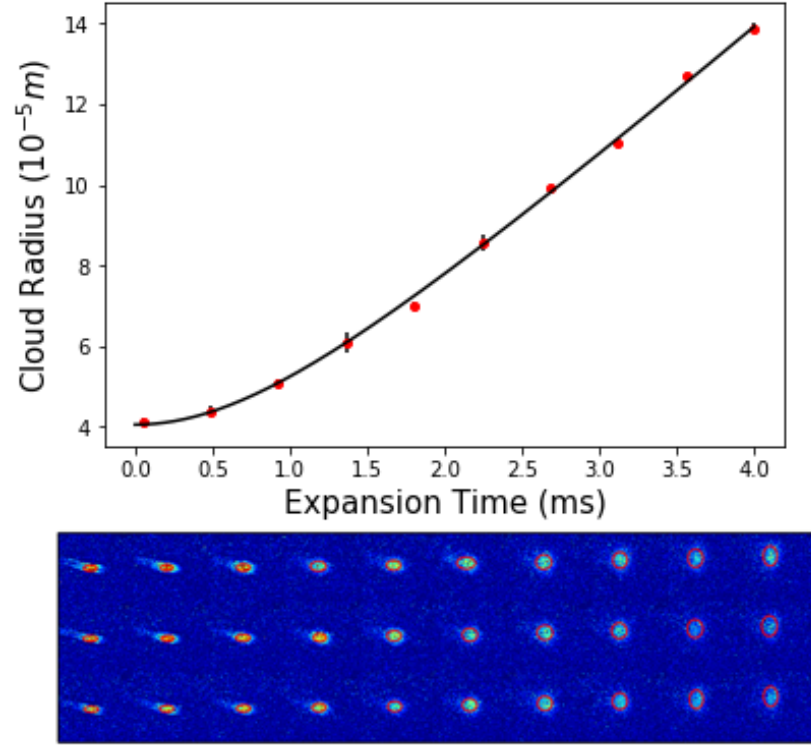


Figure 1: Cloud radius versus Time after release from the Dipole trap after forced evaporation. The TOF method of fitting has been used to determine the best fit of the cloud temperature $T = 928$ nK. The error bars correspond to the standard error of three repetitions. Below are the absorption images of the cloud for the three repetitions.

Appendix B - Reduced χ^2 Test

To test models of molecule production efficiency M_F , the models have been fitted to experimental data using Origin software that utilizes Least-Squares methods to minimize the goodness of fit parameter χ^2 . This has been done for multiple data sets for each model to determine which provides more accurate predictions. The χ^2 value for a given fit to a data set is given by [57]:

$$\chi^2 = \sum_i \frac{(y_i - y(x_i))^2}{y_{err,i}^2} \quad (1)$$

Here the y_i values correspond to the experimental data points, $y(x_i)$ values correspond to the value for the experimental point that the model predicts and the $y_{err,i}$ values correspond to the associated experimental error for the data points. The least-squares method uses algorithms to minimize this value for a given test model. This value alone does not give a good estimate on how accurate a model is, as it is dependent on the error in the experimental data, but can be used to compare how well a model fits experimental data when compared to another. For models that fit data with different numbers of fitting parameters, as well as that fit different numbers of data points, similar to in this experiment, it is more useful to define the reduced chi-squared χ_r^2 , given by:

$$\chi_r^2 = \frac{\chi^2}{\nu} \quad (2)$$

Where ν corresponds to the degrees of freedom of a model, the difference between the number of observations and the amount of fitted parameters. This value can be used to test the goodness of fit of the models to experimental data, with values closer to one corresponding to lower variance in the regression. Values less than one suggest over-estimation of the error variance and values above one show under-estimation of the error variance of the data.

Appendix C - Derivation of Molecule Dissociation rate via Fermi's Golden Rule

As an extension to the derivation of molecule dissociation rate proposed by J. E. Williams et al presented in Chapter 5.3, I will present a more intuitive derivation of the molecule decay rate based on Fermi's golden rule. The decay rate for a bound system of two atoms is given by the general form of Fermi's golden rule:

$$\hbar\gamma_d = 2\pi|\chi(E_{res})|^2 D(E_{res}) \quad (3)$$

Where γ_d is the molecule decay rate defined in Chapter 5.3. The $|\chi(E_{res})|$ term corresponds to the matrix element between the unbound atomic and bound molecular states and $D(E_{res})$ corresponds to the density of states of the system. For a three-dimensional gas, the density of energy states is given by:

$$D(E_{res}) = \frac{V}{(2\pi^2)} \frac{m^{3/2}}{\hbar^3} E_{res}^{1/2} \quad (4)$$

Here, V is the volume element for free atomic states. The atom-molecule matrix element is found by computing the energy shift of the atomic states due to coupling to a bound state in the vicinity of a Feshbach resonance. To first order, this is given by [58]:

$$\delta(E_{res}) = \frac{|\chi(E_{res})|^2}{\Delta\mu(B - B_o)} \quad (5)$$

Where $\Delta\mu$ and B_o are defined in Chapter 2.5. This can also be expressed in terms of the mean field energy of an atomic cloud held near a Feshbach resonance. in this form, the energy shift is given by [58]:

$$\delta(E_{res}) = \frac{4\pi\hbar^2}{mV} a_{bg} \Delta\mu \Delta B \quad (6)$$

Comparing Equations 5 and 6 yields an expression for the matrix element:

$$|\chi(E_{res})|^2 = \frac{4\pi\hbar^2}{mV} a_{bg} \Delta\mu \Delta B \quad (7)$$

Substitution of Equations 4 and 7 into Fermi's Golden rule gives the expression

for the molecule decay rate:

$$\gamma_d = \frac{2a_{bg}\Delta\mu\Delta Bm^{1/2}}{\hbar^2}E_{res}^{1/2} \quad (8)$$

Which is directly analogous to the form of molecule decay derived via coupled Boltzmann equations shown in Section 5.3.

Bibliography

- [1] H. K. Onnes - On the sudden rate at which the resistance of mercury disappears - *Akad. van Wetenschappen* - 14(113), 818 (1911)
- [2] S. Chu, L. Hollberg, J. E. Bjorkholm, A. Cable and A. Ashkin - “Three-dimensional viscous confinement and cooling of atoms by resonance radiation pressure” - *Phys. Rev. Lett.* **55**, 48 (1985)
- [3] E. L. Raab, M. Prentiss, A. Cable, S. Chu, and D. E. Pritchard - “Trapping of Neutral Sodium Atoms with Radiation Pressure” - *Phys. Rev. Lett.* **59**, 2631 (1987)
- [4] K. B. Davis, M. Mewes, Michael A. Joffe, Michael R. Andrews, and W. Ketterle - “Evaporative Cooling of Sodium Atoms” - *Phys. Rev. Lett.* **75**, 2909 (1995)
- [5] E. Tiesinga, B.J. Verhaar, and H. T. C. Stoof - “Threshold and resonance phenomena in ultracold ground-state collisions” - *Phys. Rev. A* **47**, 4114 (1993)
- [6] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, E. A. Cornell - “Observation of Bose-Einstein Condensation in a Dilute Atomic Vapor” - *Science* **269**, 5221 (1995)
- [7] K.B. Davis, M. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle - “Bose-Einstein Condensation in a Gas of Sodium Atoms” - *Phys. Rev. Lett.* **75**, 3969 (1995)
- [8] C.C. Bradley, C. A. Sackett, J.J. Tollett, and R. G. Hulet - “Evidence of Bose-Einstein Condensation in an Atomic Gas with Attractive Interactions” - *Phys. Rev. Lett.* **75**, 1687 (1995)
- [9] M. W. Zwierlein, C. A. Stan, C. H. Schunck, S. M. F. Raupach, S. Gupta, Z. Hadzibabic, and W. Ketterle - “Observation of Bose-Einstein Condensation of Molecules” - *Phys. Rev. Lett.* **91**, 250401 (2003)

Bibliography

- [10] J. Li, T. Fogharty, S. Campbell, X. Chen and T. Busch - “An efficient non-linear Feshbach engine” - *New. J. Phys.* **20**, 015005 (2018)
- [11] C. Sabin et al - “Phonon creation by gravitational waves” - *New J. Phys.* **16**, 085003 (2014)
- [12] Jonothan Nute - “A Quantum Integrated Light and Matter Interface” - Doctoral Thesis - University of Nottingham (2017)
- [13] Daniele Baldolini - “Insitu measurements of strongly interacting Lithium system” - MSc (by Research) Thesis - University of Nottingham (2017)
- [14] W. D. Phillips and H. Metcalf - “Laser Deceleration of an Atomic Beam” - *Phys. Rev. Lett.* **48**, 596 (1982)
- [15] C. Cohen-Tannoudji, B. Diu and F. Laloe - Quantum Mechanics, Volume 1 and 2 - John Wiley & Sons - Paris (1992)
- [16] C. J. Foot - Atomic Physics - Oxford University Press - Oxford (2005)
- [17] M. E. Gehm - “Properties of ^6Li ” - <https://www.physics.ncsu.edu/jet/techdocs/pdf/PropertiesOfLi.pdf> (2003)
- [18] G. K. Woodgate - Elementary Atomic Structure, Second Edition - Oxford university Press (1980)
- [19] M. E. Gehm - “Preparation of an optically-trapped degenerate Fermi gas of ^6Li : Finding the route to degeneracy” - Doctoral Thesis - Duke Univesity (2003)
- [20] L. D. Landau and E. M. Lifshitz - Quantum Mechanics Non-relativistic theory, Second Edition - Pergamon Press - Oxford (1965)
- [21] J. Dailibard - “Collisional dynamics of ultra-cold atomic gases” - *Proceedings of the International School of Physics "Enrico Fermi"*, Volume 140 (1999)
- [22] A. Derevianko, J. F. Babb, and A. Dalgarno - “High-precision calculations of van der Waals coefficients for heteronuclear alkali-metal dimers” - *Phys. Rev. A.* **63**, 052704 (2001)
- [23] C. Chin - “A simple model of Feshbach molecules” - arXiv:cond-mat/0506313v2 (2005)

Bibliography

- [24] G. Zürn, T. Lompe, A. N. Wenz, S. Jochim, P. S. Julienne, J. M. Hutson - “Precise Characterization of 6Li Feshbach Resonances Using Trap-Sideband-Resolved RF Spectroscopy of Weakly Bound Molecules” - *Phys. Rev. Lett.* **110**, 135301 (2013)
- [25] W. Ertmer, R. Blatt, J. L. Hall, and M. Zhu - “Laser Manipulation of Atomic Beam Velocities: Demonstration of Stopped Atoms and Velocity Reversal” - *Phys. Rev. Lett.* **54**, 996 (1985)
- [26] W. D. Phillips and Harold Metcalf - “Laser Deceleration of an Atomic Beam” - *Phys. Rev. Lett.* **48**, 596 (1982)
- [27] Sonali Warriar - “Towards an Ultracold Bose-Fermi Mixture of Caesium and Lithium Atoms” - Doctoral Thesis - University of Nottingham (2014)
- [28] R. Grimm, M. Weidemuller and Y. B. Ovchinnikov - “Optical Dipole Traps for Neutral Atoms” - *Advances In Atomic, Molecular and Optical Physics.* **42**, 2000 (2000)
- [29] D. A. Butts and D. S. Rokhsar - “Trapped Fermi Gases” - *Phys. Rev. A.* **55**, 4346 (1997)
- [30] W. Ketterle, D.S. Durfee and D.M. Stamper-Kurn - “Making, probing and understanding Bose-Einstein condensates” - arXiv:cond-mat/9904034v2
- [31] G. Reinaudi, T. Lahaye, Z. Wang and D. Guéry-Odelin - “Strong saturation absorption imaging of dense clouds of ultracold atoms” - *Optics Letters*, **32**, 21 (2007)
- [32] C. J. Pentthick and H. Smith - Bose-Einstein Condensation in dilute gases - Cambridge University Press (2002)
- [33] E. Araujo-Lopez, J. S. Lopez-Echeverry, S. Reif-Achermanc - “The Antoine equation of state: Rediscovering the potential of an almost forgotten expression for calculating volumetric properties of pure compounds” - *Chemical Engineering Science*, Volume 177 (2018)
- [34] C.L Yaws, K. P Prasad, C. Gabbula - Yaws’ Handbook of Antoine Coefficients for Vapor Pressure (2nd Electronic Edition) - Knovel (2009)
- [35] C. B. Lucas - Atomic and Molecular Beams: Production and Collimation - CRC Press (2017)

Bibliography

- [36] Asaf Paris-Mandoki - “A Single Apparatus for the Production of Ultracold Fermionic Lithium and Cold Bosonic Caesium Gases” - Doctoral thesis - University of Nottingham (2015)
- [37] Matthew Jones - “Preparation of Cold Samples of Caesium-133 and Lithium-6 in a Single Apparatus for Bose-Fermi Mixture Experiments” - Matthew Jones - Doctoral Thesis - University of Nottingham (2014)
- [38] T. Arpornthip, C. A. Sackett, and K. J. Hughes - “Vacuum-pressure measurement using a magneto-optical trap” - *Phys. Rev. A.* **85**, 033420 (2012)
- [39] S. Chaudhuri et al - “Evaporative Cooling of Atoms to Quantum Degeneracy in an Optical Dipole Trap” - *J. Phys. Conf. Ser.* **80**, 012036 (2007)
- [40] K.M. O’hara et al - “Scaling laws for evaporative cooling in time-dependent optical traps” - *Phys. Rev. A.* **64**, 051403 (2001)
- [41] R. Jauregui - “Nonperturbative and perturbative treatments of parametric heating in atom traps” - R. Jauregui - *Phys. Rev. A.* **64**, 053408 (2001)
- [42] Pierre Jouve - “Study of an interacting ^6Li molecular Bose-Einstein condensate” - Doctoral Thesis - University of Nottingham (2019)
- [43] E. Pazy, I. Tikhonenkov, Y. B. Band, M. Fleischhauer and A. Vardi - “Non-linear Adiabatic Passage from Fermion Atoms to Boson Molecules” - *Phys. Rev. Lett.* **95**, 170403 (2005)
- [44] C. Zener - “Non-adiabatic crossing of energy levels” - *Proceedings of the Royal Society A.* **137**, 883 (1932)
- [45] J. Chwedenczuk, K. Goral, T. Kohler, and Paul S. Julienne - “Molecular Production in Two Component Atomic Fermi Gases” - *Phys. Rev. Lett.* **93**, 260403 (2004)
- [46] K. E. Strecker, G. B. Partridge, R. G. Hulet - “Conversion of an Atomic Fermi gas to a Long-Lived Molecular Bose Gas” - *Phys. Rev. Lett.* **91**, 080406 (2003)
- [47] P. A. Murphy *et al* - “High-temperature pairing in a strongly interacting two-dimensional Fermi gas” - *Science* **359** (6374) (2017)

Bibliography

- [48] T.Painter, D. K. Hoffmann, M. Jager, W. Limmer, W. Schoch, B. Deissler, M. Pini, P. Pieri, G. C. Stritani, C. Chin, J. H. Denschlag - “Pair fraction in a finite-temperature Fermi gas on the BEC side of the BCS-BEC crossover” - *Phys. Rev. A.* **99**, 053617 (2019)
- [49] J. E. Williams et al - “Theory of Feshbach Molecule formation in a dilute gas during a magnetic field ramp” - *New. J. Phys.* **8**, 150 (2006)
- [50] J. Javanainen et al - “Collective Molecule Formation in a Degenerate Fermi Gas via a Feshbach Resonance” - *Phys. Rev. Lett.* **92**, 20 (2004)
- [51] S. Matyjaskiewicz, M. H. Symanska, K. Goral - “Probing Fermionic Condensates by Fast-Sweep Projection onto Feshbach Molecules” - *Phys. Rev. Lett.* **101**, 150410 (2008)
- [52] Cindy A. Regal, Christopher Ticknor, John L. Bohn and Deborah S. Jin - “Creation of ultracold molecules from a Fermi gas of atoms” - *Nature* **424**, 47-50 (2003)
- [53] C. A. Regal, M. Grenier and D. S. Jin - “Observation of Resonance Condensation of Fermionic Atom pairs” - *Phys. Rev. Lett.* **92**, 040403 (2004)
- [54] C. D’errico, M. Zaccanti, M. Fattori, G. Roati, M. Inguscio, G. Modugno and A. Simoni - “Feshbach resonances in ultracold ^{39}K ” - *New J. Phys.* **9**, 223 (2007)
- [55] P. W. Anderson - “Random-Phase Approximation in the Theory of Superconductivity” - *Phys. Rev.* **112**, 1900 (1958)
- [56] M. P. E. Lock and I. Fuentes - “Dynamical Casimir effect in curved spacetime” - *New. J. Phys.* **19**, 073005 (2017)
- [57] I. G. Hughes and T. P. A. Hase - *Measurements and their Uncertainties, A Practical Guide to Modern Error Analysis* - Oxford University Press (2010)
- [58] T. Mukayima, J. R. Ab-Shaeer, K. Xu, J. K. Chin, W. Ketterle - Dissociation and Decay of Ultracold Sodium Molecules - *Phys. Rev. Lett.* **92**, 18 (2004)