Modal Translation:

The Relevance of Worlds

by

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For my parents, David and Mary

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INTRODUCTION

"That which does not kill us

makes us stronger"

- Friedrich Nietzsche

I cannot claim that as a six year old I was ever troubled by the uncertainty of Euclid's Fifth Axiom. My introduction to philosophy was somewhat less cerebral. In 1981, I encountered the above aphorism taken from Nietzsche's *GotzenDammerung*. It was part of the opening titles of the film *Conan the Barbarian* and it hit me like a bolt of lightning. In the years that followed I sought out Nietzsche's literature but was often puzzled by the contrast between the conventional wisdom surrounding his work and the work itself. In a similar vein, many philosophers are left incredulous by Lewis' Genuine Modal Realism **(GMR)**. In particular, Lewis' claim that there is more than one world or universe. John Mackie, for example, designated **GMR** "Ptolemaic astronomy" (see Bennett 2003, page 167) in a remark intended "derisively" but which Lewis was "happy" to accept as an accurate description of his position. "Lewis neatly expounded this matter through a "Ptolemaic astronomy" – an explanatory model that portrays a system of nested "spheres" with α, the actual world, at the centre" (Bennett 2003, page 167: Lewis 1973, page 14 ff.). We are presented with a simple choice between (at least) two incompatible philosophical theses: do we believe in a single world or "universe" or do we believe in a plurality of worlds or "universes"?

Lewis' (1968) Counterpart Theory supplements the interpretation of modal language by possible world semantics. The translation of *de dicto* modal statements is secured by quantification over possible worlds. Counterpart Theory extends this practice to *de re* modal statements. That is, in addition to the practice of quantification over possible worlds, Lewis' (1968) posits Counterpart Theory in order to secure *de re* representation. Counterpart Theory consists in four conceptual primitives, eight postulates and a set of translation principles (Lewis 1968 [1983], pp. 27-31). Modal discourse in the form of Quantified Modal Logic **(QML)** is translated into the language of Predicate Logic with Identity **(PL)**. In addition to quantification over possible worlds, Counterpart Theory quantifies over an "unrestricted domain" of *possibilia* (worlds and non-worldly individuals). When we discuss what could, might or ought to happen we are talking about a possible world in which what could, might or ought to happen, did happen. The translation scheme exploits an equivalence between the modal operators of box "□" and diamond "◊" (respectively, necessity and possibility) and the universal and existential quantifiers. It avoids the problems associated with intensional language by way of a systematic translation into extensional language (§ 2.2) (Lewis 1968 [1983], page 29). The *general* scheme takes the form of a direct definition or biconditional whose *analysandum* [left side] is a sentence ϕ of **QML** and whose *analysans* [right side] is a sentence ϕ^{β} of **PL**. We can now illustrate the counterpart theoretic translation of *de re* modal sentences (see Lewis 1968 [1983], page 31):

A. Jeremy Corbyn could have been Prime Minister (**ordinary language**)
B. $\exists x \Diamond Fx$ (**QML**)
C. $\exists x (\neg Fx \ \& \ \exists y \ \exists z (Wy \ \& \ Izy \ \& \ Czx \ \& \ Fz))$ (**counterpart theoretic**) – *x* is not *F* but some counterpart of *x*, at some world, is *F*

D. Boris Johnson is essentially human (**ordinary language**)

E. $\exists x \Box Fx$ (**QML**)

F. $\exists x (Fx \ \& \ \forall y \ \forall z \ ((Wy \ \& \ Izy \ \& \ Czx) \rightarrow Fz))$ (**counterpart theoretic**) – *x* is *F* and every counterpart of *x,* in any world, is *F*

There are several theoretical benefits associated with the interpretation of modal sentences by way of the quantifier-predicate model. Advocates of Lewis' scheme claim that it is an analytic truth. They seek to justify this claim on the basis of the theoretical benefits accrued (Divers 1997, page 142). Theoretical *desiderata* include simplification and clarity in respect of the logical, semantic and expressive resources employed by the translation scheme (§ 1.1). A central presumption of our argument is that Lewis' modal translation scheme *is* an analytic truth (chapter 5). However, we justify this claim on grounds that differ from those cited by the aforementioned advocates of modal translation. We propose that the modal translation scheme is an analytic truth (partially) on the grounds that the concept of a "possible world" is "contained" in the concept of "ways things could have been". Thus the account we present is a partial defence of the analyticity of the overall modal translation scheme. That is, we restrict the scope of our examination of analyticity to the notion or concept of a possible world; one of the four primitive predicates of modal translation (Lewis 1968).

Analyticity aside, the central problem we address is that of expressive adequacy. How does Lewis' modal translation scheme "translate" extraordinary or advanced modal sentences? Advanced or extraordinary modal sentences represent a challenge to the expressive adequacy of modal translation. Specifically, modal sentences that ought to be true by the

(lights of **GMR**) are translated as false by the scheme, and *vice versa*. For example, a sentence that is true (by genuine realism), such as, "necessarily, there is a plurality of worlds" is translated as false by the schema. It is false because worlds do not overlap (Lewis 1986a, page 2); the *analysans* of this modal sentence states that at every world there is a plurality of worlds – that every world must have worlds as parts. The problem of advanced modality is generated by the world-restriction of terms (see chapter 7). The bugbear of advanced modality emerges in the genuine realist interpretation of numerous "domains of discourse" e.g. modal statements about set theory, natural properties and propositions. Absent the preservation of their "intuitive" truth-value, advanced modal sentences are apt to confound the logical, semantic and expressive resources of modal translation. Hence, the theoretical benefits secured by the quantified-predicate model are vulnerable to refutation by advanced modal claims. It is imperative, therefore, that genuine realism is able to preserve the "intuitive" truth-value of advanced modal sentences, preferably in a way conducive to the conceptual tools available to **GMR**. The primary aim of this dissertation is to preserve the "intuitive" truth-value of advanced modal sentences using the standard conceptual resources of **GMR**. In particular, we intend to preserve, as a component of modal translation, the use of the primitive predicate "*Ixy*" or "*x* is in possible world *y*" (Lewis 1968 [1983], page 27). The proposed solution will make explicit use of the predicate "at a world w". The plurality of worlds is pivotal to the solution of the problem raised by advanced modal claims. We distinguish our thesis from John Divers' Redundancy Theory recommended by him as the solution most conducive to genuine modal realism.

Lewis was inspired to postulate Counterpart Theory by reading a short story by L. Sprague de Camp (Lewis 1968 [1983], page 28 fn. 3). Coincidentally, L. Sprague de Camp worked as the "technical advisor" on *Conan the Barbarian*.

Chapter 1: An ontological commitment to a type of entity, possible world(s) is central to Lewis' modal translation scheme. The scheme adopts and expands our normal practice of seeking an "extensional" interpretation of modal sentences based on the counterpart-theoretic interpretation of **PL**. The quantifiers of **PL** are supplemented by four primitive predicates including "*x* is in possible world *y*" and employed in the translation of modal sentences. We argue that the primitive predicate "*x* is in possible world y" is essential to a genuine realist interpretation of modal translation. A solution to the threat posed by advanced modal sentences can be established by admitting sets as an ontological commitment of "total theory". Lewis subscribes to Quine's quantifier criterion of ontological commitment generally understood as identifying only the *explicit* ontological commitments of a theory. Do sets require an explicit ontological commitment or can they be subsumed as an implicit ontological commitment? We argue that Quine endorses the notion of a theory's *implicit* ontological commitments to a type of entity. We discover, however, that Lewis' commitment to set-theory is explicit. What is also explicit are Lewis' doubts about the "singularist dogma". Does a collection of, say, four apples imply the existence of a distinct type of entity, the *set* of four apples? We conclude with a cursory examination of the problem of plural predication as it applies to our proposed set-theoretic solution to the problem of advanced modality.

Chapter 2: Quine aims to regiment ordinary language into canonical notation, preferably the notation of **PL**. His preference is based on the referential transparency of **PL**. Unlike Quine, Lewis argues that in the proper context, ordinary language can reveal the *serious* ontological commitments of a theory. That is, the quantifier and predicate expressions of ordinary language can be taken seriously. We identify Lewis' criteria upon which the paraphrase of ordinary language can reveal the ontological commitments of a theory (Lewis 1970 [1983]). We test Lewis' supposition that a theory expressed in ordinary language is capable of serious ontological commitment(s). The indexical phrase of ordinary language we use to test Lewis' supposition is "ways things could have been [but are not]" paraphrased as "possible world". We conclude that use of the phrase in ordinary language does entail a serious ontological commitment to possible worlds.

Chapter 3: Lewis accepts common sense judgement(s) both as a criterion of paraphrase and as a *desideratum* of philosophical theory. What is the source of the widespread incredulity provoked by **GMR**? We utilise the notion of a "Moorean fact" in order to identify the source of the incredulous stare. The incredulity provoked by the postulation of a plurality of worlds is determined by the *type* of theoretical conservatism endorsed. If incredulity is taken as decisive objection then a "strong" form of conservatism is in play. There is, and could not be, a philosophical argument capable of dislodging the certainty of a Moorean fact, say, that there is only one world or universe. If, on the other hand, incredulity is taken as defeasible then a "weak" form of conservatism is in play. There may indeed be reason to reject a putative claim of folk knowledge that there is only one world or universe. Theoretical conservatism can be overruled by a preponderance of alternative benefits. We conclude

that Lewis' "weak" theoretical conservatism permits the incredulous stare a defeasible role in theory-choice. This conclusion is secured but only at the cost of a failure to satisfy one criterion of a genuine Lewisian paraphrase of "ways things could have been".

Chapter 4: We turn from common sense judgements to the content of those judgements. The incredulity associated with genuine realism is prompted by postulating a plurality of worlds. Lewis argues that an ontological commitment to a plurality of a *single* type of entity ought not to be counted as a theoretical cost. Methodological parsimony should be calculated not on the number of a *single* type of entity postulated but on the number of *different* types of entity postulated. In short, Lewis acknowledges the theoretical virtue of qualitative, but not quantitative, parsimony. Is Lewis' dismissal of quantitative parsimony too quick? We contrast the commitment to a plurality of worlds with a case study drawn from the history of science. We conclude that quantitative parsimony ought to play a derivative role in respect to theory evaluation. How should we determine the actual ontological commitments of rival theories? If we accept that the quantification-predicate model reflects the "structure" of reality then it is incumbent upon Lewis to provide a philosophical account of the notion of a property. Lewis' account of a "natural property" delineates the joints at which nature ought to be carved. We conclude that qualitative parsimony favours **GMR** and quantitative parsimony does not count against it.

Chapter 5: Analytic truth is traditionally a requirement of philosophical analysis. We defend the overall counterpart theoretic translation scheme as an analytic truth. That is, a translation that is true in virtue of the meaning of its terms. We distinguish the semantic

and metaphysical aspects of the scheme. Stalnaker, for example, accepts “actual” as an indexical term (semantic aspect) but rejects the Lewisian ontology of a plurality of worlds (metaphysical aspect). This distinction implies the “insubstantiality thesis”: namely, that the truth of an analytic sentence is exclusively determined in virtue of its meaning. We maintain, that as a component of the counterpart theoretic translation scheme, the predicate expression “possible world” *is* a genuine paraphrase of the ordinary language expression “ways things could have been”. This assertion, however, entails no commitment to how a possible world ought to be conceived. A “possible world” could be a discrete spatio-temporal continuity, a set of maximally consistent propositions, a collection of unactualised properties, and so on. Williamson rejects our commitment to the insubstantiality thesis. In part, his objection is based on a perceived failure to distinguish semantic from meta-semantic fact. How can Lewis’ act of paraphrase by stipulation (a meta-semantic fact) establish an analytic truth (a semantic fact)? We re-assert the insubstantiality thesis, and attempt to rebut Williamson’s critique. We do so by constructing a “bridge” that unites the meta-semantic and semantic aspects of language. We argue that the expressions “ways things could have been” and “possible world” are synonyms. As components of counterpart theoretic translation these expressions help to secure the analyticity demanded of traditional philosophical analysis.

Chapter 6: John Divers advocates a form of anti-realism in respect to the analysis of modal sentences (Divers 2004: 2006). He argues that the worldly agnostic can secure at least some of the theoretical benefits accrued by the quantificational-predicate model and yet withhold assent (commitment) to a plurality of worlds (Divers 2004, page 683). The threat of worldly

agnosticism to our main thesis is clear and obvious. Divers' first step in this endeavour is to demolish the realists' conception of possible worlds, whether a possible world is conceived as a discrete, spatio-temporal entity or as an abstract set of complete and consistent propositions. The causal isolation of each world-type entails that modal truth is beyond our epistemological and semantic grasp. In place of modal realism, Divers argues that we should adopt a moderate agnosticism in respect to non-actual possible worlds (Divers 2004, page 669). He is prepared to acknowledge, however, that the moderate agnostic is vulnerable to an alleged assertibility-belief gap (Divers 2004, page 675). Specifically, the modal realist can justify a belief in a *de re* possibility that the worldly agnostic cannot. Furthermore, the modal realist is able to assert that this modal belief is both rational *and* indispensable. In reply, Divers provides three strategies designed to show that the relevant belief is rationally dispensable (Divers 2004, page 678). We argue that each strategy falls short of its intended goal.

Chapter 7: The problem of advanced or extraordinary modal claims is one that threatens the expressive adequacy of stardard-1968 modal translation. There are some modal claims that fail to retain their pre-theoretical truth-value post translation. Divers' solution to this problem is based on the redundancy in translation of the predicate expression "*x* is in possible world *y*" and by implication "*y* is a possible world" (Divers 1999: 2002, pp 48-49). He argues that genuine realism ought to adopt a modified or non-standard-1968 translation of advanced modal claims. The debate surrounding advanced modality is ongoing. Noonan challenges the coherence of standard-1968 modal translation based on a derivation of consequences that are unpalatable to genuine realism (Noonan 2014: cf. Divers 2014). As a

standard of philosophical analysis, strict adequacy entails the necessary *de re* existence of, say, talking donkeys! (Noonan 2014, Section 2 page 3) Noonan's challenge includes but is not restricted to advanced modal claims. His argument is partially based on the work of Josh Parsons (Parsons 2011 draft). Parsons argues that genuine realism is committed to a number of theses that are collectively inconsistent (Parsons 2011 draft, page 3). Hence, Divers' attempt to distinguish advanced modal claims, as candidates for extraordinary translation, is misplaced (Parsons 2011 draft, page 10). It is a distinction without a difference. In reply, we argue that the alleged inconsistency identified by Parsons can be unlocked and that the problem posed by advanced modal claims is genuine.

Chapter 8: We offer a solution to the problem of advanced modal claims compatible with the primitive conceptual resources of **GMR**. That is, by retention of the predicate expressions of standard-1968 modal translation (esp. "*x* is in possible world *y*" and "*y* is a possible world"). Importantly, we retain the relevance of worlds as a component of the *analysans* [right side, biconditional]. Hence, as our proposed solution retains the primitive conceptual apparatus of **GMR** we argue that it ought to be preferred to Divers' Redundancy Theory. We examine a number of potential objections and replies to our solution. Does the membership relation of set theory rest on a "magical relation" against which Lewis inveighs? Are sets dispensable – do they impose an unnecessary ontological commitment? Do the antimonies of set theory generate a reason to dispense with sets altogether? We conclude that although no "knockdown argument" in favour of our proposed solution is available it nonetheless ought to be preferred on the grounds of reflective equilibrium.

Chapter One: An Ontological Commitment to Worlds

"… to be reckoned as the value of a variable." Quine 1948 [1980], page 13

1.1 Introduction

Lewis (1968) presents a modal translation scheme intended to rescue formal discourse from an apparent incongruity. With the proper adjustments we need not abandon our normal practice of conducting formal discourse in the language of extensional logic [this theoretical *desideratum* is subsequently withdrawn; see Lewis 1974 [1983]: Lewis 1983b [1983], page 46]. The exception to our normal practice is modality. In the case of modality, we favour a non-extensional Quantified Modal Logic **(QML)**. "Why this departure from our custom? Is it an historical accident, or was it forced on us somehow by the very nature of the topic of modality?" (Lewis 1968 [1983], page 26). Lewis argues that we are not compelled to adopt **QML** as an unavoidable consequence of modal discourse. In the alternative, he invites us to "translate" the modal sentences of **QML** into sentences of Counterpart Theory expressed in Predicate Logic with identity **(PL)** and a suitable domain of quantification: "The domain of quantification is to contain every possible world and everything in every world" (Lewis 1968 [1983], page 27). Lewis (1968) represents an extension to the case of *de re* modality of his general strategy of translating modal discourse by quantification over possible worlds; that is applicable to both *de dicto* and *de re* modality. Thus, our normal practice is preserved and the "incongruity" dissolved.

The modal operators of box "□" and diamond "◊" can be substituted without loss. In their place is the syntax of first-order logic (truth-functional operators, quantifiers, variables and identity) and four primitive predicates. Each predicate is to be attributed its ordinary English meaning. The predicates are supplemented by eight postulates, P1 to P8 that compose Counterpart Theory (Lewis 1968 [1983], pp 26-27) **[FN 1]**. One predicate, "W(*x*)", denotes the propositional function "*x is a possible world*" (Lewis 1968 [1983], page 27). The predicate "world" is to be interpreted as a discrete spatiotemporal continuity. As partly constitutive of the modal translation scheme, the propositional function "W(x)" incurs an ontological commitment to a type of entity, Worlds. Thus, Lewis accepts "*entia non grata*" in the form of unactualised possibilia (things "in" worlds other than the actual world) as elements of the domain of quantification. For example, Hubert Humphrey is a world-bound individual who lost the 1968 US Presidential Election. It appears reasonable to suppose that Humphrey might have won. This does not entail that Humphrey (himself) is "at" another world where he was victorious. Rather, Humphrey has a counterpart "at" a non-actual world who won the counterpart of that world's 1968 US Presidential Election (see Lewis 1968 [1983], pp 27-28: cf. Quine 1960 [2013], Section 50).

Thus, **GMR** offers in part a quantificational analysis of modality. As a philosophical technique, the quantification-predicate model can be traced back to Russell's "On Denoting". The foundation of Russell's analysis is the *variable* (Russell 1905 [2001], page 212). In Russellian terms, "W(*x*)" is a proposition, or more accurately, a propositional function, in which the variable *x* is both a constituent and wholly undetermined (Russell 1905 [2001], page 213 fn. 2). Lewis' accrues a number of theoretical advantages as a

consequence of interpreting modal discourse in quantificational terms (see Divers 1997, pp 141-142):

- Logical: the substitution of modal expressions by their "worldly" equivalents facilitates modal logic by proxy in the form of first-order logic – we thereby simplify the logical resources required to tackle modality e.g. modal operators understood as logical primitives would require special axioms and rules of inference,
- Semantic: if we are compelled to add modal operators to our reservoir of logical primitives we become needlessly enmeshed by intensional semantics. A viable proxy for a modal operator language is available in the form of a *direct* semantics (with a suitably rich model) of a "worldly", fully extensional language.
- Expressive: there are a number of related (usually conflated) advantages –

i. Restriction to the use of sentential modal operators and "actualist" quantifiers inhibits the ability to articulate modal concepts. This inhibition can be lifted by the use of quantification over a domain of worlds and their inhabitants.

ii. We acquire the ability to represent modal content in a straightforward and understandable way by the use of equivalent "worldly" expressions – we replace iterated modality by multiple quantificational generality and accessibility relations.

iii. The use of a "worldly" language allows a significant gain in expressive power over standard modal operator languages – the latter negatively entail an expansion to the number of primitive logical resources and restricts the range of expression.

In order to secure these theoretical benefits we defend the World Relevance Principle:

The World Relevance Principle **(WRP)**: The analysis of *de dicto* modal sentences ought to be based on the assumption that modal operators *both* "unite" worlds under quantification and restrict the domains of the quantifiers that fall within their scope (Lewis 1968 [1983] page 40: Lewis 1986a, pp 5-13).

This chapter concerns Quine's Quantifier Criterion of ontological commitment **(QC)**. As a source of the quantification-predicate model of philosophical analysis, we present a "standard" interpretation of Russell's "On Denoting" (Section 1.2). Inspired by Russell, Quine applies the quantification-predicate model to the notion of ontological commitment (see Quine 1948 [1980], page 5: Quine 1960 [2013], Section 37-8) (Section 1.3). We propose a solution to the problem of advanced modality that requires an ontological commitment to sets. To this end, we investigate the plausibility of identifying the *implicit* ontological commitments of a theory (Section 1.4). Lewis expresses reservations in regard to an "implicit" ontological commitment to *sets*. His reservations are articulated in the problem of a "singularist dogma" in respect of standard quantification. We address Lewis' allegation of a "singularist dogma" and offer a tentative solution by way of reply (Section 1.5). We conclude with a brief investigation in regard to the likelihood of identifying the ontological commitments of a theory expressed in the idioms of ordinary language (Section 1.6). We disregard a number of important questions. What are the subtle distinctions of interpretation in respect of the **QC** itself? How appropriate is the **QC** to the enterprise of identifying a theory's ontological commitments?

1.2 Russell: Quantification, Predication and Existence

The notion of a functional expression is adapted from the concept of a mathematical function. The value of the variable *y* in the functional expression "*y* = 3*x*" is said to be a function of *x*. We "insert" different numerals for *x* and generate different values for *y*. The variable *x* may "stand in" for different numbers that are the *arguments* of the function. Hence, for the argument 2 we get the value 6 (6 = 3 x 2), for the argument 3 we get the value 9 (9 = 3 x 3), and so on. A function can also be represented as a set of ordered pairs; where the first number represents the *argument* of the function and the second number represents the *value* generated by the function for that argument e.g. the functional expression "*y* = 3*x*" can be represented by the set of ordered pairs: {(1,3), (2,6), (3,9) …}. This set of ordered pairs is the *extension* of the function. In addition, mathematical functions may have more than one argument place e.g. two argument places "*z* = 3*x* ÷ 6*y*" can be represented by a set of ordered triples, where the first number represents the argument *x*, the second number represents the argument *y*, and the third number the value *z* generated by the function for those particular arguments. We use the word "insert" to indicate a "gap" in the functional expression subsequently "filled" by numerals used as arguments e.g. "*y* = 3( )". The brackets (or variable) of a functional expression "hold" a place for an argument of the appropriate kind – an argument in the form of an expression that will generate a value. For Russell, as we have noted, this feature indicates that functional expressions are "wholly undetermined" (Russell 1905 [2001], page 213).

Thus far, we have only examined functions that take numbers as arguments and generate numbers as values. Frege's innovation is to apply the functional model of mathematics to the semantics of predicates, connectives and quantifiers (see Frege 1891 [1980], pp 21-41) **[FN 2]**. In a series of publications, Bertrand Russell employs Frege's ideas to resolve a number of philosophical puzzles. For example, he utilises the quantification-predicate model in order to argue that "a phrase may be denoting, and yet not denote anything; e.g., "the present King of France"" (Russell 1905 [2001], page 212: Russell 1919 [1996]) **[FN 3]**. In "On Denoting" Russell subjects his theory of descriptions to four puzzles, one of which is the problem of apparent reference to non-existents **[FN 4]** (Russell 1905 [2001], page 215). Russell offers a *contextual definition* of the word "the" as that word is standardly used as part of a singular definite description e.g. "the [unique] so-and-so".

Consider the sentence:

A. "The present King of France is bald"

A number of sentences appear to be both individually true of (A) and yet collectively inconsistent (see Lycan 2000, pp 13-14):

A1: (A) is meaningful, it is not nonsense

A2: (A) has the grammatical form of a subject-predicate sentence

A3: A meaningful subject-predicate sentence can *only* be meaningful in virtue of picking out a thing and ascribing a property to that thing

A4: (A)'s subject term does not pick out a thing that exists e.g. "the present King of France"

A5: If A1, A2 and A3 are true and given the truth of A4 then either (A) is not meaningful (is nonsense) or (A) picks out a thing that does *not* exist, *but*

A6: There is no such thing as a “non-existent thing”

The main problem appears to be the *subject* or “denoting phrase” of sentence (A) e.g. “the present King of France”. The meaningful use of a “denoting phrase” appears to require a denotation and yet (A) appears to be meaningful despite the absence of a denotation (Russell 1905 [2001], page 214). In order to tackle this incongruity Russell employs the philosophical technique of paraphrase; the subject or “denoting phrase” of (A) can itself be analysed into two distinct components. First, the word “the” of the singular definite description in the *whole* sentence (A) is interpreted as an abbreviation of a conjunction of three existentially quantified sentences (Russell 1905 [2001] page 218). Second, the predicate “present King of France” of the singular definite description is understood to introduce the concept of a predicate expression. The *meaning* of the predicate expression plays a critical role in identifying the denotation of the denoting phrase. That is, in order to identify the denotation of the denoting phrase we must first understand the meaning of “present King of France”. When the meaning of the predicate expression is understood we are able to select the denotation to which the predicate can be truly ascribed. “Thus we must either provide a denotation [referent] in cases in which it is at first sight absent, or we must abandon the view that the denotation is what is concerned in propositions which contain denoting phrases” (Russell 1905 [2001], page 215). To unlock the inconsistency of A1-A6 Russell accepts the second horn of the dilemma, by denying A3. In this case, the denoting phrase is interpreted as a propositional function. The true logical structure of sentence (A), that “contains” a grammatical singular definite description as a denoting phrase, is exposed. Russell distinguishes the grammatical and logical structure of the sentence. The singular definite description is a grammatical but not a *logical* constituent of sentence (A) (Russell 1905 [2001], page 213-4). As such, the correct *logical* analysis of the

denoting phrase "*the* present King of France" is not as the grammatical subject of a subject-predicate sentence. Instead, "the present King of France" should be analysed, by way of paraphrase, as a propositional function *x is the present King of France*. The logical structure of the *whole* sentence (A) is analysed as a conjunction of three propositions involving quantification:

i. There is at least one present King of France

ii. There is no more than one present King of France

iii. Anything that is a present King of France is bald

By rule, in order to be true each conjunct must be true. Clearly, (i) is false so the whole conjunction (i-iii) *must* be false. Russell paraphrases the singular definite description in order to dissolve the apparent demand for a denotation implied by the phrase "*the* present King of France".

iv. $\exists x (Fx \,\&\, (\forall y (Fy \rightarrow y = x) \,\&\, Gx))$

In place of a grammatical subject-predicate sentence "Fa", that *must* denote in order to be meaningful, is an existentially quantified proposition whose meaning is unaffected by the lack of a denotation. "[incomplete] denoting phrases never have any meaning in themselves, but […] every proposition in whose verbal expression they occur has meaning" (Russell 1905 [2001], page 213: Russell 1919 [1996], pp 173 ff.). Therefore, "the present King of France" is not a singular term but a grammatical abbreviation of a proposition with a quantification-predicate logical structure, illustrated by (i-iii).

This is the "standard" interpretation of Russell's Theory of Descriptions of which the quantification-predicate model is a part. It is the interpretation to which W. V. O. Quine

subscribes (Quine 1960 [2013], Section 37-8). It is also an interpretation disputed in the literature (cf. Hylton 1990, Griffin 1996, Noonan 1996) **[FN 5]**. Did Russell account for the meaningful use of a denoting phrase (absent a denotation) *before* 1905? The dispute need not detain us. The "correct" exegesis of Russell's motivation is immaterial. At this juncture we are only interested in Russell's *first* employment of **PL** (modern quantification theory). Russell (1905) departs from the "fruitless complexes of [his] early theory of quantification" (Griffin 1996, page 46). The novelty of "On Denoting" (for our purposes) is Russell's use of the (broadly) Fregean quantification-predication model of **PL.** The source thus identified, we can now turn to examine how the model is applicable to Quine's quantifier criterion of ontological commitment?

1.3 The Quantifier Criterion

The identification of a theory's ontological commitments to a type of entity *K* is a task of meta-ontology **[FN 6]**. The solution to the philosophical puzzles of ontology are dependent on a prior systematic identification of a theory's ontological commitments. "We look to bound variables in connection with ontology not in order to know what there is, but in order to know what a given remark or doctrine, ours or someone else's, *says* there is; and this much is quite properly a problem involving language. But what there is is another question" (Quine 1948 [1980], pp 15-16). Hence, identifying a *criterion* of ontological commitment – however that notion is unpacked – is essential to the philosophical rigour of ontological discourse. How is the criterion of ontological commitment unpacked?

Firstly, the criterion ought to be referential: "a theory is committed to those and only those entities to which the bound variables of the theory must be capable of *referring* in order that the affirmations made in the theory be true" (Quine 1948 [1980], pp 13-14 my emphasis). The criterion ought to be descriptive rather than prescriptive. That is, although Quine's general method of ontology *is* prescriptive his meta-ontology is not. Ontology is the endeavour to regiment competing theories in the language of canonical notation (preferably **PL** (Quine 1998c [1998]); we can determine and then endorse the "best" theory on epistemic grounds, such as simplicity or fruitfulness. Ontological commitment is *not* a theoretical virtue itself but rather a means by which we assess a potential virtue, such as parsimony. We *ought* to endorse the "best" available theory, understood as the theory that is ontologically committed to those types of entity *K* that are the values of the bound variables that constitute its referential truth.

The distinction is important. If we conflate meta- and genuine ontology the general method of ontological commitment would be open to the charge of circularity. The ontological commitments that we ought to endorse depend on which theory is "best" *but* the "best" theory is determined (in part) by the theoretical *desideratum* of simplicity. Essentially, the "best" theory is the one with the fewest (*ceteris paribus*) ontological commitments but the ontological commitments we should recognise are those of the "best" theory. This potential problem is rebutted by the distinction: the criterion of ontological commitment is a criterion of meta-ontology (it facilitates theory-choice but does not determine which theory ought to be chosen). The theory we *ought* to endorse is partially dependent on theoretical parsimony but we cannot know which theory to select until we know what types of entity a theory *is* in

fact committed to. If a quantified-predicate expression, say, "∃*x* Photon(*x*)" is a component of a particular theory then the bound variable "*x*" must range over photons in the domain of discourse in order that the theory be true. In this case, the *theory* is ontologically committed to a type of entity, *photon*. With these points duly noted we can offer a definition of the quantifier criterion of ontological commitment:

The Quantifier Criterion **(QC)**:

A sentence of **PL** carries an ontological commitment to a type of entity *K* just in case *Ks must* be counted amongst the values of the variables in order for the sentence to be true (see Quine 1948 [1980], page 12-13) **[FN 7]**.

Quine's **QC** of ontological commitment represents the relation between a theory and a postulated "type of entity *K*". A theory ought to be expressed in canonical notation *interpreted* by standard Tarskian semantics. Each predicate of a theory, for instance, should be assigned a class or set as an interpretation (a monadic predicate should be interpreted as set-membership; the set whose members consist of objects in the domain of discourse that occupy the argument-place of an incomplete symbol and that when combined with a predicate expression yield a true sentence; e.g. "F(a)" is true if, and only if, the object referred to by "a" belongs to the extension of "F". A polyadic predicate, on the other hand, should be interpreted as the set-membership of ordered n-tuples; so, "F(a,b)" is true if, and only if, the related objects designated by "a" and "b" in the domain of discourse form an ordered pair as the member of a set, { (a,b) }). Quantifiers are to be assigned an objectual, rather than substitutional interpretation. An objectual interpretation of an existential quantified expression states that "∃x ϕ(x)" is true if, and only if, the open formula "ϕ(x)" is

satisfied by some *object* in the domain of discourse (see Hylton 2010, page 256). Alternative domains of discourse facilitate our ontological deliberations: that is, the referential truth of individual interpreted theories is evaluated based on how their respective bound variables range over the objects of *different* domains of discourse. A philosopher may thereby avoid an ontological commitment to a kind of 'dubious' entity *K* by a challenge to the regimented paraphrase of canonical notation in order to demonstrate that no such commitment to *Ks* is entailed (see Quine 1948 [1980], page 13: Quine 1953 [1980], pp 103-107).

The **QC** is widely accepted as the philosophical orthodoxy in respect of identifying the putative ontological commitments of a theory (Bricker 2016, Section 1.1). Quine's referential apparatus represents an expansion to the logical analysis of a "denoting phrase" constructed by Russell **[FN 8]**. In addition to singular definite descriptions, Quine includes pronouns as grist to the referential mill. "On Denoting" includes a philosophical analysis of proper names based on the supposition that each proper name abbreviates a singular definite description. Once the semantic content **[FN 9]** of a name is identified as a singular definite description we need only apply Russell's analysis and dissolve the apparent need of a denotation as an essential element of meaning. How should Russell's analysis proceed if no equivalent singular definite description presents itself? Quine's solution is to introduce or "invent" primitive predicates to canonical notation, each one uniquely true of the object or person named. This new "invented" predicate may refer to an attribute of the object or person: e.g. the name "Pegasus" can be analysed as the predicates "is-Pegasus" or *is-identical-with-Pegasus* or "pegasizes" (Quine 1948 [1980], page 8).

Hence, the analysis of:

v. "Pegasus has wings"

as an existentially quantified sentence (as at i-iii):

vi. "There is one and only one thing that Pegasizes and it has wings"

If we are engaged in serious ontic discourse, we presumably would deny the existence of a unique winged horse whose attributes are described by the predicate "Pegasizes" **[FN 10]**. The first conjunct is therefore false yielding a false truth-value for the overall sentence (vi) (cf. Cartwright 1960 [1987]: Kripke 2013). Thus, theoretical discourse is regimented and "we move closer to a stage at which all of our ontological commitments are carried by constructions which invariably express the ontological assumptions of those who employ them in serious assertions" (Hookway 1988, page 23). For Quine, the *analysans* [right side, biconditional] ought to be expressed in canonical notation, preferably the language of **PL [FN 11]**. How are sentences of ordinary language paraphrased into sentences of **PL**? Paraphrase should be determined on the grounds of pragmatism, not synonymy.

> We do not claim synonymy. We do not claim to make clear and explicit what the users of the unclear expressions had unconsciously in mind all along […] We fix on the particular functions of the unclear expressions that make it worth troubling about, and then devise a substitute, clear and couched in terms to our liking, that fills those functions. Beyond those conditions of partial agreement, dictated by our interests and purposes, any traits of the explicans come under the head of "don't cares" (§ 38). (Quine 1960 [2013], Section 53 page 238)

Quine favours a Tarskian interpretation of the quantifiers. However, thus interpreted, there is a danger that the quantifiers of **PL** may fail to meet a sufficiency condition for an accurate

determination of the types of entity to which a theory is committed. If the **QC** commits a theory to a type of entity that it ought *not* to be committed then it will over-generate a theory's ontological commitments. If the ontological commitments of a theory are skewed in this way then philosophical controversy is generated, rather than resolved. If quantifiers are susceptible to a number of different interpretations – Tarskian and non-Tarskian – then how can the **QC** accurately determine a theory's ontological commitments? Do different interpretations of the quantifier result in different ontological commitments? Take, by way of example, the problem of the empty-domain of discourse. Standard Tarskian semantics excludes the empty domain; valid well-formed formulae are those that yield true formulae under all interpretations of their non-logical symbols and over all *non-empty* domains. As a result, universal quantified sentences turn out to have existential implications: that is, "∃(x) ϕ(x)" may be logically deduced from "∀(x) ϕ(x)". Recall, **PL** neither recognises the empty domain nor mandates which *type* of entity ought to be included in the domain of discourse **[FN 12]**. In general, any predicate letter could be assigned to "ϕ(x)" and thereby generate an ontological commitment to that *type* of entity.

This existential implication can be avoided by modifying the standard Tarskian interpretation of the universal quantifier. There are interpretations in which the universal quantifier "∀(x) ϕ(x)" is true without incurring a 'dubious' ontological commitment to a type of entity associated with the predicate-place "ϕ(x)" (because there is nothing "in" the specified domain). A "domain of discourse" may relate, for example, to fictional characters, mythological creatures or non-actual *possibilia*. Adjusted in this way, a revised interpretation of the quantifiers reserves all genuine ontological commitments to the

purview of the existential quantifier *alone*. *Inclusive logic*, as the nomenclature suggests, includes the empty domain and so avoids the difficulty of the aforementioned existential entailment (cf. Quine 1953 [1980]). A new interpretation of quantification over the *empty*-domain of discourse assigns the truth-conditions "true" to all universal and "false" to all existential quantifiers. The standard Tarskian interpretation of **PL** fails to recognise the empty-domain of discourse and thus risks over-generating the ontological commitments of a theory to a type of entity, say, fictional characters. Hence, the theoretical virtues of 'rigour' and 'clarity', which motivate the regimented paraphrase of sentences by canonical notation, appear to depend on one's choice of interpretation of the quantifiers (the chicken) and existential presuppositions (the egg)!

The **QC** also imposes a distinction between a theory's ontological and ideological commitments (Quine 1951b: Quine 1983). The ontological commitments of a theory are to those types of entity that a theory *says* exists in order that it be true. The object must "inhabit" the relevant domain of discourse in order to be a value of a bound variable and ensure the theory's referential truth. The ideological commitments of a theory are generally those concepts, whether logical or not, that are expressible by the theory. They are not equivalent. A theory may commit to a type of entity *k* even though the *concept* of being a *k* exceeds the conceptual resources of the theory (is not expressible by the theory). Alternatively, a theory may draw upon the conceptual resources of, say, *relational* terms in order to provide structure to the entities postulated, without thereby adding a new type of entity to the list of ontological commitments. We will adopt Quine's approach to this facet of ontological commitment. That is, we identify the ontological commitments of a theory

wholly on the basis of the types of entity *K* postulated; and disregard the ideological (predicative) aspect of ontological commitment (cf. Sider 2011).

Not all accounts of ontological commitment are based on the **QC** and not all interpretations of the **QC** are equivalent (see Bricker 2016). Lewis' commitment to a plenitude of possible worlds is explicit and should be understood literally. We will defend a thesis in regard to advanced modality that requires an ontological commitment to sets (chapter 8). Although Lewis accepts set-theory, an ontology of sets need *not* number amongst a theory's *explicit* ontological commitments. Hence, in order to establish an *implicit* commitment to a type of entity we must first establish that the **QC** admits of a theory's *latent* ontological commitment(s). It is to this task that we now turn.

1.4 Implicit Ontological Commitment

Thus far we have limited our exploration of ontological commitment to the *explicit* ontological commitments of a theory based on the **QC**. A denoting phrase, such as "∃(x) Brother(x)", is committed to brothers and nothing else. It is true if, and only if, there is a person / object "in" the domain of discourse that satisfies the value of the bound variable, in this case "x". The problem with a restriction that acknowledges only the explicit ontological commitments of a theory is that we may under-generate the types of entity to which a theory *ought* to be committed. This deficiency relates not to a sufficiency condition (§ 1.3) but to a necessary condition of ontological commitment. If we successfully identify all

the explicit commitments of a theory we may still fail to satisfy a necessary condition of ontological commitment. We may under-generate the ontological commitments of a theory if, for example, we fail to identify all the *extrinsic* properties of an object (see Lewis 1983c [1999], page 111). The truth of "∃(x) Brother(x)", appears to imply an additional ontological commitment to a type of entity *not* identified by the bound variable *x* of the predicate "brother". This existentially quantified sentence appears to be ontologically committed (by dint of an analytic truth) to a different type of entity identified by the bound variable *x* e.g. "sibling". This second type of entity is *not* co-extensive with the first since the predicate "sibling" may also apply to a sister. Although I am both my sister's brother and sibling, she is my sibling but not my brother. By the lights of the **QC**, "∃(x) Brother(x)" fails to refer to a type of entity "in" the domain of discourse (my sister), even though that *same* object (type of entity) does satisfy a predicate implied by the former (by analytic truth); namely, "∃(x) Sibling(x)" (see Rayo 2007, page 431-432).

Two lines of defence are available to one who wishes to reserve the identification of a theory's ontological commitment(s) to those that are explicit. First, she may restrict the number of primitive predicates to those that relate to *intrinsic* properties. However, this manoeuvre simply postpones the controversy to one of identifying which predicates *are* intrinsic and which are not. Second, she might cleave to the explicit account of ontological commitment and deny that "∃(x) Brother(x)" does entail the truth of "∃(y) Sibling(y)" despite the latter being implied by the former (by virtue of the meaning of the predicates). For example, dependent on which criterion of ontological commitment is adopted, the predicate "∃(x) Parent(x)" need *not* ontologically commit a theory to *children* as a type of

entity. As a type of entity, *children* are not required as the value of a variable "x" to render the overall sentence true. (This presumes, of course, that parents are not themselves understood as children). Contrast the predicate expression "∃(x) Bachelor(x)", which *is* committed to an implied type of entity, *man*. Once again, however, this manoeuvre simply postpones the controversy in respect of which analytical entailments of a predicate expression are genuine (see Bricker 2016, Section 1.7.4).

The problem of implicit ontological commitment emerges because of the analytical links between different predicates. The problem, however, is not limited to the extrinsic properties of a type of entity. We later argue that the problems of advanced modality admits of a set-theoretical solution (chapter 8). In order to facilitate this solution we are required to establish an ontology of sets. We might be encouraged in this endeavour by the standard Tarskian semantics employed by Quine. By Tarskian semantics, every predicate of **PL** is assigned a set as a semantic value; sets must (therefore) exist as a presupposition of the quantifier criterion. We may suppose that the monadic predicate "∃(x) Horse(x)" incurs an implied ontological commitment to the set of horses, for example. This presupposition would be a mistake. Tarskian semantic theory deploys sets in order to *specify* the truth conditions of "∃(x) Horse(x)" but it does not imply that the truth conditions so specified impose a type of entity upon the universe; that it must 'contain' the *set* of all horses. By parity of reason, there is no presupposition that Tarskian semantic theory demands that the universe include horses in order to specify the truth conditions of the same predicate; contrast "∃(x) Unicorns(x)". Hence, the explicit ontological commitments of total-theory

appear not to (at least precipitately) include the apparatus of set-theory (see Quine 1953 [1980], pp 114-117).

Tarskian semantics aside, an ontological commitment to the existence of sets may also problematic on the grounds of metaphysical necessity. Metaphysical necessity appears to link (at least two) distinct types of entity, namely, a set and its membership. If we ignore the existence of the pure sets for the moment (those formed from the empty set), and stipulate that a set cannot exist absent its *actual* (this-worldly) members, then a commitment to the *actual* existence of *a* incurs a commitment to the singleton set "$\exists(x)\ x = \{a\}$": "There will be many sets that even exist from the standpoint of all worlds, for instance the numbers. Others may not, for instance the unit set of a possible individual might only exist from the standpoint of the world that the individual is in" (Lewis 1983b [1983], page 40). We will argue that this is a mistake; a set is always an abstract object even if the members of the set are not. A unitary set is no exception. We argue, therefore, that sets ought to be included amongst (at least) the *implied* ontological commitments of a theory (chapter 8). We now garner support for our argument from a surprising source.

Quine is often misconstrued as a "nominalist" although he rejects nominalism as "incoherent" (Quine 1986b [1998], page 26). We tentatively suggest that he is willing to attribute *implicit* ontological commitment(s) to a theory. Indeed, there are indications in his writing to support this contention: "*there are* perfect numbers above 100", "*there are* white rhinoceros" and "*there is* a mountain in Africa over 10000 ft." […] "Saying these things, we also say by *implication* that there are physical objects and abstract entities; for all the [white

rhinos] are physicals objects and all the [perfect numbers above 100] are abstract entities" (Quine 1951c [1976], page 205 my emphasis). An exegesis of Quine's many publications is beyond the scope of the present work but the seeds of doubt are sown. To claim that Quine recognises *only* the explicit ontological commitment(s) of a theory must account for Quine's explicit denial of that assertion. What motivates Quine to include the implicit ontological commitments of a theory? Are the implied ontological commitments "merely" ideological or objectival?

We must defer these interesting questions for another day. There are pragmatic reasons to ascribe implicit ontological commitment(s) to a theory. In respect of extrinsic properties, for example, an analytic truth may link two predicates used to describe the same object or person e.g. "*x* is a brother" and "*x* is a sibling". In respect of the apparatus of set-theory, we may wish to ascribe the "necessity" of set-membership to objects otherwise considered contingent and thereby grant an epistemic-semantic "access" to them from every possible world. We cite Quine as an advocate of the implicit ontological commitment(s) of a theory. However, one problem associated with our supposition is that Lewis explicitly rejects the thesis: to *imply* an ontological commitment to one type of entity (a set) on the basis of a commitment to another type of entity, is a pernicious dogma. We will anticipate the arguments of later chapters in order to discuss Lewis' explicit denial of the "singularist dogma" (Lewis 1991, pp 62-71). The anticipation is unfortunate but necessary given the context of our discussion viz. the implicit ontological commitment(s) of a theory to a type of entity (sets).

1.5 Plural Predication

In later chapters we argue that the problem of advanced modality admits of a set-theoretical solution. In order to facilitate this solution we are required to establish an ontology of sets. Lewis acknowledges that the **QC** could (in general) help identify the implicit ontological commitments of a theory (Lewis 1986a, pp 5-6). It is rather less likely, however, that he would number sets amongst a theory's *implicit* ontological commitments. We noted that orthodox Tarskian semantics assigns a set to every predicate of **PL** as a semantic value. That is, the extension of a predicate is understood as a single quantification over a *collection* of objects rather than an apparent plural quantification over a collection of *objects* (Lewis 1991, page 65). Predicates of **PL** are to be interpreted as a singular quantification over an object (a set or class-like entity). The distinction is not inconsequential:

> Plural quantification is irreducibly plural. It is not ordinary singular quantification over special plural things – not even when there are special plural things, namely classes, to be had. Rather, it is a special way to quantify over whatever things there may be to quantify over. Plural quantification, like singular, carries ontological commitment only to whatever may be quantified over. It is devoid of set theory and it is ontologically innocent. (Lewis 1991, pp 68-69).

Does an ontological commitment to a collection of talking donkeys, say, imply an ontological commitment to the *set* of talking donkeys? We are compelled to say "yes" (Lewis says "no"). This reply is one to which we shall cleave in order to propose a set-theoretic solution to the problem of advanced modality. We will address this problem directly in later chapters. Although not interpreted as such by standard Tarskian semantics, predicates of *plural*

quantification are familiar features of ordinary language. That is, some predicates of ordinary language may be true of some things without being true of any one of them individually e.g. “they are teammates”, “they lifted the piano”, “they were born in different countries” and “they weigh over 500lbs”. In these cases of plural predication, there is no requirement to postulate a set of individuals in order to interpret a sentence of which the predicate is a constituent. “We can say without apparent contradiction – and indeed truly – that there are some things such that for no predicate are they all and only the satisfiers of that predicate” (Lewis 1991, pp 64-65). Indeed, if a predicate cannot be true of an individual member of a set then the relation of set-membership becomes problematic (§ 8.3.1): “There are some things such that for no set are they all and only its members; and some things such that for no proper class are they all and only its members; and some things such that for no property are they all and only its instances” (Lewis 1991, page 65).

Consider by way of example the predicate expression “they lifted the piano” (see McKay 2006, page 5). By the standard semantics of **PL** this predicate is interpreted as a single quantification – say, as the *set* whose members collectively lifted the piano. Thus interpreted, the predicate is *false* of each member of the set (since no individual could lift the piano alone) and is only true *of* the set itself. *If* the predicate is false of each individual member of the set then how can an individual member of the set acquire set-membership? The putative member of a set is *both* the member of the set of which the predicate is true and yet fails to be a “member” of the set, as the predicate is false *of* it. “[Hence] to translate [a] seemingly true plural quantification into a contradictory singular quantification is to impute error – grave and hidden error” (Lewis 1991, page 65).

The fault is partially one of *distribution*: a predicate "F" is distributive if, and only if, by virtue of the meaning of "F", whenever some things are *F,* each one of them is *F* (see Lewis 1991, pp 63-65: McKay 2006, page 5 fn. 2) **[FN 13]**. Standard **PL** interprets every predicate as distributive. In the case of plural predication *not* all predicates are distributive. What is the non-standard explanation of plural predication? In his helpful exegesis of Lewis, John Burgess offers a distinction between those who are beguiled by the "singularist dogma" and those who are not. The "dogma" is a consequence of the completeness assumption of Boolean algebra that can be expressed in the terminology of second-order logic, so that "for any *class* of things there is a fusion thereof" (Burgess 2015, page 464). In the alternative, a theoretical ability to deploy plural quantification enables Lewis to exploit the interpretive resources of "monadic third-order logic". In the terminology of Hazen (Hazen 1997): "reading the formalism of that logic, a first-order variable *x* is said to range over *individuals*, a second-order variable *X* is said to range over *species*, and a third-order variable **X** is said to range over *genera*" (Burgess 2015, page 464). The distinction allows the expressive resources of Tarskian semantics to be interpreted in a non-standard way that avoids a commitment to an unnecessary ontological inflation, namely, sets.

Reply: We can perhaps mitigate the damage inflicted by plural predication with Lewisian mereology: "The simple principle of absolutely unrestricted composition should be accepted as true" (Lewis 1986a, page 212). Let us reconsider the plural predicate "they lifted the piano". We can be seduced by the 'dogma of singularism' and interpret the predicate as a set whose membership collectively lifted the piano. Let us suppose that this set has four members: Anne, Bob, Charles and Duncan. We now face the problem of non-distribution

that leaves the predicate true of the set but false of each individual member. A solution to the problem is available based on the notion of an 'unrestricted mereological sum': there are two sets, one of four members and one of a single member. In addition to the four people named there is a fifth (distinct) entity; namely, an entity of all the piano-lifting people and their parts, as its parts (say, when the group hold hands). The standard semantic value of the 'plural' predicate can now be re-interpreted as this second set whose single member is the mereological sum of those people who lifted the piano. The non-distributive predicate is true of the individual element of that second set; a single entity composed of four parts (people) together with their parts.

Lewis argues that we ought to dispense with the primitive notion of *is a member of* and replace it with a new primitive notion, *singleton* (Lewis 1991, page 61). Furthermore, that as a reflection of ordinary language the plural predicate interpretation, without a reliance on set-theory, is ontologically innocent. "[The singularist dogma] imputes ontological commitment where there is no ontological commitment" (Lewis 1991, page 68). In the alternative, we cleave to the standard Tarskian interpretation of 'plural' predicates whose extension is based on set-theory together with unrestricted mereological sums.

1.6: Conclusion

We have set the stage. We traced the development of Quine's criterion of ontological commitment through Russell's Theory of Descriptions and the latter's employment of modern **PL**. Although not extravagant, we argue that Quine does acknowledge the *implicit* ontological commitment(s) of a theory to a type of entity. Implicit ontological commitments

are to be included amongst all the ontological commitments of a theory (Lewis 1986a, page 5-6). Our endorsement of the implicit ontologically commitments of total-theory to sets raises the problem of plural predication. We were able to counter the problem by citing Lewis' notion of an 'unrestricted mereological sum'. We defer further discussion of the implicit ontological commitment(s) of a theory for later chapters in order to tackle another contentious issue: what are the ontological commitments of a theory when it is expressed in the idioms of ordinary language? Do the idioms of ordinary language carry any serious meta-ontological weight?

Footnotes

1. "World" is an ambiguous term. In the Lewisian sense the predicate designates a universe; a discrete spatiotemporal continuity. In addition to the primitive predicate "W(x)", he also introduces "Ixy" for the propositional function *x is in possible world y*, "Ax" for the propositional function *x is actual* and "Cxy" for the propositional function *x is a counterpart of y* (see Lewis 1968 [1983], page 27).
2. We should distinguish 'function' from 'functional expression', in Frege's account. A 'functional expression' is a linguistic item (e.g. a predicate) whilst 'functions' are extra-linguistic abstract entities (Frege 1891 [1980]: Miller 1998, page 306 fn. 12).
3. I do not wish to imply that Russell and Frege's philosophy of language align: Russell, for example, rejected Fregean "sense" on the grounds that it is both incoherent (Russell 1905 [2001], page 214) and superfluous (Russell 1905 [2001], page 214-215).

4. The four puzzles are (a) the apparent reference to nonexistents, (b) the problem of negative existentials, (c) Frege's puzzle about identity and (d) the problem of substitutivity. Russell argues that each can be resolved without the need to posit a Fregean "sense" (see **FN 3** above).

5. In *Principles of Mathematics* published in 1903, a *denoting concept* is invoked (in part) to account for such meaningful denoting failures as "the present King of France". The traditional interpretation of *Principles* is that although *terms* may not exist they "persist" in some form, perhaps in Russell's realm of *being*. Arguably, the traditional interpretation is superfluous since Russell explicitly states in that earlier work that not all *denoting concepts* denote: "[A] concept may denote although it does not denote anything […] These propositions appear to be true, and it would seem that they are not concerned with the denoting concepts, but with what these concepts denote; yet that is impossible, for the concepts in question do not denote anything …" (Russell 1903 [2010], Section 73 page 74).

6. See also Peter van Inwagen (1998) [2001], esp. page 13.

7. This definition includes a modal "must" incompatible with an extensional "referential" account of ontological commitment (see Quine 1953 [1980], pp 139-159). Although repugnant to Quine, **GMR** offers a solution – the 'unrestricted' domain of quantification includes types of entity that when assigned as the value of a bound variable yield true existential sentences: in this regard "∃x Unicorn(x)" is equivalent to "∃x Horse(x)". The domain of *possibilia* admits such ontological commitments as true (see Bricker 2016, Section 1.3).

8. Although Quine's philosophical *technique* is Russellian, his motivation and application of the technique is not. Quine (and Lewis), for example, aim to eliminate

all singular terms whilst Russell accepts a category of ineliminable singular terms or logically proper names (Quine 1960 [2013], Section 38: Hylton 2010, pp 383-384 fn. 2)

9. Definition: The Semantic Value of any expression is that feature of it that determines whether sentences in which it occurs are true or false (Miller 1998, page 10).

10. Quine's reluctance to include *de re* modality amongst *serious* discourse parallels his concern about true negative existential statements. "I suspect that the main motive for this expansion [possible entities] is simply the old notion that Pegasus, for example, must be because otherwise it would be nonsense to say even that he is not" (Quine 1948 [1980], page 4). It is at least questionable, however, whether Quine's injunction to reserve the paraphrase of ordinary language to those commitments that are "serious" is applicable in this case (see Quine 1953 [1980], page 103). Kripke's ontological commitment to fictional characters interpreted on the basis of the "quantifier-predicate" model is serious (Kripke 2013, page 69-70). The quantifier in question is expressed in "ordinary language" not **PL** (Kripke 2013, page 69). The legitimacy of quantification by means of ordinary language is the subject of Chapter 2.

11. The use of the word "preferably" is required at this juncture. A theory expressed in ordinary language may be assessed as ontologically committed to a type of entity but the *best* method of ascertaining a theory's ontological commitments is by regimentation into the canonical notation of **PL**: " […] I hail the relative pronoun as the crucial referential particle in ordinary language, and the variable as its mathematical version" (Quine 1998c, page 533).

12. Wilfrid Hodges, for example, notes: “First-order logic is very robust about questions like these: it doesn’t give a damn. If you think that there are fictional people and that they have or fail to have this property or that, and can meaningfully be said to be the same individuals or not the same individuals as one another, then fine, put them in your universes” (Hodges 2001, page 22).

13. The problem of distribution may be overcome by the admittance of “fuzzy sets” (Putnam 1975a [1996], pp 4, 44-45). In addition to distribution, McKay identifies the cumulative nature of the standard interpretation of predicates in **PL**. Cumulative in the following sense: “A monadic predicate F is cumulative iff the meaning of the predicate requires that whenever some things *X* are F and some things *Y* are F, then *X* and *Y* together are F” (McKay 2006, page 7).

Chapter Two: Ontological Commitment and Ordinary Language

> **Designate as S1:** I believe that there are possible worlds other than the one we happen to inhabit. If an argument is wanted, it is this. It is uncontroversially true that things might be otherwise than they are. I believe, and so do you, that things could have been different in countless ways. But what does this mean? Ordinary language permits the paraphrase: there are many ways things could have been besides the way they actually are. On the face of it, this sentence is an existential quantification. It says that there exist many entities of a certain description, to wit "ways things could have been". I believe things could have been different in countless ways; I believe permissible paraphrases of what I believe; taking the paraphrase at its face value, I therefore believe in the existence of entities that might be called "ways things could have been". I prefer to call them "possible worlds". (Lewis 1973, page 84)

2.1 Introduction

In chapter one we explored the notion of ontological commitment as it pertains to Lewis' modal translation scheme (Lewis 1968). The intensional language of **QML** is "translated" into the extensional language of predicate logic **PL** [**FN1]**. In consequence, Genuine Modal Realism **(GMR)** incurs a theoretical cost in the form of a substantive thesis, namely, the existence of a plurality of non-actual worlds. In this chapter we examine how the **QC** of

ontological commitment can be applied, not to predicate logic, but to ordinary language **[FN2]**. The shift is problematic for Quine because he denies that the serious ontological commitment(s) of a theory can be *directly* identified from ordinary language. The idioms of ordinary language are often referentially opaque. Hence, in order to identify the genuine ontological commitments of a theory, Quine argues that the sentences of ordinary language must first be explicated by paraphrase (preferably) into the canonical notation of **PL**. Lewis recognises no such methodological constraint. Persuaded by the **QC** of ontological commitment, the beliefs expressed in **S1** identify a type of entity that he refers to as "possible worlds" (as a paraphrase of the ordinary language expression "ways things could have been"). Lewis diverges from Quine's version of the **QC** to avail himself of the *serious* ontological commitments of a theory couched in the terms of ordinary language. We designate this approach "allism" **[FN3]** (Lewis 1990 [1999]: see Bricker 2016 Section 4.1).

We endorse Lewis' use of the **QC** of ontological commitment as expressed in the idioms of ordinary language (Section 2.2). We identify the criteria the satisfaction of which constitutes the paraphrase relation. Formal translation into to the canonical notation of **PL**, presumed to identify the *serious* ontological commitments of a theory, is not a criterion (Section 2.3). A common presumption of paraphrase, that it ought to result in the *explication* of concepts and thereby avoid the morass of intensional language, is examined (Section 2.4). We conclude with a brief exploration of the indexical phrase "ways things could have been". Are the phrases "ways things could have been" and "possible world" truly indexical terms? If they are, could they be used in a language stripped of all modal notions? (Section 2.5). We

conclude with remarks that relate to Lewis' third criterion of a genuine paraphrase (Section 2.6).

2.2 The Quantifier Criterion of Ordinary Language

The modal idioms of ordinary language exceed the expressive resources of standard **QML [FN 4]**. The sentences discussed in chapter one [left side, biconditional], that utilise the modal operators box "□" and diamond "◊", do not match the expressive resources of ordinary language (see Lewis 1986a, pp 13 ff.). Do counterpart theoretic sentences [right side, biconditional] also fail to match the expressive resources of ordinary language? How does Lewis account for the apparent ontological implications of the **QC** as expressed in the idioms of **S1's** ordinary language? Which segments of ordinary language – to borrow a phrase from Quine – are "ontologically serious"? In English, the existential quantifier can be expressed by a number of phrases that include "there is /are", "some", "few", "all", "most", etc. Quine's main objection to the claim that such expressions imply a *direct* ontological commitment to a type of entity is that ordinary language is referentially opaque (Quine 1998c [1998]). It falls to philosophers, therefore, to clarify which of the ontological commitments of ordinary language are genuine (and which are not) when the "existential quantifier" is used. This task is achieved by the regimented paraphrase of ordinary language into the (preferably) canonical notation of **PL**; where the serious ontological commitments to a type of entity are made perspicuous. Critically, an interlocutor may then reject the paraphrase in order to deny the ontological commitments ascribed to her theory:

> I do not advize giving up ordinary language, not even mentalistic language. But I urge awareness of its pitfalls … Ordinary language is only loosely referential, and any ontological accounting makes sense only relative to an appropriate regimentation of language. The regimentation is not a matter of eliciting some latent but determinate ontological content of ordinary language. It is a matter rather of freely creating an ontology-orientated language that can supplant ordinary language in serving some particular purposes that one has in mind. (Quine 1977 [1977], page 168: also Quine 1981a [1981], page 9)

In chapter one we identified a general doubt in regard to the sufficiency of the **QC** of ontological commitment expressed by **PL** (§ 1.3). Doubts about the necessary and sufficiency conditions of the **QC** persist: the **QC** will not be a *necessary* condition of ontological commitment if it under-generates, that is, if the criterion *fails* to entail all of the types of entity to which a theory *ought* to be ontologically committed (§ 1.4). Recall, one theoretical *desideratum* of **PL** is the fact that it is an extensional language. The idioms of ordinary language, on the other hand, are frequently couched in intensional terms so that fragments that include the logical operators "it is possible that" (modal), "it is believed that" (doxastic) or "it ought to be the case that" (normative) may conceal the ontological commitments to which our theory *ought* to adhere. The use of ordinary "intensional" language may under-generate the ontological commitments of a theory particularly *if* the quantifiers are taken to be disguised. In such circumstances, we may fail to recognise the ontological commitments of a theory to a type of entity because the quantifiers *are* disguised. In addition, the **QC** will not be a *sufficient* condition of ontological commitment if it over-generates (§ 1.3); if the criterion entails that a theory is ontologically committed to a type of entity to which it ought *not* to be committed. Again, fragments of ordinary

"intensional" language such as the aforementioned logical operators can be mistakenly identified as "disguised" quantifiers. Each "disguised" quantifier will then be understood as quantifying over a domain(s) of discourse; entailing a plethora of controversial entities *not* (obviously) entailed by the standard use of ordinary "intensional" language! Indeed, this is often taken the fundamental error of **GMR's**.

In order to be accurate, the "regimented" paraphrase of **S1** must expose the *latent* existential quantification expressed by ordinary language. However, the common belief that "things could have been different in countless ways" hardly appears to entail a belief in a plurality of concrete possible worlds where each possibility is instantiated! In a departure from Quine, Lewis appears to embark on 'eliciting some latent but determinate ontological content' from ordinary language. If this latent feature of a common belief is erroneous then Lewis is engaged in a blatant 'ontological inflation' (Daly 2010, page 96). Lewis appears to display an ambivalent attitude toward Quine's methodology: either he *is* embarked on an exploration of the ontological commitments of our common modal beliefs (contra-Quine) or he is *not* embarked on such an endeavour but has simply 'invented' an ontology to suit his theoretical purposes (pro-Quine). It appears that by accepting the **QC** Lewis' analysis of modal sentences has failed to satisfy a sufficiency condition of ontological commitment; it over-generates the types of entity to which a theory *ought* to be committed.

The application of the **QC** to the modal idioms of ordinary language requires a defence against the charge of ontological inflation. We may arm ourselves against this charge with

an orthodox interpretation of Quine's **QC**. To this end, we cite in full the passage in which the *meaning* of the existential quantifier "∃" of **PL**, is given. Quine argues:

> The given satisfaction conditions for negation, conjunction and quantification presuppose an understanding of the very signs they would explain, or of others to the same effect. A negation is explained as satisfied by a sequence when the constituent sentence is *not* satisfied by it; a conjunction is satisfied by a sequence when the one constituent sentence *and* the other are satisfied by it; and an existential quantification is satisfied by a sequence when the constituent sentence is satisfied by *some* suitably similar sequence. If we are prepared to avail ourselves thus of "not", "and", and "some" in the course of explaining negation, conjunction and existential quantification, why not proceed more directly and just offer those words as direct translations? (Quine 1986a, page 40 my underlining)

Contrary to Tarski, Quine argues that the *meaning* of the symbols of **PL** are derived from ordinary language. We might suggest, therefore, that Quine would be amenable to *direct* paraphrase from one set of ordinary language sentences to another set of ordinary language sentences – at least in principle, if not in practice. Clearly, the theoretical cost of such a principle would be high; a loss of clarity commonly ascribed to canonical notation.

However, although Quine does argue that the formal symbols of **PL** derive their meaning from ordinary language, there are well rehearsed reasons to suppose he would reject the notion of synonymy of two distinct terms (see Quine 1951a [1980], page 23: Quine 1960 [2013], Section 53 page 238, Quine 1998a [1998], page 73). Lewis has no such qualms. He argues that the synonymy of terms is a legitimate relation that might be deployed in

support of an argument. These terms include existential quantification: “The several idioms of what we call ‘existential’ quantification are entirely synonymous and interchangeable. It does not matter whether you say ‘Some things are donkeys’ […} or ‘Donkeys exist’ – you mean exactly the same thing whichever way you say it” (Lewis 1990 [1999], page 154: also Lewis 1970 [1983], pp 207-220). The use of synonyms for “existential” quantification is not restricted to the ‘dry goods’ of physical objects. They apply with equal legitimacy to more ‘controversial’ types of entity: “it does not matter whether you say ‘Some famous fictional detective uses cocaine’, ‘There is a famous fictional detective who uses cocaine’, or ‘A famous cocaine-using fictional detective exists’ – whether true or whether false, all three statements stand or fall together” (Lewis 1990 [1999], pp 154-155).

Hence. Lewis argues that discourse can be ‘ontologically serious’ even when it is conducted in the idioms of ordinary language. These idioms include the synonyms of formal existential quantification. All such expressions are legitimate and can be taken seriously, dependent on the context of their utterance (Lewis 1990 [1999], page 155). Admitting of some reservations, we conclude that Quine rejects the “allist” approach to ordinary language expressions of the **QC**. In order to identify a theory’s genuine ontological commitments Quine argues that the regimented paraphrase of ordinary language should proceed (preferably) by regimented paraphrase into **PL**. We now turn to examine the method of paraphrase deployed by both Quine and Lewis. If we accept **S1** as a philosophical technique then we are obliged to explain the nature of the paraphrase relationship. Is this relationship one of synonymy?

2.3 The Method of Paraphrase

If we accept the method of paraphrase as a philosophical technique (cf. Alston 1958: Sider 1999) then we owe an account of the relationship between paraphrased terms. Are the terms related by factual truth, logical truth, by synonymy or on pragmatic grounds? (Daly 2010, pp 89 ff.) We noted that Lewis does accept the context-sensitive synonymy of quantificational terms. How are the predicate expressions "ways things could have been" and "possible world" **[FN 5]** related? The aim of this section is to identify the general criteria the satisfaction of which generate a Lewisian paraphrase. What is Lewis' method of paraphrase? Is the Lewisian method based on the synonymy of terms?

Lewis' is initially motivated to secure the theoretical *desideratum* of a reductive analysis. To reduce the intensional language of **QML** to the extensional language of **PL** (§ 1.1). **S1** is an explication of modal discourse couched in the terms of ordinary language: for example, the meaning of the existential quantifiers of ordinary language equivalent to "∃" of **PL**. Combined with existential quantification, the paraphrase of the predicate expression "ways things could have been" by "possible world" commits **GMR** to a plurality of non-actual worlds. For Quine, an analytic statement is one that is stipulated as a matter of convention by a community of investigators to accord with their *purposes*. "But no synonymy relation is presupposed here; what matters is just that the paraphrase be acceptable to all parties for purposes of the continuing investigation" (Quine 1998b, page 95). Quine employs the **QC** as a means of identifying the ontological commitment(s) of a particular theory to a type of entity. If the regimented paraphrase of a portion of ordinary language is not committed to

*K*s then the theory can be acquitted of that particular ontological commitment. In order to assess a theory's parsimony we must first identify of the types of entity to which a theory is ontologically committed (§ 1.3).

What are Lewis' criteria of a legitimate paraphrase? In "Holes", David and Stephanie Lewis (1970) present an imagined dialogue between *Bargle*, a realist, and *Argle*, a nominalist. The paper can be used to illustrate Lewis' method of paraphrase in action. The dialogue concerns an unwelcome ontological commitment(s) to abstract objects, specifically holes. When expressed in the idioms of ordinary language, *Bargle* alleges that *Argle's* putatively 'nominalist' total theory is implicitly committed to holes. *Argle* accepts that the expressions of ordinary language *appear* to commit nominalism to abstract or immaterial objects; for example, "there are holes in this piece of cheese". This 'apparent' commitment to "immaterial" holes can be avoided, however, by the paraphrase of the relevant terms. The technique of paraphrase can obviate an ontological commitment not just to *these* holes (in this piece of cheese) but to holes *in general*. The method of paraphrase *should* (a) preserve the truth or falsity of hole-referring statements, (b) preserve the validity of any argument that employs statements about holes, and (c) be endorsed by a community of ordinary language-users (see Lewis & Lewis 1970 [1983], pp 3-9: also Bricker 2016, Section 4.2). A failure to specify the conditions of a genuine paraphrase simply defers the original problem of identifying the ontological commitments of a theory.

How do Lewis' criteria (a-c) apply in a case of an alleged ontological commitment to "holes"? *Argle's* initial attempt to avoid said commitment involves the use of a new primitive predicate, namely, "perforated" (Lewis & Lewis 1970 [1983], page 4). A sentence such as "there are holes in this piece of cheese" is paraphrased as "this cheese is perforated". The proposal allows *Argle* to 'speak with the vulgar' without incurring the burden of an ontological commitment to a type of entity, *holes*. The problem with the introduction of a new predicate is that it fails to satisfy a paraphrase in cases were 'holes' can be counted (see Lewis & Lewis 1970 [1983], page 4). In response, *Argle* adjusts the new predicate from "perforated" to "n-perforated", for each individual n: "[what] I mean by that is that either both pieces are singly-perforated, or both are doubly-perforated, or both are triply-perforated, and so on" (Lewis & Lewis 1970 [1983], page 4). Unfortunately, *Argyle's* adjusted n-place primitive predicate prompts further complications. Firstly, the paraphrase sentence (*analysans*) is now dependent on an infinite number of *primitive* n-place predicates, in clear violation of restrictions imposed by the use of *regimented* **PL**. Rather than engage in arbitrary "language chopping", primitive predicates ought to identify the joints at which "nature is carved" (§ 4.6). Secondly, the adjustment to an n-place primitive predicate fails to explain statements of comparison e.g. "how will you say that there are as many holes in my cheese as crackers on my plate?" (Lewis & Lewis 1970 [1983], page 4).

The strategy that promises nominalism the best chance of success is one of ontological reduction. A reduction in the number of ontological commitment(s) of a theory is best achieved by retaining the quantificational and referential apparatus of discourse *whilst* restricting the underlying domain to "uncontroversial" types of entity. Hence, a disputed

type of entity, such as *holes*, can be reduced by regimented paraphrase from "controversial" immaterial object(s) to "uncontroversial" material object(s). The ordinary language sentence "there are holes in this piece of cheese" is still subject to the **QC** but the assessed ontological commitments now relate to, say, material *hole-linings* not immaterial *holes*: "The lining of a hole, you agree, is a material object. For every hole there is a hole-lining; for every hole-lining there is a hole. I say the hole-lining *is* the hole" (Lewis & Lewis 1970 [1983], page 5) **[FN 6]**. Although the strategy of reduction is indicative of Quine's ontological motivation it is merely a subsidiary factor of the **QC**. Normally, the ontological commitments of a theory are identified as a precursor to the evaluation of a theory's ontological *cost*. The intention is to *avoid* an apparent ontological commitment to a controversial type(s) of entity. In the case of Lewis' passage **S1**, however, although the **QC** is used to ensure a theoretical reduction it is not to avoid an ontological commitment to a controversial type of entity, but to embrace one. As a Quinean "reductive" strategy, **S1** is deeply problematic. Not only does it appear to be at odds with the motivation behind the **QC** but also ascribes an *implicit* ontological commitment to total theory. Generally, the ontological commitments of a theory are made explicit by canonical notation (§ 1.4). The "reduction" postulated by **S1**, however, reduces "ways things could have been" to a paraphrase, "possible world". A modal expression of ordinary language is reduced to a philosophical term of art with an ontological commitment to a type of entity that is not obviously implied by the original expression. Furthermore, a reduction ought to be one that eliminates controversy. A type of controversial entity is reduced to a type of uncontroversial entity. Once again, in the case of **S1**, the shoe is on the other foot: by accepting the quantification-predicate model, a type of *uncontroversial* entity, namely, "ways things could have been" is reduced to a type of *controversial* entity, namely, a "possible world".

We identified the criteria (a-c), the satisfaction of which secures a genuine paraphrase. Lewis accepts, dependent on the context of utterance, the synonymy relation of terms. It appears that synonymy is a prime candidate for the paraphrase of "ways things could have been" by "possible world". Lewis states that personal preference guides his choice of paraphrase, perhaps on the grounds of "brevity or familiarity of expression" (Quine 1960 [2013], Section 40 page 175). We conclude that as a philosophical term of art "possible world" can be utilised for the sake of brevity as a genuine paraphrase. Does this conclusion breach the third of Lewis' criteria? We reserve the examination of this possible breach for subsequent chapters (esp. chapters 3 & 5). Meanwhile, we return to the issue of a genuine paraphrase. If they are *not* synonyms, what is the relationship between "ways things could have been" and "possible world"?

2.4 Explication

In order to answer this question we turn to a brief exposition of the philosophical method of explication. Quine's method of explication originated in the work of Rudolf Carnap. Although sympathetic to Carnap's scientific "total theory", antipathy toward the analytic / synthetic distinction led Quine to propose a distinctive account of explication **[FN 7]**:

> Explication is elimination but not all elimination is explication. Showing how the useful purposes of some perplexing expression can be accomplished through new channels would seem to count as explication just in case the new channels parallel the old ones sufficiently for there to be a striking if partial parallelism of function

> between the old troublesome form of expression and some form of expression figuring in the new method. (Quine 1960 [2013], Section 53 page 241)

Quine illustrates the method of explication with the problem of defining an ordered pair (Quine 1960 [2013], Section 53 page 236). We have encountered the notion of an ordered pair in relation to the set representation of a functional expression (in the form of a set whose members are an argument-value pair) (§ 1.2). The set {a, b} is identical to the set {b, a} since the order in which the membership of a set is represented is immaterial in respect of its identity. However, there are a number of set-theoretical purposes that require ordered pairs. Unless *a* and *b* are identical, the ordered pairs <a, b> and <b, a> are distinct. "Such entities must obey the fundamental postulate that the ordered pair consisting of *a* and *b* is identical to the ordered pair consisting of *x* and *y* if and only if $a = x$ and $b = y$" (Quine 1960 [2013], Section 53 page 237: also Hylton 2010, page 247). Should ordered pairs – as distinct from sets – be included amongst the types of entity to which a theory ought to be ontologically committed? Quine's answer is "No". He argues, based on several mathematical demonstrations by Wiener *et al*, that the resources of set-theory are sufficient to systematically account for ordered pairs (Quine 1960 [2013], Section 53 pp 238-239). The notion of an ordered-pair can be explicated in terms of set-theory.

The method of explication is linked to the enterprise of philosophy as a discipline. For Quine, philosophers begin their enquiries as physical objects embedded in a physical world. His philosophical ambition is "to ponder our talk of physical phenomena as a physical phenomena, and our scientific imaginings as activities *within* the world that we imagine" (Quine 1960 [2013], Section 1 pp 4-5 my emphasis). This is a departure from Carnap's form

of explication. Although empathetic toward Quine's predisposition in favour of the language of the empirical sciences, Carnap argues that explication is a type of philosophical clarification (Carnap 1962, pp. 3-5). For Quine, on the other hand, explication is a process by which we eliminate expressions that obstruct our understanding of ourselves as physical phenomenon (Quine 1954a [1976], pp. 228 ff.). This radical conception of the human condition informs Quine's views of philosophical analysis – genuine analysis should focus only on the observable characteristics of language: "Sentences are observable, and dispositions to assent are fairly accessible through observable symptoms … I begin with occasion sentences, indeed with observation sentences in my special sense" (Quine 1981b [1981], pp 184-5). Quine's philosophical approach to language differs from Carnap. For Carnap, language is a tool in need of clarification by philosophical analysis that in turn aids empirical science. For Quine, language is (partially) a physical "concrete" phenomenon. We qualify with the word "partially" because Quine does acknowledge that sentences are also a type of abstract object – universals. "A sentence is not an event of utterance, but a universal: a repeatable sound pattern, or repeatedly approximable norm." (Quine 1960 [2013], page 191). For Quine, sentences are analysed – explicated – in terms of sets (ordered pairs); they consist of sequences a1, a2, … , aϕ or sets of ϕ-pairs {a1, 1}, {a2, 2} … {aϕ, ϕ}. The elements of these ordered pairs are the characters and phonemes of the sentence uttered, understood as concrete instances of sound or inscription. Unuttered abstract sentences can be accommodated as sequences of ordered pairs the elements of which *are* also concrete observable phenomenon. Language is a concrete observable phenomenon because sentences are ordered pairs (sets) whose elements are concrete observable phenomenon (Quine 1960 [2013], Section 40 page 178). Quine includes sets as

abstract entities on the grounds of their "indispensability". That is, sets are an indispensable component of our best total-theory of the world (§ 8.3.2) (Quine 1954a [1976], page 244).

For Lewis, the philosophical study of modality is grounded neither in language nor thought (Lewis 1983a [1983], page xi). The metaphysical implications of the modal translation scheme are conceptually distinct from the semantic thesis (§ 5.3). As an idiom of ordinary language the phrase "ways things could have been" appears to express an indexical phrase. As such, it is doubtful that it or its paraphrase could be a component of an "eternal sentence" of canonical notation. Are indexical terms conducive to paraphrase into canonical notation? How comprehensible is a language free of indexical terms? We now examine the proposal that "ways things could have been" *is* both an indexical phrase and amenable to paraphrase into canonical notation (chapter 5).

2.5 "Ways Things Could Have Been"

Lewis accepts that the quantificational idioms of ordinary language *may* (dependent on the context of utterance) satisfy the **QC** (§ 2.2). Does this legitimacy extend to the modal idioms of ordinary language? Granting quantification, do the modal idioms of **S1**, such as "ways things could have been", actually refer to a type of entity? Could these types of modal idiom be explicated or eliminated from ordinary language? Could they be eliminated, say, in favour of terms more conducive to "eternal sentences"? We proceed on the supposition that although modal idioms are *not* eliminable they are nonetheless reducible. We adopt as

a *partial* test case the modal idiom of **S1** “ways things could have been” [but are not, **FN 8**]. If the term “actual” (and its cognates) is indexical (Lewis 1986a, pp 92 ff.) then the phrase “ways things could have been” is indexical. We are *only* able to identify the “way things could have been” [but are not] on the basis of how things actually are. If we accept a phrase of ordinary language *as* indexical then by implication it could not be constitutive in whole or in part of an ‘eternal sentence’ (Quine 1960 [2013], page 177: Kaplan 1977 [1989], page 506). As a paraphrase of “ways things could have been” the complex term “possible world” appears to be prohibited from any meaningful role in an ‘eternal sentence’.

Semantically, indexical phrases appear to be singular terms or referring expressions. Singular terms, such as pronouns, function as expressions that “pick out” particular things. Unlike most singular terms, however, the things “picked out” by indexical expressions vary with the context of utterance – the same expression can pick out different objects at different times and places. Sensitivity to the context of utterance appears to distinguish indexical expressions from other types of singular terms, such as proper names and descriptions. The same indexical phrase e.g. “I am thirsty” can be used to refer to different people depending on the context of utterance; it “picks out” me when I say it, and you when you say it. The puzzling philosophical aspects of indexical terms can be illustrated by Frege’s theory of sense and reference. In brief, the sense or “mode of presentation” of a word determines its reference. Hence, if two phrases share a sense then they must share a reference e.g. “neurologist” and “brain-doctor”. By way of contrast, if two phrases *differ* referentially then they must differ in their sense e.g. “Pope in Rome” and “Pope in Avignon”.

As a “descriptivist” (Lewis 1968 [1983], pp. 32-33) we might suppose that Lewis favours a Fregean analysis of indexical sentences. Although reasonable this supposition is problematic, however. Take the first-person pronoun “I” for instance. If the sense of a sentence that contains the word “I” determines its reference then the sense of “I” must change with the context of utterance. As in the case of the pontiff, if the referent of the indexical word “I” changes (dependent on the context of utterance) then the sense of the sentence that contains the word “I” must also change. Intuitively, however, the use of the *same* word by different people must share *some* common feature or meaning. The problem of identifying a ‘common meaning’ distinct from a Fregean sense is demonstrated by the terms “here” and “now”. The ‘common meaning’ of tokens of these indexical-types approximate to “the place of this utterance” or “the time of this utterance”. The referents of these phrases are context-sensitive e.g. “the place of this utterance” can be London, Paris or Rome depending on where the phrase is uttered. If the referents change with the context of utterance then the Fregean sense of “here” cannot be invariant, *mutatis mutandis* for the word “now”. The ‘common meaning’ of these indexical terms, therefore, should not be identified with their Fregean sense. The modal phrase “ways things could have been” appears to share the indexicality of “here” and “now”. Does the **S1** paraphrase of “ways things could have been” by “possible world” imply a ‘common meaning’ not captured by a Fregean sense?

David Kaplan (Kaplan 1977 [1989]: cf. Evans 1981 [1985]) isolates two elements of indexical sentences that constitute a shared or ‘common meaning’. Neither element corresponds to the Fregean notion of sense. The first element of ‘common meaning’ is the “proposition”

expressed by a sentence. Propositions are understood in the "Russellian" manner, that is, propositions are composed of objects and their associated properties. When I utter the phrase "I am thirsty" I express a *different* proposition from your utterance of the same phrase, because the object denoted by "I" is different in each case. However, the proposition expressed by the phrase "You are thirsty" (when uttered by you of me) is the *same* proposition expressed by my utterance of the phrase "I am thirsty". The same Russellian proposition can be expressed by the use of different words; for example, because the object denoted by "I" and "you" is the same. The second element of 'common meaning' is the "character" of an indexical type-expression.

> The character of an expression is set by linguistic conventions and, in turn, determines the content [proposition] of the expression in every context … it is natural to think of it as *meaning* in the sense of what is known by a competent language user (Kaplan 1977 [1989], page 505).

The character of an indexical expression is standardly considered to be a function from possible contexts of utterance to contents [propositions]. That is, it is the character of an expression that determines the content [proposition] of the expression in various contexts of utterance. Hence, my utterance of the indexical sentence "I am thirsty" has the *same* character as your utterance of the same sentence. Your utterance of the indexical sentence "you are thirsty" (said by you of me) has a *different* character [but the same content "proposition"] from my use of the phrase "I am thirsty" by dint of context-sensitivity. By **GMR** analysis, the indexical phrase "ways things could have been" may be used to express *different* propositions depending on the context of utterance [the only viable "context" in these circumstances being the actual world at which the phrase is uttered]. The proposition

expressed by this phrase is composed of the objects and properties of the world in which it is uttered. A change anywhere in the actual world is analysed by genuine realism as a change of world. Therefore, objects and their properties will differ depending on which world is actual. Different objects and properties entail different propositions. The character of the expression "ways things could have been" determines the content [proposition] dependent on the context of utterance. By **GMR** analysis, the context and content [proposition] of the expression relate to the actual world at which the phrase is uttered. The worldliness of the proposition implies that the relevant context is the actual world at which the relevant phrase is uttered. Just as the proposition expressed by the phrase "there is no more beer" is composed of the objects and properties of a fridge, that change with the context of utterance (from household to household), so the proposition expressed by the phrase "ways things could have been" is composed of the objects and properties of a world, that change with the context of utterance (from world to world). Kaplan argues that indexical sentences are typically those expressions whose content [propositions] vary with context (*ibid* page 506). Although Kaplan explicitly denies that "actual" is an indexical term, Lewis could avail himself of Kaplan's analysis of indexical-type sentences to argue that the phrase "ways things could have been" is an indexical term.

Quine postulates a number of different sentence-types. Eternal-sentences are those whose truth value is generally invariant. Their truth value does not change with the vagaries of our knowledge claims or beliefs. They include arithmetic sentences or sentences that specify the laws of nature. They are contrasted with standing sentences whose truth value can vary over time. Standing sentences include "George W Bush is President" (years) and "It is

Friday" (days). The third sentence-type is occasion sentences that closely resemble what we designate "indexicals". The truth values of occasion sentences change with the circumstances and context of their utterance (Hylton 2010, page 112). Occasional sentences include but are not limited to indexical expressions. Indexicals or "Indicator words" may be substituted by regimented paraphrase to canonical notation in the form of specified objects, places, times etc. These specifications may establish systems of reference to particular spaces (latitude and longitude) and times (GMT), for example. The motivation behind canonical notation is to render explicit the truth values of context dependent terms (Hylton 2010, page 271). In addition to indexical expressions, "tensed verbs" that relate to past or future events can be regimented by explicit indicators of time (Quine 1960 [2013], Section 36 page 155). "Quantifiers" may also be treated as timeless. The analysis of the quantified phrase "Aristotle is mortal" includes Aristotle in the domain of discourse regardless of whether he did, does or will exist (Hylton 2010, pp 271-72).

With these points in mind, the question is this: can the indexical phrase "ways things could have been" be explicated or eliminated from ordinary language, say, as a term of canonical notation and component of an 'eternal sentence'? "So we need to show how we could replace the context dependent sentences which are actually uttered with eternal sentences, true or false once for all" (Hylton 2010, page 271). The problem with this endeavour is generality. A language free of indexical terms can only be used to express general sentences. For example, we may wish to refer to an object by way of a predicate (see § 1.2). Unfortunately, the same predicate may equally hold of multiple objects. How do we to refer to a *unique* object in a language stripped of indexical terms? As previously suggested,

reference to a unique object might be secured by the use of terms that render the object's spatio-temporal location *explicit*. A system of reference, say, location by latitude and longitude at Greenwich Mean Time could be employed. This solution is unsatisfactory, however. Suppose we wish to refer to a unique object, namely, the Royal Observatory in Greenwich Park. The Meridian (the line of zero degrees longitude) is located at the Royal Observatory. The Royal Observatory is one of the places on the Earth's surface where the Meridian is "nailed down". We cannot, on pain of circularity, apply a system of reference based on latitude and longitude at Greenwich (GMT) in order to make explicit the location of the Royal Observatory. That is, state that the Royal Observatory is located at the Meridian *and* the Meridian is located at the Royal Observatory. Alternative means of specifying the Royal Observatory might be available but it is difficult to establish the required uniqueness of reference (Hylton 2010, pp 271-72).

Quine does *not* intend 'explication is elimination' to replace the idioms of ordinary language (§ 2.2). We find that the modal idioms of ordinary language are indexical in the sense of their sensitivity to context. As a cognate of the phrase "actual world", the content [propositions] of the phrase "ways things could have been" varies with the context of utterance. This is the mark of an indexical term. As modal idioms of ordinary language, or paraphrase thereof, these terms fail to satisfy the precepts of an 'eternal sentence'. Although not eliminable, these idioms are reducible to sentences of **PL**. The regimented paraphrase of "ways things could have been" by "possible world" preserves the context sensitivity indicative of an indexical phrase.

2.6 Conclusion

We argue that the existential quantifier may be expressed in the idioms of ordinary language. Formal translation into to the canonical notation of **PL** is not required. In a (probable) departure from Quine, we argue that a paraphrase need not be regimented but may yield translations that are trivial yet informative. We argue shortly that a paraphrase of *F* by *G* is trivial because it imposes no constraints on the world (*viz.* the "insubstantiality thesis" at § 5.4). This "pragmatic" synonymy of terms is nonetheless informative because *F's* ontological commitment to a type of entity is implicit; identified only after a conceptual reduction to a paraphrase of *F*, namely *G*. In summary, we identify the criteria (a-c) of a genuine paraphrase. Modal idioms of ordinary language can be paraphrased into the extensional idioms of ordinary language. However, *if* these extensional idioms result in an ontological commitment to a 'controversial' type of entity then we breach one of the conditions of genuine paraphrase; condition c. If a community of ordinary language users does *not* endorse an ontological commitment to Lewisian "possible worlds" then **S1** fails to satisfy one of the necessary conditions of paraphrase. We now attempt to identify the source of the incredulous stare that threatens the theoretical *desideratum* of accord with common sense (and condition c).

Footnotes

1. Quine does countenance alternatives to **PL** as the "preferred" language of canonical notation dependent on our purposes (Quine 1998d [1998], pp 533 ff.). As a

consequence, **PL** is a sufficient but not a necessary condition to "limn the true and ultimate structure of reality" (Quine 1960 [2013], Section 45, pp 198 ff.)

2. H.J. Glock argues that Gilbert Ryle identified an ambiguity in the use of the term "ordinary language": the term may refer to either a *standard* use (to be contrasted with the irregular use of the term in a specific subject) or the *everyday* use (to be contrasted with the use of a term by an expert) (see Glock (2008) *What Is Analytic Philosophy?* Cambridge University Press: 14). Actually, Ryle does not identify an ambiguity in the use of a single term "ordinary language". Instead, he sharpens a distinction too often ignored in philosophical argument (circa. 1953): a distinction between two separate phrases, "the use of ordinary language" and "the ordinary use of the expression '…'". In the former or standard case, "the edges of 'ordinary' are blurred, but usually we are in no doubt whether a diction does or does not belong to ordinary parlance" (Ryle (1953) [2009], page 315). In the latter or everyday use of the term the question of the standard use is irrelevant. "If a term is a highly technical term, or a non-technical term, there remains the distinction between its [standard] stock use and non-stock uses of it" (Ryle (1953) [2009], page 315).

3. The Lewisian "allist" may over-generate the types of entity to which a theory *ought* to be committed. The distinction between substitutional and objectual quantification can be lost, for example. Objectual quantification (Tarski) states that an existential quantified sentence "$\exists x\, \phi(x)$" is true iff the open formulae "$\phi(x)$" is satisfied by some object in the domain of discourse. Substitutional quantification states "$\exists x\, \phi(x)$" is true iff "ϕt" is true, for some singular term "t" of the language. Hence, the **QC** may over-generate if the language includes empty singular terms e.g. "Sherlock Holmes". By Quine's interpretation, all quantifiers are to be interpreted objectually (Quine

1969 [1969], page 106: Quine 1974, page 139: van Inwagen 1981 [2001]: cf. Gottlieb 1980: Marcus 1993)

4. See Lewis 1986a, pp 8-14, esp. 12-13: the expressive resources of **QML** are either "redundant" or "inadequate" (Divers 2002, page 309 fn. 26).

5. The adjective term "possible" in the phrase "possible world" may be *alienans*. If a substantive term need not or cannot apply on its own (an expectant father need not be a father and a counterfeit £10 note cannot be a genuine £10 note) then the adjective ("expectant" and "counterfeit") is said to be alienans. Hence, as a "possible world" need not be a world the adjective "possible" could be alienans in this case (cf. chapter 4).

6. Although problems with the reductive strategy persist e.g. the identity conditions of *holes* and *hole-linings* do not coincide (see Lewis & Lewis (1970) [1983], page 8) there are several prominent examples of the strategy in play: Davidson (1967) argues that action sentences are ontologically committed to *events.* Adverbs, such as "slowly", describe an *event(s)* over which the action sentences in which they are embedded *tacitly* quantify. Hence, sentences such as "John walked slowly" justify the inference that "John walked".

7. Quine's primary objection to analyticity is related to the apparent synonymy relation required as the foundation on which the analytic truth holds (Quine 1951a [1980]).

8. By **S1**, "ways things could have been" is itself a paraphrase of "things could have been *different* in countless ways" (my emphasis). Reference to a difference in the actual world is analysed (in part) by an ontological commitment to non-actual worlds. Hence, we exclude the actual world from the discussion of indexicality.

Chapter Three: Beyond Belief

"The question is this: is it crazy enough to be true?" Niels Bohr **[FN 1]**

3.1 Introduction

Lewis' third criterion of a genuine paraphrase echoes a theoretical virtue often cited as a decisive reason to reject **GMR.** The usual (and perhaps normal) reaction to Lewis' **GMR** is incredulity. The incredulity objection is articulated by Sider: "perhaps I speak for the majority when I say that I don't really know why I find the incredulous stare compelling; I only know that I do" (Sider 2003, page 194). We will examine the role played by common sense judgement – and the role it *ought* to play – in the assessment of theoretical virtues and vices. We maintain that Lewis' commitment to theoretical conservatism is a breach of his naturalist epistemology. Sider's incredulity should prompt the reply articulated by Peter van Inwagen: "it seems to be the practice of scientists (one we philosophers should adopt) not to reject the only workable theory simply because that theory is (or seems to be) incredible" (van Inwagen 1986 [2001], page 227). Naturalised epistemology maintains that there is no non-arbitrary distinction between philosophy and science. As an advocate of naturalised epistemology, Lewis *ought* to deny common sense judgement a place at the table of reflective equilibrium. He argues that the scientific picture of the world is not just *correct* but *complete* (Lewis 1994 [1999], page 292). In consequence, the only theoretical virtues that we *ought* to consider in the philosophical process of reflective equilibrium are

those of the natural sciences, especially physics. To object that the use of Bohr's quote (above) is misplaced in a philosophical thesis is to ignore Lewis' explicit 'materialism'. As a "materialist" Lewis states that "if we optimistically extrapolate the triumph of physics hitherto, we may provisionally accept that *all* fundamental properties and relations that *actually* occur are physical" (Lewis 1994 [1999], page 292 my emphasis: Lewis 1983a [1983], page *xi*) **[FN 2]**. The fact that the *completeness* of physics only pertains to *actual* world is not relevant. The important factor for our purposes is not the scope of the subject matter but the theoretical virtues that are recognised by the natural sciences: one aspect of a naturalised epistemology is the thesis that there is no "first philosophy". There is no viewpoint "outside" the natural sciences from which they can be critically assessed (Nolan 2005, page 11). In the absence of a "first philosophy" philosophers are bound to observe the theoretical standards of the natural sciences.

Lewis argues, however, that a standard to which philosophical theories *ought* to be held, but (presumably) from which scientific theories are exempt, is theoretical conservatism. For Lewis, any philosophical theory that offends against common sense judgement incurs a defeasible cost.

> [However] Common sense has no absolute authority in philosophy. It's not that the folk know in their blood what highfalutin' philosophers may forget. And it's not that common sense speaks with the voice of some infallible faculty of "intuition". It's just that theoretical conservatism is the only sensible policy for theorists of limited powers, who are duly modest about what they could accomplish after a fresh start. Part of this conservatism is reluctance to accept theories that fly in the face of

> common sense. But it's a matter of balance and judgement. Some common sense opinions are firmer than others, so the cost of denying common sense opinion differs from one case to the next. And the costs must be set against the gains. Sometimes common sense may properly be corrected, when the earned credence that is gained by making theory more systematic more than makes up for the inherited credence that is lost. (Lewis 1986a page 134, also 1983a [1983] page *x*, 1996a [1999], page 433).

What "type of case" is the theoretical postulation of a plurality of worlds? Does the "infallible faculty of 'intuition'" supply an indefeasible reason to dismiss the notion out-of-hand? Is there a more nuanced view that grants common sense judgement a defeasible plausibility? These are the questions we address in this chapter. How certain is the "intuition" that the postulation of a plurality of worlds is incredible? Do the folk know in their blood what the highfalutin' philosophers have forgotten? We intend to identify the source of Sider's incredulity and satisfy Lewis' third criterion of genuine paraphrase.

The difference between common sense judgement and **GMR** can be demonstrated by three sentences. Three types of sentence can be distinguished on the grounds of their (putative) assertion of a common sense judgement:

A1: It is possible that talking donkeys exist

A2: There are no actual talking donkeys

A3: There is a talking donkey at a possible world

A1 is an assertion of both genuine realism and common sense judgement. **A2** appears to be an assertion of genuine realism and common sense judgement, given an indexical understanding of the term "actual" (§ 2.5 & 5.3) (see Lewis 1986a, pp 97-101: cf. Soames 2007) **[FN 3]**. **A3**, on the other hand, is true by the lights of genuine realism but false by the lights of common sense judgement. The divergence of opinion at **A3** is based on the postulation of a plurality of worlds other than *this* the actual world. **A3** is an assertion by the genuine realist of what common sense judgement denies, namely, the existence of non-actual talking donkeys premised on the existence of non-actual worlds.

Lewis maintains that common sense *knowledge* can be philosophically relevant in one of two ways (Lewis 1996a [1999], page 418): firstly, in the context of everyday conversation, we all know many things. I know that pandas eat bamboo. I know that there were only four TV channels available in the UK, but now there are many more. I know that Hull FC won the 2016 and 2017 Rugby League Challenge Cup finals. We each possess a reservoir of 'context sensitive' knowledge. The "folk knowledge" that we all possess in the context of everyday life may evaporate, however, with a change in context, say, from the Clapham Omnibus to the philosophy seminar. By way of contrast, Lewis acknowledges a second type of knowledge that no philosophical analysis or argument can dislodge. Indubitable knowledge of a "Moorean fact" is immune from the vagaries of sceptical doubt by virtue of the principle of differential certainty: we are each more certain of the truth of a Moorean fact than we are of any premise of a philosophical argument that might be deployed to doubt it (cf. Unger 1975, Stroud 1984). For the purpose of exposition, we stipulate that Moorean facts constitute the core of 'total theory' whilst broader 'folk knowledge' relates to non-

Moorean facts. We distinguish, therefore, three types of common sense judgement: indubitable knowledge (by differential certainty) of Moorean facts, defeasible "folk" knowledge of non-Moorean facts and a peripheral "web of belief". To identify the source of Sider's incredulity we need to identify into which of these three categories the postulation of a plurality of worlds falls.

Lewis argues that **GMR** offers the *best* philosophical explanation of our common sense modal beliefs (cf. Stalnaker 1976/1984 [2003], page 26 ff.). We begin by identifying the type of theoretical conservatism endorsed by Lewis (§ 3.2). In order to isolate the source of Sider's incredulity, we assess the claim that "there is only one world or universe" is knowledge of a Moorean fact. The certainty associated with this type of knowledge claim is that of "negational absurdity" (§ 3.3). In the alternative, if the claim that "there is only one world or universe" is *not* knowledge of a Moorean fact, it may qualify as folk knowledge. Folk knowledge is identified as a claim that does not depend on specialised or technical terminology (§ 3.4). Finally, we argue that Lewis' philosophical interest in common sense belief / intuition is not in terms of their potential evidential value but is rather an attempt to align *his* beliefs / intuitions with reality (Lewis 1983a [1983], page *x*) (§ 3.5). We conclude that Sider's incredulity is misplaced. The claim "there is only one world or universe" is one aspect of folk knowledge, that is, knowledge of a non-Moorean fact. As such the claim is defeasible in the face of countervailing factors such as the theoretical virtue of simplicity. Folk knowledge is subject to change by, say, the introduction of new terminology (§ 3.6).

3.2 Moorean Facts and Theoretical Conservatism

The claim that *some* statements, namely, those used to express our common sense knowledge of the world, are philosophically significant can be traced to antiquity **[FN 4]**. The claim is adopted by G.E. Moore (Moore 1925 [1993], 1939 [1993]) in his attempt to demonstrate that he knew with certainty the truth of a number of contingent propositions. That is, "contingent" propositions we know with certainty to be true. The certainty ascribed to these propositions contrasts with the certainty demanded by Descartes. Certain truth in the Cartesian sense is immunity from *all* doubt. The set of contingent propositions that Moore holds with certainty to be true, on the other hand, are those insulted against philosophical theories – such as Scepticism and Idealism – that could be used to doubt them. Moore opens "A Defence of Common Sense" with a diverse list – he designates List (1) – of propositions that relate to his mind, his body and other physical objects that he claims to know with certainty to be true (see Moore 1925 [1993], pp 107-108). They include the proposition(s) that "there exists at present a living human body, which is *my* body … Ever since it was born, it has been either in contact with or not far from the surface of the earth" (Moore 1925 [1993], page 107). He then states – as single proposition (2) – that many other people share in respect of their own minds, their bodies and other physical objects a similar reservoir of propositions that they know with certainty to be true (Moore 1925 [1993], page 108-109). Moore concedes that although he knows with certainty that many of these contingent propositions are "truisms" he does not know *how* he knows them. His *defence* of common sense should not, therefore, be understood as foundational in the epistemological sense (Moore 1925 [1993], page 118).

It appears likely (see Nolan 2005, page 208), that Lewis adopts the notion of a "Moorean fact" from Armstrong (see Armstrong 1980, page 440-441 cited Lewis 1983d [1999], page 20). Lewis includes the fact that we enjoy numerous instances of "folk knowledge" as a Moorean fact (Lewis 1996a [1999], page 418). Further examples include our knowledge of the sameness of type – that two objects share the same shape, for instance (Lewis 1983d [1999], pp 20 ff.), or our knowledge that language has a broadly determinate interpretation (Lewis 1983d [1999], page 47), or our knowledge that any attempt to deny the law of non-contradiction "should be dismissed just because the hypothesis it invites us to entertain is inconsistent" (Lewis 1982 [1998], page 101). In addition, that our knowledge of the psychophysics of colour is (approximately) true; that is, we know that objects are coloured, that they retain their colour (as a property) even when they are not observed and that colour is determined by sense perception (Lewis 1997 [1999], page 333). Hence, there is a core of contingently true propositions that present themselves as candidates of knowledge of Moorean facts.

Do these candidates share a common feature or characteristic? A number of factors have been identified with Moorean facts despite a canonical definition remaining elusive. Logical truths – at least those that are knowable – are prime candidates of Moorean fact-hood. As a formal criterion, however, logical truth fails to capture the peculiarity of the contingently true propositions enumerated by Moore. We may find the claim that the Earth has *not* existed for more than 100 years rather odd, but it is not logically inconsistent. Armstrong suggests that a Moorean fact is "one of the many facts that even philosophers should not deny, whatever philosophical analysis they give of such facts" (Armstrong 1980, page 440-

441 cited Lewis 1983d [1999], page 20). Armstrong's criterion is also unsatisfactory; "… presumably, *any* truth has some claim to be something that "even philosophers should not deny"" (Nolan 2005, page 208). Elsewhere, Lewis suggests that the denial of a Moorean fact is symptomatic of "philosophical (or scientific) error" (Lewis 1997 [1999], page 333) *and* that "to doubt [a Moorean fact] would be absurd" (Lewis 1996a [1999], page 418). We will shortly distinguish the notions of "error" and "absurdity" in order to explicate the notion of a Moorean fact. We will argue that the denial of a Moorean fact is not a consequence of error but absurdity. It is the absurdity of denial that gives differential certainty epistemic bite: "[a Moorean fact is] one of those things that we know [with certainty] better than we know the premise of *any* philosophical argument to the contrary" (Lewis 1996a [1999], page 418 my emphasis).

Lewis advocates a *weak* form of theoretical conservatism. Theoretical conservatism is a principle of philosophical methodology that states our pre-philosophical judgements ought to be accorded a privileged status even *after* our philosophical deliberations are complete. Theoretical conservatism is the standard that grants a philosophical theory plausibility. Thus predisposed, a philosopher presented with the ontological commitments of **GMR** is left incredulous. That is, **GMR's** analysis of modality generates a set of beliefs in the existence of entities that have nothing to do with common sense opinion. We designate Lewis' theoretical conservatism "weak" because it is defeasible in the face of other theoretical virtues weighed in the process of reflective equilibrium.

We might object that as a virtue theoretical conservatism is flawed. The flaw pertains to the notion of reliability. The problem of reliability is best illustrated in respect of common sense *belief*. Consider the case of the opinionated coin-flipper (Christensen 1994, page 74 cited Daly 2010, page 21). The coin-flipper flips, what he believes to be a fair coin, and without looking forms the opinion that it landed on heads. The mere fact that he has formed this opinion does not justify his belief that the coin has landed on heads. He is simply dogmatic. However, theoretical conservatism states that not only does the coin-flipper have reason to hold this belief but that the reason is justified. The unreliability of the belief is generated by the process of belief formation. The coin-flipper's belief that the coin landed on heads has a cause, say, his obstinate attitude. The causal origins of the belief, however, do not generate a justification for holding it. Additionally, the traditional conception of knowledge as justified true belief, means that the coin-flipper's unjustified belief cannot qualify as knowledge and so cannot amount to knowledge of a Moorean fact (cf. Lewis 1996a [1999], page 421)**.**

As a counter-example to conservatism, the obstinate coin-flipper generates a problem of reliability. We may mitigate the potency of the "coin-flipper" objection if we turn from instances of belief to knowledge claims based on common sense judgement. Knowledge of a Moorean fact is indubitable (by differential certainty) and outweighs, so to speak, the certainty of any premise of an argument deployed to doubt it. Hence, if knowledge of a putative fact "there is only one world or universe" is knowledge of a Moorean fact then Lewis' method of reflective equilibrium is in vain. There is no philosophical or scientific premise of an argument that could be used to doubt it. However, Lewis both admits

Moorean facts and endorses theoretical conservatism as a virtue. This suggests that "there is only one world or universe" is *not* a Moorean fact, in his opinion. As a claim of folk knowledge, on the other hand, the claim may be true but defeasible. Therefore, knowledge of the putative fact that "there is only one world or universe" appears to be a candidate for "weak" theoretical conservatism (Lewis 1986a, page 134). Knowledge of this type of "fact" is defeasible and might be overturned by alternative theoretical virtues that outweigh conservatism. Hence, our examination of the incredulous stare proceeds by testing the supposition "there is only one world or universe" against three types of common sense judgement: knowledge of a Moorean fact, folk knowledge of a putative fact and as the content of a belief.

3.3 The Absurdity of Denial

All Moorean facts are indubitable. Not in the Cartesian sense of doubt as a species of thought but in the Moorean sense of differential certainty. It is time to dispense with an equivocation in the notion of a Moorean fact as either an instance of Cartesian "necessity" or Moorean "contingency" (and their associated scare quotes). Any putative Moorean fact must be epistemically necessary in order to qualify as such. Furthermore, Lewis' "weak" theoretical conservatism admits of countervailing virtues that are irrelevant in the case of a Moorean fact. Let us proceed on the supposition that **MF** is a Moorean fact:

MF: There is only one world or universe

We offer **T1** as a definition of core membership of the common sense weltbild:

T1: A proposition p is a core member of the common sense *weltbild* (and thereby S knows that p with certainty to be true) if, and only if, the truth of p is made true by a Moorean-fact **[FN 5]** (see also Moore 1925 [1993], page 119).

If the object of knowledge (that which is known) is a Moorean fact then (trivially) we cannot fail to know it (Moore 1925 [1993], page 119). This high standard of epistemic privilege secures immunity from all "reasonable doubt": we are more certain of the truth of **MF** than of any premise that could potentially be deployed in order to doubt it. Consider two lists of sentences as potential candidates for core membership of the common sense *weltbild* (see Stroll 1994, pp 33-34):

List A:

1. The Universe is very old
2. My desk is made of wood
3. I am a human being
4. Blood flows through my body
5. Human beings experience emotion: they can be happy, sad, worried, hopeful etc.

List B:

1. The Universe is 13.8 billion years old
2. There exist many different things in the Universe all of which are constructed from elements mentioned in the Periodic table
3. The Universe is expanding
4. The Magna Carta was signed in 1215
5. Human life begins 14 days after conception

In order to distinguish a Moorean fact we employ the notion of "negational absurdity" (see Wittgenstein 1953 [1969], G.E.M. Anscombe and G.H. von Wright (eds.) *On Certainty*, Blackwell: Sections 185-190). The philosophical technique of negational absurdity is employed by Wittgenstein to distinguish between a mistake and an aberration. A mistake or error occurs when we follow the normal procedure but go astray: I am asked to count the chairs in the auditorium, I complete the task and calculate that there are 299 chairs when in fact there are 300. These circumstance of a mistake or error are distinguished from the case of a paranoid who believes that her drinks have been poisoned by an unknown assailant. She has not made a mistake when she discovers that every drink is safe. Rather, she is suffering from a conceptual aberration or misunderstanding. Moore famously claims to know that the Earth has existed for many years before he was born. How does he know? Moore is unable to justify his knowledge claim. In the absence of justification, is the counter claim to know that the Earth sparked into existence 150 years ago warranted? It might be, since although we presume the counter claim is false it is not logically impossible. Moore concedes the point but maintains that it is nonetheless absurd. A knowledge claim is negationally absurd if "it runs counter to what we all know, that is, runs counter to that sort of primordial proposition we call the common sense view of the world" (Stroll 1994, page 45). We acknowledge that Stroll here restricts the phrase "we all know" to knowledge of Moorean facts. If he did not, then the denial of *any* knowledge claim that "we all know" would be negationally absurd! Instead, if any proposition – whose truth is determined by a Moorean fact – were found to be "false" the world would be incomprehensible. Hence, the denial of any proposition whose truth is determined by a Moorean fact, has this property of negational absurdity. To deny a proposition of List A, for example, is negationally absurd, so these propositions are amongst the core members of the common sense *weltbild*. The

denial of a proposition of List B is *not* negationally absurd, so these propositions are not among the *core* members of the common sense *weltbild*.

Does **MF** satisfy the criterion of negational absurdity? Consider the denial of **MF** or **dMF**;

dMF: (It is not the case) that there is only one world or universe

At first blush, **dMF** appears to be a little odd. Yet, one interpretation of theoretical physics indicates that even *if* it is odd it is nonetheless true. The Everett Interpretation of Quantum Mechanics suggests: "… when we do take it [quantum mechanics] literally, the world turns out to be rather larger than we had anticipated: indeed, it turns out that our classical "world" is only a small part of a much larger reality" (Wallace 2012, page 13). The Everett Interpretation is believed to be true albeit by a minority of physicists. One implication of the Everett Interpretation of Quantum Mechanics is that there are "many universes". Hence, the scientific postulation of many universes is at least plausible. We infer that **dMF** is not absurd even *if* the truth of the Everett Interpretation is disputed. **MF** is denied without absurdity by a faction of the scientific community so we conclude that **MF** does *not* express a Moorean fact.

Does this example of a scientific interpretation of Quantum Mechanics establish our conclusion? Are we not more certain of **MF** than of any premise of a scientific theory employed to refute it? To establish the legitimate use of a scientific theory to challenge **MF** as a Moorean fact, reconsider List A. Any scientific or philosophical theory that denied the truth of a proposition in List A would be widely *rejected*. "If a science were to develop, as

some eliminativist materialists suggest, which flatly denied that people have feelings, are or have ever been in love, and so on, such a "science" would be discredited. Here common sense is dominant" (Stroll 1994, page 35). If **MF** were a proposition at the core of the common sense *weltbild* then knowledge of its truth would be indubitable on the grounds of differential certainty. "If there is a role for science in this domain, it is to add to and not subtract from that which everyone knows" (*ibid*, page 35). Unlike eliminativist materialism, the Everett interpretation of Quantum Mechanics supplements physics with a multiverse that is coherently believable. If **MF** were a Moorean fact then this scientific belief would be incoherent.

Does the criterion of negational absurdity carry the weight we require of it? Is there a third way between mistake and aberration that confounds negational absurdity as a criterion? Crispin Wright suggests a thought experiment that might offer a safe passage between the Scylla of mistake and the Charybdis of aberration – the Cautious Man (Wright 1980). The origins of the Cautious Man are located in Wright's critique of a postulated "special" faculty used to recognise a necessary truth. The faculty under investigation is one of "a *special* certainty [enjoyed], at least in a wide class of fundamental cases, by our apprehension of necessity, it is easy enough to call it into question; rather harder to see how it might actually be demonstrated to be wrong" (Wright 1980, page 446). The cautious attitude is intended to show that this alleged faculty of recognition does not exist. To illustrate, Wright suggests that we consider a relatively straightforward case of necessary entailment – the rule of conjunction elimination (**CE**). The rule of conjunction elimination enables the deduction of a conjunct from a conjunction in which it appears; from "P & Q" we may deduce "P". Wright

invites us to consider the following conditional statement of conjunction elimination: ((P v Q) & (P v R)), therefore P v Q. “In order intelligibly *not* to accept such a proof, a person does not have to do anything which will indicate a lack of understanding or misunderstanding, or a mistake. A quite different possibility will be his adoption of [a] cautious attitude” (Wright 1980, page 452-53). Traditionally, **CE** is logically valid. There is (and indeed could not be) a counter-instance to conjunction elimination: if the premise is true the conclusion *must* be true. However, the Cautious Man is reluctant to assert the logical *necessity* of the entailment.

Is the Cautious Man’s reluctance the result of a mistake or aberration? No. The Cautious Man is aware of the correct procedure to follow in regard to **CE** (just as I was aware of the correct procedure when counting chairs in the auditorium). Hence, his reticence is not a mistake on procedural grounds. He is simply reluctant to assert the kind of modality normally associated with logical entailment. He understands the basis of the proof, the process by which we arrive at the conclusion and accepts the outcome. His “failure to follow the proof” is no more than a reluctance to assert the logical necessity that (presumably) holds between the individual steps – basis, process and outcome.

> He would [simply] refuse to regard the capacity of a proof apparently to survive arbitrary close scrutiny as a sufficient ground for saying that such a conditional expresses an *essential* truth; his acceptance of proofs would always be provisional in the way our acceptance of any hypothesis is provisional - proofs would never impress him as possessing the special cogency with which they impress other men. (Wright 1980, page 453)

If, on the other hand, there is no mistake or error could a reluctance to assert the necessity of logical entailment be an aberration? Almost certainly not. The Cautious Man is willing to accept the correctness of each individual step in the proof; he grants the obscurity of any attempt to "correct" the **CE** on procedure grounds and concedes that a repeat of the proof would yield the same conclusion. He (merely) "dispute[s] that there is anything in all that which justifies him in claiming to have apprehended any essential connection between basis, process and outcome" (Wright 1980, page 455). The Cautious Man's reluctance to assert the necessity of **CE** is not an aberration. The possibility of the cautious attitude in the case of logical necessity indicates that a similar reluctance to assert the truth of a Moorean fact need *not* result in absurdity. The criterion of negational absurdity is weakened. **MF** may indeed qualify as a Moorean fact. The reluctance to assert – rather than flatly deny – the putative necessity of logical entailment need not result in absurdity (see chapter 6).

Sider's incredulity appears to originate from an indubitable Moorean fact, namely, **MF**. As such, no premise of any philosophical or scientific argument could dislodge the certainty of **MF**. We might anticipate, therefore, that Sider's preference would be for a "strong" type of indefeasible theoretical conservatism. However, we noted that Lewis advocates a "weak" form of theoretical conservatism; a conservatism that is defeasible in light of other virtues. This is indicative of a repudiation of the proposal that **MF** is a Moorean fact. We turn, therefore, to explore a second supposition; that **MF** falls into the category of "folk knowledge". In this section, we tested the criterion of negational absurdity. The denial of **MF** should result in absurdity, but does not. The falsification of **MF** by the Everett Interpretation may be disputed by the experts but it is not intellectually absurd. In the

alternative, the Cautious Man's attitude to the logical necessity of entailment suggests that our preference for the criterion is at least questionable. If assent to examples of logical necessity can be coherently withheld then perhaps the denial of a Moorean fact – an epistemic necessity – need *not* result in absurdity. It is unlikely that Lewis would be persuaded by Wright's thought-experiment, however. Lewis' type of theoretical conservatism implies that knowledge of **MF** is defeasible and not subject to differential certainty. As an instance of "folk knowledge", **MF** is not a Moorean fact, and so Lewis is unlikely to err on the side of Caution.

3.4 The Criterion of Specialised or Technical Terms

Lewis argues that common sense knowledge is not limited to knowledge of Moorean facts. In the context of everyday life, we know many things that are not factive in the Moorean sense.

T2: A proposition p may be a member of the common sense *weltbild* (and thereby S *may* or may not know that p) if, and only if, the truth of p is *not* made true by a Moorean-fact (see Lewis 1996a [1999], page 418)

T2 can be utilised to test a new supposition; that **MF** is a non-Moorean fact. I may know a non-Moorean fact even if you do not know it (and vice versa). To reflect this change we re-designate **MF** as **FK.**

FK There is only one world or universe

FK should be understood as an instance of folk knowledge. A number of criteria have been proposed the satisfaction of which identify a fact as Moorean. We employed the criterion of negational absurdity but discovered it to be problematic. We now propose a set of criteria intended to identify non-Moorean facts. This set is derived from William Lycan's criteria of a Moorean fact. We derive criteria of defeasible knowledge of non-Moorean facts: "folk" knowledge of facts that constitute, in part, the common sense *weltbild*.

Lycan cites a number of criteria that are individually necessary and collectively sufficient to identify a Moorean-fact (Lycan 2001, pp. 49-50 cited Daly 2010, pp. 18-19)):

1. It is "about something noted by someone at a particular time and place"
2. That the claim is an instance of a more general claim that most people would accept if it were put to them
3. That neither the claim nor the generalisation use any specialised or technical terms
4. That it is difficult to give up accepting the generalisation
5. That it would sound strange to assert the generalisation because it is so obvious, and
6. That it would be peculiar to deny the generalisation

We propose to identify a *non*-Moorean fact on the basis of a criterion selected from Lycan's list. We proceed on the assumption that a claim to know that p is an instance of "folk" knowledge if, and only if, (at least) one of Lycan's criteria is *not* satisfied. We restrict our examination of "folk" knowledge to Lycan's third criterion that precludes "all specialised or technical terms" from the *analysans*. In short, in order to qualify as knowledge of a non-Moorean fact (i.e. "folk" knowledge) we must establish that the phrase "possible world" *is* either a specialised or technical term. We allow for the possibility that specialised or

technical terms may be explicable in *ordinary* language. This admits the possibility that a predicate *F* may be *derived* from expert terminology (chapter 1 and 2). Consider, once again, the true propositions of List B (§ 3.3). Some of these sentences include the use of specialised or technical terms the meaning of which is established by scientific research programmes: "the average person comes to acquire these pieces of information not by any investigation he or she has pursued but secondhand, as it were, for example, by hearing or reading news accounts of such inquiries" (Stroll 1994, page 34). The truth of these propositions is established by a community of experts (cf. Williamson 2007, pp. 94 ff.). Contrast the sentences of List A that do not employ specialised or technical terminology and whose truth is not established by experts.

Recall, Lewis' (1968 [1983]) initial motivation is to retain our normal practice of translation into **PL**. Provisionally, we can discount Lewis' modal translation scheme as a candidate of "folk" knowledge. As a normative discipline, the correct use of formal logic requires a *degree* of training and expertise (see Williamson 2007, pp 103 ff.). Formal languages, such as **QML** and **PL**, are the preserve of experts. Arguably, the interpretation of logical notation requires a degree of expertise not conducive to ordinary language. Despite the "ignorance" of the man on the Clapham Omnibus (and if we eschew fatalism **[FN 6]**) modal notions are embedded in the language and thought of everyday life. Reality is more than a collection of entities with varying properties and relations – reality includes what could or could not be, what must or must not be and what should or should not be. We fear or hope for the future, celebrate or regret the past, adopt a healthier lifestyle, develop our intellectual abilities, educate the young, construct roads and bridges etc. In the context of everyday life we

"project" a wide spectrum of modal belief and knowledge about how the world could, must or should be. Modal sentences can be expressed in the idioms of ordinary language, such as **A1**, "It is possible that talking donkeys exist" (§ 3.1). What of sentences such as **A3**, "There is a talking donkey at a possible world"? Sentences that are both couched in the idioms of ordinary language *and* that function as the *analysans* of **A1**? Do the artificial expressions of logic, such as the predicate expression "Wx", derive their meaning from ordinary language or vice versa (§ 2.2)?

GMR does appear to employ a number of specialised or technical terms that are the preserve of experts. A number of technical "primitive" concepts are employed in order to explicate the conceptual, ontological and semantic implications of possible world talk: *individual, set, part of, similar to* and *spatiotemporally related to* (see Divers 2002, page 46). If the artificial symbols of formal logic derive their meaning (interpretation) from ordinary language then Lewis does provide a "semi-technical" explication of modality conducive to folk knowledge (Lewis 1986a). For example, take the primitive theoretical concept of *similar to*. Lewis notes that the counterpart relation can be understood in terms of the concept *similar to*. Lewis also claims that our ability to recognise "sameness of type" is a Moorean fact (§ 3.2). It is tempting, therefore, to conflate the concepts of *similar to* and *sameness of type*, especially if the latter is understood as a case of numerical identity. This temptation must be resisted, however. Clearly, as an equivalence relation "sameness" is not (generally) equivalent to the counterpart relation "similar" (Lewis 1968 [1983], page 28). The (Moorean) fact that we can identify two distinct objects of the *same* type is not equivalent to the ability to identify two distinct objects of a *similar* type.

If, on the other hand, ordinary language derives meaning from the artificial languages of formal logic, then Lewis could be said to offer a "technical" explication of modality conducive to folk knowledge. That is, "technical" in the sense that the meanings of the formal symbols of logic are conceptually prior to their meaning in ordinary language (§ 2.2). Again, let the primitive concept *similar to* act as an exemplar; we find that Lewis provides a number of principles that we ought to observe whenever this concept is employed. For instance, the rule of accommodation: "what you say makes itself true, if at all possible, by creating a context that selects the relevant features so as to make it true" (Lewis 1986a, page 251). In respect to a counterfactual statement, for example, we may truly assert that Caesar would have used the atomic bomb [if he had access to one]. Alternatively, we may truly deny that he would have used the atomic bomb but assert that he would have used catapults (Lewis 1973, page 67: also Lewis 1970 [1983], Lewis 1979b [1983]). The context of utterance affects not only the precision of what is asserted but also the weight we ascribe to each point of comparative similarity. The genuine realist's reservoir of primitive concepts can be deemed either "technical" or "semi-technical". *If* the technical terms of counterpart theory do derive their interpretation (meaning) from the artificial symbols of formal logic *then* we still require a context of utterance to evaluate their use. The evaluation of **A2** "There are no actual talking donkeys" may be deemed indexical (§ 2.5), for example.

Hence, the ordinary language phrase "possible world" and the predicate expression *Wx* (*x is a possible world*) may both be interpreted in one of two ways. If the predicate expression derives its interpretation (meaning) from the ordinary language phrase then the modal translation scheme employs a semi-technical vocabulary. If, on the other hand, the complex

phrase “possible world” derives its interpretation (meaning) from the predicate expression then the modal translation scheme employs a technical vocabulary. In either case the use of the ordinary language phrase proceeds on the basis of folk knowledge. Indeed, we argue that the truth or falsity of **MF** (and **FK**) is a subject of dispute amongst the experts. In a similar vein, a non-specialist may use phrases that are derived from the vocabulary of experts but express folk knowledge. The number of universe(s) may be a point of contention among the folk just as it is among highfalutin’ philosophers!

It might be objected that scientific and philosophical inquiries are different types of enterprise. To conflate these two disciplines is erroneous. What reason do we have to suppose that the endless deliberations of armchair philosophers, say, should be accorded the plausibility of theoretical physics? Moore, for example, argues that there is “no good reason to suppose” that there are any “physical facts” logically or causally dependent on “mental facts” (Moore 1925 [1993], pp. 119-120). Idealism (broadly) invokes a reduction of the physical qualities of objects and their spatial relations to a subject’s experience of sense data. Most non-philosophers would be baffled by this analysis – what are sense data? Do sense data require the existence of physical objects? The man on the Clapham Omnibus might consider Moore’s common sense claims far more plausible than the philosophical analysis of Idealism! However, this criticism has no traction. Most journeymen on the Clapham Omnibus would also find Moore’s common sense claims more plausible than the explanation of them presented by theoretical physics; that “matter is constituted from atoms” and “atoms mostly consist of empty air”.

The philosophical-scientific dichotomy is false. Those persuaded by theoretical conservatism require a non-arbitrary distinction between philosophy and science in order to support the idea that conservatism *should* only apply in the former case. A distinction might be drawn on the basis of methodology, for example. It might be argued that the natural sciences (at least) are based on empirical investigation capable of debunking common sense "knowledge". The highly abstract arguments of philosophers, on the other hand, enjoy an immunity from empirical discovery. In the absence of empirical falsification, philosophy is obliged to appear before the tribunal of common sense judgement. The distinction, regretfully, is *ad hoc*; absent a principled reason that grants the results of empirical inquiry an epistemic authority denied to philosophical contemplation. The methods employed by natural scientists, particularly theoretical physicists, include thought experiments and highly abstract argument. Erwin Schrodinger would soon have exhausted his supply of cats were he required to replicate his thought-experiment with actual felines! By parity, Lewis' explanation of *how* we know that a non-actual cat would suffer the same fate is based on reasoning from first principles and imagination (see Lewis 1986a, pp 113-114).

"It is the reasonableness of modal beliefs which I think Lewis's extreme realism cannot account for" (Stalnaker 1976/1984 [2003], page 31). We argue that the modal translation scheme can be expressed in the idioms of ordinary language. The primitive predicate expressions of modal translation can be considered either "technical" or "semi-technical", dependent on conceptual priority. However, the use of specialised or technical terminology is a clear breach of Lycan's third criterion of a Moorean fact. We conclude that the use of specialised / technical terminology in **FK** constitutes folk knowledge of a non-Moorean fact.

If we admit common sense judgements as a theoretical virtue it would appear that Lewis' "weak" form of conservatism supports this conclusion. The source of the incredulous stare is a pre-theoretical claim to know **FK** as an instance of folk knowledge: knowledge defeasible in the light of countervailing theoretical virtues. Recall, however, that Lewis' famous declaration **S1** does not pertain to knowledge but to a shared belief. It is the shared *belief* in a type of entity, namely, the "ways things could have been" that prompts his argument from paraphrase. Is the subsequent paraphrase of "ways things could have been" by "possible worlds" ontologically innocuous? We defer an answer to this question (see chapter four). Meanwhile, we address the alleged methodological role of belief in philosophical theory. Do Lewis' shared beliefs constitute *evidence* in favour of **GMR**?

3.5 The Evidential Value of Modal Beliefs

S1 is perhaps one of the most famous – or if you prefer, notorious – statements in modern philosophy. What is the status – in respect of common sense judgement – of beliefs that pertain to the "ways things could have been"? We might be tempted to *infer* that Lewis is citing his shared belief in a type of entity as *evidence* in support of **GMR**. He argues that intuitions are beliefs (see Lewis 1983a [1983], page x). His minimalist account is supported by Peter van Inwagen: "Our "intuitions" are simply our beliefs – or perhaps, in some cases, the tendencies that make certain beliefs attractive to us, that "move" us in the direction of accepting certain propositions without taking us all the way to acceptance" (van Inwagen 1997 [2001], page 149). If Lewis is correct, then dependency on intuition or belief (henceforth, simply "intuition") is not exclusive to philosophy. Scientific progress is fuelled

by the intuitions of scientists (see Williamson 2007, page 215). Despite the commonality, the allegation that intuitions play an evidential role in an ontological commitment to a plurality of worlds, is troublesome. Why should Lewis' idiosyncratic doxastic state justify such a bizarre metaphysical claim? The answer is simple – they shouldn't.

Lewis refuses "to take language as a starting point in the analysis of thought *and of modality*" (Lewis 1983a [1983], page *xi* my emphasis). As a materialist, Lewis holds that there is a mind-independent reality composed of stuff. The metaphysical theories we construct are the result of a process of reflective equilibrium where mature "post-theoretic" intuitions are generated **[FN 7]**. To suggest that Lewis postulates a plurality of worlds because that is what his pre-theoretical intuitions tell him is to put the cart before the horse; rather, his *best* philosophical theory – composed of his own intuitions – is aligned with a pre-existing, mind-independent reality of a plurality of worlds. Hence, Williamson's general concern about a conceptual gap between the content of our intuitions and their putative truth does not apply in the case of **GMR** (see Williamson 2007, page 215).

We might weaken the evidential claim and argue that plausible philosophical theories should at least be consistent with our shared intuitions. The plausibility of a philosophical theory is measured by accord with common sense judgement (see Lewis 1986a, page 134 (as above)). Generally, "consistency" is a relation that exists between items e.g. propositions. "This propositional conception of evidence fits the discursive nature of philosophy" (Williamson 2007, page 209). However, Williamson observes that an asserted proposition may be true and yet *inconsistent* with the evidence. This is the status of **GMR**: a

"belief in" the existence of a type of entity, possible worlds, is both true and *inconsistent* with common sense judgements. If the content of a belief is the proposition P, that "there is only one world or universe" then although P is consistent with common sense judgement it is false, by the lights of **GMR**. Thus, the proposition Q, that "P is false", is true (by **GMR** lights). It might be a mistake to place any theoretical value on *consistency* with common sense judgements as a guide to veracity (Williamson 2007, page 209). We are inclined to simply dispense with the need for philosophical theories to accord with common sense intuition.

3.6 Conclusion

In this chapter, we identify the source of the incredulous stare often provoked by the postulates of **GMR**. We argue that at least two types of incredulity are possible. Firstly, the incredulity provoked by the denial of a Moorean fact, **MF**; indefeasible on the grounds of differential certainty. Ted Sider *et al*, endorse a "strong" theoretical conservatism: a degree of certainty in the truth of **MF** greater than that of any premise of a philosophical argument employed to falsify **MF**. We challenged the presumption that **MF** is a Moorean fact by arguing that the criterion of negational absurdity is not satisfied**.** The Everett Interpretation of Quantum Mechanics denies **MF** without absurdity. We acknowledge that our chosen criterion of negational absurdity is itself problematic based on Wright's thought experiment. The problem can be mitigated in part by the observation that Wright's Cautious Man does not *deny* the relevant modal claim but simply refuses to assent to it. The second type of Incredulity is prompted by claims that run counter to folk knowledge. **FK** is not insulated

against philosophical or scientific doubt and so is susceptible to revision based on further argument. We distinguish **FK** on the basis of specialised or technical language, specifically, "Wx" or "x is a possible world". Lewis' "weak" conservatism indicates that **FK** is a claim of folk knowledge. Lewis is open to the possibility that our pre-theoretic knowledge claims are false given a preponderance of other theoretical virtues. Finally, Lewis' famous **S1** declaration of a personal (and shared) belief-in a type of entity "possible worlds" should not be interpreted as *evidence* in favour of **GMR** but rather as an attempt to align his beliefs with a mind-independent reality (§ 4.6). Have we identified the true source of the widespread incredulity provoked by **GMR**? Is incredulity amongst a community of philosophers prompted by an innocuous *paraphrase* of "ways things could have been"? Sober philosophers may not even object to the indexicality of the term "actual" as a *semantic* thesis (Stalnaker 1976/1984 [2003], page 27). We argue that the objection of incredulity, that threatens criterion c of a genuine paraphrase, need not pertain to the paraphrase itself but to a particular interpretation of the term "possible world". That is, the concept of a possible world(s) interpreted as discrete "concrete" spatiotemporal continuities, much like our own world (cf. van Inwagen 1986 [2001], pp 224-225). It is to the homogeneity thesis that we now turn.

Footnotes

1. Attributed to Niels Bohr by Marcus Chown (2001) "The Universe Next Door" Headline, page xiv. Lewis' naturalistic epistemology commits philosophy to the same theoretical virtues as science (Nolan 2005, page 7). We maintain that Lewis' commitment to theoretical conservatism is a breach of his naturalist epistemology.

2. Lewis shares Quine's naturalism in respect not only of the *correctness* but also the *completeness* of the scientific world view (see Nolan 2005, page 7 ff.). Recent commentators appear to concur; Williamson argues for "a conception of theories in logic and metaphysics as scientific theories, to be assessed by the same overall standards as theories in other branches of science" (Williamson 2013, page 27). Williamson explores the implications of these preliminary remarks in Chapters 3, 6 and the Afterword of that work.

3. Alternatively, Stalnaker accepts the indexicality of the term "actual" as a semantic thesis but rejects the associated metaphysical thesis of a plurality of concrete worlds (§ 5.3) (Stalnaker 1976/1984 [2003], page 27 ff.)

4. Aristotle, for example, argues: "As with other topics we must first set out how things appear to be, and then, after developing the problems [*aporia*], go on to prove the truth, if possible, of all the common beliefs [*exdoxa*] about the matters in question, or failing that, of the majority of them and the most authoritative. For if we can solve the problems while leaving common beliefs untouched, we shall have proved the case sufficiently." (Aristotle *Nicomachean Ethics* VII. 1-2 cited J.L. Ackrill (1981) *Aristotle the Philosopher* Clarendon Press, page 12)

5. We will ignore many of Lewis' substantial contributions to philosophy in this chapter. The concern here is to identify the source of the incredulous stare expressed by critics of **GMR**. Lewis philosophical analysis of knowledge, belief, proposition or truth-maker frequently draws upon the resources of genuine realism as a component of the *analysans*. We therefore remain silent in respect of these contributions: for example, Lewis' (1998, 2001) is initially critical of the Truthmaker Principle on the grounds of an "innocent" but required assumption of a *necessary*

connection between otherwise wholly distinct existents; that a proposition P is made true by a thing T whose mere existence *necessitates* the truth of P. He subsequently embraced a *version* of the principle by way of the "qua" locution (Lewis 2003 [2003], page 30-1). The "qua" locution may generate a more demanding counterpart theoretic relation relative to which A is essentially such that A is F (See Beebee and MacBride (2015) pp 227-230 for further discussion).

6. Forbes suggests that the plausibility of fatalism is based on a failure to distinguish the (invalid) P → □P from the (valid) □(P → P) (see Forbes 1985, *The Metaphysics of Modality* Clarendon Press: Oxford, page 7)

7. The "belief-in" procedure we defend is best summarised by Quine: "My point is not that ordinary language is slipshod, slipshod though it be. We must recognize this grading off for what it is, and recognize that a fenced ontology is just not implicit in ordinary language. The idea of a boundary between being and nonbeing is a philosophical idea, an idea of technical science in a broad sense. Scientists and philosophers seek a comprehensive system of the world, and one that is oriented to reference even more squarely and utterly than ordinary language. Ontological concern is not a correction of a lay thought and practice; it is foreign to the lay culture, though an outgrowth of it." (Quine 1981a (1981), page 9)

Chapter Four: Theoretical Parsimony

entium varietates non temere esse minuendas (translated - the variety of entities should not be rashly diminished)" (Kant 1781 / 1787 [1929], page 541, A 656 / B 684)

4.1 Introduction

In chapter three we examined Lewis' third criterion of a genuine paraphrase. The paraphrase of "ways things could have been" by "possible world" must be endorsed by a community of ordinary language users (see § 2.3). We concluded that a common (shared) belief that *p*, shared by a community of ordinary language users, could satisfy the criterion and license the paraphrase (**S1)**. We now turn from the "common belief" to the *content* of that common belief. It is the belief-in concrete worlds other than the actual world that provokes "incredulity". As a materialist, we argued that Lewis should ignore the "incredulous stare" as a theoretical irrelevance. We will defend **GMR**, not from the incredulity provoked by the homogeneity thesis, but from an apparent entailment. **GMR** is challenged on the grounds that it entails that, say, talking donkeys and one million carat diamonds necessarily exist! We postpone the examination of these entailments for a later chapter (§ 7.4) but note that they can only arise by prior acceptance of non-actual worlds. *Prima facie*, **GMR** is susceptible to criticism on the grounds of an unjustified ontological inflation (§ 2.3). We argue that the ontological commitments of a theory relate to the number of *types* of entity that the theory *says* exist.

The simplicity of a theory is assessed in one of two ways: first, by the *elegance* or syntactic simplicity of a theory's basic principles, and second, by *parsimony*, which pertains to the number of the *types* of entity postulated by the theory. Parsimony pertains to the meta-ontology of a theory or to those types of things that a theory *says* exist. These two aspects can "often pull in different directions – postulating extra entities may allow a theory to be formulated more simply, while reducing the ontology of a theory may only be possible at the price of making it syntactically more complex" (Baker 2013, Section 1). In order to avoid the fallacy of "respect neglect" **[FN 1]**, this chapter is restricted to **GMR's** parsimony (or lack thereof). "Realism about possible worlds might be thought implausible on grounds of parsimony, though this could not be a decisive argument against it" (Lewis 1973, page 87).

Before we commence, we should note a further distinction in regard to theoretical parsimony. Parsimony can be interpreted as either an epistemological or methodological virtue. Epistemological parsimony states that *if* theory X is simpler (postulates fewer *types* of entity) than theory Y then it is rational to believe X over Y. Methodological parsimony states that *if* theory X is simpler (postulates fewer *types* of entity) than theory Y then it is rational to accept X as one's theory for the purpose of philosophical investigation. This chapter pertains only to methodological parsimony: given how many types of entity are postulated, which philosophical theory is it rational to accept? We define, distinguish and attempt to justify on *a priori* grounds the methodological principle of parsimony (§ 4.2). We then offer a naturalised justification of the principle based on evidence selected from case studies in the history of science. This evidence offers an *a posteriori* justification of the principle of parsimony (§ 4.3). Lewis (1973) distinguishes qualitative from quantitative

parsimony, and discounts the latter as a theoretical virtue. Is the dismissal warranted? We examine Daniel Nolan's argument that quantitative parsimony is worthy of consideration (§ 4.4). The quantifier-predicate model aims to reflect the "structure" of reality. A precursor to evaluating the ontological costs of genuine realism is that Lewis supply an explanation of the notion of a property. What are the "joints" at which nature is carved? (§ 4.5) If natural properties demarcate the joints, do the predicates expressions of the quantifier-predicate model secure agreement as to where we should carve? (§ 4.6) Lewis' conception of a property is pivotal to the determination of the *type* of parsimony he invokes (qualitative or quantitative) and the theoretical costs accrued by **GMR.** We conclude that qualitative parsimony favours **GMR** and quantitative parsimony does not count against it (§ 4.7).

4.2 Ontological Parsimony, Defined

We adopt the following criterion of ontological parsimony in respect of theory-choice (Baker 2013, Section 2 my emphasis):

Ontological Parsimony (OP): *Other things being equal*, if T1 is more ontologically parsimonious than T2 then it is rational to prefer T1 over T2

The "other things being equal" clause is critical to this definition. It renders **OP** comparatively weak as a theoretical virtue when set against other *desiderata*. We have seen that Quine's **QC** of ontological commitment is intended to identify the types of entity to which a theory is committed. Hence, if T1 and T2 are otherwise equivalent in their

satisfaction of theoretical *desiderata* but T1 is also committed to a type of entity *G* when T2 is not, then T2 ought to be accepted on the grounds of **OP**.

With all due respect to Kant's caveat, theoretical simplicity is a multi-faceted virtue. It was decisive as a criterion of theory choice amongst philosophers of science in the C20th. Theoretical simplicity can be assessed in a number of different ways, however. Is the expressive simplicity of a theory (the economy of the conceptual apparatus required for its formulation) *compatible* with a theory's instrumental simplicity (the mathematical apparatus required for its formulation)? If not, which type of simplicity should be granted precedence? Further, how are these two forms of simplicity related to a theory's pedagogical simplicity, the ease with which a theory can be taught or understood? An analogy with the manufacture of motor vehicles may prove informative (Rescher 2006, page 47). Individual motorcars vary in their complexity in respect of manufacture, maintenance, ignition systems and driver friendliness. These characteristics often conflict: a car that is easy to manufacture might be horrendous to drive. In addition, they are subject to further refinements. The characteristic of "driver friendliness" can vary with the weather conditions: a vehicle may be "friendly" in dry conditions but positively "hostile" in icy conditions. "With automobiles, simplicity is critically respectival. And the simplicity of theories is in much the same situation … Here one cannot appropriately speak of simplicity *tout court*" (Rescher 2006, page 47). Lewis resists the fallacy of "respect neglect": "[respect neglect] has the form of treating a feature *F* as a unified property that things do or do not have, whereas in fact *F* has several respects, and things can have *F* in one respect and lack it in another" (Rescher 2006, page 45). He [Lewis] unpacks the notion of theoretical simplicity

into elegance and parsimony. He further delimits parsimony in terms of quantitative and qualitative parsimony. In comments that have been criticised for their brevity **[FN 2]**, Lewis concedes that **GMR** is guilty of an ontological inflation only in respect to *quantitative* parsimony. The concession ought to have no bearing on theory choice, however: "I subscribe to the general view that qualitative parsimony is good in a *philosophical or empirical* hypothesis; but I recognise no presumption whatever in favour of quantitative parsimony" (Lewis 1973, page 87 my emphasis). The only offence against parsimony committed by **GMR** is quantitative not qualitative: it postulates a *plurality* but only of a *single* type of entity **[FN 3]**. Hence, the homogeneity thesis is a theoretical boon. Quantitative parsimony ought to be irrelevant to theory choice.

We can expand on Lewis' remarks. He acknowledges that ontological parsimony is a theoretical virtue but only in respect of how many *types* of entity are postulated. Lewis also concurs with our definition of **OP** as (in part) constitutive of rationality. Indeed, *a priori "*justification(s)" of the claim that **OP** is a theoretical virtue appear to be ubiquitous. "Justification" must suffer the indignity of scare quotes because the rationality of a methodological principle need not require that the principle be justified (see Strawson 1952). The principle may simply be taken as axiomatic (admitting of no proof) or self-evident (an obvious principle, not in need of proof). If, however, we demand that a methodological principle ought to be justified then justification on *a priori* or *a posteriori* grounds appear to be obvious candidates.

We begin with *a priori* justification. Lewis adopts the Ontological Parsimony of Kinds **(OPK)** despite the philosophical disfavour into which *a priori* justification(s) had fallen in the latter part of the C20th. The disfavour was a consequence of the rise of an empiricism ill-disposed toward metaphysical systems. Lewis' advocacy of the **OPK** is based on the wider principles of rational enquiry, however. Lewis supports methodological conservatism as a theoretical virtue (§ 3.2): "A worthwhile theory must be credible, and a credible theory must be conservative" (Lewis 1986a, page 134). Conservatism itself may be underpinned naturalistically, on the grounds of previous empirical success, or pragmatically, as a principle endorsed by the wider scientific community. In either case, as a principle embraced by a scientific community engaged in the rational enterprise of empirical investigation. The fact that the scientific community does employ the **OPK** insulates Lewis from censure by empirically minded, analytic philosophers. Unfortunately, *a priori* justifications of **OPK** are flawed: "[It is] difficult to distinguish between an *a priori* defence and no defence!" (Baker 2013, Section 3) The **OPK** can be accepted as constitutive of rationality in any of the following ways: as axiomatic (not admit of proof), or self-evident (not require proof), or pragmatic (an *a priori* principle employed by the scientific community). The acceptance of any or all of these options may reduce the **OPK** to the status of a methodological "article of faith". Absent an *a priori* justification, however, it may be difficult to persuade the sceptic of the "rational" legitimacy of the **OPK** without further controversy (see Baker 2013, Section 3: Kuhn 1977). Why is **OPK** partially constitutive of rational enquiry? How does one methodological principle qualify as rational (e.g. **OPK**) and another irrational (e.g. Kant's Principle of Plenitude)? Do methodological principles ever conflict? What meta-principles should be invoked to adjudicate these cases of conflict? Do principles like the **OPK**

constitute rational enquiry or do they merely reflect the presumptions of those engaged in the scientific or philosophical enterprise?

4.3 A Naturalised Justification

In the face of these doubts about *a priori* justification of the **OPK** alternative justification(s) might be sought. A justification based on *a posteriori* or naturalistic interpretations of the **OPK** may prove more fruitful. Naturalistic epistemology views the enterprise of philosophy as contiguous with the natural sciences (§ 3.2). Simplicity (parsimony) may be established as a theoretical virtue on the basis of historical *evidence* in the form of the acceptance or rejection of theories by the scientific community. We might justify **OPK** as a theoretical value on the basis of evidence collected from episodes in the history of science. Has the **OPK** ever been decisive in respect of theory choice? The stock example of Einstein's Special Theory of Relativity, and the role it played in the falsification of the existence of the electromagnetic ether (to use the modern spelling), is illustrative of the *relative* strength of **OPK**. However, if we set aside the **OPK** then Special Relativity still enjoys a host of other theoretical virtues over alternative explanations, such as the Lorentz-Poincare theory **[FN 4]**. The plethora of theoretical *desiderata* enjoyed by Special Relativity makes it a poor example of **OPK's** utility in regard to theory choice. In short, *ceteris non paribus*. A more relevant test of **OPK** as a theoretical virtue ought to involve a comparison of alternative theories that are otherwise equivalent in terms of their explanatory value (see Baker 2013, Section 4).

With the constraint of parity duly acknowledged we offer Buffon's Law as an illustration of **OPK** in effect (Gribbin 2002, pp 221-226). Buffon's Law relates to the diversification of animal and plant species in different geographical regions that share a common environment (see Baker 2013, Section 4). Two incompatible theories emerged to explain this pattern of species variation. The Darwin-Wallace model states that there are two causal mechanisms in effect that explain the phenomenon. Firstly, *dispersal* in which species gradually migrate to new geographic areas, and secondly, *evolution by natural selection*, in which species evolve over a period of time. As a consequence of these factors, different animal and plant species evolve in geographically distinct but similar environments. Theories of the "extensionist" tradition, on the other hand, reject a core presumption of geographic stability found in the Darwin-Wallace model. Proponents argue that areas of land or "land bridges" can be exposed by either a fall in sea level or plate tectonic shift. These are the primary causal mechanism in play. A "land bridge" is an isthmus that connects two separate geographic areas which animal and plant species may cross. As a result, the extensionist model is ontologically committed to more types of entity than the Darwin-Wallace theory. The extensionist theory postulates additional types of entity; including geographic and tectonic instability. "The debate over the more parsimonious dispersal theories centred on whether the mechanism of dispersal is sufficient on its own to explain the known facts about species distribution, without postulating any extra geographical or tectonic entities" (Baker 2013, Section 4).

Can we extrapolate from the specific case of Buffon's Law to the general framework of the **OPK** debate? Historical scientific theories that are otherwise incompatible may be exalted

and condemned in turn on the basis of **OPK**. Theories that postulate *more* types of entity are condemned for their proliferation of kinds in the absence of evidential support. Theories that postulate *fewer* types of entity are condemned for their inability to fully explain the observed facts. As such, the attempt to provide *a posteriori* justification of theoretical parsimony faces a number of cogent objections. Even *if* we grant naturalism a privileged methodological status, extracting principles from science in order to apply them to philosophy is no simple matter. How do we identify the underlying methodological principles of the scientific enterprise? Some philosophers reject the idea of a scientific method and even those who don't may still reject the notion that the method is *unified*. However, even if we could discover a way out of this impasse a greater obstacle remains. How are we to dissect 'nature at the joints' and secure an *accurate* determination of the types of entity to which total theory ought to be ontologically committed? The ability to identify a theory's ontological commitments to a type of entity presupposes a method of identification. What is to count as a "type of entity"? The quantification-predicate model appears to endorse properties as a basis of identification. But which properties? How do we distinguish the abundant from, say, the sparse properties? "The abundant properties may be as extrinsic, as gruesomely gerrymandered, as miscellaneously disjunctive, as you please. They pay no heed to the qualitative joints, but carve things up every which way" (Lewis 1986a, page 59). Before we explore the problem of "joint-selection" we must address a prior difficulty. Why does Lewis consider the *quantitative* conception of **OP** to be irrelevant to theory choice? Why is positing one token of a type of entity more *rational* that positing twelve tokens of a type of entity? Why does the number of tokens posited cast a shadow of doubt over a theory's *rationality*?

4.4 Quantitative Parsimony, Rehabilitated

Lewis' accepts a lack of *qualitative* parsimony as a theoretical cost but rejects any "cost" associated with a lack of *quantitative* parsimony (Lewis 1973, page 87). The only theoretical cost incurred by **GMR's** ontological commitments is to the *types* of entity postulated, not to the number or tokens of those types. Is there really no theoretical cost incurred by the number or tokens of each entity postulated? The proliferation of worlds ought to play some role in theory evaluation, shouldn't it?

In "Quantitative Parsimony" Daniel Nolan tests the cogency of Lewis' distinction: "I motivate the view that quantitative parsimony *is* a theoretical virtue: … [a theory] should also minimize the *number* of entities postulated that fall under these kinds" (Nolan 1997, page 329 my emphasis). Nolan does not offer an account of how competing virtues should be weighed nor does he argue that quantitative parsimony is a primitive virtue. Indeed, he argues that quantitative parsimony is essentially dependent on qualitative parsimony. It pertains not only to "just the total number of things, but [to] how many things *of each type* there are" (Nolan 1997, page 340). Nolan's primary aim is to demonstrate that quantitative parsimony ought to be amongst the *desiderata* of theory choice. He presents two case studies from the history of science in pursuit of this goal. We will examine one of these case studies. The case relates to the discovery of the neutrino during the process of beta-decay. Beta-decay occurs when the nucleus of an atom ejects an electron and is thereby transformed to an atom "adjacent" to it on the periodic table (Gribbin 2002, pp 524-526). Enrico Fermi's (1901-1954) complex mathematical model demonstrates that beta-decay is

caused by the influence of a new field of force – the weak nuclear force. “His model described how, in addition to the strong nuclear force … there was a weak, short-range force that could cause a neutron to decay into a proton and electron *plus* another, uncharged particle, which he dubbed the “neutrino” (from the Italian for “little neutron”)” (Gribbin 2001, page 525). The point germane to Nolan’s argument is that physicists’ concluded that only *one* **new** type of particle (with an observed spin of ½) is emitted from the atom during beta-decay. There are, however, several plausible hypotheses each one capable of explaining the observed phenomenon (Baker 2013, Section 6.1):

1. 1 neutrino with a spin of ½ is emitted in each case of beta-decay
2. 2 neutrino each with a spin of ¼ are emitted in each case of beta-decay
3. (More generally for any positive integer *n*) *n* neutrinos, each with a spin of 1/2*n* are emitted in each case of beta-decay

Why should physicists’ posit just one emitted neutrino rather than, say, fifty-five emitted neutrinos? Why is the former explanation more *rational* than the latter? Each hypothesis is an equally adequate explanation of the observed phenomenon. The only material difference appears to be the number of neutrinos postulated sufficient to secure explanatory adequacy. Despite the parity, physicists are drawn to hypothesis 1 as the obvious choice. Quantitative parsimony is the only methodological principle that favours hypothesis 1 over the alternatives. Therefore, quantitative parsimony *must* be a theoretical virtue.

Can we extrapolate a *general* principle of quantitative parsimony from the case of beta-decay? A preliminary suggestion is **QnP** (Nolan 1997, page 338):

QnP: We should not admit any entities into our theory that lack explanatory power.

Nolan concedes that **QnP** will require some refinement. To begin, explanatory power is a necessary but not a sufficient criterion of a theoretical virtue. Why should hypothesis 1 enjoy the privileged status if each hypothesis is equivalent in terms of explanatory power? In the 1930s, numerous experiments were conducted to investigate the phenomenon of particle decay. None yielded an example of "missing" particles with a spin of ⅕ or ⅙ or ⅛. In the absence of an observed emitted particle with less fractional spins, hypothesis 1 emerged as the candidate of choice. It is consistent with the simple explanation that no particles are emitted during beta-decay with a spin other than ½. Although hypothesis 2 is also consistent, the consistency is purchased at the price of postulating particles with a spin that have never been observed. Hypothesis 1 is able to explain the absence of particles emitted during beta-decay with a spin (less than ½) as well as the presence of particles with a spin ½. "It seems intuitive to me, and I hope to the reader, that postulating seventeen million neutrinos in each case of Beta-decay produces a theory that is less plausible and attractive than the theory proposed by Pauli and Fermi" (Nolan 1997, page 334). Nolan acknowledges, however, that **OPK** is a precondition of **QnP**: we must first identify the *kinds* of entity postulated by a theory, rather than take a simple "head count" of the *number* of each type of entity (Nolan 1997, page 341).

As a general principle, **QnP** appears to favour **GMR** as a philosophical analysis of modality. As a theory, **GMR** is ontologically committed to a type of entity, possible worlds (understood as discrete spatiotemporal continuities). There is a plurality of one type of entity – worlds. In the alternative, Actualist Realism **AR**, say, favours an ontological commitment to two types of entity, the actual world understood as a discrete spatiotemporal continuity and sets of

maximally consistent propositions. This numerical difference in the *core* ontologically commitments of each modal theory to a type(s) of entity does little to advance our thesis, however. Although not a core commitment, Lewis accepts an ontological commitment to sets (§ 8.1) and we argue (by the denial of the 'singularist dogma' (§ 1.5)) that sets *ought* to be a core commitment of total theory. Nolan's analysis of **QnP** relates to only the scientific process of beta-decay in which *one* neutrino explains the absent ½ spin. It does not pertain to the total number of neutrinos – the cardinality – of neutrinos in the universe. By parity of reason, **GMR** explains, say, modal possibility by postulating that there is at least *one* world at which that possibility is instantiated. It does not supply the total number – the cardinality – of worlds in the multiverse. In both cases, **QnP** applies only to the explanation of the process not to the cardinality of the types of entity postulated by way of explanation; firstly, beta-decay and secondly, modal possibility.

Nolan cautions against broadening the theoretical scope of **QnP**. Understood as a theoretical virtue, **QnP** should be restricted to certain subject areas, for example. "Intuitively, the strengths and weaknesses of theories about one subject matter (such as the question of the existence of neutrinos or elemental atoms) has little to do with theories about some other subject matters" (Nolan 1997, page 341). As a naturalist, Lewis is compelled to argue that Nolan's injunction is misplaced. The list of theoretical virtues and vices is identical to scientific and philosophical enquiry (§ 3.4). Ideally, therefore, we require a method that both identifies the theoretical virtues (presumed common to science and philosophy) and that applies on a case by case basis. Nolan hints at how this could be achieved: "… after all, arguing from the practice of scientists to what norms they should

follow (or what the cognitive virtues are) is by no means straightforward" (Nolan 1997, page 331). To keep faith with Lewis, we endorse **OPK** as a theoretical virtue. Qualitative parsimony is the primary theoretical virtue whilst quantitative parsimony plays a subsidiary role. We conclude that although qualitative parsimony favours **GMR** quantitative parsimony does not count against it. However, regardless of whether we accept quantitative parsimony as a *desideratum* or not we still seek a mechanism by which to "carve nature's joints". Critical to this endeavour is the quantification-predicate model that is intended to reflect the "structure" of reality. Hence, Lewis' search for a mechanism begins with properties.

4.5 The Creatures of the Night (What Sweet Music They Make!)

A consensus in regard to the nature of properties has eluded philosophers since antiquity. Initially, the notion of a property, at least the properties of material objects, appears to be unproblematic. A red apple has the properties of [redness] and [appleness]. "Properties (aka attributes, qualities, features, characteristics, types) are those entities that can be predicated of things or, in other words, attributed to them" (Swoyer and Orilia 2014 Section Introduction). Dig a little deeper, however, and problems soon emerge. The problem we shall tackle concerns the claim that properties are the "*entities* that can be predicated of things". If we suppose that the meaning of a predicate is the set whose members share a common property then that property can be defined by set-membership. Quine rejects the idea that properties can be analysed in this way on the grounds of identity conditions; by way of the assumption "no entity without identity". There are, he argues, distinct properties that have exactly the same number of instances *without* thereby being identical. The

standard examples are the properties [creature with a heart] and [creature with kidneys]. It just so happens that every creature with a heart also has kidneys, and vice versa. On the standard set-theoretic model the same set "denotes" two distinct properties and so these properties are identical! Little wonder that he dismisses the notion of properties (indeed all intensions) as "creatures of darkness" (Quine 1956 [1976], page 188). Despite Quine's misgivings, philosophers find properties to be useful in the resolution of what might otherwise be intractable problems. In addition to advantages in epistemology and semantics, there are distinct metaphysical benefits in positing the existence of properties. For example, the explanation of resemblance and recurrence can be facilitated by the notion of properties. We classify the world in objective ways based on the similarity of the objects that furnish it. For example, we can account for the resemblance of objects based on the attribution of a property or constellation of properties. Lewis accepts "the traditional realism that recognises objective sameness and difference, joints in the world, discriminatory classifications not of our own making" (Lewis 1983e [1999], page 67). The search for a cogent explanation of qualitative similarity and difference can motivate a realist account of universals.

Lewis' account of properties is reductive; he reduces properties to the sets of their *unrestricted* instances. A property "[is] the set of all its instances – *all* of them – this- and other- worldly alike" (Lewis 1986a, page 50). The inclusion of non-actual worlds blocks Quine's objection based on the same set used to identify distinct properties. Recall the phrase "it just so happens" from the previous paragraph. As a contingent matter of fact, *in this world* creatures with a heart are co-extensive with creatures with kidneys. "It is

contingent whether two properties have the same this-worldly instances. But it is not contingent whether they have the same instances *simpliciter*" (Lewis 1986a, page 51). Lewis' defence is premised on an apparent equivocation of set-membership. An equivocation between the sub-set of talking donkeys restricted to the talking donkeys of this-world, and the set of talking donkeys unrestricted to this or any other world, that is, understood *simpliciter*. Membership of the second set does not vary *simpliciter* from world to world. Membership of the set *simpliciter* is necessary. The fact that something is a talking donkey (and so whether set X is the set of actual talking donkeys) does vary between worlds and is therefore contingently true. Suppose "Eeyore" is a talking donkey who inhabits a non-actual world. It is a contingent matter whether or not he talks – some of his counterparts do whilst others do not. If Eeyore talks then he will necessarily be a member of the set of talking donkeys *simpliciter*. It is, however, a contingent matter whether he is a member of a subset of talking donkeys – understood as set-membership restricted to the possible world he inhabits.

Lewis acknowledges that his account is incomplete. "Likewise I shall reserve them for properties and relations of the kind that do not admit of degree and that are instantiated *simpliciter*, not relative to anything, and for propositions of the kind that hold or not relative only to a world" (Lewis 1986a, page 59). Hence, properties that are instantiated relative to something else, are not included in his account. In such cases, the property instantiated is *not* equivalent to the set of its instances. Hunger, for example, is a property relative to a particular time; a property that I may instantiate at one time but lack at another (§ 2.5). Despite this concession, doubts remain as to whether the move to a set-theoretic model of

properties can hold (even *simpliciter*). Some properties, say those found in mathematics, are necessarily instantiated (if at all) and thereby appear to qualify as properties *simpliciter*. Some distinct mathematical properties are instantiated by the same 'object' and are considered identical by virtue of set-membership. For example, the mathematical properties [triangularity] and [trilaterality]: "Necessarily, a plane figure bounded by line segments has the same number of angles as sides" (Lewis 1986a, page 55). Even *simpliciter*, these distinct properties are necessarily identical by virtue of the set-membership model. Lewis begins his extrication from this problem by considering the type of role fulfilled by "properties". "Which entities [he asks], if any, among those we should believe in, can occupy which versions of the property role?" (Lewis 1986a, page 55) The answer to this question is central to Lewis' strategy. A partial role can be assigned to *possibilia*: types of entity that partially fulfil the "property" role, as members of sets *simpliciter*.

Alternatively, we may align the notion of a property with the *meaning* of a predicate. This option restores the distinction between otherwise "identical" (necessarily co-extensive) properties on semantic grounds: in English, the predicate "trilateral" means to have three sides whilst the predicate "triangular" means to have three angles. To switch from 'properties' to 'meanings', however, is equivalent to jumping out of the frying pan and into the fire! One consequence of this manoeuvre is to ascribe *structure* to properties – linguistic expressions have syntactic structure so we might be tempted to ascribe a quasi-structure to properties. A quasi-structural model of properties and relations could be constructed utilising sets of possibilia *simpliciter*. In addition to properties and relations, we may also posit a higher-order *unstructured* relation (as a set of pairs) that holds between them.

Consider A and S as unstructured relations (Lewis 1986a, page 56):

- Let A be the relation of [being an angle of]
- Let S be the relation of [being a side of]
- Let T be the higher-order *unstructured* relation that holds between an unstructured property F of individuals and an unstructured relation G of individuals **iff** F is the property of being something that exactly three things bear relation G to.

"A certain unstructured property is the unique thing which bears T to A, and therefore it is the (unstructured) property of triangularity; it also is the unique thing that bears T to S, and therefore it is the (unstructured) property of trilaterality" (Lewis 1986a, page 56). Hence, the structured property of [triangularity] is the higher-order relation ‹T, A› whilst the structured property of [trilaterality] is the higher-order relation ‹T, S›. The relations A and S differ and so the distinction between necessarily co-instantiated properties is restored.

Thus far, we have ignored Lewis' views on *natural* properties. We concluded § 4.3 by noting that some properties, the natural properties, "carve nature at the joints". Natural properties constitute the objective features of reality. The objective features used to determine similarity and difference are not a matter of contingency. Hence, classification of the natural properties is not a contingent matter. Naturalness can be a matter of degree. Although, say, blue things are diverse in that they may instantiate different shades of blue, they are each more *similar* to one another than they are to grue things. Set membership is arbitrary in that it may be determined in any one of an infinite number of ways. How do we distinguish between the resemblance of objects that I thought about last Friday and the resemblance of objects that are atoms of platinum? "In systematic philosophy we constantly need the

distinction between the more and the less natural properties. It is out of the question to be without it" (Lewis 1986a, page 61). The "similarity" and "difference" of objects I thought about last Friday appears to be arbitrary – certainly, there appears to be no *natural* feature that they all share (that is, a feature not shared by the objects that I did *not* think about last Friday). Platinum atoms, on the other hand, do appear to share a natural feature. How is this demarcation explained? The idea that some properties are natural – they are constitutive of similarity and difference in a way unlike most sets of *possibilia* – is absent prior to Lewis' "New Work for a Theory of Universals". As late as Lewis (1983f), he spurns the distinction: "*Any* two things stand in infinitely many relations, share infinitely many properties, and are both included as parts of infinitely many aggregates" (Lewis 1983f [1999], page 394). Lewis' *volte-face* was prompted by conversations with David Armstrong (1978a, 1978b). Persuaded by *some* of Armstrong's arguments in favour of universals (Lewis 1983d [1999], page 8) the demarcation of natural properties became central to Lewis' theory of mind (Lewis 1983d [1999], page 52-53), to the 'exact match' of the intrinsic duplicates / properties of two worlds' spatio-temporal regions (Lewis 1983d [1999], page 27), and to the "tenability" of Humean supervenience (Lewis 1986b [1986], page *xi*).

Lewis commits to methodological naturalism as a theoretical *desiderata*. The structure of the quantification-predicate model is a reflection of the structure of reality. Lewis' 'change of heart' suggests that the "qualitative joints", at which nature ought to be carved, are those determined by the natural properties. We may presuppose a form of ontological realism but are the predicates of the quantification-predicate model truly unambiguous? If two philosophers agree on their interpretation of the quantifier (§ 1.3) must they agree on the

interpretation of the predicate? We turn from a discussion of properties to the discussion of predicates.

4.6 Ontological Realism

We return to the question posed at the end of § 4.3. We noted earlier that the decision to select the quantifier as a criterion of ontological commitment leaves open the validity of various inferences (§ 1.3). We now turn to contrast the predicate expressions of the quantification-predicate model with an alternative notion of plural predication. For our purposes, we will employ the central notion of "structure": does the structure of the quantification-predicate model reflect the "structure" of reality, specifically, in regard to predicate expressions? (Sider 2009, pp 397-402) In this section we examine the predication side of the quantification-predicate model.

In his paper "Ontological Realism" Ted Sider argues that a common presumption – perhaps the definitive presumption – of analytic philosophy is that of ontological realism. "A certain core realism is, as much as anything, the shared dogma of analytic philosophers, and rightly so" (Sider 2009, page 399). Two imagined philosophers are presented with a table. The first, DKL (David Lewis) affirms whilst the second, PVI (Peter van Inwagen) denies [**FN 5]**:

EQ: This table exists

Sider considers ontological deflationism as a potential explanation of their apparent disagreement. In brief, the deflationist argues that DKL and PVI are actually talking past one

another. **EQ** is being used to express two distinct propositions. As a result, both DKL and PVI assert (by their own lights) a true proposition on the grounds of meaning variance. DKL makes a mereological assertion of his belief that there is a table composed of simpler entities. PVI makes a non-mereological assertion of his belief that the "table" is nothing but a "collection" of simple entities, arranged *tablewise*. Hence, PVI is *apparently* able to deny what DKL affirms: that there is a table which exists *together* with a collection of simpler entities from which it is composed.

The syntactic structure of **EQ** is identical for both DKL and PVI, so the disparity of meaning must be rooted in either their use of the predicate expression "*x* is a table" or their use of the existential quantifier "exists". We can immediately set aside one possible source of their disagreement, namely, ambiguity. The interpretation of some predicates such as "set" or "bank" is open to ambiguity. The sentence "John walked by the bank" can be interpreted in one of two ways: the word "bank" could be interpreted as meaning either "riverside" or "financial institution". Ambiguity does *not* apply in the current case, however. In the present context, DKL is speaking for those "immersed in the business of ordinary life" (Van Inwagen 1990, page 1) where the truth of **EQ** is assured by the triviality of language. *If* **EQ** was ever uttered in ordinary life, then its truth is "assured" by the scope of the quantifier as determined by the domain of discourse: "They (DKL and PVI) agree on the condition ϕ that a thing *must* meet in order to count as a table; their disagreement is over whether there exists *anything* that meets that condition" (Sider 2009, page 388 my emphasis). PVI does not dispute the meaning of the quantifier but rather the existence of a type of entity, a table; understood as something *distinct* from a "collection" of simples arranged *tablewise*. This

(plural) quantification-predication of a "collection" of simples arranged *tablewise* is *not* equivalent to the additional existential claim that a table exists (see Sider 2009, pp 388-389). PVI's denial is, therefore, *consistent* with the assertion of **EQ** because the denial is, but the assertion need not be, made in an extraordinary context; the philosopher's study (see van Inwagen 1990, page 2).

Does this "context-sensitive" dispute make sense (§ 2.5)? Lewis' argument is premised on the notion of properties – specifically, natural properties – that demarcate 'the joints of nature'. In contrast, PVI *can* argue that there are no privileged lines of demarcation. *If* we concede that the joints of nature are indeed drawn on the basis of natural properties we are still left with the possibility of an arbitrary distinction: that any one group of properties is just as "natural" as any other. Nelson Goodman, for example, argues that the relation of similarity is determined by objects sharing a property, any old property (Goodman 1972, pp 443-444). Goodman denies the objective structure of reality. The problem is that *any* two objects may be judged to be "similar": an object *a* that instantiates the property *F* is "similar" to an object *b* that instantiates the property *G* because both *a* and *b* instantiate the disjunctive property *F or G*! How is *this* objection applicable to PVI? Peter van Inwagen's argument is both intricate and is illustrated with vivid analogies (van Inwagen 1990, page 104). For example, he invites us to imagine that a predicate "bliger" is used by a linguistic community (the Pluralians) to refer to the black tigers they occasionally espy moving through the jungle. After several centuries of native use, an expedition of foreign zoologists determine that bligers do not actually exist. The creatures understood by the Pluralians to be bligers are actually a "composite" of six other creatures arranged "bligerwise". Four

monkey-like creatures as “legs”, a sloth-like creature as a “trunk” and a type of owl as a “head”. Only by careful study are the zoologists able to expose the widespread Pluralist illusion of [bligerhood]. “That is, it does not follow that six animals arranged in bliger fashion compose anything, and that is what I mean to deny when I say that there are no bligers” (van Inwagen 1990, page 104). PVI’s bliger-denial, however, remains consistent with the Pluralians assertion that “there is a bliger crossing that field”. By implication, the Pluralians are ontologically committed to a type of entity, *bligers*. Does this alleged *consistency* rank PVI’s denial alongside Goodman’s argument? The *consistency* of the existential claim, asserted-in-context-a and denied-in-context-b, *appears* to render the property of bligerhood an irrelevance in respect to the objective structure of reality (see van Inwagen 1990, pp 1-2). There is no privileged set of “natural” properties that demarcate “the joints of nature”.

Lewis argues that the structure of reality (this-world) is determined by the arrangement of natural properties and relations (Lewis 1983d [1999], Lewis 1986a pp 59-69). Natural properties serve two functions; they are both “similarity-determiners” and “content-determiners”. Natural properties and relations are “similarity-determiners” in that “perfectly similar objects … are those objects over whose parts the natural properties and relations are isomorphically distributed” (Sider 2009, page 400). They also function as “content determiners” in respect of language and thought (Lewis 1983d [1999], page 53). Consider the content of my thoughts and language. How is an ideal interpreter to decipher the meaning of my sentences? What information might she draw upon in order to interpret my remarks? One way to understand my sentences is to examine how I actually *use* them:

"Think of this as determining a set of sentences, ∏, such that the interpreter ought, other things being equal, to interpret my words so that the members of ∏ come out true" (Sider 2009, page 400). This set of sentences may include those sentences I consider to be analytic or that comprise the content of those beliefs that I hold to be true. It has been cogently argued, however, that this condition – that the sentences of ∏ ought to be interpreted such that they are all true – is not a sufficient condition of interpretation **[FN 6]**. Suppose that the interpreter mistakenly accepts a sentence *F* that is intuitively false but consistent with ∏: (by this supposition) "if there are enough objects then there will be an interpretation that makes all the sentences of ∏ plus *F* as well true" (Sider 2009, page 400). An interpreter, whose resources are limited to the *facts of my use* of a particular sentence, could misinterpret the falsity of *F* as true (see § 3.5 in respect to the content of belief). In order to avoid misinterpretation, Lewis argues that in addition to *the facts of my use* the interpreter must have access to *the facts of "naturalness"*. That is the interpreter should assign natural properties and relations to my use of predicate expressions, again, all things being equal. Natural properties and relations also function as "reference magnets" (Lewis 1983d [1999], page 47) that warrant the interpretation of my sentences in novel circumstances. They prevent the over-generation of meaning to which the interpretation of my sentences may (otherwise) be susceptible.

Lewis' modal translation scheme is an application of the quantification-predicate model that reflects the structure of reality. The unrestricted quantifier has a *natural* meaning; a monadic, singular quantifier which is itself a (second-order) property of properties (Sider 2009, page 407). The meaning of the universal quantifier is the property *F* instantiated by *x*

iff everything has *F,* whereas the meaning of the existential quantifier is the property *F* instantiated by *x* iff something has *F*. "The point of metaphysics is to discern the fundamental structure of the world. That requires choosing fundamental notions with which to describe the world. No one can avoid this choice" (Sider 2009, page 420). We have made our choice, the **QC** of the quantification-predicate model.

4.7 Conclusion

Lewis endorses the Ontological Parsimony of Kinds **OPK** as a theoretical virtue. The supposed virtue – if virtue it be – of quantitative parsimony **QnP** is wholly derivative. Lewis' argues that the **OPK** is partially constitutive of rationality. It is a methodological principle whose justification may be found in the history of the natural sciences. The history of science is replete with episodes of theory choice in which the pragmatic principle of **OPK** played a pivotal role. Lewis also supports the notion of natural properties that constitute the "qualitative joints" at which nature is carved. The quantification-predicate model reflects the structure of reality. Unrestricted quantification extends beyond the boundary of the actual world to encompass everything that exists. "Our idioms of existential quantification may be used to range over everything without exception, or they may be tacitly restricted in various ways. In particular, they may be restricted to our own world and things in it" (Lewis 1973, page 86). The postulation of a plurality of worlds can be justified on the grounds of qualitative parsimony. Quantitative parsimony, on the other hand, is likely to be more problematic for the genuine realist if the content of common belief is restricted to the furniture of the actual world. In these circumstances, the postulates conflict with Lewis'

criterion c of a legitimate paraphrase. However, the detrimental effects of admitting quantitative parsimony as a theoretical virtue can be mitigated: firstly, with the recognition that the theoretical value of **QnP** is derived and secondly, that as a "materialist" the values Lewis ought to accept are those of the natural sciences. The paraphrase of "ways things could have been" by a philosophical term of art, "possible world", may strike a discordant note with common belief but the benefits are worth the earache. We now examine the claim that the modal translation scheme is an analytic truth, specifically in regard to the aforementioned paraphrase of "ways things could have been".

Footnotes

1. "Respect neglect" is defined as "a prominent instance of the broader fallacy of illicit amalgamation, which consists in treating as a single uniform unit something that in fact involves a diversified plurality of separate issues. Specifically, it has the form of treating a feature *F* as a unified property that things do or do not have, whereas in fact *F* has several respects, and things can have *F* in one respect and lack it in another" (Rescher 2006, page 45).
2. See James Woodward (2003) *Making Things Happen: A Theory of Causal Explanation* Oxford University Press
3. Daniel Nolan (1997) examines the comparative ease with which Lewis (and many others) dismiss the significance of quantitative parsimony.
4. "But there are two classes of experimental facts hitherto obtained which can be represented in the Maxwell-Lorentz theory only by the introduction of an auxiliary hypothesis, which in itself – i.e. without making use of the theory of relativity –

appears extraneous" (Einstein 2001, page 52). Hence, the addition of *ad hoc* hypotheses to match the explanatory power of Special Relativity can cloud the issue of theory choice on the basis of **OPK** alone. The mere addition of *ad hoc* hypotheses is sufficient to justify a preference for SR simply on the grounds of theoretical elegance.

5. Sider's paper is only loosely based on the actual work of Lewis and van Inwagen. For example, PVI (unlike Peter van Inwagen) rejects the existence of "composite" *living* things (see Sider 2009, page 389 fn. 17)

6. See Putnam's model theoretic argument (Putnam 1978, Part IV, Putnam 1980 [1983], Putnam 1981, chapter 2) and Kripke's interpretation of Wittgenstein (Kripke 1982).

Chapter Five: Analyticity

5.1 Introduction

We have enumerated the logical, semantic and expressive benefits accrued by **GMR's** quantification-predicate analysis of modality (§ 1.1). These are: (1) the logical benefits of a simplification of resources by the substitution of modal expressions by their "worldly" equivalents; (2) the semantic benefits of dispensing with intensional semantics by discarding modal operators as logical primitives; (3) finally, the expressive benefits that include an enhanced ability to articulate modal concepts by lifting the restriction of sentential modal operators and "actualist" quantification. It is alleged that when combined these *desiderata* entitle the genuine realist to claim that the modal translation scheme is an analytic truth (Divers 1997, page 142). This entitlement appears to be a departure from the traditional semantic account of analyticity. A philosophical analysis is true if and only if it is analytically true. *If* the analysis offered by the modal translation scheme is true it is true because of the meanings of its terms. We proceed in this chapter to examine the idea that the modal translation scheme *is* an analytic truth, not on the grounds of the theoretical benefits it secures, but by virtue of the meaning of its terms. This is not Lewis' position. Although the logical form of the modal translation scheme is that of a universalised biconditional, the truth of the sentences on each side is contingent.

We will restrict the scope of this ambitious enterprise to the claim that two of the terms of the modal translation scheme are synonyms. Specifically, the terms paraphrased in **S1**; "ways things could have been" and "possible world". We begin with an examination of analyticity as that notion is unpacked by Lewis (§ 5.2). We then contrast the genuine realist analysis of modality with that of Robert Stalnaker. Stalnaker defends a form of liberal Platonism which distinguishes the semantic from the metaphysical theses. Newly distinct, Stalnaker claims to provide a modal translation in semantic terms devoid of the onerous metaphysical implications of genuine realism (§ 5.3). What type of analyticity is operant in the modal translation scheme? We examine two forms of metaphysical analyticity explored by Williamson. We conclude that the paraphrase cited at **S1** is Frege-analytic (§ 5.4). As an instance of paraphrase, the conjoined predicate expressions face the problem of stipulation. How can the meta-semantic act of stipulation ground semantic "synonymy"? We locate an answer in the form of linguistic convention (§ 5.5). In conclusion, we return to the problem posed by Stalnaker: as a semantic thesis, can we enjoy the analytical benefit of the modal translation scheme without the metaphysical cost? (§ 5.6)

5.2 Defending a Dogma

Traditionally, philosophical analysis imposes a number of criteria the satisfaction of which legitimise the proposed analysis as philosophical (see § 7.4). One such criterion is analyticity. A philosophical analysis of the form "$\forall x$ (Fx $\leftrightarrow$ Gx)" must be an analytic truth. "Hence, if the analysis is true, it is analytically true – true in virtue of the meanings of the terms involved" (Daly 2010, page 45). Quine famously rejects the traditional account of philosophical

analysis (Quine 1951a [1980]). We might anticipate that Lewis' commitment to naturalised epistemology would entail that he shares Quine's scepticism toward the analytic / synthetic distinction. However, Lewis does accept *a* form of analyticity that grants meaning a pivotal role:

> I have tried to make sense of analyticity. Or, rather, I have tried to make sense of two different analyticities. In Chapter V.3 I defined analyticity in a possible language L; in Chapter V.5 I defined analyticity in a population *P* of language users. The latter analyticity depended on the former analyticity and on the conventional relation between a possible language and a population. If I have succeeded in rescuing the platitude that language is conventional, have I thereby succeeded in rescuing analyticity and its kin? I believe I have. (Lewis 2002, page 203: also Lewis 1975 [1983], page 163)

A linguistic community counts as speaking a language L only if it has a convention of truthfulness and trust in L (see § 5.5). But how is a given convention established within a given community? The problem is particularly acute for those who argue that conventions link possible languages to a particular language community; in order to establish a particular convention the members of a language community must first reach an accord as to which conventions to adopt and which to abandon. But how is this agreement to be reached? Presumably, by way of a language that must also be regulated by established conventions based on a prior agreement … so on *ad infinitum*. The language-agreement-convention cycle can be broken by the admittance of conventions that are themselves non-linguistic. Occasionally, conventions just 'spring up' without the need for negotiation. For Lewis, the conventions of language likely originate in this way – implicit in the sense that they are not

arrived at by explicit agreement. Generally, a convention is a regularity in the behaviour of a population that emerges in response to a co-ordination problem. Coordination problems "are situations of interdependent decision by two or more agents in which coincidence of interest predominates and in which there are two or more proper coordination equilibria" (Lewis 2002, page 24).

We argue that Lewis' modal translation scheme is based on a synonymy of linguistic "pragmatism" (§ 1.3). The translation of modal sentences into sentences of counterpart theory does not yield an analytic truth in the traditional sense of synonymy of meaning but rather in the sense of synonymy of pragmatism (cf. Beebee and McBride 2015, page 230 ff.) In order to establish this thesis we explore the notion of language (or rather languages). We discover that Lewis argues that convention is the bridge between a possible language and an actual language. Our intended destination is the overall analytic truth of the modal translation scheme. In order to arrive at our destination we must first "clear the rubble". It might be objected that, as a semantic notion, the analytic truth of the overall scheme *could* be secured without a metaphysical proliferation of worlds. Could we dispense with the controversial ontological commitments of genuine realism and yet secure the theoretical benefits acquired by Lewis' modal translation scheme? The first step would be to divorce the semantic component of genuine realism from the metaphysical implications. This is Robert Stalnaker's Liberal Platonism.

5.3 Liberal Platonism

In "On What Possible Worlds Could Not Be", Robert Stalnaker raises an objection to the thesis of indexicality as advanced by genuine realism (§ 2.5). He states that the genuine realist's interpretation of the term "actual world" conflates two distinct theses:

1. The semantic thesis is that claims about what is possible or necessary should be analysed as claims about what is true in some or all of the appropriately individuated parts of reality.
2. The metaphysical thesis is that there exists a plurality of parts of reality individuated in this way – a plurality that is full enough to make true many of our ordinary beliefs about what is possible and necessary when those beliefs are interpreted in accordance with the semantic thesis **[FN 1]**.

(Stalnaker 1996 [2003], page 40: also Stalnaker 1976/1984 [2003])

The distinction drawn, Stalnaker dissects the genuine realist's analysis of modality; how does genuine realism provide for an adequate explanation of our ordinary, practical modal deliberations. How can objects causally isolated from us, worlds or their inhabitants, be relevant in our everyday modal thoughts and language? Stalnaker's question echoes a problem faced by the Platonist in the field of mathematics. In "Mathematical Truth", Paul Benacerraf raises a dilemma for the Platonist in respect of mathematical truth and knowledge. Benacerraf exposes a tension between what mathematical statements *say*, on the one hand, and the way in which we *know* what they say is true, on the other. Statements of, say, number theory, purport to be statements about a domain of objects –

numbers – that have no spatiotemporal characteristics and hence no causal relation to us. *If* this is so, then how can the Platonist account for our knowledge that true mathematical statements are in fact true? Lewis draws a favourable analogy between mathematical and modal knowledge (Lewis 1986a, page 108 ff.). He does so by grasping one horn of Benacerraf's dilemma: we accept the truth-conditions of a "straightforward" semantic theory and deny the requirement of a causal link between knower and what is known.

> So mathematics will do as a precedent: if we are prepared to expand our existential beliefs for the sake of theoretical unity, and if thereby we come to believe the truth, then we attain knowledge … Causal accounts of knowledge are all very well in their place, but if they are put forward as *general* theories, then mathematics refutes them. (Lewis 1986a, page 109) **[FN 2]**

Stalnaker concedes that the Platonist's realm of objects is compatible with the view that knowledge of *this* particular realm need not be causal. He concurs with Lewis' dissolution of Benacerraf's dilemma but rejects the analogy with the modal case. The challenge faced by Stalnaker is to demonstrate that the proposed mathematical-modal analogy is false. In short, that there is no plausible analogy between "reference to and knowledge of numbers and reference to and knowledge of cabbages, kings, and croquet games that are alleged to exist in spatio-temporally disconnected parts of reality" (Stalnaker 1996 [2003], page 43). If Stalnaker can establish a mathematical-modal disanalogy then he can assert non-causal knowledge in the former case whilst denying it in the latter. "I will sketch a strategy for responding to the [mathematical] dilemma, and argue that this strategy does not suggest a parallel response to the epistemological objection to modal realism" (Stalnaker 1996 [2003],

page 41). What is Stalnaker's strategy; how does he intend to separate the mathematical wheat from the modal chaff?

Stalnaker's strategy of "liberal Platonism" seeks to *limit* to the mathematical domain the proposed reconciliation between "straightforward" semantics and epistemology. The strategy proceeds on the endorsement of Quine's injunction; "to ponder our *talk* of physical phenomena *as a physical phenomena*, and our scientific imaginings as activities within the world that we imagine" (Quine 1960 [2013], Section 1 pp. 4-5 my emphasis cited Stalnaker 1996 [2003], page 45) (§ 2.4). Hence, the context in which a speaker refers to a certain object or person by name and the relation in which that speaker stands to the object or person is the *subject* of empirical enquiry. The liberal Platonist maintains that in order to refer successfully to a person or object the speaker must be related to the referent in the appropriate way. In the case of mathematical discourse, any putative *act* of reference to numbers, sets or functions is guided by fairly well established rules and procedures. The rules and procedures also guide assertion, falsification, inference and mathematical argument. Stalnaker is tempted to argue that the existence of mathematical entities such as numbers is *constituted* by the fact of an operative, legitimate practice of structured discourse. The statements produced in the context of mathematical discourse satisfy the standards of correctness that are *numbers* (Stalnaker 1996 [2003], page 44). On reflection, however, Stalnaker rejects this option; *if* it were true then it would mean that the existence of non-contingent numbers would depend on the contingent practice of mathematical discourse. Instead, he revises his position. "The point is rather that the commitment to numbers is just constituted by the endorsement of the *practice* with this structure and

standards" (Stalnaker 1996 [2003], page 45 my emphasis). The *practice* of mathematical discourse is subject to empirical enquiry. The enquiry yields the conclusion that there is a domain of mathematical entities. How does this conclusion advance Stalnaker's aim to establish a disanalogy between mathematical and modal discourse? In both cases, a "straightforward" truth-conditional interpretation of semantics entails a capacity to refer to objects that stand in a non-causal relation one to another. Quine's injunction appears to be equally applicable in both the modal and mathematical case. Quine's concern about the *de re* modal status of irrational cyclists and hopping mathematicians aside, what are the operative, structural rules of modal discourse? **[FN 3]**.

Stalnaker's strategy merely begs the question. Lewis' modal translation scheme provides counterpart theory, complete with the structure of **PL** and the established rules and procedures of valid inference. The necessary schism of the mathematical and modal case is not immediately evident. The next step in Stalnaker's argument is therefore essential:

> The liberal Platonist says that the legitimacy of the practice does not presuppose that there is any story to be told, in our empirical theory of the practice, about an external relation between acts of reference and the referents … [The] point is rather that the commitment to numbers is just constituted by the endorsement of the practice with *this* structure and standards. (Stalnaker 1996 [2003], page 45 my emphasis).

Stalnaker's manoeuvre is disingenuous. It attempts to embed the relation between the *act* of reference and a putative referent *within* the *practice* of rules and procedures and thereby generate a commitment to mathematical entities. It equivocates between two positions

equally unpalatable, to the liberal Platonist. The manoeuvre equivocates between a form of realism (that isn't) and genuine realism (in all but name). Of course, Stalnaker is aware of the potential danger and attempts to dilute both sides of the oscillation. Firstly, in regard to the claim that liberal Platonism endorses a realist ontology. Stalnaker considers a number of alternatives to genuine realism more conducive to the liberal Platonist. In what sense is the liberal Platonist a Platonist? The liberal Platonist could seek safe harbour in a fictionalist account of modality, for example. The liberal Platonist may suggest an analogy between a general account of truth in fiction and modality. The truth of sentences such as "Professor Moriarty died at the Reichenbach Falls" should not be taken at face value. Rather, they should be understood as truncated sentences explicitly about a wider fictional story. We tend to assert their truth because of the context of their utterance. A context that compels their interpretation as sentences that begin with a latent operator, namely, "according to such and such fiction …". Hence, despite appearances to the contrary, there is no commitment to quantification over a domain of objects "within" the scope of the "fiction" operator. Stalnaker declines this option: "the fictionalist position requires a commitment to the intelligibility of a thesis that, if true, would make the fiction fact" (Stalnaker 1996 [2003], page 46). Alternative "realist" options, however, do appear to be in short supply (see § 6.3). Secondly, and more seriously, how can the liberal Platonist distinguish her theory from **GMR**? In what sense is the liberal Platonist a liberal? For example, how does the liberal Platonism demonstrate a mathematical-modal disanalogy? In order to be applicable in the mathematical case the non-causal account of epistemology needs to be distinguished from genuine realism. Isn't the "liberal" Platonist simply a genuine realist wolf in sheep's clothing? Stalnaker anticipates Lewis' response:

> If our most plausible semantic hypothesis tells us that a certain bit of discourse is about things that speakers are not causally connected to, and if epistemological principles that seem on reflection to be reasonable tell us that we can know about such things, then there is no reason that we should not accept both the semantics and the epistemology. Isn't that just what your liberal Platonist thinks? Why, then, should he change his tune when he turns from mathematics to modal realism? (Stalnaker 1996 [2003], page 47: Lewis 1986a, 113-15)

Stalnaker acknowledges that identifying the distinction is difficult. A preliminary step is to recognise that the liberal Platonist is allied to a diluted form of verificationism. The allegiance is "diluted" because it is limited only to the ways in which epistemology ought to constrain ontology. The genuine realist claims to know that there are people and objects that exist despite their causal isolation. People and objects that exist in space-time but not *our* space-time. How are these type of modal beliefs to be justified *if* we are causally unrelated to these types of entity? *If* we are isolated from them by regions of space-time how do our beliefs acquire content? How can we determine that these people and objects resemble us in certain respects? It is the *nature* of the entities to which the genuine realist claims to have epistemic access that thwart their claims to know. The liberal Platonist argues that in order to acquire knowledge about people and objects we must be *causally* related to them. This diluted form of verificationism offers the liberal Platonist a route to the elusive distinction: it is the very nature (*de re* contingency) of people and objects that require a causal relationship between the knower and the known. This is the epistemological constraint that is absent from the genuine realists' inflated ontology. "He [the liberal Platonist] holds that we understand what it is to be a physical object or a person

partly by understanding how one must be related to one in order to know things about it" (Stalnaker 1996 [2003], page 49: cf. Dorr 2011). **[FN 4]**

We have arrived at an impasse. Although committed to a naturalised epistemology (see chapter 3), Lewis' simply rejects the claim that ontology should be constrained by epistemology. There *are* constraints on the ontology of genuine realism but they are not epistemological. The ontological commitments of **GMR** are constrained by methodological principles. There is no conceptual link between the nature of a thing and the ways we can know about it. All *impersonal* knowledge of *de re* contingency, including non-actual people and objects, is *a priori*. Knowledge of *actual* people and objects is *personal* because it is indexical (§ 2.5). We will put an exploration of the realist distinction of the semantic-metaphysical theses on hold for the moment (see chapter six). In the meantime we turn to one consequence of the proposed distinction: *if* we segregate the semantic and metaphysical theses do we thereby reduce analytic truth to insubstantial truth?

5.4 A Metaphysical Conception of Analyticity

By **S1**, Lewis' argues that "possible worlds" is a paraphrase of "ways things could have been" (§ 2.5). It appears that Lewis stipulates a "synonymy" of predicate expressions that partially renders the modal translation scheme an analytic truth. The semantic-metaphysical distinction highlighted in the previous section appears to impose an insubstantiality thesis on any potential account of analyticity. The type of analyticity most amenable to the

insubstantiality thesis is metaphysical (Williamson 2007). We examine two forms of metaphysical analyticity. Although initially appealing as a *general* conception of analyticity the first account must be rejected. An obvious cloud of circularity lingers over the modal-analytic conception in the case of modal translation. The second account of a Frege-analytic truth, on the other hand, together with a descriptivist understanding of meaning, appears to satisfy the demands of a "synonymy" of predicate expressions. Williamson's doubts over the Frege-analytic conception of analyticity rest on the insubstantiality thesis. Illustrative of this doubt is a distinction that he argues is too often ignored between semantic and meta-sematic facts.

Williamson (2007) subjects a number of possible accounts of analyticity to scrutiny. Those that compose the metaphysical form of analyticity are united by a common thread: "they impose no constraints on the world, not even on that part of it which consists of words and concepts … [] … Analytic truths are less substantial than synthetic ones because the latter do impose constraints on the world, which it may or may not meet" (Williamson 2007, page 52). The mark of a metaphysical analytic truth is a lack of constraint that it imposes on the world. Williamson accepts the semantic notion of synonymy as a point of departure from which to explore the notion of analyticity. The real difficulty, Williamson argues, is posed not by Quine's denial of the analytic/synthetic distinction but by Kripke's "widely accepted" differentiation of analytic, a priori and necessary truth along semantic, epistemological and metaphysical grounds (Kripke 1980, page 39). Kripke argues that "analytic" entails both "a priori" and "necessary". He also argues, however, that neither "a priori" nor "necessary" entail one another. It follows (by the transitivity of entailment) that neither "a priori" nor

"necessary" entails "analytic". The notion of analyticity is thereby relieved of the epistemological burden of the *a priori* and the metaphysical burden of the *necessary*. There is good reason to suppose, however, that the notion of analyticity still has some work to do.

Let us presume that by **S1** "ways things could have been" and "possible world" are synonyms. The analytic truth of the modal translation scheme is therefore partially composed of an equivalence in meaning. The overall modal translation scheme is analytic partially because these two predicate expressions share a meaning. Contrast a synthetic truth: true both because of the meaning of the expression and that the world accords with that meaning. For example, the proposition expressed by "Rumpole is a barrister" is true because it *means* Rumpole is a barrister *and* Rumpole is, in fact, a barrister. Traditionally, in the case of an analytic truth, the second conjunct is irrelevant in respect of the truth of the sentence. "Solicitors are lawyers", for example, is true *only* because it means solicitors are lawyers. This is the crux of the insubstantiality thesis: the truth-value of an analytic sentence is wholly determined by the meaning of the words not by the nature of things. Williamson is unconvinced by the insubstantiality thesis (Williamson 2007, page 58). He argues that an element of *presumed* truth partially explains the apparently analytic truth of "solicitors are lawyers". Namely, the presumed truth of the fact that solicitors are indeed lawyers! "For any true sentence *s* whatsoever, a canonical explanation of the truth of *s* takes the overall form "*s* means that P, and P"" (Williamson 2007, page 59). If the insubstantiality thesis does not provide a necessary condition of analytic truth then perhaps it provides a *sufficient* condition. Are the meanings of the terms *alone* sufficient to secure the truth of an analytically true sentence? The meaning of an expression will be "*sufficient for truth* just in

case necessarily, in any context any sentence with that meaning is true" (Williamson 2007, page 60). Williamson designates this definition of analyticity "modal-analyticity". What are the consequences of modal-analyticity?

Let a non-indexical proposition p be a necessary truth. Any sentence s that expresses the actual meaning of p, in any context, expresses a necessary truth and so is true. The sentence s meets the sufficiency condition and is thereby a modal-analytic truth. "But how little has been achieved in so classifying it!" (Williamson 2007, page 61). Nothing has been established in regard to whether *s* is or is not:

a. a deep metaphysical necessity discovered only after detailed *a posteriori* investigation *or*
b. just "verbal" or "insubstantial" in a pre-theoretical sense *or*
c. a mathematical truth.

The paucity of theoretical ground acquired by adopting the "in virtue of meaning" clause as a sufficiency condition of analyticity is further illustrated by analogy. "Call a meaning *temporarily sufficient for truth* just in case at all times, in any context any sentence with that meaning is true" (Willamson 2007, page 61). The quantifier terms "at all times" and "in any context" are to be understood non-modally; that is, as restricted to the actual world. Any sentence that expresses an eternally true proposition (no matter how contingent) has a meaning that is sufficient to be true. We designate such a sentence "temporally-analytic" e.g. "no automobile has ever had a million seats". However, there is no valid inference from temporal-analyticity to insubstantiality: "similarly, calling a sentence "analytic" in the sense of modal-analytic does not imply that it is somehow insubstantial, in the absence of a

background connection between necessity and some sort of insubstantiality" (Williamson 2007, page 61) Hence, the route to the insubstantiality thesis is blocked by the notion of modal-analyticity.

Arguably, some modal-analytic sentences do not express necessary true propositions e.g. contingent *a priori* sentences. Further, not all necessarily true propositions are modal-analytic e.g. necessary *a posteriori* truths. However, it is reasonable to suppose that a core of true propositions is constituted by modal-analytic truths. By **S5** modal logic – the possible is non-contingently possible and the necessary non-contingently necessary – the answers to questions such as "Is it necessary or possible that P?" are all necessarily true, for example. Furthermore, if the answers can be expressed in terms that are non-indexical they will be modal-analytic. Despite this, the recognition of a sentence as a modal-analytic truth does not unlock the epistemology of that sentence's analyticity. Although the conventional meaning of such sentences may be reliably asserted, reliability is generally not sufficient for knowledge. A mathematical truth, for example, may rest on a "theorem" that is suspect or a competent language user may use a true sentence *s* to express a proposition p that is modal-analytic. Again, the correct use of *s* is dependent merely on linguistic competency in respect of the words and grammar of a particular language. This is not equivalent, however, to knowledge or justifiable belief that *s* nor does it supply an insight into the truth of *s.* We raise but do not attempt to answer these important concerns because there is another storm cloud on the horizon. We are engaged in an investigation into the plausibility of the modal translation scheme as an analytic truth. To resort to an explanation of analyticity based on the notion of modal-analytic truth is viciously circular, at least on its face. For this

reason, we (abruptly) discontinue this line of inquiry and turn to a second possibility. The possibility that the type of analyticity of the overall modal translation scheme is Fregean.

"A sentence is Frege-analytic just in case it is synonymous with a logical truth, roughly speaking because everything of the form "All F is F" is true" (Williamson 2007, page 63). For example, "all bachelors are lazy" is not equivalent to a logical truth, because not every sentence of the form "All F are G" is a logical truth. Contrast the sentence "all bachelors are unmarried men", which is equivalent to a logical truth "All F is F" after the appropriate substitution of synonyms. In "Two Dogmas" Quine concedes that analytic statements include those that are logically true (Quine 1951a [1980], page 22); so *if* we admit the notion of synonymy then we should admit the notion of a Frege-analytic truth. Lewis expresses a preference for the paraphrase of "ways things could have been" by "possible world" (Lewis 1973, page 84). Does Lewis' argument from paraphrase establish a Frege-analytic truth? Various linguistic forms of analytic truth can be distinguished by way of their syntactic category: sentence, singular term or predicate expression (see Hylton 2010, page 55). In the case of predicate expressions, the relevant linguistic form of analyticity unpacks as follows:

i. Two predicates, *F* and *G*, mean the same just in case "All *F*s are *G*s and vice versa" is not merely true but analytic (*ibid.* page 55)

The "not merely true but analytic" clause is important. Recall, Quine's famous objection by way of the identity conditions of the set-theoretic view of properties (§ 4.5) (Quine 1951a [1980]). We could attempt to explain the synonymy of predicate expressions by way of inter-substitutivity in all contexts *salve veritate* (Quine 1951a [1980], page 27). This condition would not deliver the "cognitive synonymy" required of analyticity, however. The

co-extension of predicate expressions may turn simply on the contingent facts rather than on their meaning e.g. the co-extension of heteronyms “creature with a heart” and “creature with kidneys”. As a descriptivist – with a fine grained view of meaning – Lewis’ paraphrase by stipulation need entail only linguistic competence *and* rationality. Proponents of descriptivism usually advance a theory of loose semantic connection between a proper name and a cluster of descriptions. By avoiding a strict synonymy between a proper name and a *single* description they hope to dodge a host of persuasive counter-arguments marshalled by Kripke *et al*. However, “[In] contemporary philosophy, few who propose complex analyses claim synonymy for them” (Williamson 2007, page 69). In the case of modal translation, the predicative expression (descriptive phrase) “possible world” is proposed as a synonym of the descriptive phrase “ways things could have been” or “WTCB” (based on Williamson 2007, page 68 ff.). By (i) above:

A. “All possible worlds are WTCB (and vice versa)”

Sentence (A) does not appear to be a logical truth by any standard conception of classical logic (unless we interpret “possible world” and “WTCB” as logical constants). Indeed, the logical form of (A) has instances that are false: “All Ps are Es (and vice versa)” is false in the case of “All Parisians are Ethiopians (and vice versa)”. Any trouble caused to our argument by false counterexamples is minimal, however. Frege-analytic terms include a range of expressions that include but are not limited to logical truth. Suppose we did allow “possible world” and “WTCB” the same intension (understood as a function from circumstances of evaluation to extension in all contexts of utterance). Even this generous equivalence would not supply literal synonymy. The semantic structure of the phrase “possible world” is finite and not a proper part of the semantic structure of “ways things could have been” (see

Williamson 2007, page 69). The expressions also differ in their syntactic structure. Aside from these not inconsequential difficulties, Williamson's central problem with our proposal – to establish a Frege-analytic truth by way of a stipulated synonymy – is grounded in a distinction between semantic and meta-semantic facts (Williamson 2007, pp. 71-2).

Semantic facts are those systematised by a compositional theory for any given language. Semantic facts pertain to what a given expression *means*; for example, the common noun "dog" means the species dog. Meta-semantic facts, on the other hand, are the facts on which the semantic facts supervene. Meta-semantic facts include the fact that the presence of a dog will often prompt the utterance "dog". By parity of reason, the fact (if fact it be) that the predicate expression "possible world" means the way things could have been, is semantic. However, the fact that the meaning of the phrase "possible world" is *introduced* by stipulation is meta-semantic. The manner in which a term is introduced to the lexicon of a language is semantically irrelevant. "The act of stipulation makes the sentence true by making it have a meaning on which it is, in the quite ordinary way, true" (Williamson 2007, page 72). Lewis' **S1** preference for "possible world" as a paraphrase of "way things could have been" does not make a possible world *be* a way things could have been. That would be to make a way things could have been *be* a way things could have been. Clearly, Lewis' act of stipulation *did not make* the way things could have been *be* the way things could have been. The way things could have been (literally) existed whether Lewis' stipulates the synonymy or not. "The peculiarity of the case is all at the meta-semantic level; the use of stipulative definitions as paradigms does not yield a *semantic* notion of analyticity" (Williamson 2007, page 72).

The attempt to argue that the modal translation scheme constitutes a traditional analytic truth is frustrated. We presumed that the modal translation scheme would minimally require an equivalence, "in virtue of meaning" *alone*, between two predicate expressions. As a necessary condition of analyticity the insubstantiality thesis would be confounded by the worldly assessment of truth required of *some* analytic sentences. As a sufficient condition of analyticity the insubstantiality thesis enjoys some success with the modal form of metaphysical analyticity. Unfortunately, the modal-analytic explanation is circular in respect of the modal translation scheme. An alternative variety of metaphysical analyticity is found in Fregean-analytic sentences. In relation to predicate expressions, and absent a synonymy in ordinary language, the phrases "possible world" and "ways things could have been" appear to demand stipulated synonymy. A simple declaration of preference, that by "F" I shall mean "G" (and vice versa). Quine admits that a novel term such as "possible world" could be introduced for the sake of brevity or in order to teach (Quine 1951a [1980], page 27: 1960 [2013], Section 40 page 175). The problem with Lewis' preferred paraphrase of "ways things could have been" is that synonymy cannot be established by stipulation alone. Williamson argues that synonymy by stipulation is apt to confound two different types of fact. In order to defend Lewis' preferred paraphrase, therefore, we require a bridge to link the meta-semantic fact of stipulation with the semantic fact of synonymy. Fortunately, Lewis provides the necessary materials for the construction of such a bridge in his systematic account of linguistic meaning (semantic) based on convention (meta-semantic).

5.5 Analytical Truth by Convention

We argue that Stalnaker's demarcation of the semantic from the metaphysical thesis is correct. His distinction facilitates our endorsement of the "insubstantiality thesis" denied by Williamson. An analytic truth is true by virtue of meaning *alone.* An analytic truth is true *regardless* of how the world is. The term "bachelor" is a synonym of the phrase "unmarried man" whether there are any actual bachelors or not. The term "dodo" is a synonym of the phrase "a large flightless bird, *Raphus cucullatus*, formerly native to Mauritius" whether there are any actual dodos or not. The term "unicorn" is a synonym of the phrase "animal with the body of a horse and a single straight horn projecting from the forehead" whether there are any actual unicorns or not. Hence, the predicate expression "possible world" *can* be a synonym of the phrase "ways things could have been" whether there are any ways things could have been or not. Importantly, this view entails that the complex term "possible world" has *no* metaphysical implications. A possible world could be a discrete, spatiotemporal continuity or a set of consistent and complete unactualised propositions or a recombination of actual world properties or a convenient fiction, and so forth. In this section we address Williamson's main objection to this form of analyticity. How does a meta-sematic act of stipulation establish a semantic synonymy?

Lewis' mature view expands on his preliminary remarks about the notion of a convention and which conventions must hold in order that a population P speak a language L (Lewis 1969 [2002]: 1975 [1983]). The notion of a convention is altered in two ways. First, conventions are now understood as a regularity not just in action but in *belief*. Second, an

earlier condition of conventionality is diluted. A 'preference shared by almost everyone in a community' is replaced by '(almost) every agent has a reason to conform when they believe others will also conform'. The "reason to conform" can be practical, when conformity relates to action, or epistemic, when conformity relates to belief. Lewis' earlier account of a convention related to a 'regularity' among the speakers of a language. The new account includes the hearers as well as the speakers of the language. Intuitively, Lewis' mature analysis aligns his account of linguistic convention with the *actual* practice of a population: linguistic conventions resolve coordination problems between speakers of a language *and* their hearers. As a consequence of these adjustments, a population P now use a language L only if it has a *convention* of 'truthfulness and trust' in L (cf. Hawthorne, 1990, 1993: Lewis 1992).

1. Truthfulness in L; "To be truthful in L is to act in a certain way: to try never to utter any sentences of L that are not true in L. Thus it is to avoid uttering any sentences of L unless one believes it to be true in L" (Lewis 1975 [1983], page 167).
2. Trust in L: "To be trusting in L is to form beliefs in a certain way: to impute truthfulness in L to others, and thus to tend to respond to another's utterance of any sentence of L by coming to believe that the uttered sentence is true in L" (Lewis 1975 [1983], page 167).

Lewis' earlier account took convention to be rooted in coordination between the *speakers* of a language. The English-speaking community utter English sentences in, say, London because they intend and expect their sentences to be understood by that community. Lewis' early analysis fails, however, to account for a basic form of communication: two-way coordination between a single speaker (convention to speak truly in L) and a single hearer

(convention to trust in L). Naturally, there are exceptions to the conventions of truthfulness and trust in L. People do not always intend to tell the truth. In addition to dishonesty, people can be mistaken or misapply sentences. As a result, we do not always believe what people say. However, Lewis' account of conventions in language is the default position. In a sense, the possibility of dishonesty or mistake is conceptually 'parasitic' on the predominantly honest and reliable practice of a linguistic population. As Gilbert Ryle noted, the idea of a counterfeit £10 note is conceptually dependent on the idea of a genuine £10 note.

The synonymy of terms is embedded in the conventions to speak truly and to trust in L. Lewis' initial definition of a convention repeatedly makes use of the phrase "conform to R" and "conform to R'" (Lewis 2002, page 78). Subsequently, in respect of language he cites the 'rules of composition' and 'rules verbal expression'. Provisionally, we propose that a 'conformity to R' should be interpreted as a meta-semantic component of linguistic convention, e.g. a linguistic *act* of stipulation. The meta-semantic act of stipulation is supplemented by the *rules* of 'composition' and 'verbal expression'. These rules comprise the semantic component of a linguistic convention. When combined 'conformity to R' and 'following a rule R' warrant the synonymy of our two predicate expressions. The paraphrase of "ways things could have been" by "possible world" is licensed by a bridge constructed from the distinction between conformity to a rule and following a rule. How are 'conformity to R' and the rules of composition and verbal expression combined in practice?

Consider:

(B): The Knight in chess first moves two squares in a straight line then one square to the side.

In order for a chess player to comprehend (B) she must first understanding certain practices and facts about the game of chess. What is a Knight in chess? What is a square in chess? What is a move in chess? In this context, (B) could be said to have the following canonical reading:

i. (B) formulates a rule in chess

However, (B) might be used in a number of different ways, for example:

ii. (B) is an inductive generalisation regarding the permitted movement of an object called the "knight in chess", or

iii. (B) is an empirical hypothesis about the activity of moving the knight in chess, or

iv. (B) is an expression of an indirect command to a player of chess

"That (B) has these different usages is something that continues to be relevant to its application even *after* its inception" (Juhl & Loomis 2010, page 122). A person ignorant of the difference between (i) and (iii), for example, will not be able to correct a player who mistakenly moved a knight "bishop-like" along the diagonal. If (B) is just a hypothesis (as at iii) then the movement of the knight along the diagonal is not *incorrect*, it is simply the falsification of a hypothesis. In an actual game of chess, however, a player who moves the knight along the diagonal has not merely falsified a hypothesis or refuted an empirical generalisation. They have made an illegitimate move. They are open to censure from their opponent. The stipulation-hypothesis distinction clearly rests on a difference between

following a rule and mere conformity with a rule (Wittgenstein 1958, Sections 138-242). Wittgenstein's passages on "rule-following" offer a normative account of language that is subject to a blizzard of interpretation:

> From the point of view of a normative conception of meaning such as Wittgenstein defends, a behaviouristic conception like Quine's is simply no conception of meaning at all, not even an ersatz one (fn. 19). Indeed, it is no conception of *language*, for a language stripped of normativity is no more language than chess stripped of its rules is a game. (Hacker 1996, page 15-16)

Hacker's comments demonstrate the folly of ignoring Wittgenstein's distinction (§ 2.4). The folly is amplified if there are also independent reasons that favour the distinction. There *are* independent grounds that favour the distinction; a ticking clock, a set of traffic lights and waves lapping on a seashore are amongst numerous examples of a *conformity* to a regularity. A set of traffic lights, for example, can be said to conform to the imperative "show a green light only after a red and amber light are shown together". It hardly seems appropriate to say that the set of traffic lights *follows* this rule, however. Absent Wittgenstein's distinction we are left with an invidious choice between two implausible alternatives: either *nothing* is an instance of rule-following or *every* regularity is a case of rule-following.

Mere conformity to a linguistic rule is insufficient to establish a stipulated synonymy of terms amongst a community of language users. A community must also 'follow a rule' in order to satisfy the demands of actual language use. The meta-semantic act of stipulation establishes a semantic synonymy because the convention guiding the stipulation is non-

linguistic. The stipulated synonymy warrants Lewis' preference for the paraphrase of "ways things could have been" by "possible world". However, even if this paraphrase is genuine (§ 2.3) it remains but one component of the overall modal translation scheme. The introduction of a predicate phrase "possible world" as a novel term may help to generate a number of theoretical advantages (§ 1.1). In combination, these theoretical *desiderata* may justify the claim that the modal translation scheme is an analytic truth (Divers 1997, page 142). We argue for a different route to the analyticity of the modal translation scheme. If successful, one component of the modal translation scheme is the paraphrase of an ordinary language expression. We reiterate, however, that no metaphysical implications should accompany the success of this endeavour. The truth of a traditional analytic sentence is wholly trivial. The semantic (synonymy) – meta-semantic (act of stipulation) bridge is constructed. The truth value of a sentence in L is determined by convention. The conformity to a rule of language (meta-sematic fact) is combined with following a rule of language (semantic fact), to establish a genuine paraphrase. Hopefully, we have managed to allay Williamson's primary concern.

5.6 Conclusion

Stalnaker's dislocation of **GMR's** semantic and metaphysical thesis indicates that the analyticity of the modal translation scheme need not entail an ontological commitment to a plurality of worlds. Hence, this ontological commitment must be justified on independent grounds (chapter 4). The notion of metaphysical analyticity implies that analytic truth is insubstantial. We argue that insubstantiality does not meet a necessary condition of analytic

truth. There are analytic truths whose truth is based on a presupposition about how things stand in the world. As a sufficiency condition, insubstantiality was tested against modal-analyticity. We were compelled to set aside modal-analyticity on the grounds of circularity. A second conception, Frege-analyticity was adopted. Williamson doubts the plausibility of the new analysis on the grounds that it fails to observe a meta-semantic/semantic distinction. We accept the distinction but argued that it might be bridged by Lewis' account of linguistic convention. Lewis offers a systematic account of how linguistic meaning could be generated by social convention. Appropriately adjusted to recognise "following a rule", Lewis is able to stipulate his preferred paraphrase of "ways things could have been". These gains do not banish Stalnaker's semantic-metaphysical distinction, however. Indeed, they presuppose that distinction. This is to invite danger. Do semantic explanations of modality enjoy the theoretical benefits of the quantification-predicate model without the need to postulate a plurality of Lewisian worlds? We now examine one possible alternative to **GMR** that exploits Stalnaker's semantic-metaphysical distinction, John Divers' Modal Agnosticism.

Footnotes

1. Stalnaker accepts the indexical interpretation of the phrase "actual world" as a semantic thesis but denies the metaphysical view that there is more than one world. By parity of reason, one may endorse solipsism and accept the indexical character of personal pronouns or accept the indexical analysis of tense and yet endorse the past as mere memory and the future mere anticipation (Stalnaker 1976 / 1984 [2003], page 29).

2. Lewis rejects the epistemic distinction of the *a priori* (abstract realm) and *a posteriori* (concrete realm).The distinction, he argues, is not based on the contrast between abstract and concrete objects but rather on objects that are contingent and those that are necessary. Independently, some "objects" are abstract *and* contingent: "those objects, like the Equator, whose existence is contingent upon the existence and behaviour of concrete objects" (Dummett 1991, page 239). Does the converse hold true? There are, according to **GMR**, "countless [concrete] objects causally isolated from us" that comprise the furniture of a plurality of worlds. By analogy with the mathematical case, the existence of these worlds is a matter of *necessity*. Hence, we acquire knowledge of these necessary worlds – and their *concrete* inhabitants – by non-causal, *a priori* means. This appears to licence the inference, that "objects" such as the Equator – abstract (but dependent for their existence on contingent objects e.g. the "actual" world) fall within the purview of the *a posteriori* – should be juxtaposed with the existence of objects that are concrete (but dependent for their existence on necessary objects e.g. "possible worlds") and fall within the realm of the *a priori*.

3. Beebee and McBride (2015, page 224 ff.) argue that Lewis and Quine concur insofar as the determination of the modal attributes of a thing *x* is context-dependent. However, rather than share in Quine's "bewilderment" Lewis embraced the "variety of what we say about modality and counterfactuals *de re*, [where we will find] abundant evidence that we do not have settled answers, fixed once and for all, about what is true concerning a certain individual according to a certain (genuine or ersatz) world" (Lewis 1986a, page 252) The *multiplicity* of the counterpart relation enables Lewis to accept the inconstancy of ordinary modal discourse. The practice is

still understood as rule-governed despite its inherent vagueness and inconstancy *viz.* Wittgenstein's analysis of "game": "In one context we may favour the counterpart relation that (e.g.) weighs being two-legged more heavily than being rational, but in another context we may reverse the relative weighing of these respects of similarity" (Beebee and McBride 2015, page 225: see Lewis 1986a, page 254: Lewis 1983b [1983], page 42).

4. Eventually, Stalnaker concedes that he is unable to provide a *general* principle of distinction that the genuine realist is obliged to accept. Furthermore, Lewis denies a premise on which Stalnaker's argument depends: namely, a conceptual connection of the way in which we know a thing and our conception of its nature. On Dummett's proposal of a "fifth way" (Chapter 14, *Frege: Philosophy of Language*) intended to distinguish concrete and abstract entities, Lewis comments: "Even if this fifth way succeeds in drawing a border, as for all I know it may, it tells us nothing directly about how the entities on opposite sides of the border differ in their nature" (Lewis 1986a, page 82 fn. 56).

Chapter Six: Modal Agnosticism

6.1 Introduction

The subject of this chapter is Divers' agnosticism in respect of the ontological commitment to a type of entity, possible worlds. The antirealist interpretation of modal discourse is based primarily on an ontological distinction **[FN 1]**. Broadly, the realist asserts that there is a mind-independent plurality of worlds that constitute a domain of non-modal discourse. The *type* of world can vary depending on the type of realism employed. A Genuine Modal Realist **GMR** asserts the existence of a plurality of concrete individuals (worlds) whilst the Actualist Realist **AR** asserts the existence of a plurality of abstract entities (worlds). The antirealist, on the other hand, denies the realist interpretation of the relevant domain of discourse. "There the central distinction is between *realist* interpretations that reflect acceptance of the existence of many possible worlds and *antirealist* interpretations that reflect resistance to that commitment" (Divers 2002, page 3).

Antirealism is contrasted with **GMR** analysis. The modal analysis usually presented as the viable alternative to **GMR** is **AR**. However, both varieties of realism suffer from a parallel defect. Whether worlds are concrete individuals or abstract entities they are both causally isolated from the actual world. This causal isolation prompts Divers to reject both types of realism (§ 6.2). We briefly consider the motivation behind antirealist interpretations of modal discourse (§ 6.3). We examine one variety of antirealism, Worldly Agnosticism (**WA)**,

and the criticism that it implies an assertibilty (and belief) deficit. Divers' offers three strategies available to the worldly agnostic in order to mitigate this criticism. They are loosely based on the rational dispensability of the (disputed) class of belief in *de re* possibility, that is unwarranted on agnostic grounds (§ 6.5). We conclude that although Modal Agnosticism presents a cogent alternative to **GMR** it fails to match the latter's reserve of modal assertible (and belief) sentences (§ 6.6).

We identified a number of theoretical advantages associated with the quantificational model of modal analysis (§ 1.1). Divers hopes to acquire the advantage of inferential validity without the cost of an ontological commitment to a plurality of worlds (chapter 4). The thread of inferential validity runs through the weave of Divers' argument to establish worldly agnosticism as a viable alternative to **GMR** (Lewis 1986a, page 174). If successful Divers' **WA** is able to secure the theoretical benefit of valid logical inference (§ 1.1) and criterion b of Lewisian paraphrase (§ 2.3).

6.2 Realism and Possible Worlds

We inhabit a world populated by people and objects that possess properties and stand in relations of various kinds to one another. Standard **PL** allows us to quantify over these people and other objects and assign to them properties and relations. Recall, **GMR** extends this ability to quantify over what actually exists to include those *possibilia* that do not actually exist. This literal "unrestricted" domain of quantification aims to supply a reductive

analysis of modality (§ 2.3). When we discuss what could, might or ought to happen we are talking about a possible world in which what could, might or ought to happen, did happen. We can translate modal talk of necessity and possibility into extensional talk about possible worlds where the relevant necessity or possibility is realised. The biconditional schema unfolds as follows:

A. It is necessary that P iff P is the case in all worlds
B. It is possible that P iff P is the case in at least one world
C. It is impossible that P iff P is the case in no world

Hence, **GMR** modal translation exploits a logical equivalence between the modal operators (possibility and necessity) and quantification (existential and universal). It thereby avoids the problems associated with sentences of an intensional language by translating them into sentences of an extensional language (§ 2.2). There are a number of significant theoretical benefits associated with this philosophical technique: for example, "we are entitled to calculate the modal consequences of modal premises by applying first-order quantificational logic to their possible-world 'translations'" (Divers 2004, page 665).

Many philosophers consider the price attached to these benefits too high. As a result, they have sought alternative routes to the same destination: enjoy the inferential power of **GMR** whilst eschewing the ontological commitment to a plurality of worlds. One route is that of Worldly Agnosticism **(WA)**: "Worldly Agnosticism offers us the replication of the conceptual and inferential benefits afforded by genuine realism but without the ontological cost, and the associated epistemological cost, of commitment to a plurality of worlds" (Divers 2004,

page 683). Divers' alternative to **GMR** is *moderate* **WA** "(which is) agnostic about the existence of any entities that are characterized as unobservables, *except when some further characterization gives grounds for disbelief in the entities in question*" (Divers 2004, page 669). Moderate **WA** is contrasted with *radical* agnosticism which retains its agnostic stance about unobservable entities *no matter how they are otherwise described* **[FN 2]**. However, despite a licence to share many of the modal claims of the genuine realist, **WA** suffers from a deficit of warranted modal belief. The charge of an assertibility (and belief) deficit against worldly agnosticism is made on logical grounds.

As a preliminary, we explore the reasons Divers enumerates to reject two influential varieties of modal realism. The realist-antirealist demarcation is drawn on the basis of their respective ontological commitments to a type of entity. Realism is marked by a commitment to a type of entity, non-actual worlds; although the ontological nature of those world(s) differs dependent on the type of realism. The postulates of **GMR** state that possible worlds are mind-independent, discrete spatiotemporal individuals. By implication each world is causally isolated. Consider the following translation (1968) of a modal sentence:

GMR1: It is possible that dragons exist iff there is a world at which a dragon exists

The *analysans* [right side, biconditional] of the *analysandum* [left side, biconditional] is:

GMR2: ∃x ∃y (Wx & Iyx & Dragon y)

Pitted against genuine realism is a view espoused by Robert Adams *et al* Divers designates Actualist Realism **(AR)**. Philosophers perplexed by an ontological commitment to a *plurality*

of concrete worlds suggest a different conception of "possible world". Adams, for example, argues that possible worlds are determinate but abstract, maximally consistent sets of propositions. Worlds are "determinate" when they satisfy two conditions:

> [That] (1) For every possible world, w, and every pair of contradictory propositions, one member of the pair is true in w and the other member is false in w. (2) Each possible world, if temporally ordered at all, is a complete world history and not a momentary stage of one (Adams 1979, page 191).

The genuine realist *appears* to enjoy a number of theoretical advantages over her **AR** counterpart. On the presumption that theoretical reduction is a *desideratum* (Divers 2002, page 181 fn. 1: cf. Stalnaker 2012, page 4) the attempt by **AR** to provide a reductive analysis of modality must be in vain. The *analysans* of **GMR** translation is satisfied (in part) by a plurality of worlds that exist with or without our knowledge or belief (§ 3.5). This objectivity renders them immune to the vagaries of our doxastic state – in terms of what we believe to be necessarily or possibly true. Worlds are not dependent on our notions of modality and are themselves described in non-modal terms (§ 1.1). **GMR** analysis supplies a reductive, non-circular account of modality. By way of contrast, the preliminary accounts of **AR** considered by Divers have no reductive account of modality to offer. Adams' account, for example, is based on the *consistency* of a set of propositions that constitute a possible world. Hence, it must be possible for *all* of the propositions of the relevant set to be collectively true. Clearly, to explicate the notion of a possible world based on the "consistency" of a set of propositions is to employ a modal notion. We may justifiably conclude, therefore, that the **AR** account of modality is circular.

On the other hand, **AR** does enjoy the theoretical advantage of a "safe and sane ontology" (see Divers 2002, page 227). If we are already predisposed to accept propositions as amongst the ontological commitments of total theory we may avoid ontological inflation. However, Divers is not persuaded that the ontological commitments of **AR** are innocuous. An ontological commitment to either sets of propositions *or* a plurality of concrete worlds betrays a common flaw: causal isolation. **GMR** postulates a plurality of discrete, concrete worlds that are by implication causally isolated from each other (§ 5.3). How do we acquire modal knowledge of possible worlds and their inhabitants? Modal knowledge could not be *a posteriori* (causal) since by stipulation each world is causally isolated. The epistemological problem is particularly acute for *de re* possibilities. Humphrey's defeat in the 1968 US Presidential Election could have been otherwise. Humphrey might have won. **GMR** analysis states that we know that Humphrey might have been victorious because he has *other*-worldly counterparts who defeat a counterpart of, say, Nixon. For the genuine realist *possible* people and physical objects *are* people and physical objects. The problem of causal isolation originates in the genuine realists' requirement of different epistemic (causal) relations for entities of the same kind. Why should there be any epistemological difference based on how we are related to Humphrey and his counterparts, when they are both contingent people? In respect of Humphrey, our *a posteriori* knowledge of his electoral defeat is based on a causal relation. In respect to Humphrey's counterpart, our *a priori* knowledge of his electoral victory is based on a non-causal relation. To understand what it *is* to be a person or physical object is *in part* to understand how one is epistemically *related* to that person or physical object (see Stalnaker 1996 [2003], pp 48-9). How should epistemology constrain a theories ontological commitments? **[FN 3]** For the genuine realist, they shouldn't and they don't (§ 4.1).

AR fares little better. **AR** postulates the existence of abstract entities (e.g. propositions and sets of propositions) which are themselves causally isolated. Again, how do we acquire knowledge of abstract types of entity? "**AR** ontology is not safe and sane because its causal isolation from us renders it semantically and epistemologically queer – we do not stand in the kind of relations to **AR** worlds that are (normally) considered necessary for us to be able to refer to things and to know about them" (Divers 2002, page 230). In terms of their causal isolation from the actual world, Divers argues that both **GMR** and **AR** are tarred with the same brush. Each postulates an ontology that is difficult to reconcile with our ordinary claims to know or debate modal truths. In these circumstances, why should we endorse either type of realism? Why not simply abstain in response to a belief in possible worlds that are causally isolated? Abstention is the doxastic stance taken up by the Worldly Agnostic **WA**: "The epistemological thesis of the worldly agnostic (i.e. one who is agnostic about the existence of other Lewisian worlds) is that we have no warrant for believing in the existence of any possible world other than the actual world" (Divers 2004, page 668).

6.3 Antirealism and Possible Worlds

Antirealism in the modal sphere is based on a general strategy of securing the benefits of **GMR** whilst avoiding an ontological commitment to a plurality of worlds. "Modal antirealists, in this context, are those who wish to lay claim to (certain of) the benefits associated with talking as if many possible worlds existed without being committed to the existence of many possible worlds" (Divers 2004, pp 662-63). The strategy is illustrated by

the modal translation of **GMR 1**, [right side, biconditional]: “there is a world at which a dragon exists”. *Prima facie* when we assert this sentence – thereby implying its truth – we are committed to the existence of at least one non-actual world which has a dragon as a part (granting, of course, that there are no dragons at the actual world). In order to avoid asserting the realist *analysans* of modal translation, thereby avoiding an ontological commitment to non-actual worlds, the antirealist has traditionally employed one of two stratagems: either deny that the sentence means what it appears to mean or accept the apparent meaning of the sentence but deny that it is true **[FN 4]** (Divers 2004, page 663). The problem with the first stratagem is that denying the apparent meaning of a sentence is to risk sacrificing the theoretical benefits – such as inferential validity – that a *prima facie* interpretation of meaning secures. In the alternative, adopting the second stratagem does secure the antirealist the theoretical benefits of **GMR**. That is, a *prima facie* interpretation of the meaning of the relevant sentence provides an analysis in the form of truth conditions and conceptual content central to inferential validity. By the second stratagem the “antirealist interpretation is identical to the realist interpretation, and with possible-world talk so interpreted (i.e. at face value) there is no mystery about the (first-order) consequence relations that she takes ultimately to underlie modal validity” (Divers 2004, page 666). The antirealist accepts the *prima facie* meaning of the sentences but rejects the truth of the existential *assertion* made by the realist and thus avoids a commitment to non-actual worlds.

The second stratagem of non-assertion may be pursued in one of two ways. Firstly, the antirealist can allege that the existential *assertions* made in possible world discourse are

systematically false – that is, they are not really assertions at all. Secondly, the antirealist can refuse to acknowledge that *any* of the existential assertions of possible world discourse are true **[FN 5]**. **WA** "offers a vindication of our modalizing without requiring that we abandon the view that our modal beliefs frequently satisfy the full-blooded cognitive norms of truth and knowledge" (Divers 2004, page 667). The difficulty with the second stratagem is one of an assertibilty / belief gap: if we hold that the biconditional [right side] is not true this must entail that the biconditional [left side] is not true (cannot be truly asserted).

6.4 Worldly Agnosticism and Possible Worlds

Divers' begins his advocacy of Worldly Agnosticism **WA** with a distinction:

1. The Epistemological Thesis: there is no warrant to believe in any world other than the actual world
2. The Semantic Thesis: no sentence that entails the existence of any non-actual world is assertible and no sentence (of this type) can be *held true* even though *some* are true (for all we know)

(Divers 2004, page 668)

Combined, (1) and (2) express a natural epistemological attitude toward modality. Lewis presents a utilitarian justification for his shared belief-in non-actual worlds which implies an *a priori* justification for their existence (chapter 3). In the alternative, Divers advocates a *moderate* form of **WA** is *not* committed to a comprehensive agnosticism about modality.

The moderate agnostic may legitimately make claims to modal knowledge based on necessity or impossibility, for example. These claims may be justified on the grounds of logical validity, language or the laws of nature. For instance, Divers claims to characterise necessity and impossibility by the application of unrestricted negative existential content (Divers 2004, page 670).

Hence,

GMR Necessity – It is necessary that A iff at all worlds A

Becomes

WA Necessity – It is necessary that A iff there is no world at which not-A

(Divers 2004, page 669)

We will confine our examination of Divers' antirealist argument to moderate agnosticism. Parallel types of logical inference guide a raft of agnostic claims to modal knowledge. "What the agnostic about other worlds knows to be true of *no* individual, she knows – a fortiori – to be true of no part of any world and, crucially, she knows these things without knowing whether there exists any world other than our own" (Divers 2004, page 670). Additional agnostic claims to modal knowledge can be warranted by analytic truth; that nothing is both male and a vixen, for example. Agnostic claims to know beyond the realms of logic and analyticity become more controversial. A worldly agnostic, persuaded by Kripke *et al*, might claim to know modal truths warranted by the laws of nature; to know that nothing is water and hydrogen free, for instance. Where should the worldly agnostic draw the line between

knowledge and ignorance? Does moderate **WA** really enjoy epistemic parity with **GMR**? If so, does **WA** incur the same epistemic costs?

In the previous section we noted that worldly agnosticism is open to criticism on logical grounds: "The outcome, then, is that when compared with the realist, and with the folk, the worldly agnostic has a limited, but initially shocking, modal assertibility (and belief) deficit" (Divers 2004, page 675) (§ 6.4). Despite impressive claims to modal knowledge the **WA** does *not* enjoy epistemic parity with her realist counterpart. There is an epistemic deficit or shortfall. Recall, modal translation is materially adequate in that it preserves the truth or falsity of the (modal sentence) *analysandum*.

GMR 1: It is possible that dragons exist iff there is a world at which a dragons exist

GMR1 is a biconditional sentence of the form P iff Q. If the **WA** is to profess agnosticism in respect of the *analysans* "Q" [right side, biconditional], specifically, a reluctance to assert the existence of a non-actual world, she is obliged – on pain of logical inconsistency – to profess agnosticism in respect of the *analysandum* "P" [left side, biconditional]. That is, the **WA** is obliged to be agnostic about the truth of the claim that dragons are possible. This obligation is incurred by the truth-conditions required of all true biconditionals: the sentences *must* either both be true or both be false, otherwise the biconditional must be false. "But to hold of either truth-value [true or false] that it is the truth-value of Q is contrary to *agnosticism* about Q" (Divers 2004, page 673 my emphasis). As a normative discipline, logic appears to debar the agnostic from asserting the truth of P [left side, biconditional] without an entailment to the truth of Q [right side, biconditional]!

The remainder of this chapter is reserved for the analyses of modal sentences of *de re* possibility. The assertion (and belief) of *de re* possibility appears to offer the most fertile ground for discussion. Divers argues that the **WA** does possess a warrant to assert certain modal truths of *de re* possibility. For example, she may deny that A is a *de re* possibility by the assertion that there is no world at which A. Furthermore, she may assert that A is a *de re* possibility on the grounds that A can be asserted of the actual world. These assertions are licenced on shared grounds *viz.* an argument of *de re* representation advanced by Lewis in *Plurality*: "What is the case *at* a world is what the world represents, and a world represents (de re) of *x* that it is F by having a part which is a counterpart of *x* and is F" (Divers 2004, page 674: see Lewis 1986a, pp 194-96). For example, by **GMR** and **WA** analysis, Michael Dummett is a philosopher in the actual world since the actual world has parts which are both Michael Dummett counterparts and philosophers **[FN 6]**. However, the denial is also true: Michael Dummett might *not* have been a philosopher since the actual world has parts that are both counterparts of Michael Dummett and *not* philosophers.

> So, to reach the crucial point, the worldly agnostic faces no assertibility deficit over the wide and interesting class of de re claims of the type, possibly A, for which it is false that A but it is the case (in virtue of appropriately this-worldly counterparts) that at the actual world, A. (Divers 2004, page 674)

However, matters are apt to become more opaque in other cases of *de re* possibility. For example, Divers' concedes that **WA** does face an assertibility (and belief) deficit in the case of possibly A when *either* there is no warrant to assert that there is no world at which A *or* no warrant to assert that, at the actual world, A (Divers 2004, page 674). Although the worldly agnostic enjoys *some* degree of epistemological parity with her realist counterpart –

e.g. she may assert the possibility of A on the grounds that there is a warrant to assert that A in the actual world – Divers acknowledges that a deficit remains.

In order to be a decisive objection against **WA**, however, Divers argues that the genuine realist is obliged to show that the deficit represents a significant omission. "So it is up to the realist to present examples of how the stunted and lop-sided modalizing (as she sees it) of the worldly agnostic has (unwelcome) practical consequences" (Divers 2004, page 676). The putative examples are significant only if the deficit "has implications for what we have reason to do, either practically or intellectually" (Divers 2004, page 684). Modal beliefs are 'rationally efficacious' because our assessment of what is possible has an impact on our thoughts, language and actions. Divers aim is to demonstrate that "all of the *disputed* beliefs that are rationally efficacious are also rationally dispensable" (Divers 2004, page 678 my emphasis). In short, Divers argues that the disputed class of beliefs in *de re* possibility can be abandoned without practical or intellectual loss. The genuine realist, on the other hand, must claim that the disputed class of beliefs represent an insuperable obstacle for the moderate agnostic. Failure to supply a warrant to those disputed beliefs in *de re* possibility leaves the moderate agnostic with a practical and intellectual gap. "One important and commonplace instance in which we take ourselves and others to have reason to ϕ is where ϕ is (presented as) an attempt to ψ" (Divers 2004, page 680). How can the agnostic warrant the belief that an attempt to ψ is rational? Any *rational* ϕ should in some regard be constrained by the modal belief set of the agent. For example, provisionally, an agent should not attempt to ψ if she believes that ψ is impossible. Any attempt to prove Goldbach's Conjecture (that every even integer greater than 2 can be written as the sum of two prime

numbers), say, is impossible. The absence of a belief that the attempt to ψ is impossible, is one the agnostic can easily meet [it is impossible that ϕ iff there is no world at which ϕ]. However, as a provisional requirement this option is not very robust – it amounts to a demand that disaster is not inevitable. Difficulties begin to mount when we require *not* a belief that the attempt to ψ is impossible but that the practical cost of an attempt could be prohibitive. In these cases, the rational burden placed on the modal content of a belief is increased. Divers selects a homely example by way of illustration: "Grant that in many everyday situations, drivers have reason to carry out various manoeuvers, and indeed that in many cases they (practically) ought to do as they do" (Divers 2004, page 681). Imagine that in normal road conditions a driver is faced with a decision whether or not to overtake. A more robust belief in the *de re* possibility of a successful manoeuvre is required than one secured by the warrant that an attempt is not impossible. This stronger requirement – a belief that overtaking is possible – is precisely the type of modal belief that the realist needs in order to press home the deficiency objection. That is, it is a belief in the disputed class of *de re* possibility *and* it is rationally indispensable.

6.5 In Defence of Worldly Agnosticism

In order to block the significance of this rationally *in*dispensable belief of *de re* possibility, Divers' suggests that the worldly agnostic can adopt one of three responses.

Option 1: The relevant type of belief *is* warranted from the agnostic standpoint

The **WA** can reply that a warranted belief in the *de re* possibility to attempt to ψ is compatible with agnosticism on the basis of a warranted belief in a (this-worldly) counterpart of the agent. I do have a rational [warranted] belief in my ability to attempt to successfully overtake because of my warranted belief in my (this-worldly) counterpart, who has done so. "**[WA]** often have a warranted belief that something is possible for a given individual because she has a warranted belief that the individual has an actual counterpart who actualizes (directly, non-vicariously) the possibility in question" (Divers 2004, page 681). Although initially promising, Divers does acknowledge a potential flaw in this strategy. The reply places an onerous demand on the agents' warranted beliefs about their counterparts in the actual world. "We face just too many new, and even one-off, cases to be in a position to rely, in general, on – what is typically – knowledge of actual precedent to warrant belief in the relevant possibility" (Divers 2004, pp 681-82).

This drawback does not apply to **GMR** analysis which enjoys a reservoir of *a priori* beliefs in the existence of counterparts at some world, actual or not. "So while beliefs about actual counterparts may occasionally, or even often, supply the worldly agnostic with a rationally efficacious possibility belief, this consideration alone is insufficient to meet the present kind of realist challenge in *general*" (Divers 2004, page 682 my emphasis). Megan Blomfield (2008) argues that in addition to over-generating cases (is not a sufficiency condition) the strategy under-generates cases (is not a necessary condition). Blomfield argues that the

practical and intellectual implications of 'rationally efficacious modal beliefs' impose a restriction that is too narrow to serve as a general analysis of the disputed class. That is, Divers has significantly underestimated the size of the disputed class. Specifically, he ignores beliefs in a *de re* possibility, such as "possibly A", when we know that "at the actual world, not-A". "By concentrating on rationally efficacious beliefs, Divers neglects to discuss this problem" (Blomfield 2008, page 75).

Consider the following biconditional modal translation:

MB1: It is possible for [my mind] to exist without a body iff there is a world in which [my mind] exists without a body

MB2: It is possible for [my mind] to exist without a body

MB3: There is a world in which [my mind] exists without a body

(Blomfield 2008, page 76)

If MB2 is a member of the disputed class then **WA** is obliged to profess agnosticism about the truth value of MB2 and MB3. However, if we are persuaded that MB2 is actually true – based on a well thought out Cartesian epistemology – then MB2 expresses a belief in *de re* possibility warranted by a 'clear and distinct idea' of my non-actual counterpart. In these circumstances, **WA** might argue that MB1 is warranted on agnostic grounds; perhaps on the basis of a this-worldly counterpart. It might be supposed that my mind will continue to exist in *this* world after the death of my body. However, given the actual laws of nature we can block the agnostic's reply and affirm that such a "possibility" is an *actual* (nomological) impossibility. The Cartesian supposition that asserts the *de re* possibility of [my mind]

counterpart existing without a body is a logical truth – [my "bodiless" mind] counterpart exists at a non-actual world. As a result, there is no *this* worldly counterpart to warrant the agnostic belief in the *de re* possibility. The relevant modal belief is unwarranted on agnostic grounds of a this-worldly counterpart but is a member of the disputed class. The membership of this class extends beyond Divers' criterion of 'rationally efficacious' belief in a *de re* possibility.

We now turn to the second option suggested by Divers to avoid the unwelcome consequence of an agnostically "unwarranted" rationally efficacious modal belief that is *in*dispensable.

Option 2: That dispensability is shown by the sufficiency of some distinct and warranted modal belief

Divers' second option "is that in many, if not all, cases where an appropriate possibility belief is not available to the agnostic, a related belief with counterfactual content will be available and rationally efficacious" (Divers 2004, page 682). To return to the example of a successful attempt to overtake, the belief of a *de re* possibility could be one warranted by counterfactual content; namely, that if he [Divers] were to attempt to overtake then he would be successful. The truth conditions of the counterfactual are met by the absence of a world in which Divers' counterpart's attempt to overtake is unsuccessful. Divers' argues that the invocation of a counterfactual conditional, to supply the content of a belief of *de re*

possibility, leaves **WA** and **GMR** in the same boat: "[both must use] the relations of *similarity* that are invoked implicitly (via the notion of selection) do substantial work if either is to earn the right to assert the counterfactual, and the entitlement to do so does not depend on belief in other worlds" (Divers 2004, page 682 my emphasis). A counterfactual belief in a *de re* possibility invokes the similarity relation twice over; firstly, through the selection relation and secondly, through the counterpart relation. The warrant to assert the counterfactual belief of *de re* possibility is satisfied both on worldly agnostic and genuine realist grounds. *If* there is a world at which an individual who attempts to overtake is unsuccessful *then* the world in question is not relevantly similar to this one (the genuine realist) or the individual is not relevantly similar to him [Divers] (the worldly agnostic) (Divers 2004, page 682). A belief in the truth of a counterfactual conditional offers the agnostic a rationally efficacious surrogate for the content of a belief in a *de re* possibility. This approach could supply **WA** with a rich vein from which to mine examples that rebut the realist challenge. "If so, the agnostic's entitlement to the relevant counterfactual beliefs may form the very crux of the success of the agnostic programme" (Divers 2004, page 682).

Does the second option of a belief with counterfactual content as a surrogate for a belief in a *de re* possibility block Blomfield's objection? In order to do so, the **WA** requires an *x* such that:

MB4: If *x* had been the case, then [my mind] would have existed without a body

(Blomfield 2008, page 76)

Blomfield argues that the logical possibility of her mind existing without a body is compatible with her ignorance about what *x* could be. It appears that the **WA** is compelled to remain agnostic about the truth value of modal claims, such as MB2 [left side, biconditional]. "The same goes for all de re possibility claims which are translated into contested [disputed] sentences" (Blomfield 2008, page 76). A reluctance to assert the truth of a modal claim (*analysandum*) that is a member of the disputed class leaves **WA** open to an "obvious and serious objection … [namely, that **WA**] delivers radically revisionary … consequences for the range of modal statements that we are entitled to hold true" (Divers 2006, page 190 fn. 6).

Blomfield examines a central assumption of **GMR**; that genuine realism is ontologically committed to the existence of a plurality of worlds. It is Divers adherence to *this* assumption of **GMR** that leads the agnostic into difficulty over MB2 and the pre-theoretic propensity we have to declare a sentence of *de re* possibility true. In order to suspend judgement about the existence of non-actual worlds the agnostic is obliged to suspend judgement about the intuitive truth of modal sentences of *de re* possibility, in the disputed class (Blomfield 2008, page 76). Further, the agnostic does not merely contend that we do not know of the existence of non-actual worlds but that we *could* not know of their existence. If we did know the truth of a modal sentence of *de re* possibility (in the disputed class) then by **GMR** translation we also know of the existence of a non-actual world. "But I do not know that we should give up trying to decide whether we are correct in thinking that things might have been different … just because we cannot know whether or not the corresponding possible world exists" (Blomfield 2008, page 77).

Divers' philosophical task is to explain how the inferential benefits accrued by genuine realism are compatible with a denial of the existence of a plurality of non-actual worlds. He argues that the modal translation scheme functions by matching the truth value of modal sentences with the truth value of existential sentences relating to a multitude of worlds. Blomfield's criticism is (mainly) concerned with the truth value of the *analysandum* [left side, biconditional] "assigned - by and large - on the basis of our prior modal beliefs" (Divers 2002, page 107). Although a clash with intuitive modal opinion need not be a decisive objection to **WA** (chapter 3), in the absence of 'prior modal beliefs' we have "(arbitrarily) redefined [the] subject matter" (Divers 2002, page 107, fn. 3). In respect of the *right* side of the biconditional, the truth value of the *analysans* relates (in part) to the existence of a plurality of worlds. Divers' paper "Possible-World Semantics" is concerned with a worldly agnostic account of the syntactic structure of modal sentences that inform how they are used in logical argument. He is therefore primarily concerned with the notions of validity, completeness and soundness. How fruitful can the **WA** analysis of modal sentences be given her denial of non-actual worlds? Divers is optimistic:

> We have a new, and philosophically well-motivated argument to the effect that such genuine benefits as there are to be had from applied possible-worlds semantic theories can be had without either devious re-interpretation of the Kripkean semantics or commitment to realism about possible worlds. (Divers 2006, pp 223-224).

The criticism we make of worldly agnosticism does not relate to the syntactic features of declarative modal sentences and their use in logical inferences. Instead, we ask whether the absence of non-actual worlds of the *analysans* unduly affects the representational aspect of

the modal sentence or *analysandum*. Of course, many philosophers are not persuaded by Lewis' translation scheme and the associated ontological commitment to a plurality of worlds, but we allege that Divers *is* so committed.

Divers hopes to acquire the inferential benefits of genuine realism for worldly agnosticism but in order to do so he must be fully committed to the whole biconditional. Specifically, the agnostic must acknowledge as *true* the [right side, biconditional] with an ontological commitment to non-actual worlds, which partially license that truth. If the agnostic rejects an ontological commitment to this type of entity then she owes us an independent account of the "worldly" primitive predicate of the quantificational-predicate model (§ 1.3). In the actual world, the name-bearer relation is perhaps paradigmatic of the type of semantic relation demanded of the agnostic. The indicative sentence "the cat sat on the mat" can be translated into a sentence that states: there is an *x* and there is a *y* such that *x* is a cat and *y* is a mat and *x* sat on *y*. The truth or falsity of this sentence is determined by how the world is (§ 4.6). By parity of reason, Lewis' unrestricted use of the **QC** of ontological commitment is satisfied (in part) by individuals that may or may not inhabit the actual world. To deny or refuse to commit to a plurality of worlds places the agnostic in an invidious position: true sentences of belief in a *de re* possibility lead the agnostic into an impasse, especially when the belief in question is not warranted by a this-worldly counterpart or by counterfactual content.

We now conclude our examination of Divers' defence of **WA** with his final option intended to grant the agnostic the rational dispensability of the disputed belief in a *de re* possibility.

Option 3: The dispensability of the disputed belief is shown by the sufficiency of a distinct and warranted non-modal belief

Divers argues that the content of many of our beliefs is non-modal [left side, biconditional] and non-extensional [right side, biconditional]. This observation nullifies the problem of generality associated with option 1 (above). Recall, we concluded that the agnostic's warrant to a belief in the relevant *de re* possibility is based on her *a posteriori* existential beliefs about the actual world. Option 3 replaces an *a posteriori* existential belief about the actual world with a "belief that in all, or more cautiously, most such cases, if overtaking is attempted it will be completely safe" (Divers 2004, page 683). A shift in emphasis has now occurred. The agnostic's warrant to believe in a relevant *de re* possibility is based not on a modal belief but on a distinct and warranted non-modal belief. In these circumstances, the *factors* that determine the selection of a counterpart ought to be made explicit – that the driver is sober, the road conditions are good, there is a sufficient gap in oncoming traffic etc. However, Divers claims that to impose a greater theoretical demand on the agnostic, non-modal decision maker – compared to her modal counterpart is "unfair" (Divers 2004, page 683). These (potentially) unfair demands include greater precision, clarity or specificity in regard to the similarity relation of counterpart selection. "So here we have at least the beginnings of an actualist case for declaring that the belief that it is possible for me to

overtake successfully is rationally dispensable" (Divers 2004, page 683). The criticism made of worldly agnosticism, namely, that the truth conditions of each side of the biconditional cannot be simultaneously satisfied by grounding the *analysans* in a non-modal, actualist ontology, is thereby mitigated. In 2006, Divers revised his strategy in order to avoid the possibility of a modal collapse into fatalism **[FN 7]**. He sought to distance **WA** from one-worlders: "once we distinguish the position of the agnostic antirealist, who is neutral about the existence of other worlds, from the out and out one-worlder who denies the existence of other worlds" (Divers 2006, page 190).

Does the manoeuvre to replace a modal warrant to believe in a *de re* possibility, with a non-modal warrant (and so dispense with the "rationally efficacious" belief of the disputed class), bear fruit? Initially, Divers' third option *appears* to place the worldly agnostic between the hammer and the anvil. Either he accepts an actualist – and presumably one-worldly – account of the truth conditions of the [right side, biconditional] where modal collapse looms large *or* he accepts the genuine realist interpretation – and presumably a plurality of worlds – of the truth conditions of the [right side, biconditional] where an unsafe and insane ontology awaits. His preferred third way – of claiming agnosticism about the [right side, biconditional] – is scuppered by his warranted agnostic belief in the *de re* possibility of successfully overtaking. We prefixed this assessment with the words "initially" and "appears". Divers' may rebut these criticisms on the basis of an argument to the effect that the analysis of modal discourse does *not* rest on questions of ontology. Recall, Wright's thought experiment about the 'cautious man' who is unwilling to commit himself to the *modal* aspect of an assertion (§ 3.3). If the 'cautious attitude' can be defended in regard to

the *truth* of a modal sentence (*anlysandum*) then it is probable that a cautious attitude could be defended in respect of the *truth* of a non-modal sentence (*analysans*). The application of (an admittedly) tentative strategy of the 'cautious man' in regard to the truth of these sentences may warrant an agnostic belief in a *de re* possibility. A refusal to assert the truth of the *analysandum* would potentially liberate the worldly agnostic from a logical obligation to assert the truth of the *analysans*. Wright's thought experiment in respect of the "cautious man" is an option Divers could exploit in defence of the logical entailments of **WA**. The common sense judgements in regard to the truth of a belief in a *de re* possibility (*analysandum*) would be compromised but the logical entailment of a biconditional (*analysans*) would be secured. The worldly agnostic could remain agnostic about the truth of both sides of the biconditional analysis. The violence done to our common sense modal judgements, however, may be a price Divers is unwilling to pay.

6.6 Conclusion

In general, we believe that we perceive objects whose properties are independent of our perceptions of them. This presumption might lead us to question whether thought and language are *related* to something independent of thought and language. Ontological autonomy implies that the object or property represented by thought and language can function as an *independent* standard of correctness. An independent standard renders thought and language correct if, and only if, we are appropriately related to the object or property. Realists frequently allege that antirealists sacrifice independence on the altar of

this relation. Antirealists allege that realists sacrifice the relation on the altar of independence.

Divers' published papers on **WA** are not presented as a complete theory of modality. Although highly sophisticated, his arguments are merely a provisional exploration of a new type of modal theory. They are not intended to be exhaustive. For example, at the conclusion of his 2004 paper Divers' notes that:

> The application of the strategy (rational dispensability) has been illustrated all too briefly to convince anyone (myself included) that the agnostic programme will succeed. However, I do believe that worldly agnosticism is a new and interesting option in modal philosophy which merits further attention. (Divers 2004, page 684).

In consequence, Blomfield's critique of Divers – that he *only* considers 'rationally efficacious' beliefs – is not entirely justified. Despite credentials in favour of antirealism, such as the common sense denial of non-actual worlds – we argued that the worldly agnostic suffers an assertibility (and belief in) *deficit*. That is, the realist enjoys access to a plethora of assertible (and belief in) modal claims that are not warranted on agnostic grounds. The realist can have her cake and eat it. She enjoys the theoretical advantages accrued by an ontological commitment to a plurality of worlds *and* an assertion (belief in) bonanza of modal claims that relate the asserter to what is asserted. She is able to secure the theoretical benefit of valid inference (§ 1.1) and the second criterion of Lewisian paraphrase (§ 2.3). Is this conclusion premature? Is there a disputed class of assertible (and belief in) modal claims not conducive to **GMR** analysis? The next two chapters will address the problem of advanced

modality. Advanced modal claims are those that the genuine realist *ought* to assert (and believe) but that produce (post-translation) an assertibility (and belief in) *deficit*. By the lights of genuine realism, modal sentences that are true are translated into non-modal sentences that are false. In chapter 7 we will set out the problem of advanced modality together with proposed solutions. These solutions often depend on radical adaptations of the standard translation scheme. In chapter 8 we will propose a solution to the problem that retains (most of) the standard translation scheme.

Footnotes

1. The ontological basis for the distinction is not unanimously endorsed. Grayling argues that since mind-independence is the crux of the matter the distinction should be epistemological. “Realists and their opponents do not disagree about what exists, but about how we are epistemologically related to what exists” (Grayling 1997, page 288)
2. The contrast of moderate-radical agnosticism allows for a further characterisation that could supply a warrant to deny / disbelieve the content of an otherwise “agnostic” assertion (and belief). For example, if we were agnostic about the existence of God and discovered a fatal contradiction derived from the presupposition of His existence the *moderate* agnostic could change her agnosticism to denial.
3. Recall § 5.3; for Lewis, knowledge of contingent physical objects is acquired in a way that does not require a causal relationship between knower and what is known. Knowledge of *actual* contingent physical objects is distinguished not by the nature of

the thing [contingency] but by the fact that it is *indexical* [personal]. Indexical in the sense that we stand in a particular relation to it (§ 2.5). "… [hence] *all* impersonal ontological beliefs are to be justified on a priori grounds" (Stalnaker 1996 [2003], page 49).

4. For example, Fictionalism (Rosen 1990), Modalism (Forbes 1985, 1989) and Expressivism (Blackburn 1984, 1986)

5. The first sub-stratagem is associated with Error Theory and the second with agnosticism about the existence of non-actual possible worlds.

6. Postulate 5 of Lewis' original paper prohibits this-worldly counterparts (Lewis 1968 [1983], page 27). The prohibition is subsequently relaxed when Lewis' allowed – dependent on context – this-worldly counterparts (Lewis 1983b [1983], page 43).

7. Modal Collapse: "Most obviously, and dramatically, when such theories are restricted to one-world models, they seem bound (invariably) to validate modal collapse as represented by the formulae (MC) "If P then □ P"" (Divers 2006, page 190)

Chapter Seven: The Problem of Advanced Modality

7.1 Introduction

We argue that the theoretical benefits accrued by Genuine Modal Realism **GMR** are a consequence of the quantification-predicate model of Lewis' modal translation scheme. A potential threat to the truth-value of sentences translated by the scheme is posed by advanced modal claims. That is, there are **QML** sentences, whose "intuitive" pre-theoretical truth is lost in a translation scheme that yields **PL** sentences that are false, by **GMR** lights **[FN 1]** (§ 7.2). Divers' solution to this problem is premised on the redundancy (in translation) of the modal operators of **QML** sentences. The redundancy renders the domain of the quantifiers within the scope of the said operators an unrestricted interpretation; the content of **PL** sentences are interpreted as true / false without reference to worlds (§ 7.3). Divers is thereby able to extricate the genuine realist from the impasse generated by advanced modal claims by discarding our central thesis, **WRP**: Recall,

World Relevance Principle **WRP**: The analysis of *de dicto* modal sentences ought to be based on the assumption that modal operators both unite worlds under quantification and restrict the domains of the quantifiers that fall within their scope.

The problem of advanced modality is an ongoing controversy within philosophy. Divers' contribution is pivotal and he recently developed his original arguments (§ 7.4). We subject the problem of advanced modality to a criticism raised by Josh Parsons; namely, that the "problem" of advanced modality is a pseudo-problem, a consequence of the misapplication of the quantifier(s) of the *analysans* (§ 7.5). The application of the **QC** of ontological

commitment makes Parsons' remarks particularly pertinent to our central thesis, **WRP** (see §1.3). We conclude that Parsons has misdiagnosed the "apparent" problem of advanced modality which remains a genuine philosophical concern (§ 7.6). Although Divers' Redundancy Theory is both elegant and coherent we argue that an alternative solution to the problem of advanced modality is available. That is, a "standard" solution that is able to preserve the "intuitive" truth-value of advanced modal claims under **GMR** analysis. The proposed solution is "standard" in the sense that it is based solely on the conceptual apparatus already available to the genuine modal realist. This solution warrants a continued (indispensable) ontological commitment to a type of entity, possible worlds. By implication, we secure **WRP** (see Chapter 8).

7.2 Advanced Modal Claims

Advanced modal claims are defined as "modal claims about entities other than spatiotemporally unified individuals (perhaps, then, spatiotemporally disunified individuals, sets, numbers, properties, propositions and events)" (Divers 1999, page 217). For **GMR,** the *prima facie* problem of advanced modal claims is their apparent reference to entities that are not unified individuals, are not "at a world w" and so cannot be the subject of a **PL** sentence of the *analysans*. By **GMR** standard translation, the intuitive truth-values of *some* modal sentences are not preserved. "The intention is that *all* GR [GMR] analyses should terminate in *analysans* that feature only the primitive proper concepts of individual, set, part-of, spatiotemporal relatedness and similarity" (Divers 2002, page 50 my emphasis). The

standard translation of a modal sentence or *analysandum* [left side, biconditional] that states the possibility of P, is:

(P) It is possible that P if, and only if, there is some world w, such that at w, P.

The content of the *analysans* [right side, biconditional] is restricted to individuals that are themselves spatiotemporally related. The bugbear of advanced modality emerges when a pre-theoretically (intuitively) true *analysandum* is translated to an *analysans* that is false (by **GMR** lights): for example, the truth value of modal sentences that pertain to sets, say, "each set has its members essentially" is *not* preserved by standard **GMR** translation. Importantly, the truth of **WRP,** that entails an ontological commitment to a plurality of worlds, falls into the category of an advanced modal claim **[FN 2]**. In these "advanced" circumstances, modal sentences call for a distinctive "non-standard" form of translation. Preferably, this distinctive translation ought to accord with standard analysis as far as possible in order to remain a viable option for the genuine realist. Divers' argues that the preferred option ought to be a non-standard translation that simply omits talk of worlds:

(**AM**) It is possible that P if, and only if, P

The standard translation of modal discourse adopts the "normal" practice of ordinary language by imposing a restriction on the scope of the "domain of discourse" as determined by the context of utterance. The truth-value of a declarative sentence "that there is no beer" is, say, restricted to the contents of the fridge, or kitchen, or house, or city, or country or "world". The truth-value of an ordinary language sentence used to assert the existence of, say, swans is usually interpreted as an existential claim restricted by the context of utterance in numerous ways; for example, as one whose truth is restricted to the actual world (see Lewis 1970 [1983], Lewis 1975 [1983], Lewis 1979b [1983]):

1. ∃x (Ix@ & Sx) where "@" designates the actual world

However, advanced modal claims are not conducive to the standard world-restricted interpretation. A sentence used to assert the actual existence of a plurality of worlds, for example, though entailed by **GMR** analysis must be interpreted as false. The truth-value shift in this case is a consequence of a prohibition against the postulation of a world that has distinct worlds as parts (Lewis 1986a, see Index "overlap of worlds" page 274: also Noonan 1994, page 138).

PW: Necessarily, there is a plurality of worlds (discrete spatiotemporal regions)

PW*: ∀x (Wx → ∃y ∃z (Wy & Wz & Iyx & Izx & ¬ (y = z)))

How does the genuine realist square this circle? The analysis of modality is dependent on the "intuitive" pre-theoretical truth of **PW** and yet by the lights of counterpart theory, **PW*** is false! The modal status of the postulates and the implications for counterpart theory are deeply problematic.

7.3 A Solution: The Redundancy Theory

Divers argues that any (putatively) adequate philosophical explanation of modality ought (at least) to accommodate two principles. The absence of either principle incurs a significant loss of explanatory power:

PI: Possibility Introduction – p ⊢ It is possible that p, and

MU: Modal Ubiquity – For every true statement p, it is true that it is necessary that p or it is true that it is contingent that p

(Divers 1999, page 217)

Divers' solution to the problem of advanced modality is to abandon standard practice and interpret advanced modal statements as quantifications that are *not* world-restricted (see Divers 2002, page 48). The genuine modal realist ought, therefore, to distinguish between standard and advanced modal claims. Standard modal claims ought to be interpreted with quantifiers whose domain *is* populated by individuals whose place within the scope of the modal operator [left side, biconditional] is held by non-modal world-restricted terms [right side, biconditional].

Hence,

A. "there are swans" if, and only if, ∃x (Ix@ & Sx)
B. "it is possible that there are swans" if, and only if, ∃x (Wx & ∃y (Iyx & Sy))
C. "it is necessary that there are swans" if, and only if, ∀x (Wx → ∃y (Iyx & Sy))

Advanced modal claims, on the other hand, should *not* be interpreted with quantifiers whose domain *is* populated by individuals whose place within the scope of the modal operator [left side, biconditional] is held by non-modal world-restricted terms [right side, biconditional]. That is, in order to preserve the pre-theoretical truth-value of advanced modal claims the genuine realist ought to discard the principle that the content of non-modal terms is world-restricted. The primitive predicate "Ixy (x is in possible world y)"

(Lewis 1968 [1983], page 27) ought to be abandoned. The genuine realist is thereby able to accommodate the principle of **PI** in circumstances that she otherwise would not:

D. "It is possible that there is a plurality of worlds" if, and only if, ∃x ∃y (Wx & Wy & ¬ (x=y))

"There is no world-restricting element afoot in the content of [D] to sustain subsequent existential or universal generalisation. In the alternative, GR can say that in such modal cases, the modal modifiers are redundant, hence the schema [**AM**] for extraordinary [advanced] possibility" (Divers 2002, page 49). In the case of advanced modal claims the implicit modal operator is redundant. The redundancy theory can be expanded beyond **PI** to include the cognates of possibility, namely, necessity and contingency (Divers 2002, page 49):

(**AN**) It is necessary that P, iff P

(**AC**) It is contingent that P, iff P and not-P (so no advanced claim is contingent) **[FN 3]**

We ought to take a moment and reflect on the principle of quantifier restriction abandoned by Divers. Lewis argues that the restricting modifier "at W", "works mainly by restricting the domains of quantifiers in its scope, in much the same way that the restricting modifier 'in Australia' does" (Lewis 1986a, page 5). In other words, "at W" has a dual-function: it *unites* the worlds under quantification and *restricts* the domain of quantification to the scope of the modal operator. Lewis demonstrates this duality by way of an analogy: the restricting modifier "in Australia" can be compared to the restricting modifier "at a world w". The phrase "in Australia, all swans are black" is normally interpreted to mean that if we ignore everything not in Australia (or unite the "individuals" of Australia) – that is, if we restrict

quantification to the "domain of discourse" of Australia – then all swans are black. *Mutatis mutandis* "at world w, all swans are magenta" is normally interpreted to mean that if we ignore everything not at world w (or unite the "individuals" of world w) – that is, if we restrict quantification to the "domain of discourse" of world w – then all swans are magenta. The phrase "at world w" is included in standard **GMR** analysis in order to account for the possibility (or necessity etc.) that all swans are magenta (see Noonan 1994, page 138).

The remarks of the previous paragraph were qualified by the phrase "is normally interpreted". This qualification is necessary because Lewis notes two exceptions to the general rule (Lewis 1986a, page 6). First, not every quantifier falling within the scope of the modal operator *is* so restricted. The phrase "in Australia, there is a yacht faster than any other" would fall somewhat short of its actual (immodest) meaning if the modifier ("in Australia") restricted *both* quantifiers, rather than just the first. The yachts against which the "fastest yacht" is compared are not limited to those in Australia! Second, there are restrictions on the meanings of sentences uttered in a context that can "normally" be ignored. The undoubted existence of immigrant white swans in Australian zoos can be ignored in the interpretation of "in Australia, all swans are black", for example. These exceptions certainly mitigate the potential damage inflicted by the Redundancy Theory on standard **GMR** analysis.

7.4 The Redundancy Theory (Revisited)

Harold Noonan (2014) offers a *general* critique of **GMR** analysis on the grounds that it does not supply a traditional, and by implication, genuine *philosophical* analysis of modality. "Traditional" philosophical analysis must do more than satisfy the condition of material adequacy: "p" is true if, and only if, p. That is, the *interpretation* of a language L must assign the value "true" to *all and only* those sentences of L that are already "intuitively" true. However, given the weak truth conditions for "P materially implies Q" and the paradoxes generated by material implication, a further condition is usually demanded of traditional philosophical analysis **[FN 4]**. Strict equivalence or $\Box (P \leftrightarrow Q)$; hence, for a truth-functional biconditional to be strictly adequate both $\Box (P \rightarrow Q)$ *and* $\Box (Q \rightarrow P)$ must be true. By system **K** of **QML**, if $\Box (P \rightarrow Q)$ then $(\Box P \rightarrow \Box Q)$ and *mutatis mutandis* of the second conjunct; if $\Box (Q \rightarrow P)$ then $(\Box Q \rightarrow \Box P)$. This is, to employ Divers' term, an "indirect" deduction. By rule, the necessity of the consequent is inferred on the grounds of the necessity of the antecedent (and vice versa for the second conjunct). Strict adequacy demands the *de dicto* necessity of each side of the bi-conditional. Philosophical analysis ought to be of the general form (see Daly 2010, pp 45-48):

E. $\forall x (Fx \leftrightarrow Gx)$

Philosophical analysis should take the form of a universally quantified biconditional that is not only true but necessarily true. A traditional analysis of language, for example, ought to apply not only to every actual language but to *any* possible language. The type of necessity Noonan invokes is *de dicto* necessity (Noonan 2014, Section 1 page 2 fn. 2). Further, Noonan's remarks should be understood as directed at Lewis' (1968) translation scheme

supplemented by Divers (1999). Lewis acknowledges that the expressive resources of **QML** are limited (see **FN 4**, chapter 2). Noonan presents the genuine realist with a dilemma: either cleave to the 1968 counterpart theoretic interpretation of **QML** *or* the strict adequacy demanded of traditional philosophical analysis. He argues that the only escape route available to the genuine realist is to deny *de dicto* contingency (Noonan 2014, Section 3).

Noonan's overall argument is illustrated by way of examples (Noonan 2014, Section 2). Both Lewis and the "common man" agree that what is possible extends beyond what is actual. The England football team might have won the World Cup in 2018, for example. However, Lewis and the "common man" diverge in terms of what exists. Lewis asserts what the "common man" denies; the existence of non-actual *possibilia*, such as talking donkeys. This schism with common sense judgement is the result of an "inflated" ontological commitment entailed by the modal translation scheme. Lewis argues that the possibility of a talking donkey, say, can be translated from a true sentence of **QML** or English to a true sentence of counterpart theory. For Noonan, the central problem of Lewis' analysis is that the *mere* possibility of a talking donkey, is premised on the *de dicto necessary* existence of a talking donkey. That is, if the "standard" modal translation scheme (1968) is to satisfy the demands of strict adequacy then any modal sentence that asserts the possibility of a talking donkey entails the *de dicto* necessary existence of a talking donkey! Broadly, by the strict adequacy of standard modal translation (1968) the mere possibility of a talking donkey must be false *unless* there is at least one talking donkey, at a world *w*. Hence, by **GMR** lights, the sentence "it is possible that there is a talking donkey (iff) but there are none" is incoherent. We can unpack Noonan's argument. **GMR** analysis assigns, to the predicates "possible world" and

"donkey", satisfaction conditions that entail the existence of *possibilia* that occupy a spatiotemporal location (Noonan 2014, Section 2 page 3):

a. It is possible that there is a talking donkey
b. It is possible that there is a talking donkey iff there is a world at which there is a talking donkey
c. There is a world at which there is a talking donkey iff there is a talking donkey

When we apply the demands of strict adequacy to each side of these bi-conditionals (a-c) asserted or *implied* by **GMR** we may deduce:

d. It is necessary that it is possible that there is a talking donkey (*implied* by a)

The modal translation scheme 1968 ought to assign "false" to (d). This is despite the fact that (d) is a logical consequence of sentences that (by that same scheme) are not only true, but are (by strict adequacy) *necessarily* true! (Divers 2014, Section 4 page 5-6). We can illustrate Noonan's reasoning formally (by Modus Ponens at step 16): note the implied premise at step 4.

1. ◊ ∃x Fx Premise: (Contingent Entity)
2. □ (◊ ∃x Fx) Premise: (Necessary Possibility)
3. ◊ ∃x Fx ↔ ∃y (Wy & ∃x (Ixy & Fx)) (by Modal Translation, 1)
4. ∃y (Wy & ∃x (Ixy & Fx)) ↔ ∃x Fx (by Standard Logic 3)
5. □ (∃y (Wy & ∃x (Ixy & Fx)) ↔ (∃x Fx)) (by Strict Adequacy, 4)
6. □ ((∃y (Wy & ∃x (Ixy & Fx)) → (∃x Fx)) **&** (∃x Fx) → (∃y (Wy & ∃x (Ixy & Fx))) Def ↔, 5
7. □ ((◊ ∃x Fx) ↔ ∃y (Wy & ∃x (Ixy & Fx) (by Strict Adequacy, 3)
8. □ ((◊ ∃x Fx) → ∃y (Wy & ∃x (Ixy & Fx))) **&** (∃y (Wy & ∃x (Ixy & Fx)) → (◊ ∃x Fx)) Def ↔, 7
9. □ (◊ ∃x Fx) → ∃y (Wy & ∃x (Ixy & Fx)) **&** □ (∃y (Wy & ∃x (Ixy & Fx)) → (◊ ∃x Fx)) □ Distrib, 8
10. □ ((◊ ∃x Fx) → ∃y (Wy & ∃x (Ixy & Fx))) & E, 9
11. □ (◊ ∃x Fx) → □ (∃y (Wy & ∃x (Ixy & Fx))) by Rule **K**, 10
12. □ (∃y (Wy & ∃x (Ixy & Fx))) Modus Ponens: 2, 11
13. □ (∃y (Wy & ∃x (Ixy & Fx)) → (∃x Fx)) **&** □ (∃x Fx) → (∃y (Wy & ∃x (Ixy & Fx)) □ Distrib, 6
14. □ ((∃y (Wy & ∃x (Ixy & Fx)) → (∃x Fx)) & E, 13
15. □ (∃y (Wy & ∃x (Ixy & Fx))) → □ (∃x Fx) by Rule **K**, 14
16. □ (∃x Fx) Modus Ponens: 12, 15

If sound, Noonan's argument demonstrates that the demands of traditional philosophical analysis (strict adequacy) yield a conclusion that is "intuitively" false by standard-1968 translation. This is despite the "intuitive" truth of the premises. "Intuitive" is here used in the sense of intuitively true or false by **GMR** lights. How can a *de dicto* contingent statement of ontological commitment imply the *de dicto* necessary existence of an object? Noonan argues that the dilemma is equally applicable regardless of the type of interpretation – standard or advanced – that is applied to the *analysandum*. "I [Noonan] argue that even when the rules of the translation scheme are modified as Divers proposes modal realist

analysis of statements of modal discourse will be strictly *inadequate* (though materially adequate) in the eyes of the modal realist" (Noonan 2014, Section 1 page 2 my emphasis). A modal realist, committed to the standard-1968 translation scheme, is compelled to assert – by strict adequacy – both sides of the biconditional (at step 15) whilst holding that only one of them is true. "Whether the key sentence (15 Left) / (15 Right) is one that merits the redundancy interpretation or one that does not, *one* of the strict biconditionals that [GMR] is constrained to hold true (…) will not come out true" (Divers 2014, Section 4 page 7).

Noonan suggests that the only recourse open to a modal realist, committed to the strict adequacy of traditional philosophical analysis, is to abandon the notion of *de dicto* contingency (Noonan 2014, Section 1 page 2). Of course, the modal realist is not compelled to accept Noonan's solution. He may simply reject the demands of traditional philosophical analysis (strict adequacy), say, on the grounds of paradox (Daly 2010, pp 62-66), limited success (Daly 2010, pp 73-74) or the intractable problem of multiple analyses (Daly 2010, pp 67-68). How is Divers able to extract **GMR** analysis (standard and advanced) from Noonan's dilemma of strict adequacy? Divers begins by challenging the motivation attributed to the Redundancy Theory by Noonan (Divers 1999, 2002): "In its unqualified form, the proposal would be that a sentence type merits the redundancy interpretation of an outer modal modifier if (and only if) its subject matter is something other than ordinary world bound individuals" (Divers 2014, Section 5 page 8). Divers' *actual* motivation, on the other hand, is "to identify systematically – which is not to say, mechanically – the range of cases across which a *charitable* construal of the modal assertion of Lewis* required the redundancy interpretation of those sentences on the occasion of their use" (Divers 2014, Section 5 page

8) **[FN 5]**. Hence, the true motivation of the Redundancy interpretation echoes the flexibility of the genuine realist interpretation of modal sentences. This flexibility is clearly *not* available to a modal realist attempting to meet the high demands of strict adequacy. Divers moves to confront the apparent "dilemma" on two broad fronts: first, he denies a requirement (implicit in traditional philosophical analysis) that **GMR** ought to provide an invariant interpretation of any sentence *type* and, second, he re-affirms the entitlement to apply either of two interpretations to what are putatively "different" (in terms of their semantic value) sentences. Combined, these two presumptions licence the claim that both sides of the biconditional can be interpreted as true, by **GMR** analysis.

Divers argues that the conceptual resources employed by the Redundancy Theory to the analysis of extraordinary (advanced) modal sentences are *not* intended as a general panacea. In this light, the interpretation of sentence *types* should not be seen as invariant. Take, by way of illustration:

e. It is possible that there are many worlds

On a standard-1968 interpretation (e) is false (sentence D above, §7.3): that there is a world that has many worlds as parts, is false. By the redundancy interpretation, however, (e) simply states that there are many worlds, which is true. The pre-theoretic truth-value of an advanced modal statement can be preserved by adopting the redundancy interpretation. So interpreted, a *token* modal statement is charitably determined to be true, *as* the domain of quantification is not interpreted as world-bound (see §7.3). Hence, (e) is assessed without reference to a world, and is true by **GMR** lights. An abrupt change to the truth-value of advanced modal statements post-translation is avoided. Noonan's general concerns are

partially ameliorated. Divers also employs a further subtlety: “For there are things that are true in (of) the space that we cannot get at when we confine ourselves to expressing thoughts about what is true in (of) each and every one of its segments, even when we further think of those segments as exhausting that space” (Divers 2014, Section 5 page 9). The distinction can be applied to *token* modal statements such as “necessarily p” in one of two ways: as either the sum of segments that exhaust modal space (the totality of worlds) or as the totality of modal space *itself* (see Divers 2014, Section 5 page 9).

For example, consider:

(f) It is necessary that ((it is possible that there is a talking donkey) iff there is a world that has a talking donkey as a part)

In the case of (f), the modal realist could employ Divers’ distinction to interpret the clause “It is necessary that …” as true based on the presumption that it is true *of modal space itself*. In other words, the full sentence, (f), need not be a claim about the sum of all the *segments* (worlds) *of* logical space. Lewis* exercises a choice on how best (given the principle of linguistic charity) to interpret the modal operator (Divers 2014, Section 5 page 9).

The second prong of Divers’ strategy relates to Noonan’s aforementioned “indirect” assessment of the truth-conditions of the biconditional (Divers 2014, Section 4 page 5). There is an inherent ambiguity both in the interpretation of the modal operators of **QML** and the modal phrases of ordinary language. There are two interpretations available to the modal realist (i.e. standard and redundancy). However, both interpretations can be further understood in one of two ways (i.e. weak or strong) (Divers 2014, Section 5 page 10). Noonan’s “indirect” interpretation of the strict biconditional has both a weak and a strong

form. The “weak” or “univocal” form, “requires only preservation of truth under every uniform interpretation of all relevant necessity operators” (Divers 2014, Section 5 page 10). The “strong” or “multivocal” form is “[true] under every permutations of interpretations of every relevant expression of necessity: the bi-conditional is true if and only if it comes out true under every permissible assignment of truth-value to each of its parts” (Divers 2014, Section 5 page 10). Divers argues that the weak or univocal “test” is the correct one. Armoured with charitable flexibility, Lewis* may re-assert the truth of (f) since it relates to modal space – what is true *of* it. Lewis* can assert the truth of (f, Left) *if* he applies a redundancy interpretation which in turn is dependent on his willingness to assert (f, Right) *of* modal space: that it is *there* possible that there is a talking donkey. ”For what is true of modal space in matters of *de dicto* possibility just is a matter of what is true (*de dicto*) of some part of modal space” (Divers 2014, Section 5 page 11).

Noonan’s dilemma is dispelled. “So we can justify the application of the effective redundancy interpretation of modal modifiers in cases where modalizing is *about* the theses of counterpart theory or **GMR**” (Divers 2014, Section 5 page 12 my emphasis). This conclusion is provisional, however. The debate that surrounds the correct interpretation of advanced modal sentences is ongoing **[FN 6]**. We now set the debate aside in order to examine a claim made by Josh Parsons. Parsons questions the legitimacy of the distinction that separates “standard” from “advanced” modal sentences. Is there a genuine philosophical problem to be solved?

7.5 Against Advanced Modal Claims

What is the modal status of the modal translation scheme? Suppose, for the sake of argument, that Lewis* translation of modal sentences is necessarily true (strictly adequate). If we presume "it is necessary that it is possible that P" as a premise, and if blue swans are possible, it must be a logical consequence of Lewis* theory that there are blue swans. The thesis that there are blue swans, designated "*Swans"*, is therefore a logical consequence of Lewis* theory, designated "*PW*". These premises collectively entail "if *PW* then *Swans*". However, if *PW* is a necessary truth then *Swans* is a necessary truth. It then follows that necessarily there are blue swans: "and that is absurd" (Parsons 2011 draft, page 2). Contrast ordinary modal claims. Ordinary modal claims, such as "blue swans are possible" do *not* entail the existence of blue swans since they can, at least in principle, be challenged. We may wish to deny the possibility of blue swans. However, *if* blue swans or talking donkeys are possible then together with Lewis* analysis their existence must be asserted.

GMR assigns a *de re* contingency to entities such as blue swans; they exist in some but not all worlds (Lewis 1986a, page 11).

1.1 *PW* logically entails *Swans*

1.2 (the N rule) from "p logically entails q" you may infer $\Box (p \rightarrow q)$

1.3 (the K rule) from $\Box p$, $\Box (p \rightarrow q)$ you may infer $\Box q$

1.4 $\Box$ *PW*

1.5 $\neg \Box$ *Swans*

Parsons argues that **GMR** is committed to five inconsistent theses (1.1 to 1.5): "My [Parsons] thesis is there is a contradiction in the large bundle of theses that philosophers associate

with modal realism … and that Divers' Advanced Modalising does not succeed in removing that contradiction" (Parsons 2011 draft, page 3). The contradiction is located at premises 1.4 and 1.5: if *PW* is necessary then we can infer *Swans* is necessary (by premise 1.3) but *Swans* is contingent since blue swans exist at some worlds but not others. Hence *Swans* is both a necessary and not a necessary (contingent) thesis of modal realism, a contradiction. If the argument is valid, in order to deny the contradiction the modal realist must deny at least one premise, but which one? Parsons' suggests that premise 1.5 is the weakest: "Many modal realists (or their fellow travellers) will think that, in some subtle way, this is the premise to deny" (Parsons 2011 draft, page 5).

We will attempt to defuse the contradiction. We dispute the validity *in this case* of the N-rule (thesis 1.2). The N-rule is applied to thesis 1.3 in order to infer □ q. However, there is an equivocation in the N-rule cited at thesis 1.3. Suppose □ *PW* refers to the **set** of possible worlds that exists at every world as an abstract object. Hence, although in normal circumstances the N-rule is valid, in this *particular instance* the inference □ (*PW* → *Swans*) is not valid: *PW* is a thesis about an abstract object that exists at every world whilst *Swans* is a thesis about concrete objects (and their counterparts) that exist at some but not all worlds. Again, suppose *BS* is a thesis about the **set** of blue swans (that, as a set, exists at every world) and *Swans* is a thesis about blue swans (that exists at some worlds but not others). In this *particular case* the inference from □ *BS* (a set), and □ (*BS* (a set) → *Swans* (an object)) to □ *Swans* (an object) is not valid and so there is no contradiction at thesis 1.5. Of course, this diagnosis depends on the prior distinction of advanced and standard modal claims, a distinction that Parsons appears to deny. We say "appears to deny" since he occasionally

slips into advanced modal talk: “My conclusion is that modal realists should regard their theory of modality as *contingent*” (Parsons 2011 draft, page 16 my emphasis).

What are the options available to a counterpart-theoretic translation when faced with advanced modal statements that confound it, such as *PW*. “… the most obvious and simple analyses of modality have serious problems when turned on the subject matter of modal realism itself” (Parsons 2011 draft, page 6). Firstly, **GMR** could be advised to “bite the bullet” and accept that *PW* is both true (by advanced translation) and necessarily false (by standard translation)! Parsons attributes this view to Noonan (1994) although I do not share his assessment. Noonan argues that advanced modal claims yield, not the truth value *false*, but rather an *abortive* truth value – neither true nor false. Noonan argues that, despite appearances, advanced modal claims do not involve modal operators and so are not subject to standard translation. The modal statement that there is “necessarily, a plurality of worlds” is not conducive to **GMR** analysis because the term “necessarily” *in this case* is *not* a modal operator.

> Nevertheless, the function of a restrictive modifier is to restrict the domain of quantifiers within its scope, and hence an expression is not functioning as a restrictive modifier if it restricts the domain of *no* quantifier within its scope. Consequently, an expression is not functioning, on Lewis’s account, as a modal operator unless it both introduces quantification over possible worlds *and* restricts the domain of at least one quantifier within its scope. (Noonan 1994, page 138-39)

In the alternative, Parsons argues that in order to satisfy the demands of a genuine analysis of modality, **GMR** must satisfy three criteria (Parsons 2011 draft, page 7):

2.1 That the analysis be compatible with the simple and literal truth of *PW* and its consequences.

2.2 That the analysis be compatible with validity of basic patterns of ordinary modal inference (such as the N and K rules)

2.3 That the analysis, in conjunction with *PW*, be compatible with modal common sense.

To align *PW* with these three criteria Parson's advocates a T-preserving analysis that retains the K- and N-rules (Parsons 2011 draft, page 7):

S is possibly-true iff S, or for some world w, [S]w

S is necessarily-true iff S, and for every world w, [S]w

We concur with Parsons insofar that the K- and N-rules should *generally* be preserved but find them inapplicable (on the grounds of equivocation) to the collectively inconsistent theses set out above (1.1-1.5). In order to satisfy criteria (2-1 – 2.3) Parsons argues that *PW* ought to be asserted but on the presumption of a multiverse; that is, understood as a mega-universe containing all "worlds" as island universes (Parsons 2011 draft, page 7). Island universes remain spatially, temporally and causally isolated and thus secure the theoretical benefits of **GMR** together with the intuition that "everything is actual" (see Divers 2002, pp 93-99). A T-preserving analysis enjoys the consequence of rendering *PW* possibly-true (a pluriverse). However, when a sentence of *PW* contains quantifiers restricted to a single world, *PW* is false (a single island world). As the assertion and denial of *PW* are equally possible, dependent on the restriction of the quantifiers, *PW* must be a contingent truth.

We argue that advanced modal sentences are amenable to standard translation *if* we understand the subject-matter of those sentences to be abstract sets rather than concrete individuals. As a thesis, *PW* refers not to a plurality of concrete worlds but to the single **set** of possible worlds that exist "at a world" as a single abstract entity i.e. a set. The problem of an ontological commitment to a plurality of concrete worlds "at a world" (single world) is no longer applicable. We can test this new explanation with Parsons' statement 2.4 (Parsons 2011 draft, page 8):

2.4 That the analysis be compatible with the necessary-truth of *PW*.

Parsons maintains that:

> No analysis can satisfy (2.4) while also satisfying (2.2) and (2.3). That is what my [Parsons] argument of the previous section showed. For any analysis that made *PW* out to be necessarily-true would either violate (2.2) by omitting to validate the N and K rules, or violate (2.3) by making "there are blue swans" necessarily true. Contingent truth is the best that the modal realist can consistently hope for. (Parsons 2011 draft, page 8).

Actually, we can hope for rather more than that! In order to account for every genuine possibility **GMR** posits the existence of a plurality of worlds where those possibilities are realised. We interpret *PW* as an advanced modal claim and therefore as a sentence about the *set* of worlds that exists at every world. We subject this advanced modal claim (*analysandum*) to the standard translation scheme to yield an *analysans* – that observes the world relevant analysis **WRP** – that is true. That is, the ontological status (abstract or concrete) of the membership of the set does *not* determine the ontological status of the set (abstract). The *set* of all worlds exists *as a set* at every world. Individual concrete worlds also

exist necessarily in order to account for every possibility. Statement 2.4 is satisfied. Statement 2.2 is also upheld since although the N and K rules of normal modal inference are recognised as valid, the N-rule is simply not applicable *in this case*. We are in the position of accepting the validity of the N-rule but claim an exception to it in the case of advanced modality. We recognise the standard-1968 analysis of advanced modal claims (but "advanced" modal claims are subject to a set-theoretic analysis). The analysis is "standard-1968" in the sense that the conceptual resources employed by the *analysans* are identical in both the advanced and standard cases. There is no extraordinary analysis required on the *analysans* side of the equation – and so no world redundancy. Thesis 1.4 refers to the set of possible worlds each of whose members exist of necessity. Some members of the set will enjoy the presence of blue swans and some will not. This satisfies thesis 1.5 – the contingent existence of blue swans. Every possible world exists of necessity, as does the possibility that blue swans exist; necessarily, blue swans are possible. This validates the K-rule inference made by thesis 1.3.

Divers (1999, 2002, 2014) argues that different "types" of *de dicto* modal sentence require different types of translation – *ordinary* cases are subject to standard (1968) translation while *extra-ordinary* or advanced cases are subject to a *redundancy* analysis. Parsons argues that *this* putative distinction can be understood in one of two ways. Either as a distinction based on ambiguity or as a distinction based on a disjunction. The disjunction is associated with the following analysis of modality (Parsons 2011 draft, page 9):

A. S is possibly-true iff either a) S contains some vocabulary relating to worlds, sets, properties, propositions, or other predicates of modal realist metaphysics and S or b) S contains no such vocabulary, and there is some world *w,* such that [S]*w*.
B. S is necessarily-true iff either a) S contains some vocabulary relating to worlds, sets, properties, propositions, or other predicates of modal realist metaphysics and S or b) S contains no such vocabulary, and for each world *w*, [S]*w*.

The disjunctive view can be illustrated by way of a disanalogy. The meaning of the English word "jade" is established by cases – a substance is "jade" iff it is jadeite or nephrite (see Putnam 1975a [1996], page 25). A competent member of the English speaking community need not recognise or even be aware of the distinction and yet still use the word "jade" correctly. Parsons notes that Divers does not explicate his view of advanced modality to include the types of distinction(s) employed (Parsons 2011 draft, page 10). In any event, both types of distinction suffer from decisive objections. We will examine the objection against ambiguity. The ambiguity distinction is one subject to empirical refutation (see Parsons 2011 draft, page 10). A competent English speaker will understand the different uses of the word "bank". Dependent on the context of utterance, the word could mean either "riverside" or "financial institution". Now contrast this "obvious" (see Parsons 2011 draft, page 10) case of ambiguity with that of the modal phrase "is possibly-true". A competent English speaker does not surrender their competency when they fail to recognise the putative ambiguity of this modal phrase (in the sense of the standard / advanced distinction). The distinction is drawn only after philosophical investigation: "That was not a discovery of an ambiguity in the meaning of ["is possibly-true"] – at best, it was a discovery of the need for one" (Parson 2011 draft, page 11).

Fortunately, our set-theoretic approach neatly side-steps this objection. Parsons' objection is motivated by the need for a novel re-interpretation of modal sentences that fail to preserve truth under standard translation: "To say that there is some other, technical, extraordinary, advanced, sense of possibility that does not have this problem does not remove it from the ordinary sense" (Parsons 2011 draft, page 11). However, the set-theoretic interpretation makes no attempt to subject advanced modal claims to extraordinary analysis. Advanced modal sentences are recognised as such and then subject to a *standard* world-restricted translation, based on sets. The thorny issue of a necessarily-false outcome to a standard analysis of *PW* is a consequence of the Lewisian notion of individual worlds as spatiotemporally isolated – worlds that do not overlap. We submit that the content of *PW* does not refer to a collection of concrete entities but to an abstract *set* of concrete entities which could indeed, *must* exist at every world.

Parsons' chief objection against the distinction of standard-advanced modal claims relates to the *type* of analysis employed. Advanced modal claims such as *PW* demand a special type of redundancy analysis wherein all reference to worlds is abandoned. Furthermore, the distinction that demarcates the line between standard and advanced analysis appears to be wholly *ad hoc*: that is, a distinction acknowledged by Divers only in order to resolve an internal difficulty of genuine realism related to the modal status of counterpart theory. We will argue that these difficulties are not insurmountable. Indeed, we can meet them and retain the **WRP** that leaves the *type* of analysis constant: the distinction (though genuine) applies only to the *analysandum* or [left side, biconditional] translation scheme. If the *analysandum* is an advanced modal claim then the *analysans* refers to sets but if the

analysandum is a standard modal claim the *analysans* may refer either to concrete objects or sets **[FN 7]**.

7.6 Conclusion

We have identified the problem of truth preservation posed by advanced modal claims under standard-1968 analysis. The theoretical truth of the modal *analysandum* yields a theoretical false non-modal *analysans* (by **GMR** lights). Divers' solution to the problem of advanced modal claims is the Redundancy Theory. Modal operators are "redundant" in the sense that the quantifiers that fall within their scope are no longer interpreted as world-restricted. The "intuitive" truth-value of advanced modal claims is thereby preserved but at the cost of an ontological commitment to a type of entity, possible worlds. As a solution conducive to the genuine realist we allege that the cost is too high. Noonan's general concern is based on traditional philosophical analysis in which the universal bi-conditional is *de dicto* necessary. If **GMR** embraced Divers' Redundancy Theory it would still fall short of strict adequacy, as a condition of genuine philosophical analysis. Noonan's argument is based (in part) on Josh Parsons' paper "Against Advanced Modalizing". We argue that Parsons' objection (1.1-1.5) is based on presumptions that do not withstand scrutiny. Although, in *general* we endorse the N-rule as valid, *in this case* it is invalid – the inference from $\Box p$, $\Box p \rightarrow \Box q$ to $\Box q$ does not hold. We now expand on earlier remarks in relation to the set-theoretic solution to the problem of advanced modality.

Footnotes

1. The threat pertains to Lewis' first criterion of a genuine paraphrase, namely, the truth preservation of modal sentences under paraphrase (see § 2.3).
2. Noonan argues that the genuine realist is committed to the claim that a plurality of worlds is a necessary truth but that is false by standard translation (Noonan 1994, page 138). Intuitively, what holds of logical space would appear to be a non-contingent matter. Unfortunately, I have been unable to locate clarification on the modal status of a plurality of worlds endorsed by Lewis as a necessary truth. There are passages in Lewis' work that appear to imply this conclusion, however. Challenged to provide an account of how we *know* that a plurality of worlds exists, Lewis cites mathematics as a precedent (Lewis 1986a, page 111). Lewis appears to align the non-contingency of modality with the non-contingency of mathematics (see also Lewis 1996b [1999], esp. page 222) (§ 5.3).
3. Divers (Divers 2002, fn. 16 pp 307-308) distinguishes two senses in which "It is contingent *that* P" can be understood. **AC** should be understood as contingent in the *factive* sense. Factive in *that* P is the case, granting the possibility of not-p. In the alternative, contingency could be interpreted in a non-factive sense. Non-factive in the sense of *whether* P is the case (P is neither necessary nor impossible).
4. The paradoxes of material implication are well rehearsed: for example, for any statement P, if P is false then "P → Q" is always true. Therefore, both "if grass is magenta then skunks are cuddly" and "if grass is magenta then skunks are not cuddly" are true.
5. Lewis* is represented as an imaginary philosopher convinced by Lewis' arguments (Divers 2014, Section 1 page 1). That is, Lewis* is persuaded by the counterpart

theoretic interpretation of modal discourse (Lewis 1968 [1983], Lewis 1971 [1983]) the metaphysics of genuine modal realism (Lewis 1986a) and a conception of what qualifies as a successful analysis of modality (Lewis 1973, page 88: Lewis 1986a, page 3-4). Both Divers and Noonan appear to concede to Lewis* the possibility of an ontological reduction of modality but reject the possibility of Lewis* conceptual reduction of modality (Divers 2002, Ch. 7: Noonan 2014, Section 2 page 5). The distinction between "what there is" and "what we think/say there is" is evaluated by Hale (Hale 1997: Hale 2013 pp 63-69).

6. Although the debate is ongoing, I restrict my remarks to papers published prior to 2016 (the end of the research phase). This decision has left unmentioned many subsequent contributions. For example, Alex Steinberg (2018).

7. "I would suggests instead that the restricting of modalities by accessibility or counterpart relations, *like the restricting of quantifiers generally*, is a very fluid sort of affair: inconstant, somewhat indeterminate, and subject to instant change in response to contextual pressure. Not anything goes, but a great deal does. And to a substantial extent, saying so makes it so: if you say what would only be true under certain restrictions, and your conversational partners acquiesce, straightway those restrictions come into force" (Lewis 1986a, my emphasis).

Chapter Eight: A Set-Theoretic Solution

8.1 Introduction

Lewis' "total theory" consists of two distinct components (Lewis 1983b [1983], page 40). The first is **GMR** – there is a plurality of worlds each a discrete spatiotemporal region populated by concrete individuals. The second is iterated set-theory. Sets are constructed from concrete particulars, or from mereological aggregates of such particulars (Rosen 2015, page 382). Clearly, one may accept set-theory without thereby committing oneself to **GMR**. Lewis' commitment to the former must be independently motivated. That is, the reasons that compel acceptance of set theory are independent of those that motivate acceptance of **GMR**. Lewis' general criterion of acceptance is the theoretical role that the putative entity fulfils. "We find the wherewithal to reduce the diversity of notions we must accept as primitive, and thereby to improve the unity and economy of the theory that is our professional concern – total theory, the whole of what we take to be true" (Lewis 1986a, page 4). Lewis accepts set-theory as an indispensable component in the field of mathematics. "Their [mathematicians'] thesis of plurality of sets is fruitful; that gives them good reason to believe that it is true" (Lewis 1986a, page 4: Lewis 1991, pp 36-38). Lewis is ontologically committed to a type of entity, sets. However, Lewis appears to endorse two distinct accounts of set-theory (see Rosen 2015, page 383). Lewis' "simple view" is expressed in the body of *Parts of Classes* (1991) but it is superseded by the "structuralist view" both in the appendix of that work and in a later paper "Mathematics as Megethology" (1993). Lewis explores the interrelated topics of mereology (the general theory of parts) and

set-theory (the general theory of collections). His mature explanation of the metaphysical underpinning of set theory became known as Structuralism. We decline an invitation to follow Lewis' exploration of the metaphysics of set-theory. The reason for this reticence is a simple one – I lack the necessary expertise. Hence, I set aside the task of assessing the "structuralist" view of set theory as a potential solution to the problem of advanced modality to those more mathematically qualified **[FN 1]**. We decline a sojourn into the metaphysical foundations of mathematics and simply acknowledge that: "When God made the world(s), he made the concrete individuals; he laid down the principles of set-theory, and then he stopped" (Rosen 2015, page 382).

We proceed on a "simple view" of set theory as presented by Steinhart (Steinhart 2009, Chapter 1) **[FN 2]**. The view is "simple" in the sense that we will leave the axioms of set theory largely unexplored (Steinhart 2009, page 1). In chapter seven we examined the notion of advanced modality and the challenge it represents to the expressive adequacy of **GMR** analysis. Advanced modal claims are a threat to the theoretical benefits associated with the quantification-predicate model of modal translation (§ 1.1). Divers' Redundancy Theory offers the genuine realist a solution to this problem. In the alternative, we propose a different solution to the problem of advanced modality that draws upon the conceptual resources of *standard* modal translation. We propose a solution based on set theory. Sets are among the standard conceptual resources of **GMR**. A set-theoretic solution to the problem of advanced modality is therefore more conducive to genuine realism. We recommend that the genuine realist ought to prefer our alternative solution to Divers' Redundancy Theory. A preference over the Redundancy Theory is justified on the grounds

that the alternative solution does not require an atypical evaluation of standard-1968 translation. That is, the predicate expressions of standard-1968 translation are retained (esp. "*x* is in possible world *y*" and "*y* is a possible world"). The conceptual resources upon which the alternative solution is based are already available to the genuine realist. Importantly, we offer a translation of advanced modal claims that retains the relevance of worlds as a component of the *analysans* [right side, biconditional]. We expand on earlier comments (§ 7.5) where the ontological status of sets is mentioned. In brief, we employ the conceptual resources of **GMR** to preserve the standard-1968 translation of modal sentences. There are two critical aspects to our proposal. Firstly, the recognition that sets have *no* spatiotemporal location. Secondly, and as a consequence, every set is "at world w" (Lewis 1983b [1983], page 40) (§ 8.2). Advanced modal claims are assessed in a standard-1968 manner "at a world w" and so the primitive predicate *Ixy* (*x* is in possible world *y*) remains an essential component of the translation scheme. We offer a number of objections to our set-theoretic solution to the problem of advanced modality together with replies (§ 8.3). Potential objections include the magical nature of the set membership relation (§ 8.3.1), the disputed ontological commitment to sets (§ 8.3.2) and the antinomies of proper classes (§ 8.3.3). We tackled the additional problem of plural predication in chapter 1 (§ 1.5). We conclude that although their "indispensable" status in the field of mathematics (and thus empirical science) has been challenged, an ontological commitment to sets ought to be part of 'total theory'. They are amongst the conceptual resources available to the genuine realist and offer a viable solution to the problem of advanced modality (§ 8.4).

8.2 A Set Theoretic Solution to the Problem of Advanced Modality

Lewis utilises the ontological and conceptual resources of **GMR** combined with set theory to embark on a systematic reduction of several important philosophical entities. We have witnessed some of these reductions in action: “properties” that are reduced to arbitrary classes (§ 4.6), “relations” reduced to classes of ordered pairs, “propositions” reduced to classes of possible worlds (§ 2.5) and “events” reduced to classes of spatiotemporal regions (Lewis 1986a, page 185 ff.: see Rosen 2015, page 382).

The conceptual resources of **GMR** are exhausted by: individual, set, part-of, spatiotemporal relatedness and similarity. For Lewis, as there are no non-individual non-sets, the furniture of each world can be divided into individuals and sets. In order to avoid the introduction of new conceptual primitives we argue that the problem of advanced modality can be resolved by their interpretation as claims about sets. If our interpretation is correct, standard-1968 modal translation remains the appropriate interpretation of *all* modal sentences. All *analysans* [right side, biconditional] will continue to employ the primitive predicate “*Ixy*” *(*x is in possible world *y*). We accept Divers’ argument in respect of flexibility when identifying which modal *analysandum* [left side, biconditional] are advanced and which are standard (§ 7.4). Divers offers the Redundancy Theory as an escape route conducive to genuine realism from the impasse generated by advanced modal sentences. The price of escape is the loss of world-restricted quantification-predication in the *analysans*. The primitive predicate “*Ixy*” is abandoned by the Redundancy Theory. This, Divers argues, is a price the genuine realist ought to be willing to pay. In contrast, we argue that it is a price the genuine realist ought

not to pay. As a solution to the problem of advanced modality we should avoid paying this price *if* a plausible alternative is available. The alternative solution is a set theoretic approach whose plausibility we now examine. There is a problem, however, with a set-based interpretation of advanced modal claims that threatens to scupper the manoeuvre before it is made. Lewis accepts the existence of sets. However, by the standard analysis of world-restricted quantification the existence of sets appears to be logically impossible. If we endorse the primitive predicate "*Ixy*" or "*x* is in possible world *y*" then at first blush the existence of sets is conceptually dependent on some world having set(s) as parts (see Divers 1999, page 224 fn. 7). Orthodox set theory, on the other hand, states that no set is part of any world, and Lewis concurs: "There are some abstract entities, for instance numbers or properties, that inhabit no particular world but exist alike from the standpoint of all worlds," (Lewis 1973, page 39: also Lewis 1986a, page 94 fn. 60). How do we reconcile standard-1968 modal translation with orthodox set theory? How do we include sets amongst the conceptual resources of **GMR** when they are *ex hypothesi* not a "part of" any world?

Rather than abandon orthodoxy, Lewis' response is to embrace the dilemma and re-interpret what it means to be "at a world w". "Then no set is in any world in the sense of being a part of it. Numbers, properties, propositions, events – all these are sets, and not in any world. Numbers *et al* are no more located in logical space than they are in ordinary time and space" (Lewis 1983b [1983], page 40). This quote is central to our proposed set-theoretic solution to the problem of advanced modality. The phrase "at a world w" can be interpreted in one of several ways:

1. An entity can be “at a world w” by being a part of world w, or
2. An entity can be “at a world w” by having a part that is part of that world w, hence, extraordinary trans-world individuals would be part of many worlds *if* they were possible but they are not possible (Lewis 1983b [1983], page 39) or
3. An entity can be “at a world w” by existing from the standpoint of a world: “Let us say that an individual exists *from the standpoint of* a world iff it belongs to the least restricted domain that is normally – modal metaphysics been deemed abnormal – appropriate in evaluating the truth at that world of quantifications” (Lewis 1983b [1983], page 40)

There are reasons to accept a modified interpretation of “at a world w”. Firstly, as actual existence pertains to *this*-worldly existence the strategy implies the *actual* existence of set-based entities, such as numbers. Secondly, the strategy appears to close the explanatory gap posed by the existence of impure sets; the sets whose membership exist at some but not all worlds (see Divers 2002, page 88-89). The second justification is one that we must decline. The intuition that motivates the notion of a “contingent” impure set is flawed because it implies that the ontological status of a set is dependent on the ontological status of its membership. What is the ontological status of an impure set? Each set is an abstract object whose membership may or may not consist of contingent individuals. We maintain that a set *qua* set is always a “necessary” object even *if* the existence of the members of that set is contingent – the existence of an impure *set* of red apples is “necessary” even though the existence of the *red apples* is contingent. This is a bold claim whose justification is based on the quote from Lewis (cited above). In this context, the word “necessary” must endure the indignity of quotation marks since the type of necessity invoked is that of the

third interpretation (3) of “at a world w”, namely, “from the standpoint of a world”. We will elaborate on these preliminary remarks shortly. It is important to avoid the implication, however, that the proposed “necessity” of sets is derived from the claim that sets are abstract objects. Not all abstract objects are necessary. The Equator is an abstract object but if the Earth did not exist then intuitively neither would the Equator (see Dummett 1991, page 239: see **FN 3,** Chapter 5). We must reconcile the claim that all sets (pure and impure) are “necessary” objects with the recognition that the members of some sets are contingent individuals (see § 8.3.2). The “necessity” of impure sets commits us to the view that sets constitute an *additional* type of entity “above and beyond” the existence of their members (§ 1.4). Lewis’ “simple view” is that every set is a fusion of singletons (sets with one member); that is, every non-singleton set is just the fusion of the singletons of its members. Hence, set-membership is a consequence of the fusion of the singletons of the putative members of the set. For example, the set of actual red apples is constituted not by the apples themselves but by the singletons of each apple. As subsets, singletons enjoy a necessary ontological status. Of course, to cite Lewis’ singleton solution is simply to postpone addressing our original question: what is the ontological status of a singleton set, do singleton sets constitute an additional type of entity “above and beyond” the individuals that form their membership?

By **GMR** analysis, an object is necessary if it has a counterpart at every world. One drawback of Lewis’ three-fold distinction of “at a world, w” is the phrase “from the standpoint of a world” (option 3). The introduction of this phrase is problematic since it appears to be an *ad hoc* adjustment. Worse still, it appears to introduce a new conceptual primitive to the **GMR**

repertoire. Divers argues that the imposition of a new conceptual primitive is unwarranted since the phrase "from the standpoint of a world" can itself be explicated without significant loss.

> [**GMR**] can recognise effective *variety* within the being-in-a-world genus without resorting to the suspect notion of existing from the standpoint of a world. Thus consider the following explication:
>
> (D) ∀w [∀x [x is in w iff
>
> (i) (x is an individual Λ ∃y (y is a part of x Λ y is a part of w) V
>
> (ii) (x is a set Λ ∀y [(y is an individual Λ ((y is a member of x) V (y is a member of a member of x) V … → y is a part of w)]]] (Divers 2002, page 89 my emphasis)

Divers' explication of "at a world, w" is based on the distinction of two primitive concepts, individual and set. Individuals (D i) are "at a world, w" in the sense that each individual is a spatiotemporal part of a world, w. Sets (D ii) are "at a world, w" in a different sense. They have no location and so are *not* a spatiotemporal part of a world, w (although their members might be). He notes that (D) is neither vague nor introduces a conceptual primitive beyond the prior commitments of **GMR**. Instead, (D) deploys the conceptual resources already acknowledged by **GMR**. Furthermore, (D) is not *ad hoc* since it explicitly cleaves to the phrase "at a world w" used to express the ontological commitments of **GMR**. Formula (D) is a disjunction that falls within the scope of a universal quantification. Two mutually exclusive and collectively exhaustive disjuncts account for the existence of (i) individuals and (ii) impure sets. Individuals are composed of parts which are, in turn, both a part of an

individual and of the world at which they exist. Disjunct (ii) pertains to sets that have individuals as members. These sets themselves are the subject of iteration since they may form the membership of "higher" ranking sets. The initial universal quantifier relates to "for all worlds w" and so the first disjunct pertains to all world-bound individuals that exist whether their existence is actual or not. The second disjunct pertains to all sets that also exist but whose existence is not "actual" in the same sense.

We will add to Divers' formulae (D). This addition will account for the case of a set that does *not* have individuals as members, that is, pure sets. Pure sets are iterations of the empty set **[FN 3]**.

(D*) (iii) x is a pure set (i.e. all members of all parts of x are sets)

This addition is not trivial. As noted, we decline Divers second justification. Divers concurs with Lewis in that the existence of some impure sets is intuitively a contingent matter. "For impure sets whose members exist contingently are good candidates for existing from the standpoint of some but not all worlds – indeed, existing from the standpoint of exactly all of those worlds according to which all of their members exist" (Divers 2002, page 89). We dispute Divers' claim that the contingency of an individual / member of a set implies the contingency of the set. Postulate 2 of Lewis' (1968) states that "nothing is in two worlds", hence each individual is world-bound (see Lewis 1968 [1983], page 27). Postulate 2, therefore, appears to imply the metaphysical thesis of extreme essentialism (Plantinga 1973 [1979], page 157 ff.: Salmon 2004, page 236: cf. Stalnaker 1987 [2003], page 118-19). In brief, **GMR** explains the possibility that an individual might have been different in some way by the existence of a *counterpart* of that individual in some *other* world who instantiates an

unrealised (in *this*-world) property **[FN 4]**. The possibility of property variance is denied to each world-bound individual so each *de re* individual "necessarily" instantiates all the properties that it actually instantiates **[FN 5].** If a tree had fallen in Sherwood Forest five minutes before it actually fell then *you* and *I* (as world-bound individuals) would no longer instantiate *all* of the properties that we actually instantiate. The possibility of difference is analysed by way of the counterpart relation that posits a different individual causally disconnected from the subject of the possibility (*if* this argument is sound, and interpreted as world-restricted: if ~◊ ~P then □P). The metaphysical thesis of extreme essentialism is one that we shall reject. Lewis argues that sets are abstract entities which suggests that P2 is not applicable. Pure sets have no concrete individuals as members and are not threatened by contingency. Impure sets have concrete individuals as members which may imply that some impure sets exist contingently. However, individuals are invariably located in space-time and are therefore part of a world in the sense of "at a world, w" (interpretation Di). Impure sets *qua* sets, on the other hand, do not have a spatiotemporal location and therefore cannot be part of a world but are nonetheless "at a world, w" (interpretation Dii). We conclude the "necessary" (interpretation Dii) existence of impure sets. The *set* of all dragons exists "necessarily" (interpretation Dii) even if there are worlds at which no dragon (interpretation Di) has ever existed: {x I x is a dragon}. The "necessity" of every set is facilitated by the ambiguity of the phrase "at a world, w".

We propose to render advanced modal claims into modal claims about sets. Under a standard interpretation ordinary modal statements draw upon the conceptual resources of individuals and sets with the former understood as world-bound. We argue that advanced

interpretation of extra-ordinary modal statements ought to draw upon the conceptual resources of either impure sets of individuals or pure sets of sets. Hence, advanced interpretation would include impure sets with individuals as members {x I x is a blue swan} or pure sets {*x* I *x* is {}} with sets as members (see Steinhart 2009, page 20).

We are now able to illustrate the set theoretic interpretation of modal sentences by way of example:

(A) Possibly, there are talking donkeys

(B) Necessarily, there is a plurality of worlds

(C) Possibly, all numbers are perfect numbers

(A) is a true sentence about non-actual individuals yielding a true *analysans* by standard-1968 translation. There is a possible world at which talking donkeys exist. The theoretical truth of sentence (B) is preserved by a true *analysans* interpreted as a translation of a modal sentence about individuals considered as members of an impure set. The proposed set theoretic interpretation of (B) employs a standard-1968 translation of (B) understood as an advanced modal sentence whose content pertains to the set of all worlds. We say "standard-1968" translation since the *analysans* is assessed "at a world, w". The set of all possible worlds {x I x is a world} exists "at a world, w" (interpretation Dii) but not in the same sense as an individual (interpretation Di). The ambiguity of the phrase "at a world, w" facilitates our employment of primitive conceptual resources of the proposed set-theoretic solution [see (D)]. As a result, the problem of advanced modal sentences is side-stepped and

the subsequent need for an adjusted translation scheme that renders world-restricted analysis redundant, is avoided. The final sentence (C) refers to perfect numbers or numbers that are the sum of their divisors. Sentence (C) is false – not all numbers are perfect numbers – so the standard-1968 translation of (C) demonstrates that the set of perfect numbers is merely a subset of the set of all numbers, so the *analysans* is false by standard-1968 translation. Again, standard-1968 translation in the sense of an *analysans* assessed "at a world, w".

The proposed solution to the problem of advanced modality posits the existence of sets whose membership is determined by the content of the advanced modal claim. For example, modal claims that relate to the plurality of possible worlds are claims about the set of all possible worlds. This example unpacks in the language of **PL** [right side, biconditional] as:

A*: $\forall x (Wx \rightarrow \exists y (Sy \ \& \ Iyx \ \& \ y = \{x \ I \ x \text{ is a possible world}\}))$

Or alternatively:

A**: $\forall x (Wx \rightarrow \exists y (Sy \ \& \ Iyx \ \& \ \exists t \ \& \ \exists u (Wt \ \& \ Wu \ \& \ (t \in y) \ \& \ (u \in y) \ \& \ (t \neq u))))$

Let the predicate letter "S" designate a set. Each set exists "at" every world as an abstract object understood as an instance of the formulae (D) above; hence, the equations A* and A** begin with a universal quantifier whose scope includes all possible worlds. By A*: at any

arbitrary world “Wx” there exists a set “∃y Sy” that is the set of all possible worlds. The set y exists “I*yx*” or “*y* is in possible world x”, again, in the sense outlined by Dii (above). Finally, the set y is the set of all possible worlds: in set theory notation, the set represented by curvy brackets wherein for all x, such that (vertical stroke) x is a possible world. Of course, the number of possible worlds is disputed. In this case, the dispute relates to the *set* of all possible worlds; one that boils down to a dispute over the cardinality of that set.

The standard or world-restricted interpretation of modal statements is retained throughout. The ambiguous phrase “at a world, w” remains as part of the *analysans* of modal translation and **WRP** is preserved since the phrase is an expression of world-restricted quantification. Advanced modal sentences are distinguished from standard modal sentences in the manner outlined by Divers (see § 7.4). The Lewisian notion “from the standpoint of a world” is abandoned together with the disadvantages associated with it. We deploy (D*) in order to demarcate the types of entity under analysis – individuals and sets – with the proviso that whenever advanced modal claims are made they refer to the sets of those entities. Lewis accepts that sets are not a part of any world because they are abstract entities. Sets do exist, although not as “a part” of either logical or spatiotemporal “space”.

8.3 Objections and Replies

We now turn to a number of potential objections to our thesis. We argue that the problem of advanced modal sentences can be solved by the application of set theory. Many of these objections are general – directed against set theory itself. We have already encountered one objection in respect of the "singularist dogma" in which a single quantification is erroneously applied to multiple objects (§ 1.5). We now turn to other concerns.

8.3.1 A Kind of Magic?

The first challenge to our thesis is based on a famous argument articulated by Peter van Inwagen (1986 [2001]); a reply to an objection made by Lewis to the magical ersatzer's account of possible worlds (Lewis 1986a, pp 174 ff.). Van Inwagen's argument is a *tu quoque*: "by reasoning parallel to Lewis's, we can show that one could grasp set-theoretical membership only by magic; and Lewis's ontology requires at least some set-theoretical constructions" (van Inwagen 1986 [2001], page 238).

In *Plurality* Lewis inveighs against abstractionism on the grounds that it renders the *selection* or *makes-true* relation wholly mysterious. How does the magical ersatzer secure the "representational property" of a world required to explain, say, the possibility of a

talking donkey? How is a world selected as one at which a donkey talks? For **GMR**, the explanation is based on at least one spatiotemporal world that has a talking donkey as a part; these worlds enjoy a "representational property" as an intrinsic property. Contrast the magical ersatzer, who must posit "abstract worlds" that lack an internal structure (Lewis 1986a, pp 174-75). That is, these "worlds" or abstract simples are comprised from a subset of a larger class of abstract simples or "elements" e.g. propositions. Some elements are "selected" (made true) by the actual world. Hence, some propositions are "made true" by how things stand in the actual world. If a donkey talks (at the actual world) then one subset of elements (propositions) is selected (made true), whilst if a cat philosophizes (at the actual world), another subset of elements (propositions) is selected (made true), and so on. Clearly, the notion of "selection" or "made true" stands in need of explanation. Selection might be understood, for example, as a primitive concept; it facilitates the definition of implication on the elements (propositions), so that, *p* implies *q* iff, necessarily, if the actual world selects *p*, it selects *q*. If a "world" is a "maximal" subset of abstract simples or elements (propositions) complete under implication (via Boolean algebra). "[The worlds] are the elements that imply, but are not implied by, other elements (except the "null" element that implies all of the elements). *w* is a world *at which* donkeys talk iff (it is selected iff donkeys talk)" (Rosen 2015, page 392).

Lewis' critique of magical ersatzism is directed principally at the use of *selection* or the *made true* relation understood as a primitive concept. "Then it is a mystery how anyone could have understood the predicate "selects", which is supposed to express an *internal* relation that involves these properties. If the ersatzer has understood his own primitive, he must

have done it by magic" (Lewis 1986a, page 178 my emphasis) **[FN 6]** For van Inwagen, this is all a bit rich. As we have seen, Lewis draws on the resources of set theory in order to explicate a raft of philosophical entities. How can Lewis hope to avail himself of the primitive notion *is a member of* except by magic? If two sets, say {a, b} and {b, c}, can only be individuated by the objects that comprise them (a, b and c) then the *is a member of* relation must employ an equally "magical" explanation: "There is no way to pick out *any* set except by somehow specifying the things that bear membership to it" (van Inwagen 1986 [2001], page 238). The genuine realist finds himself in the same precarious position as the magical esatzer; as in the case of the primitive notion of 'set membership' so in the case of the primitive notion of 'selection'; it's a kind of magic!

In *Parts of Classes* Lewis concedes to van Inwagen's *tu quoque* (Lewis 1991, pp 36-38). He argues that van Inwagen has ignored a relevant distinction of cases, however (cf. van Inwagen 1986 [2001], pp 232-242). The set-membership relation is "indispensable" to the foundation of mathematics; if we reject the primitive relation *is a member of* then the whole edifice of present day mathematics must fall. However, to dispense with the primitive notion of *selects* or *makes true* leaves **GMR** as a viable option. No such viable alternative presents itself if we abandon *is a member of* relation as a conceptual primitive of mathematics.

Reply: There appears to be a theoretical impasse in respect of the *makes true* and *is a member of* relations as conceptual primitives. Problems that beset the former beset the

latter. Lewis acknowledges that on the presumption of an *internal* (based on intrinsic properties) primitive relation van Inwagen's *tu quoque* holds (Lewis 1991, page 37). On the other hand, if the primitive relation is understood as *external* (based on extrinsic properties) we can mitigate the force of van Inwagen's *tu quoque* (Lewis 1991, page 37). For our thesis, Lewis' attempt to deflect the force of van Inwagen's criticism by invoking the *type* of necessity required by each notion can only enjoy partial success. The proposed distinction is based on the type of necessity associated with the *identity* of the set membership relation (of singletons) and the *qualitative character* of the necessity of the *makes true* relation (see Lewis 1991, pp 37-38). However, on the "simple view" of set theory the distinction is arbitrary: that is, we defend a "dogma" that Lewis denies. The membership of a non-unitary set is based on an ability to identify a certain relation that all the members have in common. We are unable to individuate a set *except* by way of the objects that bear an *is a member of* relation to it.

8.3.2 Who Mentioned Sets?

The assertion that "P" is stronger than the assertion that "possibly P" yet the latter entails an ontological commitment to *more* types of entity. How can a weaker claim impose a greater ontological commitment? By **GMR** analysis, that "P" commits us to an ontology of individuals whilst "possibly P" commits us to an ontology of individuals and sets (Rosen 2015, page 382). As previously noted, Lewis accepts both individuals and sets as components of 'total theory'. Those unconvinced by **GMR**, however, may puzzle about the

inflated ontological commitments associated with a weaker claim. Those philosophers who prefer 'desert landscapes', for instance. The inclusion of sets in the analysis of *all* advanced modal claims exacerbates the problem of ontological inflation. *Prima facie*, an advanced modal claim about the plurality of possible worlds relates to possible worlds, not the set of possible worlds (except perhaps by implication) **[FN 7]**. As George Boolos' (1984) has noted, the fact that I had cornflakes this morning for breakfast does not entail that I also consumed an additional entity, the *set* of cornflakes in my bowl! (cited Bennett 2009, page 59) The set-theoretic analysis of advanced modality inflates the ontological commitments of total-theory beyond what is necessary for a fruitful discussion of the subject matter. Indeed, it can be argued that sets are wholly dispensable. Bennett (2009) argues that plural quantification enables the metaphysician to avoid an "ontological commitment to anything more than the first order individuals themselves – not to sets, nor composite objects of any kind" (Bennett 2009, page 59). Plural quantification enables the denial of composite objects whilst accepting claims such as "there are some simples arranged table-wise" (§ 1.5 and § 4.6). The predicate "arranged table-wise" is non-distributive (§ 1.5) and is not intended to entail satisfaction by a *single entity*. Hence, sets appear to be wholly dispensable (see Linnebo 2003, 2004: Bennett 2009).

Reply: The objection can be blocked. We argue that there is parity, in respect of the ontological commitments of a theory to a type of entity *K*, necessitated by the truths that "P" and "possibly P". In short, both statements incur the same ontological costs. Mathematics is "indispensable" to the empirical sciences and so to 'total theory' (Quine 1948 [1980], 1951a [1980], 1954b [1976], 1981a [1981], 1981c [1981]: Putnam 1975b

[1979], 1975c [1979]). *If* mathematics is indispensable to the empirical sciences and *if* sets provide the foundation of mathematics, then sets are indispensable to the empirical sciences. Sets, therefore, are an indispensable component of 'total theory' **[FN 8]**. Broadly, the Quine-Putnam argument is:

Premise A: We ought to have an ontological commitment to all and only those types of entity that are indispensable to our best scientific theory

Premise B: The types of entity posited by mathematics are indispensable to our best scientific theory

Therefore,

Conclusion C: We ought to have an ontological commitment to the types of entity posited by mathematics (Colyvan 2015, Section 1)

The type of entity indispensable to mathematics and therefore to the empirical sciences is sets (cf. Feferman 1998: Weaver 2009). For Lewis, mathematical "fruitfulness" is a theoretical benefit enjoyed by postulating a plurality of sets (see § 8.1). Unfortunately, although "fruitful" the "indispensability" of set theory in regard to the foundations of mathematics is widely rejected by mathematicians. Number Theory and functions are able to fulfil this foundational role without the unwarranted theoretical costs associated with set theory. For example, "[the] classical paradoxes of naïve set theory particularly cast doubt on

the idea of a well-defined canonical universe of sets in which all set-theoretic questions have definite answers" (Weaver 2009, page 1).

Although the "indispensability" of sets in the field of mathematics and empirical sciences is disputed we argue that set theory remains mathematically useful or "fruitful". Certainly, in the alternative the coherent application of plural quantification in the field of empirical science is beset by problems. How does an advocate of plural quantification preserve the *compositional* structure of "objects"? Empirical science is often engaged in the project of explaining how big things are made of smaller things. Are the big "things" actually simples arranged big-thing-wise? Are the small "things" actually simples arranged smaller-thing-wise? To which type of entity do we grant conceptual and ontological priority, the big things or the little things? Relatively simple predicates, such as "arranged *tablewise*" mask a fiendishly complicated structure. Take, for example, a phrase about a table constructed from a soft wood species, "there are simples arranged *tablewise*" unpacked as:

((((((there are simples arranged *sub-atomicwise*) arranged *atomwise*) arranged *cellulose-wise*) arranged *hemicellulose-wise*) arranged *lignin-wise*) arranged …)

(Bennett 2009, pp 59-60)

The ontological status of sets, as "indispensable" to mathematics and science, may be precarious but sets serve other theoretical purposes (Lewis 1986a, pp 3-4). Although rejoinders are available, the advocate of plural quantification appears to lack an explanation

of the inherent structure of reality (§ 4.6). How are the proposed *simples* arranged? In the absence of a cogent explanation of *structure* the plural quantification model holds scant appeal for the empirical sciences. Sets have "fruitful" non-foundational applications in mathematics and plural quantification is problematic in the field of empirical science – hence, we retain set theory as the most viable option.

8.3.3 The Antinomies of Proper Classes

Recall that the Redundancy Theory (in part) is motivated by a desire to provide **GMR** with a strategy to meet the demands of modal ubiquity **MU** and possibility introduction **PI**, in the context of advanced modal sentences (§ 7.3). These are "strong default principles – principles such that it is expected a theory of modality will be at least consistent with both and is significantly disadvantaged otherwise" (Divers 1999, page 218). We will now extrapolate from an argument constructed by Divers *against* a solution to the problem of advanced modality, proposed by Menzies and Pettit (1994). We apply the extrapolation to our proposed set-based theoretic solution to the problem of advanced modal claims. Menzies and Pettit argue that "it is true at2 *w* that *p* holds just when *p* holds true of *w* in virtue of the relations of *w* to other things – in particular, to other worlds" (Menzies and Pettit 1994, page 33 cited Divers 1999, page 225). In the case of an advanced modal claim of possibility introduction **PI**, say, that the plurality of worlds is true iff there is some world *w* at2 which there is a plurality of worlds. In other words, to say that there are many worlds is true of *w* in virtue of the *relation* of *w* to other things (in this case other worlds).

By **PI**:

1: There are many worlds

P1: It is possible that there are many worlds

(Divers 1999, page 221)

The truth conditions of a **PI** at P1 [left side, biconditional] is P1* [right side, (biconditional]:

P1* that there is some world *w* that has the relational property of being a member of a set of worlds, one of which contains many worlds

(Divers 1999, page 226)

P1* suffers from the now familiar problem of advanced modal sentences. By standard-1968 translation, there is no world that "contains" (in whole or in part), many worlds (§ 7.2) (Divers 1999, page 226). Divers considers adapting Menzies and Pettit's general formulae to accommodate more favourable truth-conditions on the proviso "that there are many worlds holds true of *w* in virtue of *w* standing in the relation of set membership to a set which has many worlds as members" (Divers 1999, page 226).

However, he concludes that the adapted formula delivers only partial success. There are intuitively true advanced modal claims that yield false *analysans* by standard-1968 translation. Consider:

P5: It is possible that there are proper classes

(Divers 1999, page 226)

A proper class is not a member of anything and so cannot be a member of a set required by the Menzies-Pettit account. The point is illustrated by P5.1, where "C" is a predicate letter designating a proper class:

P5.1: $\exists x \exists y \exists s (Wx \& (x \in s) \& Cy \& (y \in s))$

(Divers 1999, page 227)

> A uniform solution to the problem of interpreting advanced possibility claims is desirable and the worry about the Menzies-Pettit proposal is that far from delivering a uniform solution, it can offer only a piecemeal and ad hoc approach in which we are to seek in each case some relations or other involving worlds that will generate a truth-condition that is amenable to the genuine realist. (Divers 1999, page 227)

In subsequent work, Divers exorcises the ghost that haunts the relation between worlds and non-individuals (see D above). This leaves the residue of accounting for those potential objects that are prohibited from set membership and therefore excluded from any set-

based account of a solution to advanced modal claims. Clearly, proper classes are a prime candidate. Proper classes are those classes that are not reducible or equivalent to sets. The set / class distinction was first made by Frege in the late nineteenth century. A logical collection (or class) is the extension of a property that contains all and only those things which are the extension of that property. As each class is conceptually *dependent* on a prior predicate it cannot qualify as a conceptual primitive. A combinatorial collection (or set), on the other hand, is the result of combinatorial activity codified formerly by the axioms and theorems of set-theory.

Reply: Our strategy in respect of P5 is to deny the possibility of proper classes – proper classes do not, indeed, could not exist. Clearly, in order to establish grounds for this denial we must show that it is more than a convenient truth. We may deploy one of two options in pursuit of this strategy: firstly, cleave to the standard practice of set theory and deny all classes or, secondly, employ the Neumann-Bernays-Godel **NBG** axioms of set theory that admit classes (except for proper classes). We argue that the strategy is in a similar vein to denying other intuitively plausible but contentious entities; for example, zombie worlds, private languages and contingent identity statements. The strategy of denying proper classes can provide an avenue of escape from Divers' objection. We will utilise Russell's famous paradox in order to test the viability of the strategy. The paradox unfolds as follows: Suppose R is the set of all sets. If R exists then we face Russell's Paradox. It is an axiom of standard set theory that no set *x* can be a member of itself, that is, *x* excludes itself. *Ex hypothesi*, since every set is self-excluding, R is actually the set of all self-excluding sets.

R = {*x* I *x* is a set} = {*x* I *x* not-∈ *x*}

The problem is one of self-reference. Does the "set" R adhere to the axioms of standard set theory? Is R self-excluding or not? Assume that R is self-excluding, R not-∈ R. But we have established by axiom, for every *x,* if *x* is not a member of *x* then *x* is a member of R (if *x* not-∈ *x* then *x* ∈ R). Therefore, if R not-∈ R then R ∈ R! The second horn of the dilemma is equally unappealing. Assume that R *is* a member of itself, R ∈ R. But we have established by axiom, for every *x*, if *x* is a member of R then *x* cannot be a member of *x* (if *x* ∈ R then *x* not-∈ *x*). Therefore, if R ∈ R then R not-∈ R! When each "if-then" avenue of set membership is combined we discover R ∈ R iff R not-∈ R. That is a paradox. The *flavour* of Russell's Paradox is illustrated by analogy, the Barber's Paradox. Suppose there is a barber who lives in a town where he cuts the hair of all and only those people who do not cut their own. The question is this: who cuts the barber's hair? It cannot be himself, since that would mean he doesn't *only* cut the hair of those people who do *not* cut their own, excluded *ex hypothesi*. It cannot be someone else, since that would mean he does *not* cut the hair of *all* the people who do not cut their own, again, excluded *ex hypothesi*. Hence, the "paradox": "The antinomies of intuitive set theory pose the problem of providing a theory of sets which is free of contradictions" (Stoll 1979, page 289).

How should we answer Russell's Paradox? We could restrict predicates in respect of their definitional role in regard to sets. Indeed, this is how standard set theory avoids Russell's Paradox. By standard set theory, we cannot simply form a set with an arbitrary predicate P. A second set is required. Hence, the representation of the extension of a property (the

predicate P) by a single set is ill-formed {*x* I *x* is P}. We must first specify a second set *y* from which *x* is selected. The predicate P is now represented by the set {*x* ∈ *y* I *x* is P}. Hence, we define {*x* ∈ *y* I *x* not-∈ *x*} – which is simply an alternative way to represent the set *y* (Steinhart 2009, page 62). We can avoid both Russell's Paradox and Divers' inclusion of the possibility of *classes* (P5) simply by accepting the practice of standard set theory.

Intuitively, this solution is unsatisfactory. Intuitively, *every* property has an extension or a collection of all and only the objects that instantiate it. For example, the extension of the property is-a-cat is intuitively the set {*x* I *x* is a cat}. To account for our intuitions, an alternative non-standard solution to Russell's Paradox is available based on a distinction between those collections that are sets and those that are not. Let us assume for the moment that classes are possible and *do* exist. There are some collections (classes) that are not sets but play a prominent role in mathematics and the philosophy of language. Class Theory states that all collections are classes. Any arbitrary class is either a set or a proper class. Hence, every set is a class but not all classes are sets. Sets are distinguished from classes on the basis of the membership relation. A set is a member of some other *class* e.g. if X is a set then there is a distinct class Y such that X ∈ Y. A set may also be the member of another *set* e.g. its own power set. A proper class, on the other hand, is neither a member of another class nor a set. If X is a proper class then there is no class or set Y such that X ∈ Y. Importantly, there is a class of all sets. In order to avoid Russell's Paradox the class of all sets cannot itself be a set so it must be a proper class. The proper class of all sets is designated "V".

V = {*x* I *x* is a set}

Russell's Paradox is thereby dissolved. As a proper class, V cannot be the member of a set or another class so it cannot be a member of itself. Class Theory is able to dodge Russell's Paradox whilst preserving the intuition that every property has an extension, the class of a predicate P. The difference is one of *generality*. For *all* properties, the extension of that property is a class e.g. the extension of the property [is-a-set] is a class. *Occasionally*, the extension of a property is a set e.g. the extension of the property [is-a-perfect number] is a set (see Steinhart 2009, page 63). *If* we choose to accept classes, we can avoid Russell's Paradox and satisfy the intuition that the extension of *every* property is represented by a class of a predicate P. On the other hand, an ontological commitment to classes would restrict the scope of our set theoretic solution to the problem of advanced modality i.e. by Divers' P5.

Is there a solution to the impasse generated by the inclusion of non-standard set theory? A solution that can secure the theoretical benefits associated with an ontological commitment to classes but does not incur a theoretical cost to the proposed set-theoretic solution to the problem of advanced modality? As we have seen, the axioms of standard set-theory restrict the type of objects that qualify as a set; they postulate that for every set *x*, *x* is not a member of itself, for example. The non-standard account of set-theory entails an adjustment to our preferred strategy of denying the existence of classes. The resolution to the impasses involves adopting the axioms of von Neumann-Bernays-Godel (**NBG**) that take classes as the basic objects of set theory. However, although "basic" the **NBG** axioms restrict the domain of quantification to sets rather than *all* classes. That is, proper classes are excluded. By definition, this ontologically conservative extension of the Zermelo-Fraenkel

axioms of standard set theory does not posit the existence of proper classes. The axioms of **NBG** are appealing since we noted earlier that classes do not qualify as a conceptual *primitive* of **GMR**. We may either adopt the practice of standard set theory and exclude classes or the axioms of **NBG** that exclude proper classes. Whichever choice we are inclined to make we avoid an ontological commitment to proper classes and Divers' objection to a set theoretic solution to advanced modality. However, we have now strayed into a discussion of the axioms of set theory and breached our promise to confine our discussion to the "simple view" of sets.

8.4 Conclusion

We accept Divers' method of identification in respect to advanced modal claims. We reject Divers' solution to said problem on the grounds that – as a proposal conducive to **GMR** – a more appropriate solution is available. The set-theoretic solution to the problem of advanced modality is neither ad hoc nor piecemeal. An ontological commitment to a type of entity, sets, is compelling regardless of one's prior commitments to a plurality of worlds. Lewis' ontological commitment to sets is based on their theoretical utility, to the explication of a number of contentious philosophical "entities". Despite these theoretical advantages problems remain. We have shamelessly endorse sets on the grounds of their theoretical "fruitfulness" and yet reject proper classes despite their theoretical "fruitfulness"! The only fig-leaf we offer to cover this embarrassment is an argument from the "indispensable" nature of sets to the empirical science via the field of mathematics. Although disputed, this

"indispensability" provides ample motivation to include sets as a component of 'total theory'.

Footnotes

1. A philosopher – or philosophy student – ought to concede whenever a solution to a particular problem is elusive or beyond his or her intellectual abilities. In these circumstances, we ought to defer the problem to those with the expertise to tackle it. Indeed, it appears that such intellectual honesty was a personal trait of David Lewis himself (see Jackson and Priest 2004, page 2).
2. For example, we assume that all sets are abstract entities that necessarily exist – that is, every set is "accessible" from every possible world. This is not Lewis' position. Lewis notes that we usually restrict quantification when evaluating the truth of a quantified sentence; sets do not exist in either logical or ordinary "space" but *some* sets exist from the *standpoint* of every world (unrestrictedly) e.g. numbers. On the other hand, some sets can only exist from the *standpoint* of a particular world (restrictedly) e.g. the unit set of an individual who / which only exists in that world. (Lewis 1983b [1983], page 40, Lewis 1986a, pp 94-5) The latter case is one in which an abstract object (a set) exists contingently e.g. the Equator.
3. Alternatively, a pure set can be generated from iterations of the empty set.
4. As a first-order predicate "existence" is notoriously problematic. Properties should be construed herein as "extrinsic" in the broadest sense – an inclusive relation linking every actual individual (past, present or future) in a spatial-temporal region.
5. A modal claim that is counterintuitive. **GMR** may counter the claim on the basis that in this context "necessarily" means "all counterparts".

6. An external relation, for example, is one that does not depend on the intrinsic properties of an object, like the spatiotemporal relation of distance. An internal relation is one that does depend on the intrinsic properties of an object, say, intrinsic similarity (see Lewis 1991, page 34)

7. For a discussion of the *implied* ontological commitments of a theory to a type of entity see (§ 1.4)

8. An argument premised on the indispensability of set-theory to mathematics is unlikely to placate our protagonist. The constitution of mathematical objects is a contentious matter; although some writers (e.g. von Neumann) argue that all mathematical objects can be reduced to or identified by pure sets, others, famously Benacerraf (1965), argue that no such reduction is possible. Benacerraf argues that a multiplicity of incompatible methods to "reduce" numbers to pure sets invalidates the strategy of reduction (e.g. von Neumann with Zermelo techniques). Several replies have been made to Benacerraf's objection. One response is the *structuralist* account in the philosophy of mathematics. Alternatively, we could deny the claim that both types of reduction are equally valid; for example, Steinhart argues that the von Neumann reduction enjoys many advantages over the Zermelo reduction (Steinhart 2010, page 20)

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