Learning to Teach Mathematics through Inquiry: A Case Study of Continuing Professional Development in Malta

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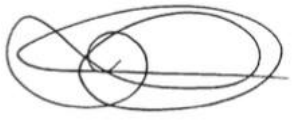
Statement of Authenticity

I, the undersigned, declare that I am the author of this thesis entitled Learning to Teach Mathematics through Inquiry: A Case Study of Continuing Professional Development in Malta

I further confirm that this study presented is authentic and has never been published in any other institution before.

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August 2018



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James Calleja

97,160 words

Dedication

To the wonderful memories of my late father

JOHN

my inspiration and guide,

an outstanding teacher.

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Along this journey, I was fortunate to find the support of many people who have, in many ways, contributed to making this a meaningful learning experience. I start with due gratitude to Professor Malcolm Swan, my main supervisor during the first three years of this study, who sadly passed away. I am grateful for the time he dedicated to listen to and discuss my ideas, and for his guidance, insights and motivation in the research design. Professor Jeremy Hodgen for accepting to take over Malcolm's work as my main supervisor. He made sure to be supportive but at the same time critical of my ideas during a time when I was analysing and writing up the thesis – something that helped me delve deeper into thinking about research. Dr Colin Foster, my second supervisor, who was always there when I needed help and his words of encouragement kept me going. I am indebted to his timely and thorough manner in which he supervised and offered feedback about my work. I feel greatly privileged to have had the opportunity to get to know and work so closely with academics such as these.

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Abstract

In the past decade, local policy documents envisage a shift towards more student-centred and inquiry-based pedagogies. At the same time, the OECD (2009) report shows that Maltese teachers, particularly those of mathematics, while exhibiting constructivist beliefs describe using teacher-centred practices. It appears that, rather than demonstrating unwillingness to adopt student-centred approaches to teaching, Maltese teachers are unprepared to take on the challenge. The provisions of CPD, which have also seen documents proposing the development of school-based learning communities, become crucial towards addressing this issue and hence in preparing teachers for this change.

This study focuses on the design of a CPD programme aimed towards supporting teachers in learning to teach mathematics through inquiry (LTMI). LTMI, which offers participation in summer workshops followed by ongoing follow-up meetings, is based upon research-informed principles of effective CPD, namely: offering practice-based learning, the provision of CPD materials, a community of practice component and long-term engagement. In addition, to facilitate the shift to IBL and to make it meaningful to teachers, CPD is designed around four IBL features. This means that the IBL features being promoted as essential for student learning – a task-oriented approach, creating opportunities for collaborative learning, using purposeful questioning and shifting agency and responsibility onto learners – are integrated within the CPD experiences offered to teachers.

While investigating the impact that the designed CPD has on teacher development and change towards more IBL practices, this research explores case studies that delve deep into the challenges that Maltese teachers face. Conducted over a period of 14 months, this study looks into contextual factors to explore how these influence the way teachers enact IBL.

This study adopts a case study methodology and use five key research instruments to collect data about each case study: semi-structured interviews, teacher questionnaires, lesson observations, teacher lesson journals and a focus group discussion. These data sources were administered at different stages: before, during and at the end of teachers' participation in LTMI. With this data, the research sought to identify how teachers interpret and enact IBL in their classrooms, the ways in which they structure their IBL lessons and the changes in beliefs and practices that take place over the course of the CPD programme.

Data from this research show that teachers' interpretation of IBL is context-specific and influenced by system restrictions, and the way they operationalise IBL is influenced by how students engage with and respond to the inquiry teaching practices that they offer. In the change process, teachers are likely to go through a phase where they adopt a discovery teaching approach. Moreover, teachers structure their lessons in atypical ways. This appears to be influenced by a number of factors but particularly

the tasks they use. Modifications to lesson structure are found to be inevitable as teachers learn about ways of providing purposeful interventions to facilitate the learning process for their students. Over the course of LTMI, teachers also report changes towards more connectionist beliefs – these being more consistent with their inquiry practices. Teachers attribute changes in practice and learning about IBL implementation as emanating and enhanced by their ongoing engagement within a community of practice. The collegial, safe and informal setting contrasted with the isolated and restricting contexts of learning within teachers' own schools. In supporting the enactment of reform, this thesis offers recommendations towards providing 'just-in-time' collaborative learning opportunities for teachers in supporting them with *enabling* change. Enabling change requires empowering teachers to become autonomous learners in creating their own professional workplace learning networks and assuming professional learning responsibilities.

Key words

Inquiry-based learning; lesson structure; community of practice; beliefs and practices; continuing professional development; learning contexts; 'just-in-time' teaching and learning

Professor Jeremy Hodgen
Main Supervisor

Dr Colin Foster
Second Supervisor

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Glossary of Abbreviations

CoP	Community of Practice
CPD	Continuing Professional Development
CPS	Collaborative Problem-solving
CTP	Council for the Teaching Profession
GCE	General Certificate of Education
HoD	Head of Department
IBL	Inquiry-based Learning
IBSE	Inquiry-based Science Education
INSET	In-service Education and Training
ITE	Initial Teacher Education
JITL	'Just-in-time' Teaching and Learning
LTMI	Learning to Teach Mathematics through Inquiry
MEDE	Ministry for Education and Employment
MTL	Master in Teaching and Learning
MUT	Malta Union of Teachers
NCF	National Curriculum Framework
PRIMAS	Promoting Inquiry in Mathematics and Science across Europe
SEC	Secondary Education Certificate
SLT	Senior Leadership Team
TALIS	Teaching and Learning International Survey
ZPD	Zone of Proximal Development

Chapter 1: Background of the study

1.0 Introduction

This thesis presents a study of teacher participation in a continuing professional development (CPD) programme designed to support teachers in *Learning to Teach Mathematics through Inquiry* (LTMI). LTMI is a 14-month voluntary CPD programme, carried out during the period of April 2015 to June 2016, consisting of summer workshops and ongoing follow-up meetings during one scholastic year (see Section 4.3). The aim of the study is to identify, explore and understand teacher engagement in LTMI, their professional learning experiences and teachers' understanding and enactment of inquiry-based learning (IBL). This thesis describes the professional learning journeys of teachers, but it also focuses on my learning, alongside teachers, in becoming a researcher.

This chapter presents the reader with accounts drawn from personal experiences that were influential for me in pursuing this doctoral research. Personal experiences, dating back to my years of primary and secondary schooling, have shaped my beliefs and values, and the way I see teacher education and professional learning. As the research instrument (see Stake, 2010), I choose to provide an account written in the first person. In describing my own experiences along the years, I see myself as a learner but also as a researcher looking through a window into the worlds entered and the events experienced. As a writer, I become the subject of the text (Denzin & Lincoln, 1994) because what I write is based on my own perceptions and interpretation of my experiences and encounters. The aim of this chapter is thus intended to help the reader understand the research problem and contextualise the study undertaken. Although carried out in Malta, a country with a relatively small geographical size, structure and organisation influenced by the British education system, the research context is likely to resonate with others elsewhere: particularly with contexts where a high-stakes assessment system and an isolated teacher learning culture are prevalent.

1.1 Personal learning experiences

In pursuing this study, I begin by describing and drawing on personal experiences as a student of mathematics, a teacher and a head of department (HoD). Eventually, midway through this study, I left my HoD post to assume a teacher educator role at the University of Malta – a role that involves developing and providing nation-wide professional learning programmes to support in-service teachers. In the following account, I focus on how my personal philosophy of learning and teaching mathematics, and of teacher learning evolved over the years. I aim to help the reader gain an understanding into how my experiences have nurtured a passion for IBL and the design of CPD, and how these eventually influenced the type of research I engage in to study teachers' enactment of IBL practices and their learning journeys.

1.1.1 As a student of mathematics

At primary level, I attended a school which used an 'ability' based strategy according to measures of students' prior attainment (including national examinations and school-based tests and assessments). According to Taylor et al. (2017), 'ability grouping' appears to complicate attainment in education with the notion that one can set groups of students possessing some innate academic 'ability'. Indeed, the idea that mathematical ability can be measured using standardised testing may be oversimplified – hence my preference to use 'attainment grouping' to refer to such streaming practices. Students, in the school I attended, were streamed and those in the top streams, as I was, were encouraged to do well in examinations.

Looking back on my experiences as a student, both at primary and secondary levels, I can say that school mathematics was almost always presented through teacher exposition followed by drill and practice exercises. Lessons were textbook-driven and classwork activity followed the same repetitive and predictable pattern. The students in my class, particularly during my secondary school years, engaged in what Skemp (1986) identifies as 'habit learning'. Predominantly this type of learning

involved memorising by rote the separate chunks of mathematical knowledge that were passed on to us by the teacher. As a learner, I experienced a 'lone-fighter' culture (Krainer, 2001) learning to work on my own and in competition with my classmates, and behaving in what Mason and Johnston-Wilder (2006) refer to as an 'accepting mode'. In other words, I was happy to sit back and wait to be told what to do to learn mathematics. Teachers adopted an approach that demanded 'silent learning' (Boaler, 2009) of rules and procedures, and performing quick calculations as if by automation. This approach promoted a competitive culture among students which our teachers used to see as necessary for success in mathematics.

1.1.2 My first encounter with active learning

During my final year of secondary studies, my school ran short of mathematics teachers and it was up to a teacher of French to take our mathematics classes. As soon as Mr Borg (pseudonym) entered class, he would put a problem on the board or read a question from the textbook, with no explanation whatsoever. 'Is there anyone who has an idea about this?' was the recurrent question Mr Borg would put forward to invite us to contribute ideas. With students motivated to do mathematics as we were, his approach seemed to work. Mr Borg never minded the noise caused by our animated discussions. He simply managed the class by asking those who '*had an idea*' to stand at the front. Then, Mr Borg preferred to sit by the side allowing us to share and discuss our thoughts.

In Mr Borg's class, I recall experiencing a high degree of ownership, responsibility and agency. For the first time I was in an 'asserting mode' (Mason & Johnston-Wilder, 2006), that is, in control of my own learning, taking risks and getting to know mathematics in terms of my own operations upon it (Glynn, 1985). Although at that time I did not as yet understand the pedagogical importance of creating responsive classroom environments that support and promote independent learning, I appreciated the value of active autonomous learning. I recall, for instance, sorting out the remainder theorem, feeling really proud of myself that I managed to do it without the help of my teacher. There I experienced

'intelligent learning' (Skemp, 1986) because for the first time I was thinking deeply about something I was doing and recognising important relationships with the other topics I had studied. This, in turn, developed more confidence in my own ability to understand, explore and build mathematical knowledge. In this class, we took turns to go up to the board presenting ideas, discussing steps and procedures, and explaining why the methods offered made sense. I would say that Mr Borg had managed to create a particular learning community where we (as learners) took agency over our learning. This experience had a positive impact on my beliefs as a learner of mathematics that opened up new ways of conceptualising learning.

1.1.3 As a trainee teacher of mathematics

My experience with Mr Borg instilled in me a belief that learning mathematics can take on a different perspective to the teacher-controlled one I had been accustomed to. It is with this belief in mind that I continued studying mathematics at the two-year post-secondary level education and then at undergraduate level as part of my four-year initial teacher education (ITE) course at the University of Malta. With hindsight, my 'proven' ability to teach myself mathematics had somewhat channelled me towards teaching – I felt that if I could teach myself mathematics, I could also help others to assume more responsibility in learning the subject.

At University, I found myself once again learning in a competitive environment until I got to know Mark (pseudonym), a fellow student, who was also very keen on the teaching and learning of mathematics. We soon discovered that our philosophy of teaching and learning had a social constructivist outlook (Ernest, 1991). Our philosophy directed us towards engaging students in meaningful and rich mathematical activities through social interactions. Without actually planning it, during our ITE course and later as practising teachers, Mark and I had set up a community of practice (Wenger, 1998) in which we sought to improve our teaching practices through the sharing of ideas and mutual support. As part of our collaboration, we planned and designed mathematical tasks that included

practical activities and investigations**.** We could continue this collaboration when we were posted in the same school during our final six-weeks of practicum.

Our teaching experience in this school was exceptionally fruitful. Our collaborative work with the students gave us important understandings into the teacher's role, the students and how to create responsive learning environments. Activity areas were set around the four corners of the classroom, and students were generally organised in groups and discussion lessons prevailed. To deepen our emerging understandings, we observed each other teaching, video recorded lessons and interviewed students about their experience. We found ourselves learning about teaching as much as students were learning about mathematics.

1.1.4 As a teacher of mathematics and head of department

Mark and I believed that our collaborative experience had enriched our professional development as future mathematics teachers. Possibly even without realising we had embarked on a personal journey of reflective practice. To further develop our fruitful collaboration, once we were accepted in September 1993 to teach as full-time mathematics teachers, we made a formal request (duly granted) to be posted in the same school.

During this period, Mark and I started to experience teacher-research, in the sense that we were looking critically at issues that were central to our work, building in the process a stock-of-knowledge that was grounded in our professional knowledge and experience (see Roulston, Legette, Deloach, & Buckhalter Pitman, 2005). In the process, I learned a lot in terms of researching my own practice and the benefits of having a critical friend. Our stay at this school, however, was short-lived as it was closed in 1995. Although we still met occasionally to discuss our teaching, my direct collaboration with Mark was discontinued when we had to resume our teaching duties in separate schools. For the next thirteen years, I taught mathematics and physics at another secondary school. My previous positive collaborative experience with Mark encouraged me to seek out people with

a like-minded philosophy. Luckily, I managed to find individuals with whom I could further develop my teaching ideas.

I had been teaching at secondary level for 14 years when I started my master's degree. This experience provided me with the possibility to engage more systematically in the kind of classroom research I had been doing since my earlier teaching days. The decision to conduct an action research study for my dissertation during the 2008-2009 scholastic year was consequently the most natural thing for me. In 2010, when I was reaching the final year of my postgraduate degree, I was promoted to a HoD. That post meant that I had to be deployed to a new secondary school.

It soon emerged that, at this school, teachers generally worked in isolation and adopted transmission teaching practices (Buhagiar & Murphy, 2008). At first, many seemed sceptical and resistant to change. Motivated by my social constructivist theories of learning, I sought to primarily create a collaborative department engaged in improving the learning experiences provided to our students. By sheer coincidence, however, the arrival of PRIMAS (*Promoting Inquiry in Mathematics and Science Education across Europe*) in 2010 was a well-timed opportunity for a number of colleagues who showed a disposition to embark on new ways of teaching. Six of the eleven teachers in the mathematics department were willing to participate in this EU-funded project that sought to promote IBL (see http://primas.mathshell.org/pd.htm). In PRIMAS, which was implemented over a three-year period (2010-2013), I was one of the project's so-called 'multipliers' who were responsible for leading and supporting school-based communities of teachers. This role involved providing a CPD experience for teachers through discussion, planning and evaluation of inquiry lessons. That was the start of my journey in creating and designing collaborative CPD opportunities with teachers at this school.

With the onset of PRIMAS, I witnessed teacher change in different ways. Some had placed more importance on students becoming more actively involved in the learning process and viewed the need of assessment practices becoming more formative; others valued the shift from working

in isolation and sought ways of learning about teaching with others. With hindsight, this experience provided that extra push needed at the time to start reading for a PhD. I was intrigued by the following aspects.

- *Some teachers were willing to change and eventually implemented IBL. Could others working in different school contexts do the same?*
- *How do teachers learn to teach mathematics through inquiry?*
- *What aspects from PRIMAS and my own personal experiences working with teachers could I draw on to design an effective CPD programme?*

As a social scientist, I embraced a role where the researcher "gathers information about actions and interactions, reflects on their meaning, arrives at and evaluates conclusions, and eventually puts forward an interpretation" (Marshall & Rossman, 2014, p. 21). With this in mind, I considered participants with a range of teaching experiences and possibly working in different school contexts. I thought this would offer the possibility of creating a community of practice with more diverse interests, challenges and professional learning needs and experiences. Furthermore, reflecting on and understanding the implications of my role as a HoD and professional development provider, that I had at the time, mostly during In-service Education and Training (INSET) was critical in designing this study. I soon recognised that doing this research was going to be "confusing, messy, intensely frustrating, and fundamentally nonlinear" (Marshall & Rossman, 2014, p. 21). Hence, although the participation call I issued was open to all state and non-state secondary mathematics teachers, I chose to research the practices of those who did not teach within the state College I was in charge of at the time (see Section 1.2.1 for more details regarding 'Colleges'). I sought to attract participants who had a willingness to explore possibilities without feeling obliged to take part. Moreover, given the generally mandatory, short-term and unappealing nature of CPD offered locally (see Section 1.4.2), I recognised the importance of teacher 'buy-in'. Enrolling teachers, hence, required more than just offering a voluntary

course. It necessitated a well-developed and attractive programme with relevant content that addressed teachers' professional learning needs and opportunities for ongoing collegial support (see Chapter 3).

1.2 The context of the study

1.2.1 Malta and its education system

Malta is an archipelago that includes the islands of Malta, Gozo and Comino. Spread over an area of 316 square kilometres and situated in the middle of the Mediterranean, the Maltese Islands are inhabited by approximately 430,000 people (National Statistics Office, 2016). Malta has Maltese and English as official languages, but the national language is Maltese. Malta became part of the British Empire in 1814 until it gained its independence in 1964. It is not surprising, hence, that our education system still has strong British overtones (see Sultana, 1997). As with many countries with a colonial heritage, up to 2006, Malta had a highly centralised and selective compulsory education system.

There are two kinds of schools in Malta: state and non-state. State education is free of charge from pre-primary to tertiary education (see Table 1). The non-state education sector is sub-divided into Church schools and Independent private schools. Church schools are predominantly Roman Catholic schools and are heavily subsidised by the government. As a result, Church schools do not charge any tuition fees. Independent schools are fee paying and set up by individuals or non-profit parents' foundations.

Level	Provider	Age (Years)
Primary	State, Church and Independent schools	5 to 11
Lower Secondary		11 to 16
Higher Secondary	Upper Secondary (Sixth Form Colleges)	16 to 18
	Vocational schools	16 to 20
Tertiary	Mainly the University of Malta	18+

Table 1: The education system in Malta

In Malta, the Ministry for Education and Employment (MEDE) is responsible for the organisation, administration and the human and financial resources in state schools at all levels of education. Furthermore, MEDE also monitors the functioning of non-state schools. As from 2005, state schools have been grouped into ten College networks – nine in Malta with the tenth being Gozo. Each College, headed by a Principal, is composed of a number of feeder primary schools and at least two secondary schools. Within secondary schools, for all subjects except the core subjects (i.e.: mathematics, English and Maltese), students are in mixed-attainment classes, even if some classification may occur on the basis of the subjects chosen. For the core subjects, students are set in classes according to their performance in the annual examinations.

In the secondary sector the average class size is 18.0: with 17.1 in state schools, 21.6 in church schools and 15.0 in independent schools (National Statistics Office, 2011). Secondary schools generally dedicate five lessons per week for mathematics, with lesson periods ranging from 38 to 45 minutes. Teaching in state schools normally commences on the fourth Monday in September and ends on the day prior to the commencement of the annual examinations (second week in June). A typical school year comprises about 32 teaching weeks, which is about 160 teaching days. These do not include examination periods and staff development days/INSET courses.

Throughout lower secondary education students follow the same pattern of assessment as that of the primary cycle – two assessments and two examinations (Grima & Chetcuti, 2003). The two examinations, half-yearly and annual, are carried out in February and in June of each scholastic year. Assessments, on the other hand, are carried out within two specific periods during the year: October to January and March to June, with teachers generally gathering marks on classwork, homework and tests. At the end of compulsory education, most students sit for the Secondary Education Certificate (SEC) examination – which act as an entry qualifications to post-secondary education – set by the University of Malta.

1.2.2 Reforms and priorities

Over the past six years, there has been a restructuring and a reorganisation of the local education system. The National Curriculum Framework (NCF) (Ministry for Education and Employment, 2012), introduced to review the National Minimum Curriculum (Ministry of Education, 1999), projects the vision, the principles, the main objectives and values and establishes the framework of the curricular aspects of Maltese education. The NCF also takes into account the eight key competences for lifelong learning of the European Reference Framework (European Communities, 2007) – one of which is mathematics. A number of themes, namely critical thinking, creativity, initiative, problem-solving, risk assessment, decision-making and constructive management of feelings are embedded in the eight key competences.

In its implementation, the NCF (Ministry for Education and Employment, 2012, p. 25) proposes that, it will

> not only demand pedagogical reform as traditional ways to teaching will now be replaced by a more student centred and inquiry-based approach to learning but also a cultural change process as school leaders, teachers, educators and parents are imbued with a new value system.

The NCF provides a strategy towards improving the quality of education and raising the level of student achievement; departing from traditional curriculum structures and practices, and providing flexible learning programmes for diverse and integrated learning experiences for all learners. In particular, the NCF provides guidelines aimed towards the restructuring of (i) learning programmes, (ii) assessment policies and (iii) professional development. Within the learning programmes' dimension, the focus is on understanding and emphasising the process of learning and the active co-construction of meaning rather than the mere acquisition of content. Learning programmes are also aimed at empowering teachers to implement innovative teaching and learning strategies. With regards to the assessment policy, the NCF values and assesses both the processes and the products of learning – hence, there is a move towards more formative ways

of assessing student learning. Within the professional development dimension, the document proposes developments that are congruent with the availability of opportunities for the development of the school as a learning community of reflective practitioners.

1.2.3 Becoming a teacher of mathematics

Up to 2015, ITE involved a four-year Bachelor of Education (Honours) degree course or a Bachelor's degree in one or more subject areas followed by a one-year Post-Graduate Certificate in Education. Both routes were offered by the Faculty of Education at the University of Malta. As from 2016, these two routes have been replaced with a two-year Master in Teaching and Learning (MTL) degree course. The MTL course may be pursued only after a candidate has obtained a first-cycle degree in a subject area or a related area of the curriculum. Besides being professionally qualified, teachers in Malta need to hold a teachers' warrant issued by the Council for the Teaching Profession (CTP). During the first two years of employment, teachers go through an induction and mentoring phase. On successful completion, teachers are then recommended for a teacher's warrant (see Eurydice, 2018).

1.2.4 Teacher CPD obligations

At the time of writing this thesis, a new sectoral agreement between the Government and the Malta Union of Teachers (MUT) has been signed (Ministry for Education and Employment, 2017). Therefore, to better foreground the context of the present study, I will focus on teacher CPD obligations prior to this new agreement (i.e.: those *in vigore* at the time the study). In Chapter 8, where I discuss the implications of the findings and provide recommendations for future CPD opportunities, I will however outline the new CPD structure and obligations put into effect as from January 2018.

Until 2016, teachers in Malta were provided with up to 30 hours of CPD each year (see Table 2). This training time is equivalent to approximately 7.3 days, which is well below the TALIS (Teaching and Learning International

Survey) average of 15.3 days dedicated to CPD in Europe (OECD, 2009). CPD opportunities for teachers usually occurred at school level, but they were also provided by MEDE, which was at the time the main agent for providing INSET (see Ministry for Education and Employment, 2012), most of which was mandatory. Teachers in the non-state sector could choose to attend training which was also organised in-house (Attard Tonna & Calleja, 2010). Apart from these CPD obligations, teachers could also undertake post-graduate courses offered by tertiary institutions, among which is the University of Malta.

CPD Activity	Period	Duration
Staff development sessions	After school hours	*3 two-hour sessions (6 hours)*
School development meetings	Within school hours	*3 two-hour sessions (6 hours)*
School development day	Within school hours	*1 full-day (6 hours)*
INSET training	In July (following end of school year) or in September (before school year) *	*3 half-day sessions (12 hours)*

* *Scholastic year ends after the first week of July and starts in the last full-week in September.*

Table 2: CPD opportunities for teachers in Malta

Describing the Maltese CPD context, Attard Tonna and Calleja (2010, p. 50) claim that

> The way the training is structured does not allow for any relationships of collegiality and trust to be developed with the trainers and the rest of the teachers; moreover, training is often disconnected from real teaching experience as teachers are not given the opportunity to experiment and return with feedback, or receive support. The training objectives and knowledge imparted do not originate from the teachers themselves and more often than not, these reflect policy directives which are not necessarily congruent with the challenges teachers are experiencing at present.

Generally speaking, CPD activities have so far been informed by a 'deficit model' (see Brown & McIntyre, 1993) with the assumption that educators have deficiencies and CPD would serve to correct these. Training is usually

provided by outside experts, and most sessions tend to be led through power-point presentations. In such cases, CPD takes a top-down approach of knowledge transfer to participants who, in turn, end up having to listen for most of the time with little or no input on their part. While disregarding teacher motivation and agency in learning and development, this model is found to be ineffective (Little, 1993; Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010) for the lack of transferability of knowledge that teachers may take into their classrooms.

1.3 Teaching and learning mathematics

1.3.1 Views of mathematics

The way mathematics is presented in official documents may not necessarily convey what doing mathematics involves, and this may in turn influence the way in which it is taught. There appears to be two distinct views of mathematics: an 'absolutist' and a 'fallibilist' (Ernest, 1991). Within an absolutist view, mathematics is an objective, certain and unchallengeable body of knowledge that rests on deductive logic. Moreover, it consists of pure isolated knowledge that is value-free and culture-free. Consequently, this propagates an image of mathematics as fixed, rigid, objective, logical, pure, abstract and inhuman. Within a fallibilist view, mathematical knowledge is understood as fallible and open to revision. Hence, it becomes the product of social processes and practices. This view rejects the absolutist image of mathematics and embraces the notion that mathematics has a 'front' and a 'back' (Greiffenhagen & Sharrock, 2011; Hersh, 1991). Using the analogy of the restaurant, Hersh (1991) argues that at the 'front' the public is served with neatly presented mathematical dishes with products in their finished form. This mathematics is presented in a neatly ordered and precise form to the public in classrooms, textbooks and journals. Yet, at the 'back', mathematicians cook up, discuss and create new knowledge. At the 'back', mathematics is fragmented, informal, tentative and incomplete.

Conceiving mathematics within an objectivist 'front' is likely to lead teachers to adopt a pedagogy based on transmission and textbook teaching (Ernest, 1991; Greiffenhagen & Sharrock, 2011). In the Maltese context, this kind of view may be further accentuated by examination syllabi that present mathematical content as made up of a set of isolated, disconnected, fixed and sequential units that need to be covered within a pre-determined time (Buhagiar & Murphy, 2008). This, I argue, directs teachers into creating traditional classroom settings that project a fixed and rigid image of mathematics.

Now, while local documents have, to some extent, proposed a more humanised view of school mathematics incorporating fallibilist dimensions to learning the subject (Ministry for Education and Employment, 2012; Ministry of Education, 2012), a more absolutist view of mathematics is still predominantly reinforced through mathematical texts (syllabi, textbooks, worksheets and examination papers) that make their way into local classrooms (see, for example, Ministry of Education, 2012, 2014; University of Malta, 2018). Such texts present mathematics as defined, fixed and unchanging, often prompting teachers to "seek an authority on the mathematics they present in their classrooms … reinforced through traditional testing and examinations" (Jaworski, 2010, p. 13) which are still part and parcel of the mathematical diet that teachers feed their students. Indeed, within traditional classrooms, teachers of mathematics appear to operate in the 'exercise paradigm' (Skovsmose, 2001) with the textbook representing the given text for practising mathematics. A move towards an inquiry approach, as proposed by Skovsmose (2001), would open up mathematics for exploration and creativity. But this may "be threatening for teachers who feel insecure without an authority to sanction their judgments" (Jaworski, 2010, p. 13).

Notwithstanding this, teachers in Malta predominantly see their role as facilitators of active learning (OECD, 2009). They seem to exhibit constructivist beliefs of engaging students in seeking out solutions for themselves, as opposed to a role of transmitting knowledge and providing

correct solutions. Particularly revealing in the OECD (2009) report is the fact that the same teachers, but particularly those teaching mathematics, describe using practices aimed at ensuring that learning is well-structured rather than using student-oriented practices (see also Buhagiar & Murphy, 2008). Teacher-centred practices go against, to some extent, the dominant constructivist beliefs reported by teachers. This suggests that Maltese teachers are unprepared rather than unwilling to take up the challenges of using IBL. Promoting mathematical inquiry in class may offer a possibility that Maltese teachers might wish to explore, yet it might appear ambitious to implement within the local education system. Moving away from the 'exercise paradigm' towards an inquiry-based approach would require teachers to make changes in, for example, the forms of communication they use – from being top-down and fixed to the nurturing of classroom discourse emanating from the students.

1.3.2 Inquiry-based learning

IBL is a contested term, often conflated and used interchangeably with other terms that describe similar approaches to teaching and learning, such as, active, experiential, problem-based, project-based, investigative, inductive, and dialogical approaches (Anderson, 2002; Engeln, Mikelskis-Seifert, & Euler, 2014). However, there are also commonalities between each of these approaches. These approaches support learning that is driven by questions, problems or situations, is active with students assuming more responsibility for their learning, and is collaborative with the teacher acting more as a facilitator than a provider of knowledge (Artigue & Blomhøj, 2013; Engeln et al., 2014; PRIMAS, 2012; Swan, Pead, Doorman, & Mooldijk, 2013; Towers, 2010; Walker, 2007). IBL has its roots in constructivism – cognitive (Piaget, 1985) and social (Vygotsky, 1978). Within the constructivist theory of learning, IBL embraces self-directed learning, collaboration and experiences through which students build on the knowledge they already have. As a more knowledgeable other, the teacher has a key role to play in scaffolding student learning (see Section 2.4.2). Scaffolding refers to the way a teacher structures lessons and intervenes to

help students engage with tasks which would otherwise be unachievable, and then gradually removing this support as students become more autonomous (Wood, Bruner, & Ross, 1976).

1.3.3 Teaching mathematics in the Maltese context

The mathematics curriculum (see Ministry of Education, 2012, 2014; University of Malta, 2018) compartmentalises the subject into isolated topics, and teachers tend to plan the order of their teaching according to this conceptual layout. A local study by Buhagiar and Murphy (2008, p. 173) has also depicted the much-diffused teaching and learning style in Malta:

> The teaching style of the teachers studied, apart from generally adhering to the traditional 'talk and chalk' approach, can be subdivided into three easily recognisable separate segments, with one phase following the other. These are *exposition* (i.e., teacher presents theory and solution methods, and students take down notes), *practice* (i.e., students work questions – either on their own or else, as more frequently happens, guided by the teacher – to practise the solution methods) and *consolidation* (i.e., teacher clarifies students' learning and difficulties). This irrefutable cycle of exposition, practice, and consolidation ... presents 'teaching as transmission' and 'learning as practice'.

As already outlined, this traditional view of doing mathematics is supported by two main features that prevail in local schools: (i) the prescribed content-based syllabi that teachers are expected to cover; and (ii) the dominant role that examinations have in 'measuring' how successful students are in learning mathematics. Given this emphasis on content coverage and knowledge testing, mathematics teachers are more likely to become transmitters of knowledge during lessons (Agudelo-Valderrama, 2007). Perhaps this also explains why mathematics teachers continue to find it difficult to place, as Schoenfeld (2006) suggests, more emphasis on 'teaching for understanding' than 'teaching for skills'.

In recent years, Malta participated in the TIMSS, PIRLS and PISA international studies. These confirmed that whilst our top achievers compare well with those of other countries, we have a high level of low

achievers. In the TIMSS 2015, for example, results for mathematics show that Maltese 13-year old students ranked 20th out of the 39 participating countries with a mean score of 494 (6 scale points lower than the international average). In this survey, our students scored relatively well in *knowing*, just within average in *applying* but demonstrated below average *reasoning* skills (Ministry for Education and Employment, 2016). The PISA 2010 assessment, which considers students' mathematical ability to tackle non-routine problems set in real-world contexts, had already signalled that *applied reasoning* was our students' main deficiency; with students scoring 463 points which is 33 scale points below the OECD average (Walker, 2011).

1.4 Identifying the research study

This study involves the design of a CPD programme for teachers based on both the research literature and personal experiences. The study, however, focuses on understanding teachers' CPD experience and their journeys in learning and enacting IBL. As described above, my interest in IBL dates from 25 years ago; back to my ITE experiences. Moreover, during formal and informal discussions with teachers, both as a teacher and HoD, they generally valued the benefits of IBL approaches – a positive orientation, reported by the PRIMAS baseline study, showing Maltese teachers endorsing IBL (see Euler, 2011). Nevertheless, teachers still appear hesitant to try out inquiry, and especially with their higher attaining classes. In contrast, they seem restricted to transmit mathematical knowledge and skills to their students because of an education system that values testing and examinations, content coverage and teaching skills, procedures and techniques. Hence, they shy away from active learning approaches to teaching on such restrictions. Choosing to transmit knowledge in view of the constraints in which they operate, these teachers reveal their 'sense of practicality' (see Hargreaves, 1994) which, as Buhagiar (2004) argues, allows them to operate 'successfully' within the local education system. Despite this prevalent conception of mathematics teachers towards IBL, I came across teachers who had managed to successfully engage students in inquiry. This instilled in me a desire to think about the design of a CPD

programme that could support those teachers who sought to develop and make changes to their classroom practices but did not appear to have the necessary assistance and confidence to do this.

While I recognised the social influences emanating from my experiences and beliefs, I was mindful of the need to create and maintain a 'disinterested interest' necessary for research (Bourdieu, 1996). This necessitated that, in researching IBL and teacher learning, I act as objectively as possible to researching a subjective area of interest. While I sought to and wanted the CPD design to be effective and work for teachers, I also remained cautious and questioned it. In my pursuit for acting reflexively (Ball, 1990), I remained alert to the social influences so that these do not, as much as possible, influence my judgements. I was conscious throughout of the fact that ignoring these would have an effect on the findings I report. To address this, I problematised the research instruments and the purpose they were serving, engaging in a process of ongoing self-questioning in order to make reflexive use of the findings. I also avoided possible distortions that are embedded within the very structure of the relationships created with the participants (Bourdieu, 1996). I did this by presenting myself and the research in a way that does not prevent participants from freely voicing their opinions. Building and maintaining trustworthy relationships was important to minimise influences on participants' actions and voices.

The design of LTMI was based upon research-informed principles of effective CPD. Besides investigating the impact that the designed CPD could have on teacher development and change, this research explores case studies that delve deep into the challenges reported by Maltese teachers (see Engeln, Euler, & Maaß, 2013). This study, moreover, looks into school contexts and system restrictions to explore the impact that these may have on the way teachers enact and develop their IBL practices. CPD thus takes a practice-based approach offering ongoing opportunities for teachers to share, reflect upon and address issues of practice. Creating and cultivating a community of practice (CoP) was a necessary component within LTMI, as

it provided ongoing support for teachers to discuss, evaluate and develop practice-based learning about inquiry teaching.

The significance of communities of practice and the design of professional development builds on my work, initially as a HoD but also later as a teacher educator at the Faculty of Education, University of Malta. Along the years, I met teachers of mathematics (both within my school and across other schools nationwide) on a regular basis and collaborated with them on various initiatives. Stemming from this work, I have developed an understanding that Maltese teachers need and value experiences as participants in communities of practice (see Attard Tonna, Murphy, & de Paor, 2018) in which they are afforded agency to experiment in reforming practices and strategies with the support of colleagues (Putman & Borko, 2000; Stein, Smith, & Silver, 1999).

1.4.1 Goals of the study

When I started this journey, I sought to embark on a project through which to: (i) develop my personal professional development; and (ii) contribute towards improving education policymaking and practice, on a contemporary area of research, both locally and internationally. I thus pursued a study within a research area that I had a passion for; one that would fulfil me professionally, intellectually and emotionally.

In undertaking this study, I had three goals in mind: personal, practical and intellectual (Maxwell, 2008). My personal goal was motivated by a personal interest in IBL and CPD design. Through this study, I attempted to design an innovative CPD programme that works for teachers in integrating more IBL in their secondary mathematics classrooms. The programme design had to challenge the existing situation of how professional development courses are offered locally. The practical goal, therefore, was to create a CPD programme that is collaborative, practice-based, context-sensitive and, hence, effective in bringing about teacher change. My intellectual goal, on the other hand, was to understand teachers' knowledge of IBL, their IBL enactment within different classrooms and its development over time.

1.4.2 Significance of the study

This research has important contributions to make both nationally and internationally. Locally, this study is important in investigating how teacher learning may be supported and sustained through communities of practice. A more significant contribution will be provided at an international level. This study is aimed to problematise IBL, the design of IBL-related CPD, and teacher practice and beliefs change.

At a national level, CPD through learning communities seemed to be gaining momentum with the publication of the National Minimum Curriculum document (Ministry of Education, 1999) that emphasised the need to develop and maintain support structures that will enhance the on-going professional growth of teachers. Yet, nothing seems to have changed over the years and, as Bezzina (2002, p. 65) claims, "the underlying feeling one gets is that the authorities may be assuming that it can just happen". The Maltese situation still does not provide the necessary structures for supporting ongoing CPD and the concept of creating and sustaining learning communities is still awaiting the shift envisaged by Bezzina (2002; 2006). Both the NCF (Ministry for Education and Employment, 2012) and the new sectoral agreement (Ministry for Education and Employment, 2017) refer to this particular lack of development.

At an international level, this study aims to make contributions in critical and contemporary areas – learning to teach mathematics through inquiry. As reviewed in the next chapter, while there is extensive literature on these research areas, there are gaps in knowledge on how teachers interpret and enact IBL, IBL lesson structure, beliefs and practice change, and 'just-in-time' teaching and learning embedded within CPD designs.

1.5 Structure of thesis chapters

This thesis is structured into eight chapters.

Chapter 1 provides background information about me as a student of mathematics, teacher, HoD, teacher educator and as a research-oriented practitioner. By disclosing who I am, my interests, beliefs and practices, I intend to help the reader understand my choice for the design of CPD and why IBL and teacher learning are fundamental areas of interest to me. In this chapter I also provide the background and scope for my research, and information about the research context.

Chapter 2 presents a review of literature related to my area of study. I start by presenting the theoretical framework upon which I base my study. Besides giving a critical account of theories and perspectives emerging from the literature, I position myself in the field by re-defining IBL through four features. I also distinguish between IBL in science and mathematics, discuss teachers' interpretation and their role in enacting inquiry, and delve into teachers' beliefs and practice change. I end this chapter by identifying the gaps in knowledge and by presenting the research questions.

Chapter 3 presents the LTMI programme offered to teachers. Drawing on other CPD programmes supporting IBL, I explain the modifications applied in designing the practice-oriented activities. A key discussion relates to how features of IBL as enacted in the classroom informed the design and implementation of CPD, arguing that incorporating features of IBL into CPD is essential in modelling IBL for teachers.

Chapter 4 focuses on the research methodology. It starts by discussing my personal ontological and epistemological beliefs in order to justify the choice of the research design. I also present ethical considerations, give details about the participants and explain the selection of my three case studies. I then discuss the research instruments, the piloting phase and describe the thematic approach used in analysing the data. I end this chapter by elaborating on my dual roles, designer and researcher, and also refer to the boundaries and restrictions of the study.

Chapter 5 presents three case studies. These case studies focus on teachers' beliefs and practices prior to their engagement in LTMI and the particular school contexts that the three chosen teachers operated within. This chapter looks into teachers' planning and the teaching of a 'typical' and an 'inquiry' lesson. By contrasting these two types of lessons, a discussion of teachers' perceived challenges and opportunities for inquiry teaching is presented within the theoretical framework informing this study.

Chapter 6 offers the analysis of data within two emergent themes: (i) teacher interpretation and enactment of IBL and, (ii) teacher development, learning and change. Findings show how sociocultural factors influence teachers' understanding and enactment of IBL. An important aspect that enabled teachers to make changes to their practices was the social dimension of learning offered by LTMI.

Chapter 7 serves to answer the research questions while providing links to the literature. This chapter is divided into four sections – teacher interpretation and enactment of IBL, lesson structure, changes in teachers' practices and beliefs, and 'just-in-time' teaching and learning (JITL) opportunities. Each section provides a detailed discussion of the main findings.

Chapter 8 concludes this thesis by presenting contributions to knowledge and exploring implications, emerging from the findings, towards supporting teachers in learning to teach mathematics through inquiry. In this chapter I also provide recommendations that centre on the premise that enabling teachers will enable change. I end this thesis by discussing personal learning experiences related to how this research journey contributed to my knowledge in becoming a teacher educator and researcher.

Chapter 2: Review of related literature

2.0 Introduction

I start this chapter by discussing sociocultural theories of learning with a focus on zones of enactment (see Spillane, 1999) as a theory that frames this research. The literature related to IBL and teacher classroom enactment is then presented. Conceptualising and, eventually, framing a working definition for IBL is seen as important in understanding how teachers may eventually put IBL into practice. This review focuses on distinguishing IBL in mathematics from science, and also presents the theoretical foundation of this study. Throughout the discussion, knowledge related to the research area and theoretical understandings of teacher enactment, which underpins this study, will be articulated. This will enable the identification of gaps in knowledge that the research questions, presented at the end of this chapter, will then attempt to address.

In reviewing related literature, I browsed the mathematics education and professional development literature for peer-reviewed journals, books, handbooks, and proceedings relevant to the topic. I first identified articles, chapters and reports in which IBL, CPD, teacher collaboration and CoP were an explicit part of the research study and/or design, and in which their influence on mathematics teachers' learning and professional practices were mentioned, discussed and problematised.

2.1 Sociocultural theories of learning

Sociocultural theories of learning originated in Vygotsky's (1978) work on child development, particularly following his work on the concept of zone of proximal development (ZPD). ZPD gravitates around the notion of what the child (learner) can achieve alone and with the help of a more knowledgeable other. With the shift of mathematics education research from constructivist approaches towards a more 'social turn' (Lerman, 2000), these emerging theories started to conceptualise learning as taking place with the individual's changing participation in social practice. Lerman

(1996, p. 4) had earlier defined sociocultural approaches to mathematics teaching and learning as involving "frameworks which build on the notion that the individual's cognition originates in social interactions ... and therefore the role of culture, motives, values, and social and discursive practices are central, not secondary". Applying this interpretation, researchers such as Jaworski (2006b) and, more recently, Potari (2013), Skott (2013) and Goos (2014) studied teacher learning looking into the role of community and the context of teacher learning within education settings. Jaworski (2006b), for example, argues that inquiry is a fundamental theoretical principle in teacher learning. She extends community conceptualisation by adding two elements: *critical alignment* and *inquiry*. With the former, teachers are not just seen to align with practice, as established by the community, but to look critically at that practice while aligning themselves with it. However, teachers may also take a critical stance and then align themselves with opposing forms of that practice. Within community engagement, teaching is seen as a social learning process of teachers engaging in inquiry as a mode of practice to develop their knowledge of practice.

2.1.1 Zones of enactment theory

Teachers encounter challenges, constraints and dilemmas in their attempts to change their practices (Anderson, 1996; Dorier & Garcia, 2013; Menezes, Oliveira, & Canavarro, 2015). The ways in which CPD and contextual factors enable or hinder teachers' knowledge, beliefs and practices in embracing IBL is central to this study. Asking teachers to reform their practices towards inquiry teaching is an arduous task (Anderson, 2002; Chapman, 2011) because it involves significant transformations in teachers' thinking and in operationalising practice (Chapman & Heater, 2010; Walshaw & Anthony, 2007). Moreover, teachers tend to enact reform practices differently (Martinie, Kim, & Abernathy, 2016) for different reasons: personal beliefs about subject-matter, teaching and learning, their knowledge and dispositions, and the school environments and conditions

that they work in. However, the influences of teachers' enactment extend beyond the individual school (Spillane, Reiser, & Reimer, 2002).

Spillane (1999) argues that the extent to which teachers reconstruct their practice depends on the characteristics of their *zones of enactment*. Zones of enactment are defined as the space in which teachers "make sense of, and operationalize for their own practice, the ideas advanced by reformers" (Spillane, 1999, p. 159). Enactment zones support Vygotsky's (1978) emphasis on the social dimension of learning and the extent to which a teacher changes in practice, provided that there is support to do this. According to Spillane (1999, p. 144), *zones of enactment* delineate zones in which "teachers notice, construe, construct and operationalize the instructional ideas advocated by reformers". Ideas about reform in the curriculum are usually communicated to teachers in different ways: by the policy, professional (the working context), private (textbook and curriculum publishers) and public (parent and community) sectors. Opportunities for teachers to learn about and to reconstruct their practice are, according to Spillane (1999), mobilised by these sectors. Moreover, the consistency in conveying ideas about reform practice among these sectors would influence teacher enactment. Spillane (1999, p. 144) considers

> the interplay of incentives and opportunities for teachers to learn about practice as mobilised by the school system (broadly construed to include agencies and institutions both in and beyond the formal education governance system) with teachers' capacity and will to construct their mathematics practice.

Successful enactment depends on teachers and their *will* and *capacity* to become agents to changing classroom practice (Martinie et al., 2016; Priestley, 2005). Spillane (1999) refers to *will* as the motivation of teachers to change their practice while *capacity* is the ability to practise in ways suggested by reformers. This notion of *will* to change may be elusive and should not be confused with the will to learn. The 'will to learn' refers to a desire to learn, and someone willing to learn is typically alert, fully mindful and with a desire to experience something new (Van Eekelen, Vermunt, &

Boshuizen, 2006). Having a will to learn implies having a psychological state of openness. Teachers with a will to learn may, however, engage in what Hargreaves (1999) calls a 'tinkering' process – that is, a process of testing and modifying practices. When teachers engage in a continuing process of 'tinkering', they may well become knowledgeable about teaching as recommended by reformers. For me, this process may lead to Spillane's notion of *capacity*. *Capacity*, in Spillane's (1999) terminology, refers to the ability of teachers to understand and to teach in ways recommended by reformers. This is conceptually similar to *pedagogical design capacity* (PDC), coined by Brown (2012), to refer to the ability of teachers "to perceive and mobilize existing resources in order to craft instructional contexts" (Brown, 2012, p. 24). PDC, like Spillane's (1999) capacity, captures teachers' active role in deploying and mobilising curriculum materials and personal resources – skills, dispositions, knowledge and beliefs – to change the core dimensions of their teaching and assist students in learning worthwhile mathematics. According to Brown (2012), PDC develops over time and through PDC teachers may accomplish new things. It is related not only to what teachers know about practice, but to their ability to craft instruction that is pedagogically robust.

2.1.2 Zones of enactment model

Drawing on the study of Spillane and Zeuli (1999), Spillane (1999) proposes a model to account for the way teachers respond to and enact mathematics reform. This model (see Figure 1) positions the *personal* resources that teachers have for learning about practice – their existing knowledge, beliefs and dispositions – as central to the learning process. Teachers notice opportunities for learning or stimuli for change depending on their *personal* resources. Such noticing is, of course, not automatic – some teachers may notice many opportunities for learning while others may only notice a few. But learning opportunities also arise from within their environment – *pupils*, *professional*, *policy*, *private* and *public* sectors.

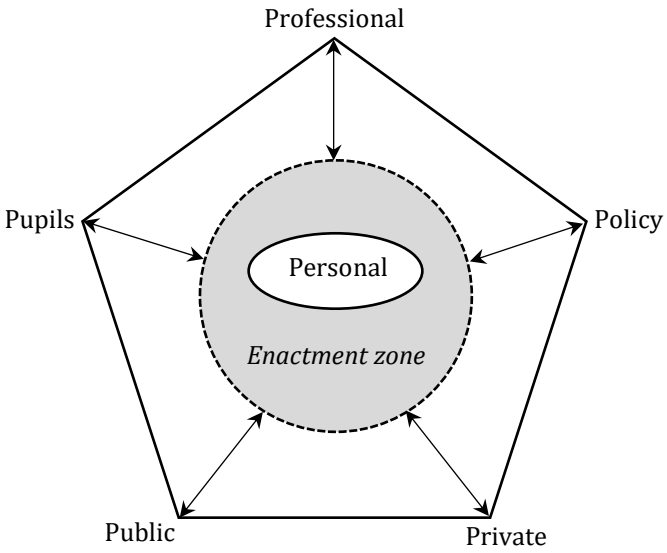


Figure 1: Zones of enactment model adapted from Spillane (1999)

Teachers encounter new ideas about practice from within the environment they work in and process these ideas through *personal* resources. In this model, the *personal* is at the centre as influences from *public*, *pupils*, *professional*, *private* and *policy* are mediated through teachers' personal resources. Hence, for example, the way in which the *policy* sector influences teachers' learning about practice depends, to a large extent, on what they notice in relation to their existing beliefs, knowledge and dispositions. The two-way arrows linking the *personal* to the *pupils*, *policy*, *professional*, *public* and *private* represent the influence that the environment has on the *personal* resources of teachers in enacting reform practices, but also on what teachers notice and their influence of practice as mediated through their beliefs, knowledge and dispositions.

The model depicts the array of external factors that impinge on local enactment. The five outer factors of the pentagon represent those organisations, associations and individuals, and the opportunities and incentives they mobilise from which teachers might learn about enacting reform practices. One external factor refers to the *policy* sector which includes government and school policies. This includes both formal policies and informal policy talk. Another represents the *professional* sector, including both formal associations and informal contacts among educators. Professional interactions may occur with colleagues at and within other

schools. Usually, for Maltese teachers, this happens as a result of subject departmental meetings, staff development sessions, informal discussions in staffrooms and participation in INSET courses. This also comprises conversations with other professionals, for example, education officers from MEDE when visiting to observe and discuss lessons with teachers. Another factor is *pupils*, and the influence that their responses to schooling has on teachers. Teachers' perceptions of students have an important influence on teachers' practice and their response to implementing inquiry-based practices (Hunter, 2010). The fourth factor denotes the *public*, which involves the concerns of parents and community towards IBL. The fifth factor denotes the *private* sector, particularly textbook and curriculum publishers.

These external factors bear on the enactment of reform from which teachers might learn about changing practice, and they might therefore support or hinder teacher opportunities and incentives to learn about and change their practice. Spillane (1999) argues that the influence of 'external' incentives and opportunities for learning in changing practice impinges, to a large extent, on teachers' *zones of enactment*.

2.1.3 Enactment theory as a social construct

Enactment zones serve a powerful mediating function between promoted and enacted practice, and the quality of teachers' enactment zones is enhanced by both personal and social resources. Spillane (1999, p. 170) states that:

> The extent to which teachers' enactment zones extend beyond their individual classrooms to include rich deliberations about the reforms and practising the reform ideas with fellow teachers and other experts, the more likely teachers are to change the core of their practice.

This is where CPD, in my view, has an important role to play in facilitating teachers' social enactment zones. Encouraging and assisting teachers to change the core of their practice depends, in a significant way, on an environment that supports ongoing collaborative inquiry about practice

and its improvement (see Lotter, Yow, & Peters, 2014; Meirink, Meijer, & Verloop, 2007). Enactment zones that are social do not only promote changes in the personal resources of teachers. Indeed, more importantly, they also promote changes in the way teachers enact their learning in practice. This means that CPD has the potential to create enactment zones for teachers that extend beyond individual classrooms to include collaborative deliberations between teachers that eventually support learning and the enactment of reform ideas (Spillane, 1999).

According to Spillane (1999), enactment zones that have strong social connections are an indispensable condition for profound teacher change. I would therefore argue that by focusing exclusively on the individual teacher, the significance that the wider social contexts and resources have on their enactment zones is likely to be overlooked. For Spillane (1999), teachers' zones of enactment have three key characteristics. First, enactment zones have a social dimension built on ongoing deliberations with colleagues and experts. Second, deliberations are supported through discussion and, finally, deliberations are enabled by expert teachers, university academics and material resources. Hence, social resources are critical in mediating between policy incentives and teachers' personal resources.

Teachers construct different understandings of reform policy, even when offered the same policy texts or CPD experiences (Spillane et al., 2002). This happens in view of their prior knowledge and beliefs about subject matter, students, teaching and learning. Differences in enactment zones become critical in understanding teachers' efforts to change and, hence, in understanding the ways they interpret and enact IBL. If teachers are offered the same CPD experiences, it would be important to examine how they construct notions of IBL practice from their engagement in the programme. Moreover, the influence of school contexts is central towards understanding teachers' sense-making (Blignaut, 2008; Spillane et al., 2002) of IBL and why they do or do not change their core practices. Hence,

both external and personal sectors of their zones of enactment are critical in understanding teachers' learning about practice (Blignaut, 2008).

From their studies drawing on social enactment zones, Boreham and Morgan (2004) and Golding (2017) found that the richness of teachers' interactions and their inclination to learn within a collaborative environment are conditions that support successful curriculum enactment and the development of capacity for change. Hodgen and Johnson (2003) extended and developed the notion of zones of enactment from their longitudinal study that explored the professional change of six teacher-researchers. Through their participation in a variety of settings – as teachers, learners of mathematics, curriculum designers, tutors and researchers – two of these teacher-researchers developed multiple *identities* (see Goos, 2013; Holland, Lachicotte, Skinner, & Cain, 1998; Wenger, 1998) and, as a result, created broad and heterogeneous zones of enactment. Like Hodgen and Johnson (2003, p. 231), I would therefore argue that enabling teacher change "is not so much a matter of telling teachers to reflect nor simply of providing access to a wide variety of mathematical experiences, but rather one of nurturing teachers' rich participation in a variety of settings". In facilitating change, a crucial influence is the existence of a genuinely collaborative environment and dialogue between professionals. Yet, for this to happen, there is a need for the creation of structures, both within and across schools, that provide spaces and tools that facilitate constructive and productive dialogue to emerge (Boreham & Morgan, 2004).

2.1.4 Teacher identity and change

In recognition of the important role played by a collaborative environment, I concur with the view that research on teacher learning and practice should not only focus on individual teachers, but also on their participation within their social education setting. Within this approach, "teachers' learning is understood as changing participation in sociocultural practices that develop their *identities* as teachers" (Goos, 2013, p. 522). Identity formation is an ongoing process of becoming and involves both internal and external

factors. Identity is, hence, the interplay between the individual and the social dimensions of learning. From the individual perspective, a tension between the present image of oneself as a teacher and that of the 'ideal' teacher drives the change process. From a social perspective, identity develops through learning by being part of a community of practice (Wenger, 1998). Wenger (1998) describes identity as "a way of talking about how learning changes who we are" (p. 5) because "such participation shapes not only what we do, but also who we are and how we interpret what we do" (p. 4). The notion of teacher identity is a situated phenomenon that exists in relation to contextual factors and considers learning as directly shaping teachers' self-perception of their own role, their agency and relationship with others.

A change in identity implies a different way of thinking about practice and a sense of empowerment that transforms one's overall teaching. Chapman and Heater (2010) identify three types of change that a teacher may go through in adopting IBL practices – *instrumental*, *conceptual* and *foundational* change. These three types represent three different levels in the process of teacher change. *Instrumental* change is seen to encompass a set of techniques or activities that a teacher has learned to adapt to his or her teaching, but which does not impact on changing practice. This type of change focuses on the 'what' and 'how', without an understanding of the 'why' or 'self'. This seems more of a surface change. *Conceptual* change resides at a deeper level. It includes not only an understanding about why an IBL technique works, but it is also transferred and applied beyond the situation in which it was initially enacted. At a more profound level, there is *foundational* change. This involves not only a change in teachers' practice, but also a change in their orientation of self and, hence, their identity.

Identity, in the sense suggested by Wenger (1998), is not a static entity. Rather, it is constantly changing and evolving in response to the challenges and tensions of the social and cultural circumstances, personal experiences and histories (Holland et al., 1998). For Peressini, Borko, Romagnano, Knuth, and Willis (2004) identities are created and recreated through a

process of accommodation. Identities become central in determining how teachers deal with concerns and manage classroom dilemmas. Teachers' professional identities are shaped through their participation and interactions in social practices (Goos, 2013; Hunter, 2010), and develop in response to the doubts and tensions generated by social, cultural and personal conflicts.

2.1.5 Situated and distributed cognition

A situative perspective considers teacher learning "as a process of increasing participation in the practice of teaching, and through this participation, a process of becoming knowledgeable in and about teaching" (Adler, 2000, p. 37). Taking a situative position thus implies that knowledge, beliefs and the practices they influence are inextricable from the situations in which they are enacted (Borko & Putman, 1996). Contexts are thus critical in understanding teacher learning and change (see for example Cobb, Wood, & Yackel, 1990; Lave, 1988; Lave & Wenger, 1991). It follows that the study of teacher learning and change needs to take into consideration the individual teacher-learner as well as the physical and social systems in which teachers participate (Putman & Borko, 2000).

I concur with situative theorists who posit that learning occurs when knowledge is socially constructed (Lave & Wenger, 1991; Wenger, 1998). The work of Lave (1988) shows that mathematical skills are formed and shaped by interactions taking place between individuals and the resources available within that context. Situated knowledge is thus inextricably bound to social, cultural and material contexts. Hence, teachers' acquisition of knowledge resides within an engagement in contextualised activity. Moreover, when diverse groups of teachers with different types of knowledge, experience and expertise come together, community members can draw upon and incorporate each other's expertise while negotiating new insights into teaching and learning. This is one way in which cognitive processes may be distributed across members of a social group.

Distributed cognition, which is perhaps best known for the work of Hutchins (1995), refers to knowledge that is not necessarily held across a

single community, but distributed through cognitive processes across communities through interaction with their material environment (e.g., technology) and across time. It views knowledge as residing in negotiations between networks of people and objects. In essence, cognition is viewed as leveraging between communities through social and material resources that act as mechanisms for sharing and developing knowledge. As a result, culture, context and history are all interlinked within the core concepts of cognition (Hutchins, 1995).

2.2 Inquiry-based learning

The emergence of inquiry-based education is generally attributed to John Dewey (1859-1952). Dewey (1938), for whom inquiry was the basis of learning, attributed two main features to his inquiry model: everyday life experiences and inductive learning. For Dewey, learning subject content was subordinate to learning through life experiences. Apart from seeing inquiry as the basis of discovery and learning, he conceptualised learning as discovery guided by mentoring. Inductive learning – the second feature of Dewey's inquiry education model – involves a learner's journey from the known to the unknown. Through inquiry pedagogies, learners encounter and engage with challenging and intriguing familiar situations to discover the unknown. Learners engage in exploring, questioning, reflecting, explaining and communicating understandings (Artigue & Blomhøj, 2013; Chapman & Heater, 2010).

In the literature about IBL, discovery and connectionist approaches often appear conflated. In discovery learning, it is essential to provide a stimulating environment. To facilitate student exploration, a task is gauged at a level at which students can make progress and extend their learning. Discovery learning is often associated with independent activity (Aulls & Shore, 2008; Piaget, 1970), with the teacher generally standing back to allow exploration (Kirschner, Sweller, & Clark, 2006). With connectionist approaches, teachers have a more proactive role. They work alongside students and intervene to stimulate thinking so that meanings and

connections may be explored and misconceptions are explicitly addressed. A connectionist approach views mathematical content as a connected set of topics and concepts, and values students' methods and solution strategies (Askew, Brown, Rhodes, Johnson, & William, 1997). Through discussion with peers and the teacher, students may notice and explore connections between the work they are doing and the mathematical ideas that they have learned in the past. To be effective, teachers are expected to facilitate the activities and the discussion by offering challenges so that mathematical concepts and connections become explicit. A connectionist approach is collaborative and involves providing extensive scaffolding and guidance (e.g., through questioning) to facilitate student learning (Hmelo-Silver, Duncan, & Chinn, 2007). It is not minimally guided or unguided instruction and, hence, it is not a discovery approach. The discovery teacher creates an environment that structures and facilitates students' discoveries. However, in contrast to a connectionist approach, students have little or no awareness of what this structure is and have to find their own way of discovering new knowledge for themselves. In a connectionist approach, the structure for inquiry is evident to the learner – it is both explained and modelled by the teacher thus enabling students to enact it. This difference explains why the role of the teacher is so crucial when adopting these approaches. Connectionist approaches provide forms of scaffolding, and research shows that they are more effective in bringing about learning when compared to approaches offering minimal guidance (Askew et al., 1997; Swan, 2006).

2.2.1 Effectiveness of IBL towards learning

A common trend has emerged in recent years for the promotion of paradigms in education that support active learning approaches, particularly in mathematics and science. Learning science and mathematics through inquiry has been recognised as crucial in developing understanding, reasoning and attitudes towards life (Harlen, 2013). For society, in general, inquiry is seen to support people in making informed decisions about leading a healthy life and making career choices (Echazarra, Salinas, Méndez, Denis, & Rech, 2016; OECD, 2009). In response to reform

documents – locally (see Ministry for Education and Employment, 2012) and elsewhere (see NCTM, 2000; Rocard et al., 2007) – CPD projects and programmes such as COMPASS (2009), PRIMAS (2012), Fibonacci (2012), Mascil (2013) and, more recently, MaSDiV (2017) commissioned by the European Union, were implemented to encourage teachers to shift away from the dominant transmission-based teaching approaches (see Maaß & Artigue, 2013; OECD, 2009) towards more inquiry-based pedagogies.

Research on the benefits of IBL gives an inconsistent picture of its effects (Maaß & Artigue, 2013), and doubts do exist regarding its efficacy (Aulls & Shore, 2008). Obtaining reliable evidence from empirical studies seems an arduous task given the vast spectrum of IBL and the contrasting conceptualisations in the literature (see, for example, the contention of Hmelo-Silver et al. (2007) towards Kirschner et al. (2006) for defining IBL as a minimally guided form of instruction). In addition, research is highly contextual (Alfieri, Brooks, Aldrich, & Tenenbaum, 2011; Anderson, 2002), and the ways by which researchers measure its effectiveness are varied and broadly defined, namely, through increased motivation, improved performance or enhanced thinking competences (Artigue & Blomhøj, 2013). Different authors report in fact a number of benefits. These include: increased attainment levels in mathematics and science with a strong impact on students with low self-confidence (Rocard et al., 2007); improved understanding of scientific knowledge (Walker, 2007); increased higher-order thinking (Anderson, 2002; Marshall & Horton, 2011); and wider range of collaborative, problem solving and thinking skills (Borasi, 1992; Rocard et al., 2007).

While IBL is seen as a fundamental approach to improving science and mathematics education (Rocard et al., 2007), its effectiveness is arguably still to be established (Bruder & Prescott, 2013; Lazonder & Harmsen, 2016). For example, Kirschner et al. (2006) argue that inquiry-based methods are ineffective and inefficient compared to more guided approaches because they put high demand on the working memory (the cognitive structure in which conscious processing occurs), leaving limited

capacity for the accumulation of new information in long-term memory. The significance of providing students with guidance during inquiry emerged in a study by Alfieri et al. (2011). They reported that, across domains and settings, minimally guided or unguided inquiry-based methods are less effective than expository methods. On the other hand, students learned the same content better when they were offered guided inquiry over expository teaching methods. In another study with 102 middle school mathematics and science teachers, Marshall and Horton (2011) found that when guided exploration is offered before giving explanations, students think more deeply about the content. When the focus, however, is on developing lower cognitive skills, inquiry-based learning approaches may not be helpful.

There appears to be convincing evidence showing that guided inquiry methods are more effective when compared to expository methods of instruction (Lazonder & Harmsen, 2016). Meta-analyses reports support this positive outlook towards IBL (Alfieri et al., 2011; Hattie, 2009). Hattie (2009, pp. 209-210), for example, concludes that "inquiry-based instruction was shown to produce transferable critical thinking skills as well as significant domain benefits, improved achievement and improved attitude towards the subject". On the other hand, when compared to other forms of teacher instruction, IBL was found to produce only moderate effects to improving students' performance. A possible reason for this, I argue, is that the traditional nature of examinations is likely to better 'measure' the mathematics learning of students prepared using traditional teaching methods than using inquiry-oriented approaches.

2.2.2 Distinguishing between inquiry in mathematics and in science

Mathematics and science provide powerful insights into the world and enable us to construct knowledge about the world (Harlen, 2013). In both fields, a prevailing mode of knowledge building is through inquiry. However, one issue that seems problematic with the term inquiry is that, while it defines an *activity*, it may also be seen to describe a *process*. In science education, for example, learning through inquiry is seen as the

process of building understanding by collecting evidence and testing ideas about the natural world, its objects and phenomena (Harlen, 2013). In science, real-world phenomena and questions are considered, and models emerge as a result of the process of observation, experimentation, exploration and interpretation (NRC, 1996). Inquiry in science is the act of building and testing knowledge, drawing on what is already known with the aim of going beyond that knowledge. Inquiry-based science education (IBSE) encourages students to develop their scientific skills, independently or in collaboration, in such a way that they can appreciate the processes that many scientists use in their everyday lives. In IBSE structured or guided inquiry is generally used to teach scientific concepts and content. On the other hand, more open inquiry is used to teach cognitive development and scientific reasoning (NRC, 1996).

There are both similarities and differences between mathematics and science. Like science, learning mathematics is about "knowing a conceptual domain" (Greeno, 2006, p. 543). It involves more than just following a recipe; it is about "really cooking" (Schoenfeld, 1998, p. 299), that is, "knowing what resources are available in the domain, knowing where to find them, knowing how to use them, and anticipating the results of using them in different circumstances" (Greeno, 2006, p. 543). Inquiry concerns students developing an understanding through processes that originate as a result of curiosity. In both mathematics and science, while IBL is used to teach the process of investigation, it is also used to introduce students to content.

In distinguishing between inquiry-based approaches to mathematics and science, the Rocard report (2007) identifies IBL for science and problem-based learning for mathematics. Mathematics deals with patterns, relationships and abstract constructions such as those related to numbers, shape and algebraic structures. Doing mathematics involves engaging in processes that develop mathematical reasoning in order to help understand and explain abstract concepts and structures (Devlin, 1994). Establishing the connections between these underlying concepts and structures,

requires inquiry through problems. Problem solving is thus seen as a vehicle for engaging in the process of inquiry and, hence, an essential component towards developing mathematical reasoning (Cai, 2010; Hiebert et al., 1996; Schoenfeld, 1992). Inquiry in mathematics starts with a problem or from a question, and involves mathematicians in problem posing and mathematical modelling. Unlike science, mathematics deals more with the abstract than with the concrete and is not restricted to real world situations (Harlen, 2013). Hence, mathematical inquiry places less emphasis on observation and experimenting (Harlen, 2013; Rocard et al., 2007), and deals more with developing deep links between different areas of the subject matter, conjecturing and generalising. Inquiry in mathematics also involves articulating or elaborating questions; exploring, explaining, arguing and proving; defining and structuring; connecting, representing and communicating (Fibonacci, 2012). Unlike scientific inquiry, mathematical inquiry is not necessarily initiated with a real-life experience or phenomenon – indeed, a stimulus to inquiry can be a mathematical statement, prompt, question or equation (see, for example, the work of Blair, 2014; Foster, 2013; Swain & Swan, 2007; Swan, 2006). The stimulus can emerge explicitly from within mathematics and may include the use of a wide variety of strategies, such as, images or artefacts used to provoke questions about a situation, errors and mistakes, a statement that prompts ambiguity and conflict (Borasi, 1992) that compel students to ask. In mathematics, IBL consequently takes on a problem-based approach involving problem-solving.

2.2.3 Towards conceptualising IBL

Inquiry-based practice is recognised as one way of engaging students in learning mathematics (Towers, 2010). More specifically, inquiry practice is learner-centred, investigative, problem-oriented, collaborative and question-driven (Goodchild, Fuglestad, & Jaworski, 2013; Jaworski, 2006b; Swan, 2006). IBL is seen to engage learners in thinking, starting off as a mediating tool through the use of tasks to then shift over time to become more "as a way of being" (Jaworski, 2006a, p. 204). More recent outlooks, in

particular, 'dialogical inquiry' is considered by Wells (2004, p. 121) as "a willingness to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to make answers to them". Within an IBL classroom culture, learners are invited to work like mathematicians do (Maaß & Artigue, 2013) – asking questions and seeking answers; recognising problems and seeking solutions; identifying patterns, conjecturing and seeking generalisations. The aim of mathematical inquiry is to generate students' thinking so that they, in turn, use their informed understandings responsibly to construct mathematical knowledge.

Inquiry is a multifaceted activity (NRC, 1996) based on student-centred, collaborative, constructivist theories and the development of higher-order reasoning. Consequently, the spectrum of IBL conceptions is rather extensive and can be placed along a continuum ranging from teacher-directed inquiry at one end to student-centred inquiry at the other end. A number of authors (e.g., Tafoya, Sunal, & Knecht, 1980; Walker, 2007), have attempted in fact to differentiate IBL according to the type, structure and complexity of the task, and the degree of engagement and responsibility shouldered by students. On the same lines of Tafoya et al. (1980), Staver and Bay (1987) classify IBL lessons into *structured*, *guided* and *open* inquiry. Within the *structured* type, students are provided with the problem, the method and the materials to solve it. For a *guided* inquiry approach, students are given the problem and the necessary materials, but they have to find the appropriate problem-solving strategies and methods. *Open* inquiry involves students in deciding about the problem, the methods and the materials that they would like to use. Aulls and Shore (2008) propose that student and teacher roles and responsibilities also exist along a continuum. In teacher-directed (*structured*) inquiry, the teacher assumes responsibility for students' learning; in teacher-guided (*guided*) inquiry the teacher shares responsibility with the students; and in student-centred (*open*) inquiry responsibility is on the students with the teacher acting as a consultant and facilitator (see Section 2.4.2). Teachers structure their inquiry lessons depending on a number of factors, amongst which are their own competence in inquiry teaching (Leikin & Rota, 2006), student

readiness for inquiry and the intended pedagogical goals (Chapman, 2011; Russo & Hopkins, 2018). Research suggests that *open* inquiry offers better opportunities for students' cognitive development because students are more likely to work independently of the teacher and take ownership of their inquiries (Boaler, 1998; Rocard et al., 2007). In addition, *structured* and *guided* inquiry have been found to produce better results when the teacher needs to teach content knowledge, and hence learning is more oriented towards conceptual development (Alfieri et al., 2011; Lazonder & Harmsen, 2016).

2.3 Re-defining IBL

Across the literature, inquiry comes across as an equivocal concept – used in different ways and contexts to interpret and describe similar teaching and learning approaches (Engeln et al., 2014). A common misunderstanding is to confuse and conflate IBL with, but also to restrict it to, hands-on activities. The interchangeable and loose use of the term (see Hmelo-Silver et al., 2007; Kirschner et al., 2006) makes it difficult to discern and elicit clear characteristics of inquiry practice.

In mathematics, IBL is used to engage students in a process of problem-solving with others (Luckin, Baines, Cukurova, Holmes, & Mann, 2017), and thus has similarities with collaborative problem-solving (CPS). CPS brings together two key elements: collaboration and problem-solving. It is worth noting here that critical thinking, problem-solving and collaboration are indeed skills necessary for future careers (OECD, 2015) that employers are nowadays seeking (see Echazarra et al., 2016; Hart, 2013).

While discovery approaches advocate unguided instruction (Alfieri et al., 2011), an IBL approach adheres to more thoughtful support and scaffolding (see Hmelo-Silver et al., 2007) by the teacher at different stages of the inquiry process. This understanding is informed by what I consider as four essential research-based features that define IBL and inform the design of LTMI (see Section 3.1.2), namely:

- Using inquiry-oriented mathematical tasks (Smith & Stein, 1998);
- Fostering collaborative learning (Swan, 2006);
- Using purposeful questioning (Mason, 2000);
- Shifting agency and responsibility onto students (Schoenfeld, 2013).

2.3.1 Using inquiry-oriented mathematical tasks

The tasks teachers select and the way students negotiate mathematical meaning by working on tasks largely determine students' classroom experiences and their learning of mathematics (Hiebert et al., 1997; Shimizu, Kaur, Huang, & Clarke, 2010; Sullivan & Clarke, 1991). According to Doyle (1983, p. 161), tasks are "defined by the answers students are required to produce and the routes that can be used to obtain these answers". Additionally, it is important for teachers to carefully consider cognitive demands, gauging tasks to an appropriate and productive level (see, for example, Willis, 2010). Gradual increase in the level of task difficulty, challenge and structure is likely to facilitate student engagement in IBL. Nonetheless, questions concerning 'which tasks?' and 'what actions?' still need consideration, understanding, and addressing.

In addressing the first question, Swan (2005) and Willis (2010) present key insights on task design. Swan (2005), for example, devised a design framework for providing students with mathematical tasks that foster both conceptual development and problem solving. His 'rich' collaborative tasks involve students in: evaluating mathematical statements; classifying mathematical objects; interpreting multiple representations; creating and solving problems; and analysing reasoning and solutions. Willis (2010, p. 17) recommends achievable-challenge tasks that "require students to exert mental effort, performing a task that is just difficult enough to hold their interest but not so difficult that they give up in frustration". Along the same lines, Blair (2014, p. 33) offers prompts, set just above students' current knowledge, as stimuli to "encourage students to rise above themselves".

To understand the role of inquiry in the mathematics class, it is important to distinguish between a mathematical task and the mathematical activity generated by the task. Drawing on the work of Christiansen and Walther (1986), a 'task' is represented by what the teacher offers students, whereas what students do in response to the task would denote the 'activity'. In fostering inquiry, mathematical tasks need not necessarily be open-ended. While open tasks invite students to assume more ownership of the problem being investigated, closed tasks tend to be limited in this aspect but not necessarily in stimulating cognitive processes. Foster (2013) mitigated this by drawing on the notion of musical études to develop mathematical tasks that embed extensive practice of essential techniques within a rich and exploratory context. Through mathematical études, fluency and inquiry are more closely connected.

In Table 3, I present a taxonomy of IBL tasks drawn on the works of Tafoya et al. (1980) in differentiating between inquiry tasks, the theoretical model of Yeo and Yeap (2009) in delineating phases in an inquiry lesson and the responsibility of the teacher and students along each phase of each particular inquiry lesson (see Fradd, Lee, Sutman, & Saxton, 2001).

	INQUIRY	**STRUCTURED**	**SEMI-STRUCTURED**	**UNSTRUCTURED**
		Closed Tasks	Guided Tasks	Open Tasks
LESSON PHASES	**TASK PRESENTATION**	*Teacher presentation*	*Teacher initiates conversation*	*Teacher or students initiate conversation*
	INDIVIDUAL WORK *Problem-Posing*	*Teacher supports students*	*Teacher challenges students*	*Students pose their own problems*
	SMALL GROUP WORK *Problem-Solving*	*Students work collaboratively with support from the teacher through interactions initiated by both students and the teacher*		
	PLENARY	*Teacher leads the presentations of students*	*Teacher challenges the presentations of students*	*Students lead their presentations with support from teacher*

Table 3: A taxonomy of the role of tasks for mathematical inquiry

Semi-structured inquiry in which the teacher takes on a supportive role, appears to be the most effective in supporting student learning (Kirschner et al., 2006). In particular, *guided tasks* provide extended problem solving

opportunities (Bruder & Prescott, 2013). I agree with Hiebert et al. (1997, p. 163) when they advise teachers to "select tasks with goals in mind". Effective inquiry teaching, indeed, goes beyond the nature of tasks, to include teachers' implementation and how students "come to think about, develop, use and make sense of mathematics" (Breen & O'Shea, 2010, p. 44).

2.3.2 Fostering collaborative learning

Inquiry may be achieved in classrooms that welcome students' ideas and value a collaborative questioning attitude to mathematical sense making. Doing mathematics is increasingly seen as a social and collaborative endeavour (Schoenfeld, 1992) and collaborative learning is currently more than ever before at the heart of mathematics learning (Dekker, Elshout-Mohr, & Wood, 2006; Swan, 2006; Tatsis & Dekker, 2010). Collaborative learning is "a process that combines the knowledge of group members into something that could not be created by individuals working alone" (Summers, 2006, p. 277). Within such a classroom environment, students are organised in small groups to work on challenging tasks as they receive support and encouragement to construct shared understandings through a process of discussion and negotiation. Through social processes cultivated by the teacher, students develop taken-as-shared meanings (Cobb, Yackel, & Wood, 1992) as they engage in developing social norms through immersion and commitment (Wood, 1999).

Despite the body of research on the problematics of teaching students how to work collaboratively (Mercer & Littleton, 2007; Summers, 2006), evidence shows that collaborative learning, in comparison to competitive and individualistic settings, is an effective approach to learning (Slavin, 1989; Swan, 2006). In addition, it has strong effects reported on achievement, socialisation, motivation and self-development. Research by Lee and Johnston-Wilder (2013) also suggests that students value working on mathematical tasks as a group and enjoy working with their peers. Moreover, in this study, students accept their active involvement and value it as an approach that helps them become successful learners. These students also indicated that they became more mathematically resilient

when they were actively involved in lessons and suggested to work with others, collaborate, think, discuss and argue in developing their mathematical knowledge. However, one must remain cautious of the fact that students, accustomed to work in competitive environments, may feel threatened and hence offer resistance to working collaboratively. According to Dekker et al. (2006), as students engage in solving a mathematical task, they also need to regulate their own collaborative learning. For IBL, this implies that learners are actively involved in taking decisions, sharing and selecting strategies in solving a problem.

2.3.3 Using purposeful questioning

A powerful strategy for student-centred learning is the use of questions that stimulate student thinking and facilitate the process towards knowledge construction (Hiebert & Wearne, 1993; Menezes, Guerreiro, Martinho, & Ferreira, 2013). However, in classrooms where communication and discussion are central for the learning of mathematics, teachers also encourage students to ask questions. Foster (2011, p. 28) claims that "unless students learn to ask questions, they cannot operate as mathematicians" and when teachers support students in learning to ask questions, students would be in a better position to inquire. Within a collaborative inquiry culture, "questioning is an activity shared by all the classroom actors" (Menezes et al., 2013, p. 52). Yet, while essential, undertaking and cultivating a questioning attitude is challenging for teachers (Boaler & Brodie, 2004).

Questions serve many purposes (Menezes et al., 2013) and several researchers have classified the questions that teachers ask by creating a number of dichotomies, such as, 'convergent' and 'divergent' (Smith, 1969), 'low order' and 'high order' (Hiebert & Wearne, 1993) or 'closed' and 'open' (Blosser, 2000) questions. Watson (2002) argues that such distinctions are unhelpful particularly when, in analysing their use in inquiry classrooms, a classification of 'bad' versus 'good' is made. For Sahin and Kulm (2008) questions serve three purposes: *factual*, *guiding* and *probing*. *Factual* questions can be classified as 'low order', 'convergent', 'closed' recall

questions asking to verify specific facts, definitions, answers and the next step in a procedure. *Guiding* questions usually focus students' attention and make potential mathematical ideas visible, hence helping to guide them to discuss problems. Teachers usually ask guiding or focusing questions when students appear stuck or confused. Asking for the next step acts as a guide, lead or focus to pursue the completion of a procedure or understanding a concept. *Probing* questions require higher order thinking because they encourage students to elaborate on, explain and justify their reasoning. More important is the purpose of using probing questions to extend inquiry and promote deeper thinking. Considering teachers' prevalent use of factual questions (Myhill & Dunkin, 2002; Vacc, 1993), this classification may indeed illuminate changes over time about the when, the type and the frequency of teachers' use of questions. Like Sahin and Kulm (2008), Capraro et al. (2010) found that when teachers demonstrate increased knowledge of questioning and pay attention to the questions they asked, they are more likely to influence the development of students' awareness of mathematical thinking.

2.3.4 Shifting agency and responsibility onto students

Adopting IBL has significant implications for students. When students are central in the learning process, they have a key role to play – they take control of their learning, are involved and become more independent. In active learning environments, students "see themselves as doers of mathematics" (Schoenfeld, 2013, p. 617) and develop a sense of agency. Agency refers to students' identity and disposition (Schoenfeld, 2013), a perception of themselves that they can do mathematics and make progress on challenging mathematical problems. According to Mason and Johnston-Wilder (2006), active learning places students in an *asserting* mode because they use their initiative and take risks. There appears to be general agreement in literature that active learning empowers students to assume more responsibility (Carpenter & Pease, 2012; Drew & Mackie, 2011; Engeln et al., 2014). In an IBL classroom, students are given substantial responsibilities that are uncommon in more traditional learning settings

(Artigue & Blomhøj, 2013; Boaler, 2016); they assume responsibility for what they do and how they do mathematics (Ollerton & Watson, 2001). To experience transformation to IBL, students need to move from a passively *accepting* to an actively *asserting* mode (Mason & Johnston-Wilder, 2006), learn to communicate mathematical meanings and make assertions. However, teachers need to support students to take responsibilities for thinking mathematically and accept invitations to inquire (Skovsmose, 2001).

Teachers need to stimulate students to take responsibility for learning (Carpenter & Pease, 2012; Dekker et al., 2006), but this cannot be separated from issues related to supporting students to work collaboratively, offering choice, autonomy and ownership (Carpenter & Pease, 2012; Lee & Johnston-Wilder, 2013). Mason and Johnston-Wilder (2006, p. 39) contend that handing responsibility to students is a process that "begins with the teacher asking learners to 'do things', but it can become more sophisticated as learners are encouraged or inveigled into becoming less dependent".

2.4 IBL in the mathematics class

2.4.1 Inquiry teaching

Inquiry teaching is complex and adopting this practice is challenging for teachers (Towers, 2010). Important to the development of CPD materials promoting IBL is having a shared understanding of inquiry teaching. Inquiry teaching refers to the practice of teachers in providing students with opportunities to actively engage in constructing mathematical knowledge (Swan, 2006). This may include teaching strategies using, for example, a variety of prompts, questions and students' suggestions for investigation, providing mathematical challenges through tasks and problems, encouraging collaborative engagement, showing genuine interest in students' alternative methods and solution strategies, and encouraging mathematical reasoning, exploration and communication. In using such strategies, teachers do not leave students on their own to discover, but guide and support them in learning to work independently

(Hmelo-Silver et al., 2007; Maaß & Artigue, 2013). Inquiry teaching thus rests upon a set of teacher competencies (Towers, 2010), personal understandings and contextual factors (Ireland, Watters, Brownlee, & Lupton, 2012).

2.4.2 The role of the teacher in cultivating an environment for IBL

Orchestrating and facilitating the learning processes is a skill that teachers need to learn and develop for IBL to function well. A common term recurrent in the literature is the role of the *teacher as a facilitator* (Maaß & Artigue, 2013; Walker & Shore, 2015). But what does the *teacher as a facilitator* role entail? How can a teacher facilitate student learning through inquiry, to work collaboratively, to explore and to communicate their work? It is crucial at this point, not only to define what the term *facilitator* implies but, more importantly, to clarify how undertaking a facilitator's role unfolds in the inquiry classroom. Swan (2005), for example, identifies the teacher's role as a challenger and an intervener; one who asks questions to encourage and stimulate student thinking and reasoning. Such teacher assistance does not take away any agency from the students in determining the outcome of their learning. Instead, the teacher acts as an initiator, an instigator and an enabler of thinking processes. Teacher assistance brings with it the interplay between the use of questioning strategies and teacher telling (Hmelo-Silver, 2006). In a classroom as a community of mathematical inquiry (see Goos, 2004), students are not left alone, but are guided and supported by the teacher to learn to work independently (Maaß & Artigue, 2013). Establishing a consensus about the nature of inquiry teaching is a demanding task for teachers, particularly since the dilemma between exploration and telling remains unresolved (Towers, 2010). Striking a balance between the challenge offered and the assistance provided (Engeln et al., 2014) is something that teachers need to learn.

From a socio-constructivist perspective, teachers provide scaffolding when learning outcomes are comprehensible to students but not easily achievable without support (Goos, 2004; Vygotsky, 1978). Similarly, Mercer (1995, p. 48) describes this role as "the sensitive, supportive intervention

of a teacher in the progress of a learner, who is actively involved in some specific task, but who is not quite able to manage the task alone". For this to happen, the teacher must hone skills to scaffold student learning through modelling and coaching (Hmelo-Silver et al., 2007). Foster (2014, p. 149) argues for minimal interventions with the teacher "being fully present, interested, engaged, listening, accepting – while actively avoiding committing ideas". The teacher does not provide answers, hints towards an answer or judgements to student responses, but encourages, promotes and provokes cognitive challenges (Hmelo-Silver, 2006). The teacher thus offers the space for students to think, share, discuss, make decisions and come up with sound arguments and plausible solutions. Yet, the teacher also draws student attention to significant ideas by providing "content knowledge on a just-in-time basis" (Hmelo-Silver et al., 2007, p. 100). Clearly, this may bring about some level of uncertainty for both the teacher and the students. The challenge for teachers is how to adopt this approach and how to integrate it as a basis of their teaching (Chapman, 2011).

2.4.3 Constraints and dilemmas for teachers in using IBL

Introducing inquiry approaches to teaching into traditional classrooms requires teachers to change their ways of planning, teaching and assessing. Inquiry teaching is not teaching done through a rehearsed lesson plan; it does not involve teaching a single method through practice and does not necessarily follow a single structure; instead it is complex and unpredictable. While teachers may face constraints external to them – such as the curriculum and assessment practices, high-stakes examinations and lack of resources – much of the difficulties they encounter also seem associated with their beliefs and values about mathematics, teaching and learning (Anderson, 2002; Maaß, Swan, & Aldorf, 2017). Maaß et al. (2017) infer that lack of collaboration with colleagues and support received from school administrations are limitations that may influence teachers' use and views of IBL. School contexts matter for teacher learning. For teachers, the extent of resources available, support offered, and the reactions to

innovation of students and parents are likely to influence their work (Chin & Lin, 2013; Joubert & Sutherland, 2008).

In a cross-site analysis of school case-studies, Anderson (1996) identified that teacher constraints and dilemmas can be grouped under three dimensions: the technical, political and cultural dimension. The technical matters include the limited ability to teach constructively, abandoning preferred teaching styles, challenges to using group work and the difficulties of adapting to new teacher and student roles. A lack of an explicit understanding of IBL and an apparent gap in knowledge about IBL practice may leave teachers feeling vulnerable to criticism from students, parents, the school administration and education officials (Chin & Lin, 2013; Niemi, 2002). However, Niemi (2002) also reports that teachers are concerned about behaviour management issues believing that behaviour will deteriorate when students are in a position to take control of their learning. The political aspects include limited ongoing training, resistance from parents, lack of resources and syllabus constraints. Speaking about the prevalent 53-minute lesson period in Scottish secondary schools, Priestley (2010) identifies lesson time as a constraint for IBL, while others (e.g., Chin & Lin, 2013; Maaß et al., 2017; Niemi, 2002) cite the additional time, work and resources needed to plan and adopt this pedagogy. The cultural dimension regards the beliefs and values about, for example, the role of textbook teaching and preparing students for examinations, and views of assessment, teaching and learning (Chin & Lin, 2013; Maaß et al., 2017; Niemi, 2002; Priestley, 2010). Anderson (2002) claims that the cultural dimension potentially presents the biggest hurdle. Even if the technical aspect is addressed, changing beliefs and values, and preparing teachers for inquiry teaching requires an intervention that addresses the political and cultural context of the schools in which teachers work.

This implies that for IBL to make its way into classrooms, one needs to give careful consideration to the political and cultural contexts in which teachers work. Even if teachers believe in the goals and effectiveness of inquiry teaching, education systems may not encourage and adequately support

such a choice. Hargreaves (1994) argues that teachers' decisions are based on their 'sense of practicality' which involves a sense of mindfulness in operating 'successfully' within their local context. The priority for education systems may rest on prescribed content knowledge that students need to be 'fed' rather than developing students' thinking skills. For a teacher, the most effective and efficient way of addressing this could be that of adopting a transmission approach to teaching.

2.5 Teacher professional development

The literature reviewed reveals that there are many and varying definitions of CPD. For the purpose of this study, CPD is taken to encapsulate the personal and the professional learning of the teacher (Earley & Bubb, 2004), in other words, "those processes and activities designed to enhance the professional knowledge, skills, and attitudes of educators so that they might in turn, improve the learning of students" (Guskey, 2000, p. 16).

While there are various CPD models (see Kennedy, 2014), there are two distinctive approaches to providing CPD – the traditional 'training model' and the 'sociocultural model' (Mansour, Albalawi, & Macleod, 2014). While the 'training model' views learning as the acquisition of skills that teachers may take from a course and apply into their classrooms, the 'sociocultural model' construes knowledge, teaching and learning as being socially created and culturally enacted. From the vast literature of studies on mathematics teachers' CPD, it is clear that there has been a shift towards programmes that model inquiry-based pedagogies (Back, Hirst, De Geest, Joubert, & Sutherland, 2009; Loucks-Horsley et al., 2010) and *authentic activities*, that is, CPD activities that are similar to what teachers could be doing in their classrooms (see Brown, Collins, & Duguid, 1989).

Research studies on mathematics teachers' CPD show a well-defined shift towards programmes that model effective IBL pedagogies (e.g., Luft, 2001). According to Jaworski (2007, p. 1693), CPD activities need to bring in "a critically questioning attitude towards practice and knowledge in practice that allows critical reflection on the practice of teaching". From this

perspective to which I adhere, CPD should not only engage teachers to simply align themselves with IBL, but instead to look critically at IBL while aligning themselves with it.

2.5.1 What we know about effective CPD

There is widespread consensus about what constitutes effective professional development (Guskey, 2000; Loucks-Horsley et al., 2010; Putman & Borko, 2000). According to Loucks-Horsley et al. (2010, p. 5):

> It is directly aligned with student learning needs; is intensive, ongoing, and connected to practice; focuses on the teaching and learning of specific academic content; is connected to other school initiatives; provides time and opportunities for teachers to collaborate and build strong working relationships; and is continually monitored and evaluated.

Professional development is an on-going and long-term process (Loucks-Horsley et al., 2010). To be effective, CPD needs to provide collaborative opportunities for teachers to design, implement, share, discuss and reflect on their classroom practices (Guskey, 2002; Putman & Borko, 2000). Despite what we know about what renders CPD effective, the challenges and barriers towards successful implementation need still to be addressed (Bubb & Earley, 2013; Guskey, 2002; Putman & Borko, 2000). Generally, CPD providers (institutions and schools) still appear to conceive teacher professional development as done *on* teachers *just-in-case* they want to use it, rather than *with* teachers 'just-in-time' when they need it. In other words, rather than isolated off-site training workshops disconnected from practice, CPD needs to offer ongoing collaborative on-site experiences of practice-oriented learning. Research shows that, for this to happen, schools need to minimise teacher isolation and instead instil a collaborative learning culture (Fullan, 2007; Loucks-Horsley et al., 2010). Besides depending on a collaborative learning culture, on-site CPD strategies like lesson study (see Stigler & Hiebert, 1999) depend on the supportive provision of time and space for teachers to meet regularly. This is more so in the case of externally-driven CPD that require schools to not only embrace the intervention but to support its dissemination within and

across schools (Krainer, 2001; Owston, Wideman, Murphy, & Lupshenyuk, 2008).

CPD is effective when it is practice-based and involves the negotiation of meanings, collaboration, and reflective practice (Loucks-Horsley et al., 2010; Mewborn, 2003; Stoll, Harris, & Handscomb, 2012). Researchers interested in teacher professional communities draw on the community of practice perspective (see Jaworski, 2006b; Lave & Wenger, 1991; Wenger, 1998) – also referred to as professional learning communities (DuFour, 2004) – to explain the social processes shaping teacher learning. Communities of practice are dynamic learning communities in which participants engage "in actions whose meaning they negotiate with one another" (Wenger, 1998, p. 73).

2.5.2 Introducing IBL through a 'task-first' approach to CPD

One important principle of CPD is for teachers to develop a meaningful interpretation of mathematical inquiry and to implement IBL effectively in their classroom (Boston & Smith, 2011; Swan, 2007). This means that CPD, designed to be task-oriented, introduces teachers to inquiry through a first-hand experience working on mathematical tasks. The intent of these 'task-first' CPD designs, like LTMI (see Section 3.1), is that teachers "can experience for themselves at their own level something of what their learners might experience and hence become more sensitive to their learners" (Watson & Mason, 2007, p. 208). By working on mathematical tasks, teachers develop their understanding of pedagogical knowledge for inquiry teaching while, at the same time, becoming "more analytic and critical readers of instructional materials" (Ferrini-Mundy, Burrill, & Schmidt, 2007, p. 315).

Literature shows that if teachers are to be effective in implementing IBL, then they first need to experience inquiry through professional learning tasks (Boston & Smith, 2011; Swan, 2007). A 'task-first' approach is underpinned by the view that in learning about inquiry teaching one needs to experience the process by learning through inquiry (see, for example, Silver, Clark, & Ghousseini, 2007). Watson and Mason (2007, p. 206) argue

that to engage teachers in learning through inquiry, a task could be "on a spectrum between an open invitation and direct instruction". The crucial aspect is not the task in itself, but its role in what it contributes to evoking mathematical inquiry. This position has, I would argue, implications about how a task-delineated CPD is presented and structured so that teachers may make sense of mathematical inquiry and its role in developing student learning. The ultimate aim of this process is to develop teachers' ability in transferring IBL into their classrooms. It is questionable whether presenting and engaging teachers with challenging inquiry tasks is the only way for them to approach inquiry. Indeed, Watson and Mason (2007, p. 213) posit that for teachers, within transmission teaching dispositions, inquiry "with traditional tasks in traditional settings would be a more effective way of changing teaching within such contexts". This suggests that professional learning tasks need to involve teachers in thinking deeply about what and how they teach, and how tasks and lessons might be sequenced so that students can make sense of the mathematics presented to them.

A 'task-first' approach to CPD is usually embedded within a practice-based model that views learning tasks and activities as purposefully connected to the curriculum that teachers are teaching (Chapman, 2011). Moreover, tasks need to address the mathematical content with activities modelling an effective pedagogy through a collaborative working environment. This process of learning to teach through inquiry offers tasks within realistic contexts of classroom situations so that teachers have the opportunity to explore the mathematical and pedagogical ideas being promoted.

CPD designers need teachers to distinguish between tasks for their personal development and those they can take to their classroom. The key issue is for teachers to understand that it is not the task that will bring about the change in their classroom, but the mathematical activity which results as the teacher supports students in trying to make sense of it. The use of tasks in CPD is, hence, central to help teachers understand the importance of

making in-the-moment decisions and providing students with 'just-in-time' teaching which is responsive to student learning.

2.5.3 CPD offering 'just-in-time' teaching and learning opportunities

The importance of community, collaboration, negotiation, support and feedback in professional development is well documented (e.g., Goos, 2014; Meirink et al., 2007; Putman & Borko, 2000; Stoll et al., 2012). Teacher collaboration is known to offer a powerful learning environment because ongoing engagement is likely to feed into the professional learning journeys and practices of teachers. Feedback and support are necessary for teachers to persist with their implementation of reform practices, but it is crucial that this assistance is provided within teachers' working contexts and in a timely manner (Cavanagh, 2006; Goos, Dole, & Makar, 2007; Lerman & Zehetmeier, 2008). In other words, if CPD programmes are to benefit teachers' professional development, they need to offer ongoing and 'just-in-time' learning spaces for teachers.

Just-in-time teaching and learning (JITL) is usually associated with technology-based learning spaces, including participation in online communities. While technological tools have a key role to play in transforming teacher learning, tools that offer 'just-in-time' collaboration, interaction and feedback are essential for job-embedded learning (Glazer, Hannafin, Polly, & Rich, 2009). JITL draws on learning principles incorporated within games and game-based learning. In his book, *What Video Games Have to Teach Us about Learning and Literacy*, Gee (2003) discusses 36 learning principles and argues that games give players assistance 'on demand' and 'just in time'. This argument challenges the notion of learning that occurs out of context of actual use or that is independent from the needs and goals of the learner. Indeed, people are more likely to effectively apply knowledge when this is provided in time and in context.

JITL involves making content immediately and readily available for CPD participants to use at their own discretion – usually through online access. Besides content, participants are offered access to resources, support

materials and assistance. JITL promotes the self-determination and self-directedness of people engaging in learning when needed. In teacher professional learning, JITL embraces job-embedded learning. According to Glazer et al. (2009), on-site teacher communities offer ongoing and 'just-in-time' support that address teachers' emerging needs. As the research by Mouza (2003) shows, teachers benefit from in-classroom administrative assistance and feedback – which is one form of 'just-in-time' support.

2.6 Teacher enactment of IBL

Research on teacher enactment has shown that teachers come to understand new ideas about practice through their own pre-existing knowledge, beliefs and practices, often interpreting and transforming these ideas as they put them into practice (Spillane et al., 2002). Furthermore, enactment occurs within a process influenced by the sociocultural setting of teachers' workplaces (Spillane, 1999). More recently, Maguire, Braun, and Ball (2015) argue that teacher enactment is influenced by whether new ideas are mandated or recommended and the extent to which they are valued and dominant within a school setting. Hence, depending on the dominance and priority attributed by schools, enactments are influenced by time and space and "are contingent, fragile constructions" (Maguire et al., 2015, p. 488). What works for teachers depends on both the personal and sociocultural factors within which new ideas are to be enacted. It follows that teacher enactment is a result of how personal theories, values and understandings are transformed by the context in which practice occurs.

2.6.1 Teachers' interpretation of IBL

Teachers construct their understandings of IBL from classroom experiences (Chin & Lin, 2013) and the contextual conditions shaping their teaching. But I would argue like Anderson (2002, p. 8) that "the matter must [also] be addressed … at a level that includes central attention to beliefs and values". This builds on the understanding that the way teachers view, speak about, interpret, characterise and enact IBL is also based on their preconceptions and beliefs about mathematics, teaching and learning.

Professional development opportunities must hence provide teachers with opportunities to change their practice while, at the same time, developing an understanding of IBL.

Kang, Orgill, and Crippen (2008) report, for instance, that science teachers interpreted inquiry based on their classroom experiences. In defining IBL, these teachers mentioned features that were consistent with those for science education and their understanding developed over time. Similarly, the study by Chin, Lin, and Tuan (2016) shows how, over a one-year long CPD programme, mathematics teachers' understanding changed from being initially theoretical to eventually become more practical. In other studies (e.g., Lotter, Harwood, & Bonner, 2007; Wee, Shepardson, Fast, & Harbor, 2007), however science teachers' conceptions of inquiry remained relatively unchanged. Wee et al. (2007, p. 82), for example, contend that "classroom implementation of inquiry did little to improve teachers' individual understanding of inquiry". This contrasts sharply with the finding reported by Chin et al. (2016) that teachers' improved knowledge of inquiry influences their corresponding teaching practice. Hence, it remains unclear whether improved knowledge of inquiry teaching is a result of its successful implementation. Notwithstanding the fact that a clear conceptualisation of inquiry teaching appears to be necessary for successful classroom implementation, researchers have yet to come to an understanding of how teachers develop their interpretation of inquiry and how this is eventually translated into classroom practice.

In reporting on the PRIMAS project, Farrugia (2014) states that Maltese mathematics and science teachers' interpretations of IBL demonstrated a limited understanding. For some teachers, IBL involved asking questions and eliciting from students rather than providing information. For others, IBL entailed offering hands-on activities and investigations with students generally working on their own with no guidance. Reporting on the same project, Maaß et al. (2017) reveal that teachers of mathematics and science, from 12 European countries, characterised IBL in three different ways. Some developed the view that IBL is a peripheral activity used mainly to

motivate students. For these, IBL was recreational and hence detached from the rest of their teaching. Others saw IBL as a collection of pedagogical practices designed to promote students' thinking and reasoning. These teachers associated IBL with using open questioning, discussion and collaborative learning. The remaining teachers viewed IBL as a combination of activities and pedagogies intended to develop students' capacity to engage in scientific processes and learn content. This third group appeared to have a more sophisticated concept of IBL recognising that content and process may be taught concurrently. These different interpretations appeared closely linked to how teachers perceived their IBL enactment.

2.6.2 Lesson phases within student-centred approaches

There is an implicit assumption in the literature that for an inquiry-oriented lesson to be successful, it must include a cognitively demanding task and follow a well-established lesson structure. In addressing lesson structure, I reviewed literature on lesson organisation across a range of student-centred approaches. This was important primarily to understand and identify similarities and potential differences in lessons purporting to promote reasoning and discussion in the mathematics class. Five teaching and learning approaches were identified (see Table 4), with evidence of much overlap between them.

Based on data about the effectiveness of IBL (see Section 2.2.1), it seems reasonable to find a well-defined structure for lessons involving cognitively demanding tasks. Within all five approaches, I noted a lesson structure that is based on either three or four phases. The first phase – referred to as the 'introduction', 'launch', 'task' or 'problem presentation' – involves a sequence of teacher actions aimed at presenting the task while establishing students' readiness for undertaking the task. The next phase – referred to as the 'explore', 'development', 'investigation' or 'inquiry' – is the stage during which students tackle the task or problem posed, individually or in small-groups. The third phase – referred to as the 'summary', 'discussion' or 'summing up' – usually involves a whole-class discussion during which students present their ideas and their work on the task. During this phase,

the teacher may also summarise the main teaching points arising from students' work. In line with the instructional dynamics of phase three, Bishop and Goffree (1986) had suggested that the plenary is the most suitable phase to expose mathematical connections. To be noted that in some models the 'discussion and summary' is considered as one phase (see Lampert, 2001; Stein, Engle, Smith, & Hughes, 2008). For some others (e.g., Menezes et al., 2015; Stigler & Hiebert, 1999), however, the 'summing up' represents the fourth and final phase of the lesson during which the teacher synthesises the main mathematical ideas emerging from the discussion. An exemplar of such four-phase lesson categorisation is provided by Stigler and Hiebert (1999) in their report of Japanese problem-solving lessons.

Mathematical Approaches		**Lesson Phases**			
		1	**2**	**3**	**4**
Problem Solving	*Lappan et al. (1998)*	Launch	Explore	Summarise	
	Van de Walle (2004)	Before	During	After	
Japanese Lesson Study	*Shimizu (1999a)*	Hatsumon	Kikan-shido	Neriage	Matome
	Stigler and Hiebert (1999)	Task Presentation	Work on Task	Discussion	Summary
Investigative and Exploratory	*Christiansen and Walther (1986)*	Start	Development	Summing up	
	Ponte et al. (2003)	Task Presentation	Student Work on Task	Presentations & Discussion	
	Ponte et al. (2014)	Task Presentation	Development	Discussion	
Inquiry-based Learning	*Leikin and Rota (2006)*	Introduction	Inquiry	Summary Discussion	
	Stein et al. (2008)	Launch	Explore	Discussion & Summary	
	Ponte (2011)	Introduction	Student Work on Task	Presentations & Discussion	
	Menezes et al. (2015)	Task Launch	Exploration	Discussion	Systematization of Learning
Reform-oriented	*Lampert (2001)*	Task Launch	Exploration	Discussion & Summary	

Table 4: Lesson phases within student-centred approaches

Researchers (e.g., Shimizu, 1999b; Sullivan et al., 2015) report that in learning to introduce challenging tasks teacher find the proposed lesson structure as feasible. In their study with seven and eight-year old students, Russo and Hopkins (2017) investigate the respective strengths of using a 'task-first' and 'discussion-first' approach. A 'task-first' approach generally follows the launch-explore-discuss lesson phases (see, for example, Stein et al., 2008) with a challenging task being presented to students at the beginning of a lesson. In contrast, a 'discussion-first' approach starts with a teacher-facilitated discussion of key mathematical ideas and content. Students are then assigned routine tasks as consolidation work, with the cognitively demanding task being offered toward the end of the lesson. The latter approach is more aligned to traditional lesson structures, similar to the *exposition-practice-consolidation* teaching style found prevailing in Maltese classrooms (Buhagiar & Murphy, 2008). Proponents of guided approaches (see Kirschner et al., 2006) argue that it provides more carefully scaffolded teacher instruction.

Inquiry-oriented lesson structures present demands on teachers because enacting inquiry is complex (Russo & Hopkins, 2017) and teachers require particular skills to successfully orchestrate the lesson while encouraging mathematical thinking and reasoning (Stein et al., 2008). From their quasi-experimental design with groups of primary school students receiving different interventions of 'task-first' and 'discussion-first' approach by one of the authors, Russo and Hopkins (2018) conclude that while both approaches improved problem-solving performance, the 'discussion-first' approach is more efficacious in improving fluency. They suggest, as a result, that "the decision to endorse a particular instructional approach may depend on the learning objectives and associated learning outcomes valued by educators" (Russo & Hopkins, 2018, p. 11). This position questions the idea that it is necessary to begin an inquiry-oriented lesson with a cognitively demanding task.

2.6.3 Teacher lesson structure and the use of inquiry tasks

According to Leikin and Rota (2006), the three-phase lesson structure is an indicator of teachers' proficiency in enacting IBL. In their analysis of three lessons, taught with the same class, Leikin and Rota (2006, p. 53) argue that "changes in lesson structure were not task dependent because the three tasks were designed for similar lesson organisation". They identify two interrelated reasons for the changes in the way the teacher structured lessons: (i) increased confidence in granting students more independence, and (ii) growing expertise in managing inquiry-based lessons. This means that the students' improved ability to cope with inquiry tasks facilitates the teacher's development in managing inquiry-based lessons.

As already noted, Leikin and Rota (2006) do not link changes in lesson structure to the kind of tasks that the teacher presented. However, it appears that the task used in the first lesson required a higher degree of mathematical understanding and application than the other two (see Figure 2). In the first lesson, students were asked to identify the common property of sets of discrete points from observing the graphs presented to them. Students were required thus to read the set of points presented in the different lines and then describe them. The context of using graphs and connecting the points on the graph to a relationship between the coordinates of the points may have presented a higher cognitive demand on the children. For this reason, the teacher may have needed to intervene and address the whole-class repeatedly. In fact, the teacher had to introduce the task on four occasions as "it was not clear to them" (Leikin & Rota, 2006, p. 52), but perhaps also because the graphical representations presented a constraint to the children in understanding and achieving the intended goal – that is, of identifying the common property of the points and investigating relationships.

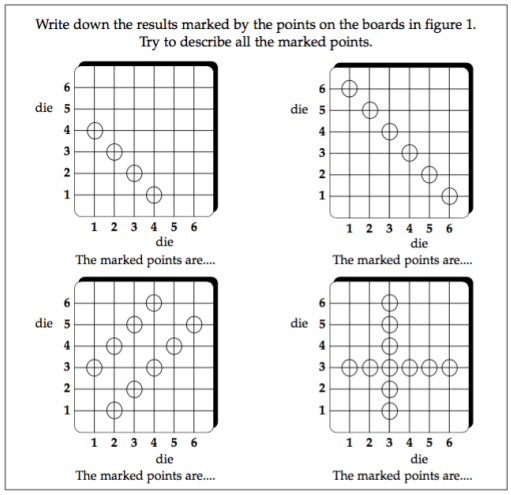


Figure 1. Task for the first lesson.

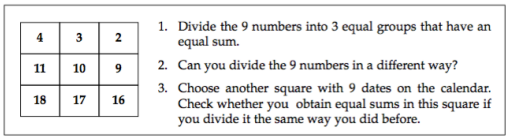


Figure 2. Task for the second lesson.

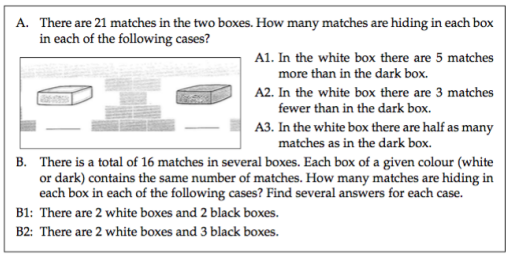


Figure 3. Task for the third lesson.

Figure 2: The three tasks reported in Leikin and Rota (2006, p. 49-50)

While teacher proficiency in lesson structure may improve with experience in managing inquiry-based lessons, lesson structure may also be dependent on the kind of task that teachers use. For me, there exists a link between students' familiarity with the mathematical content inherent within the tasks and teachers' lesson structure. Williams and Clarke (1997, p. 414), in their paper on task complexity, remark that "the practical act of task selection can only be undertaken in the knowledge of the students for whom the task is intended and the purpose for which the task is being chosen". In other words, teachers select to use certain types of tasks based on their learning goals for students, their knowledge of students and the subject knowledge they plan to teach. It appears that the way teachers structure their IBL lessons, incorporating cognitively demanding tasks, depends on the anticipated, perceived and/or emerging struggles that students encounter when working on the task. I add to this that students' familiarity or novelty of the mathematical content inherent within the task may also determine lesson structure.

2.7 Characterising teachers' beliefs and practice change

Teachers' beliefs about mathematics, teaching and learning are recognised as relevant to teachers' practices (Ernest, 1991; Thompson, 1992). Beliefs are the foundations of personal thought and behaviour (Pajares, 1992;

Swan, 2006) and "act as a filter through which new phenomena are interpreted" (Swan, 2006, p. 171). The study of beliefs seems to be a 'messy construct' (Pajares, 1992) because of the different definitions and characteristics attributed to it. Referring to terms found in the literature, Pajares (1992, p. 309) denotes beliefs as:

> … attitudes, values, judgements, axioms, opinions, ideology, perceptions, conceptions, conceptual systems, preconceptions, dispositions, implicit theories, personal theories, internal mental processes, action strategies, rules of practice, practical principles, perspectives, repertoires of understanding, and social strategy …

According to Pajares (1992), such myriad of terms show a distinction between beliefs and knowledge, which both are deeply personal, and distinguishing between the two becomes a difficult task. For example, two teachers may have similar knowledge of IBL but use it differently in their classrooms. Such disparity in these teachers' practices of IBL may be a result of their beliefs about mathematics, teaching and learning. However, it may also depend upon the situative nature of their practice – perceptions of the context – the school, the students, resources available and the education system. Teachers' beliefs represent personal theories (Schoenfeld, 1985), deeply rooted values, judgements and implicit assumptions about curriculum, education, students, teaching and learning acting as cognitive filters (Wilson & Cooney, 2002) through which new knowledge and experience is interpreted and enacted by teachers.

CPD programmes attempting to bring about teachers' learning and changes in practice also focus on changing teacher beliefs. Changing beliefs is difficult, yet, attending to beliefs is important in developing teaching practices (Beswick, 2005; Pajares, 1992). There appears to be two main perspectives in the literature on the interplay between beliefs and practices. One perspective suggests that teachers' beliefs influence classroom practices and, thus, changing teachers' practices requires a change to their beliefs (Thompson, 1992). The second perspective views teachers' practices as influencing their beliefs. This would suggest that

changing teachers' beliefs requires changes in teacher practices (Swan, 2006; Wilson & Cooney, 2002). While the first perspective resonates with traditional models of professional development, where ideas and theories precede implementation, the second perspective considers context-specific practices as shaping beliefs. I favour the latter perspective and hold that changes in beliefs are more likely to come about following teachers' take-up of classroom practices enacted and shaped by the contextual constraints they face. Put differently, for me, practice change involves a process shaped by the agentive roles afforded to teachers within CPD and the workplace context (Clarke & Hollingsworth, 2002).

Literature also shows that there are, however, two other emerging viewpoints. One suggests that beliefs and practice influence one another and develop together (Cobb et al., 1990), while the other proposes that teacher change may not necessarily reveal changes in beliefs. According to Nathan and Knuth (2003, p. 202), changes in teachers' practices occur within a consistent belief system and when the new practices "are compatible with their old (and still) beliefs". Indeed, teachers may reveal dissatisfactions with their practices because these are incompatible with their beliefs. These different perspectives show the subtle and complex nature of the connection between beliefs and practices.

2.7.1 Belief change and belief systems

Teachers' beliefs change not because of the CPD but as a result of teachers' experience of successful implementation (Swain & Swan, 2007; Swan, 2006; Wilson & Cooney, 2002). Like Beswick (2005), I would argue that context has relevance to beliefs development. Indeed, beliefs are situated because they are influenced by the constraints and possibilities under which teachers work. In her study of 25 secondary school mathematics teachers, Beswick (2005) reports that students' perceived ability level, class grade level and syllabus pressures shaped the teachers' beliefs. Hence, taking a *person-in-practice* approach to researching CPD interventions would be important as it gives consideration to the setting in which teachers work,

their practical knowledge and their development of identities (Goos, 2013). Goos (2013, p. 531) argues that:

> Teacher beliefs do not unequivocally determine the impact of an educational change project; nor is the professional environment simply the backdrop for practice. Beliefs and contexts both matter, but not in ways that deny individual agency or possibilities for teachers to change themselves.

Belief systems are descriptions of the organisation of a set of beliefs that undergoes change while the individual reflects on experience (Swan, 2006). Teachers' beliefs can be classified into *transmission*, *discovery* and *connectionist* (Askew et al., 1997; Swain & Swan, 2007; Swan, 2006). To give an example, a *transmission* belief views learning as an individual activity based on watching, listening and replicating; a *discovery* belief views learning as an activity based on practical exploration and reflection; and a *connectionist* belief views learning as a social activity where learners are challenged and learn by negotiating meanings. It is possible, however, that a teacher's conception of mathematics, teaching and learning will enclose a combination of the above three seemingly conflicting aspects, though perhaps with different degrees of belief (Thompson, 1992). As the study by Askew et al. (1997) shows the most effective teachers are those characterised by connectionist beliefs. While the study by Askew et al. (1997) focuses specifically on the teaching of numeracy, I would argue that teachers demonstrating change towards connectionist orientations are likely to be more effective in inquiry teaching (see also Swan, 2006).

2.7.2 Considering beliefs in professional development programmes

Attending to beliefs is recognised as crucial in CPD programmes. Consequently, apart from taking into consideration the social context of learning, in order to influence practice CPD must focus on teacher beliefs, and reflective thought (Ernest, 1991). Over the years, professional development has shifted from trying to persuade teachers to change their beliefs to providing them with opportunities "to doubt, reflect and reconstruct" knowledge about practice (Wilson & Cooney, 2002, p. 132) in unhurried and non-threatening environments. This leads me to argue that

cultivating and sustaining an environment where teachers can safely make their existing beliefs and practices explicit is fundamental (Swan, 2006; Swan et al., 2013). Within such a culture, teachers discuss their beliefs and practices, particularly in situations when they are asked to consider contrasting practices. Such deliberations are intended to create conflicts (see Section 3.1.3), and in their attempt to resolve conflicts as they act in new ways, teachers are offered support (Swan & Swain, 2010).

Effective CPD programmes value 'distributed cognition' (see Hutchins, 1995). Hence, in CPD learning develops not just from each participant but also between participants because each participant brings distinctive knowledge and beliefs to a learning community. Indeed, through social interactions, teachers exchange knowledge about their students, the particular settings in which they teach, their teaching practices and their beliefs. While engaging with such deliberations, teachers may develop knowledge about practice and eventually experience change in their beliefs.

2.8 Identifying gaps in the literature

Following a thorough investigation and analysis of the literature related to the area of study, I have identified a number of gaps in research. This study, which primarily focuses on teachers' journey in learning to teach mathematics through inquiry, will attempt to address these gaps related to teachers' interpretation of IBL, their enactment and lesson structure, changes in teachers' practices and beliefs, CPD design and the support needed for CPD to be effective for teachers.

2.8.1 Teachers' interpretation and enactment of IBL

While CPD programmes are designed to enhance both teachers' understanding and their implementation of inquiry teaching (e.g., Chapman & Heater, 2010; Chin & Lin, 2013; Swan et al., 2013), few studies focus on teachers' interpretation of IBL and its influence on classroom practice. In reviewing the related literature, most studies were on science (e.g., Ireland et al., 2012; Kang et al., 2008; Lotter et al., 2007; Wee et al., 2007), one on

mathematics (Chin et al., 2016) and two targeting mathematics and science teaching (Farrugia, 2014; Maaß et al., 2017).

Research in mathematics education specifies two methods for identifying teachers' interpretation of inquiry: interviews held at the end of a CPD programme and lesson observations (Farrugia, 2014; Ireland et al., 2012; Maaß et al., 2017). Such data, although valuable in understanding the way in which teachers interpret inquiry as a result of CPD, do not address how teachers develop this understanding. As a way of addressing this lacuna, I analyse teachers' written reflections of IBL at different instances during their CPD journey as this would help in identifying their conceptualisations of IBL and understanding its development. In trying to capture the essence of what inquiry teaching means for teachers attempting to implement it, this study contrasts pre- and post-CPD definitions of IBL supported by data from interviews and the ongoing teacher reflections of classroom enactment as documented in their lesson journals (see Section 6.1.1). Understanding teacher interpretation of IBL and its enactment is essential in identifying what teachers see as important, and how they conceptualise it and put it into practice. This understanding may provide teacher educators with: (i) knowledge of inquiry ideas teachers may bring with them to professional development, (ii) ways in which teachers implement IBL in practice, and (iii) teacher conceptualisation of inquiry teaching through practice.

2.8.2 The development of teachers' IBL lesson structure

Literature on reform-oriented approaches shows a prevalence on adopting a 'task-first' approach (Russo & Hopkins, 2017), which involves applying a three or four phase lesson structure, recommended for teachers using IBL. Research generally focuses on an analysis of what happens during these phases with critique on teachers' roles during each lesson phase and how these influence students' mathematical activity (see, for example, Menezes et al., 2015; Ponte et al., 2014). The gap in knowledge identified is the lack of research that problematises whether it is necessarily a 'task-first' lesson structure that facilitates inquiry teaching.

In the study by Russo and Hopkins (2017), in which one of the researchers was responsible for developing the lesson plans and for doing the teaching, teachers acting as observers perceived the 'task-first' approach as suitable to foster student mathematical creativity. These teachers, on the other hand, described the 'discussion-first' as a "highly focussed and efficient approach to learning … providing an opportunity for lower-achieving and less confident students to be successful" (p. 43). Russo and Hopkins (2017) conclude that learning with a cognitively demanding task can support student learning when prior knowledge is stimulated using a teacher-facilitated discussion. This indicates that challenging tasks can be introduced using a 'discussion-first' lesson structure.

It is important to investigate teachers' potential flexibility in altering this 'task-first' structure as they learn to teach through inquiry. There is, indeed, no evidence to show how teachers, particularly secondary school teachers, develop their learning of inquiry by studying lesson structures that deviate from the 'task-first' model proposed in the literature. Only one study, conducted by Leikin and Rota (2006, p. 46), was found to investigate the development of a teacher's IBL lesson structure as an indicator of "the teacher's proficiency in the classroom". This case study focused on a beginning primary school teacher of mathematics – one of the authors – engaged in implementing IBL over a period of 16 months. Leikin and Rota (2006) base their findings on three of 21 videotaped lessons during which IBL was implemented. A major conclusion from this study is that changes in lesson structure were not task-related, but dependent on the teacher's growing expertise and confidence in managing inquiry lessons and granting students more autonomy. The present study questions and investigates this conclusion by considering two aspects related to the kind of tasks teachers use – the *mathematical content* inherent within tasks and the *cognitive challenge* that students encounter.

It seems reasonable for teachers, particularly those adopting transmission approaches, to encounter challenges when adopting a 'task-first' approach. While LTMI offered teachers a 'task-first' model for structuring inquiry

lessons, it endorsed teacher flexibility to select tasks and develop their own teaching approach. The conjecture is that a 'task-first' approach is not the only way of structuring inquiry-oriented lessons. Such an investigation focuses on identifying when teachers deviate from this structure, how and why they do it. Findings about these aspects would be critical towards achieving a better understanding of how teachers develop lesson structure and, as a result, learn to enact IBL. Such a finding also has implications to the design of CPD programmes supporting teachers in learning to teach through inquiry.

2.8.3 Changes in beliefs and practices

The study of CPD programmes in attempting to bring about teacher learning and practice change usually involve investigations on whether and how teachers' beliefs change as a consequence (Beswick, 2005; Swan, 2006; Wilson & Cooney, 2002). Generally, changes in practice appear to influence changes in beliefs (Swan, 2006; Wilson & Cooney, 2002) rather than vice-versa (Thompson, 1992). Whenever shifts in beliefs are reported, these tend to progress along three trajectories: from transmission to discovery, transmission to connectionist and discovery to connectionist (Swain & Swan, 2007; Swan, 2006; Swan & Swain, 2010). This implies that teachers who, at the beginning of the CPD programme, hold transmission beliefs are likely to shift to discovery and/or to connectionist. Swain and Swan (2007, p. 54) posit that this happens because "some teachers recognise the limitations of transmission methods and move from a 'telling' role into a 'not telling' role". To address this limitation, teachers appear to stand back and allow students to think on their own, taking on a passive role allowing students to make their own discoveries. As Askew et al. (1997) illustrate, discovery teaching is often less effective than transmission and connectionist teaching. Teachers might, hence, eventually recognise limitations with this passive role and adopt a more proactive role.

The study of teachers' beliefs is not independent of their classroom actions. This study attends to this notion and in addition considers the perspective suggested by Nathan and Knuth (2003, p. 202) that while teachers'

practices may change, their beliefs might not. In addressing this more closely, one of the case studies in the present research considers a teacher who joined LTMI with predominantly connectionist beliefs.

Literature shows that data on beliefs' rating reported by teachers are collected at the beginning and at the end of their CPD experience (e.g., Polly et al., 2014; Swain & Swan, 2007; Swan, 2006; Swan & Swain, 2010). Such investigations overlook the fact that CPD designs may offer multiple contexts for teacher learning. For example, in the study by Polly et al. (2014) teachers attended a summer institute followed by sessions held during the following scholastic year. Similarly, for LTMI (see Chapter 3) teachers were offered an *off-the-job* component consisting of summer workshops followed by *on-the-job* ongoing meetings held during in the following scholastic year. For CPD programmes that incorporate multiple contexts for teacher learning, the nature of the CPD experiences provided may potentially influence their changes in beliefs. As a result, in the present study the data to categorise teachers' beliefs include a third set collected in between these two CPD components. This triad data collection is intended to investigate not only how beliefs change but also potential influences emerging as a result of each CPD component.

2.8.4 CPD design models offering 'just-in-time' teaching and learning

Research shows that teacher practices are more likely to be transformed by CPD that is sustained, coherent and intense (Guskey, 2002; Supovitz, Mayer, & Kahle, 2000). This implies that for teachers to change and improve their practices, they must engage into a personally transformative experience that can only occur over time. In designing CPD with a community of practice component, the literature reviewed also focuses on four key aspects: the social aspect of learning and the negotiation of meanings, creating a safe learning environment, engagement in reflective practices, and sustained ongoing engagement. LTMI was designed to offer support for teacher development and change by embedding these four aspects together with four features of IBL (see Sections 2.3 and 3.1.2).

Evidently, to be successful, CPD must be undertaken as a process, rather than an event (Loucks-Horsley et al., 2010). Moreover, along the process, teachers need to be offered ongoing support and 'just-in-time' feedback to persist with their enactment of reform practices. Luft (2001), for example, investigated the support provided to teachers in implementing IBL. She reports that while participants valued observers discussing their inquiry lessons, they found it also useful to observe peers to gather ideas about inquiry teaching. Her conclusion is that teachers who receive such experiences are likely to implement inquiry more frequently. This result sustains the notion that in understanding and enacting reform, teachers need job-embedded in-house support. While literature on CPD designs focus on providing support, collaboration and feedback for teacher learning (Fullan, 2007; Putman & Borko, 2000), the notion of JITL has not been investigated by the wider research community.

LTMI utilised a practice-based model drawing on research-based principles (see Chapter 3). As an intervention programme, LTMI was intended to support teachers in schools within a collaborative learning context. Although contextual factors impinging on teachers' uptake of inquiry teaching were taken into consideration, schools were not involved in its design. In fact, this research study explores two learning spaces for teachers – one provided by schools and the other by the CoP within LTMI. LTMI, including the resources made available for teachers, provided readily available negotiation, support and feedback. CPD materials and tools offered teachers JITL and autonomy to take control over their learning. Indeed, teachers had access to materials to use in context, when needed and at their own learning pace.

2.8.5 The research questions

Four questions were subsequently formulated in response to the gaps identified in the literature.

1. How do teachers interpret and enact IBL in their classrooms?
2. In what ways do teachers structure their inquiry lessons and how does this develop over time?
3. What are the changes in teachers' practices and beliefs over the course of the CPD programme?
4. How does 'just-in-time' teaching and learning embedded in a CPD programme support teacher change?

These research questions are guided by a theoretical framework as a conceptual tool to understand teacher learning and their CPD journey. The zone of enactment theory (see Spillane, 1999) frames the research by outlining the factors to which teachers respond in enacting IBL. In turn, answering the research questions will provide links to theories and contribute to knowledge about teacher interpretation, lesson structure and beliefs and practice change in enacting IBL over time.

2.9 Conclusion

For teachers to understand and attend to the core features of IBL, they need to question and reconsider their deeply-rooted understandings of teaching, learning and mathematics. While appreciating the inadequacies of their current understandings, teachers need to reconstruct and change their practices. According to Little (2003), teacher learning and their ability to enact practice are influenced by personal resources, that is, skills, prior knowledge, dispositions and beliefs. However, efforts to change practices have an important social dimension (see, for example, Golding, 2017; Hodgen & Johnson, 2003) and, hence, understanding these requires investigation in both the personal and the social resources of enactment zones (Blignaut, 2008; Spillane et al., 2002).

The approach undertaken in this study of teacher change takes into account teachers' sense-making of IBL and their actions based on the ideas about practice that they construct, together with the human and material resources provided by the school and those offered by LTMI. Walshaw and Anthony (2007, p. 18) suggest that the school community alone cannot fully determine how teachers construct ideas, and socially constructed CPD embracing reform practices "will not necessarily guarantee that teachers will work in ways fully consistent with reform intents". Teachers make sense of new ideas about practice in unique ways as they "often bring their own personal habits, thoughts, sentiments and predispositions to the act of teaching, while at the same time being constrained by institutional arrangements" (Blignaut, 2008, p. 120). Recognising how reforms are understood by individual teachers, working within particular school structures and professional communities, is therefore crucial in understanding how reforms are mediated and enacted.

This chapter focused on a review of the literature relevant to IBL, teacher professional development, teacher interpretation and enactment of IBL and, changes in practices and beliefs, and JITL opportunities. In this review, gaps in knowledge were identified and research questions that attempt to address these gaps were presented. Supporting teachers in learning about IBL and enacting it in their classrooms requires the design of a CPD programme that offers a practice-based and feasible framework through which IBL could be modelled for teachers. The next chapter delves into the CPD programme design by presenting an in-depth discussion of four research-informed design principles while also integrating the four IBL features essential to define IBL. The discussion will show how these four IBL features can be used in CPD to model the effective implementation of inquiry teaching in the mathematics classroom.

Chapter 3: The CPD programme design

3.0 Introduction

In this chapter, I describe how the four IBL features, outlined and discussed in the previous chapter, were embedded as I designed and researched the *learning to teach mathematics through inquiry* (LTMI) programme. LTMI is innovative as it challenges traditional models of CPD where teachers tend to be passive recipients of context-detached best practices provided by experts. Its purpose is transformative (Kennedy, 2014) and, rather than simply transmitting knowledge, it draws on knowledge from large-scale projects to offer opportunities for teachers to develop knowledge and dispositions through a community of practice (Lave & Wenger, 1991). The LTMI programme design, described in this chapter, was intended to provide teachers with practice-based experiences for collaborative thinking, planning and reflection through their CPD journey in developing IBL practices.

3.1 The LTMI design

Changing core features of teaching, namely the knowledge represented in the mathematical tasks that teachers use, classroom discourse practices, and roles and responsibilities that teachers and their students assume during instruction seems arduous (Dorier & Garcia, 2013; Engeln et al., 2014; Menezes et al., 2015). Moreover, features underlying IBL are complex and adopting these is usually undertaken gradually by teachers (Swan et al., 2013). LTMI is designed to provide ongoing support for teachers within this process of change. The CPD tasks, activities, structure and content are designed to engage teachers to think differently about their current practices, becoming critical and subsequently enabling them to consider and construct alternative practices. Indeed, if teachers are to grasp the core features of IBL, then they would need to challenge, question, rethink and eventually reform their prevailing deductive teaching style. This CPD design invites teachers to identify inadequacies in their current understandings

about teaching, learning and classroom practices and, as a result, enables them to see a need to learn about IBL.

3.1.1 Designing CPD for LTMI

Knowing a conceptual domain (see Greeno, 2006) – the design of professional development in supporting teachers' implementation of IBL – implies not just knowing what IBL means and what effective CPD looks like. It requires integrating the two domains together so that CPD is presented as a modelling experience of inquiry teaching. In designing an effective and meaningful CPD, I offered a programme that included: (a) practice-oriented learning (Smith, 2001), (b) the provision of CPD materials also available online, (c) a community of practice experience (Coburn & Stein, 2006), and (d) long-term engagement (Guskey, 2002).

Adopting a 'task-first' approach, I designed CPD activities with the belief that teachers learn just as students do, participating within collaborative support structures to build knowledge of inquiry teaching while inquiring into practices. LTMI aimed to shape the way teachers develop their identities as teachers of mathematics – by internalising new ways of speaking, acting and thinking. Research into CPD gives considerable attention to the impact that community plays within teacher development and learning. In particular, CPD taking the form of professional learning community is an appropriate way to initiate, support and sustain teacher change over time (Lewis, Perry, & Hurd, 2009; Little, 1990). Creating 'just-in-time' opportunities for teachers to share and question approaches to teaching was thus at the core of the 14-month CPD programme.

3.1.2 Incorporating IBL features in LTMI

In enhancing the shift to IBL, and to be meaningful for teachers, LTMI was designed around these same four IBL features (see Table 5), thus sending a coherent message of IBL (Schoenfeld, 2015). The goal of CPD is to present pedagogies that stimulate teachers to act, speak and think within an IBL perspective.

Applying Features	IBL for Students in Schools	IBL for Teacher Development
Mathematical Tasks	Introduce mathematics to students through rich tasks that provide access and opportunities to experience both mathematical content and connections (Smith & Stein, 1998)	Present tasks that provide teachers with access to explore, interpret, experience and develop knowledge about mathematical inquiry (Swan, 2007)
Collaborative Learning	Integrate small-group activities for students to share ideas, develop mathematical arguments and challenge views (Swan, 2006)	Organise collaborative activities for teachers to reflect and evaluate practices, and providing new challenges and tensions (Chapman, 2011)
Purposeful Questioning	Ask questions that foster students' reasoning, scaffold learning and stimulate them to communicate articulated mathematical ideas (Mason, 2000)	Cultivate a questioning mind-set that engages teachers in deep thinking about practices, taking more active roles in developing their knowledge (Jaworski & Huang, 2014)
Shifting Agency and Responsibility	Provide opportunities for students to assume more responsibility over their learning, arousing confidence to engage actively with mathematics (Schoenfeld, 2013)	Provide experiences that support the development of confidence in teacher capabilities to plan, implement and evaluate inquiry-based practices (Little, 1990)

Table 5: IBL features for student learning and teacher development

By immersing participants in the process of inquiry, LTMI provided teachers with modelling experiences of inquiry teaching. Translating and, hence, enacting the same four features by which I define IBL, into the structural design of LTMI, was important to model IBL in CPD (Farmer, Gerretson, & Lassak, 2003) and, consequently represent the teaching and learning of IBL being promoted. This alignment between what was being offered (IBL for teachers in CPD) and what was being promoted (IBL for students in classrooms) – was an intentional design decision for enhancing teacher learning.

3.1.3 A conflict discussion approach

A key aspect is the role that CPD activities play in creating "opportunities for teachers to ponder pedagogical problems and their potential solutions through processes of reflection, knowledge sharing, and knowledge

building" (Silver et al., 2007, p. 262). Through CPD activities teachers are made aware of their existing practices and challenged to share these through conflict discussions. Conflict discussions serve to generate conceptual change as teachers explore alternatives (Peled & Suzan, 2011). Following the work of Swan (2011), my own approach draws on deliberately creating cognitive conflict opportunities (see Piaget, 1985). As a result of teachers' dissatisfaction with existing practices, a need to reconsider their existing concepts and practices arises. In LTMI, cognitive conflict follows a three-phase structure of activities – exposition, discrepancy and resolution.

PHASE 1: THE EXPOSITION PHASE

Teachers are invited to work on an inquiry task (see Calleja, 2016 for an example), first individually and then within small groups. A whole-class discussion then follows, with prompts and questions such as:

- *Comment on your experience working on this task.*
- *Comment on the task characteristics.*
- *What do you anticipate would be the challenges for students working on this task? Why? How would you address these?*
- *Would you consider using this task with one of your classes? Why?*
- *Would you present the task in a similar way as it was presented to you? Would you do it differently? Why? How?*

These questions are intended to uncover teachers' values, beliefs and practices, while the ensuing discussion provides the means for teachers to share insights and experiences.

PHASE 2: A DISCREPANT EVENT

Teachers then watch a video of a teacher using the same task in a local classroom. Alternatively, they are provided with vignettes of two teachers offering contrasting ways of using the task. Teachers are again presented with a set of questions intended to challenge and help them reflect upon ways of thinking and acting in their class. For example:

- *What are your comments about the lesson?*
- *Would you structure the lesson as the teacher did or would you do it differently? Why?*
- *Who generates the mathematical ideas that get discussed? Who evaluates and/or responds to these ideas?*
- *How deeply do students get to explain their ideas?*
- *How does the teacher respond to students' struggles?*

The video and the contrasting situations usually generate a conflict between teachers' existing practices and beliefs and the promoted IBL practices. The questions thus challenge teachers to reconsider pedagogical approaches and attempt to resolve their issues through discussion.

PHASE 3: THE RESOLUTION PHASE

Teachers are finally asked to plan an inquiry lesson that they would eventually use with one of their classes. Through this collaborative exercise, teachers work at resolving inconsistencies between their own pedagogical approaches and those observed.

It is important to mention that each workshop usually includes more than one cycle of the conflict discussion approach. This approach values the *practice-based* facet of CPD (see Smith, 2001), which is purposeful in connecting pre-existing values, beliefs and practices to new ways of promoting inquiry in the mathematics class. Within the conflict discussion approach, teachers experience tensions in their thinking. While seeking to rectify the conflict, they develop their own new meanings, hence becoming better equipped to use inquiry more effectively in their classrooms.

3.1.4 The role of CPD facilitators

Teacher-leaders, here referred to as CPD facilitators, need to have the knowledge and abilities to model for teachers the appropriate processes and pedagogical actions that teachers would then use when engaging their students with IBL. The term *facilitator* is used to specify undertaking the key role of empowering people in knowledge creation. Facilitators do not

impart knowledge directly to the participants, but provide challenging and appealing situations for participants to wrestle with and construct their knowledge of inquiry. They need to be skilful at probing, stimulating and supporting thinking, thus modelling IBL by immersing participants in the process of inquiry.

The CPD facilitators leading the LTMI programme – summer workshops and follow-up meetings (see Section 3.1.5) – were a head of department and two teachers with experience in IBL and teacher training. The head of department was a well-read practitioner involved in initial teacher education courses at the University of Malta and INSET training. One of the two teachers participated in the PRIMAS project and had since continued to develop his IBL practices. The other teacher was Kevin one of the five participants in the pilot project (see Section 3.4). At the beginning of every session, I introduced the facilitator and reminded participants about my researcher role – as a non-participant observing teachers during the different activities they engaged in.

The role of CPD facilitators involves guiding discussions, linking arguments to the content of the CPD. Facilitators support participants through tasks, questions and prompts inviting them to take an inquiry stance to learning. They elicit teachers' varying solutions to the problems and situations posed, and invite them to question one another in a supportive learning environment (Loucks-Horsley et al., 2010). CPD facilitators need to be sensitive in identifying key moments when more responsibility may be shifted onto the participants. When this occurs, such scaffolding is gradually removed and the community becomes its own resource for learning.

Besides developing content knowledge, facilitators work towards building community. Their role becomes pivotal in establishing and fostering a community of learners. For a community to flourish, building trust becomes key in sustaining productive long-term learning. Facilitators pay attention to the events unfolding and act without judging, thus creating an atmosphere where teachers feel safe to share their beliefs, values and practices (Fullan, 1993).

3.1.5 The LTMI Programme Structure

Working within the Maltese context of INSET teacher training, I draw on a CPD design framework suggested by Putman and Borko (2000, p. 7). They claim that:

> One promising model for the use of multiple contexts combines summer workshops that introduce theoretical and research-based ideas with ongoing support during the year as teachers attempt to integrate these ideas into their instructional programmes.

This approach to the design of creating opportunities to learn (see Goos, 2014) combines *learning-off-the-job* with *learning-on-the-job* opportunities (see Table 6). The LTMI programme provides CPD experiences for teachers first through summer workshops and follow-up meetings held during the scholastic year.

Summer Workshops	**Follow-up Meetings**
Introducing four features of IBL	*Reflecting on classroom practices*
Four workshops One held in July and three in September (3½ hours each)	Ten meetings Held October to May during school hours (1¼ hours each)
Offering a visual image of high-quality inquiry teaching pedagogies	*Creating a learning community for practice-based inquiry teaching*

Table 6: The LTMI Programme for CPD

The *off-the-job* workshops take a learner-centred approach where teachers are exposed to modelling experiences of IBL. The *on-the-job* meetings then offer ongoing support where teachers work as a professional community to share and construct classroom experiences of IBL.

3.2 The summer workshops

3.2.1 Designing the content

In designing the content for LTMI, I drew mostly on two CPD initiatives promoting IBL: **PRIMAS** and **Bowland Maths** (see http://bowlandmaths.org.uk). I participated in the PRIMAS project, as a CPD facilitator (multiplier) leading a group of Maltese teachers, and this

was instrumental in guiding my design of the CPD materials. My first-hand experience using the CPD materials with teachers helped me gain knowledge about the potential of using such carefully designed resources, and what could possibly work better for teachers. In Table 7, I present similarities between the modules offered by PRIMAS, Bowland Maths and the LTMI programme (shaded modules).

CPD Project	PRIMAS	Bowland Maths	LTMI
	Tackling unstructured problems	*Tackling unstructured problems*	*Mathematical tasks*
	Students working collaboratively	*Fostering and managing collaborative work*	*Collaborative learning*
	Asking questions that promote reasoning	*Questioning and reasoning*	*Purposeful questioning*
	Self and peer assessment	Involving students in self and peer assessment	Shifting agency and responsibility
	Learning concepts through IBL	Assessing key processes	
	Building on what students already know	ICT: Using resources effectively	
	Student-led inquiry	The case studies and Mathematics	

Table 7: Linking LTMI modules with PRIMAS and Bowland Maths

Each summer workshop was designed to focus on one of the four IBL features. The session in July considered *mathematical tasks* for inquiry while the three September sessions addressed the role of *purposeful questioning*, *collaborative learning*, and *student agency and responsibility*. In designing these workshops, I drew on the session structure and some content as provided by the PRIMAS project materials. To this I added lesson videos taken from local classrooms, published YouTube videos, contrasting lesson vignettes and mathematical tasks. To explain in more detail the CPD tools design, I now draw on examples from two modules – *collaborative learning* and *purposeful questioning* – to illustrate how each module was developed.

3.2.2 Structure of the workshops

Summer workshops followed a consistent pattern of activities. In the introductory activity, teachers worked collaboratively to solve a mathematical task and later discussed their experience working on the task. This strategy was led by a personal belief that, to understand IBL, teachers need to experience it first. For example, in the *collaborative learning* module, teachers were introduced to the 'Human Tower' tradition in Barcelona and then invited to collaborate in small groups to design a suitable structure for building a human tower made up of three to four people (depending on the number of people in the group). This task encouraged creativity and the investigation of design possibilities. It promoted decision-making processes and valued explorations and explanations.

A video from a local classroom was then introduced. In CPD programmes, incorporating videos as artefacts of practice is a valuable tool providing visual representations of the teaching strategies endorsed (Borko, Jacobs, Eiteljorg, & Pitman, 2008; Groschner, Seidel, Pehmer, & Kiemer, 2014). Videos also have a distinctive ability in capturing the richness and complexity of classrooms. Brophy (2007, p. 289) argues that best videos for analysis "show teachers with whom viewers can identify implementing a curriculum similar to the one they use… in a classroom similar to the classroom in which they teach". With this understanding, I created, produced and field tested videos of lessons taken from the local context supported by a selection of YouTube clips – for example, Dylan Wiliam (https://www.youtube.com/watch?v=TqBNWEQmBRM) discussing how teachers may set up collaborative learning in the classroom.

To stimulate further discussion, I offered teachers contrasting scenarios. This technique of exposing teachers to opposing practices is used in two ways: (1) through teacher classroom vignettes, and (2) by presenting informal discussions between two teachers (see Calleja, 2016 for examples). The example I present is taken from the *purposeful questioning* workshop. Participants are presented with contrasting vignettes of how

two teachers – Mark and Amy – use questioning to introduce a problem to their class. Participants were asked to examine the questions that Mark (factual and seeking correct answers) and Amy (probing and seeking strategies) use. A whole group discussion was then introduced through leading discussion questions. Such scenarios challenged teachers to think and to position themselves. I found this situation to be quite revealing in terms of stimulating teachers in genuinely saying what they do in practice, and how they managed to identify and articulate their own gaps within the conflicting practices of others.

In a follow-up activity teachers were provided with individual reading and analysis of narrative lesson cases. Participants were also provided with theoretical knowledge that provided insights when they watched lesson videos, interpreted, analysed and planned lessons. At the end of each workshop, teachers collaboratively planned a lesson using the activities presented and the ideas generated. The lesson planning activity offered teachers the opportunity to discuss and reflect upon the issued raised, and then translating their thoughts into a lesson design.

3.2.3 Online access to workshop materials

The *Teaching Mathematics through Inquiry* website (see www.iblmaths.com) was set up with a two-fold aim:

1. *To provide teachers with direct access to the CPD materials, and*
2. *To support facilitators in running CPD workshops with teachers*

The website offered JITL and support as teachers could access the online materials and learn from it whenever they felt the need. The intended benefit was self-directed learning. Hence, teachers could access materials in any order rather than in the order offered in the summer workshops. Besides a number of links to videos, literature readings and websites related to IBL, this platform offers access to the PowerPoint presentations, lesson videos, eight CPD booklets for teachers and one CPD facilitator guide used during the workshops.

Both PRIMAS and Bowland Maths CPD materials provide access to module guides and teacher handouts. I took up this idea to design a session workbook for each workshop referred to as the '*Thinking about*' booklet. The wording '*Thinking about*' was chosen to reflect an initial "mindfulness" (see Groschner et al., 2014) – a state of being attentive and being aware – that participants experienced as they engaged with the IBL features promoted.

The '*Thinking about*' session booklets assisted teachers in documenting their reasoning while working on mathematical tasks, in taking personal notes and reflections, and in responding to the discussion questions presented and raised during the workshop activities. Meanwhile, a second set of booklets, referred to as '*Planning for*', was intended as an information guide. Since it was a guidebook for planning IBL lessons, this booklet served as a resource and reference tool for teachers that included references to literature. All four sets of the '*Thinking about*' and the '*Planning for*' booklets can be accessed at https://issuu.com/iblmaths/docs.

3.3 The follow-up meetings

3.3.1 Purpose and structure of the teacher meetings

In the follow-up meetings held during the scholastic year, teachers were asked to share experiences of IBL lessons, discuss challenges, communicate insights, and plan and reflect upon lessons. In what follows, I provide an outline of how these meetings with teachers were conducted. The session structure consisted of an opening, a main and a concluding activity.

OPENING ACTIVITY: IDENTIFYING TEACHERS' 'JUST-IN-TIME' KNOWLEDGE

Teachers were provided with a question offering opportunities to reflect on their inquiry practices while, at the same time, providing the session facilitator with access to the 'just-in-time' knowledge that teachers bring in. The following were typical questions used:

- *What are your experiences so far using IBL in your class? (use a short phrase to describe these)*

- *What IBL aspects are working well for you and which aspects do you still find challenging to implement?*

Teachers were asked to write their responses on sticky notes and then invited to either place these on a board with the facilitator reading the responses given, or to communicate their response to the whole group.

MAIN ACTIVITY: EXPLORING AN ASPECT OF TEACHING

Participants were provided with the following opportunities to work collaboratively:

- *Reporting back on practices, challenges and successful stories;*
- *Planning lessons together; and*
- *Sharing IBL tasks and lessons.*

CONCLUDING ACTIVITY: ELICITING LEARNING AND SUGGESTING AGENDA FOR FUTURE MEETINGS

Participants described what they learned or what they intended to do/try differently in their classrooms next time. This was also an opportunity for them to contribute ideas on aspects that they wished to discuss in future meetings.

3.3.2 The schedule and content

Follow-up meetings, coordinated by a facilitator (see Section 3.1.4), were 75 minutes long and were planned to take place once every four weeks. Early on, because of teachers' inexperience with inquiry, the facilitator provided guidance to support teachers in learning about their inquiry practices.

The agenda for each meeting was not predetermined by the facilitator – it either emerged from teachers' conversation during the *concluding activity* or as a result of the *opening activity* (see Section 3.3.1). For example, in the *opening activity* of meeting 8 (see Table 8), teachers were presented with four flashcards.

MEETING	DATE	GOALS	BRIEF DESCRIPTION
1	15/10/15	Reflective writing	*Facilitator discussed skills for reflective writing – describing the lesson plan, its significance and opportunities for learning. The aim was to support participants in writing about IBL practices in their lesson journals.*
2	12/11/15	Sharing initial experiences	*Facilitator supported teachers in sharing initial experiences – dilemmas, concerns and challenges – in their use of IBL in their classrooms.*
3	10/12/15	Sharing IBL tasks	*Participants were encouraged to bring a task that they had used with their students. They shared experiences including benefits with a focus on student learning.*
4	28/01/16	Teacher's role in managing group work	*Teachers identified a struggle with the use of questioning in leading small-group discussions. Facilitator returned to the CPD materials related to the summer workshop on collaborative learning.*
5	03/03/16	Looking into tasks and planning an IBL lesson	*A number of mathematical tasks were offered to teachers for discussion. Based on the year groups taught, teachers then worked in groups to plan a lesson around a task of their choice.*
6	17/03/16	Watching and critiquing a lesson	*One of the participants presented a video of a lesson taught with his class. After watching the lesson, participants discussed how the teacher enacted IBL.*
7	07/04/16	Reporting back on the planned IBL lesson	*Teachers who had implemented the lesson planned in meeting 5 shared their classroom experiences with the whole group discussing strategies that worked and possible improvements.*
8	28/04/16	Sharing critical incidents	*Participants were encouraged to focus on critical incidents that enhanced or hindered their use of IBL. Some shared entries from their lesson journal.*
9	12/05/16	Using unstructured Tasks	*Facilitator presented videos of two teachers using unstructured tasks. Discussion focused on possibilities and challenges in using such tasks with students.*
10	26/05/16	Sharing learning experiences	*Teachers discussed experiences using IBL, reporting on their learning and development as teachers of mathematics. They focused on their CPD journey and being part of a community in their learning about IBL.*

Table 8: The schedule of follow-up meetings

Each flashcard had a phrase representing an experience, namely:

- *A positive experience in using IBL*
- *An unresolved challenge in using IBL*
- *A critical incident that encouraged you to use IBL*
- *A personal doubt about the use of IBL*

Teachers were asked to choose one aspect and write about it in some detail. The facilitator then invited teachers to share their writings with the rest of the group and a discussion followed. Such an activity afforded teacher participation but, more importantly, it supported teachers to learn from others and build on their existing knowledge.

3.4 Piloting the CPD programme

LTMI was not just about offering a CPD programme but one that supports teacher learning. Hence, an important role I assumed was to research the features of LTMI that facilitated teacher learning. Piloting, held during the period May 2014 to February 2015, was primarily intended to examine the content, the structure, the learning experiences and the feasibility of the programme. Participants in the pilot study were five secondary school teachers who chose to join this CPD for a number of reasons. They were either intrigued to know more about inquiry teaching, needed help to support their 'low-achieving' students and/or were dissatisfied with their traditional approaches to teaching mathematics. Table 9 shows data about the participants – all names are pseudonyms.

	Kevin	**Fiona**	**Lara**	**Nadia**	**Amanda**
Age Range (Years)	20 – 25	20 – 25	30 – 35	30 – 35	30 – 35
Teaching experience	Less than 5 years	Less than 5 years	Between 5 and 10 years	Between 10 to 15 years	Between 10 to 15 years
School	Non-state	Non-state	Non-state	State	State
Classes (Year)	7 and 8	7 and 8	9	7 and 11	10 and 11

Table 9: Participating teachers in the pilot study

To gather data about the CPD programme and materials, I (1) collected written and oral feedback received from critical friends and the CPD facilitators; (2) gathered teachers' written reflections and comments on the '*Thinking about*' booklets; and (3) conducted two interviews with teachers – one following their participation in the summer workshops (September 2014) and the other held at the end of the CPD programme (February

2015). Since I sought responses on the effectiveness of the CPD materials on teacher learning and the feasibility of the CPD programme, I felt that the participants would be the most important contributors. Understanding that it could be awkward for participants to be critical about the CPD design, I emphasised that their feedback would be crucial for me to make the necessary changes to eventually improve the programme for my main study.

Critical friends who attended the summer workshops provided feedback from a non-participatory observer position. My critical friends were two education officers of mathematics, a doctoral student who was, at the time, conducting a similar study with science teachers and the CPD facilitators – as these were having a first-hand experience using the CPD materials. I knew my critical friends well and had established a very good professional working relationship to trust that they would provide sincere analytical comments.

3.4.1 Testing the feasibility of the CPD intervention

SUMMER WORKSHOPS PROGRAMME

Feedback from participants on the end-of-session evaluation sheet was built into my field notes and the observations made by my critical friends. Timing of sessions seemed appropriate, with participants working, discussing and reflecting in an unhurried atmosphere. However, participants remarked that they benefitted from videos explicating more inquiry practices. Changes to two videos were carried out – one of the videos was eventually produced from a lesson taught by Kevin.

Rethinking and redesigning of the '*mathematical tasks*' workshop was necessary as teachers expected more opportunities to look into a range of tasks, reflect, discuss and select those that supported their planning of future lessons. Being held in July, this session and the materials presented seemed to offer teachers the opportunity to plan and do some research on tasks during the summer recess.

FOLLOW-UP MEETINGS

In the piloting phase, follow-up meetings were initially scheduled to take place once every three weeks. Participants however felt that this period offered limited time for planning especially at the beginning of the scholastic year when teachers need more time to settle in.

Moreover, in creating and cultivating a community, some form of external leading early on was needed to drive the discussions. To stimulate this, it was necessary for CPD facilitators to encourage teachers to (a) plan and discuss lessons using a non-routine task, (b) bring and share journal entries discussing classroom experiences, and (c) exchange 'open cases' involving unresolved dilemmas (see Nichols, Tippins, & Wieseman, 1997).

PAIRS RATHER THAN INDIVIDUAL PARTICIPANTS

Participants in the pilot study commented that they would have benefitted from support at school, possibly from a colleague. One possible strategy to begin building such communities in the participating schools was to request that pairs of teachers, rather than individuals, are involved in LTMI. The follow-up meetings could serve as ongoing occasions to bring all teachers together to share ideas and challenges, and seek solutions amongst colleagues from their own and other schools (see Goos et al., 2007). For the main study, inviting pairs of teachers from different schools was a deliberate strategy adopted to encourage participants to connect with a colleague from their own school while extending their experience beyond their own context.

3.5 Concluding note

Developing and implementing a CPD intervention programme like LTMI was a stimulating venture and a learning experience throughout. For countries like Malta, where the culture of cultivating learning communities and CPD for teachers is still relatively new and under researched (Attard Tonna & Shanks, 2017), implementing this CPD design brought with it challenges. The design of the CPD materials, the piloting of the programme

and the actual implementation phase involved a long, demanding process particularly in designing and trialling materials, gathering data and making the necessary improvements. Moreover, this also involved much time dedicated to finding teachers who would be willing to participate in either the pilot or the main study, and then speaking to their respective heads of schools about the CPD programme and coordinating timetable arrangements for a common time-slot for teachers to meet during the year. Moreover, I had to identify and then obtain approval from CPD facilitators, and then hold meetings with them to discuss the LTMI design principles.

In ensuring the feasibility of conducting LTMI, operating within the INSET training days was a deliberate choice to ensure that teachers could attend (Bubb & Earley, 2013). Having summer workshops that offered an intensive immersion in IBL followed up by ongoing meetings during the scholastic year was both a feasible and successful model in bringing to awareness the pedagogical strategies for promoting effective IBL.

Chapter 4: Research methodology

4.0 Introduction

This chapter describes the methodology used in gathering data about teachers' understanding and implementation of IBL as they participated in LTMI. Hence, the aim is to explain the process involved in constructing knowledge and understanding the phenomena under study. I start by discussing personal ontological and epistemological beliefs that define how I engage with research. I move on to explain my approach to collecting qualitative data and how this was used to generate theory about teacher learning. A thematic analysis approach and process is then described and justifications for using a case study methodology are provided.

A case study was used to describe the professional development of a group of secondary school teachers of mathematics. The study used both inductive and deductive methods within a thematic analysis approach to analyse, interpret and develop theory. I show how a rigorous thematic approach coupled within triangulation and reflexivity enhanced the quality of the research and helped to produce an insightful analysis to answer the research questions.

In addressing the research questions, I used five research instruments with methods for coding and analysing data that I explain below. I also describe how I selected participants and the ethical guidelines I adhered to throughout the study. Finally, I discuss dilemmas, boundaries and restrictions that I faced as a researcher having a dual designer-researcher role.

4.1 Ontological and epistemological beliefs

This study is guided by my ontological and epistemological beliefs which invariably had implications on methodological issues in understanding the phenomena under study. Ontological beliefs are concerned with the nature of reality within the world, while epistemological beliefs represent how we gain an understanding of what we know about the world and the

relationships between the knower and what is to be known. These beliefs are crucial, not only in understanding the approach implemented in undertaking this research, but also in uncovering knowledge of social behaviour from the way I, as a researcher, see the world (Cohen & Manion, 1989). In generating and developing theory, I draw on the constructivist paradigm using a thematic analysis theory (Boyatzis, 1998; Braun & Clarke, 2006) as a methodology to analyse my case studies.

A central tenet for constructivist researchers is gaining an understanding of individual meanings and realities (Guba & Lincoln, 1994). Blumer (1969) contends that the actions of humans towards things are a result of what things mean to them. Such meanings arise from social interactions that humans engage in and are developed within an interpretative process that humans use to deal with the things encountered. To understand what is happening one needs to understand what the actors themselves believe about the world (Charon, 1998), how they see and interpret it. As a constructivist, I acknowledge that multiple realities exist and these are inherently unique because they are constructed by individuals with unique beliefs and experiences that are also context-specific and time-related. I also operate within the assumption that, because of limitations of human inquiry, the inherent dimensions of how the world works cannot be discovered completely (Denzin & Lincoln, 1998). From an epistemological point of view, I worked to apprehend constructions and close approximations of reality in the world as experienced by individuals. In this study, it was teachers' views, experiences, development and learning about IBL that I was interested in. Participants were regarded as constructors of their own reality, and an understanding of their behaviour emerged from the practices, reasons and beliefs they revealed.

Qualitative research was conducted with the main data-gathering instrument being the researcher (Bogdan & Biklen, 1992). The researcher-as-instrument approach (Hatch, 2002; Patton, 2002) required observing participants in their contexts and talking to them to capture what they claimed. As a researcher, I sought to understand the complexities of

teachers' realities, their development and learning about IBL along their CPD journey. In order to do this, data about teachers were collected through multiple sources seeking highly detailed and rich descriptions of human behaviours, beliefs and understandings in an effort to reconstruct the constructions of how participants engaged in LTMI.

4.2 Case study methodology

While it seems difficult to describe case study (Bassey, 1999), my research design fits within the definition provided by Yin (2003, p. 13) who defines case study in terms of its scope as an:

> empirical inquiry that investigates a contemporary phenomenon within the real-life context, especially when the boundaries between the phenomenon and context are not clearly evident.

In case study, the researcher identifies the unit of analysis by specifying the boundaries of the case (Merriam, 1988; Yin, 2003). My unit of analysis is defined by a small group of Maltese secondary school mathematics teachers as they learn to enact IBL while participating in a one-year long CPD programme. This case study involved observations and interviews with seven teachers, of whom three were purposefully selected (see Section 4.5) for an in-depth investigation of contemporary phenomena focusing on teachers' understanding and enactment of IBL. This methodology served as an all-embracing strategy to research how participation in CPD may support teachers in learning to implement inquiry practices in their classrooms over time. Contextual detail and the representation of teachers' voices were key elements to focus on within the research design. The significance of the context – Maltese secondary schools and their mathematics classrooms – on teacher learning was also a highly relevant issue for selecting case study (Yin, 2003).

I chose case study because I sought to understand the voluntary participation of teachers teaching in state and non-state Maltese school as they come together and learn about IBL in a community of practice designed within LTMI. It was in these settings – the community of practice within the

CPD and teachers' classrooms – that teacher learning evolved through planning, discussions, classroom practices, reflections and evaluations. This study investigated the changes that three of the seven teachers made to their practices, and how they interpreted and used IBL. Research questions also addressed propositions emerging from the literature related to how teacher interpretation and enactment of IBL, lesson structure, changes in beliefs and practices, and JITL opportunities over the course of the programme (see Chapter 2).

4.3 Case study design

The purpose of this research was to examine the learning experiences of teachers in LTMI. This required an intensive process of data gathering and analysis of individual teachers working within their real-life contexts (Yin, 2003). In my role as a researcher, I interviewed seven participants at three different stages during their CPD journey, and observed their practices within the community and in their classrooms over the course of the CPD programme (see Figure 3). Case reports, constructed on "careful probing and thoughtful analysis" (Bassey, 1999, p. 87), followed a descriptive reporting style drawing a detailed picture of each teacher – their perceptions, beliefs and practices; their classrooms, students and observed lessons; their participation in the CPD and response to this experience.

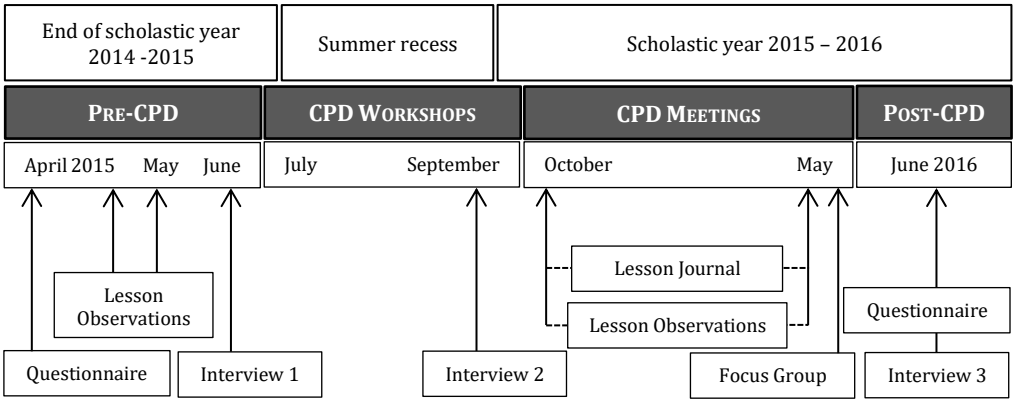


Figure 3: Timeline in using the data collection instruments

For the main study, twelve teachers volunteered to participate in the LTMI programme. Of these, seven provided consent for data collection with

access to their classrooms for observation purposes. Data were collected about each teacher and their professional contexts at different stages of the research. In this thesis, I choose to report on three purposefully selected cases (see Section 4.5). The choice of three cases offers broader interpretative perspectives and stronger analytical conclusions (Patton, 1990). The nature of multiple cases presented in this thesis is exploratory, building on existing knowledge and theories, but also being receptive to any new or emerging relationships or phenomena.

In case study design, a logical plan was used to link the data to be collected and analysis with the conclusions to be drawn to the research questions. Planning such a design involved thinking about the questions to study, the data to collect, and ways of analysing the results. I began the study with partially preconceived ideas about what the research questions could be. However, as Lincoln and Guba (1985) contend, the research design emerges as the research progresses. Indeed, a question on teachers' interpretation of IBL features evolved into one related to how they interpret and enact IBL over time. Through a reflexive iterative process, I reconsidered and modified my design decisions, and the boundaries of what I wanted to study, in response to new understandings. I simultaneously found myself engaged in a process that, according to Lincoln and Guba (1985), involves collecting and analysing data, developing and modifying theory, refining and refocusing the research questions, and identifying and addressing trustworthiness issues – with each activity influencing all others. The research questions somewhat evolved as the study did, and this process suited a 'theory-seeking' approach (Flyvbjerg, 2006).

4.4 Ethical considerations

Sociocultural contexts matter in all kinds of research but particularly in a qualitative study. This research delved into the 'privacy' of schools and classroom contexts. Gaining access to research sites, collecting, analysing and reporting data were issues of paramount importance. From the outset, participants were informed about the study, the commitments involved, the

right to freely decide whether to participate, and the right to withdraw at any time without risks and prejudice. This research was carried out in accordance with the British Education Research Association's Ethical Guidelines for Educational Research (BERA, 2011).

4.4.1 Entering the field and gaining access

Before entering the field, I obtained ethical approval to proceed with research from the School of Education ethics committee at the University of Nottingham (Reference: 2014/12/JG) and Maltese education authority officials. Following approval, teachers were informed about the CPD programme and research through an information letter (see Appendix 1). The information letter, which briefly explained the aim and structure of LTMI, was sent to schools via email in January 2015. In this letter, I also offered the possibility to meet interested teachers and discuss the CPD programme and the research study I intended to conduct.

Ten meetings with groups of teachers, from state and non-state schools, were held followed by seven meetings with heads of school of teachers who had accepted to participate. During meetings with heads of school, I explained the programme, outlined participants' and researcher's roles and responsibilities, and explored possibilities for timetable arrangements for teachers to attend the follow-up meetings. Following the recommendation of Hatch (2002, p. 46), I prepared a document with an outline of a "research bargain" in which I specified and explained the roles and responsibilities of the researcher and participants. Included in this were ethical measures through which potential participants would be safeguarded. Consent for participation was negotiated by discussing these in the face-to-face meetings held with each participant. I also specified what this CPD programme would be contributing to teachers and what they could gain from their engagement (see Section 4.4.2). An informed consent document was signed by participants, and both researcher and participants retained separate copies of it.

4.4.2 Respecting participants, minimising risks and maximising gains

When the focus of research is on people, researchers need to be cautious about minimising risks and maximising what people may take from their participation in the study. Consent and access were negotiated with the seven participants through a trusting ongoing relationship (Kvale, 1996). Malta being a small island and within my teacher training role, made it inevitable that I knew most of the participants. Hence, building and establishing a trusting relationship with each participant was potentially easier for me and the participants. I described that their contribution was important to my research and pointed out that pressure would not be placed on them to do more than they believed or felt they could. I also informed them that there would be no risk or prejudice should they choose not to continue in the project or withdraw at any time. Participants were provided with a timeline showing the important stages of data collection. For on-site data collection, I always offered participants to choose a suitable date and time for my interviews and lesson observations.

Reciprocity – what participants got out of their involvement in this research – was an ethical issue I considered. Participation in research did not just imply giving time and exposing one's practices and views to the researcher. Through LTMI, participants were exposed to gains that were greater than risks. CPD was intended to empower people, exposing them to innovative practices and supporting them to become reflective practitioners while engaging in a community of practice. Moreover, participants in this study had access to a range of online resources to support their CPD journey.

4.4.3 Reporting data and disseminating findings

In reporting findings, I attempted to capture as faithfully as possible the views narrated by teachers, their descriptions of activities, their learning and beliefs. I used "member check" (Koelsch, 2013, p. 12) as a means to validate what I had captured (Lincoln & Guba, 1985). Member checking involved presenting transcripts of lesson observations and interviews to participants and asking for correction and clarifications (Stake, 2010). Participants were in a position to determine whether I, as a researcher, had

accurately reported the events. I used member checking for two reasons: to seek correctness in recording their views and actions, and to ensure that respondents would not be upset or unhappy with what was being reported.

Data generated by the research were kept in a safe and secure location (an external hard disc kept in a safe place at home) and was used purely for the purposes of the research. Nobody other than the researcher had access to any of the data collected. All necessary steps were taken to protect the privacy and ensure the non-traceability of participants. To safeguard their identity I used pseudonyms, for both individual and institutional participants, in any written report of the research and other forms of dissemination. I also agreed with participants to provide the video recordings of their lessons at the end of the research, and that all data would be destroyed after the thesis has been successfully examined.

4.5 The participants

At the end of the data collection period, three participants (1 male and 2 females) were purposefully selected as case studies from a total of seven (2 males and 5 females). Case selection drawn from the five data sources available was not clear-cut. Given the large data set about each of the seven teachers, I used the sampling framework presented by Patton (1990) for guidance in purposefully selecting information-rich and different cases. Intensity and maximum variation sampling strategies led to the selection of three case studies – Chris, Greta and Sarah (see Table 10).

Intensity sampling: this involved selecting information-rich cases that had a lot to say about learning and teaching through IBL providing ample data for investigation. In identifying participants, I looked into transcripts and selection was based on those participants that provided significant text units to the coded data. Text units were regarded as significant not only with regard to length but also in providing relevant information in a concise manner.

Maximum variation sampling: this entailed identifying cases with key dimensions of variations. The cases varied from each other by including a

wide range of characteristics. My aim was to study how IBL was experienced, interpreted and implemented by teachers with different teaching experiences, holding different beliefs, working in different schools, and teaching different year groups. With this heterogeneous sample, I sought to identify common patterns that captured core experiences of the entire group. According to Patton (2002, p. 235), with a small sample of great diversity, data analysis would "yield important shared patterns that cut across cases and derive their significance from having emerged out of heterogeneity". Maximum variation sampling was hence used to construct as much of a holistic understanding of learning and teaching through IBL as possible.

Teacher	School	Prior Knowledge of IBL	Beliefs pre-CPD	Teaching Experience (Years)	Year Group Taught
Chris	Non-state	ITE	Transmission	1 – 5	7
Greta	Non-state	Course	Connectionist	16 – 20	8
Sarah	State	Course	Transmission	16 – 20	9
Tania	State	ITE	Discovery	1 – 5	10
Janet	State	None	Connectionist	11 – 15	8
Colin	Non-state	ITE	Transmission	1 – 5	9
Jackie	Non-state	None	Connectionist	16 – 20	10

Table 10: Information about the participants

Purposeful selection of multiple cases (see Patton, 1990) allowed for comparisons between the different cases investigated and to develop theory. Contrasting cases were selected for representation of different teacher prototypes (particularly related to their beliefs prior to their participation in LTMI), school contexts but also because of the distinct ways that these three teachers interpreted and enacted IBL in their classrooms (see Chapter 6). For example, teachers enrolled in CPD with different knowledge of IBL (see Chapter 5). Despite their different teaching experiences, Chris and Greta had limited exposure to IBL. Chris first encountered IBL in a session during his ITE training and Greta during an INSET course workshop. Sarah, on the other hand, had previously

participated in an IBL-related CPD and had also used IBL in her classroom. However, she still reported transmission beliefs, unlike Greta who had similar years of teaching experience, no prior IBL-related CPD and, yet, held connectionist beliefs.

4.6 Data collection phases

Data collection, spread over a 14-month period from April 2015 to June 2016, were designed to capture the beliefs, practices, experiences and understandings of IBL of teachers at different stages during the study. The research instruments and the rationale for designing, using and analysing the data collected were thus intended to capture teacher development.

To gather this data, five research instruments were used: semi-structured interviews, teacher questionnaires, lesson observations, teacher lesson journals and a focus group discussion. The interviews were intended to capture data at three critical points during the study – before teacher participation in the CPD, following their participation in the summer workshops, and at the end of the CPD programme.

The research design and data collection took place in four phases as shown in Table 11.

Research Phase			**Data Collection Instrument**
1	Pre-CPD	April 2015 to June 2015	**Questionnaire 1**: to gather baseline data **Lesson observations**: one 'typical' and one 'inquiry' lesson offered by teachers **Interview 1**: teaching of lessons observed
2	Post-CPD Workshops	July 2015 to September 2015	**Interview 2**: teachers' participation in summer workshops and what they may take and implement
3	CPD Meetings	October 2015 to May 2016	**Reflective journal**: to record and reflect on their inquiry lessons **Lesson observations**: four lesson observations with each teacher **Focus group**: a discussion with all participants was held at the end of May
4	Post-CPD	June 2016	**Questionnaire 2**: filling in the same pre-CPD questionnaire **Interview 3**: very similar to interview 2

Table 11: Research design and data collection phases

4.7 The research instruments

4.7.1 Semi-structured interviews

In designing semi-structured interviews, I focused on addressing teachers' knowledge, contexts, practices and professional learning experiences related to IBL. A semi-structured interviewing technique was used and guided by an outline of questions where probes and follow-up questions were not necessarily pre-determined. Within this outline, as an interviewer, I was free to follow leads and thus "to deviate from the interview questions as needed to pursue serendipitous findings and fruitful directions" (Vanderstoep & Johnston, 2009, p. 225). This technique offered some control over data collection, and data from one interview to another were likely to be quite comparable.

Interviews were intended to capture data at three critical phases during the study – before teachers' participation in CPD, just at the beginning of the scholastic year, and at the end of the project (see Table 12).

Interview	Description	Administered
1	*Views, practices and knowledge of IBL with a focus on the 'typical' and 'inquiry' lessons observed*	June 2015
2	*The summer workshops experience; what teachers may take and implement*	October 2015
3	*Retrospective reflective analysis of teachers' CPD journey*	June 2016

Table 12: The interviews

The first interview (see Appendix 5) established the background details of teacher experiences within the context of their work and explored what they valued in their teaching. Besides gathering more in-depth understandings of teachers' perspectives, this interview provided "avenues into events and experiences that have not been observed" (Hatch, 2002, p. 91) during the two pre-CPD lesson observations. A card sorting activity then challenged teachers to justify their choices, views and beliefs of the features offered in LTMI. The purpose of card sorting was to stimulate the discussion thus focusing on eliciting meanings from those salient aspects

transpired from my initial observations and questionnaire data analysis. Data triangulation was eventually used to generate personal profiles for each participant (see Chapter 5).

The second interview (see Appendix 6), conducted at the beginning of the scholastic year, asked teachers about ideas they could take from the workshops and use in their classrooms. Questions were directed to prompt teacher reflection and discussion around their familiarity with four IBL features (see Section 3.1.2) and the potential challenge they anticipated these would offer when attempting to implement them in their classrooms.

The third and final interview (see Appendix 7) was intended to elicit meaningful reflection of past, present and future practices. To this end, teachers were provided with their responses to interview 2 and asked to reflect on whether they noted and/or could identify changes in their responses to the same questions.

Interviews generally took between 30 and 45 minutes and were held in a quiet room (a classroom or office) at each teacher's respective school, except for the third interview with Chris held in the school yard, which lasted 80 minutes. The interviews were audio recorded using a laptop computer, transcribed verbatim and presented to participants for member checking (see Section 4.4.3). All interviews were held in the mother language (Maltese) and only the parts used in this thesis were translated. To ensure translation reliability, once translated to English, I asked a colleague (a University member of staff) to translate them back to Maltese. Upon cross-checking this version with the original, I addressed the minor modifications required.

4.7.2 Teacher questionnaires

To gather data about changes in teachers' beliefs and practices as a result of their participation in the CPD programme offered, teachers filled in the same questionnaire twice – before their participation in LTMI (April, 2015) and again at the end (June, 2016). The post-CPD questionnaire, however, had a

slight change to three questions in the 'mathematical inquiry' section (see Appendices 4). The questionnaires included the sections shown in Table 13.

Section	Description	Administered
Biography	*Teachers' name, school, teaching experience, qualifications, email and contact number.*	April 2015
Beliefs	*Teachers give weightings to nine statements regarding mathematics, teaching and learning.*	April 2015, October 2015 and June 2016
Lessons	*Questions related to how teachers teach a new topic, their typical classroom arrangement and resources used.*	April 2015 and June 2016
Classroom actions	*Teachers rank lists of 'classroom action' statements in order to demonstrate how they structure a typical mathematics lesson.*	April 2015 and June 2016
Inquiry	*Questions related to when teachers first heard about IBL, their definition of IBL, perceived challenges in implementing IBL and skills needed.*	April 2015 and June 2016
Pen portraits	*Teachers read descriptions of four teachers and underline phrases that they think most closely resonate with their own practices and beliefs.*	April 2015 and June 2016

Table 13: Outline of the questionnaire

The beliefs section of the questionnaire was administered with teachers at three different stages in the teachers' CPD journey: before, during and after. With this I sought to gather data and categorise teacher beliefs under *transmission*, *discovery* and *connectionist* (see Askew et al., 1997; Swan, 2006). I used the nine statements provided by Swan (2006) asking teachers to position their own beliefs about mathematics, teaching and learning by giving each one a percentage weighting. The three percentages in each section had to add up to 100. The first statement in each section of three corresponded to a *transmission*, the second to a *discovery* and the third to a *connectionist* belief. However, teachers were not made aware of this classification. Their ratings were analysed by working out the mean of the percentage weighting for statements under *transmission*, *discovery* and *connectionist*. These nine statements were also given to 58 mathematics teachers attending a separate three-day obligatory INSET training course in July 2015. Beliefs data of LTMI teachers before engaging in the CPD programme were compared with that of INSET teachers. This was carried

out to compare the beliefs of LTMI teachers with those attending a compulsory CPD course.

To elicit teacher practices for structuring a lesson, a list of 'classroom action' statements was provided. Teachers had to arrange each statement in order to demonstrate how they typically structure their mathematics lessons. Responses to these 'teacher actions' were analysed by looking at the order indicated. Finally, four teacher pen portraits adapted from Swan (2006) offered descriptions of teacher practices, views, concerns, challenges and constraints related to their classroom practices. Teachers were invited to underline phrases that resonated with their own practices. Underlined phrases and comments were collated to create detailed descriptions of each teacher's views and practices. By collecting together underlined phrases, comments and 'classroom actions' statements, 'pen portraits' were developed for each participating teacher.

The second questionnaire was intended to identify potential changes in teachers' views, understanding of IBL, practices and beliefs. The same procedure for analysing the first questionnaire was used and responses were compared. To analyse teachers' beliefs changes, triangular plots were used (see Graham & Midgley, 2000). These represented tri-variate data in which the three variables – in this case, *transmission*, *discovery* and *connectionist* beliefs – represented proportions of a whole.

4.7.3 Lesson observations

Over the course of the study, teachers were observed teaching on six occasions – twice prior and four times during the CPD programme. The two pre-CPD lesson observations were audio recorded and field notes were taken. Field notes included records of timing and content of the main phases of the lesson, the resources used and anything that the teacher and the students wrote on the whiteboard. The field notes and the audio recordings were combined to produce detailed transcripts and accounts of each lesson.

During the scholastic year, teachers offered four lesson observations during which they implemented IBL. The four inquiry lesson observations, which

were between 35 and 45 minutes long, were spread over the year (see Table 14). All lessons were video recorded, and later transcribed. A fixed mini wide-angle video camera with rotatable lens, placed at height of 2 m, usually at the top of a cupboard, captured the whole-class activity while a portable tablet camera was used to focus on small-group interactions.

Lesson Observations		**Period**	
Typical lesson Inquiry lesson	Audio recorded Field notes	Pre-CPD	May 2015
Inquiry lesson 1	Video recorded	During CPD	November to December 2015
Inquiry lesson 2			January 2016
Inquiry lesson 3			April 2016
Inquiry lesson 4			May 2016

Table 14: Conducting lesson observations

For lesson analysis, the time-line presentation to problem-solving research by Schoenfeld (1985) was applied. The diagrams (see Figure 12) provided lesson structure and organisation analysis with lessons divided into 1-minute intervals. The four phases (teacher presentation, individual work, small-group activity and whole-class discussion) are presented on the vertical axis and their occurrence during the lesson is shown along the time-line (1 minute as a time unit on the horizontal axis).

Video recordings were viewed in their entirety to get a sense of the content, with each lesson transcribed by writing short, time-coded descriptions of the content of the recordings (see Appendix 10). Emerging issues from this preliminary analysis were then investigated further for a more detailed account of lesson structure, IBL features enacted, teacher questioning and activities taking place. Teacher questioning was investigated using the classification provided by Sahin and Kulm (2008) involving three types of questions: *factual*, *guiding* and *probing* (see Section 2.3.3). To this I add a fourth type which I call *confirming* questions. Confirming questions included teacher interventions used to check upon students' work or to verify students' understanding confirming a correct step, a procedure or an answer to a question. The place and frequency of questions supported

analysis of the types of questions asked within each phase of the lesson (see Appendix 10 for examples).

4.7.4 Lesson journal

Of the various materials and experiences available for developing critically reflective teachers (see Nichols et al., 1997), journal writing was used. Eliciting responses in journals was however dependent on what teachers chose to reveal of their thinking. The teacher lesson journal (see Appendix 8) prompted teachers to comment about the lesson and describe what they wanted to attain, what challenges they encountered in planning for and doing the lesson, what they had learned, what was the cause of the experience, and what they would like to improve on or do differently.

The lesson journal was provided to teachers as a booklet in which they could write post-lesson reflections of their inquiry practices. Over the period of the study, journal entries ranged in both number and the amount of detail that teachers included. On average, teachers described between eight and nine learning experiences each, with a total of 27 collected from the three case-study teachers.

4.7.5 Focus group

As a qualitative method for gathering data, the focus group brings together the researcher and participants to discuss a research theme. According to Krueger and Casey (2015), the research theme is explored collaboratively through the attitudes, perceptions, feelings and ideas that participants share, observing them as they engage in discussion.

Through the focus group discussion, held with the seven participants in May 2016, I collected the shared understandings of individual teachers within the community, yielding data about the meanings that lie behind teachers' CPD experiences and their use of IBL. The focus group participants were led through the discussion by the researcher, acting as a moderator, using questions as probes and prompts for participants to elicit experiences, meanings and insights (see Appendix 9). The discussion, held in Maltese,

was video-recorded and later transcribed verbatim using pseudonyms. The same procedure used for translating the interviews was followed.

4.7.6 Piloting the Research Instruments

The pilot study, carried out with five participants in the period May 2014 to February 2015 (see Section 3.4), was used as a procedure to pre-test the quality of the research instruments (Baker, 1994). As a procedure, I tested the research instruments to see if the planned data collection instruments accomplished what they were envisioned to. In this test-run, I followed an iterative cycle of designing, testing and refining the research instruments. For lack of space, I will here focus on piloting the interviews – the main data collection instrument for this study.

The three interviews were trialled to test that they captured the process and the changes that teachers made. In the piloting phase, the final interview offered a retrospective analysis with questions focused on eliciting teachers' thinking and practices before and after. Yet, it appeared that respondents struggled to get into the depth needed in identifying potential changes they had made and particularly in identifying reasons for these changes. This set me thinking about a modification that could facilitate teachers' reflections to provide more profound data. Instead of operating the interview with new questions, as I had originally planned, I decided to provide teachers with a transcript of their second interview and ask them to reflect about the same questions. The starting question I posed was – *would you give the same answer to this question now or would you provide a different response, and why?* This eventually worked better as participants appeared to muse deeply over their responses, and came up with reasons why they had responded the way they did then, and in identifying reasons for potential changes that had occurred or for maintaining invariable points of view and practices.

According to Patton (2002), piloting also serves to immerse the researcher in the researcher-as-instrument role. Trialling, hence, helped to check about the time I was allowing participants to respond before intervening, and to test that the interview time was reasonable and within that planned and

agreed with participants. As part of this analysis, I asked participants for feedback so that they could identify potential ambiguous questions. This feedback focused on participants' experiences, emotions and thoughts through questions such as: *Which questions were unclear or awkward to answer? Did you feel that I was somehow expecting a particular answer from you, different from the one you wanted to provide? Are there any questions you would add?* Based on participants' responses to these questions, some questions were reworded for clarity and focus.

4.8 A thematic analysis approach to theory

Thematic analysis, first named as an approach by Merton (1975), involves a search for themes that are key in describing and interpreting phenomena (Hatch, 2002). A thematic approach to analysis is a suitable method for "identifying, analysing, and reporting patterns (themes) within data" (Braun & Clarke, 2006, p. 79) gathered from teachers' interpretation and enactment of IBL along their CPD journey. Braun and Clarke (2006, p. 82) state that "a theme captures something important about the data in relation to the research question, and represents some level of *patterned* response or meaning within the data set". Using a thematic approach requires pattern recognition within the data, and emerging themes become the categories for analysis. Themes emerge from the data as a result of the active engagement that the researcher plays in coding, identifying patterns and themes, selecting these and eventually reporting them (Patton, 1990). In other words, themes do not reside anywhere except in the thinking that the researcher engages in to make sense of the data and creating links.

I went into my research with a partial framework of concepts and theories, acquired from my reading of research literature coupled with my own experiential practice of working and learning with teachers (see Chapter 1). With this initial knowledge, I could be "theoretically sensitive" (Glaser & Strauss, 2006, p. 46) in conceptualising and formulating theories based on the emerging data about teacher practice. It was a developmental process that progressed towards addressing the gaps in research framed by my

research questions. I approached my data open to concepts that could emerge, but also with preconceived categories derived from theory (Fereday & Muir-Cochrane, 2006). I started by collecting data and then searched for theoretical constructs, patterns and themes within the data. The method of analysis chosen for this study was a hybrid approach to qualitative methods of thematic analysis, incorporating both a data-driven inductive approach (see Boyatzis, 1998) and a deductive approach. The deductive approach draws on zones of enactment (Spillane, 1999) as a sociocultural theoretical framework (see Section 2.1). For example, 'IBL tasks' was a code generated inductively from teachers' descriptions of their planning and enactment of IBL practices. On the other hand, descriptions related to students' experiences, responses and reactions were coded as 'pupils' – a deductive code informed by the zone of enactment theory referring to one of the five external factors impinging on teachers' enactment of reform. This process involved analytic procedures of comparing inductive and deductive codes that eventually complemented the emergence of themes that served to answer the research questions.

Since I was interested in thematic analysis at an interpretative level, I identified themes at a latent rather than a semantic level (see Boyatzis, 1998). A semantic approach considers the surface meanings of data – descriptions and events – without looking at anything beyond what participants say. My analytical process progressed from first describing data and showing patterns and then interpreting these with an attempt towards theorising (Patton, 1990), and relating this to research literature. At a latent level, I attempted to examine underlying meanings, assumptions and conceptualisations that teachers developed about IBL and their enactment of it. With thematic analysis, I sought to theorise the sociocultural contexts through teachers' personal resources (existing knowledge, beliefs and dispositions) and resources within their environment that enabled or hindered their enactment of IBL.

4.9 Coding data and generating themes

Braun and Clarke (2006) provided a six-phase guideline to thematic analysis that I followed. These phases are: (1) familiarising yourself with the data; (2) generating initial codes; (3) searching for themes; (4) reviewing themes; (5) defining and naming themes; and (6) producing a report. After collecting data and transcribing it, I made repeated readings of it. Both the process of transcription and my subsequent readings of data sets (usually done at least twice) provided familiarity that was crucial before I started coding (Bird, 2005). Developing the transcripts, consisting of verbatim accounts (see Figure 4 as an example), was a time-consuming process that provided a thorough understanding of events, descriptions and meanings as conveyed by participants.

	A	B	C	D	E
1	Teacher	Transcript	Code 1	Code 2	Code 3
2	Ch2(1)	A nice experience because I felt I could share my ideas with others and learn from others. I felt comfortable working with other teachers and I do believe I learned a lot.	Experience	Share Ideas	Community
3	Ch2(1)	Since speakers leading the sessions were teachers themselves, I could easily relate it to what they were saying.	Speakers	Teachers	CPD
4	Ch2(1)	I also feel that the mix of teachers coming from state and non-state schools was beneficial	Mix of teachers	Networking	Community
5	Gr2(1)	We were constantly involved in hands-on activities, just like a maths lesson should look like. Really enjoyed that.	Activities	Modelling	CPD
6	Gr2(1)	I really like the fact that the sessions and the tasks we were presented with mirrored what we can do with our students. I got the message that the PD session structure provided a model for participants of how an IBL lesson should look like.	Activities	Modelling	CPD
7	Gr2(1)	I really appreciate something that I do not usually do in my class, that is, giving some time for us to think on the task individually, then in pairs and then as a group	Ideas for Practice	Modelling	CPD

Intrvs2&3 | Community | PD | Personal | Knowledge Learning | Knowledge | Learning | Beleifs Change | Students | Knowledge Cont. | Analysis | Intrv2 | Coding | Intrv3 | +

Figure 4: Using data codes

Data collected through interviews, lesson journals and the focus group were divided into chunks – usually short paragraphs of between 20 to 60 words – applying an open-ended coding technique to label chunks and assign codes on the margin. The coding process developed on the basis of inductive codes grounded in the content of the data, and then moved onto more theoretically driven codes informed by research in the area. I looked for meanings and patterns and noted these down as short notes, initially starting off coding the data using words and descriptions provided by the participants (codes 1 and 2). However, theoretical and epistemological predispositions informed the next level of coding (code 3). Data chunks, cross-referenced to codes were then copied and pasted into a spreadsheet,

with category codes appearing in adjacent columns next to each block of text.

To ensure intercoder reliability, Miles and Huberman (1994) suggest involving an independent coder in coding the data. However, I believe that it is unlikely that two people can apply the same subjective perspective to the same text (see Joffe & Yardley, 2004), particularly when coding was deductively informed by a theoretical framework. An independent coder, I believe, is likely to interpret and categorise codes differently. Hence, once I was satisfied with my category codes, I discussed these with a critical friend selecting parts of one interview, a lesson observation and the focus group transcript. My critical friend was a colleague who taught mathematics and had ample experience working with teachers. Deliberations with my critical friend were considered essential to refine codes. No modifications were made to my predetermined codes.

Each time a block of text was category coded, I compared it with the previous one in the same and different groups coded within the same category. Essential to this process was the 'constant comparative' method (Glaser & Strauss, 2006; Strauss & Corbin, 1998) with which I engaged in analysing data from the different data sources. Constant comparison was employed to ensure that empirical data, rather than personal impressions, guided the findings. Constant comparison entailed detailed analytical processes using both inductive and deductive thinking (Hatch, 2002). In coding the data, I constantly looked for examples of similar cases and repeated patterns of meaning. This approach facilitated my sense making of what participants conveyed.

In coding blocks of text, I usually found conflicts in the way I was thinking about coding categories. More often than not, I mused over theoretical notions while concurrently trying to concentrate on studying the next incident, description or event. The conflicts I encountered were related to the subjective nature of my analysis, finding myself trying to determine alternative ways of coding and comparing subsequent incidents. That generally signalled a stage when I had to stop coding and start writing a

memo of the emerging ideas, trying to make sense of what the data were telling me. Memo writing served as a reflective exercise to illustrate the ideas that emerged, eventually helping to clarify my thinking. Analytical memos provided an avenue for focusing on a theme to examine data at a greater level of abstraction, exploring hypotheses and relationships but also being inquisitive about meanings and properties delineated within particular categories. Through memos I could make conceptual leaps from raw data to abstractions (Birks, Chapman, & Francis, 2008) that explained research phenomena in the context in which I was studying them.

The following phase involved searching for possible themes. Using the filtering tool available on the MS Excel 2013 spreadsheet, I copied and pasted the coded data in order to be able to sort the different codes into potential themes. With a focus on generating themes, I collated all the relevant data within an identified theme. This was my first step in analysing my codes, and considering how different category codes could combine to form a particular overarching theme. At this point, I constructed a visual representation of an initial thematic representation consisting of different codes. As Figure 5 illustrates, I ended up with a number of themes with arrows indicating links to sub-themes. This representation provided a means for making sense of how these themes may be connected while also indicative of the possible emerging themes. Hence, I reviewed the coded data chunks collated to check that they formed a coherent pattern within an overarching theme. Validating themes (Miles & Huberman, 1994) was essential and required reviewing and refining. The result was a refined thematic representation (see Figure 6) consisting of two overarching themes related to teachers' use of IBL.

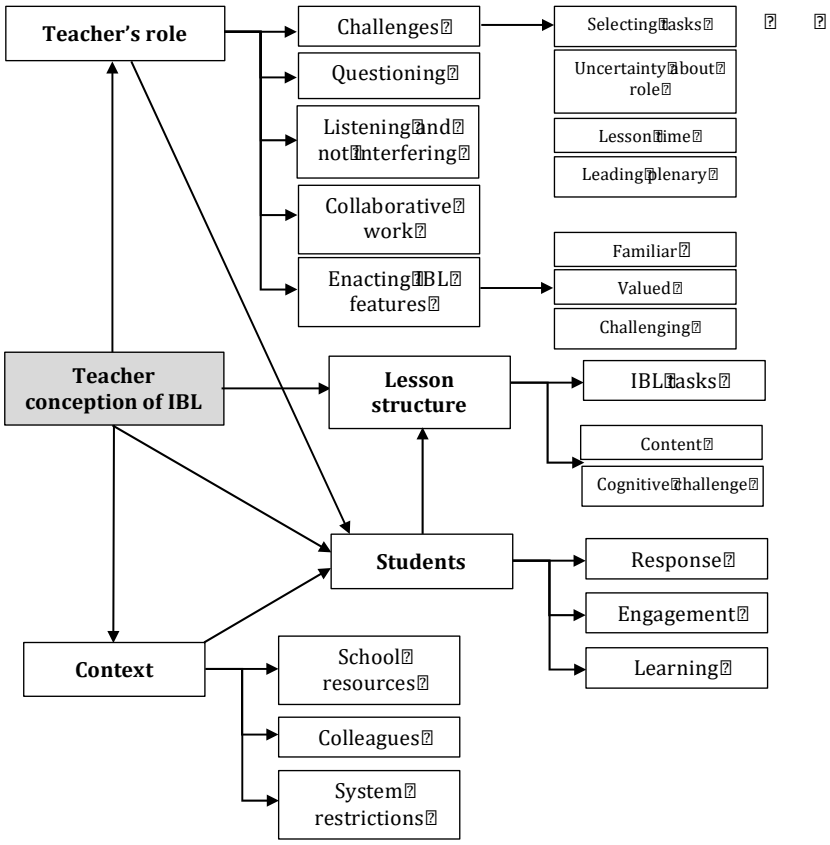


Figure 5: Initial thematic representation

The next process was linking these themes to the entire data set. This involved rereading the data set to make sure that themes work, but also to code any additional themes that may have been missed in earlier coding. Having an established thematic representation of my data, I defined and named each theme to determine what aspect of the data it captured. While there might be an element of overlap between the themes, these were still distinct and the final thematic representation (see Figure 7) provided an illustration to answer a specific research question.

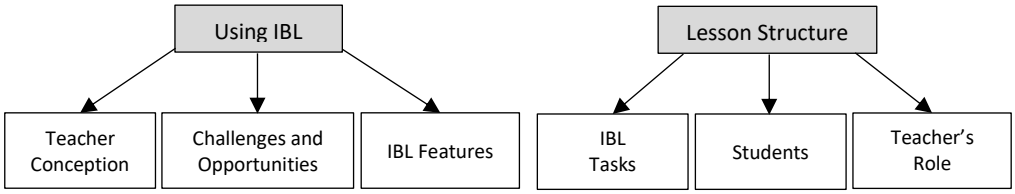


Figure 6: Refined thematic representation

The final step was hence the writing up of this thesis, showing how thematic analysis facilitated answering the research questions going beyond the descriptive element of the data, making arguments and providing interpretations grounded in and supported by data.

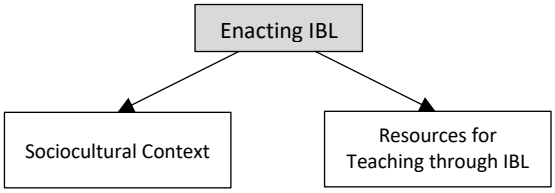


Figure 7: Final thematic representation

4.10 Undertaking dual roles

Approaching my work as a designer-researcher, I faced a dilemma in balancing these two roles. In designing and initiating this CPD intervention, I dedicated considerable thinking and effort to offer the best design possible (see Chapter 3), and making this CPD experience a successful one towards supporting teacher learning. I sought success from within a well-engineered programme that could be delivered by others – CPD facilitators (see Section 3.1.4).

As a designer-researcher, my roles evolved through a reflexive process involving self-doubt (Ball, 1990). I kept a reflective journal that helped me to problematise my role at the different stages of the research. For example, I assumed different roles from the piloting to the main phase of the study. In piloting the programme I was deliberately involved. For instance, I facilitated some workshops to get a first-hand experience using the designs. On numerous occasions, I asked participants for their feedback and discussed suggestions with them. However, once the designs had been developed, I assumed a different role in the main study. Lutz and Iannaccone (1969) describe three different roles of a participant-observer: (1) the participant as observer; (2) the observer as a limited participant; and (3) the observer as a non-participant. Deciding about my level of involvement implied considering how participation in the particular context could influence the natural flow of events. In conducting my

research, I had a 'disinterested interest' (Bourdieu, 1996) and assumed a non-participant observer role. I was present for all summer workshops and follow-up meetings, but refrained from participating in conversations, staying on the margins of community interactions. This brought with it critical decisions about my role.

Guest, Namey, and Mitchell (2013) suggest that the qualitative researcher's role is based on three key elements: conducting research in situ, building rapport with participants, and spending time interacting with them. Getting into the location was discussed in Section 4.4.1. As Ball (1990) claims, entry does not guarantee access. Indeed, access to teachers' classrooms was perhaps the most awkward to deal with. An important decision was the way I presented myself to students when observing lessons. In agreement with teachers, they introduced me as a person leading a CPD project which they were participating in. They explained that my role was not to focus on students but on how teachers took up ideas from the project to plan and teach lessons. The idea, behind this brief introduction focusing on observing the teacher rather than students, was to make myself acceptable (Ball, 1990) and minimise students' negative perceptions that could be created by my entry – as these could potentially inhibit or distort their behaviour. The next important decision was where to stay in order to be in the least possible observable position, yet remaining able to capture the activities taking place in the classroom. I found it helpful to sit down during teacher explanations, whole-class discussions and presentations in a corner at the back of the classroom. Video-recordings of whole-class activity afforded taking this peripheral positioning. However, in accordance with the teacher and the students, during group work I usually followed a group more closely using a portable tablet to video-record discussions.

In understanding teacher development and learning, it was important to develop a close relationship with the LTMI teacher community and with each participant. This was an important step in understanding each individual as well as the activities they engaged in. This presented a complex endeavour. Having a 'disinterested interest' involved looking from

a standpoint which was disconnected from personal, contingent interest in the object under study. In other words, it is an interest disconnected from personal feelings, biases and preferences. This was indeed difficult because as Schwandt (1993, p. 8) contends, it is impossible to observe and describe "the way things really are, free of any prior conceptual scheme or theory ... without some theory of what is relevant to observe, how what is to be observed is to be named, and so on". Indeed, personal biases were inevitable and engaging in a reflexive process was crucial in remaining aware of social influences.

Building rapport with the participants was an important step as I sought to build trust. In cultivating trust, I dedicated time to informal interactions before and after interviews, CPD sessions, and lesson observations. However, for two participants, my non-participant role was difficult to understand. They identified me as the 'expert' – someone who had designed a CPD programme to support their implementation of IBL, and who was 'refraining to help' when the programme was being implemented. Particularly during the initial follow-up meetings, I had to constantly remind teachers that my presence was not to provide views, judgements, support or guidance towards their learning of inquiry – this was the facilitator's role and learning was their responsibility.

4.11 Conclusion

This chapter provided an overview of the case study research process, personal beliefs and approach to generating theory within this study of teacher learning and enactment of IBL practices. From now onwards, the study will focus on the professional learning journeys of three teachers purposefully selected as my case studies. These case studies show teachers working within unique yet comparable contexts. Moreover, their learning to teach experiences generated important insights into teachers' conceptualisations, enactment and beliefs about IBL and its practice in their classrooms. The scope of the next chapter is to introduce the details of each case prior to their engagement in the CPD programme. Being an exploratory

study, the prior experiences, beliefs and perceptions of teachers becomes crucial in eventually understanding how these teachers developed through their LTMI learning journey.

Chapter 5: Introducing the case studies

5.0 Introduction

In this exploratory research, I chose a case study approach to study human learning (Flyvbjerg, 2006) through an in-depth analysis of three of the seven CPD participants. In this chapter, I present an analysis of each individual case, focusing on teachers' beliefs and practices before they engaged in CPD. The three teacher-participants – Chris, Greta and Sarah – were purposefully selected as my case studies, based on the schools in which they taught, the year group with whom they chose to implement IBL, their teaching experiences, prior knowledge of IBL and their beliefs about mathematics, teaching and learning.

Moreover, in describing and analysing the data, I use zones of enactment theory (Spillane, 1999) to explore and understand how their teaching was influenced by the *pupils*, *policy*, *professional*, *private* and *public* sectors. Understanding teachers' practices and their understanding of IBL prior to their participation in LTMI is essential to explore their enactment of IBL and how this developed through their CPD journey. This theory further informs the cross-case analysis that will be discussed in the subsequent chapters. As a theoretical framework, enactment zones serve to identify commonalities as well as differences in teachers' learning and enactment of IBL.

5.1 The three case studies

The three case studies that I report on were selected from a total of seven participants using intensity and maximum variation sampling (see Section 4.5). Sampling was done purposefully (see Patton, 1990) to select information-rich and different cases of teachers that allowed for representation and comparison of different teachers.

Chris was in his late twenties and had been teaching mathematics at a non-state school for less than five years. During his first years of teaching, the school encouraged Chris to attend INSET training courses, but none of these related specifically to the use of IBL. I chose to present this case because

Chris was working in, what appeared to be, a supporting environment. However, the education system was a factor that essentially influenced his practices and beliefs. To me, Chris represented the 'typical' Maltese teacher. He was motivated to make changes to his practices, yet struggled to do as much as he wanted because of system restrictions. Chris' case is also worth exploring because of his willingness to learn and experiment, his agency to act and the moments of uncertainty, doubt and disequilibrium that he encountered. Chris' case is selected for theoretical reasons as well. It illustrates a particular kind of transition from his early transmission beliefs that allows analysis of significant changes in relationships between his physical environment – the school and the group of students he implemented IBL with – and the CPD activities that he was exposed to.

Greta was in her early forties and had been teaching mathematics for nearly 20 years. At university, Greta followed an ITE course and then enrolled in a post-graduate degree course in mathematics education. Greta had heard about IBL in a one-off INSET professional development session but had not participated in any IBL-related CPD. I chose the case of Greta because she had been teaching for quite some time and, over the years, she had been implementing end-of-topic investigations, that to her, stimulated students to inquire. Additionally, it was important to explore this case as Greta held predominantly connectionist-oriented beliefs (see Section 5.2.1) which is quite uncommon to find, even compared to INSET teachers (see Table 16).

Sarah was in her late thirties and had been teaching mathematics for over 15 years. At university, she followed an ITE course and subsequently gained knowledge about IBL through her 18-month participation in a CPD programme a couple of years back. The case of Sarah is important for a number of reasons. Sarah together with a number of mathematics teachers at her school had already participated in a CPD programme promoting the use of IBL pedagogies. During this CPD, Sarah used IBL but only occasionally. Sarah explained that although this experience was important in exposing teachers to IBL, after the project terminated none of the mathematics teachers at her school appeared to continue using IBL. As a

result, she could not rely on support from her school colleagues when it came to implementing IBL. Hence, she saw LTMI as an opportunity to gain continuing support in resuming with IBL practices. Prior to her participation in LTMI, Sarah described that she was "fed up teaching the same way" (Interview 1, June 2015). Over the years, teaching mathematics for her involved talking, explaining and demonstrating. She felt this was by now unexciting yet necessary given the exam syllabus she was required to 'cover'. Sarah was motivated to reconsider her beliefs and practices and LTMI appeared to offer her this possibility.

5.1.1 The school contexts

Table 15 outlines the school contexts in which the three case study teachers worked. Although there are some differences between Maltese state and non-state schools, all schools are generally well resourced and offer technological tools. Moreover, all schools endorsed active learning pedagogies and supported teacher participation in CPD.

Teacher	School	Mathematics curriculum	Classroom setting	Students
Chris	St Cecilia girls' non-state	Exam-driven with teachers having control on content and assessment	Three rows facing the interactive board	Set in mixed-attainment groups
Greta	St George boys' non-state	Exam-driven with teachers having control on content and assessment	Pairs of desks set in columns and rows facing the interactive board	Set in mixed-attainment groups
Sarah	St Simon boys' state	Exam-driven with teachers having no control on content and assessment	Five rows facing the interactive board	Set by their attainment levels

Table 15: School contexts

5.2 Teachers' beliefs and practices pre-CPD

In the following section, I provide details about teachers' beliefs and teaching practices, snapshots of their classroom practices prior to LTMI, their planning and teaching of an inquiry lesson, reasons for choosing a

particular class for my classroom observations, and their perceived challenges and opportunities in using IBL. Data were generated by the pre-CPD teacher questionnaire, interview and two lesson observations.

5.2.1 Beliefs about mathematics, teaching and learning

In rating their beliefs about mathematics, teaching and learning, data revealed that Chris and Sarah held predominantly transmission-oriented beliefs while Greta held connectionist beliefs (see Table 16). For example, Chris and particularly Sarah rated that teaching involves structuring a linear curriculum for the students; giving verbal explanations, practising questions and correcting misunderstandings. With regards to learning, while Chris believed that it involves a mix of individual and interpersonal activities based on exploration, discussion and practice, Sarah believed it involves an individual activity based on watching, listening and practice for fluency. Moreover, data from her pen portraits showed that, while Sarah valued student discussion her practices were teacher-directed and driven by teacher talk. Similarly, Chris' beliefs in learning through discussion were not reflected in his classroom practices. The percentage ratings given by Chris and Sarah on the discovery and connectionist orientations were quite low compared to those of LTMI and INSET teachers. Their strong orientation to transmission beliefs ratings contrasted with those provided by LTMI and INSET teachers, suggesting dominance of a firm view of mathematics, teaching and learning.

Greta, on the other hand, rated that teaching involves a non-linear dialogue between teacher and learners where connections are explored verbally. She did not see learning as an individual activity but rather as interpersonal in which learners are challenged. Greta also viewed mathematics as an interconnected body of ideas that the teacher and students create through discussion. Drawing from data on her pen portraits, Greta claimed that discussion and pair work were common activities in her lessons. Yet, she notes that her connectionist beliefs are inconsistent with her classroom practices. Moreover, her strong orientation to connectionist beliefs ratings contrasted highly with those provided by LTMI and INSET teachers.

Teachers	Percentage Beliefs Ratings		
	Transmission	Discovery	Connectionist
Chris	42	33	25
Greta	18	15	67
Sarah	67	23	10
LTMI (n = 7)	35	31	34
INSET (n = 58)	28	36	36

Table 16: Teachers' beliefs about mathematics, teaching and learning

5.2.2 Snapshots of classroom practices prior to LTMI

When discussing lesson planning and teaching, teachers stated that they use a rather consistent and predetermined process. For example, Greta preferred to start her lessons with a whole-class discussion using questioning to involve students. This was usually followed by providing an explanation, examples and notes before students moved on to work on an exercise individually or in pairs. Likewise, Sarah starts her lessons with a discussion to revise work done and evaluate students' knowledge. Lessons usually always include a 10-minute part during which students copy down notes and end with work on more challenging questions. Chris reported a similar lesson structure.

> My teaching style is like the one you observed and according to the scheme of work. I look for exercises that are purely related to the topic, connected to what I would have said during the previous lessons. It is quite a linear process… So, I usually start with an activity to remind students about the work done in the previous lesson, trying to elicit these from the students, so that then I add and introduce new material.
>
> (Interview 1, June 2015)

On observing a 'typical' lesson that Chris offered, his practices seemed very traditional and resonated with the description of classroom practices that he provided. By and large, his lesson on circle theorems was teacher-directed. Chris did much of the talk (see Table 17) and led during most parts of the lesson. During class correction, he provided students with answers and explanations to correct their mistakes. Chris rarely asked questions. In

the few occasions where he did, the questions required factual knowledge – asking for specific facts, definitions, answers to a question or to provide the next step in a procedure he was performing. Occasionally, Chris did attempt to probe for students' understanding but generally tended to demonstrate, guide and explain.

Time Interval	Lesson Phase		Teacher talk	Student talk
00:00 – 05:00	**1**	*Class correction*	402 (85%)	68 (15%)
05:00 – 17:00	**2**	*Revise circle theorems done in previous lessons*	962 (81%)	226 (19%)
17:00 – 25:00	**3**	*Introduces new theorems*	830 (95%)	44 (5%)
25:00 – 35:00	**4**	*Students work on questions in small groups*	525 (79%)	140 (21%)

Table 17: Analysis of Chris' 'typical' lesson

In analysing classroom talk, I counted the number of words uttered by teachers and students (using the Microsoft Word 'Word Count' option on the lesson transcript) during each phase of the lesson. Chris spoke for most of the time. Even when students worked in small-groups (lesson phase 4), he still dominated the conversations uttering over three-quarters of the words spoken. When most of the groups called an incorrect answer, Chris preferred to offer his own method for working out the question. His questions were mainly factual, for example, "*We are told that this is a diameter. So, the angle is?*". On other occasions, Chris used questions that sought to confirm students' response, for example, "*Answer C. Are there others who selected C?*"

The 'typical' lesson that Greta offered also demonstrated an approach that was very teacher-directed. The lesson had distinct phases (see Table 18). Greta did most of the talk during each phase of the lesson during which she explained, asked questions and answered students' queries.

Time Interval	Lesson Phase	Teacher talk	Student talk
00:00 – 10:00	**1** *Class correction*	671 (86%)	108 (14%)
10:00 – 28:00	**2** *Explanation and questioning*	1444 (85%)	253 (15%)
28:00 – 36:00	**3** *Individual work*	Not applicable	
36:00 – 39:00	**4** *Assigning and explaining homework*	117 (98%)	2 (2%)

Table 18: Analysis of Greta's 'typical' lesson

Moreover, teacher-student interactions were dominated by factual questions seeking answers to close-ended questions. The lesson observed was about recognising straight-lines with positive and negative gradients. The following excerpt demonstrates a typical interaction that Greta engaged in with her students (pseudonyms).

On the interactive board, Greta displayed the graphs of $y = 2\boldsymbol{x}$ and $y = -3\boldsymbol{x}$.

(01) Teacher: We have this graph ($y = 2\boldsymbol{x}$) where the number next to x does not have a minus in front of it. If I look at it from this way the graph goes uphill. Can you notice this?

(02) Mark: Yes

(03) Teacher: Pay attention please… Luke, and what happened to the line when the number is negative?

(04) Luke: It goes downhill

(05) Teacher: Very good. This one ($y = -3\boldsymbol{x}$) which has a minus in front, this goes downhill. If I look at it this way, from left to right, I see it going downwards. Do we agree?

(06) Students: Yes

(07) Karl: So, if the line goes upwards from left to right it will be positive

(08) Teacher: That is what we are saying Karl! The number will have a minus in front of it is going downhill, and when it does not, it would be positive and it then goes up.

Interventions (01), (05) and (08) show that Greta explained things and, although she had described using whole-class discussions, discussions in this particular lesson were generally with individual students. She seldom allowed students to take initiative and they appeared to wait for direction from the teacher. It seemed important for Greta that her students paid attention, followed her explanations and responded to the factual and closed questions she offered.

My observation of the 'typical' lesson offered by Sarah, and the subsequent analysis, reflected a transmission approach. Over the years, Sarah learned that students preferred to assume a passive role, to be guided and told what to do. She described that in her lessons she would "guide students and tell them what and how much work they should do" (Interview 1, June 2015). The lesson, with her Year 8 class, was about solving equations from story problems. Following a short explanation of the work carried out in the previous lesson, Sarah moved on to work four graded textbook examples with students. In the final part of the lesson, students were assigned three further problems to work out individually (see Table 19).

Time Interval	Lesson Phase		Teacher talk	Student talk
00:00 – 05:00	**1**	*Explanation*	591 (91%)	56 (9%)
05:00 – 25:00	**2**	*Working examples of problems*	1776 (87%)	264 (13%)
25:00 – 36:00	**3**	*Individual work*	Not applicable	

Table 19: Analysis of Sarah's 'typical' lesson

The following excerpt is taken from a typical interaction that Sarah engaged in with her students (pseudonyms) while solving one of the textbook problems that she was using as an example. Sarah first read the problem and then explained each sentence using drawings on the board. She addressed questions to the class and students raised their hands to give an answer.

The Problem: *Joanne buys three packets of mints. There are m mints in each packet. She gives 32 mints away to her students in her class and has seven mints left. Write an equation for m and solve it.*

(01) Teacher: Now listen to this carefully… She gives 32 mints away (reading the third sentence of the problem). Tell me

(02) Mark: 3m – 32

(03) Teacher: Well done! If she gives away 32 sweets will her sweets increase or decrease?

(04) Philip: Decrease

(05) Teacher: Decrease, so which operation shall I use?

(06) Ronald: Minus

(07) Teacher: Minus. How much will she give away? 32. She has 7 mints left… equals?

(08) Jake: Equals 7

(09) Teacher: 3m – 32 = 7, well done! Now our next step is to solve it.

Sarah directed students to the key words in the problem (01) and led students through in forming the equation. She directed students using questions that required short answers – (04), (06) and (08). Sarah did not delve deep or ask for justifications to the answer provided by Mark. With her intervention (03), she sought to make sure that all the other students in class had understood how the expression 3m – 32 was formed and, eventually, how the resulting equation was derived.

5.2.3 Planning and teaching the pre-CPD 'inquiry' lesson

In the 'inquiry' lessons I observed, all three teachers accessed the internet to find ideas about inquiry tasks and used a 'task-first' approach structuring their lesson over three or four main phases (see Table 20). The task offered by Chris presented students with the question: "*My shoe size is 28 cm. How tall am I?*". Greta presented students with a task entitled "Odds and Evens" to conclude the topic of probability. The task, accessed from the NRICH website – a University of Cambridge affiliated website providing mathematical thinking tasks (see https://nrich.maths.org/4308 for task) presented challenges in getting even and odd totals. Similarly, the task offered by Sarah related to presenting students with a set of probability statements for discussion.

Teachers	**Lesson Phases (minutes)**			
	Introduction	**Group work**	**Plenary**	**Summary**
Chris	7	23	10	0
Greta	8	20	14	2
Sarah	3	14	13	0

Table 20: Inquiry lesson phases pre-CPD

In the introductory phase, while Chris and Sarah presented the task, Greta included questions that required students to recall work done in the

previous lessons. An important feature related to the teachers' roles during the small-group activity emerged. Teachers generally went around observing and listening to the small-group discussions. They rarely intervened and the only times they did was to encourage participation (Sarah) and when students asked a question (Chris). However, when Greta recognised that a group was struggling she provided help making sure to correct any mistake that she spotted. When speaking about her IBL lesson, Greta mentioned a dilemma she faced.

> I was not sure about the amount of information I should provide my students with. For example, I was unsure whether or not I should help them with identifying things in the problem that they did not notice.
>
> (Interview 1, June 2015)

Chris and Sarah both avoided engaging in a conversation, responding with statements such as, "*As you wish. If you think it helps do so*." Teachers were conscious of their lack of involvement, and the role undertaken appeared aligned with a discovery approach – students were not given any information and had to solve the problem independently and with only the provided materials. Teachers assumed a passive role, observing what students were doing, listening to discussions and avoiding interactions. There may be at least three reasons why Chris and Sarah avoided interactions and did not provided support: (1) students progressing well through the task; (2) an uncertainty on the role to assume during collaborative work; or (3) their preconceived ideas that in IBL students need to be left alone to discover things independently.

During the plenary part, teachers invited students to share their work with the rest of the class. In the case of Chris, the plenary was teacher-directed, asking each group to report on their work in finding a method to answer the question posed. Each group took turns and the ensuing conversations involved the interaction between the teacher and a student from each group. Students generally provided short answers and Chris then explained and interpreted their thoughts. In Greta's class, students found it difficult to explain their work ending up calling out and writing answers on the board.

Greta usually intervened to support students by describing what the students had done. Greta also posed questions to each group, generally engaging in a one-to-one discussion with the student at the board. On the other hand, Sarah asked students to explain their reasons. Whenever a group contributed an idea she involved others to comment and asked whether they agreed or not. In general, she avoided judging responses and made sure students came to an agreement. At that stage, she further explained and clarified the situation supporting (in most cases) or refuting students' lines of thinking.

Chris reported a positive feeling in managing the learning environment and received positive feedback from his students. In the interview following this lesson (June 2015), Chris spoke at length about a number of aspects. For example, the task "stimulated their thinking"– something that seemed uncommon in the lessons he typically offered. Chris was also "confident presenting the task and having students involved … happy with the lesson closure and the way I managed to bring their results together". In particular, Chris was "satisfied seeing students actively engaged with the task, using their own methods" and "surprised that they were quite self-sufficient during the activity and eventually managed to do the task without any help".

For teachers, lesson planning was time-consuming. In the interview following this lesson, Greta said that it was "problematic to find an IBL activity" because she sought a task related to the topic she was doing. Greta reiterated this point in the second interview.

> If I design an IBL lesson not directly related to a topic, I wouldn't say that I would be wasting time but I would worry that students would question why we are doing such a task if they cannot see its relevance to the topic just covered. What I intend to do is to include an IBL lesson at the end of every topic, as a revision.
>
> (Interview 2, October 2015)

Greta expressed a preference to use IBL as consolidation and emphasised that the tasks selected embedded mathematical content. Moreover, she

made sure that students saw a clear link between the inquiry task she offered and previously learnt mathematical content.

Similarly, Sarah elaborated on this aspect. While she recognised the benefits of IBL for life, she felt that time dedicated to inquiry "means lessons taken away from the syllabus because students will not use IBL in their exam". She also appeared uncertain whether the purpose of IBL was for "testing whether they know the answer or whether they can discuss and develop an argument for justifying their answer". For Sarah, her role during each phase of the lesson was crucial. She believed that during group work she learned to "push students so that they do not give up, to refuse giving answers away and to avoid talking to the groups while they are working". However, she expressed concerns regarding the plenary part about "whether to lead the discussion or not, and being particularly uneasy that students come up with something which I am not prepared for and cannot handle".

5.2.4 Selecting a group of students for IBL

Following the summer workshops, teachers were asked to identify a class for my lesson observations and to provide a description of the class giving reasons for their selection. Table 21 outlines the characteristics, as identified by the teachers, of the class chosen for my lesson observations.

Teacher	**Information about students**			
Chris	Year 7	20 students	High-attainment	Well behaved, willing to learn and receptive to group and hands-on activities
Greta	Year 8	17 students	Mixed-attainment	A relatively quiet group, keen to discuss and participate actively
Sarah	Year 9	22 students	Low-attainment	A noisy class, lack motivation and interest to learn mathematics

Table 21: Characteristics of the class chosen for IBL as identified by teachers

Greta confessed that "chaos in class was not tolerated" (Interview 1, June 2015). She disliked noisy and chaotic classrooms and hence, based her choice "on the least chaotic class" she had. Greta opted for a class that she

perceived receptive to IBL tasks, relatively quiet and active to learn. She stressed, however, that she would still do the same IBL tasks with her other classes.

Sarah declared that whenever she used IBL she did this with her high-attaining groups "as they get involved in the tasks I present them with and tend to participate more". She also claimed to enjoy herself doing IBL with such students as they give her "much joy watching them debate and discuss". However, she wanted to overcome her everyday struggles in motivating her low-attaining Year 9 group she was assigned to teach. The positive experiences shared by a participant and one of the CPD facilitators during the summer workshops further convinced her that IBL could work for this class. CPD facilitators and one participant spoke about the benefits that IBL brought in the attitudes, motivations and attainments of students placed in their lower sets. Sarah, hence, decided to "give it a go" knowing that she had to win her students over first. She felt she had to cultivate "a thinking class" with students who would be expected to think, changing what these students were usually expected to do and how they worked and learned mathematics.

In the first month of the new scholastic year, Chris sought to get to know his students well before choosing his class – making sure that they were receptive to different teaching strategies. Chris anticipated a number of opportunities but also challenges in using IBL with the class he chose. Chris believed that he had to undertake change slowly, starting gradually and reassuring students at each stage. He argued that this approach may help to gradually empower his students in taking the initiative and a more active part in learning. Chris also explained that this change required professional growth as a teacher, particularly in moving away from anticipating the problems and telling students about potential mistakes towards allowing them to make mistakes and correct them.

5.3 Challenges and opportunities for inquiry teaching

Before joining the project, teachers identified a range of challenges and opportunities in using IBL. The examination syllabus, parents' expectations of success in examinations, students' disposition for IBL and increased school activities were the main limitations for teachers in doing IBL.

5.3.1 The mathematics curriculum and IBL

The three teachers mentioned limitations and inconsistencies between the mathematics curriculum and IBL. For example, Greta pointed out that examination and assessment methods do not resonate with IBL. For Sarah, the syllabus was there to complete and she was concerned about the number of lessons she would potentially miss during the year due to the numerous outings and activities organised by the school. These, she felt, caused her to fall behind in the scheme of work. When this occurred, syllabus pressures restricted her use of IBL and "inquiry is unfortunately eliminated" opting then for "graded exercises where students work and move at a faster pace than they would with IBL" (Interview 1, June 2015). Sarah also linked this traditional teaching approach to the expectations of parents. Irrespective of the class and year group she taught, annual exams were very important for Sarah. Sarah argued that covering the exam syllabus is always of concern to her, and this usually created pressures in managing to teach all content in the time available.

Similarly, Chris felt restricted by the system and the content he had to teach. Chris explained:

> At times, I feel I want to do things differently but I am restricted by the system… this frustrates me a bit. I know there are other possibilities for delivering mathematical content, but time constraints and other pressures related to administrative roles do not allow space for me to experiment and be adventurous in my teaching.
>
> (Interview 1, June 2015)

5.3.2 The school context and IBL

Chris spoke about hindrances within his professional context and tensions between aspects of the context. He perceived tensions between what the school leadership team (SLT) at St Cecilia was promoting and supporting, and the challenges related to system restrictions and students' readiness to embrace learning through inquiry.

With regards to opportunities and possibilities, Sarah mentioned having double lessons, her own class, an additional lesson every week, a low-attainment group where emphasis on past papers is minimal, a less demanding syllabus, and fewer mathematical topics to teach. Instead of the usual five lessons per week, the SLT at St Simon granted Sarah an 'extra' lesson with her Year 9 low-attainment class. She planned to use this 'extra' lesson for IBL. Sarah claimed that she could afford to do an IBL lesson per week because the syllabus content was also less and not as demanding on students when compared to those in higher sets. On the other hand, Greta did not have her own class and identified the classroom setting consisting of individual desks placed in rows and columns as a disadvantage because of time-wasting in having to move desks together. For Greta, however, having a double lesson in her timetable was perceived as an opportunity for using IBL with her Year 8 class.

5.3.3 The influence of school colleagues and IBL

For LTMI, teachers were encouraged to attend in pairs to facilitate community building within their schools. However, this was not possible for all participants. Sarah, who was the only teacher attending from her school, perceived no openings to discuss IBL practices with her colleagues. Although they had participated as a team in her previous IBL-related CPD, they neither implemented IBL nor were interested to pursue the LTMI professional learning journey. Moreover, while she was mentored by her then head of department, now she had to do it all on her own. In the case of Greta, a colleague with experience implementing IBL attended the summer workshops but did not continue with the follow-up meetings. On the contrary, Chris' colleagues within the mathematics department were all

enthusiastic about participating in LTMI. Indeed, his five colleagues attended the summer workshops, and one of them, Alan (pseudonym), also continued attending the follow-up meetings with Chris. However, following the summer workshops, Chris still described how he rarely collaborated with colleagues towards the development of teaching.

> We tend to discuss things related to syllabus coverage because of our race to get through our scheme of work in time for the exams. To be honest, I never discuss my practices with others. We rarely talk about our classroom practices, and when we do we tend to focus on the negative aspects related to students' behaviour or motivation.
>
> (Interview 2, October 2015)

A sense of competition rather than collaboration appeared to prevail within schools. Before joining this CPD, Greta had previously used challenging tasks to promote inquiry with her Year 8 classes. However, during that year, Greta faced the challenge emerging from the competitive element with one of her colleagues who taught the other Year 8 classes at a faster pace.

5.3.4 Students and IBL

For teachers, IBL was going to offer a new way for students to learn mathematics. Hence, they were both concerned and curious to see how students would react to this approach. Greta believed that "in maths you need to concentrate, and hence the classroom environment needs to be calm and quiet" (Interview 1, June 2015). She believed that, to help students concentrate on doing mathematics, cultivating a quiet environment was a priority. Greta, hence, perceived it to be "quite difficult to include IBL at the beginning". Chris also felt it would be a struggle "to convince students that learning through inquiry is not a waste of time" (Interview 1, June 2015). He believed that students were too oriented to think in terms of exams and, whenever he assigned work from the textbook they questioned which problems would feature in the exam. He considered that this mindset would be more challenging to overcome as "students are not prepared for IBL and not trained to think in this manner" (Interview 1, June 2015).

5.3.5 Parents' expectations and IBL

Teachers also mentioned parents as a challenge in their implementation of IBL. Parents, according to Sarah, expected a certain type and amount of work from teachers. This tended to restrict teachers to provide graded examples, exercises and practice for students.

Chris also believed that parents would object to IBL as a teaching approach. He communicated that the parents in his school were very demanding regarding the notes and homeworks given, and that it was not unusual for them to call the SLT complaining about such issues. Each of these factors were perceived by Chris as constraints limiting his teaching actions.

5.3.6 CPD as a learning opportunity with possibilities for IBL

For Sarah, LTMI offered an opportunity to continue the work she had begun during the previous IBL-related CPD experience. Moreover, she expressed the desire to change her transmission-oriented approach. Greta engaged in LTMI because she sought to use IBL to vary her lessons and include a fun element for students in learning mathematics.

Unlike Sarah and Greta, Chris had only heard about IBL in a session as part of his ITE course. Yet, he still saw LTMI as offering possibilities for him to learn new things. Notwithstanding the system restrictions, Chris still wanted to explore the possibility of allocating more time for IBL.

5.3.7 Teachers' zones of enactments pre-CPD

Drawing on the zones of enactment theory, I present teachers' challenges and opportunities in enacting IBL. The representation (see Figures 8, 9 and 10) shows the interplay between teachers' personal resources and the external factors and learning opportunities influenced by the *pupils*, *professional*, *policy*, *private*, and *public* sectors. These teachers worked within school contexts that shaped and restricted their approaches to teaching. Prior to their participation in CPD, the enactment zones of teachers were individualistic, where deliberations with other teachers on creating resources for teaching and developing practices were missing.

These teachers perceived challenges emanating from the *private*, *public*, and *policy* sectors as a result of the exam-oriented system, with their willingness to enact IBL mainly driven by personal and students' needs.

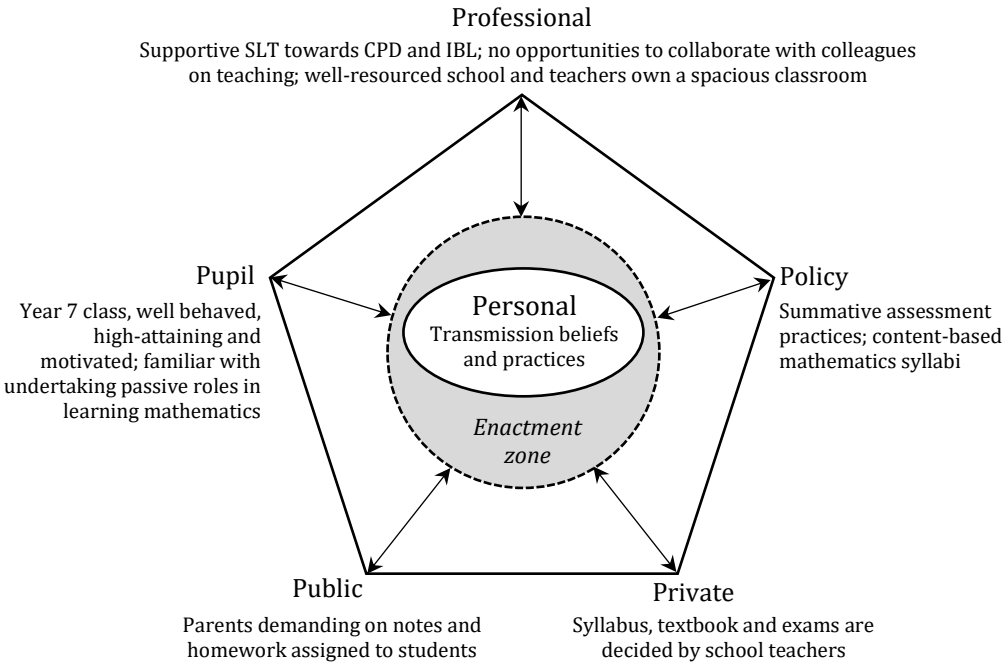


Figure 8: Chris' zones of enactment prior to CPD

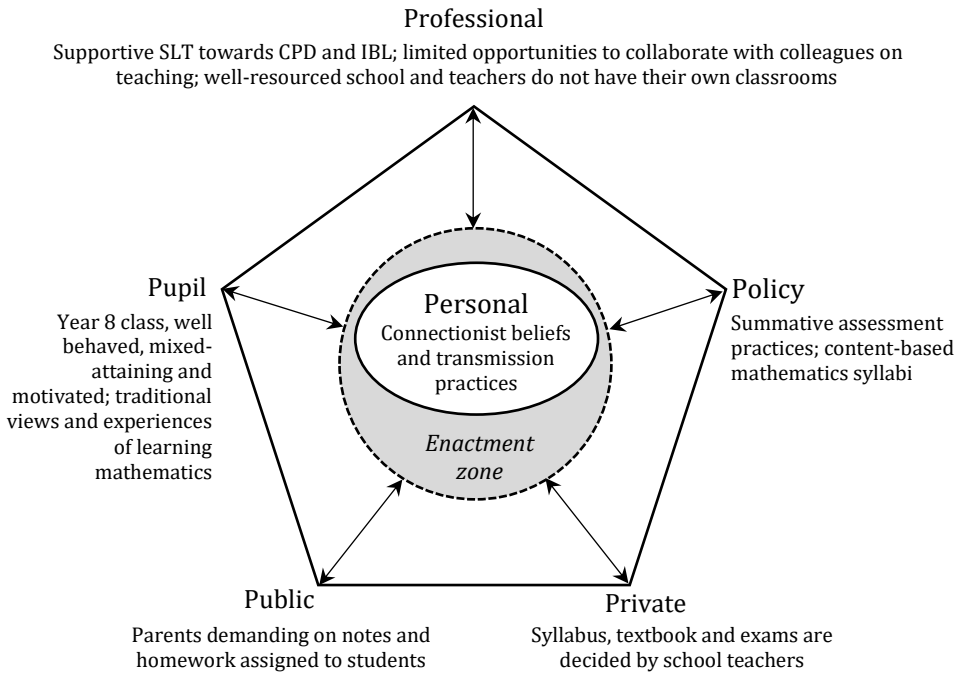


Figure 9: Greta's zones of enactment prior to CPD

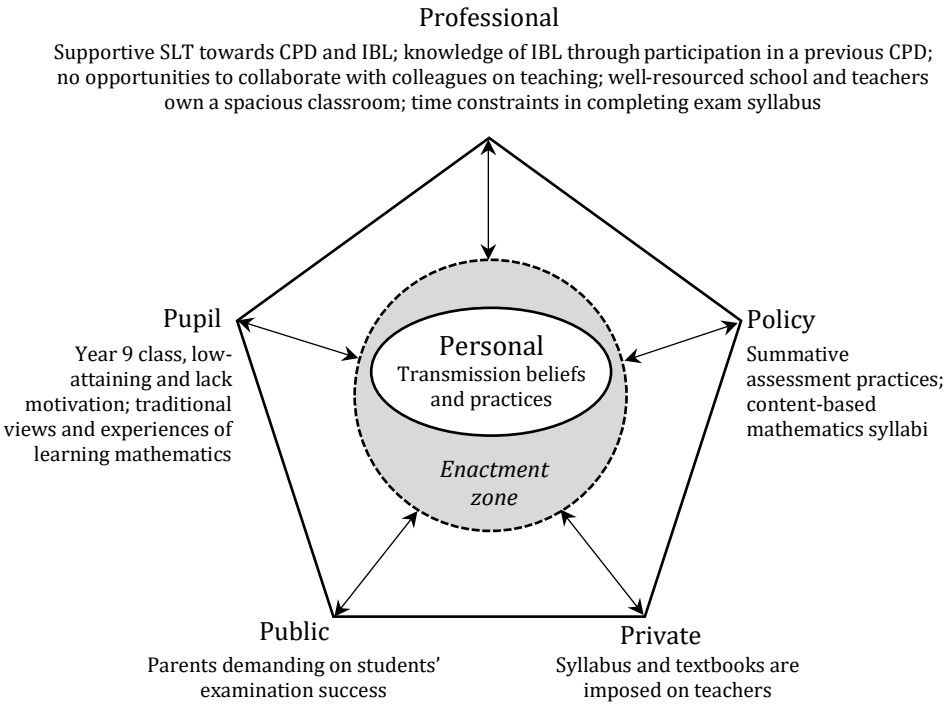


Figure 10: Sarah's zones of enactment prior to CPD

5.4 Conclusion

In this chapter, the three cases were presented for an understanding of teachers' professional contexts, their practices and knowledge of IBL, and their beliefs about mathematics and pedagogy. Drawing on the theory of enactment zones, I sought to outline factors communicated by these three teachers. These factors – the *personal*, *policy*, *public*, *private*, *professional* and *pupil* sectors – were identified and discussed because they are seen to influence teachers' enactment of practices (see Table 22).

The *professional*, *public* and *policy* sectors, reinforcing a competitive exam-oriented education system (Cachia & Ferrari, 2010; Eurydice, 2011), appeared common to all teachers and viewed by them as restrictive towards enacting IBL. While non-state schools afforded Chris and Greta some authority over the content to teach, exams and choice of textbooks, Sarah had to abide by instructional materials that were imposed on her. Moreover, albeit school leadership teams appeared supportive for these three teachers participating and employing IBL practices, a *lone-fighter*

culture (Krainer, 2001) was dominant within schools. Notwithstanding this isolation, teachers identified possibilities for working with students and undertake the challenge of transforming their transmission classroom practices.

<table>
<tr><th rowspan="2">Enactment Zones</th><th colspan="3">Teachers</th></tr>
<tr><th>Chris at St Cecilia</th><th>Greta at St George</th><th>Sarah at St Simon</th></tr>
<tr><td>Personal</td><td>Transmission beliefs and transmission practices</td><td>Connectionist beliefs and transmission practices</td><td>Transmission beliefs and transmission practices</td></tr>
<tr><td>Policy</td><td colspan="3">Summative assessment practices; content-based syllabi</td></tr>
<tr><td>Public</td><td colspan="3">Parents demanding notes, homework and success in examinations</td></tr>
<tr><td>Private</td><td colspan="2">Syllabus, exams and textbooks are decided between teachers</td><td>Syllabus and textbooks are imposed on teachers</td></tr>
<tr><td>Professional</td><td colspan="3">Supportive SLT towards CPD and IBL; well-resourced school but no culture of collaboration between colleagues on teaching and learning</td></tr>
<tr><td>Pupils</td><td>Year 7, high-attaining, well behaved, motivated and have traditional experiences learning mathematics</td><td>Year 8, mixed-attaining, well behaved and have traditional experiences learning mathematics</td><td>Year 9, low-attaining, lack motivation and have traditional experiences of learning mathematics</td></tr>
</table>

Table 22: Teachers' enactment zones prior to LTMI

The aim of this chapter was to describe each teacher's enactment zones prior to their CPD journeys. These teachers engaged in LTMI to increase and sustain their capacity to enact IBL by changing their core practices. In the next chapter, the theoretical framework will guide my understanding and interpretation of how teachers sought and managed to make changes towards IBL practices through their enactment zones.

Chapter 6: Cross-case analysis

6.0 Introduction

The previous chapter focused on presenting three case studies identifying features related to the teachers' professional contexts and aspects of teachers' practices and beliefs prior to their participation in LTMI. The three case studies are representative of the main themes that emerged across the seven cases of participants in this research. In this chapter, the analysis of data will move onto a cross-case analysis to highlight the two main themes that emerged from the coding process and thematic analysis (see Table 23).

During analysis, cross-case and within-case analytic strategies made it possible to develop themes in a way that took into account particular factors shaping the journey and experiences of each case study teacher. This chapter will be structured into two main sections, each of which represents a theme with subsections dedicated to subthemes.

Theme	Subthemes	Data Sources
1 Teacher Interpretation and Enactment of IBL	• Teachers' conception of IBL • Lesson time restrictions • Structuring IBL lessons • The teacher's role in an IBL classroom • IBL features valued and enacted in class • Student engagement with IBL	Interviews Questionnaires Lesson observations
2 Teacher Development, Learning and Change	• Effective strategies for IBL • Changed practices and views about teaching mathematics • Teacher learning in implementing IBL • Changes in teachers' beliefs • Changes in teachers' beliefs in relation to reported practices	Lesson journals Focus group

Table 23: Emergent themes from data analysis

6.1 Theme 1: Teacher interpretation and enactment of IBL

In this section, I focus on teachers' interpretation and enactment of IBL over the period of the project. This theme incorporates aspects related to teachers' conceptions of IBL, their planning, structuring, time restrictions, the IBL features that teachers valued and promoted in class, the role they assumed and their perceptions of students' engagement. This section is divided in subsections that provide the context and experiences as reported by teachers using the five data collection sources.

6.1.1 Teachers' conceptions of IBL

In order to identify potential changes, I explored teachers' conceptions of IBL by comparing their definitions of IBL before and after their participation in LTMI. Teachers' definitions are presented word-for-word in Table 24. However, I also refer to their lesson journals and interviews to elicit instances where they also mention IBL in order to gain a better understanding of the way conceptions evolved during their CPD journeys.

	Defining IBL	
	Pre-CPD	**Post-CPD**
Chris	An approach to engage students in the learning process such that they learn through exploration and discovery.	A process through which students explore concepts and knowledge, with the aim to extend their capabilities through inquiry, exploration and personal experiences.
Greta	Learning through problem-solving and open-ended activities where students apply the mathematics learnt. It can also be an activity which helps students discover new concepts.	An opportunity where students discuss a task in a group and apply the mathematics they learnt to arrive at a solution. The task should be challenging in such a way that they can challenge each other during discussion.
Sarah	It is when the students are engaged in tasks in which they investigate, try to solve and arrive at conclusions alone.	An opportunity for students to work together to investigate, discuss and challenge statements; clarify misconceptions; realise that there exist various methods for one solution; there exists more than one solution for one problem.

Table 24: IBL definitions provided by teachers in questionnaires

Prior to their participation in CPD, teachers defined IBL as a way of learning involving independent discovery. In comparing the definitions, in the post-CPD questionnaire teachers provided longer and broader descriptions. In analysing these, I compare teachers' use of words to identify changes. For example, in his post-CPD definition Chris was more specific in explaining the learning process as students engaged in IBL. Rather than simply referring to "exploration and discovery", Chris specified that IBL involves the exploration of "concepts and knowledge" that supports students to "extend their capabilities". In addition, he appeared to acknowledge the role that "personal experiences" play in engaging students with IBL. Like Chris, Greta defined IBL as learning arising as a result of the activities that students engage in. In fact, in both her definitions she described IBL as offering the opportunity for students to "apply the mathematics learnt". However, in her post-CPD definition, Greta highlighted the role of discussion and tasks. Following LTMI, these two features became crucial and she viewed the use of challenging tasks as useful in students' social construction of mathematical knowledge. Post-CPD, Sarah also viewed IBL as a collaborative activity based on an open approach to solving mathematical problems.

Analysing instances where teachers referred to IBL, supported the conceptions teachers had before they joined CPD. Referring to his pre-CPD 'inquiry' lesson, Chris described IBL in terms of his role in class.

> I was literally doing nothing, simply going around observing and seeing what was going on. When they asked questions, I threw the ball back at their feet with the aim of not helping them.
>
> (Chris, Interview 1, June 2015)

Similarly, Greta spoke of IBL as an activity where she sought to interfere as little as possible and, hence, which students could do independently of the teacher. Sarah also held the view that she should not speak or give answers to students. Her role was to encourage students to keep trying. During LTMI, teachers' conceptions evolved in different ways. Sarah still saw her role as passive, opting to stop transmitting knowledge and encourage

students to solve the task collaboratively. On the other hand, Greta saw the role of questioning as crucial during student inquiry while Chris perceived inquiry teaching in terms of the experiences he sought to provide.

> I see students starting a bit lost. They eventually connect ideas and come up with a way of solving the task when presenting their work within the final whole class discussion. I see the lesson concluded by confirming the correct method and then giving them some practice as homework.
>
> (Chris, Interview 2, October 2015)

These data were further triangulated with teachers' journal writings. For each written reflection teachers were asked to provide their reasons for choosing to do a lesson. Responses to this question provided a rich array of purposes (see Table 25). The purpose for teachers in using IBL related to: (1) planning lessons – how teachers saw IBL fitting into their scheme of work, (2) a teaching approach, and (3) engaging students in learning mathematics. It is worth noting that, over the course of their implementation, the purpose for using IBL for these three teachers did not change. In the case of Chris and Sarah, IBL served as a means of introducing students to a topic or concept and offered students opportunities to explore and investigate these through discussion. For Greta, IBL was similar to the practical tasks she had been using prior to her participation in LTMI. She generally implemented these tasks at the end of a topic or unit for revision purposes for students to practise and apply the knowledge learned.

	Using IBL		
	When planning to	*As an approach to*	*To engage students in*
Chris	Introduce a topic	Discover new content	Discussion and thinking
Greta	Revise a topic	Practice, revise and apply knowledge learned	Collaborative work and discussion
Sarah	Introduce, explore and investigate concepts	Challenge student thinking	Discussion

Table 25: Teachers' use of IBL in their lesson journals

The three teachers also developed conceptions of IBL drawing on the learning experiences they offered to students. During interviews, teachers mostly mentioned the roles envisaged through IBL. Greta and Sarah defined IBL as a collaborative activity initiated by a challenging or open-ended task during which they allowed students to work independently, while Chris considered IBL as a flexible pedagogy for engaging students in learning mathematics. Chris moved away from his initial position focusing on undertaking an observer role in assisting students' discovery. Over the course of the CPD, Chris developed his understanding further. For him, IBL offered a flexible pedagogy – providing different ways of engaging students in thinking and discussion about a problem. IBL offered possibilities for creating learning opportunities aimed towards, and that worked for, the particular group of students he taught.

> I cannot say that doing one thing is inquiry and doing another is not. IBL is so flexible. It seems to me that you form your own idea of inquiry based on the particular students you teach. However, it is based on important features that we discussed during the CPD, but there are also different methodologies for implementation.
>
> (Interview 3, June 2016)

According to Chris, IBL is context specific. While it depended on the mathematical content that he had to teach, it was also specific to the particular group of students. It appears that with a different class, Chris would possibly implement IBL differently.

The conceptions of Greta and Sarah appeared to converge. Initially, they both saw IBL as initiated by tasks that promote investigation and, similar to Chris, with the teacher undertaking an observer role. Sarah still viewed her role as refraining from intervening while Greta found the use of questioning as critical. Ultimately, a discovery-oriented conception persisted.

> When doing IBL, I simply give a problem and let students come up with ideas and they need to solve it. Instead of telling them what and how to do things, I just let them work. They need to identify the mistake and correct it.
>
> (Greta, Interview 3, June 2016)

> In doing IBL, I realised that I should speak less and not provide a hint straight away. I listen to what students are saying and, as much as possible, I do not say anything.
>
> (Sarah, Interview 3, June 2016)

For both Greta and Sarah, tasks and collaborative learning also emerged as essential features for IBL. While Greta mentioned the use of challenging tasks to engage students in discussion, Sarah referred to open-ended tasks for investigation and discussion. Through IBL, these teachers sought to shift their role from telling and leading to observing and listening, while engaging students in collaboration, discussion and independent exploration. Their conceptions of IBL were also shaped by students' reactions to the learning situations that they offered and what worked within their professional context.

6.1.2 Lesson time restrictions

Besides mentioning system restrictions, for using IBL, related to the summative content-oriented assessment practices, a recurrent issue for teachers was the limited lesson time available. Secondary schools generally allocated 40 to 42-minute lessons in their timetable. However, this was effectively less due to teachers or students having to change classes between lessons and additional time used to settle down the class (see Table 26). The first lesson I observed Greta was longer than this because she could avail herself of a double lesson on a particular day.

	Observed Inquiry Lesson Time (minutes)			
	1	**2**	**3**	**4**
Chris	33	32	38	39
Greta	45	33	37	36
Sarah	32	38	35	32

Table 26: Effective lesson time for the observed inquiry lessons

Teachers reported difficulties in assigning adequate time for the different lesson phases and this was a consistent concern for them. For example,

Chris and Greta found it more difficult than Sarah to allow time for the plenary, as they generally extended time for small-group activity. Organising the classroom environment was also an issue for teachers. Greta, for instance, had to change classrooms for her lessons, organising the classroom – connecting her laptop to the interactive board, setting up desks and organising the resource materials – involved essential time taken away from the lesson. Moreover, she felt uneasy when students did not know what to do or did not understand the aim of the task she presented. Greta also contrasted her classroom organisation with those in the lesson videos shown during the summer workshops. She perceived her classrooms as less organised and noisier. Prior to LTMI, she had also communicated her belief that students learn best within calm and quiet environments (see Section 5.2.4). By the end of LTMI, Greta still held to this belief but recognised that this was not always possible when discussion was central to the IBL lessons she implemented.

Like Chris, Sarah had her own classroom and she considered this as an opportunity for implementing IBL. Her lessons were limited to an average of 34 minutes because students took some time to arrive to class. However, during this time, Sarah had the opportunity to set up and organise her own class according to the activity she had planned.

6.1.3 Structuring IBL lessons

In this section I present and discuss how teachers structured their inquiry lessons drawing on data from the six lesson observations and teachers' journal entries.

Pre-CPD, I observed a 'typical' and an 'inquiry' lesson that teachers offered. In the previous chapter, teachers' planning for teaching these two lessons was investigated (Sections 5.2.2 and 5.2.3). An emerging feature that I focus on is the way teachers structured these two lessons (see Figures 11, 12 and 13) – including teachers' roles, lesson phases and time allocated for each phase. 'Typical' lessons seemed dominated by the teacher. Chris, for example, devoted 25 minutes for teacher presentation, Greta 22 minutes and Sarah 25 minutes. This phase usually included the teacher explaining

work to the whole-class – from explaining concepts, showing examples and giving notes to correcting homework and clarifying student difficulties. In this phase, the teacher usually dominated the talk with little evidence of discussion taking place. In the first part of the lesson, both Chris and Sarah included a long stretch of teacher presentation. Greta followed a similar structure, yet engaged the class in two short discussions followed by a longer discussion based on a problem that students worked on individually. A common pattern emerges with the way these three teachers structured their 'typical' lessons. Such lessons were dominated by the teacher explaining things and at some point – which was usually half-way through the lesson – shifting to an activity where students did some practice on what was presented and explained.

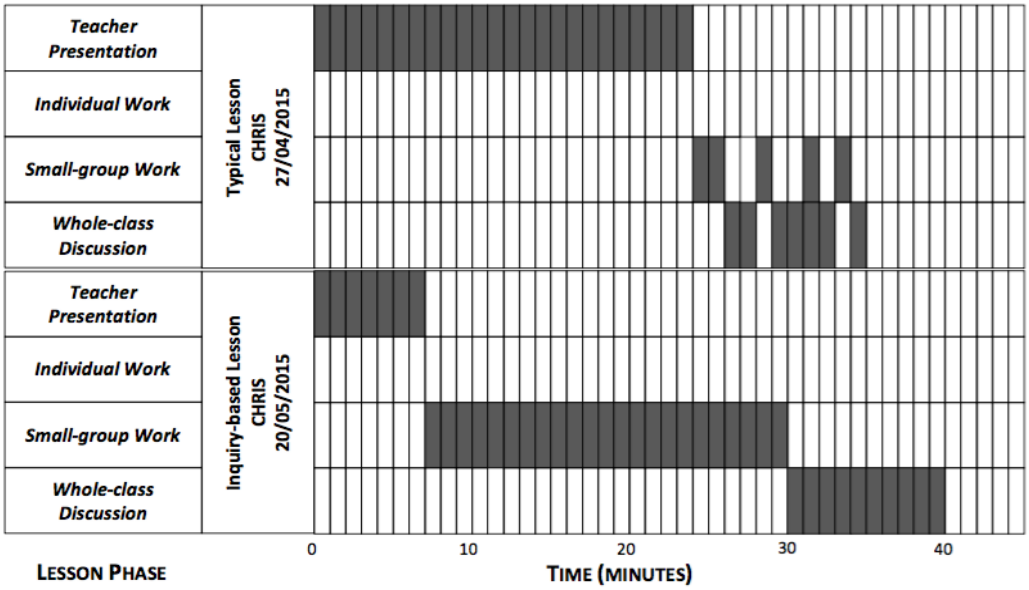


Figure 11: Chris' structure of lessons pre-CPD

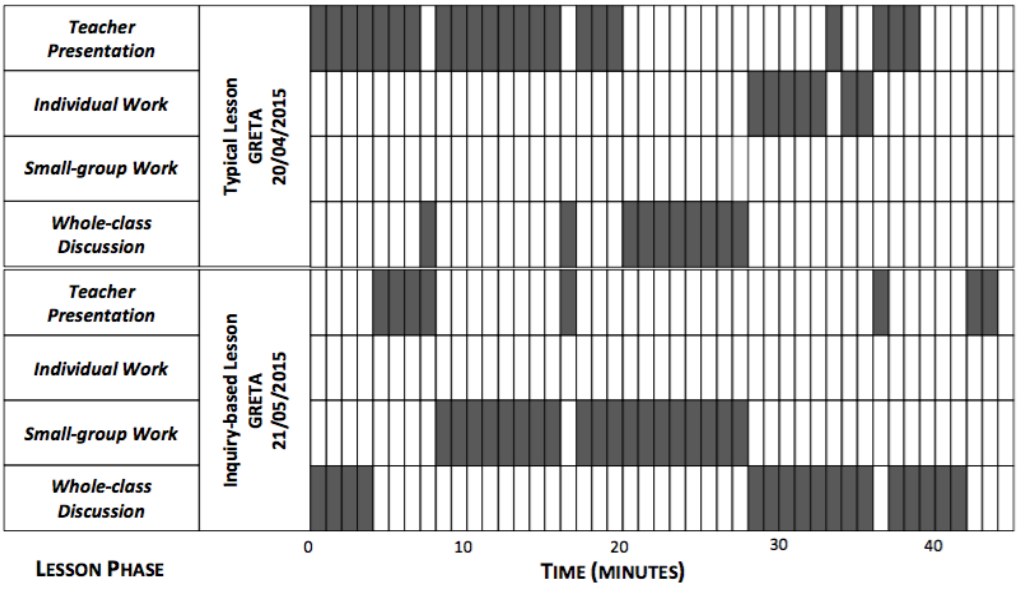


Figure 12: Greta's structure of lessons pre-CPD

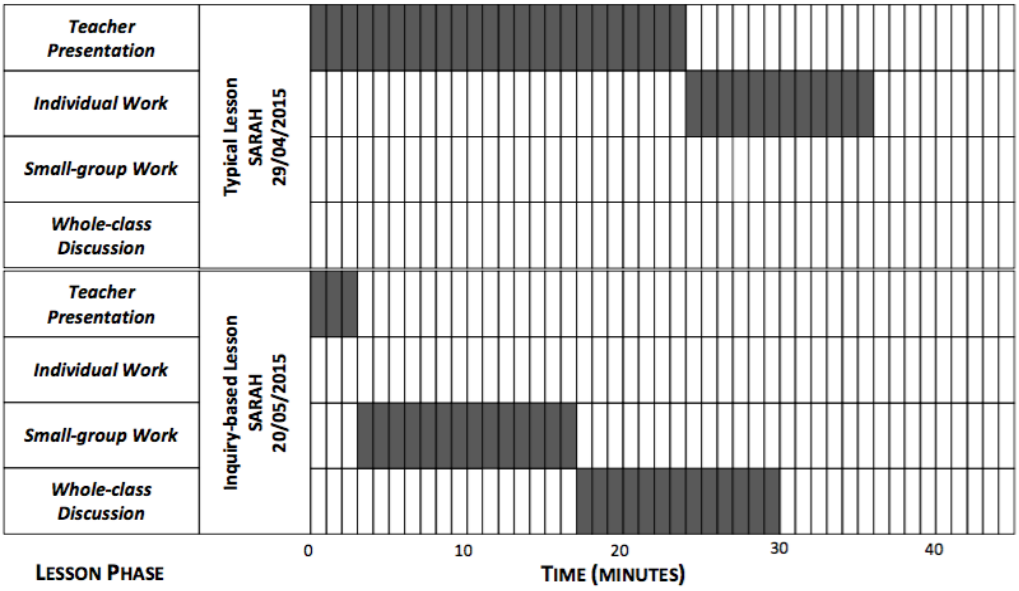


Figure 13: Sarah's structure of lessons pre-CPD

A pattern also emerged for their 'inquiry' lesson. All adopted a three-phase lesson structure consisting of a task presentation, collaborative work and a plenary. Chris devoted extensive time for collaborative work with Sarah allowing practically the same amount of time for small-group work and the final whole-class presentation and discussion. Unlike Chris and Sarah, Greta intervened twice (17th and 37th minute) addressing the whole-class about an aspect that seemed unclear. It appeared that prior to their engagement in CPD these three teachers already conceived of inquiry lessons as

consisting of three main phases, similar to what was presented and modelled in LTMI. Indeed, teachers were exposed to this same structure during the summer workshops. Yet, analysis of the inquiry lessons observed during the CPD period provided an inconsistent pattern in teachers' inquiry lesson structure (see Figures 14, 15 and 16). This will be discussed next.

During CPD, Sarah maintained the three-phase lesson structure in all four lessons observed, while Chris and Greta seemed to adopt variations to it for reasons that will be investigated. Chris, for instance, incorporated a sequence of two or three tasks within the first three lessons observed. With each task he introduced, he applied iterations of a three-phase structure – presenting the task, allowing time for students to work collaboratively in groups and finally doing a short plenary. Lesson 4 (see Figure 14) was eventually more consistent with his pre-CPD inquiry lesson structure (see Figure 11). For both lessons, Chris spent 7 minutes presenting the task. The only significant difference that emerged was the time Chris dedicated to small-group work and whole-class discussion. For while in the pre-CPD lesson the ratio was 23:10, in lesson 4 this ratio was 16:16. Figure 14 also shows a notable modification in the way Chris structured the first three inquiry lessons. During these lessons, Chris alternated between teacher presentation and small-group work, ending with a very short whole-class discussion in both the first and the third lesson, but not in the second one. In the second lesson, which incorporated two tasks related to classifying polygons, Chris included a whole-class discussion after each task. Still, he did not do this in the first lesson that incorporated three tasks related to the number 24, and for the third lesson that involved students working on two tasks related to multiple representations (see Appendix 15 for the tasks Chris used in his IBL lessons).

Over the scholastic year, the lesson structure of the three teachers became more aligned to a three-phase structure. Yet, while for Sarah this seemed to be a standard practice, Greta and Chris experimented with different lesson structures. This shift could be a result of teachers' growing expertise in managing IBL lessons (see Leikin & Rota, 2006). However, analysis of lesson

structure of each teacher's pre-CPD 'inquiry' lesson shows that the three teachers already had this preconceived notion. Changes to lesson structure were therefore determined by other factors, possibly the tasks that teachers used to engage students in IBL. In exploring this, I classified tasks using the descriptions provided by teachers in their lesson journals and the task sheets teachers offered during lesson observations. The nature of mathematical tasks was classified within two dimensions: the *mathematical content* inherent within the task and the *cognitive challenge* that students would encounter while engaging with a task. The possible relationship between the nature of the task selected by these teachers and lesson structure was investigated.

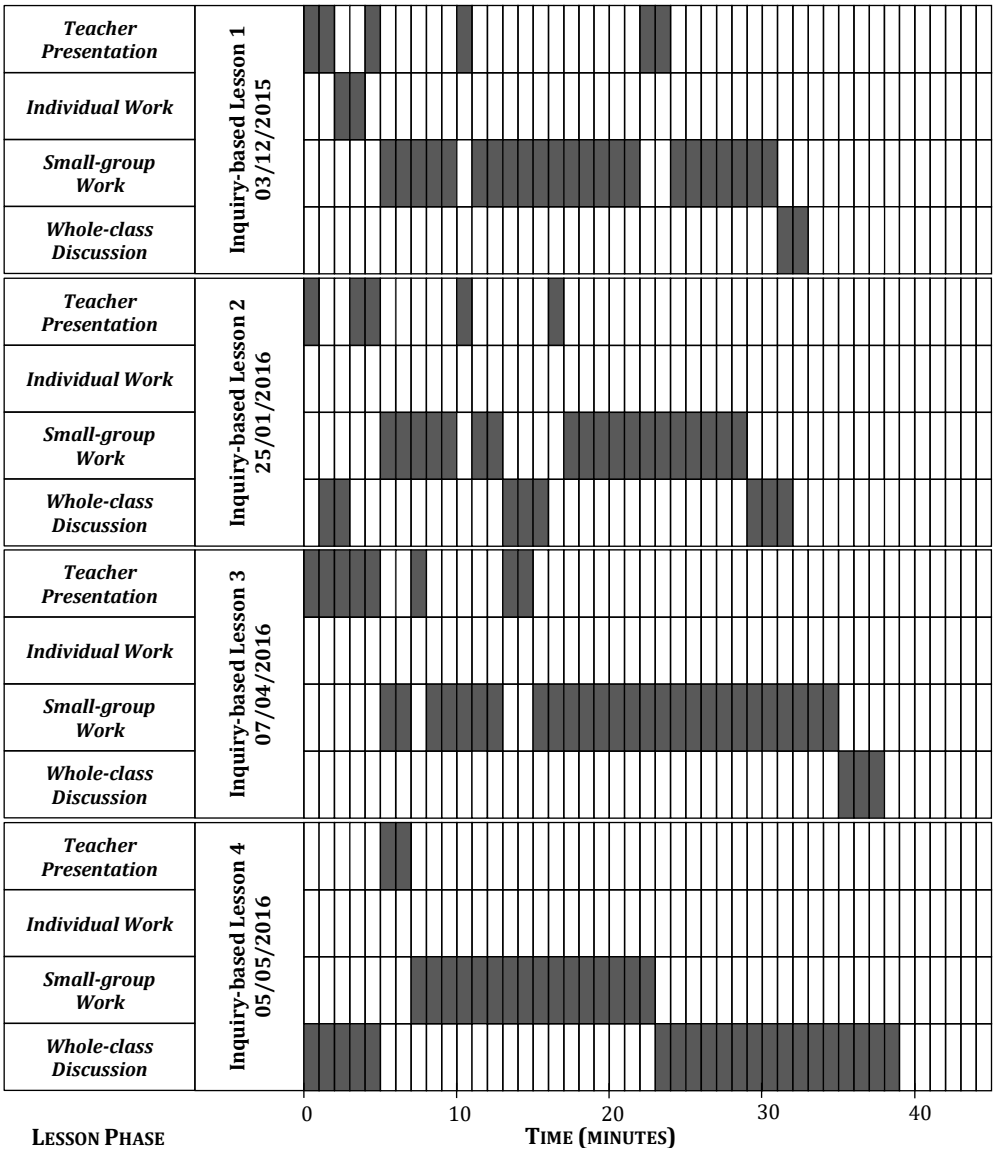


Figure 14: Chris' IBL lesson structure during the CPD

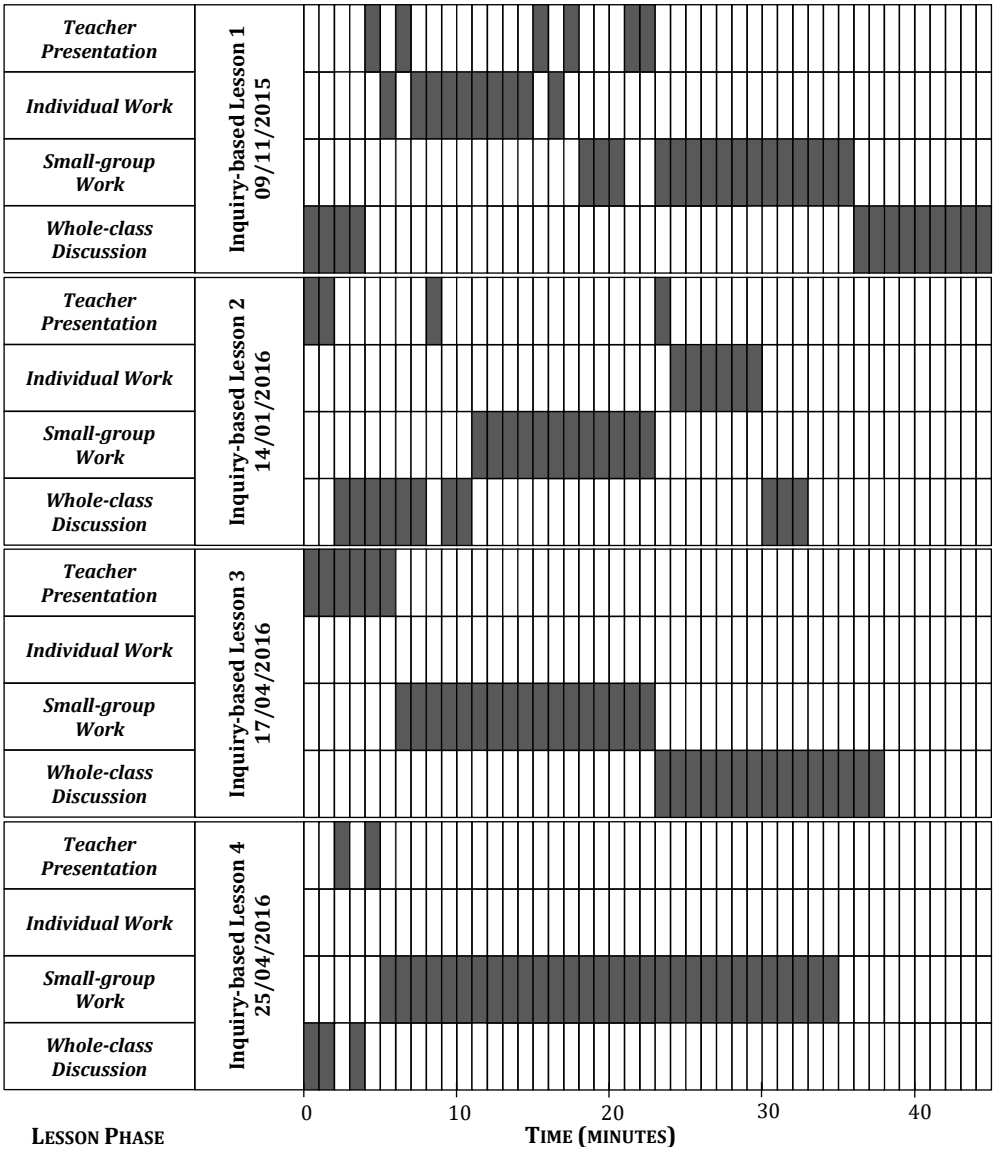


Figure 15: Greta's IBL lesson structure during the CPD

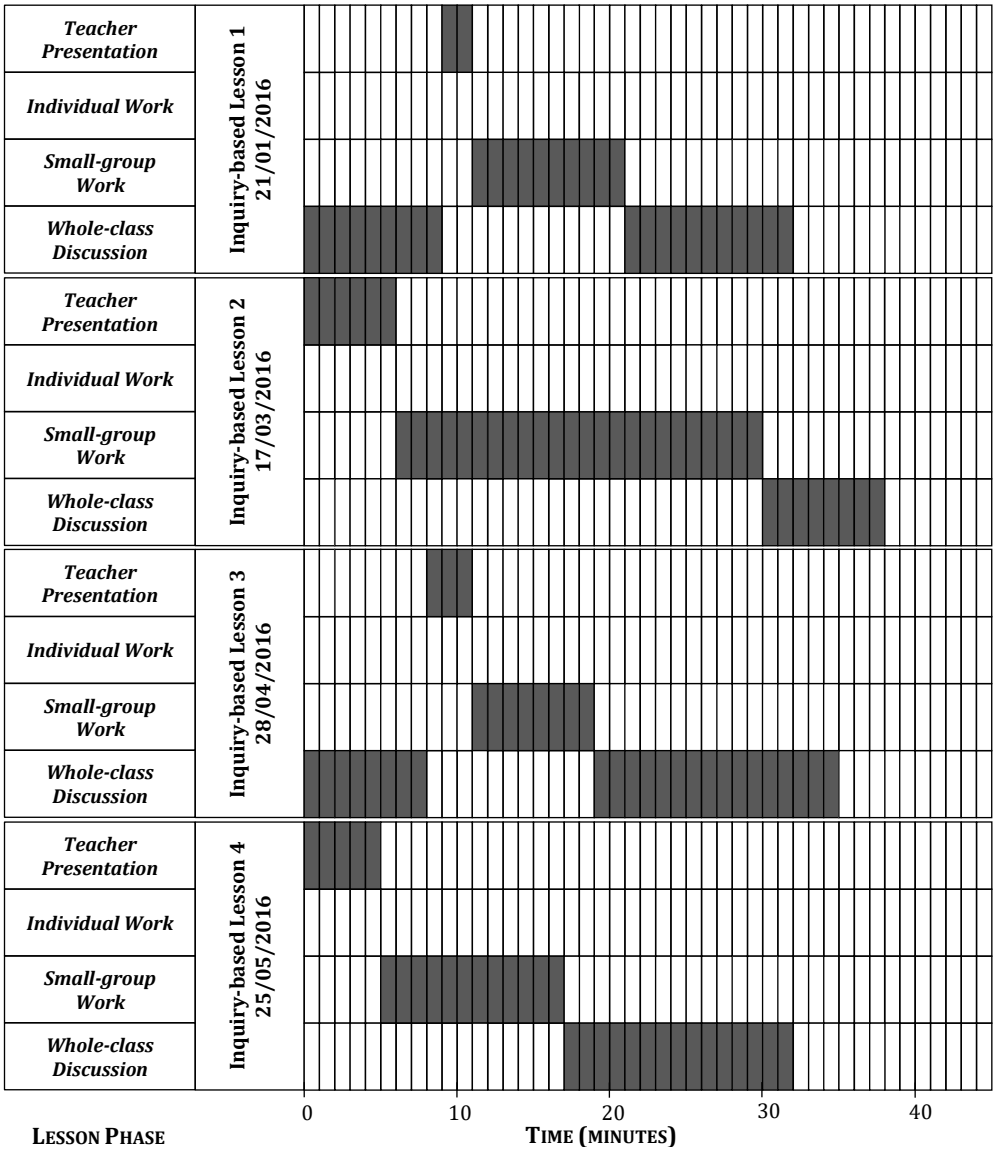


Figure 16: Sarah's IBL lesson structure during the CPD

In their inquiry lessons, teachers generally chose to use tasks that offered students a high cognitive demand (see Table 27), understandably because they interpreted these as lending themselves more to inquiry. I classified these tasks using the framework provided by Stein, Smith, Henningsen, and Silver (2000). A task involving only memorisation or procedures without connection was classified as requiring a low cognitive demand, whereas one requiring students to make connections between and among mathematical ideas in new ways as a high cognitive demand task. This classification exercise was carried out following analysis of lesson observations with a focus on whether the nature of the task and cognitive demands changed during instruction – the task Greta used in the second lesson was a case in

point (see Table 30). Hence, a task was classified as having a high cognitive demand when it required students to use processes involving decisions to be taken about how to use mathematical knowledge and skills in particular circumstances to interpret, analyse and solve problems (Doyle, 1988; Stein et al., 2000). The focus of high cognitive demand tasks is that they provoke higher levels of thinking.

The inquiry tasks that teachers used also incorporated mathematical content knowledge. In nine of the twelve lessons observed, teachers used tasks that incorporated mathematical content that students were already *familiar* with because it had already been studied in class (based on what teachers reported in their lesson journal). The term *familiar* does not imply that the task was easy for students to do, as this also depended on the way mathematical content was used (the cognitive demand/challenge). Mathematical content inherent within a task was usually cross-referenced to the learning objectives listed in the exam syllabus. It appeared that, for these teachers, mathematical content was a driving feature of the instructional tasks that they selected when planning IBL lessons. Hence, of particular interest are the decisions teachers make in relation to their choice of tasks for their inquiry lessons.

Analysis of these 12 lessons (see Table 27) illustrates that the tasks that these three teachers used, incorporated a high cognitive challenge or unfamiliar content or both (as in lesson 3 for Chris). In their lesson implementation, Chris and Greta, generally adopted a multiple-phase structure. This would imply that teachers were likely to deviate from the three-phase structure when a task incorporated both these dimensions – a high cognitive demand and/or unfamiliar content. Chris and Greta used different intervention strategies to guide students towards task completion. Chris structured his lessons to facilitate students' progression with IBL by gradually increasing the cognitive demand. He did this by planning lessons incorporating a number of short tasks (in his first three lessons). Greta, on the other hand, addressed students' challenges as they arose during lessons. She intervened to minimise students' struggle and avoid them giving up.

This could imply that teachers generally modify the typical three-phase IBL lesson structure to provide help. In the case of Chris, this help appeared aimed at facilitating the process of engaging with IBL, while for Greta, help appeared targeted at making tasks more accessible to students. Both teachers intervened and stopped group activity at some point. In the case of Chris this was to introduce a new task (refer to Figure 14 and his interventions in lessons 1, 2 and 3) while Greta stopped group activity and conducted a short whole-class discussion to address student difficulties (refer to Figure 15 and her interventions during lessons 1 and 2).

	Lesson 1		Lesson 2		Lesson 3		Lesson 4	
	Task	**Structure**	**Task**	**Structure**	**Task**	**Structure**	**Task**	**Structure**
Chris	High Cognitive Familiar content	Multiple Phase	High Cognitive Familiar content	Multiple Phase	High Cognitive Unfamiliar content	Multiple Phase	High Cognitive Familiar content	Three Phase
Greta	High Cognitive Familiar content	Multiple Phase	Low Cognitive Unfamiliar content	Multiple Phase	High Cognitive Familiar content	Three Phase	High Cognitive Familiar content	Multiple Phase
Sarah	Low Cognitive Unfamiliar content	Three Phase	High Cognitive Familiar content	Three Phase	High Cognitive Familiar content	Three Phase	High Cognitive Familiar content	Three Phase

Table 27: IBL tasks and lesson structure

Sarah, unlike Chris and Greta, adopted a three-phase structure in all her lessons. She encountered similar challenges but adopted a different intervention strategy – addressing these by modifying the time and the amount of work allocated to the small group work (see also Appendix 14). For example, when Sarah recognised that the plenary part required more time for discussion, she amended her tasks in such a way that the small-group activity involved thinking about one or two situations rather than say six different ones. Hence, in lessons 3 and 4, each group worked on different problems dealing with just one or two problems (or statements). This modification worked well and discussions improved during these lessons. Sarah shifted her role from requesting answers to asking for reasons. For her, following the three-phase lesson structure seemed an accepted and inevitable strategy with IBL and that she felt unavoidable to work within.

These teachers had a disposition to select tasks that incorporated content to address the time-constraint that IBL posed towards 'covering' the syllabus. Similar to what Priestley (2010) found in Scottish secondary schools, the 40-minute lesson time-frame available was a constraint to the implementation of collaborative inquiry-based methods. For Greta and Chris, tasks with a high cognitive challenge seemed difficult to implement within a three-phase lesson structure because they considered that students needed extensive time to unravel the task within the group activity. As a result, group work tended to take a large proportion of their lesson time (see Table 28), and hence, in some cases, allowing limited time for the plenary. For this reason, it seems that teachers were also less likely to select tasks that presented students with novel mathematical content. Whenever they did – Chris in lesson 3 and Greta in lesson 2 – teachers guided students at an early stage during the lesson. In the case of Chris, he intervened addressing the whole-class after two minutes of group activity (see Figure 15), while Greta adopted a different approach providing hints to students as they engaged with the task. One plausible explanation could be that, in both cases, teachers sought that students would manage to reach the intended learning outcomes within the lesson time available.

	Lesson 1		**Lesson 2**		**Lesson 3**		**Lesson 4**	
	Phase	Time (min)	Phase	Time (min)	Phase	Time (min)	Phase	Time (min)
Chris	Presentation	6	Presentation	7	Presentation	8	Presentation	7
	Individual	2	Individual	0	Individual	0	Individual	0
	Group-work	23	Group-work	19	Group-work	27	Group-work	16
	Plenary	2	Plenary	6	Plenary	3	Plenary	16
Greta	Presentation	10	Presentation	11	Presentation	6	Presentation	5
	Individual	10	Individual	6	Individual	0	Individual	0
	Group-work	16	Group-work	12	Group-work	17	Group-work	31
	Plenary	9	Plenary	3	Plenary	14	Plenary	0
Sarah	Presentation	11	Presentation	6	Presentation	11	Presentation	5
	Individual	0	Individual	0	Individual	0	Individual	0
	Group-work	10	Group-work	24	Group-work	8	Group-work	12
	Plenary	11	Plenary	8	Plenary	16	Plenary	15

Table 28: Time allocated for each lesson phase

Sarah, who held to a three-phase lesson structure, appeared more consistent and able in managing time allocated to each phase. Sarah did not dedicate time to individual activity – a strategy that Chris and Greta used but evidently less consistently over time (data cross-referenced to teachers' lesson journals – see Appendix 14). As already mentioned, Sarah also managed to allocate a higher proportion of time to the plenary part of the lesson – an aspect that Chris and Greta struggled with. Based on these considerations, the next section will investigate the roles teachers assumed during lessons. This analysis is intended to shed light on the actions undertaken by teachers in implementing IBL under conditions – particularly the content to teach and the time available for inquiry – that did not appear favourable for promoting IBL.

6.1.4 The teacher's role in an IBL classroom

Prior to LTMI, teachers assumed traditional roles based on transmitting knowledge to students (see Section 5.2.2). Teachers appeared orientated to tell more and involved students through close-ended questions seeking factual knowledge. In IBL, the teacher is seen as a facilitator, supporter or guide. This shift appears to conflict with the professional responsibility of transmitting knowledge that teachers appeared to assume. Moving from transmission to a facilitator role may potentially increase the challenge for teachers in enacting IBL practices. Indeed, the three teachers recognised this challenge as soon as they embarked with this approach.

> I usually anticipate problems for students and tell them where they need to be careful. It is like telling them to avoid making mistakes. I believe I need to learn and grow out of this and instead allow my students to make mistakes.
>
> (Chris, Interview 2, October 2015)

> I am currently focusing on the choice of tasks because I think that IBL depends on that eventually. I see questioning, collaborative work and giving responsibility to students are all incorporated within the tasks a teacher selects.
>
> (Greta, Interview 2, October 2015)

> I will be changing the way I am used to teach, promoting student thinking and allowing them to come up with their learning. But I am not sure if I will be able to do this when time might work against me.
>
> (Sarah, Interview 2, October 2015)

Chris and Sarah anticipated that they needed to shift their roles and the actions they typically assumed in class. In investigating this during lesson observations, I looked into teacher actions, their verbal interactions with students, and the manner in which they supported students' participation. Teachers' roles were examined within four dimensions: presenting the task, interacting with students during group work, leading students' presentations and the type of questions used. These dimensions will now be analysed to show each teacher's development over the scholastic year.

In enacting IBL, Chris presented tasks through a whole-class discussion. In managing collaborative learning, Chris moved from initially supervising student work (see lessons 1 and 2 in Table 29) in which he observed and encouraged student inquiry to a facilitator role in which he promoted mathematical thinking, reasoning and argumentation (lessons 3 and 4). In facilitating student learning, Chris developed a more active role. He made thoughtful interventions by questioning and probing. Table 29 illustrates the shift in the type of questions Chris used over the four lessons.

Actions	**CHRIS**			
	Lesson 1	**Lesson 2**	**Lesson 3**	**Lesson 4**
During task presentation	Explains task Explains roles during group work	Initiates a whole-class discussion to revise work done	Short whole-class discussion to introduce task	Whole-class discussion through a problem related to task
During group work	Observes students	Observes students Encourages students	Observes students Asks questions	Observes students Asks questions
During plenary	Requests answers	Requests answers and reasons	Requests answers	Requests reasons Shifts decisions onto students
Questions asked during the lesson	Factual 1 Guiding 0 Probing 0 Confirming 1	Factual 6 Guiding 5 Probing 8 Confirming 0	Factual 4 Guiding 2 Probing 20 Confirming 0	Factual 6 Guiding 15 Probing 16 Confirming 14

Table 29: Actions taken by Chris in during inquiry lessons

In transforming his role from transmission to inquiry teaching, it appears that Chris initially adopted an unguided discovery approach where he allowed students to discover things on their own. Over time, he shifted towards adopting a more scaffolded and collaborative approach that allowed for more purposeful interventions and interactions with his students. In lessons 1 and 2, Chris demonstrated a sense of uncertainty about his role during group work. He spent between 5 to 10 seconds observing each group, occasionally moving towards his desk and back to a group. These actions, characterised by a lack of interaction with students, seemed to suggest that Chris was unsure about his roles during this phase of the lesson. Over time, he overcame this uncertainty. In lessons 3 and 4, Chris assumed different roles during group work. He appeared more responsive to group discussions listening to student interactions and eventually spending more time with each group, usually between 40 to 60 seconds. Chris first observed students and then interacted with them using probing questions. As lesson 4 shows, he asked a mix of factual, guiding and probing questions. Over time, the roles Chris assumed were more purposeful towards supporting student learning, and appeared more assertive in managing his class during group work.

This shift was also evident in the way he led the plenary sessions over these four lessons. In the first three lessons, Chris requested short answers from students and rarely sought justifications. One possible reason could be the limited time available for this part of the lesson (see Table 26). In lesson 4, where he dedicated significantly more time for the plenary (16 minutes), he adopted a different approach. This shift is illustrated in Figure 17 using two interactions taken from the plenaries of lessons 3 and 4.

Plenary 3

Teacher projects an interactive slide showing cards. He drags and groups the cards according to what students mention.

Teacher:	Now, what list of numbers?
Group 1:	2, 4, 6, 8, … (Teacher drags card down)
Teacher:	Formula?
Group 1:	2n
Teacher:	You chose 2n. Why?
Group 1:	Because they are even numbers
Teacher:	Ok, and the last group… the function?
Group 1:	Number times 2
Teacher:	Ok. Now because time is up, what we will do is I will collect your charts and tomorrow I will ask for an explanation to all the other groups please.

Plenary 4

After all groups had presented ways of showing examples of triangles with perimeter 30 cm, the teacher makes an observation.

Teacher:	According to two group presentations, not any three numbers that add up to 30 necessarily form a triangle. It seems there is a rule. What could this be?
Student 1:	You cannot have sides with 1 cm or 2 cm
Teacher:	She is saying that you cannot have sides with 1 cm or 2 cm. Why?
Student 1:	Because these are too small
Teacher:	Because 1 cm and 2 cm are small according to her. Do you agree?
Students:	Yes
Teacher:	So, we cannot have a triangle with side 1 cm?
Student 2:	What we need is for the other two sides to be big numbers
Teacher:	So, if one of them is 1 cm and the others are large … you can draw a triangle?
Student 3:	You cannot have that
Teacher:	What if I find a triangle with 1cm?
Student 1:	You cannot
Teacher:	They are saying that we cannot. Can we be sure? Think a bit about this.

Figure 17: Extracts from Chris' lesson 3 and lesson 4 plenaries

In lesson 4, instead of asking for answers and facts, Chris requested reasons and eventually engaged students in further thinking when he extended the task for students to investigate rectangles with a perimeter of 30 cm. Over the course of these four lessons, Chris used more questions and varied the type of questions he asked. Chris commented on this shift particularly with the tasks he used and how these were enacted by students in class.

> I believe I used a variety of tasks. There were those that ended up being closed because students converged to the same idea and conclusion. However, there were times when this did not happen. Students either did not reach the conclusion I had predicted or else took different routes. Because of time limitations, I continued with this the next day in order to conclude the activity. Sometimes, I even returned to a task weeks later when I felt students had gained the knowledge required to discuss it again. I showed photos of their previous solutions and that really worked.
>
> (Chris, Interview 3, June 2016)

This extract shows changes in the role Chris took up. It contrasts with his previous transmission teaching approach characterised by transferring content and presenting exercises to students that built on earlier work. Previously, his lessons had a clear well-rehearsed structure (see Section 5.2.2). However, this was not necessarily the case anymore. At least for some of his lessons, Chris integrated more challenging tasks that required students to actively and collaboratively negotiate mathematical meanings rather than passively accepting and practising the knowledge he transmitted to his students. He shifted responsibility onto students during these lessons and he did not necessarily conclude lessons, but at times left them open and returned back to them when he felt students were in a better position to tackle them again. Chris was mindful of his new way of teaching, encouraging student thinking and using specific occasions to cultivate it.

> Now I encourage and expect students to think. If a student is thinking out of the box, then I bring this to the attention of others. I ask this student to share her reasoning and explain her thinking. I am now educating students to think and communicate it.
>
> (Chris, Interview 3, June 2016)

Like Chris, Greta chose to start her IBL lessons through a whole-class discussion, usually to revise work done in the previous lessons, before introducing the main task. She used tasks for IBL making sure that content was incorporated and 'visible' to students. In the task presentation phase, Greta usually asked students to recall the work they had covered in class and, in three of the four lesson observations, she told students that the task

required them to apply content already known (lessons 1, 3 and 4). Like Chris, Greta made a significant, yet different, change in her role, particularly in lessons 3 and 4 (see Table 30). It is worth mentioning that, following the summer workshops, Greta believed that her 'old' practices – the occasional end-of-topic tasks she assigned – resonated with IBL. Her conception was that inquiry tasks involved students in discussion and thinking. However, in lessons 1 and 2 Greta adopted an interventionist role that contrasted the unguided discovery approach that Chris had assumed. Once students set off working Greta would go around and observe. In observing students, she tended to identify students' mistakes and speak about them. During lesson 1 she directed questions to students specifically to draw their attention to the mistakes she had spotted. Greta seemed to assume a role where she made sure that students were moving on the right track and, whenever they seemed struggling, she would intervene to resolve their issues. She also offered this kind of support in lesson 2 when students were asked to discover the relationship between the diameter and circumference of circles (see Appendix 15). As Greta went around the groups she intervened and suggested ways of measuring these dimensions of circles. Moreover, when students asked questions she provided answers that resolved the uncertainties that students had.

Actions	**GRETA**			
	Lesson 1	**Lesson 2**	**Lesson 3**	**Lesson 4**
During task presentation	Whole-class discussion to revise work done and introduce task	Whole-class discussion followed by task presentation	Explains task and reads set of probability statements presented	Whole-class discussion to revise work done and present task
During group work	Observes students Asks questions Resolves issues	Observes students Provides suggestion Answers questions	Observes students Asks questions	Observes students
During plenary	Requests answers Guides students to reasons	Requests answers Guides students to reasons	Requests reasons Shifts decisions onto students	No plenary
Questions asked during the lesson	Factual 14 Guiding 9 Probing 8 Confirming 4	Factual 19 Guiding 3 Probing 6 Confirming 5	Factual 5 Guiding 15 Probing 24 Confirming 3	Factual 6 Guiding 3 Probing 1 Confirming 5

Table 30: Actions taken by Greta during inquiry lessons

However, in lessons 3 and 4 a shift occurred. Greta initially observed students and did not interact with them as quickly as she had done in previous lessons. She appeared attempting to understand their thinking. Once a group reached a conclusion she questioned it. In lesson 3 Greta asked a total of 47 questions, 13 of which occurred during group work. She asked more questions than in the previous two lessons and these were now used to probe and guide student thinking. Another notable change emerged in lesson 4. Although the group activity lasted 31 minutes, the proportion and type of questions Greta asked reduced drastically, with confirming questions used to verify that students' matching of cards was correct. An extract of one short conversation with students is provided below.

(01)	**Greta**:	Have you matched these set of cards? (Pointing to one set of cards that students placed on one side of the table)
(02)	**Student**:	Yes (Pointing to the coordinates card), the y and x are the same, the equation tells that y is equal to x and the coordinates on the graph are the same
(03)	**Greta**:	Ok. So, are you sure about this then?
(04)	**Students**:	Yes
(05)	**Greta**:	Then you can stick them on your poster

This was a typical exchange she had with students in lesson 4. It contrasted sharply with the interactions she had with students in lesson 3. One plausible explanation for using confirmation questions, as in (01) and (03), could be the fact that the task had one particular solution. It was about revising material related to multiple representations. Moreover, groups were generally matching cards correctly demonstrating that they could apply the mathematical content they had previously learned. So, Greta may have felt the need to just check on students' work before they put the cards on their poster. Greta seemed to extend the time for group work until all groups were ready. Indeed, the fact that no plenary was held may indicate that the teacher may have felt that there was no need for further discussion.

Greta's role during plenary sessions also suggested that initially she assumed much of the responsibility taking decisions to resolve issues students had. In lesson 3, Greta dedicated 14 minutes to the plenary to discuss probability statements with students. Her role in orchestrating

student presentations and discussion was different than in the previous two lessons. She now encouraged students to explain their reasoning making sure they provided convincing arguments for deciding whether statements were true, false or sometimes true. In the following extract, Greta reflects upon this change in her role.

> I learned to allow students to talk about mathematics. I realised that I should talk less and not provide any hints as soon as I see students struggling. First, I try to listen to what students are saying and, as much as possible, I do not say anything. I had to do an effort because I tend to speak a lot to my students.
>
> (Greta, Interview 3, June 2016)

Within this change of role, Greta sought to take a step back yet, doing this was difficult for her. She encountered challenges in finding balance between talking and listening, supporting and providing a hint, and extending and concluding the activity. Initially, it appears that, in using IBL, she enacted only the shallow or 'superficial similarities' (Spillane et al., 2002) between her 'old' practices of using end-of-topic tasks to engage students in inquiry. Having initially understood IBL as being determined by the choice of task and eventually seeing similarities between the tasks offered within LTMI and those she had previously used, Greta relied on her 'old' end-of-topic tasks for inquiry teaching. Over time, she included other 'less familiar' tasks – those she found from her research in textbooks and the internet, and others shared by colleagues during the follow-up meetings. This move appeared supported by a gradual shift in her role – from intervening instantly to correct mistakes towards intervening more purposefully to facilitate student discussion and thinking.

Unlike Chris and Greta, Sarah structured her IBL lessons in three phases. In some aspects, the roles Sarah assumed during the different phases were similar to those of Chris and Greta. However, her role during group work seemed to remain unchanged (see Table 31). In the task presentation phase, Sarah alternated between initiating a whole-class discussion and starting off with an explanation of the task. This explains why overall, she used more

questions in lessons 1 and 3 than in lessons 2 and 4. To present the task, Greta used 28 questions in lesson 1 and 18 in lesson 3.

Actions	SARAH			
	Lesson 1	**Lesson 2**	**Lesson 3**	**Lesson 4**
During task presentation	Initiates a whole-class discussion to revise work done Presents task	Explains task	Initiates a whole-class discussion to revise work done Presents task	Explains task
During group work	Observes students Answers questions	Observes students Allows students to resolve uncertainties	Observes students Encouraged discussion	Observes students Encouraged discussion
During plenary	Requests answers Guides students to reasons	Requests answers Guides students to reasons	Requests reasons Shifts decisions onto students	Requests reasons Shifts decisions onto students
Questions asked during the lesson	Factual 31 Guiding 11 Probing 11 Confirming 7	Factual 15 Guiding 1 Probing 5 Confirming 1	Factual 22 Guiding 12 Probing 19 Confirming 6	Factual 3 Guiding 2 Probing 5 Confirming 1

Table 31: Actions taken by Sarah during inquiry lessons

During three of the four lessons, Sarah assumed two key roles: observing students and making sure that they were on task, and answering questions. Sarah made sure to go around each group more than once, observing and monitoring their work. She did not intervene but seemed to be there to clarify any difficulties students had, related to what they were required to do. Her main function during group work seemed to be that of supervising students. The only time she interacted with students was when students asked questions. In such cases, Sarah guided students to consider alternative solution strategies. Generally, she encouraged further discussion and thinking. A different scenario occurred in lesson 2 and this will be investigated next.

Incidentally, Sarah used the same task that Greta did in lesson 4 – one of the tasks made available as a resource in LTMI. The only difference was that Sarah provided her students with 24 multiple representation cards while Greta used 32 cards. When matching cards within their group, one particular group ended up guessing and Sarah intervened.

(01) Teacher: What are you struggling with?
(02) Alan: This one miss (pointing to the equation card x = –2)
(03) Teacher: What are you trying to match it with a graph or a table of values?
(04) Alan: A table
(05) Teacher: Try starting with a graph first

Alan looks for a graph from the remaining 4 and checks each one of them with the equation.

(06) Teacher: What information can you get from the graph?

In the meantime, Alan starts reading the coordinates of some points on the line.

(07) Teacher: You are reading the points on the graph. Try to find a matching table then.

Initially, Sarah sought to understand the difficulty and later attempted to direct students' attention to particular aspects to support student inquiry. She provides suggestions – (05) and (07) – as starting points towards matching a pair of cards. Sarah's question (06) revealed that she sought to guide and engage students in thinking. These kinds of interventions were specifically done when students appeared confused or struggled. Otherwise, Sarah would allow students to discuss, observing their work while listening to their conversations.

It appeared that generally Sarah assumed a supervising role during group work and then later, during the whole-class discussion, she challenged students to explain their thinking. In the plenary of the first two lessons, Sarah asked more factual questions requesting answers. In both cases, she also guided students in providing reasons for their answers. There appeared to be a shift in her role in managing the plenary part in lessons 3 and 4. Sarah made very little use of the factual questions and seemed more interested in the reasons provided by students. She delved deeper and allowed students to assume responsibility in deciding whether they agreed or not.

> When doing IBL I simply throw a problem at students and I let them come up with ideas. Instead of telling them how, I let them solve it in groups on their own. If their working or method is wrong I do not speak. Then when they present it to the whole-class I challenge them and we come up with the correct method for it.
>
> (Sarah, Interview 3, June 2016)

For Sarah, in IBL, collaborative work offered students the possibility of exploring a task independent of the teacher. It was then through the plenary that she challenged students' thinking and eventually expected the class to reach agreed-upon solutions. These well-defined roles might explain why Sarah consistently used a three-phase lesson structure. Sarah also allowed more time for the plenary part than Chris and Greta did. For Sarah, it appeared that the plenary part was crucial in concluding her inquiry lessons.

The roles teachers undertake during each phase of an inquiry lesson is potentially related to a number of factors – their conceptions of IBL, tasks, student engagement and lesson structure. It appeared that, when teachers dedicated more time for the plenary part this shaped the kinds of conversations taking place. Over time, when teachers dedicated around a quarter of an hour – Chris (lesson 4), Greta (lesson 3) and Sarah (lessons 3 and 4) – they were more likely to seek reasons and delve deep into students' thinking than when time was limited.

6.1.5 IBL features valued and enacted in class

In analysing the development of IBL features as enacted by teachers in class, data from the third interview were triangulated with teachers' lesson journals and data from their pre- and post-CPD questionnaires. By the end of their CPD journey, teachers valued the shifting of responsibility as a key feature that represented engaging students in IBL (see Table 32). Initially, Chris and Greta were mostly familiar with questioning. In the case of Chris, his engagement with IBL provided him with opportunities through which he became more familiar with selecting tasks. Sarah, for instance, was familiar with collaborative work and sought to engage students in IBL by shifting more responsibility onto them. This was something that she valued but found most challenging to implement, at least initially.

IBL FEATURES	Chris		Greta		Sarah	
	Before	After	Before	After	Before	After
Most familiar with	*Questioning*	*Questioning and tasks*	*Questioning*	*Selecting tasks*	*Collaborative work*	*Collaborative work*
Most valued	*Shifting responsibility*	*Shifting responsibility*	*Selecting tasks*	*Shifting responsibility*	*Shifting responsibility*	*Shifting responsibility*
Most challenging	*Selecting and using tasks*	*Collaborative work*	*Collaborative work*	*Shifting responsibility*	*Shifting responsibility*	*Questioning*

Table 32: Enacting IBL features as reported by teachers

Through their engagement with IBL practices, teachers reported to have made progress on those features that they had initially identified as most challenging. Chris, for example, identified selecting and using tasks as a challenge. By the end of the CPD, he became more familiar with this and eventually identified it as a strength. Table 33 shows teachers' perceived progression on IBL features from the start of their LTMI journey (light grey shading) up to the end (dark shading). Features with no perceived improvement are shaded in black.

Teachers valued and rated their development in implementing IBL mostly by shifting more responsibility onto students. Yet, teachers believed to be least effective in enacting this particular feature of LTMI. It appears that 'shifting responsibility onto students' represented, for teachers, the essence of productive inquiry-based lessons. At the end of the CPD journey, Chris had identified managing group work as the most challenging IBL feature and eventually believed that he did not make any progress on this by the end of the CPD. On the other hand, Greta had identified questioning as her most familiar feature in teaching. However, she did not feel she had improved her practice on this feature possibly because she perceived her questioning as already effective in stimulating student inquiry. Questioning was, however, a challenge for Sarah, and a feature she reported least progress on.

IBL Features		Rating (Perceived progress before to after participation in CPD)				
		Very ineffective	Ineffective	Average	Effective	Very effective
Choosing a task for inquiry	**Chris**					
	Greta					
	Sarah					
Shifting responsibility onto students	**Chris**					
	Greta					
	Sarah					
Managing small-group activity	**Chris**					
	Greta					
	Sarah					
Using questioning to stimulate thinking	**Chris**					
	Greta					
	Sarah					

Note: Light grey shading indicates before; dark grey shading indicates after; black shading indicates no change

Table 33: Teacher ratings of enacting IBL through four features

6.1.6 Student engagement with IBL

For teachers, students' reactions, dispositions and involvement in the activities provided appeared critical in persisting with IBL. Student engagement with IBL is investigated using data from the third interview and lesson observations.

Chris and Sarah, in particular, repeatedly mentioned students' engagement in reflecting about their enactment of IBL. Since IBL was a new pedagogy for their students, teachers were cautious about winning them over. Chris first sought to get to know students well and made sure they were receptive to innovative learning approaches before immersing them in taking on more challenging tasks. For example, Chris requested students to discuss questions in groups before they addressed queries to him. Like Greta, he also regarded student behaviour and motivation as key conditions for offering IBL. Chris was careful in introducing IBL to students. He was mindful of students' reactions and responses and so introduced IBL gradually. He started off by providing students with short 10-minute tasks.

He stressed how important it was for him to create and nurture a classroom culture based on trust. He sought to win students over and he felt that he needed to convince them that he was confident with inquiry teaching.

> The most important thing is building trust between the teacher and the students. Once you create that then students will pass on positive messages to their parents. As a teacher, you can then experiment with anything you like. But you do not get this in the first lesson. Building trust takes time. I believe that having a very good relationship with students and working within a friendly atmosphere, where mistakes and asking questions were part of the learning process, were crucial. As a teacher, you need to convince them that all this may lead to something greater.
>
> (Interview 3, June 2016)

Chris' students responded positively to IBL. He could notice that they were thinking deeply and this inspired him to pursue with his IBL practices. Chris started to compare his 'old' transmission approach of providing his students with the method for working out a problem with the 'new' IBL approach where students explored, created and discussed their own methods. Chris was seeing students' cognitive development within the kind of questions that they started to pose.

> The questions they asked were more inquisitive. They started asking "What if" questions, and students were carried away by these questions delving deep, thinking further and extending on the work involved in the tasks I presented them. The classroom environment probably supported them to explore and, on many occasions, I could notice that students were reasoning and thinking.
>
> (Interview 3, June 2016)

Likewise, students' engagement with the tasks Sarah presented offered her direct feedback to persist with and reflect upon her IBL practices. She noted students' positive experiences and increased motivation each time she offered them a collaborative learning experience. Her students looked forward to group activities and their evolving positive response helped Sarah to trust students more in their capabilities to undertake challenging tasks. However, Sarah and Greta were initially troubled with students'

behaviour and their undertaking of the task they presented to them. Unlike Chris, Sarah implemented IBL with her class from day one. She faced classroom management issues that she had to address. With her class, Sarah discovered that she had to be careful in managing activities. Greta also had a similar experience and felt that she could be more ambitious if she had a less noisy class.

Teachers mentioned students' positive engagement with IBL. The three teachers related student engagement to a number of aspects: enjoyment, thinking and learning. Sarah was, however, unsure whether students' enjoyment of the activities she provided were related to IBL or to other external factors. It appeared that the students that Sarah was teaching engaged with IBL because it appealed to them – they disliked writing and, in the inquiry tasks that Sarah offered, they were expected to discuss things but not required to write much. She referred to her students seeing IBL as "group activities" and that these offered them a space where they could work together and chat. It seemed as if IBL liberated her students from the rigidity of the traditional classroom setting. But Sarah also saw how this evolved and, over time, students learned to talk about mathematics.

> I could see that they became more capable to speak mathematically. I recall that, at times, I tried to rush things and even told them how to do things. But they stopped me… they asked me why. I had missed a lesson the previous week and I was tight with the syllabus and tried to hurry things up, but they did not allow me to do that. They asked and required explanations.
>
> (Interview 3, June 2016)

Such a demand from students showed how discussion, reasoning and thinking were an integral part of their learning of mathematics. Sarah noted not only improved thinking skills but that students started to make links between aspects discussed in other lessons. She experienced instances when students identified their own mistakes and, through IBL, incorrect assumptions were revealed. The following is an excerpt of one such discussion with students (pseudonyms) that took place during the plenary of the third lesson observation.

Teacher projects a statement and calls Alan to the board.

Statement: *The outcome in a football match is win, lose or draw. The probability of winning is one-third.*

Teacher: Alan, what did you do?

Alan: We said it is always true

Teacher: Always true.

John: I agree, I agree.

Alan: Because there are three possibilities

Teacher: Is there anyone who can argue the same as Alan or maybe against this argument?

Paul: It is always true because you cannot have a loss and a draw at the same time. It is either one or the other.

Teacher: Yes, sure. It is either one or the other.

Paul: It is one-third. It can never be two-thirds.

Teacher: Ok, so let us go back to my race with Matthew (referring to an introductory activity). I can win or lose, but we can also draw. In that case, the probability that I win will be one-third?

Paul: Wrong, wrong

Alan: Sometimes true

Teacher: Why is it now sometimes true?

Alan: Because sometimes true means that it is probable that you will win, lose or draw. You don't know what is going to happen

Teacher: What makes a team win, then?

Alan: To win?

Teacher: Yes. If I had to mention two football teams …

Paul: Manchester United

Teacher: Valletta, a Maltese team, and Manchester United. What is the probability that Manchester United win against Valletta? Is it one-third?

Alan: One whole

Teacher: So, now you are going to change it from one-third? What do you think, Kyle? The probability that Manchester United win against Valletta

Christian: We need to consider other things

Teacher: Similar to the race I was going to have with Matthew. You had mentioned age and height as factors affecting the result, right? What do you think?

John: Sometimes true

Teacher: In this football game, who would you think will win?

John: Sometimes true

Teacher: John, tell us why?

John: Because although Manchester United will probably win, you still cannot tell who will win.

Teacher: So, what should we also consider in teams?

Michael: The very good players they have

Teacher: Ok, what else?

Kyle: The form they are in

Teacher: Yes, all these factors must be taken into consideration.

(Sarah, Lesson 3, Plenary discussion – 28/04/2016)

This excerpt shows that students were open for discussion and ready to be challenged on their claims. With the prompts Sarah provided, students were able to think more deeply and consider mathematics as applied to a real-life situation – something that appeared to be missing in initial discussions. Chris also mentioned how IBL helped student thinking when they were presented with a real-life situation. He recalled a best buy situation that he used with his students to engage them with IBL, and how impressed he was with the range of responses that his students provided.

> Students came up with so many ideas and some of the solutions were really impressive. I would have never come up with these myself, and these things really inspired me to do more. Had I done this lesson the traditional way, I would have told them my method and explained each step… that is how I would have done this lesson before. By presenting a problem and opening up the discussion, I learned a lot as well and students keep on mentioning it when similar problems arise.
>
> (Interview 3, June 2016)

Through students' active engagement, Chris experienced the benefits of IBL. According to Chris, students were not "afraid of mathematics and of engaging with the novel situations" and he valued IBL because students showed "improved understanding of mathematical knowledge through discussions they had with others" (Interview 3, June 2016). The shift from transmission to a more collaborative and inquiry-based approach appeared to support student learning. For Chris, this learning went beyond mathematical knowledge as students learned to think, communicate their thinking clearly, developed critical skills and learned how to apply knowledge to novel situations.

> I need to be realistic though. If you ask me "will my students do better in exams?", I do not think that students' progress through IBL can be quantified. But I am sure there has been an evolution in student thinking and maths has become something they look forward to.
>
> (Interview 3, June 2016)

Student engagement with the IBL activities served as a driving force for teachers to persist with IBL. In implementing IBL, teachers could rely on direct observation and gain 'just-in-time' feedback on students' engagement, their response and learning through IBL.

6.1.7 Concluding note

This theme focused on teachers' interpretation and enactment of IBL. Initially, teachers interpreted IBL as non-interventionist teaching offering students with opportunities to explore mathematics on their own. This unguided discovery approach persisted, at least up to midway through the CPD, until teachers recognised inconsistencies and attempted to address these through the use of more purposeful interventions. Table 34 shows that, in enacting IBL, teachers with prior knowledge of IBL relied on perceived similarities with their 'old' practices (Greta) or CPD experiences (Sarah). Indeed, the three-phase approach promoted in LTMI was only adopted by Sarah. Chris and Greta experimented with multiple-phase scaffolded approaches. While lesson structure may also be related to teachers' adaptation of new roles, it appears determined by two factors inherent within the mathematical tasks they used – the cognitive challenge and the mathematical content that they were required to teach.

Teacher	**Inquiry Teaching Practices**				
	Pre-CPD			*During CPD*	
	Teaching (years)	Exposure to IBL	Inquiry teaching experience	Similarities with prior practices	Lesson structure
Chris	1 – 5	Session during ITE course	None	None	Multiple-phase
Greta	16 – 20	Number of sessions during INSET	End-of-topic tasks	With end-of topic tasks	Multiple-phase
Sarah	16 – 20	Participation in 18-month CPD programme	Occasional lessons during IBL-related CPD	With lesson structure promoted by CPD	Three-phase

Table 34: Comparing teaching experience

In enacting IBL, teachers perceived limitations related to the system and lesson time restrictions. However, they attributed their persistence in

implementing IBL to students who, through their engagement and response to IBL, offered teachers immediate feedback related to developing and improving classroom activities.

6.2 Theme 2: Teacher development, learning and change

In this section, I now look into teacher development, learning and change over the course of LTMI. This theme incorporates aspects related to strategies that teachers found effective for enacting IBL, changes in practices, views about teaching and learning that teachers developed following their use of IBL, and changes in their beliefs about mathematics, teaching and learning.

6.2.1 Effective strategies for IBL

Teachers mentioned a number of implementation strategies that they found effective for IBL. In describing effectiveness, teachers related these to the particular class with whom these were implemented. Moreover, strategies were lesson specific, meaning that teachers believed that these only applied to particular kinds of lessons that incorporated specific learning outcomes. Post-CPD data, from the focus group, final interview and questionnaire, were used in analysing this subtheme.

For Chris, IBL was an approach that provided flexibility about how, when and the extent to which students are engaged in collaborative work. A strategy that worked well for Chris was to first set students to think about a problem individually before sharing ideas in a group. At times, he found that this individual phase generated enough student ideas for a whole-class discussion to introduce a topic or return to revise a concept. Greta took a similar approach of introducing a topic through a question and then allowing students to think about it in pairs. For Greta, this strategy did not necessarily apply to lesson introductions. She used this strategy even midway through the lesson whenever she felt that students needed to delve deeper into ideas and concepts. Sarah offered 'why' questions to challenge students and stimulate their thinking about mathematical problems.

Teachers also mentioned that implementing effective strategies required creating, cultivating and sustaining a particular classroom culture for IBL. Chris believed that a strategy overlooking inquiry-based classrooms was establishing a healthy relationship with and between students.

> I feel it is fundamental. Students should not be afraid of the teacher or of each other. You need to show them that you respect their ideas and educate them to respect the ideas of their peers. If this is missing, then students would probably be less inclined towards inquiry.
>
> (Interview 3, June 2016)

IBL, according to Chris, necessitated establishing relationships – respecting ideas, accepting mistakes, recognising efforts, and celebrating thinking. Sarah also felt strongly about this. She claimed that, in such environments, it would be students driving the learning and not necessarily the teacher.

> Looking at those videos I am amazed by the classroom environment. It is such a calm atmosphere. I wish my lessons could be like that in the future and, as much as possible, I see students driving the learning. I had this dream about the environment I wanted to create. I did not reach the level intended, but I still learned a lot about how I should go about it next time. I now feel better prepared to introduce students to IBL.
>
> (Interview 3, June 2016)

Sarah's reflection demonstrated the challenges teachers are likely to face when they introduce IBL to their students. She believed that she was too quick in introducing IBL to her students and argued that she would do it more gradually next time. Sarah also discovered that her students were not used to working in groups and she had to prepare them and nurture their skills before engaging them further with IBL. Through her CPD journey, it seemed that Sarah had drawn on strategies and ideas taken from videos, but also from her own approaches. One strategy that appeared to have worked for Sarah was managing student time during group work.

> If a task had four items, problems or questions, rather than giving all these to each of the groups I preferred to give one or two to each group. In that way, I could dedicate about 10 minutes to the group work and have enough time for students to present and discuss their work. You need to plan for this depending on the class and time available.
>
> (Interview 3, June 2016)

Over time, Sarah dedicated about 10 minutes for group work and around 15 minutes for the plenary. She believed that it was not the amount of work offered to students that was important but the level of their engagement in thinking. Evidence from the lesson observations with groups showed that when a task had a number of problems or questions, students tended to answer these quickly in order to get their work ready in the time available. Sarah's strategy, to limit the amount of work during group work and assign different problems to each group, seemed to work better – both in engaging students in more thoughtful discussions and in managing plenaries.

Greta also found other strategies. With her class, pair work evidently worked better and became an everyday strategy. Her students looked forward to it and she believed it supported their learning. This strategy also helped Greta in improving the way she conducted class corrections.

> In my typical lesson, class corrections usually involved explaining methods and then assigning further exercises. I changed this by first asking students to work on the exercises in pairs and then we discussed this as a whole class.
>
> (Interview 3, June 2016)

This strategy helped transform class corrections into more student-directed activities. Greta appeared to be using strategies that she had developed through IBL and was now implementing them to change her more transmission-oriented lessons. Chris also changed his own class correction strategy. Rather than indicating where students had made mistakes in homeworks, he started to create his own worked examples that included typical mistakes he encountered while correcting students' work. Chris started lessons by projecting these on the interactive board and asked students to help him identify mistakes. He believed this strategy was

inquiry-based as it involved students in discussing their own ideas while he only intervened occasionally. His class corrections now involved students in spotting mistakes, explaining why and correcting them in a collaborative manner with students learning "from each other and not only from the teacher" (Interview 3, June 2016).

6.2.2 Changed practices and views about teaching mathematics

Over time, lesson observations revealed the changes that teachers implemented. In examining the role of the teacher (see Section 6.1.4), differences between teachers' 'old' and 'new' ways of teaching mathematics emerged. These data were also corroborated with data provided by teachers themselves. Prior to and, again, at the end of the CPD, teachers were asked to put in order a list of teacher actions to show the typical sequence of activities during lessons (see Appendix 3). Table 35 shows the changes as reported by teachers. Highlighted text shows movements in the sequence of certain actions undertaken, text in bold specifies new actions that teachers introduced while text in italics represents written comments included by teachers to explain or justify specific actions used.

These teachers reported a number of changes that were consistent with observations reported in Section 6.1.4. As a result of the CPD, Chris and Sarah dedicated time for group work in their lessons. Sarah also provided opportunities for students to share and explain their work. This modification was also consistent with her changes in beliefs reported in Section 6.2.4. Teachers also modified their actions during lessons. For Chris, devoting time for students' questioning was not something that necessarily followed his explanations, notes giving and questioning anymore. Chris transformed this, becoming more flexible to allow students to intervene and pose questions at any stage rather than at a specific time during the lesson. Greta also reported changes to her previous approach. She identified developing notes and discussing examples while negotiating mathematical content with students rather than providing these herself. For Sarah, including collaborative experiences was not limited to group work but included class discussions with opportunities for students to initiate and

contribute to this. These teachers also reported shifting certain activities. Of particular interest are shifts (highlighted) indicating a move towards student-centred approaches.

	Pre CPD	Post CPD
Chris	1. Provide notes 2. Give an explanation 3. Pose oral questions to the students 4. Devote time for student questioning 5. Give an easy question 6. Demonstrate examples 7. Give a challenging question 8. Assign work to be done individually	1. Give a challenging question 2. Pose oral questions to the students 3. Give an explanation 4. Provide notes 5. Give an easy question 6. Assign work to be done individually 7. **Assign work to be done in groups** • *No specific instance in the lesson when I devote time for student questioning. I do it all the time.*
Greta	1. Pose oral questions to the students (*to discuss a mathematical concept*) 2. Devote time for student questioning 3. Give an explanation 4. Demonstrate examples 5. Provide notes (*during discussion*) 6. Give a challenging question 7. Assign work to be done individually 8. Assign work to be done in groups	1. Pose oral questions to the students 2. Devote time for student questioning 3. Give a challenging question 4. Ask students to explain their work 5. Assign work to be done individually 6. Give an explanation 7. Assign work to be done in groups • *Notes are built with students during the lesson* • *I provide examples when discussing content with students*
Sarah	1. Pose oral questions to the students 2. Devote time for student questioning 3. Give an explanation 4. Provide notes 5. Assign work to be done individually	1. Pose oral questions to the students (*to present a task or introduce a problem*) 2. **Assign work to be done in groups** 3. **Ask students to explain their work** 4. Devote time for student questioning 5. Provide notes 6. Assign work to be done individually *Sometimes, instead of group work, we do a class discussion. Students answer questions directed by the teacher or posed by the students to arrive at a solution based on students' methods and not the teacher's.*

Note about text: Highlighted shows pre-to post changes; bold shows new actions; italics shows teachers' comments

Table 35: Reported actions undertaken in typical lessons

Chris made a key change starting lessons with a challenging question as opposed to starting lessons by providing notes. Over time, providing notes featured at a later stage during lessons and it followed student engagement. This marks a sharp difference from his teaching prior to CPD. Greta

exhibited a similar shift. While lesson introductions remained unchanged, she started to offer a challenging question prior to providing explanations. The post-CPD actions demonstrated a change towards less transmission teaching. In contrast, the case of Sarah shows a change in the central part of the lesson. Rather than relying on giving explanations and notes, Sarah incorporated collaborative work to enhance students' active contribution.

Lesson observations support these data showing that, through IBL, teachers engaged students in deeper thinking. Collaborative learning and discussion were central features in teaching mathematics. The following analysis explores how teachers' enactment, of these relatively new practices and strategies, could have potentially influenced and transformed both their practices and views about teaching. The LTMI journey produced changes related to four aspects of inquiry teaching: (1) from revising content towards exploring content, (2) from a one-off activity to a more integrated approach, (3) from transmission towards more collaborative learning, and (4) from changes in teaching to changes within self.

When teachers joined LTMI they were inclined to use IBL as an approach to elicit mathematical meanings from students. Over the course of CPD, IBL served as an assessment tool to identify students' current knowledge and potential strengths and weaknesses in their understanding. Teachers also reported that, with the use of IBL over time, they eventually started transforming their lessons developing an "awareness for the need that teaching mathematics had to take a different direction other than the traditional one" (Chris, Focus Group Discussion, May 2016). Through his immersion in IBL, Chris saw himself working to inculcate in students a different way of thinking based on a collaborative culture for doing mathematics. Sarah echoed a similar view.

> I like a lively class where students are receptive to what I present them with and their share ideas. IBL provided this opportunity in my teaching which I enjoyed doing. Having students work collaboratively and contribute ideas motivated me, and I would not go back to a teacher-centred approach.
>
> (Interview 3, June 2016)

For Sarah, using IBL brought about changes in the role students assumed and this eventually changed her.

> I gained so much from my participation in this CPD, this means more to me than winning the lottery. It was such a great experience. After many years teaching in the same manner, I wanted a change. I was bored teaching in classes where students were not responsive and where students were uninterested and unmotivated to learn. I needed something to refresh myself and that is what this CPD did to me.
>
> (Interview 3, June 2016)

Sarah commented that this experience transformed her as a teacher because of the way students responded to IBL. Their engagement appeared to be a determining factor for her changes in teaching and within herself. Chris also felt this kind of change in his teaching.

> I am noticing a change in the way I ask questions, when I ask a question, whether I expect an answer or not, and in the amount of time I allow for students to think. It seems that I am finding myself doing things and acting in ways that are not typical of the ways I used to act in class before.
>
> (Interview 3, June 2016)

Chris seemed to be experiencing a change driven by his immersion in IBL. Over time, Chris valued the role of inquiry for student learning, and this led him to "appreciate the beauty within teaching mathematics" (Interview 3, June 2016). Through IBL, Chris saw potential in his students. In this change process, Chris faced personal challenges and repeatedly expressed these as "frustrations" between what he believed was important to do for his students and how the system restricted him to operate. Chris made an effort to integrate thinking with his teaching of mathematical content. He described how rather than answering the questions that students directed to him and move on, Chris was now more likely to offer it back to students to explore it and involve other students in answering the question. Still, the exam syllabus remained a restriction and a burden for him, and there were recurring instances during the year when he questioned the evidence-based education system he had to cope with. Chris compared the Maltese system

with “an input-output machine where the knowledge delivered by the teacher is constantly being tested to make sure that students learned it” (Interview 3, June 2016). For Chris, this measurement-oriented system was at odds with his conception of IBL. Through IBL students were offered the space and time to develop their learning, but the system was restraining all this from happening as much as he desired. Frustrations persisted during his IBL implementation journey. Nevertheless, these frustrations were “neutralised by the positive vibes generated by students’ thinking and contributions, and the feel-good factor experienced” (Interview 3, June 2016).

Greta faced similar dilemmas. Prior to her participation in LTMI, she did not see IBL as an appropriate pedagogy for her Year 10 and 11 groups because of examination pressures. Of the three classes she was teaching, Greta chose to do IBL with her Year 8 class. However, following her CPD experience, Greta mentioned how she intended to change her preconceptions about IBL.

> Perhaps my mindset is what I will change… how I consider IBL in Year 10 and 11. It is true that at Year 11 it is a race against time for the exam. But nowadays I am not saying not even once, as if IBL is for the Year 7 and 8 only. If I would be teaching the higher years next year, I would not say that IBL will not feature in my lessons.
>
> (Interview 3, June 2016)

Greta’s experience provided a new prospect for using IBL. IBL was not seen as only limited for lower secondary students (11 – 13 years), but it could also feature in her teaching with higher forms (14 – 16 years). Using IBL required her to dedicate more time for learning and offered a challenge in completing the exam syllabus. However, she saw her implementation of IBL as a success story, and this appeared to have influenced Greta to challenge and modify her initial views about teaching mathematics to students in higher forms.

6.2.3 Contexts for teacher learning in implementing IBL

During the scholastic year, teacher learning in enacting IBL occurred through an ongoing process involving two parallel teacher activities: classroom implementation and participation in a CoP. In implementing IBL, teachers planned, taught and reflected upon a number of inquiry lessons. Within the CoP, teachers met together to share experiences, discuss the pedagogical implications of using IBL and reflect on their development of new practices and beliefs. While during implementation teachers worked in the privacy of their classroom, during the CoP meetings teachers collaborated and shared knowledge. Data from these two distinct contexts were analysed to understand situations and conditions that facilitated or hindered teachers in learning about IBL implementation.

Classroom learning opportunities were recorded by teachers, in writing, in the final section of their lesson journals. Data presented in Table 36 are taken word-for-word from these post-lesson reflections. In analysing and coding these reflections, three forms of learning emerged: learning from lesson planning and management, learning from students' engagement, and learning about developing their role in class.

For Chris and Greta, learning about classroom practice took the following pattern. Initial entries focused on their planning and implementation of inquiry tasks reporting on the importance of using a range of tasks to support inquiry – providing meaningful, spontaneous, simple and hands-on activities. Eventually, reflections focused on students' role and their active engagement during these lessons. Over time, teachers wrote on learning about their role and the knowledge they developed as a result of enacting IBL.

LEARNING FROM IBL IMPLEMENTATION		
Chris	Lesson	• Using a meaningful task and being creative when presenting it to engage students in inquiry • Planning extensions for early finishers
	Students	• Are creative and can explore different scenarios • Can make connections and extend knowledge to other topics not yet covered in class
	Teacher	• Delegating responsibility onto students leads to increased discussion, participation and learning • Good time management helps in reaching lesson objectives
Greta	Lesson	• Spontaneous, simple and hands-on activities work well with students • Planning and managing time during each phase of the lesson
	Students	• Own and understand new material when they discover it themselves • Appreciate doing something different in which they are responsible • May come up with ideas that the teacher would not have thought of while planning
	Teacher	• Questions are indispensable in situations where students get stuck • Learning to listen rather than intervening
Sarah	Lesson	• Grouping students requires planning and thinking • Managing the amount of work to assign in the time available
	Students	• Takes them time to take risks and think independently of the teacher • May come up with ideas that the teacher would not have thought of • Need more opportunities to develop their thinking skills
	Teacher	• Persist despite initial challenges • Never underestimate student capabilities

Table 36: Teacher learning as reported in their journal entries

Greta went through a process reflecting upon her role in interacting with students, eventually realising the importance to listen more to students' talk and, hence, to intervene less frequently. Sarah seemed to reflect upon her learning in a different way. Her initial reflections focused on students. This is possibly because of the initial challenges she encountered in her enactment of IBL. The class I was observing her teach seemed demanding in terms of attention and behaviour. Hence, initially, Sarah had to resolve issues related to class control and organisation. While Chris and Greta focused more on implementing a new pedagogy, Sarah also needed to

nurture students' motivation to learn. These teachers learned that, through IBL, students developed their creativity, engaged in deeper thinking and came up with ideas that were novel to them.

> I did not learn all these considerations about IBL in my ITE course. I learned through practice and by seeing new ways in which students were learning mathematics.

(Chris, Interview 3, June 2016)

Chris highlighted the practice-based approach and how he developed a different notion of learning.

> In one of the meetings, I had mentioned something I am now convinced about. As humans, we do not learn in a linear manner but through experiences. For example, we learn by trial and error... today I may not be successful but I might be successful tomorrow, what I know today for a fact might turn out to be false the next day and what I do not know now I might discover something about it later on in life.

(Chris, Interview 3, June 2016)

For Chris, learning took place over time and was not a linear process – it was evolutionary and ever changing. Data showed that Chris developed this notion as a result of both his engagement with enacting inquiry practices in his classroom and the ongoing interactions and negotiations he had with colleagues during the CoP meetings. This also appeared to be the case of Greta and Sarah. In the focus group discussion, Sarah mentioned that the learning environment created by the CoP provided "the opportunity to demonstrate" their work and practices. The environment appeared to allow agency and freedom for teachers to make as many contributions as they liked.

> Had it not been for these meetings I would surely have given up hope. The meetings helped me improve on the strategies I was using, getting tips from what was discussed. Meeting others was for me crucial in persisting with IBL.

(Sarah, Focus Group Discussion, May 2016)

For Sarah, engaging with and learning about IBL within the school context was characterised by isolation – no teacher used IBL to teach mathematics and, hence, she could not seek any assistance there. This was also true for Greta who described a rather competitive atmosphere between teachers as a "race to cover the (exam) syllabus" (Interview 2, October 2015). Chris, on the other hand, occasionally discussed tasks and lessons with Alan, a colleague who also attended the follow-up meetings and who consistently used IBL. Albeit this experience, Chris still felt that a lone culture prevailed within the school.

> The situation at school has remained the same. Most teachers have their own ideas and you become aware that your ideas would not be recognised. Sometimes, it feels like they are thinking that I am trying to introduce ideas that they would have to adapt to…. It feels like they are telling me to keep my ideas to myself because then they would be made to adopt them. So, in a subtle way, these experiences bring you to a point where you do not seek colleagues for help.
>
> (Chris, Interview 3, June 2016)

For Chris, the perceived feeling was that teachers sought to maintain the status quo. Within their schools, these teachers were lone-fighters in their journey to innovate their practices. For these teachers, the school system restricted their enactment zones. School administrations did encourage teachers to participate in CPD and undertake innovative practices. However, collaborative supporting structures and a culture of sharing and negotiation were lacking. Teachers found this support in the CoP created within LTMI. Learning to teach within the CoP contrasted with that of school life, with qualities of collegiality emergent from the sharing of practices. Collegiality and the potential of collaborative learning within the CoP contrasted the 'modus operandi' within schools. For Chris, for instance, the CoP environment also afforded a particular kind of freedom in his journey towards learning.

> One of the things that helped make this course such a success was that this was not something imposed, but we all seem to be doing this for our own learning and also because we believe in IBL. It was not because somebody was checking on me that I did this, but because I wanted to grow as a teacher and for my students to learn... and I think that gives you more motivation, the fact that it is something desirable for us to do. We came to these meetings not because someone forced us to but because we wanted to. I think that made all the difference. This course was something I wished to do. And if there was a time when I could not do as much, there was nothing wrong with that... so I was free... free from the control of having things imposed on me.
>
> (Chris, Focus Group Discussion, May 2016)

Chris spoke about the "control" that teachers seem to be under in their daily professional work. He argued that teachers are not trusted by their superiors. According to Chris, it appeared that, for school administrations, it was not enough for teachers to teach 'good' lessons. Teachers appear to be overwhelmed with preparation workload related to keeping and presenting schemes of work, detailed lesson plans and up-to-date assessment records. While these responsibilities are important, they appeared to be taking most of the teachers' time; time that they could dedicate towards planning more reform-oriented lessons. But, more importantly, such an obligation is viewed by Chris as a sign of lack of trust. What Chris seemed to be saying was that trust is essential in enabling him to reform his practices at his own pace, and without control and impositions.

These teachers looked forward to the CoP meetings because they did not spend hours listening to a speaker, as was common in most of the INSET training courses they attended. Through LTMI, they shared experiences, worked together and designed useful and relevant materials that they could apply to their particular classroom setting. Meetings promoted active involvement where teachers collaboratively planned and reflected upon their lessons. Chris' account of this experience captures the importance of active learning and the common traits that emerged within the CoP.

> I feel this was a course that didn't just speak about theory, although we did tackle that. This could have been a course delivered by experts and we just stood passively listening to them teaching us about inquiry. Instead, we got our hands dirty, we found challenges and difficulties, we understood that the challenges we encountered were common to all and the most valuable aspect was that we shared and learned from that.
>
> (Chris, Focus Group Discussion, May 2016)

The benefits of being part of a CoP were recognised by teachers. Collegiality within the CoP was not valued simply because it offered opportunities for the sharing of experiences. Teachers listened to and provided their own reflections on each other's experiences whilst appreciating the diverse working contexts framing their respective work places and classrooms. Sarah and Greta mentioned the community as an essential feature for their persistence in using IBL. Both teachers valued the sharing of experiences and the discussions through which they could address the challenges they faced. For Greta, the CPD community was a "support group" (Interview 3, June 2016) for her to exchange ideas with others. Similarly, Sarah mentioned the opportunity where to discuss tasks and strategies she used for IBL. Initially Greta and Sarah faced more challenges than Chris with introducing their class to IBL and both considered the CPD community as the space where they could share dilemmas, seek support and develop their inquiry teaching.

Engagement in CoP shaped the identity of teachers, that it, their self-perception of their own role, their agency and relationship with others (Wenger, 1998).

> **Chris**: Through this course and my use of IBL, I found that IBL is very much close to the way people naturally learn. We do not learn in a systematic way but learn by discovering small pieces and then seeing how these fit together... I feel we learn things this way, not only mathematics but also life skills.
>
> **Greta**: Being part of this group of teachers really helped me. It would have been difficult this year using IBL without being part of this group because when I used IBL the previous year and encountered difficulties I gave up. I feel more confident now to accept and take on new challenges.

> **Sarah**: I really feel changed this year... I am not the same teacher I was last year. I find myself now integrating features of IBL like discussion and allowing students to have a go at it first... and these have now become part of me, and my teaching.
>
> (Focus Group Discussion, May 2016)

Sarah, Greta and Chris spoke about developing themselves as teachers in different ways. Sarah had changed in her self-perception of her role as a teacher. She experienced a change from being a transmission teacher to one who was now more receptive to employing active learning strategies with students. Greta revealed more agency in taking challenges that she would not have done prior to this course. On the other hand, Chris developed a new understanding of inquiry and learning. His experience of students learning through inquiry helped him understand how the process of inquiry resonated with the natural way in which people learn skills in everyday life.

6.2.4 Changes in teachers' beliefs

To gather data about changes in teachers' beliefs, participants filled in the beliefs section of the questionnaire at three different stages of their CPD journey – before (April 2015), midway (October 2015) and at the end (June 2016). Pen portraits from the pre- and post-CPD questionnaires were used for data triangulation.

Table 37 displays the professed beliefs of each of the seven LTMI teachers at three different stages of their CPD journey – dark shading indicates predominant transmission, discovery or connectionist beliefs while light shading shows an equal mixture of more than one belief. Data collected prior to LTMI showed that Sarah held predominantly transmission beliefs while Greta held predominantly connectionist beliefs. Chris and Colin held transmission beliefs, Tania held discovery beliefs while Jackie and Janet held connectionist beliefs. However, the beliefs of these five teachers were less prevalent. Following the summer workshops (during CPD), the three teachers who had held a predominantly connectionist orientation at the beginning retained these beliefs more strongly. However, different changes occurred for the three teachers who had held predominantly transmission beliefs. Chris, for instance, shifted his beliefs towards discovery while Colin

shifted towards connectionist. Sarah, instead, retained her predominantly transmission beliefs but these were then less strong. By the end of the CPD programme, no teacher held predominant transmission beliefs. Five held predominantly connectionist beliefs, Tania held an equal degree of discovery and connectionist, while Chris retained his discovery beliefs.

	BEFORE CPD			DURING CPD			AFTER CPD		
	Tran	Disc	Conn	Tran	Disc	Conn	Tran	Disc	Conn
CHRIS	42	33	25	27	38	35	25	38	37
GRETA	18	15	67	10	20	70	5	40	55
SARAH	67	23	10	43	37	20	17	37	46
JACKIE	30	33	37	28	25	47	20	37	43
COLIN	42	30	28	28	25	47	27	30	43
JANET	25	33	42	27	28	45	23	33	44
TANIA	20	47	33	20	43	37	26	37	37
Mean	**34.9**	**30.6**	**34.6**	**26.1**	**30.9**	**43.0**	**20.4**	**36.0**	**43.6**
SD	**17.1**	**9.9**	**17.6**	**9.9**	**8.5**	**15.2**	**7.7**	**3.4**	**6.1**

Table 37: Beliefs at three different stages of teachers' CPD journey

There were noticeable aspects to how teachers' professed beliefs changed following each stage of their CPD experience. Following participation in the summer workshops, the transmission beliefs of four of the seven LTMI teachers were less prevalent, scoring higher on their connectionist beliefs. Moreover, Chris, Greta and Sarah reported higher scores on their discovery beliefs. For most teachers, the workshops seemed to have lessened their reliance on transmission beliefs, particularly for Chris, Sarah and Colin. The mean scores show that there was a movement towards connectionist beliefs following the summer workshops with discovery beliefs scores remaining consistent. At the end of the CPD, teachers appeared to have experienced a different shift with increased means in discovery beliefs (from 30.9 to 36.0) while retaining their predominantly high connectionist beliefs (from 43.0 to 43.6). Moreover, the smaller standard deviation measures (3.4 and 6.1) suggest that participants were then more in agreement with one another related to their professed beliefs shifts.

Overall, shifts in teacher beliefs from transmission to connectionist occurred when teachers were exposed to IBL pedagogies through the summer workshops. On the other hand, overall shifts from transmission to discovery occurred while teachers implemented IBL in their classrooms.

For the three case study teachers, there seemed to be different patterns related to connectionist beliefs that they claimed to hold at the end of the CPD programme. Following the summer workshops up to the end of the CPD, the connectionist beliefs of Chris remained unchanged, those of Sarah became stronger while Greta claimed these to a lesser degree. In analysing how and when these shifts occurred, I focused on beliefs' changes occurring as a result of participation in each of the two components of the CPD. During the summer workshops, teachers were *learning-off-the-job* about IBL and its implementation. They worked on tasks and analysed lesson videos of teachers enacting IBL in Maltese classrooms. On the other hand, *learning-on-the-job* occurred when teachers implemented IBL in their classrooms and attended CoP meetings to share, discuss and reflect upon their implementation.

In observing and analysing changes in teachers' beliefs, triangular plots (see Figure 18) were used to represent the tri-variate data (see Graham & Midgley, 2000) collected in which the three variables – in this case, *transmission*, *discovery* and *connectionist* beliefs – represent proportions of a whole. Figure 18 shows the beliefs' trajectories for each of the three case study teachers with each point representing the overall professed beliefs of one teacher at a specific point in time – before, during and after CPD.

Sarah and Chris appeared to follow a similar trajectory. Initially, both held transmission beliefs. While the beliefs of Chris developed to rely more towards a combination of discovery and connectionist, those of Sarah shifted to become predominantly connectionist. Greta, who initially held predominantly connectionist beliefs, changed and her beliefs converged to became more aligned with those of the rest of the group. It appeared that following participation in LTMI, teachers' overall beliefs converged to a mix of discovery and connectionist beliefs with a predominance of the latter.

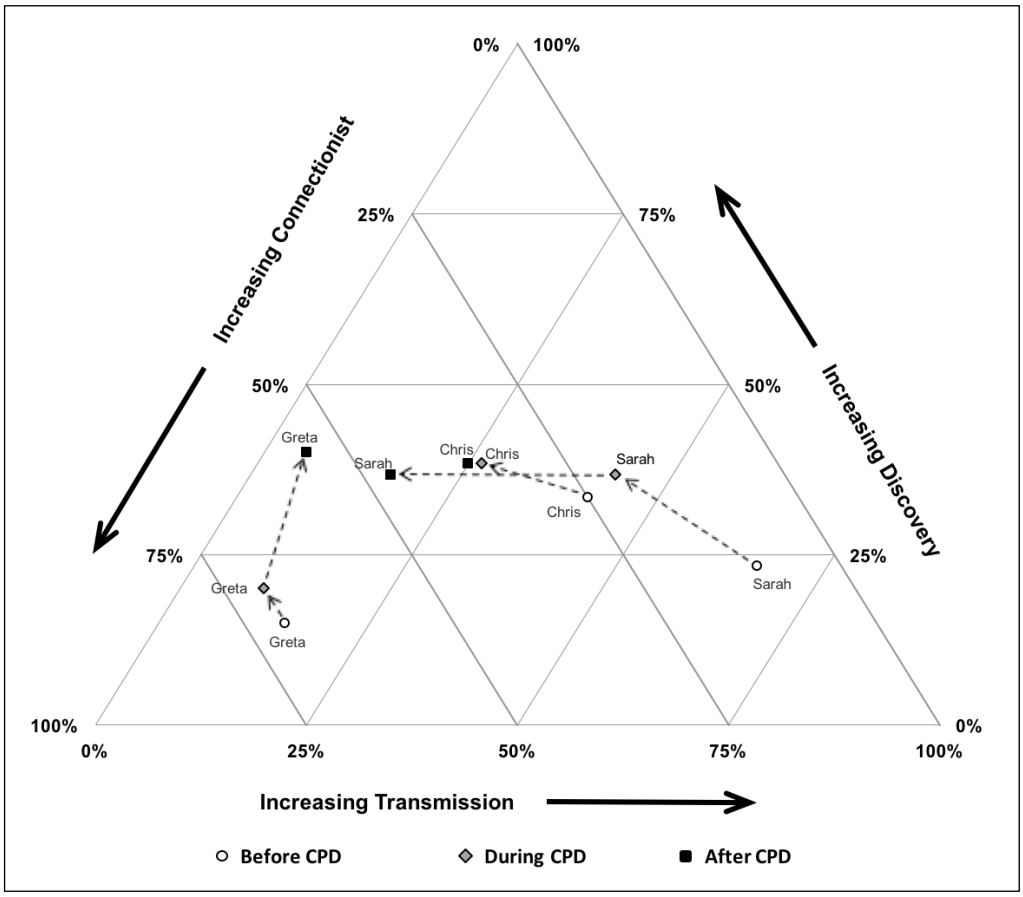


Figure 18: Triangular plots showing changes in beliefs

6.2.5 Changes in teachers' beliefs in relation to reported practices

Chris enrolled in LTMI with predominantly transmission beliefs, but his pen portraits show that he also valued learning through discussion and inquiry (Figure 19). Due to system restrictions, he found it difficult to practise his beliefs. Chris demonstrated a disposition to change his practices and saw LTMI as an opportunity to do this. Following participation in the summer workshops, the self-reported beliefs of Chris shifted towards discovery, and increased connectionist beliefs. At the end of the CPD, Chris declared practically the same beliefs that he had professed following the summer workshops. His discovery and connectionist beliefs had almost the same rating. His pen portraits show that Chris maintained much of what he believed prior to LTMI particularly his teaching style, his relationship with students, how students see themselves as learners of mathematics and

system restrictions (highlighted in italics), and identified changes related to integrating inquiry, discussion and thinking in his class (in bold).

Like Chris, prior to her participation in LTMI, Sarah held predominantly transmission beliefs but more strongly. Her pen portraits (see Figure 20) also show her teacher-directed practices and her reliance on doing the talking, explaining and demonstrating methods. Following the summer workshops, she still held transmission beliefs but to a lesser extent. Sarah reported higher ratings on her discovery and connectionist beliefs. Sarah was the only teacher who still reported transmission beliefs following the summer workshops. One explanation could be the strong transmission beliefs she held at the beginning. It appeared that for Sarah, the *learning-off-the-job* component of the CPD was not enough to bring about changes in her beliefs. This may be indicative of the effect that training provided through an INSET course alone may have on changing the transmission beliefs of teachers, particularly if these are predominant. Indeed, Sarah reported profound changes in her beliefs following the practice-based component of LTMI. While she reported the same discovery beliefs, her transmission beliefs shifted to connectionist. Her post-CPD pen portrait show that she maintained the same beliefs about her students (highlighted in italics) and reported changes related to classroom practices (highlighted in bold).

Pre CPD	Post CPD
My explanations are clearly delivered and include questions directed at individual students. I keep introductions short and do not waste much time on ideas that are 'out' of the syllabus. *I only ask a few questions and then provide extended periods where students work on exercises from the textbook or a worksheet. The exercises I assign start with a few easy questions and then gradually moves to harder examples. I set students working and deal with student difficulties individually while I go around the class.* **When students are working, I prefer to tell students where they have gone wrong and show them where and how to correct mistakes.** *I stop the class from time to time to clarify questions that I identify as particularly difficult*.	**It is important to do as many problems and questions as possible and carefully plans lessons so that nothing is left out.** *I only ask a few questions and then provide extended periods where students work on exercises from the textbook or a worksheet. The exercises I assigns start with a few easy questions and then gradually moves to harder examples. I set students working and then deals with student difficulties individually while she goes around the class. I stop the class from time to time to clarify questions that I identify as particularly difficult.* **I tend to adopt this approach balanced with a 'dose' of inquiry. In my class, desks are arranged in pairs, but students work individually and only occasionally in pairs. Lessons begin with giving short introductions and maybe one or two examples.**
I have a formal, yet cheerful and positive relationship with students. I make sure to get to know my students and try to establish a relationship with them first. *Students in my class need to be motivated and prompted all the time.* I am not too hard on them. *Students seem to like to be organised and prefer to be told what to do. They have the idea that mathematics is about working lots of problems from the textbook and trying to get as many of them correct as possible.*	*I have a formal, yet cheerful and positive relationship with students.* I am kind, caring and positive. *Students in my class need to be motivated and prompted all the time. Students seem to like to be organised and prefer to be told what to do. They have the idea that mathematics is about working lots of problems from the textbook and trying to get as many of them correct as possible.*
I believe that students learn best when their own ideas are discussed and investigated, but there is little evidence of this in my classroom. Although I believe that it is more important for students to develop conceptual understanding rather than computational skills, I find it very difficult to do because of many constraints. *I feel much pressure to 'cover' the syllabus and I want to make sure that students do well in their exams. I feel constrained and frustrated by the scheme of work. I also claim that overloaded syllabi, high-stakes examinations and pressure from parents are constraints that I find hard to overcome.*	**I make sure to involve students in thinking about the question and to engage them in the discussion.** **I am unhappy with my approach as this in inconsistent with my personal philosophy of teaching and learning.** *I feel much pressure to 'cover' the syllabus and I want to make sure that students do well in their exams. I feel constrained and frustrated by the scheme of work. I also claim that overloaded syllabi, high-stakes examinations and pressure from parents are constraints that I find hard to overcome.*
Note: Words in *italics* show no changes; those in **bold** show changes in beliefs and/or; normal text shows no direct link from pre CPD to post CPD.	

Figure 19: Pen portraits of Chris before and after his CPD journey

Pre CPD	**Post CPD**
I think it is very important to get to know your students. I try to understand their situation and help them to overcome their challenges.	*I think it is important to get to know your students. I try to understand their situation and help them to overcome their challenges.*
It seems that if I do not do the talking, then there is not much teaching going on. This is parents' and students' views and expectations of a good teacher.	
I start each topic by explaining and demonstrating methods and procedures particularly with very low ability students. I re-teach each topic from the beginning just to check to what extent students know about the topic.	**I follow up ideas arising from the class. There is no 'best' way to organise and teach topics. I am willing to spend time discussing mistakes and errors when they arise, even if it takes considerable time. I spend time explaining and leading discussion. During this time, I go around prompting and guiding students.**
I believe that students learn best when their own ideas are discussed and investigated. Meanings and connections are explored and refined through discussion. My practices are not consistent with my beliefs about learning.	*I believe that students learn best when their own ideas are discussed and investigated. Meanings and connections are explored and refined through discussion.* **I see mathematics as an interconnected network of ideas which I and my students must create and work on together.**
Students have the idea that mathematics is about working lots of problems from the textbook and trying to get as many of them correct as possible. I want to make sure that students do well in their exams, and I believe that every teacher wishes that their students do well in exams!	If questions are easy for some students, they can skip to the more challenging ones.
I feel constrained and frustrated by the scheme of work. Sometimes yes, schemes of work restrict me from opening up more on certain topics.	
Note: Words in *italics* show no changes; those in **bold** show changes in beliefs and/or practices; normal text shows no direct link from pre CPD to post CPD.	

Figure 20: Pen portraits of Sarah before and after her CPD journey

Greta, on the other hand, entered LTMI with predominantly connectionist beliefs. Her pen portraits (see Figure 21) show that she valued an interconnected view of mathematics, collaborative learning and conceptual understanding. However, like Chris, she revealed that her practices did not reflect her beliefs. Following the summer workshops, Greta maintained her strong connectionist beliefs, indicating a strong position aligned with IBL.

Pre CPD	Post CPD
I see mathematics as an interconnected network of ideas which I and my students must create and work on together. There are many links between topics. I see teaching as a non-linear dialogue in which meanings and connections are explored and refined through discussion. This means that, more often than not, I follow up ideas arising from the class. I cannot always predict in which direction the lesson will take beforehand. I see learning as a collaborative activity based on exploration and discussion.	*I see mathematics as an interconnected network of ideas which she and her students must create and work on together. There are many links between topics* and **there is no 'best' way to organise and teach topics.** *I see teaching as a non-linear dialogue in which meanings and connections are explored and refined through discussion. This means that, more often than not, I follow up ideas arising from the class. I cannot always predict in which direction the lesson will take beforehand. I see learning as a collaborative activity based on exploration and discussion.*
Students seem to like to be organised and prefer to be told what to do. They are not responsible enough and need my guidance and the structure I provide. I think it is important to get to know your students and try to establish a relationship with them first. I am not too hard on them. I also know what students should learn in each lesson. I try to understand their situation and help them to overcome their challenges.	
It is more important for students to develop conceptual understanding rather than computational skills. I am willing to spend time discussing mistakes and errors.	*It is more important for students to develop conceptual understanding rather than computational skills.*
I spend a third of the time explaining and leading discussion, with the rest of the time devoted to students working on tasks in pairs or in small groups. I make sure to involve students in thinking about the question and to engage them in the discussion. I use whole-class discussions in which I try to involve the students. I deal with student difficulties individually while I go around the class. When problems crop up, I stop the class to focus on specific questions.	
Desks are arranged in pairs. In my class, students work hard, collaborate and are always ready to participate and do the work I assign. However, *I feel much pressure to 'cover' the syllabus* and I want to make sure that students do well in their exams.	**I just use the textbook to assign homework.** *I am not relaxed about syllabus coverage.*
My practices are not consistent with my beliefs about learning.	
Note: Words in *italics* show no changes; those in **bold** show changes in beliefs and/or practices; normal text shows no direct link from pre CPD to post CPD.	

Figure 21: Pen portraits of Greta before and after her CPD journey

A noteworthy change seemed to have occurred during the implementation phase. Rather than sustaining her predominantly connectionist beliefs, although these were still prevalent, her beliefs' ratings showed a shift towards discovery. One plausible explanation could be the change from her transmission approach of directing student learning and imparting knowledge, to one that was more inquiry-based. Greta, for example, struggled to adapt her role while students worked in small groups. In initial lessons, she intervened and resolved issues for students. On other occasions, she observed students, occasionally questioning students' work. This dilemma from initially being actively involved in helping students in their inquiry and then shifting to provide students with less teacher-guidance to make their own discoveries, may have triggered these self-reported changes. Her post-CPD pen portraits show that she maintained connectionist beliefs (highlighted in italics). However, the lack of information provided related to her students and aspects regarding her lessons could possibly be an indication of her dilemma.

6.3 Conclusion

The themes emerging from the analysis of the data, provided in this chapter, show that there were sociocultural factors influencing teachers' understanding and enactment of IBL. The construct of 'enactment zones' (see Spillane, 1999) captured how teachers' capacity, will, beliefs and practices interacted with LTMI, as a CPD incentive, but also with the learning opportunities offered by external factors.

Within the professional sector, teachers' isolated practice often saw them retreating into their own classrooms and keeping professional exchanges with school colleagues to a minimum. These teachers reported teaching in rather traditional ways before joining the CPD programme (see Chapter 5), and eventually made important changes to their practice – both in terms of their new ways of thinking about teaching and learning, and also in integrating IBL. To understand these changes, it was necessary to examine their zones of enactment that extended beyond their individual classrooms.

Their efforts to enact IBL were supported and sustained within a social dimension provided by the CPD. Ongoing deliberations with colleagues from different schools, supported by a CPD facilitator, were central to their efforts in learning about and enacting IBL. Their enactment zones were grounded within discussions and considerations of inquiry teaching ideas and their simultaneous implementation of IBL in their classrooms. An important characteristic was that their deliberations were enabled by the resources they created and the materials provided in the CPD. While these materials were important for teachers to teach through IBL, they enabled conversations about not just what to use but how to use them effectively. Teachers did not just put into practice the resource material offered; they questioned the material, discussed, adapted and evaluated them during meetings. Ongoing follow-up meetings served as opportunities for teachers to feed each other with knowledge about practice-based experiences in enacting IBL. Essentially, in their enactment zones, teachers engaged in a norm of collaboration and deliberation that contrasted sharply with the norm of privacy that still dominates Maltese schools (Attard Tonna & Shanks, 2017). These teachers learned about IBL in an environment that offered and supported ongoing support through discourse that centred on classroom practice. For these teachers, translating IBL into practice was mediated by enactment zones that were more social, and offered support and feedback eventually enabling them to change elements of their practice over time.

Besides the social dimension for learning about IBL, a key aspect that motivated teachers to proceed with their efforts were students – their interest, response, engagement and success with IBL. Although students' engagement appears to be a primary source and a lever for change, this is rarely capitalised by reformers who are more likely to use the *public* and the *policy* sectors to press for changes in schools and classrooms (Spillane et al., 2002). In the following two chapters, these findings will be used to answer the four research questions and eventually delve deep into ways teachers may be better supported to learn about and teach mathematics through inquiry.

Chapter 7: Discussion of findings

7.0 Introduction

Having explored in the previous chapter the themes and subthemes that emerged from the analysis of the data, I now move towards answering the four research questions that I identified in Section 2.8.5. This chapter is hence divided into four main sections, each of which provides an in-depth discussion of the findings specifically related to one of the four research questions. The arguments presented in each section, which shed light on issues arising from the data analysis and findings, are moreover linked to the wider relevant picture that emerges from the literature.

In discussing the findings, I decide to follow the same sequence of the research questions as presented in Chapter 2. I find that this helps the reader to better keep his or her focus on what this research study set out to explore and the corresponding 'answers' suggested by my meaning making that built on the analysis of the data. Before presenting these 'answers' to the four research questions, however, I need to make an important clarification. While all data were gathered in the same rigorous manner, the evidence I provide in answering each research question depends on the kind of responses received and reactions observed from the participants. Put differently, although I always did my best to obtain the richest possible data, there were issues about which the participants seemed more eager to discuss and elaborate upon than others. As a result, I find myself having more to say for some findings than for others. Nevertheless, I would argue that the quality of the evidence and analysis is guaranteed throughout, and that consequently all the findings presented hereunder are equally important regardless of the amount of words dedicated to answering particular research questions.

7.1 How do teachers interpret and enact IBL in their classrooms?

LTMI was designed to support teachers who were willing and motivated to make changes to their classroom practices. Yet, as Spillane (1999) argues, changing the core dimensions of teaching, such as the knowledge represented in mathematical tasks, classroom discourse patterns, roles and responsibilities in the classroom, is both arduous and complex for teachers to undertake. Similar to the findings of Spillane and Zeuli (1999), Greta, who held strong connectionist beliefs, initially attended to, interpreted, practised and enacted only the superficial similarities between her 'old' practices and IBL. For most of her journey, she held on to the epistemological similarities of the 'old'. For example, while she recognised that her 'old' end-of-topic tasks were potentially inquiry oriented she did not give the necessary attention to address pedagogical aspects that diverged from the familiar, particularly the shift in role in assisting students during inquiry.

Blignaut (2008) reports that teachers' contexts or enactment zones determine to a great extent how they make sense of and ultimately enact reform practices. This study also shows that even though school contexts offered teachers the same type of constraints (working in isolation, restricted lesson time and exam-driven system), teachers responded differently to how they viewed, interpreted and enacted IBL. IBL was interpreted on the basis of teachers' knowledge, practices and views of mathematics, of students and of teaching mathematics.

In understanding teachers' interpretation is important to identify, not only what teachers notice and value but also how conceptions may change over the course of a CPD journey. In addition, the way teachers conceive of and develop their understanding of IBL is likely to shape their inquiry teaching and, hence, the inquiry opportunities teachers offer to students. Bringing about change in teaching practices – whether *instrumental*, *conceptual* or *foundational* (see Chapman & Heater, 2010) – is seen as a developmental process during which teachers experiment, reflect upon, deliberate and

evaluate their practices. Development in the way teachers interpret IBL emerges, from this study, as a combination of both an individual and a social phenomenon. It is this connection between what teachers did in practice, within the confines of their classrooms, and the way they negotiated meaning with colleagues within a CoP embedded in LTMI that conceptions of IBL emerged and developed. In what follows, I review the development of teachers' conceptions before, during and after their CPD experience.

For these teachers, learning and interpreting inquiry developed during their classroom implementation rather than following the summer workshops. In the process, their views and conceptions of IBL fluctuated and changed. More importantly, for the three teachers, major shifts were observed and reported towards the end of the scholastic year. Research based on data gathered at the end of the PRIMAS CPD by Maaß et al. (2017) shows that teachers' views of IBL can be classified under one of three categories (see Section 2.6.1). Evidence from my research supports these categorisations and shows that, at particular points in their CPD journey, teachers may be classified under one specific category. However, such classification is limited to a particular point in time. Classifying teachers' views and interpretations of IBL into categories, as in the study by Maaß et al. (2017), is seen as a limitation to the developmental process of teachers' LTMI journey. Since learning about and implementing IBL were context-related, culturally-influenced and CPD supported, teachers' interpretations were self-motivated, dynamic and changing.

7.1.1 Teachers interpreted IBL on the basis of what they noticed as familiar and/or contrasting to their existing practices and made changes based on ongoing implementation and reflection with others

Evidence from this study shows that teachers' interpretation of IBL is a function of the individual, contextualised and socially constructed. Greta, for example, initially interpreted IBL as familiar to her end-of-topic lessons that she taught before joining LTMI. She interpreted IBL lessons as similar to the practical revision lessons she had been using – a narrow

understanding of engaging students in active learning (Drew & Mackie, 2011; Priestley, 2010). Initially, Greta noticed and attended to familiar ideas promoted by the CPD, such as collaborative work and using open questions. In conceptualising and enacting IBL, Greta seemed to rely on similarities between her 'old' and 'new' practices that lacked deep critical engagement. She found support within the CoP in addressing the challenges she faced and eventually in refining her thinking and practices about IBL. Similarly, Sarah relied on the similarities she perceived in the three-phase IBL lesson structure presented to her in a prior CPD programme she had attended and then again offered in LTMI. Indeed, convinced as she was that this lesson structure suited inquiry lessons, she abided by it whenever she implemented IBL.

Unlike Greta and Sarah, Chris could not relate to any preconceived similarities of IBL with his prior practices. Chris engaged in a sense-making process looking at IBL as a different way of learning, not only for his students, but also for him. Through CPD, he recognised and realised the potential of inquiry and collaboration towards learning to teach. Interpreting, enacting and conceptualising IBL was, for Chris, an active process guided by ongoing personal reflections and deliberations with colleagues during the CoP. In his attempt to develop his practices, Chris sought to address his emerging frustrations between his growing beliefs that IBL was important for student learning and an education system that was restricting him from providing students with more access and opportunities to inquire. The distinction in the ways in which Greta, Sarah and Chris interpreted and engaged in classroom enactment stems from their teaching experience and perceived similarities with their IBL practices. Greta and Sarah taught mathematics for nearly 20 years each and recognised similarities between what was proposed and their preconceptions of inquiry teaching. Chris, on the other hand, had less than 5 years teaching experience and had no inquiry teaching experience before. However, he engaged in deeper thinking about the ideas proposed by LTMI than Greta and Sarah did. His critical alignment to IBL was the result of a developmental process involving interpretation, enactment and critical

reflection that eventually shaped the way he looked at his teaching, the students and the system he was part of. Unlike Chris, Sarah accepted the limitations offered by the education system and attempted to make changes that worked for her. Within the system restrictions she faced, Sarah developed a more interventionist role during IBL, integrated tasks that were less broad so that students had time to delve deeper in their thinking and offered students more time to communicate their work to the whole class.

An emerging finding in the data was that the preconceived ideas about inquiry lessons that Greta and Sarah had as they joined LTMI, restricted their initial understandings of what enacting IBL involved. Indeed, data show that the similarities they saw between their 'old' practices and conceptions of IBL and those promoted by LTMI inhibited their implementation to make noticeable changes in their practices. Chris, who was less familiar with IBL, assumed a more gradual cautious approach in introducing IBL. He made sure that the strategies he used gradually supported students' learning through inquiry, cultivated an inquiry environment and, in the process, supported his own personal and professional development of IBL practices. During classroom implementation, the three teachers used IBL to engage students in discussing mathematical knowledge. In their lessons, IBL was conceived of as a strategy to either introduce or revise a topic. Chris, for example, thought of IBL as a means to introduce new content knowledge while Greta and Sarah generally used IBL at the end of a topic as revision, application and consolidation of previously learned material.

Evidence shows that teachers' views of IBL evolved as a result of a learning process that teachers experienced during the CPD. The three teachers in this study initially saw IBL as a set of peripheral activities to motivate the students. However, over the course of the programme, this view was transformed. For Greta, IBL was mainly intended to develop student motivation and used as a bolt-on approach to the traditional mathematics curriculum; an additional activity at the end of every topic she taught.

Teaching content held a high priority for her. It appeared that working within a system that prioritises content coverage and examination success had relegated IBL to be marginalised from her way of teaching mathematics. Initially, Greta's prior knowledge and practice influenced her interpretation and implementation of IBL. She saw the ideas promoted by the CPD as more familiar than they actually were. IBL was interpreted as essentially the same as her end-of-topic hands-on activities that she had been implementing in the past, prior to LTMI. It appeared that Greta interpreted IBL as familiar "without sufficient attention to aspects that diverge from the familiar" (Spillane et al., 2002, p. 398). She recognised that the tasks she had been previously using were potentially inquiry-oriented but did not attend to effectively transform her role when engaging students in inquiry. However, over time and particularly near the end of the CPD, Greta started to see possibilities for integrating IBL within the curriculum. This denoted an emerging shift that Greta started to experience – towards viewing IBL as *pedagogical practices that promoted thinking and reasoning* (Maaß et al., 2017). It appears that Greta moved from initially making an *instrumental* change and eventually moving towards a more *conceptual* change (see Chapman & Heater, 2010) – initially focusing on what and how to enact IBL and later on why IBL worked for her students. This shift appeared to be related to the way she initially interpreted IBL based on similarities she saw with her past practice, associating it and enacting it more as a discovery approach rather than a connectionist one. Once Greta overcame her concerns with syllabus coverage, pedagogical features such as the use of open questions and collaborative discussion started to be incorporated more often and more purposefully in her teaching.

Like Greta, Sarah used IBL with her class to motivate her students. However, since this group was low-attaining, the syllabus was less restricting and less demanding to cover. Sarah went through a similar process to Greta with her change taking place as slowly as that of Greta. Initially, Sarah encountered challenges in enacting IBL through a three-phase structure due to lesson time limitations (see Section 6.1.2). While this appeared to initially hinder Sarah from making progress, her persistence and ongoing reflection

eventually enabled her to make changes within the lesson structure itself. For example, she encountered challenges as her students found difficulties in managing to complete tasks and then later having enough time to discuss these as a whole class. Over time, she managed to address these issues. She minimised the work assigned by offering one or two problems or discussion statements for the small group activity that allowed students to explore these more deeply and eventually leave more time for the plenary part. Moreover, Sarah also refined her role making increased interventions to support students' exploration during small group activity and more direct interventions to challenge and add knowledge during the plenary.

Unlike Greta and Sarah, Chris did not immerse himself instantly in engaging students with IBL. He implemented IBL more gradually and only provided more challenging tasks when he recognised students' readiness for it – through their increased active engagement, positive response and articulation of thinking. One plausible explanation for this could be related to the transmission-oriented practices he was accustomed to prior to engaging in LTMI (see Section 5.2.2) and his unfamiliarity with inquiry-oriented teaching approaches. Shifting to inquiry teaching, as modelled by LTMI, was perceived by Chris as a challenge both for him and his students. The IBL practices of Chris moved from initially getting to know students and motivating them to discuss mathematics to then engage them in thinking and reasoning. Students' positive response to IBL augmented his initial frustrations towards an education system fixated with continuous measurement and testing. As a result, Chris took an inquiry stance to practice that enabled a more *conceptual* change. By the end of the CPD programme, Chris expressed an initial change from viewing IBL as pedagogical practices to improve students' mathematical experiences towards viewing IBL as offering opportunities for *learning mathematical processes and content* – a more complex and sophisticated view of IBL (Maaß et al., 2017). With hindsight, Chris communicated how, by the end of the CPD, he was then integrating features of IBL (collaboration, questioning and thinking) during his other lessons as well. He found out that he did this instinctively, without having the need to think about it. He said that he

developed and learned this through ongoing implementation, reflection and negotiations with colleagues about classroom practices.

The practice-based approach, adopted by LTMI, supported Chris in developing a new way of conceptualising learning as a non-linear, evolutionary and developmental process through mistakes and failures (see Section 6.2.3) – one that contrasted with his pre-CPD understanding. Over time, with his developments in designing and using inquiry tasks, Chris became more committed to use IBL as an approach to teaching while encouraging students to act more responsibly within the inquiry classroom community that he sought to create. Chris eventually manifested a *foundational* change in his orientation (see Chapman & Heater, 2010) of self, identity and practice. He was thinking in new ways, empowered by his new understanding of inquiry and learning (see Sections 6.2.2 and 6.2.3) to transform his teaching. Still, in becoming inquiry-oriented, teachers need "to experience or attend to a tension involving self and practice" (Chapman & Heater, 2010, p. 457). Chris experienced ongoing frustrations and tensions between his new conceptualisation of learning mathematics through IBL and how the system was restricting him from doing as much as he wanted to. With the acquired knowledge, experience and confidence in enacting IBL, by the end of LTMI, Chris was more open to accept and learn from his uncertainties and frustrations. His role, as a teacher, was to work alongside students to model his newly refined view of learning mathematics. Rather than withdrawing and allowing students to discover, as he did initially, by the end of the programme Chris became a motivator and an instigator for students to pursue with inquiry using open questions to stimulate thinking, uncover misconceptions and learn through mistakes.

The limited teaching experience and unfamiliarity with IBL appears to have supported the gradual approach implemented by Chris in introducing IBL to his students and in developing a more refined conceptualisation of IBL. On the other hand, Greta's and Sarah's interpretation of IBL influenced by their previous classroom experiences are likely to have prompted these two teachers to start off with distorted preconceptions of inquiry teaching.

Starting off with preconceptions of IBL required more time on their part to better understand, modify and eventually improve their inquiry practices.

7.2 In what ways do teachers structure their inquiry lessons and how does this develop over time?

In implementing inquiry, teachers did not necessarily follow the three-phase 'task-first' lesson structure promoted in the literature and offered by LTMI. Analysis of data show that the way teachers structured their IBL lessons, incorporating inquiry tasks, was informed and shaped by two key features of the tasks: (1) the *cognitive challenge* provided, and (2) students' *familiarity with the mathematical content* inherent within the task.

Maltese teachers working within summative content-oriented systems (Cachia & Ferrari, 2010; Eurydice, 2011), selected tasks that incorporated content to address the time-constraint that IBL was perceived to pose in 'covering' the examination syllabus. These teachers also tended to provide students with tasks that incorporated content that students had already covered and, hence, were familiar with. When tasks offered high cognitive challenge and/or novel content knowledge, teachers were not likely to structure their lessons using a three-phase structure. Data showed that teachers also intervened in ways that supported the completion of tasks. By intervening and/or structuring their actions, teachers appeared to support students in achieving the challenge within the task. When using tasks with high cognitive challenge and/or unfamiliar content knowledge, teachers adopted a different approach to the three-phase structure offered in LTMI. In supporting students, teachers either offered as a sequence of graded smaller tasks (as in the case of Chris) or intervened in such a way to direct students to the outcome of the activity (the case of Greta). A critical aspect emerging from teachers' lesson structure was the way they orchestrated classroom activity. In response to students' actions, teachers managed and organised their IBL lessons in ways that ensured successful task completion. In doing so, they were likely to seek ways in which they could facilitate the work of students within the 40-minute lesson time.

For teachers, tasks with a high cognitive challenge appeared difficult to implement within a three-phase lesson structure. Teachers believed that students needed extensive time to unravel the task, with small-group activity taking a large proportion of the lesson time (at times 20 minutes or even more). As a result, very little time was usually left for the plenary. For the same reason, teachers were also less likely to select tasks that presented students with novel mathematical content. Whenever they did, teachers tended to scaffold student learning at an early stage during the lesson because they could not 'afford' not to reach the intended learning outcomes.

7.2.1 Teachers structured their IBL lessons using approaches that included adaptations to the 'task-first' approach

Investigation into teachers' lesson structure indicates that using a 'task-first' approach did not necessarily facilitate their implementation of IBL. I draw on the study by Leikin and Rota (2006) who attributed a teacher's growth in expertise as the reason for successfully adopting a three-phase structure. Besides confirming this finding, this research adds other context-specific reasons. Whether teachers adopt a three-phase lesson structure also depends upon their preconceptions of inquiry lessons and the tasks they use. For instance, prior to their participation in LTMI, the three teachers adopted a three-phase structure for the IBL lesson they offered for observation (see Section 6.1.3). Sarah, who was already familiar with this structure prior to her participation in CPD, maintained it during LTMI. However, Chris and Greta, for whom LTMI was the first IBL-related CPD they participated in, structured their inquiry lessons differently.

The findings show distinct ways in which teachers undertook IBL lessons. Teachers' approaches to lesson structure appear determined by constant contextual concerns arising from the limited lesson time available, the amount of content they were required to teach and using challenging tasks for inquiry. While recognising the importance of shifting more responsibility onto students, these three teachers appeared to encounter challenges in structuring and managing their IBL lessons and adopted distinct ways for doing and developing this over time. In planning for IBL,

Chris structured his lessons using a series of tasks that gradually introduced students to more challenging work. Chris introduced IBL by adopting a scaffolded approach consisting of increased challenges, usually offering two to three graded tasks with increased cognitive challenges. Similarly, Greta's lesson structure addressed the emerging struggles that students encountered. Her lessons generally started off with a whole-class discussion. Using a 'discussion-first' approach (see Russo & Hopkins, 2018), Greta presented the challenging task through a whole-class discussion reviewing prior mathematical content that was inherent within the task. As students worked collaboratively on the task, Greta would then offer support generally through direct interventions – spotting and indicating mistakes, questioning methods used at arriving to an answer or stopping the activity and conducting a short whole-class discussion that addressed emergent difficulties. Midway through the scholastic year, Greta gradually removed such scaffolding, moving away from these immediate interventions and was more likely to listen and challenge student thinking. In both cases, teachers appeared mindful of not presenting tasks with an overload of cognitive demand, as characterised by the 'task-first' approach, but offered initial support through increased challenges (Chris), discussion and direct feedback (Greta).

Sarah, on the other hand, was consistent in using a three-phase lesson structure in enacting IBL. Moreover, when challenges appeared, she learned to make adaptations to the tasks she presented without changing her lesson structure. Over time, she reduced the amount of work assigned (for example, the number of statements to discuss or cards to sort out) and limited collaborative work to about ten minutes which, in turn, allowed more time for the plenary. Sarah found that in reducing the quantity of work assigned, students eventually engaged in more focused in-depth discussions and found herself with more time to challenge students' reasoning during the plenary.

7.2.2 In enacting IBL, teachers structured their lessons in ways to incorporate the ideas promoted in CPD, but with adaptations in order to address their contextual constraints

In viewing lesson structure, task selection and implementation become key to the success of teaching mathematics (Doyle, 1988; Hiebert & Wearne, 1993). Teachers take key decisions linked not only with the choice of task, that is, mathematics content and cognitive processes (Lappan & Briars, 1995; Shimizu et al., 2010; Sullivan, 2011), but also with the way they structure their lessons to promote and support inquiry. Teachers, in the present research, appeared to adapt their lesson structure based on the cognitive demand of tasks (promoted by LTMI) and the mathematical content inherent within the task (that they were required to teach).

Figure 22 illustrates the transition that took place from the planning phase, which tends to be task-oriented, to the enactment phase in inquiry teaching, which is more activity and goal oriented. The first two arrows corresponding to the lesson planning phase indicate that, in selecting and designing inquiry tasks, teachers focused on attending to both the mathematical content they had to teach and seeing that the task offered an 'achievable' cognitive challenge (see Willis, 2010) as promoted by LTMI. These two components, content knowledge and cognitive challenge, appeared to determine the way teachers then visualised and planned the intended lesson structure – as indicated by the two vertical arrows. As their lesson plan made its way into the classroom, teachers either adhered to it as planned or otherwise changed it based on the ensuing classroom activity. In the enactment phase, lesson structure was adapted based on a range of circumstances – (i) students' response to task presentation; (ii) challenges encountered by students during group activity and a teacher's response in supporting engagement; and (iii) lesson time limitations. Following task presentation, teachers observed and attended to how both content and cognitive challenge are encountered and negotiated by the students. In attending to students' engagement with the task, teachers either maintained the level of challenge during enactment through questioning

(see Boaler & Staples, 2008), or reduced it (Doyle, 1988; Stein, Grover, & Henningsen, 1996). Their actions and the roles undertaken, in turn, influenced the way the lesson unfolded and, hence, the way lesson structure was adapted so that teachers supported students in reaching the intended learning outcomes.

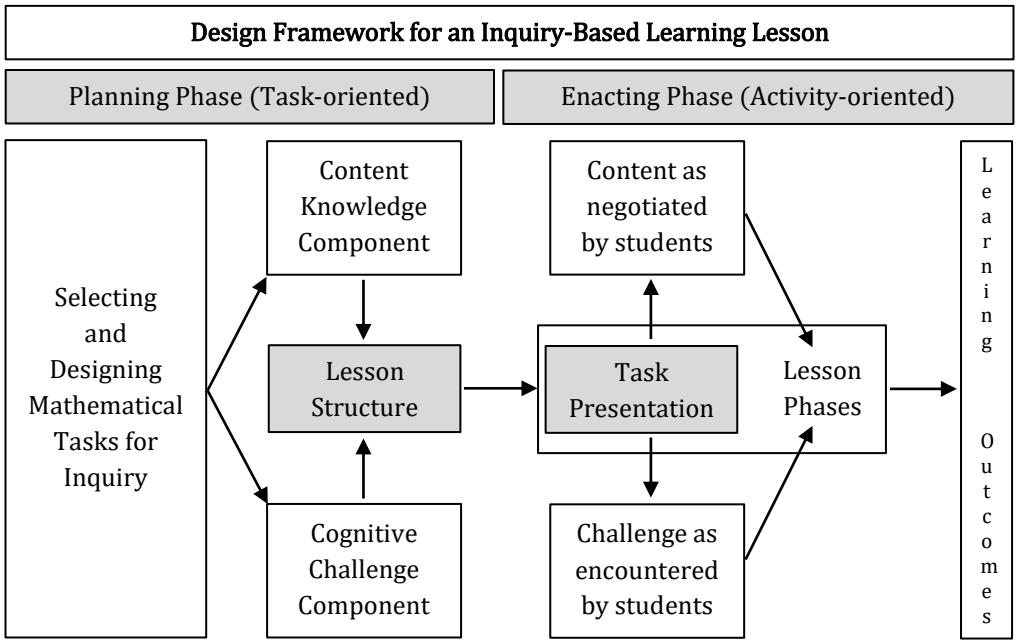


Figure 22: A framework for planning and implementing IBL

The process of finding and selecting tasks for inquiry entailed making decisions about learning goals, learning activities, and the thinking and learning that students might engage in. This process required teachers to design lessons planning subject content, making a prediction of the content to be learned through the lesson (specific knowledge and skills) and how students' thinking might evolve as they engage with the activity. All this culminated itself into a teacher's plan – a lesson structure for organising student activity around the selected mathematical task/s. As such, in planning for IBL, teachers selected learning tasks and visualised a structure through which tasks – with their intended content and cognitive challenge – are presented and eventually enacted with students. However, teachers also anticipated the challenges that students may find. In addressing this, teachers were likely to decide to adhere to this structure or adapt it either in the planning or the enacting phase. When following a particular lesson

structure – as in the case of Sarah – the teacher focused on changing task characteristics rather than the structure of the lesson phases.

7.3 What are the changes in teachers' practices and beliefs over the course of the CPD programme?

Teachers' beliefs are likely to determine classroom practices (Beswick, 2005; Ernest, 1991; Spillane, 1999; Thompson, 1992) and literature has shown that enacting reform practices may bring about changes in teachers' beliefs (Swain & Swan, 2007; Swan, 2006; Wilson & Cooney, 2002). This study delved into teacher's beliefs and their trajectories at three different stages during the CPD journey. Noticeable changes occurred following both the summer workshops and then their participation in follow-up meetings, providing insights upon the impact that professional development may produce upon teachers' beliefs. Following the former, the connectionist beliefs of all seven participants grew stronger. For the three case study teachers, their discovery beliefs also increased. After the follow-up meetings, these teachers reported different changes which, however, appeared to show a convergence towards connectionist beliefs. Indeed, by the end of the CPD programme five teachers held predominantly connectionist beliefs with the remaining two holding a combination of discovery and connectionist beliefs.

7.3.1 In shifting to inquiry teaching, teachers passed through a phase of discovery teaching with practices becoming more aligned with their changing beliefs

Findings from present study about teachers' beliefs reveal distinct trajectories arising from two phases within LTMI: data at the beginning and midway, and data midway to the end of CPD. This contrasts with other studies (e.g., Swan, 2006; Swan & Swain, 2010) in which data were gathered at the beginning and the end of teachers' CPD. These studies show that teachers' beliefs evolve along three trajectories: from transmission to discovery; transmission to connectionist; and from discovery to connectionist. The present study adds to this analysis because, while it

reveals changes toward more discovery beliefs from beginning to end, it shows these teacher belief changes following two particular phases within CPD: *learning-off-the-job* in summer workshop and *learning-on-the-job* following ongoing implementation and community meetings.

The belief trajectories of the seven teachers participating in LTMI show that beliefs evolved along two trajectories: from transmission to connectionist or to a combination of discovery and connectionist; and from discovery to a combination of discovery and connectionist. Following the summer workshops, those teachers with primarily either discovery or connectionist beliefs maintained these, with connectionist teachers holding to their beliefs more strongly. Mean scores show that while transmission beliefs were less prevalent following each stage of LTMI, teachers' discovery beliefs remained unchanged following the summer workshops but then increased after the implementation phase. On the other hand, teachers' connectionist beliefs grew stronger following the summer workshops and were then held as strongly after the follow-up meetings. Overall, irrespective of their starting beliefs, by the end of LTMI teachers held either connectionist beliefs or a combination of discovery and connectionist beliefs.

Prior to their participation in CPD, the three teachers defined IBL as discovery learning involving independent work unaided by the teacher. Standing back and expecting students to discover things for themselves was a common feature of teachers' initial descriptions of their roles. Teachers described their role within inquiry lessons as involving monitoring students' work, refraining from transmitting any knowledge, allowing students to struggle, and encouraging them to persist with their inquiry and not give up. Teachers seemed to interpret their role in inquiry teaching as 'stepping out' (see Lampert & Blunk, 1999) from their usual conversations with students, allowing them to work on their own.

In enacting IBL, they moved from a 'telling' into a 'not telling' role. Initially, lesson observations revealed that Chris and Sarah allowed students to wonder without providing any direction at all. They appeared to recognise

the limitations that their transmission methods did not allow students to think for themselves (Swain & Swan, 2007). Hence, they did not provide students with support for interactions by 'stepping in' (Lampert & Blunk, 1999) when it was opportune to stimulate thought and further inquiry. They observed group activity leaving students working on their own. Based on teachers' descriptions and my lesson observations, prior to LTMI and in their first attempts to implement inquiry, teachers' interpretations of IBL indicated a discovery approach to learning. On the other hand, Greta was initially quick to provide students with suggestions about solving a task. However, midway through the year, she also adopted a non-interventionist role allowing students to solve tasks on their own.

Over time, Chris and Sarah managed to balance these two roles – allowing time for independent work while at the same time guiding students to look for support when needed. In 'stepping in', which Sarah managed more successfully in the plenary part of the lesson, teachers carefully monitored classroom interactions, evaluated and supported them while reinforcing and sustaining collaborative discourse to prevail. Nathan and Knuth (2003, p. 203) claim that such discourse "does not come about simply because the teacher creates the space for it", but by supporting and scaffolding "student learning of content during classroom interactions". In enabling student interactions and productive inquiry, teachers need to learn the when, the how and the why of stepping in and out. Chris eventually made this shift providing students a more supportive role by responding more proactively during student inquiry. He was then more inclined to make constructive use of students' contributions, manage small group and whole class discussions, encourage students to discuss alternative perspectives, and helping them to make connections between their ideas and relate them to important mathematical concepts and methods. Greta and Sarah demonstrated this less consistently however. Indeed, of the four IBL features, shifting responsibility onto students was identified by the three teachers as the feature upon which they had made the least progress. Teachers appeared to recognise the challenges in allowing more agency for students to explore and communicate their mathematical meanings.

These three teachers started off with different beliefs about mathematics, teaching and learning. Still, their belief trajectories appeared quite similar. Sarah entered the CPD with predominantly transmission beliefs that she held, to a lesser degree, following the summer workshops and that eventually changed to become more connectionist by the end of the programme. Her belief trajectories show that as her transmission beliefs diminished, both her discovery and connectionist beliefs grew stronger. In the case of Chris, his belief trajectories show that his predominantly transmission beliefs changed and he now held a combination of discovery and connectionist beliefs. On the other hand, Greta held her initial connectionist beliefs but less strongly showing a convergence that included more discovery beliefs. Findings show that this change was potentially caused by her dilemmas in shifting her role in supporting students' explorations. In changing from her more transmission teaching practices to IBL, she assumed a more passive non-intervening role allowing students to discover things on their own.

Shifting from transmission to inquiry teaching requires teachers' timely, purposeful and skilful use of interventions. Teachers are likely to interpret this change as a shift from 'telling' to 'not telling' and, as a result, interpret it as adopting a discovery approach. To me this move appears both natural and logical for teachers to go through, but possibly also for their students. In learning to inquire, students need time to think and discuss ideas independently of the teacher. Similarly, teachers need time to observe and listen to students' interactions. Adopting discovery teaching appears a necessary stepping stone in teachers' journey towards LTMI. As teachers offer an unguided approach, they go through a phase during which they recognise its limitations for facilitating student learning and, hence, the need to offer timely interventions.

7.4 How does 'just-in-time' teaching and learning embedded in a CPD programme support teacher change?

LTMI was designed to offer 'just-in-time' teaching and learning (JITL) opportunities for teachers that were recognised as missing or limited within schools. The CPD programme incorporated two phases: off-the-job and on-the-job learning. While on-the-job, teachers received support from the CoP embedded within LTMI. Within this on-the-job phase of implementing IBL, teachers participated in and experienced two contrasting learning contexts: on-site isolated practices and off-site collaborative support. As a result, the JITL opportunities available for teachers – including collaboration, feedback, reflection, access to online resources and opportunities for negotiating ideas – emerged as essential to their continuing professional development in enacting IBL. In particular, JITL opportunities supported teachers' enactment zones to pursue with their IBL practices, development and change.

7.4.1 Enactment zones that offered 'just-in-time' feedback and collaboration supported teachers in their efforts to pursue change

Enactment zones refer to how CPD initiatives and the learning opportunities they offer are encountered, interpreted, constructed and operationalised by teachers. Spillane (1999) argues that, while influenced by teachers' personal resources (knowledge, beliefs, dispositions, capacity and will to reform their practice), zones of enactment are also shaped by *pupils* (students' responses to IBL), the *professional* sector (contact with colleagues in and out of school), the *private* (textbook and curriculum publishers), the *public* (parents and community), and the *policy* sector (policies, curriculum and assessment practices). Enactment zones are context dependent and, as Spillane (1999) argues, vary on a continuum – from individualistic to social. Enactment zones that extend beyond teachers' individual classrooms, as offered by LTMI through a CoP, are more

likely to support teachers both in developing a deep understanding of reform and in enacting it.

The three teachers in this study valued enactment zones that provided JITL, particularly, 'just-in-time' feedback, collaboration, support and negotiations. In particular, teachers referred to students' engagement with IBL and the collaborative learning created within the CoP as supportive in their journey towards change. Sarah attributed her change to students' increased engagement and direct response to inquiry tasks. Students' developed capability to discuss mathematics encouraged her to pursue with her efforts to promote discussion, thinking and reasoning. Chris experienced the same kind of shift driven by students. Through IBL he saw potential in his students and the experienced benefits, compared to his prior transmission teaching style, led him to develop a new conception of learning. The other sectors – the *professional*, *private*, *public*, and *policy* – were perceived by teachers as limiting and hindering their implementation of IBL because these sectors were imposing on them non-negotiable curricular practices. These sectors allowed no negotiation and, hence, did not provide JITL opportunities in their LTMI journeys. Students were a reassuring aspect. As direct clients of their teaching, students provided teachers with the most honest, telling and 'just-in-time' feedback on their inquiry experiences. Their positive response to IBL activities, and students' development in thinking skills and the way they communicated their mathematics learning were highly powerful for teachers in persisting with IBL.

Similar to the findings of Spillane and Zeuli (1999), Hodgen and Johnson (2003), Boreham and Morgan (2004) and Golding (2017), teachers also reported that IBL enactment was enhanced and sustained by their collaborative deliberations within the CoP. The CoP, created through ongoing follow-up meetings, emerged as essential for teachers in this study to persist with IBL. The monthly meetings afforded JITL for developing their inquiry practices. Community offered, what Golding (2017, p. 504) describes, "a supportive change environment" for teachers. The CoP

meetings offered teachers the opportunity so that their personalised enactment zones become shared with others. Through participation in the CoP, teachers experienced co-learning as they interacted and supported each other. It was through their rich deliberations about planning, implementing and evaluating IBL that teachers increased their knowledge and found the 'just-in-time' support needed to persist with experimenting and changing their practices. The CoP provided a supportive environment for change distinct from that experienced within their own schools, and a safe and supportive space to share their IBL practices while learning with and from others.

7.4.2 The off-site collaborative engagement by teachers during CPD supported teachers' on-site isolated practices, but did not enhance teacher collaboration within their school

Teachers reported that LTMI offered the opportunity for them to experience a change from on-site learning taking place in isolation within their classrooms to collaborative off-site learning within a CoP. At their schools, these teachers did not draw on their colleagues for support and resources in using IBL – they were lone-fighters (see Krainer, 2001). The 'norm of privacy' (Spillane, 1999) dominating classroom practice in the three schools warranted individualistic enactment of IBL. IBL enactment was, however, supported by teachers' active agency cultivated within the CoP. Through the CoP, teachers could share and negotiate learning emanating from classroom practices and eventually profit from the practice-based deliberations to then return back to their classrooms with more informed understandings. Learning for teachers about IBL progressed from personal reflections on lesson planning, to student engagement and eventually on developing their role in supporting student learning. Within the CoP, teachers discovered an environment that offered agency and freedom, was nonthreatening, and provided JITL opportunities to develop their practices at their own pace.

This contrasted with school contexts that seemed to control and isolate teachers, and only provide just-in-case support. Teachers did not consult

school colleagues for support in implementing active learning pedagogies. Hence, learning to teach was a sink-or-swim affair (Lortie, 1975). Teachers described feeling in isolation within their context. Evidently, with their attempts to use inquiry, most of them anticipated that isolation would now perpetuate as their colleagues preferred to adopt traditional approaches. There was also little room for teachers' autonomy and agency as a result of an education system that is prescriptive, standardised and outcomes-driven supported by summative assessment methods. Moreover, teachers' active engagement in the CoP meetings also contrasted with the passivity assumed during training courses including those held within schools (Attard Tonna & Calleja, 2010). Their 'new' active ways of learning within the CoP in contributing knowledge, collaborating, and autonomous learning contrasted with the passive, isolated, accepting and controlled roles that framed their learning in schools. Teachers revealed how they developed their knowledge of inquiry teaching within the CoP and how engagement in the CoP shaped their identities and their self-perception of their roles in learning about teaching.

Belonging to a CoP, hence, offered teachers the means to develop their identities (Wenger, 1998). Although they shared common goals in learning about IBL, over time teachers developed their own unique identities in thinking about practice, and these identities developed in response to personal conflicts generated by contextual, social and cultural factors. Teachers engaged in IBL practices and attempted to understand how reforming their practices would fit into their existing school and education systems. They also sought possibilities and opportunities for development and, in the process, looked critically at the practice promoted by the CoP while aligning with it (Jaworski, 2006b).

For these teachers, engagement in LTMI offered JITL that influenced their teaching of mathematics. However, teachers experienced an opposing force coming from their colleagues within schools (see the case of Chris in Section 6.2.3) who were pushing to maintain the status quo. Even though pairs of teachers from the same school as that of Chris and Greta participated in

LTMI, communities and communications between teachers within the school context remained lacking. School contexts appeared unprepared towards such dialogues. The insularity that prevailed within schools restricted LTMI teachers from sharing their knowledge and building communities with colleagues.

7.5 Conclusion

The aim of this chapter was to explore and discuss the research findings. Data revealed that teachers interpreted IBL based on what they noticed as familiar or contrasting to their existing practices. Their views and conceptions of IBL were dynamic and changed as teachers became more reflective and gained knowledge of what works for them in practice. Teachers developed their interpretation of IBL in a two-fold manner: individually and socially. IBL conceptions emerged and developed as a result of what teachers were able to do with students in their classrooms, and the way they negotiated meanings with colleagues within CoP meetings.

Initially teachers interpreted IBL as discovery learning involving students working independently unaided by the teacher. Descriptions and conceptions about their roles in assisting students in class were mainly to stand back allowing students to discover things for themselves. 'Stepping out' from their usual conversations with students was an approach teachers adopted; moving from a 'telling' into a 'not telling' role while monitoring students' work. It seemed that teachers interpreted their role in inquiry teaching as allowing students to work on their own, refraining from transmitting any knowledge, accepting that students need to struggle with the task, and encouraging them to persist with their inquiry. However, over time, teachers' interpretation and enactment of IBL evolved in different ways. Initially, teachers appeared to conceptualise IBL as additional and peripheral activities. However, these conceptions changed and IBL was seen more as a set of pedagogical practices for promoting thinking and reasoning. By the end of the CPD programme, teachers provided broader

descriptions showing how IBL offered students challenges and opportunities to explore and discuss mathematics.

Their journey in learning to teach and enacting IBL took place within two contrasting contexts. Data revealed that, for these teachers, enactment zones that offered 'just-in-time' feedback resulted mostly from students' engagement and responses to IBL, and the ongoing support received from colleagues during the CPD. When teachers' personal enactment zones became social, they experienced new ways of thinking and acting about practice. This shows that if teachers are entrusted to implement and sustain change in enacting reform ideas, they need to be allowed and empowered to develop their knowledge socially. Ongoing follow-up meetings provided teachers with JITL opportunities to share concerns and develop their learning in effectively enacting IBL features. Of the four IBL features offered during LTMI, teachers initially rated themselves lowest (ineffective) in *shifting responsibility onto students* (see Section 6.1.5). By the end of the CPD, teachers also viewed *shifting responsibility* as the most valued feature in creating opportunities for IBL and also the least that they had made progress on.

This study looked into the way teachers structured their inquiry lessons. In shifting from transmission to inquiry teaching a 'task-first' structure offered challenges for teachers particularly in reforming their new roles. Moreover, structuring lessons and enacting IBL required addressing contextual constraints related to content-based examinations. Teachers' lesson structure was, hence, determined by two key task features: *cognitive challenge* and *familiarity with mathematical content*. With tasks offering a high cognitive challenge or unfamiliar content teachers were more likely to deviate from using a three-phase lesson structure. The 40-minute time slot was an additional challenge for teachers in managing to orchestrate the lesson using a 'task-first' approach. Over time, data revealed that, in enacting IBL, teachers did not necessarily need to adopt a 'task-first' structure commonly reported in the literature and offered during LTMI. Indeed, there is more than one approach for structuring IBL lessons.

Teachers in this study also showed changes in their beliefs following both the summer workshops and the follow-up meetings. Through their journeys and irrespective of their beliefs, on their way to enacting IBL, teachers moved to discovery teaching. This was also evident in how their beliefs changed following the summer workshops and then by the end of LTMI. In visualising and transforming their roles, teachers initially shifted from 'telling' to 'not telling', and once they recognise inconsistencies in this approach they eventually valued the use of more purposeful interventions to facilitate student learning.

The next and final chapter will provide an overview and a discussion of the opportunities and boundaries of interpreting and using the research findings, the hypotheses arising from the analysis of the data, and their contribution to the field of learning to teach mathematics through inquiry.

Chapter 8: Contributions, recommendations and conclusions

8.0 Introduction

This final chapter focuses on the contributions to knowledge, implications and recommendations that this study has to offer. Although this study – like all research – has its own limitations that arise from the particularities and boundaries that frame the CPD design, its findings offer analytical generalisations (see Patton, 1990) that have the potential to add to established theories. In other words, this study contributes to the accumulation and crystallisation of knowledge. In particular, I use the findings discussed in the previous chapter to contribute analytical generalisations as supplements or alternatives (Flyvbjerg, 2006) to what we know about teacher interpretation and enactment of IBL, lesson structure, beliefs and practice change, and JITL in CPD design. I conclude this chapter by presenting the personal learning experiences that I gained as a result of undertaking this research study with teachers.

8.1 Boundaries and restrictions to the study

This study has its boundaries and restrictions that derive from its qualitative case study nature, its context and limitations to generalisability. Still, I would argue that these boundaries and restrictions are strengths rather than 'limitations' because they lead to highly contextualised findings that map out an authentic reality for the reader. What this study lacks in breadth and numbers, it certainly makes up for this through the depth and authenticity of understandings gained, which admittedly are highly contextualised. In a sense, the reader is being asked to engage in an explorative exercise of a somewhat 'unique' situation in order to widen and deepen his or her knowledge of a universal phenomenon – that is, teachers' learning to enact IBL through participation in CPD.

This case study was chosen because it is an area of interest to me both as a researcher and practitioner. My immersion, as a researcher, in the context might be seen to pose limitations, as personal interests and interpretations are known to impinge and influence the research process. Still, this is one of the strengths of qualitative research provided that one is aware of it and acts reflexively (see Ball, 1990) throughout the research process. My insider role has, moreover, provided me with a better understanding of the ways in which teachers' experiences and practices evolve and how their learning to teach through inquiry occurs. Understanding the factors that influence teacher knowledge, practices and beliefs, in my opinion, were better understood through my personal immersion in the field, 'living' the experiences with participants and 'closing in' on phenomena as they unfolded in practice (Flyvbjerg, 2006). Ball (1990) contends that both as an analytical decision-making process and as a social process, one should expect different researchers to pick their way through fieldwork differently. As a result, the presence, the effect, the selections and preferences of the researcher cannot be removed from research, be it of a quantitative or qualitative nature. Indeed, I chose what to observe and what was important based on the four research questions and the data that started to come in.

This research presents a case that explores teachers' LTMI as implemented in a particular context. This case study includes the following boundaries:

- A single case representing Maltese secondary school mathematics teachers;
- The voluntary participation of teachers in a one-year CPD programme;
- A CPD programme designed by me but delivered by others (see Section 3.1.4);
- Qualitative data based on five research instruments collected over an intensive 14-month period (see Section 4.7); and
- Data initially collected from seven teachers, but later focused on data from three purposefully selected case studies (see Section 4.5).

Teachers, in this study, participated voluntarily in CPD and they showed a personal need and willingness to learn about IBL. Results are likely to have been different had the teachers been 'made to attend' and to use IBL in their classrooms. The structure of the programme – in particular, its ongoing and collaborative nature – does not resonate with the Maltese culture of designing teacher professional development courses. This could have presented additional challenges and opportunities within the participants' learning journeys.

A restriction to this research regards the data collection period. Findings from the research are bound by the 14-month time window during which all the data were collected. It cannot therefore be argued that teachers' engagement, in using IBL, continued to develop further, weakened or ended following their participation in CPD. I cannot extend my claims about aspects, experiences and developments in teacher learning about IBL beyond this time-frame.

During the study, I collected data from seven teachers. This sample is probably not representative of the teachers who might be interested to take on such a CPD programme and even less so of the whole population of mathematics teachers in Malta. However, my interest was to uncover how teachers, who are willing to take on CPD, engage in reforming their practices. So, I was primarily concerned with obtaining the specifics of each individual situation and detailed descriptions of teacher engagement as a participant of the LTMI programme. Consequently, the findings arising from the study may not be generalised to a wider population of teachers. However, I argue that they may well speak of the conditions under which keen and committed teachers may develop their knowledge and practices of IBL. I say this in the knowledge that the generalisability of case studies is "increased by the strategic selection of cases" (Flyvbjerg, 2006, p. 231). I would thus contend that my findings not only strengthen the relationship between the social, cultural and contextual factors that emerge from this study, but, as Yin (2003) points out, were useful in generating theory.

Case studies have limitations to generalisations from findings. However, they contribute to analytical generalisations in which previously developed theory or observations are used to compare the empirical results of a particular case study (Yin, 2003). When conducted with rigour, case studies may indeed serve towards theory development. As in this research, cross-case analysis (see Chapter 6) provides ways for analytical generalisations. Analytical generalisations, as the ones presented in this chapter, offer what Lincoln and Guba (1985) call 'working hypotheses'. Since the present findings are based on rich data, the working hypotheses developed here are applicable and, hence, transferable to other cases. I would therefore argue that the evidence presented (see Chapter 6), offers a sound representation of typical secondary school mathematics teachers working within schools that are typical (but not representative) of Malta and beyond. Transferability to other cases, in the sense meant by Lincoln and Guba (1985), was done by comparing the degree of similarity between the data presented in this research and data reported in other studies and countries.

8.2 Contribution to knowledge

This thesis has important contributions to make to the literature with implications on teachers' use of IBL, the structure of inquiry lessons, the design of CPD initiatives and the way IBL is offered through CPD. I summarise these contributions on basis of the gaps in knowledge identified in Chapter 2. As I argue, these contributions challenge some of the current notions of IBL enactment and IBL-related CPD.

8.2.1 Teachers interpret and enact IBL on the basis of their prior teaching experiences and the familiarity between their existing practices and those promoted by CPD

Besides showing that teachers construct their understanding of IBL from classroom experiences, this knowledge adds to the body of research showing that contextual conditions (Fullan, 2007) including the challenges they encounter (Euler, 2011; OECD, 2009; Swan et al., 2013) and their beliefs and values (Anderson, 2002) shape teachers' interpretation and

enactment of IBL. To enact IBL, teachers consider and address numerous contextual factors. As a result, teachers constructed a personal understanding of IBL based on such considerations that are also socially negotiated and shaped over time.

Much of the research appears to have addressed the issue of teachers' interpretation of IBL based on interviews with teachers (Farrugia, 2014; Ireland et al., 2012; Maaß et al., 2017) or, as in the case of Chin et al. (2016), the drawing of concept-maps and follow-up interviews held at the beginning and end of the CPD. This study uses other forms of data collection in order to gain a more comprehensive understanding about how teachers interpret IBL and the way this develops over time. To uncover conceptual and practice-based developments, teachers were asked to provide a written definition of IBL, discuss understandings during interviews and write reflections in their lesson journals. These data were gathered through multiple instruments (questionnaires, interviews and lesson journals) at the beginning, during and the end of LTMI. Besides providing teachers' IBL interpretations, understandings and practices at different stages during the study, this approach also enabled multiple perspectives into how teachers interpret, use and enact inquiry. While initially interpreting IBL as discovery learning, teachers developed their understanding of IBL over time through their ongoing implementation process and interactions within the CoP. The underlying statement here is that, because this transformation is individual (based on previous knowledge, beliefs and practices), context-dependent (depending on the school system and the particular group of students) and socially constructed (opportunities to interact with others offered by the CoP), it is difficult to categorise teachers' conceptualisations and practices as, for example, the study by Maaß et al. (2017) does.

Moreover, the findings reported by Maaß et al. (2017) show that teachers with some or a great deal of prior IBL experience exhibit a more sophisticated interpretation of IBL. Findings from my research expand on this and show that the way teachers interpret and enact IBL depends on prior teaching experiences and, in particular, the similarities they see

between their existing practices and those promoted by CPD. Teachers with some prior experience of inquiry teaching, and specifically those who perceive similarities between their 'old' and the 'new' proposed practices, are likely to attempt a more rapid immersion in inquiry teaching. With the preconceptions they bring with them, teachers may overlook the need for reflection that hinders the implementation and their developmental process.

8.2.2 Teachers make adaptations to the 'task-first' lesson structure in order to address contextual constraints

An important finding emerging from this study is that the tasks teachers used integrate mathematical content and cognitive challenge. While teachers sought cognitively challenging tasks to stimulate inquiry, they addressed the issue of syllabus coverage by using tasks that included the content they were required to teach. Teachers also recognised the benefits of using such tasks, and were aware of the need to address both personal issues (e.g.: learning to teach through inquiry by integrating challenging tasks) within contextual factors (particularly, teaching mathematical content, fostering active learning, supporting student learning and lesson time limitations). The present study shows that addressing these challenges had a direct impact on teachers' lesson structure.

Modifications to lesson structure were made to make tasks more accessible and facilitate the process for students in engaging with IBL. Such changes were intended to reduce the cognitive load (see Ward & Sweller, 1990) that teachers perceived IBL tasks were presenting. However, teachers experienced other debilitating factors including prior teaching experiences, their knowledge about IBL and their perceived learning needs of students. As teachers face such complex situations, adopting a 'task-first' lesson structure, as modelled in LTMI, appeared problematic. To facilitate learning and subsequent inquiry, Ward and Sweller (1990) suggest the use of appropriately structured worked examples. Similar to graded tasks (Chris) and teacher interventions (Greta), worked examples are purposefully

introduced to serve as a form of 'telling actions' (Lobato, Clarke, & Ellis, 2005) in supporting students' task completion.

Contrary to the study by Sullivan et al. (2015), this research reveals that the 'task-first' lesson structure may not necessarily be feasible for teachers. The CPD programme offered by Sullivan et al. (2015) has similarities to LTMI. Their intervention incorporated the provision of tasks, student-centred pedagogies and a suggested lesson structure. However, there are also important differences related to the CPD offered. The study by Sullivan et al. (2015) involved a two-day induction programme during which teachers were given specific lessons to use with tasks that matched the mathematical content that teachers intended to teach. On the other hand, those in LTMI were free to use the tasks offered and encouraged to collaboratively and individually design their own tasks and lessons over a one-year period. I argue that the CPD design – its prescriptive nature, short-term nature and lack of teacher ownership and space for reflections on teaching – are likely to account for the fact that "teachers found the proposed lesson structure workable" (Sullivan et al., 2015) and, hence, did not deviate from it. Indeed, as reported by Owston et al. (2008) a highly controlled CPD programme imposed on teachers leaves little flexibility for them to experiment with activities in the classroom.

Moreover, the present study also challenges the results reported in the study by Leikin and Rota (2006). The present study shows that a teacher's ability in using a 'task-first' lesson structure, consisting of three to four phases, does not necessarily indicate proficiency in managing an inquiry-based classroom. The study by Leikin and Rota (2006), which describes a mathematics teacher's development in using a 'task-first' inquiry-based lesson structure, focuses on a case study of a teacher-researcher. This teacher had a two-year teaching experience before engaging in a further 15-month period implementing inquiry with her second-grade students. This case study has similarities with Chris who had a short teaching experience before joining LTMI. As in the study by Leikin and Rota (2006), Chris did show increased confidence and growing expertise over time. However, his

proficiency in managing IBL cannot be attributed to his ability in using the 'task-first' approach. Instead, the present study shows that, irrespective of the lesson structure that a teacher uses during inquiry teaching, his or her increased confidence and expertise develops his or her ability to better facilitate student learning.

Moreover, while concurring with Leikin and Rota (2006) that positive changes towards IBL in both teachers' and their students' actions and behaviours occur over time, this study adds that modifications to lesson structure are expected because they depend on the tasks teachers decide to use. Teachers did not just implement the tasks that LTMI offered; they adapted and designed their own, playing an active part in task design, planning and enactment. They scaffolded students' work and thinking "by providing a predictable structure for inquiry" (Goos, 2004, p. 282), withdrawing this support over time. For example, Chris favoured a sequence of tasks rather than just one cognitively challenging task and this required him to adopt a different kind of lesson structure. Providing a sequence of tasks required students to focus their attention to solve separate tasks eventually leading them to be better equipped in undertaking the more challenging task presented at the end of the lesson.

In the process of learning to teach through inquiry, teachers seek a balance between integrating IBL and, at the same time, addressing contextual challenges. While serving a useful guideline, the 'task-first' approach may offer limitations to particular teaching situations. Within a context influenced by numerous constraints (restricted lesson time, content coverage and an exam-oriented system) using a 'task-first' approach may offer a too rigid lesson structure that teachers are likely to find difficult to adhere to. In addressing this, teachers use their 'sense of practicality' (see Hargreaves, 1994) and adapt lesson structure on the basis of what appears to work for them in practice.

As a result, an alternative lesson structure emerged – the 'scaffolded inquiry' approach. Like 'discussion-first' (see Russo & Hopkins, 2018), this approach may start off with a discussion of key mathematical ideas and

include appropriately selected teacher telling interventions such as the use of worked examples (see Ward & Sweller, 1990). Otherwise, inquiry lessons may incorporate a sequence of small tasks leading to the main inquiry task. For teachers with limited prior knowledge of IBL, this approach may be an expected uptake. Yet, over time, this approach may transform itself to become more aligned to 'task-first', once teachers gain the required confidence, skills and knowledge of their roles in enacting and supporting students using inquiry teaching.

8.2.3 Teachers experience discovery teaching along their journey towards inquiry and their practices and beliefs become more aligned over time

Literature shows that beliefs shape teachers' decisions about what mathematical knowledge is relevant, how knowledge is learned and what teaching approaches are effective to achieve this (Askew et al., 1997; Swan, 2006). Given the nature of the instruments used, this study presented both professed and attributed beliefs and practices (see Speer, 2005). In other words, the reported findings about beliefs and practices include both data as stated by teachers and as drawn from observations.

This study shows that as teachers implemented changes in their classroom practice they also reported changes towards connectionist beliefs. In addition, the move from transmission to discovery teaching was also evident in their changes in beliefs following their participation in the summer workshop. For Fullan (2007) changes in beliefs are more difficult to bring about than changes in practices. While data from the present study cannot contest this conclusion, it suggests that changes in beliefs are closely tied to teachers' practice change and their CPD experiences. Following the summer workshops, both discovery and connectionist beliefs of teachers grew stronger. This indicates that changes in beliefs were evident even within a period during which teachers were not implementing the ideas promoted by the CPD.

The present study on beliefs was undertaken with the understanding that changes in teachers' beliefs are likely to occur with changes in practices. In

order to investigate this, data on teachers' beliefs were collected at three different stages (see Section 4.7.2). Previous research (Swain & Swan, 2007; Swan, 2006; Wilson & Cooney, 2002) had indicated that beliefs change not because of CPD per se but due to successful classroom implementation. Findings from this research agree only in part with this conclusion. Beliefs may change as a result of CPD, particularly when LTMI focused on and attended to teachers' beliefs (see Section 3.1.3). In addition, classroom implementation of the ideas promoted by CPD shaped teachers' beliefs, eventually revealing to be more consistent with classroom practices. Hence, context is also critical towards developing beliefs (Beswick, 2005; Putman & Borko, 2000; Speer, 2005). While changes in beliefs are not necessarily a result of classroom practice, but may also emerge as a result of teachers' participation in summer workshops, over time changes in beliefs were found to converge and become aligned with teachers' practices. Over time, beliefs are likely to undergo further changes and become refined as teachers take up ideas from the CPD and put them into practice.

8.2.4 Teachers' zones of enactment are supported by 'just-in-time' teaching and learning opportunities

Research has shown that external factors, such as policy, curriculum and assessment practices, over which teachers generally have no control, impinge on teachers' enactment of reform (Anderson, 2002; Chin & Lin, 2013; Niemi, 2002; Priestley, 2010). This study supports this body of research but shows further that, when teachers recognise that these external factors are unchangeable, they rely on JITL opportunities for negotiating understandings of IBL practices. Since teachers in this study recognised their own limitations of influencing the *policy*, *private* and *public* sectors (see Spillane, 1999), they were more strongly motivated to implement changes within their classrooms driven by the JITL opportunities created within LTMI.

It appears that, in LTMI, the students and the classroom context provided the space for ongoing implementation, feedback and reflection that facilitated teachers' IBL enactment. Teachers in this study noticed

opportunities for learning mainly from within the classroom context and, particularly, from their students. The *professional* context was limiting in providing JITL opportunities for teachers to share and discuss their challenges and dilemmas with colleagues at their own schools – a lack of collaborative culture prevailed. These teachers also appeared to see no learning opportunities arising from the other sectors within their environment – *policy*, *public* and *private*. These sectors seemed to represent the status quo offering no potential for enhancing and supporting their implementation of and learning about IBL. Teachers joined LTMI with the knowledge that these system restrictions coupled with societal influences about what good teaching is about (ensuring student success in examinations) would offer a challenge in integrating IBL. However, rather than focusing on how these sectors were limiting, they recognised these limitations and relied specifically on how students engaged and responded to classroom activities in making sense of and operationalising IBL. Students, and teachers' ongoing interactions with them, offered enactment zones with JITL opportunities for teachers to notice, interpret, construct and operationalise IBL.

In making changes to their practices, teachers' enactment zones were not limited to their individual classrooms but extended beyond and included deliberations within the LTMI community. Although the CoP meetings were enabled by a CPD leader and facilitated by ongoing deliberations about practice-based experiences using a range of material resources, it appears that coming together and talking to other teachers was transformative in its own sense. However, even though pairs of teachers from schools joined LTMI, occasions for 'just-in-time' collaboration within their school context were limited. Schools did not appear to offer opportunities and provide structures for teacher collaboration, which might explain why such interactions were rare. Within the professional sector, privacy was a dominating feature in schools and enacting IBL was highly individualistic and limited to teachers' negotiations with students.

This study shows that enactment zones offering JITL were mostly influential in supporting teacher development and change. Hence, I argue that given the contextual constraints of teachers' work, their incentives and opportunities to learn and change are stimulated by three sectors (see Figure 23). The first sector influencing teachers' enactment zones is *policy*. Policy, here, refers to the design, implementation and evaluation of education policies. The two-way arrow linking policy with a teacher's enactment zone signifies a two-way relationship that practitioners may have in their efforts to influence and eventually change it. Policy, in this sense, incorporates also the private sector (textbook and curriculum publishers) considered here as an integral part of policy implementation and practice. The *professional* sector, as the second sector, incorporates both formal and informal encounters that teachers have with other educators in and outside the school. For teachers, formal encounters include staff and departmental meetings, and CPD opportunities offered by the school but also by other education institutions. Informal encounters include 'just-in-time' discussions held online but also with colleagues during break generally in staff rooms, corridors and the school yard. Channels for professional encounters and communities have an important influence on teachers' practice (Coburn & Stein, 2006; Jaworski, 2003; Little, 1993; McLaughlin & Talbert, 2001). The third sector, *pupils*, pertains to the influence that students' responses to schooling and to the reform practices introduced in class have on teachers. Research suggests that teachers' perceptions of students have an important influence on teachers' practice, their reaction and will to enact the proposed reforms (Blignaut, 2008; Golding, 2017; McLaughlin, 1990). Lessons offer a learning space in which teachers enact reform but, as this study shows, where they also notice and are eventually transformed by the lived experiences of their students.

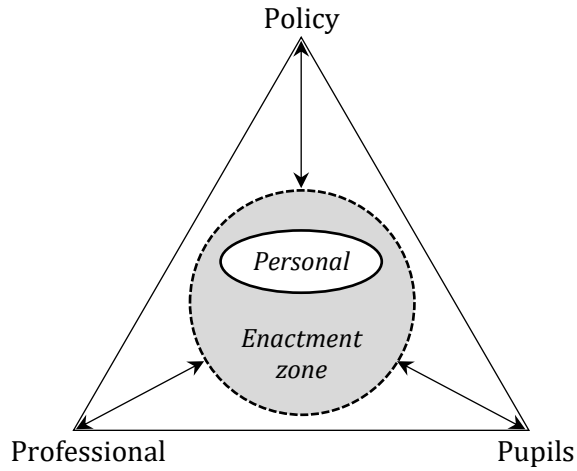


Figure 23: Redefined model of zones of enactment

8.2.5 Externally-driven practice-based CPD models benefit from 'just-in-time' teaching and learning opportunities, but their effectiveness is likely to suffer when there is a lack of this kind of support from schools

Through LTMI, which offered JITL opportunities through online CPD materials and a CoP component, teachers developed practical knowledge of IBL. For instance, the ongoing collaborative engagement and negotiations, which teachers considered as crucial to persist with their use of IBL, was also found to support their learning and pedagogical development. This finding confirms the extensive research literature on the importance of the social aspect of learning for effective CPD (Back et al., 2009; Jaworski, 2006a, 2008; Levine & Marcus, 2010; Lewis et al., 2009; Little, 1990; Loucks-Horsley et al., 2010) to support teacher change. In LTMI, the social dimension was a result of the embedded CoP component integrated into a practice-based model. This confirms research showing that teachers make sense of inquiry teaching from practice (Anderson, 2002; Chapman, 2011). Indeed, teachers profited from ongoing meetings to share and reflect upon classroom ideas and practices. Over time, this practical knowledge facilitated their planning and enactment of IBL. Hence, the present study adds to the body of research on CPD design. It shows that the kind of IBL knowledge that teachers develop is likely to depend on the nature of the CPD intervention offered and on what is promoted. In LTMI, teachers

developed practical knowledge of IBL possibly because LTMI offered a practice-based CPD design. On the other hand, in the study by Chin et al. (2016) in which CPD engaged teachers in reading literature, the authors claim that "teachers tended to understand mathematics inquiry from a theoretical perspective" (p. 859). Hence, this study indicates that teachers develop their knowledge of IBL based on what is promoted in CPD.

Similar to the findings reported by Owston et al. (2008), the opportunity for teachers to share teaching experiences face-to-face supports community building while it also addresses the feeling of isolation experienced within their schools. Indeed, schools have a key role to play in embracing and supporting externally-driven CPD designs. LTMI was a CPD intervention designed externally from schools: as schools were not directly involved in the design but offered to endorse teachers' participation. For example, before the CPD, I only consulted school administrations to provide teachers with time to attend the CoP meetings (see Section 4.4.1). While LTMI, within its research-based design principles, considered contextual factors, it was not internally owned by or co-constructed with the schools in which participants worked. This aspect led to the two contrasting scenarios that teachers experienced within their learning journeys – enacting IBL in isolation within the school and negotiating their inquiry experiences within the CPD community. Hence, asking teachers to implement IBL was hard for them to achieve without including the direct involvement and ownership of schools. For these teachers, JITL opportunities in their schools were limited to their classrooms. According to Fullan (2007), reform is likely to fail when it focuses on the development of innovation without the necessary attention paid to 'reculturing' schools. This study shows that the lack of a collaborative learning and sharing culture within schools contrasted the 'just-in-time' CPD opportunities promoted by LTMI. Schools lacked a culture where teachers could share knowledge and classroom experiences, and this limited teachers' development of IBL to the CoP meetings, access to the online CPD materials and their own classrooms.

Ideally, teacher development and school improvement work hand in hand (Fullan, 2007). Referring to lesson study as one effective CPD model involving outside experts working with schools to support teacher learning, Stigler and Hiebert (1999) report that improving teaching requires changes in the culture of teacher learning. In creating and sustaining this culture, schools in Japan co-design initiatives and interventions with outside experts (CPD designers and providers). This is an effective way of providing 'just-in-time' on-site support for teachers to address their professional development needs while cultivating collaboration – an element that was limiting in LTMI to further sustain teacher change. Given the presence of a collegial school culture, in-classroom support by colleagues or outside experts (see Mouza, 2003) is one example of in-school JITL opportunities that CPD designs may incorporate to support teacher learning.

8.3 Implications for CPD supporting inquiry teaching

In this section, I now summarise each finding and discuss the implications on IBL and the design of CPD.

8.3.1 Teachers interpret IBL on the basis of what they notice as familiar and/or contrasting to their existing practices and make changes based on ongoing implementation and reflection with others

According to Spillane et al. (2002, p. 395), different teachers will construct different understandings as they tend to see what is offered "in terms of what is already known and believed". Chris saw IBL as contrasting to his transmission practices while Greta and Sarah recognised similarities between what was offered and their preconceived ideas and prior practices.

Over the course of CPD, Greta and Sarah had the 'capacity' that Spillane (1999) regards as critical to practise in ways recommended by reformers but less so the pedagogical design capacity (PDC, see Brown, 2012) that enables teachers to become critical, reflective and designers of IBL. Eventually they reconstructed their IBL practices when they encountered challenges and recognised the need to question, unlearn and discuss their

practices with others – something that Chris demonstrated from the start. To attend to the ideas promoted by LTMI, teachers had to appreciate the inadequacies of their current understandings and practice, thereby seeing a need to learn.

In the process of addressing inconsistencies emerging as a result of their preconceptions, teachers initially recognise only shallow similarities to eventually modify their practices to align these to IBL. Teachers with limited teaching experience and unfamiliar with IBL may act more cautiously in introducing IBL, as a new pedagogy to students, and act reflectively to notice opportunities for learning. Hence, as in the case of Chris, they introduce IBL gradually, engage in deep reflection and develop an ongoing critical stance to address the contextual constraints in enacting IBL.

This finding has implications to the design of IBL-related CPD. CPD programmes need to support teachers' development of their PDC, that is, help teachers develop their ability to design classroom instruction that is pedagogically robust and aligned with IBL. CPD that recognises teachers as agents also supports them to become researchers of their practices by constantly helping them to contrast their 'old' and 'new' ways of teaching. This shift is not only about developing a teacher's capacity to practise IBL but, more importantly, to question, challenge, research and eventually endorse robust practice-informed IBL practices. In supporting this shift, CPD programmes need to introduce and offer new knowledge in the form of suggestions open for discussion rather than 'rules' to abide by. In other words, as in LTMI, CPD promoting a specific model of inquiry teaching should promote, encourage and support teachers to introduce their own practice-based knowledge. While inviting teachers to use and question their IBL understandings, they may grow professionally when they are asked to contribute their own practice-informed knowledge.

8.3.2 Teachers structure their IBL lessons using approaches that include adaptations to the 'task-first' approach

Two of the teachers in this study used a scaffolded approach to introduce tasks gradually, either through a sequence of graded tasks (Chris) or through a whole-class discussion (Greta). This latter type of lesson structure blends the 'discussion-first' approach, usually associated with transmission teaching (Russo & Hopkins, 2018), with the 'task-first' approach promoted in reform-oriented practices. This study shows that, for teachers new to IBL, a 'scaffolded inquiry' approach "using predictable dialogue structures" (Goos, 2004, p. 262) may be more achievable in bridging the gap from transmission to inquiry teaching. Using a 'scaffolded approach' affords teachers to engage in interactions with students and to structure tasks in ways that "allow students to participate in joint activities that would otherwise be beyond their reach" (Goos, 2004, p. 262). Offering and expecting teachers to adopt the 'task-first' approach is likely to present too high a challenge given the instructional change to IBL (Stein et al., 2008) and the contextual constraints that teachers need to simultaneously address. Besides facilitating teachers in shifting more responsibility of learning onto students, a 'scaffolded inquiry' approach is more likely to support students in the transformation of their roles to become more independent and active learners.

CPD programmes promoting and supporting the use of IBL tend to promote a 'task-first' approach to lesson structure. This study questions this approach and supports the claim made by Russo and Hopkins (2018) that there is more than one way to incorporate cognitively challenging mathematical tasks. I argue that offering a 'task-first' approach consisting of three stages – launch, explore and discuss – would be complex for teachers to implement especially when they are more accustomed to the reversed type of lesson structure sequence (see Buhagiar & Murphy, 2008). Within more traditional lesson structures, teachers are likely to be familiar with a sequence starting with discussion and exposition followed by work

on graded tasks and ultimately engaging students with a challenging task – a 'discussion-first' approach.

Transformative CPD allows teachers to adapt their lesson structure. Through CPD, teachers need to be provided with the freedom to experiment, negotiate, transform and develop their own understandings. The implications for CPD would be to offer teachers different approaches to lesson structure, identifying and discussing differences while supporting them to make informed decisions about which approach to adopt. Such decisions also need to be based on the learning objectives and the associated learning outcomes that teachers intend to achieve through inquiry.

8.3.3 In enacting IBL, teachers structure their lessons in ways to incorporate the ideas promoted in CPD, but with adaptations in order to address their contextual constraints

To integrate IBL, teachers generally opted to offer high cognitive demand tasks that integrated familiar content. This process involved an investigation into a range of instructional materials (textbooks, syllabus, schemes of work, online resources, CPD materials and the consideration of students' prior knowledge and understandings). This is an important feature of lesson study, referred to as *kyozaikenkyu* (Watanabe, Takahashi, & Yoshida, 2008) and involves the study of curriculum materials, content and its teaching prior to lesson planning. Research shows that this sophisticated process enables teachers to envision how students might approach the mathematical tasks they select and, hence, anticipate students' reactions, responses and solutions to tasks (Schoenfeld, 1998; Stigler & Hiebert, 1999) within an IBL process.

The present research suggest that teachers prefer to design or adapt tasks that cater for the particular students they have in class and address the content they are required to teach. Indeed, tasks play a key role in lesson planning and the careful selection of tasks is crucial for engaging students in thinking about and discussing mathematics (Hiebert et al., 1997). Task selection was informed by time limitations and teachers attempted to

overcome this dilemma by a rethinking of both the tasks they used for IBL and their lesson structure. In an education context where teaching content is a priority, teachers found it difficult to introduce challenging tasks within their teaching, and addressed this dilemma by incorporating two key dimensions: cognitive demand and mathematical content. Open-ended tasks, similar to those offered by LTMI are likely to provide teachers with inquiry experiences. Yet, due to system restrictions, teachers are also likely to seek and use more guided tasks (see Section 2.3.1) that provide more structure and where content is visible to both teachers and students. This suggests that mathematical tasks such as the five types of collaborative tasks by Swan (2005), the inquiry prompts of Blair (2014) and the mathematical études offered by Foster (2013) are likely to be more useable and serve the purpose of promoting inquiry as they embed both cognitive demand and content.

For teachers who wish to introduce IBL, offering mathematical statements, prompts and études may serve as a stepping-stone to initiating inquiry. Starting off with such tasks may be a deliberate and sensible action for teachers, yet teachers may extend these tasks to challenge students inviting them to test their methods and question their answers. Using tasks that embed practice within thought-provoking mathematical activity may serve as springboards to introduce students (and teachers during CPD) in developing confidence and an ability to inquire. This study concurs with the claim made by Blair (2014) that the spirit of inquiry may be achieved by trusting that even simple problems with a closed goal and answer may lead students to something greater and not yet evident.

8.3.4 In shifting to inquiry teaching, teachers pass through a phase of discovery teaching with practices becoming more aligned with their changing beliefs

Inquiry teaching requires teachers to be prepared for a range of student responses and to respond to and follow-up students' ideas in a manner that is both meaningful and productive towards their learning. This is usually quite difficult for teachers to do particularly if they hold onto more

transmission-oriented approaches. This study further shows that, as a result of this, in transforming their roles towards inquiry, teachers shift from 'telling' to a 'not telling' role, demonstrating that 'stepping out' (see Lampert & Blunk, 1999) from student conversations is initially a natural thing to do. The route to inquiry teaching passes through a phase of discovery teaching, and teachers remain committed to this approach until they recognise its limitations when the need to support students arises.

In the cases reported in this research, teachers appeared to shift from 'telling' to 'not telling' roles – initially withdrawing themselves completely from student activity and later finding a better balance in undertaking a more supportive role. Indeed, because telling featured as a central characteristic of teachers' transmission teaching methods, the move away from transmission teaching to IBL was interpreted as a move away from telling (Swain & Swan, 2007; Swan, 2006). However, in inquiry teacher telling is instructionally important (Lobato et al., 2005; Swain & Swan, 2007), should not be downplayed and is seen as taking different forms particularly in initiating, eliciting and challenging student thinking. These telling actions (see Lobato et al., 2005) are aimed to broaden the roles that teachers can take up in an inquiry-oriented classroom. After having mastered the knowledge of *what* and *how* to tell, for inquiry teaching teachers need to learn when to tell.

The role of telling in IBL has important implications towards designing CPD for IBL. In offering inquiry lessons for review and discussion, teachers need to be supported to notice the most significant changes taking place – particularly the change in teachers' roles and how this influences classroom activity. Purposefully designed CPD activities and lessons demonstrating teachers assuming a non-interventionist role, or offering teachers themselves opportunities to teach by deliberately avoiding telling, is likely to create a cognitive conflict (see Section 3.1.3). During CPD, as teachers seek to rectify the conflict (Peled & Suzan, 2011) between highly interventionist and non-interventionist teaching, they are likely to develop new meanings and understandings aligned to inquiry teaching. The central

notion is that, rather than avoiding teachers from moving into discovery teaching, make this a deliberate path so that teachers can more deeply recognise, negotiate and eventually address the inconsistencies of this approach.

8.3.5 Enactment zones that offer 'just-in-time' feedback and collaboration support teachers in their efforts to pursue change

The CoP was initially restricted to monthly meetings. Midway through the year, however, teachers requested to increase the face-to-face support and met every two or three weeks (see Section 3.3.2). These face-to-face meetings and the website resources (see www.iblmaths.com), readily offered JITL opportunities for teachers. Enactment zones were positively influenced by ongoing deliberations between teachers from different schools, supported by the professional development materials provided, and the resources that teachers created.

Similar to the findings reported by Spillane (1999), Hodgen and Johnson (2003), and Golding (2017), data from this research shows that enactment zones that are social are indispensable for teacher learning, development and change. Moreover, this research shows that ongoing change is realised when teachers are offered 'just-in-time' assistance and feedback when needed.

Offering JITL has implications to designing CPD. This study shows that rather than offering ongoing support, CPD needs to create opportunities through which teachers may access and get assistance when needed. Admittedly, CPD needs to transform teacher learning from being 'just-in-case' and instead creating 'just-in-time' opportunities (Schrum, 1999). Teachers rarely benefit from expert-driven CPD offered 'just in case' they might want to use in the future (Loucks-Horsley et al., 2010). They value job-embedded learning that offers new forms of discourse communities with opportunities for 'just-in-time' conversations on present-day matters. Through engagement in within-school professional networks (Goos, Stillman, & Vale, 2017) teachers may transform their insular work environments to address their 'just-in-time' needs. Schools, as

organisations, need to support this ongoing learning by strategically creating considerable time for collaboration and time that is purposefully directed and structured for teacher learning (Garet, Porter, Desimone, Birman, & Yoon, 2001).

8.3.6 The off-site collaborative engagement by teachers during CPD supports teachers' on-site isolated practices, but does not enhance teacher collaboration within their school

This study uncovered the work and the kind of learning opportunities that arise for teachers both in the context of their workplace and within the CoP created by LTMI. Teachers' autonomy within their school appeared to be exercised in a context of isolation whereas their autonomy within the CoP developed as a result of rich, meaningful and ongoing professional dialogues. Due to lack of collaboration between teachers in schools on matters that relate to teaching and learning, teachers reported limited learning-on-the-job that could better sustain the learning-off-the-job created within LTMI.

Contexts may offer isolated or social learning spaces for teachers. The two contrasting learning contexts presented in this study have implications on the way countries like Malta offer professional learning opportunities for teachers. In particular, CPD programmes need to recognise that the learning space they offer is likely to contrast with the learning space that teachers find themselves working on a day-to-day basis within their schools. Addressing this or bridging this gap requires that not only teachers take up CPD to reform their practice but that schools also immerse themselves in creating social learning spaces for teachers. Dorier and Garcia (2013, p. 848) call for "a change in mentality and culture". To achieve this, school settings need a rethinking in enabling teachers to act productively towards improving their own practice.

For schools to become learning spaces where teachers can work and act collaboratively in developing their own knowledge about practice, they need to co-create their professional learning programmes with support from outside experts. As learning centres, schools need to constantly work

towards creating and supporting network opportunities to enhance and sustain teacher quality (Lerman & Zehetmeier, 2008).

8.4 Recommendations for enabling change

Effective school systems encourage teachers to experience CPD opportunities within their everyday activities in the workplace. More importantly, schools are likely to be more effective when they *enable* teachers to drive their own learning through JITL opportunities for reflection, community building and networking (Krainer & Zehetmeier, 2013; Lerman & Zehetmeier, 2008). The term *enabling* is used to refer to a process whereby externally-designed CPD programmes and in-school CPD initiatives provide teachers with the autonomy and means to co-create their own professional learning networks. The aim is to empower teachers to assume professional responsibility and eventually influence change in others.

In designing CPD for IBL, I provide the following considerations that can be embedded in the activities promoted by CPD. While these considerations are intended to provide ongoing support for teachers, they are also aimed towards cultivating collaborative engagement and self-directed learning.

- Engage teachers in ongoing reflections and negotiations about their 'old' and 'new' practices as this is likely to better address any misconceptions that they may bring with them as a result of prior practices;
- Include CPD activities with classroom scenarios showing teachers using a discovery approach and avoiding telling. This helps teachers to understand that telling has a place in IBL when purposefully used;
- Offer a range of IBL lesson structures (e.g.: 'task-first', 'discussion-first' and 'scaffolded inquiry') rather than a single approach as this is likely to open up possibilities for teachers to introduce IBL;
- Promote and ask teachers to bring a range of tasks so that discussions on the organisation of lessons can centre around different levels of

cognitively challenging tasks including those incorporating mathematical content;

- Support teachers to take a proactive role in reforming their practice, taking control of their learning, finding their own ways of implementation and having opportunities to share their knowledge with others within and beyond the CPD community;
- Seek within and across school networks and in-classroom administrative assistance that provide teachers with 'just-in-time' support and feedback required for effective implementation.

Further to these considerations, below I provide recommendations that span three aspects arising from this research: the active roles teachers assume in IBL-related CPD designs, collegial professional learning contexts and CPD designs offering JITL opportunities.

8.4.1 CPD designs should allow teacher agency and self-directed learning to enhance the uptake of inquiry teaching

To promote teachers' capacity to learn and act as agents of innovation and change in their transition to IBL, CPD providers, school administrators and policy makers must look critically into zones of enactment, particularly the *policy* and *professional* sectors. In the absence of dialogue, interaction and negotiation, these sectors do not appear to support a move towards inquiry teaching. Any planned intervention must take into account that it is teachers who need to own the reform and, in the process, control the destiny of their change (Chapman & Heater, 2010). Teacher agency becomes a crucial transitional factor and teachers in this study, particularly Chris, lamented their potential to influence change. Rather than seeing learning opportunities arising from different zones of enactment, teachers see these zones as stumbling blocks because they are both detached from the reality of the classroom they are living and restricting access to communicate and mediate their understandings. While the *policy*, *professional* and *private* sectors may be communicating to teachers about the *what* to do, they are not negotiating the *how* or recognising the

challenges within the change process. Rarely do Maltese teachers appear consulted and, hence, recognised for the expertise they have (Attard Tonna & Shanks, 2017). Teachers need to be involved, listened to, believed and treated as professionals to bring about the desired change. Without this understanding, it would be a mistake to think that changes in teaching can just happen.

The practices required from teachers challenge them to reconsider their existing knowledge, experiences and beliefs. To do this, teachers need to be freed from the control of an education system that restricts self-directed initiatives, action research and collaborative learning. What matters is that teachers are treated as key agents of change, because when it comes to changing classroom practice, teachers are the final policy brokers (McLaughlin, 1990). They are, indeed, the fundamental intermediaries to the change process that will either inspire or cease the enactment of reform.

8.4.2 Schools need to cultivate networking through a collegial environment in order to enable teacher learning

Embedded within everyday school learning for teachers are formal and informal opportunities for teacher-directed learning. The workplace is a site that should support professional learning and teachers need more informal opportunities for 'professional socialisation' (Goos et al., 2017) to meet, talk about issues of practice and engage in sharing their work with others. Schools need to enable these encounters for two reasons: for facilitating the change towards engaging in more collegial relationships and; for building the capacity for school improvement. Schools have a key role to play in creating professional learning opportunities that challenge teachers to think, be critical and proactive while supporting them to experiment, take risks and innovate. Developing supportive and interactive communities of practice has become one of the leading approaches in initiating and sustaining reform within and across schools (Levine & Marcus, 2010).

Schools need to recognise and build on teachers' current knowledge of mathematical content, pedagogical practices and their rationale for

decision-making in planning, teaching and evaluating instructional practices. When professional development considers teachers as designers they can becomes agents of change holding agency to inform what works in practice. One possible way to do this is to engage them in reflecting upon and researching their own practice and that of others. In Malta, there appears to be an encouraging move in this direction. With the new sectoral agreement signed in December 2017 (Ministry for Education and Employment, 2017) and the setting up of the Institute for Education in 2016 (see http://ife.gov.mt) aimed at offering various CPD programmes for educators, more emphasis is now being placed on creating professional learning communities, within and across schools, that facilitate professional discussions while also encouraging educators to take part in more self-sought CPD sessions. These self-sought initiatives, defined as school initiated including self-reflection and peer-to-peer learning are, as from 2018, becoming recognised forms of teacher CPD.

Cultivating communities and networking are key towards fostering sustainability of structures for developing teaching and learning (Lerman & Zehetmeier, 2008). Drawing on prior research (Boaler, 1999; Coburn & Stein, 2006; Garet et al., 2001; Jaworski, 2007; Lotter et al., 2014) and data from this study, I provide three suggestions for creating, cultivating and sustaining learning communities for teachers based on constructive dialogue.

As a start, learning communities need to foster a climate of respect, well-being and positive working relationships (Borko, Koehler, & Jacobs, 2014) that motivate teachers to commit themselves towards collaboration. For those accustomed to working in isolation, learning communities may offer a threat to their personal experiences, insights, beliefs and practices. Trusting others requires time (Attard, 2012) but, just as importantly, a safety net for them to move out of their comfort zone; a protective environment in which they can share their concerns, dilemmas and difficulties. Secondly, fostering sustained and meaningful engagement with a community is not something straightforward to achieve and requires the

presence of a leader and knowledgeable other within the group (Jaworski & Huang, 2014; Shabani, 2012). Knowledgeable others with expertise in leading learning communities act in a mindful way (see Groschner et al., 2014) to foster engagement, interaction and collaboration. The knowledgeable other, here referred to as a leader-facilitator is also a participant who acts mindfully, from the outset, in creating a welcoming and safe environment for teachers. Creating a non-judgemental environment is a reassurance for participants to make contributions that become the lifelines within which the community thrives. Finally, learning communities need to cultivate a sense of ownership, agency and appreciation amongst teachers. Achieving this requires nurturing respectful relationships enabling teachers' freedom and openness to speak about their strengths, demonstrate their work and classroom practices but also to communicate their fears, dilemmas, weaknesses and concerns.

8.4.3 CPD designers should provide teachers with 'just-in-time' teaching and learning opportunities along their journey to enact IBL

IBL, as conceptualised in this study, involves a collaborative undertaking and the active engagement of students in the learning process. Working as a community of inquirers, students become immersed in mathematical sense making, wrestling with constructing, exploring and explaining mathematical meanings around a task set by the teacher or a question originating from the students (Smith & Stein, 1998). The tasks teachers use need not necessarily be open-ended but need to provide an 'achievable challenge' (Willis, 2010) – a challenge falling within Vygotsky's (1978) ZPD. This leads to the importance of the supportive role of the teacher who needs to determine the potential of the student in making progress in learning.

Stemming from this definition and the research findings, I draw similarities between CPD and IBL. Just as teachers need 'just-in-time' feedback and support for learning to teach, students require scaffolding offered 'just-in-time' by teachers to support their learning of mathematics through inquiry. Offering a 'scaffolded inquiry' approach involves using a range of teacher

intervention strategies – discussion, collaboration, structured tasks, questioning and telling – deemed to enhance both teachers' enactment of IBL as well as student engagement. Through CPD teachers will experience a change and require support in undertaking more self-directed roles for inquiry teaching. These 'new' roles, simultaneously experienced by students in their classrooms, require teachers to understand, own and offer 'just-in-time' support to students in their inquiry-based classroom environments.

Creating opportunities for teachers to reconsider their practices and eventually change themselves and their teaching requires that a number of key elements are in place. Indeed, pursuing "the road less travelled" (Bezzina, 2006, p. 166) involves a cultural change process which needs to be lengthy, slow, supportive and resilient.

Teachers develop new meanings, understandings, skills and behaviours when they feel motivated to change (Fullan, 2007), have a 'will to learn' (Van Eekelen et al., 2006) and have access to support (Day, 1999). Within an environment promoting social learning experiences, teachers may accumulate knowledge not only from their personal reflections on classroom practices but also from co-learning experiences (Jaworski, 2006b) with colleagues and knowledgeable others. Drawing on the Maltese context, I here elicit one example to show how schools can create 'just-in-time' collaboration and job-embedded co-learning experiences.

It is common for teachers to receive visits from their respective education officers and to then obtain oral feedback on their classroom practice following observation. The most common situation sees the 'knowledgeable other' (the education officer in this case) giving comments and offering suggestions to the teacher. As Mouza (2003) shows, this in-classroom feedback is likely to be beneficial for teachers. However, teachers may view comments as judgements that position them on the defensive. This situation, I argue, is likely to limit negotiation and, rather than allowing for the exchange of ideas between professionals, confines teachers to having to defend their work. Instead, teachers should be the ones involved in talking

about their teaching and learning experiences within the context. Perceptions of contexts by outsiders are likely to be distorted when they are unfamiliar with the school context and the class they are observing, particularly the students' learning needs. My argument is that such visits should be reviewed if they are to serve as co-learning experiences. Indeed, a different kind of relationship also needs to be established. Within this relationship, the 'knowledgeable other' has a key role in enabling teachers to take a critical stance to practice by stimulating thinking while providing 'just-in-time' support. Creating 'critical friendship' (see Stoll et al., 2012) requires establishing positive co-learning relationships and agreements (Jaworski, 2003) where both parties respect the expertise of each other. For instance, the lesson follow-up discussion can then serve to link to and build on the contextual experiences described by the teacher and the outcomes of practice. A teacher might ask questions about ways in which teaching is approached and how learning is experienced by students. The role of the 'knowledgeable other' can serve to stimulate 'just-in-time' thinking in pursuit of engaging teachers in exploring and inquiring about their practice, rather than just defending it. Ultimately, the goal should be that, through interaction and reflection, teachers obtain assistance on demand and 'just-in-time' (see Gee, 2003) from the education officer. Such assistance is also in-context of actual use and directly related to their professional needs, and teaching and learning goals.

8.5 Directions for further research

While the findings reported in this research provide in-depth answers to the research questions, nonetheless, further research with a wider sample of teachers would generate a more comprehensive understanding of the phenomena under study. For example:

1. In understanding how teachers learn to teach through inquiry, further investigation about the way teachers structure their inquiry lessons, why and how they modify lessons needs to be carried out. Moreover, one needs to investigate this with a wider sample, widespread across

teachers with varying teaching experience, experience in using IBL, teaching at different year groups and in different education settings and contexts. The main focus would be to investigate not only what changes in practices occur but also how beliefs develop over the course of a CPD programme.

2. Emerging from this research is the finding that, in shifting from transmission to inquiry teaching, transmission teachers go through a phase of discovery teaching. Would a CPD programme like LTMI, that aims first at supporting teachers to shift to discovery teaching facilitate their eventual transition to inquiry? Research would hence investigate how CPD, aiming to support teachers in teaching through discovery, eventually facilitates the change process towards inquiry. Investigation would specifically address teachers' and students' learning experiences while adopting a discovery approach.

3. Involving schools in co-design and implementation is one way of achieving close collaboration. Schools do not simply need to recognise the benefits of CPD but, more importantly, they need to own it and make it work for their teachers. With CPD involving more schools, investigating the influence of contexts on teacher learning and understanding the conditions and the structures provided and needed to facilitate teacher learning would be crucial. An investigation of how schools implement IBL-related CPD and support teachers in implementing IBL should be the focus. How schools implement CPD is important in understanding conceptualisations and ways in which they translate programmes in practice for sustained learning opportunities.

4. However powerful, LTMI had its limitations related to the process through which teachers gained knowledge and managed to transfer ideas about practice from the CoP setting to their classroom. Possible alternatives to this model are lesson study (see Stigler & Hiebert, 1999) or the sandwich model (see Swan et al., 2013) – the latter model involves an iterative process that starts with a phase of exploring features of IBL, followed by implementation and ending with a reflective

phase. Hence, another aspect of interest would be to investigate how these different CPD models would work in practice. From a teacher educator perspective and the role of CPD designer, a study comparing various CPD designs implemented within relatively similar contexts is required to investigate how different CPD structures impact on the way beliefs and practices change over time.

5. This study was motivated to understand teacher development and learning to teach. Although the data sources (particularly the interviews, classroom observations and questionnaires) provided a comprehensive picture of teachers' learning journeys as they engaged in this CPD programme, attention could also be dedicated to the students – their experiences, feelings and understanding of mathematics while learning through IBL. Students' perspectives would have been highly valuable in supporting data to show the extent to which teachers' enactment zones were highly influenced by students. It would be noteworthy to investigate students' experiences and any potential dilemmas and challenges they encounter.

8.6 Reflections upon my journey alongside teachers

Over my 23-year teaching career, I collaborated with teachers in planning, implementing and evaluating IBL lessons. That experience, which included an action research study (see Section 1.1.4), provided insights related to how working in a team may benefit each individual in overcoming the challenges encountered. Learning with teachers involved being one of them, a member of a team and, hence, with the possibility of contributing my insights, beliefs and experiences.

Doing this research with teachers offered a different experience, a new perspective and role in researching and understanding how teachers translate IBL into practice. Designing LTMI, its structure, content and materials involved an onerous task of translating my personal knowledge and experiences into the materials that were eventually offered to teachers. Researching teachers' practices implied listening to, interacting with and

working alongside teachers. Rather than supporting teachers along the way, I was now looking into how teachers were making sense of the content and eventually enacting IBL in their class. While observing lessons, at times I could relate to the difficulties teachers were encountering. On other occasions I was informed by new ways in which teachers were operating in making IBL a successful experience for themselves and the students. As a researcher, looking into the practice of teachers revealed insights that were previously unnoticed. While collecting and analysing data, I found myself deconstructing teachers' engagement and practice and, along this process, new understandings about the phenomenon surfaced.

8.6.1 Reflexivity in becoming a researcher

This doctoral journey has had parallels with my journey as a teacher and teacher educator. Being a novice researcher required thinking, designing and ongoing reflection, similar in many ways to my first years of teaching. As much as it was a learning journey for the teachers, this was also a learning venture for me – a journey of becoming a researcher. Along this journey I learned, through reading and grounded experience, about the complexities of undertaking a research study with rigour and consciousness. In this section I draw on aspects focusing on the importance of a researcher's reflexivity as self-doubt (see Ball, 1990) in terms of my role, the research settings and the data collection instruments.

As a researcher, I attempted to assume a role of non-participant observer (see Section 4.10) but I was "always and inevitably a part of the scene" (Ball, 1990, p. 159). Hence, undertaking this role brought about uncertainties about how I was perceived and identified by participants and their students. I was indeed mindful, meandering through and grappling with questions. What impression am I giving to teachers? Are they seeing me as 'the IBL expert' and hence as someone judging their comments, ideas, practices and reflections? An additional important and recurrent consideration was that of working towards sustaining an ongoing trusting and mutually respectful relationship (Bryk & Schneider, 2002). Building a trusting relationship, I believe, is likely to ensure trust in the data. When asking teachers for lesson

observations, I usually offered particular periods (at the beginning, midday and end of the scholastic year) but preferred to attend the class times that they offered. Respecting this decision was important, I feel, in underlining my non-judgemental role and eventually build trust.

To add to this complexity is the fact that my researchers' roles changed depending on the research setting, and decisions about these are not trivial. When observing CPD meetings, I assumed a fixed position in order to lessen the influence of my perception on the contributions participants made, whereas during lesson observations there were times when I moved around the class observing students. During interviews, I assumed a completely different role interacting with and probing into teachers' responses and contributions. Decisions about where and when to hold the interview partially determined the data collected. Reflexivity, in the sense expressed by Ball (1990), has opened up new opportunities for learning about research in school settings.

This brings me to a second and important consideration, taking a reflexive stance to research and to problematising the research instruments. Two key questions arose as I designed, developed and used the data collection instruments – whom are they serving? and what purpose are they serving? The obvious answer is that the research instruments serve the researcher to collect data that answer the research questions. However, a potential function of the research instruments was that of constituting an additional element of CPD for the participants. Over time, this aspect became more evident. I refer here to examples extracted from teachers' data during interviews in which it was clear that the interview served as a reflective exercise. To the questions I posed, teachers replied using the following typical starting phases: "I am realising that…", "Now that you are asking about it…", "When I reflect back on the lesson I did…", and "What I would do is…". Such expressions demonstrate how the dialogue created facilitated reflection and the social aspect of learning, not only for me as a researcher but also for the participants. I argue that these interviews, the lesson journal, questionnaires and the focus group discussion provided an

additional element of JITL for teachers, at specific points during their CPD journey.

This kind of thinking, I believe, and the consideration of alternatives afforded choices that eventually also influenced my approach to data analysis. These lived experiences and ongoing considerations were influential in making deliberate choices and reflexive decision-making (Ball, 1990). Reflexivity brought me closer to my fieldwork transforming, in the process, the way I now look at the delicate receptive role that researchers needs to undertake, not only in establishing trusting research relationships but particularly in situating the data collected.

8.6.2 Supporting teachers' implementation of IBL

Even though participants accepted my role as an observer, it was clear that they sought 'just-in-time' feedback following lesson observations. On many occasions, teachers hunted ways to discuss lessons generally by directly asking for feedback but also by starting a discussion on something that they believed did not unfold as planned. In order to support teachers with this, I encouraged them to reflect and write about these experiences and then share them with others during follow-up meetings. However, at times immediate feedback appeared vital for these teachers. It appears that teachers considered post-lesson encounters as learning opportunities, breaking away from their isolated practice. They demonstrated a desire to talk and seek advice.

With hindsight, I believe that teachers would have benefitted from having a critical friend or a mentor both for lesson preparation and post-lesson discussions. Asking pairs of teachers to participate in LTMI (see Section 3.4.1) was intended to initiate collaboration and communities within participating schools. However, school contexts revealed unproductive towards such dialogues. Ongoing support incorporating observations, the provision of critical feedback and opportunities for reflections (Stoll et al., 2012) are likely to facilitate teachers further in introducing new ideas and practices in their everyday teaching. More recently, CPD programmes have taken up this idea to use an 'educating the educator' or 'train the trainer'

model for capacity-building through networks and scaling up by educating CPD leaders to create and support teacher communities (see Mascil, 2013; PRIMAS, 2012) within and across schools.

8.7 Concluding comments

This research problematises and challenges the notions of IBL and CPD designs intended to support teachers' enactment of IBL. The present findings suggest that presenting IBL through the use of challenging tasks and the application of a 'task-first' lesson structure is likely to offer what might prove to be an unachievable challenge for teachers – particularly when teachers are 'new' to IBL and have a disposition to transmission teaching approaches. Hence, teachers' tendency of relying on more scaffolded approaches to IBL appears to be inevitable: Teachers are likely to see it as more achievable, practical and more effective towards both student learning and their own personal development in learning to teach through inquiry. For teachers to realise the pedagogical benefits of inquiry they need to be supported by a CPD design that provides learning spaces with 'just-in-time' negotiation and feedback. CPD designs need to be flexible, be responsive to teachers' needs and offer a collaborative learning environment during teachers' professional learning journeys. Yet, this 'new' form of learning to teach may contrast sharply with the school learning context that teachers experience on an everyday basis.

In professional development, the teacher, the school organisation and the education system are closely connected and impinge on the teacher's personal, social and professional growth. As Krainer (2001, p. 290) puts it:

> teachers, who work in an educational system with narrow regulations on curriculum and assessment, who have no influence on those regulations in the past, and who will not be having any in the future, who were educated at universities where lecturing was the dominant teaching method…, currently teaching at a school with a low level of communication among the teachers, and who are now confronted with in-service courses oriented only towards their weaknesses, need a very strong motivation not to regard students as "received knowers," in the way they themselves have been socialised.

The recurrent theme in this concluding chapter was enabling teachers to enable change. LTMI was an intervention aimed to support teachers within their school context. However, schools offered unfavourable conditions for teacher learning of IBL. The factors reinforcing the status quo were systemic and held together by isolated classroom practices that support and reinforce individualistic enactment zones. I would therefore argue that the challenge that lies ahead is to design and implement CPD interventions that take into consideration what Fullan (2007) calls a systemic approach. In other words, these are interventions that blend top-down reforms with support for bottom-up initiatives. To implement reform, there is the need to design CPD that stimulates one's thinking about practice and enables collaborative growth, not only for teachers, but at all levels of the education system – including students, school administrators and leaders, education officials, teacher educators, CPD providers and policymakers alike.

Professional learning needs to be conceptualised as authority and responsibility that is bestowed upon teachers to own it and lead it with others – particularly when CPD blends off-the-job and on-the-job learning spaces. In the present study, talking to colleagues across different schools, exchanging views and sharing classroom related dilemmas were valued opportunities that were made possible through teachers' participation in a CoP. The CoP offered networking opportunities for teachers to participate actively, take ownership and feel safe to attempt changes (Bryk & Schneider, 2002). This is the transformative purpose (see Kennedy, 2014) that schools should seek when promoting CPD. CPD with a transformative purpose sees teachers as inquirers creating opportunities for themselves to address their own professional development needs. Just as students enhance their learning of mathematics within an inquiry environment, this shifting of responsibility onto teachers is likely to promote agency and autonomy in teachers to take control of and develop their own professional practice.

Cultivating a professional learning culture is complex and requires a systemic undertaking (Fullan, 2007). CPD providers need to empathise with

the learning environment of teachers to be able to collaborate closely with teachers, schools and the wider education system. Negotiations and deliberations need to take place *in situ*, carried out over time and within supportive, unhurried and non-threatening environments for teachers. An environment is likely to be more productive when it recognises that learning has a strong and ingrained social component. Moreover, it needs to value learning as a journey that encompasses a myriad of complex and uneasy struggles that are best addressed in a creative manner that aims to empower and transform teachers. Prolonged engagement in such a community enables teachers to look at challenges as opportunities, turn constraints into possibilities and dilemmas into reflective openings for learning rather than as stumbling blocks within the change process.

Appendices

APPENDIX 1 – TEACHER INFORMATION LETTER

10th January 2015

Learning to Teach Mathematics through Inquiry

Dear Teacher,

My name is James Calleja and I am a head of mathematics department. Through this short note, I would kindly like to present to you a professional development project experience for secondary school teachers interested in *inquiry-based learning* (IBL).

Your participation in the project would involve attending four INSET workshops and a number of follow-up meetings to be held during the next scholastic year. Through the summer workshops we will get to experience and discuss four main aspects of IBL, namely: *mathematical tasks for inquiry*, *purposeful questioning*, *collaborative learning*, and *student agency and responsibility*. An important aim of this professional development program is to create and foster a teacher learning community. The follow-up sessions are intended to support teachers in sharing, discussing and reflecting on their classroom experience. Hence, the meetings serve to support teachers in *learning to teach mathematics through inquiry*.

If you or a group of teachers at your school are interested to know more about this project or if you would like to gain more information about it, please do not hesitate to contact me. I would be happy to visit your school to discuss and share with you further ideas about this project.

Thanks for your time and attention.

Best Wishes

James Calleja

Contact details

Email: ims.jamescalleja@gmail.com

Mobile: 79065354

APPENDIX 2 – END OF WORKSHOP EVALUATION SHEET

1. *Briefly describe your experience during today's session.*

2. *What did you feel un/comfortable doing during the session?*

   Comfortable:

   Uncomfortable:

3. *I used to think... but now I know…*

   I used to think

   Now I know

4. *What will you take with you and try to implement in your class?*

5. *Any other comments/suggestions that you would like to add.*

Thank you for your participation and reflections.

Your Beliefs and Practices

You are presented with three statements about mathematics, learning and teaching.

For each statement give a percentage (%), so that the three percentages in each section add up to a 100.

Mathematics is…

- a given body of knowledge and standard procedures. A set of universal truths and rules which need to be conveyed to students. %
- a creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods. %
- an interconnected body of ideas which the teacher and the student create together through discussion. %

Learning is…

- an individual activity based on watching, listening and imitating until fluency is attained. %
- an individual activity based on practical exploration and reflection. %
- an interpersonal activity in which learners are challenged and arrive at understanding through discussion. %

Teaching is…

- structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when learners fail to 'grasp' what is taught. %
- assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences. %
- a non-linear dialogue between teacher and learners in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on. %

Your Mathematics Lessons

How do you typically start teaching a new topic?

_____

_____

_____

How is your classroom arranged? _____

_____

What resources do you usually use? _____

_____

Now imagine one of your usual lessons…

In the table below, you are provided with a list of ***teacher actions***.

Tick ( ) **only** those actions that you usually do and then arrange them in order (1, 2, 3, …) to show the structure of your typical lesson.

You can add other actions to the list if you would like, and you may also provide additional comments if necessary.

Teacher Action	Tick	Order	Comments
Pose questions to the students			
Assign work to be done individually			
Devote time for student questioning			
Provide notes			
Give a challenging question			
Assign work to be done in groups			
Give an explanation			
Demonstrate examples			
Ask students to explain their work			
Give an easy question			

In the table below, you are now provided with a list of ***student actions***.

Tick ( ) **only** those actions that your students usually do and again arrange them in order (1, 2, 3, …).

You can add other actions to the list if you would like, and you may also provide additional comments if necessary.

Student Action	**Tick**	**Order**	**Comments**
Ask questions to the teacher			
Explain work to the whole class			
Solve a challenging problem			
Listen to teacher explanations			
Work with others			
Copy notes from the board			
Work on their own			
Pose questions to other students			
Solve an easy problem			

Mathematical Inquiry

How did you hear about inquiry-based learning?

_____

_____

_____

How would you define inquiry-based learning?

_____

_____

_____

What, in your opinion, are the major challenges that teachers face in trying to teach mathematics through inquiry?

_____

_____

_____

What skills do teachers need to have to use inquiry-based learning practices effectively in their class?

_____

_____

_____

Which of these teacher skills do you believe you have and which are those that you believe you need to learn more about and develop?

I am able to _____

_____

I need to learn _____

_____

How I see myself as a Teacher…

Read the following descriptions of how four teachers (Mark, Diane, Joanne and Jonathan) see themselves.

<u>Underline</u> the things that you see yourself doing in class and those you believe in. You can also add comments and/or notes next to particular phrases.

Mark

I think I tend to be a bit traditional. I prefer to do a lot of teaching from the front. Students seem to like to be organised and prefer to be told what to do.

It seems that if I do not do the talking, then there is not much teaching going on.

Mark sees mathematics as a well-defined body of knowledge and procedures that needs to be covered. His teaching involves structuring the work into a logical linear sequence. He prefers to start by explaining and demonstrating methods and procedures. Students learn by listening to his detailed explanations, copying down notes and then working exercises individually from the textbook using the methods demonstrated in his worked examples. Mark is very well organised and knows what students should learn in each lesson.

Mark believes that the most effective way to 'cover' the syllabus is to 'deliver' the content through clear explanations and drill and practice exercises. He feels that it is important to do as many problems and questions as possible and carefully plans lessons so that nothing is left out. Mark starts each topic assuming students know nothing, re-teaching each topic from the beginning.

Mark's class is organised in rows of tables with students sitting individually facing the front. He spends most of the time giving explanations and dictating notes that then serve as guides for students as they work on textbook exercises. Mark does encourage students to copy his examples from the board and to do as many examples as possible. The exercises he assigns start with a few easy questions and then gradually moves to harder examples. His explanations are clearly delivered and include questions directed at individual students. He does not waste time dealing with errors and alternative methods. Mark feels under pressure from the syllabus so he focuses on correct methods and answers. Due to time constraints, his lessons flow at a brisk pace and not much time is wasted on ideas that are 'out' of the syllabus.

During Mark's explanations, students are generally quiet and attentive, as they know that they will be asked questions which they need to be able to answer. Mark believes that most of his students do not have enough maturity to work without his help. He believes they are not responsible enough and need his guidance and the structure he provides. Mark has a formal, yet cheerful and positive relationship with students.

Diane

I think it is important to get to know your students and try to establish a relationship with them first. I am not too hard on them.

I try to understand their situation and help them to overcome their challenges.

Diane sees mathematics as an interconnected network of ideas which she and her students must create and work on together. She claims that there are many links between topics and there is no 'best' way to organise and teach topics. Diane sees teaching as a non-linear dialogue in which meanings and connections are explored and refined through discussion. This means that, more often than not, she follows up ideas arising from the class. Diane says that she cannot always predict in which direction the lesson will take beforehand. She sees learning as a collaborative activity based on exploration and discussion.

Diane is quite relaxed about syllabus coverage, claiming that it is more important for students to develop conceptual understanding rather than computational skills. She is willing to spend time discussing mistakes and errors when they arise, even if it takes considerable time. Diane prefers a classroom with desks arranged in a U shape. She considers this flexible arrangement allowing whole-class discussion and also small-group work. She spends about a third of the time explaining and leading discussion, with the rest of the time devoted to students working on tasks in pairs or in small groups. During this time, Diane goes around prompting and guiding students.

Diane rarely uses the textbook, but when she does she usually suggests an exercise and then allows students to select other questions they will work on. Diane occasionally also offers open questions and investigations in which students can choose the direction and the methods to use. Her class is rarely quiet and students are quite free to work at their own pace. She has a friendly and informal relationship with students. Diane appears relaxed and unhurried and is positive and encouraging in her approach.

Joanne

Students have the idea that mathematics is about working lots of problems from the textbook and trying to get as many of them correct as possible. Students do not like whole-class discussions and prefer to work individually for most of the time.

Joanne, like Mark, sees mathematics as a given body of knowledge and procedures which must be 'covered'. She sees teaching as structuring a series of exercises and problems that students work individually while she goes around giving support. She gives less emphasis to whole-class demonstrations and explanations, as students in her class do not seem to concentrate for long periods. She thus keeps introductions short, sets students working and then deals with student difficulties individually while she goes around the class. Like Mark, she sees learning as an activity based on listening and practicing.

In Joanne's class, desks are arranged in pairs, but students work individually and only occasionally in pairs. Lessons begin with Joanne giving short introductions and maybe one or two examples. She only asks a few questions and then provides extended periods where students work on exercises from the textbook or a worksheet. She stops the class from time to time to clarify questions that she identifies as particularly difficult.

When students are working, Joanne prefers to tell students where they have gone wrong and shows them where and how to correct mistakes. If a student cannot do a question, she shows them how to do it. She does not spend much time with individuals because she believes that each individual student needs her attention.

Students do not necessarily have to complete the whole exercise that she assigns. If questions are easy for some students, they can skip to the more challenging ones. Joanne feels that the students in her class need to be motivated and prompted all the time. At times students' noise is so high that she feels like losing control and finds it hard to gain their attention when needed. Joanne is kind, caring and positive. She admits that at times she needs to be harder on students. However, more often than not, she fails to get students' attention and feels that during these instances she tends to lose class control.

Jonathan

In my class, students work hard, collaborate and are always ready to participate and do the work I assign. However, I feel much pressure to 'cover' the syllabus and I want to make sure that students do well in their exams.

Jonathan's practices are not consistent with his beliefs about learning. While he claims to share Diane's beliefs, he feels constrained and frustrated by the scheme of work. He believes that students learn best when their own ideas are discussed and investigated, but there is little evidence of this in his classroom.

Students sit in rows and for most of the time they work alone. His lessons are dominated by worksheets which he prepares for his students. Jonathan prefers to use his worksheets rather than the textbook. He claims that he is the expert and knows exactly what his students need.

Jonathan starts lessons alternating explanations with whole-class discussions in which he tries to involve the students. This usually takes a quarter of the lesson which then proceeds with individual work on the worksheets he provides on a regular basis. Students are usually quiet, passive and undemanding. When problems crop up, he stops the class to focus on specific questions which he presents on the board. Jonathan makes sure to involve students in thinking about the question and to engage them in the discussion.

Jonathan is unhappy with his approach as this in inconsistent with his personal philosophy of teaching and learning. Indeed, he claims that overloaded syllabi, high-stakes examinations and pressure from parents are constraints that he finds hard to overcome.

Which of these teachers represents you best?

_____

Thanks a lot for your time!

James Calleja

This is the same as the pre-CPD Questionnaire except for the "Mathematical Inquiry" sections which had the following questions.

Mathematical Inquiry

How would you define inquiry-based learning?

_____

_____

_____

What were the major challenges you faced in trying to teach mathematics through inquiry?

_____

_____

_____

What skills do teachers need to have to use inquiry-based learning practices effectively in their class?

_____

_____

_____

Which skills do you believe you have and which are those that you believe you still need to learn more about and work on?

I am able to _____

_____

I need to learn _____

_____

Part 1: Background information

Name and surname	
Age	
Qualifications	
Teaching experience	
School	
Year Groups Teaching this Year	

Part 2: Understanding inquiry

1. The aim of this research project is to support teachers' use of inquiry practices through ongoing professional development. You seemed quite interested to take part in this project. What motivates you to participate?

2. For my lesson observations, I had asked you to plan a 'typical' and an 'inquiry' lesson. Can you talk to me through how you went about planning these lessons?

3. Which aspects of the inquiry-based lesson were you most confident about and least confident about? Why?

4. What were the main differences between planning for these two lessons?

5. What were the challenges in planning and doing the inquiry-based lesson?

6. When you focus on your professional context, what obstacles do you foresee to implanting IBL in your classroom?

Part 3: Thinking about mathematical tasks

7. Here are three mathematics tasks which I would like you to take some time to have a look at.

a) Describe any differences that you see between the three tasks.

b) Place the tasks in order, according to how confident you feel about using them with a high attaining class:

TASK 1

Jane wants to make wristbands for charity.

The cost of a kit for making the bands is €50.

With this kit she can make 60 wristbands.



TASK 2

Jane wants to make wristbands for charity.

The cost of a kit for making the bands is €50.

From this kit she can make many wristbands.

She will sell the bands for €4 each.



TASK 3

Jane wants to make wristbands for charity.

She can buy a kit for making the bands.

She wants to know how much profit she will make.



I feel *most confident* using task …

I feel *less confident* using task …

Explain why you say this.

c) Place the tasks in order, according to how confident you feel about using them with a low attaining class:

I feel *most confident* using task …

I feel *less confident* using task …

Explain why you say this.

Part 4: A focus on beliefs

8. I would like you to read the following pairs of statements. For each pair, decide which statement you agree more with. Provide a comment upon your decision.

In a lesson it is better to provide students with just one or two challenging tasks.	In a lesson it is better for students to solve many graded tasks.

Students learn best when they choose what problems to do and how to go about them.	Students learn best when the teacher selects the problems they attempt and the methods to use.

Collaborative group work is ineffective because students do not stay focused.	Collaborative group work is effective because students learn from each other.

When students are struggling I prefer to help by asking a question.	When students are struggling I prefer to help by providing a specific suggestion.

Thank you for your time. Is there anything else you would like to add?

1. How would you describe your experience participating with other teachers in the PD sessions?

2. Which PD activities did you find most valuable in supporting your inquiry-based practices? Elaborate on these.

3. Following this experience, do you see IBL featuring in your scheme of work? Why? How do you intend to plan and incorporate it?

4. How often do you see yourself using IBL with your class? What type of IBL lessons do you intend to try out? Why?

5. Which aspects related to your place of work may potentially help or restrict you in your choice for more IBL?

6. How do you intend to overcome these constraints and restrictions?

7. The PD sessions were based on four research-based principles:
   (1) mathematical tasks; (2) collaborative learning; (3) purposeful questioning; and (4) student agency and responsibility.
8. Which of these principles were you familiar with and already implemented in your class? With which classes and to what degree?
9. Which of the principles do you most highly value? Why?
10. Which of the principles do you least highly value? Why?
11. Which of these principles do you think you need to work on for your own professional development? Why?
12. Which of these principles do you think would be the least challenging for you to implement in class? Why?
13. Which of these principles do you think would be the most challenging for you to implement in class? Why?

14. From the experience and knowledge gained through the professional development sessions, how do you see ***yourself*** and ***your class*** changing? In other words, I used to… now I will…

15. In order to develop your inquiry practices, do you intend to rely on learning exclusively from within your PD participation or do you intend to seek other sources or learning opportunities? If yes, what sources do you think might be useful and why?

16. Thank you for your time. Is there anything else that you would like to add?

In Part 1, you will be provided with the same questions and offered the responses you provided in your second interview. You will be asked to discuss whether you would make any changes to those responses provided prior to your experience of inquiry-based learning practices in your class.

You are encouraged to reflect on your prior responses to see whether you would provide the same or different views now.

In Part 2, you are provided with additional questions directed to reflect on the whole CPD experience.

Part 2: Reflecting on your experiences

1. Describe what a typical day looks like in your classroom. For example, how do you begin and end the class? What would one see you and your students doing on most days?
2. In what ways is this different from the lessons you used to plan and teach before joining the project?
3. What are some of your successful strategies that you are using now?
4. What do you feel you have gained professionally from your participation in this project?
5. Looking back on your experience, did this have any impact on you at a personal level? If yes, how?
6. From your experience so far, how do students benefit from their engagement with inquiry-based learning? How is this evident in your class?
7. Rate your ability in effectively implementing the PD principles.

The PD Principles	Very Effective	Effective	Average	Ineffective	Very Ineffective
Choose a task for inquiry					
Use questions to stimulate thinking					
Manage small-group activity					
Shift responsibility on students					

8. Thank you for your time. Is there anything else you would like to add?

Date: _____ **Class**: _____ **Time**: _____ min

Topic: _____

Approximate time spent

Teacher explanation to the whole class _____ min

Students working individually _____ min

Students working in small-groups _____ min

Whole-class presentation of work _____ min

Other (please specify) _____ _____ min

Reflections on the lesson

Briefly describe the lesson plan.

_____

_____

_____

_____

What motivated you to do this lesson and what were your lesson objectives?

_____

_____

_____

_____

_____

Describe challenging aspects (e.g.: any constraints, doubts, worries) that you encountered in planning for this lesson.

_____

_____

_____

_____

Your ability in implementing the PD principles

How effective were you in …

The PD Principles	Very Effective	Effective	Average	Ineffective	Very Ineffective
Choosing a task for inquiry					
Using questions to stimulate thinking					
Managing small-group activity					
Shifting responsibility on students					

Describe any challenges that you encountered during the lesson.

_____

_____

_____

_____

Reflections on your own learning

From this lesson I learned that …	I feel I need to improve on …
_____	_____
_____	_____
_____	_____
_____	_____

Your experience participating in CPD

- Why did you attend this CPD programme? What were your expectations?
- Did the activities and tasks you engaged in meet your expectations? How?
- Describe your experience participating in CPD and attending the follow-up meetings. Did this have any impact on you as a teacher? In what ways?
- What did you get/learn from this experience of working with other teachers?
- In what ways did this CPD support your use of IBL in the classroom?
- What activities did you learn most from? What would you change (include and/or eliminate)?

Greta – Year 8 – St George School – 25/04/2016 – Inquiry Lesson 4

The class is set in with desks in pairs.

The teacher started the lesson by explaining what the lesson is about. She then displayed the set of six probability statements on the IWB.

… pause of 3 or more seconds

		Questions
	Lesson Presentation Phase	
Teacher:	These past few weeks we were doing graphs. Can you please tell me what you remember from what we did in this topic?	Factual
Student:	About the axis	
Teacher:	Yes, we have the axis that we had also done last year as well	
Mark:	We learnt about the gradient and the equation	
Teacher:	So, we know the gradient which measures what?	Factual
Mark:	The steepness of the graph	
Teacher:	The steepness of a graph. And what else?	Guiding
Student:	The y-intercept	
Teacher:	The y-intercept, what is it?	Factual
Student:	Where the line of the graph meets the vertical line.	
Teacher:	Ok good.	
Alan:	We learnt how to plot the y-axis and the x-axis	
Teacher:	You had also done that the year before and this year we did	
Alan:	Straight line graphs	
Teacher:	Yes, straight line graphs… Matthew	
Matthew:	Conversion on graphs	
Teacher:	Yes, we did conversion graphs, graphs to convert one value to another. For example, from miles to kilometres true	
John:	We did graphs going different directions	
Teacher:	Very good. And how do you know the graph is going uphill?	Factual
John:	From the gradient	
Teacher:	From the gradient, it would be what?	Factual
John:	Plus	
Teacher:	Yes, it would be positive. A graph going uphill has a positive gradient and one going downhill has a negative gradient	
John:	It can also have a gradient of zero if it is like this (indicating horizontal)	

Teacher: Yes, true we said that a horizontal line has a gradient zero and one vertical has a gradient of infinity which we cannot find. Now today we are going to

2 minutes

Mark: Miss, we also used an equation to get the coordinates

Teacher: Yes, it's true. Last year you worked on coordinates and this year we included the equation, and from it you took out the coordinates that you were going to plot. Very good.

Task Presentation Phase

Teacher: Now today, to close up this topic, we are going to do an activity in groups. I am going to give you a pack of 32 cards which you should match in fours. You are going to have a poster like this one. I want you to hold it in a portrait position not landscape. Each four cards that you match need to be fixed on the poster like this (indicating rows one under another).

Paul: So, we will have eight rows

Teacher: Yes, eight rows. But please listen here as I am not finished yet. Your cards are going to be like this (teacher displays one of the cards $y = x + 1$ on the IWB). Some of them are equations, so we can have equations and then plot straight-line graphs from it. Here we have the ready-made graph (Teacher now displays the graph of y=x+1). So, you need to match the graph with the right equation. We also have, some of the cards, have a table of values on them (teacher displays a table, but not the one for y=x+1). Can you, for example, say something about this table? Guiding

Nathan: There are some missing values in the table

Teacher: Yes, true but, for example, can you tell me what the equation is? Guiding

Miguel: y = 2x

Teacher: We are not going to say why, ok. Maybe it's right, maybe it's wrong. So, if you can do what Miguel said, and come up with an equation, you can then match the table to an equation. And then we have some ready-made coordinates (teacher displays one coordinate (40, 120) on the board), and again you can say the relationship between x and y.

So, you have all these different cards. For each one you are going to match an equation, a graph, a coordinate and a table of values. Then, those which you match, only when you are really sure, stick them to the poster. Is that clear? Confirming

Students: Yes (chorus)

Teacher: Now each group takes a poster and I will also give you your set of cards.

5 minutes

Small-group work Phase

Students work in groups of three and four. While visiting the groups, the teacher observes and seems to intervene whenever she spots a wrong matching. It is there when she challenges students to explain their reasoning.

Group 1 **Teacher is listening to the group**

Student 1: (Selects the equation y = *x* and puts it to one side)

I will look for the table now

Student 2: Here is the graph

Student 1: (Puts the graph card next to the equation y=x and keeps on looking for the table of values)

Teacher: Are these ready? — Confirming

Student 1: Yes miss

Teacher: Can you explain why you matched these cards? (In the set of cards there is also (42, 42)) — Probing

Student 2: (Pointing to the coordinates card) the y and *x* are the same, the equation tells that y is equal to *x* and the coordinates on the graph are the same

Teacher: Ok then are you sure about this? — Confirming

Student 2: Yes

Teacher: If you are sure you can stick them on your poster

Type of question	Presentation	Group-work	Plenary	Total
'Factual/Answer'	5	1	0	6
'Guiding/Procedure'	3	0	0	3
'Probing/Reasoning'	0	1	0	1
'Confirming/Checking'	1	4	0	5
Total	9	6	0*	15

* *Lesson had no plenary part*

Question:

Which PD activities did you find most valuable in supporting your inquiry-based practices? Elaborate on these.

Initially he spoke about the mathematical task presented and discussed during LTMI and how these offered opportunities for students to think, discuss and pose their own mathematical questions. Our conversation then moved onto what had changed over the year in terms of the practices he adopted.

Time	Discussion	Code 1	Code 2
0:03:50	Even in my more traditional lessons, adopting an inquiry approach helps, and I am finding myself doing this regularly. **I am noticing a change in the way I ask questions, when I ask a question, whether I expect an answer or not, and in the amount of time I allow for students to think. It seems that I am finding myself doing things and acting in ways that are not typical of the ways I used to act in class before.** I do not plan questions beforehand, these have become natural to me and the way I relate to students is different from the past. (Quote in bold used in section 6.2.2)	Classroom practices Questioning	Teacher change Classroom actions
Question: *What are some of your successful strategies that you are using now?*			
1:04:55	I think a strategy overlooking IBL is the very good relationship I had with my students. **I feel it is fundamental. Students should not be afraid of the teacher or of each other. You need to show them that you respect their ideas and educate them to respect the ideas of their peers. If this is missing, then students would probably be less inclined towards inquiry.** (Quote in bold used in section 6.2.1)	Relationship with students	Classroom culture
1:05:30	Without identifying who is right and who is wrong, **I encourage and expect students to think. If a student is thinking out of the box, then I bring this to the attention of others. I ask this student to share her reasoning and explain her thinking. I am now educating students to think and communicate it.** If they come up with an idea that I am happy with, I tell them... not necessarily a good idea but an idea that shows that students are thinking. So, indirectly I am educating my students to think. (Quote in bold used in section 6.1.4)	Encouraging thinking	Teacher's role

Teachers were discussing their learning experiences participating in LTMI				
Time	**Teacher**	**Discussion**	**Code 1**	**Code 2**
05:00	Chris	**I feel this was a course that didn't just speak about theory, although we did tackle that. This could have been a course delivered by experts and we just stood passively listening to them teaching us about inquiry. Instead, we got our hands dirty, we found challenges and difficulties, we understood that the challenges we encountered were common to all and the most valuable aspect was that we shared and learned from that**. (Quote used in section 6.2.3)	Practice-based learning Learning with others	Teachers as active learners Community collaborative practices
06:10	Sarah	**Had it not been for these meetings I would surely have given up hope. The meetings helped me improve on the strategies I was using, getting tips from what was discussed. Meeting others was for me crucial in persisting with IBL.** (Quote used in section 6.2.3)	Learning with others	Community collaborative practices
06:30	Jackie	I had a vague idea about IBL from my teacher education course at university and I only knew the method I was trying to implement, the one method that I thought made sense. When I came here I learned about other ideas from my colleagues, I tried them out and saw the results, then came back and picked other ideas always improving on my previous knowledge of inquiry.	Isolated practices Learning with others	School context Teacher learning

Skills teachers need for inquiry teaching		
Teacher	**Pre-CPD**	**Post-CPD**
Chris	• *Develop open-ended tasks* • *Time management* • *Delegate responsibility to students*	• *Teacher flexibility* • *Time management* • *Give responsibility to students*
Greta	• *Ability to organise groups well* • *Select appropriate tasks* • *Not interfere too much*	• *Class management/organisation* • *Select tasks addressing students' abilities*
Sarah	• *Managing group work* • *Refrain from helping or giving hints* • *A lot of patience and ability to urge students to keep trying*	• *Managing group work* • *Preparing tasks that are not too challenging* • *Time management* • *Thinking 'on the spot'*

Skill I have for inquiry teaching		
Teacher	**Pre-CPD**	**Post-CPD**
Chris	• *Engage students through questioning*	• *Questioning to stimulate thinking* • *Create tasks that students are intrigued to engage in*
Greta	• *Choose appropriate tasks*	• *Choose appropriate tasks*
Sarah	• *Managing group work* • *Being patient*	• *Managing group work* • *Lesson time management* • *Choose adequate tasks for students*

Skill I need to learn for inquiry teaching		
Teacher	**Pre-CPD**	**Post-CPD**
Chris	• *Integrating IBL in the scheme of work while adhering to timeframes for teaching content*	• *Using lesson time more effectively* • *Integrate IBL in the scheme of work*
Greta	• *Managing small-group activity* • *Avoid providing too much help*	• *Listening more to students' reasoning while avoiding unnecessary intervention*
Sarah	• *Re-directing student questions for students to answer their own queries*	• *Questioning techniques* • *Thinking 'on the spot' to redirect student questions onto others*

Teacher		Reported time dedicated to each lesson phase (minutes)				
		Teacher explanation	*Individual work*	*Small-group work*	*Whole-class presentation*	*Practice*
Chris	Lesson 1	5	5	10	10	5
	Lesson 2	5	0	20	10	0
	Lesson 3	5	5	20	5	0
	Lesson 4	5	0	25	10	0
	Lesson 5	5	10	0	0	20
	Lesson 6	5	0	30	5	0
	Lesson 7	5	5	15	15	0
Greta	Lesson 1	5	0	10	5	0
	Lesson 2	2	0	15	5	18
	Lesson 3	5	5	10	10	0
	Lesson 4	2	0	15	3	0
	Lesson 5	4	0	20	5	0
	Lesson 6	4	0	15	5	0
	Lesson 7	5	5	20	10	0
	Lesson 8	2	3	20	15	0
	Lesson 9	2	5	15	20	0
	Lesson 10	5	0	20	5	0
	Lesson 11	5	0	35	0	0
Sarah	Lesson 1	5	0	10	20	0
	Lesson 2	5	0	10	15	0
	Lesson 3	5	0	10	20	0
	Lesson 4	5	0	15	10	0
	Lesson 5	5	0	10	15	0
	Lesson 6	5	0	10	15	0
	Lesson 7	10	0	15	10	0
	Lesson 8	5	0	10	20	0
	Lesson 9	10	0	10	15	0

Chris Lesson 1 (03/12/2015)

Chris presented a sequence of three short tasks all related to the number 24. The first required students to work on their own and write all they know about the number 24. During this two-minute activity, he went around the class checking that everyone was focused and working. Then, Chris allowed some time for students to share their thoughts with the rest of the group. Following this, he moved on to present a second task. Chris asked students to pretend they are mathematics teachers and create questions whose answer would be 24. Students were asked to work in pairs and then later as a group. Following this 15-minute activity, Chris presented the third task with the following problem.

> *There was this guy named David. David was mad about motorbikes and limousines. Once, during a boring Maths lesson, he looked out of the window and was amazed to see a number of motorbikes and limousines. Now he knows that a motorbike has 2 wheels while a limousine has 6. If in total he counts 24 wheels, how many motorbikes and limousines did he see?*

Chris Lesson 2 (25/01/2016)

Chris started the lesson with a whole-class discussion focused on properties of flat shapes. Following this short 2-minute discussion, he presented students with a set of polygons, each labelled with a letter, and a 2 by 3 grid with properties labelled on each side. Students were required to classify each polygon accordingly. Following this activity, Chris asked groups to exchange their grids to check the work of another group. He later led a whole-class discussion involving different groups in explaining their disagreements regarding the work of others. Next, Chris presented a second task. He gave each group an envelope containing different properties of polygons. From these properties, he asked each group to select five at random and to place them on a second 2 by 3 empty grid that he provided. For the plenary part, Chris asked students to share one doubt that they had when classifying their polygons.

Chris Lesson 3 (07/04/2016)

The lesson was about multiple representations. Chris structured the lesson into smaller tasks. He started with a whole-class discussion posing the following situation.

> *In our lives, there are many things that seem different, look different but mean exactly the same. Can you give me some examples from everyday life?*

After a very short class discussion, Chris presented each group with a description card (for example: *We are one more than the multiples of 3*.). He urged students to come up with different ways and representations of their description card. Later, he provided each group with an envelope containing other description cards and picture cards. He asked students to match their own description card with a picture card. Finally, Chris gave students more cards – number, formula and function cards – asking students to match these to their description and the picture cards. Following this 25-minute group activity, he asked students to present their work during the plenary part which was 3 minutes long.

Chris Lesson 4 (05/05/2016)

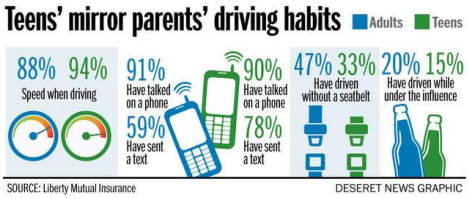
The lesson started with a whole-class discussion on the concept of perimeter. Following this 7-minute discussion, Chris introduced the main task through the following question:

> *How many different triangles are there whose perimeter is 30 cm?*

To reduce the complexity of this task, Chris limited the possibilities for students to consider and work with whole numbers only. Students were engaged for 16 minutes working in small-groups and then a further 16 minutes to present their work in the plenary session.

Greta Lesson 1 (19/11/2015)

Students, working in groups, were presented with five different situations related to percentages in everyday life. Each group had a particular situation and their task was to come up with a question related to the particular situation presented through pictures (see one example below).



The teacher challenged students to act like teachers in coming up with an original question related to the work they had covered, but also to be creative in finding questions that may be different to the ones they had done in class. Initially, students work on the task individually and later in groups. A whole-class discussion followed at the end of the lesson.

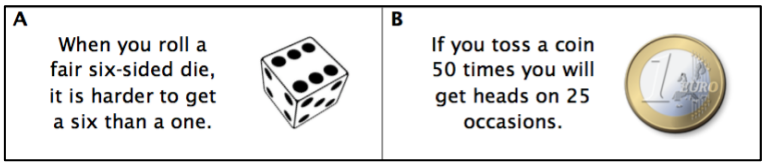
Greta Lesson 2 (14/01/2016)

This lesson involved students in discovering the relationship between the diameter and the circumference of circles. For this lesson, each student had to bring a round object and a piece of string. Each group, made up of four students, had four different objects. The teacher demonstrates what students had to do, using a round sharpener as an example. She also provided a handout that included a table in it and encouraged students to use their calculator to discover the relationship.

The lesson involved students working in a group, then writing their work on their notebook with a short whole-class presentation at the end.

Greta Lesson 3 (07/04/2016)

The lesson focused on the topic probability. The teacher presented students with six probability statements. Students had to classify these as true, false or sometimes true. Students were set in groups of three to discuss each of these statements and eventually provide reasons and explanations for their answers. The lesson ended with a whole-class presentation and discussion.

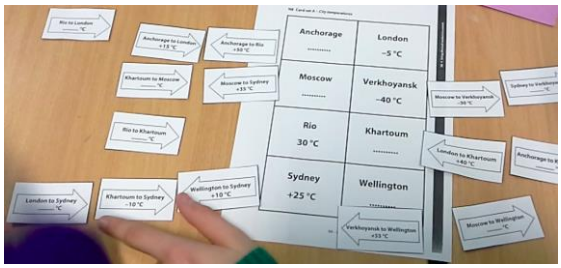


Greta Lesson 4 (25/04/2016)

This lesson was about multiple representations. Students, working in groups of four, were provided with a set of 32 cards. The cards included equations of straight lines, tables of values, straight line graphs and coordinates of points. Students had to match these and present their work on a poster. Following an introduction to the task, students spent the rest of the time working in groups matching cards and working on the poster presentation.

Sarah Lesson 1 (19/11/2015)

Students, working in groups, were presented with a sheet with different cities; some cities had given temperatures while others were unknown. Students were also provided with an envelope containing clues to help them find the unknown temperatures. Students had 10 minutes to finish the task following which a whole-class discussion was held.

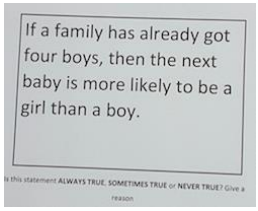


Sarah Lesson 2 (17/03/2016)

This lesson was about multiple representations. Students, working in groups of four, were provided with a set of 24 cards. The cards included equations of straight lines, tables of values and straight line graphs. Students had to match these and present their work on a chart. Following an introduction to the task, students spent 14 minutes working in groups and then 8 minutes for the whole-class presentation.

Sarah Lesson 3 (28/04/2016)

The lesson focused on the topic probability. The teacher presented students with six probability statements. Students had to classify these as true, false or sometimes true. Students were set in pairs to discuss one statement providing a reason and an explanation for their answer. Following the 7-minute small-group activity, the lesson ended with a whole-class presentation and discussion.



Sarah Lesson 4 (25/05/2016)

Teacher presents the following two situations related to percentages and asks students, working in pairs, to discuss whether these are always true, sometimes true or never true.

> *Situation 1: John gets a pay rise of 15%. Ann gets a pay rise of 10%. So, John gets a bigger pay rise. You need to discuss, as usual, whether this statement is always true, sometimes true or never true. Remember that, as usual, you need to provide reasons for your answers.*
>
> *Situation 2: In a sale, the price of every item was reduced by 25%. After the sale, all the prices were increased by 25%. So, the prices went back to where they started.*

Students have 12 minutes to discuss these and then 15 minutes dedicated to a whole-class discussion.

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