

*Vibration Analysis and Investigation of
Synchronous Reluctance Motor*



UNITED KINGDOM • CHINA • MALAYSIA

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ABSTRACT

In the last decade, the interest in synchronous reluctance motors has increased considerably due to several key advantages in comparison to other types of motors. It offers good torque density, wide speed range, and can deliver higher efficiency when compared to induction motor for the same frame size. Even though the torque density of synchronous reluctance is lower than the permanent magnet synchronous motors, it is compensated by a lower cost of the rotor that is free from active excitation. As for synchronous reluctance motors, their electromagnetic torque is entirely developed by the reluctance torque. Hence, the rotors of these machines are usually designed as a relatively complex structure, made by flux barriers and iron paths forming an anisotropic structure to produce high reluctance torque. This leads to a new challenge with respect to the mechanical performance of the synchronous reluctance motor, as the retaining iron ribs, if not designed well can lead to a fragile rotor and excessive vibrations.

In order to investigate the relations between the geometrical parameters and the mechanical performance of the synchronous reluctance motors, this thesis takes a 5 kW synchronous reluctance motor as a case study to develop the research, and then propose a 15 kW synchronous reluctance prototype to validate the analyzing methods. The main contributions of the thesis are shown below.

In Chapter 2, an analytical model based on the magnetic equivalent circuit is proposed. This model studies the magnetic characteristics of the synchronous reluctance motor, followed by the analytical derivation of the air-gap flux density, electromagnetic torque and force. A sensitivity analysis is conducted to investigate

the impact of the key geometrical parameters on the electrical performance, from where the air-gap flux density, electromagnetic torque and force are highlighted.

In Chapter 3, the mechanical characteristics of the synchronous reluctance motor are analysed by performing the modal analysis, stress analysis, and vibration analysis, from where the modal shape, natural frequency, stress tensor tensile, and vibration level are presented, respectively. There are several sensitivity studies following each of the analysis, that is the impacts of the motor geometry on the mechanical characteristics are explored and analysed.

In Chapter 4, the impact of permanent magnets inserted into the rotor structure are investigated. The equivalent magnetic circuit is developed by adding the influence of permanent magnet, therefore the magnetic characteristics of the permanent magnet assisted synchronous reluctance motor are calculated. Then, the mechanical characteristics of permanent magnet assisted synchronous reluctance motor are analysed through the modal analysis, stress analysis, and vibration analysis, followed by their corresponding sensitivity analysis, in order to deeply dig out the influence of permanent magnet. Furthermore, the vibration level of permanent magnet assisted synchronous reluctance motor is compared to that of synchronous reluctance motor by considering the electromagnetic force harmonics.

In Chapter 5, the synchronous reluctance motor is studied in case of eccentric rotor position. The air-gap length of synchronous reluctance motor is usually very small for getting a better electrical performance. Hence, a tiny eccentricity, within the permissible manufacturing tolerances, will have a significant effect on the motor performance. The analytical model is developed by applying the eccentric condition,

from where the air-gap distribution is uneven. Then, the electrical performance and mechanical characteristics are analysed in case of eccentricity.

In Chapter 6, a 15 kW synchronous reluctance motor is analysed by using the same analytical model and Finite-Element Analysis (FEA) methods, then it is prototyped to experimentally validate the analytical modelling and FEA research method. This prototype is operated under various power mode, including light load (5 kW) condition, mild load (10 kW) condition, and nominal load (15 kW) condition. Then the electrical and mechanical characteristics of this prototype are tested and compared with the FEA results.

List of Publications

- J. Li et al., "Vibration Analysis of Permanent-Magnet-Assisted Synchronous Reluctance Machines," 2019 22nd International Conference on Electrical Machines and Systems (ICEMS), Harbin, 2019.
- J. Li et al., "Impact of Vibration of Eccentric Permanent Magnet Assisted Synchronous Reluctance Machine," 2020 XXVI International Conference on Electrical Machines (ICEM), Gothenburg, 2020.

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Chapter 1

Introduction

1.1 Background

Synchronous reluctance (SynRel) motors are employed in many applications due to its several merits over other types of the electrical motors. Firstly, the SynRel motors offer higher efficiency and/or torque density comparing with the induction motors (IMs) of the same frame size [1]. Even though the torque density of SynRel motor is lower than permanent magnet synchronous motors (PMSMs), it is compensated by the reduced cost of the rotor and useful in specific applications where the cost reduction is fundamental factor in the decision-making process, or when it is mandatory not to use magnets [2]. Rare-earth permanent magnets (PMs) price is unstable and fluctuating, and the global reserves are decreasing. Hence, it is critical to minimise the utilization of PM or even replace the rare-earth magnets, and consequently SynRel motor is becoming an alternative approach. Besides, the SynRel motors are robust, there are no problems of short-circuit current and without induced voltage at no-load condition [3], which are usually associated with PM motors. In addition, the SynRel motors have a robust structure and a wide constant power speed range (CPSR) [4].

However, these machines present some drawbacks, such as low power factor and high torque ripple, which limit the application of the SynRel motor to some areas. As for the power factor, it could be significantly improved by inserting ferrite PM

into the flux barrier, leading the SynRel motor to permanent magnet assisted synchronous reluctance (PMaSynRel) motor [5, 6]. Whilst the mitigation of the torque ripple is more challenging [7-13] and it is mainly addressed through the optimisation of the rotor structure.

The electromagnetic torque of the SynRel motors is mainly consisted of the reluctance torque [14] which is determined by the saliency ratio that depends on the inductances on d -axis and q -axis, respectively. As a result, the rotor structure of the SynRel motors always appear a relatively complex design for maximizing the saliency ratio [15-17] to achieve a high electromagnetic torque density. However, it also brings extra torque ripple and more electromagnetic force harmonics. All of the above characteristics will lead to significantly acoustic and vibrational issues in SynRel motors.

The noise and vibration of SynRel motors can be originating from three sources, which are the aerodynamic, the mechanical, and the electromagnetic source [18]. As the electromagnetic source is the main reason of vibration in small and medium SynRel motor, it is mostly studied by considering the radial and tangential electromagnetic force produced in the air gap. Some of these investigations focus on analytical method for calculating the radial and tangential forces of the PMSM by using Maxwell tensor [19-22]. Other studies use the concept of global force and local force via the numerical simulating, to explore their impacts on the vibration [23-25]. Also, there are some scholars focusing on the structural optimization for improving the electrical characteristics but resulting in increased vibration [26-30].

To analyse the vibration characteristics of the SynRel motors, firstly a precise model for finite element analysis (FEA) is essential to investigate its electrical

performance, especially the electromagnetic torque and the electromagnetic force. Secondly, a structural model should be built precisely to explore its mechanical characteristics, including natural frequency, stress distribution, modal shape, and vibration level.

Generally, the mechanical features of the PMaSynRel motor is more complicated because of the influence from the added PM, although it benefits the improvement of power factor of SynRel motor obviously. There is a number of studies on the impact of PM on electrical performance of the PMaSynRel motor [31-36]. However, those are rarely considering the mechanical characteristics, and the mechanical differences present between SynRel and PMaSynRel motors, which is also a vital aspect to consider during the preliminary design of the SynRel motor.

To achieve a high output, the air-gap of SynRel motor is usually designed with a quite small size. As a result, the eccentricity will have of much more remarkable effects on motor torque quality and vibration, even though such eccentricity lie within the permissible region of manufacturing tolerance. It is meaningful to understand the mechanism of how the eccentricity affects the magnetic reluctance torque and the vibration as well. Currently, most of the research is aimed at the electrical performance in case of the SynRel motor [37-39], rather than on its torque and vibration characteristics of these machines.

Looking at the mechanical features of the SynRel motors, specifically the vibration, the investigations in this thesis is performed to find the mechanisms, principles, and rules of vibration variation, that are caused by rotor geometry, PM, and eccentricity. To achieve this research, the electrical and mechanical features of a 5 kW SynRel motor is theoretic analyzed, and a precise model to determine the

vibration in it is proposed. Related experimental studies are carried out on the prototype to validate the hypothesis. All the obtained results could provide a basic reference for investigation and design of the SynRel motor in term of vibration.

1.2 Electrical Characteristics of SynRel Motor

1.2.1 Operating Principles

The SynRel motors operate based on the principle of reluctance. The main idea of reluctance concept is illustrated by Fig. 1.1 [40], from where an anisotropic rotor is placed in a magnetic field with a defined direction, as shown in Fig. 1.1 (a), and then the reluctance torque is produced if there is an angle between the d -axis and the field ($\theta_{rel} \neq 0$). This torque is due to the interaction of the magnetic reluctance in both d - and q -axes of the anisotropic rotor and the magnetic field. The magnetic reluctance in the d -axis direction is much lower than it is in the q -axis direction. Therefore, for the sake of reducing the whole system potential energy to reduce the field distortion, the reluctance torque is produced to align the d -axis of the rotor with the magnetic field.

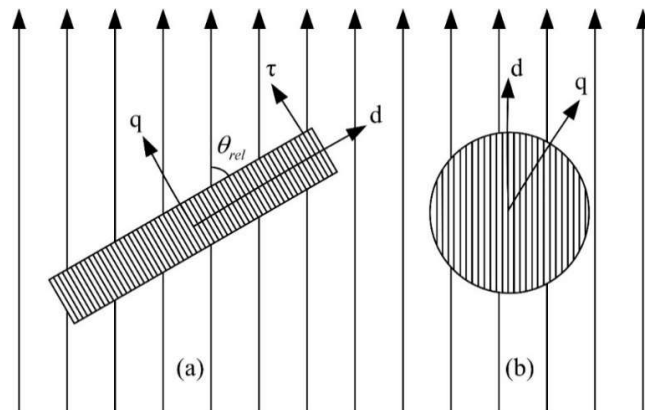


Fig. 1.1: Magnetic field and torque production based on (a) anisotropic geometry; (b) isotropic geometry.

On the other hand, from Fig. 1.1 (b), it can be seen that with an isotropic rotor [41], the magnetic reluctance is uniform. Thus, the reluctance torque is zero.

Furthermore, the magnetic field is produced by a sinusoidal distributed winding in slots of stator, linking the stator and rotor through an air-gap [42]. Consequently, the field is rotating at synchronous speed and be assumed to distribute sinusoidally.

Then, the operating region of the SynRel motor can be explained by Fig. 1.2 [3], from where the operating regions of the SynRel motor can be classified to 3 parts, which are:

Region 1 (Constant torque region): In this region, the speed of the motor is below the base speed (ω_B), making it possible to produce the maximum torque by Maximum Torque per Ampere (MTPA) control. In Fig. 1.2, it is at the point B, where the torque reached its maximum value T_N at the base speed.

Region 2 (Constant volt-ampere region): In this region, the speed of the motor is above the base speed, and the current vector is controlled by Flux Weakening (FW) control. Due to a constant voltage and an increasing speed, the flux is reduced. By changing the current angle, the current vector moves from point B to point B' alongside the current limit circle. During this process, with an increasing current angle, i_d is reduced while i_q is increased, and therefore, the flux linkages on both d - and q - axes are changed. But since the extent of the reducing λ_d is more than it of the increasing λ_q , the total flux is reduced, and the flux weakening is achieved.

Region 3 (Decreasing volt-ampere region): In this region, the speed of the motor is above ω_B , which is at the point B' in Fig. 1.2. And then, the optimum current vector is reached by using the Maximum Torque per Voltage (MTPV) control.

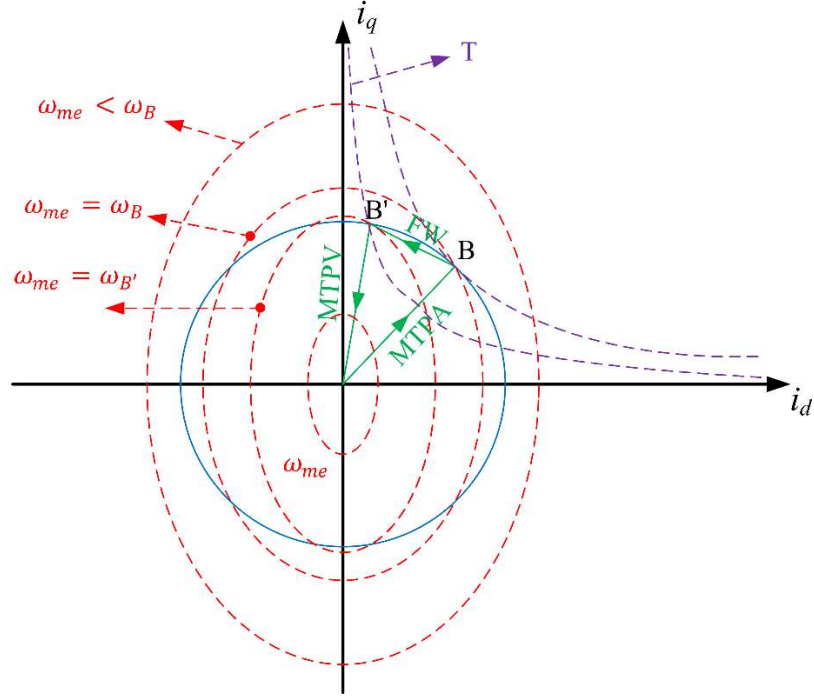


Fig. 1.2: Operating limits and working points of SynRel motor.

By applying the torque versus speed curve of the SynRel motor to present the operating regions, which are highlighted in Fig. 1.3.

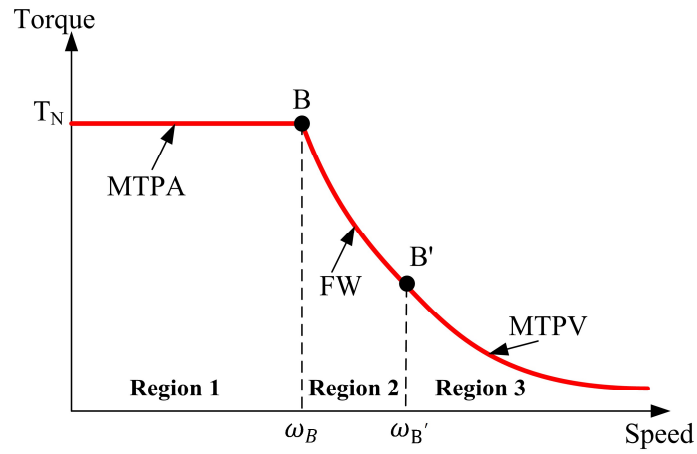


Fig. 1.3: Operating regions on T_n curve.

1.2.2 Electrical Features of SynRel Motor

Fig. 1.4 shows two sketches of a SynRel and a PMaSynRel motor, respectively [44]. These two motors are with four poles and characterized by two and three flux barriers per pole, respectively. The PMaSynRel motor shown in Fig. 1.4 (b) is

developed from the SynRel motor shown in Fig. 1.4 (a) by inserting PM into the middle of flux barrier. The PM adopted here is for saturating the rotor bridges to increase the electromagnetic torque, as well as the power factor.

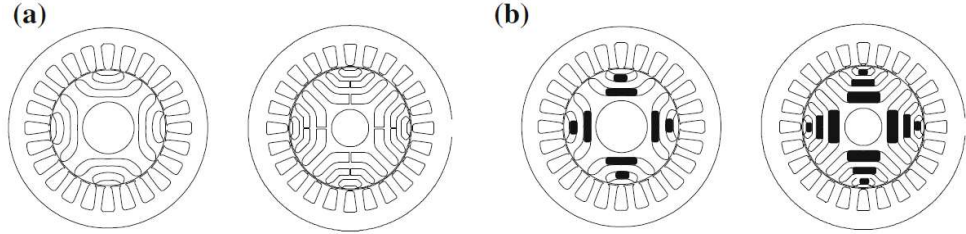


Fig. 1.4: Sketch of (a) SynRel motor, and (b) PMaSynRel motor with two and three flux barriers per pole, respectively [44].

Taking the SynRel motor with three layers flux barriers per pole as a studying object, the analytical model is based on its equivalent magnetic circuit which is shown in Fig. 1.5 [45].

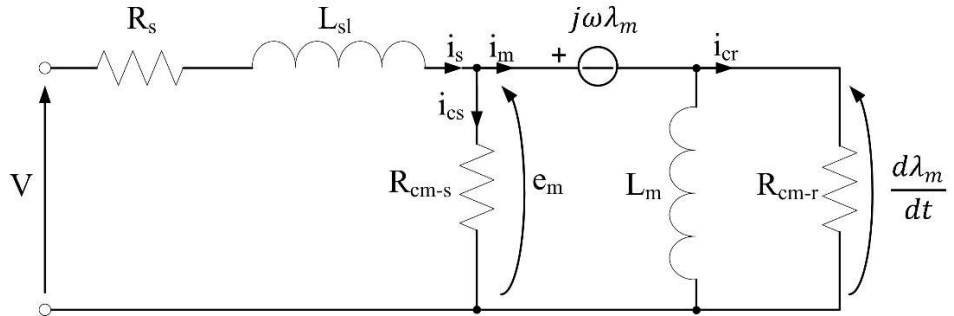


Fig. 1.5: Equivalent circuit of SynRel machine.

Based on Fig. 1.5, the field and damper winding equations are neglected, therefore, the SynRM vector equations in the d - and q - axis [46] are expressed as

$$v = e_m + R_S i_S + j\omega L_{sl} i_S, \quad (1.1)$$

$$e_m = \frac{d\lambda_m}{dt} + j\omega\lambda_m, \quad (1.2)$$

where, v represents the terminal voltage of the machine, λ_m represents the air gap flux linkage, L_{sl} represents the total winding leakage inductance, R_s represents the winding resistance, i_s represents the stator current, ω represents the electrical angular speed under reference frame, and e_m represents the electromotive voltage.

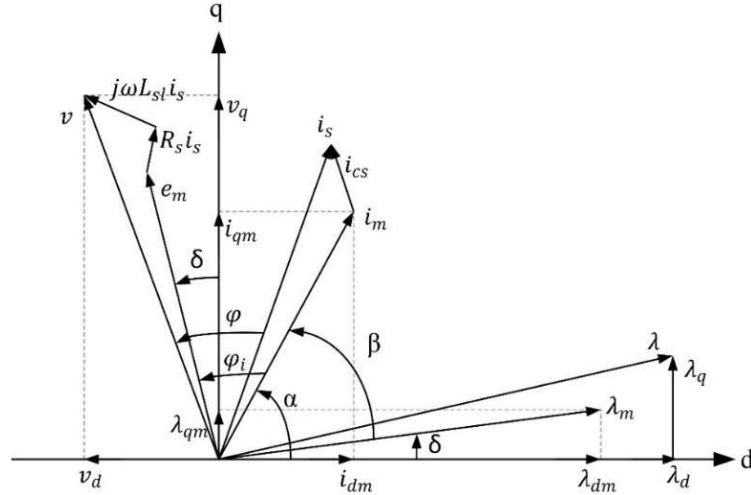


Fig. 1.6: Vector diagram of SynRel machine in steady state.

By applying Eq. (1.1) and Eq. (1.2), the vector diagram of the SynRel motor in steady state can be drawn as shown in Fig. 1.6. Then, the internal power factor of the SynRel motor in steady state can be calculated through

$$\text{IPF} = \cos(\varphi_i) = (\xi - 1) \sqrt{\frac{\sin(2\alpha)}{2(\tan \alpha + \xi^2 \cot \alpha)}}, \quad (1.3)$$

from where the coefficient ξ is defined by

$$\xi = \frac{L_{dm}(i_{dm})}{L_{qm}}. \quad (1.4)$$

When the internal power factor is maximized, it equals to

$$\text{IPF}_{\max} = \frac{\xi - 1}{\xi + 1}. \quad (1.5)$$

Then, by omitting the losses from Eq. (1.1) and Eq. (1.2), the equivalent circuit can be simplified from Fig. 1.5 to Fig. 1.7 as shown below [47].

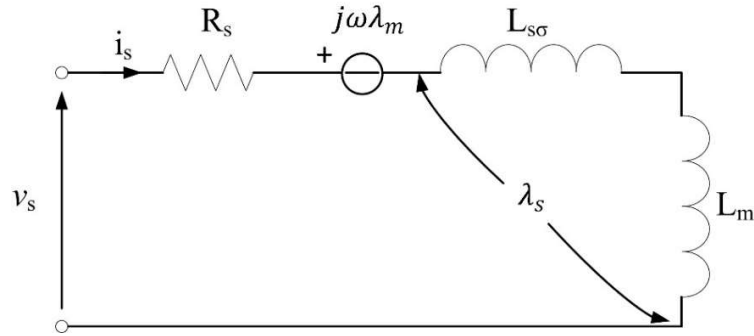


Fig. 1.7: Simplified equivalent circuit based on stator stationary reference frame.

In Fig. 1.7, λ_s represents the amplitude of the stator flux linkage vector. $L_{s\sigma}$ and L_m are representing the stator leakage inductance and magnetizing inductance per phase, respectively. Furthermore, λ_s is comprising the leakage flux linkage of the stator $\lambda_{s\sigma}$ and the flux linkage of the air gap λ_m as shown in the following equations.

$$v_s = R_s i_s + \frac{d\lambda_s}{dt}, \quad (1.6)$$

$$\lambda_s = (L_{s\sigma} + L_m) i_s = \lambda_{s\sigma} + \lambda_m. \quad (1.7)$$

Since the rotor of the SynRel motor has different inductances on d - and q - axis, respectively, and the salient pole machine without excitation, the model can be presented in the synchronous reference frame. The equivalent circuit can be split into

two circuits according to d - and q - axis, respectively, based on which v_s , i_s and λ_s are referring to both d - and q - axis, respectively, as shown in the following equations.

$$v_s = v_d + jv_q, \quad (1.8)$$

$$i_s = i_d + ji_q, \quad (1.9)$$

$$\lambda_s = \lambda_{sd} + j\lambda_{sq}. \quad (1.10)$$

Then, the voltage can be expressed as

$$v_d = R_s i_d + \frac{d\lambda_{sd}}{dt} - \omega_{me} \lambda_{sq}, \quad (1.11)$$

$$v_q = R_s i_q + \frac{d\lambda_{sq}}{dt} + \omega_{me} \lambda_{sd}, \quad (1.12)$$

from where ω_{me} is the electrical angular speed of the machine. In addition, λ_{sd} and λ_{sq} are representing the flux linkage factors in d - and q - axis, respectively, which are expressed as

$$\lambda_{sd} = (L_{s\sigma} + L_{md})i_d = L_d i_d, \quad (1.13)$$

$$\lambda_{sq} = (L_{s\sigma} + L_{mq})i_q = L_q i_q. \quad (1.14)$$

Fig. 1.8 shows an example of the flux linkages on d - and q - axis, respectively.

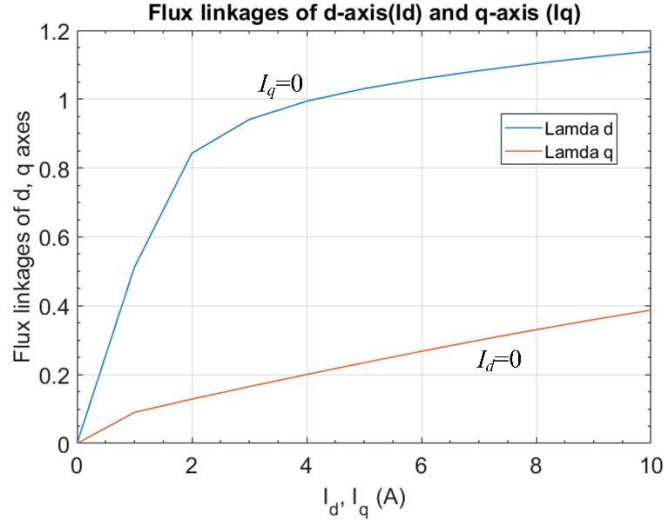


Fig. 1.8: Flux linkages of d - and q - axes.

Fig. 1.8 presents the saturation effect of SynRel motor, which notes that λ_d has a bigger reduction comparing to its linear result due to the saturation effect. On the contrast, λ_q is slightly decreasing with the increasing i_q , which means the saturation effect has a lighter effect on the linear relation on q - axis. Besides, the flux linkages variation in Fig. 1.8 is not including the effect of cross saturation.

Then, the electromagnetic torque is calculated by using [48]

$$T = \frac{3}{2} p (L_d - L_q) i_d i_q, \quad (1.15)$$

It can be seen from Eq. (1.15) that the electromagnetic torque is highly depending on the inductances difference, which is $(L_d - L_q)$. It is also be known as the saliency ratio, which is a vital factor to regard the SynRel motor [15].

1.2.3 Electromagnetic Force in SynRel Motor

The analytical method for calculating the electromagnetic force is derived from Maxwell tensor equations, which are based on the expressions of the air-gap flux density in both radial and tangential direction.

The analytical derivation of the air-gap flux density of surface mounted permanent magnet synchronous (SPM) motor and brushless DC motor are investigated in [49-53]. It starts with replacing the stator with a current sheet and assuming the relative permeability of the rotor core to be infinite, then linearly considering the magnetomotive force of both stator and rotor to analytically compute the air-gap flux density. As discussed in [54-56], the motor is divided into different areas to calculate the air-gap flux density distribution separately for the inserted permanent magnet synchronous (IPM) motors. After computing the expression of the air-gap flux density, the Maxwell tensor equation is adopted to calculate the electromagnetic force. Hence, it can be concluded that utilizing the analytical method to calculate electromagnetic force is precise for surface mounted PM motors without saturation. But for motors with unconventional rotor geometry, such as SynRel or PMaSynRel motors, it is not straightforward to accurately compute the electromagnetic force analytically.

There are some other research focusing on the electromagnetic force from FEA, which are based on the establishment of a 2D electromagnetic model, followed by FEA through Maxwell stress method and virtual work method [57-62]. In [58], Biot-Savart Law is adopted to compute the electromagnetic force acting on the boundaries of each unit, then combine all the electromagnetic forces of all the units inside the area. In [59], an equivalent magnetic load method is used, but it lacks accuracy and

time consuming. Comparing to the equivalent magnetism methods used in [58] and [59], Maxwell stress method and virtual work method are more precise to obtain the electromagnetic force by combining with FEA [60-62].

1.3 Vibration Characteristics of SynRel Motor

As compared to the vibration caused by flowing air and mechanism, the vibration caused by electromagnetic force is the major contributing component to the total vibration in SynRel Motor [63, 64]. Hence, the electromagnetic force is considered as the sole source of vibration in this thesis to evaluate the vibration level of the SynRel motor.

1.3.1 Multi-degrees Vibrating System of SynRel motor

For studying the multi-degrees vibrating system, it starts with the single particle. However, in the case of the vibration study of the SynRel motor, which consists of rigid bodies, the Newton's second law can be derived as the motion equations of rigid and deformable bodies by deriving the equations for system of particles [65].

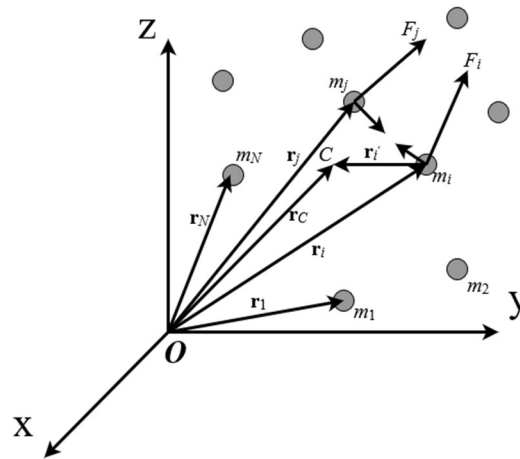


Fig. 1.9: Particles system of SynRel motor.

Assume the object system of the SynRel motor consists of a group of particles, in which each particle has its own mass represented by m_i ($i = 1, 2, 3, \dots, N$). The particle system of SynRel motor is shown in Fig. 1.9.

In Fig. 1.9, $\mathbf{F}_i$ represents the force acting on m_i , and $\mathbf{F}_{ij}$ represents the force exerted by m_j on m_i ($i, j = 1, 2, 3, \dots, N; i \neq j$). By considering Newton's second law, the motion equation for a single particle m_i is expressed as

$$\mathbf{F}_i + \sum_{j=1}^N \mathbf{F}_{ij} = m_i \mathbf{a}_i, \quad (1.16)$$

where, $\mathbf{a}_i$ is the acceleration of particle m_i . By combining all the particles inside this system, and omitting the internal forces $\mathbf{F}_{ij}$ and $\mathbf{F}_{ji}$ which are neutralize in pairs, the motion equation for the system is described as

$$\mathbf{F} = \sum_{i=1}^N m_i \mathbf{a}_i. \quad (1.17)$$

Hence, it is stated that the system of particles is equivalent to the motion of a single mass centre point. After it, by also considering the stiffness and damping coefficients of the SynRel motor, its motion equation can be derived. For a better understanding on the motor system, Fig. 1.10 shows a sketch of the freedom system.

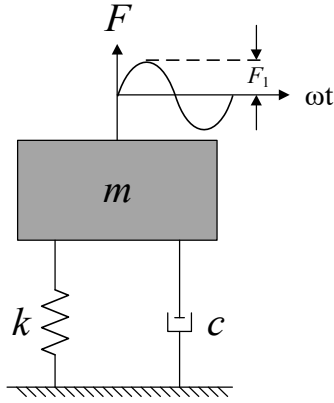


Fig. 1.10: Sketch of a freedom system.

In Fig. 1.10, k represents the stiffness coefficient, while c represents the damping coefficient. The motion equation of the system can be expressed as [66],

$$F = ma + cv + kx , \quad (1.18)$$

from where v represents the velocity of the particle, x represents the displacement of the particle. Then, by applying the differential relations between the acceleration, the velocity, and the displacement, it leads to

$$F = m\ddot{x} + c\dot{x} + kx . \quad (1.19)$$

from where $\dot{x}$ and $\ddot{x}$ are representing the first order and second order differential of x , respectively. By assuming the vibrating displacement is showing a sinusoidal waveform, which is described as

$$x = A \sin(\omega t + \varphi) , \quad (1.20)$$

where, A , ω , and φ represent the amplitude, frequency, and phase degree of the vibrating displacement, respectively. Then, the parameters shown in Eq. (1.19) can be expressed through the differential function, which is

$$\begin{aligned} x &= A \sin(\omega t + \varphi) , \\ \dot{x} &= \omega A \cos(\omega t + \varphi) , \\ \ddot{x} &= -\omega^2 A \sin(\omega t + \varphi) . \end{aligned} \tag{1.21}$$

From Eq. (1.19) and Eq. (1.21), it is noted that the vibrating displacement can be discovered by obtaining the distribution of the exciting force in advance. Also, the velocity and acceleration of the vibrating system can be computed through Eq. (1.21).

1.3.2 Influence of Electromagnetic Force on Vibration

After the distribution of electromagnetic force of the SynRel motor is obtained through analytical method or FE simulation, it can be seen that the tangential component of the electromagnetic force produces the electromagnetic torque, and the radial component of electromagnetic force acts on the stator for its deformation and vibration. Many works and studies focusing on the electromagnetic torque, including the improvement of torque quality and the reduction of torque ripple have been carried out earlier [67-70]. However, there are relatively fewer studies concentrating on the radial component of the electromagnetic force.

The electromagnetic force of electrical motor is produced by the mutual interaction of the armature current (on stator side) and the rotor magnetic potential

(on rotor side), therefore the harmonic of the electromagnetic force is consequently including the time harmonics and space harmonics [71, 72], which states that the vibration caused by the exciting force are proportional to the amplitude of the exciting force, while the amplitude of vibration displacement is inversely proportional to the value of 4th mode number of the exciting force. Furthermore, the exciting force comprise nine group of the waves, by setting r and f_r as the mode number and frequency of the exciting force wave, respectively, the components of the exciting force are shown in Tab. 1.1. It is noted that the 4th group of harmonics consist of extra slotting permeance fields and various orders of field waves.

By analyzing the harmonics of the exciting force, which is the electromagnetic force acting on tooth tips of the stator core, it aids to further investigate the relations between the electromagnetic force and the vibration level. Hence, it establishes the direction to mitigate the vibration level by reducing or optimizing the corresponding harmonic order of the electromagnetic force.

Hence, the links between air-gap flux density, electromagnetic force, and vibration level can be investigated in terms of their harmonics [73], from where an analytical method for calculating the temporal and spatial orders of the radial electromagnetic force is established. After that, the 2-D electromagnetic model and 3-D structural model are built for computing its vibration level, which investigates that the higher amplitude of the electromagnetic force order produces extra vibration comparing to other electromagnetic force harmonic orders. Hence, the vibration reduction can be optimized by improving the harmonics of both the air-gap flux density and the electromagnetic force.

Table 1.1: Main parameters of the exciting force waves.

Group	Mode Number	Frequency
1 st	$2\mu p$	$2\mu f_1$
2 nd	$(\mu_2 + \mu_1)p$	$(\mu_2 + \mu_1)f_1$
	$(\mu_2 - \mu_1)p$	$(\mu_2 + \mu_1)f_1$
3 rd	$2(\mu_p + kZ_s)$	$2\mu f_1$
	$2(\mu_p - kZ_s)$	$2\mu f_1$
4 th	$(\mu_2 + \mu_1)p + (k_2 + k_1)Z_s$	$(\mu_2 + \mu_1)f_1$
	$(\mu_2 + \mu_1)p + (k_2 - k_1)Z_s$	$(\mu_2 + \mu_1)f_1$
	$(\mu_2 + \mu_1)p - (k_2 + k_1)Z_s$	$(\mu_2 + \mu_1)f_1$
	$(\mu_2 + \mu_1)p - (k_2 - k_1)Z_s$	$(\mu_2 + \mu_1)f_1$
	$(\mu_2 - \mu_1)p + (k_2 + k_1)Z_s$	$(\mu_2 - \mu_1)f_1$
	$(\mu_2 - \mu_1)p + (k_2 - k_1)Z_s$	$(\mu_2 - \mu_1)f_1$
	$(\mu_2 - \mu_1)p - (k_2 + k_1)Z_s$	$(\mu_2 - \mu_1)f_1$
	$(\mu_2 - \mu_1)p - (k_2 - k_1)Z_s$	$(\mu_2 - \mu_1)f_1$
5 th	$(\mu_2 + \mu_1)p + kZ_s$	$(\mu_2 + \mu_1)f_1$
	$(\mu_2 + \mu_1)p - kZ_s$	$(\mu_2 + \mu_1)f_1$
	$(\mu_2 - \mu_1)p + kZ_s$	$(\mu_2 - \mu_1)f_1$
	$(\mu_2 - \mu_1)p - kZ_s$	$(\mu_2 - \mu_1)f_1$
6 th	$(\mu + \nu)p$	$(\mu + 1)f_1$
	$(\mu - \nu)p$	$(\mu - 1)f_1$
7 th	$\mu p + kZ_s + \nu p$	$(\mu + 1)f_1$
	$\mu p - kZ_s + \nu p$	$(\mu + 1)f_1$
	$\mu p + kZ_s - \nu p$	$(\mu - 1)f_1$
	$\mu p - kZ_s - \nu p$	$(\mu - 1)f_1$
8 th	$2\nu p$	$2f_1$
9 th	$(\nu_1 + \nu_2)p$	$2f_1$

Besides the general condition, there is another kind of the electromagnetic force, which is the unbalanced electromagnetic force. In [74] and [75], the SynRel model is analysed with an eccentric rotor case. An analytical model of the eccentric SynRel

motor is proposed for exploring the electromagnetic features of the motor. First the air-gap distribution is established based on an uneven air-gap length, which is shown in Fig. 1.11 [3].

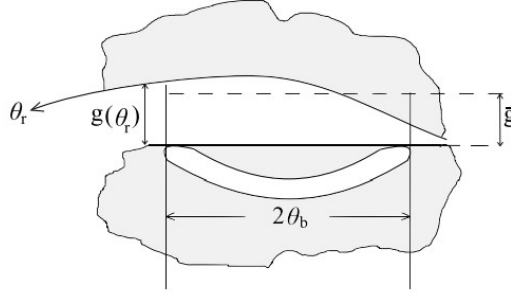


Fig. 1.11: Average air-gap length in front of each flux barrier in case of eccentricity.

After applying the analytical model for eccentricity case, the rotor magnetic potential, the air-gap flux density, and the electromagnetic torque are computed. Then, the analytical calculation is validated by FEA. Therefore, revealing that the eccentricity affects the motor adversely in terms of torque ripple. Furthermore, the different rotor types are compared for the eccentric cases, which reports that the “Machaon” rotor geometry is helpful in the torque ripple reduction.

Another source of unbalanced electromagnetic force is slots and poles combination. For example, authors in [76] discussed that significant unbalanced electromagnetic forces occur in 9 slots/ 8 poles and 3 slots/ 2 poles machine. This component of unbalance electromagnetic forces increased with the rising load. Hence, these fractional slots must be avoided or paid more attention when minimizing the vibration level is the main goal.

Furthermore, the unbalanced electromagnetic force also appears in case of the SynRel motor with eccentric rotor. It is generally acknowledged that the air-gap length of SynRel motor is usually designed as small as possible to deliver a better

electrical performance [77]. In this case, a tiny change in the air-gap length influences the air-gap flux density distribution quite significantly as compared to those motor with larger air-gap length. Furthermore, when the eccentricity occurs, the air-gap length shows an uneven distribution [78], accordingly its air-gap flux density and electromagnetic force distributions are performing uneven waveforms. This type of unbalanced electromagnetic force also influences the vibration of the SynRel motor.

1.4 Thesis Outline

Chapter 1 briefly introduces the background, the electrical characteristics, and the vibration characteristics of the SynRel motor.

Chapter 2 presents the linear analytical model of a concentric SynRel motor with three layers of flux barriers per rotor pole. The model is developed from the equivalent magnetic network, where the electric loading, the stator magnetic potential, and the rotor magnetic potentials are analytically derived. Then the distributions of the air-gap flux density, the electromagnetic force, and the electromagnetic torque are analytically computed. Besides, a campaign of sensitivity analyses is conducted by parameterising the barrier geometry of the rotor structure in order to have optimized electrical performance of the SynRel motor.

Chapter 3 focuses on the mechanical characteristics of the SynRel motor, which starts from the modal analysis by applying and deriving the motion equation of simple harmonic motion (SHM), followed by the stress analysis to investigate the stress distribution on the rotor which is acted by the centrifugal force. Then, the vibration harmonic response of the SynRel motor is analysed.

Chapter 4 compares the SynRel motor with the PMaSynRel motor for their vibration level to explore the impact of inserted PM on vibration characteristics. The analytical model of the PMaSynRel is developed from it of the SynRel motor analyzed in Chapter 2. Then, the air-gap flux density, the electromagnetic torque, and the electromagnetic force of the PMaSynRel motor are presented and compared with that of SynRel motor, as well as their mechanical characteristics.

Chapter 5 investigates the impact of eccentric rotor on the vibration level of the SynRel motor. The cases of eccentricities are proposed and discussed at first, followed by the derivation of the analytical model of the eccentric SynRel motor. Then, the SynRel motor is analyzed under both those eccentric cases.

Chapter 6 establishes an experimental validation by using a 15 kW SynRel motor to evaluate the analytical model and research method proposed, including its electrical performance and mechanical characteristics.

Chapter 7 summarises the conclusions of this work and proposes the possible future work.

Chapter 2

Analytical Derivation of Synchronous Reluctance Motor

This chapter proposes an analytical model of a SynRel motor, which is based on its equivalent magnetic circuit. It is aiming to analytically compute the air-gap flux density for different rotor position, and determine the electromagnetic torque, electromagnetic force, and torque ripple. By doing so, the motor characteristics can be studied and analyzed much more convenient, which saves a lot of time and efforts in the process of preliminary design.

As for the vibration analyse, the excitation of the motor vibration is mainly coming from the electromagnetic force produced in air-gap, which is principally determined by the air-gap flux density. So, the proposed analytical model of the SynRel motor is focusing on the air-gap flux density and the derivation of the electromagnetic force. Therefore, the analytical model is also beneficial to optimizing the electrical and mechanical performance.

2.1 Equivalent Magnetic Circuit Analyses

For getting a better electrical performance, SynRel motors are usually demanding a rather complex rotor design, therefore, this section proposes an equivalent magnetic circuit of the SynRel motor, and then its analytical model is built up, which

leads to the analytical computation of the stator magnetic potential, rotor magnetic potential, air-gap flux density, electromagnetic torque and electromagnetic force.

2.1.1 Stator Magnetic Potentials

In order to simplify the analyses of the stator magnetic potentials, an equivalent current sheet which is placed in the inner surface of the stator, is used to replace the conventional distributed windings.

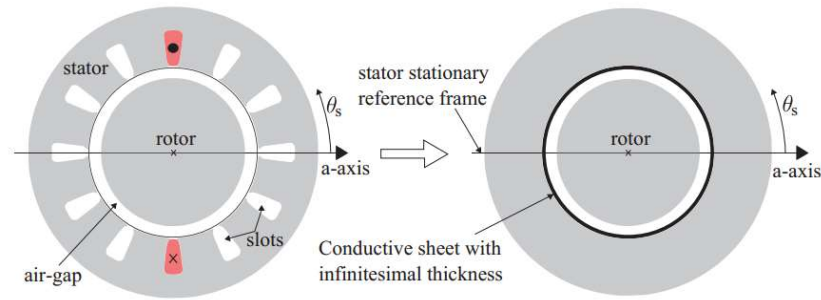


Fig. 2.1: Stator windings and simplified distribution using current sheet.

In Fig. 2.1, the current sheet is representing the current density distribution along the sheet, starting from one coil with single turn. Besides, as the tangible slots are replaced by the current sheet, the slot effects are ignored in this case.

The conductive sheet using current sheet shown in Fig. 2.1 is furtherly simplified, as shown in Fig. 2.2.

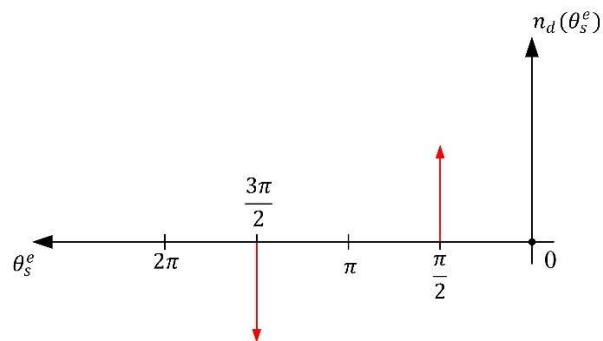


Fig. 2.2: The conductor distribution function corresponding to the stator windings.

In Fig. 2.2, $n_d(\theta_s)$ represents the conductor density distributing on the current sheet, which can be calculated by

$$n_d(\theta_s^e) = \frac{2}{D} \left\{ \delta(\theta_s^e - \frac{\pi}{2}) - \delta(\theta_s^e - \frac{3\pi}{2}) \right\}, \quad (2.1)$$

where, θ_s^e is the electric angle in the stator reference frame, D is the rotor diameter.

Since $n_d(\theta_s^e)$ is a periodic function, which is odd and half-wave symmetric. Its Fourier coefficients b_v can be expressed as

$$b_v = \frac{1}{\pi} \int_0^{2\pi} n_d(\theta_s^e) \sin(v\theta_s^e) d\theta_s^e = \frac{4}{\pi D} \sin v \frac{\pi}{2} \quad (2.2)$$

where v represents the electric space harmonic orders, that $v=6k+1$ ($k=0, \pm 1, \pm 2, \pm 3 \dots$).

From Eq. (2.2), it is noted that the even harmonics are deleted by the coefficient factor, which is $\sin(v\pi/2)$. Then, the distribution function of phase B and C can be derived from Eq. (2.1) and Eq. (2.2), leading to

$$\begin{aligned} n_{dA}(\theta_s^e) &= \frac{4}{\pi D} \sum_{v=1}^{+\infty} k_w^v \sin v \frac{\pi}{2} \sin v \theta_s^e, \\ n_{dB}(\theta_s^e) &= \frac{4}{\pi D} \sum_{v=1}^{+\infty} k_w^v \sin v \frac{\pi}{2} \sin v(\theta_s^e - \frac{2\pi}{3}), \\ n_{dC}(\theta_s^e) &= \frac{4}{\pi D} \sum_{v=1}^{+\infty} k_w^v \sin v \frac{\pi}{2} \sin v(\theta_s^e - \frac{4\pi}{3}), \end{aligned} \quad (2.3)$$

where, k_w^v represents the winding factor.

Then, the electric loading of the SynRel motor can be computed by

$$K_s(\theta_s^e, t) = n_{dA}(\theta_s^e) i_A(t) + n_{dB}(\theta_s^e) i_B(t) + n_{dC}(\theta_s^e) i_C(t), \quad (2.4)$$

where i_A , i_B , i_C are representing the currents in windings of phase A, B, C, respectively, which are a system of symmetric sinusoidal value, are given by

$$\begin{aligned} i_A(t) &= \hat{I} \cos(\omega t + \alpha_i^e), \\ i_B(t) &= \hat{I} \cos(\omega t + \alpha_i^e - \frac{2\pi}{3}), \\ i_C(t) &= \hat{I} \cos(\omega t + \alpha_i^e - \frac{4\pi}{3}). \end{aligned} \quad (2.5)$$

After that, by substituting Eq. (2.3) and Eq. (2.5) into Eq. (2.4), it leads to

$$K_s(\theta_s^e, t) = \frac{6}{\pi D} \hat{I} \sum_{\substack{v=6k+1 \\ k \in \mathbb{Z}}}^{\infty} \sin v \frac{\pi}{2} \sin(v\theta_s^e - \omega_{me} t - \alpha_i^e), \quad (2.6)$$

where, ω_{me} is the electric rotating speed of the rotor.

Then, it is to compute the previous single turn winding, which has 2 conductors, with the single coil equivalent winding of a distributed and chorded one. In this equivalent winding, the number of series conductors is N_s , and its winding factor is which can be defined as k_w^v , and the coefficient of $K_s(\theta_s^e, t)$ can be set as K_v , which is given by

$$K_v = k_w^v \widehat{K}_s \sin v \frac{\pi}{2}, \quad (2.7)$$

where, $\widehat{K}_s$ is given by

$$\widehat{K}_s = \frac{3N_s \hat{I}}{\pi D}. \quad (2.8)$$

Then, by substituting Eq. (2.8) into Eq. (2.7), it leads to

$$K_v = \frac{3N_s \hat{I} k_w^v}{\pi D} \sin v \frac{\pi}{2}. \quad (2.9)$$

Finally, the electric loading of the winding which has symmetric three phases can be calculated by

$$K_s(\theta_s^e, t) = \sum_{\substack{v=6k+1 \\ k \in \mathbb{Z}}}^{\infty} K_v \sin(v\theta_s^e - \omega_{me}t - \alpha_i^e). \quad (2.10)$$

Then, by integrating the electric loading with the pole pitch λ_p , it leads to the stator magnetic potential which is given by

$$U_s(\theta_s^e, t) = \int K_s(\theta_s^e, t) \lambda_p, \quad (2.11)$$

where, λ_p is calculated through

$$\lambda_p = \pi D \cdot \frac{d\theta_s}{2\pi}. \quad (2.12)$$

By substituting Eq. (2.10) and Eq. (2.12) into Eq. (2.11), it results in

$$U_s(\theta_s^e, t) = -\frac{D}{2} \sum_{v=1}^{+\infty} \frac{K_v}{pv} \cos(v\theta_s^e - \omega_{me}t - \alpha_i^e). \quad (2.13)$$

2.1.2 Rotor magnetic Potentials

It is foresaid that the rotor of SynRel motor is usually appearing a rather complex structure for a better electrical performance, therefore, it is critical to deeply study and understand the impact of rotor. It firstly starts with the linearized geometry for a rotor pole as shown in Fig. 2.3.

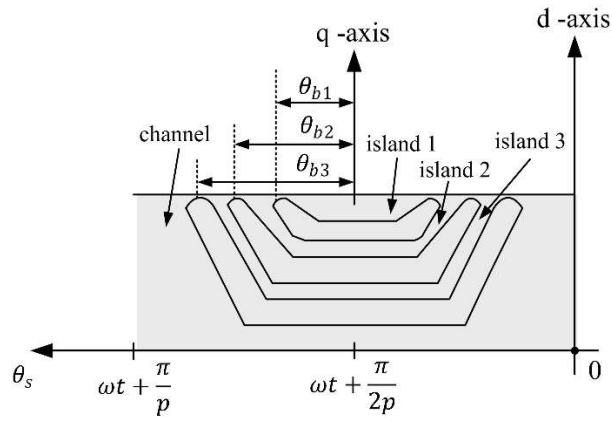


Fig. 2.3: Linearized geometry of a rotor pole.

In Fig. 2.3, θ_{b1} , θ_{b2} and θ_{b3} are the half of the end angle of first, second and third flux barrier, respectively. Besides that, a single pole of the SynRel motor with three layers of flux barriers divides the pole area into islands and channels as shown. As for the analytical analysis, the flux barrier is considered as a constant in terms of the barrier width.

Based on the linearized geometry of a rotor pole, assuming the equivalent magnetic circuit can be built as shown in Fig. 2.4.

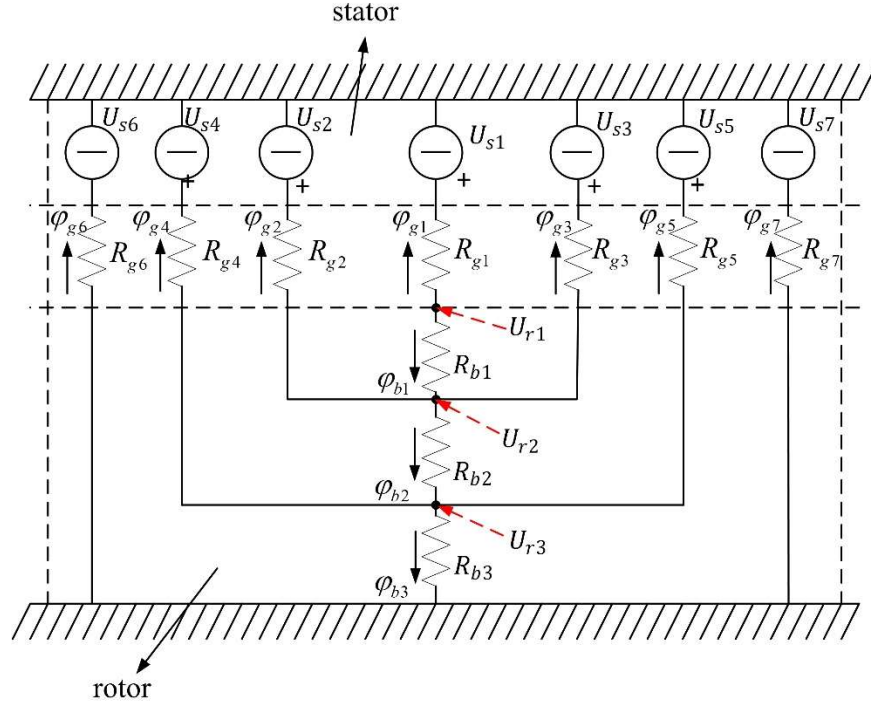


Fig. 2.4: Equivalent magnetic circuit of a rotor pole.

It is noted from Fig. 2.4 that the rotor magnetic potential of the i -th flux barrier can be computed by

$$U_{ri} = \varphi_{bi} R_{bi}, \quad (2.14)$$

where, the reluctance R_{bi} at each different area can be calculated by using

$$R_{bi} = \frac{t_{bi}}{\mu_0 l_{bi} L_{stk}}, \quad (2.15)$$

where,

R_{bi} is the magnetic reluctance of the i -th flux barrier;

t_{bi} is the thickness of the i -th flux barrier;

l_{bi} is the length of the i -th flux barrier;

L_{stk} is the stack length of the motor.

Again, it can also be obtained from Fig. 2.4 that

$$\varphi_{b3} = \varphi_{b2} - \varphi_{g4} - \varphi_{g5}. \quad (2.16)$$

By substituting (2.16) into (2.14), the magnetic potential U_{r3} is computed through

$$U_{r3} = (\varphi_{b2} - \varphi_{g4} - \varphi_{g5}) \cdot R_{b3}, \quad (2.17)$$

where, φ_{b4} and φ_{b5} can be expressed as

$$\begin{aligned} \varphi_{g4} &= - \int_{\omega t + \frac{\pi}{2p} - \theta_{b3}}^{\omega t + \frac{\pi}{2p} - \theta_{b2}} \mu_0 \frac{U_s(\theta_s) - U_{r3}}{L_g} L_{stk} \frac{D}{2} d\theta_s, \\ \varphi_{g5} &= - \int_{\omega t + \frac{\pi}{2p} + \theta_{b2}}^{\omega t + \frac{\pi}{2p} + \theta_{b3}} \mu_0 \frac{U_s(\theta_s) - U_{r3}}{L_g} L_{stk} \frac{D}{2} d\theta_s. \end{aligned} \quad (2.18)$$

By substituting (2.15) and (2.18) into (2.17), it leads to

$$U_{r3} = \left[\frac{(U_{r2} - U_{r3})l_{b2}}{t_{b2}} + \int_{\omega t + \frac{\pi}{2p} - \theta_{b3}}^{\omega t + \frac{\pi}{2p} - \theta_{b2}} \frac{U_s(\theta_s) - U_{r3}}{L_g} \frac{D}{2} d\theta_s + \int_{\omega t + \frac{\pi}{2p} + \theta_{b2}}^{\omega t + \frac{\pi}{2p} + \theta_{b3}} \frac{U_s(\theta_s) - U_{r3}}{L_g} \frac{D}{2} d\theta_s \right] \cdot \frac{t_{b3}}{l_{b3}} \quad (2.19)$$

Then, by letting

$$\begin{aligned} S &= \int_{\omega t + \frac{\pi}{2p} - \theta_{b3}}^{\omega t + \frac{\pi}{2p} - \theta_{b2}} U_s(\theta_s) d\theta_s + \int_{\omega t + \frac{\pi}{2p} + \theta_{b2}}^{\omega t + \frac{\pi}{2p} + \theta_{b3}} U_s(\theta_s) d\theta_s \\ &= - \sum_v^{\infty} \frac{K_v D}{(vp)^2} \cos \lambda_v [\sin(vp\theta_{b3}) - \sin(vp\theta_{b2})] \end{aligned} \quad (2.20)$$

where, λ_v is a coefficient determined by

$$\lambda_v = \frac{v\pi}{2} + p(v-1)\omega t - \alpha_i^e. \quad (2.21)$$

and changing the components which includes U_{r3} to the left side, it results in

$$\left[1 + \frac{t_{b3}l_{b2}}{t_{b2}l_{b3}} + \frac{Dt_{b3}}{L_g l_{b3}}(\theta_{b3} - \theta_{b2}) \right] U_{r3} = \frac{t_{b3}l_{b2}}{t_{b2}l_{b3}} U_{r2} + \frac{Dt_{b3}}{2L_g l_{b3}} S. \quad (2.22)$$

After computing the rotor magnetic potential U_{r3} , it is similar to compute the rotor magnetic potential produced by first and second flux barrier, which is U_{r1} and U_{r2} , that is expressed as

$$U_{r2} = \phi_{b2} R_{b2} + U_{r3}, \quad (2.23)$$

$$U_{r1} = \phi_{b1} R_{b1} + U_{r2}, \quad (2.24)$$

From where a , b , c , and d are the coefficients for simplifying the expression of rotor magnetic potential, and they are determined by

$$a = \frac{\frac{Dt_{b1}}{2L_g l_{b1}}}{1 + \frac{Dt_{b1}}{L_g l_{b1}}(\theta_{b1})}, \quad (2.25)$$

$$b = \frac{1}{1 + \frac{Dt_{b1}}{L_g l_{b1}}(\theta_{b1})}, \quad (2.26)$$

$$c = \frac{a \frac{t_{b2}l_{b1}}{t_{b1}l_{b2}}}{1 - \frac{t_{b2}l_{b1}}{t_{b1}l_{b2}}(b-1) + \frac{Dt_{b2}}{L_g l_{b2}}(\theta_{b2} - \theta_{b1})}, \quad (2.27)$$

$$d = \frac{\frac{Dt_{b2}}{2L_g l_{b2}}}{1 - \frac{t_{b2} l_{b1}}{t_{b1} l_{b2}} (b-1) + \frac{Dt_{b2}}{L_g l_{b2}} (\theta_{b2} - \theta_{b1})}. \quad (2.28)$$

Then, by substituting (2.22) into (2.23), from which U_{r3} is expressed by U_{r2} , and considering $\varphi_{b2} = \varphi_{b1} - \varphi_{g2} - \varphi_{g3}$, it leads (2.23) to

$$U_{r2} = (c-d) \sum_{v,k}^{+\infty} -\frac{K_v D}{(vp)^2} \cos \lambda_v \sin(vp\theta_{b1}) + d \sum_{v,k}^{+\infty} -\frac{K_v D}{(vp)^2} \cos \lambda_v \sin(vp\theta_{b2}) + z U_{r3}, \quad (2.29)$$

from where z is another coefficient which is given by

$$z = \frac{1}{1 - \frac{t_{b2} l_{b1}}{t_{b1} l_{b2}} (b-1) + \frac{Dt_{b2}}{L_g l_{b2}} (\theta_{b2} - \theta_{b1})}. \quad (2.30)$$

To a further simplification of the expressions of the rotor magnetic potentials, there are some more coefficients defined, which are given by

$$m = \frac{c \frac{t_{b3} l_{b2}}{t_{b2} l_{b3}}}{1 - (z-1) \frac{t_{b3} l_{b2}}{t_{b2} l_{b3}} + \frac{Dt_{b3}}{L_g l_{b3}} (\theta_{b3} - \theta_{b2})}, \quad (2.31)$$

$$n = \frac{d \frac{t_{b3} l_{b2}}{t_{b2} l_{b3}}}{1 - (z-1) \frac{t_{b3} l_{b2}}{t_{b2} l_{b3}} + \frac{Dt_{b3}}{L_g l_{b3}} (\theta_{b3} - \theta_{b2})}, \quad (2.32)$$

$$q = \frac{\frac{Dt_{b3}}{2L_g l_{b3}}}{1 - (z-1) \frac{t_{b3} l_{b2}}{t_{b2} l_{b3}} + \frac{Dt_{b3}}{L_g l_{b3}} (\theta_{b3} - \theta_{b2})}. \quad (2.33)$$

Hence, the rotor magnetic potentials can be simplified and expressed as

$$U_{r3} = - \sum_{v,k}^{+\infty} \frac{K_v D}{(vp)^2} \cos \lambda_v \rho_3, \quad (2.34)$$

$$U_{r2} = - \sum_{v,k}^{+\infty} \frac{K_v D}{(vp)^2} \cos \lambda_v \rho_2, \quad (2.35)$$

$$U_{r1} = - \sum_{v,k}^{+\infty} \frac{K_v D}{(vp)^2} \cos \lambda_v \rho_1, \quad (2.36)$$

from where ρ_1 , ρ_2 , and ρ_3 are the symbols of U_{r1} , U_{r2} , and U_{r3} , respectively, representing their calculating coefficients that are given by

$$\rho_3 = (m-n) \sin(vp\theta_{b1}) + (n-q) \sin(vp\theta_{b2}) + q \sin(vp\theta_{b3}), \quad (2.37)$$

$$\begin{aligned} \rho_2 = & [c-d+z(m-n)] \sin(vp\theta_{b1}) \\ & + [d+z(n-q)] \sin(vp\theta_{b2}) + qz \sin(vp\theta_{b3}) \end{aligned} \quad (2.38)$$

$$\begin{aligned} \rho_1 = & \{a+b[c-d+z(m-n)]\} \sin(vp\theta_{b1}) \\ & + \{b[d+z(n-q)]\} \sin(vp\theta_{b2}) + bzq \sin(vp\theta_{b3}) \end{aligned} \quad (2.39)$$

2.1.3 Air-gap Flux Density Distribution

Fig. 2.5 presents the distribution of the stator magnetic potential and the rotor magnetic potential aligning in the air-gap. Also, the relations between the stator and the rotor are highlighted in Fig. 2.5.

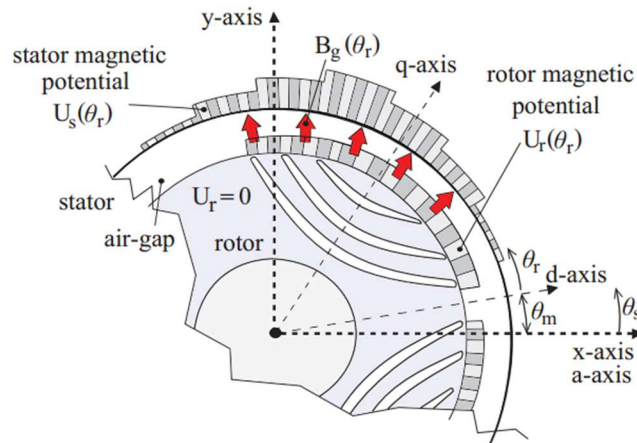


Fig. 2.5: Distribution of the magnetic potentials of stator and rotor [3].

Since the magnetic flux flows through the rotor because of the stator magnetic potential, the rotor islands subsequently produce magnetic potential proportional to the magnetic reluctance of each barrier and the flowing flux. By ignoring the magnetic saturation existing both in the iron material of stator and rotor, the air-gap flux density is calculated through

$$B_g(\theta_s) = -\mu_0 \frac{U_s(\theta_s) - U_r(\theta_s)}{L_g}, \quad (2.40)$$

where, μ_0 is the free space permeability.

The stator magnetic potential is computed analytically by applying Eq. (2.13), which is shown in Fig. 2.6. Then, the air-gap flux density of the SynRel motor is also analytically calculated by applying Eq. (2.40), from where the stator magnetic potential $U_s(\theta)_s$ is already obtained, and the rotor magnetic potential $U_s(\theta)_s$ is calculated by applying Eq. (2.34), Eq. (2.35), and Eq. (2.36). The air-gap flux density of the SynRel motor is shown in Fig. 2.7. Besides that, the current is set as 1 A, and the current angle is 45 electric degrees.

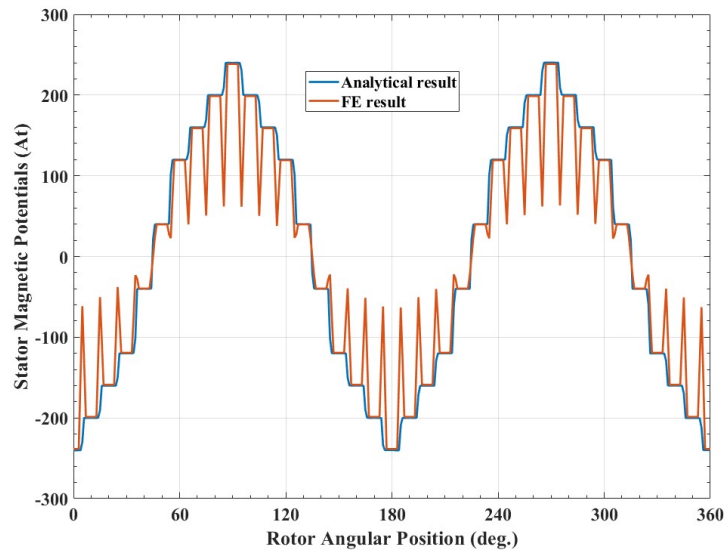


Fig. 2.6: Analytical and FE stator magnetic potentials.

In Fig. 2.6, it compares the stator magnetic potentials of the analytical and FE results. The peak stator magnetic potential is reaching at 220 At, and it shows a good “sinusoidal” waveform. Also, it notes that there is a good agreement between the FE result and the analytical result.

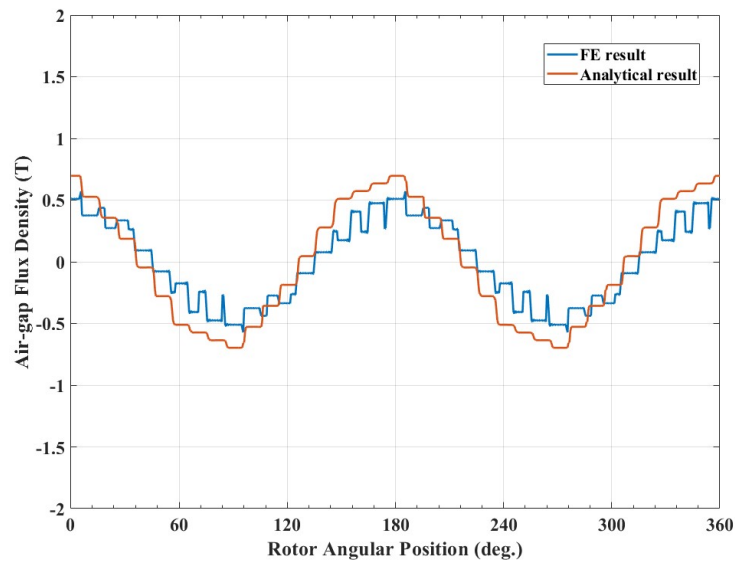


Fig. 2.7: Analytical and FE air-gap flux density.

Fig. 2.7 shows the air-gap flux density comparison between the analytical result and FE result. As the analytical result is obtained by omitting the stator, which means that the slotting effect is ignored, while the slotting effect can be observed from the

dips of the FE waveform of the air-gap flux density. Besides that, the absolute peak value of the analytical result is about 0.6 T, while it of the FE result is about 0.5 T.

2.2 Analytical Modelling for Torque Derivation

2.2.1 Electromagnetic Torque Model

The electromagnetic torque is obtained and computed by integrating Lorentz force density τ_m which is given by $B_g(\theta_s)K_s(\theta_s)$ along the outer surface of the rotor, and then multiplying the rotor radius. Since the scalar magnetic potentials of the stator and the rotor are achieved, respectively, in last sections, then it leads the computation expression of electromagnetic torque to

$$\tau_m = -\frac{D}{2} \int_{\omega t}^{\omega t+2\pi} B_g(\theta_s) K_s(\theta_s) L_{stk} \frac{D}{2} d\theta_s. \quad (2.41)$$

Then, by substituting (2.40) into (2.41), it becomes

$$\tau_m = \frac{\mu_0 D^2 L_{stk}}{4} \left[\int_{\omega t}^{\omega t+2\pi} \frac{U_s(\theta_s) K_s(\theta_s)}{L_g} d\theta_s - \int_{\omega t}^{\omega t+2\pi} \frac{U_r(\theta_s) K_s(\theta_s)}{L_g} d\theta_s \right]. \quad (2.42)$$

It is noted from Eq. (2.42) that the first part of the integration is consisting of $U_s(\theta_s)$ and $K_s(\theta_s)$, which are orthogonal functions in their Fourier series expansions. Hence, the electromagnetic torque is only computed by the interaction of the electric loading and the magnetic scalar potential of the rotor, and therefore, Eq. (2.42) is simplified as

$$\tau_m = \frac{-\mu_0 D^2 L_{stk}}{4} \int_{\omega t}^{\omega t + 2\pi} \frac{U_r(\theta_s) K_s(\theta_s)}{L_g} d\theta_s. \quad (2.43)$$

As for the SynRel motor with three layers of flux barriers and four poles, the total electromagnetic torque is consisting of it produced by each flux barrier, which are represented by τ_{m1} , τ_{m2} and τ_{m3} , are expressed as

$$\tau_{m1} = 2p \cdot \frac{-\mu_0 D^2 L_{stk}}{4} \int_{\omega t + \frac{\pi}{2p} - \theta_{b1}}^{\omega t + \frac{\pi}{2p} + \theta_{b1}} \frac{U_{r1}(\theta_s) K_s(\theta_s)}{L_g} d\theta_s, \quad (2.44)$$

$$\tau_{m2} = 2p \cdot \frac{-\mu_0 D^2 L_{stk}}{4} \left[\int_{\omega t + \frac{\pi}{2p} - \theta_{b2}}^{\omega t + \frac{\pi}{2p} - \theta_{b1}} \frac{U_{r2}(\theta_s) K_s(\theta_s)}{L_g} d\theta_s + \int_{\omega t + \frac{\pi}{2p} + \theta_{b1}}^{\omega t + \frac{\pi}{2p} + \theta_{b2}} \frac{U_{r2}(\theta_s) K_s(\theta_s)}{L_g} d\theta_s \right], \quad (2.45)$$

$$\tau_{m3} = 2p \cdot \frac{-\mu_0 D^2 L_{stk}}{4} \left[\int_{\omega t + \frac{\pi}{2p} - \theta_{b3}}^{\omega t + \frac{\pi}{2p} - \theta_{b2}} \frac{U_{r3}(\theta_s) K_s(\theta_s)}{L_g} d\theta_s + \int_{\omega t + \frac{\pi}{2p} + \theta_{b2}}^{\omega t + \frac{\pi}{2p} + \theta_{b3}} \frac{U_{r3}(\theta_s) K_s(\theta_s)}{L_g} d\theta_s \right]. \quad (2.46)$$

Then, the final electromagnetic torque is calculated through

$$\tau_m = \tau_{m1} + \tau_{m2} + \tau_{m3}. \quad (2.47)$$

By applying Eq. (2.47), the analytical result of the electromagnetic torque is computed and shown in the following figure, from where it is also compared with the FE result.

By setting the current as 1 A, current angle is 45 electric degrees, the analytical and FE results of the electromagnetic torque of the SynRel motor, which has three flux barriers per pole, are shown in Fig. 2.8. It notes that the maximum analytical result is occurring at 1 N·m, while it of the FE result is at 0.8 N·m. The torque ripple

of both analytical result and FE result is 18.68%. It informs that a satisfactory agreement of the torque ripple and waveform is stated, except the average torque from the analytical model is about 17% higher than it from FE model. It is mainly due to the reduced flux in the rotor path, which is caused by the finite iron permeability in FE model.

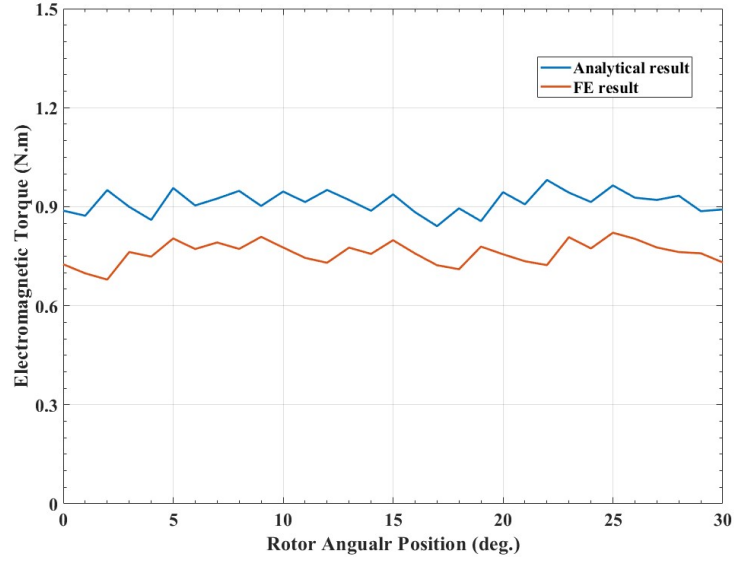


Fig. 2.8: Analytical and FE electromagnetic torque.

2.2.2 Electromagnetic Force Model

The electromagnetic force produced in the air gap is playing a critical role that it is the excitation of the noise and vibration. So, it is important to study the electromagnetic force before doing the acoustic and vibrational analyses. In addition, the harmonics of electromagnetic force could be analysed for deeply understanding its impact on the acoustic stress level and vibrational response.

To compute the electromagnetic force, the Maxwell tensors are applied and expressed as [79]

$$\sigma = \frac{1}{2\mu_0}(B_r^2 - B_\theta^2), \quad (2.48)$$

$$\tau = \frac{1}{\mu_0}B_r B_\theta, \quad (2.49)$$

where, σ and τ are representing the electromagnetic force density in radial and tangential direction, respectively, B_r and B_θ are representing the air-gap flux density in radial and tangential direction, respectively.

Furthermore, as the air-gap flux density in the tangential direction is much smaller with respect to it in the radial direction [80], therefore, the tangential air-gap flux density is neglected from Eq. (2.48) and Eq. (2.49), which leads the electromagnetic force density to

$$f_m = \frac{B_r^2}{2\mu_0}. \quad (2.50)$$

Then, the electromagnetic force acting on the first three tooth tips are shown in Fig. 2.9. Due to the symmetric motor geometry, and the number of phases, all the electromagnetic forces acting on the rest of tooth tips are repeating them of the first three tooth tips.

In Fig. 2.9, it shows that the peak electromagnetic force acting on the first three tooth tips is around 210 N, which occurs at the third tooth tip. Besides, the forces acting on each tooth tip performs a similar waveform. Due to the number of slots per pole per phase ($q = 3$) in this case, the electromagnetic forces acting on every three tooth tips are the symmetric results as shown in Fig. 2.9.

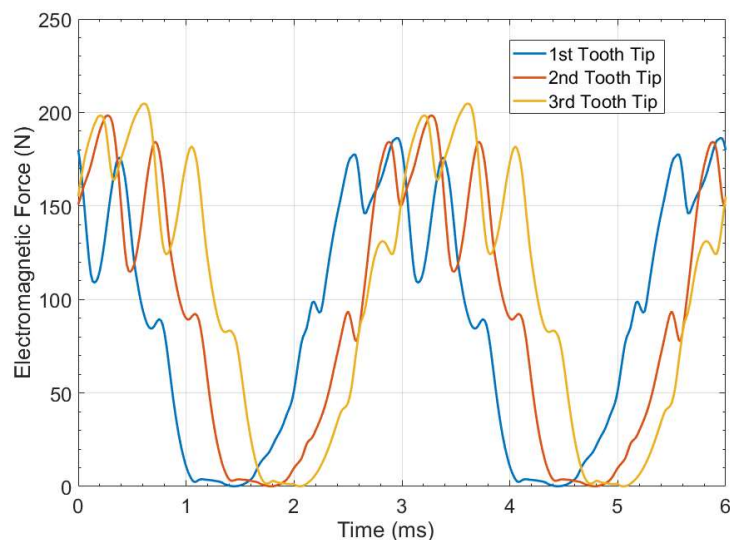


Fig. 2.9: Radial electromagnetic forces.

2.3 Impact of Rotor Geometry on Motor Performance

By realizing a large proportion of reluctance torque, the rotor of the SynRel motor presents a relatively complex design, when the torque ripple needs to be minimized [81]. Large torque ripple and electromagnetic force fluctuations can become sources of significant noise and vibration. This is bringing a new design challenge for these types of motors and their vibration analysis, which is a very important aspect to consider. Therefore, the rotor geometry sensitivity analysis is carried out to understand the relations between the rotor geometry and its electrical performance.

Firstly, the geometric data of the SynRel motor are listed in Tab. 2.1.

Table 2.1: The main geometric data.

Parameters	Symbol	Value
Outer diameter	OD	200 mm
Inner stator diameter	D_s	125 mm
Rotor diameter	D	124.3 mm
Shaft diameter	D_{ri}	44 mm

Air-gap length	L_g	0.35 mm
Stack Length	L_{stk}	40 mm
Number of pole pairs	p	2
Number of stator slots	Q_s	36

After the geometry is determined in Tab. 2.1, the rotor geometry is analyzed by setting a few sensitivity parameters, including the number of flux barrier layers, the end angle of each flux barrier, and the barrier ribs, in the following subsections.

2.3.1 Impact of Barrier Layers

With different layers of flux barrier, the electrical performance of the SynRel motor is different. As for getting a better electrical performance, it always demands an optimized rotor design. In this section, it starts with one flux barrier layer per pole, then extend to two and three flux barrier layers, respectively. The geometrical data of each flux barrier are listed in Tab. 2.2.

Table 2.2: Geometrical data of each flux barrier layer per pole.

Parameters	Symbol	Value
1 st flux barrier angle	$2\theta_{b1}$	28°
2 nd flux barrier angle	$2\theta_{b2}$	52°
3 rd flux barrier angle	$2\theta_{b3}$	76°
Thickness to length ratio of 1 st barrier	t_{b1}/l_{b1}	0.21
Thickness to length ratio of 2 nd barrier	t_{b2}/l_{b2}	0.23
Thickness to length ratio of 3 rd barrier	t_{b3}/l_{b3}	0.33

As discussed in last sections, the current sheet is adopted in the analytical model, the FEA analysis in this section is also using the current sheet. For a case study of the comparison between the analytical result and the FEA result, the number of

conductors is set as 67, the peak current is 1 A, and the current electric angle is 45 degrees ($\alpha_i^e=45^\circ$).

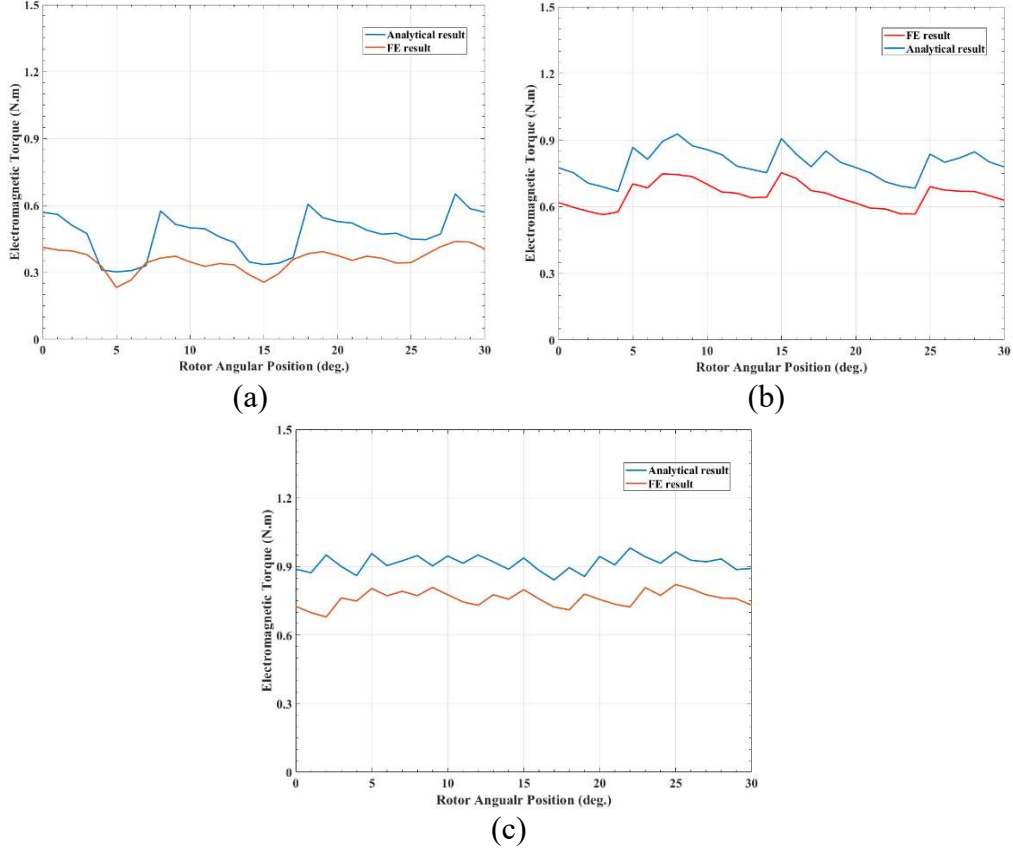


Fig. 2.10: Electromagnetic torque of SynRel motor with (a) one flux barrier layer; (b) two flux barrier layers; (c) three flux barrier layers.

Fig. 2.6 shows the electromagnetic torque against the rotor position of the SynRel motor with one, two and three flux barriers per pole, respectively. Due to the three-phase system, the electromagnetic torque is repeating every 30 mechanical degrees. It shows that the average analytical torque is higher than it from FEA, it is mainly coming from the finite iron permeability which slightly reduces the flux in the rotor path. Besides the difference of the values, the analytical results have a good agreement in terms of the torque ripple and changing trend.

After that, it can be drawn that three layers of flux barrier per pole is the best option comparing to the other two types. Besides, considering the limited space in the rotor, the layer number of flux barrier per pole is determined as three.

2.3.2 Impact of Barrier End Angles

By keeping the main geometry of the SynRel motor, the end angles of each barrier are set as a sensitivity parameter, and therefore the impact on the performance of the SynRel motor can be figured out. The sensitivity values of the barrier end angle are shown in Tab. 2.3.

Table 2.3: Sensitivity end angle of each flux barrier.

Parameters	Symbol	Sensitivity values		
1 st flux barrier angle	$2\theta_{b1}$	27°	28°	29°
2 nd flux barrier angle	$2\theta_{b2}$	51°	52°	53°
3 rd flux barrier angle	$2\theta_{b3}$	75°	76°	77°

By keeping all the simulation conditions as the same as last section, the sensitivity FE analysis of the barrier end angle is conducted.

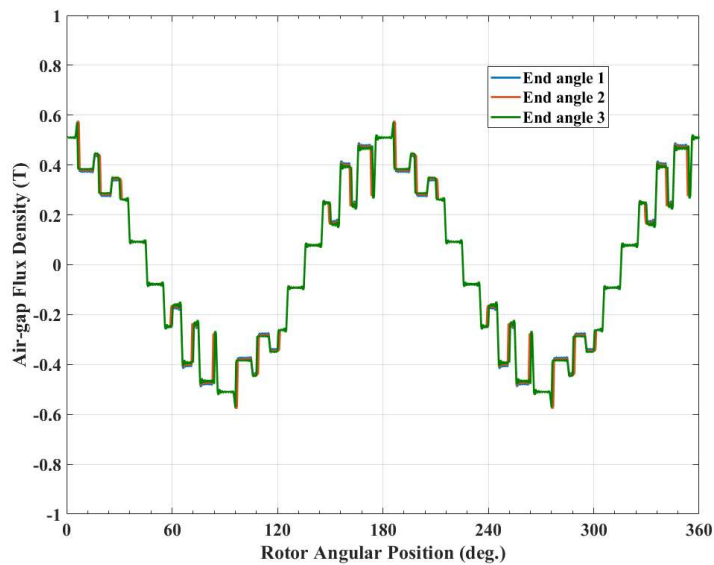


Fig. 2.11: Air-gap flux density of SynRel motor varying end angle.

Then, Fig. 2.11 shows the air-gap flux density of the SynRel motor with a sensitivity end angle. It informs that the air-gap flux densities under the cases of different flux barrier end angle have a well agreement. According to the magnetic circuit shown in Fig. 2.4, and the analytical derivations of the stator magnetic potentials, rotor magnetic potentials, and air-gap flux density, it can be noted that the air-gap flux density waveform is strongly relating to the corresponding positions between each two flux barriers. However, since the thickness and the length of the flux barrier is fixed in this case, the amplitude of the air-gap flux density is not that sensitivity to the flux barrier end angle. Furthermore, as the electromagnetic force is relating to the air-gap flux density which can be known from Eq. (2.50), and therefore the electromagnetic force is not considered here.

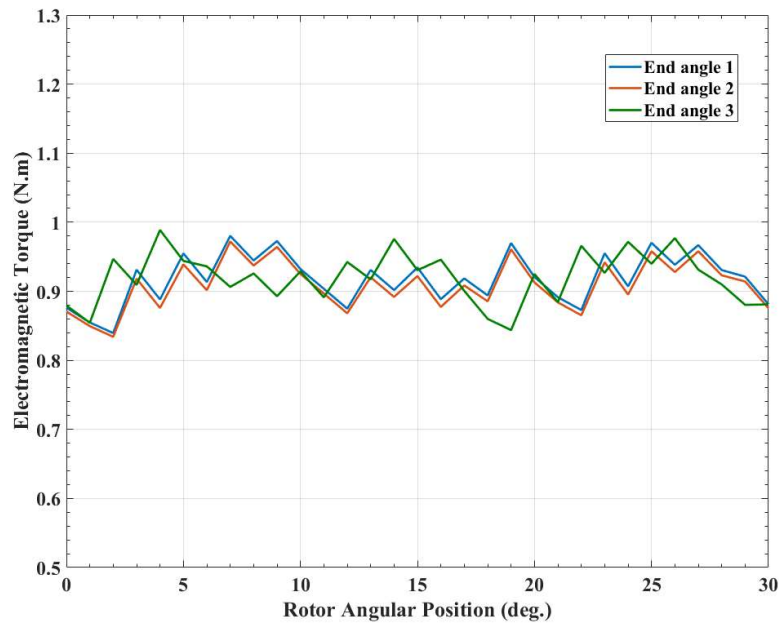


Fig. 2.12: Electromagnetic torque of SynRel motor varying end angle.

Fig. 2.12 shows the electromagnetic torque characteristics of the SynRel motor with a sensitivity end angle. The model is simulated for rotating 30 mechanical degrees, which means the same electromagnetic configuration appears every 60 electric degrees. It can be seen that the electromagnetic torques are performing

different waveforms with various flux barrier end angles. Besides, it can be computed that the torque ripple of these three cases is 15.33%, 15.17% and 15.77%, respectively. Hence, the flux barrier end angle can be set as sensitivity parameter for improving the torque ripple.

2.3.3 Impact of Barrier Rib

In this section, the barrier rib lengths are studied for its impact on the performance of the air-gap flux density, the electromagnetic force, and the electromagnetic torque. Besides, since the analytical model cannot reflect the influence of saturations appearing on the rib area between the barriers, and hence the FE simulation is applied here. For a better understand on the electromagnetic force comparison, and the extension to further vibration analysis, the FE model is analyzed by setting the motor at rated working condition, that the current amplitude is 5 A, the rotating speed is 5000 rpm. Then, the varying rib lengths of each flux barrier are listed in Tab. 2.4, followed by the FE results of air-gap flux density, electromagnetic torque, and electromagnetic force acting on the first tooth tip are presented. Hence, the impact of rib length is investigated and analyzed.

Table 2.4: Sensitivity barrier rib of each flux barrier.

Parameters	Symbol	Sensitivity values		
1 st Barrier rib	r_{b1}	0.8 mm	1.0 mm	1.2 mm
2 nd Barrier rib	r_{b2}	0.8 mm	1.0 mm	1.2 mm
3 rd Barrier rib	r_{b3}	0.8 mm	1.0 mm	1.2 mm

By considering the stator effects, Fig. 2.13 shows the air-gap flux density distribution under the cases of the sensitivity barrier ribs.

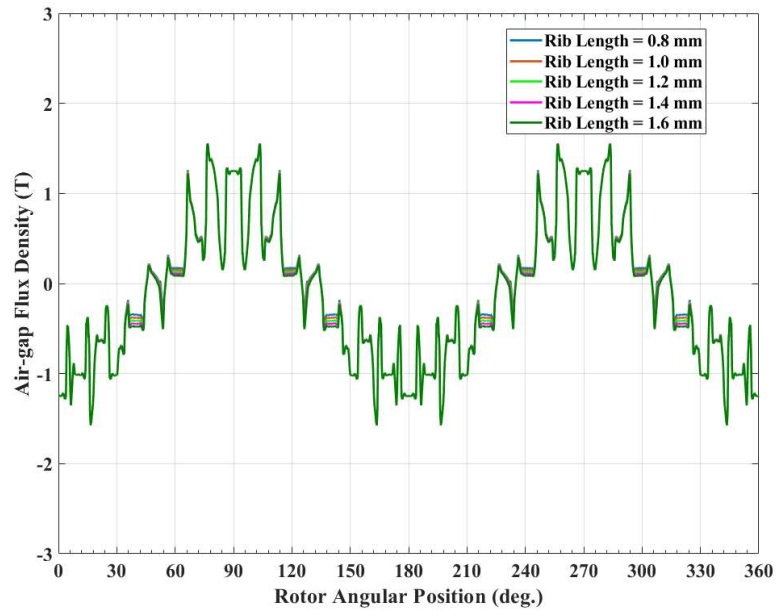


Fig. 2.13: Air-gap flux density of SynRel motor varying rib length.

Fig. 2.13 informs that the rib length has a slight influence on the distribution of air-gap flux density. Around the positions in the air-gap which are corresponding to the ribs, the air-gap flux density is decreasing with the rising rib length. It is mainly because of the saturation at the rib is lesser when the rib is getting wider. Besides that, the amplitudes in cases of all these rib lengths are performing a similar value, which is 1.25 T.

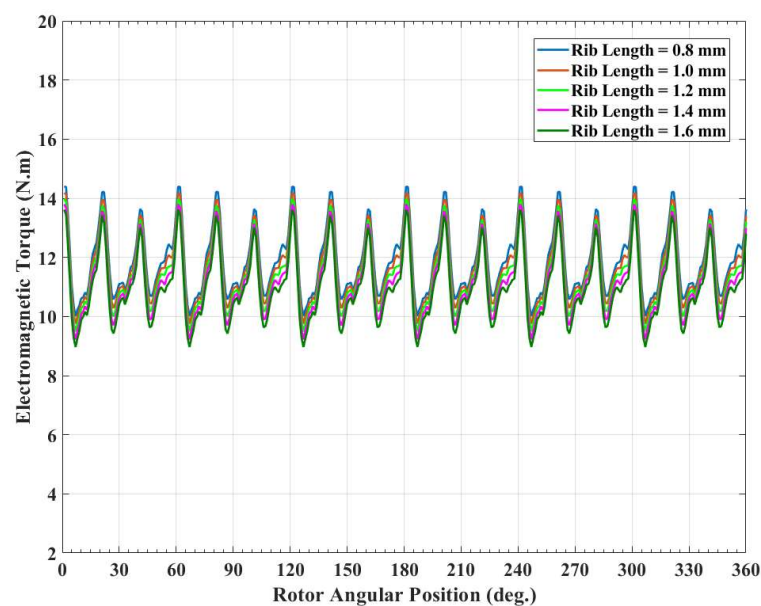


Fig. 2.14: Electromagnetic torque of SynRel motor varying rib length.

Then, Fig. 2.14 presents the torque versus the varying rib lengths. It is noted that among all these five conditions, the average torque is bigger with a smaller rib length. But the waveforms of the electromagnetic torques are performing similar distribution, which means their torque ripples are similar as well. Therefore, for obtaining a better electromagnetic torque, the rib length should be designed as small as possible when it is allowed by the rotor stiffness and saturation.

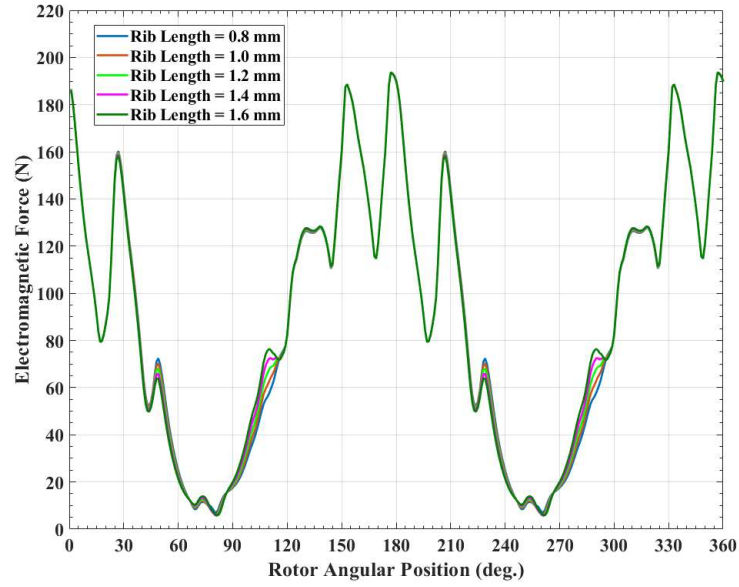


Fig. 2.15: Electromagnetic force of SynRel motor varying rib length.

Fig. 2.15 presents the electromagnetic force distribution, that the force acting on the first tooth tip is taken as an example to show the impact of rib length. It can be noticed that difference of the electromagnetic force distribution is occurring at the areas corresponding to the ribs. Besides, the average torque is bigger with a smaller rib length. Then, by considering the expression of electromagnetic force calculated in Eq. (2.50), the distribution of electromagnetic force is relating to it of the air-gap flux density. It can also be noted from Fig. 2.15 that it is responding to Fig. 2.13, which is the distribution of air-gap flux density against the varying rib length.

2.4 Summary

This chapter proposed an analytical model of the concentric SynRel motor based on its equivalent magnetic network, followed by the derivation and computation of the electrical characteristics of which the air-gap flux density, electromagnetic force and electromagnetic torque were highlighted. Then the analytical results were compared to them from FEA, it informs that they were well meeting with each other, that the proposed analytical model was precise.

By utilizing the analytical model, the key parameters of the rotor geometry were analyzed. It illustrated that the number of flux barrier per pole was mostly influencing the average torque, that three layers of flux barrier per pole produced more torque with respect to one and two layers of flux barrier per pole. Besides, the end angle and the rib length of each flux barrier were affecting the distributions of the electromagnetic torque and force. The amplitudes were close, while the difference was occurring at the rotor angular position corresponding to the sensitivity rotor geometry.

Chapter 3

Investigation of Vibration Level in SynRel Motor

This chapter is aiming to investigate the vibration phenomena of the SynRel motor through FEA. As for SynRel motor, which normally has a more fluctuant waveform of electromagnetic force and torque distributed in the airgap, it usually has larger noise and vibration with respect to the other types of electrical machines. The SynRel motor used in Chapter 2 is taken as a case study to discover the vibration characteristics, including the natural frequency, modal shape, stiffness, vibration, and noise level. It is aforesaid that these characteristics are strongly relating to the motor structure, specifically for the SynRel motor, in which the rotor is usually designed as a relatively complex structure for getting a better electrical performance. Particularly, the sizes of the ribs, bridges, and the flux barriers inside the rotor are generally set as very small values. Therefore, it brings a new challenge into the geometrical optimization of the SynRel motor. Then, for a better understanding of the mechanical performance, the analyses of modal, stress, vibration, and noise are performed.

Therefore, aiming to explore the relations between the geometrical parameters and its vibration characteristics, and improve the preliminary design process, the sensitivity analyses are developed in this chapter to find out the relations between the motor structure and the mechanical characteristics.

3.1 Modal Analysis

Modal analysis is the foremost critical factor to be considered during the mechanical design for exploring its intrinsic structural properties and improving its mechanical performance. As for the modal analysis results, the most crucial parts are the modal shape and natural frequency which are used for predicting the resonance, evaluating the structural characteristics, investigating the vibration resource, and recognizing the design bug. Moreover, the modal analysis results are greatly relating to the mechanical properties of the material used in motor, for instance the mass and stiffness. Hence, it is important to determine the impact of geometrical parameters on modal shape and natural frequency. Basically, there are two methods to perform the modal analysis, which are analytical computation and FEA. As the analytical computation is lack of computing accuracy [82], the FEA method is adopted in this section. Besides, the rotor geometry is relatively complex, and its geometrical parameters are usually set as small values, therefore the change of rotor geometry will marginally influence the whole mechanical properties. As a result, this section is focusing more on the stator geometry by considering the outer stator diameter, back-iron thickness, tooth width, and slot height.

3.1.1 Mathematic Derivation of Modal Equation

It is aforesaid the modal analysis is for investigating the intrinsic characteristics of the motor, therefore the analyzing system is in a free mode which means there is no exciting force acting on this system. Hence, by applying Eq. (1.19) in the modal analysis of the SynRel motor, and letting the exciting force as 0, Eq. (1.19) goes to

$$\mathbf{M}\ddot{x} + \mathbf{C}\dot{x} + \mathbf{K}x = 0, \quad (3.1)$$

from where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ represent the matrix of the mass, the damping, and the stiffness coefficient, respectively. As for the stator core and the rotor core of the motor, their structural damping of is normally quite low, therefore it is ignored which means $\mathbf{C} = \{0\}$. Then, by substituting Eq. (1.19) into Eq. (3.1), it leads to

$$(\mathbf{K} - \omega^2 \mathbf{M})\{x\} = 0. \quad (3.2)$$

For solving the equation above, there are two cases:

(1) $\{x\} = \{0\}$. In this case the studied object has no vibration since there is not any vibrating displacement, therefore this case is not discussed here.

(2) $\det(\mathbf{K} - \omega^2 \mathbf{M}) = 0$. For this characteristic polynomial, it can be solved with i ($i = 1, 2, 3, \dots$) eigenvalues as $(\omega_1^2, \omega_2^2, \omega_3^2, \dots, \omega_i^2)$ which is depending on an n - order matrix. Hence, the natural frequency of the system can be calculated as

$$\{f_i\} = \frac{\{\omega\}}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{\mathbf{K}}{\mathbf{M}}}. \quad (3.3)$$

3.1.2 Modal Analysis of Stator

The main geometrical parameters of the stator are listed in Tab. 3.1, and the initial stator model is shown in Fig. 3.1. Then, the sensitivity modal analysis is developed based on those geometrical parameters to investigate the impact of the geometry on the modal shape and natural frequency of the stator.

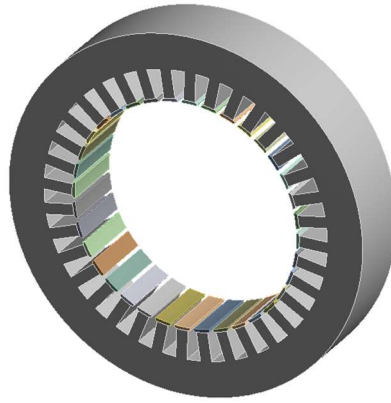


Fig. 3.1: 2D cross section of stator geometry.

Table 3.1: Main geometrical parameters of stator.

Parameter	Value
Outer diameter	200 mm
Inner stator diameter	125 mm
Number of slots	36
Slot opening width	2.2 mm
Slot opening height	1 mm
Slot height	16.5 mm
Back-iron thickness	20 mm

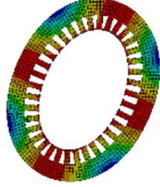
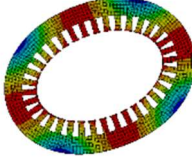
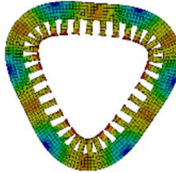
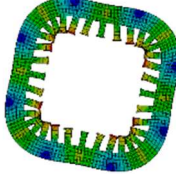
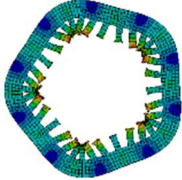
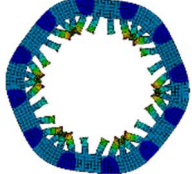
The 3D model of the stator is built as shown in Fig. 3.1. Besides, Tab. 3.2 presents the material properties used in the stator core and the rotor core. After that, the mesh is mandatory set as Hex dominant, which means the shape of every mesh is hexagon.

Table 3.2: Material properties of stator and rotor core.

Parameter	Value
Mass density	7600 kg/m ³
Young's modulus	200 GPa
Poisson ratio	0.3

Then, Tab. 3.3 presents the modal analysis results of this stator, including the mode deforming shape and the natural frequency.

Table 3.3: Modal analysis results of SynRel motor stator core.

Mode number	Mode shapes	Natural frequency (Hz)
1		1273.9
2		1274.1
3		3363
4		5883
5		8331
6		10098

Tab. 3.3 shows the modal shapes, and their natural frequencies at first six orders. The 1st order natural frequency is very close to it of the 2nd order, which is around 1274 Hz, but their modal shapes are displaying different style. From the 3rd order to 6th order, the natural frequency keeps increasing and the structure deforming becomes more obvious.

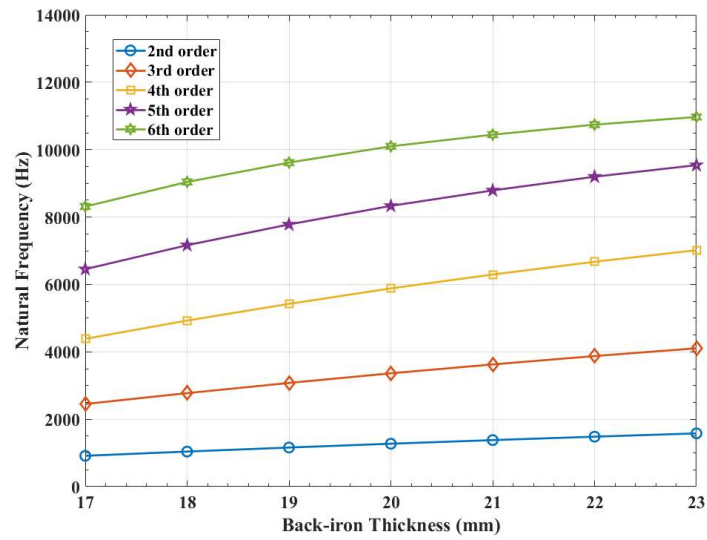


Fig. 3.2: Natural frequency of SynRel motor varying back-iron thickness.

In Fig. 3.2, it illustrates the natural frequency of the stator core against the back-iron thickness. It informs that the natural frequencies on each order are increasing with the increasing back-iron thickness. Because when the back-iron becomes thicker, the mass of the whole motor increases, as well as the stiffness of the stator. But the increase of stiffness is much larger than the increase of mass, so the natural frequency rises with the growing back-iron thickness.

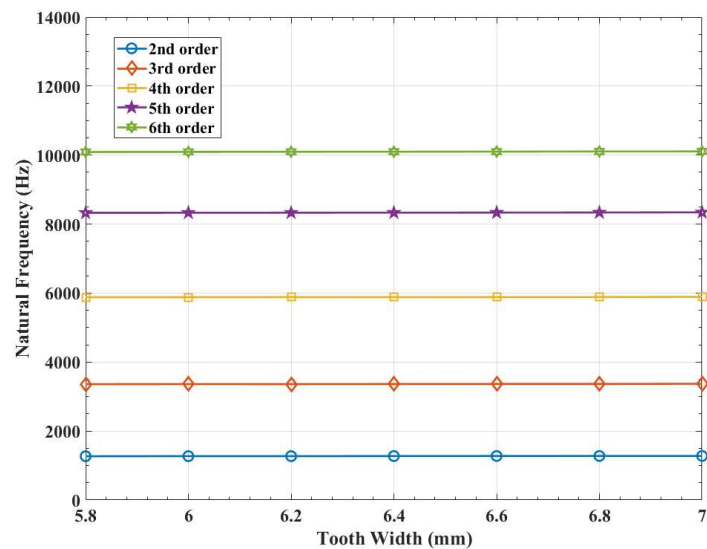


Fig. 3.3: Natural frequency of SynRel motor varying tooth width.

Fig. 3.3 describes the relations between the natural frequency and the tooth width. When the tooth width increases, the i^{th} order natural frequency ($i = 2, 3, 4, 5, 6$) is slightly increasing. However, the tooth width has an insignificant influence on the natural frequency. It is mainly because of the slight increase of the mass and stiffness with the increasing tooth width.

In Fig. 3.4, it shows the relations between natural frequency against slot height, that the natural frequency is not that sensitivity to the slot height. When the slot height rises, the back-iron thickness is accordingly reducing. Hence, the mass and the stiffness of the whole system are decreasing. Hence, the natural frequency does not change too much.

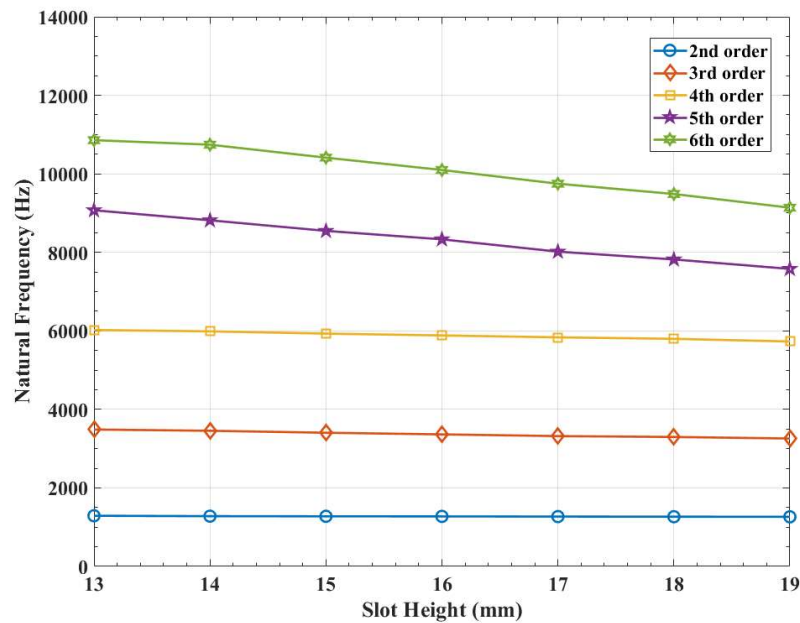


Fig. 3.4: Natural frequency of SynRel motor varying slot height.

Above all, the natural frequency of the stator is principally decided by the back-iron thickness. For two different stators having the same outer diameter, the natural frequency is higher with a thicker back-iron. However, the change of tooth width and slot height have a tiny influence on the natural frequency. As a result, to avoid

the serious resonance at some sensitivity frequency point, the back-iron thickness is the best parameter to optimize the motor and prevent the resonance.

3.1.3 Modal Analysis of Rotor

After the modal analysis of the stator, the rotor is analyzed to investigate its modal shape and natural frequency. For the rotor of SynRel motor, it is much more complex and fragile with respect to the rotors of other types of electrical motors. The geometrical parameters of the rotor are listed in Tab. 3.4. Fig. 3.5 shows the FE model of the rotor. Tab. 3.5 shows the modal analysis results of the rotor core.

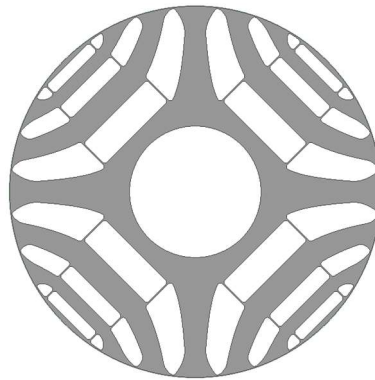
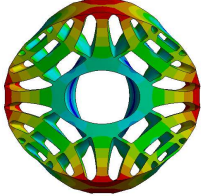
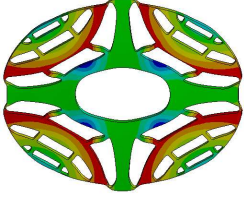
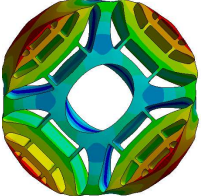
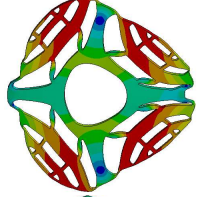
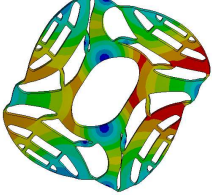
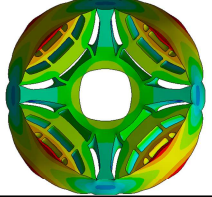


Fig. 3.5: 2D cross section of rotor geometry.

Table 3.4: Main geometrical parameters of rotor.

Parameter	Value
Rotor diameter	124.3 mm
Shaft diameter	44 mm
1 st barrier end angle	28°
2 nd barrier end angle	52°
3 rd barrier end angle	76°
1 st barrier thickness	3 mm
2 nd barrier thickness	6 mm
3 rd barrier thickness	10 mm

Table 3.5: Modal analysis results of SynRel motor rotor core.

Mode number	Mode shapes	Natural frequency (Hz)
1		3538
2		4347
3		5845
4		6729
5		7028
6		8422

Tab. 3.5 displays the modal shapes and natural frequencies of the rotor on i^{th} order ($i = 1, 2, 3, 4, 5, 6$). Unlike the modal analysis results of the stator, the modal shape of the rotor does not have too much deforming, which is due to the lower stiffness and mass comparing to the stator.

3.1.4 Modal Analysis of Whole System

After analyzing the stator and rotor separately, the whole motor is analyzed for its natural frequency which is shown in Tab. 3.6.

Table 3.6: Natural frequency of stator and the whole motor.

Orders	Natural Frequency (Hz)	
	Stator	Whole Motor
2	1274	2140
3	3363	5724
4	5883	9218
5	8331	11763
6	10098	13741

From Tab. 3.6, it can be recognized that the natural frequency of the whole motor is much higher than it of the stator. Besides, after the stator, rotor, and all the other components are assembled together as a whole part, the natural frequency of the motor is increasing. Hence, the resonance frequency of the motor cannot be determined by the modal analysis of the stator only.

3.2 Stress Analysis

3.2.1 Stress Distribution on Rotor

When the motor is working, the rotor is suffering a group of external forces, including the centrifugal force, electromagnetic force, fluid air force, and thermal stress. Among the sources of the external forces, the centrifugal force is playing a dominate role in most of the cases, in terms of the rotor stress analysis. Hence, the centrifugal force is taken as the studying object in this section to investigate the stress distribution of the SynRel rotor.

Due to the narrow bridges and ribs inside the rotor of the SynRel motor, the rotor geometry is relatively fragile. When the rotor is rotating at different speed, the internal parts of the rotor is mainly affected by the centrifugal force. Hence, in this section, the whole rotor is analyzed for its stress distribution under different rotating speeds. Firstly, the stress distribution of the original rotor geometry under rated speed, which is 5000 rpm, is shown below.

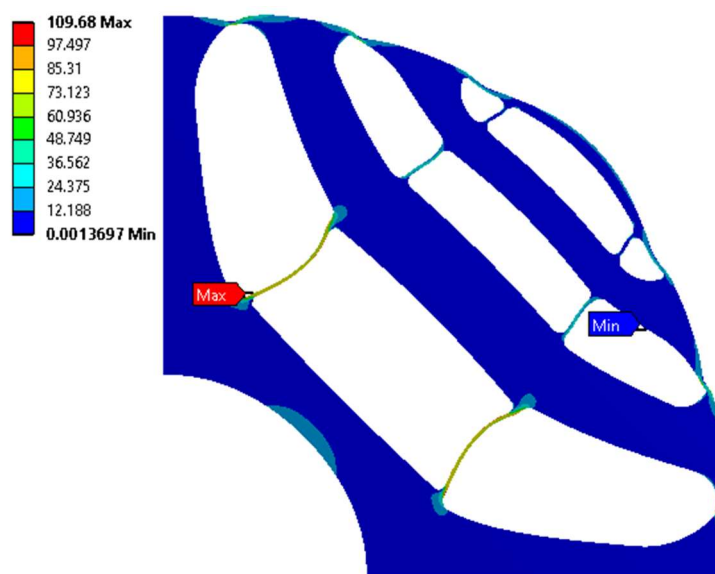


Fig. 3.6: Stress distribution of the rotor at 5000 rpm.

From Fig. 3.6, it illustrates that the maximum stress is 109 MPa, occurring at the middle area of the third flux barrier. Some other serious stresses are occurring at the other ribs of the first and second flux barriers, respectively. Considering the ribs are critical in the electrical performance, it reveals that the lengths of ribs must be principally considered during the preliminary design for its stress safety.

3.2.2 Sensitivity Analysis Varying Barrier Rib Length

It is discovered that the maximum stresses are appearing at the ribs in the middle of barrier. Hence, the ribs of each flux barrier are set as sensitivity parameter to investigate its impact on the stress performance.

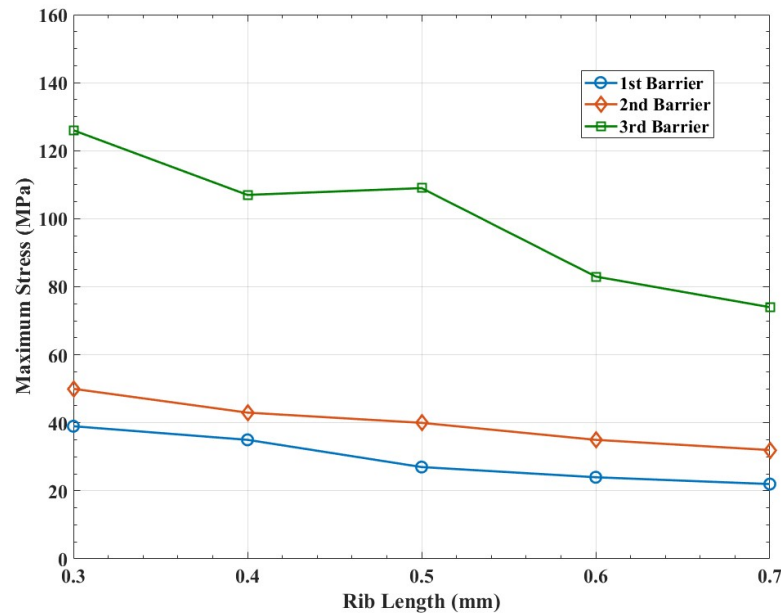


Fig. 3.7: Maximum stress of SynRel motor varying rib length.

From Fig. 3.7, it informs that the area closer to the rotor centre suffering the larger stress. When the rib length is 0.5 mm, the maximum stress of the 3rd flux barrier is 109 MPa, while it of the 1st and 2nd flux barrier is 27 MPa and 40 MPa, respectively. Besides, with an increasing rib length, the stiffness of the rotor is increasing as well, and therefore the maximum stress is decreasing at all these flux barriers. Especially for the 3rd flux barrier, the decrease of the maximum stress is more obvious comparing to the other flux barriers. Above all, in term of the stress performance, the flux barrier, which is closer to the rotor centre, should be paid more attention and consideration during the preliminary design process.

3.2.3 Sensitivity Analysis Varying Rotating Speed

Then, as the stress of the rotor is mainly influenced by the centrifugal force which is determined by the rotating speed and the mass of the rotor. The rotating speed is set from 1000 rpm to 10000 rpm, with a step as 1000 rpm to find out the maximum stress under each specific rotating speed.

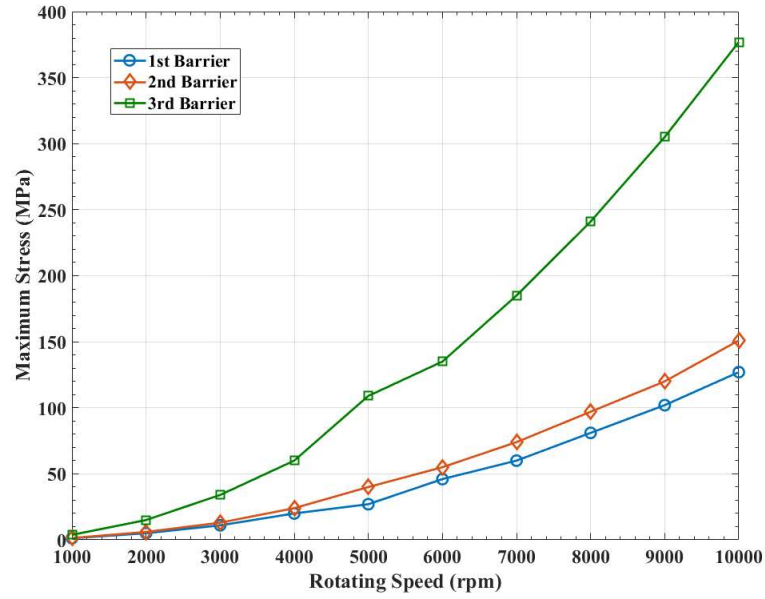


Fig. 3.8: Maximum stress of SynRel motor varying rotating speed.

Fig. 3.8 shows the maximum stress on each rib against the rotating speed. It illustrates that the maximum stress is exponentially increasing with the rotating speed. When the rotor is rotating at a low speed, the stress on the rib of i^{th} barrier ($i = 1, 2, 3$) is similar to each other. However, at the full rotating speed range, the stress on the rib of the 3rd barrier is increasing much more significant with respect to the stress on the ribs of the 1st and 2nd barrier, respectively. It also proves that the area of the rotor closer to the rotor centre suffers a more serious stress comparing to the other parts of the rotor. Furthermore, the stress on the ribs of 1st and 2nd barrier have a similar increasing rate. Hence, the parameters strongly relating to the stress, which are the mass, mechanical properties of the rotor core, and the designed rotating speed, have to be considered during the design of the SynRel motor, especially for the high-speed ones.

3.3 Harmonic Response of Vibration

Besides the centrifugal force acting on the rotor, the electromagnetic force produced in the airgap is playing another critical role in the mechanical performance

of the SynRel motor. As discussed in Chapter 1, the radial electromagnetic force is the main source of the motor vibration, particularly for this analyzing SynRel motor which has a small size and a lower stiffness. Besides, for the main parts of the motor, their damping coefficient is nearly 0. Hence, the smaller motor, which has a smaller stiffness coefficient, will have a more significant vibration.

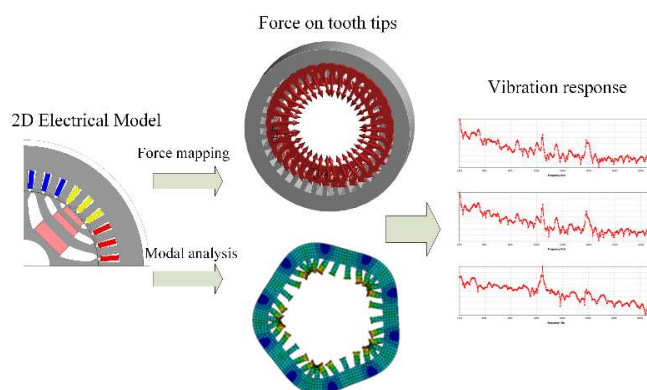


Fig. 3.9: Flowchart of vibration investigation.

To investigate the vibration level, a multi-physic model of the SynRel motor is proposed. As shown in Fig. 3.9, the electromagnetic forces acting on the inner surface of the stator are computed through a 2D FEA. Then, the resulted electromagnetic forces are transferred automatically into the mechanical FE model to investigate the harmonic response of the vibration.

As the mechanical FEA must be aligning with the time step set in electromagnetic FEA, the sampling frequency of the mechanical FEA is double considered to ensure the accuracy of the mechanical simulation. Since the time steps in electromagnetic simulation is 360, and the rotating speed is 5000 rpm, a time step is $0.006/360 = 16.7 \mu\text{s}$, and therefore the sampling frequency in the mechanical FEA is 30000 Hz.

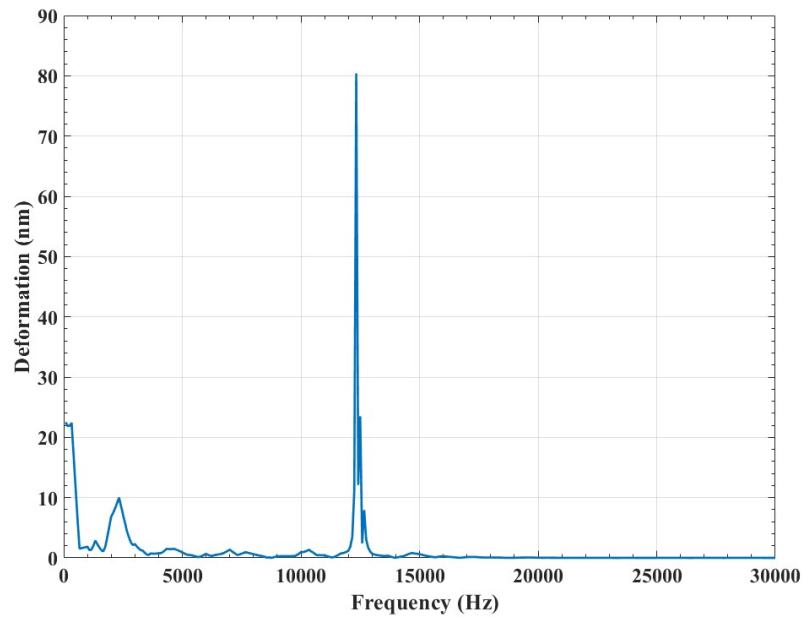


Fig. 3.10: Deformation of the SynRel motor at 5000 rpm.

Fig. 3.10 shows the deformation displacement of the motor housing at 5000 rpm. The maximum deformation appears around 12500 Hz, which meets with the resulted natural frequency. At the high frequency range, which is over 15000 Hz, it is noted that the deforming displacement is very small and stable. It proves that the motor is vibrating more serious at low frequency range, but more stable at high frequency range, and therefore the electromagnetic force harmonics at frequency range higher than 15000 Hz can be ignored in the study of motor vibration.

Fig. 3.11 notes that these 5 peak accelerations, which are 0.88 g, 0.96 g, 0.35 g, 1.23 g, and 0.39 g ($g = 9.8 \text{ m/s}^2$), are occurring around 6000 Hz, 9200 Hz, 12000 Hz, 13700 Hz, and 20000 Hz, respectively. By considering the natural frequencies resulted in Tab. 3.6, they are well agreed with the natural frequency where the peak vibration happens. Among these 5 peak accelerations, the worst housing acceleration is 1.23 g occurred around 13700 Hz, which is the 6th order natural frequency.

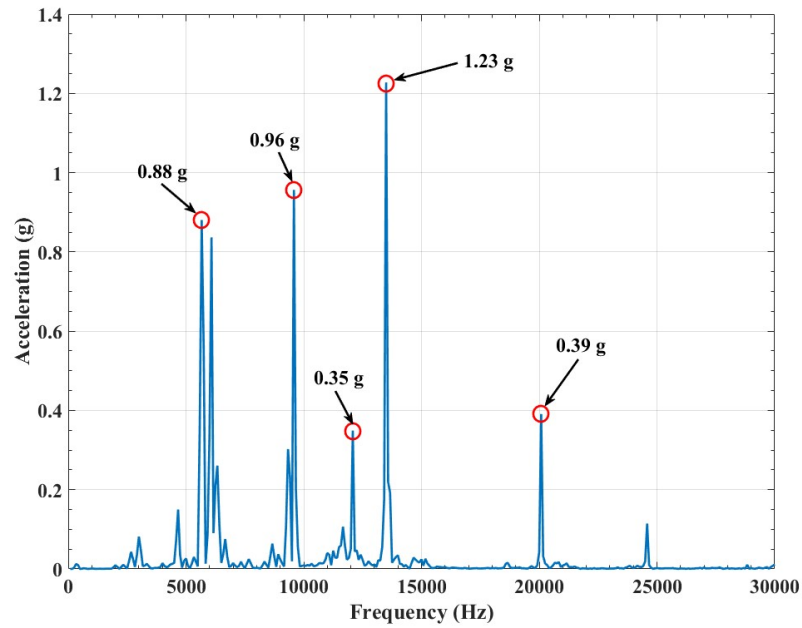


Fig. 3.11: Housing acceleration of the SynRel motor at 5000 rpm.

3.4 Summary

In this chapter, a generic approach for analyzing the mechanical characteristics, which included the modal shape, natural frequency, stress tensor, and vibration level, of the SynRel motor was developed. The modal analysis was conducted for the stator, the rotor and the whole motor, respectively, to find their modal shape and natural frequency. A sensitivity analysis was performed by varying back-iron thickness, tooth width and slot height, which explored that the back-iron thickness was holding the largest influence on the natural frequency. Then, the stress analysis was conducted to check the tensile strength of the rotor structure. By sensitivity analyzing the rib length and rotating of the rotor, it revealed that the maximum stress was smaller with wider rib length, while it was exponentially increasing with the rising speed. Finally, a general methodology of the vibration analysis on the SynRel machine was introduced. The electromagnetic forces obtained from 2D electromagnetic simulation were imported to be acting on each tooth tip as an excitation of the vibration. The housing acceleration was adopted to represent the

vibration level of the whole system, and the peak vibration (1.23 g) was occurring around the 6th order of the natural frequency.

Chapter 4

Impact of PM on Motor Performance

In the PMaSynRel motors, low-energy permanent magnet (PM) is adopted and placed in flux barriers in order to improve the electrical performance and keep a relatively low cost of the reluctance motors.

As the electromagnetic characteristics are changing with the inserted PM, which also brings a potential issue of the vibration to this kind of motors. Hence, this chapter is aiming to investigate the impact of PM on the SynRel motor in terms of its electrical and vibrational performance. Then, the SynRel motor and the PMaSynRel motor are compared for their electromagnetic characteristics and the vibration level.

It starts with an electromagnetic performance analysis, from where the electromagnetic torque and the electromagnetic force of PMaSynRel motor are compared with them of SynRel motor to investigate the impact of PM on the electromagnetic performance on SynRel machine. After that, the vibration analysis of the PMaSynRel motor is conducted to investigate the impact of PM on their vibration characteristics. Finally, a sensitivity analysis is proposed to understand the impact of PM with different size on the vibration characteristics.

4.1 Electrical Performance of PMaSynRel Motor

As for keeping both a relatively low cost, and a high efficiency of the PMaSynRel, ferrite is adopted and inserted inside the flux barriers to improve the electrical performance. The material properties of ferrite have an effect on the motor performance, for instance the mass density, relative permeability, and so on. In this section, the electrical performance of the PMaSynRel motor is studied, and therefore its B - H curve is shown in Fig. 4.1 as a reference for the following studies.

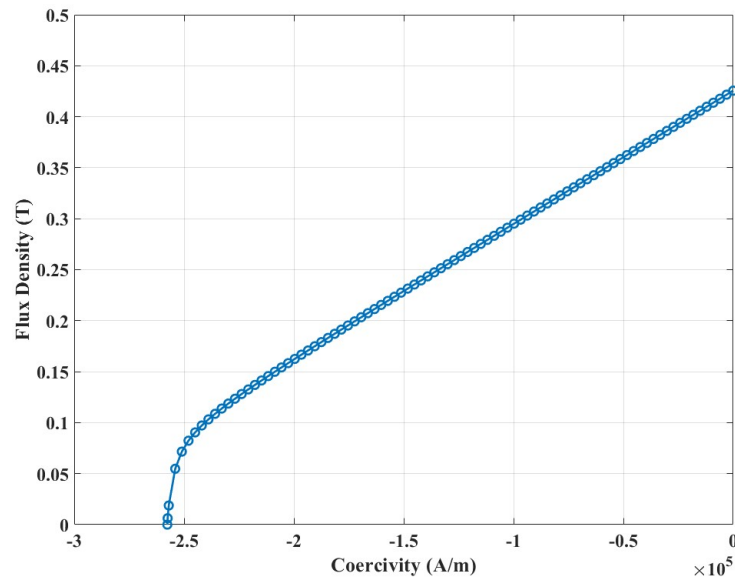


Fig. 4.1: B - H curve of the ferrite PM.

4.1.1 Analytical Modelling Considering Permanent Magnet

The geometrical data of the studied PMaSynRel motor used in this case study are listed in Table 4.1.

Table 4.1: Geometrical data of the PMaSynRel motor.

Parameters	Symbol	Value
Stator outer diameter	OD	200 mm
Stator inner diameter	D_s	125 mm

Rotor diameter	D	124.3 mm
Stack length	L_{stk}	40 mm
Number of slots	Q_s	36
Number of pole pairs	p	2
Airgap	L_g	0.35 mm
End angle of 1 st barrier	$2\theta_{b1}$	25.8°
End angle of 2 nd barrier	$2\theta_{b2}$	51.4°
End angle of 3 rd barrier	$2\theta_{b3}$	77.2°
Thickness of 1 st PM	t_{m1}	3 mm
Thickness of 2 nd PM	t_{m2}	6 mm
Thickness of 3 rd PM	t_{m3}	10 mm
Width of 1 st PM	w_{m1}	20 mm
Width of 2 nd PM	w_{m2}	26 mm
Width of 3 rd PM	w_{m3}	30 mm

By considering the PM used in SynRel motor, the proposed analytical model of the SynRel motor is developed by adding the consideration of PM, which is shown in Fig. 4.2.

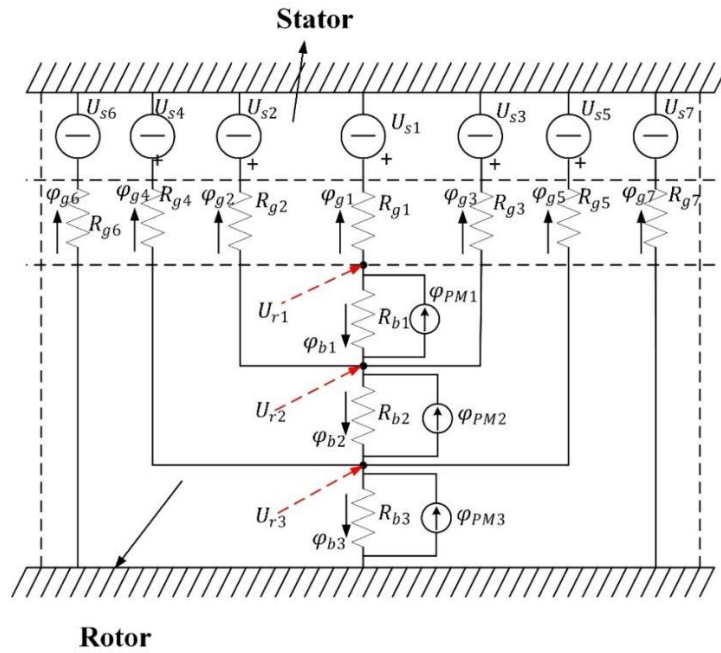


Fig. 4.2: Equivalent magnetic circuit of PMSynRel motor.

In Fig. 4.2, U_s and U_r are the magnetic potential of stator and rotor, respectively; φ_g is the flux flowing through the airgap, while φ_b is the flux flowing through the flux barrier; φ_{PM} is the flux produced by PM, and R_g is the magnetic reluctance of the airgap, while R_b is the magnetic reluctance of flux barrier.

Fig. 4.3 shows a sketch of a single layer flux barrier with inserted PM. Unlike it is in SynRel motor, the flux barrier is divided into two three parts by the inserted PM, that the left and right part are parallel connected with the PM.

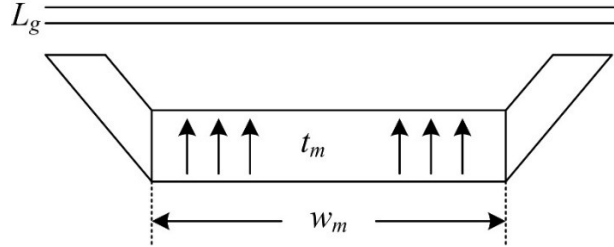


Fig. 4.3: Sketch of a single layer flux barrier of PMaSynRel motor.

Similar to the analytical derivation of the SynRel motor, the lumped parameter of the PM, including the flux source φ_{PM} and the PM reluctance R_{PM} . In addition, once the size and shape of the PM is set up, the no load flux density of each PM can be derived and calculated. Besides, the sizes of the flux barriers inserting the PMs are assumed the same. Then, the flux source φ_{PM} and PM reluctance R_{PM} can be given by

$$\varphi_{PM} = B_{rem} w_m L_{stk}, \quad (4.1)$$

$$R_{PM} = \frac{t_m}{\mu_{rem} \mu_0 w_m L_{stk}}. \quad (4.2)$$

Then, according to Fig. 4.3, the reluctance of the barrier except the PM is given by

$$R_{bm} = \frac{t_b}{\mu_0(l_b - w_m)L_{stk}}. \quad (4.3)$$

As for the analytical model at no load condition, the equivalent magnetic circuit can be simplified and led to Fig. 4.4.

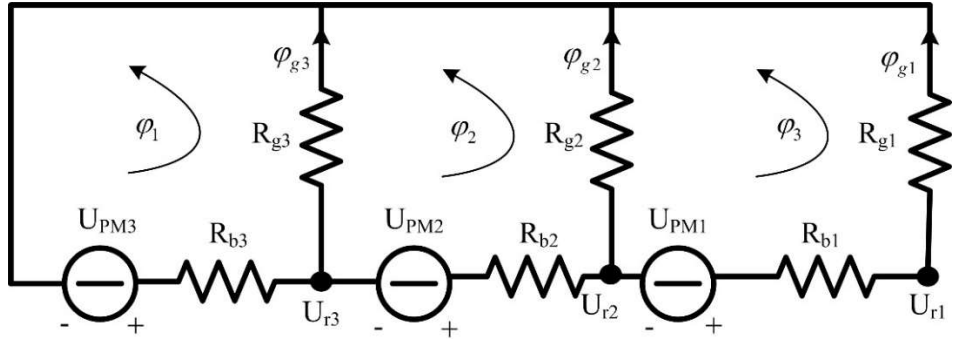


Fig. 4.4: Simplified equivalent magnetic circuit of PMaSynRel motor at no load.

Based on the magnetic circuit shown in Fig. 4.4, by applying the Kirchhoff's voltage law, the fluxes ϕ_1 , ϕ_2 , and ϕ_3 are calculated by using

$$\begin{bmatrix} U_{PM1} \\ U_{PM2} \\ U_{PM3} \end{bmatrix} = \begin{bmatrix} R_{g1} + R_{g2} + R_{b1} & -R_{g2} & 0 \\ -R_{g2} & R_{g2} + R_{g3} + R_{b2} & -R_{g3} \\ 0 & -R_{g3} & R_{g3} + R_{b3} \end{bmatrix} \cdot \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}, \quad (4.4)$$

where, R_{gi} represents the magnetic reluctance of the airgap part corresponding to i -th flux barrier, R_{bi} represents the magnetic reluctance of i -th flux barrier including the PM part. Then, R_{gi} and R_{bi} can be expressed as

$$R_{g_i} = \frac{L_g}{\mu_0(\theta_{b_i} - \theta_{b_{i-1}})DL_{stk}}, \quad (4.5)$$

$$R_{b_i} = \frac{R_{bm_i} R_{PM_i}}{R_{bm_i} + R_{PM_i}}. \quad (4.6)$$

By assuming μ_{rem} equals to 1, and then substituting (4.2) and (4.3) into (4.6), it is given by

$$R_{b_i} = \frac{t_{b_i}}{\mu_0 l_{b_i} L_{stk}}. \quad (4.7)$$

Furthermore, according to Fig. 4.4, the flux φ_{g_i} of the airgap corresponding to i -th flux barrier is given by

$$\varphi_{g_i} = \varphi_i - \varphi_{i-1}. \quad (4.8)$$

Then, the rotor magnetic potential U_{r_i} is given by

$$U_{r_i} = \varphi_{g_i} R_{g_i}. \quad (4.9)$$

The flux flowing through i -th flux barrier can be derived and calculated based on Fig. 4.4 as

$$\varphi_{b_i} = \frac{U_{r_i} - U_{r_{i+1}}}{R_{bm_i}}. \quad (4.10)$$

Hence, the no load flux density of i -th PM is calculated through

$$B_{mo_i} = \frac{\varphi_{PM_i}}{w_{m_i} L_{stk}}, \quad (4.11)$$

where, φ_{PM_i} is the PM flux which is calculated in Eq. (4.1).

Fig. 4.5 and Fig. 4.6 show the flux density map and airgap flux density against rotor position of the PMaSynRel motor, respectively.

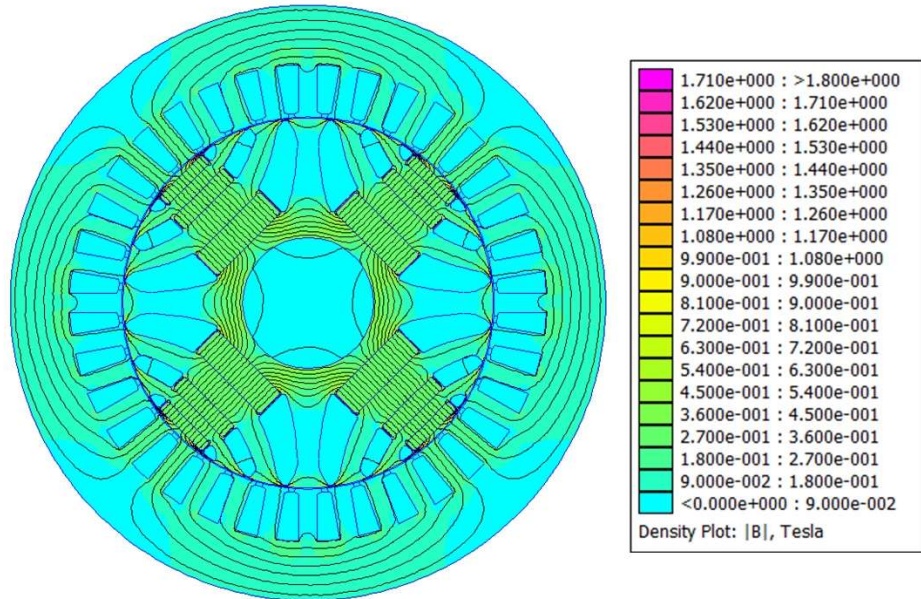


Fig. 4.5: Flux density map of PMaSynRel motor at no load.

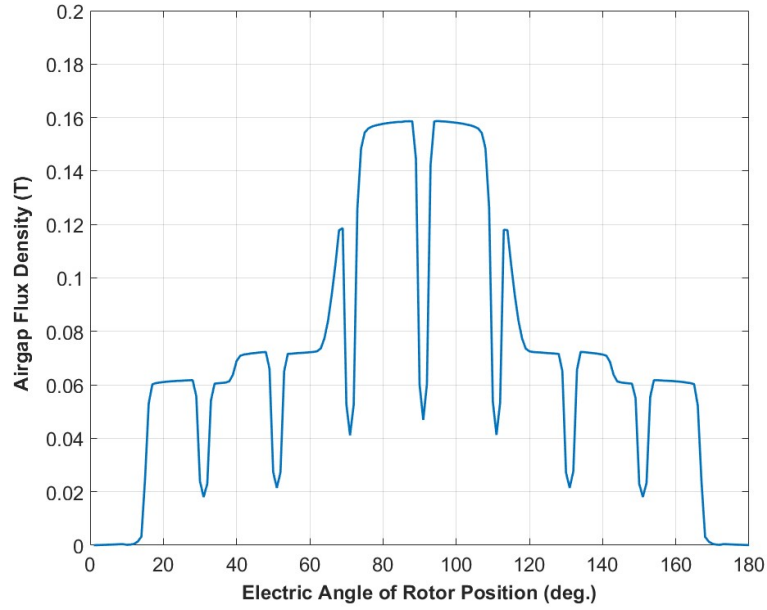


Fig. 4.6: Airgap flux density distribution of PMaSynRel motor at no load.

After the computation of the no load flux density, the influence of the stator magnetic potential is added into the analytical model to predict the completed airgap flux density. The equivalent magnetic circuit of a single pole of the PMaSynRel motor can be extended from Fig. 4.4 to Fig. 4.7, by adding the stator magnetic potential into consideration.

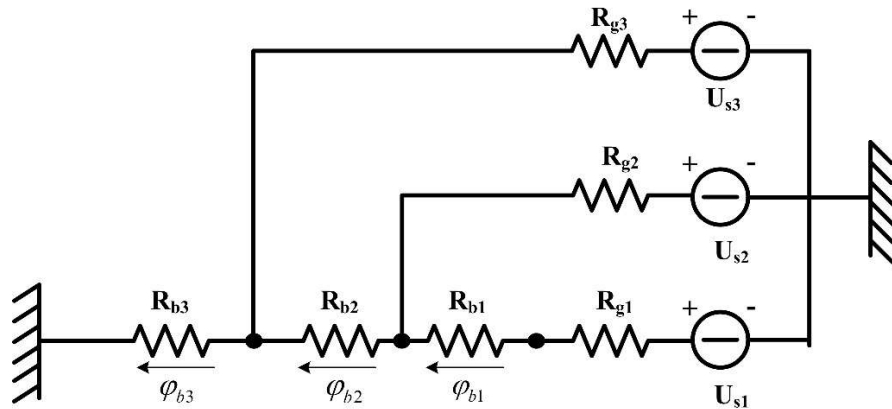


Fig. 4.7: Simplified equivalent magnetic circuit of PMaSynRel motor.

Similar to the derivation of rotor magnetic potential, the stator magnetic potential can be derived from Fig. 4.7 as shown below.

$$\begin{bmatrix} U_{s_1} - U_{s_2} \\ U_{s_2} - U_{s_3} \\ U_{s_3} \end{bmatrix} = \begin{bmatrix} R_{g_1} + R_{g_2} + R_{g_3} & -R_{g_2} & 0 \\ -R_{g_2} & R_{g_2} + R_{g_3} + R_{b_2} & -R_{g_3} \\ 0 & -R_{g_3} & R_{g_3} + R_{b_3} \end{bmatrix} \times \begin{bmatrix} \varphi_{b_1} \\ \varphi_{b_2} \\ \varphi_{b_3} \end{bmatrix}, \quad (4.12)$$

where, U_{s_i} represents the average value of the stator magnetic potential corresponding to the i -th flux barrier, which is expressed as

$$U_{s_i} = \frac{\sin \theta_{b_i} - \sin \theta_{b_{i-1}}}{\theta_{b_i} - \theta_{b_{i-1}}} \cdot \hat{U}_s, \quad (4.13)$$

where, $\hat{U}_s$ is the peak value of the stator magnetic potential, which is introduced and calculated in Chapter 2.

Then, Fig. 4.8 and Fig. 4.9 show the airgap flux density map and the airgap flux density against the rotor position, respectively, of the PMaSynRel motor. Again, the airgap flux density B_{mg} can be calculated by Eq. (2.40), by using the rotor magnetic potential calculated in Eq. (4.9), and the stator magnetic potential calculated in Eq. (4.13) of the PMaSynRel motor.

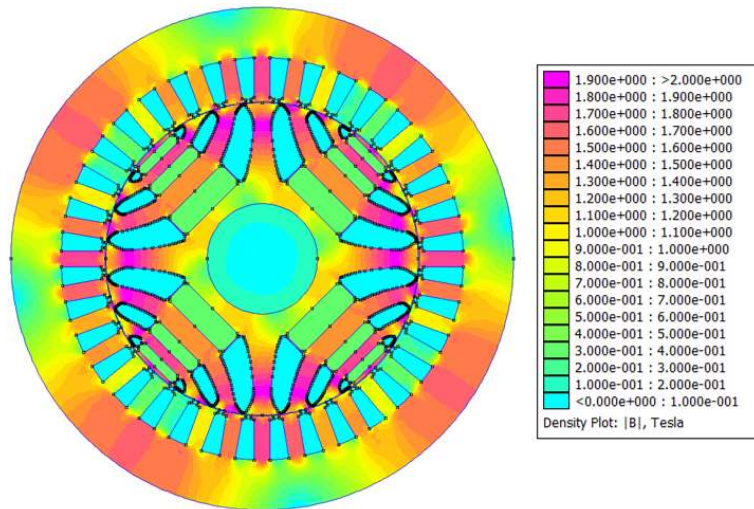


Fig. 4.8: Flux density map of PMaSynRel motor.

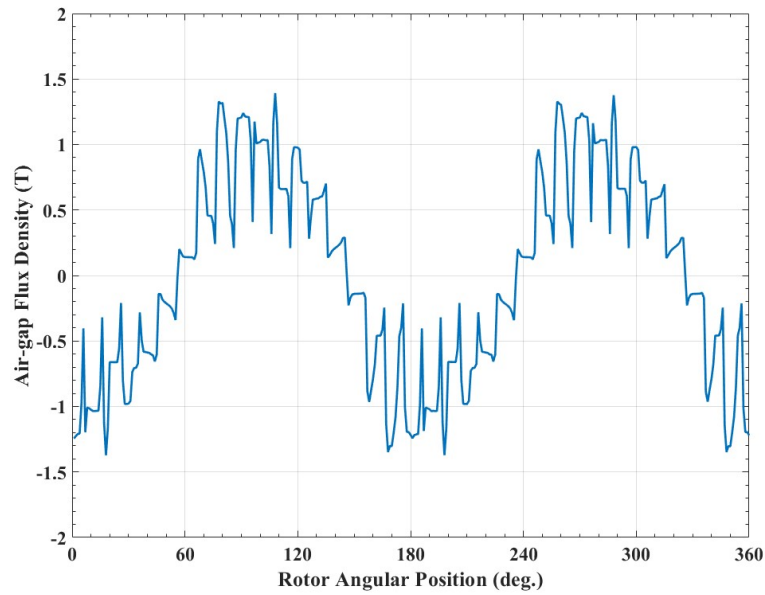


Fig. 4.9: Air-gap flux density distribution of PMaSynRel motor.

In Fig. 4.8 and Fig. 4.9, the flux density of the PMaSynRel motor is presented by adding the influence of the stator current, which is developing from Fig. 4.5 and Fig. 4.6. It can be seen that the peak flux density appears at the areas between each two flux barriers, which is about 2 T. Hence, these places should be double checked and considered for avoiding the saturation. From Fig. 4.9, which is the distribution waveform of the air-gap flux density under loading condition. It informs that the peak value is around 1.4 T, and the waveform is much more fluctuant with respect to it under no load condition.

4.1.2 Impact on Electromagnetic Torque

To understand and analyze the impact of ferrite PM on the output torque, the electromagnetic torque of the PMaSynRel motor is analyzed and presented in this section, and then compared with the electromagnetic torque produced in the SynRel motor.

After calculating the airgap flux density of the PMaSynRel motor is the last section, it can be substituted into the analytical equation of the electromagnetic torque Eq. (2.41), which goes to

$$\tau_{PM} = -\frac{D^2}{4} \int_{\omega t}^{\omega t + 2\pi} B_{mg}(\theta_s) K_s(\theta_s) L_{stk} d\theta_s, \quad (4.14)$$

where, τ_{PM} represents the Lorentz force density.

Then, Fig. 4.10 shows the electromagnetic torque of the PMaSynRel motor.

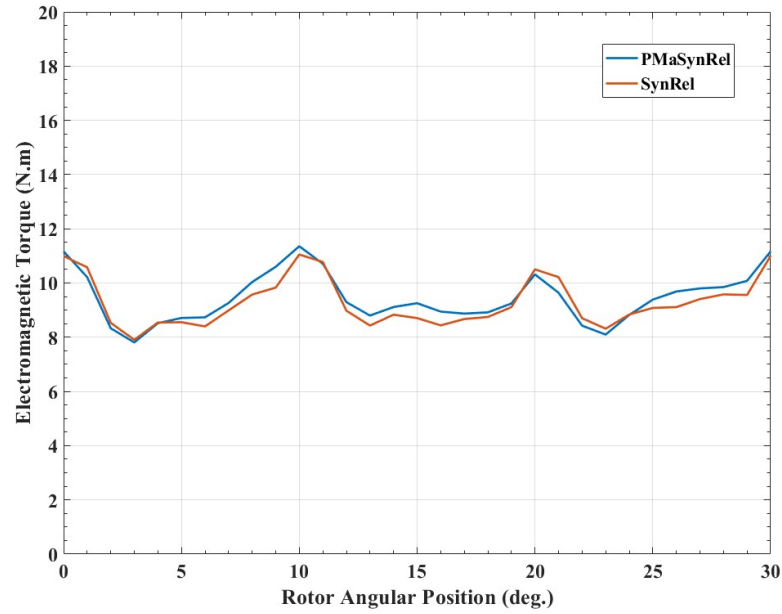


Fig. 4.10: Electromagnetic torque of the PMaSynRel motor.

It is noted from Fig. 4.10 that the average electromagnetic torque of the PMaSynRel motor is about 0.2 N·m higher than it of the SynRel motor, and their electromagnetic torque are performing a similar distribution and waveform.

4.1.3 Impact on Electromagnetic Force

It is aforesaid that the electromagnetic force is the main excitation of the motor vibration, therefore, it is principally analyzed and discussed in this section to understand the impact of PM and the difference between the SynRel motor and the PMaSynRel motor in terms of their electromagnetic force distribution.

Fig. 4.11 shows the electromagnetic force acting on the first three teeth tips of the stator at rated speed. Since the number of slots per pole per phase is 3, so all the other electromagnetic forces acting on the rest of teeth tips are repeating the same as they are on the first three teeth tips.

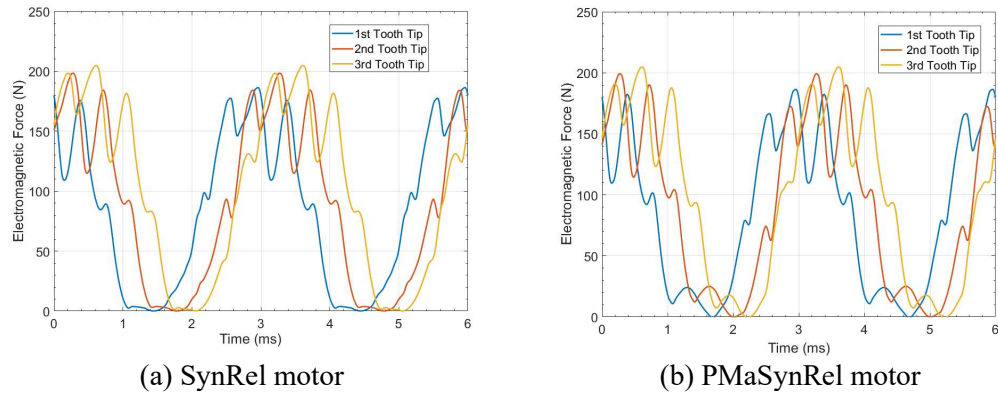


Fig. 4.11: Electromagnetic force acting on first three teeth tips.

It is noted from Fig. 4.11 that the amplitude of the electromagnetic force produced from PMaSynRel motor is similar to it from SynRel motor. However, at the full-time range, the electromagnetic force produced from the PMaSynRel motor is higher than it produced from the SynRel motor. For a better understanding on the difference of the electromagnetic forces of the SynRel motor and the PMaSynRel motor, the harmonic analysis of these two electromagnetic forces are furtherly studied.

By putting the electromagnetic forces distributing on all the teeth tips in one figure, it goes to Fig. 4.12.

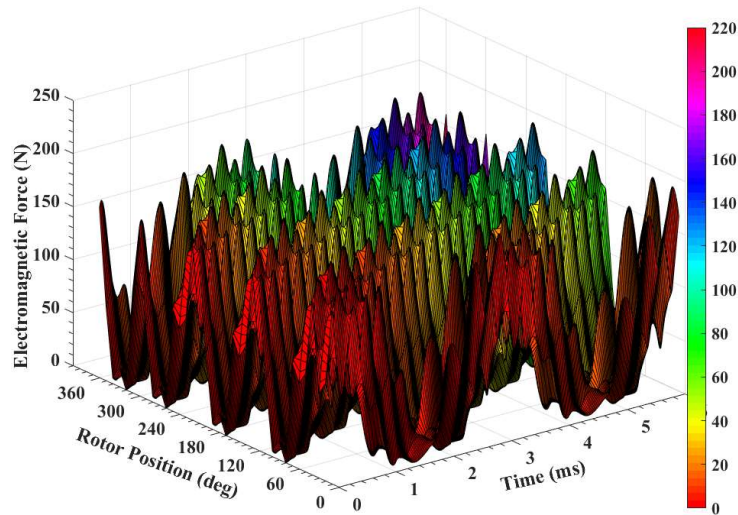


Fig. 4.12: Electromagnetic forces acting on all teeth tips.

Then, the electromagnetic force acting on the first tooth tip is taken as an example to deliver a harmonic analysis, in order to deeply understand the relations between the electromagnetic force and the vibration. Fig. 4.13 shows the FFT of the electromagnetic force acting on first tooth tip showing wide harmonic range.

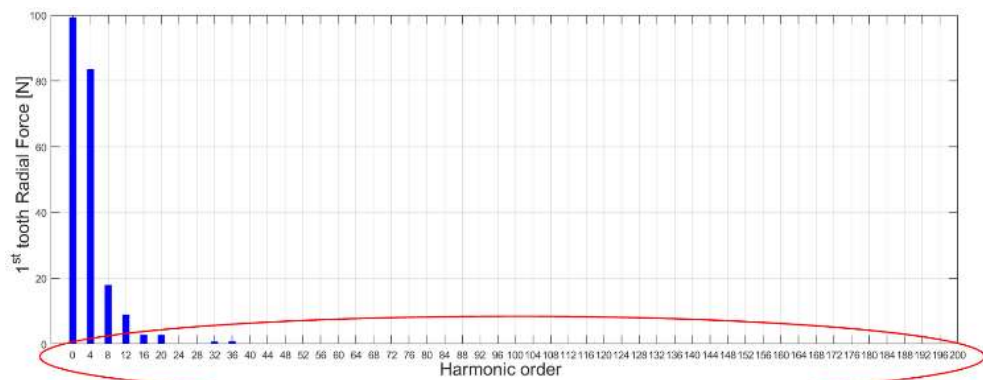


Fig. 4.13: FFT of the electromagnetic force on 1st tooth tip.

By focusing on the harmonic orders lesser than 64, Fig. 4.13 is changing to Fig. 4.14 as shown below.

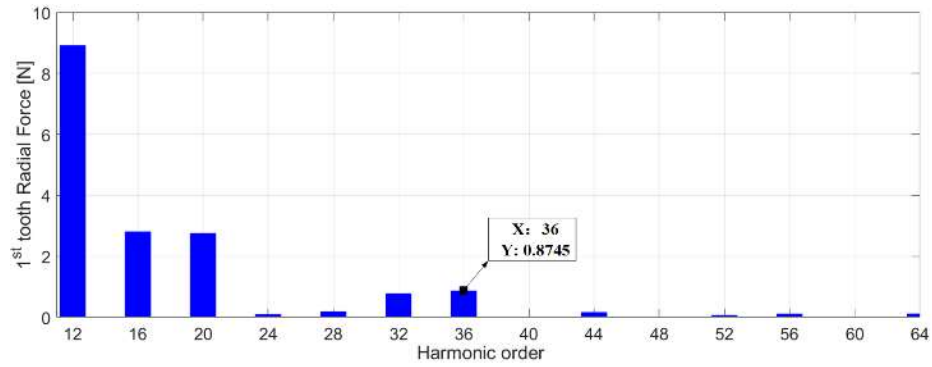


Fig. 4.14: FFT of the electromagnetic force focusing on orders lesser than 64.

Then, the FFT analysis is going to be connected to the vibration modes and natural frequency in the following sections to help deeply understanding the relations between the electromagnetic forces and the vibration level. Furthermore, it also helps to get robust electromagnetic against the vibration problems.

4.2 Vibration Investigation of PMaSynRel Motor

After the analysis on the electrical performance of the PMaSynRel motor, this section is focusing on the vibration analysis, and comparing the SynRel and PMaSynRel motor in terms of their vibrational level.

Firstly, the modal analysis is conducted to capture the modal frequencies and the modal shapes. Then, the impact of PM on the vibration is analyzed. Also, the comparisons between SynRel and PMaSynRel motor are established following the modal analysis and vibration analysis.

4.2.1 Modal Analysis

The material of the stator core, and the geometry of the PMaSynRel is kept the same as it of the SynRel motor, which are shown in Tab. 3.2 and Fig. 3.1, respectively. Besides that, the material of ferrite PM is added and shown in Tab.

4.2, which lists the main properties that will influence the mechanical performance of the motor.

Table 4.2: Material properties of ferrite PM.

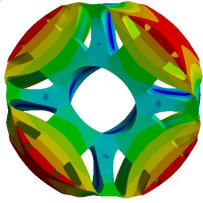
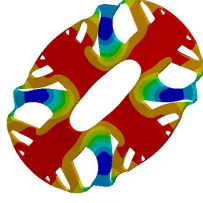
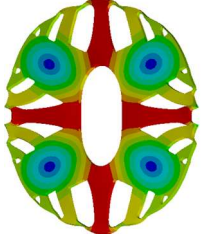
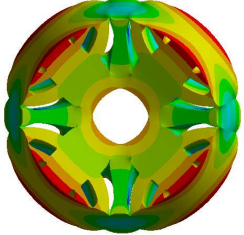
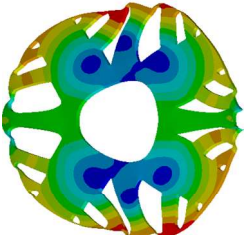
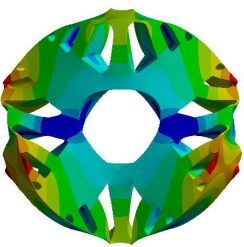
Property	Value
Mass density	5000 kg/m ³
Young's modulus	180 GPa
Poisson ratio	0.28

The modal analysis is conducted again with the PMaSynRel motor. Unlike the SynRel motor described in the former chapters, the PM is inserted inside each flux barrier of the PMaSynRel motor. On one hand, the total mass of the motor is increasing, which consequently changes its mechanical characteristics. On the other hand, the contact between the PM and the rotor core may have an unexpected effect on the final performance.

Because the stator of the PMaSynRel motor keeps the same geometry and laminations as them of the SynRel motor, the modal shape and the natural frequency of the stator are not changed. Hence, the modal analysis of the stator core is omitted in this section.

As for the rotor of the PMaSynRel motor, it is aforesaid that its geometry and mechanical characteristics are changed due to the inserted PM. Besides, the modal shape and natural frequency of a system is principally determined by the total mass and structural shape. Therefore, the modal analysis is performed in this section to investigate the modal shape and natural frequency of the rotor, which is then compared with the modal results of SynRel motor.

Table 4.3: Modal analysis results of PMaSynRel motor rotor core.

Mode number	Mode shapes	Natural frequency (Hz)
1		5533
2		7634
3		9681
4		11009
5		11408
6		12155

By comparing Tab. 3.5 and Tab. 4.3, which represents the modal result of SynRel motor and PMaSynRel motor, respectively. It can be noted that the natural frequency of PMaSynRel motor on each mode order is much higher than if of

SynRel motor. It is due to the inserted PM, which significantly increases the stiffness coefficient of the rotor laminations. Therefore, even though the inserted PM makes the mass of the whole system bigger, the natural frequency is still rising. It also provides another solution to avoid the resonance of SynRel motor, that an insulator which is very light in mass can be inserted into the flux barrier of SynRel motor to deliver a momentous change of the natural frequency.

4.2.2 Impact of Permanent Magnet on Vibration

With the ferrite PM inserted in the flux barriers, the mechanical characteristics of the PMaSynRel is different to the SynRel motor, for example the weight, stiffness, Young's modulus, and the Poisson's ratio. As a result, the vibrational level of the PMaSynRel is consequently changing with different mechanical characteristics. Hence, it is important to study and analyze the impact of ferrite PM on the vibration level of synchronous reluctance machines.

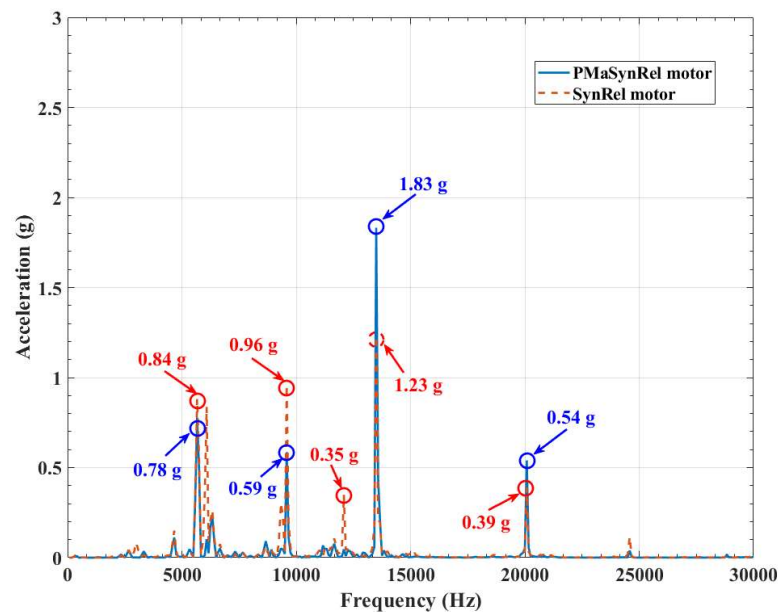


Fig. 4.15: Vibrational level of the SynRel and PMaSynRel motor.

As for the vibration analysis of the PMaSynRel motor, it firstly starts with mapping the electromagnetic forces into the structural model as the excitation. Then, the PMaSynRel motor is analyzed at the rated working condition for its vibrational level as shown in Fig. 4. 15.

It is noted from Fig. 4.15 that the housing accelerations of both SynRel and PMaSynRel motor are reaching a peak value around each natural frequency. In addition, due to the same geometry and working conditions, the peak vibrations of both SynRel and PMaSynRel motors are appearing around the same frequency point. Furthermore, Fig. 4.15 also informs that the vibration of PMaSynRel motor is having a larger peak value, while lesser harmonics. As shown in Fig. 4.15, the peak vibration of SynRel and PMaSynRel motor is 1.23 g and 1.83 g ($g = 9.8 \text{ m/s}^2$), respectively, at the same frequency point which is 13500 Hz. Besides, there is another peak vibration of SynRel occurred around 12500 Hz, while the vibration of the PMaSynRel motor is smooth nearby this frequency. Furthermore, when the frequency is lesser than 13500 Hz, the peak vibration of the SynRel motor is higher than it of the PMaSynRel motor. Hence, in some situations when the vibration level is not the principle factor, the inserted PM can be a possible solution to mitigate the vibration harmonics.

The vibration levels of both SynRel and PMaSynRel are collected and listed in Tab. 4.4, from where they are compared and related to the electromagnetic force harmonics which are shown in Fig. 4.13 and Fig. 4.14.

Referring to the housing acceleration response, which gives an indication of the vibration transfer through the stator core surface to the other structure parts, it can be noted that there are many peaks of the response. Tab. 4.4 reports the peaks of

the acceleration response and their corresponding frequency. It can be concluded that those peaks occurred at the slot harmonic (36th) order and its multiples. The most significant peak is caused by the 84th harmonic order.

Table 4.4: Peak vibration on natural frequency.

Peak vibration (g)		Frequency (Hz)
SynRel motor	PMaSynRel motor	
0.84	0.78	6000 (36 th harmonic)
0.96	0.59	9000 (54 th harmonic)
0.35	0.07	12000 (72 nd harmonic)
1.23	1.83	13500 (80 th harmonic)

4.3 Sensitivity Analysis and Comparison

After the investigations of the impact of the PM on vibration are conducted, the study is extended to the size of the PM. In order to get a good electrical performance and avoid a big vibration issue, this section is focusing on the selection of the PM, including the width and the thickness. To this aim, two sensitivity analyses are proposed in the following sections to investigate the impact of the PM width and thickness, respectively.

4.3.1 Sensitivity Analysis Varying PM width

According to Eq. (4.3), the width of the PM is inversely proportional to its magnetic reluctance, then the flux density and the flowing flux are consequently influenced. Besides, the changing size of the PM is also having an effect on the mechanical characteristics and the vibrational level. In this section, a sensitivity analysis of the PM width is conducted to find out the relations between the PM size and the vibrational level.

According to the initial design of the width selections listed in Tab. 4.1, a sensitivity analysis of the PM width is conducted by changing the PM width of each flux barrier with a step as 1 mm, which are listed in Tab. 4.5.

Table 4.5: Sensitivity variation of PM width

Order of PM layer	Symbol	Sensitivity value (mm)		
1	w_{m1}	18	19	20
2	w_{m2}	22	23	24
3	w_{m3}	28	29	30

By changing the PM width according to Tab. 4.5, a group of the FE simulations are conducted to present the electromagnetic and vibrational characteristics in the following figures. From Fig. 4.16 to Fig. 4.18, they are showing the comparison on airgap flux densities, electromagnetic torques, electromagnetic forces, and vibrational level, respectively.

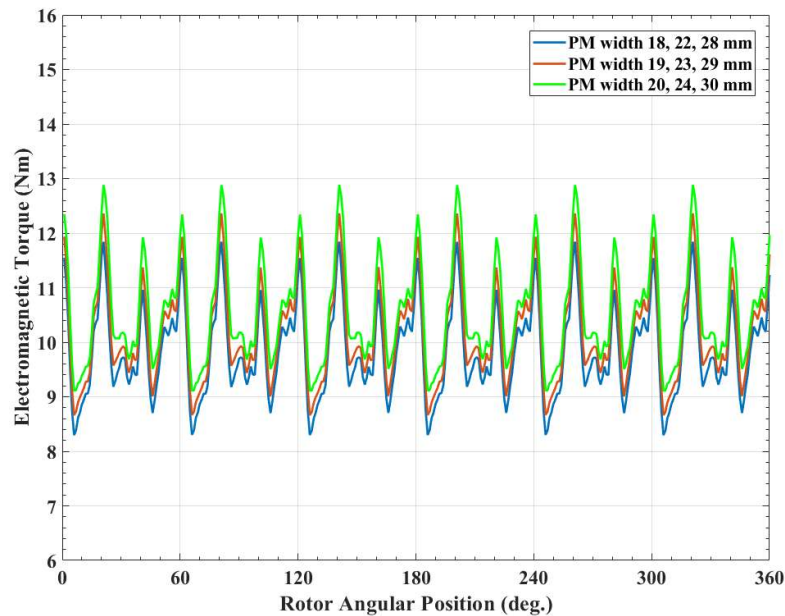


Fig. 4.16: Electromagnetic torque of PMaSynRel motor varying PM width.

Fig. 4.16 presents the electromagnetic torque distribution of the PMaSynRel motor varying PM width. As for the sensitivity PM width groups shown in Tab.

4.5, the average torque is 9.85 N·m, 10.17 N·m, and 10.55 N·m, respectively, and the torque ripple is 36.02%, 36.38%, and 35.78%, respectively. It can be seen that with a bigger PM width, the average torque is increasing, whilst the torque ripple is not linear changing with the PM width. Besides, the electromagnetic torques under each case are performing a similar waveform, while it has a larger amplitude when the PM width is larger, which is due to the bigger proportion of the PM usage. Hence, the torque could be improved by sensitivity optimizing the PM width to obtain a good average electromagnetic torque, and a better quality of the torque ripple.

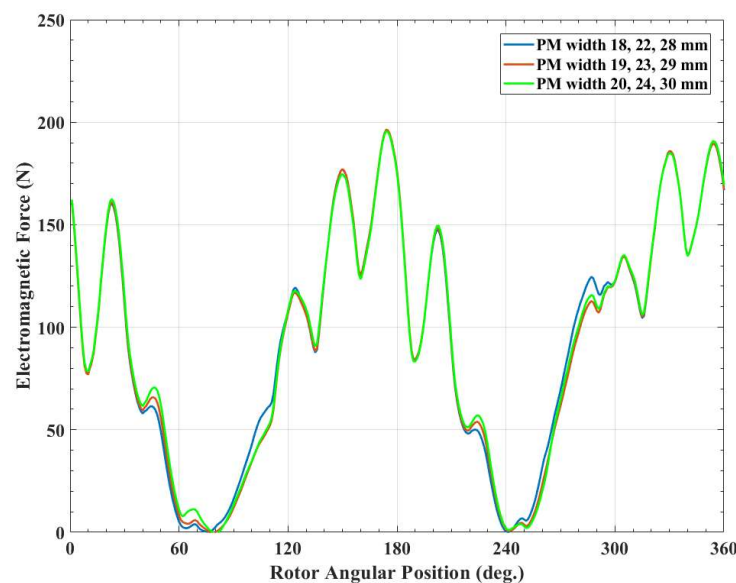


Fig. 4.17: Electromagnetic force of PMaSynRel motor varying PM width.

Fig. 4.17 illustrates the distributions of the electromagnetic force acting on the first tooth tip of the PMaSynRel motor varying the PM width, respectively. In this case, the electromagnetic force with different PM width set shows a various performance. The fluctuant waveforms are similar, from where the amplitudes of each electromagnetic force curve are very close to each other. However, around the rotor angular positions corresponding to the place of inset PM, the value of the electromagnetic force is fluctuating. Unlike the comparison of the electromagnetic

torque varying the PM width, in which case the electromagnetic torque is increasing with a larger PM width, the electromagnetic force is lesser sensitivity to the change of PM width. Besides, between 40 degrees to 80 degrees of the rotor angular position, the electromagnetic force of PMaSynRel motor with bigger PM width is performing a larger value. However, between 80 degrees to 110 degrees, it shows an opposite comparison.

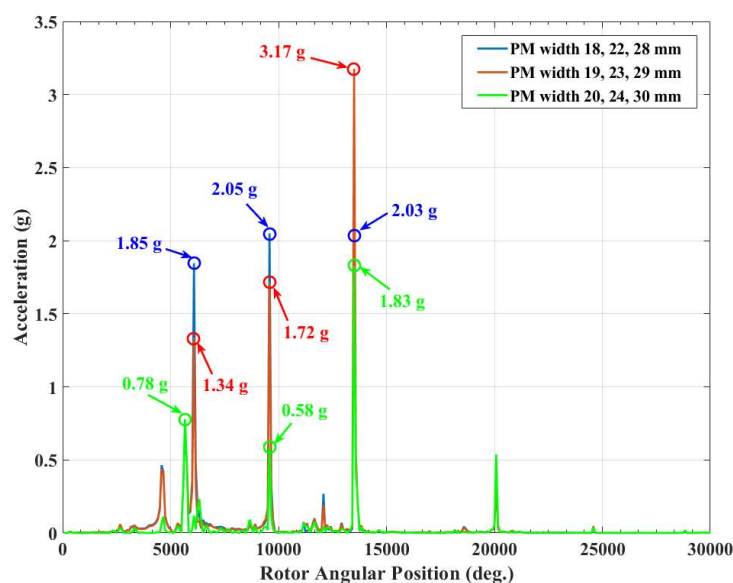


Fig. 4.18: Vibration level of PMaSynRel motor varying PM width.

Fig. 4.18 compares the vibration levels of the PMaSynRel motor with various PM widths. When the PM width is getting larger, the usage of PM is also increasing, which leads the whole mass of the PMaSynRel motor to consequently increase. However, the natural frequency is not that sensitive to the change of PM width as shown in Fig. 4.18, that the peak vibrations under each PM width group are occurring around the same frequency point, except the green line. At the second peak vibration order highlighted in Fig. 4.18, the green line occurs a peak (0.78 g , $g = 9.8 \text{ m/s}^2$) at 5660 Hz, while the other two lines occur the peak (1.85 g and 1.34 g , respectively) at 6083 Hz. However, in terms of the other two peak vibrations

highlighted in the figure, both three lines are occurring at the same frequency point (9580 Hz and 13500 Hz, respectively). Furthermore, the largest vibrating accelerating is 3.17 g, which is under the case of the medium PM width case, at 13500 Hz. It informs that the peak vibration is not linearly increasing with the increase of the PM width. Therefore, it is helpful to investigate the impact of the PM size on vibration, which provides a method to optimize the design of the PMaSynRel motor by considering its mechanical performance.

4.3.2 Sensitivity Analysis Varying PM Thickness

The thickness of PM is a key parameter of the design of PMaSynRel motor, which is modified to limit the flux density variation and to avoid an irreversible demagnetization of the PM itself. Thus, it is an important aspect to be considered for the electrical and the mechanical performance of the PMaSynRel motor.

According to Eq. (4.3), the thickness of the PM is proportional to its magnetic reluctance, then the air-gap flux density and the flowing flux are consequently influenced. Hence, this section conducts a sensitivity analysis varying the PM thickness to investigate the impact of PM thickness on vibration of the PMaSynRel motor.

Table 4.6: Sensitivity variation of PM thickness.

Order of PM layer	Symbol	Sensitivity value (mm)					
1	t_{m1}	2	3	4	5	6	
2	t_{m2}	4	5	6	7	8	
3	t_{m3}	8	9	10	11	12	

By changing the PM width according to Tab. 4.6, a group of the FE simulations are conducted to present the electromagnetic and vibrational characteristics in the

following figures. From Fig. 4.19 to Fig. 4. 21, they are showing the comparison on airgap flux densities, electromagnetic torques, electromagnetic forces, and vibrational level, respectively.

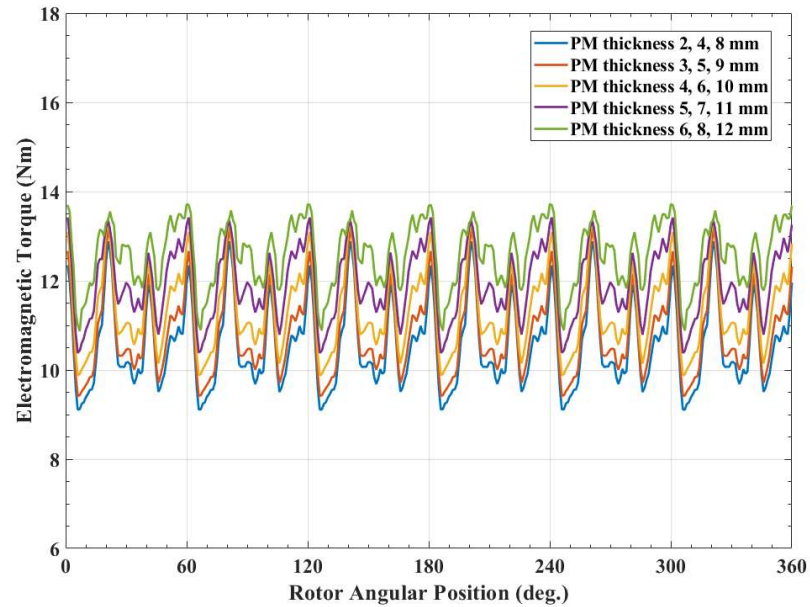


Fig. 4.19: Electromagnetic torque of PMaSynRel motor varying PM thickness.

Fig. 4.19 presents the distributions of the electromagnetic torque under the cases of the varying PM thickness. It informs that the electromagnetic torque is not linearly changing with the increasing PM thickness. When the PM is getting thicker, the average torque under each cases of varying PM thickness is increasing, while the torque ripple is slightly decreasing. The torque ripple under these five cases are 35.78%, 35.58%, 34%, 31.17% and 31.34%, respectively. Besides, it is noted that with more PM inset in the flux barrier, the electromagnetic torque is bigger due to the increased proportion of magnetic torque with respect to the reluctance torque. Hence, it is an option for improving the electrical performance of the PMaSynRel motor by properly adopting and designing the PM material and size.

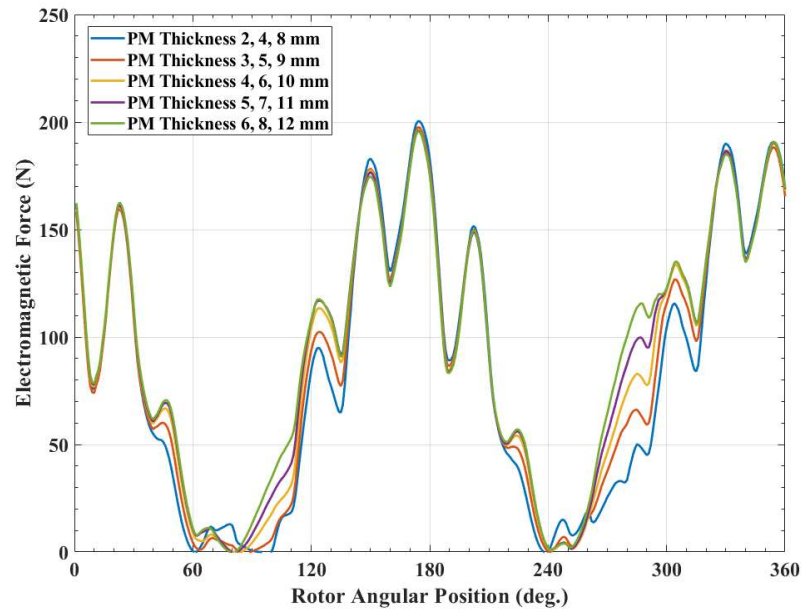


Fig. 4.20: Electromagnetic force of PMSynRel motor varying PM thickness.

In Fig. 4.20, the electromagnetic force acting on the first tooth tip on the inner stator core is taken as an example to compare these varying cases in term of the electromagnetic force. It is presented that when the PM is thicker, its corresponding electromagnetic force is bigger. Besides, the amplitudes of the electromagnetic force under each case are performing a similar value. Furthermore, it is also noted that the difference of each electromagnetic forces is appearing around the positions corresponding to the PM. As the electromagnetic force is exciting the vibration of stator, therefore the impact of PM width on electromagnetic force is critical to be considered during the design.

Fig. 4.21 illustrates the vibration level, which is represented by the housing acceleration ($g = 9.8 \text{ m/s}^2$), under each case of the varying PM thickness. It can be seen that for both of the cases, the peak vibrations are appearing around the same frequency. At 13500 Hz, the amplitudes of the peak vibration are 1.83 g, 3.01 g, 2.83 g, 1.14 g and 1.99 g, respectively, responding to each group of the PM thickness. In terms of the mechanical properties, such as the mass density and

Young's modulus, the added PM is smaller than the rotor core and the stator core, and therefore the change of PM size slightly changes the whole system mass, consequently the mechanical characteristics of the whole system, for instance the natural frequency and the modal shape will not be influenced too much. So, the peak vibrations are appearing at 13500 Hz. However, the change of the PM thickness will have an impact on the distribution and the harmonics of the electromagnetic force, therefore the vibration level under each case are performing a different amplitude. Besides, at the frequency points around 5600 Hz, 9500 Hz, and 12000 Hz, which meet with the 2nd, 4th, and 5th order natural frequency listed in Tab. 3.6, there are other peak vibrations occurred.

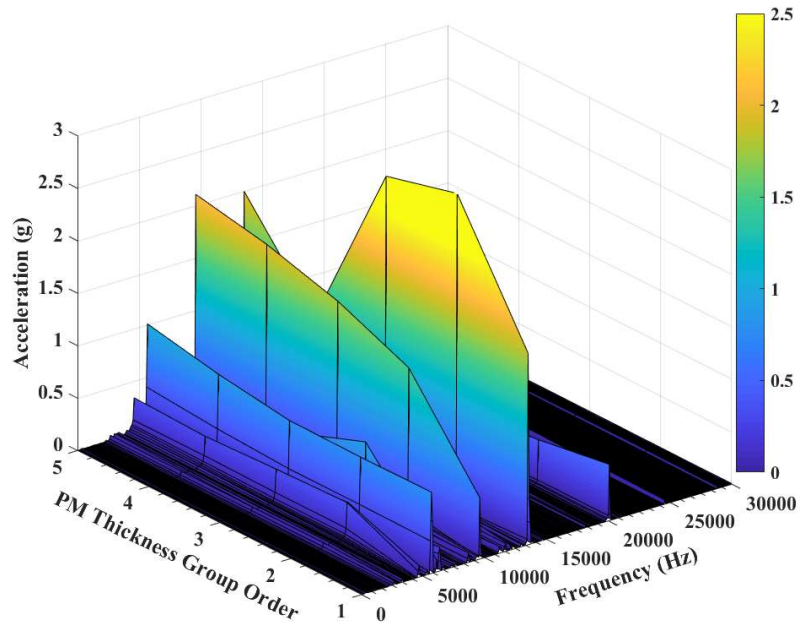


Fig. 4.21: Vibration level of PMSynRel motor varying PM thickness.

4.4 Summary

This chapter investigated the impact of inserted PM on the SynRel motor for its vibration level. An equivalent magnetic circuit, which considered the flux of PM, was derived from the proposed equivalent magnetic circuit of SynRel motor. The

electrical performance of the PMaSynRel motor was analytically calculated, and then validated by FEA computation. The impact of the inserted PM on the electromagnetic torque and force were highlighted, which informed that the PMaSynRel motor provided similar average torque by using smaller phase current. The electromagnetic force acting on the first three tooth tips of the PMaSynRel motor was exacted to present its distribution, which shows that the electromagnetic force of the PMaSynRel motor was slightly higher than it of the SynRel motor.

The impact of inserted PM on the mechanical characteristics was investigated. As the stator of PMaSynRel motor was the same as it of SynRel motor, the rotor was highlighted with the stress analysis. Even though the inserted PM made the total mass bigger, it increased the stiffness by supporting the ribs of the flux barrier. Therefore, the maximum stress on the rotor of PMaSynRel motor was 47 MPa, while it of SynRel motor was 109 MPa. Then, the vibration analysis was conducted to investigate the impact of PM on vibration. The peak vibration of PMaSynRel motor was 1.83 g, while it of SynRel motor was 1.23 g.

Chapter 5

Impact of Eccentric Rotor on SynRel Motor Performance

During the manufacturing process of a motor, there is a possible eccentric rotor allowed by the manufacturing tolerance. However, a slight shift of the rotor position may have a significant effect on the motor performance, specifically for the SynRel motor which is usually having a small airgap length. Therefore, this chapter is looking at the vibration level of a SynRel motor with eccentric rotor position (eccentric SynRel motor), comparing to those of concentric cases. Then, in case of the rotor with magnets, which is the PMaSynRel motor, the vibration resulted by eccentric rotor is properly more observed. Furthermore, it could be providing a new solution to check the status of rotor eccentricity by a vibration testing.

It starts with an asymmetric magnetic model deducing from the analytical model described in the former chapters. Then, the electrical performance, specifically the electromagnetic torque and force, under both the concentric and eccentric conditions are analyzed. Then, the investigation is furtherly performed on the PMaSynRel motor case to identify the effects of magnets. Finally, the eccentric SynRel and PMaSynRel motors are compared for their vibration level.

The obtained results above all could help to understand and compare the vibration level in a SynRel or a PMaSynRel motor in case of the eccentric rotor position.

5.1 Electrical Performance of Eccentric SynRel Motor

As the centre of the rotor is shifted from the rotating centre, the airgap circumferential length is led to an uneven distribution, which makes the airgap flux density more fluctuant. In this case, the electrical performance, specifically the electromagnetic torque and force are crucial for the vibration investigation. Therefore, the electrical performance of eccentric SynRel motor is firstly studied here.

5.1.1 Asymmetric Magnetic Model

The eccentric cases include static eccentricity and dynamic eccentricity [83]. To understand the difference between the healthy (concentric rotor), static eccentric and dynamic eccentric rotor position, a brief comparison shown in Fig. 5.1 is proposed. Fig. 5.1 (b), it is a static eccentricity, and the rotor is shifted from the centre point to the stator for 0.15 mm, but still rotates around its own centre. As shown in Fig. 5.1 (c), the rotor is similarly shifted from the centre point to the stator for 0.15 mm and rotates around the stator centre. Both these two cases are having an uneven airgap, that there is a smaller airgap on the positive x -axis direction. The healthy rotor is also considered in this case study as a comparison, which is shown in Fig. 5.1 (a).

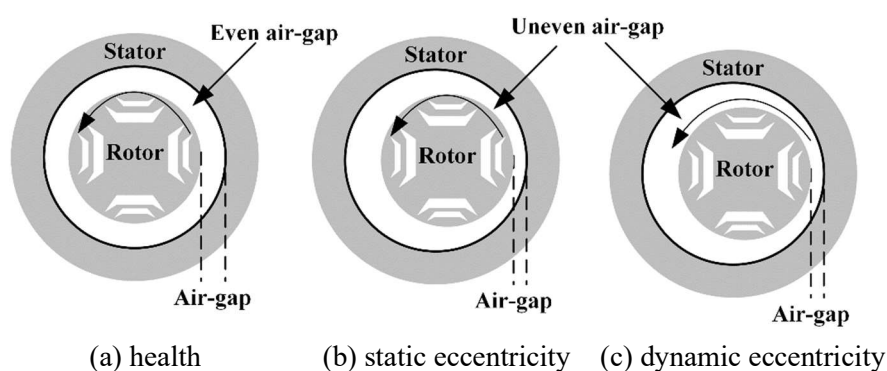


Fig. 5.1: Sketches of the rotor centre with respect to the stator centre.

As for these two eccentric cases, their magnetic equivalent circuits are accordingly changed due to the uneven airgap. Fig. 5.2 shows the diagram of static eccentric SynRel motor that the airgap length at each position is fixed with rotating rotor, which means the airgap length is not influenced by the rotor position.

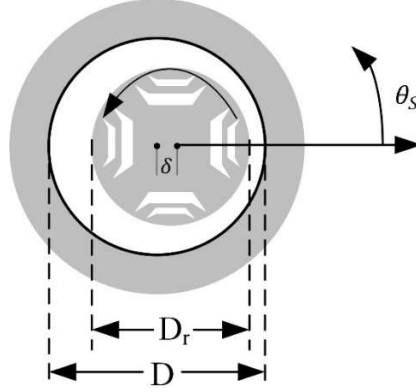


Fig. 5.2: Airgap length variation with rotor position in static eccentricity.

In Fig. 5.2, δ represents the eccentric distance between the stator axial centre and the rotor axial centre; θ_s represents the rotating angle of the rotor. Then, according to Fig. 5.2, the airgap length at each circumferential position based on d -axis is expressed as,

$$L_g(\theta_s) = L_g - \delta \cos(\theta_s). \quad (5.1)$$

In case of the dynamic eccentricity, Fig. 5.3 shows the airgap length distribution varying on the rotor position.

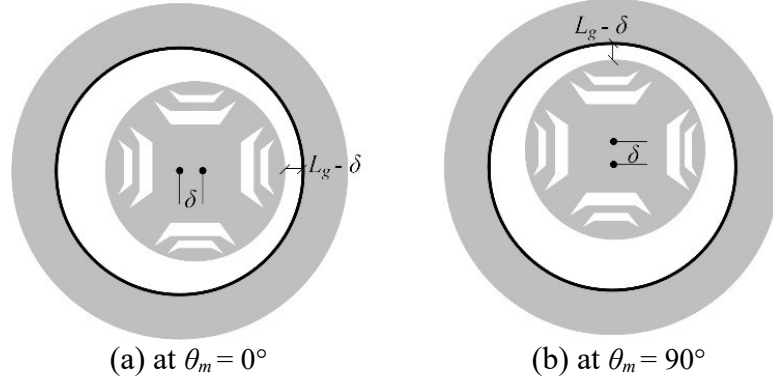


Fig. 5.3: Airgap length variation with rotor position in dynamic eccentricity.

Fig. 5.3 illustrates that the circumferential airgap length of dynamic eccentric SynRel motor changes with the rotating rotor. In Fig. 5.3 (a), the shortest airgap length appears aligning with x -axis, then in Fig. 5.3 (b), the shortest airgap length aligns with y -axis when the rotor has rotated 90 mechanical degrees. Then, the airgap length variation of dynamic eccentric SynRel motor can be presented as a function of θ_s in the stator reference frame, which is expressed as,

$$L_g(\theta_s) = L_g - \delta \cos(\theta_s - \theta_m). \quad (5.2)$$

By applying Eq. (5.1) and Eq. (5.2) in Eq. (2.40), the airgap flux density of the eccentric SynRel motor can be given by

$$B_g(\theta_s) = -\mu_0 \frac{U_s(\theta_s) - U_r(\theta_s)}{L_g(\theta_s)}. \quad (5.3)$$

By setting the model at rated working condition, the airgap flux density distribution is obtained by means of FEA. Fig. 5.4 shows the results of airgap flux density at both static eccentricity and dynamic eccentricity.

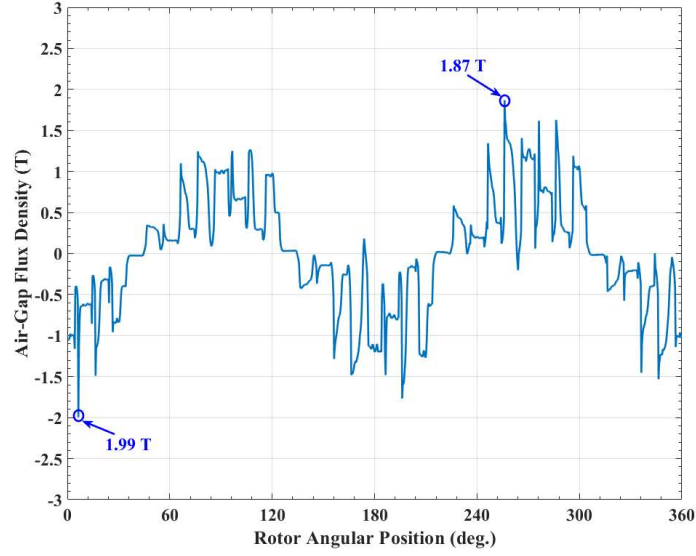


Fig. 5.4: Airgap flux density distribution of eccentric SynRel at $\theta_m = 0^\circ$.

According to the static eccentric and dynamic eccentric motor cases which are shown in Fig. 5.2 and Fig. 5.3 (a), by assuming these positions as the initial position ($\theta_m=0$), they are appearing exactly the same structure. Hence, as shown in Fig. 5.4, their airgap flux densities are displaying the same distribution. Besides, it is obvious that this motor has four poles, but the amplitudes of each pole are various. By ignoring the vectors, the amplitudes of the air-gap flux density waveform are highlighted, while they are 1.87 T and 1.99 T, respectively.

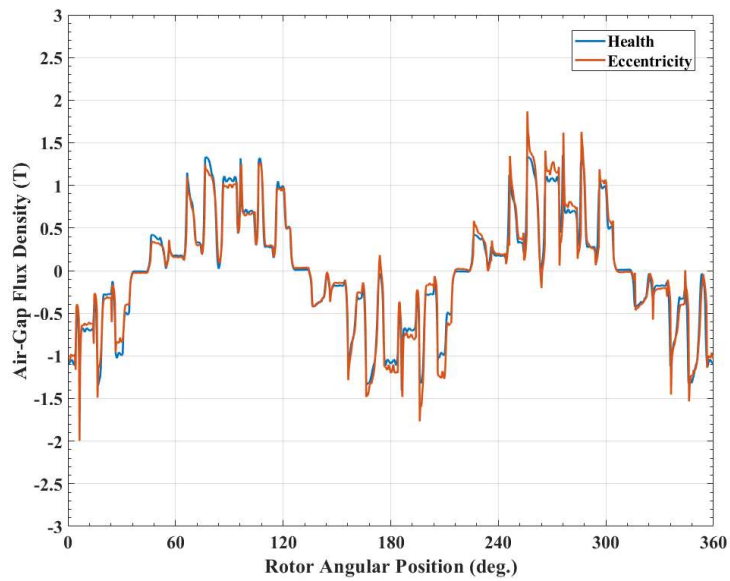


Fig. 5.5: Comparison on the airgap flux density distribution when $\theta_m = 0^\circ$.

Fig. 5.5 compares the airgap flux density distribution between healthy and eccentric SynRel motor at $\theta_m = 0^\circ$. It illustrates that the trends of the airgap flux density curves are well aligning with each other. However, the amplitude of the airgap flux density under eccentric case is fluctuating comparing to it under healthy case, which is mainly caused by the inconstant airgap length variation. It can be seen that the peak airgap flux density of healthy motor is about 1.3 T, while the peak airgap flux density of eccentric motor is reaching 2 T. Furthermore, the airgap flux density of healthy motor is evenly distributing at each pole pitch by considering the absolute value of airgap flux density.

Then, by anticlockwise rotating the healthy, static eccentric, and dynamic eccentric motor, respectively, in 90 mechanical degrees ($\theta_m = 90^\circ$), which makes Fig. 5.3 (a) to Fig. 5.3 (b). In this case, the motor structures of healthy motor and static eccentric motor are keeping the same. Fig. 5.6 shows the airgap flux density distributions under those three cases.

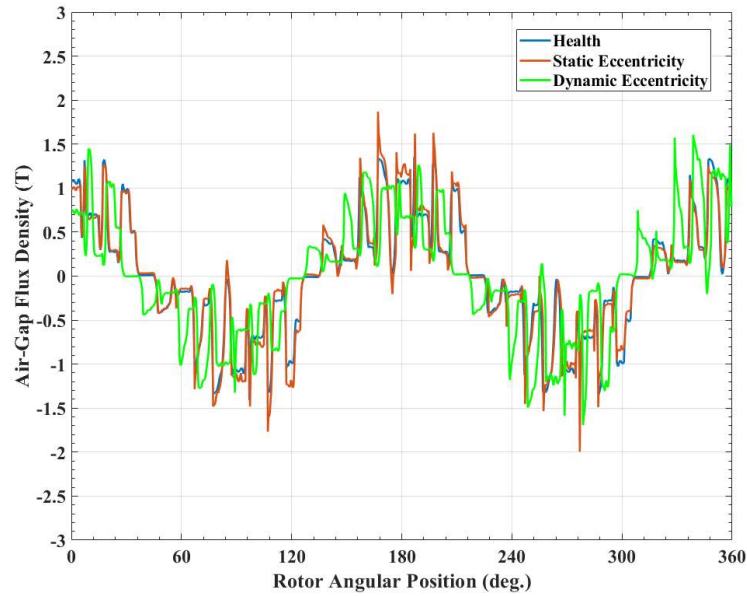


Fig. 5.6: Comparison on the airgap flux density distribution when $\theta_m = 90^\circ$.

It is described by Fig. 5.6 that the airgap flux density curves under both three cases are keeping the similar trend. However, the peak values of healthy, static eccentric, and dynamic eccentric cases are appearing at different amplitudes, respectively. The air-gap flux density waveforms of both static eccentric and dynamic eccentric SynRel motor are performing an asymmetric distribution.

5.1.2 Electromagnetic Force

In both static eccentricity and dynamic eccentricity cases, they are analyzed through FEA for their electromagnetic forces distributing on all the teeth tips. From Fig. 5.7 to Fig. 5.9, they present the distribution of the electromagnetic force of concentricity, static eccentricity, and dynamic eccentricity, respectively.

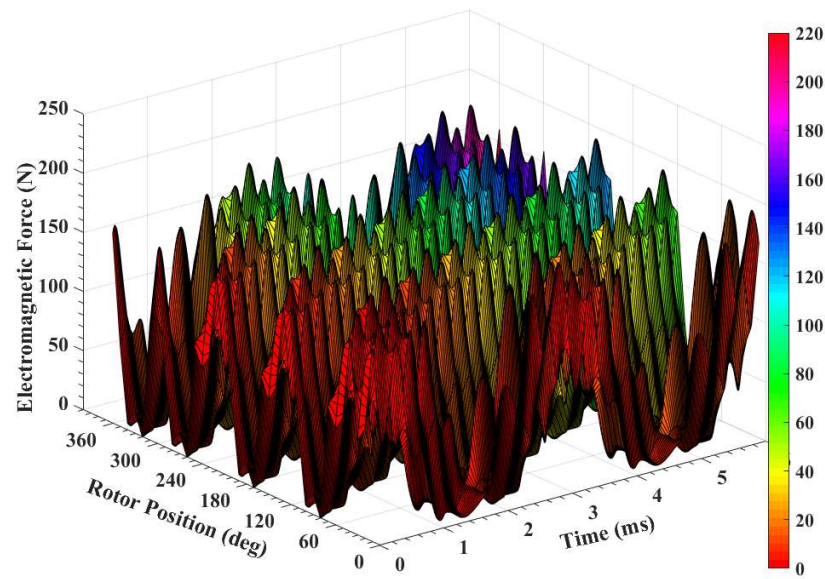


Fig. 5.7: Radial electromagnetic forces (healthy).

Fig. 5.7 presents the distribution of the radial electromagnetic forces acting on all the tooth tips of the stator core. It can be seen that the electromagnetic forces acting on every three tooth tips are performing a similar waveform with the same amplitude.

It is because of the even distribution of the air-gap, that the air-gap flux density is regularly performing when the rotor is rotating.

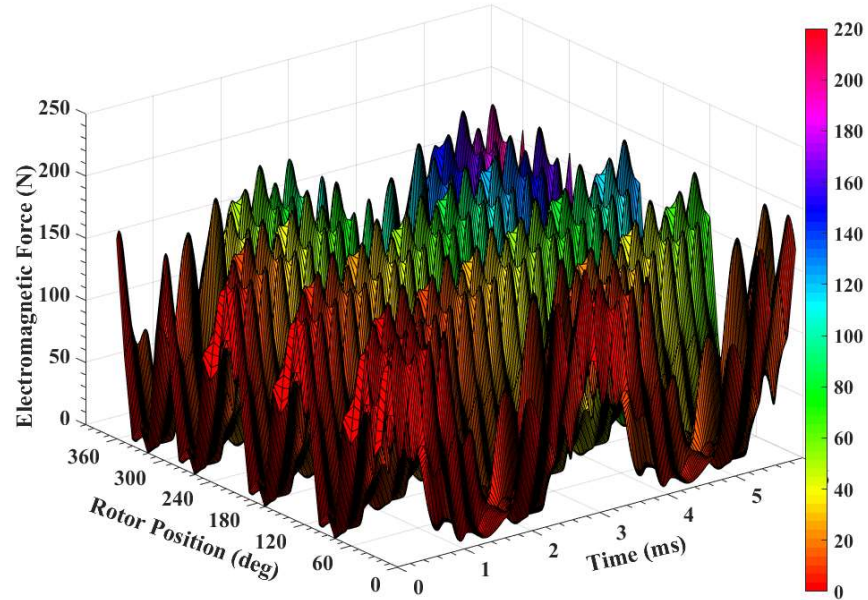


Fig. 5.8: Radial electromagnetic forces (static eccentricity).

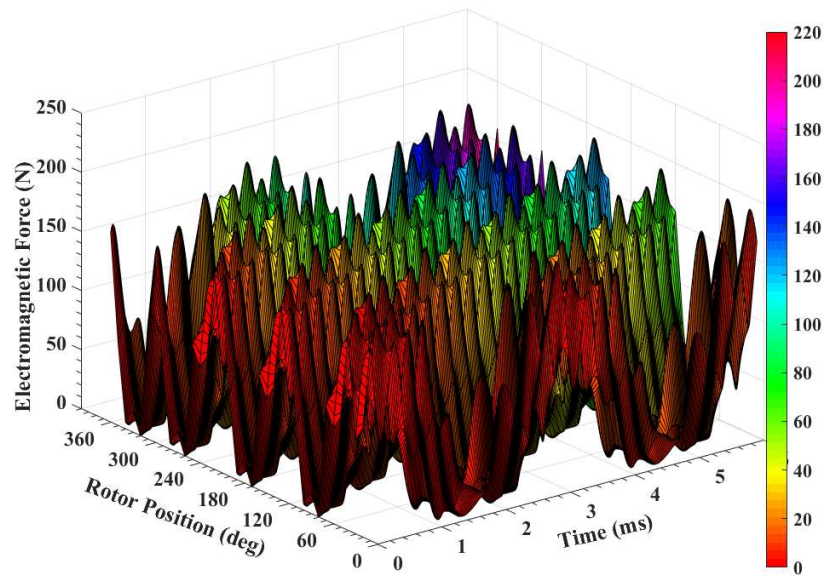


Fig. 5.9: Radial electromagnetic forces (dynamic eccentricity).

Fig. 5.8 and Fig. 9 present the electromagnetic force acting on all the tooth tips on stator core of static eccentric and dynamic eccentric SynRel motor, respectively. Unlike the electromagnetic force distribution of healthy SynRel motor, both of these

two eccentric cases are showing an uneven distribution. It is mainly due to the uneven distribution of the air-gap, which leads to the unregular air-gap flux density distributions as shown in Fig. 5.6. Besides, the highest amplitudes of both these two eccentric SynRel motors are much higher than it of the healthy case.

5.1.3 Electromagnetic Torque

After computing and analyzing the air-gap flux density and the electromagnetic force, the electromagnetic torque of the eccentric SynRel machine is presented and analyzed in this section. In Fig. 5.10, the comparison on electromagnetic torque between concentric and eccentric SynRel motor is shown.

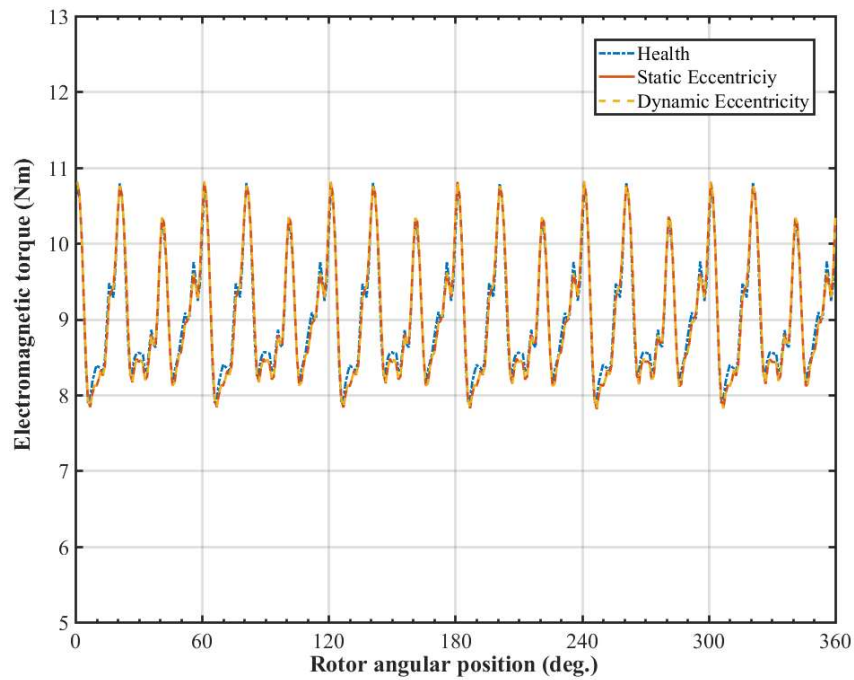


Fig. 5.10: Comparison on electromagnetic torque of SynRel motor.

It is noted from Fig. 5.10 that the electromagnetic torques of the three cases are similar to each other. As for the concentric case, the maximum electromagnetic torque reaches $10.79 \text{ N}\cdot\text{m}$, and the minimum torque is about $7.89 \text{ N}\cdot\text{m}$. In this case, the average torque of this motor is around $9.06 \text{ N}\cdot\text{m}$, and the torque ripple is 32.09%.

Besides, as for the static eccentric case and the dynamic eccentric case, their electromagnetic torque characteristics are more similar, the maximum values are 10.82 N·m and 10.79 N·m, the minimum values are 7.81 N·m and 7.80 N·m, the average values are 9.03 N·m and 9.02 N·m, and the torque ripples are 33.31% and 33.15%, respectively.

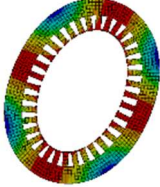
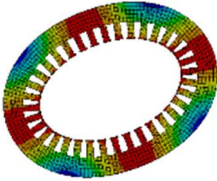
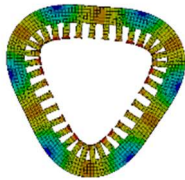
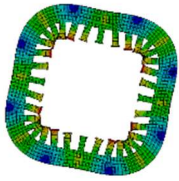
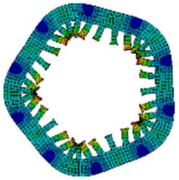
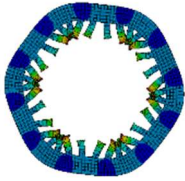
Based on the study on the electromagnetic torque characteristics of the concentric, static eccentric and dynamic eccentric SynRel motor, it can be drawn that under the two eccentric cases, they have almost the same output torque, but their torque ripple are slightly higher than that of the concentric case. After that, it can be inferred that the torque characteristics are not enough to identify their vibrational level, for their difference among the three cases is not that significant. Therefore, the electromagnetic force investigation is necessary and helpful to distinguish the impact factors on the vibration level in eccentric SynRel motor.

5.2 Modal Analysis of Eccentric SynRel Motor

As the mechanical characteristics of the motor is fully depending on its physic properties and construction, the eccentric rotor position changes the structure of the whole system, and therefore the modal analysis is conducted to investigate the effects on its mechanical characteristics.

In case of static eccentricity and dynamic eccentricity, they are appearing a same geometry and structure as described in last sections. Hence, their modal shapes and natural frequencies, which are fully depending on the mechanical properties and the structures, are conducted and analyzed by using the same model.

Table 5.1: Modal analysis results of SynRel motor stator core.

Mode number	Mode shapes	Natural frequency (Hz)
1		1273.9
2		1274.1
3		3363
4		5883
5		8331
6		10098

In terms of the modal shape and the natural frequency, the difference between concentric case and eccentric cases is not that big, which means this parameter cannot be used as a symbol to represent the eccentricity. It is mainly due to the same geometry and mass of the motor, from which the eccentricity has a minor effect on the mechanical characteristics. As a result, the vibration analysis is conducted in the

following section to potentially provide another method to the validation of the manufactured motor.

5.3 Vibration Investigation on Eccentric SynRel Motor

In this section, the vibration analysis is conducted to understand the vibrational difference between concentric and eccentric SynRel motors. As the electromagnetic force distributions of the healthy, static eccentric, and dynamic eccentric cases are obtained and analyzed, respectively, the vibration analysis of each case is shown. Fig. 5.11 uses the acceleration values of the motor housing as a symbol to represent the vibration level.

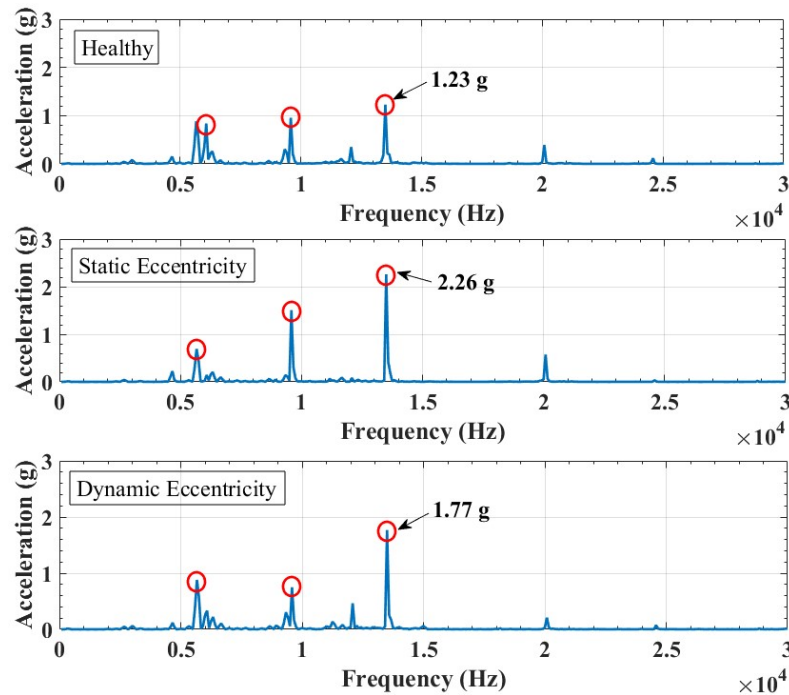


Fig. 5.11: Comparison on vibration level of SynRel motor.

From Fig. 5.11, it can be seen that the peak vibrations are occurred around 13500 Hz, which is a natural frequency. Among these three cases, the peak vibration of static eccentricity is the highest, which is at 2.26 g, whilst the peak vibration of concentricity and dynamic eccentricity are at 1.23 g and 1.77 g, respectively. Hence,

the peak vibration of static eccentric case is 83.74% higher than health motor, and the peak vibration of dynamic eccentric case is 43.9% higher than health motor. Also, it can also be validated by the electromagnetic force distribution, that the static eccentric motor produces bigger and more fluctuant electromagnetic forces. Furthermore, the vibration analysis provides a possible solution for identifying the eccentric status of the manufactured SynRel motors.

5.4 Impact of Inserted Permanent Magnet on Eccentric PMSynRel Motor

It is aforesaid that the PM is broadly adopted in many industrial applications for obtaining a better electrical performance, PM is adopted in many industrial applications. By inserting the PM into the flux barrier layer, it changes the mass and the structure of the whole system, leading its mechanical characteristics to be changed. Hence, this section is focusing on the inserted PM to find out its impact on vibration. To better understand the vibration level in a PMSynRel motor in case of eccentric rotor position, both static and dynamic eccentricity conditions are analyzed and compared.

5.4.1 Electromagnetic Torque

The electromagnetic torque is analyzed and compared to understand to what extent the motor performance is affected in case of eccentricity. Fig. 5.12 shows the electromagnetic torque characteristics of the health, static eccentric, and dynamic eccentric cases, respectively, from where the impact of eccentricity can be found.

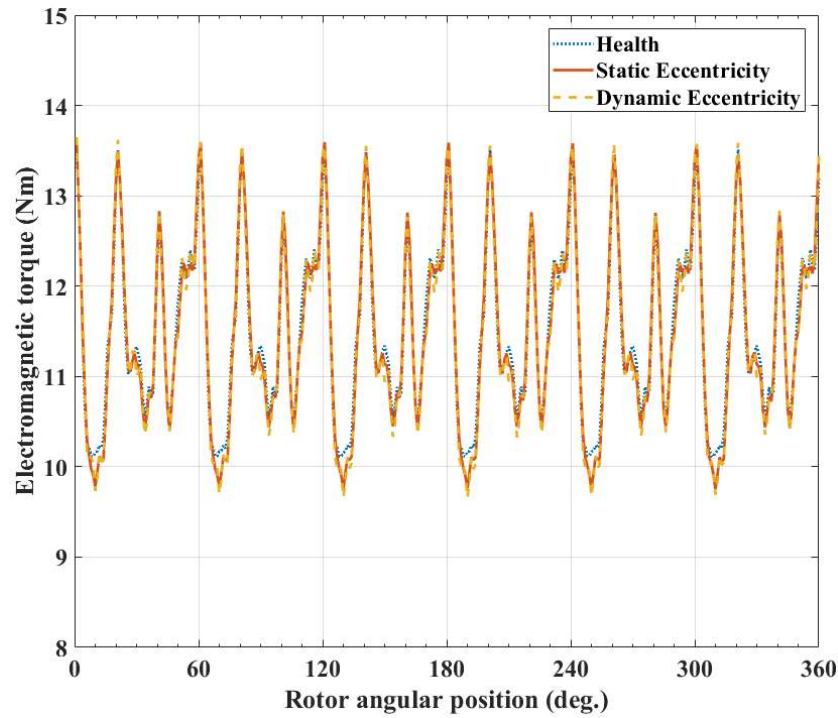


Fig. 5.12: Comparison on electromagnetic torque of PMaSynRel motor.

It is noted from Fig. 5.12 that the electromagnetic torques of both the three cases are similar from each other. As for the concentric case, its electromagnetic torque has a maximum value at $13.51 \text{ N}\cdot\text{m}$, a minimum value at $10.11 \text{ N}\cdot\text{m}$, the average value is $11.51 \text{ N}\cdot\text{m}$, and the torque ripple is 29.56%. Besides, as for the static eccentric case and the dynamic eccentric case, their electromagnetic torque characteristics are more similar, the maximum values are $13.6 \text{ N}\cdot\text{m}$ and $13.65 \text{ N}\cdot\text{m}$, the minimum values are $9.75 \text{ N}\cdot\text{m}$ and $9.65 \text{ N}\cdot\text{m}$, the average values are $11.48 \text{ N}\cdot\text{m}$ and $11.45 \text{ N}\cdot\text{m}$, and the torque ripples are 33.54% and 34.89% respectively.

Based on the conducted simulation results of the electromagnetic torque characteristics, the conclusion can be drawn that both the health, static eccentric, and dynamic eccentric cases are having a similar maximum output torque and torque ripple. However, the torque ripple of health motor is 3.98% lower than static eccentric motor, and 5.33% lower than dynamic eccentric motor. Comparing to the electromagnetic torque distribution of SynRel motor, the inserted PM improves the

electrical performance. However, in case of eccentric rotor position, the inserted PM deteriorates the performance, for instance the torque ripple.

5.4.2 Electromagnetic Force

The electromagnetic force distributing on all the teeth tips are analyzed, in case of healthy, static eccentric, and dynamic eccentric rotor. Fig. 5.13 to Fig. 5.15 show the results of the electromagnetic forces of the three cases, respectively.

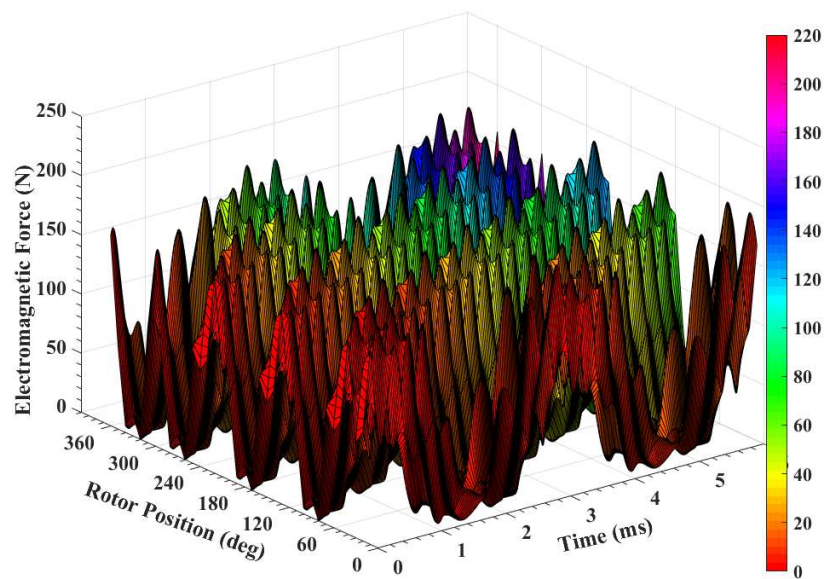


Fig. 5.13: Radial electromagnetic forces distribution (healthy) of PMaSynRel.

Fig. 5.13 shows the distribution of the electromagnetic force in healthy machine. It is seen that the electromagnetic forces acting on the other teeth tips are repeating them of first three teeth tips. Furthermore, the electromagnetic force acting on each tooth tip has a similar amplitude.

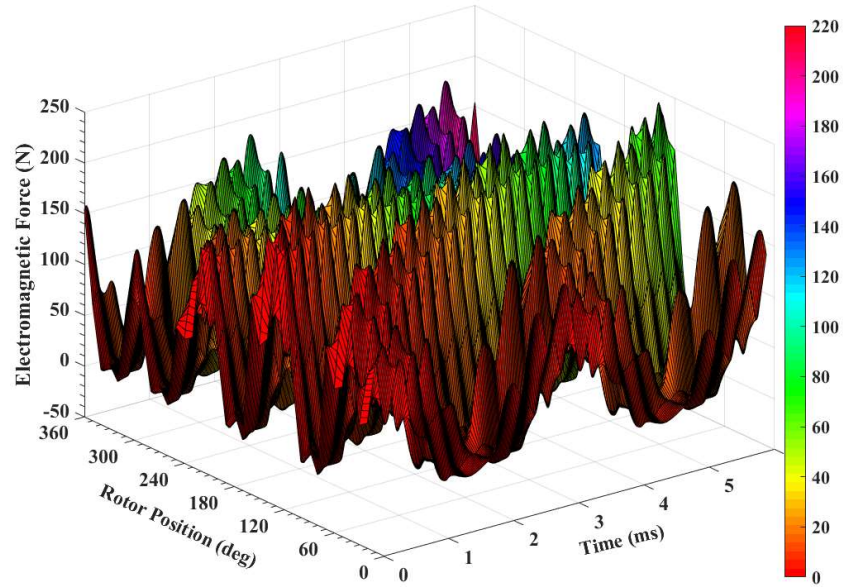


Fig. 5.14: Radial electromagnetic forces distribution (static eccentricity) of PMaSynRel.

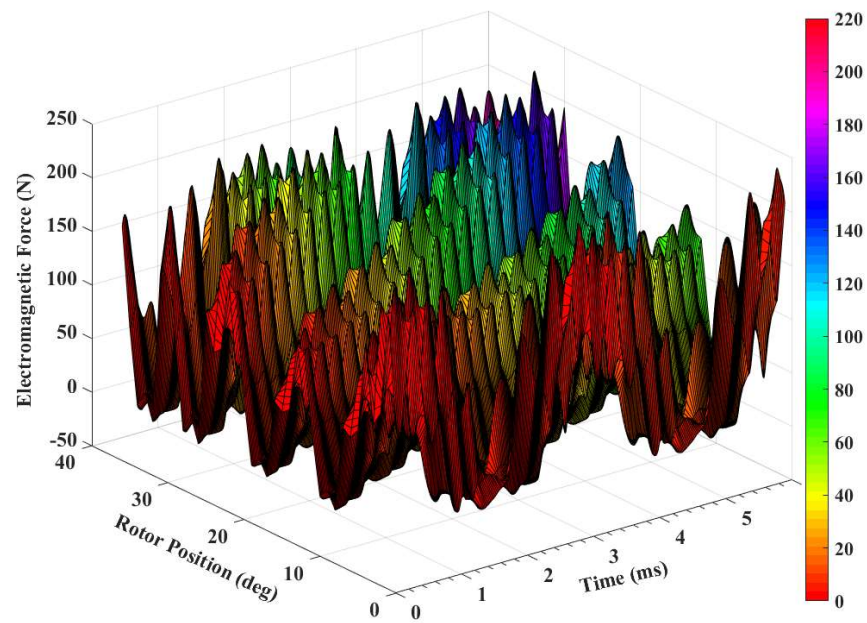


Fig. 5.15: Radial electromagnetic forces distribution (dynamic eccentricity) of PMaSynRel.

In Fig. 5.14 and Fig. 5.15, the electromagnetic forces are from the static eccentricity and the dynamic eccentricity, respectively. It is seen that they have different amplitude, and distribution. The force amplitude in case of healthy rotor is around 160 N, while in both static and dynamic eccentricity, respectively, they are about 240 N, which is 50% higher.

For an obvious comparison of the electromagnetic force, the force amplitudes of each tooth tip are put together as shown in Fig. 5.16.

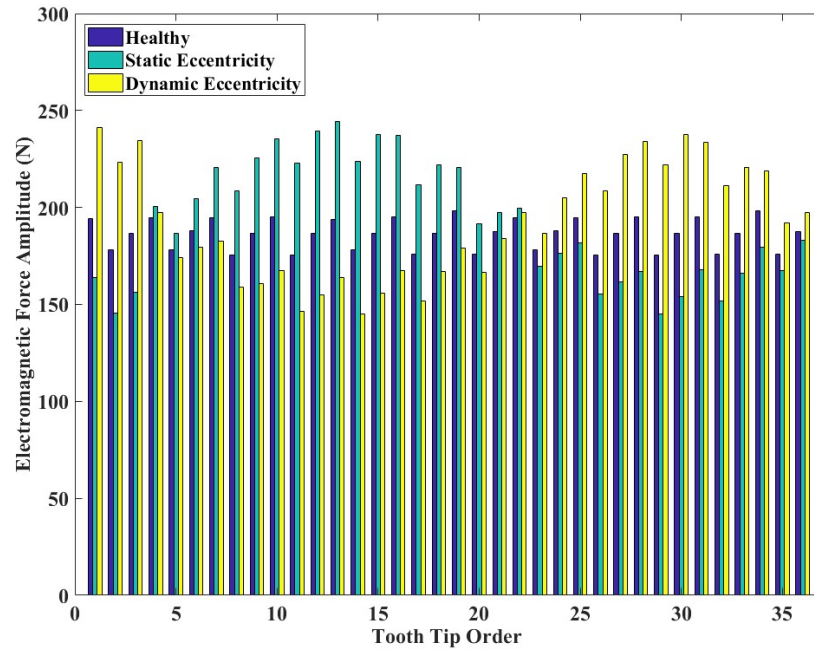


Fig. 5.16: Amplitudes of electromagnetic force acting on each tooth tip.

In Fig. 5.16, it shows the amplitudes of the electromagnetic force from healthy machine is presenting the same trend and value repeating every three teeth tips. However, in both of the eccentric cases, their force amplitudes are fluctuating.

5.4.3 Vibrational Level

The vibrational level of healthy rotor, static eccentric rotor, and dynamic eccentric rotor are obtained through their FE model. Fig. 5.17 shows the comparison on the acceleration response of the three cases.

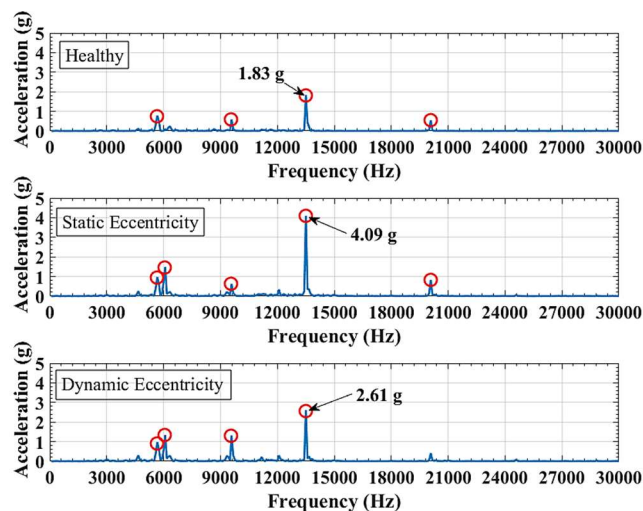


Fig. 5.17: Comparison on vibration level of PMSynRel motor.

From Fig. 5.17, it can be seen that the static eccentricity has the biggest vibration, the amplitude is 4.09 g, while it is 1.83 g and 2.61 g in healthy and dynamic eccentricity, respectively. Hence, the conclusion is drawn that when the eccentricity is 0.15 mm, the peak vibration of the static eccentricity is 123.5% higher than healthy motor, and the peak vibration of the dynamic eccentricity is 42.62% higher than healthy motor. Then, it is obvious that in both static eccentricity and dynamic eccentricity, there are more harmonics in the full frequency range, which means the motor is having more vibrations with respect to the healthy motor. The richer vibration harmonics are mainly caused by the uneven electromagnetic force distribution.

5.5 Summary

In this chapter, both the SynRel and the PMSynRel motors were analyzed for their vibration level in cases of the healthy (concentric condition), static eccentricity, and dynamic eccentricity. In both static eccentricity and dynamic eccentricity cases, their airgap distributions were uneven, which resulted an unbalanced distribution of the electromagnetic force, and consequently a worse vibration. In the healthy

condition, the amplitude of the electromagnetic force acting on every three tooth tips were repeating the same as they were shown on every three tooth tips. While in static eccentricity and dynamic eccentricity, their electromagnetic forces varied with fluctuant but different amplitudes. Consequently, it can also be reflected in the peak vibration, that the largest acceleration in healthy condition was 1.23 g, while it was 2.26 g and 1.77 g in the cases of static eccentricity and dynamic eccentricity, respectively. After that, the analysis was extended to PMaSynRel motor, from where the impact of PM on the eccentric cases was investigated. The largest acceleration captured in healthy condition was 1.83 g, while it was 4.09 g and 2.61 g in cases of static eccentricity and dynamic eccentricity, respectively. It informed that the inserted PM would increase the vibration level of the SynRel motor, and the increasing proportion was larger when the eccentricity occurred.

Chapter 6

Analyses and Experimental Validation of 15 kW SynRel Motor for Vibration

A 15 kW SynRel motor is designed and tested in this chapter to evaluate the analytical and FE modelling methods proposed in the former chapters. Firstly, the 15 kW SynRel motor is built up and analyzed for its electrical performance, from where the air-gap flux density, the electromagnetic torque, and the electromagnetic force are highlighted. Then, the 15 kW SynRel motor is delivered to the modal analysis, the stress analysis, and the harmonic response analysis for its vibration level. Furthermore, the FEA results of the studied SynRel motor are compared with the experimental results to validate the analytical and FEA methods are effective.

6.1 Electrical Performance of 15 kw SynRel Motor

6.1.1 FEA Modelling

It is aforesaid that an electromagnetic FEA model is essential to capture the electromagnetic force distribution of the 15 kW SynRel motor. Therefore, this model is built up using Maxwell Ansys, as shown in Fig. 6.1. Its geometrical data of the 15

kW SynRel motor are concluded in Tab. 6.1. The technical specifications are listed in Tab. 6.2.

Tab. 6.1: Geometrical parameters of the 15 kW SynRel motor.

Parameter	Value
Outer diameter	260 mm
Inner stator diameter	170 mm
Number of slots	48
Slot opening width	3.5 mm
Slot opening height	0.82 mm
Slot height	20.3 mm
Back-iron thickness	23 mm

Tab. 6.2: Technical specifications of the 15 kW SynRel motor.

Parameter	Value
Line-Line voltage	380 V
Rated power	15 kW
Rated torque	95 N·m
Rated speed	1500 rpm
Maximum phase current	75 A

By applying the data listed in Tab. 6.1 and Tab. 6.2, the 15 kW SynRel motor is built up, and its electromagnetic model for FEA is shown in Fig. 6.1.

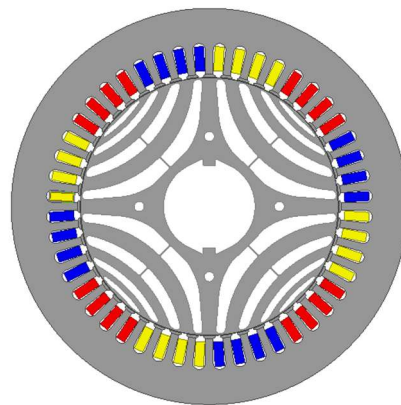


Fig. 6.1: 2D cross section of 15 kW SynRel motor.

It is presented in Fig. 6.1 that this SynRel motor is with 48 slots and 4 poles, therefore the number of slots per phase per pole is 4. Inside the rotor, there are 4 layers of flux barrier per pole. Besides, as the Maxwell stress tensor method is adopted here to compute the electromagnetic force acting on tooth tip, the full electromagnetic model of the 15 kW SynRel motor is created and simulated for better performing the electromagnetic force acting on all the tooth tips.

6.1.2 Air-gap Flux Density

The FEA model is set to be working under the maximum torque per amper (MTPA) condition, in which case the current angle (α_i^e) is equal to 61 electric degrees. Besides, the electromagnetic model is based on a concentric motor, which is without the consideration of eccentricity. Then, the air-gap flux density distribution is obtained through FEA as shown in Fig. 6.2 and Fig. 6.3, that the air-gap flux density distribution is simulated at different rotor position at $\theta_m=0^\circ$ and $\theta_m=60^\circ$, respectively.

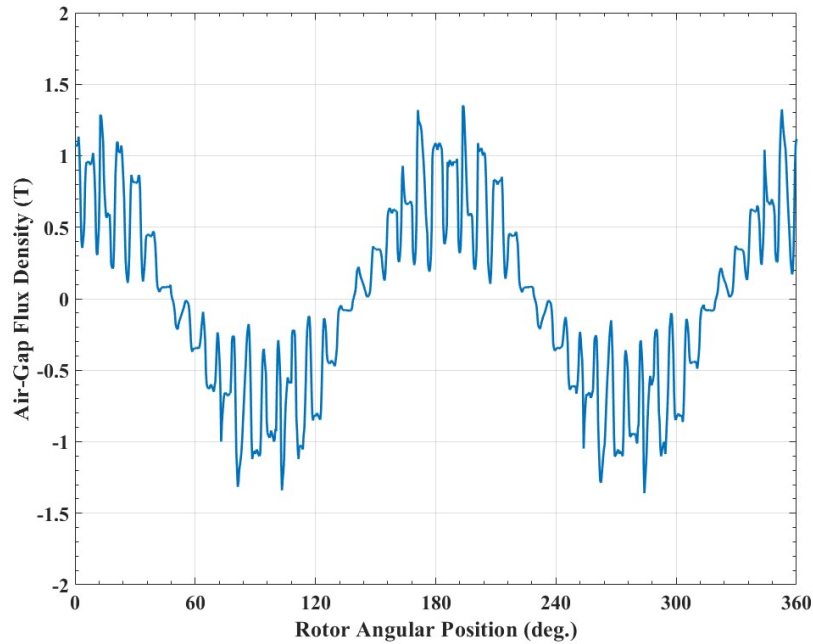


Fig. 6.2: Air-gap flux density distribution of the 15 kW SynRel motor at $\theta_m=0^\circ$.

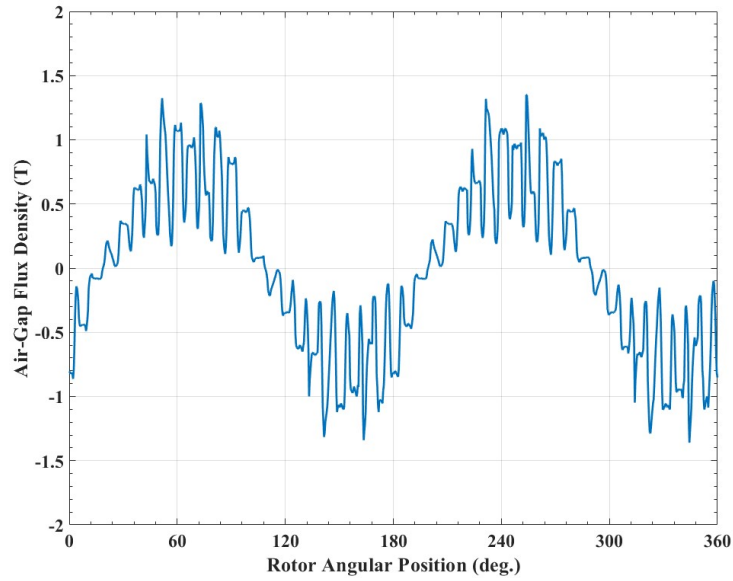


Fig. 6.3: Air-gap flux density distribution of the 15 kW SynRel motor at $\theta_m=60^\circ$.

From Fig. 6.2 and Fig. 6.3, it is noted that the air-gap flux density distributions for all poles are symmetrically performing. The absolute peak value is 1.3 T.

6.1.3 Electromagnetic Force

As there are four slots per phase per pole, the electromagnetic force acting on the first four tooth tips are taken as an example to present the electromagnetic force distribution. Then, Fig. 6.4 presents the electromagnetic force distribution under at rated rotating speed, which is 1500 rpm.

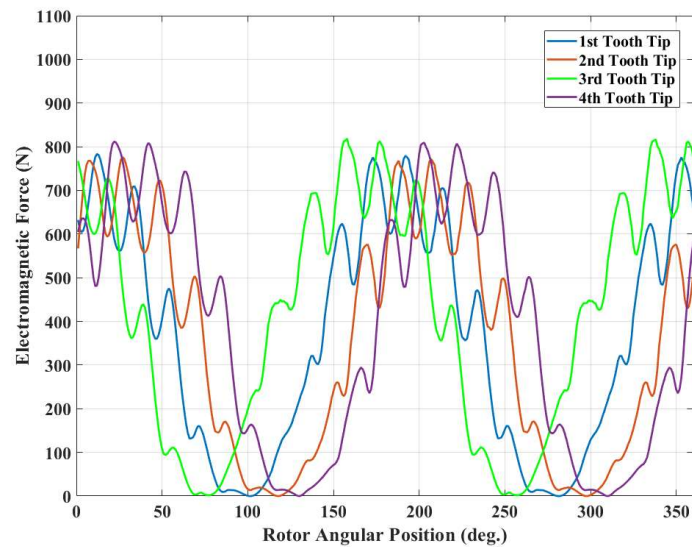


Fig. 6.4: Distribution of electromagnetic force acting on first 4 tooth tips at 1500 rpm.

It is illustrated by Fig. 6.4 that the amplitudes of the electromagnetic force acting on each tooth tip are around 800 N. Besides, their waveforms are performing a similar fluctuating trend. After that, as the armature current is constant, and the magnetic characteristics of the motor under 500 rpm and 1000 rpm conditions are not changing, therefore the distributions of the air-gap flux density and the electromagnetic force under 500 rpm and 1000 rpm are lesser sensitivity to the rotating speed, which means their waveforms are similar to it of under 1500 rpm.

By taking the electromagnetic force acting on the first tooth tip as an example, the difference of electromagnetic force under various load condition is investigated as shown in Fig. 6.5.

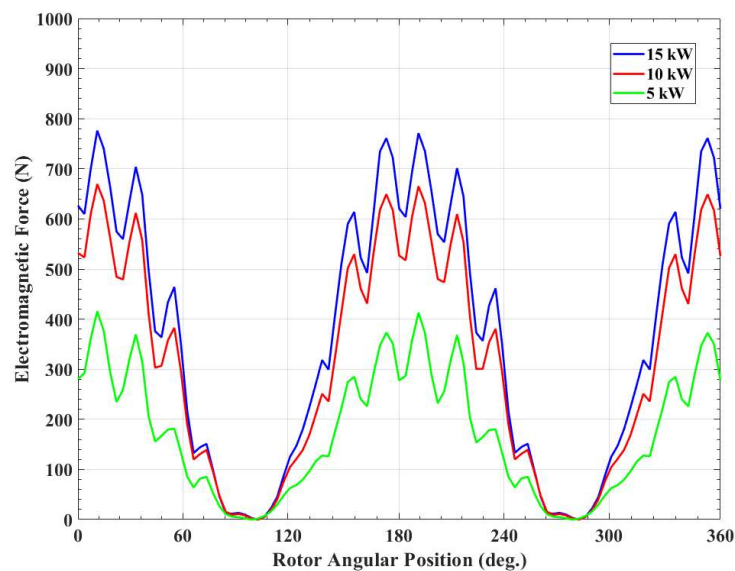


Fig. 6.5: Comparison on electromagnetic force acting on 1st tooth tip varying load conditions at 1500 rpm.

It is noted from Fig. 6.5 that the amplitude of the electromagnetic is not linearly sensitive to the load condition, while the amplitude is higher with a larger load.

6.1.4 Electromagnetic Torque

After analyzing the electromagnetic force, the electromagnetic torque is computed. Similarly, the studied SynRel motor is simulated at the rated condition at

first, and then it is extended to 500 rpm and 1000 rpm, respectively. Fig. 6.6 shows the distribution of the electromagnetic torque at 1500 rpm.

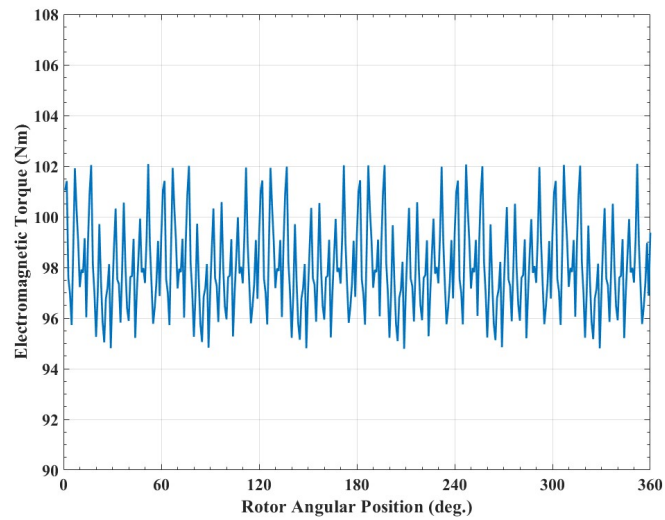


Fig. 6.6: Distribution of electromagnetic torque at 1500 rpm.

From Fig. 6.6, it informs that the maximum torque is at 102.09 N·m, the minimum torque is at 94.79 N·m, and the average torque is at 98.03 N·m. Therefore, the torque ripple is 7.45%. According to Tab. 6.2, the average torque is well meeting the requirement. Then, by simulating the studied SynRel motor at 5 kW and 10 kW, respectively. Fig. 6.8 compares the electromagnetic torques at various power condition, which is 5 kW, 10 kW, and 15 kW, with the speed as 1500 rpm.

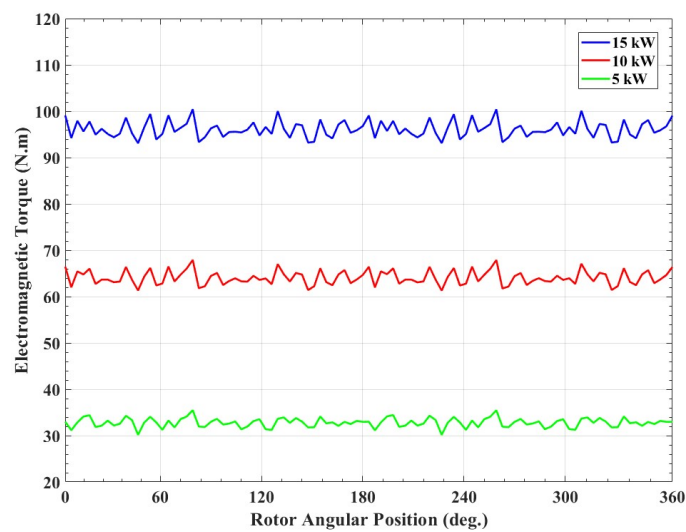


Fig. 6.7: Comparison on electromagnetic torque varying load conditions.

From Fig. 6.7, it can be calculated that the torque ripple under 15 kW, 10 kW, and 5 kW load condition is 7.63 %, 9.42 %, and 16.22 %, respectively. It illustrated that the torque ripple is reaching the minimum value when it is at nominal load condition. Meanwhile, the average torque is also reaching at the highest.

6.2 Mechanical Performance of 15 kW SynRel Motor

As a main part of the mechanical performance, the material used in the stator core and rotor core is M270-50A of which the properties are listed in Tab. 6.3.

Then, a 3D structural model of the studied SynRel motor is built in Ansys Mechanical for the modal analysis, the stress analysis, and the vibration analysis. Again, as the electromagnetic force is considered as the sole exciting force in this work, therefore the influence of controller is ignored, which means the power supplier is assumed as an ideal one.

Table 6.3: Material properties of M270-50A.

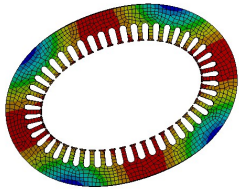
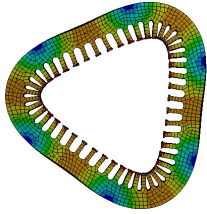
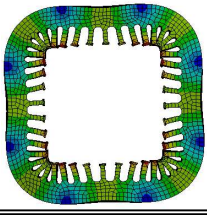
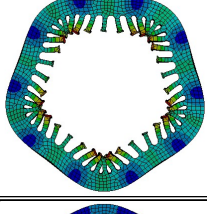
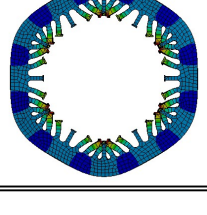
Property	Value
Mass density	7600 kg/m ³
Young's modulus	175 GPa
Poisson's	0.27

6.2.1 Modal Analysis

As the modal analysis method described in the former chapters, it is utilized in this section to investigate the modal shape and the natural frequency of the 15 kW SynRel motor. For a better understanding on each component of the motor, the modal analysis is performed with the stator and the rotor solely, respectively, in

advance. Tab. 6.4 and Tab. 6.5 presents the modal analysis results of the stator and the rotor, respectively.

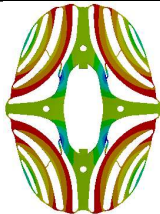
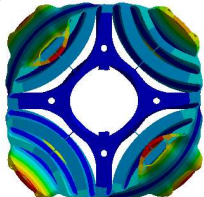
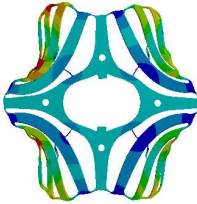
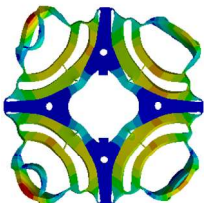
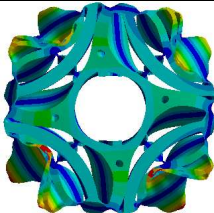
Table 6.4: Modal analysis results of 15 kW SynRel motor stator core.

Mode number	Mode shapes	Natural frequency (Hz)
2		892
3		2387
4		4250
5		6133
6		7498

By comparing the modal analysis result of the 15 kW SynRel motor stator core to it shown in Tab 3.3, it informs that the stator core of the 15 kW SynRel motor reaches a lower natural frequency on each order with respect to it of the 5 kW SynRel motor. It can be identified by the size of these two SynRel motors, that the 15 kW SynRel motor is bigger than the 5 kW SynRel motor, in terms of the outer stator

diameter, the rotor diameter, and the stack length. By considering the mass density and the stiffness coefficient of the materials used in each SynRel motor, the total mass of the 15 kW SynRel motor is much bigger than it of the 5 kW SynRel motor. Then, it can be known from Eq. (3.3) that the natural frequency on each order of the 15 kW SynRel motor is lower than it of the 5 kW SynRel motor. Then, Fig. 6.5 presents the modal analysis results of the rotor core.

Table 6.5: Modal analysis results of 15 kW SynRel motor rotor core.

Mode number	Mode shapes	Natural frequency (Hz)
2		2079
3		3264
4		4575
5		5451
6		6451

Similar to the modal analysis of the stator core, the natural frequency of the rotor core of the 15 kW SynRel motor is lower than it of the 5 kW SynRel motor due to the difference of their total mass.

Then, Tab. 6.6 presents the natural frequency of the whole motor, including the stator core, rotor core, and the housing.

Table 6.6: Natural frequency of whole motor.

Orders	Natural Frequency (Hz)	
	Stator	Whole Motor
2	892	1080
3	2387	2520
4	4250	4320
5	6133	6480
6	7498	7920

6.2.2 Stress Analysis

As the tensile stress acting on the rotor is mainly influenced by the centrifugal force, which is strongly relating to the mass and the rotating speed, the 15 kW SynRel motor is used in this section to deliver a stress analysis to compare to the stress distribution of the 5 kW SynRel motor used in former chapters. Therefore, the difference between these two SynRel motors can be investigated, in terms of their tensile stress of the rotor.

The rotor of the 15 kW SynRel motor is simulated under the rated speed, which is 1500 rpm, to find out its stress distribution. Then, the rotating speed is increasing to 3000 rpm, which is the expected maximum speed, to compare with it under 1500 rpm condition. The stress distribution in cases of 1500 rpm and 3000 rpm is presented in Fig. 6. 8 and Fig. 6. 9, respectively.

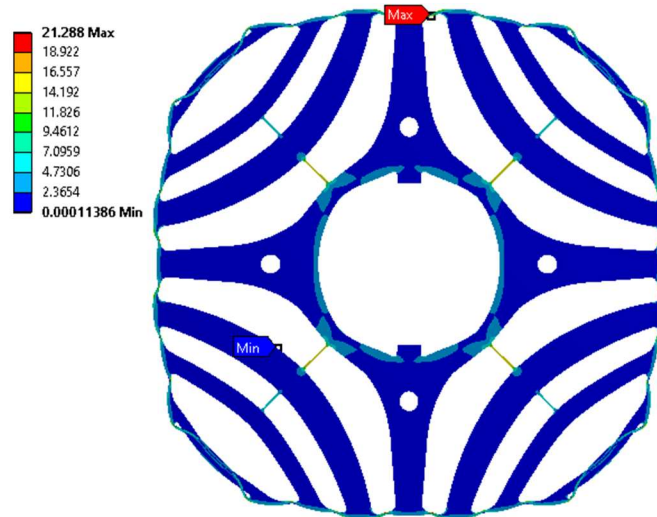


Fig. 6.8: Stress distribution of the 15 kW SynRel motor under 1500 rpm.

In Fig. 6. 8, it illustrates that the maximum stress under 1500 rpm is appearing at the boundary between the 4th flux barrier and the rotor edge, which is around 21.28 MPa. Besides, the overwhelming stress occurs at each position of the rotor is at a safe value. Furthermore, the expected deformation of each flux barrier is highlighted by setting a zoom in (50 times), from where it can be seen the 1st flux barrier is the most vulnerable one to be deformed.

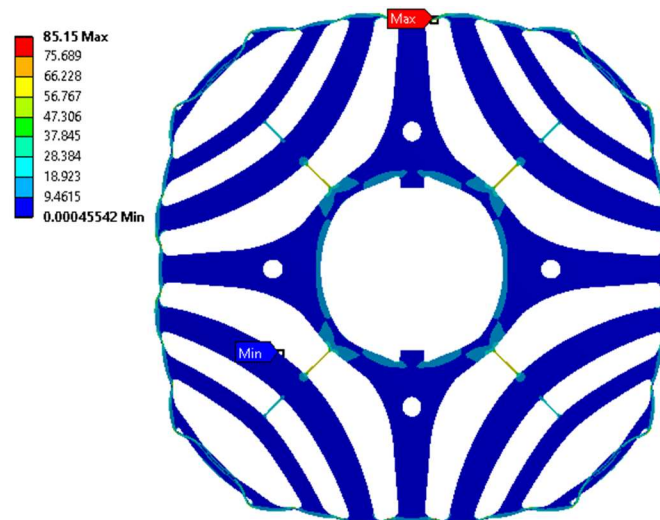


Fig. 6.9: Stress distribution of the 15 kW SynRel motor under 3000 rpm.

Fig. 6.9 presents the stress distribution under 3000 rpm, from where it is noted that the maximum and minimum stresses are occurring around the same positions as they are under 1500 rpm, while the maximum stress is 85.15 MPa in this case. Since

the tensile stress of the material used here is 470 MPa, by setting the safety factor as 2, the maximum stress is still far below. Therefore, this rotor structure is well designed to be working safely from the mechanical side.

6.2.3 Vibration Analysis

The electromagnetic forces obtained from the electromagnetic FEA are imported into the mechanical model to be acting on the tooth tips of the stator inner surface. Then, the vibration analysis is conducted by means of Ansys Mechanical harmonic response analysis. By taking the vibration level on z -axis as an example, Fig. 6.10 presents the deforming and housing acceleration at 1500 rpm under nominal load condition, which is 15 kW.

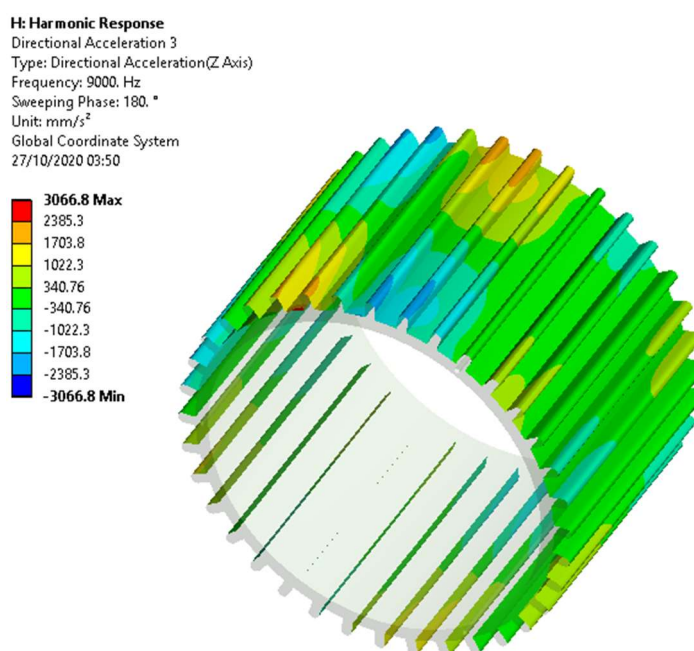


Fig. 6.10: Housing acceleration on z -axis.

From Fig. 6.10, it is noted that the maximum acceleration on the outer surface of the housing is 3066.8 mm/s², which is 3.06 m/s². By collecting the housing acceleration on each axis, Tab. 6.7 presents the vibration level, which is represented by the housing acceleration, of the SynRel motor under 15 kW load condition.

Table 6.7: FEA results of vibration level under 15 kW.

Rotating speed (rpm)	Housing acceleration on z -axis (m/s^2)
500	0.58
1000	0.65
1500	3.06

As Tab. 6.7 presents the total acceleration on z -axis, the acceleration is furtherly studied on its frequency domain. Fig. 6.11 shows the harmonic response of acceleration of the SynRel motor at 1500 rpm.

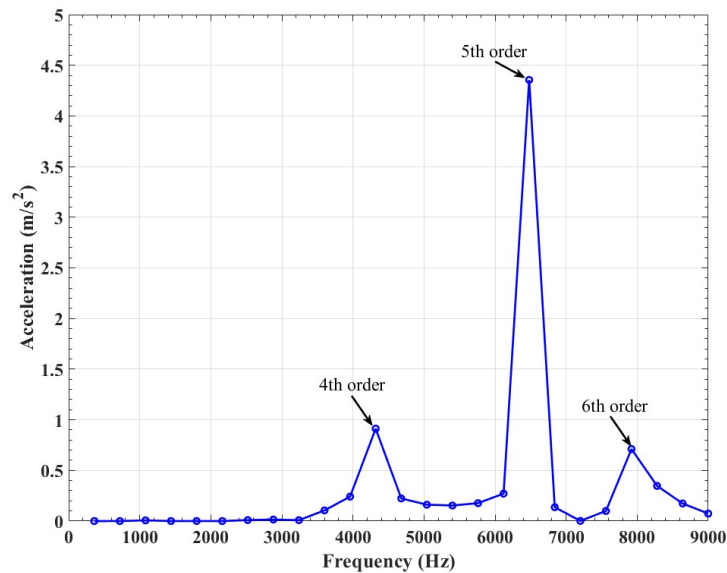


Fig. 6.11: Harmonic response of acceleration for 15 kW SynRel motor at 1500 rpm.

From Fig. 6.11, it is described that there are 3 peak vibrations captured among the frequency range, which is around 4300 Hz, 6500 Hz, and 8000 Hz, respectively. All these frequency points are referring to the natural frequencies shown in Tab. 6.6. The amplitudes of each peak vibration are represented by the housing acceleration, which is 0.91 m/s^2 , 4.35 m/s^2 , and 0.71 m/s^2 , respectively.

6.3 Experimental Validation

After analyzing the 15 kW SynRel motor from the electrical and the mechanical side, the prototype of this studied SynRel motor is manufactured for characterizing and testing its operation, which intends to get the motor performance when driven by a commercial driver, but mainly focuses on the vibration characteristics under various operating. The tests include the no load test, nominal test (15 kW), lower load test (10 kW), and light load condition (5 kW).

6.3.1 Test Specifications and Equipment

In this section, the test specifications and the needed equipment are introduced.

Fig. 6.12 presents the test bench.

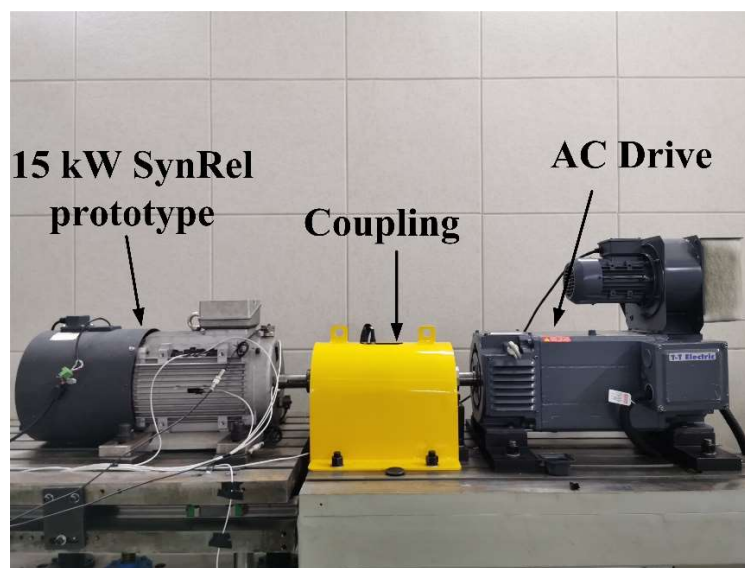


Fig. 6.12: Test bench and 15 kW SynRel prototype.

Then, the torque-speed curves under various power are shown to present the operating cases in test as illustrated in Fig. 6.13.

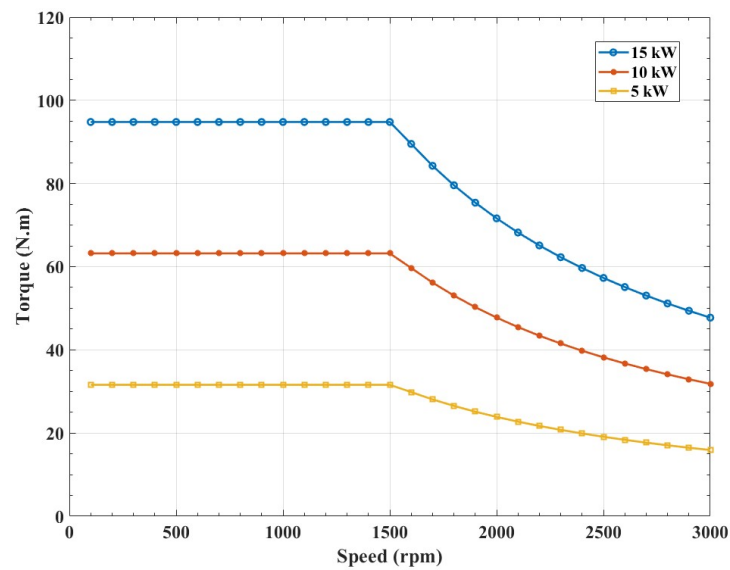


Fig. 6.13: Torque-speed curves under various power of 15 kW SynRel motor.

The tests are operated under the three conditions shown in Fig. 6.13, then Tab.

6.8 lists the measures and equipment used in the tests.

Table 6.8: Measuring methods and Equipment.

No.	Items	Methods	Equipment
1	No load test	- Measure of the line-line voltage, the torque, and the constant K_T .	- Power analyser - Speed meter - Accelerometer
2	Load test: Nominal load (15 kW)	- The motor is operated under nominal load, several measurements are taken: speed, input power, torque, output power, efficiency, housing acceleration. - Calculation of constant K_T under nominal speed conditions - Curves of torque, efficiency and K_T against speed are obtained. - Housing acceleration on each axis.	- Power analyser - Speed meter - Driving machine (Load) - Accelerometer
3	Load test: Light load (5 kW)	- The motor is operated under nominal load, several measurements are taken: speed, input power, torque, output power, efficiency, housing acceleration.	- Power analyser - Speed meter - Driving machine (Load)

		- Calculation of constant KT under nominal speed conditions	- Accelerometer
		- Curves of torque, efficiency and KT against speed are obtained.	
4	Load test: Lower load (10 kW)	- Housing acceleration on each axis.	
		- The motor is operated under nominal load, several measurements are taken: speed, input power, torque, output power, efficiency, housing acceleration.	- Power analyser - Speed meter
		- Calculation of constant KT under nominal speed conditions	- Driving machine (Load)
		- Curves of torque, efficiency and KT against speed are obtained.	- Accelerometer
		- Housing acceleration on each axis.	

Then, Tab. 6.9 introduces the specifications of the measuring equipment used in the tests.

Table 6.9: Specifications of measuring equipment.

Equipment	SN	Performance
Power analyser	YOKOGAWA WT 1800.01	
Scope	KEYSIGHT DSOX2024A	
Load machine	TT-Electric 132-4C	IM 4p 41 kW
Driver	ABB ACS880-01-061A-3	

Fig. 6.14 shows the test conditions, and Fig. 6.15 shows the connection of the vibration test.



Fig. 6.14: Test conditions.

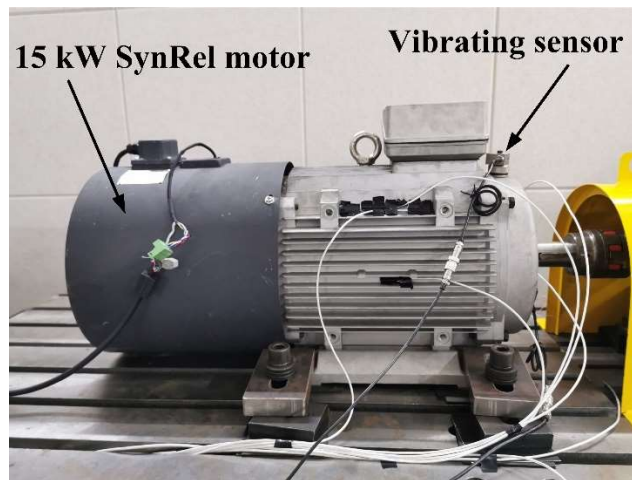


Fig. 6.15: Connections of vibration test.

6.3.2 No Load Test

In this section, the tested motor is operated by the power converter at different speeds while the shaft is connected with the load. The magnitude of the line-line voltage is evaluated. The torque constant K_T is calculated here considering the voltage applied by the inverter and the rotor speed. The experimental results are collected and listed in Tab. 6.10.

Table 6.10: Experimental results of tested motor at no load condition.

Parameter		Measuring value					
Speed (rpm)		500	1000	1500	2000	2500	3000
Line-line voltage (V)		93.43	179.7	268.9	356.2	378	376.1

Current (A)	8.24	8.24	8.2	8.14	6.95	5.95
Input power (W)	0.23	0.38	0.57	0.75	1.05	1.38
Frequency (Hz)	16.66	33.34	50	66.67	83.34	100
Torque (N·m)	1.61	1.57	1.84	2.02	2.24	1.96
Efficiency (%)	36.65	43.27	50.71	56.41	55.85	44.62
KT	0.195	0.191	0.224	0.248	0.322	0.33

Then, Fig. 6.16 and Fig. 6.17 present the figure of the line-line voltage and the efficiency.

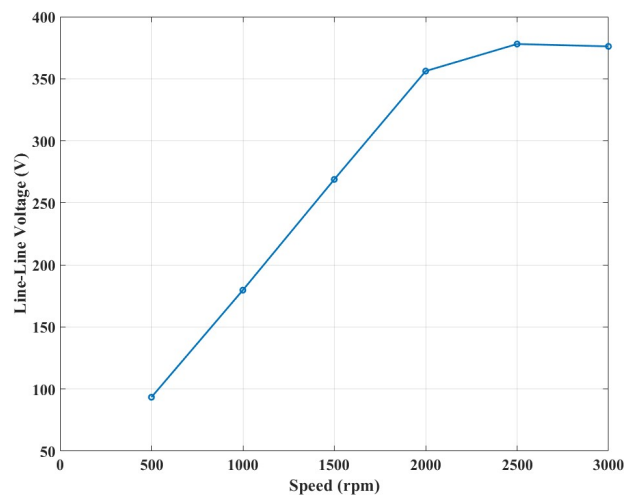


Fig. 6.16: Line-line voltage of tested motor at no load condition.

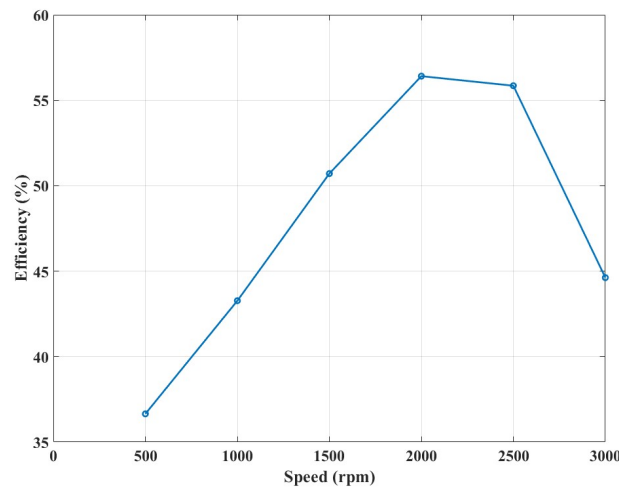


Fig. 6.17: Efficiency of tested motor at no load condition.

From Fig. 6.16 and Fig. 6.17, they inform that the line-line voltage under the maximum speed (3000 rpm) is 376 V, which meets the demand of the designed line-

line voltage 380 V as shown in Tab. 6.2. Besides, from 500 rpm to 2000 rpm, the line-line voltage and the efficiency are about linearly increasing with the rising speed. Then, at the speed range from 2000 rpm to 3000 rpm, the line-line voltage is slowly increasing and then decreasing, while the efficiency is slightly decreasing, then carry on decreasing sharply.

6.3.3 Load Test: Light Load Condition (5 kW)

In this section, the tested motor is operated by the power converter at different speeds while the shaft is connected with the load. The aim is to measure the input power, torque, output power and calculate efficiency under several speed conditions (from 0 to 3000 RPM) when the motor is operating with light load as 1/3 of the nominal condition. Then, the torque constant K_T is calculated here considering the voltage applied by the inverter and the rotor speed. Furthermore, the housing acceleration on each axis is measured by the vibrating sensor. Tab. 6.11 records the experimental results.

Table 6.11: Experimental results of tested motor at light load condition.

Parameter	Measuring value					
Speed (rpm)	500	1000	1500	2000	2500	3000
Line-line voltage (V)	97.04	192.5	291	361.9	365.4	364
Current (A)	14.69	15.03	15.61	13.35	12.1	11.93
Input power (W)	1.79	3.59	5.51	5.62	5.65	5.79
Frequency (Hz)	16.66	33.34	50	66.67	83.34	100
Torque (N·m)	30.07	31.01	31.9	23.97	19.15	16.29
Efficiency (%)	88.19	90.5	90.87	89.32	88.71	88.37
K_T	2.047	2.063	2.044	1.796	1.583	1.365
Housing acceleration on x -axis (m/s ²)	0.66	1.24	2.15	2.59	1.51	2.31

Housing acceleration on y -axis (m/s²)	0.68	1.21	1.76	2.6	2.13	1.88
Housing acceleration on z -axis (m/s²)	0.64	0.91	2.66	2.65	1.48	2.85
Total acceleration 1(m/s²)	1.05	1.36	4.78	6.26	3.17	4.66
Total acceleration 2(m/s²)	1.53	1.6	6.84	6.81	4.22	4.32
Total acceleration 3(m/s²)	1.51	2.21	4.82	3.93	4.32	5.24

For a better and clearer understanding on the experimental results above, the line-line voltage and efficiency are plotted as shown in Fig. 6.18 and Fig. 6.19, respectively. Then, the torque curve is presented and compared with the FEA results as shown in Fig. 6.20.

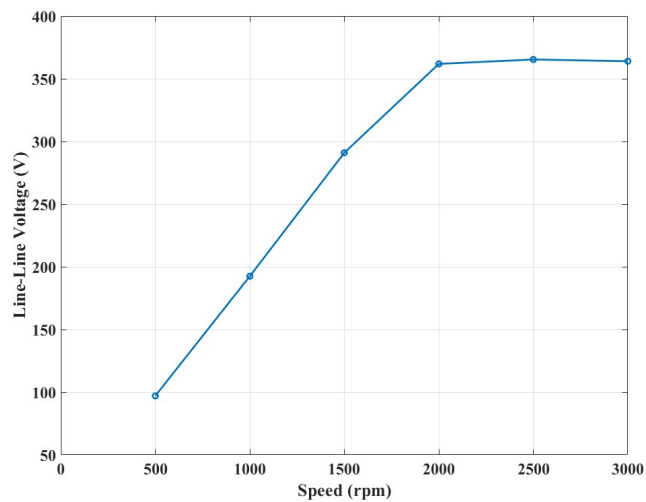


Fig. 6.18: Line-line voltage of tested motor at light load condition.

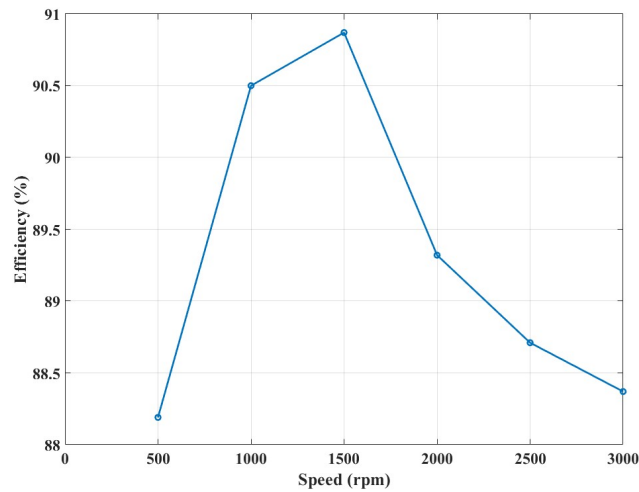


Fig. 6.19: Efficiency of tested motor at light load condition.

From Fig. 6.18 and Fig. 6.19, it is illustrated that the line-line voltage is linearly increasing with the rising speed from 500 rpm to 2000 rpm, the lowest value is occurred at 500 rpm, which is 97.04 V, then keeping constant over 2000 rpm, with the amplitude around 360 V. The efficiency is increasing with the rising speed as well, it gets the lowest value at 500 rpm, which is 88.19%, but reaches the peak at 1500 rpm, with the amplitude as 90.87%. From 2000 rpm to 3000 rpm, the efficiency is decreasing. Under maximum speed, the efficiency is 88.37%.

Then, Fig. 6.20 compares the torque-speed curve of experimental result to it of FEA result under light load condition.

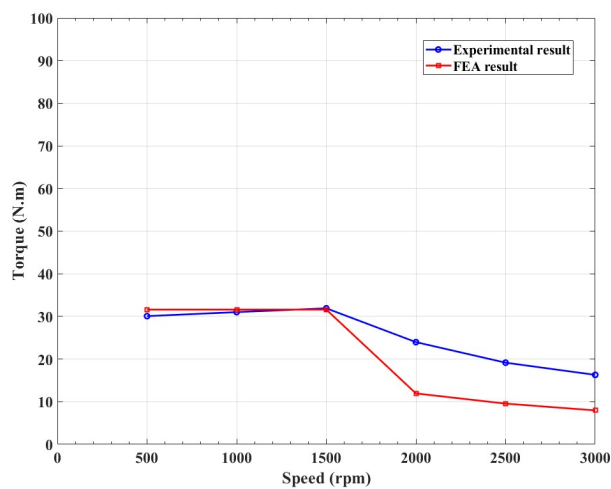


Fig. 6.20: Comparison on torque curve of tested SynRel motor at light load.

Fig. 6.20 shows that the value of the average torque is about 30 N·m at the constant torque region, then decreases with the rising speed. The average torque of FEA result is lower than it of the experimental result, while their waveforms are showing a similar trend.

Then, Fig. 6.21 presents the vibration level at light load condition.

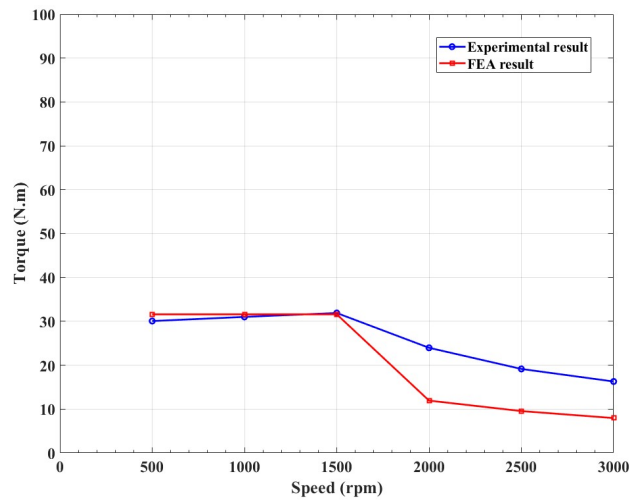


Fig. 6.21: Vibration level at light load condition.

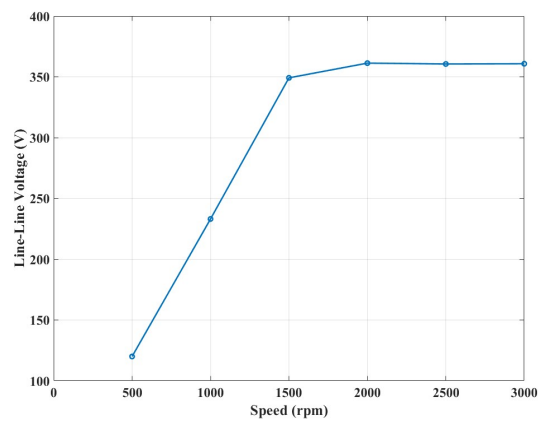
6.3.4 Load Test: Lower Load Condition (10 kW)

This section is aiming to measure the input power, torque, output power and calculate efficiency under several speed conditions (from 0 to 3000 RPM) when the motor is operating at 2/3 of the nominal load. Besides, the load is controlled to maintain a nominal torque from 0 to 3000RPM. Then, the housing acceleration on each axis is measured and collected. Finally, all the experimental results are listed in Tab. 6.12.

Table 6.12: Experimental results of tested motor at lower load condition.

Parameter	Measuring value					
Speed (rpm)	500	1000	1500	2000	2500	3000
Line-line voltage (V)	120	233.1	349.2	361.3	360.6	360.8
Current (A)	23.39	23.62	24	21.4	21.04	21.45
Input power (W)	3.65	7.08	10.66	10.71	10.88	10.88
Frequency (Hz)	16.66	33.34	50	66.67	83.34	100
Torque (N·m)	62.01	62.68	63.67	47.54	38.3	31.54
Efficiency (%)	88.92	92.73	93.78	92.97	92.12	91.06
KT	2.651	2.654	2.653	2.221	1.82	1.47
Housing acceleration on x -axis (m/s ²)	0.86	1.5	2.03	2.81	1.68	2.11
Housing acceleration on y -axis (m/s ²)	0.85	1.21	1.59	2.97	2.25	2.18
Housing acceleration on z -axis (m/s ²)	0.77	1.25	3.11	2.44	1.6	2.92
Total acceleration 1 (m/s ²)	1.35	2.09	3.81	4.43	3.28	5.23
Total acceleration 2 (m/s ²)	1.78	1.97	4.57	5.3	3.99	5.24
Total acceleration 3 (m/s ²)	1.79	2.56	3.81	3.87	5.97	11.11

According to Tab. 6. 12, the line-line voltage and efficiency are plotted as shown in Fig. 6.22 and Fig. 6.23, respectively. Then, the torque curve is presented and compared with the FEA results as shown in Fig. 6.24.

**Fig. 6.22: Line-line voltage of tested motor at lower load condition.**

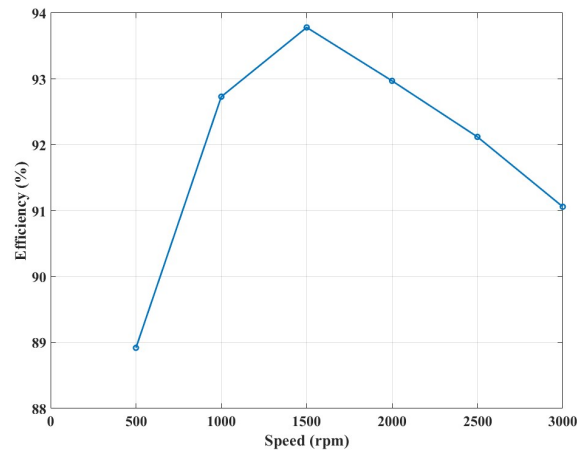


Fig. 6.23: Efficiency of tested motor at lower load condition.

From Fig. 6.22 and Fig. 6.23, it is illustrated that the line-line voltage is linearly increasing with the rising speed from 500 rpm to 2000 rpm, the lowest value is occurred at 500 rpm, which is 120 V, then keeping constant over 2000 rpm, with the amplitude around 360 V. The efficiency is increasing with the rising speed as well, it gets the lowest value at 500 rpm, which is 88.92%, but reaches the peak at 1500 rpm, with the amplitude as 93.78%. From 2000 rpm to 3000 rpm, the efficiency is decreasing. Under maximum speed, the efficiency is 91.06%.

Then, Fig. 6.24 compares the torque-speed curve of experimental result to it of FEA result under light load condition.

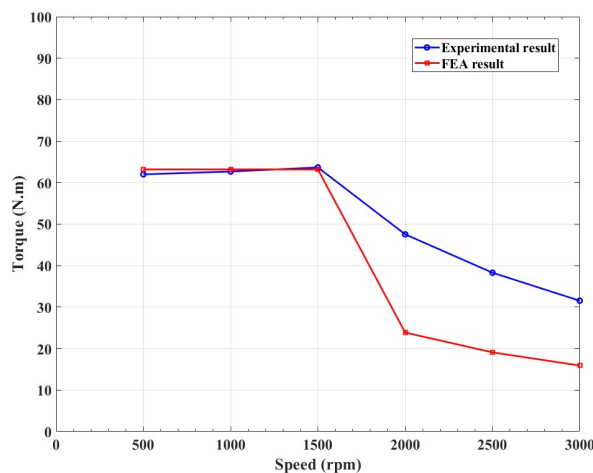


Fig. 6.24: Comparison on torque curve of tested SynRel motor at lower load.

From Fig. 6.24, it illustrates that the value of the average torque at the constant torque region is around 62 N·m, which is aligning with the designed specifications. The FEA result is well agreed with the experimental result for their waveform trend, however the FEA value is much lower than the experimental value.

6.3.5 Load Test: Nominal Load Condition (15 kW)

In this section, the studied motor is tested under various speed, then the magnitude of the line-line voltage is evaluated, the torque, input power, output power, and the housing acceleration on each axis are measured. The efficiency and the torque constant K_T are calculated. Finally, the experimental results are collected and listed in Tab. 6.13.

Table 6.13: Experimental results of tested motor at nominal load condition.

Parameter	Measuring value					
Speed (rpm)	500	1000	1500	2000	2500	3000
Line-line voltage (V)	133.4	255.9	358.5	357.4	358.2	359.3
Current (A)	32.37	32.55	32.74	31.49	32.59	37.32
Power (W)	5.67	10.81	15.92	16.13	16.3	16.78
Frequency (Hz)	16.66	33.34	50	66.67	83.34	100
Torque (N·m)	95.32	95.33	95.33	71.81	57.34	48.05
Efficiency (%)	87.83	92.31	94.28	93.23	92.09	89.96
K_T	2.942	2.929	2.919	2.28	1.759	1.288
Housing acceleration on x -axis (m/s ²)	1.24	1.48	2.06	2.72	1.91	3.02
Housing acceleration on y -axis (m/s ²)	1.29	1.43	1.56	3.3	2.14	1.85
Housing acceleration on z -axis (m/s ²)	1.06	1.17	3.3	2.73	2.11	3.25

Total acceleration 1 (m/s ²)	1.95	2.87	3.22	3.85	4.14	5.62
Total acceleration 2 (m/s ²)	2.58	2.36	3.64	4.61	4.28	6.45
Total acceleration 3 (m/s ²)	2.13	2.78	4.37	4.98	8.42	32.09

For a better understanding and a comparison on the experimental data shown in Tab. 6.10, the curves of the line-line voltage and efficiency under the conditions of light load, lower load, and nominal load, are plotted in Fig. 6.25 and Fig. 6.26, respectively. After that, the comparison on torque curves under these three load conditions is conducted and shown in Fig. 6.27.

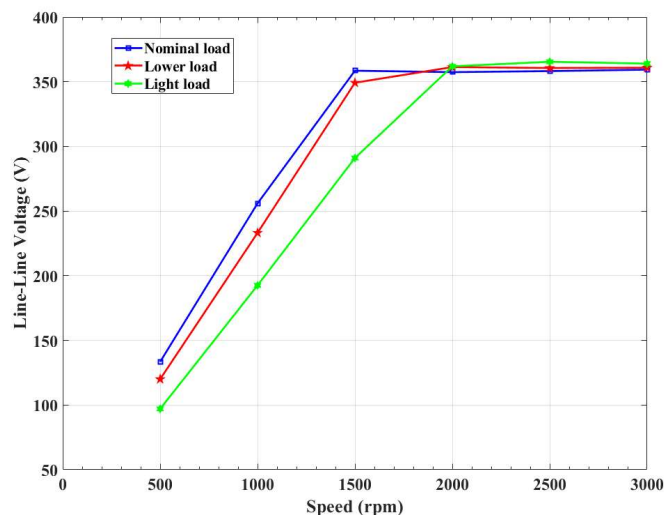


Fig. 6.25: Comparison on line-line voltage of tested motor varying load condition.

From Fig. 6.25, it compares the line-line voltage under various load condition. It shows that the line-line voltage under nominal load condition is always higher than it of under the other two conditions in the region of constant torque. However, at the speed of the motor is over 1500 rpm, the line-line voltages of under the nominal load condition and the lower load condition are aligning with each other around 360 V,

which is at the region of constant power. But the line-line voltage of under light load condition is reaching 360 V over 2000 rpm.

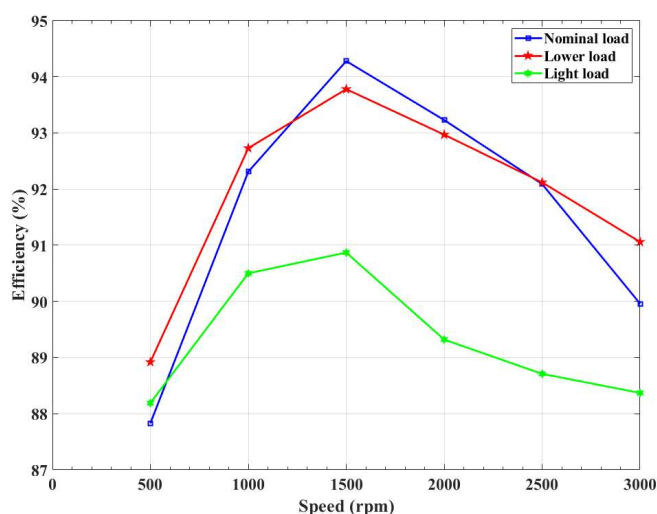


Fig. 6.26: Comparison on efficiency of tested motor varying load condition.

From Fig. 6.25 and Fig. 6.26, it is illustrated that the line-line voltage is linearly increasing with the rising speed from 500 rpm to 2000 rpm, the lowest value is occurred at 500 rpm, which is 133.4 V, then keeping constant over 2000 rpm, with the amplitude around 360 V. The efficiency is increasing with the rising speed as well, it gets the lowest value at 500 rpm, which is 87.83%, but reaches the peak at 1500 rpm, with the amplitude as 94.28%. From 2000 rpm to 3000 rpm, the efficiency is decreasing. Under maximum speed, the efficiency is 89.96%. At the rated working point, the efficiency under nominal load condition is the highest with respect to it of under light load and lower load condition. As for the rest of the speed region, the efficiencies of under nominal load and lower load condition are well aligning with each other, while they are always higher than the efficiency of under light load condition.

Then, two comparisons are conducted and shown in Fig. 6.27. Firstly, the torque curves under each load condition are compared with each other. Secondly, all the

torque curves drawn from the experimental data are compared to them of the FEA results. Therefore, this SynRel motor is well studied in terms of its torque characteristics and the research method for electromagnetic FEA.

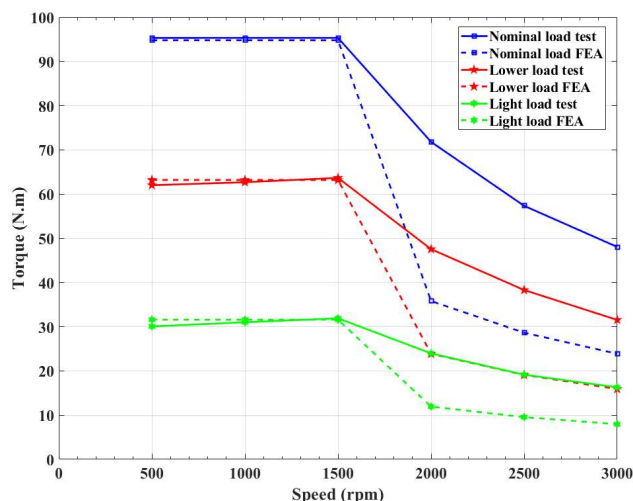


Fig. 6.27: Comparison on torque curve of tested SynRel motor varying load conditions.

Fig. 6.27 compares the torque-speed curve under various load conditions, which is 5 kW, 10 kW, and 15 kW, respectively. At the constant torque region, the FEA results are well aligning with the experimental results. Besides, the experimental average torque of the aforesaid load conditions is 95.33 N·m, 63.67 N·m, and 31.01 N·m, respectively. It can be seen that the average torques under various load conditions are proportional to the load power. Also, they are meeting the requirement demanded in Tab. 6.2.

6.4 Summary

In this chapter, a 15 kW SynRel motor was prototyped for the experimental validation of the analytical method and FEA conducted in the former chapters. Firstly, the electromagnetic model was built, that the full model was required for obtaining the electromagnetic force acting on all the tooth tips, by modelling in FEA software. Then, the electrical performance was simulated and studied, from where

the air-gap flux density, electromagnetic force, and electromagnetic torque were highlighted. It informed that the FEA results of the electrical performance well agreed with it required by the designing demands, and aligned with the analytical value. Secondly, the mechanical performance, which included the modal analysis, stress analysis, and vibration analysis, were performed through FEA by utilizing Ansys Mechanical. Hence, the natural frequency, stress distribution, and vibration level were obtained through FEA, which were meeting with the analytical laws and equations. Finally, the 15 kW SynRel prototype was tested for its electrical performance and mechanical performance, from where the analytical derivation and FEA method were effectively validated.

Chapter 7

Conclusion and Future Works

In this work, the SynRel motors were studied for its vibration level. The main objective was to investigate the impact of the motor geometry and features on the vibration level of SynRel motors.

In Chapter 1, a brief introduction to the SynRel motor of which the basic reluctance rule and the operating method were presented. An overview of the electrical performance of SynRel motor was presented with prime focus on the computation of the electromagnetic torque and force. As the main excitation of the motor vibration, the methodology for analyzing and calculating the electromagnetic force was listed and discussed, from where the Maxwell stress tensor method was highlighted with its advantages and disadvantages. Following that, the multi-degrees vibrating system was introduced, based on which the motion equation was derived. Then, the influence of the electromagnetic force on vibration was furtherly reviewed and studied, including the electromagnetic force harmonics, the impact of inserted permanent magnet, and the eccentricity.

In Chapter 2, an analytical model of the SynRel motor was proposed by utilizing a 5 kW SynRel motor, with 36 slots, 4 poles, and 3 flux barriers per pole. An equivalent magnetic circuit was developed by dividing the rotor into various regions based on the magnetic reluctance on each flux path. By ignoring the stator slot effect, a current sheet was adopted to perform the electrical characteristics, where the air-

gap flux density, the electromagnetic torque, and the electromagnetic force were obtained through the analytical calculation, and then validated by FEA results. Furthermore, a group of sensitivity analyses of the key geometrical parameters were performed to investigate the impact of the rotor geometry on the electrical characteristics of SynRel motor.

In Chapter 3, a generic approach for analyzing the vibration response of the SynRel motor was presented. A general methodology of the vibration analysis on the SynRel machine was introduced. It started with the modal analysis to find the structural characteristics of the SynRel motor, including the natural frequencies, and the modal shape corresponding to each natural frequency. Then sensitivity analyses were performed by varying the back-iron thickness, tooth width, and slot height for investigating their impacts on the natural frequency, from where it was illustrated that the back-iron held the largest influence. Therefore, the back-iron thickness should be the first option in case of shifting natural frequency.

After that, the stress distribution of the rotor of SynRel motor was analyzed under the rated rotating speed of 5000 rpm, followed by a sensitivity analysis with varying rib length and rotating speed, respectively. It discussed that the flux barrier closer to rotating centre suffered more serious stress tensor with respect to the other flux barriers. Besides, the maximum stress was smaller with a wider rib length, therefore the rib length should be carefully considered during the preliminary design by considering both the electrical performance and the mechanical characteristics. Furthermore, the maximum stress exponentially increased with the rising rotating speed, which is well defined in the equation of centrifugal force. Hence, the

operating speed range should be determined by considering both the electrical performance and the mechanical characteristics of the motor.

Then, the vibrational harmonic response analysis was conducted by means of FEA to investigate the vibration characteristics of SynRel motor. The exciting force of the vibrating system mainly consisted of the electromagnetic force, which was obtained from the 2D electromagnetic FEA. Then, the housing acceleration was adopted as the representative of the vibration level. The peak vibration under 5000 rpm was 1.23 g, which occurred around the 6th order natural frequency of 13700 Hz.

In Chapter 4, the impact of inserted PM on the SynRel motor for its vibration level was investigated. An equivalent magnetic circuit, which considered the flux of PM, was derived from the proposed equivalent magnetic circuit of SynRel motor. The electrical performance of the PMaSynRel motor was analytically calculated, and then validated by FEA computation. The impact of the inserted PM on the electromagnetic torque and force were highlighted, which informed that the PMaSynRel motor provided similar average torque by using smaller phase current (lesser electrical loading). The electromagnetic force acting on the first three tooth tips of the PMaSynRel motor was plotted to present its distribution, which shows that the electromagnetic force of the PMaSynRel motor was slightly higher than it of the SynRel motor.

The impact of inserted PM on the mechanical characteristics was investigated. As the stator of PMaSynRel motor was the same as it of SynRel motor, the rotor was highlighted with the stress analysis. Even though the inserted PM made the total mass bigger, it increased the stiffness by supporting the ribs of the flux barrier. Therefore, the maximum stress on the rotor of PMaSynRel motor was 47 MPa,

while it of SynRel motor was 109 MPa. Then, the vibration analysis was conducted to investigate the impact of PM on vibration. The peak vibration of PMSynRel motor was 1.83 g, while it of SynRel motor was 1.23 g.

In Chapter 5, both the SynRel and the PMSynRel motors were analyzed for their vibration level in cases of the concentric, static eccentric, and dynamic eccentric conditions. In both static eccentricity and dynamic eccentricity cases, their airgap distributions were uneven, which resulted in an unbalanced distribution of the electromagnetic force, and inadvertently worsening the vibration. In the healthy condition, the amplitude of the electromagnetic force acting on every three tooth tips were repeating the same as they were exhibited on every three tooth tips. While in static eccentricity and dynamic eccentricity, their electromagnetic forces fluctuated with different amplitudes. Consequently, it can also be reflected in the peak vibration, that the largest acceleration in healthy condition was 1.23 g, while it is 2.26 g and 1.77 g in the cases of static eccentricity and dynamic eccentricity, respectively. After that, the analysis was extended to PMSynRel motor, from where the impact of PM on the eccentric rotor cases. The largest acceleration captured in healthy condition was 1.83 g, while it was 4.09 g and 2.61 g in cases of static eccentricity and dynamic eccentricity, respectively. It showcased that the inserted PM would increase the vibration level of the SynRel motor, and the increase in proportion was larger when the eccentricity occurred.

In Chapter 6, the experimental verifications of the vibration analysis in this thesis were carried out using a 15 kW SynRel prototype. It was shown that the peak vibrations were happening around each order natural frequency. However, the experimental results of the vibration were higher than them from FEA result. As one

of the reasons, the current harmonics from the AC drive and controller were ignored in the FEA. Also, the manufacturing and assembling errors such as dynamic eccentricity would produce extra vibration.

The works conducted in this thesis provided a comprehensive analysis of the SynRel motor, and successfully investigated the key geometrical parameters which were strongly relating to the mechanical characteristics of the SynRel motor. Besides, the impact of permanent magnet on the performance of SynRel motor was developed. The effects of eccentricities on both SynRel and PMaSynRel motor were investigated, which also provided a method for evaluating the eccentric error during the manufacturing of the SynRel motor. Based on the findings of this thesis, following consideration should be made while designing the SynRel or PMaSynRel motor for avoiding resonance or a low vibration level.

1. The rotor geometry should be designed taking both electrical and mechanical performance into account. Especially, some key geometrical parameters such as the rib length have opposite impacts on electrical performance and mechanical characteristics.
2. As the electromagnetic force is part of the exciting factors of the motor vibration, the current harmonics existing in controller should be also considered, specifically in case the current distribution is not sinusoidal waveform.
3. The inserted PM significantly increase the electrical performance of SynRel motor, however, the peak vibration of PMaSynRel motor is about 50% higher than it of SynRel motor. As a result, the inserted PM should

be carefully designed by integrated considering both the cost and the mechanical characteristics of the motor.

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