RECOVERY RATE, DEBT STRUCTURE AND VALUATION WITHIN U.S. BANKRUPTCY LAW

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Abstract

The U.S. Bankruptcy Code is a frequently used channel to resolve corporate financial distress. In the code, liquidation (Chapter 7) and reorganization (Chapter 11) are two most crucial processes. My PhD thesis discusses several important issues around the U.S. Bankruptcy Code, including recovery rate determination in bankruptcy, debt valuation in bankruptcy under a two-class debt structure and determination of an optimal bankruptcy threshold. With the aim of linking corporate finance and asset pricing, new models of credit risk are developed in this thesis and fruitful empirical implications are generated.

Specifically, the first main chapter is "Default and Recovery Rate under Chapter 11 with Multiple Debts". This studies both theoretically and empirically the influence of debt structure on the outcome of debt renegotiation under Chapter 11. I investigate the trilateral negotiation in court-supervised formal bankruptcy. The model demonstrates how loans and bonds differ in terms of concentration level of debt owner and how this disparity impacts the action of the debtor company both before and after bankruptcy. The model developed in this chapter predicts that creditors' ultimate recovery is higher for firms with more bank debt and less bargaining frictions and, despite the bank's involvement improving total recovery, bondholders are still disadvantaged by the presence of senior

bank creditors. Using a sample of 439 U.S. firms that filed for Chapter 11 during 1987-2014, I present evidence on the link between bank debt share and recovery rates that is supportive of the model's prediction.

The second main chapter is "Debt Structure and Valuation in U.S. Bankruptcy Code". In this chapter I discuss the impact of bankruptcy procedure on security valuation by developing a credit risk model. As in the first chapter, the debt structure is in the form of two-class debt. A structural model of credit risk is built where default and liquidation are represented by two boundaries and a grace period is granted prior to liquidation. Within the setup, corporate debt is viewed as quasi Parisian corridor option and valuations are obtained via a partial differential equation formulation solved using a finite difference approach. The model can generate a credit spread for corporate debt which is more quantitatively consistent with the market credit spread. In this chapter I also show how the debt valuation is affected by several bankruptcy-related factors such as length of grace period.

The last chapter, "Boundary Determination and Optimal Control Right Allocation in Financially Distressed Firms", analyzes the determination of optimal default and liquidation boundaries in bankrupt entities. Compared with the previous chapter, this chapter reexamines the issue of debt valuation but allows the liquidation and renegotiation boundaries to be determined endogenously by valuing the maximization decision of involved parties. The

model results show that different claim holders choose different default and liquidation boundaries to maximize the value of securities they hold, which leads to conflicts of interest between borrowers and lenders and also between different borrowers. The first-best outcome can be achieved if bondholders determine the liquidation boundary. Finally, the model shows that the optimal length of grace period, in the sense of firm value maximization, is roughly 6 months.

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List of Notations

Notation	Definition
b	Continuous coupon rate of bank loan
$B_A(X_t)$	Value function of bank obligation in non-bankruptcy states
$B_P(X_t)$	New bank debt value after renegotiation
c	Continuous coupon rate of corporate bond
d	The grace period—maximal amount of time the firm is allowed to stay in Chapter 11
D	Total face value of all debts outstanding
D_J	Junior debt value
D_S	Senior debt value
E	Equity value
$E_A(X)$	Equity value in the non-bankruptcy region
G	renegotiation surplus
$i \in (b, m, e)$	Player in the negotiation game
k	Proposer of the renegotiation plan
K	Fixed liquidation costs
$M_A(V_t)$	Value function of bond debt in the non-bankruptcy states

p_i	Probability of being chosen as the proposer for player i
P_b	Principal of senior debt
P_c	Principal of junior debt
q	Probability that the procedure moves to the following date. On the contrary, $1-q$ is renegotiation failure probability
r	Constant risk-free interest rate
RR_{T}	Firm recovery rate
RR_{M}	Bondholder recovery rate
$s(V_u)$	Reduced debt service in Chapter 11
$s^S(V)$	Reduced bank debt service in Chapter 11
$s^J(V)$	Reduced bond debt service in Chapter 11
T	Maturity date of junior and senior debt
$v(V_t)$	The value of firm
v_b^L	Liquidation value of bank debt
v_m^L	Liquidation value of bond debt
v_i	The continuation payoff of player i, or, the expected payoff of player i from the subgame staring next period
V_t	Value of unlevered assets
V_B	Bankruptcy (Chapter 11) boundary (V_S in Chapter 4)
V^L	liquidation value of the firm
V_L	Liquidation (Chapter 7) boundary

W_t	Standard Brownian motion under the risk-neutral probability
x_i^k	The amount of the firm's value allocated to player i under the proposal submitted by k
X_S	Endogenous default threshold
X_t	Operating earnings before interest and taxes (EBIT)
α	Proportional liquidation costs
β	Discount factor
δ	Firm's payout rate
$\boldsymbol{\eta}$	Bargaining power in renegotiation
θ^d	Liquidation time
μ	Firm's constant growth rate
σ	Volatility of the value of unlevered assets
τ	Tax rate
τ_t^d	The first time that the length of the excursion in the corridor reaches given level d, given that the excursion starts from the upper barrier V_B
φ	Distress costs
ω	The percentage of bank portion in total debt
Subscript b	Bank lender
Subscript m	Bondholder
Subscript e	Equity holder
Subscript A	Valuation function before bankruptcy

Subscript P	Valuation function after bankruptcy
$\mathbf{1}_{\{x\}}$	Indicator function of the event x, which takes the value of 1 if x is true and zero otherwise.

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CHAPTER 1

Introduction

1.1 Research Questions

I begin with debt pricing based upon structural credit risk models developed in the Black-Scholes-Merton contingent claim framework. This will be explained later but a key point is that, in much of the literature (e.g., Merton (1974), Black and Cox (1976), Leland and Toft (1996), Carey and Gordy (2016), etc.), default is taken as equivalent to liquidation: once a company is in default, it is modelled as instantly in liquidation.

In practice, companies which default generally go into a period of reorganization and may or may not be subsequently liquidated. For instance, the biggest bankruptcy in American history is the case of Lehman Brothers with the value of $691 billion at bankruptcy on September 2008. As the financial meltdown accelerated, the government made the now-controversial decision not to bail out the giant investment bank. Lehman was eventually liquidated. In another famous bankruptcy case, Chrysler, file for Chapter 11 at the Federal

Bankruptcy Court of the Southern District of New York, on April 30, 2009. Chrysler expressed hope for a "surgical" bankruptcy lasting 30 to 60 days, with the result of reducing the company's liabilities and post-bankruptcy emergence in stronger financial shape. The submitted court documents indicated that there would be a reorganization plan presented to the court in 120 days, on August 28, 2009. The United Automobile Workers were given control of the company, with the federal government and Italian carmaker Fiat as minority stakeholders. Two years out of bankruptcy, Chrysler was profitable again. In the list below, I give the big bankruptcy cases happened in American. All of these firms initially file under Chapter 11. Some of them eventually successfully emerged from Chapter 11 while others are liquidated or acquired.

Name	**Chapter**	**Date Confirm**	**Date Filed**	**Emerge**
Lehman Brothers Holdings Inc.	11	12/06/2011	09/15/2008	no
Washington Mutual, Inc.	11	02/23/2012	09/26/2008	no
Worldcom, Inc.	11	10/31/2003	07/21/2002	yes
General Motors Corporation	11	03/29/2011	06/01/2009	yes
Enron Corp.	11	07/15/2004	12/02/2001	yes
Conseco, Inc.	11	09/09/2003	12/17/2002	yes
Refco Finance Inc.	11	12/15/2006	10/17/2005	no
Texaco Inc.	11	03/23/1988	04/12/1987	yes
First Republic Bank Corp	11	10/30/1990	07/30/1988	no
General Growth Properties, Inc.	11	10/21/2010	04/16/2009	yes
UAL Corporation (United Airlines)	11	01/20/2006	12/09/2002	yes
Pacific Gas & Electric Co.	11	12/22/2003	04/06/2001	yes

This thesis investigates debt pricing, separating the notions of default and liquidation. Although more recent researches such as Fan and Sundaresan (2000), Broadie, Chernov and Sundaresan (2007) and Valta (2016) have started to notice the possibility of debt renegotiation before liquidating the firm, most studies cover corporate bonds only, ignoring the interplay between bond holders and banks. Thus, in either empirical or theoretical work on credit risk models, scholars usually follow the assumption that a firm only issues a single class of renegotiable consol debt whereas, in practice, the most common scenario is for a firm to have both bank debt and market debt (bonds) outstanding simultaneously. Now, in financial distress, the firm might choose different strategies towards banks and bondholders in order to optimally resolve the distress. The models in this PhD thesis investigate whether the behavior of the firm would be expected to change with different debt structures and how to value the debt in these cases.

By generalizing the debt structure, I analyze the debt renegotiation under the U.S. Bankruptcy Code and discuss several important problems around the U.S. Bankruptcy Code within the contingent claim framework. In the decades since the founding work of Merton (1974), there have been very many studies endeavoring to develop better structural credit risk models. In this thesis, I attempt to contribute to the strand of debt renegotiation by offering a much richer modelling of different renegotiation behavior of debt holders. My thesis

contributes to three important topics: debt renegotiation; debt valuation; corporate governance in Chapter 11.

Specifically, my aim is to answer the following research questions:

1. How are debts issued by the defaulted company renegotiated when bank debt and market debt coexist during the Chapter 11 period? Does the debt structure affect the total recovery of the firm in bankruptcy? How can we model this impact?
2. How will the debt structure factor finally affect the defaulted bond pricing and how can debt structure information be incorporated into recovery prediction models and bond pricing models?
3. If we assume the bank and market debt claims have finite maturities, how should they be priced within the context of the U.S. Bankruptcy Code? What option pricing techniques should be used to get the valuation?
4. How are the valuation results influenced by bankruptcy factors such as the length of grace period?
5. Based on the asset pricing results, what is the optimal control right allocation policy in Chapter 11 in order to achieve the social best result? Additionally, what is the optimal length of grace period?

In summary, I study the role of a mixed debt structure in a structural model of

credit risk with the focus on correct modelling and investigating results. Debt structure is expected to affect both recovery and debt valuation. It is pointed out in Chapter 4 that the composition of debt is an important determinant of the ultimate outcome of bankruptcy, i.e. it is expected that some debt structures are prone to better ultimate recovery outcomes. The interaction between creditors and determination of recovery rates is explored through this chapter. The findings might be useful in recovery rate prediction as well as for bond pricing models. Chapter 5 covers pricing results for two-layer finite maturity debts. Particularly, this chapter discusses the impact of bankruptcy procedure on the valuation of corporate securities. With a similar approach, I then proceed in Chapter 6 to discuss the optimal boundary determination in bankruptcy by value maximization function.

1.2 Research Motivations and Contributions

This thesis is motivated by recent studies in asset pricing (including but not limited to François and Morellec (2004), Galai, Raviv and Wiener (2007) and Broadie, Chernov and Sundaresan (2007)) that highlight the important role of the bankruptcy code, especially the reorganization provisions.[1] Other models

[1] Additionally, Rodano, Serrano-Velarde and Tarantino (2016) documents that their results highlight the importance of identifying the distinct effects of liquidation and reorganization, as these procedures differently address the tension in bankruptcy law between the continuation of viable businesses and the preservation of repayment incentives.

such as Fan and Sundaresan (2000), Hackbarth, Hennessy and Leland (2007) and Valta (2016) are developed for out-of-court debt renegotiation. This thesis strives to study the U.S. Bankruptcy Code and formal renegotiation because bankruptcy option serves as a benchmark for private renegotiation. As debt restructuring under bankruptcy code is the last chance that the firm can resolve their financial distress without incurring liquidation, it is expected that the payoffs from bankruptcy will impact the behavior of claimholders in earlier period like out-of-court renegotiation.

In reading the theoretical and empirical literature on firm defaults and debt restructuring, we find that the impact of debt structure is missing. A few economic papers such as Eraslan (2008) and Annabi, Breton and François (2012) have attempted to model the interaction of multiple debtholders under bankruptcy within a game-theoretical setting, but their models are static and discrete in time. Most existing structural models of default follow the assumption of single perpetual debt outstanding at the date of bankruptcy. Hackbarth, Henessy and Leland (2007) consider the mix of market and private debt in the firm's capital structure but assume that renegotiation through private workout is only possible for bank debt. This assumption is plausible in private renegotiation but not in formal renegotiation as Chapter 11. Valta (2016) models

debt renegotiation with mixture of secured and unsecured debt, but only unsecured debt is renegotiable.[2]

This over-simplified assumption regarding debt structure leaves problems regarding bankruptcy unsolved which this thesis addresses. For instance, how to value the bond properly for the firms that also have banks loans outstanding? Is the firm's ultimate renegotiation outcome in Chapter 11 affected by the debt composition? This thesis attempt to relax the assumption and adopt a two-class debt structure. Firstly, it is consistent with the purpose and principle of Chapter 11 to decrease the coordination problems among creditors and allow all the parties to get involved in the renegotiation.[3] Secondly, theoretically, Park (2000) suggest optimal debt structure is two-class. In practice, most of the firms rely on the mixture of bank debt and corporate bond debt. Thirdly, this practice can potentially address the credit spread puzzle.[4]

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[2] My model differs from his in the following three aspects: (1) In renegotiation, they are following a traditional two-party game while my model features in three parties involved in renegotiation. I believe this is more consistent with what happens in Chapter 11 renegotiation. (2) His model assumes no taxes are incurred but I introduce the tax advantage which justifies the use of debt. (3) The purpose of his model is to investigate how equity return is affected when taking into account such renegotiation possibilities while my research interest is in recovery rate.

[3] For instance, under Section 1102, the Bankruptcy Code requires that "an official committee of unsecured creditors, composed of the debtor's largest unsecured creditors, be appointed to represent the interests of unsecured creditors with the court".

[4] The credit spread yielded from my model is higher than that from previous strategic models and is quantitatively more consistent with market credit spread.

The key contribution of this thesis is to rectify the bond valuation modelling and to introduce the idea of multiple debt structure under the context of formal bankruptcy. This is the first structural model that incorporate a trilateral debt renegotiation following bankruptcy filing. This is the first systematical analysis on the impact of debt structure on recovery rate. This is the first time that the single consol debt structure assumption is relaxed in bond pricing models within the context of bankruptcy.

Specifically, Chapter 4 is motivated by the work of Acharya, Bharath and Srinivasan (2007) which investigates the firm-level recovery rate from the perspective of industry distress. This work inspires me to dig deeper in the topic of recovery determination, especially the recovery rate for bankrupt firms. I update and develop the work of Acharya, Bharath and Srinivasan (2007) to include the impact of debt structure and bankruptcy code. Therefore, the recovery rate from my thesis mirrors the results of the bargaining process under Chapter 11 instead of the liquidation value under Chapter 7. It is a more direct reflection of the bargaining outcome of all creditors. Jenkins and Smith (2014) studies whether the firm reorganizes or liquidates eventually. The result is incomplete as the choice of channel only tells part of the story. Additionally, recovery from this thesis is a measure of bankruptcy efficiency.

The contribution of Chapter 4 is threefold. Firstly, this chapter incorporates the two-class debt structure in strategic default models. The existing models (Fan and Sundaresan (2004), Hackbarth, Hennessy and Leland (2007), etc.) assume that only bank debt can be renegotiated. This assumption might be true for out-of-court renegotiation. But for Chapter 11 filings, this assumption is actually conflicting with the principles of Chapter 11. Provision in Chapter 11, like the unsecured creditor committee, is to reduce the hold-out problem and get bondholders involved. It is sensible to relax the assumption. Secondly, this is the first systematical analysis on firm-level recovery rate theoretically and empirically. Debt structure information is neglected in earlier papers on recovery rate, for instance, Acharya, Bharath and Srinivasan (2007) and Jankowitsch, Nagler and Subrahmanyam (2014). Thirdly, this chapter contributes to the application of strategic default models. This group of model has been used to explain credit spread in Davydenko and Strebulaev (2007), stock returns in Favara, Schroth and Valta (2012) and Valta (2016). My thesis shows that the model can also be used to explain recovery rate, as long as being adapted to a more realistic debt structure.

Chapter 5 is motivated by an advanced bond pricing model of Broadie, Chernov and Sundaresan (2007). Before this work, the majority of bond pricing models such as Merton (1974), Leland and Toft (1996) and Fan and Sundaresan (2000) either completely rule out the possibility of renegotiation or are developed for our-of-court renegotiation. Formal renegotiation however, is fundamentally

very different from out-of-court renegotiation.[5] The model of Broadie, Chernov and Sundaresan (2007) determines the optimal debt and equity values in the presence of Chapter 7 and Chapter 11 for firms with only one consol renegotiable bond outstanding. I extend their work by introducing a two-layer debt structure (bank debt plus bond debt) with finite maturity date and allow the two classes of debts to be negotiated in Chapter 11. The modification and extension to a two-layer debt structure is motivated by the limitations of empirical tests of structural models such as Eom, Helwege and Huang (2004) and Huang and Huang (2012). They have to have to restrict their samples to firms with one class of debt, which generates an unnecessary sample bias as well as reduced sample size.

The model in Chapter 5 discusses the term structure of credit spread for bonds subject to bankruptcy risk, showing systemically how bankruptcy and debt structure affect the shape and magnitude of the term structure of credit spread. Different combinations of bankruptcy-related factors and debt structure result in different sensitivities of the credit spread to asset volatility, length of grace period, and bargaining power. Additionally, some provisions in Chapter 11 of the U.S. Bankruptcy Code are included and modelled explicitly for the first time.

5 Court-supervised debt renegotiation is usually costly and time-constraint. Additionally, the purpose of the Code is to mitigate the holdout problem and let junior bondholders to get involved who are typically excluded from the out-of-court renegotiation, as shown in Hackbarth, Hennessy and Leland (2007).

In summary, in this chapter I contribute to the literature as I extend the structural model of credit risk to the case of two-class debt structure and finite maturity. Moreover, the type of option I use in this chapter is Parisian corridor option, which is a very new concept. No paper has explained the finite difference methods to get valuations of Parisian-corridor options. This is the first attempt to accomplish it.

Chapter 6 is motivated by the debate on functioning of existing bankruptcy procedures and bankruptcy reform, such as the debate on efficiency of a procedure in Eckbo and Thorburn (2009), the debate on creditor-friendly or debtor-friendly regime in Ayotte, Hotchkiss and Thorburn (2012) and Ayotte and Morrison (2009). Most of these works are published in law journals and there is no model or data in these works. Chapter 5 has extended the work of Broadie, Chernov and Sundaresan (2007) and allowed two-layer debt structure, which facilitates the discussions on the conflicts of debt holders in Chapter 6. Stick to the single debt structure means that the only conflict in renegotiation is between the creditor and debtor. Broadie, Chernov and Sundaresan (2007) find that, in order to restore the first-best outcome, creditors should be given the right to take control of the reorganization. Chapter 6 further discusses the problem of: which class of creditor should be given the control right in order to obtain the first-best outcome?

Therefore, Chapter 6 contributes to the literature as it provides guidance on the distribution of control rights in bankruptcy. The agency problem is severe in defaulted firms. We find that the firm value maximization can be obtained by transferring the control right from shareholder to bondholders. The model provides support for the opinion that the bondholder represents the first-best solution and should be entitled to the control right. This chapter thus sheds light on the design of bankruptcy law as the results recommend a creditor-friendly regime in a normative sense, which coincides with the direction of U.S. bankruptcy law reform -- switching from a debtor-friendly regime to creditor-friendly regime. Additionally, this model suggests some fresh findings in the determination of an endogenous default boundary; for instance, the liquidation cost has no impact on the shareholders' liquidation decision when renegotiation is introduced. Furthermore, this chapter suggests that the optimum length of period for firm to recover (the grace period) is 6 months.

1.3 Thesis Structure

The remainder of the thesis proceeds as follows. In Chapter 2, I introduce the U.S. bankruptcy code, concentrating on the provisions that are important for modelling in later chapters. Chapter 3 reviews the literature related to the subsequent three main chapters. Note that this chapter starts with reviews on

theoretical corporate bond valuation models,[6] but I put extra effort on a narrower branch, strategic default models, since this branch is closest to the work in the thesis. Valuation approaches for bonds with additional features (option-like bonds) are reviewed separately, in Chapter 5.

Chapter 4 is the first main chapter in this thesis. I extend a continuous-time model of credit risk by introducing a trilateral renegotiation of debt contracts. Theoretically and empirically, the share of bank loan outstanding alters the timing of default and ultimate debt reduction in renegotiation. In Chapter 5, I analyze the role of provisions of the U.S Bankruptcy Code in debt valuation by developing a model with multiple renegotiable debts. I attempt to investigate how the characteristics of the law will impact on the credit spread of corporate bond. The model generates a more reliable credit spread for a corporate bond. In Chapter 6, I link debt valuation with optimal default and liquidation decision making for bankrupt firms. I discuss the optimal renegotiation and liquidation boundaries in distressed companies based on the valuation results in the previous chapter.

In Chapter 7, results from previous chapters are summarized. The implications and limitations of the PhD study are also discussed. The final section of the

[6] Bond valuation models are categorized in two groups: structural models and reduced-form models. I only consider structural models in this thesis.

thesis includes the mathematical derivation for Chapter 4 in Appendix A, data collection details are in Appendix B, as well as pseudo-code in Appendix C.

CHAPTER 2

Introduction to the U.S. Bankruptcy Code[7]

The term "bankruptcy" refers to the "statutory procedure by which a debtor obtains financial relief and undergoes a judicially supervised reorganization or liquidation of the debtor's assets for the benefit of creditors."[8] In the case of bankruptcy, the debtor is the company[9] that has gone bankrupt and owes money to various creditors. In this chapter, I begin in section 2.1 with a brief introduction to the bankruptcy process. In section 2.2 and 2.3 two procedures, liquidation and reorganization, are discussed in more detail.

[7] This chapter is compiled and written by reviewing Bankruptcy Basics. Bankruptcy Basics is published by the Administrative Office of the U.S. Courts, which provides basic information to stakeholders and the general public on different aspects of federal bankruptcy laws. Source: http://www.uscourts.gov/services-forms/bankruptcy/bankruptcy-basics

[8] Garner, B. A. Black's law dictionary.

[9] The U.S. Bankruptcy Code also deals with individual bankruptcy but it is outside the scope of this thesis.

2.1 The Process

The U.S. Bankruptcy Code is the uniform federal law that governs all bankruptcy cases. The current Bankruptcy Code was enacted in 1978 by §101 of the Bankruptcy Reform Act of 1978. It has been amended several times since its enactment, with the most significant recent changes enacted in 2005 through the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005.

A fundamental goal of the federal bankruptcy laws enacted by Congress is to give debtors a financial "fresh start" from burdensome debts. The Code is divided into different "chapters". In the context of bankruptcy proceedings, a "chapter" refers to the section of the U.S. Bankruptcy Code that describes the situation and procedure to be followed, based on the type of debtor seeking bankruptcy protection. Entities seeking relief under the Bankruptcy Code may file a petition for relief under six basic different chapters of the Code, depending on circumstances. The key chapters of the Code which are most commonly used and will be discussed in this PhD thesis are chapter 7 (liquidation chapter) and chapter 11 (reorganization chapter). Besides court-supervised bankruptcy, out-of-court agreements with creditors may provide an alternative to a bankruptcy filing.

2.2 Chapter 7 Liquidation

Chapter 7 deals with “liquidation bankruptcy”. As that name implies, Chapter 7 is for companies whose sole intent is to enter the bankruptcy process to pay off their creditors to whatever extent they are able to and then to dissolve the firm at the completion of the process. A Chapter 7 trustee converts the debtor’s non-exempt assets into money and distributes this money to the debtor’s unsecured creditors.

The case officially starts to be regulated under Chapter 7 when the debtor files a petition with the bankruptcy court serving the area where the business debtor is organized or has its principal place of business or principal assets. In addition to the petition, the debtor also needs to file the following documents with the court: (1) schedules of assets and liabilities; (2) a schedule of current income and expenditures; (3) a statement of financial affairs; and (4) a schedule of executory contracts and unexpired leases. Filing a petition under chapter 7 brings the “automatically stays” provision into effect immediately which means most collection actions against the debtor or the debtor’s property are automatically stops. Between 21 and 40 days after the petition is filed, the case trustee will hold a meeting of creditors. The debtor is required to attend the meeting and answer questions regarding the debtor’s financial affairs and property.

2.3 Chapter 11 Reorganization

Alternatively, debtors may prefer to remain in business and avoid liquidation. Such debtors should consider filing a petition under chapter 11 of the Bankruptcy Code. Chapter 11 filings are known as reorganization filings. When a company files for Chapter 11 bankruptcy protection, this indicates that, while recognizing its dire financial condition, the company feels that it still has valuable assets and a viable business plan, and thus intends to keep the business in operation at the completion of the proceedings. Thus, while Chapter 11 is concerned with protecting the creditors by trying to ensure that those creditors are paid off over time, its design is also to give the debtor company some "breathing room" in the form of extra time and reduction of debt load, in order to allow the company to get back on its feet. Under chapter 11, the debtor may seek an adjustment of debts, either by reducing the debt or by extending the time for repayment, or may seek a more comprehensive reorganization.

A chapter 11 case begins with the filing of a petition with the bankruptcy court serving the area where the debtor has a domicile or residence.[10] A petition can be initiated either voluntarily by the debtor, or it may be an involuntary petition, which is filed by creditors that meet certain requirements. Once the petition is

[10] The U.S. Bankruptcy Code gives flexibility to debtors in choice of venue for filing. An interesting topic here is "forum shopping", which refers to the case where a debtor chooses the court in which the bankruptcies will have the greatest chance of success.

filed, the debtor has 20 days to respond to the petition. If the debtor does not respond, the court will allow the bankruptcy and the debtor will have no choice but to participate. If the debtor does respond, a hearing will be set, and if the judge decides based on the evidence that the petition was filed in good faith and that the debtor is not paying the debts, the judge will order that the bankruptcy go forward. If the judge finds in the debtor's favour, however, he or she will dismiss the case and require the creditors that filed the case to pay the debtor's costs and fees.

Unlike Chapter 7 liquidation, Chapter 11 bankruptcy requires a reorganization plan to keep a business going. In reorganization cases, creditors are usually repaid from post-bankruptcy revenues as provided for in the plan as the debtor continues its operation and not through liquidation of the debtor's estate. Chapter 11 cases may, by design or by circumstances, become liquidation cases. As for the formulation of the renegotiation plan, bankruptcy law gives the debtor an exclusivity period of 120 days, and is often extended to propose a plan about how to restructuring the debts.[11]

[11] "Generally, a written disclosure statement and a plan of reorganization must be filed with the court. The disclosure statement is a document that must contain information concerning the assets, liabilities, and business affairs of the debtor sufficient to enable a creditor to make an informed judgment about the debtor's plan of reorganization. The contents of the plan must include a classification of claims and must specify how each class of claims will be treated under the plan. Creditors whose claims are "impaired," *i.e.*, those whose contractual rights are to be modified or who will be paid less than the full value of their claims under the plan, vote on the plan by ballot." --- Source: Bankruptcy Basics.

"Who may file a plan? Except as otherwise provided in this section, only the debtor may file a plan until after 120 days after the date of the order for relief under this chapter. Any party in interest, including the debtor, the trustee, a creditors committee, an equity security holders committee, a creditor, an equity security holder, or any indenture trustee, may file a plan if and only if (1) the debtor has not filed a plan before 120 days after the date of the order for relief under this chapter; or (2) the debtor has not filed a plan that has been accepted, before 180 days after the date of the order for relief under this chapter, by each class of claims or interests that is impaired under the plan."

Source: U.S. Code: Title 11 Bankruptcy

https://www.law.cornell.edu/uscode/text/11

In contrast with Chapter 7, Chapter 11 does not require the Absolute Priority Rule to be followed in the reorganization plan. Additionally, the provisions above indicate that bankruptcy filings under Chapter 11 imply the "automatic stay" of assets that prevents bondholders from appropriating firm value by liquidating the firm's assets. Specifically, the automatic stay provision provides a period of time in which all judgments, collection activities, foreclosures, and repossessions of property are suspended and may not be pursued by the creditors on any debt or claim that arose before the filing of the bankruptcy petition. As with cases under other chapters of the Bankruptcy Code, a stay of creditor actions against the chapter 11 debtor automatically goes into effect when the

bankruptcy petition is filed. Additionally, Chapter 11 also imposes time and cost constraints on the renegotiation process.

The court ultimately approves (confirms) or disapproves the reorganization plan. Under the confirmed plan, the debtor can reduce its debts by repaying a portion of its obligations and discharging others. The debtor can also terminate burdensome contracts and leases, recover assets, and rescale its operations in order to return to profitability. Under chapter 11, the debtor normally goes through a period of consolidation and emerges with a reduced debt load and a reorganized business. However, a case can also be converted into chapter 7. A debtor in a case under chapter 11 has a onetime absolute right to convert the chapter 11 case to a case under chapter 7. A party in interest may file a motion to dismiss or convert a chapter 11 case to a chapter 7 case "for cause."

CHAPTER 3

Literature Review

This thesis covers several broad areas: structural models of credit risk, particularly strategic default models; recovery rate determination; debt structure and corporate governance. In this chapter I review the most relevant papers in these fields. This literature review chapter is organized as follows: Reviews on general structural models and strategic default models are in Section 3.1 and Section 3.2 respectively. I picked models in this field and grouped them regarding their debt structure assumption and distress resolution channel[12]. Section 3.3 covers theoretical and empirical studies in determination of recovery rates in defaulted debt. The general methodology, major assumptions, contributions and disadvantages in these papers are fully disclosed here. Section 3.4 focuses on the literature about debt structure theories. Section 3.5 reviews papers discussing corporate governance in financially-distressed companies.

[12] I use these two criteria to group the models because the contributions of this thesis are multiple debts and formal bankruptcy.

3.1 Structural Model of Credit Risk

Credit risk models can be divided into two main types: structural and reduced-form models. The basic idea of structural models is that a default event is deemed to occur when the firm's assets reach a sufficiently low level compared to its liabilities. On the contrary, reduced-form models assume that default is driven by a default intensity, instead of an economic process. As opposed to reduced-form models, the main advantage of structural models is that they provide an intuitive picture, as well as an endogenous explanation for default.

All structural models share a common theoretical foundation stemming from the model of Merton (1974). The general idea is that corporate bonds represent contingent claims on the underlying assets of a firm. In this setting, given a particular diffusion process of asset value, the firm defaults if the value of its assets falls below the amount of debt at debt maturity date. Contingent claim valuation technology, particularly option pricing, is applied to the valuation of debt claims of the firm. Although this is a great break-through in the finance area of defaultable bond pricing, it has natural disadvantages in unrealistic assumptions. First, default can only happen on the maturity date of the debt, a scenario that is at odds with reality. Second, the bond is typically assumed to be zero-coupon, which is clearly not true for corporate bonds in reality. Also, this framework assumes that the Absolute Priority Rules are strictly adhered to upon default in that debts are paid off in the order of their seniority. Actually, the

Absolute Priority Rule is often violated in bankruptcy, especially in reorganization bankruptcy. [13]

One conventional criterion to group the subsequent pricing models based on Merton (1974) is their assumptions regarding the default trigger. A model may be classified as exogenous or endogenous. Exogenous models assume the default barrier is determined solely by the capital structure of the firm while endogenous models assume that equity holders can strategically default whenever it is optimal for them to stop servicing the firm's debt.[14] In the strand of exogenous models, Black and Cox (1976) use the same method to price zero-coupon bonds. However, instead of admitting only the possibility of default at maturity time T, they assume that default occurs the first time that the firm's asset value drops below a certain time-dependent 'barrier', $K(t)$. An extension by Kim, Ramaswamy and Sundaresan (1993) is much more advanced as they consider the stochastic interest rate process of Cox, Ingersoll and Ross (1985) and incorporate a callable feature and non-zero coupon rate into corporate bond valuation.

[13] Sundaresan (2013) gives an excellent review of Merton's model and its application in his paper titled "A Review of Merton's Model of the Firm's Capital Structure with its Wide Applications" published on The Annual Review of Financial Economics 2013.

[14] Since generally a lot of mathematical derivatives are involved in structural models, to avoid confusion in this literature review I omit the mathematics in these models. Instead, I summarize the default barrier assumptions and contributions of these models. See Table 3.1.

As for endogenous barrier models, the first attempts can be found in Leland (1994) and Leland and Toft (1996). Bankruptcy is determined by the smooth-pasting condition when the amount required for debt service exactly matches the new equity capital which can be raised. Leland (1994) generalizes structural models of default by explicitly introducing taxes and bankruptcy costs. This allows, for the first time, a formal characterization of optimal capital structure, debt capacity, and credit spreads in a classic trade-off model. Leland and Toft (1996) further relax the assumption of perpetual debt in Leland (1994).

A specific branch of endogenous default barrier models is strategic default models which also endogenizes the firm's bankruptcy point. The rationale behind this type of model is that liquidation is not the only possible outcome from default. The major attribute of strategic default models is that they treat financial distress resolution as a bargaining process among parties involved and explicitly model this renegotiation game. As this PhD thesis falls directly into this group of models, I will discuss it in detail in the following Section 3.2. Also, in the main body of the first chapter where I build my model, I pick several papers and analyze in more depth for the purpose of theoretical comparison.

3.2 Strategic Default Model

As mentioned earlier, strategic default models extend the Merton (1974) model to reconcile the fact that financially distressed firms in real life do not have to

file for Chapter 7 under the Bankruptcy Code and thus resort to liquidation. This assumption is maintained in Merton (1974), Black and Cox (1976), and Leland (1994). Hart and Moore (1994) noted the concept of strategic default for the first time, as a comparison with liquidity default in which the cash flows generated from the operation of the firm are insufficient to honor the debt contract. Strategic default describes the scenario in which the firm fails to pay the amount specified in the debt contract even though it possesses the resources to do so. Upon default, if the liquidation value is lower than the going concern value, creditors may prefer to forgive some of the debts if doing so allows the firm to survive. This creates incentives for equity holders to default opportunistically in order to obtain debt concessions. Structural credit risk models with debt renegotiation are introduced by Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997).

Strategic default models are developed such that the lender and the borrower can resolve financial distress through filing for Chapter 11, or Chapter 7, or by renegotiating their original debt claims under the shadow of either Chapter 7 or Chapter 11 (out-of-court renegotiation).[15] Since the objective of my model is to incorporate debt structure consideration and features of bankruptcy code into current bond pricing models, in what follows I will discuss strategic default

[15] For a brief summary of the models in this strand, readers are suggested to refer to Table 3.2.

papers at three levels: origin of the model, models with multiple debts, and models under the context of bankruptcy procedure.

3.2.1 Origin of Strategic Default Models

The first paper to model such debt renegotiation in the setting of structural models is presented by Anderson and Sundaresan (1996) using a binomial process for the cash flow generated by the firm. The debt contract requires a constant coupon payment out of the generated cash flows. In the case that the generated cash flow is not enough to repay the debt, the firm is not automatically thrown into bankruptcy (liquidation). The model allows the equity holders to make a "take-it-or-leave-it offer" to the creditors. The creditors therefore face a decision node with two alternative options to choose from: (1) liquidate the firm and receive the liquidation value less the liquidation costs, or (2) accept the proposed payment. In a game theory setting, the equity holders will reduce the coupon payment to the minimum level such that the creditors will be indifferent between accept and reject. Thus, the equilibrium debt service and the bankruptcy trigger arise endogenously as outcomes of the non-cooperative game between lenders and borrowers. They also demonstrate that accounting for bankruptcy costs leads to credit spreads that are closer to the observed ones, relative to models that do not account for strategic debt service.

While equity holders receive the exclusive right in bargaining in Anderson and Sundaresan (1996), Fan and Sundaresan (2000) develop a more generalized bilateral bargaining model that can accommodate varying bargaining powers between debt holders and shareholders. In addition, the tax advantage of debt is considered in this model. In the presence of tax shield, bargaining on the firm value (which differs from the value of assets) becomes possible and its value becomes endogenous, since it depends on the optimal reorganization policies. When in distress, the creditors and borrowers renegotiate to reduce the coupon payments. The ultimate amount of debt service thus is an equilibrium outcome of the bargaining process.

3.2.2 Strategic Default Models with Multiple Debts

Most strategic default models assume the firm issues only a single class of debt and thus are naturally silent on the impact of debt structure on debt renegotiation. Only a few recent studies (Hackbarth, Henessy and Leland (2007), Bourgeon and Dionne (2013) and Valta (2016)) start to point out the difference between private (bank) and public debt in renegotiation.

Hackbarth, Henessy and Leland (2007) consider the mix of market and private debt in the firm's capital structure. They distinguish between bank and public debt by the assumption that renegotiation through private workout is only possible for bank debt. Since the borrower is always the beneficiary from

renegotiation, this possibility to be renegotiated makes bank debt more attractive to the borrower. However, at the same time, it limits bank debt capacity for firms with high bargaining power as banks are reluctant to lend considering the possible renegotiation. When the maximum bank debt capacity is reached, the firm complements bank debt by corporate bond to benefit from the increased tax shield. The model therefore explains why bank debt is senior to bonds, and why small (weak) firms rely exclusively on bank loan while mature (strong) firms usually have a structure of mixture of debts. The renegotiation in their model is conducted between the bank lender and the firm. For instance, in the case of a strong firm, the firm has the exclusive ability to propose the offer and push down the value of the bank debt until the bank is indifferent between liquidating the firm and accepting the offer. This means that, although the model explicitly assumes a complex debt structure, the renegotiation still takes place between two parties. The existence of a bondholder matters only in that it distorts the private workout process, as market lenders capture some of the benefit attributable to bank concessions. The essence of renegotiation does not change compared with earlier models.

Bourgeon and Dionne (2013) extend the Hackbarth, Henessey and Leland (2007) model to allow banks to adopt a mixed strategy in which renegotiation is sometimes refused by banks ex post (even when renegotiation is also in their immediate financial interest) in order to retain reputation and raise debt capacity ex ante.

The most relevant research is conducted by Valta (2016) which proposes a model of strategic default and debt renegotiation with a mixture of different debts. He considers the case of a firm financed with equity, straight debt, and secured debt. Secured debt holders require the firm to pledge a part of the firm's assets as collateral, which cannot be sold without the agreement of debt holders. Suppose the debt contract specifies that debt holders get a fraction $\pi \in [0,1]$ of the unlevered firm value for sure in renegotiation.[16] At the renegotiation point V_S, the residual value available for renegotiation thus drops to $v(V_S)(1-\pi)$ while the value sharing rule still follows the Nash equilibrium solution. Therefore, in renegotiation, the equity holder is expected to get $\eta v(V_S)(1-\pi)$ rather than $\eta v(V_S)$ as in other models. Furthermore, the optimal renegotiation boundary derived from this model also drops as equity holders are reluctant to default early since they benefit less from renegotiation now. The treatment of debt structure and renegotiation is rather brief compared with the model in this thesis.

In summary, probably owing to the lack of a realistic bargaining game for three parties in the Chapter 11 renegotiation, existing papers stick to the traditional two-party game in analyzing the debt renegotiation. Luckily, a recent published economic paper modelling the Chapter 11 procedure by Eraslan (2008) encourages and enables me to extend strategic default models with complex

[16] This amount of collateral directly relates to the proportion of secured debt.

debt structure.

3.2.3 Strategic Default Models under Bankruptcy Code

The strategic default models discussed in the previous two sections focus on non-bankruptcy defaults and out-of-court renegotiations of debt contracts. Nevertheless, the U.S. bankruptcy law allows for a court-supervised debt renegotiation under Chapter 11 filing. A number of other papers look at these bankruptcy payoffs. I am particularly interested in this group because I believe understanding of bankruptcy payoffs is an important step in understanding non-bankruptcy defaults. Bargaining taking place out of court is very likely to be influenced by expectations about bankruptcy timing and outcomes.

François and Morellec (2004) are first to extends the Fan and Sundaresan (2000) model to incorporate the possibility of Chapter 11. Under this court-supervised renegotiation, the court grants the survival of the defaulting firm for an observation period. To incorporate this breathing space, equities are modeled as a Parisian down-and-out option on the firm's asset. The firm is liquidated (i.e. the equity holders' option to repurchase the firm's asset dies) when the value of the firm's assets reaches a default threshold and stays below that boundary continuously throughout the observation period. Similar to the approach of Fan and Sundaresan (2000), François and Morellec (2004) consider a Nash bargaining game between equity and debt holders and denote their bargaining

power as η and $1-\eta$ respectively. Similarly, the firm renegotiates its debt obligations once the asset value touches the constant default threshold from above. In François and Morellec (2004), court-supervised renegotiation is costly. A proportional costs ϕ are incurred by the company during the renegotiation process under Chapter 11. While in the model of Fan and Sundaresan (2000), the renegotiation costs are ignored due to the private renegotiation setup.

Moraux (2002) extends the François and Morellec framework to account for the total time, instead of only the latest episode spent by the firm's asset value, below the renegotiation boundary. In this paper, liquidation is triggered when "the accumulated excursion time of the asset's value below the distress threshold exceeds a pre-determined grace period". Therefore, this model capture the entire history of the firm's financial distress, instead of only the last default.

Galai, Raviv and Wiener (2007) develop a model where more severe and more recent distress events are more influential compared to older and less profound financial troubles by making the renegotiation threshold time dependent as in Black and Cox (1976). The firm liquidates when the weighted average of the distances from V_t to V_B during the history of the firm's defaults exceeds a certain value.

To demonstrate the impact of depth of financial distress, Broadie, Chernov and Sundaresan (2007) develop a model that also treats default and liquidation as distinctive events. They include two barriers for default and liquidation separately and determine the optimal debt and equity values in the presence of the two boundaries. The main contribution of Broadie, Chernov and Sundaresan (2007) compared with previous models is to incorporate a second mechanism for liquidation: when asset value deteriorates rapidly during the observation period. Thus, liquidation could happen as the firm value either reaches the liquidation barrier or stays under the bankruptcy barrier for longer than the grace period.

Generally speaking, the input information for a structural model (including strategic default models) include: (1) the process of asset value, which is usually modelled as a geometric Brownian motion; (2) the process of risk-free interest rate; (3) the issuer's capital structure, typically referred as the leverage ratio; (4) terms and conditions to define the bankruptcy process. The leverage ratio is considered in structural models. This implies that existing structural models are based on the total amount of debt but provide no information on debt composition or debt structure.[17] My research tries to extend this by analyzing

[17] As far as I know, Carey and Gordy (2016) present the only theoretical model that explicitly touches the debt heterogeneity consideration in structural models. However, even in their paper, the debt structure is important simply because it endogenously affects the lower reorganization

how debt structure would affect ultimate recovery and debt valuation after declaring bankruptcy.

3.3 Recovery Rate Determination

In the literature, by definition, recovery rate is the ratio of the debt recovery to the exposure at default (EAD), i.e. the debt value when default occurs. However, different opinions are held on the interpretations of the numerators and denominators (Calabrese and Zenga 2010). There are three main measures for recovery rate. Market recovery is direct observed via defaulted debt prices in the market, usually computed as the ratio of post default trading price of debt instrument 30 days after default to face value. This measure is the most suitable for investors who sell their debt immediately after default but not necessarily an accurate measure of loss if the debt is worked out by the bank to eventual resolution. Workout recovery is the present value of ex-post realized cash-flows back to the time of default. It is particularly suited for bank loans since a liquid secondary market of defaulted loans typically does not exist. Both market recovery and workout recovery are physical (realized) recovery rates. The third is implied recovery rate. This measure is derived from risky bond or other relative financial instrument prices using a theoretical asset pricing model.

boundary of the firm, which is incomplete compared with the model in this thesis. Also, how the renegotiation bargaining is conducted among the claim holders is missed in their paper.

These are used intensively as input parameters for fixed income products and credit derivatives pricing (Schuermann 2004).

Recovery rate on default plays a critical role in bond pricing models since it determines the expected losses in case of default. In this literature review I focus on papers concerning realized recovery rates. These papers clarify factors determining the realized (physical) recovery rates among different industries and different types of debts. The theoretical part will be introduced in Section 3.3.1 and the development of the empirical group will be introduced in Section 3.3.2.

3.3.1 Theoretical Models of Recovery Rate

As far as I know, there is no model that explicitly addresses the problem of recovery rate determination, though several structural models can produce implications for it. I focus only on strategic default models here. This is because other structural models do not allow debt renegotiation while the research interest here is the recovery rate in Chapter 11 which is a renegotiation process. I reviewed this type of model in the previous section. Bear in mind that we are not investigating all the important papers in strategic default models. Here I concentrate on those that can produce direct predictions on recovery rates for illustration. Later, when I build my model, I will review this field again from the perspective of model setup.

As noted earlier, strategic default models extended the Merton (1974) model to reconcile the fact that default is not tantamount to liquidation. Due to the existence of liquidation costs, direct liquidation tends to be the most costly approach. Therefore, it leaves a region where it is in debt holders' interest to concede a part of coupon payment to equity holders through renegotiation of debt. Strategic models are developed such that the lender and the borrower can resolve financial distress through filing for Chapter 11, or Chapter 7, or by renegotiating their original debt claims under the shadow of either Chapter 7 or Chapter 11 (out-of-court renegotiation).

Since it is a renegotiation process, previous models are developed based on the game-theoretical setting to analyze the result of the renegotiation. Anticipating the renegotiation outcome, this outcome will be reflected in the decision of when to default as well as the relevant debt values ex ante. Although Anderson and Sundaresan (1996) is the first paper modelling debt renegotiation, Fan and Sundaresan (2000) develop a more generalized bilateral bargaining model that can accommodate varying bargaining powers between debt holders and equity holders. When in distress, the creditors and borrowers renegotiate to reduce the coupon payments. Borrowers offer a debt service that is less than the contractual amount as an equilibrium outcome of the bargaining process. Based on the setup, their model actually implies a recovery rate of

$$RR_{FS} = \left[(1-\tau) + \frac{\eta K}{c/r}\right]\frac{-\lambda_-}{1-\lambda_-} - \frac{\eta K}{c/r}. \qquad (3.1)$$

In (3.1), τ is the tax rate, η is the bargaining power of the equity in the two-party game, c is the coupon payment of the single debt and r is the risk-free rate. K represents the fixed liquidation cost. $\lambda_- = \left[0.5 - \frac{r-\beta}{\sigma^2}\right] - \sqrt{\left[0.5 - \frac{r-\beta}{\sigma^2}\right]^2 + \frac{2r}{\sigma^2}}$ is calculated in a similar way as in Merton's model.

This implies that the recovery rate is independent of proportional liquidation cost. In addition, when the fixed liquidation cost is negligible, the recovery rate is independent of the equity holder's bargaining power. The model of Anderson and Sundaresan (1996) can been seen as a special case of Fan and Sundaresan (2000) where there is no fixed liquidation cost and the shareholders possess the whole bargaining power. This yields the recovery rate

$$RR_{AS} = (1-\tau)\frac{-\lambda_-}{1-\lambda_-}. \tag{3.2}$$

In the later work of Davydenko and Strebulaev (2007), renegotiation friction is introduced, which means that renegotiation is not always possible. Sometimes the firm might still enter into liquidation even when renegotiation is in everyone's interest. Denote q the renegotiation frictions, the recovery rate in this model can be expressed as

$$RR_{DS} = \left[1 - \frac{q\alpha}{1-(1-q)\alpha}\right]\frac{-\lambda_-}{1-\lambda_-} \tag{3.3}$$

where α is the proportional liquidation cost. The model assumes no fixed liquidation cost and no tax is incurred.

Christensen, Flor and Lando (2014) allow for callable debt and reissuance of debt. Hence, the model extends this strand to incorporate the dynamic capital structure. The combination of callability and renegotiation increases the tax advantage of debt. This model also distinguishes credible threats from non-credible threats such that, in debt renegotiation models, debt holders will only make interest rate concessions (or other forms of debt reduction) on credible threats in the sense that liquidation is indeed in the interest of shareholders. Since the setup and mathematical derivation is complicated in this model, I just point out the implication on recovery rates of the model: the equity holders' bargaining power and the bankruptcy costs have very little impact on debt recovery. Earnings (and thereby asset) volatility, which has a significant impact on the optimal capital structure choice, has virtually no impact on the size of debt recovery.

3.3.2 Empirical Study on Determinants of Recovery Rate

Basically speaking, researchers consider four general types of factors are relevant for recovery on defaulted debts. They are: macro-economic conditions, industry conditions, firm-specific fundamentals and instrument-specific variables. Table 3.3 is a summary of elements considered in the existing literature as driving forces of recovery rates. As shown in the table, all elements can be classified into one of the four groups. Some of elements in the table have

been proved to be more significant in influencing recovery rate.

Debt Characteristics The main determinants of instrument-level recovery rates are seniority and security of the debt instrument. Seniority refers to the order of repayment in the event of default. Bank loans are the most senior and thus have the highest recoveries. In the research of Benzschawel, Haroon and Wu (2011) they show loans have average recovery rate of 65.2% while junior bond recovers only 25.8% on average.

Macroeconomic Factors Recoveries are systematically lower during recessions. Altman, Brady, Resti and Sironi (2005) find that recoveries are lower when the real GDP growth is below long-term average. Giesecke, Longstaff, Schaefer and Strebulaev (2011) reach a similar conclusion that stock market returns, stock market return volatility and changes in GDP have significant forecasting power for realized default rates. However, Bruche and Gonzlez-Aguado (2010) find that macroeconomic variables in general are significant determinants of default probabilities but not so for recovery rate distributions. They propose an econometric model in which the joint time-variation in default rates and recovery rates distributions is driven by an unobserved Markov chain, which they interpret as the credit cycle.

Industry Factors Industry is also proven to matter. Altman and Kishore (1996) show significant differences in recovery between different industries. Public

utilities have the highest recovery rates while manufacturers have the lowest average recoveries. Moreover, in general, the variability of average recovery rates is quite high. Another more important industry variable that has been considered only recently is industry distress. Acharya, Bharath and Srinivasan (2007) find that industry-wide distress affects creditor recoveries and they attribute the reason to the economic-downturn effect as well as the anticipation or realization of asset fire-sales. Similar results are also found in a more recent paper of Elkamhi, Jacobs and Pan (2014) using the risk-neutral recovery rates implied in CDS prices.

Firm-specific Factors Firm-specific factors are important. Elkamhi, Jacobs and Pan (2014) find that firm characteristics such as leverage, asset specificity and tangibility of firm assets significantly affect recovery rates. Despite almost all current papers covering leverage ratio in their research, few touch the analysis of debt structure.

Besides the typical framework above, some recent papers endeavor to analyze the recovery determination from some fresh new perspectives. For instance, Jankowitsch, Nagler and Subrahmanyam (2014) investigate the underlying microstructure of trading activity for a broad set of default event types by examining the transaction data of defaulted U.S. corporate bonds over the time period from 2002 to 2010. Based on quantified liquidity proxies, it studies the price formation for individual defaulted bonds, which is an innovation relative

to the prior literature. Eckbo and Thorburn (2009) take the perspective of different bankruptcy regime. They present evidence to suggest that the Swedish auction bankruptcy system is a surprisingly efficient restructuring mechanism for small firms compared with U.S. Chapter 11. Interestingly, in their paper, the overall debt recovery rates for the firm are increasing in the proportion of secured debt in the firm's capital structure and the authors attribute this to the possible lower liquidity for intangible assets (Thorburn 2000, Eckbo and Thorburn 2009). Merrick (2001) considers the problem from the perspective of an investor. From empirical study of the 1998 crash of Russian Federation Eurobond prices, he proposes that significant downward revisions in the markets assessment of recoveries played a significant part. Shifts in recovery ratio perceptions can be as quantitatively important for bonds. Donovan Frankel and Martin (2015) and Carrizosa and Ryan (2013) consider accounting conservativeness. Their finding is that creditors of firms with more conservative accounting prior to default have significantly higher recovery rates. They attribute this higher recovery to three reasons. First, conservative firms file for bankruptcy protection more quickly following negative economic performance. Secondly, duration of bankruptcy resolution is significantly shorter for conservative firms. Lastly, conservative firms experience lower asset write-downs after entering bankruptcy.

The race for the remaining value of firm is dependent on the identities and relative strength of different creditors from the economic rationale. However,

current papers pay more attention to leverage ratio alone (which only shows the amount of total debt). In addition, current papers have contrary opinions in which kind of factors is more important in determining recovery rates. For instance, Khieu, Mullineaux and Yi (2012) assert that loan-related factors are more significant than borrower characteristics. Bastos (2010) supposes that borrower characteristics are more influential in explaining determinants of loan recovery while Dermine and Neto de Carvalho (2006) argue that macro-economic factors are major and significant explanatory factors of loan recovery in lending institutions. I suspect that these differences arise because current papers take the instrument-level angle. If we analyze the problem from the perspective of firm-level recovery, these differences might disappear. It is expected that firm-specific variables are the primary explanatory factors for determination of total recovery while instrument-specific variables are more influential in explaining the distribution of total recovery.

3.4 Debt Structure

The first main chapter of the thesis studies the debt structure and its relationship with the bankruptcy outcomes, particularly on ultimate recovery rates. Debt structure is a comprehensive term which can be interpreted in various ways. For instance, Rauh and Sufi (2010) discuss three kinds of structure: (1) Debt by type: bank debt, bonds, program debt, private placements, mortgage or equipment debt, convertible debt, other debt. (2) Debt by priority: secured, senior

unsecured, and subordinated. (3) Debt by covenants and debt maturity. Due to the research objective of this PhD thesis, the focus will be the source structure (identity of creditor and concentration of creditors) as well as the composition structure (share owned by each creditor). Other debt structure, such as maturity structure, covenants structure can be promising extensions of this study.[18]

However, debt structure research is typically placed within the domain of capital structure selection. The objective of those researches is to rationalize the formulation of an optimal debt structure rather than trying to clarifying the implication of a particular debt structure. An important exception is Asquith, Gertner and Scharfstein (1994), which proposes the potential effects debt structure plays in financial distressed firms. Their finding is that debt structure has real effects on the way financially distressed firms restructure. Firms with complex public debt structures are less likely to be forced into bankruptcy. Different from their work, this thesis goes further by analyzing how debt structure would eventually determine the recovery results and debt valuation in the bankruptcy process.

Although banks doubtlessly perform other functions, bank debt presents in a more concentrated fashion than public debt. Hence, lack of dispersion is a good

[18] In Chapter 5 and 6 I do consider finite maturity debt but assuming the bank debt and bond debt mature at the same date, which rules out discussion of maturity structure.

characterization of one feature of a bank debt. I first review the papers on the debt concentrations in Section 3.4.1 and then see how this characteristic is specifically reflected in renegotiation and bankruptcy in Section 3.4.2.

3.4.1 Debt Concentration and Number of Creditors

The classical Bolton and Scharfstein (1996) paper examines the optimal number of creditors a company borrows from. The basic point of their theory of debt structure is that debt structure affects the negotiations that follow default. An optimal debt structure should be capable of balancing two concerns: it should deter defaults; it should not make unavoidable defaults too costly. Bris and Welch (2005) take a different approach to the coordination problem between multiple creditors in ex post bargaining. They note that the measure of debt dispersion (the number of creditors) are correlated positively with the entrepreneur's retention of the firm in bankruptcy, and correlate negatively with the in-equilibrium claims collection costs. With regard to debt recovery, both papers send us the same message: two creditors receive less (not more) than one creditor.

One disadvantage of these two papers is that they concentrate completely on the number of creditors rather than full analysis of the sources and features of the debt instruments. It is not easy to find in real life a firm that has only one or two creditors as in their model. Although the number of creditors could be

interpreted as representative of selection between bank debt and public debt sometimes, that would be misleading when bank debts are syndicated, tradable or when public debt is owned by institutional investors (Bolton and Scharfstein 1996). In this thesis, I am trying to figure out the differences in nature of debt structure rather than simply the number of creditors. More importantly, these normative studies are based on the objective to find the optimal number of creditors but I am interested in the recovery rate given a fixed debt structure. Lastly, even in Bris and Welch (2005), banks issuing loans are different from public bondholders because they are highly concentrated and concentrated debt structure can generate higher liquidation values for creditors. The impact is through the changes of liquidation value but not through a renegotiation process as I want to analyze.

Traditionally, bank loans are seen as different from publicly placed debt because of informational advantages. James (1987) points out that a bank has a cost advantage over other outsiders in producing and transferring information. Lummer and McConnell (1989) revisit the role of banks as transmitters of information in capital markets and conclude that the bank loan review and renewal process is crucial in transmission of information. However, banks that enter new credit agreements possess no information advantage over others. Bolton and Freixas (1997) suppose that the key distinction between bonds and bank debt is the monitoring ability of banks. If current returns are low and default is pending, banks can investigate a borrower's future profitability,

whereas bond holders always liquidate the borrower.

The papers above all focus on the information collection and procession at the origination of debt rather than in the bankruptcy process. However, it is reasonable to deduce that banks are still special in the bankruptcy process and it is promising to enrich current models by taking into account this consideration. For instance, Carey and Gordy (2016) reported that over 60 percent of bank debt receives approximately a full recovery. They attribute it to the reason that private debt usually has covenants that give the bank the right to force a distressed firm into bankruptcy while the firm's public debt is junior and has no material covenants. Roberts and Sufi (2009) observe at least one financial covenant in 97 percent of a sample of private credit agreements. Chava and Roberts (2008) shows that these covenants are customized to be relatively tight and violations are very likely to take place. With a sample of Japanese firms, Hoshi, Kashyap and Scharfstein (1990) find that for firms with less close financial relationships to their banks, financial distress is more costly because of free-rider problems and information asymmetries make it difficult for these firms to renegotiation with their creditors than when they are concentrated.

Some theoretical papers in debt structure also admit the special role played by banks in bankruptcy. Bulow and Shoven (1978) put forward explicitly that bank's special role lies in its ability to extend new credit in a bankruptcy. Diamond (1991) and Chemmanur and Fulghieri (1994) argue that bank lenders

have the specific expertise to implement a liquidation since they devote more resources to evaluating whether to liquidate or continue a firm in Chapter 11 compared with public debt due to a free-rider issue with public debts.

Hackbarth, Hennessy, and Leland (2007) develop a continuous-time optimal model of debt renegotiation with multiple creditors. They assume that there are two types of debt, market debt and bank debt. The only unique difference is that a bank is able to negotiate in a costless and efficient private workout, while market debt cannot be renegotiated at all. Ideally, firms would only prefer to issue bank debt but that amount is limited by the firm's collateral value. In order to make full use of the tax shield benefit, firms must take on market debt. These authors show in particular that optimally both types of debt coexist.

Generally speaking, diversified creditors make it more difficult to coordinate among creditors. Gobbi and Sette (2013) propose that concentrated borrowing increases creditors' incentives to gather information and monitor borrowers while diversified borrowing may be costly in the event of bankruptcy since many lenders must be coordinated. My research is built on the difference of bank debt and bonds - diversified bond debt is prone to holdout problems. Diversification weakens incentives to restrict the firm from opportunistically stealing value in the debt renegotiation. I will show in detail in later chapters how this is presented in the model.

3.4.2 Debt Structure and Bankruptcy

Existing papers show the importance of creditors' identities in bankruptcy from various research angles. For instance, von Thadden, Berglöf and Roland (2010), for the first time, jointly integrates the problem of designing corporate bankruptcy and optimal debt structure in the theoretical field. In their model, there are two interesting and important equilibrium results. The first is that the model predicts that debt claims should be "under-secured" which means creditors should get less than the face value of their claims in bankruptcy even in a multi-creditor model. Second is that an equilibrium result is individual claims to be designed such that creditors have the same recovery rates in bankruptcy.

From the empirical strand, using a sample of 484 UK firms, Poletti-Hughes and Ozkan (2014) investigate the relevance of corporate ownership structure in determining the outcome of financial distress. The evidence suggests that the characteristics of an ultimate controller, like its type and its cash flow stake, potentially increase the predictive power of bankruptcy prediction models. Firms controlled by financial institutions are more likely to become insolvent when in financial distress. A recent paper by Demiroglu and James (2015) examines the relationship between the number and type of lenders that are involved in loan activities and the troubled debt restructuring process. Their main finding is that the likelihood of a successful out of court restructuring is

significantly related to the identity of the firm's lenders.

The two papers mentioned earlier share a basic idea with this thesis: the choice of debt structure influences what happens in bankruptcy. However, the major difference between the papers and this research is that they analyze the aspects of bankruptcy outcomes such as resolution type, bankruptcy duration or the likelihood of an out of court restructuring rather than the recovery. I am more interested in recovery for the following reasons. First, recovery is a more direct reflection of the bargaining outcomes of all creditors. Secondly, recovery is an important measurement of the bankruptcy efficiency. Finally, recovery is also a key input variable in credit risk models. The work by Jiang, Li and Wang (2012) is to some extent similar to this thesis. They also discuss the creditor identity's impact on bankruptcy outcome (total debt recoveries) by focusing on hedge funds' specialness. They find that the presence of hedge fund unsecured creditors is associated with higher total debt (including secured and unsecured) recovery.

A recent work of Rodano, Serrano-Velarde and Tarantino (2016) provides novel evidence on how the design of financial contracts and firm investment depend on the two major instruments in bankruptcy: reorganization and liquidation. They find that bankruptcy reforms that strength borrower rights to renegotiate outstanding financial contracts produce an increase in interest payments on bank financing while the increased creditor rights in liquidation resulting from the

new liquidation procedure lead to a significant reduction in the cost of bank financing. Another work by Mankart and Rodano (2015) reveal that bankruptcy provides entrepreneurs with insurance against the financial consequences of failure at the cost of worsened credit conditions. If secured credit is available the optimal law is lenient since entrepreneurs rationed out of the unsecured credit market can still obtain secured credit, lowering the costs of worse credit conditions.

Combining current theoretical models as well as the empirical results, I think that the bank's role in determining the firm-level recovery is still under-researched. Do they really actively change the outcome of bankruptcy and hence the value of the underlying assets as mentioned in those theoretical papers? Another uncertainty arises from regulatory restrictions that constrain banks' capacity for getting involved in the management of their portfolio firms due to insider trading considerations.

3.5 Corporate Governance in Chapter 11

> ""There isn't really any good way to do effective corporate governance in insolvency," Mr. Bebchuk said. "Most of the mechanisms that you hope would do the work outside the insolvency are not there." Stocks and stock options, for example, which are traditional mainstays of executive pay, offering incentives to produce profits, become virtually

> worthless when a company is in bankruptcy. In addition, Mr. Bebchuk said, outsiders can no longer take over the company by buying up its stock – a powerful way of enforcing investors' will."
>
> Chapter 11? Or Time to Close the Books?
>
> The New York Times, December 15, 2002

The last main chapter discusses the issue of transfer of control right from debtor to creditors for bankrupt firms, which touches the broad area of corporate governance. The corporate governance in insolvency is a big problem. Different parties have different tools to exert their control right in defaulted companies. They can retain some influences over outcomes in distress through several mechanisms. I summarize the most frequently scenarios in Table 3.4.

Generally speaking, in Chapter 11, the debtor remains in possession, in other words, the debtor's pre-bankruptcy managers can continue to manage the firm through the reorganization process. Allowing management to stay in control is thought to provide continuity and remove a disincentive to delay filing until the last minute. The bankruptcy court can remove management in favor of a trustee, but such appointments are rare in practice and tend to occur only in cases of fraud or extreme mismanagement. Bankruptcy law does not, however, deprive the debtor's board of directors of the power to replace management, and managerial turnover in and around bankruptcy is commonplace (Ayotte, Hotchkiss and Thorburn 2012).

At the same time, significant changes can be observed in corporate governance resulting from financial distress as the senior creditors (typically banks) frequently become major stockholders and play a key role in the whole process of workouts. As pointed out by Banerji and Bose (2004) and Gilson (1990), "banks hold equity in distressed firms, negotiate claims with the other stakeholders including public bondholders and shareholders, occupy positions in boards of directors in the restructured firms and very often make several concessions to public bondholders and shareholders". The ultimate objective of such proposals is to provide the distressed firm with some debt relief with respect to both bank and public debt, necessary for continuation of the firm as an ongoing entity. Gilson (1990) finds empirical evidence regarding how corporate governance and decision making power transfers from shareholders to banks. On an average in the study, banks receive 36% of the common stocks, and creditors in private work-outs collectively receive 79.2% of the bankrupt firm's equity.

Ayotte and Morrison (2009) analyses the large privately and publicly held companies that filed Chapter 11 bankruptcy petitions during 2001. In contrast with traditional views of Chapter 11, equity holders exercise little or no leverage during the reorganization process: 70 percent of CEOs are replaced in the two years before a bankruptcy filing; very few reorganization plans deviate from the Absolute Priority Rule in order to distribute value to equity holders.

Despite these diverse findings, the common theme shared by the papers above is that debtor versus manager no longer characterize the principal tension in large corporate reorganizations. Instead, creditor vs. debtor seems to be a more important consideration in the corporate governance of distressed a firm. Recent papers (Ayotte and Morrison (2009) and Jenkins and Smith (2014)) also pointed out that the story might go beyond this. These papers start to investigate the conflicts among creditors in bankruptcy. Practically, bank creditors have substantial power to control bankruptcy outcomes with regarding to the decision for liquidation or reorganization. Ayotte and Morrison (2009) find that bargaining between secured and unsecured creditors can distort the reorganization process. A Chapter 11 case is significantly more likely to result in liquidation if secured creditors are over-secured, consistent with a secured creditor-driven fire-sale bias. Liquidation is less likely if secured lenders are under-secured or when there is no secured debt at all. Jenkins and Smith (2014) develop a bargaining model that assumes a senior creditor can exert strong control over whether a firm reorganizes as a going-concern or liquidates during the bankruptcy process. The conflicts between creditors are demonstrated in the model to gauge the efficiency of bankruptcy outcomes.

It can be concluded that, in addition to shifting the focus away from equity and managerial control in Chapter 11, attention has also been directed away from the unified, single creditor framework. However, the two papers mentioned above focus on the decision of firm's fate. The final main paper hopes to give a

normative research on the optimal decision-making process in Chapter 11 by understanding the determination of default and liquidation boundary in Chapter 11 as well as the duration of grace period. In this sense, this research is more in line with the work of Broadie, Chernov and Sundaresan (2007). They address the issue of whether the first-best outcome (i.e., the one that maximizes total firm value subject to the limited liability of equity and debt) can be restored. They show that debt maximization leads to a strategy that is qualitatively very similar to the first best. I extend their model and consider the optimal control right allocation when the senior and junior debt creditors coexist.

Table 3.1 Exogenous and Endogenous Default Models

Exogenous Default		
Paper	**Default Barrier**	**Extension**
Merton (1974)	Nominal value of the debt D	Starting point for studying credit risk
Black and Cox (1976)	A certain time-dependent barrier $K(t)$	Relax the assumption that firm can only default at the maturity date of bond
Kim, Ramaswamy and Sundaresan (1993)	Default barrier $V_B = c/\delta$ where c is coupon rate and δ is net cash outflow rate from the firm	Default is triggered by cash flow shortage Sort-term interest rate follows a CIR process Callable feature and non-zero coupon bond

Endogenous Default		
Paper	**Optimal Default Barrier**	**Extension**
Leland (1994)	$V_B^* = \frac{(1-\tau)c}{r+\sigma^2/2}$	Introduce tax shield and bankruptcy costs
Leland and Toft (1996)	$V_B^* = \frac{\frac{C}{r}\left(\frac{A}{rT}-B\right)-\frac{AP+\tau CxT}{rT}}{1+\alpha x-(1-\alpha)B}$	Leland and Toft relax the assumption of perpetual debt in Leland (1994)

Table 3.2 Strategic Default Models

Strategic Default Models-Origin of Strategic Default Models	
Anderson and Sundaresan (1996)	A discrete time model where all bargaining power belongs to shareholders. In a two-player game theory setting, the equity holders determine the minimum coupon payment such that the creditors will be indifferent between accepting and rejecting.
Fan and Sundaresan (2000)	Bilateral bargaining in a game-theoretic setting that can accommodate varying bargaining powers between debt and equity holders. In the presence of tax, bargaining on firm value (different from value of assets) becomes possible.
Strategic Default Models with Multiple Debts	
Hackbarth, Henessy and Leland (2007)	Considers the mix of market and private debt in the firm's capital structure where renegotiation through private workout is only possible for bank debt.
Bourgeon and Dionne (2013)	Allows banks to adopt a mixed strategy in which renegotiation is sometimes refused ex-post in order to raise debt capacity ex-ante. Market debt is non-renegotiable.
Valta (2016)	Models strategic default and debt renegotiation with a mixture of different debts (straight bond and secured bond). Secured debt is collateralized thus total renegotiable assets are reduced by the amount of secured debt.
Strategic Default Models for Bankruptcy Procedure	
François and Morellec (2004)	Incorporates the possibility of Chapter 11 filings (the court grants the survival of the defaulting firm for an observation period). Equities are modelled as a Parisian down-and-out option on the firm's asset.
Moraux (2002)	Liquidation is triggered when the accumulated excursion time of the asset's value below the distress threshold exceeds a predetermined grace period
Broadie, Chernov, and Sundaresan (2007)	Optimal debt and equity values are determined in the presence of both Chapter 7 and Chapter 11. Two distinct barriers for default and liquidation and considered.
Galai, Raviv and Wiener (2007)	Introduces the notion of a dynamic grace period, which depends on the severity of the distress period.

Table 3.3 Four Types of Determinants

Macroeconomic Factors	**Industry Factors**
Country-specific	Industry distress
Business cycle	Industry liquidity
Stock market returns	Industry characteristics
GDP	Tangible assets
Credit cycle	

Firm-specific Factors	**Debt Characteristics**
Types of default	Seniority
Reorganization	Collateral
Default probability	Absolute Priority Rule
Capital structure	Credit ratings
Asset specificity	

Table 3.4 Corporate Governance in Financial Distress

Equity Holder
• Shareholders retain their formal rights to replace the board and management inside and outside bankruptcy • Shareholders can represent their interests in bankruptcy through official committees, and they retain the rights to vote on a reorganization plan • The delay inherent in confirming a cram-down plan can give shareholders bargaining power
Bank Lender
• Before entering into Chapter 11, bank initiates the management changes and aligns managerial incentives by compensation; influence the selection of directors in distressed firms; use covenants rights to influence firm's policy. Banks play a role in the decision to restructure out-of-court versus under Chapter 11 • Banks help reduce information asymmetries and thus holdout problems among public creditors • Bankruptcy' automatic stay makes secured lenders no longer enforce rights followed violation. Banks have become more adept in controlling a firm • Senior secured lenders exert significant control over the bankruptcy process through provisions attached to post-petition loans: debtor-in-possession (DIP) loan or a cash collateral order • Bank lenders also could play an important role when firms are auctioned (by providing financing)
Junior Creditors
• Coordination problem is mitigated as claims are commonly acquired and consolidated by active investors • Vulture investors play an important role as they usually gain a blocking position • The valuation of the bankrupt firm plays an integral role for the distribution of value under the reorganization plan

CHAPTER 4

Default and Recovery Rate under Chapter 11 with Multiple Debt Structure[19]

This chapter develops a model to investigate the scenario of trilateral negotiation in the context of Chapter 11. I break down the conventional setting of bilateral debt restructuring which is widely used in existing strategic default models as it violates the principle and purpose of Chapter 11 in U.S. Bankruptcy Code. The model clearly demonstrates how the difference between loan and bond in terms of concentration level impact the behavior of the debtor both before and after bankruptcy. Eventually, the model derives how the ultimate outcome of debt re-contracting (measured by recovery rate for creditors) is shifted mathematically as an integrated result of the behaviors of involved parties. From an analysis of a sample of 444 U.S. firms that filed for Chapter 11 during 1987-2016, my empirical result shows that the firm-level recovery rate

[19] This chapter has been presented in the World Finance Conference 2016 in New York.

after restructuring is closely related to the debt structure of the defaulted firm, which reinforces the validity of the model.

4.1 Introduction

Default does not necessarily have to lead to liquidation. Financially-distressed firms may first resort to creditors for debt concession in order to resolve their crises. In practice, such debt renegotiation can be undertaken either under Chapter 11 of the U.S. Bankruptcy Code or through informal out-of-court renegotiation. [20] Furthermore, this reorganization rarely coincides with liquidation, not least because the high costs associated with liquidation leave considerable scope for debt renegotiation (Denis and Wang, 2014). Most structural credit risk models omit the distinction between reorganization and liquidation (e.g. Leland, 1994; Goldstein, Ju and Leland, 2001; Titman and Tsyplakov, 2007). In these models, default and liquidation are interchangeable, except in the set of structural models classed as strategic default models, which do include the possibility of debt renegotiation after default.

Strategic default models are introduced by Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997). As mentioned in Chapter 3, some subsequent models have incorporated the possibility of strategic renegotiation.

[20] For a discussion of Chapter 11 bargaining see Hackbarth, Haselmann and Schoenherr (2015).

These are within the setting of out-of-court renegotiation. However, Chapter 11 procedures include components that render the renegotiation system fundamentally different from out-of-court renegotiation and there is only a superficial resemblance between the two systems. The starting point of this chapter is to build a strategic default model imitating formal bankruptcy and consolidating these Chapter 11 attributes.

The models of François and Morellec (2004) and Broadie, Chernov and Sundaresan (2007) are alternatives to the model in this thesis. In François and Morellec (2004), under supervised renegotiation a court grants the survival of the defaulting firm for an observation period. To incorporate this provision, equities are modelled as a Parisian down-and-out option on the firm's assets.[21] Broadie, Chernov and Sundaresan (2007), determine optimal debt and equity values in the presence of both Chapter 7 and Chapter 11 and consider two distinct barriers for default and liquidation. As in Fan and Sundaresan (2000) and other major work in the area, both François and Morellec (2004) and Broadie, Chernov and Sundaresan (2007) follow the assumption of one class of debt and, thus, a bilateral bargaining game framework between creditor and debtor. This means that a Nash solution is readily available for formulating the outcome of debt renegotiation.

[21] The "Parisian" feature is such that the firm is liquidated when the value of its assets both crosses the default threshold and stays below that threshold for the complete pre-set observation period.

This assumption of one and only one type of debt obligation in renegotiation neglects the essence of Chapter 11. The first and probably the most noticeable distinction between formal debt renegotiation under Chapter 11 and out-of-court renegotiation is in the parties involved. Under Chapter 11, all claimholders are actively engaged in the bargaining process but until now this has not been reflected in the theoretical models. Since, by and large, bondholders are numerous and anonymous, it is challenging and costly for a firm to renegotiate with them and there can be a problem of hold-out, so the assumption of a solitary negotiable debt is reasonable when modelling private/out-of-court renegotiation. However, provisions under Chapter 11 are exactly intended to decrease coordination problems among creditors. For instance, under Section 1102, the Bankruptcy Code requires that "an official committee of unsecured creditors, composed of the debtor's largest unsecured creditors, be appointed to represent the interests of unsecured creditors with the court". It becomes unreasonable to assume that bondholders do not participate in the renegotiation. Thus, models tailored for private renegotiation are inapplicable in the scenario of multilateral formal negotiations under Chapter 11.

The point of this chapter is precisely to relax the assumption of single negotiable debt structure. To this end, I develop a continuous-time model of credit risk including both bond and bank debt and, crucially, trilateral renegotiation of debt contracts among shareholders and creditors, hence permitting analysis of the

role of debt structure on creditors' recovery under Chapter 11. This model shows how the bank loan alters incentives to default and affects ultimate debt reduction in the renegotiation process. Some provisions of Chapter 11 (e.g. generation of proposer) are considered and modelled for the first time. This setup allows me to answer the question of whether heterogeneous debt structure alters the incentive to default and the size of debt depreciation. The continuous-time model of credit risk in this chapter derives the equity and debt values, together with the optimal default boundary selected by the firm and the ultimate recovery for creditors. To the best of my knowledge, this is the first attempt to address that question.

Gilson (1990), Gilson, John and Lang (1990) and Asquith, Gertner and Scharfstein (1994) also investigate the impact of debt structure on bankruptcy outcome but in terms of the choice of procedure – either the firm chooses out-of-court renegotiation, Chapter 7 or Chapter 11. Jenkins and Smith (2014) find that a secured creditor such as a bank tends to liquidate a financially troubled firm even when renegotiation would be optimal. However, the attention on recovery rates, of all the lenders and with trilateral negotiations, in this thesis separates it from earlier research. I am particularly interested in recovery rate of all the lenders as it mirrors the result of Chapter 11. There is a growing body of literature that studies the determinants of realized debt recovery. Examples include Acharya, Bharath and Srinivasan (2007) and Jankowitsch, Nagler and Subrahmanyam (2014). Instead of following these papers and only looking at a

single debt recovery rate, this chapter targets firm-level recovery rate and investigates the losses of both bond and bank creditors. The thesis interprets the impacts of creditor composition on the renegotiation process, which is ignored in past empirical papers on recovery rates.

In the theoretical portion of this chapter the bank, as a solitary lender, holds a more advantageous bargaining position in restraining wealth transfer from lenders to borrowers. Thus, the more bank debt is on a firm's balance sheet, the weaker is its bargaining power in renegotiation. With the expectation of such containment by bank lenders in Chapter 11 bargaining, the firm will delay renegotiation as long as is allowed. Reflected in my model, late renegotiation refers to a lower total asset value for involved parties to share. However, the model shows that the toughness of the bank in renegotiation dominates the effect of late renegotiation. Albeit the firm strategically postpones renegotiation (and leaves creditors fewer assets to reclaim), the ultimate recovery rate for all the creditors is still increased in the presence of a bank lender. The magnitude of this effect is expected to be associated with other renegotiation-related factors. Specifically, the sensitivity of creditors' recovery rate to bank loan percentage diminishes with the face value of total debt amount and the renegotiation frictions.

Empirically, there is a robust and statistically significant positive relationship between firm-level recovery and share of bank debt in total debt at default. The

existence of a bank lender does help to improve total recovery for all creditors. The theoretical and empirical results straightforwardly refute the supposition by Christensen, Flor, Lando and Miltersen (2014) whose model suggests that none of equity's bargaining power or bankruptcy costs have any major impact on the debt recovery. At the same time, bondholders find their payoff from Chapter 11 is not necessarily reduced if there is more bank debt, regardless of their being junior to bank creditors, since they may experience a windfall gain due to the side-effect of the strong bargaining power of the bank. Consequently, the ultimate recovery is an integrated result of the two effects. My empirical work reinforces the validity of strategic default models. These have been utilized to clarify credit spreads as in Davydenko and Strebulaev (2007) and stock returns as in Favara, Schroth and Valta (2012) and Valta (2016). It is shown in this chapter that strategic models can also be used to explain the severity of losses to creditors upon default, as long as they are adapted to a more realistic debt structure with associated trilateral not bilateral bargaining.

This chapter contributes to the strand of strategic default models by breaking the paradigm of a two-party game in debt renegotiation universally followed in previous models. Modelling a bilateral bargaining game is convenient because a Nash equilibrium solution for the negotiation is readily available. Extension to a trilateral haggling setting is not so straightforward, especially when customized for Chapter 11 renegotiation. For this I appeal to the recent game theoretic and economics literature. Eraslan (2008) develops a game theoretic

model to demonstrate the debt renegotiation under Chapter 11. She then employs the model to measure the unobserved liquidation value of a firm that relies on the information contained in the allocations that are agreed upon in Chapter 11 negotiations. By adapting the highly useful game theoretical modelling of Eraslan (2008) to structural modelling[22], a strategic debt service model is developed within the context of Chapter 11 and multiple involved claimholders in the bankruptcy process. This model differs from hers in several respects. First, Eraslan's model only captures post-bankruptcy renegotiation results but excludes pre-bankruptcy valuation of the claims. It cannot be used to investigate the strategic default behavior I require in this paper. The result in her model is the sharing rule in Chapter 11 given the total firm value to be shared is fixed and exogenously determined. While in this paper, the total value to be distributed to all involved parties are endogenous. Therefore, her model cannot generate any predictions regarding recovery rate of the debt, which is the aim of this paper. Secondly, in Eraslan's model the bank loan and bond only have different priorities in receiving payment (bank debt is senior) without other divergences in negotiation. In practice, bank and bond debts differ in several dimensions.[23] A crucial element of bank debt is that the bank is a single highly

[22] I note that the description "structural" has a different meaning in the game theoretic and economics literature. See Merlo and Tang (2012) for a helpful review of strategic (non-cooperative) bargaining theory.

[23] Section 3.4.1 in the literature review chapter points out that: due to the high concentration level and absence of free-rider issue compared with public debt, bank has informational

concentrated creditor (Demiroglu and James, 2015). The model is motived in light of the role played by creditor concentration in post-default firm restructuring[24].

This model reveals more sophisticated strategic behavior from the firm and its creditors. In particular, it features the bank, as a concentrated debt holder, reducing opportunistic behavior from the troubled firm by restricting equity's bargaining power in renegotiation after default. This mechanism enhances the total benefit to all creditors. So far as I am aware, this is the first model to incorporate explicitly into a strategic default model this ubiquitous property possessed exclusively by private debt.

The rest of the chapter proceeds as follows. Section 2 introduces the theoretical framework and the model. Section 3 discusses the empirical implications of the model, particularly regarding recovery rate, spill-over effect and the default boundary. Sections 4 and 5 contain an empirical analysis of the model. Section 6 concludes. Appendix A at the last section of the thesis gives proofs of the

advantage as well as monitoring function both at and after issuance of debt, if compared with bondholders.

[24] Aslan and Kumar (2012) also address theoretically and empirically the impact of control concentration on debtholders' welfare in post-default firm restructuring. Instead of concentrating on the debt side as in my model, their paper studies the equity side's concentration when dominant shareholders can influence post-default debt and restructuring negotiations against the interests of creditors and minority shareholders.

propositions and hypotheses.

4.2 The Model

Few papers abandon the presumption of single debt and, hitherto, none has moved beyond bilateral negotiation. Only the bank and the firm are modelled in renegotiation yet, in practice, firms can address their financial difficulties by seeking debt reduction from all the lenders. In this section, I offer a fuller model for reconciliation between different creditors. In my model, the firm funds projects with bank loan and bonds, and, when facing financial difficulties, obtains partial debt forgiveness from both banks and bondholders so as to resolve its distress. The uniqueness of the bank emanates from the property of creditor concentration, which implies a greater bargaining power in Chapter 11.

I then show how debt structure influences the renegotiation process and, eventually, the default decision of the firm as well the firm-level recovery rates. My model sheds light on the decision of optimal debt reduction in formal renegotiation and reconciles the observed recovery rates for different debt holder.

4.2.1 The Setup

Following Broadie, Chernov and Sundaresan (2007), I choose the operating

earnings before interest and taxes (EBIT) as our state variable. The EBIT starts from X_0 and follows a geometric Brownian motion with a constant growth rate $\mu > 0$ and a constant volatility σ under the risk-neutral measure **Q**:

$$\frac{dX_t}{X_t} = \mu dt + \sigma dW_t \tag{4.1}$$

where W_t is a standard Brownian motion under the risk-neutral probability. The term structure is flat with risk-free rate r at which investors can borrow and lend. The firm pays taxes at rate of τ on corporate income.

The fundamental idea rooted in trade-off theory is the tax shield advantage of debt: debt payments are deductible before paying taxes, offering an incentive to issue debt. In this model, the firm has two types of debt obligation outstanding: a private bank loan, with promised coupon payment b and public market debt in the form of bonds with coupons c. I rule out information asymmetry in the model, so all bodies have full information about the EBIT characteristics, μ and σ, and they are able to observe the values of EBIT persistently.[25]

The investment policy of the firm is settled and EBIT is used to pay off the debt obligation. There is an option to default on the firm's liabilities, which is exercised if earnings fall beneath an endogenous default threshold, X_S. There is

[25] For readers who are interested in how information asymmetry and debt renegotiation are related, please refer to theoretical work by Shibata and Tian (2012) and empirical work by Roberts (2015).

no default when company's EBIT is above the barrier and, in that situation, the bank and bondholders receive their coupons b and c. The remaining earnings, $(X_t - b - c)(1 - \tau)$, are distributed to shareholders as dividends. If earnings touch the barrier from above, the firm defaults (files for Chapter 11) prior to completely destroying its equity value.[26] Thus far, default is interchangeable with filing for Chapter 11. However, in practical implementation, the claimholders may not reach consensus even under supervision of the court in Chapter 11, in which case the filing may be converted to Chapter 7 liquidation. Such renegotiation frictions will be included in the model, as I demonstrate in more detail later.[27]

If liquidation is triggered, costs are incurred. The liquidation value of the firm is

$$V^L \equiv V_S - \alpha V_S - K = (1 - \alpha)V_S - K \qquad (4.2)$$

where α and K represent the proportional and fixed liquidation costs.[28]

With the introduction into the model of a tax shield benefit and distress cost associated with debt financing, the value of the firm is no longer identical with

[26] I admit that in reality the firm has an option to restructure its debt either through a private workout in the case of bank debt, or a distressed exchange in case of public debt. I focus on the value of the bankruptcy option since this provides a benchmark for private renegotiation.

[27] The way I include this bargaining friction differs considerably from existing strategic default models.

[28] Examples of fixed liquidation costs include legal fees and fees for asset auction. Examples of proportional costs include lost business, the loss of valuable employees and asset depreciation.

the value of the firm's assets. The former, denoted by $v(V_t)$, is equal to the value of firm's unlevered assets V_t plus the present value of the tax saving benefit subtracting the present value of financial distress costs and liquidation costs.[29]

Two provisions are incorporated in the model, which are clearly stated in Chapter 11 but have been ignored in earlier papers. First, after filing for bankruptcy, the debtor firm has an exclusive right to offer a debt renegotiation arrangement on how to reallocate the firms' resources among stakeholders. Secondly, given no agreement being achieved on a plan, a proposer is chosen for the next step in negotiation. In what follows, I show how the post-default renegotiation is conducted with multilateral parties when these two provisions are explicitly considered. I derive the equilibrium asset allocation in Chapter 11 and, based on the renegotiation results after filing for bankruptcy, I then determine the equity and debt values prior to default.

4.2.2 Post-Bankruptcy Renegotiation

Under Chapter 11, a formal renegotiation among firm claimants is organized by the court. Since three parties are involved, the usual Nash bargaining framework

[29] The unlevered value of firm's assets can be determined by its one-to-one relationship with EBIT which yields $V_t = \mathbf{E}^{\mathbb{Q}}\left(\int_t^{\infty} e^{-r(s-t)} X_s \, ds | \mathcal{F}_t\right) = \frac{X_t}{r-\mu}$. In compliance with the formula, the value of assets at renegotiation is $V_S = X_S/(r-\mu)$, which differs from the firm value at renegotiation $v(V_S)$. Future tax benefits are retained if the firm operates as an ongoing entity.

no longer applies and payments allocated to claimants are altered. The value-sharing results of trilateral negotiation are distinct from those in a bilateral setting. A natural question is: what is the equilibrium post-bankruptcy debt forgiveness in this context? How much can the creditors actually get back?[30] Eraslan (2008) is the first paper displaying multilateral bargaining in the Chapter 11 environment. Secured and unsecured creditors together with equity holders argue over two issues: the destiny of the firm (liquidation or reorganization) and the split of the firm's assets. The setting is similar to mine in the sense of three participants but I advance the approach and accommodate it to my structural model.

Three claimants take part in the negotiation: bank lender b, bondholder m and equity holder e. In the U.S. Bankruptcy Code, under Chapter 11 the equity holder is entitled to submit a proposal during the initial 120 days after filing for bankruptcy. In the event that no unanimous consent is accomplished, a party (we'll call a 'player') is selected to offer a proposal that specifies whether to liquidate or reorganize the firm and how to split the value among the claimants.[31] I assume in this stage stage the probability of being chosen as the proposer for player $i \in (b, m, e)$ is stationary and given by $p_i \geq 0$. The

[30] The answer to this question will be used as the numerator when calculating recovery rate.

[31] The Code allows any claimholder who are impaired to propose a renegotiation plan, as long as the manager of the defaulted firm do not propose a plan that is universally agreed upon. Details are revealed in Chapter 2 when I introduce the U.S. bankruptcy code.

summation of total probability is equal to one: $p_b + p_m + p_e = 1$. Note that the likelihood of being chosen as a proposer here depends upon the number of players, the identities of players and the sizes of the claims. Particularly, the probability of equity holder proposing a plan, p_e, depends on the composition of the participants in the negotiation. Bondholders, as dispersed lenders, are prone to the problem of holdout.[32] The bank, as a sole lender, is more likely to act as a proposer than bondholders, who have fewer incentives for active involvement in renegotiation.[33]

Renegotiation failure, where the court converts the case from Chapter 11 to Chapter 7, is mathematically modelled by probability $1 - q$. I expect that, whenever consensus upon a proposal is not reached, there is a chance that the case is changed to liquidation at the beginning of the following period. If the case is converted to Chapter 7, the settlements to the players follow the Absolute Priority Rule and equity is worthless upon liquidation. The proceeds are used to repay the bank first, implying payment to player b of

$$v_b^L = \min\left\{(1-\alpha)V_S - K, \frac{b}{r}\right\}. \tag{4.3}$$

In the event that the liquidation value of the firm after subtracting liquidation

[32] Although the renegotiation costs is not explicitly shown in the model, it is both sensible and consistent with the literature to expect that participation in the renegotiation process is an outcome of trade-off.

[33] Ivashina, Iverson and Smith (2016) and Demiroglu and James (2015) provide up to date findings on creditor dispersion and its impact on Chapter 11 renegotiation.

cost $(1-\alpha)V_S - K$ is sufficient to cover the principle amount of bank debt b/r, the bank will get the full reimbursement b/r and the residue will be distributed to the other creditors. Otherwise, all assets will be seized by the bank. Therefore, player m (unsecured debt holder) receives

$$v_m^L = \max\left\{(1-\alpha)V_S - K - \frac{b}{r}, 0\right\}. \tag{4.4}$$

The renegotiation process is illustrated in Figure 4.1. At the starting date 0, player e (equity holder) is automatically selected to propose. Lenders respond by either accepting or rejecting the offer. If both bank and bondholder accept, the negotiation ends and the proposal is implemented. If no agreement being reached, the procedure moves to the following date with probability q, while with probability $1-q$, bargaining breaks down, the case is converted to Chapter 7 and players receive their liquidation allocations.

I first examine what happens at a later date (date 1) when the proposal at date 0 is refused by at least one creditor. Suppose the proposer at date 1 is k. Let x_i^k represent the amount of the firm's value allocated to player i under the proposal submitted by k. The continuation payoff of player i, or, the expected payoff of player i from the subgame staring next period, is

$$v_i = \beta q \sum_k p_k x_i^k + \beta(1-q)v_i^L \tag{4.5}$$

Apparently, the continuation value is the present value of the payoffs in the following period under different scenarios.[34] The time value of money has been taken into account by multiplying the payoff by the discount factor β. v_i here can also be regarded as the payment anticipated by player i if he declines the current plan. For this reason, the optimal strategy for proposer k is to offer player i a share equal to v_i such that the non-proposer members will be indifferent between approval and opposition. According to this logic, the remaining portion $x_k^k = v(V_S) - \sum_{i \neq k} v_i$ is left to the proposer himself.

Proposition 1. *In equilibrium, the proposer k himself will receive*

$$x_k^k = v(V_S) - \frac{\beta q(1-p_k)\big((1-\beta)v(V_S) + \beta(1-q)G\big)}{1-\beta q} - \frac{\beta(1-q)\big(V^L - v_k^L\big)}{1-\beta q} \tag{4.6}$$

and other players i receive v_i

$$v_i = \frac{\beta q\left(p_i\big((1-\beta)v(V_S) + \beta(1-q)G\big)\right) + \beta(1-q)v_i^L}{1-\beta q} \tag{4.7}$$

[34] Specifically, for bank, bondholder and equity respectively, the expected payoff from the game next period will be

$$v_b = \beta q \sum_k p_k\, x_b^k + \beta(1-q)v_b^L$$

$$v_m = \beta q \sum_k p_k\, x_m^k + \beta(1-q)v_m^L$$

$$v_e = \beta q \sum_k p_k\, x_e^k + \beta(1-q)v_e^L = \beta q \sum_k p_k\, x_e^k.$$

The last equation holds since we assume equity is worthless in liquidation.

where $G = v(V_S) - V^L = v(V_S) - (1-\alpha)V_S + K$ *as the renegotiation surplus.*

Since at date 0 the equity holder has the exclusive right to file the reorganization plan first, the equity holder can maximize his payoff by offering player $i \in \{b, m\}$ a share equal to $x_i^e = v_i$ and keeping $x_e^e = v(V_S) - v_b - v_m$ for himself. Combining with Proposition 1 and the liquidation values of bank debt and bond as specified in equation (4.3) and (4.4), equity holder will propose

$$x_b^e = \frac{\beta q\left(p_b\left((1-\beta)v(V_S)+\beta(1-q)G\right)\right)+\beta(1-q)\min\left\{(1-\alpha)V_S-K,\frac{b}{r}\right\}}{1-\beta q} \tag{4.8}$$

to bank lender and

$$x_m^e = \frac{\beta q\left(p_m\left((1-\beta)v(V_S)+\beta(1-q)G\right)\right)+\beta(1-q)\left\{(1-\alpha)V_S-K-\frac{b}{r}\right\}^+}{1-\beta q} \tag{4.9}$$

to market lender and leaves the remainder to himself

$$x_e^e = v(V_S) - \frac{\beta q(1-p_e)\left((1-\beta)v(V_S)+\beta(1-q)G\right)}{1-\beta q} - \frac{\beta(1-q)V^L}{1-\beta q}. \tag{4.10}$$

4.2.3 Pre-Bankruptcy Valuation

Next, this section will show how the renegotiation outcomes determined in section 4.2.2 are reflected in the *ex ante* valuation of the claims, together with the determination of timing for strategic default. Assume agents are risk-neutral and market is complete. Consider a time-homogeneous asset $F(X_t)$ which makes cash payment $mX_t + k$ after all taxes. The expected instantaneous return of holding this asset must equal the return on the safe asset r, to avoid arbitrage.

Thus [35]

$$rF(X_t) = mX_t + k + \mu X_t F'(X_t) + \frac{1}{2}\sigma^2 {X_t}^2 F''(X_t). \tag{4.11}$$

The general solution to this ordinary differential equation (4.11) is

$$F(X_t) = A_1 {X_t}^{\lambda^-} + A_2 {X_t}^{\lambda^+} + \frac{mX_t}{r-\mu} + \frac{k}{r} \tag{4.12}$$

where $\lambda^- < 0$ and $\lambda^+ > 1$ are the roots of the quadratic[36]

$$\frac{1}{2}\sigma^2 x^2 + \left(\mu - \frac{1}{2}\sigma^2\right)x - r = 0. \tag{4.13}$$

This formula will be used frequently later, with suitable boundary conditions from the post-bankruptcy renegotiations.

For scenario $X \geq X_S$, we say the borrower is in the 'normal' region without motive to default.[37] The firm will pay the contractual coupon payments b to the bank and c to the bondholder. On the premise of equation (4.12), the value

[35] The right-hand side of the equation is the total return on the asset $F(X_t)$. Gain from holding the assets is the sum of the after-tax "dividend'' plus the expected capital gain. The last two terms represents the expected capital. This is derived by applying Ito's lemma

$$\frac{dF(X_t)}{dt} = \mu X_t \frac{\partial F}{\partial X_t} + \frac{1}{2}\sigma^2 {X_t}^2 \frac{\partial^2 F}{\partial {X_t}^2}.$$

[36] This implies $\lambda^- = -\frac{1}{\sigma}\left[\frac{\left(\mu-\frac{1}{2}\sigma^2\right)}{\sigma} + \sqrt{\left(\frac{\mu-\frac{1}{2}\sigma^2}{\sigma}\right)^2 + 2r}\right]$, $\lambda^+ = -\frac{1}{\sigma}\left[\frac{\left(\mu-\frac{1}{2}\sigma^2\right)}{\sigma} - \sqrt{\left(\frac{\mu-\frac{1}{2}\sigma^2}{\sigma}\right)^2 + 2r}\right]$.

[37] As clarified earlier, the endogenous threshold value V_S is the decisive boundary that distinguishes the solvency and insolvency state of the firm and the corresponding EBIT value is $X_S = V_S/(r-\mu)$.

function of bank obligation[38] in the non-bankruptcy states $B_A(X_t)$ must satisfy the format

$$B_A(X_t) = B_1 {X_t}^{\lambda^-} + B_2 {X_t}^{\lambda^+} + \frac{b}{r} \qquad \forall X \in [X_S, \infty) \tag{4.14}$$

To solve for unknowns (B_1 , B_2), I use value matching and no bubble (asymptotic) conditions as follows:

(1) Value-matching condition: $B_A(X_S) = B_P(X_t)$;

(2) No-bubble condition: $\lim_{X_t \to \infty} B(X_t) = \frac{b}{r}$.

Intuitively, the value-matching condition means that, at the point where the condition is satisfied, the bank is indifferent between allowing the firm to continue as a going concern and renegotiating under bankruptcy. From the value-matching condition, the new bank debt value after renegotiation $B_P(X_t) = x_b^e$ in debt renegotiation. With x_b^e plugged into the value-matching condition, the solution of bank debt valuation is produced and displayed in Proposition 3. Using a similar technique, the bond values can be derived.

In terms of equity value, given the dynamics of cash flow, the after-tax cash flow distributed to shareholders is $e(X_t) = (1-\tau)(X_t - b - c)$ in the normal state. Applying the same technique, the equity value in the normal region, $E_A(X)$, satisfies

$$E_A(X_t) = E_1 {X_t}^{\lambda^-} + E_2 {X_t}^{\lambda^+} + (1-\tau)\left(\frac{X_t}{r-\mu} - \frac{b+c}{r}\right) \tag{4.15}$$

[38] Subscripts A and P indicate pre- and post-bankruptcy values respectively.

and is subject to the value-matching and no-bubble conditions as follows:

$$\lim_{X \downarrow X_S} E_A(X_t) = x_e^e \tag{4.16}$$

$$\lim_{X \uparrow \infty} E_A(X_t)\,/X \leq \infty \tag{4.17}$$

The second boundary condition (4.16) implies that $E_2 = 0$. Using the value-matching condition (4.16) in combination with the general solution (4.15) yields the value of equity in Proposition 3.[39]

It remains to address the renegotiation barrier V_S. This can be determined by characterizing the default policy that maximizes equity value. Mathematically, this entails the smooth-pasting condition

$$\lim_{X \to X_S} E_A{}'(X_S) = 0. \tag{4.18}$$

Calculations show that the renegotiation threshold V_S depends on the renegotiation surplus G, while G is associated with the value of the firm as a going-concern. To unravel this, we must first ascertain the firm value $v(V_t)$. This leads to Proposition 2.

Proposition 2. *Given the trigger point for strategic debt service V_S, the total value of the firm is*

[39] To keep the parameter consistent, we express values of claims in respect of asset value rather than cash flow in Proposition 3.

$$\begin{cases} v_A(V_t) = V_t + \dfrac{\tau(b+c)}{r}\left[1 - \dfrac{\lambda^+}{\lambda^+ - \lambda^-}\left(\dfrac{V_t}{V_S}\right)^{\lambda^-}\right] & \text{when } V > V_S \\ v_P(X_t) = V_t - \dfrac{\lambda^-}{\lambda^+ - \lambda^-}\dfrac{\tau(b+c)}{r}\left(\dfrac{V_t}{V_S}\right)^{\lambda^+} & \text{when } V \le V_S \end{cases} \tag{4.19}$$

In the light of (4.19), at the trigger point V_S, the total value of the firm is

$$v_A(V_S) = v_P(V_S) = V_S - \frac{\lambda^-}{\lambda^+ - \lambda^-}\frac{\tau(b+c)}{r} \tag{4.20}$$

and the renegotiation surplus is

$$G = v(V_S) - (1-\alpha)V_S + K = \alpha V_S - \frac{\lambda^-}{\lambda^+ - \lambda^-}\frac{\tau(b+c)}{r} + K. \tag{4.21}$$

We can now characterize the optimal renegotiation point and the valuation of debt and equity under negotiated debt reductions in Proposition 3 as follows.

Proposition 3.

(i) The optimal renegotiation point for strategic debt service is

$$V_S = \frac{\lambda^-}{\lambda^- - 1}\left[\frac{q(1-p_e)\dfrac{\lambda^-}{\lambda^+ - \lambda^-}\dfrac{\tau(b+c)}{r}(1-\beta q)}{q(1-p_e)(1-\beta+\beta(1-q)\alpha)+(1-q)(1-\alpha)}\right. \\ \left. + \frac{\big(1-\beta q(1-p_e)\big)(1-q)K + \left(\dfrac{1}{\beta}-q\right)\dfrac{b+c}{r}}{q(1-p_e)(1-\beta+\beta(1-q)\alpha)+(1-q)(1-\alpha)}\right]. \tag{4.22}$$

(ii) The equity value before bankruptcy is

$$E_A(X_t) = \left[x_e^e - \left(V_S - \tfrac{(1-\tau)(b+c)}{r}\right)\right]\left(\frac{V_t}{V_S}\right)^{\lambda^-} + V_S - \tfrac{(1-\tau)(b+c)}{r}. \tag{4.23}$$

(iii) The bank debt value before bankruptcy is[40]

[40] The equation can also be written in the form of

$$B_A(V_t) = \left(x_b^e - \frac{b}{r}\right)\left(\frac{V_t}{V_S}\right)^{\lambda^-} + \frac{b}{r} \tag{4.24}$$

(iv) The market debt value before bankruptcy is

$$M_A(V_t) = \frac{c}{r} + \left(x_m^e - \frac{c}{r}\right)\left(\frac{V_t}{V_S}\right)^{\lambda^-} \tag{4.25}$$

where $G = \alpha V_S - \frac{\lambda^-}{\lambda^+ - \lambda^-}\frac{\tau(b+c)}{r} + K$ *and* x_e^e, x_b^e *and* x_m^e *are the values of the claims after renegotiation derived from previous section.*

4.3 Empirical Implications

This section exploits the model in order to explore how the welfare of creditors[41] is affected by the Chapter 11 renegotiation procedure. To start with, I examine the question of whether the presence of a bank lender can enhance the aggregate recovery ratio of all creditors. The calculations that follow show that the bank does bring welfare improvement in the sense that it is associated with a higher ultimate firm-level recovery rate. Next, I investigate whether this improvement spills over to bondholders and, finally, I compare how the default threshold derived from our model is shifted relative to other strategic default models.

$$B_A(V_t) = \frac{b}{\square}\left[1 - \left(\frac{V_t}{V_S}\right)^{\lambda^-}\right] + x_b^e\left(\frac{V_t}{V_S}\right)^{\lambda^-}$$

in which the bank debt value is the weighted discounted value of the pay-off under going concern and bankruptcy. The term b/r captures the former and the liquidation value captures the latter. This rule applies to other valuations as well.

[41] The welfare is measured in terms of the ultimate recovery rate. The definition of measurement of ultimate recovery rate follows the Moody's Ultimate Recovery Rate database and will be introduced later, in the empirical analysis section.

4.3.1 Firm-Level Creditor Recovery

Total welfare of the creditors is quantified by total recovery rate for all the creditors. To simplify the exposition, I let discount factor $\beta = 1$ and denote face value of all debts outstanding as D which is equivalent to $(b + c)/r$. I denote the percentage of bank portion in total debt as ω which implies that the bank debt face value $b/r = \omega D$ and bond debt face value $c/r = (1 - \omega)D$. The aggregate recovery rate for all the creditors is

$$\begin{aligned} RR_{\mathrm{T}} &= \frac{B_P(V_t) + M_P(V_t)}{b/r + c/r} \\ &= \frac{V_S}{D} - \frac{\alpha V_S + K}{D}[1 - q(1 - p_e)] - q(1 - p_e)\frac{\lambda^- \tau}{\lambda^+ - \lambda^-}. \end{aligned} \tag{4.26}$$

where V_S follows equation (4.22).

In contrast with past models (Fan and Sundaresan (2000), Broadie, Chernov and Sundaresan (2000), Valta (2016), etc.), this model features the differentiation of bank debt from market debt by their distinctive possibilities to offer a renegotiation plan. First, the bank behaves as a more potent dissenter than bondholders when facing an unfavorable plan from the firm. Higher bank percentage corresponds to greater recovery rate.[42] The economic rationale

[42] Mathematically, I demonstrate that higher bank percentage corresponds to higher recovery rate by calculating the signs of $\partial RR/\partial\omega$. Details of the calculations are in an appendix. Table 4.1 summarizes hypotheses and implications.

behind this prediction is the uniqueness of the bank compared with other creditors. Traditional studies on debt structure hold to the theoretical premise that banks creditors have superior monitoring ability. Diamond (1984, 1991), for instance, contends that banks have scale economies and comparative cost advantages in information production that enable them to undertake superior debt-related monitoring. Further, diffused public debt ownership and the associated free-rider problem diminish bondholders' incentives to engage in costly information production and monitoring.

Hypothesis 1: Lower bank percentage results in a higher bargaining power of equity, which is further associated with early renegotiation and lower creditors' recovery as $\frac{\partial V_S}{\partial \omega} < 0$ *and* $\frac{\partial RR}{\partial \omega} > 0$.

In this way, if the composition of debt is biased toward bank obligation, the firm is expected to default at a lower asset value level but the recovery rate to all creditors is still improved. Hypothesis 1 entails two mechanisms through which that debt structure can potentially manipulate the ultimate recovery. One is the influence on default time (negative effect), the other is alteration of the equity holders' strength in renegotiation (positive effect). Calculations show that the second mechanism is predominant. Further insights on the bank's ability to improve recovery are illustrated in the following hypotheses.[43]

[43] Please refer to an appendix on how we arrive at these predictions.

Hypothesis 2: The sensitivity of creditors' recovery rate to bank loan percentage diminishes with the amount of face value of total debt and the renegotiation frictions (the probability of renegotiation failure and conversion to Chapter 7 liquidation).

The bank's specialness in the model - its advantageous bargaining position compared with dispersed creditors relies on the condition of the firm entering into Chapter 11. The bank does not maintain any specialness in liquidation. Therefore, when the possibility of Chapter 7 liquidation increases, the power of the bank to lift the recovery rate is weakened, as reflected in Hypothesis 2. The effect of debt size D in our model needs to be analyzed in tandem with fixed liquidation costs K. With a lower level of total debt D, the fixed liquidation costs K will be a significant cash outflow to creditors if the process ends with liquidation. Consequently, creditors have an extra incentive to agree to renegotiate their claims so as to avoid the potentially large expenses of liquidation. Shareholders will take this opportunity to default at an early stage in order to ask for debt reduction, which implies a higher amount of renegotiation surplus for involved parties to divide. Therefore, with smaller debt size, the bank can better exhibit its ability to restrict shareholders' opportunistic behaviors.

Hypothesis 3: Higher bargaining friction leads to lower recovery rates.

Bargaining friction has been repeatedly mentioned as critical in determining the outcome of default. However, existing papers generally focus on how bargaining friction determines the firm's choice between Chapter 11 and Chapter 7.[44] In contrast, what hypothesis 3 tries to clarify is that liquidation is not a necessary condition to depress the recovery rate. Even the threat of liquidation can itself result in a lower recovery rate in renegotiation. As far as I know, no paper has compared the recovery rates within the sample of Chapter 11 cases and analyzed the effect of bargaining friction on renegotiation results for those companies that effectively avoid liquidation.

Hypothesis 4: Higher total liquidation costs leads to a higher renegotiation boundary, and lower recovery rates.

The importance of liquidation costs has been well documented. Blazy, Chopard and Nigam (2013) compare the differences in bankruptcy costs for several different bankruptcy systems. They argue that the divergences in total recovery rates might reflect different bankruptcy costs. Acharya, Bharath and Srinivasan (2007) show empirically that, when an industry is in distress, defaulting firms in the industry experience lower recoveries. Mechanism for this is the fire-sales

[44] It is commonly believed in existing research that, if the firm is subject to higher bargaining frictions, it is more likely to be ended via Chapter 7 liquidation, leading to a lower final recovery for all creditors.

effect: reduced ability of the distressed firm to sell assets to competitors in the same industry, as discussed by Shleifer and Vishny (1992). This is in line with this model because a distressed industry implies higher liquidation costs for the defaulted firm exactly because peer firms in the same industry are the most likely buyers of the assets.

4.3.2 Bond Recovery: Is There Spill-Over Effect?

Following the analysis of firm-level recoveries, this section turns attention to the distributions of recovery rates at the obligation level. Given the recovery improvement caused by involvement of the bank, the question remains whether bondholders can profit from this enhanced aggregate recovery. One possibility is that the bank grabs all advantages while bondholders gain nothing from the enhancement. Addressing this question may further broaden our knowledge on the role of banks.[45]

[45] The impact of bank debt percentage increase is not well addressed in the literature. The most relevant paper is Ongena Roşcovan, Song and Werker (2014) which investigates the effect of bank loan announcements on the borrowing firm's bond prices. Theoretically, the impact on the current firm bondholders is ex ante ambiguous. On the one hand, new bank loans may provide an additional and timely certification that the firm is still of an acceptable credit quality. On the other hand, new bank loans affect the firm's capital structure, increasing not only the value of its assets but also its leverage ratio and, consequently, the expected loss given default for bondholders. In addition, the frequent seniority of bank debt over public debt further disadvantages the current bondholders in case of default, exacerbating their expected losses.

We find that, for the spill-over effect to be realized, the following two conditions must hold simultaneously[46]

Condition (1)
$$\left|\frac{\partial p_e}{\partial \omega}\right| < \frac{1-\alpha+q(1-p_e)\alpha}{q\alpha(1-\omega)} \frac{\frac{\alpha V_S}{D}+\left(\frac{K}{D}-\frac{\lambda^-}{\lambda^+-\lambda^-}\tau\right)}{\frac{\alpha V_S}{D}+\frac{\lambda^-}{\lambda^- -1}\left(\frac{K}{D}-\frac{\lambda^-\tau}{\lambda^+-\lambda^-}\right)}; \tag{4.27}$$

Condition (2)
$$\omega > \left(\frac{1-\alpha}{\alpha}\right)\frac{\lambda^-}{\lambda^- -1}\left[\frac{q(1-p_e)\frac{\lambda^-\tau}{\lambda^+-\lambda^-}+1+\frac{1K}{\alpha D}}{1/\alpha-\left(1-q(1-p_e)\right)}-\frac{K}{D}\right]-\frac{K}{D}. \tag{4.28}$$

Therefore, whether or not a bondholder can benefit from the improved creditor recovery is based on the percentage of bank debt (from condition (2)) and the sensitivity of equity's bargaining power with respect to change of debt composition (from condition (1)). Spill-over happens when the percentage of bank loan is sufficiently large (as implied in condition (2)) and the equity's bargaining power is marginally reduced by involvement of bank lender (implied by condition (1)). Otherwise, the two conditions will not hold and, thus, the bondholders are expected to be worse off from this increase in bank debt percentage.

Along this line, from condition (1), I anticipate that spill-over happens when the specialness of the bank decreases as it means that bank debt and bonds are more likely to move towards equality.[47] For instance, if bank debt is held by several

[46] An appendix contains proofs of the two conditions.

[47] This is because condition (1) means that percentage of bank debt in total debt significantly reduces the bargaining power of shareholders.

banks, we expect a decrease in the ability of the bank debt to restrict the firm in proposing. This means that equity still holds high bargaining power in renegotiation and so the default threshold will increase[48] implying an early default for the firm. If the firm defaults at higher asset value, it is more likely that the bondholders can gain higher recovery. Conversely, if the bank debt is owned by a single bank, then the presence of this bank greatly lowers the bargaining power of the firm, which then defaults at a later date, leaving bondholders to recover nothing.

Hypothesis 5: Generally an increase in bank percentage of total debt is associated with a decrease in bond recovery rate, but the magnitude of this decrease depends on $\partial p_e / \partial \omega$.

My main argument regarding bondholders' recovery is in the Hypothesis 5. The magnitude of this decrease in bond recovery rate is dependent on loan characteristic or different bankruptcy law. When a bank has the expertise to weaken the firm, there is no spill-over effect and so senior and junior debt exhibit a severe conflict of interest. The spirit is similar to that of Ivashina, Iverson and Smith (2016) who find that higher ownership concentration within a debt class is associated with higher recovery rates to that class.

[48] The ratio of default threshold V_S / D will also be higher.

4.3.3 Firm-Specific Bankruptcy Boundary

The default boundary is usually defined as the level of state variable that separates non-defaulting firms from those whose severity of distress makes default unavoidable. Structural models contend that the market value of a firm's assets reflects its prosperity or economic distress and, therefore, should be used as a state variable to capture fully the firm's default risk (Davydenko 2012). Agreement has not been achieved among scholars on the exact level of the default boundary. In fundamental structural models, starting with Black and Cox (1976), default (liquidation) occurs as the value of a firm's assets moves lower than that of its liabilities. Although such a 'zero net-worth' boundary is often a natural choice, Huang and Huang (2012) and Eom, Helwege and Huang (2004) find that a boundary of 60% of debt face value is more consistent with observed debt recovery rates. Several authors accordingly offer extensions to the basic Merton model where liquidation is triggered when the value of the firm's assets reaches a lower threshold barrier. Another strand of structural models argues that the default boundary is neither exogenous nor fixed. Default is treated as a decision made by the firm to maximize equity value, and the boundary is derived as a function of other parameters.[49]

[49] A conspicuous example of this group, Leland and Toft (1996), infers the default boundary as

$$V_{LT} = \frac{(C/r)(A/(rT) - B) - AP/(rT) - \tau Cx/r}{1 + \alpha x - (1 - \alpha)B}.$$

This model is in the spirit of endogenous structural models. The usual practice in value-based structural models is to express the default boundary as the ratio of the asset value compared with the face value of debt. Correspondingly, the model yields the following expression for the default boundary[50]

$$\frac{V_S}{D} = \frac{\lambda^-}{\lambda^- - 1}\left[\frac{q(1-p_e)\left(\frac{\lambda^-\tau}{\lambda^+ - \lambda^-} - \frac{K}{D}\right) + 1 + \frac{K}{D}}{q(1-p_e)\alpha + 1 - \alpha}\right]. \tag{4.29}$$

Given expression (4.29), I provide a numerical example of the default boundary in Table 4.3. The input parameter values are those commonly used in current papers, as shown in Table 4.2. Table 4.3 shows that model's output, the theoretical default boundary, is compatible with observed values found in Davydenko (2012). In the baseline case (the third column of Table 4.3), the estimated default boundary from the model is 64.7%, while Davydenko estimates the default boundary as 66% of the face value of the debt based on market values of defaulting firms. In addition, the numerical example shows that the default boundary is lower when asset volatility is high, which is likewise reflected in Davydenko's empirical findings According to the theoretical paper of Leland and Toft (1996) and the empirical paper of Davydenko (2012), the

[50] Equation(4.29) is a simplified version of

$$\frac{V_S}{D} = \frac{\lambda^-}{\lambda^- - 1}\left[\frac{\left(\frac{\beta q(1-p_e)\lambda^-\tau}{\lambda^+ - \lambda^-} + 1\right)\frac{1-\beta q}{\beta} + \left(1 - \beta q(1-p_e)\right)\frac{(1-q)K}{D}}{q(1-p_e)(1-\beta+\beta(1-q)\alpha) + (1-q)(1-\alpha)}\right]$$

through assuming the discount factor is one.

boundary is increasing in the cost of liquidation, which is also evident in my example.

4.4 Empirical Analysis

There is no direct market measurement of loan values and so tests on the third aspect of the model, the default boundary, are impossible. The empirical analysis here covers the first two aspects: firm-level ultimate recovery rate and bondholders' recovery rate under Chapter 11. The aim is to discover the impact of debt structure on the ultimate recovery rate for creditors in bankruptcy.[51] The key variable relating debt structure and bankruptcy outcome here is the probability of a player proposing a plan. As explained in the model description, this probability is determined by the quantity and the composition of the players as well as the sizes of claims. Specifically, the probability of equity acting as the proposer depends on the percentage of bank debt versus market debt. The strength of borrower in bargaining is determined not only by the size of debt, but also on how that debt is composed.[52]

[51] Up to now, I focus on Chapter 11 cases. The recovery rates in out-of-court restructurings are affected by the values creditors can obtain in Chapter 11.

[52] An extreme example is two firms with the same debt-to-equity ratio but one is fully financed by a bank whereas the other is funded by thousands of bondholders.

4.4.1 Data Description

Data used in this chapter are primarily from the Default and Recovery Database (DRD) and Ultimate Recovery Database (URD) developed by Moody's. In addition, I obtain the firm and industry variables by crossing-matching the defaulted companies with the CRSP-Compustat merged database. For each bankruptcy filing, Moody's collects information about the defaulted issuer, the bankruptcy case and the debt instruments that were outstanding at the time of filing.[53] I draw detailed information on 444 issuers who filed for Chapter 11 bankruptcy in the U.S. between 1987 and 2016. Table 4.4 shows the distribution of filings over time and across industry. The data cover three credit default cycles, including the 1990-1991 recessions, the 2002-2003 technology bubble, and the 2008-2009 financial crisis.

The URD database measures recoveries using three separate methods: (1) The Settlement Method - the value of the settlement instruments is taken at or close to emergence; (2) The Liquidity Method - the value of the settlement instruments is taken at the time of a liquidity event, such as the maturity of the instrument, the call of the instrument, or a subsequent default event; (3) The Trading Price Method - the value based on the trading price of the defaulted

[53] The data on the capital structures of the defaulted firms include a detailed description of each debt instrument in the capital structure at the time of default, including the type of instrument (e.g., revolving credit facility, term loan, or bond), whether not it was secured, the size of the claim at filing and the recovery rate.

instrument taken at or after emergence. Moody's also provide the preferred method, which varies for each default. The preferred method is the most representative of the actual recovery. The most common preferred method is the settlement method.[54] Variables used in this study are included in Table 4.5.

4.4.2 Sample Statistics

Creditors' Total Recovery: The dependent variable here is the ultimate debt recovery rate for all the creditors. [55] Table 4.6 summarizes the descriptive statistics for recovery values. The average ratio of reclaimed amount to face value of debt is marginally higher than 50%. Decomposing the total recovery by type of debt instrument, loan owners recover more than twice the amount obtained by bondholders (73.28% and 32.91% respectively). Figure 4.2 plots the recovery rates for three sub-groups of firms, divided by the borrower's source of funding. The first bar speaks to the companies that are financed purely by bank debt, while the third represents those entirely funded by issuing bonds.

[54] Acharya, Bharath and Srinivasan (2007) and Jankowitsch, Nagler, and Subrahmanyam (2014) use trading prices of loans and securities to calculate recovery rate, which is unsuitable for this study for two reasons. First, trading prices of defaulted debt instruments are easily affected by market forces. Secondly, defaulted debts usually have relatively low liquidity, implying that the trading prices are poor indicators of actual recovery.

55 This is equivalent to one minus proportional change in value of total debt over the re-contracting period. Creditor status, bargaining frictions, firm fundamentals as well as industry conditions are hypothesized to affect the dollar change in debt. The dollar change in debt is divided by the initial level of debt to reduce heteroskedasticity in the error terms.

The residuals collectively constitute those with multiply-raised debts. The plot shows the first type, financed purely by bank debt, has the highest recovery rate (over 70%).

Debt Composition: Moody's URD stores information on each firm's debt obligations at the time of default, including its origination date, maturity date, face value, type, etc.[56] I calculate the share of bank loan. Table 4.7 outlines the summary statistics of independent variables. I winsorize the variables at the level of 1% to remove the impact of outliers. Gilson (1997) computes the percentage of institutional debt as those held by banks and insurance companies. Since I exclude insurance companies here, the share of bank obligation in my sample (at mean of 38.89%) is less than that of the private debt in that in Gilson (1997) (47.5%).

Bargaining Frictions: I use three aspects in choosing measures to capture the bargaining frictions in Chapter 11. (1) Debt concentration, for which I use: number of classes involved, number of debt issues outstanding, a dummy variable which equals to one if the firm obtains at least 99% of its debt from one debt issuance, and the Herfindahl-Hirschman Index of debt.[57] (2) Time spent

[56] Moody's classify different types of debt obligations as "Revolver", "Term Loan", "Subordinated Bonds", "Senior Unsecured Bonds", "Senior Secured Bonds", "Junior Subordinated Bonds", and "Senior Subordinated Bonds". Amongst these, "Revolver" and "Term Loan" are bank-owned debt while the other five are corporate bonds.

[57] According to bankruptcy law, debts are grouped into different classes and, within the same class, debt holders are treated equally. Therefore, the higher the number of classes in

in the Chapter 11 procedure. The duration of a bankruptcy reflects the level of complexity and the difficulty of reaching consensus in the case. [58] (3) Prepackaged Chapter 11. Some firms have already achieved a tentative plan of reorganization with creditors before filing. For these prepackaged cases, debt re-contracting is anticipated to be confirmed in less time and with less expense than a standard Chapter 11 case.[59]

Firm Fundamentals: Bankruptcy and recovery are related to the financial health of the firm (see, for instance, Acharya, Bharath and Srinivasan, 2007, Khieu, Mullineaux and Yi, 2012, Blazy, Chopard and Nigam, 2013, Jankowitsch, Nagler and Subrahmanyam, 2014, and Ivashina, Iverson and Smith, 2016). Following firm-specific characteristics are considered in the specification. (1) Profit margin, which is the ratio of EBITDA divided by sales for the defaulting firm. Previous papers such as Acharya, Bharath and Srinivasan (2007) and Jankowitsch, Nagler and Subrahmanyam (2014) also apply this measure to control the impact of profitability of the bankrupt firm. (2)

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renegotiation, the more severe is the holdout problem. Carey and Gordy (2016) use the number of separate debt obligations of the firm at the time of bankruptcy is filed as well as the number of different class labels assigned by the court to measure such bargaining frictions.

[58] Moody's URD report the date on which the issuer files a petition for Chapter 11 (Issuer Default Date) and the date for which a default resolution becomes official (Emergence Date). I calculate the distance between the two dates as the length of the procedure.

[59] Prepackaged bankruptcies exploit Chapter 11 process to moderate the holdout problem and pre-filing negotiations to speed the process to completion. Therefore, prepackaged Chapter 11 bankruptcy filings are regarded as a smoother system.

Leverage. Leverage is measured as long-term debt to assets ratio. I use long-term debt since it is a more stable funding source. (3) Tangibility; which is the percentage of property, plant, and equipment (PP&E) in total assets. (4) Log(Assets). [60]

Industry Conditions: Using instrument-level data, Acharya, Bharath and Srinivasan (2007) show that creditors of defaulted firms recover significantly lower amounts in present value terms when the industry of defaulted firms is in distress. This is economically sound since other firms in the same industry are potential buyers of a defaulted firm's assets and their financial condition strongly affects the value of the assets. Since asset liquidation is an alternative to debt restructuring, it is natural to expect that industry conditions will affect the firm-level recoveries too. I identify the industry of a bankrupt firm as the set of firms with the same initial 2 digits in their SIC code. All industry variables are computed using data from Compustat and CRSP. I use the following two measures to capture industry environment. (1) The industry is "distressed" if the median stock return for the industry of the defaulting firm in the year of default is less than or equal to -30%. (2) Med. Ind. Q - the median of the ratio of the market value of the firm (estimated as book value of total assets book value of

[60] For each firm, I retrieve accounting information just prior to default from Compustat. If this information is not available, I use the accounting data one year before. If there are still no data available, I delete the firm from my sample.

equity market value of equity) to the book value of the firm. This proxy captures the growth prospects of assets which also should affect recovery rates positively.

4.5 Presentation of Results

The final sample includes only firms entering Chapter 11 for the first time, excluding cases where a firm has previously entered into Chapter 11.

4.5.1 Firm-level Recovery Rate

I begin by testing Hypothesis 1. The first regression results are reported in Table 4.8. Through models (1) to (4) in the table, bank percentage, firm fundamentals, industry conditions, and renegotiation frictions are included as explanatory variables. Consistent with predictions[61], the bargaining friction, liquidation cost and degree of operational deterioration depress recovery under Chapter 11.[62] Regressing bank percent on firm-level recovery rate, controlling for bargaining frictions[63], firm fundamentals and industry conditions, the coefficient is approximately 0.22 as persistently shown through model (5) to model (8). Thus,

[61] Equations (A.24), (A.25), (A.31) and (A.35) in Appendix shows $\frac{\partial \mathrm{RR_T}}{\partial \mathrm{p_e}} < 0$, $\frac{\partial \mathrm{RR_T}}{\partial \omega} > 0$, $\frac{\partial \mathrm{RR_T}}{\partial (1-\mathrm{q})} < 0$ and $\frac{\partial \mathrm{RR_T}}{\partial \mathrm{K}} < 0$.

[62] The adjusted R^2 for model (4) is relatively low compared with that in other specifications. The reason for the low R^2 is the absence of an accurate measure for bargaining friction.

[63] We use different measures for bargaining friction through models (5) to (8) in Table 4.8. Section 4.2.2 gives the details of those measures.

holding other variables fixed, if the bank debt ratio is increased by one percentage point, the ultimate total recovery rate increases by 0.22 of a percentage point. With the debt structure now incorporated, firm fundamentals and industry conditions still matter. As shown in Table 4.8, firms that are more financially sound show higher recovery. Specifically, recovery rates exhibit a positive relationship with profit margin and negative relationship with leverage ratio of the firm. In model (8) of Table 4.8, holding other variables fixed, if the profit margin is increased by one unit, the ultimate total recovery rate increases by 7.25 percentage point. Leverage ratio matters as well. Holding other variables fixed, one-unit increase of the leverage ratio result in 6.98 percentage point decrease of ultimate total recovery rate.

As predicted in Hypothesis 3, Table 4.8 also shows higher recovery rate is associated with shorter duration under Chapter 11, prepackaged filings, fewer debt classes and concentrated debt. Davydenko and Strebulaev (2007) find that recovery is higher for firms with lower renegotiation frictions. They attribute this to firms with higher renegotiation frictions being more likely to end in liquidation, which unavoidably results in large liquidation costs. This model shows that, even during the renegotiation process, the threat of a higher possibility of liquidation will ultimately result in a lower recovery.

Additionally, as expected from Hypothesis 4, the liquidation cost, which can be measured by industry conditions as well as asset tangibility, lowers the recovery

rate.[64] It can be observed in Table 4.8 that industry distress increases the liquidation costs and so reduces the recovery ratio. Firms that are identified in distressed industry exhibit 12.93 percentage point lower in recovery rate compared with firms that are not in distressed industry. Also, firms with more tangible asset have higher recovery rates, since tangible assets are associated with lower liquidation costs. Holding other variables fixed, if the tangibility assets (compared with total assets) is increased by 1%, the ultimate total recovery rate increases by 12.09%.

In Table 4.9, I repeat the regression for firms with mixed types of debt only; and the results continue to hold. Altman, Brady, Resti and Sironi (2005) argue that recovery rate and default rate might co-move as the impact of macroeconomic condition. Table 4.4 shows a cluster of Chapter 11 filings around the period when the macro-economy is gloomy. To remove the potential impact of the economic environment, we add year dummies in the regression in Table 4.10. My conclusions still hold after taking year of default into account. Based on the notion that, compared with bondholders, banks can apply more restrictions on the rights of equity holders, the empirical analysis above is consistent with the first hypothesis that higher bargaining power of equity is associated with creditors' lower recovery. To be specific, holding other variables fixed, if the

[64] When the industry of the defaulted firm is in distress, the potential buyers of the assets of the defaulted firms are more likely to be financially constrained. Liquidation costs increase as the assets can be only sold at a fire-sale prices.

bank debt ratio is increased by one percentage point, the ultimate total recovery rate increases by 0.13 percentage point.

I proceed to verify the conditions under which the effect of bank debt percentage on recovery rate is likely to be more pronounced, as I describe in Hypothesis 2 (see Section 3.1). I test the hypothesis by incorporating interaction terms of bank debt percentage multiplied by bargaining friction and debt size, respectively. The results of these tests are reported in Tables 4.11 and 4.14.[65] Table 4.11 reports regression results on renegotiation frictions alongside interaction terms. Through model (1) to model (4) in Table 4.11, the signs of the interaction terms are consistent with our predictions: negative for *Bank percent* × *Number of class* and *Bank percent* × *Number of issues* while positive for *Bank Percent* × *Excl90* and *Bank Percent* × *Debt Concentration.* Thus, the positive impact of bank debt proportion on recovery rate is stronger when the numbers of issues and classes are small, there exists a dominant debtholder, and debts are more concentrated. I also find that the impact of bank debt is stronger when duration is larger in model (5) of Table 4.11. This is consistent with our predictions. The rationale is that, as bargaining friction increases, the probability of liquidation also increases. Thus, the positive effect brought by the bank to improve recovery diminishes since this efficiency must be associated with the renegotiation

[65] To avoid the potential problem of multicollinearity, I also try to remove the proxies used in constructing a particular interaction term in regressions and use alternate proxies for the same factors instead. The results still hold.

process. Holding other variables fixed, if the number of issues outstanding is increased by one, the sensitivity of the relationship between ultimate total recovery rate and bank debt will decrease by 0.03 percentage point, as shown in model (1) of Table 4.11. Despite being a measure of bargaining friction, longer duration also implies that the banks are now more likely to be actively involved and alter the results. Again, I repeat the regression by limiting to mixed debt issuers only, in Table 4.12, and add year dummies in Table 4.13. All conclusions remain unchanged.

In Tables 4.14 and Table 4.15, I use the absolute values of the amount of total debt and leverage ratio to proxy debt size in calculating the interaction term. Hypothesis 2 also states that the sensitivity of creditors' recovery rates to bargaining power decreases with the debt amount. Results in Tables 4.14 and 4.15 confirm that this is the case with my sample: the impact of bank debt percentage on recovery rate decreases when debt amount increases, measured either by absolute value or relative value. The coefficients of interaction terms *Bank Percent* × *Total Debt* and *Bank Percent* × *Leverage* are -0.02 and -0.18. Both are negative and significant, regardless of the measure of debt concentration level I use. Given that equity bargaining power is negatively correlated with bank percent, this is equivalent to a positive sensitivity effect between debt amount and bargaining power impact on recovery rate. This finding is economically reasonable since, when a firm has a high leverage ratio,

it is expected that the equity holder is deeply out of money and thus not influential in the renegotiation process.

4.5.2 Bondholders' Recovery Rate

As predicted in Hypothesis 5, generally the seniority effect dominates any potential benefit brought by the bank debt. Thus, bondholders are usually worse off if the bank loan percentage increases. Recent work by Li, Purda and Wang (2016) finds that yield spread of corporate bond at issuance is higher if the senior loan lender exerts more control. These results combine to suggest that bondholders are negatively affected by the control rights of the senior loan lender, despite the benefits provided by the bank.

In Hypothesis 5, the model further predicts that this negative effect is related to the structure of the bank loan. A tough bank, with greater capability to restrict shareholders, strengthens the negative relationship. In my empirical work, I use three variables measuring the level of concentration (dispersion) of bank loans to capture the toughness of a bank. They are: number of bank debt issues outstanding, number of classes containing bank debt only, and, Herfindahl-Hirschman Index of bank debt[66]. I then generate an interaction term of bank

[66] Ivashina, Iverson and Smith (2016), use the share of bankruptcy claims owned by the top-10 creditors in the firm as well as the HHI and the correlation between the two measures is 0.61. HHI is computed as

percentage multiplied by the three measures to indicate whether the negative relationship is stronger with concentrated bank debt. The first regression result is shown in Table 4.16. As noted by Acharya, Bharath and Srinivasan (2007), bond recovery rate is affected by debt characteristics. I add coupon rate, issue size, seniority dummies and collateral dummies in the controls. The negative effect between a bondholder's recovery rate and bank percentage is supported in the data through model (1) to model (7) in Table 4.16. For example, model (6) shows that, controlling for debt characteristic, bargaining frictions, firm fundamentals and industry conditions, if the bank debt ratio is increased by one percentage point, the bondholders' recovery rate drops by 0.251 percentage point. Additionally, through model (1) to (4), overall debt concentration level is no longer important in determining the individual bondholders' recovery rates. Instead, the concentration level of bank debt is significant in all regressions through model (5) to (7).

To test the condition under which the bondholders are more vulnerable to increase in bank debt, I further construct interaction terms of Bank Percent × Number of Bank Debt, Bank Percent × Number of Bank Debt Class and Bank

$$\text{HHI} = \sum_{\text{i}} \left(\frac{\text{Amount of Bank Debt i}}{\text{Amount of All Bank Debts}} \right)^2$$

Thus, if a firm employs exclusively one single debt type, HHI equals one, while if a firm simultaneously employs several bank lenders in equal proportion, HHI approaches zero. Higher HHI values indicate firms' tendency to specialize in fewer bank lenders.

Percent × HHI of Bank Debt. As reported in Table 4.17, significant differences exist between concentrated and dispersed bank debt. Specifically, the bondholders' recovery rates are depressed further by concentrated bank debt while, when there are multiple bank debts or bank debt classes, the bondholders' recovery rates are less damaged. This is consistent with my prediction for a spill-over effect in Hypothesis 5.

4.6 Conclusion

This chapter derives and tests a new model including features exclusive to negotiation under formal bankruptcy, with the engagement of both bondholders and banks as required by Chapter 11. Current strategic default models are unsuitable since they are limited to two-sided bargaining. To address this, I build a trilateral debt renegotiation model and tailor it in line with the provisions of Chapter 11. Intuitively, owing to the seniority of bank loans, bondholders ought to recover less when banks are also creditors. This model joins this discussion by quantifying the variation of bondholders' recovery when debt composition changes. The impact of bank on bondholders' recovery is found to be complicated and is contingent with the specific capital structure and other related firm fundamentals.

Renegotiation evolves differently when bondholders are included. I show how equity responds and eventually how much of their capital the creditors can

regain. The share of bank debt in the total liability of the firm decides the ability of shareholders to act opportunistically in renegotiation. For firms that rely primarily on bank loans as their source of funding, the participation of a bank puts them at a disadvantage and deferring the default date so as to avoid debt renegotiation is a sensible action. My model allows two paths through which the debt structure can influence the final recovery of all creditors. One is the dominance of creditors over shareholders in bargaining when a bank is involved, producing a higher recovery rate for creditors. The other is a late default time decided by the firm, which leaves limited assets for involved parties to share. In this case all creditors are worse off as measured by recovery rate. Mathematically, my model shows the positive mechanism dominating, with bank involvement expected to boost the overall recovery rate. Empirically, I find exactly that, with a significant positive relationship between firm's recovery and bank percentage.

Figure 4.1 The Renegotiation Plan Generating Process

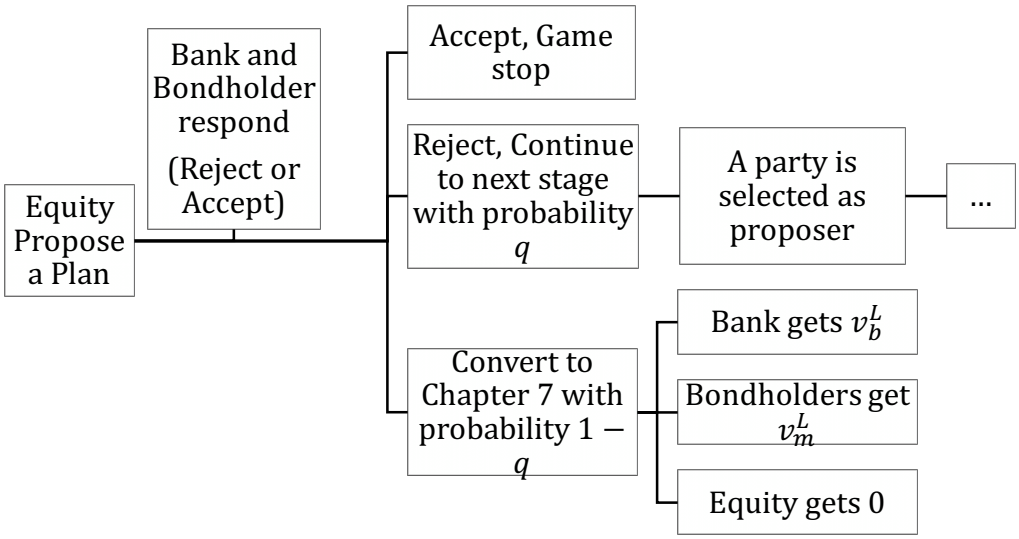


Figure 4.2 Recovery Rates for Three Groups of Firm



Table 4.1 Hypotheses and Implication

Variable	Predicted Sign	Implication
$\frac{\partial RR_T}{\partial(1-p_e)}$	>0	Higher bargaining power of creditors is associated with later renegotiation and higher creditors' recovery.
$\frac{\partial\left(\frac{V_S}{D}\right)}{\partial(1-p_e)}$	<0	
$\frac{\partial RR_T}{\partial(1-q)}$	<0	Higher bargaining friction leads to lower recovery rates.
$\frac{\partial RR_T}{\partial(\alpha V_S+K)}$	<0	Higher liquidation cost is associated with early default and lower total recovery rates.
$\frac{\partial\left(\frac{V_S}{D}\right)}{\partial(\alpha V_S+K)}$	>0	
$\frac{\partial^2 RR_T}{\partial(1-p_e)\partial(1-q)}$	<0	The positive impact of creditor bargaining power on recovery rate decrease as bargaining friction increases.
$\frac{\partial^2 RR_T}{\partial(1-p_e)\partial D}$	<0	The positive impact of creditor bargaining power on recovery rate decrease as debt amount increases

Table 4.2 Inputs of Related Researches

Input		Moraux and Silaghi (2014)	Hackbarth, Henessy Leland (2007)	Broadie, Chernov Sundaresan (2007)
Tax Rate	τ	0.35	0.31	
Drift of EBIT	μ		0.01	0.01
Volatility of EBIT	σ	0.25	0.25	0.2
Risk-free Rate	r	0.06	0.06	0.05
Liquidation Cost (Proportional)	α	0.4	0.5	0.5
Liquidation Cost (Fixed)	K	1~20	0	
Equity Bargaining Power	p_e	0 or 1	0 or 1	0 or 0.5
Debt Amount	D	116	242.21	60

Table 4.3 Numerical Example of Default Boundary

Inputs											
Tax Rate	τ	0.35	0	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
Drift of EBIT	μ	0.01	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Volatility of EBIT	σ	0.25	0.25	0.25	0.4	0.25	0.25	0.25	0.25	0.25	0.25
Risk-free Rate	r	0.05	0.05	0.05	0.05	0.03	0.05	0.05	0.05	0.05	0.05
Liquidation Cost (Proportional)	α	0.5	0.5	0.5	0.5	0.5	0.25	0.5	0.5	0.5	0.5
Liquidation Cost (Fixed)	K	0.33	0.33	0.33	0.33	0.33	0.33	0.66	0.33	0.33	0.33
Bargaining Power	p_e	5	5	5	5	5	5	5	20	5	5
Debt Amount	D	100	100	100	100	100	100	100	100	200	100
Liquidation friction	q	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.5
Outputs											
Default Boundary	$\boldsymbol{\frac{V_S}{D}}$	0.647	0.688	0.714	0.426	0.543	0.548	0.801	0.701	0.638	0.730

Table 4.4 Description of firms filing for Chapter 11 bankruptcy

Sample distribution	Number	Percent
Panel A: By filing year		
1987	2	0.45
1988	2	0.45
1989	5	1.13
1990	10	2.25
1991	17	3.83
1992	18	4.05
1993	16	3.6
1994	8	1.8
1995	10	2.25
1996	7	1.58
1997	5	1.13
1998	13	2.93
1999	22	4.95
2000	26	5.86
2001	42	9.46
2002	45	10.14
2003	37	8.33
2004	27	6.08
2005	18	4.05
2006	12	2.7
2007	8	1.8
2008	16	3.6
2009	36	8.11
2010	13	2.93
2011	10	2.25
2012	6	1.35
2013	6	1.35
2014	3	0.68
2015	4	0.9
Panel B: By Industry		
Agriculture, Forestry and Fishing	2	0.45
Mining	20	4.5
Construction	11	2.48
Manufacturing	173	38.96
Transportation, Communications, Electric, Gas	81	18.24
Wholesale Trade	15	3.38
Retail Trade	60	13.51
Finance, Insurance and Real Estate	11	2.48
Services	71	15.99

Table 4.5 Variable Definitions in Empirical Study

Variables	Definition	Proxies	Data Source
RR_{T}	Firm recovery rate	Weighted average recovery rate of all creditors	Moody's URD
RR_{M}	Bondholder recovery rate	Individual recovery rate of bondholder	Moody's URD
α	Proportional liquidation cost	Industry distress	WRDS Compustat
		Tangibility	
		Industry median Q ratio	
q	Bargaining frictions	Number of different issues	Moody's URD, DRD
		Number of different classes	
		Excl90_Issue	
		HHI of all debts	
		Duration of the procedure	
		Prepackaged Chapter 11	
$\frac{\partial p_e}{\partial \omega}$	Bank debt concentration	Number of different bank debts	Moody's URD
		Number of classes for bank debts	
		HHI of bank debts	
p_e	Bargaining power	Bank Percent	Moody's URD
μ	Firm's financial status	Profit Margin	WRDS Compustat
		Leverage	
		Equity value	
		Log(Assets)	

Table 4.6 Summary Statistics of Creditor Recoveries

Variable	Obs	Mean	Std. Dev.	Min	Max
Total Recovery	444	53.670	27.191	0.469	144.290
Bank Recovery	444	73.284	36.581	0	121.661
Bond Recovery	444	32.907	30.887	0	154.126

Table 4.7 Summary Statistics of Independent Variables

Variable	Obs	Mean	Std. Dev.	Min	Max
Bank Percent	444	38.891	31.604	0	100
Number of Bank Debt	444	2.027	1.498	0	9
Bank Debt Concentration	444	0.601	0.336	0	1
Number of Bank Class	444	1.061	0.729	0	5
Log(Assets)	424	6.331	1.364	-1.604	11.551
Profit Margin	421	0.030	0.392	-4.000	0.748
Leverage	421	0.468	0.469	0	3.207
Tangibility	421	0.384	0.245	0	0.943
Med. Ind. Q	444	1.390	0.362	0.867	2.848
Industry Distress	444	0.142	0.349	0	1
Med. Ind. Return	444	0.009	0.263	-0.623	0.752
Number of Issues	444	5.126	5.518	1	61
Number of Classes	444	2.660	1.222	1	8
Excl90_Issue	444	0.099	0.299	0	1
Debt Concentration	444	0.445	0.238	0.036	1
Duration	444	1.080	1.168	0.071	12.849

Table 4.8 OLS Estimates of Regression of Firm-level Ultimate Recovery Rate

For description of variables, refer to Table 4.5. Model 1 includes bank percent only. Models 2 and 3 regress the firm-level ultimate recovery rate on firm fundamentals and industry variables respectively. Model 4 examines the relation between total recovery and variables that capture the frictions in the bargaining game. Models 5 to Model 8 includes all the variables, with different measures of debt concentration levels respectively. All models are estimated using linear least squares. ***, ** and * indicates statistical significance at the 1%, 5% and 10% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Bank Percent	0.23***				0.22***	0.23***	0.22***	0.22***
	(5.76)				(5.24)	(5.57)	(5.40)	(5.42)
Log(Assets)		0.30			0.82	-0.22	0.92	0.90
		(0.29)			(0.74)	(-0.18)	(0.76)	(0.84)
Profit Margin		10.40***			7.25**	7.00**	7.28**	7.25**
		(3.11)			(2.27)	(2.20)	(2.28)	(2.27)
Leverage		-7.31**			-7.07**	-7.40***	-7.07***	-6.98**
		(-2.58)			(-2.54)	(-2.73)	(-2.59)	(-2.56)
Tangibility		2.31			11.86**	11.70**	11.94**	12.09**
		(0.42)			(2.23)	(2.20)	(2.24)	(2.27)
Med. Ind. Q			9.85***		9.09***	9.07***	9.15***	9.18***
			(2.77)		(2.60)	(2.60)	(2.61)	(2.63)
Distress1			-13.54***		-13.06***	-13.03***	-12.98***	-12.93***
			(-3.68)		(-3.53)	(-3.54)	(-3.52)	(-3.50)
Duration				0.79	-0.29	-0.39	-0.31	-0.32
				(0.64)	(-0.24)	(-0.32)	(-0.25)	(-0.26)
Prepackage				3.48	4.65	4.90*	4.58	4.56
				(1.15)	(1.57)	(1.66)	(1.53)	(1.54)
Number of Class				-1.20	-0.11			
				(-1.13)	(-0.10)			
Number of Issues						0.40		
						(1.47)		
Debt Concentration							1.41	
							(0.23)	
Excl90_Issue								2.25
								(0.54)
const	44.83***	53.91***	41.90***	54.78***	26.65***	30.73***	24.92**	25.29***
	(22.68)	(7.74)	(8.01)	(14.95)	(2.98)	(3.30)	(2.17)	(2.75)
N	444	417	444	444	417	417	417	417
R-sq	0.07	0.04	0.06	0.01	0.16	0.17	0.16	0.16

Table 4.9 Firm-level Ultimate Recovery Rate for Firms Financed by Mixed Types of Debts

For description of variables, refer to Table 4.5. The table examines the determinants of firm-level recovery rate for firms that have both debt loan and corporate bond debt outstanding. Model 1 includes bank percent only. Models 2 and 3 regress the firm-level ultimate recovery rate on firm fundamentals and industry variables respectively. Model 4 examines the relation between total recovery and variables that capture the frictions in the bargaining game. Models 5 to Model 8 includes all the variables, with different measures of debt concentration levels respectively. All models are estimated using linear least square. ***, ** and * indicates statistical significance at the 1%, 5% and 10% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Bank Percent	0.14***				0.14***	0.15***	0.14**	0.13**
	(2.63)				(2.60)	(2.79)	(2.51)	(2.39)
Log(Assets)		1.91*			2.28*	1.53	1.772	2.11*
		(1.76)			(1.91)	(1.13)	(1.35)	(1.87)
Profit Margin		11.13***			9.07**	8.93**	8.864**	9.07**
		(3.02)			(2.55)	(2.52)	(2.49)	(2.56)
Leverage		-5.42*			-5.89**	-6.23**	-6.134**	-6.18**
		(-1.85)			(-2.01)	(-2.20)	(-2.16)	(-2.18)
Tangibility		12.31**			18.61***	18.55***	18.549***	18.51***
		(2.17)			(3.34)	(3.33)	(3.33)	(3.32)
Med. Ind. Q			6.99*		7.25**	7.28**	7.160**	7.25**
			(1.90)		(2.00)	(2.02)	(1.98)	(2.01)
Distress1			-13.19***		-14.13***	-13.90***	-14.185***	-14.14***
			(-3.35)		(-3.60)	(-3.55)	(-3.62)	(-3.62)
Duration				1.82	0.09	0.07	0.17	0.13
				(1.45)	(0.07)	(0.05)	(0.14)	(0.10)
Prepackage				7.84**	7.71**	7.83**	8.03**	8.01**
				(2.47)	(2.46)	(2.50)	(2.54)	(2.54)
Number of Class				-0.14	-0.14			
				(-0.12)	(-0.11)			
Number of Issues						0.27		
						(0.90)		
Debt Concentration							-5.17	
							(-0.66)	
Excl90_Issue								-5.37
								(-0.89)
const	48.47***	38.82***	45.99***	49.68***	18.22*	20.89**	23.48*	19.65**
	(19.58)	(5.19)	(8.43)	(12.17)	(1.90)	(2.09)	(1.87)	(2.03)
N	352	329	352	352	329	329	329	329
R-sq	0.02	0.06	0.05	0.02	0.16	0.16	0.16	0.16

Table 4.10 Determinants of Firm-level Ultimate Recovery Rates

For description of variables, refer to Table 4.5. The table examines the determinants of firm-level recovery rate with year dummies included. Models 1 to 4 are regression results for all firms in the sample. We use number of class, number of issues outstanding, debt concentration, which is measured by HHI index, and excl90_isse, a dummy variable equal to 1 if there is a single dominant issue in the debt structure that accounts for more than 90% of total debt respectively to proxy debt concentration level. Models 5 to 8 are regression results for firms that have both debt loan and corporate bond debt outstanding. All models are estimated using linear least square. ***, ** and * indicates statistical significance at the 1%, 5% and 10% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Bank Percent	0.21***	0.22***	0.21***	0.21***	0.17***	0.17***	0.15***	0.14**
	(4.91)	(5.16)	(5.02)	(5.08)	(2.78)	(2.94)	(2.59)	(2.47)
Log(Assets)	0.51	-0.53	0.67	0.70	0.93	0.05	0.51	0.87
	(0.41)	(-0.38)	(0.49)	(0.59)	(0.70)	(0.04)	(0.35)	(0.69)
Profit Margin	6.81**	6.48**	6.87**	6.83**	7.21*	7.02*	7.04*	7.24**
	(2.06)	(1.97)	(2.08)	(2.07)	(1.97)	(1.92)	(1.92)	(1.98)
Leverage	-5.55*	-5.97**	-5.45*	-5.23*	-4.93	-5.20*	-5.01*	-4.96*
	(-1.89)	(-2.09)	(-1.89)	(-1.82)	(-1.61)	(-1.75)	(-1.68)	(-1.68)
Tangibility	11.91**	11.84**	11.99**	12.31**	20.10***	20.15***	20.04***	19.95***
	(2.16)	(2.15)	(2.17)	(2.23)	(3.44)	(3.45)	(3.43)	(3.41)
Med. Ind. Q	3.76	3.86	3.82	3.84	3.65	3.82	3.58	3.73
	(1.00)	(1.03)	(1.01)	(1.02)	(0.94)	(0.98)	(0.92)	(0.96)
Distress1	-6.06	-6.06	-5.97	-5.79	-8.42*	-8.26*	-8.60*	-8.71*
	(-1.32)	(-1.32)	(-1.30)	(-1.26)	(-1.73)	(-1.70)	(-1.77)	(-1.79)
Duration	1.41	1.36	1.38	1.36	1.72	1.70	1.82	1.76
	(1.06)	(1.03)	(1.03)	(1.02)	(1.23)	(1.22)	(1.29)	(1.26)
Prepackage	6.62**	6.74**	6.54**	6.47**	9.54***	9.56***	9.75***	9.81***
	(2.14)	(2.19)	(2.10)	(2.09)	(2.89)	(2.90)	(2.94)	(2.95)
Number of Class	-0.04				0.26			
	(-0.04)				(0.20)			
Number of Issues		0.38				0.34		
		(1.34)				(1.11)		
Debt Concentration			1.64				-5.20	
			(0.25)				(-0.64)	
Excl90_Issue				3.76				-4.96
				(0.88)				(-0.81)
const	22.52	26.85	20.48	20.81	17.38	21.22	23.59	19.00
	(1.14)	(1.35)	(0.96)	(1.05)	(0.91)	(1.10)	(1.11)	(0.99)
Year Dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	417	417	417	417	329	329	329	329
R-sq	0.24	0.25	0.24	0.25	0.26	0.26	0.26	0.26

Table 4.11 Recovery Rate Sensitivity – Effect of Bargaining Friction

This table examines the condition under which the effect of bank percentage on recovery rate is more pronounced. Models 1 and 2 use the interaction term of Bank Percent * Number of Class and Bank Percent * Number of Issues. Model 3 and 4 uses Bank Percent * Excl90 and Bank Percent * Debt Concentration respectively. Finally, in Model 5 we use Bank Percent * Duration. For description of variables refer to Table 4.5. All models are estimated using linear least squares. ***, ** and * indicates statistical significance at the 1%, 5% and 10%, respectively.

	(1)	(2)	(3)	(4)	(5)
Bank Percent	0.33***	0.39***	0.19***	0.04	0.13**
	(5.58)	(4.28)	(4.24)	(0.41)	(2.56)
Log(Assets)	0.32	0.89	0.92	0.60	1.31
	(0.26)	(0.81)	(0.86)	(0.49)	(1.24)
Profit Margin	7.26**	7.41**	7.40**	7.26**	7.22**
	(2.29)	(2.32)	(2.32)	(2.28)	(2.28)
Leverage	-6.79**	-6.82**	-7.03**	-6.89**	-6.65**
	(-2.51)	(-2.46)	(-2.58)	(-2.54)	(-2.46)
Tangibility	11.61**	12.23**	11.80**	11.60**	11.64**
	(2.20)	(2.31)	(2.21)	(2.19)	(2.20)
Med. Ind. Q	9.73***	9.53***	9.32***	9.30***	7.73**
	(2.80)	(2.74)	(2.67)	(2.67)	(2.20)
Duration	-0.19	-0.46	-0.37	-0.31	-5.39**
	(-0.16)	(-0.38)	(-0.30)	(-0.25)	(-2.30)
Prepackage	5.38*	5.05*	4.76	5.00*	2.75
	(1.83)	(1.71)	(1.61)	(1.68)	(0.91)
Distress1	-13.47***	-12.50***	-13.08***	-12.98***	-13.80***
	(-3.68)	(-3.39)	(-3.55)	(-3.53)	(-3.76)
Number of Issues	0.89***				
	(2.62)				
Bank Percent * Number of Issues	-0.03**				
	(-2.41)				
Number of Classes		2.60			
		(1.51)			
Bank Percent * Number of Class		-0.08**			
		(-2.11)			
Excl90_Issue			-2.19		
			(-0.40)		
Bank Percent * Excl90			0.12		
			(1.29)		
Debt Concentration				-11.48	
				(-1.38)	
Bank Percent * Debt Concentration				0.34**	
				(2.32)	
Bank Percent * Duration					0.08**
					(2.54)
const	24.30**	18.98**	26.18***	33.80***	30.89***
	(2.52)	(1.97)	(2.84)	(2.80)	(3.44)
N	417	417	417	417	417
R-sq	0.18	0.17	0.17	0.17	0.18

Table 4.12 Recovery Rate Sensitivity for Mixed Debt Issuer – Effect of Bargaining Friction

This table examines the condition under which the effect of bank percentage on recovery rate is more pronounced for firms that have both debt loan and corporate bond debt outstanding. Models 1 and 2 use the interaction term of Bank Percent * Number of Class and Bank Percent * Number of Issues. Models 3 and 4 uses Bank Percent * Excl90 and Bank Percent * Debt Concentration respectively. Finally, in model 5 we use Bank Percent * Duration. For description of variables refer to Table 4.5. All models are estimated using linear least square. ***, ** and * indicates statistical significance at the 1%, 5% and 10% level, respectively.

	(1)	(2)	(3)	(4)	(5)
Bank Percent	0.35***	0.25*	0.15***	0.01	0.08
	(3.84)	(1.66)	(2.64)	(0.06)	(1.22)
Log(Assets)	2.02	2.22*	2.05*	1.77	2.60**
	(1.48)	(1.86)	(1.83)	(1.35)	(2.29)
Profit Margin	9.12**	9.21**	9.05**	8.71**	8.94**
	(2.59)	(2.58)	(2.55)	(2.44)	(2.52)
Leverage	-5.46*	-5.83**	-5.81**	-6.12**	-5.72**
	(-1.93)	(-1.98)	(-2.04)	(-2.16)	(-2.03)
Tangibility	18.58***	18.84***	18.39***	18.67***	18.24***
	(3.37)	(3.37)	(3.31)	(3.35)	(3.28)
Med. Ind. Q	7.99**	7.33**	7.26**	7.13**	6.46*
	(2.23)	(2.02)	(2.01)	(1.97)	(1.77)
Duration	0.53	0.02	0.11	0.22	-3.98
	(0.42)	(0.02)	(0.08)	(0.17)	(-1.35)
Prepackage	8.59***	7.78**	7.63**	8.24***	6.13*
	(2.76)	(2.48)	(2.42)	(2.60)	(1.86)
Distress1	-13.81***	-13.94***	-14.24***	-13.76***	-14.45***
	(-3.56)	(-3.54)	(-3.65)	(-3.50)	(-3.70)
Number of Issues	1.23***				
	-2.65				
Bank Percent * Number of Issue	-0.04***				
	(-2.69)				
Number of Classes		1.24			
		(0.56)			
Bank Percent * Number of Class		-0.04			
		(-0.76)			
Excl90_Issue			-0.34		
			(-0.05)		
Bank Percent * Excl90			-0.40		
			(-1.47)		
Debt Concentration				-13.01	
				(-1.25)	
Bank Percent * Debt Concentration				0.30	
				(1.14)	
Bank Percent * Duration					0.06
					(1.53)
const	11.56	14.48	19.36**	27.24**	21.49**
	(1.10)	(1.35)	(2.01)	(2.10)	(2.21)
N	329	329	329	329	329
R-sq	0.18	0.16	0.17	0.16	0.16

Table 4.13 Recovery Rate Sensitivity with Year Dummies – Effect of Bargaining Friction

For description of variables, refer to Table 4.5. This table examines the condition under which the effect of bank percentage on recovery rate is more pronounced with year dummies included. Models 1 and 2 use the interaction term of Bank Percent * Number of Class and Bank Percent * Number of Issues. Models 3 and 4 uses Bank Percent * Excl90 and Bank Percent * Debt Concentration respectively. Finally, in model 5 we use Bank Percent * Duration. All models are estimated using linear least square. ***, ** and * indicates statistical significance at the 1%, 5% and 10% level, respectively.

	(1)	(2)	(3)	(4)	(5)
Bank Percent	0.32***	0.37***	0.19***	0.02	0.12**
	(5.29)	(4.05)	(3.89)	(0.22)	(2.21)
Log(Assets)	0.01	0.48	0.64	0.18	1.15
	(0.01)	(0.39)	(0.54)	(0.13)	(0.97)
Profit Margin	6.84**	7.01**	6.94**	6.85**	6.63**
	(2.09)	(2.13)	(2.11)	(2.09)	(2.03)
Leverage	-5.16*	-5.26*	-5.16*	-5.24*	-4.97*
	(-1.81)	(-1.80)	(-1.80)	(-1.83)	(-1.76)
Tangibility	11.80**	12.20**	12.06**	11.76**	11.92**
	(2.16)	(2.22)	(2.18)	(2.14)	(2.19)
Med. Ind. Q	4.44	4.13	4.01	3.91	2.38
	(1.19)	(1.10)	(1.06)	(1.04)	(0.63)
Duration	1.55	1.30	1.36	1.48	-4.57*
	(1.17)	(0.98)	(1.02)	(1.11)	(-1.84)
Prepackage	7.37**	7.09**	6.87**	7.18**	4.44
	(2.40)	(2.29)	(2.22)	(2.31)	(1.41)
Distress1	-6.40	-5.39	-5.95	-5.92	-6.45
	(-1.40)	(-1.17)	(-1.29)	(-1.29)	(-1.42)
Number of Issues	0.86**				
	(2.48)				
Bank Percent * Numer of Issue	-0.03**				
	(-2.37)				
Number of Classes		2.59			
		(1.46)			
Bank Percent * Numer of Class		-0.08**			
		(-1.99)			
Excl90_Issue			-1.06		
			(-0.19)		
Bank Percent * Excl90			0.14		
			(1.35)		
Debt Concentration				-11.84	
				(-1.37)	
Bank Percent * Debt Concentration				0.35**	
				(2.35)	
7Bank Percent * Duration					0.09***
					(2.85)
const	20.12	16.35	22.70	30.15	26.08
	(1.01)	(0.82)	(1.15)	(1.40)	(1.34)
Year Dummies	Yes	Yes	Yes	Yes	Yes
N	417	417	417	417	417
R-sq	0.26	0.25	0.25	0.26	0.26

Table 4.14 Recovery Rate Sensitivity – Effect of Debt Amount

For description of variables, refer to Table 4.5. This table examines the condition under which the effect of bank percentage on recovery rate is more pronounced regarding debt amount. We use the interaction term of Bank Percent * Total Debt. Model 1 includes all the variables except the measures of debt concentration levels. Models 2 to Model 5 include different measures of debt concentration levels respectively. All models are estimated using linear least squares. ***, ** and * indicates statistical significance at the 1%, 5% and 10% level, respectively.

	(1)	(2)	(3)	(4)	(5)
Bank Percent	0.24***	0.25***	0.24***	0.24***	0.24***
	(5.66)	(5.90)	(5.52)	(5.66)	(5.69)
Log(Assets)	1.63	0.56	1.59	1.82	1.77
	(1.42)	(0.43)	(1.34)	(1.38)	(1.51)
Profit Margin	7.41**	7.17**	7.38**	7.48**	7.45**
	(2.33)	(2.26)	(2.31)	(2.34)	(2.34)
Leverage	-6.72**	-6.98**	-6.80**	-6.63**	-6.54**
	(-2.48)	(-2.57)	(-2.44)	(-2.43)	(-2.39)
Tangibility	11.91**	11.73**	11.90**	12.02**	12.18**
	(2.25)	(2.22)	(2.24)	(2.26)	(2.29)
Med. Ind. Q	9.76***	9.83***	9.77***	9.84***	9.86***
	(2.79)	(2.81)	(2.79)	(2.80)	(2.81)
Duration	-0.47	-0.61	-0.47	-0.49	-0.51
	(-0.38)	(-0.50)	(-0.38)	(-0.40)	(-0.41)
Prepackage	4.80	5.09*	4.83	4.67	4.67
	(1.63)	(1.73)	(1.63)	(1.56)	(1.58)
Distress1	-13.12***	-13.13***	-13.09***	-13.06***	-13.00***
	(-3.57)	(-3.58)	(-3.55)	(-3.54)	(-3.53)
Bank Percent * Total Debt	-0.02*	-0.02*	-0.02*	-0.02*	-0.02*
	(-1.72)	(-1.95)	(-1.72)	(-1.73)	(-1.74)
Number of Issues		0.48*			
		(1.73)			
Number of Classes			0.15		
			(0.13)		
Debt Concentration				1.85	
				(0.30)	
Excl90_Issue					2.50
					(0.60)
const	20.28**	24.34**	20.09**	18.07	18.79*
	(2.12)	(2.47)	(2.07)	(1.49)	(1.90)
N	417	417	417	417	417
R-sq	0.17	0.17	0.17	0.17	0.17

Table 4.15 Recovery Rate Sensitivity – Effect of Leverage Ratio

For description of variables, refer to Table 4.5. This table examines the condition under which the effect of bank percentage on recovery rate is more pronounced regarding debt amount. We use the interaction term of Bank Percent * Total Debt. Model 1 includes all the variables except the measures of debt concentration levels. Models 2 to Model 5 include different measures of debt concentration levels respectively. All models are estimated using linear least squares. ***, **, and * indicates statistical significance at the 1%, 5% and 10% level, respectively.

	(1)	(2)	(3)	(4)	(5)
Bank Percent	0.29***	0.31***	0.29***	0.29***	0.30***
	(5.40)	(5.56)	(5.29)	(5.39)	(5.41)
Log(Assets)	1.10	0.08	1.10	1.14	1.20
	(1.04)	(0.06)	(0.99)	(0.93)	(1.12)
Profit Margin	7.50**	7.28**	7.50**	7.52**	7.53**
	(2.36)	(2.29)	(2.35)	(2.36)	(2.37)
Leverage	-0.68	-0.89	-0.68	-0.68	-0.58
	(-0.16)	(-0.21)	(-0.16)	(-0.16)	(-0.14)
Tangibility	11.22**	11.05**	11.22**	11.25**	11.44**
	(2.12)	(2.09)	(2.11)	(2.11)	(2.15)
Med. Ind. Q	8.35**	8.32**	8.35**	8.37**	8.43**
	(2.39)	(2.39)	(2.39)	(2.39)	(2.41)
Duration	-0.53	-0.64	-0.53	-0.54	-0.56
	(-0.43)	(-0.52)	(-0.43)	(-0.44)	(-0.46)
Prepackage	5.01*	5.25*	5.01*	4.98*	4.91*
	(1.70)	(1.78)	(1.69)	(1.67)	(1.66)
Distress1	-13.33***	-13.33***	-13.33***	-13.31***	-13.23***
	(-3.63)	(-3.64)	(-3.62)	(-3.62)	(-3.60)
Bank Percent * Leverage	-0.18**	-0.18**	-0.18**	-0.18**	-0.18**
	(-2.07)	(-2.09)	(-2.06)	(-2.05)	(-2.05)
Number of Issues		0.41			
		(1.50)			
Number of Classes			-0.01		
			(-0.01)		
Debt Concentration				0.44	
				(0.07)	
Excl90_Issue					2.00
					(0.48)
const	22.97**	27.19***	22.98**	22.46*	21.86**
	(2.55)	(2.88)	(2.53)	(1.95)	(2.35)
N	417	417	417	417	417
R-sq	0.17	0.18	0.17	0.17	0.17

Table 4.16 OLS Estimates of Regression of Bondholder Recovery Rate

For description of variables, refer to Table 4.5. This table examines the determinants of bondholders' recovery rate. Models 1 to 4 include measures of overall debt concentration level. Models 5 to 7 include measures in bank debt concentration level. All models are estimated using linear least squares. ***, **, and * indicates statistical significance at the 1%, 5% and 10% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Bank Percent	-0.17***	-0.22***	-0.22***	-0.22***	-0.16***	-0.25***	-0.05
	(-4.23)	(-5.49)	(-5.32)	(-5.25)	(-3.70)	(-6.33)	(-1.04)
Effective coupon rate	-1.27***	-1.26***	-1.31***	-1.32***	-1.38***	-1.08***	-1.39***
	(-3.85)	(-3.79)	(-3.93)	(-3.94)	(-4.15)	(-3.26)	(-4.22)
Log(Issue Size)	-2.05***	-3.41***	-3.47***	-3.45***	-3.22***	-3.13***	-2.79***
	(-3.47)	(-6.25)	(-6.33)	(-6.31)	(-5.85)	(-5.78)	(-5.09)
Log(Assets)	-1.22	3.23***	2.59***	2.34***	1.99***	2.72***	3.71***
	(-1.33)	(4.26)	(3.05)	(3.32)	(2.83)	(3.95)	(5.18)
Profit Margin	3.07	3.74	3.12	3.05	3.14	2.66	4.70**
	(1.32)	(1.58)	(1.32)	(1.29)	(1.34)	(1.14)	(2.02)
Leverage	-8.25***	-4.95**	-7.38***	-7.50***	-6.30***	-6.97***	-5.75***
	(-3.82)	(-2.11)	(-3.36)	(-3.43)	(-2.85)	(-3.23)	(-2.66)
Tangibility	14.93***	14.90***	16.95***	16.73***	17.16***	15.18***	13.61***
	(3.66)	(3.59)	(4.08)	(4.04)	(4.17)	(3.72)	(3.33)
Med. Ind. Q	3.07	1.08	1.41	1.43	0.60	2.30	2.66
	(1.10)	(0.39)	(0.50)	(0.51)	(0.21)	(0.83)	(0.96)
Distress1	-5.80*	-7.89**	-6.98**	-7.06**	-7.75**	-6.18*	-8.36***
	(-1.80)	(-2.43)	(-2.14)	(-2.17)	(-2.39)	(-1.92)	(-2.61)
Prepackage	12.73***	13.18***	13.28***	13.37***	13.00***	14.04***	14.10***
	(5.59)	(5.73)	(5.73)	(5.78)	(5.65)	(6.16)	(6.21)
Duration	1.80**	2.00**	2.18**	2.23**	2.15**	2.45***	2.05**
	(2.06)	(2.28)	(2.47)	(2.53)	(2.45)	(2.82)	(2.37)
Number of Issues	0.58***						
	(5.79)						
Number of Classes		-2.07***					
		(-2.92)					
Debt Concentration			3.24				
			(0.54)				
Excl90_Issue				0.65			
				(0.12)			
Number of Bank Debt Class					-3.97***		
					(-2.99)		
Bank Debt Concentration						13.93***	
						(5.74)	
Number of Bank Debt Issues							-3.94***
							(-6.50)
const	58.78*	45.20	44.08	47.02	49.04	28.69	29.22
	(1.66)	(1.27)	(1.21)	(1.31)	(1.37)	(0.81)	(0.83)
Seniority Dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Collateral Dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year Dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	1300	1300	1300	1300	1300	1300	1300
R-sq	0.46	0.44	0.44	0.44	0.44	0.46	0.46

Table 4.17 Test of Bank Percentage on Bondholder's Recovery Rate

For description of variables, refer to Table 4.5. This table examines the determinants of bondholders' recovery rate. Models 1 to 4 include interaction term of Bank Percent × Number of Bank Debt. Model 5 to 8 use interaction term of Bank Percent × Number of Bank Debt Class. The last four models include interaction term of Bank Percent × HHI of Bank Debt. In every four model set we use different measures of debt concentration: number of issues, number of classes, debt concentration, and excl90_issue. All models are estimated using linear least square. ***, **, and * indicates statistical significance at the 1%, 5% and 10% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Bank Percent	-0.114	-0.234***	-0.221***	-0.238***	-0.171*	-0.387***	-0.369***	-0.376***	-0.032	-0.073	-0.073	-0.041
	(-1.37)	(-2.81)	(-2.68)	(-2.87)	(-1.70)	(-3.90)	(-3.87)	(-3.91)	(-0.25)	(-0.57)	(-0.57)	(-0.32)
Effective coupon rate	-1.244***	-1.203***	-1.240***	-1.220***	-1.352***	-1.335***	-1.373***	-1.374***	-1.174***	-1.079***	-1.158***	-1.147***
	(-3.29)	(-3.11)	(-3.22)	(-3.16)	(-3.50)	(-3.38)	(-3.50)	(-3.49)	(-3.05)	(-2.76)	(-2.97)	(-2.93)
Log(Issue Size)	-1.418**	-2.863***	-2.765***	-2.844***	-2.130***	-3.468***	-3.434***	-3.447***	-1.686***	-3.134***	-2.979***	-3.089***
	(-2.23)	(-4.81)	(-4.63)	(-4.77)	(-3.32)	(-5.76)	(-5.69)	(-5.72)	(-2.62)	(-5.18)	(-4.89)	(-5.09)
Log(Assets)	-0.198	4.128***	3.326***	4.037***	-2.809***	1.245	0.629	0.891	-1.265	3.632***	2.091**	3.028***
	(-0.19)	(4.95)	(3.63)	(5.11)	(-2.72)	(1.33)	(0.66)	(1.13)	(-1.18)	(4.31)	(2.25)	(3.87)
Profit Margin	3.762	2.627	2.380	2.630	2.175	0.904	0.636	0.757	3.755	2.876	2.419	2.707
	(1.31)	(0.90)	(0.82)	(0.90)	(0.74)	(0.30)	(0.21)	(0.25)	(1.29)	(0.97)	(0.82)	(0.92)
Leverage	-7.620***	-6.362***	-6.824***	-6.537***	-8.085***	-6.151**	-6.774***	-6.696***	-8.859***	-6.168**	-8.168***	-7.786***
	(-3.41)	(-2.60)	(-3.01)	(-2.89)	(-3.48)	(-2.46)	(-2.88)	(-2.85)	(-3.93)	(-2.49)	(-3.56)	(-3.40)
Tangibility	20.005***	20.036***	19.377***	19.938***	25.791***	25.933***	26.267***	26.389***	23.779***	23.286***	23.451***	23.969***
	(4.49)	(4.40)	(4.26)	(4.39)	(5.81)	(5.62)	(5.81)	(5.85)	(5.35)	(5.12)	(5.18)	(5.29)
Med. Ind. Q	4.335	2.402	2.649	2.393	2.188	0.280	0.212	0.200	4.047	2.332	2.675	2.370
	(1.46)	(0.80)	(0.88)	(0.80)	(0.74)	(0.09)	(0.07)	(0.07)	(1.38)	(0.79)	(0.90)	(0.80)
Distress1	-5.935	-9.085**	-9.166**	-9.139**	-6.667*	-10.301***	-10.292***	-10.343***	-3.879	-7.200*	-6.772*	-6.837*
	(-1.60)	(-2.42)	(-2.45)	(-2.44)	(-1.75)	(-2.70)	(-2.70)	(-2.71)	(-1.02)	(-1.88)	(-1.77)	(-1.79)
Prepackage	12.987***	13.800***	14.064***	13.929***	11.980***	12.850***	12.959***	12.977***	13.208***	13.852***	14.493***	14.321***
	(5.32)	(5.56)	(5.67)	(5.60)	(4.78)	(5.06)	(5.10)	(5.10)	(5.32)	(5.49)	(5.75)	(5.66)
Duration	2.511**	2.381**	2.542**	2.418**	3.012***	3.081***	3.200***	3.171***	2.644***	2.384**	2.710***	2.579**
	(2.51)	(2.33)	(2.49)	(2.37)	(2.92)	(2.93)	(3.05)	(3.03)	(2.61)	(2.30)	(2.62)	(2.50)
Number of Issues	0.696***				0.695***				0.707***			

	(5.85)				(5.45)				(5.84)			
Number of Classes		-0.106				-0.543				-1.271		
		(-0.13)				(-0.57)				(-1.59)		
Debt Concentration			-11.296				-4.192				-14.737**	
			(-1.59)				(-0.59)				(-1.97)	
Excl90_Issue				-3.368				-2.972				-5.675
				(-0.49)				(-0.42)				(-0.81)
Number of Bank Debt	-6.583***	-7.545***	-7.675***	-7.628***								
	(-5.23)	(-5.79)	(-6.05)	(-6.00)								
Bank Percent * Number of Bank Debt	0.042	0.059**	0.056**	0.060**								
	(1.58)	(2.19)	(2.10)	(2.23)								
Number of Bank Debt Class					-6.736**	-11.532***	-11.971***	-11.984***				
					(-2.26)	(-3.91)	(-4.17)	(-4.17)				
Bank Percent * Number of Bank Class					0.030	0.141*	0.134*	0.137*				
					(0.41)	(1.96)	(1.88)	(1.92)				
HHI of Bank Debt									23.950***	28.123***	30.570***	30.710***
									(3.35)	(3.86)	(4.25)	(4.23)
Bank Percent * HHI of Bank Debt									-0.007	-0.085	-0.033	-0.106
									(-0.04)	(-0.49)	(-0.19)	(-0.61)
const	35.518	28.554	36.326	29.087	57.565	53.670	57.552	54.625	8.582	-4.432	1.324	-6.946
	(1.02)	(0.81)	(1.02)	(0.82)	(1.62)	(1.49)	(1.58)	(1.52)	(0.24)	(-0.12)	(0.04)	(-0.19)
Seniority Dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Collateral Dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year Dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	1036	1036	1036	1036	1036	1036	1036	1036	1036	1036	1036	1036
R-sq	0.534	0.518	0.519	0.518	0.511	0.497	0.497	0.497	0.520	0.504	0.505	0.503

CHAPTER 5

Debt Structure and Valuation in U.S. Bankruptcy Code

This chapter investigates the impact of the U.S. bankruptcy procedure on the valuation of corporate securities under a two-class debt structure. A structural model of credit risk is built when default and liquidation are represented by two different boundaries and the grace period for recovery is granted. In this context, corporate debt (including bank loan and corporate bond) and equity values can be viewed as Parisian type options. Their valuations are then obtained by using PDE formulation and finite difference approach. I find that, when both Chapter 11 and Chapter 7 are embraced in the model, the predicted credit spread can better reflect the observed one. The chapter also shows how the bankruptcy-related factors such as length of grace period, bargaining power in debt renegotiation affect the debt valuation.

5.1 Introduction

For almost half century, the modelling and pricing of defaultable bonds have attracted numerous scholars' efforts, the majority of whose work adopts the methodology of Black, Scholes, and Merton (BSM hereafter). The popularity of the BSM framework is partly due to timing - the 1970s was a turbulent decade with tremendous amounts of economic uncertainty and financial market volatility. Now, after the internet bubble and financial crisis, a new phenomenon is the increasing volume of bankruptcies filed as well as a regulatory revolution regarding bankruptcy, which inspires me to develop the bond pricing models with the provisions of bankruptcy law embodied.

Earlier discussions on bankruptcy code and bond pricing can be found in François and Morellec (2004) and Broadie, Chernov and Sundaresan (2007), both of which examine the impact of U.S. bankruptcy procedure on the valuation of corporate securities and capital structure decisions. Conclusions from these two papers serve as intellectual foundation for this paper. However, both papers assume only perpetual bonds are issued by the firm. This simplification produces insights but limits application in empirical tests, not least by excluding investigation of the term structure of credit spreads, so I choose finite-maturity debt. In François and Morellec (2004), liquidation occurs when the time that the firm's asset value remains continuously under default

barrier exceeds a grace period. This implies that the firm might be able to survive even when the asset level is extremely low.

I follow Broadie, Chernov and Sundaresan (2007) by introducing a second, lower barrier which, if reached, triggers immediate liquidation. I also extend the model to a two types of debt. The model in this chapter is theoretically more reasonable since multiple debt types and finite debt maturity is allowed. The same as other structural models of credit risks, corporate debt is viewed as option with firm's asset as underlying. The distinction of this model is the type of option I shall use become more complicated if bankruptcy procedure is taken into account. More precisely in mathematical finance terms, they are quasi-Parisian-corridor options.[67] As mentioned earlier, this chapter applies option theory to model the debt instruments to consider the effect of bankruptcy law. Parisian corridor option is a particularly new concept and no paper has explained the finite difference methods to get the valuations of Parisian-corridor options. This is the first attempt to apply option pricing techniques to get the valuation.

[67] This is because the two boundaries (default boundary and liquidation boundary) form a corridor, the time spent within the corridor is recorded as an indicator of state of the firm. But, different from ordinary corridor option, in this particular bond pricing problem, only the price entering into the corridor from above is considered in this story. The scenario of price going up and entering into the range is excluded.

The credit spread of corporate debt generated from this approach is consistent with the market credit spread quantitatively. I also perform a number of comparative static analyses to investigate the effects of different parameter on credit spread. Particularly, the value of debt is found to be affected by factors associated with the bankruptcy process, such as the length of the grace period.[68]

As mentioned in the literature review chapter, research on the effect of debt structure in structural modelling is limited. Early discussions include Black and Cox (1976), Moraux (2002) and Dumitrescu (2007)[69]. Hackbarth, Hennessy and Leland (2007) examine optimal capital structure in a model in which firms can issue bank debt, public bonds, and equity. Valta (2016) models debt renegotiation with mixture of different debts and investigate how debt structure affect the stock prices. Carey and Gordy (2016) incorporate debt structure into structural model by allowing the bank to set the endogenous asset value threshold below which corporations declare bankruptcy. However, all the three papers are not conducted under bankruptcy procedure thus the unique features of the procedures are omitted in those work. The Parisian features are not displayed in these models.

[68] The grace period is the time period during which the firm is allowed to take measures and hopefully, recover from the bankruptcy. The firm is allowed to be survived from liquidation during grace period.

[69] While Dumitrescu (2007) applies game theory to a discrete two-period time structure, the model in this chapter is under a contingent claim framework.

In spite of the mathematical difficulty incurred, allowing a complex debt structure is necessary in structural models. Most empirical tests have to restrict their samples to firms with one class of debt, which generate an unnecessary sample bias as well as reduced sample size, the root of such comprise is exactly the simple capital structure in existing models. Theoretically, by introducing a layer of bank loan debt, the corporate bond price is expected to decrease and, hopefully, this will help to explain the credit spread puzzle in bond pricing. Additionally, Park (2000) offers an optimal contracting explanation and shows that optimal contract is a two-class debt contract. This further reinforce the necessity to extend the debt structure assumption in structural models. Finally, on the operational level, this work provides us guidance on the distribution of control rights in bankruptcy. The agency problem is severe in defaulted firms. The ultimate goal of US bankruptcy law is to align the stakeholders to achieve the total firm value maximization, as mentioned in Hart (2006). The work of Broadie, Chernov and Sundaresan (2007) partially answers the question. For a single class of debt, Broadie, Chernov and Sundaresan (2007) find that, in order to restore the first-best outcome, creditors should be given the right to take control of the reorganization. However, in the more common cases involving both bank and bond debt, a natural question is: which class of creditors should be given the control right in order to obtain the first-best outcome?[70]

[70] This problem will be discussed separately in Chapter 6.

The chapter is arranged as follows. Section 5.2 presents the model and discusses the valuation issues. Distinctions between my model and the related work are also explained here. Section 5.3 explains the pricing methodology and numerical scheme. Section 5.4 presents the pricing results in a series of comparative statics. Discussions on the determinants of valuations are fully displayed in this section. Section 5.5 investigates the optimal leverage ratio when bankruptcy procedure is considered and Section 5.6 concludes.

5.2 The Model

In this section I present my valuation model for corporate debt. I firstly put forward the assumptions of the model in section 5.2.1, in which the fate of the defaulted entity is clearly depicted. Then I show how the equity, debt and firm values should be computed within this model in section 5.2.2.

The critical divergence among structural models is in the definition of default. The original Merton (1974) model only allows for default - and immediate liquidation - at the point of maturity of a single zero-coupon debt and was quickly followed by the 'first-passage' model of Black and Cox (1976) which allows for default / liquidation at any time if the asset value falls below a certain default boundary (in options terminology: a barrier option). Since, in practice, liquidation is not instantly triggered in this way, in a further set of first-passage

models default is triggered as the asset value falls below a default boundary but liquidation occurs only if it remains below that level for a specified time (in financial terms: the 'grace period'; in mathematical modelling terms: the 'window'). The time below the barrier is recorded, as if by a clock, either continuously, whereby the clock resets to zero if the asset value re-crosses the boundary moving upwards (a 'Parisian' option: François and Morellec 2004, Broadie, Chernov and Sundaresan 2007, Galai, Raviv and Wiener 2007) or cumulatively, whereby the clock stops but does not resets to zero if the asset value re-crosses the boundary moving upwards and starts again if the assets value falls back below the boundary (a 'ParAsian' option: Yu 2004). When the clock first starts, the firm enters bankruptcy and is then liquidated after the grace period unless, meantime, its asset value recovers to levels above the default boundary. These models emulate Chapter 11 in so far as a typical the grace period granted by a court is included in the calculations. These assume perpetual debt which renders the valuation time-independent, considerably simplifies the mathematics and enables analytical solution. An exception is Yu (2004), who models senior and junior debt with finite maturity and stochastic interest rate, finding spreads for senior and junior debt switching dynamically according to maturities and weights. While her focus is on stochastic interest rate and default history (ParAsian feature), I am more interested in the impact of legislation.

Broadie, Chernov and Sundaresan (2007) distinguish Chapter 11 default from Chapter 7 liquidation by adding a second barrier to the Parisian setup. My thesis

generalizes this, modelling the default process using two boundaries but including both bank debt and bond debt with finite maturity. If asset value falls below the lower boundary, the company is liquidated but if it falls only below the higher boundary then liquidation only occurs if the asset value remains below that boundary for the grace period and, if it rises above the higher boundary, the company has emerged from bankruptcy.

5.2.1 Assumptions

(1) The market is arbitrage-free and complete. The instantaneous riskless interest rate is a constant r.

(2) The unlevered asset value of the firm follows the process:

$$dV_t = (r - \delta)V_t dt + \sigma V_t dW_t \tag{5.1}$$

where δ and σ represent the firm's payout rate and volatility. There is no information asymmetry about the characteristics of the value of the firm's assets δ and σ.

(3) The firm raises cash to finance its projects by (issuing one bond and engaging in a bank loan) with principal P_b and P_c, continuous coupon rate b and c, respectively, and the same finite maturity date T.[71] As a result, the

[71] This assumption rules out the discussions on maturity structure. Geske (1977) and Yu (2004) investigate the debt with different maturities which complement this work.

principals P_b and P_c together determine the firm's debt structure as well as capital structure.[72] The use of debt brings a tax shield benefit to the firm with the amount of $\tau(bP_b + cP_c)$ where τ is the tax rate.

(4) Management acts in the best interests of equity holders. If the firm is financially distressed, the management files for protection under Chapter 11 of the U.S. bankruptcy code. Upon bankruptcy, the firm is granted an exclusivity period (grace period) d during which the firm is permitted to restructure its debt through renegotiation with the debtholders. This means that the company is allowed to stay in Chapter 11 for no more than the duration of d. The life of the company is terminated in form of liquidation if it spends too long under the Chapter 11. However, if the firm's asset values depreciate severely before the end of grace period, the firm will be liquidated immediately as well. Accordingly, there are two bankruptcy boundaries - Chapter 11 boundary and Chapter 7 boundary - to describe three possible fates of the firm. 1) The firm's asset value falls below the Chapter 7 boundary before the maturity of grace period. 2) The firm recovers from the bankrupt state as its value of assets touches the Chapter 11 boundary from below before the end of grace period. 3) The firm is liquidated at the end of the grace period as it shows no sign of recovery.

[72] The coupon rate and loan rate are selected optimally in the presence of taxes and bankruptcy costs, from which the optimal capital and debt structure can be derived.

(5) While in Chapter 11, the firm is exposed to the continuously accruing proportional distress cost φ. This cost can represent legal fees, lost business, and the loss of valuable employees. This cost ensures that the firm would not want to drag out the bankruptcy forever. It also implies the bankruptcy results in some economic distress as well. Upon liquidation, the firm incurs costs that represent a proportion α $(0 < \alpha < 1)$ of the value of remaining assets V_L. After deducting the liquidation costs, the remaining assets are distributed with strictly obeyed APR.

(6) When the value of an unlevered firm is under the bankruptcy boundary, the firms is said to be in financial distress during which the debt is serviced strategically and the tax benefits are lost. Debt service in the bankrupt state is determined in a trilateral bargaining game. This assumption is consistent with the goal of the bankruptcy procedure (see Hart, 2006).

5.2.2 Determining Equity, Debt and Firm Values

I model the bankruptcy process via two boundary levels for the firm's asset value. The grace period, d, allowed for negotiation in Chapter 11 triggers liquidation if the firm's asset value both falls below and remains below the higher boundary for that period. Alternatively, liquidation is triggered immediately if the asset value falls so low that it reaches the second boundary while in Chapter 11. The value of a claim is the discounted expectation of the

future cash flows under the martingale measure. Denoting the liquidation time of the bankrupt firm as θ^d, [73] we can determine the total equity value as

$$\begin{aligned} E = \quad & \mathbf{E}^{\mathbb{Q}}\left\{\int_0^{\theta^d \wedge T} e^{-ru}\{\delta V_u - (1-\tau)(bP_b + cP_c)\mathbf{1}_{\{V_u > V_B\}} \right. \\ & \left. - [\varphi V_u + s(V_u)]\mathbf{1}_{\{V_L < V_u < V_B\}}\}du\right\} \\ & + \mathbf{E}^{\mathbb{Q}}\left\{e^{-rT}(V_T - P_b - P_c)\mathbf{1}_{\{\theta^d > T\}}\right\} \end{aligned} \tag{5.2}$$

where $\mathbf{1}_{\{x\}}$ is the indicator function of the event x, which takes the value of 1 if x is true and zero otherwise. The first term of the above expression means that equity holders receive the payment of $\delta V_u - (1-\tau)(bP_b + cP_c)$ in liquid state. After filing for Chapter 11, distress costs are incurred and firm starts to service the debt strategically. The last term means that if the firm is healthy at maturity, then the firm pays out the principal.

The values of bank debt (denoted with S, as senior) and bond debt (denoted with J, as junior) are:

$$\begin{aligned} D^S = \mathbf{E}^{\mathbb{Q}}\left\{\int_0^{\theta^d \wedge T} e^{-ru}\left[bP_b 1_{\{V_u > V_B\}} + s^S(V_u)1_{\{V_L < V_t < V_B\}}\right]du\right\} \\ + \mathbf{E}^{\mathbb{Q}}\left\{e^{-rT}P_b\mathbf{1}_{\{\theta^d > T\}}\right\} \\ + e^{-r\theta^d}\mathbf{E}^{\mathbb{Q}}\left[\min\{(1-\alpha)V_{\theta^d}, P_b\}\,\mathbf{1}_{\{\theta^d \le T\}}\right] \end{aligned} \tag{5.3}$$

and

[73] In section 5.3 I give will details of the mathematical expression of liquidation time.

$$D^J = \mathbf{E}^{\mathbb{Q}}\left\{\int_0^{\theta^d \wedge T} e^{-ru}\left[cP_c 1_{\{V_u > V_B\}} + s^J(V_u)1_{\{V_L < V_t < V_B\}}\right]du\right\}$$
$$+ \mathbf{E}^{\mathbb{Q}}\left\{e^{-rT}P_c \mathbf{1}_{\{\theta^d > T\}}\right\}$$
$$+ e^{-r\theta^d}\mathbf{E}^{\mathbb{Q}}\left\{\left[(1-\alpha)V_{\theta^d} - P_b\right]^+ \mathbf{1}_{\{\theta^d \leq T\}}\right\}. \quad (5.4)$$

Each includes: interest / coupon payments received by the creditors before and in Chapter 11; creditors recovering the firm's remaining assets, after subtraction of liquidation costs, if the firm is unable to recover from bankruptcy; payment of principal if the firm is healthy at maturity.

The total value of the firm is the summation of debt and equity value as

$$v = \mathbf{E}^{\mathbb{Q}}\left\{\int_0^{\theta^d \wedge T} e^{-ru}\left[\delta V_u + \tau(bP_b + cP_c)\mathbf{1}_{\{V_u > V_B\}} - \varphi V_u \mathbf{1}_{\{V_L < V_u < V_B\}}\right]du\right\}$$
$$+ \mathbf{E}^{\mathbb{Q}}\left\{e^{-r\theta^d}(1-\alpha)V_{\theta^d}\mathbf{1}_{\{\theta^d \leq T\}}\right\} + \mathbf{E}^{\mathbb{Q}}\left\{e^{-rT}V_T \mathbf{1}_{\{\theta^d > T\}}\right\}. \quad (5.5)$$

The first term on the right-hand side of (5.5) accounts for the intermediate cash flows generated by the firm until liquidation. This term reflects the fact that (i) the firm generates cash flow δV_u, (ii) tax benefits are akin to coupons $\tau(bP_b + cP_c)$ that are paid as long as the firm is healthy; and (iii) renegotiation costs are incurred whenever the firm files Chapter 11. The second and third terms on the right-hand side of (5.5) account for liquidation value and terminal payoff at debt maturity, respectively.

This model generalizes the work in Broadie, Chernov and Sundaresan (2007). Therefore, default time is defined as the first instant at which the firm value

process either remains a certain time below the default threshold or hits a lower barrier. This corresponds, for instance, to a real situation in which a firm is allowed temporarily to be short of funds, but enters default immediately when the financial distress becomes severe and a critical level, defined as a threshold set below the classical default boundary, is reached. I model the claims as a quasi Parisian corridor option. There are two boundaries in the model. The option is knocked out when the asset value drops down to the higher boundary and stays in the region in between the two barriers for enough time[74], or, the knockout can be triggered immediately if the lower boundary is touched[75].

5.3 Numerical Implementation

Reviewing the relevant literature, two approaches to obtain the Parisian option valuations are available. The first one - Laplace transform – was introduced by Chesney, Jeanblanc-Picqué and Yor (1997). This approach can yield the analytical solution of Parisian options, but involves complicated mathematical derivations. The second approach - PDE formulation – is more widely applied in practice. It is first developed in the work of Haber, Schnbucher and Wilmott (1999) by using the Black-Scholes PDE with boundary conditions adapted to the Parisian contract. The modified PDE is then solved by numerical methods.

[74] This is similar to the knock out feature of Parisian option.

[75] This is the characteristics of barrier option.

The advantage of this method is that the pricing framework is flexible, i.e. it is possible to deal with discrete dividends and early exercise.

I use partial differential equations to price quasi Parisian corridor option for two reasons. First, the Laplace transform involves a lot of mathematical derivation which is not of the interest in this research. Second, although the first approach can yield analytical solution, visualization of the results from the analytical solution still involves inversion of the results, thus incurring cumbersome numerical work. In this section I introduce the methodology used to get the debt valuation. Since the technique is complex, I give detailed derivations here. In what follows I introduce the Parisian options and clarify the similarity between the debt claims and Parisian option in this context in section 5.3.1. The PDE formulation is discussed in Section 5.3.2 and the detailed algorithm is explained in Section 5.3.3.

5.3.1 Introduction on Parisian Options

Parisian options are introduced to the literature in this area by Chesney, Jeanblanc-Picqué and Yor (1997). They are path-dependent options whose payoff depends not only on the final value of the underlying asset, but also on the trajectory of the underlying above or below a predetermined barrier. For example, the owner of a Parisian down-and-out call option loses the option

when the underlying asset price reaches the level L and remains constantly below this level for a time interval longer than d.

Double-barrier Parisian options are a new kind of Parisian option. Dassios and Wu (2011c) consider two examples of such options: the double-barrier Parisian max-out option and the double-barrier Parisian min-out option. In the first type, the owner of option loses the option if the underlying asset price process S has both an excursion above the upper barrier for longer than a continuous period d_1 and below the lower barrier for longer than d_2 before the maturity of the option. While in the second example, the owner loses the option if either one of these two events happens before the maturity. Moreover, similar to basic Parisian options, double-barrier Parisian options can be classified into two types: continuous options and cumulative options. We adopt the continuous option as the continuous double-barrier Parisian option reflects what happens in Chapter 11 more accurately. Another reason is its relative simplicity in algorithm.

A double-barrier Parisian option touches the areas outside the corridor formed by the two boundaries while the corridor option looks at the region within the two barriers. However, as defined in Dassios and Wu (2011a, b), denote τ_1 as the first time that the length of the excursion in the corridor reaches the given level d, given that this excursion starts from the upper barrier l_1; and denote τ_2 as the first time that the length of the excursion in the corridor reaches d, given

that this excursion starts from the lower barrier l_2. Then the first time that the length of the excursion inside the corridor reaches given level d should be the smaller of τ_1 and τ_2. That is: $\tau = \tau_1 \wedge \tau_2$. This simply means, for in a corridor option, it does not matter whether the asset price enters into the corridor from above or from below. Therefore, the equity value in Chapter 11 is similar to the Parisian corridor option except that we only allow the knock-out feature to be triggered when the direction of the asset price movement is downwards. This is the reason why I call it quasi-Parisian-corridor option.

5.3.2 PDE Formulation

The State Space and the Barrier Time

The crucial point to note about Parisian options is that they are strongly path dependent – the payoff does not only rely on the value of the underlying at expiry but also on the path taken to get there. Define

$$\mathrm{g_B} \coloneqq \sup\{s \le t | V_s = V_B\} \tag{5.6}$$

$$\mathrm{g}_L \coloneqq \sup\{s \le t | V_s = V_L\} \tag{5.7}$$

$$\mathrm{d_B} \coloneqq \inf\{s \le t | V_s = V_B\} \tag{5.8}$$

$$d_L \coloneqq \inf\{s \le t | V_s = V_L\} \tag{5.9}$$

Therefore, it can be expressed that

$$\tau_{\mathrm{t}}^{\mathrm{d}} \coloneqq \inf\{\mathrm{t} > 0 | \mathbf{1}_{\{V_L < V_t < V_B\}} \mathbf{1}_{\{g_B > g_L\}} (\mathrm{t} - \mathrm{g_B}) \ge \mathrm{d}\} \tag{5.10}$$

which is the first time that the length of the excursion in the corridor reaches given level d, given that the excursion starts from the upper barrier V_B. This is the barrier time in the work of Haber, Schnbucher and Wilmott (1999). Denote

$$\theta^d \coloneqq \tau_t^d \wedge d_L \tag{5.11}$$

as the smaller of τ_t^d and d_L. This is the liquidation time of the distressed firm.

Two boundaries V_B and V_L here form a corridor, the barrier time dictates the length of time that the asset price has been within the corridor in the current excursion. We can imagine this state variable as a clock that starts ticking as soon as the underlying price cross the barrier level V_B, and is immediately reset when the underlying asset returns above V_B.

$$d\mathcal{T}(t) = \begin{cases} 0 & \text{if } V_t > V_B \\ -\mathcal{T}(t-) & \text{if } V_t = V_B \\ dt & \text{if } V_L < V_t < V_B \\ d - \mathcal{T}(t-) & \text{if } V_t = V_L \end{cases} \tag{5.12}$$

Compared with the dynamics of barrier time introduced in Haber, Schnbucher and Wilmott (1999), barrier time here features the immediate jump to d when the liquidation boundary is touched from above. With the expression of barrier time in (5.12), the knock-out feature of the option is activated as long as the length of the current excursion exceeds a given level ($\mathcal{T} > d$). The scenario of immediate liquidation once the lower boundary is reached is implicitly implied in the setup.

The Partial Differential Equations of Debt, Equity and the Firm

Suppose firm value function is $v = v(V, t, \mathcal{T})$. Construct the portfolio Π by buying one unit of firm value, and short selling Δ units of the assets of the firm

$$\Pi = v - \Delta V \tag{5.13}$$

where Δ is the hedging ratio, i.e., the shares of underlying (value of the firm's assets) in our portfolio. From Ito's lemma that the dynamics for the value of the firm are represented by

$$\begin{aligned} dv &= \frac{\partial v}{\partial t}dt + \frac{\partial v}{\partial \mathcal{T}}d\mathcal{T} + \frac{\sigma^2 V^2}{2}\frac{\partial^2 v}{\partial V^2}dt + \frac{\partial v}{\partial V}dV \\ &= \left(\frac{\partial v}{\partial t} + \frac{\partial v}{\partial \mathcal{T}}\frac{d\mathcal{T}}{dt} + \frac{\sigma^2 V^2}{2}\frac{\partial^2 v}{\partial V^2}\right)dt + \frac{\partial v}{\partial V}dV \end{aligned} \tag{5.14}$$

In order to completely eliminate the randomness, the optimal hedging ratio is selected as $\Delta = \partial v/\partial V$. And, the return of the risk-free portfolio should equal the risk-free return. The dynamics of the portfolio can be also written as

$$d\Pi = r\Pi dt. \tag{5.15}$$

Without cash payment during the lifetime of the option, the instantaneous return of the portfolio can be expressed as simply as $d\Pi = dv - \Delta dV$. While in this case, the underlying asset (the value of the firm's assets) has a cash payout of $\delta V_t dt$. Since we receive $\delta V_t dt$ for every asset held and hold $-\Delta$ of the underlying, the portfolio changes by an amount $-\Delta\delta V_t dt$. In addition, from equation (5.5) we know that the cash payment to the "option" (firm value v) is

$$[\delta V + \tau(bP_b + cP_c)]\mathbf{1}_{\{V>V_B\}} + [\delta V - \varphi V]\mathbf{1}_{\{V_L<V<V_B\}}.$$

Therefore, the earlier dΠ must be adjusted with the intermediate cash flows and the dynamic of the portfolio should be

$$d\Pi = dv - \Delta dV + [\delta V + \tau(bP_b + cP_c)]1_{\{V>V_B\}}dt + [\delta V - \varphi V]1_{\{V_L<V<V_B\}}dt - \delta\Delta V dt. \quad (5.16)$$

The final general partial differential equation can be achieved by equalizing the dynamics of portfolio

$$\frac{\partial v}{\partial t} + \frac{\partial v}{\partial \mathcal{T}}\frac{d\mathcal{T}}{dt} + \frac{\sigma^2 V^2}{2}\frac{\partial^2 v}{\partial V^2} + (\mathrm{r} - \delta)\mathrm{V}\frac{\partial v}{\partial V} - rv + [\delta V - \varphi V]1_{\{V_L<V<V_B\}} + [\delta V + \tau(bP_b + cP_c)]1_{\{V>V_B\}} = 0. \quad (5.17)$$

Notably, compared with the standard Parisian option partial differential equation, my version contains the source term $[\delta V + \tau(bP_b + cP_c)]1_{\{V>V_B\}} + [\delta V - \varphi V]1_{\{V_L<V<V_B\}}$. As introduced in Wilmott (2010), the source term represents a cash flow. The term will appear in the PDE if the option contract involves earning, or paying money throughout its life, which is exactly what happens in our case. In addition, the term is a function of underlying V reflecting the fact that the cash flow is V dependent. Next, plug in the dynamics of $\mathcal{T}(\mathrm{t})$ indicated in equation (5.12), the PDE system (5.17) can be further reduced as

$$\begin{cases} \frac{\partial v}{\partial t} + \frac{\sigma^2 V^2}{2}\frac{\partial^2 v}{\partial V^2} + (r - \delta)V\frac{\partial v}{\partial V} - rv + \delta V + \tau(bP_b + cP_c) = 0 & V > V_B \quad (5.18) \\ \frac{\partial v}{\partial t} + \frac{\partial v}{\partial \mathcal{T}} + \frac{\sigma^2 V^2}{2}\frac{\partial^2 v}{\partial V^2} + (r - \delta)V\frac{\partial v}{\partial V} - rv + \delta V - \varphi V = 0 & V_L < V < V_B \quad (5.19) \end{cases}$$

Boundary Conditions

The pricing domain of the problem is

$$\mathrm{D} = \{(V, t, \mathcal{T}) | 0 \leq V < \infty, 0 \leq t \leq T, 0 \leq \mathcal{T} \leq d\} \tag{5.20}$$

should be further divided into three domains I, II and III where

$$D_I = \{(V, t, \mathcal{T}) | V_B \leq V_t < \infty, 0 \leq t \leq T, \mathcal{T} = 0\}$$

$$D_{II} = \{(V, t, \mathcal{T}) | V_L < V_t < V_B, \tau \leq t \leq \tau + T - d, 0 \leq \mathcal{T} \leq d\}$$

$$D_{III} = \{(V, t, \mathcal{T}) | 0 \leq V_t < V_L, 0 \leq t \leq T, \mathcal{T} = d\} \tag{5.21}$$

Now, let v_1, v_1 and v_1 denote the option prices in the region I, II and III.

Region I: Clearly, in region I, the variable $\mathcal{T}$ remains unchanged, as the underlying is above the barrier.

At maturity (terminal condition 1): The terminal condition in this region is given by the payoff function of a European call option, i.e.

$$v_1(V_T, T) = V_T. \tag{5.22}$$

Besides the terminal condition, a set of boundary conditions along the V direction is also needed to solve for v_1.

Asset value is infinite positive: the fact that the bankruptcy cost becomes zero when the underlying price approach infinity gives

$$\lim_{V_t \to \infty} v_1(V_t, t) = V_t + \frac{\tau(bP_b + cP_c)}{r}\left(1 - e^{-r(T-t)}\right) \tag{5.23}$$

which means that debt reduces to a riskless bond when the value of an unlevered firm is sufficiently high.

Asset value is V_B: whereas the continuity of option price across the barrier V_B demands

$$\lim_{V_t \to V_B} v_1(V_t, t) = \lim_{V \to V_B} v_2(V_t, t, 0). \tag{5.24}$$

Region II: On the other hand, in region II, the barrier time $\mathcal{T}$ starts to accumulate.

At maturity (terminal condition 1): the terminal condition in this region is given by the payoff function of a European call option, i.e.

$$v_2(V_T, T, \mathcal{T}) = \begin{cases} V_T & \text{for all } \mathcal{T} < d \\ (1-\alpha)V_T & \text{for } \mathcal{T} = d \end{cases} \tag{5.25}$$

At the end of window (terminal condition 2): appropriate boundary condition along the $\mathcal{T}$ direction is needed to close the PDE system. From the definition of the "trigger clock", it is clear that when the variable $\mathcal{T}$ reaches the trigger value d, the option becomes worthless, i.e.

$$\lim_{\mathcal{T}\to d} v_2(V_t, t, \mathcal{T}) = (1-\alpha)V_t. \tag{5.26}$$

Asset value is V_L: the fact that the firm is liquidated as soon as the liquidation boundary is touched yields another boundary condition as follows

$$\lim_{V_t\to V_L} v_2(V_t, t, \mathcal{T}) = (1-\alpha)V_L. \tag{5.27}$$

Asset value is V_B: moreover, the boundary condition at $V_t = V_B$ is specified by the so called "reset condition", i.e.

$$\lim_{V\to V_B} v_2(V, t, \mathcal{T}) = v_2(V, t, 0) = \lim_{V\to V_B} v_1(V, t). \tag{5.28}$$

which indicates that $\mathcal{T}$ is reset to zero every time the underlying V_t rises back to the barrier V_B from below.

Region III, the firm is liquidated and

$$v_3(V_t, t, d) = (1-\alpha)V_t. \tag{5.29}$$

Define operator $\mathbb{L} = \frac{1}{2}\sigma^2 V^2 \frac{\partial^2}{\partial V^2} + (r-\delta)V\frac{\partial}{\partial V} - rI$ with I being the identity operator. Now summarizing what has been explained, the PDE system for pricing the value of the firm under Parisian option pricing framework can be written as:

$$\mathcal{A}_1 \begin{cases} \dfrac{\partial v_1}{\partial t} + \mathbb{L}v_1 + \delta V_t + \tau(bP_b + cP_c) = 0 \\ v_1(V_T, T) = V_T \\ \lim\limits_{V_t \to \infty} v_1(V_t, t) = V_t + \dfrac{\tau(bP_b + cP_c)}{r}\left(1 - e^{-r(T-t)}\right) \\ \lim\limits_{V_t \to V_B} v_1(V_t, t) = \lim\limits_{V_t \to V_B} v_2(V_t, t, 0) \end{cases} \tag{5.30}$$

$$\mathcal{A}_2 \begin{cases} \dfrac{\partial v_2}{\partial t} + \dfrac{\partial v_2}{\partial \mathcal{T}} + \mathbb{L}v_2 + \delta V - \varphi V = 0 \\ v_2(V_T, T, \mathcal{T}) = \begin{cases} V_T & \text{for all } \mathcal{T} < d \\ (1-\alpha)V_T & \text{for } \mathcal{T} = d \end{cases} \\ v_2(V_t, t, d) = (1-\alpha)V_t \\ \lim\limits_{V_t \to V_L} v_2(V_t, t, \mathcal{T}) = (1-\alpha)V_L \\ \lim\limits_{V_t \to V_B} v_2(V_t, t, \mathcal{T}) = \lim\limits_{V \to V_B} v_2(V_t, t, 0) \end{cases} \tag{5.31}$$

with the connectivity condition:

$$\lim_{V_t \to V_B} \frac{\partial v_1}{\partial \mathrm{V}}(V_t, t) = \lim_{V_t \to V_B} \frac{\partial v_2}{\partial \mathrm{V}}(V_t, t, 0). \tag{5.32}$$

In order to get the valuation of debt and equity, it is necessary to know how the debts are serviced upon bankruptcy. I follow Chapter 4 to adopt the result from a trilateral bargaining game. In Chapter 4 I put large efforts on the derivation of the sharing rule in debt renegotiation. Note that, after renegotiation, the bank and bondholder will get a reduced coupon payment of $s_S(V_t)$ and $s_J(V_t)$ respectively, which represents a reduced-form solution form Chapter 4.

Once the cash outflow to the creditors during Chapter 11 is known, the equity value can be calculated by the same reasoning given above. The PDE system for equity value, thus, follows

$$\mathcal{A}_1 \begin{cases} \frac{\partial E_1}{\partial t} + \mathbb{L}E_1 + \delta V - (1-\tau)cP = 0 \\ E_1(V_T, T) = V_T - P_b - P_c \\ \lim\limits_{V_t \to \infty} E_1(V_t, t) = V_t + \frac{(\tau-1)(bP_b + cP_c)\left(1-e^{-r(T-t)}\right)}{r} - e^{-r(T-t)}P \\ \lim\limits_{V_t \to V_B} E_1(V_t, t) = \lim\limits_{V \to V_B} E_2(V_t, t, 0) \end{cases} \tag{5.33}$$

$$\mathcal{A}_2 \begin{cases} \frac{\partial E_2}{\partial t} + \frac{\partial E_2}{\partial \mathcal{T}} + \mathbb{L}E_2 + (\delta - \varphi)V - s(V) = 0 \\ E_2(V_T, T, \mathcal{T}) = \begin{cases} [V_T - P]^+ & \text{for all } \mathcal{T} < d \\ 0 & \text{for } \mathcal{T} = d \end{cases} \\ E_2(V_t, t, d) = 0 \\ \lim\limits_{V_t \to V_L} E_2(V_t, t, \mathcal{T}) = 0 \\ \lim\limits_{V_t \to V_B} E_2(V_t, t, \mathcal{T}) = \lim\limits_{V_t \to V_B} E_2(V_t, t, 0) \end{cases} \tag{5.34}$$

Regarding the debt value, the bank debt and bond debt are subject to the same PDEs, with the differences are their terminal conditions. For bank debt,

$$\mathcal{A}_1 \begin{cases} \frac{\partial D_1^S}{\partial t} + \mathbb{L}D_1^S + bP_b = 0 \\ D_1^S(V_T, T) = P_b \\ \lim\limits_{V_t \to \infty} D_1^S(V_t, t) = e^{-r(T-t)}P_b + \frac{bP_b}{r}\left(1 - e^{-r(T-t)}\right) \\ \lim\limits_{V_t \to V_B} D_1^S(V_t, t) = \lim\limits_{V \to V_B} D_2^S(V_t, t, 0) \end{cases} \tag{5.35}$$

$$\mathcal{A}_2 \begin{cases} \frac{\partial D_2^S}{\partial t} + \frac{\partial D_2^S}{\partial \mathcal{T}} + \mathbb{L}D_2^S + s^S(V) = 0 \\ D_2^S(V_T, T, \mathcal{T}) = \min[V_T, P_b] \quad \text{for all } \mathcal{T} < d \\ D_2^S(V_t, t, d) = \min[(1-\alpha)V_t, P_b] \\ \lim\limits_{V_t \to V_L} D_2^S(V, t, \mathcal{T}) = \min[(1-\alpha)V_L, P_b] \\ \lim\limits_{V \to V_B} D_2^S(V, t, \mathcal{T}) = \lim\limits_{V_t \to V_B} D_1^S(V_t, t) \end{cases} \tag{5.36}$$

Bond Debt

$$\mathcal{A}_1 \begin{cases} \dfrac{\partial D_1^J}{\partial t} + \mathbb{L}D_1^J + cP_c = 0 \\ D_1^J(V_T, T) = P_c \\ \lim\limits_{V_t \to \infty} D_1^J(V_t, t) = e^{-r(T-t)}P_c + \dfrac{cP_c}{r}\left(1 - e^{-r(T-t)}\right) \\ \lim\limits_{V_t \to V_B} D_1^J(V_t, t) = \lim\limits_{V \to V_B} D_2^J(V_t, t, 0) \end{cases} \tag{5.37}$$

$$\mathcal{A}_2 \begin{cases} \dfrac{\partial D_2^J}{\partial t} + \dfrac{\partial D_2^J}{\partial \mathcal{T}} + \mathbb{L}D_2^J + s^J(V) = 0 \\ D_2^J(V_T, T, \mathcal{T}) = \min[(V_T - P_b)^+, P_c] \quad \text{for all } \mathcal{T} < d \\ D_2^J(V_t, t, d) = [(1-\alpha)V_t - P_b]^+ \\ \lim\limits_{V_t \to V_L} D_2^J(V, t, \mathcal{T}) = [(1-\alpha)V_L - P_b]^+ \\ \lim\limits_{V \to V_B} D_2^J(V, t, \mathcal{T}) = \lim\limits_{V_t \to V_B} D_1^J(V_t, t) \end{cases} \tag{5.38}$$

The conditions of the above PDE system can be understood as follows. The terminal condition at maturity of the debt means that the payoff of the debt is the minimum of firm asset's value at expiry and the principal if the firm is not liquidated after the grace period. While whenever the Parisian feature is triggered, creditors seize the firm and realize a payoff of $(1-\alpha)V_t$ as the cost of liquidation is borne by creditors under this circumstance. The boundary condition when asset value is infinite shows that creditors are guaranteed their claims without suffering loss. Condition $\lim\limits_{V_t \to V_B} D_1(V_t, t) = \lim\limits_{V \to V_B} D_2(V_t, t, 0)$ is the pathwise continuity condition, which means the option price does not jump at a barrier. The other boundary condition at barrier $\lim\limits_{V \to V_B} D_2(V, t, \mathcal{T}) = \lim\limits_{V_t \to V_B} D_1(V_t, t)$ is the reset condition as the variable $\mathcal{T}$ is reset to zero when the underlying returns to the barrier V_B. Table 5.1 summarizes the PDE systems and boundary conditions for the securities I am evaluating.

5.3.3 Numerical Algorithm

The first thing to do is to get rid of the awkward V and V^2 terms multiplying $\frac{\partial \mathrm{D^J}}{\partial \mathrm{V}}$ and $\frac{\partial^2 D^J}{\partial \mathrm{V}^2}$. Set the transformation

$$\begin{aligned} x &= \ln(V_t) \\ u(x,t,\mathcal{T}) &= D^J(V_t,t,\mathcal{T}) \end{aligned} \tag{5.39}$$

and PDE for corporate bond in (5.35) and (5.36) becomes

$$\frac{\partial u}{\partial t} + \frac{\sigma^2}{2}\frac{\partial^2 u}{\partial x^2} + \left(r - \delta - \frac{\sigma^2}{2}\right)\frac{\partial u}{\partial x} - ru + cP_c = 0 \tag{5.40}$$

as $x > 0$, and

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial \mathcal{T}} + \frac{\sigma^2}{2}\frac{\partial^2 u}{\partial x^2} + \left(r - \delta - \frac{\sigma^2}{2}\right)\frac{\partial u}{\partial x} - ru + s^J(e^x) = 0 \tag{5.41}$$

as $x < 0$. Partition the domain $D = \{(V,t,\mathcal{T}) | x \in \mathbb{R}, 0 \le t \le T, 0 \le \mathcal{T} \le d\}$ as follows:

$$\begin{aligned} x_i &= i\Delta x, \quad (i = 0,1,2,\dots I) \\ t_j &= j\Delta t, \quad (j = 0,1,2,\dots,J) \\ \mathcal{T}_k &= k\Delta\mathcal{T}, \quad (k = 0,1,2,\dots,K) \end{aligned} \tag{5.42}$$

where $\Delta t = \Delta\mathcal{T} = \frac{T}{J} = \frac{d}{K}$. The transformed asset price x_t, time t and barrier time $\mathcal{T}$ are discretised as Δx, Δt and $\Delta\mathcal{T}$, respectively, where Δt is set to be equal to $\Delta\mathcal{T}$ to simplify the calculation. Denote with $u_{i,j}^k$ the numerical approximation to the value $u(x,t,\mathcal{T})$ at asset price $x_t = i\Delta x$, time $t = j\Delta t$, and barrier time $\mathcal{T} = k\Delta\mathcal{T}$.

Following the approach of Haber, Schönbucher and Wilmott (1999), I use explicit finite difference scheme to get the numerical solution to equations in (5.40) and (5.41). In the region I, the asset price is above the default boundary ($i > 0$), the clock $\mathcal{T}$ remains zero ($k = 0$). Denote $u_{i,j}^0 = u(x_i, t_j, 0) = u(x_i, t_j)$. Then, the PDE in this region is in accordance to the time-stepping scheme

$$\begin{aligned} &\frac{u_{i,j}^0 - u_{i,j-1}^0}{\Delta t} + \frac{\sigma^2}{2}\left[\frac{u_{i+1,j}^0 - 2u_{i,j}^0 + u_{i-1,j}^0}{\Delta x^2}\right] \\ &+ \left(r - \delta - \frac{\sigma^2}{2}\right)\left[\frac{u_{i+1,j}^0 - u_{i-1,j}^0}{2\Delta x}\right] - r u_{i,j}^0 + cP_c = 0. \end{aligned} \tag{5.43}$$

Rearranging the expression in (5.43) yields

$$\begin{aligned} u_{i,j-1}^0 = \; & \frac{\Delta t}{2\Delta x}\left[\frac{\sigma^2}{\Delta x} - \left(r - \delta - \frac{\sigma^2}{2}\right)\right] u_{i-1,j}^0 + \left[1 - \frac{\sigma^2 \Delta t}{\Delta x^2} - r\Delta t\right] u_{i,j}^0 \\ & + \frac{\Delta t}{2\Delta x}\left[\left(r - \delta - \frac{\sigma^2}{2}\right) + \frac{\sigma^2}{\Delta x}\right] u_{i+1,j}^0 + cP_c \Delta t. \end{aligned} \tag{5.44}$$

Region II is the area in which the clock is running and the $k > 0$. I use the same reasoning as in Jiang (2005), first I use the finite difference scheme with characteristic line, draw a characteristic line in $(t_j, \mathcal{T}_k)$ $\mathcal{T} = t - t_j + \mathcal{T}_k$, then write the PDE (5.41) as

$$\begin{aligned} &\frac{d}{dt} u(x, t, t - t_j + \mathcal{T}_k) \\ &+ \left[\frac{\sigma^2}{2}\frac{\partial^2 u}{\partial x^2} + \left(r - \delta - \frac{\sigma^2}{2}\right)\frac{\partial u}{\partial x} - ru + s^J(e^x)\right]_{\mathcal{T} = t - t_j + \mathcal{T}_k} = 0 \end{aligned} \tag{5.45}$$

Discretize (5.45) yields the scheme

$$\frac{u_{i,j}^{k}-u_{i,j-1}^{k-1}}{\Delta t}+\frac{\sigma^2}{2}\left[\frac{u_{i+1,j}^{k}-2u_{i,j}^{k}+u_{i-1,j}^{k}}{\Delta x^2}\right]$$
$$+\left(r-\delta-\frac{\sigma^2}{2}\right)\left[\frac{u_{i+1,j}^{k}-u_{i-1,j}^{k}}{2\Delta x}\right]-ru_{i,j}^{k}+s^{J}(e^{x_i})=0. \tag{5.46}$$

which can be further simplified as

$$u_{i,j-1}^{k-1}=\frac{\Delta t}{2\Delta x}\left[\frac{\sigma^2}{\Delta x}-r+\delta+\frac{\sigma^2}{2}\right]u_{i-1,j}^{k}+\left[1-\frac{\sigma^2\Delta t}{\Delta x^2}-r\Delta t\right]u_{i,j}^{k}$$
$$+\frac{\Delta t}{2\Delta x}\left[\frac{\sigma^2}{\Delta x}+\left(r-\delta-\frac{\sigma^2}{2}\right)\right]u_{i+1,j}^{k}+s^{J}(e^{x_i})\Delta t. \tag{5.47}$$

Denote the index for the boundaries as $\bar{i}=\frac{1}{\Delta x}\ln(V_B)$ and $\bar{l}=\frac{1}{\Delta x}\ln(V_L)$. The boundary conditions and terminal conditions can be expressed in the scheme as:

$$u_{i,J}^{0}=P_c \quad \text{for all } i>\bar{i} \tag{5.48}$$

$$u_{I,j}^{0}=e^{-r(T-t)}P_c+\frac{cP_c}{r}\left(1-e^{-r(T-t)}\right) \tag{5.49}$$

$$u_{\bar{i},j}^{k}=u_{\bar{i},j}^{0} \tag{5.50}$$

$$u_{i,J}^{k}=\min[(e^{x_i}-P_b)^{+},P_c] \quad \text{for all } k<K \text{ and } i<\bar{i} \tag{5.51}$$

$$u_{i,j}^{K}=[(1-\alpha)e^{x_i}-P_b]^{+} \text{ for all } i<\bar{i} \tag{5.52}$$

$$u_{\bar{l},j}^{k}=\left[(1-\alpha)e^{\bar{l}\Delta x}-P_b\right]^{+} \tag{5.53}$$

$$u_{i,j}^{k}=[(1-\alpha)e^{x_i}-P_b]^{+} \text{ for all } i\le\bar{l} \tag{5.54}$$

Table 5.2 summarizes the explicit scheme for securities I am evaluating. We can calculate the values following the sequence listed as follows (Figure 5.1):

1) Calculate the values of the points $u(x_i, t_J, \mathcal{T}_k) = u_{i,J}^k$ using the terminal condition (5.48) and (5.51).
2) Get the values of points using boundary condition (5.49): $u(x_I, t_j, \mathcal{T}_0) = u_{I,j}^0 = e^{-r(T-t)}P_c + \frac{(bP_b+cP_c)}{r}\left(1 - e^{-r(T-t)}\right)$.
3) Applying the formula (5.44) to get the valuation of the interior points $u(x_i, t_j, \mathcal{T}_0) = u_{i,j}^0$ when $k = 0$.
4) Find the index $\bar{l}$ of x such that asset value is V_L, therefore, $x(\bar{l}) = \ln(V_L)$. Based on this, calculate the values of points $u(x_I, t_j, \mathcal{T}_k) = u_{I,j}^k = (1-\alpha)e^{x_i}$ for all $i \leq \bar{l}$ using boundary condition (5.53).
5) Get the values at the barriers $u(x_0, t_j, \mathcal{T}_k) = u_{0,j}^k$ from the values obtained in step 2 by the continuity condition (5.50).
6) Calculate the points at the terminal condition of barrier time ($k = K$) $u(x_i, t_j, \mathcal{T}_K) = u_{i,j}^K = (1-\alpha)e^{x_i}$ with the terminal condition in (5.52).
7) Find the interior points of $u(x_i, t_{j-1}, \mathcal{T}_{k-1}) = u_{i,j}^{k-1}$ based on values of $u_{i-1,j}^k$, $u_{i,j}^k$ and $u_{i+1,j}^k$ from equation (5.47), starting from $k = K - 1$ to $k = 1$.

5.4 Numerical Results

In this section I display the valuation results from the model. I present the results for a base case, together with static analysis of credit spread. Finally, I discuss

the relevant factors that affect the credit spread, including asset volatility, leverage ratio, duration of grace period, etc.

5.4.1 The Base Case: Parameter Value Choices

For simplicity, I assume $s^S(V) = \eta^S(\delta - \varphi)V_t$ and $s^J(V) = \eta^J(\delta - \varphi)V_t$ to proceed the calculation where $\boldsymbol{\eta} = [\eta^S, \eta^J, \eta^E]$ is a reduced-form result of the renegotiation process. The Matlab code for the debt pricing is provided separately[76]. The Matlab code for equity pricing is similar to the debt except some modification to the boundary conditions and calculations of interior points. In the numerical experiment that follow, the parameters of the model are set to the following values[77]: $\sigma = 25\%$, $r = 5\%$, $\delta = 5\%$, $b = 8\%$, $c = 10\%$, $T = 10$, $V_0 = 200$, $P_b = P_c = 50$, $\alpha = 0.5$, $\varphi = 0.02$, $\boldsymbol{\eta} = [1/3, 1/3, 1/3]$, $\tau = 0.35$, $d = 1$. The predetermined boundaries are $V_B = 70$ and $V_L = 40$.

Figure 5.2 and Figure 5.3 plot the bank debt value and market debt value at the date of issuance within the explicit method. The debt values are express as

[76] The only paper that investigates Parisian option pricing with finite difference method is Haber, Schonbucher and Wilmott (1999). However, they did not disclose the codes they used for the option pricing. In the Matlab codes provided, I first try to reproduce their results to validate the correctness of our codes in Appendix C.1. Additionally, I extend the algorithm to American Parisian option with the codes and results in Appendix C.2.

[77] To make the results comparable with the results in other papers, the values for the parameters in the numerical examples hereafter follow the work of Galai, Raviv and Wiener (2007), François and Morellec (2004), Leland and Toft (1996) and Huang and Huang (2012).

functions of value of the firm's assets and barrier time, i.e., the 'clock' mentioned earlier.

5.4.2 The Static Analysis of Credit Spread

Table 5.3 reports all the yield spreads for bank and bond debt from my model with different input values, including the base case above. At the first step, it can be observed that the credit spread from the model is comparable with the average market spread in Huang and Huang (2012). Considering the leverage ratio we used is 50% in the base case, the most similar group is Ba-rated corporate bond and Aa-rated for the loan. Since credit risk premium is only one component of the yield spread between corporate bonds and Treasury bonds with the same maturity[78], the results of my model are reasonable to be lower than the market yield spread. According to Huang and Huang (2012), the historical average yield spread for ten-year bond is approximately 60, 100, 150, 200, 350 and 450 for Aaa, Aa, A, Baa, Ba and B-rated bond. To better understand the meaning of the model's credit spread, I compare our base case to results derived from previous structural models with same parameter values.[79] The Leland (1994) model yields a credit spread of 92 bps. In another

[78] Other factors such as liquidity, call and conversion features also contribute to the yield spread as found in Huang and Huang (2012).

[79] Since the two papers I compare are based on single debt structure, I add the bank and bond debt together in application of their models. Another difference is that the two papers assume a consol bond while my model has a finite-maturity debt.

related study, François and Morellec (2004) report the computed credit spreads ranging from 65 bps to 166 bps. Obviously, credit spreads from the two papers are lower than results in my thesis.

Next I turn to analyze the determinants of the levels of credit spreads. In all scenarios, bondholder requires a higher credit spread. This is straightforward as bondholders require a higher return considering the fact of being junior in liquidation. When looking at the impact of leverage ratio on credit spreads, an intuitive finding from Table 5.3 is that credit spread increases with leverage ratio, regardless of the type of the debt. More importantly, the effect of leverage ratio seems to be stronger for bond compared with bank loan. This point will be further illustrated in the section 5.4.5.

Consider next the impact of the exclusivity period on credit spreads. Table 5.3 indicates that credit spreads increase with the length of the exclusivity period. The effect of duration allowed to stay in Chapter 11 (size of d) seems to be controversial and depends on other parameters. Intuitively, debt forgiveness leads to welfare reduction of lenders. However, my model indicates that debt forgiveness may also be in the interest of the lenders and the firm as a whole because costly liquidation could be avoided. Therefore, the ultimate sign depends on the aggregated effect of the two contrary mechanisms. A detailed explanation for this is presented in the footnote of the table as well as in section 5.4.4. Finally, data in Table 5.3 also reveals that for both bank and market debt,

credit spreads are positively correlated with the liquidation costs, which is now a well-documented phenomenon of corporate bonds, as shown in Huang and Huang (2012).[80] This effect seems to be stronger for bank debt, which can be attributed to the reason that liquidation costs are more likely to be borne by the bank.

5.4.3 The Impact of Asset Volatility

Figure 5.4 shows the value of the debts as a function of the volatility of the firm's assets. The value of debt goes down with increase of σ as expected. For both bank loan and corporate bonds, they will worth more if the underlying assets are less volatile.

Figure 5.5 displays the credit spread of bank and market debt as functions of asset volatility with different leverage ratios and time to maturity. The top panels indicate the credit spread with three year to maturity while the bottom ones have ten years to expire. From Figure 5.5, it can be seen that creditors require a higher rate of return with higher asset volatility and higher leverage ratio. This is straightforward as high volatility and debt ratio are associated with greater credit risks. With regarding debt maturity, for bank debt, as long as the

[80] Basically, credit spread is the multiple of default probability and loss in default. The liquidation cost is an important component of loss in default as the liquidation cost are typically borne by the creditors.

leverage is not extremely high, longer the maturity date, higher the credit spread (upward sloped term structure). However, it is not always the case for corporate bond. When leverage ratio is moderately high (50% in my case), the short term credit spread is higher than the long-term credit spread. Another observation is that the sensitivity of impact of leverage ratio on credit spread is more prominent for short-term debt.

Galai, Raviv and Wiener (2007), by displaying the value of equity as a function of the volatility of the firm's assets, find that the Merton's model implies the strongest incentive for asset substitution to the shareholders. In this chapter, I also investigate the asset substitution issue. The two-class debt structure allows a deeper analysis beyond their paper and studies the shareholders' incentive to asset substitution (1) in different debt maturity and; (2) under different debt structure. Figure 5.6 shows that, stronger incentive for asset substitution when 1) debt outstanding has longer maturity date; 2) leverage ratio is higher; 3) bank percentage is smaller.

5.4.4 The Credit Spread and Length of Grace Period

Figure 5.7 gives more insights on the impact of asset volatility. It shows how the values of debts changes with the length of grace period under different asset volatility and bargaining power allocations. The top panels are low volatility scenario ($\sigma = 20\%$) and the bottom one is the high volatility scenario ($\sigma =$

40%). First, consistent with the findings above, the debt values are negatively associated with volatility. The debt values in the bottom panels are lower than the values in the top one. Another more important finding here is that the impacts of the length of the exclusivity period on debt values are to some extent ambiguous. This is because in the framework I present, the length of d has two major effects on debt values with opposite directions. First, larger d implies that the probability of liquidation is reduced. Therefore, debt values increase as the liquidation costs, which are at the expense of creditors (ex post), are less likely to be incurred. Second, larger d might cause more damage to the creditors as during this period the debt service is significantly reduced. Simulations results indicate that which effect dominates is dependent on the debt type and related parameters. Therefore, we can observe bond values increase with duration of window period regardless of the asset volatility. Finally, when look at the impact of bargaining power (or, the split of earnings in observation period), three entries of vector $\boldsymbol{\eta}$ correspond to bargaining powers of bank, bondholders and shareholders respectively. I consider three scenarios in the numerical example $\boldsymbol{\eta} = [0.6, 0.1, 0.3]$, $\boldsymbol{\eta} = [0.3, 0.6, 0.1]$, $\boldsymbol{\eta} = [0.1, 0.3, 0.6]$ in which each of the three parties dominates the renegotiation process. As expected, the values of the debts increase with the holders' bargaining power. Bank debt value is highest in scenario 1 and scenario 2 represents the highest bond value.

5.4.5 The Credit Spread and Leverage Ratio

The impact of leverage ratio[81] can be found in Figure 5.8. Without any doubt, all creditors require a higher credit spread with higher leverage ratio, holding other things unchanged. Moreover, parameter α is positively related with credit spread, as the debt holders will require more compensation ex ante if it is costly to liquidate the assets. Figure 5.8 also shows that this effect is more evident for bank debt. This can be explained by the Absolute Priority Rule. Since bondholders will only be repaid once bank debt is fully recovered, the liquidation costs are more likely to be at the expense of bank creditors, it is not surprising that parameter α has only marginal effect on bond credit spread. This finding is more evident when bank percentage is high. Figure 5.8 also provides comparisons of credit spread for different debt structure. Higher bank percentage implies higher credit spread for not only bank debt itself but also the bond debt. Once again, the impact of debt structure is more prominent for bank debt, especially when leverage ratio is high.

Figure 5.9 depicts the term structure of credit spread for both types of debts. The term structure is upward sloping in most of the cases, except for the bond debt when leverage ratio is 80%. The downward sloped term structure under this circumstance might because of the following reasoning. When the leverage ratio is as high as this level, the short-term risk of default is higher than long-

[81] The leverage ratio I use here is the face value of all the debts as a percentage of asset value.

term default risk and bondholders are highly likely to undergo a loss for the bonds that will mature in near future. Leland (1996) also plots the term structure of credit spread with different leverage ratios. The shape from their paper is not perfectly the same as in this model but the most important idea here is that: when leverage ratio is extremely high, the term structure of credit spread can be convex.

5.4.6 The Credit Spread and Debt Structure

The innovation of this model is the two-layer debt structure, which makes discussions on the debt structure and its impact on credit spread possible. In this section I look at how credit spread changes with different debt structure. To start with, I investigate the credit spread of debts under different capital structure (leverage ratio) and different debt structure. The result is displayed in Figure 5.10. I calculate three different debt maturities: one year, five years and ten years. Following findings can be obtained from the figure. (1) Credit spread increase with leverage ratio, which has been covered in previous section. (2) For low leverage ratio firm (which corresponds to high-rated issuer), the bankruptcy risk is marginal such that the credit spread is almost zero. (3) When leverage ratio is low, the impact of bank percentage on bond price is small while at high leverage ratio, the change of debt composite significantly affect the prices of bond.

Figure 5.11 further shows the term structure of credit spread with different debt structure. I use the leverage ratio of 50%, which is typical in other papers. The figure manifests that a complex debt structure matters. The junior debt performs the function of a cushion for senior debt. When a senior debt takes up a smaller percentage of total debt, cushion effect is increased. Additionally, Figure 5.12 demonstrates how the asset volatility and debt structure interact and impact on the bond valuation. When the majority of the total debt is secured bank debt, the humped shape of term structure is more likely to happen for both debts. The shape of bank debt is because of the stunted cushion effect of bond debt, while the performance of bond debt is due to the increased risk for bondholders as the seniority effect.

5.5 Firm Value and Leverage Ratio

This section discusses the optimal leverage ratio that maximize the value of the firm v in the scenario of exogenously-determined default and liquidation boundary.[82] Notably, both François and Morellec (2004) and Broadie, Chernov and Sundaresan (2007) discuss the optimal capital structure. Since in their models the debt is infinite, the debt face value can be determined by coupon payment every year divided by the risk-free interested rate. Therefore, their

[82] François and Morellec (2004) explain why the optimal leverage is determined ex ante to maximize firm value instead of equity value in section III(B).

optimal capital structure ultimately is essentially to compute the optimal coupon rate that maximizes the firm value ex ante. However, since the debt in this PhD study is finite, coupon rate alone does not tell the whole story. I follow the methods in Leland and Toft (1996) as it also addresses finite maturity debt. In what follows, at the initial asset value ($V = 200$), I find the coupon payment for both bank and bond debt such that the newly-issued debt sells at par value. I then examine how the par value of bank and bond debt will affect the value of firm and the debt structure that maximize the firm value for alternative choices of debt maturity.

Panel (a) of Figure 5.13 shows the relationship between firm value and leverage ratio for firms issuing debt with maturities 1 year, 5 year 10 years and 30 years. The finding in Leland and Toft (1996) that "leverage ratio which maximizes firm value is larger for debt with longer maturity" is not observed in this model when bankruptcy procedure in considered. The optimal leverage ratio is roughly 60% for median to long term debt, which is higher than that of Leland and Toft (1996). This is because when bankruptcy renegotiation is allowed, the cost associated with bankruptcy is mitigated and higher debt ratio is preferred. As illustrated in Panel (b), the optimal leverage ratio increases with the length of grace period. This is consistent with the idea in Panel (a) that the possibility of formal renegotiation after default allowing for renegotiation reduces the present value of liquidation cost, thus increases optimal leverage ratio. While in Leland

and Toft (1996), default results in an immediate liquidation, which implies a higher liquidation cost.

5.6 Conclusion

In much of previous continuous-time debt pricing literature, it has been assumed that debt issued by the firm is single class and has infinite maturity date. In practice this is not the case. In this chapter I develop a continuous time asset pricing model of debt to price the two-layer debts with finite maturity date. Moreover, I separate the notions of default and liquidation, by defining liquidation to result when a firm spends enough time in default or when firm's asset value deteriorates severely. These attempts capture the features of Chapter 11 and Chapter 7 in the U.S. bankruptcy code.

In the model setup, debt holders do not liquidate immediately upon default. As liquidation can be triggered for the two reasons above, the debt is tantamount to a quasi-Parisian corridor option. I use finite difference approach to obtain numerical solutions for the bank debt and bond debt. Compared with other models, the bond credit spreads from this model are closer to market credit spreads. With the assumption of finite debt, this model renders discussions on term structure of credit spread possible. It is shown in the model that debt value is affected by bankruptcy-related factors including length of grace period, bargaining power in debt renegotiation, asset volatility, etc. In this chapter, I

assume the boundaries and duration of grace period to be fixed and exogenous. In Chapter 6 I will relax the assumption and discuss on the optimal boundaries and optimal decision making in Chapter 11.

Figure 5.1 Figure Illustration of Algorithm

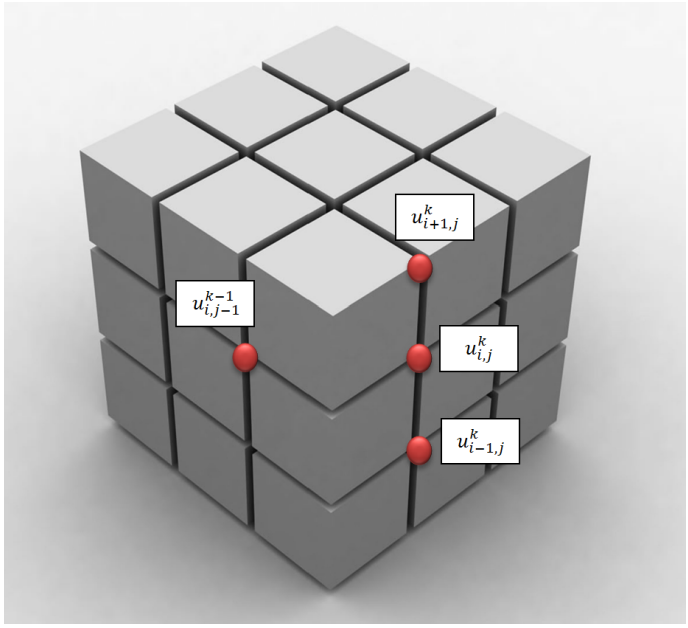


Figure 5.2 The Bank Debt Value at Date of Issuance within Explicit Method

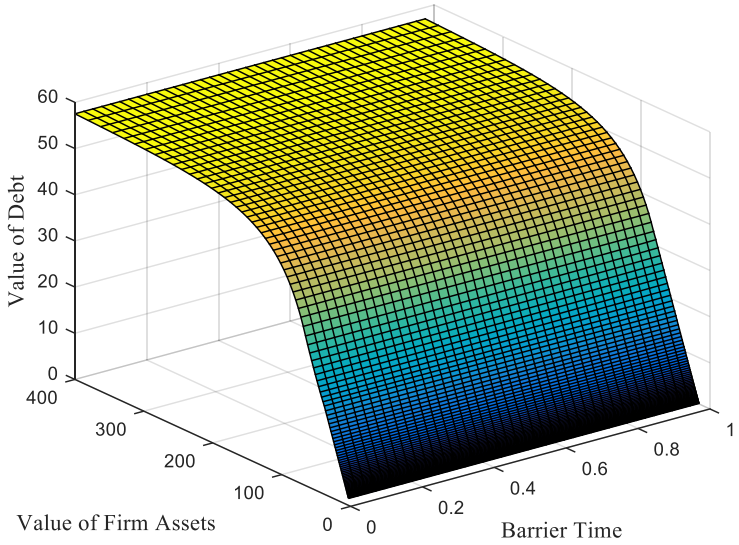


Figure 5.3 The Bond Debt Value at Date of Issuance within Explicit Method

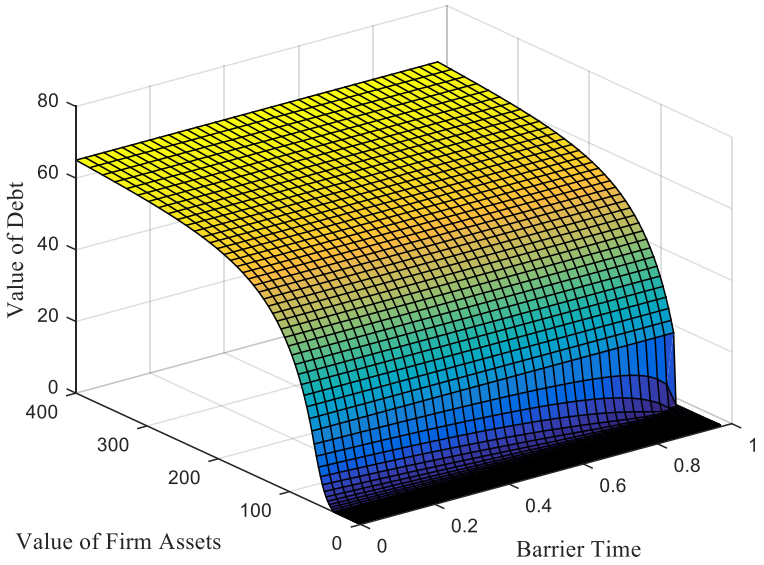


Figure 5.4 Debt Values as Function of Asset Volatility

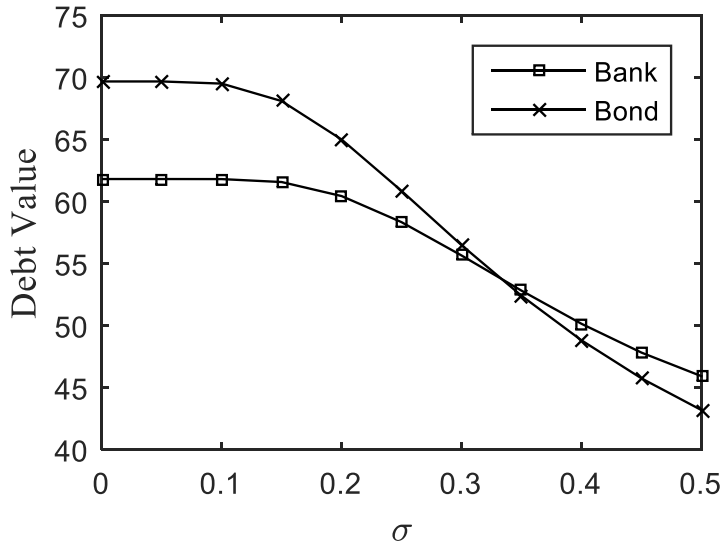
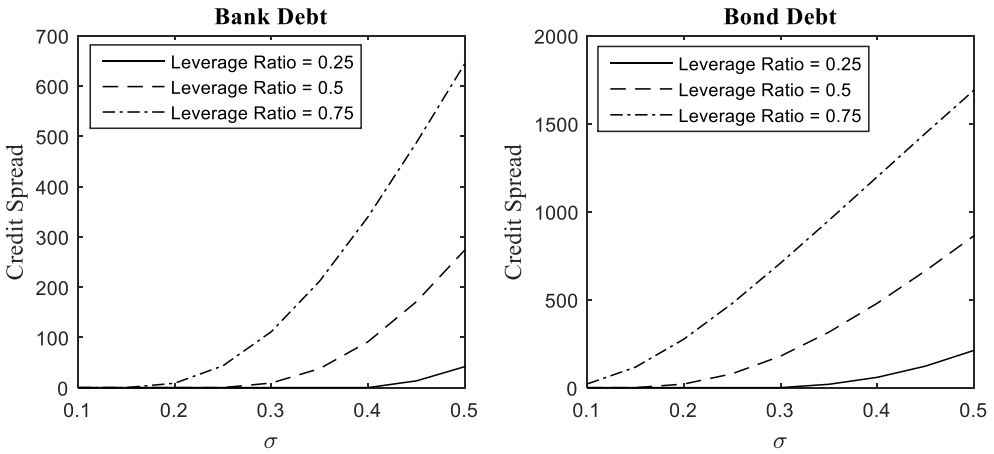


Figure 5.5 Credit Spread as Function of Asset Volatility and Debt Maturity

(a) T = 3 years



(b) T = 10 years

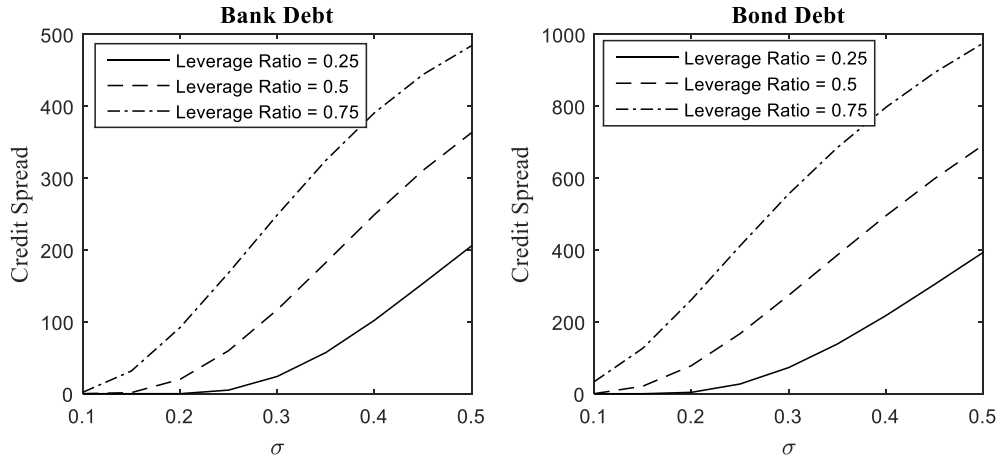


Figure 5.6 Security Values with Different Asset Volatility

Figure 5.6 displays the value of equity as a function of the volatility of the firm's assets and investigate shareholders' incentive to asset substitution (1) in different debt maturity and; (2) under different debt structure. Figure 5.6 shows that, stronger incentive for asset substitution when 1) debt outstanding has longer maturity date; 2) leverage ratio is higher; 3) bank percentage is smaller.

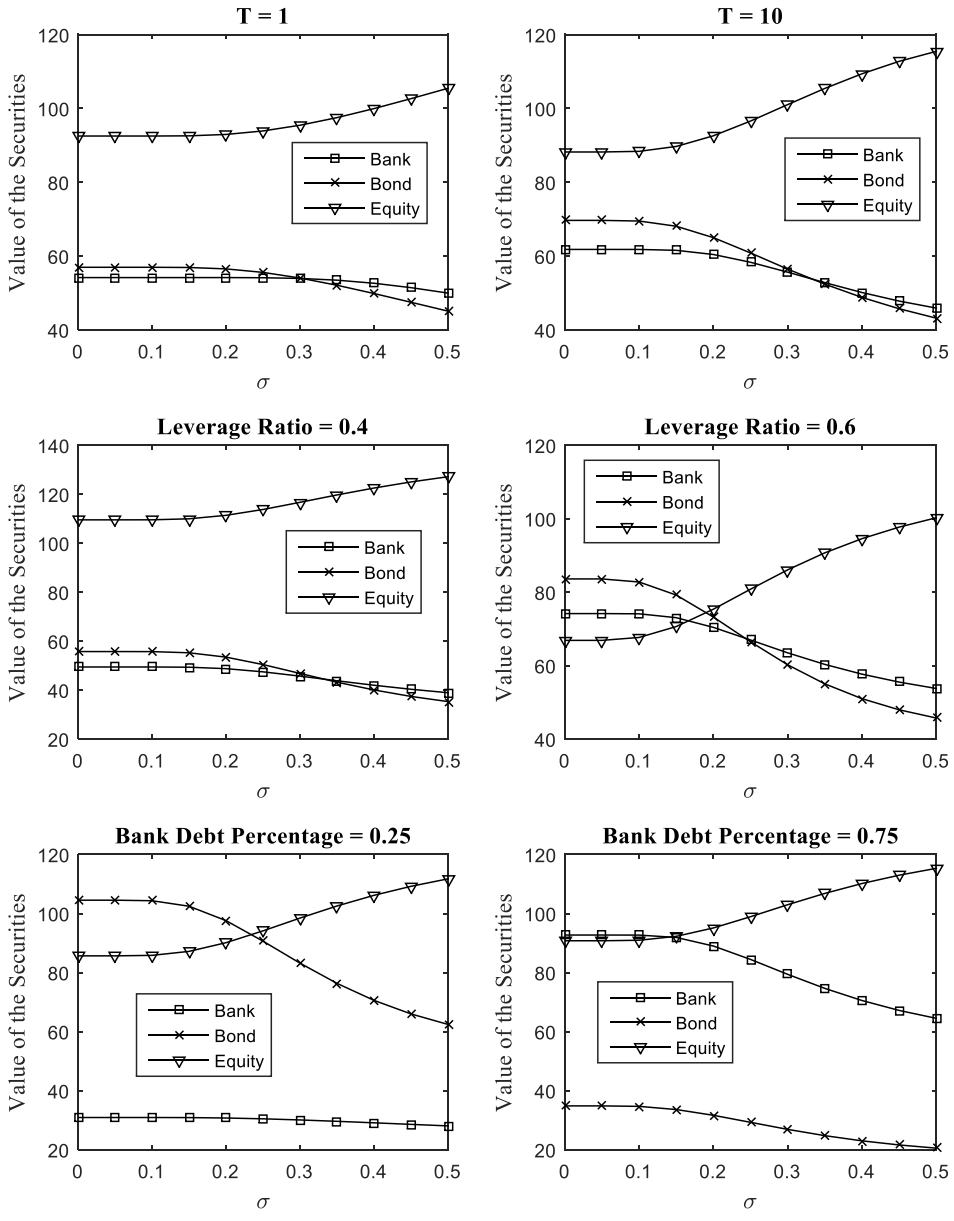
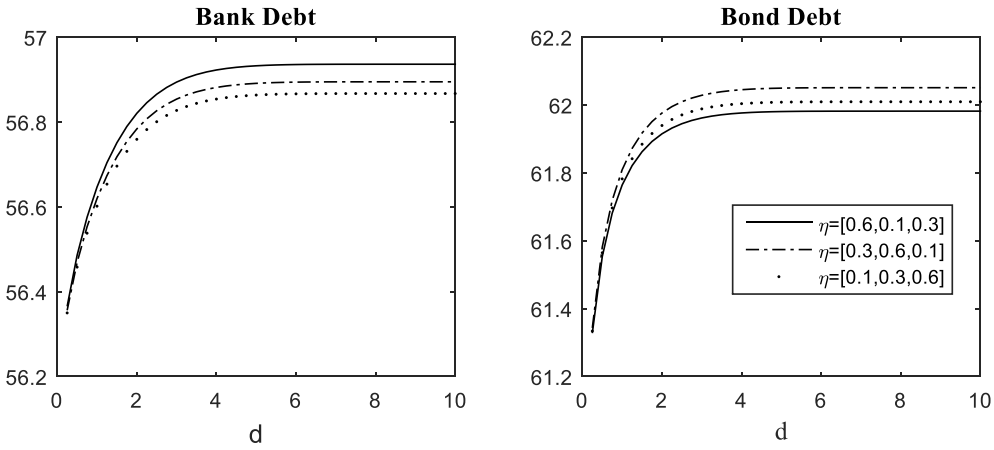


Figure 5.7 Debt Values with Different Asset Volatility and Bargaining Power

Figure 5.7 shows how the values of debts changes with the length of grace period under different asset volatility and bargaining power. The top panels are low volatility scenario ($\sigma = 20\%$) and the bottom one is the high volatility scenario ($\sigma = 40\%$). The solid line, dashed line and dotted line corresponds to different combinations of bargaining powers. I consider three scenarios $\boldsymbol{\eta} = [0.6, 0.1, 0.3]$, $\boldsymbol{\eta} = [0.3, 0.6, 0.1]$, $\boldsymbol{\eta} = [0.1, 0.3, 0.6]$ in which each of the three parties dominates the renegotiation process.

(a) Volatility = 20%



(b) Volatility = 40%

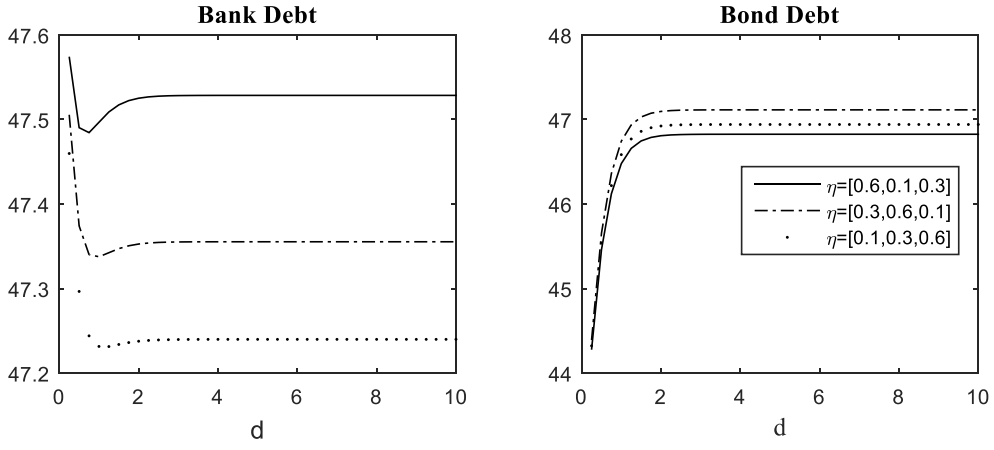


Figure 5.8 Credit Spread with Different Leverage Ratio

Figure 5.8 displays the values of credit spread as functions of leverage ratio with different combinations of liquidation cost and bank debt percentage. The liquidation cost considered is $\alpha = 0.25$ and $\alpha = 0.75$. The debt structure that are considered includes higher bank debt scenario ($\omega = 0.75$) and low bank debt scenario ($\omega = 0.25$).

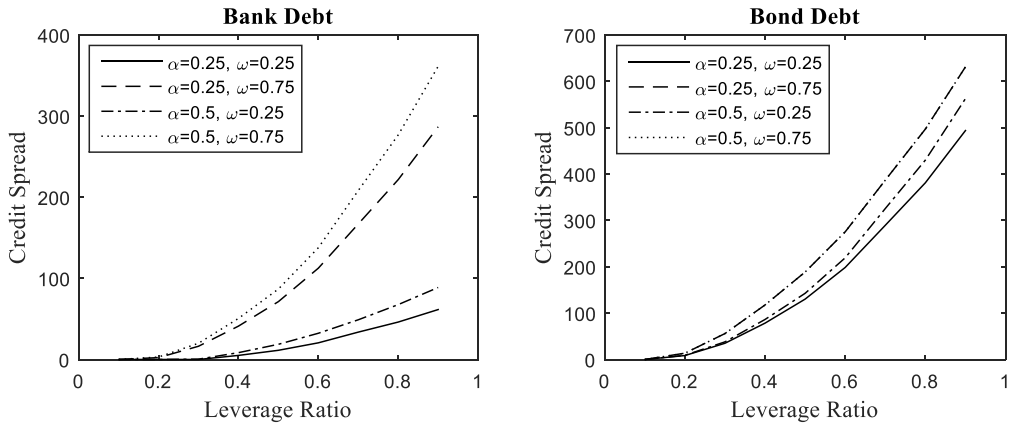


Figure 5.9 Term Structure with Different Leverage Ratio

Figure 5.9 depicts the term structure of credit spread for both bank debt and bond debt when leverage ratio is 0.2, 0.4, 0.6, 0.8. The term structure is upward sloping in most of the cases, except for the bond debt when leverage ratio is 80%.

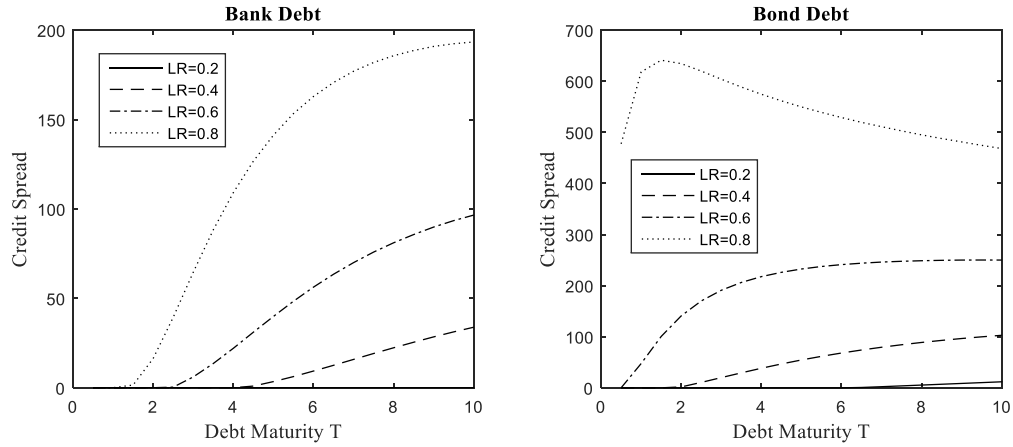


Figure 5.10 Credit Spread with Different Capital and Debt Structure

Figure 5.10 investigates the credit spread of debts under different capital structure (leverage ratio) and different debt structure. I calculate the credit spread for both bank debt and bond debt with three different debt maturities: one year, five years and ten years. In each subfigure I consider four different leverage ratio: 0.2, 0.4, 0.6 and 0.8.

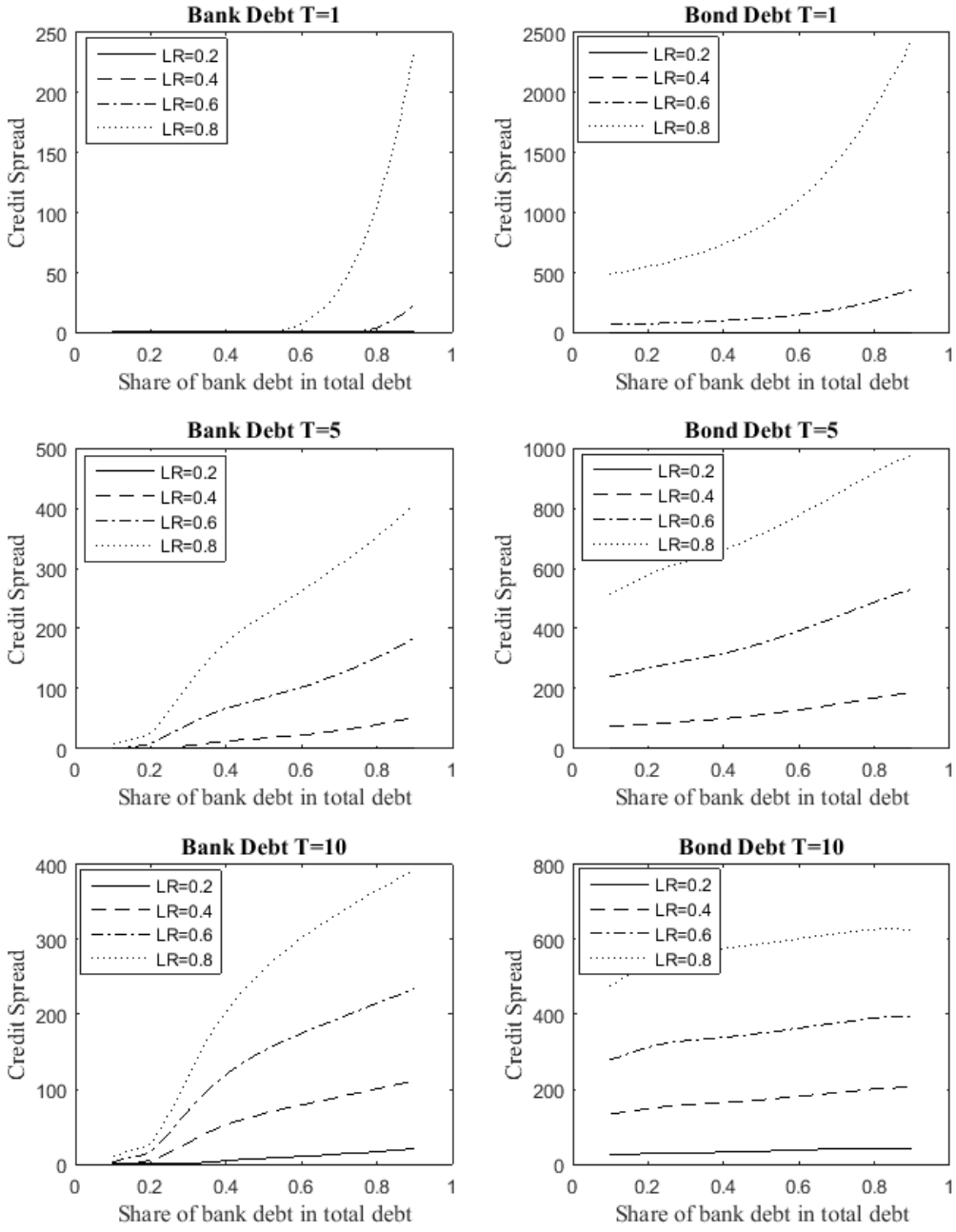


Figure 5.11 Term Structure with Different Debt Structure

Figure 5.11 shows the term structure of credit spread with different debt structure for both types of debt. Leverage ratio is set to be 50%, which is typical in other papers. The four lines in each subfigure corresponds to different debt structure: $\omega = 0.8$, $\omega = 0.6$, $\omega = 0.4$ and $\omega = 0.2$.

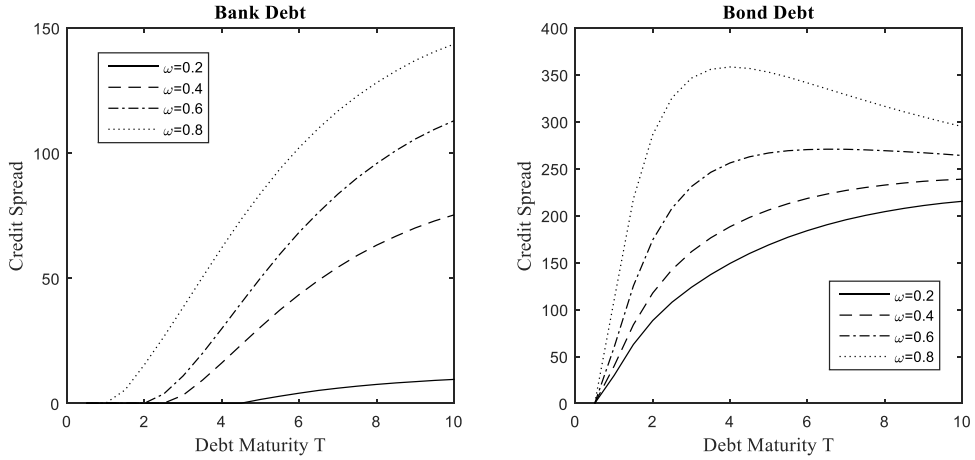
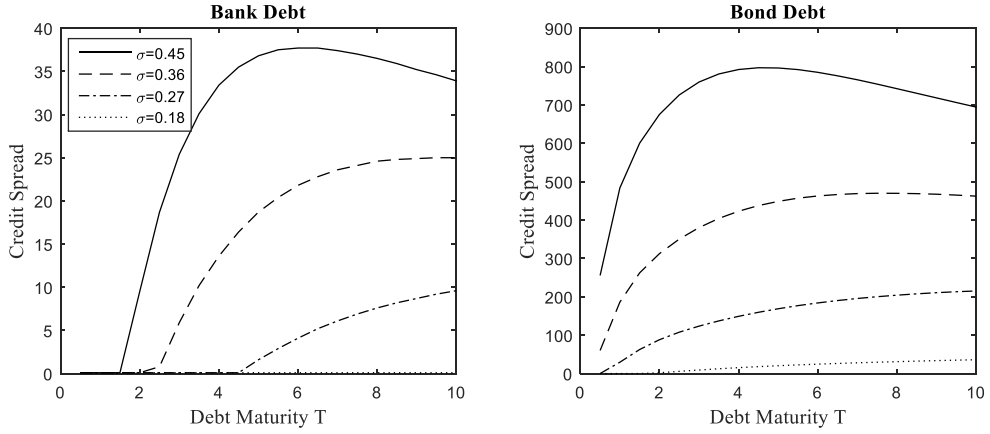


Figure 5.12 Term Structure with Different Asset Volatility and Debt Structure

Figure 5.12 demonstrates how the asset volatility and debt structure interact and impact on the term structure. Panel (a) represents the case of secured debt making up 20% of total debt and panel (b) shows the result when secured debt makes up 80% of the total debt. When the majority of the total debt is secured bank debt, the humped shape of term structure is more likely to happen for both debts.

(a) Secured debt making up 20% of the total debt



(b) Secured debt making up 80% of the total debt

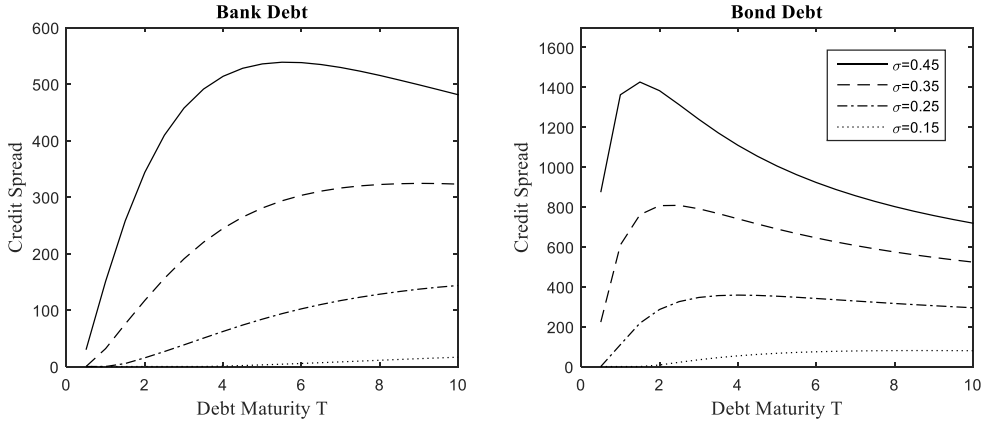
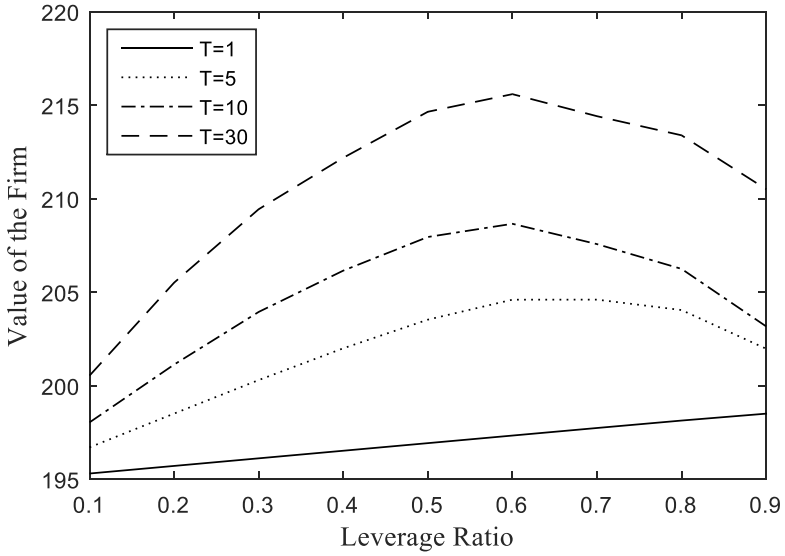


Figure 5.13 Firm Value as a Function of Leverage

Figure 5.13 demonstrates the changes of firm value as function of leverage ratio. Panel (a) shows the relationship between firm value and leverage ratio for firms issuing debt with maturities 1 year, 5 year 10 years and 30 years. The optimal leverage ratio is roughly 60% for median to long term debt. Panel (b) displays the optimal leverage ratio with different length of grace period.

(a) Different maturity



(b) Different length of grace period

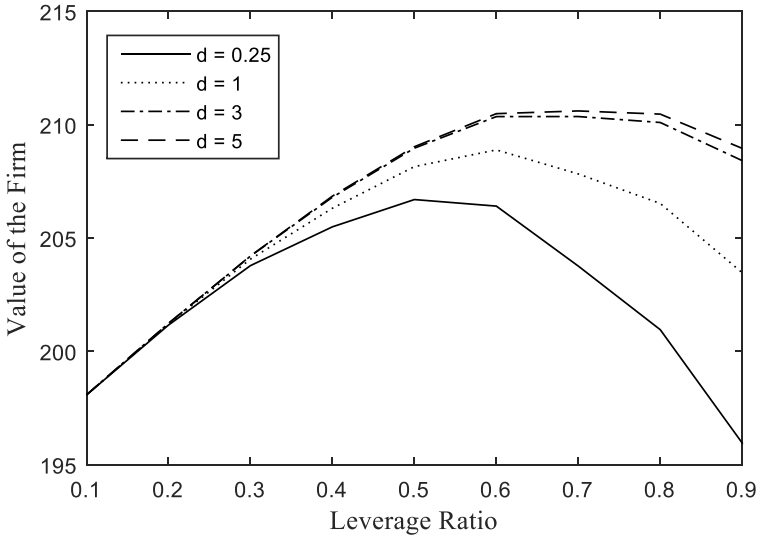


Table 5.1 Partial Differential System for Security Valuation

PDE	Firm Value	Equity Value
D_I	$\frac{\partial v_1}{\partial t} + \mathbb{L}v_1 + \delta V_t + \tau(bP_b + cP_c) = 0$	$\frac{\partial E_1}{\partial t} + \mathbb{L}E_1 + \delta V - (1-\tau)(bP_b + cP_c) = 0$
D_{II}	$\frac{\partial v_2}{\partial t} + \frac{\partial v_2}{\partial \mathcal{T}} + \mathbb{L}v_2 + \delta V - \varphi V = 0$	$\frac{\partial E_2}{\partial t} + \frac{\partial E_2}{\partial \mathcal{T}} + \mathbb{L}E_2 + (\delta - \varphi)V - s(V) = 0$
Boundary Conditions:	$v_1(V_T, T) = V_T$	$E_1(V_T, T) = V_T - P_b - P_c$
D_I: at debt maturity	$\lim_{V_t \to \infty} v_1(V_t, t) = V_t + \frac{\tau(bP_b + cP_c)}{r}\left(1 - e^{-r(T-t)}\right)$	$\lim_{V_t \to \infty} \mathrm{E}_1(\mathrm{V_t}, \mathrm{t}) = V_t + \frac{(\tau - 1)(bP_b + cP_c)}{r}\left(1 - e^{-r(T-t)}\right) - e^{-r(T-t)}(P_b + P_c)$
D_I: asset value is infinite	$\lim_{V_t \to V_B} v_1(V_t, t) = \lim_{V \to V_B} v_2(V_t, t, 0)$	$\lim_{\mathrm{V_t} \to \mathrm{V_B}} \mathrm{E}_1(\mathrm{V_t}, \mathrm{t}) = \lim_{\mathrm{V} \to \mathrm{V_B}} \mathrm{E}_2(\mathrm{V_t}, \mathrm{t}, 0)$
D_I: asset value is V_B	$v_2(V_T, T, \mathcal{T}) = V_T \quad \text{for all } \mathcal{T} < d$	$\mathrm{E}_2(\mathrm{V_T}, \mathrm{T}, \mathcal{T}) = [\mathrm{V_T} - P_b - P_c]^+ \quad \text{for all } \mathcal{T} < \mathrm{d}$
D_{II}: at debt maturity	$\lim_{\mathcal{T} \to d} v_2(V_t, t, \mathcal{T}) = (1-\alpha)V_t$	$\mathrm{E}_2(\mathrm{V_t}, \mathrm{t}, \mathrm{d}) = [(1-\alpha)\mathrm{V_t} - P_b - P_c]^+$
D_{II}: at the end of window	$\lim_{V_t \to V_L} v_2(V_t, t, \mathcal{T}) = (1-\alpha)V_L$	$\lim_{\mathrm{V_t} \to \mathrm{V_L}} \mathrm{E}_2(\mathrm{V_t}, \mathrm{t}, \mathcal{T}) = [(1-\alpha)\mathrm{V_L} - P_b - P_c]^+$
D_{II}: asset value is V_L	$\lim_{V \to V_B} v_2(V, t, \mathcal{T}) = v_2(V, t, 0) = \lim_{V \to V_B} v_1(V, t)$	$\lim_{\mathrm{V_t} \to \mathrm{V_B}} \mathrm{E}_2(\mathrm{V_t}, \mathrm{t}, \mathcal{T}) = \lim_{\mathrm{V_t} \to \mathrm{V_B}} \mathrm{E}_2(\mathrm{V_t}, \mathrm{t}, 0)$
D_{II}: asset value is V_B	$v_3(V_t, t, d) = (1-\alpha)V_t$	$\mathrm{E}_3(\mathrm{V_t}, \mathrm{t}, \mathrm{d}) = [(1-\alpha)\mathrm{V_t} - P_b - P_c]^+$
D_{III}	$v_1(V_T, T) = V_T$	$E_1(V_T, T) = V_T - P_b - P_c$

PDE	Bank Debt	Bond Debt
D_I	$\frac{\partial \mathrm{D_1^S}}{\partial \mathrm{t}} + \mathbb{L}\mathrm{D_1^S} + \mathrm{bP_b} = 0$	$\frac{\partial \mathrm{D_1^J}}{\partial \mathrm{t}} + \mathbb{L}\mathrm{D_1^J} + \mathrm{cP_c} = 0$
D_{II}	$\frac{\partial \mathrm{D_2^S}}{\partial \mathrm{t}} + \frac{\partial \mathrm{D_2^S}}{\partial \mathcal{T}} + \mathbb{L}\mathrm{D_2^S} + \mathrm{s^S(V)} = 0$	$\frac{\partial \mathrm{D_2^J}}{\partial \mathrm{t}} + \frac{\partial \mathrm{D_2^J}}{\partial \mathcal{T}} + \mathbb{L}\mathrm{D_2^J} + \mathrm{s^J(V)} = 0$
Boundary Conditions:		
Region I: at debt maturity	$D_1^S(V_T, T) = P_b$	$D_1^J(V_T, T) = P_c$
Region I: asset value is infinite	$\lim_{\mathrm{V_t} \to \infty} \mathrm{D_1^S(V_t, t)} = e^{-r(T-t)}P_b + \frac{bP_b}{r}\left(1 - e^{-r(T-t)}\right)$	$\lim_{\mathrm{V_t} \to \infty} \mathrm{D_1^J(V_t, t)} = e^{-r(T-t)}P_c + \frac{cP_c}{r}\left(1 - e^{-r(T-t)}\right)$
Region I: asset value is V_B	$\lim_{\mathrm{V_t} \to \mathrm{V_B}} \mathrm{D_1^S(V_t, t)} = \lim_{\mathrm{V} \to \mathrm{V_B}} \mathrm{D_2^S(V_t, t, 0)}$	$\lim_{\mathrm{V_t} \to \mathrm{V_B}} \mathrm{D_1^J(V_t, t)} = \lim_{\mathrm{V} \to \mathrm{V_B}} \mathrm{D_2^J(V_t, t, 0)}$
Region II: at debt maturity	$\mathrm{D_2^S(V_T, T, \mathcal{T}) = \min[V_T, P_b]} \quad \text{for all } \mathcal{T} < \mathrm{d}$	$\mathrm{D_2^J(V_T, T, \mathcal{T}) = \min[(V_T - P_b)^+, P_c]} \quad \text{for all } \mathcal{T} < \mathrm{d}$
Region II: at the end of window	$\mathrm{D_2^S(V_t, t, d) = \min[(1-\alpha)V_t, P_b]}$	$\mathrm{D_2^J(V_t, t, d) = [(1-\alpha)V_t - P_b]^+}$
Region II: asset value is V_L	$\lim_{\mathrm{V_t} \to \mathrm{V_L}} \mathrm{D_2^S(V, t, \mathcal{T}) = \min[(1-\alpha)V_L, P_b]}$	$\lim_{\mathrm{V_t} \to \mathrm{V_L}} \mathrm{D_2^J(V, t, \mathcal{T}) = [(1-\alpha)V_L - P_b]^+}$
Region II: asset value is V_B	$\lim_{\mathrm{V} \to \mathrm{V_B}} \mathrm{D_2^S(V, t, \mathcal{T})} = \lim_{\mathrm{V_t} \to \mathrm{V_B}} \mathrm{D_1^S(V_t, t)}$	$\lim_{\mathrm{V} \to \mathrm{V_B}} \mathrm{D_2^J(V, t, \mathcal{T})} = \lim_{\mathrm{V_t} \to \mathrm{V_B}} \mathrm{D_1^J(V_t, t)}$
Region III	$\mathrm{D_3^S(V_t, t, d) = \min[(1-\alpha)V_t, P_b]}$	$\mathrm{D_3^J(V_t, t, d) = [(1-\alpha)V_t - P_b]^+}$

Table 5.2 Numerical Scheme for Security Valuation

Bond Debt	Bank Debt
$u_{i,j-1}^{0} = Au_{i-1,j}^{0} + Bu_{i,j}^{0} + Cu_{i+1,j}^{0} + cP_c\Delta t$	$u_{i,j-1}^{0} = Au_{i-1,j}^{0} + Bu_{i,j}^{0} + Cu_{i+1,j}^{0} + bP_b\Delta t$
$u_{i,j-1}^{k-1} = Au_{i-1,j}^{k} + Bu_{i,j}^{k} + Cu_{i+1,j}^{k} + s^J(e^{x_i})\Delta t$	$u_{i,j-1}^{k-1} = Au_{i-1,j}^{k} + Bu_{i,j}^{k} + Cu_{i+1,j}^{k} + s^S(e^{x_i})\Delta t$
$u_{i,J}^{0} = P_c \quad \text{for all } i > \bar{i}$	$u_{i,J}^{0} = P_b \quad \text{for all } i > \bar{i}$
$u_{i,J}^{k} = \min[\left(e^{i\Delta x} - P_b\right)^+, P_c] \quad \text{for all } 0 < k < K \text{ and } i < \bar{i}$	$u_{i,J}^{k} = \min[e^{i\Delta x}, P_b] \quad \text{for all } 0 < k < K \text{ and } i < \bar{i}$
$u_{I,j}^{0} = e^{-r(T-j\Delta t)}P_c + \frac{cP_c}{r}\left(1 - e^{-r(T-j\Delta t)}\right)$	$u_{I,j}^{0} = e^{-r(T-j\Delta t)}P_b + \frac{bP_b}{r}\left(1 - e^{-r(T-j\Delta t)}\right)$
$u_{\bar{i},j}^{k} = u_{\bar{i},j}^{0}$	$u_{\bar{i},j}^{k} = u_{\bar{i},j}^{0}$
$u_{i,j}^{K} = \left[(1-\alpha)e^{i\Delta x} - P_b\right]^+ \text{ for all } i < \bar{i}$	$u_{i,j}^{K} = \min\left[(1-\alpha)e^{i\Delta x}, P_b\right] \text{ for all } i < \bar{i}$
$u_{i,j}^{k} = \left[(1-\alpha)e^{i\Delta x} - P_b\right]^+ \text{ for all } i \le \bar{l}$	$u_{i,j}^{k} = \min\left[(1-\alpha)e^{i\Delta x}, P_b\right] \text{ for all } i \le \bar{l}$
Firm Value	**Equity Value**
$u_{i,j-1}^{0} = Au_{i-1,j}^{0} + Bu_{i,j}^{0} + Cu_{i+1,j}^{0} + \left[\delta e^{i\Delta x} + \tau(bP_b + cP_c)\right]\Delta t$	$u_{i,j-1}^{0} = Au_{i-1,j}^{0} + Bu_{i,j}^{0} + Cu_{i+1,j}^{0} + \left[\delta e^{i\Delta x} - (1-\tau)(bP_b + cP_c)\right]\Delta t$
$u_{i,j-1}^{k-1} = Au_{i-1,j}^{k} + Bu_{i,j}^{k} + Cu_{i+1,j}^{k} + (\delta - \varphi)e^{i\Delta x}\Delta t$	$u_{i,j-1}^{k-1} = Au_{i-1,j}^{k} + Bu_{i,j}^{k} + Cu_{i+1,j}^{k} + \left[(\delta - \varphi)e^{i\Delta x} - s^S(e^{x_i}) - s^J(e^{x_i})\right]\Delta t$
$u_{i,J}^{0} = e^{i\Delta x} \quad \text{for all } i > \bar{i}$	$u_{i,J}^{0} = e^{i\Delta x} - P_b - P_c \quad \text{for all } i > \bar{i}$
$u_{i,J}^{k} = e^{i\Delta x} \quad \text{for all } 0 < k < K \text{ and } i < \bar{i}$	$u_{i,J}^{k} = \left[e^{i\Delta x} - P_b - P_c\right]^+ \quad \text{for all } 0 < k < K \text{ and } i < \bar{i}$
$u_{I,j}^{0} = e^{i\Delta x} + \frac{\tau(bP_b + cP_c)}{r}\left(1 - e^{-r(T-j\Delta t)}\right)$	$u_{I,j}^{0} = e^{i\Delta x} + \frac{(\tau-1)(bP_b + cP_c)}{r}\left(1 - e^{-r(T-j\Delta t)}\right) - e^{-r(T-j\Delta t)}(P_b + P_c)$
$u_{\bar{i},j}^{k} = u_{\bar{i},j}^{0}$	$u_{\bar{i},j}^{k} = u_{\bar{i},j}^{0}$
$u_{i,j}^{K} = (1-\alpha)e^{i\Delta x} \text{ for all } i \le \bar{i}$	$u_{i,j}^{K} = \left[(1-\alpha)e^{i\Delta x} - P_b - P_c\right]^+ \text{ for all } i < \bar{i}$
$u_{i,j}^{k} = (1-\alpha)e^{i\Delta x} \text{ for all } i \le \bar{l}$	$u_{i,j}^{k} = \left[(1-\alpha)e^{i\Delta x} - P_b - P_c\right]^+ \text{ for all } i \le \bar{l}$

where:

$A = \frac{\Delta t}{2\Delta x}\left[\frac{\sigma^2}{\Delta x} - \left(r - \delta - \frac{\sigma^2}{2}\right)\right]$, $B = \left[1 - \frac{\sigma^2\Delta t}{\Delta x^2} - r\Delta t\right]$ and $C = \frac{\Delta t}{2\Delta x}\left[\frac{\sigma^2}{\Delta x} + \left(r - \delta - \frac{\sigma^2}{2}\right)\right]$

$\bar{l} = \frac{1}{\Delta x}\ln(V_L)$ and $\bar{i} = \frac{1}{\Delta x}\ln(V_B)$

Table 5.3 Static Analysis of Credit Spread

Leverage ratio	α	d	Loan credit spread (bps)	Bond credit spread (bps)	Loan spread change	Bond spread change
$LR = 0.25$ $(P_S = P_J = 25)$	0.25	0.25	38.7	81.6		
		1	38.7	78.5	↓	↓
		3	38.1	77.2		
	0.5	0.25	43.4	81.6		
		1	40.8	78.5	↓	↓
		3	38.5	77.3		
	0.75	0.25	48.2	81.6		
		1	42.9	78.5	↓	↓
		3	38.9	77.3		
$LR = 0.5$ $(P_S = P_J = 50)$	0.25	0.25	69.7	225		
		1	71.6	201	-[83]	↓
		3	69.4	188		
	0.5	0.25	103	225		
		1	89.7	201	↓	↓
		3	74.9	188		
	0.75	0.25	138	225		
		1	108	201	↓	↓
		3	80.4	188		
$LR = 0.75$ $(P_S = P_J = 75)$	0.25	0.25	133	506		
		1	144	425	-[84]	↓[85]
		3	145	375		
	0.5	0.25	226	507		
		1	200	425	↓	↓
		3	167	376		
	0.75	0.25	327	507		
		1	258	426	↓	↓
		3	189	377		

[83] The two scenarios happens both at $\alpha = 0.25$. In all the other cases, the credit spread declines as d increases. (1) larger d reduces liquidation probability, thus reduce credit spread (dominant effect); (2) larger d results in more debt forgiveness, thus increases credit spread. My results indicate that the effect (2) will dominate (1) when leverage ratio is high and liquidation cost is low. My understanding is that when liquidation cost is low, the cost saving from effect (1) is marginal. When leverage ratio is high the liquidation probability is high and the effect of d is marginal.

[84] When the alpha is small, the cost saving from avoidance of liquidation is limited. Therefore, the credit spread does not decrease with increase in d.

[85] But for bondholders, the increment of d reduces the credit spread. This means that, the liquidation is detrimental to the bondholders and they prefer to avoid it despite the debt forgiveness during the grace period.

CHAPTER 6

Boundary Determination and Optimal Control Right Allocation in Financially Distressed Firms

Chapter 5 demonstrates how a more accurate evaluation of corporate securities can be achieved when the features of the U.S. bankruptcy procedure are considered. I develop a credit risk model and find solutions for corporate debt (including bank loan and corporate bond) and equity values when both Chapter 11 and Chapter 7 are allowed and the grace period for recovery is granted. In this chapter I revisit the issue but allow the default and liquidation boundaries to be determined endogenously. Still using a contingent claim model, this chapter shows that (1) conflicts exists not only between borrowers and lender, but also between different borrowers, with regard to the determination of the threshold values; (2) the optimal outcome can be rebuilt if the liquidation boundary is determined by bondholders; (3) the optimal length of grace period, in the sense of socially optimum, is approximately 6 months.

6.1 Introduction

The properties of the default boundaries are fundamental but essential in structural models of credit risk since they have significant implications in debt pricing. As introduced in the literature review chapter, before Leland (1994), structural models adopted an exogenous approach. The level of the default boundary is exogenously specified, normally in relation to the amount of debt. Since Leland (1994), more and more work diverts to the alternative approach and assumes that the boundary is selected strategically by equity holders in order to maximize the value of equity. By taking the debt as a consol bond, the default boundary can be expressed as explicit formulae, as functions of variables such as asset volatility, payout ratio, tax rate, etc.

However, this assumption of shareholders-determined boundary to maximize the equity value does not receive substantial empirical support. It is regarded as unsuitable especially for firms in countries with creditor-friendly bankruptcy regimes, or in the case of secured debt. Therefore, recent theoretical developments in structural modelling consider alternative mechanisms and explanations for the determination of default and liquidation boundaries. For instance, in the continuous time asset pricing model of debt and equity by Bruche and Naqvi (2010), the default decision and liquidation decision are made by equity holders and creditors, respectively. Carey and Gordy (2016) assume that, upon covenant violation, the bank creditor optimally decides whether to

ignore it or force default. The default boundary in their model thus maximizes the value of bank debt instead of equity value.

Papers such as François and Morellec (2004), Moraux (2002) and Galai, Raviv and Wiener (2007) which have been introduced in Chapter 5 incorporate the Parisian or ParAsian feature of formal bankruptcy. In these papers, default is still determined by equity holders but liquidation is triggered by an exogenous event in regards to grace period. Once again, as mentioned in the previous chapter, these theoretical papers use a simplified debt structure, represented by a single negotiable debt, generally of infinite maturity. This compromise is a sensible one in bond pricing as a trade-off between mathematical complexity and economic realism and sheds light on the creditor-debtor tension. However, when it comes to discussion of the bankruptcy code, it conceals conflicts between creditors.

Therefore, the objective of this chapter is threefold. First, as in the literature, I study the characteristics of equity-determined default boundary, but under the setup of two-class debt structure and bankruptcy procedure. Will equity holders postpone or accelerate the default timing when asset volatility is higher, or debt has longer maturity date? Secondly, I explore the optimal boundary to maximize the firm value. The last goal is to find the optimal length of grace period that maximize the value of the firm as a whole.

In this chapter I generalize the conclusion from these papers by looking at the impact of different boundaries, if they are determined by different claimholder. More importantly, by comparing the consequences of these different scenarios[86], I revisit the issue of optimal boundary level from the perspective of socially optimal considerations. Is equity value maximization consistent with firm value maximization? If not, how can the first-best outcome[87] be restored? As explained in the literature review chapter, there are a few discussions on the best practice in resolving financial distress for the defaulted companies in law journals. Every involved party strives to make full use of their control right and maximize their payoffs during the firm's restructuring process. In Leland (1994), in the presence of limited liability, equity value maximization coincides with total firm value maximization. Later, in Broadie, Chernov and Sundaresan (2007), they are no longer consistent when the agents are presented with the options to reorganize or to liquidate. Moreover, they show that debt maximization leads to a strategy that is qualitatively very similar to the first best. We further dig into the problem and investigate which class of the creditor best represents the firm.

In this chapter, default barrier and liquidation barrier are endogenously determined by one of the involved parties, rather than fixed boundaries as in the

[86] The scenarios here mean who determine the boundaries.

[87] Following Broadie, Chernov and Sundaresan (2007), the first-best outcome refers to the one that maximizes total firm value subject to the limited liability of equity and debt.

previous chapter. This setting enriches the current research as it allows creditors or debtors to demonstrate their strategies in dealing with defaulted entities, which is in the spirit of Chapter 11 of U.S Bankruptcy Code. The results indicate that bond value maximization has most resemblance to firm value maximization, implying best practice is to entitle the bond lender to determine the liquidation barrier. This chapter sheds light on the control right allocation in a financially distressed firm as it is able to determine the overall firm value along with the values of equity and different classes of debts.

Another issue touched upon in this chapter is the duration of the grace period. The values of securities vary with different length of grace period. It is shown later in this chapter that the ex ante firm value is highest when the duration is set to be approximately six months.

The chapter is arranged as follows. Section 6.2 and Section 6.3 looks at default boundary and liquidation boundary respectively. Section 6.4 solves the optimization problem simultaneously and discuss the optimal default boundary determination that maximize the total value of the firm as a whole. Section 6.5 investigates the optimal duration of grace period that maximize the firm value ex ante. Section 6.6 concludes.

6.2 Determinants of the Default Boundary

To start with, I investigate the 'barrier effect', that is, how the valuations for debt, equity and firm change with respect to the default boundaries in section 6.2.1. I then study the characteristics of the optimal default boundary, especially one that maximize equity value in section 6.2.2. Section 6.2 focuses on default boundary V_B while section 6.3 deals with liquidation boundary V_L.

6.2.1 Barrier Effects

In Figure 6.1 I let V_B vary between the fixed liquidation barrier $V_L = 40\%$ and the face value of whole debt. The figure reports the values of bank debt, bond, equity and firm separately under different default boundary. Several interesting conclusions can be drawn from the figure.[88]

(1) Looking at the values on the y-axis, the values calculated from this model are not highly sensitive to the choice of boundaries (liquidation boundary in Figure 6.1).[89] For comparison, I calculate and demonstrate the valuation results from two related models - Leland (1994) and François and Morellec (2004) in Figure 6.2. The weak barrier effect is a strong advantage of this type of

[88] I also change the value of the barrier and replot the figures. The conclusions remain unchanged.

[89] In Figure 6.8 a similar pattern can be observed when default boundary is allowed to vary when holding liquidation boundary fixed.

modelling in calibration as the boundary determination is still an unsolved problem in empirical study. Additionally, in the base case, the value of bond debt is relatively more sensitive to the change of default boundary, when compared with bank debt or equity.

(2) Debt value and firm value decreases as the default boundary increases, while the equity values is a concave function of default boundary. This concavity is the theoretical grounding for the 'smooth pasting condition' or 'low contact condition' in Leland (1994). Simply speaking, in credit risk models with consol debt, equity is treated as a perpetual option. The optimal default level for the equity holder is the optimal exercise point of the option.

(3) Regarding the default boundary V_B, the figure seems to imply that creditors should be entitled to decide as they are more likely to restore firm value maximization. For banks and bondholders, the best practice is to delay Chapter 11 filing as late as possible, while the equity holders are not as patient and it is in their interest to file for Chapter 11 at a higher asset value level in the hope of reducing the debt load. This is consistent with the low frequency of involuntary filings. As documented in the UCLA-LoPucki Bankruptcy Research Database, within the 1001 defaulted companies, only 42 cases are involuntary.[90]

[90] The data provider gives the following definition of voluntary case: A case is "voluntary" if it was filed by the debtor, "involuntary" if it was filed by creditors. If some entities file voluntarily and others are brought in involuntarily, indicate "both". If the debtor and creditors

6.2.2 Characteristics of Default Boundary

This section discusses the characteristics of default boundary, with a focus on the equity-determined optimal default boundary, under a full choice of parameter values.

Different debt maturity: Figure 6.3 demonstrates that, regardless of the maturity of the corporate debt, the firm's optimal renegotiation boundary aligns with the bondholders. Consistent with the finding in the base case, equity's optimal renegotiation threshold is concave and higher than that of bank and bondholder. Additionally, the impact of the renegotiation threshold is stronger for long-term debt since we observe the values are more sensitive to the threshold when debt maturity is longer. Finally, the optimal default barrier that maximizes the equity value is decreasing with the debt maturity. This negative relationship is also predicted by Leland and Toft (1996) under a default-equals-liquidation framework. Figure 6.4 implies that the conclusion remains unchanged even when renegotiation is allowed and a grace period is granted.

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file roughly simultaneously, characterize the case by the first to file, even if that case is dismissed in favour of the other. If the involuntary case is transferred, so indicate in the Transferred field. If the voluntary and involuntary cases are before the same judge and the times they are pending overlap, characterize the case by the first to file, even if that case is dismissed in favour of the other.

Different asset volatility: In basic endogenous structural models where there is no renegotiation region or Parisian features, when asset volatility is high, it is worth keeping the option alive for shareholders. Thus, the default boundary, if determined by equity holder, is lower when the asset volatility is high. Figure 6.3 shows the values of securities with respect to various default thresholder under different asset volatility (20% and 40%). As with other structural models, the shareholders are expected to postpone to trigger default when the asset volatility is high. Davydenko (2012) likewise report a negative relationship between asset volatility and default boundary.

Bankruptcy cost: Next I look at the impact of bankruptcy costs. In the model, two types of bankruptcy cost are included: liquidation cost, represented by α in the model, which is associated with Chapter 7 bankruptcy, and financial distress cost ϕ, which will be incurred once the firm enters into Chapter 11. Interestingly, from Figure 6.5, the optimal boundary chosen by shareholders, given a fixed liquidation boundary, remains the same regardless of the choice of liquidation cost. A reason to explain this it is that, in liquidation, the equity holder is excluded from the distribution of remaining assets by the Absolute Priority Rule. Therefore, the liquidation cost does not impact on an equity holder's decision making. This results are different from other models which do not distinguish default and liquidation. For instance, in Anderson and Sundaresan (1996), the optimal default barrier increased with liquidation cost. This is because, in their model, the shareholders can request a substantial debt concession from creditors

when liquidation is more costly. However, this is not the case when we differentiate the event of default and liquidation and shareholders are excluded from the allocation of the liquidated assets.

Figure 6.6 shows the impact of the second type of bankruptcy cost, i.e., the distress cost that captures the loss of businesses under bankruptcy. The optimal default threshold to maximize the equity value is higher when distress cost is $\phi = 0.02$, compared with the case of $\phi = 0.05$. The explanation is straightforward: that the shareholders would be reluctant to trigger bankruptcy if the costs associated with renegotiation is high. At first glance, the result here seems to contradict the interpretation above since distress cost should likewise give equity holders greater bargaining power in renegotiation, which encourages an early default. This contradiction emerges because the distress cost is not included in the formulae of the reduced-form strategic debt service. Results might change if a more accurate expression for the debt service under Chapter 11 is given to reflect the role of distress cost.

I also examine the influence of debt structure in Figure 6.7. The variation of optimal default is negligible from the figure. The precise calculations from my programming code reveal that the default boundary determined by shareholders is lower when there is more bank debt in the total debt. However, the reason for this relationship in this model is not economically meaningful. It is simply because the shift of debt structure modifies the boundary condition as the cost

saving associated with tax shield changes. The positive relationship between default threshold and bank debt percentage in Carey and Gordy (2016) is economically sound.

6.3 Liquidation Boundary

6.3.1 Base Case: barrier effects

In Figure 6.8 the default boundary V_B is fixed (70%) while V_L changes from zero to V_B. In the figure I report the values of bank debt, bond, equity and firm separately. Different from the determination of V_B, the determination of optimal liquidation boundary is based on the values of securities upon filing for Chapter 11.

In terms of liquidation boundary V_L, it can be observed directly from the figure that value of secured creditors (banks) decreases with liquidation boundary. This conclusion is important as it implies the bank creditor will have the incentive to liquidate the firm earlier than optimum. Within a bargaining model, Jenkins and Smith (2014) shows the incentives of senior claimants to force inefficient liquidations, which is consistent with findings here under the contingent claim framework. Moreover, an intuitive guess from the figure is that the firm value might be maximized if the bondholder is chosen to determine the level of V_L in bankruptcy code. To be more specific, in the base case as

plotted in the figure, the bond would choose to liquidate the firm when asset value falls down to approximately 40, which almost corresponds to the first-best outcome.

Bruche and Naqvi (2010) shows theoretically that debtholders have an incentive to liquidate the firm prematurely relative to the liquidation boundary that maximizes firm value, taking the default boundary as given. The model in this chapter further shows that this "liquidation motivation" only applies to the bank debtholder. We cannot observe similar pattern in bondholder's behaviour from Figure 6.8.

6.3.2 Characteristics of Liquidation Boundary

Different debt maturity[91]: In the model of Leland and Toft (1996) where default leads to immediate liquidation of the firm, the authors show a decrease of optimal default boundary when the debt maturity increases from 5 years to 20 years in Table II Panel C in the paper. Figure 6.9 demonstrates how the valuation can be expressed as a function of liquidation boundary under different debt maturity from this model. A similar negative relationship can be observed, even when renegotiation is permitted. The lower left panel suggests that long term debt ($T = 10$) corresponds to a lower optimal liquidation boundary, if

[91] T here is different from T in default boundary discussions. The T here represents the time to maturity of the debt when the firm files for bankruptcy protection under Chapter 11.

chosen by shareholder. While looking at the other panels, Figure 6.9 further implies that this negative relationship does not hold for other security owner.

Different asset volatility: As securities under the model are option-like, the asset volatility is expected to increase the value of the securities. Thus, Figure 6.10 clearly shows a decrease in optimal liquidation boundary when the asset volatility is higher ($\sigma = 0.4$) compared with the low volatility environment ($\sigma = 0.2$). It is in the interests of security owners to keep the option alive with volatile underlying assets.

Different debt structure: When comparing Figure 6.11 with Figure 6.8 in the base case, it can be directly observed that the optimal default boundary by maximizing the debt value increases with the share of bank debt in total debt. When the percentage of bank debt decrease from 50% (in Figure 6.8) to 20% (in Figure 6.11), the bank determined liquidation boundary falls from approximately 65% to 55%. This decline can be attributed to the thicker cushion to the loan owner provided that the asset level is the same but the amount of bank loan is smaller.

Bankruptcy cost: The first type of bankruptcy cost, liquidation cost, is most likely to lead to a direct welfare reduction to the bank and so the bank should have a strong incentive to liquidate the firm early so as to secure their benefit. As expected, Figure 6.12 shows a higher liquidation boundary to maximize

bank debt value when liquidation cost is higher (dashed line). On the contrary, the liquidation cost has completely no impact on the optimal liquidation threshold to maximize equity value. The bondholders are somewhat in between the two extreme cases because they are only moderately subject to the liquidation cost.

Figure 6.13 exhibits the securities values as a function of liquidation boundary with different distress cost. When renegotiation is costly (dashed line), both equity holders and bondholders have the tendency to trigger liquidation early to avoid those costs associated with the Chapter 11 process. However, for a bank debt holder, the inclination to early liquidation is not affected by amount of distress costs.

6.4 Endogenous Boundary Determination

Reflected in the valuation model, up until now the values of default boundary V_B and V_L are exogenous and fixed. Inspired by the study of Davydenko (2012), the V_B is set to be 70% of the debt face value. V_L is set to be 40%, which is based on the calculation of Moody's Ultimate Recovery Database. This helps to keep the analysis concise and focused on the pricing issue.[92] In this section, I look at

[92] Setting default boundary at a fixed level has to some extent received empirical support, which can be found in Davydenko (2012).

how those boundaries are determined if the bankruptcy decision is made by shareholders, bondholders or bank lenders respectively. I discuss the optimal default threshold chosen by equity holders and debt holders, respectively. A natural point of comparison will turn out to be the default boundary at which equity holders would choose to default if default implied immediate liquidation as in Leland (1994). A pre-specified boundary exhibits simplicity in calculation. However, when we allow the bankruptcy barriers to be chosen optimally, fruitful insights can be generated from the model, especially in terms of control right allocation in bankruptcy. The results provide guidance on who should decide the two levels here to improve overall benefit. Endogenizing variables V_B and V_L adds substantial computational complexity to the problem and so I explain in more detail how we address them.

Suppose that given the value of V_B, the liquidation boundary V_L is determined by maximizing the value of security value X.[93] Furthermore, given the value of V_L, the bankruptcy boundary V_B maximize the value of security value Y.[94]

[93] As I will discuss who should control the decision of boundary later, I use X and Y here to make a general reference.

[94] Broadie, Chernov and Sundaresan (2007) write 'liquidation boundary V_L is similar to the determination of the optimal exercise boundary in American option pricing' and 'the liquidation boundary is regarded as the optimal stopping time of an American option'. The resulting free boundary from American option pricing is a curve if mapped using a binomial tree or finite difference grid. However, the liquidation boundary here should be a straight line such that below this level the option is exercised immediately.

The tricky part in solving the problem is as follows. The liquidation time is dependent upon the bankruptcy time. It is determined by the holder of security X, given the firm defaults at asset level V_B. Therefore, the default boundary will affect the value of security Y by two paths: firstly, the choice of V_B directly affects the values of all securities, as shown in earlier sections; secondly, the value of V_B affects the liquidation timing which, in turn, affects the valuation of security Y. When the holder of security Y decides whether to default or not, the impact of choice of V_B on liquidation threshold V_L must be taken into account. In other words, the two boundaries are determined simultaneously rather than separately.

I use loops to find the optimal boundaries here. The process is illustrated in Figure 6.14. Using this process, optimal default boundary V_B can be calculated by choosing a particular party to maximize his wealth as well as the optimal liquidation level V_L. In the following result, the threshold values are incremented on 1 unit due to the computer capacity. A more accurate result can be obtained by reducing the step size in each increment. A distinction in my model is that we are not limited to the scenario of only shareholders can decide when the firm is supposed to enter into the default and liquidation state. Creditors are also allowed to file for bankruptcy to the court.

First, I look at the optimal liquidation timing. The question can be reduced to: given different values of V_B, what the optimal V_L will be if chosen by different party? The results are displayed in Table 6.1, which support my initial guess in terms of control right allocation.[95] For a given level renegotiation threshold V_B (ranging from 20% to 100% of aggregated debt face value), the table shows what the optimal liquidation boundary V_L will be if it is set to maximize the security value hold by bank creditor, bondholders, the shareholders respectively.[96] The optimal boundary to maximize the firm value is also calculated and reported in the table.

Now it is manifest from the table that neither the bank creditors' nor an equity holder's optimal liquidation threshold are perfectly in line with an ideal boundary to maximize the total value of the firm. It turns out that after the filing for Chapter 11, if the control right to liquidate the firm is given to bondholders, the firm value maximization is most likely to be achieved. For both banks and the firm, there exists an incentive to early liquidation. The conclusion is clearer in Figure 6.15 which plots the optimal liquidation boundary to maximize the value of bank debt, bond debt, equity and firm respectively. The dotted line

[95] Appendix C.4 gives the Pseudo-code for solution of the optimization problem for liquidation threshold.

[96] I have to reduce the size of the claims proportionately here due to the computation capacity limitation. The initial asset value is 100 now, with face value of bank and market debt 25 and equity 50 respectively.

(represented the optimal liquidation boundary is determined by bondholders) and the solid line (the optimal liquidation boundary to maximize firm value) almost coincide with each other, regardless of the default boundary. Another point worth mentioning from Table 6.1 is that optimal liquidation boundary V_L, chosen by debt holder, is positively associated with the default boundary V_B. In an infinite maturity case in Bruche and Naqvi (2010), they arrive at the same conclusion.[97]

Given the optimal liquidation boundary, the optimal default threshold can be calculated now. Recall from Figure 6.1 that bond values are a decreasing function of renegotiation threshold. Therefore, they would postpone the renegotiation until as late as possible. Contrarily, the equity value is a concave function of default threshold and its value peaks at some point in between the liquidation boundary and the amount of total debt. The question thus becomes: anticipating the behaviour of the 'killer' of the firm, when should the manager choose to file for Chapter 11 in order to maximize the shareholders' value? Table 6.2 gives the equity values under different combination of default boundaries and optimal liquidation boundary by bank, bondholder and equity. Column (1), (3) and (5) is the optimal liquidation boundaries extracted from

[97] Bruche and Naqvi (2010) provides an economic explanation as well as a mathematical proof for this positive relationship when the liquidation boundary is determined by the debtholders under the assumption of consol debt. They give mathematical proof for the conclusion in Appendix C.1.

Table 6.1. Column (2), (4) and (6) shows the equity values for a given default boundary and endogenously determined liquidation boundary. Therefore, if the liquidation decision is made by bank creditor, the optimal default threshold for equity holder is $V_B = 33$. In the other two cases when the liquidation boundary is determined by bondholders or by equity holder, correspondingly, the equity holders are expected to default when $V_B = 42$. It is worth noting that the combination of $[V_B, V_L] = [33, 24]$ is quantitatively similar with the empirical findings in Davydenko (2012) as $V_B = 33$ in our case implies a default boundary of $V_B/D = 33/50 = 66\%$, which is exactly the boundary they found with real data. This may suggest in current implementation of bankruptcy code, the bank is influential in determining the liquidation timing of the firm.

Several structural models with endogenous barrier have calculated the optimal default boundary determined by shareholders. In Leland (1994) where default leads to liquidation, he gives the closed-form solution as follows:

$$V_B = \frac{(1-\tau)C}{r + 0.5\sigma^2} \implies \frac{V_B}{C/r} = \frac{(1-\tau)r}{r + 0.5\sigma^2}. \tag{6.1}$$

With the parameter values in the base case plugged into the formula, the boundary from their model is 42.74%, which is considerably lower than the default boundary in my model. In François and Morellec (2004)[98], the optimal bankruptcy boundary is

[98] Proposition 4 in the paper gives full descriptions of the equation.

$$V_B = \frac{(1-\tau)C}{r}\frac{\xi}{\xi+1} \quad \Longrightarrow \quad \frac{V_B}{C/r} = (1-\tau)\frac{\xi}{\xi+1}. \tag{6.2}$$

In the paper the authors compare different choices of parameter values and report their default boundaries ranging from 32.09% to 55.59%. Therefore, as shown in this model, when liquidation boundary and finite debt is introduced, the optimal default boundary determined by the shareholder is increased.

6.5 Duration of Grace Period

I mentioned in Chapter 4 the fact that the duration of defaulted firms stays in Chapter 11 varies significantly by individual case. The shortest one in Moody's URD is 32 days while the longest case is 4690 days. Despite the 120 days specified in the code to allow the renegotiation plan to be proposed, in the past the bankruptcy court in the U.S tends to extend the length of grace period. However, this action seems to be less and less likely to happen. The shift of bankruptcy court's attitude inspires me to use the model in this thesis to investigate what the optimal duration would be to maximize the firm value.

Holding other variables fixed, I show how the values of firm, debt and equity change with d in Figure 6.16. From the figure, I obtain several new empirical implications of the model with regards to the expected time in bankruptcy as a function of different firm characteristics. For firms with higher fraction of tangible assets, bank debt value decrease with d, indicating the bank's

preference for early liquidation. However, both bank and bond debt value increase with d for firms with higher liquidation cost. The shape of equity value and firm value is the most interesting. The values peak at $d = 3$ months for equity value and approximately $d = 6$ months for firm value. The U.S bankruptcy law specifies a grace period of 3 months, which can be clearly supported by my model.[99]

6.6 Conclusion

Chapter 11 of the U.S Bankruptcy Code allows economically sound but financially distressed firms to restructure and improve the value of their overall operations. Rather than wealth transfer to only one stakeholder group, ideally, the shareholders and all types of creditor should benefit from this restructuring. In this chapter, I examine how this ideal is achieved by investigating the determination of default and liquidation boundary in a continuous time asset pricing model with all the characteristics of bankruptcy law incorporated. Compared with early models, this model suggests some fresh findings in the determination of an endogenous default boundary; for instance, the liquidation

[99] Additional findings predicted by the models can be accessed by running the programming code, including (1) the optimal d is insensitive to the volatility of asset value and; (2) firms with lower average maturity of debt in their capital structure should spend more time in Chapter 11. The author omits these as the economic driving forces for the relationship is unclear.

cost has no impact on the shareholders' liquidation decision when renegotiation is introduced.

Furthermore, this chapter finds that the firm value arrives at maximization if the control right[100] is reallocated from shareholder to bondholders. Despite various models developed either under creditor-friendly or debtor-friendly bankruptcy law, few theoretical papers ask which is better. The results in this chapter recommend a creditor-friendly regime in a normative sense, which sheds light on the design of a bankruptcy code.[101]

[100] The control right here refers to the rights to file for bankruptcy (Chapter 11 or Chapter 7), which is reflected by the determination of the default barrier and the liquidation barrier.

[101] The work of Mankart and Rodano (2015) also touches up the topic of bankruptcy law reform. They show that secured credit significantly changes the trade-off between insurance and credit market conditions. Therefore, the analysis on the effect of bankruptcy law in models without secured credit yields misleading results because it overstates the worsening of credit market condition. These models thus overstate the negative effects of a generous bankruptcy law.

Figure 6.1 Bankruptcy Barrier Effect

Figure 6.1 shows the valuations for debt, equity and firm as function of default boundary. The liquidation boundary is fixed and set as $V_L = 40\%$. Figure 6.1 allows the default boundary V_B vary between the fixed liquidation boundary V_L and the face value of total debt.

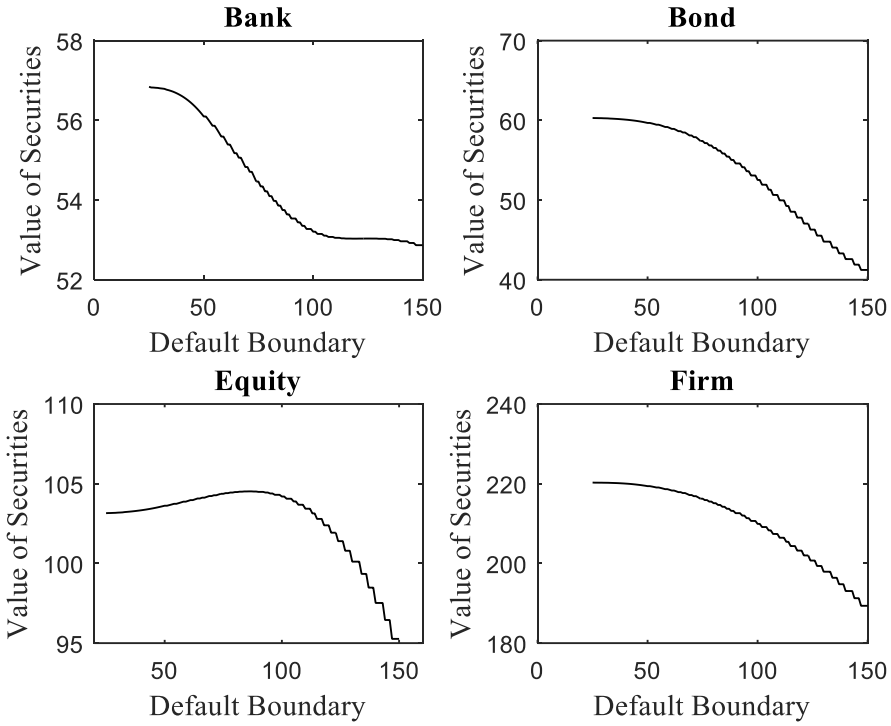


Figure 6.2 Barrier Effect in Related Models

Figure 6.2 shows the valuation results from two related papers Leland (1994) and François and Morellec (2004) as function of default boundary. The liquidation boundary is fixed and set as $V_L = 40\%$.

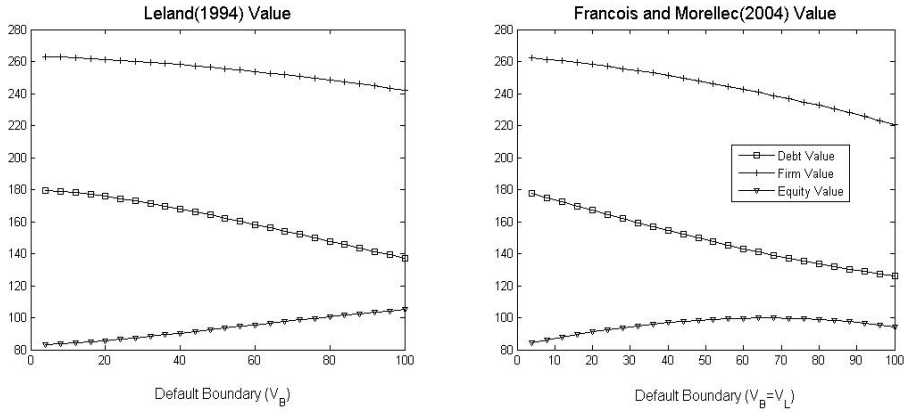


Figure 6.3 Values as Function of Default Barrier with Different Maturity

Figure 6.3 shows the valuation results from my model as function of default boundary when debt maturity is five years or ten years. The liquidation boundary is fixed and set as $V_L = 40\%$.

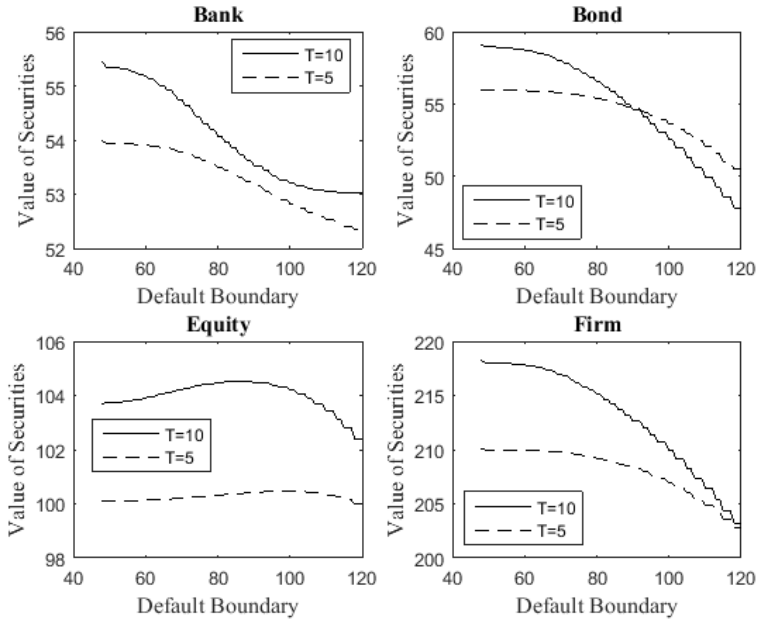


Figure 6.4 Values as Function of Default Barrier with Different Volatility

Figure 6.4 shows the valuation results from my model as function of default boundary when asset volatility is 0.2 or 0.4. The liquidation boundary is fixed and set as $V_L = 40\%$.

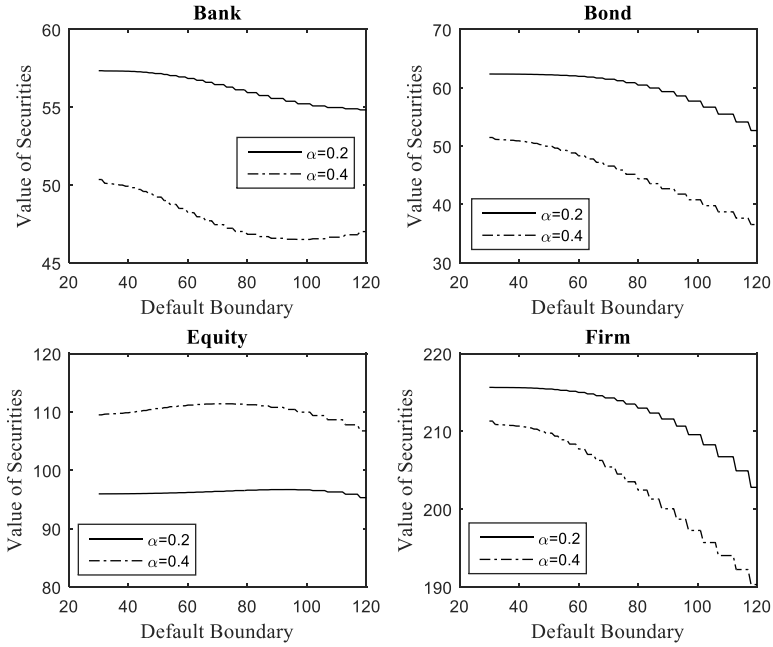


Figure 6.5 Values as function of default barrier with different liquidation cost

Figure 6.5 shows the valuation results from my model as function of default boundary when proportionate liquidation cost is 0.2 or 0.4. Liquidation boundary is fixed and set as $V_L = 40\%$.

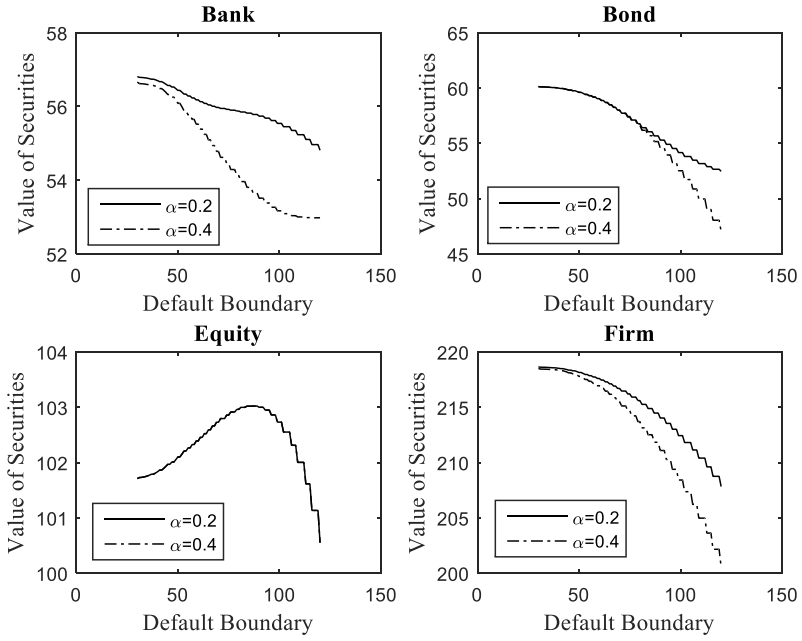


Figure 6.6 Values as Function of Default Barrier with Different Distress Cost

Figure 6.6 shows the valuation results from my model as function of default boundary when proportionate distress cost is 0.02 or 0.05. Liquidation boundary is fixed and set as $V_L = 40\%$.

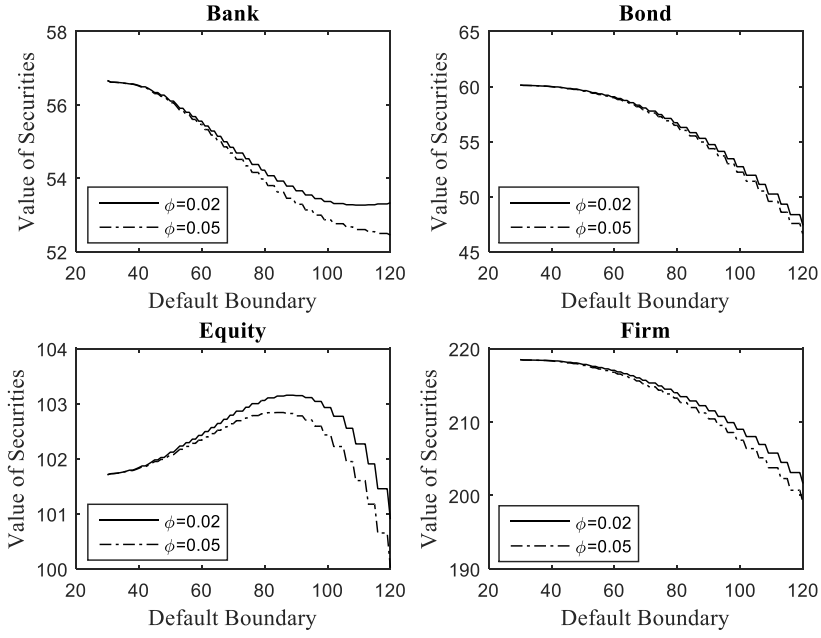


Figure 6.7 Values as Function of Default Barrier with Different Debt Structure

Figure 6.7 shows the valuation results from my model as function of default boundary when secured debt makes up 20% or 80%. Liquidation boundary is fixed and set as $V_L = 40\%$.

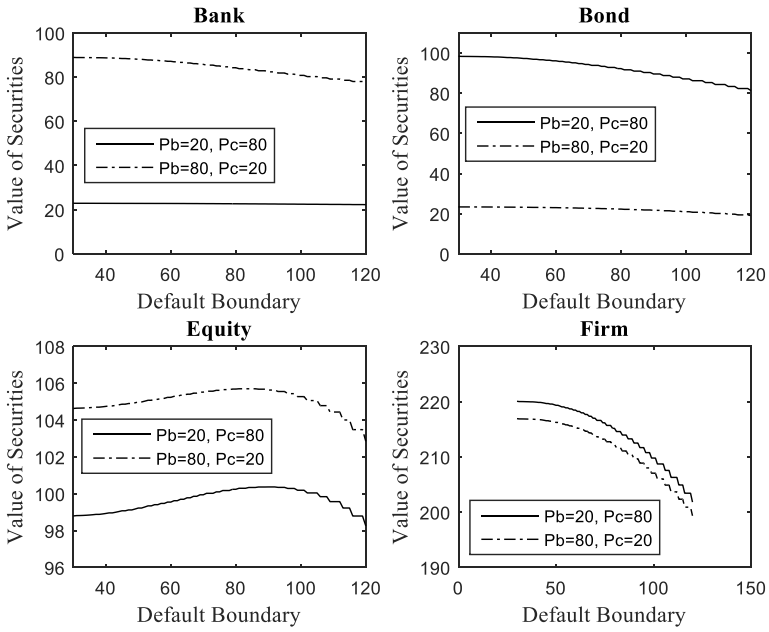


Figure 6.8 Liquidation Barrier Effect (Secured debt making up 50%)

Figure 6.8 shows the valuations for debt, equity and firm as function of liquidation boundary. The default boundary is fixed and set as $V_B = 70\%$. Figure 6.8 allows the liquidation boundary V_L vary between five and fixed default boundary V_B. Secured debt makes up 50% of total debt.

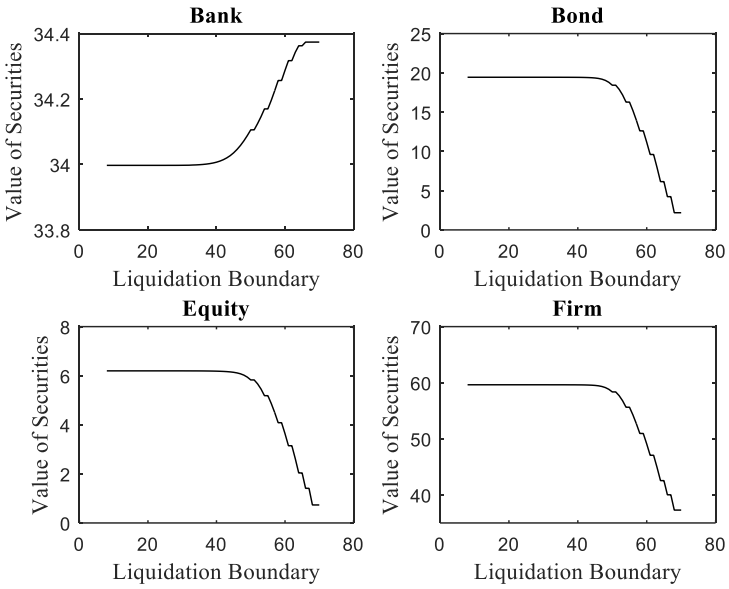


Figure 6.9 Values as Function of Liquidation Barrier with Different Maturity

Figure 6.9 shows the valuation results from my model as function of liquidation boundary when debt maturity is five years or ten years. Default boundary is fixed and set as $V_B = 70\%$.

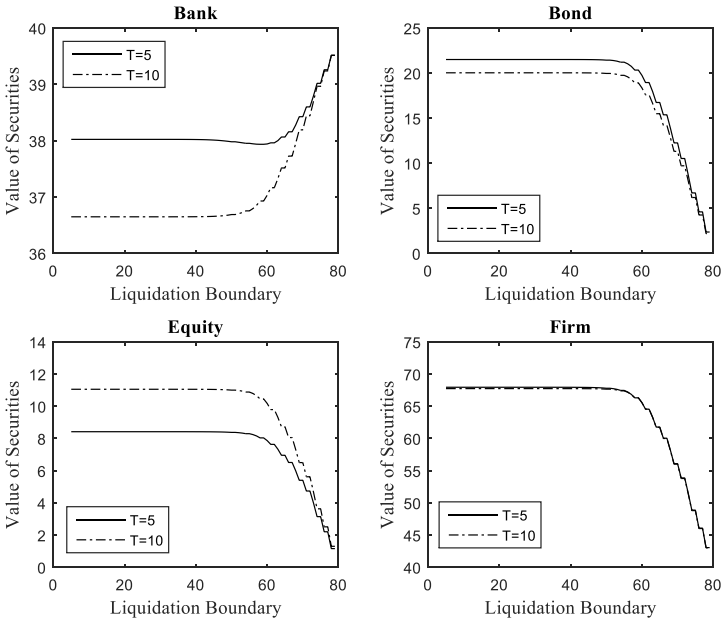


Figure 6.10 Values as Function of Liquidation Boundary under Volatility

Figure 6.10 shows the valuation results from my model as function of liquidation boundary when asset volatility is 0.2 or 0.4. Default boundary is fixed and set as $V_B = 70\%$.

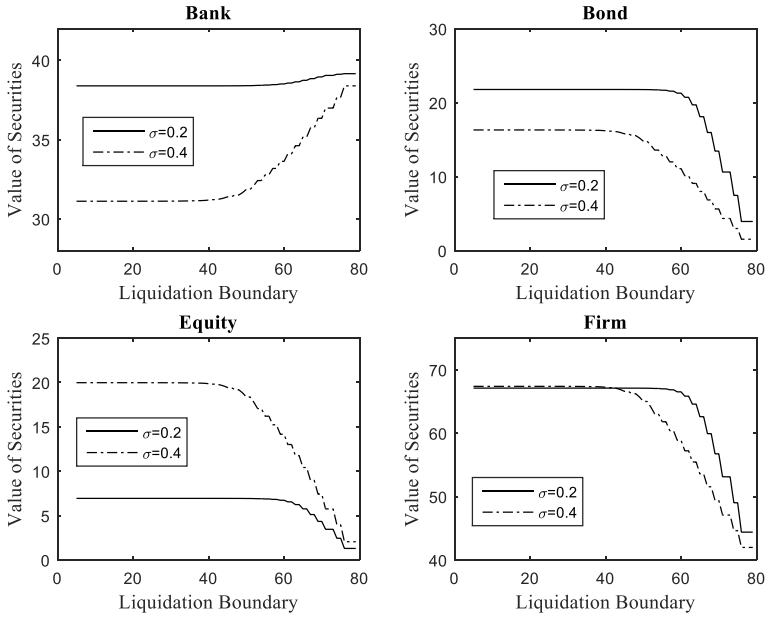


Figure 6.11 Values as Function of Default Barrier with Different Debt Structure (Secured debt making up 20%)

Figure 6.11 shows the valuation results from my model as function of liquidation boundary when secured debt makes up 20% of total debt. Default boundary is fixed and set as $V_B = 70\%$.

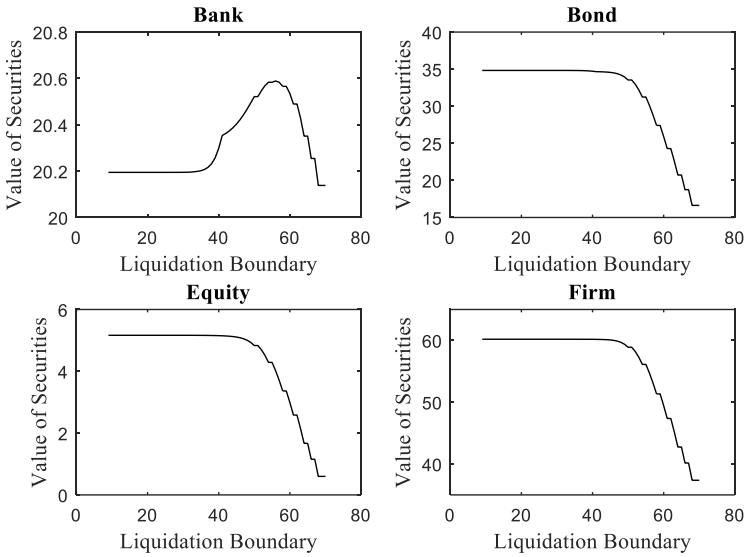


Figure 6.12 Values as function of default barrier with different liquidation cost

Figure 6.12 shows the valuation results from my model as function of liquidation boundary when proportionate liquidation cost is 0.2 or 0.4. Default boundary is set as $V_B = 70\%$.

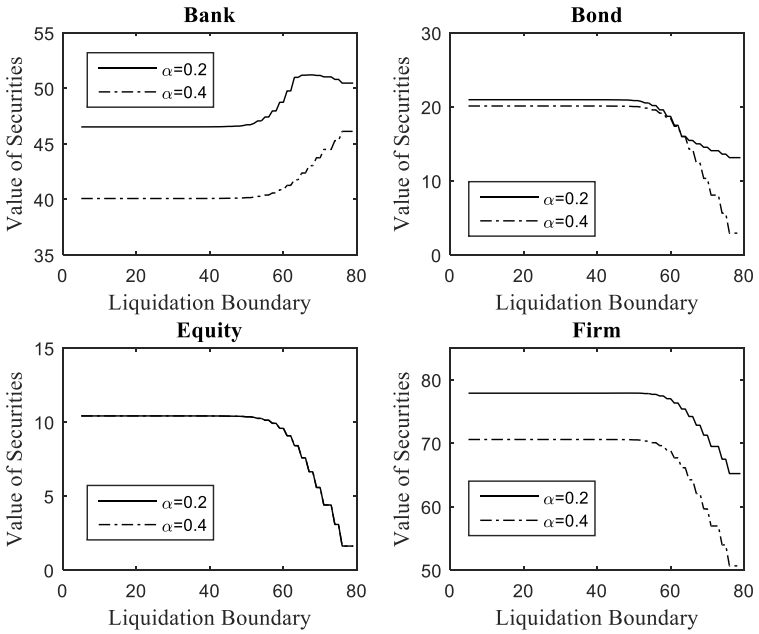


Figure 6.13 Values as Function of Default Barrier with Different Distress Cost

Figure 6.13 shows the valuation results from my model as function of liquidation boundary when distressed cost is 0 or 0.05. Default boundary is fixed and set as $V_B = 70\%$.

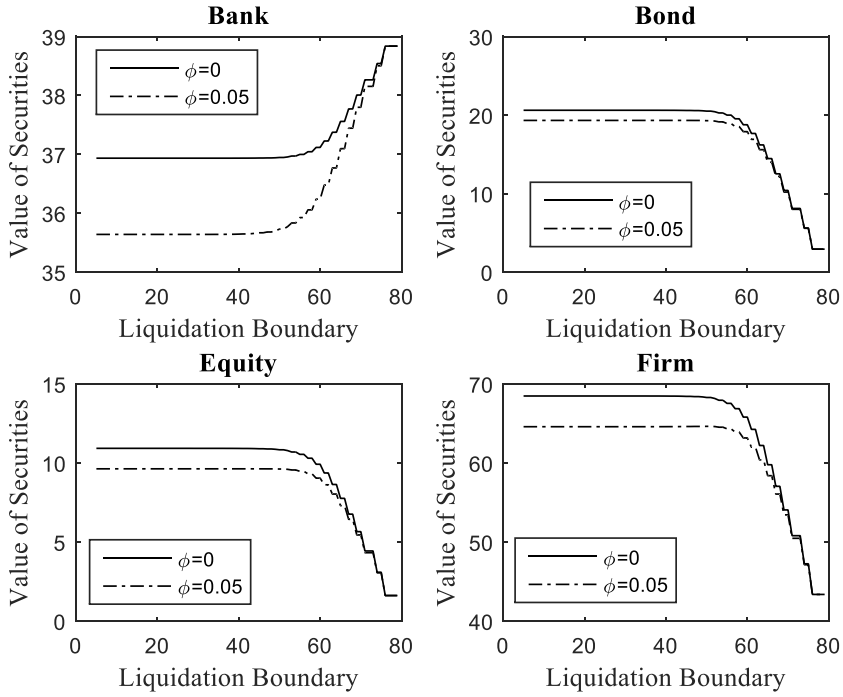


Figure 6.14 The Computing Process of Optimal Default and Liquidation Boundaries

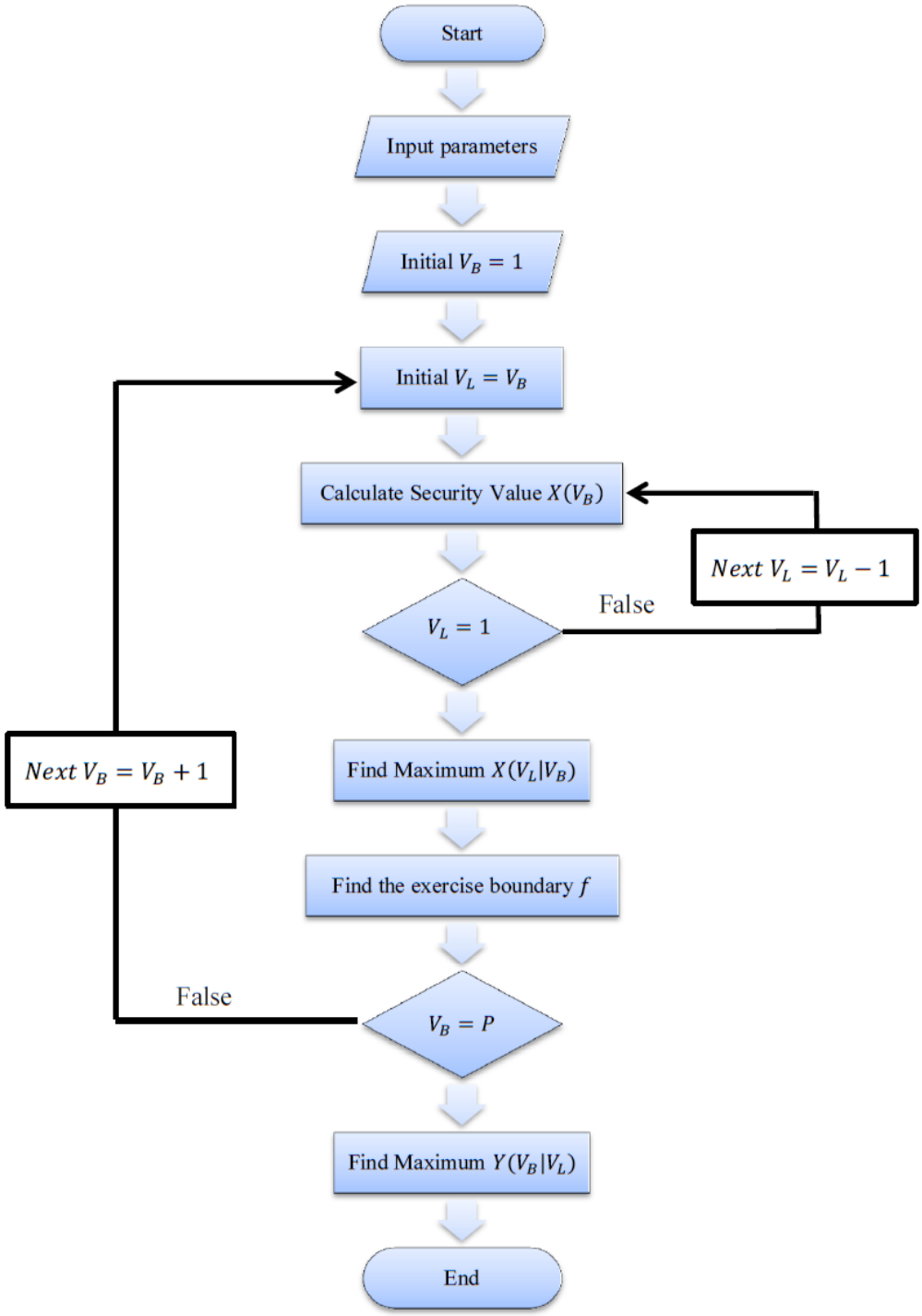


Figure 6.15 Optimal Endogenously-determined Liquidation Boundaries

Figure 6.15 plots the optimal liquidation boundary to maximize the value of bank debt, bond debt, equity and firm respectively, given default boundary ranging from 10 (20% of total debt) to 50 (100% of total debt).

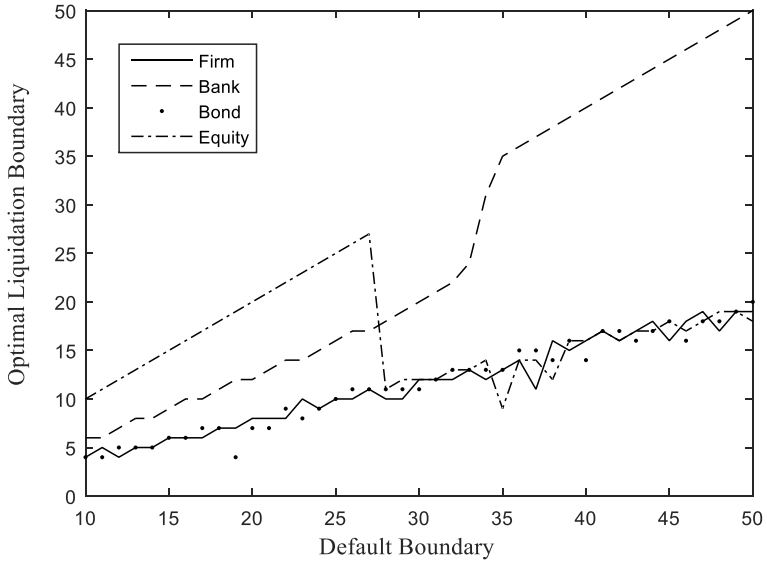


Figure 6.16 Valuations as Functions of Length of Grace Period

Figure 6.16 shows how the values of firm, debt and equity change with d - the duration of grace period, under different liquidation cost.

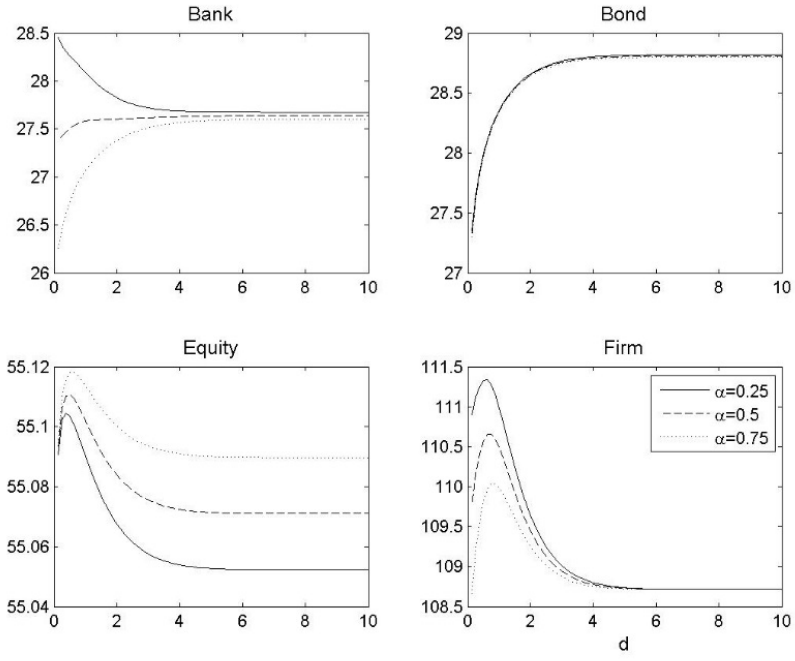


Table 6.1 Optimal Liquidation Level Chosen by Different Parties

This table shows the optimal liquidation boundary if it is chosen by the firm, bank, bondholders or shareholders respectively, to maximize their own benefit. The parameters we use are: $V_0 = 100$, $P_b = P_c = 25$, $b = 0.08$, $c = 0.1$, $r = 6\%$, $\alpha = 0.5$, $\sigma = 0.25$, $\phi = 0.02$, $T = 10$, $d = 1$, $\eta = [1/3, 1/3, 1/3]$, $\delta = 0.05$, and tax rate is 35%.

	Optimal Liquidation Boundary (V_L)					**Optimal Liquidation Boundary (V_L)**			
V_B	**Bank**	**Bond**	**Equity**	**Firm**	**V_B**	**Bank**	**Bond**	**Equity**	**Firm**
10	6	4	10	4	31	21	12	12	12
11	6	4	11	5	32	22	13	13	12
12	7	5	12	4	33	24	13	13	13
13	8	5	13	5	34	31	13	14	12
14	8	5	14	5	35	35	13	9	13
15	9	6	15	6	36	36	15	14	14
16	10	6	16	6	37	37	15	14	11
17	10	7	17	6	38	38	14	12	16
18	11	7	18	7	39	39	16	16	15
19	12	4	19	7	40	40	14	16	16
20	12	7	20	8	41	41	17	17	17
21	13	7	21	8	42	42	17	16	16
22	14	9	22	8	43	43	16	17	17
23	14	8	23	10	44	44	17	17	18
24	15	9	24	9	45	45	18	18	16
25	16	10	25	10	46	46	16	17	18
26	17	11	26	10	47	47	18	18	19
27	17	11	27	11	48	48	18	19	17
28	18	11	11	10	49	49	19	19	19
29	19	11	12	10	50	50	20	18	19
30	20	11	12	12					

Table 6.2 Optimal Default Boundary Chosen by Equity Holders

V_B	(1) Optimal V_L by Bank	(2) Equity Value	(3) Optimal V_L by Bond	(4) Equity Value	(5) Optimal V_L by Equity	(6) Equity Value
10	6	51.2181	4	51.2181	10	51.2236
11	6	51.2226	4	51.2226	11	51.2307
12	7	51.2269	5	51.2269	12	51.2376
13	8	51.2339	5	51.234	13	51.2484
14	8	51.2416	5	51.2417	14	51.2601
15	9	51.2514	6	51.2514	15	51.2746
16	10	51.261	6	51.2611	16	51.2886
17	10	51.2726	7	51.2726	17	51.305
18	11	51.2862	7	51.2863	18	51.3238
19	12	51.3023	4	51.3024	19	51.3451
20	12	51.3162	7	51.3163	20	51.3627
21	13	51.3316	7	51.3318	21	51.3816
22	14	51.3549	9	51.3552	22	51.4083
23	14	51.3748	8	51.375	23	51.4291
24	15	51.3889	9	51.3893	24	51.443
25	16	51.4118	10	51.4123	25	51.4633
26	17	51.4366	11	51.4373	26	51.4823
27	17	51.4544	11	51.455	27	51.4935
28	18	51.4822	11	51.4831	11	51.4831
29	19	51.5013	11	51.5027	12	51.5027
30	20	51.5314	11	51.5331	12	51.5331
31	21	51.5516	12	51.5539	12	51.5539
32	22	51.5712	13	51.5749	13	51.5749
33	**24**	**51.5884**	13	51.5958	13	51.5958
34	31	51.5254	13	51.6164	14	51.6164
35	35	51.4388	13	51.6363	9	51.6363
36	36	51.3986	15	51.6551	14	51.6551
37	37	51.3469	15	51.6724	14	51.6724
38	38	51.2821	14	51.6875	12	51.6875
39	39	51.2443	16	51.694	16	51.694
40	40	51.1567	14	51.7047	16	51.7047
41	41	51.0514	17	51.7114	17	51.7114
42	42	50.9916	**17**	**51.713**	**16**	**51.713**
43	43	50.856	16	51.7121	17	51.7121
44	44	50.6976	17	51.7048	17	51.7048
45	45	50.6091	18	51.6984	18	51.6984
46	46	50.412	16	51.6792	17	51.6792
47	47	50.3028	18	51.666	18	51.666
48	48	50.1861	18	51.6503	19	51.6503
49	49	49.9287	19	51.6103	19	51.6103
50	50	49.7874	20	51.5856	18	51.5856

CHAPTER 7

Conclusion

This thesis focuses on several questions in chapters 4, 5, and 6. It contains both empirical analysis as well as theoretical construction, with a theoretical-oriented focus.

First, in Chapter 4, I investigate how post-default renegotiation is conducted using a trilateral negotiation model and how equity value and debt value pre-default will be in consideration of all the following renegotiation outcomes. Fruitful predictions can be generated from the model. Particularly, I discuss the determination of creditors' recovery on their defaulted debts. The empirical study in this chapter provides support on the role of debt structure. Consistent with the predictions, I find: (1) In Chapter 11, bank debt share in total debt is positively related to the ultimate firm-level recoveries for all creditors. (2) The sensitivity of recovery rate to bank debt share diminished with the amount of debt and bargaining friction. (3) Despite overall recovery is improved, for bondholders there is no clear-cut spill-over benefit.

In Chapter 5, I recreate a structural model of credit risks with bankruptcy procedures embodied and assumption of single consol debt relaxed. I calculate the value of the claim using finite difference method and investigate the term structure of credit spread, especially the impact of asset volatility, debt structure, capital structure, bargaining power, etc. With respect to the shape of term structure of credit spread, the model presents a variety of results with different combination of debt structure and duration of grace period. The sensitivity study in this chapter further displays unique characteristics of the term structure in the presence of Chapter 11 reorganization, Chapter 7 liquidation and grace period.

Chapter 6 attempts to contribute to policy reforms regarding bankruptcy code through analysis of interest conflicts in bankruptcy. I examine the optimal decision making in distressed companies. Within the framework proposed in Chapter 5, I study the characteristics of optimal renegotiation and liquidation boundary, if they are determined by shareholder, bank or bondholders. This chapter also reveals that the total firm value can be maximized if the length of grace period is set to be six months.

However, there are some weaknesses and limitations of this thesis. The model in Chapter 4 relies heavily on the assumption that bond debt is more dispersed and has less bargaining power vis-à-vis bank creditor. This assumption only holds when the bonds are diversely held. In the case of private placements or when there is a bondholder's representative, this assumption does not

necessarily hold. Empirically, to address the limitation, further studies could distinguish different bond ownership structure if the bond trading data or detailed bankruptcy filing data are available. Additionally, workers and state may have priority over bank debt and bond debt, which might potentially impact the firm-level recovery rate in the empirical analysis.

In Chapter 5, an explicit finite difference scheme is used in the numerical implementation. This approach is numerically stable and convergent only if a certain mathematical condition is met. This could be addressed by using more advanced schemes such as Successive Over-Relaxation or Crank–Nicolson method, but at the cost of a little more mathematical difficulty. This is beyond the scope of the thesis so I choose the basic approach. Furthermore, in this chapter I use a reduced-form solution for the valuation of strategic debt service during the Chapter 11 period. The process of renegotiation is not fully displayed in this chapter, as what I have done in Chapter 4.

In the third empirical chapter, ideally, the endogenous determined boundaries are generated as the outcomes that maximize total firm value, debt value or equity value, ex ante upon filing for Chapter 11. However, this type of problem cannot be solved by the author. I follow the method in François and Morellec (2004) in which the boundary is determined to maximize the values ex ante at the date of debt issuance. In addition, this chapter lacks an empirical study with

regarding to the actual default threshold and liquidation threshold to support the validity of the model in this chapter.

There are several pathways for further research on this topic. (1) In this model, the two types of debts, bank loan and bonds differ in seniority and the number of holders. Accordingly banks exhibit greater powers in restricting equity holders from opportunistic behavior. Since most bank loans are secured obligations, in future work it would be promising to add the collateral consideration in modelling, especially how the type of collateral may influence the renegotiation. (2) The lifetime of the firm can be moved back to the time when it needs to make investment decisions. How strategic default will impact on firm's ex ante investment decisions is also an interesting topic. Sundaresan and Wang (2007) has made some pioneering effort in this direction but it is limited to the bilateral setting. Furthermore, in all the three chapters, the capital structure and the debt structure are taken as fixed and given exogenously. It is interesting to build a model in which the fraction of bank debt is endogenous. (3) In debt renegotiation, there are some non-financial debts. For instance, workers and state may have priority over bank- and bond- debt. It would be interesting to know how this affects the renegotiation outcome.

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Appendices

Appendix A Proofs of the Model in Chapter 4

In this section, I show the detailed derivation needed for Chapter 4.

Proof of Proposition 1

Substitute $x_i^k = v_i$ and $x_k^k = v(V_S) - \sum_{i \neq k} v_i$ into equation (6) yields:

$$\begin{aligned} v_i &= \beta q \sum_k p_k \, x_i^k + \beta(1-q) v_i^L \\ &= \beta q \left(\sum_{k \neq i} p_k \, x_i^k + p_i x_i^i \right) + \beta(1-q) v_i^L \\ &= \beta q \left(\sum_{k \neq i} p_k \, v_i + p_i x_i^i \right) + \beta(1-q) v_i^L \\ &= \beta q \left(v_i \sum_{k \neq i} p_k + p_i \left(v(V_S) - \sum\nolimits_{j \neq i} v_j \right) \right) \\ &\qquad + \beta(1-q) v_i^L \\ &= \beta q \left((1-p_i) v_i + p_i \left(v(V_S) - \sum\nolimits_{j \neq i} v_j \right) \right) \\ &\qquad + \beta(1-q) v_i^L \end{aligned} \tag{A.1}$$

Summing the above equation (A.1) over all i, I obtain

$$\begin{aligned} \sum\nolimits_i v_i = \; & \beta \sum\nolimits_i \left[q p_i \left(v(V_S) - \sum\nolimits_{j \neq i} v_j \right) + q(1-p_i) v_i \right. \\ & \left. + (1-q) v_i^L \right] \end{aligned} \tag{A.2}$$

$$
\begin{aligned}
&= \beta \sum_i \left[q p_i \left(v(V_S) - \sum_{j \neq i} v_j \right) + q(1-p_i) v_i \right] \\
&\quad + \beta \sum_i (1-q) v_i^L \\
&= \beta q \sum_i \left[p_i v(V_S) - p_i \sum_i v_i + v_i \right] \\
&\quad + \beta (1-q) \sum_i v_i^L \\
&= \beta q v(V_S) + \beta (1-q)[(1-\alpha)V_S - K]
\end{aligned}
$$

Substituting back in (A.1) for $\sum_{j \neq i} v_j$,

$$
\begin{aligned}
v_i = \;& \beta q \left((1-p_i) v_i + p_i \left(v(V_S) - \sum_{j \neq i} v_j \right) \right) + \beta (1-q) v_i^L \\
= \;& \beta q \left(v_i + p_i \left(v(V_S) - \sum_i v_i \right) \right) + \beta (1-q) v_i^L \\
= \;& \beta q \big(v_i + p_i v(V_S) - p_i (\beta q v(V_S) + \beta (1-q) V^L) \big) \\
& + \beta (1-q) v_i^L \\
= \;& \beta q \left(v_i + p_i \big((1-\beta) v(V_S) + \beta (1-q) G \big) \right) + \beta (1-q) v_i^L
\end{aligned}
\tag{A.3}
$$

Denote $G = v(V_S) - V^L = v(V_S) - (1-\alpha)V_S + K$ as the renegotiation surplus, (A.3) is equivalent to

$$
\begin{aligned}
v_i = \;& \beta q \left(v_i + p_i \big((1-\beta) v(V_S) + \beta (1-q) G \big) \right) \\
& + \beta (1-q) v_i^L
\end{aligned}
\tag{A.4}
$$

Rearranging (A.4) yields

$$
v_i = \frac{\beta q \left(p_i \big((1-\beta) v(V_S) + \beta (1-q) G \big) \right) + \beta (1-q) v_i^L}{1 - \beta q} \tag{A.5}
$$

Thus, in equilibrium, the proposer k himself will receive

$$
\begin{aligned}
x_k^k = \;& v(V_S) - \sum_{i \neq k} v_i \\
= \;& v(V_S) - \sum_{i \neq k} \left[\frac{\beta q p_i \big((1-\beta) v(V_S) + \beta (1-q) G \big)}{1 - \beta q} \right. \\
& \left. + \frac{\beta (1-q) v_i^L}{1 - \beta q} \right] \\
= \;& v(V_S) - \frac{\beta q (1-p_k) \big((1-\beta) v(V_S) + \beta (1-q) G \big)}{1 - \beta q}
\end{aligned}
\tag{A.6}
$$

$$
\begin{aligned}
& -\frac{\beta(1-q)(V^L - v_k^L)}{1-\beta q} \\
= \quad & v(V_S) - \frac{\beta q(1-p_k)\big((1-\beta)v(V_S) + \beta(1-q)G\big)}{1-\beta q} \\
& -\frac{\beta(1-q)(V^L - v_k^L)}{1-\beta q}
\end{aligned}
$$

and other players i receives v_i as in (A.5).

Proof of Proposition 2

$$
\begin{cases}
rv(V) = (r-\mu)V + \tau(b+c) + \mu V v'(V) + \frac{1}{2}\sigma^2 V^2 v''(V) & V > V_S \\
rv(V) = (r-\mu)V + \mu V v'(V) + \frac{1}{2}\sigma^2 V^2 v''(V) & V > V_S
\end{cases} \tag{A.7}
$$

with boundary conditions

$$
\begin{aligned}
& \lim_{V\uparrow\infty} v(V) = V + \tau\frac{b+c}{r} \\
& \lim_{V\downarrow V_S} v(V) = \lim_{V\uparrow V_S} v(V) \\
& \lim_{V\downarrow V_S} v'(V) = \lim_{V\uparrow V_S} v'(V) \\
& \lim_{V\downarrow 0} v(V) = 0 \\
& V_A(V_t) = A_1 {V_t}^{\lambda^-} + V_t + \frac{\tau(b+c)}{r} \\
& V_P(V_t) = B_1 {V_t}^{\lambda^-} + B_2 {V_t}^{\lambda^+} + V_t \\
& V_A(V_t) = \frac{\lambda^+}{\lambda^- - \lambda^+}\frac{\tau(b+c)}{r}\left(\frac{V_t}{V_S}\right)^{\lambda^-} + V_t + \frac{\tau(b+c)}{r} \\
& V_P(V_t) = B_1 {V_t}^{\lambda^-} + \frac{\lambda^-}{\lambda^- - \lambda^+}\frac{\tau(b+c)}{r}\left(\frac{V_t}{V_S}\right)^{\lambda^+} + V_t.
\end{aligned} \tag{A.8}
$$

Proof of Proposition 3

$$
E_A(V_t) = \quad (1-\tau)\left(V_t - \frac{b+c}{r}\right) \tag{A.9}
$$

$$
\begin{aligned}
&+\left[\left(1-q(1-p_e)\right)\alpha-(1-\tau)\right]V_S\left(\frac{V_t}{V_S}\right)^{\lambda^-}\\
&+\left[(1-\tau)\frac{b+c}{r}-\left(1-q(1-p_e)\right)\right.\\
&\left.\left(\frac{\lambda^-}{\lambda^+-\lambda^-}\frac{\tau(b+c)}{r}-K\right)\right]\left(\frac{V_t}{V_S}\right)^{\lambda^-}\\
=\;&(1-\tau)\left(V_t-\frac{b+c}{r}\right)\\
&+\left[\left(1-q(1-p_e)\right)\alpha-(1-\tau)\right]{V_t}^{\lambda^-}{V_S}^{1-\lambda^-}\\
&+\left[(1-\tau)\frac{b+c}{r}-\left(1-q(1-p_e)\right)\right.\\
&\left.\left(\frac{\lambda^-}{\lambda^+-\lambda^-}\frac{\tau(b+c)}{r}-K\right)\right]{V_t}^{\lambda^-}{V_S}^{-\lambda^-}
\end{aligned}
$$

The first-order condition with respect to V_S gives

$$0=(1-\lambda^-)a{V_S}^{-\lambda^-}+(-\lambda^-)b{V_S}^{-\lambda^--1} \tag{A.10}$$

with

$$a=\left[\left(1-q(1-p_e)\right)\alpha-(1-\tau)\right]{V_t}^{\lambda^-} \tag{A.11}$$

$$
\begin{aligned}
b=\;&\left[(1-\tau)\frac{b+c}{r}\right.\\
&\left.-(1-q+qp_e)\left(\frac{\lambda^-}{\lambda^+-\lambda^-}\frac{\tau(b+c)}{r}-K\right)\right]{V_t}^{\lambda^-}
\end{aligned} \tag{A.12}
$$

The solution to this equation is reported in Proposition 3 as

$$
\begin{aligned}
V_S\;&=\frac{\lambda^-}{(1-\lambda^-)}\frac{b}{a}\\
&=\frac{\lambda^-}{(1-\lambda^-)}\frac{(1-\tau)\frac{b+c}{r}-\left(1-q(1-p_e)\right)\left(\frac{\lambda^-}{\lambda^+-\lambda^-}\frac{\tau(b+c)}{r}-K\right)}{\left(1-q(1-p_e)\right)\alpha-(1-\tau)}.
\end{aligned} \tag{A.13}
$$

Proof of Hypotheses

When $\beta = 1$,

$$RR_{\mathrm{T}} = (1 - \alpha + q(1 - p_e)\alpha)\frac{V_S}{D} - \big(1 - q(1 - p_e)\big)\frac{K}{D} - q(1 - p_e)\frac{\lambda^- \tau}{\lambda^+ - \lambda^-} \tag{A.14}$$

and

$$\frac{V_S}{D} = \frac{\lambda^-}{\lambda^- - 1}\left[\frac{q(1 - p_e)\frac{\lambda^- \tau}{\lambda^+ - \lambda^-} + 1 + \big(1 - q(1 - p_e)\big)\frac{K}{D}}{1 - \alpha + q(1 - p_e)\alpha}\right] \tag{A.15}$$

$$\begin{cases} \lambda^- = -\dfrac{1}{\sigma}\left[\dfrac{\left(\mu - \frac{1}{2}\sigma^2\right)}{\sigma} + \sqrt{\left(\dfrac{\mu - \frac{1}{2}\sigma^2}{\sigma}\right)^2 + 2r}\right] \\ \lambda^+ = -\dfrac{1}{\sigma}\left[\dfrac{\left(\mu - \frac{1}{2}\sigma^2\right)}{\sigma} - \sqrt{\left(\dfrac{\mu - \frac{1}{2}\sigma^2}{\sigma}\right)^2 + 2r}\right] \end{cases} \tag{A.16}$$

$$\lambda^+ - \lambda^- = \frac{2}{\sigma}\sqrt{\left(\frac{\mu - \frac{1}{2}\sigma^2}{\sigma}\right)^2 + 2r} \tag{A.17}$$

Therefore,

$$\frac{\partial\left(\frac{V_S}{D}\right)}{\partial\omega} = \frac{\partial\left(\frac{V_S}{D}\right)}{\partial p_e}\frac{\partial p_e}{\partial\omega} < 0 \tag{A.18}$$

$$\frac{\partial\left(\frac{V_S}{D}\right)}{\partial\alpha} = \frac{1 - q(1 - p_e)}{1 - \alpha + q(1 - p_e)\alpha}\frac{V_S}{D} > 0 \tag{A.19}$$

$$\frac{\partial\left(\frac{V_S}{D}\right)}{\partial K} = \frac{\lambda^-}{\lambda^- - 1}\frac{1}{D}\left[\frac{1 - q(1 - p_e)}{1 - \alpha + q(1 - p_e)\alpha}\right] > 0 \tag{A.20}$$

$$\frac{\partial\left(\frac{V_S}{D}\right)}{\partial q} = \frac{\lambda^-(1 - p_e)}{\lambda^- - 1}\left[\frac{\frac{\lambda^- \tau}{\lambda^+ - \lambda^-} - \frac{K}{D} - \alpha\frac{V_S}{D}}{1 - \alpha + q(1 - p_e)\alpha}\right] < 0 \tag{A.21}$$

$$\frac{\partial\left(\frac{V_S}{D}\right)}{\partial D} = \frac{V_S}{D} = -\frac{\lambda^-}{\lambda^- - 1}\left[\frac{1-q(1-p_e)}{1-\alpha+q(1-p_e)\alpha}\right]\frac{K}{D^2} < 0 \tag{A.22}$$

Moreover,

$$\begin{aligned} RR_{\mathrm{T}} = & \frac{\lambda^-}{\lambda^- - 1}(1-\alpha \\ & + q(1 \\ & - p_e)\alpha)\left[\frac{q(1-p_e)\frac{\lambda^-\tau}{\lambda^+ - \lambda^-} + 1 + \left(1-q(1-p_e)\right)\frac{K}{D}}{1-\alpha+q(1-p_e)\alpha}\right] \\ - & \left(1-q(1-p_e)\right)\frac{K}{D} - q(1-p_e)\frac{\lambda^-\tau}{\lambda^+ - \lambda^-} \\ = & \left(\frac{q(1-p_e)}{\lambda^- - 1}\right)\frac{\lambda^-\tau}{\lambda^+ - \lambda^-} + \frac{\lambda^-}{\lambda^- - 1} + \left(\frac{1-q(1-p_e)}{\lambda^- - 1}\right)\frac{K}{D} \end{aligned} \tag{A.23}$$

Therefore,

$$\frac{\partial RR_{\mathrm{T}}}{\partial p_e} = \left(\frac{q}{\lambda^- - 1}\right)\left[\frac{K}{D} - \frac{\lambda^-\tau}{\lambda^+ - \lambda^-}\right] < 0 \tag{A.24}$$

$$\frac{\partial RR_{\mathrm{T}}}{\partial \omega} = \frac{\partial RR_{\mathrm{T}}}{\partial p_e}\frac{\partial p_e}{\partial \omega} > 0 \tag{A.25}$$

$$\frac{\partial(V_S/D)}{\partial p_e} = \frac{q}{(1-\alpha+q(1-p_e)\alpha)}\left[\frac{\lambda^-}{\lambda^- - 1}\left(\frac{K}{D} - \frac{\lambda^-\tau}{\lambda^+ - \lambda^-}\right) + \alpha\frac{V_S}{D}\right] > 0 \tag{A.26}$$

$$\frac{\partial\left(\frac{V_S}{D}\right)}{\partial \omega} = \frac{\partial\left(\frac{V_S}{D}\right)}{\partial p_e}\frac{\partial p_e}{\partial \omega} < 0 \tag{A.27}$$

Furthermore, it can be computed that

$$\frac{\partial^2 RR}{\partial p_e \partial(1-q)} = -\frac{\partial^2 RR}{\partial p_e \partial q} = -\left(\frac{1}{\lambda^- - 1}\right)\left[\frac{K}{D} - \frac{\lambda^-\tau}{\lambda^+ - \lambda^-}\right] > 0 \tag{A.28}$$

$$\frac{\partial^2 RR}{\partial \omega \partial(1-q)} = \frac{\partial\left(\left(\frac{1-(1-q)}{\lambda^- - 1}\right)\left[\frac{K}{D} - \frac{\lambda^-\tau}{\lambda^+ - \lambda^-}\right]\right)}{\partial(1-q)}\frac{\partial p_e}{\partial \omega} \tag{A.29}$$

$$= \left(\frac{1}{1-\lambda^-}\right)\left[\frac{K}{D} - \frac{\lambda^-\tau}{\lambda^+ - \lambda^-}\right]\frac{\partial p_e}{\partial \omega} < 0$$

and

$$\frac{\partial^2 RR}{\partial \omega \partial D} = \left(\frac{q}{1-\lambda^-}\right)\left[\frac{K}{D^2}\right]\frac{\partial p_e}{\partial \omega} < 0. \tag{A.30}$$

For hypothesis 3, I look at the bargaining friction $1-q$,

$$\begin{aligned}\frac{\partial RR_T}{\partial(1-q)} &= -\frac{\partial RR_T}{\partial q} \\ &= \left(\frac{1-p_e}{\lambda^- - 1}\right)\left(\frac{K}{D} - \frac{\lambda^-\tau}{\lambda^+ - \lambda^-}\right) < 0.\end{aligned} \tag{A.31}$$

Next I investigate the impact of liquidation costs on the timing of default as well as the recovery rates of creditors. Again, I calculate the partial derivatives of V_S with respect to α and K.

$$\frac{\partial\left(\frac{V_S}{D}\right)}{\partial \alpha} = \frac{1-q(1-p_e)}{1-\alpha+q(1-p_e)\alpha}\frac{V_S}{D} > 0 \tag{A.32}$$

$$\frac{\partial\left(\frac{V_S}{D}\right)}{\partial K} = \frac{\lambda^-}{\lambda^- - 1}\frac{1}{D}\left[\frac{1-q(1-p_e)}{1-\alpha+q(1-p_e)\alpha}\right] > 0 \tag{A.33}$$

and

$$\frac{\partial RR_{\mathrm{T}}}{\partial \alpha} = 0 \tag{A.34}$$

$$\frac{\partial RR_{\mathrm{T}}}{\partial K} = \left(\frac{1-q(1-p_e)}{\lambda^- - 1}\right)\frac{1}{D} < 0. \tag{A.35}$$

It can be seen that both higher proportional and fixed liquidation costs will lead to an early default because this actually increases the renegotiation surplus. The total recovery rate is lower if the total liquidation cost is higher.

Proof of the Spill-Over Effect Condition

The recovery rate of the bondholders is

$$RR_{\mathrm{M}} = \frac{M_P(V_t)}{c/r} = \left[qp_m G + \left\{(1-\alpha)V_S - K - \frac{b}{r}\right\}^+\right]\frac{r}{c}$$

$$= \begin{cases} \frac{qp_m G}{(1-\omega)D} & \text{if } (1-\alpha)V_S - K < \frac{b}{r} \\ \frac{qp_m G}{(1-\omega)D} + \frac{(1-\alpha)V_S}{(1-\omega)D} - \frac{1}{1-\omega}\frac{K}{D} - \frac{\omega}{1-\omega} & \text{if } (1-\alpha)V_S - K \geq \frac{b}{r} \end{cases}$$

$$= \begin{cases} \frac{qp_m}{1-\omega}\left(\frac{\alpha V_S}{D} - \frac{\lambda^-}{\lambda^+ - \lambda^-}\tau + \frac{K}{D}\right) & \text{if } (1-\alpha)V_S - K < \frac{b}{r} \\ \frac{[1-\alpha(1-qp_m)]\frac{V_S}{\boldsymbol{D}} - \left(qp_m\frac{\lambda^-}{\lambda^+ - \lambda^-}\tau + \omega\right) - (1-qp_m)\frac{K}{D}}{1-\omega} & \text{if } (1-\alpha)V_S - K \geq \frac{b}{r} \end{cases} \quad \text{(A.36)}$$

When $(1-\alpha)V_S - K \geq \frac{b}{r}$,

$$RR_{\mathrm{M}} = \frac{1}{1-\omega}\left\{[1-\alpha(1-qp_m)]\frac{V_S}{\boldsymbol{D}} - \left(qp_m\frac{\lambda^-}{\lambda^+ - \lambda^-}\tau + \omega\right) - (1-qp_m)\frac{K}{D}\right\} \quad \text{(A.37)}$$

$$\frac{\partial(RR_{\mathrm{M}})}{\partial\omega} = \frac{[1-\alpha(1-qp_m)]\frac{\partial(V_S/D)}{\partial\omega} - 1 + RR_{\mathrm{M}}}{1-\omega} < 0 \quad \text{(A.38)}$$

as

$$\frac{\partial(V_S/D)}{\partial\omega} = \frac{\partial(V_S/D)}{\partial p_e}\frac{\partial p_e}{\partial\omega} < 0. \quad \text{(A.39)}$$

Therefore, when $(1-\alpha)V_S - K \geq \frac{b}{r}$, the spill-over effect never happens.

When $(1-\alpha)V_S - K < \frac{b}{r}$,

$$\frac{\partial(RR_{\mathrm{M}})}{\partial\omega} = \frac{qp_m}{(1-\omega)^2}\left[\alpha(1-\omega)\frac{\partial(V_S/D)}{\partial\omega} + \frac{\alpha V_S}{D} + \frac{K}{D} - \frac{\lambda^-}{\lambda^+ - \lambda^-}\tau\right] \quad \text{(A.40)}$$

For spill-over to be effective, condition $\frac{\partial(RR_{\mathrm{M}})}{\partial\omega} > 0$ must be hold which is equivalent to

$$\alpha(1-\omega)\frac{\partial(V_S/D)}{\partial\omega} + \frac{\alpha V_S}{D} + \frac{K}{D} - \frac{\lambda^-}{\lambda^+ - \lambda^-}\tau > 0. \quad \text{(A.41)}$$

Since

$$\frac{\partial\left(\frac{V_S}{D}\right)}{\partial\omega} = \frac{\partial\left(\frac{V_S}{D}\right)}{\partial p_e}\frac{\partial p_e}{\partial\omega} \quad \text{(A.42)}$$

The inequality (A.41) is equivalent to

$$\frac{\partial(V_S/D)}{\partial p_e}\frac{\partial p_e}{\partial \omega} > \frac{-1}{\alpha(1-\omega)}\left[\frac{\alpha V_S}{D} + \frac{K}{D} - \frac{\lambda^-}{\lambda^+ - \lambda^-}\tau\right]. \quad \text{(A.43)}$$

We already get

$$\frac{\partial\left(\frac{V_S}{D}\right)}{\partial p_e} = \frac{q}{1-\alpha+q(1-p_e)\alpha}\left[\frac{\lambda^-}{\lambda^- - 1}\left(\frac{K}{D} - \frac{\lambda^-\tau}{\lambda^+ - \lambda^-}\right) + \alpha\frac{V_S}{D}\right]. \quad \text{(A.44)}$$

Thus,

$$\frac{\partial p_e}{\partial \omega} > -\frac{1-\alpha+q(1-p_e)\alpha}{q\alpha(1-\omega)}\frac{\frac{\alpha V_S}{D} + \left(\frac{K}{D} - \frac{\lambda^-}{\lambda^+ - \lambda^-}\tau\right)}{\frac{\alpha V_S}{D} + \frac{\lambda^-}{\lambda^- - 1}\left(\frac{K}{D} - \frac{\lambda^-\tau}{\lambda^+ - \lambda^-}\right)}. \quad \text{(A.45)}$$

and

$$\left|\frac{\partial p_e}{\partial \omega}\right| < \frac{1-\alpha+q(1-p_e)\alpha}{q\alpha(1-\omega)}\frac{\frac{\alpha V_S}{D} + \left(\frac{K}{D} - \frac{\lambda^-}{\lambda^+ - \lambda^-}\tau\right)}{\frac{\alpha V_S}{D} + \frac{\lambda^-}{\lambda^- - 1}\left(\frac{K}{D} - \frac{\lambda^-\tau}{\lambda^+ - \lambda^-}\right)}. \quad \text{(A.46)}$$

Following condition $(1-\alpha)V_S - K < \frac{b}{r}$, it can be observed that the minimum bank debt percentage can be computed as follows

$$\begin{aligned} & (1-\alpha)V_S - K < \frac{b}{r} \\ \Leftrightarrow\ & \omega D > (1-\alpha)V_S - K \\ \Leftrightarrow\ & \omega > \left(\frac{1-\alpha}{\alpha}\right)\frac{\lambda^-}{\lambda^- - 1}\left[\frac{q(1-p_e)\frac{\lambda^-\tau}{\lambda^+ - \lambda^-} + 1 + \frac{1}{\alpha}\frac{K}{D}}{1/\alpha - \left(1-q(1-p_e)\right)} - \frac{K}{D}\right] \end{aligned} \quad \text{(A.47)}$$

Appendix B Data Collection and Final Sample

Step 1. Identify the initial sample

Moody's Ultimate Recovery Database report over 1000 defaulted firms which include both bankruptcy cases (Chapter 7 and Chapter 11) as well as Distressed

Exchange. Moody's reports the outcome of a defaulted case as: emerged, liquidated or acquired. I delete the cases of liquidated or acquired. My Chapter 11 sample contains 635 cases. For every firm, Moody's provide the company name, CUSIP, ticker and SIC code. However, by comparing the Moody's and Compustat and SEC filing, I find that some of Moody's records are incorrect. I correct the information according to Compustat and SEC.

Step 2. Obtain accounting information for our sample

The accounting information I need include:

Long-term debt, Book value of assets, Book value of equity, Non-intangible assets, Intangible assets, PPandE, Total liability, Short-term debt, Total Debt, Total receivables, EBITDA, Total sales, Market value of equity etc. I need the information just prior to the default time. After this step, I have 454 defaulted events in my sample. The accounting information is obtained from WRDS Compustat.

Step 3. Identify Industry and Accounting Information for Industry

Moody's provide the SIC code for the defaulted firms. I obtain stock price and accounting information of the industry based on the same first two-digit SIC code. After this step, I have 454 defaulted events in my sample. The industry information is obtained from WRDS Compustat.

Step 4. Identify Default Times

Several firms in the sample enter into Chapter 11 several times. I only keep the firms that firstly enter into bankruptcy. After winsorization at 1%, my final sample contains 444 Chapter 11 cases.

Appendix C Pseudo-code

In this appendix I give the most important programming code for valuation.

1. European Parisian Up-and-Out Option

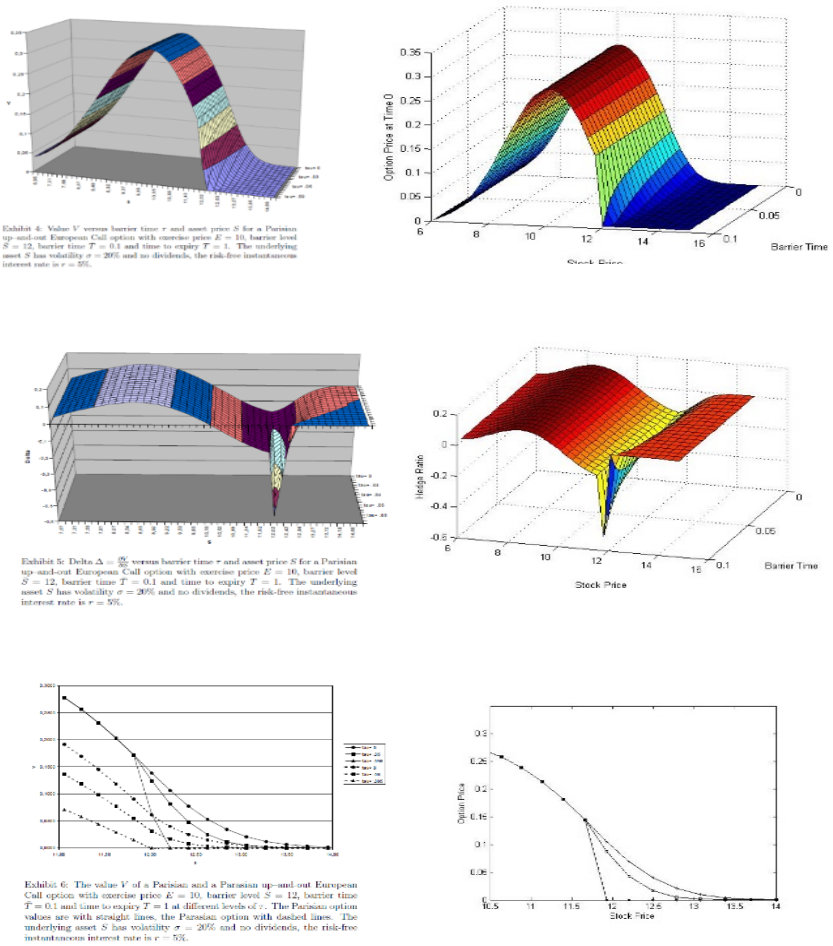
This is my code to price the European Parisian option in Haber's paper

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Setting the Mesh:
K=fix(d*J/T); % Numer of barrier time steps
              % The number is determined such that dt=dK
xMax=log(SMax);
xMin=log(SMin);
dx=(xMax-xMin)/I; % The step size of the stock price
dt=T/J;           % The step size of the time
dK=dt;
x=xMin:dx:xMax;
S=exp(x);        % The sequence of the stock price
t=0:dt:T;        % The sequence of time
tau=0:dK:d;      % The sequence of barrier time
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Determining the Matrix Coefficients of the Explicit
Algorithm:
a=dt/(2*dx);
b=(sigma^2)/dx;
c=r-delta-0.5*(sigma^2);
A=a*(b-c);
B=1-2*a*b-r*dt;
C=a*(b+c);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Initializing the Matrix & Terminal and Boundary Conditions
u=zeros(I+1,K+1,J+1);
Payoff=max(S'-E,0);        % terminal condition 1 --
u(:,1:K,end)= Payoff*ones(1,K);% values at the expiration
u(:,end,:)=0;    % terminal condition 2 --
                 % values when the knock-out is triggered
u(1,:,:)=0;      % Boundary condition 1 --
                 % When underlying approaches left limit
u(end,1:end-1,1:end-1)=0;  % Boundary conditions 2 --
                 % When underlying approaches right limit
ii=floor((log(SBar)-xMin)/dx); % index x(ii)=ln(Sbar)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Implement the Explicit Scheme to Compute the Interior Points
% The calculation is conducted backwards
for j=J:-1:1
    u(ii,1,j)=A*u(ii-1,1,j+1)+B*u(ii,1,j+1)+C*u(ii+1,1,j+1);
    u(ii,2:end-1,j)=u(ii,1,j);
```

```
    u(2:ii-1,1:end-1,j)=A*u(1:ii-2,1:end-1,j+1)+B*u(2:ii-
1,1:end-1,j+1)...
        +C*u(3:ii,1:end-1,j+1);
    u(ii+1:end-1,1:end-1,j)=A*u(ii:end-
2,2:end,j+1)+B*u(ii+1:end-1,2:end,j+1)...
        +C*u(ii+2:end,2:end,j+1);
end
```

Result of the Pricing and Comparison with the Result in the Paper of (Haber, Schönbucher and Wilmott 1999). The three figures in the left panel are extracted from their paper and the threes on the left are produced by the code given above.

Figure A.1 Replication of Haber, Schönbucher and Wilmott (1999)



2. American Parisian Up-and-Out Call Option and Optimal Exercise Boundary

This file computes and visualizes the price of an American Parisian up and out call option and the corresponding optimal exercise boundary. The algorithm is based on Haber's paper (1999) on European Parisian Option.

```
function [S,t,tau,u]=Haber_American
(SMax,SMin,SBar,I,J,T,d,sigma,r,delta,E)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Implement the Explicit Scheme to Compute the American
Options
for j=J:-1:1
    u(ii,1,j)=max(A*u(ii-
1,1,j+1)+B*u(ii,1,j+1)+C*u(ii+1,1,j+1),...
        u(ii,1,end));
    % This represents the option value u should be greater than
or equal to
    % the amount to be received at expiry
    u(ii,2:end-1,j)=u(ii,1,j);
    % The clock is reset to zero at the barrier point
    u(2:ii-1,1:K,j)=max(A*u(1:ii-2,1:K,j+1)+B*u(2:ii-
1,1:K,j+1)...
        +C*u(3:ii,1:K,j+1),u(2:ii-1,1:K,end));
    u(ii+1:I,1:K,j)=max(A*u(ii:I-
1,2:end,j+1)+B*u(ii+1:I,2:end,j+1)...
        +C*u(ii+2:end,2:end,j+1),u(ii+1:I,1:K,end));
    % Compare the payoff if wait until next period or exercise
the option
    % immediately
end

function [f]=Haber_FreeBoundary(S,t,tau,u,E)
f=zeros(length(tau)-1,length(t));
myeps=E*1e-5;

for j=1:length(t)
    for k=1:length(tau)-1
        f(k,j)=S(find(abs(u(:,k,j)+E-S')<myeps,1,'first'));
    end
end
```

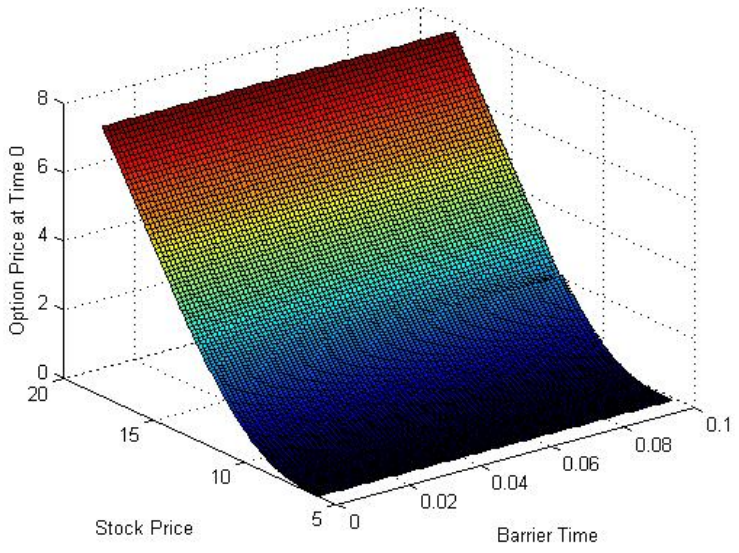
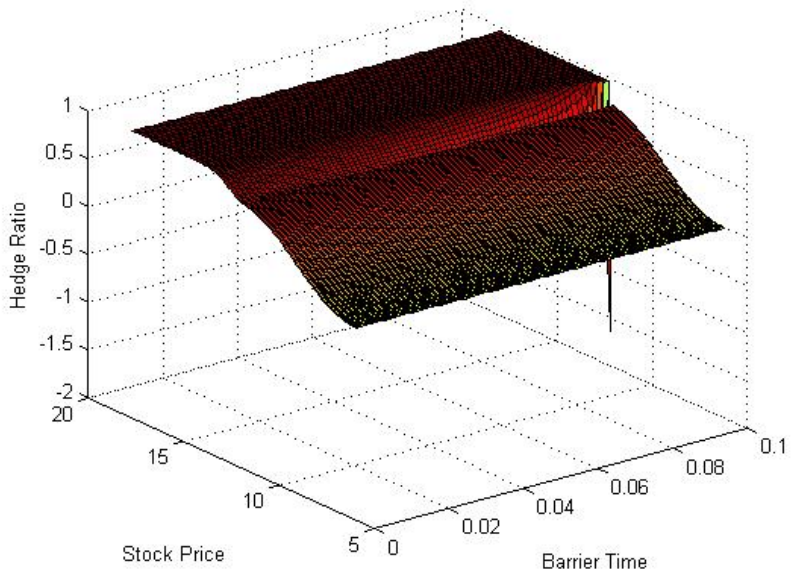


Figure A.2 The American Parisian Up-and-Out Call Option Prices



FigureA.3 The American Parisian Up-and-Out Call Option Hedge Ratios

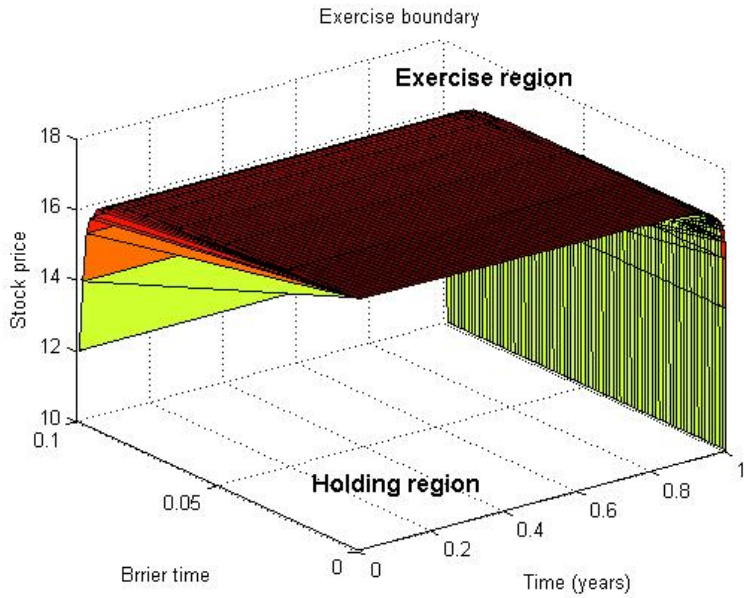


Figure A.4 The American Parisian Up-and-Out Call Option Optimal Exercise Boundary

3. Code for valuation in Chapter 5

```
function [u,U]=explicit_fd(payoff_fn,m_inf_fn,p_inf_fn,...
    knockout_fn,cashflow_fn,cashflow_default_fn,VL,VB,T)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
input_parameters;
% Setting the Mesh:
xMax=log(VMax);   % Change of variables -> x=ln(V)
xMin=log(VMin);   % Change of variables -> x=ln(V)
K=fix(d*J/T);     % Numer of barrier time (tau) points
                  % The number is computed such that dt=dK
dx=(xMax-xMin)/I;% Underlying step
dt=T/J;           % Time step
dK=dt;            % Barrier time step
x=xMin:dx:xMax;   % Get the mesh points of underlying
V=exp(x);         % Change of variables -> V=exp(x)
t=0:dt:T;         % Get the mesh points of the time
tau=0:dK:d;       % Get the mesh points of the barrier time
index=floor((log(V0)-xMin)/dx)+1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Determining the Matrix Coefficients of Explicit Algorithm:
aa=dt/(2*dx);  % Refer to formulas (4.23-26) in Xiyang'paper
bb=(vol_sigma^2)/dx;
cc=r-delta-0.5*(vol_sigma^2);
A=aa*(bb-cc);
B=1-2*aa*bb-r*dt;
```

```
C=aa*(bb+cc);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Find the threshold points when asset value is VL and VB.
ii=floor((log(VB)-xMin)/dx)+1;
% Find ii such that x(ii)=ln(VB)
ll=floor((log(VL)-xMin)/dx)+1;
% Find ll such that x(ll)=ln(VL)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Initializing the Matrix & Terminal and Boundary Conditions
u=zeros(I+1,K+1,J+1); % Initializing the matrix u=u(x,tau,t)
u(:,1:(end-1),end)=payoff_fn(V,Pb,Pc,u);
u(:,end,:)=knockout_fn(V,alph,Pb,Pc,u);
u(1,:,:) = m_inf_fn(V(1),Pb,Pc,alph); % min((1-alph)*VMin,Pb);
for j=1:J+1
time = T-(j-1)*dt;
u(end,1:(end-1),j)=p_inf_fn(V,Pb,Pc,tax,time,r,b,c);
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for j=J:-1:1
    for i=2:ll
        u(i,1:end-1,j)=m_inf_fn(V(i),Pb,Pc,alph);
    end
    u(ll+1:ii-1,1:end-1,j)=A*u(ll:ii-2,2:end,j+1)+...
        B*u(ll+1:ii-1,2:end,j+1)+C*u(ll+2:ii,2:end,j+1)+...
        dt*cashflow_default_fn(V(ll+1:ii-1),...
        alph,ita,delta,phi,u);
    cf_normal=cashflow_fn(Pb,Pc,b,c,delta,V,tax);
    u(ii+1:end-1,1:end-1,j)=A*u(ii:end-2,1:end-1,j+1)+...
        B*u(ii+1:end-1,1:end-1,j+1)...
        +C*u(ii+2:end,1:end-1,j+1)...
        +dt*cf_normal(ii+1:end-1)*ones(1,K);
    u(ii,1,j)=A*u(ii-1,1,j+1)+B*u(ii,1,j+1)+...
        C*u(ii+1,1,j+1)+dt*cf_normal(ii);
    u(ii,2:end-1,j)=u(ii,1,j);
end

U=u(index,1,1);
%Matrix=u(:,1:end-1,1);
%figure
%surf(tau(1:end-1),V,Matrix)
```

4. Code for optimal boundary in Chapter 6

This function computes the optimal boundary of liquidation. The input of the function is vl_determination, which is the party who determine the boundary. The output is a matrix g which gives the optimal liquidation boundary given the renegotiation boundary.

```
function g = optimal_vl(vl_determination)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
input_parameters; T=10; P=Pb+Pc; Table =zeros(81,2);
for index_VB=20:100
    VB=index_VB; Matrix=zeros(VB-6+1,2);
    for index_VL=6:VB
        VL=index_VL; Matrix(index_VL-5,1)=VL;
        switch vl_determination
            case 1   % bank decide vl
                [u,X,X_B] =
explicit_fd_barrier(@payoff_bank,...
                    @m_inf_bank, @p_inf_bank,...
                    @knockout_bank,@cashflow_bank,...
                    @cashflow_default_bank,VL,VB,T,Pb);
            case 2   % bondholders decide vl
                [u,X,X_B] =
explicit_fd_barrier(@payoff_bond,...
                    @m_inf_bond,@p_inf_bond,...
                    @knockout_bond,@cashflow_bond,...
                    @cashflow_default_bond,VL,VB,T,Pb);
            case 3  % shareholders decide vl

[u,X,X_B]=explicit_fd_barrier(@payoff_equity,...
                    @m_inf_equity,@p_inf_equity,...
                    @knockout_equity,@cashflow_equity,...
                    @cashflow_default_equity,VL,VB,T,Pb);
            case 4   % bank decide vl
                [u,X,X_B] =
explicit_fd_barrier(@payoff_firm,...
                    @m_inf_firm,@p_inf_firm,...
                    @knockout_firm,@cashflow_firm,...
                    @cashflow_default_firm,VL,VB,T,Pb);
        end
        Matrix(index_VL-5,2)=X_B;
    end
    vec=Matrix(:,2); indexmax=find(max(vec)==vec);
    index=max(indexmax);
    VL=index+5;
    Table(index_VB-19,1)=VB; Table(index_VB-19,2)=VL;
end
```