

# Methodologies for evaluation of high strain rate properties of composite material constituents

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# **Abstract**

Virtual testing using high strain rate material data can play a major role in realising an optimum design of composite parts for impact and crash resistance. The objective of this thesis was to develop methodologies for generation of high strain rate properties of composite material constituents and demonstrate their use with an already developed multi-scale modelling platform [1] for prediction of the probabilistic strength of composite materials. Two different approaches were used for generation of the high strain rate data of the fibres and resins. The first one is the direct evaluation of properties through experimental testing and the other is an inverse modelling approach which uses the test data of composites to provide information on the constituent data, using the micro-scale models [1], through an iterative optimisation study. A split Hopkinson tensile bar was developed with an innovative clamping methodology which was used to generate the high strain rate tensile characteristics of carbon fibres. An inverse modelling methodology was used in conjunction with the high strain rate data from the longitudinal and transverse compression of unidirectional composites to generate the longitudinal and transverse compressive properties of the fibre. The constituent's data generated was then further used in the multi-scale model for probabilistic prediction of respective composite strength through Monte Carlo simulations. This study was primarily focussed on generating data for carbon fibre composites.

The contribution to knowledge from this study is the demonstration of the use of hybrid approach of generation of data in conjunction with multi-scale models for prediction of the strength properties of composites. This approach can be further applied for different multi-scale models relating to different composite architecture for evolution of new optimum designs for impact and crashworthy applications.

Specific areas of novelty related to this thesis can be summarised as follows:

- 1) Development of an innovative clamping methodology for high strain rate tensile testing of fibres.
- 2) Development of an inverse modelling approach for generation of longitudinal and transverse compressive properties of fibres.
- 3) Use of analytical Jacobian for improved efficiency during optimisation for inverse modelling studies
- 4) Generation of high strain rate data of standard carbon fibres such as T800, T700 and IM7.

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## Abbreviations

|       |                                                    |
|-------|----------------------------------------------------|
| CFRP  | Carbon Fibre Reinforced Plastic                    |
| CTE   | Coefficient of Thermal Expansion                   |
| FAR   | Federal Aviation Regulations                       |
| SHTB  | Split Hopkinson Tensile Bar                        |
| SHPB  | Split Hopkinson Pressure Bar                       |
| TSHB  | Torsional Split Hopkinson Bar                      |
| GFRP  | Glass Fibre Reinforced Plastic                     |
| GLARE | GLass Aluminium REinforced laminates               |
| FML   | Fibre Metal Laminate                               |
| SPH   | Smoothed Particle Hydrodynamics                    |
| EOS   | Equation of State                                  |
| CEL   | Coupled Eulerian-Lagrangian                        |
| FE    | Finite Element                                     |
| UD    | Uni-Directional                                    |
| UTM   | Universal Testing Machine                          |
| EMF   | Electromotive force                                |
| 3D    | Three-Dimensional                                  |
| EDM   | Electro Discharge Machining                        |
| ASTM  | American Society for Testing and Materials         |
| SEM   | Scanning Electron Microscope                       |
| LVDT  | Linear Variable Differential Transformer           |
| TSA   | Time Speed Acceleration                            |
| EPSRC | Engineering and Physical Sciences Research Council |
| BSD   | Berkeley Source Distribution                       |
| CCD   | Charge – Coupled Device                            |
| FEMU  | Finite Element Model Updating                      |
| DIC   | Digital Image Correlation                          |
| RAM   | Random Access Memory                               |
| CPU   | Central Processing Unit                            |
| ELS   | Equal Load Sharing                                 |
| LLS   | Local Load Sharing                                 |
| FEM   | Finite Element Method                              |
| TIFF  | Tagged Image File Format                           |
| PDF   | Probability Distribution Function                  |
| CDF   | Cumulative Distribution Function                   |
| QS    | Quasi-Static                                       |
| HSR   | High Strain Rate                                   |
| RoM   | Rule of Mixtures                                   |

## Notations

|                       |                                                     |
|-----------------------|-----------------------------------------------------|
| $c^*$                 | Cunniff velocity                                    |
| $\sigma_f$            | Tensile failure strength of fibre                   |
| $\varepsilon_f$       | Tensile failure strain of fibre                     |
| $\rho_f$              | Density of the fibre                                |
| $E_f$                 | Tensile modulus of the fibre                        |
| $A_{RT}$              | Rate adjusted parameter                             |
| $A_0$                 | Quasi-static parameter / Area of the striker        |
| $C_A$                 | Strain rate scaling constant for parameter A        |
| $\dot{\varepsilon}$   | Strain rate                                         |
| $\dot{\varepsilon}_0$ | Quasi-static reference strain rate                  |
| $\sigma$              | Stress / dynamic stress                             |
| $\rho$                | Density / Weibull modulus                           |
| $L$                   | Initial length of specimen                          |
| $X$                   | Undeformed length of specimen                       |
| $L_1$                 | Final length of specimen                            |
| $u$                   | Displacement in x direction                         |
| $t$                   | time                                                |
| $c$                   | Speed of sound                                      |
| $E$                   | Elastic modulus                                     |
| $v$                   | Particle velocity in the bar/specimen/striker       |
| $A_A$                 | Area of section A                                   |
| $A_B$                 | Area of section B / Area of Bars                    |
| $\sigma_I$            | Incident stress on an interface                     |
| $\sigma_R$            | Reflected stress from an interface                  |
| $\sigma_T$            | Transmitted stress from an interface                |
| $v_I$                 | Incident particle velocity                          |
| $v_R$                 | Reflected particle velocity                         |
| $v_T$                 | Transmitted particle velocity                       |
| $T$                   | Ratio of mechanical impedance of section of A and B |
| $V_0$                 | Striker velocity                                    |
| $C_o$                 | Speed of sound in striker                           |
| $L_0$                 | Length of the striker                               |
| $T$                   | Duration of the incident pulse                      |
| $\varepsilon_I$       | Incident strain pulse                               |
| $\varepsilon_R$       | Reflected strain pulse                              |
| $\varepsilon_T$       | Transmitted strain pulse                            |
| $E_B$                 | Modulus of incident bar / transmission bar          |
| $A_S$                 | Area of specimen                                    |
| $\varepsilon_S$       | Strain observed in the specimen                     |
| $L_S$                 | Length of the specimen                              |

|                |                                                                                          |
|----------------|------------------------------------------------------------------------------------------|
| $c_0, C$       | Speed of sound in bars                                                                   |
| $\sigma_0$     | Stress in the striker due to hollow striker impact / characteristic strength             |
| $A_1$          | Area of incident bar                                                                     |
| $\sigma_1$     | Stress in the incident bar due to hollow striker impact                                  |
| $A_2$          | Area of incident bar flange                                                              |
| $\sigma_2$     | Stress in the incident bar flange due to hollow striker impact                           |
| $BF$           | Back out factor                                                                          |
| $E_{11}$       | Tensile modulus of UD composite in fibre direction                                       |
| $E_{22}$       | Tensile modulus of UD composite in transverse direction                                  |
| $\nu_{12}$     | Poisson's ratio in 1-2 direction                                                         |
| $F(\sigma)$    | Cumulative probability of failure                                                        |
| $m$            | Weibull modulus / Total number of measurement in data vector                             |
| $n$            | Total number of specimens / Total number of model parameters in column vector            |
| $i$            | Datum value                                                                              |
| $u_1$          | Velocity of incident bar-specimen interface                                              |
| $F(\beta)$     | Governing function definition in a physical problem                                      |
| $\beta$        | Column vector of n model parameters                                                      |
| $r_i$          | Residuals                                                                                |
| $S$            | Sum of the squares of the residuals / Strength parameters for each fibre in fibre bundle |
| $N(\sigma, L)$ | Number of flaws of fibre length, $L$ which can fail at or below stress, $\sigma$         |
| $\alpha$       | Additional parameter in Weibull probability distribution function                        |
| $\mu_n^*$      | Mean bundle strength of n fibres                                                         |
| $\mu^*$        | Asymptotic mean bundle strength                                                          |
| $V_f$          | Volume fraction of the fibres in a fibre bundle                                          |
| $\delta_e$     | Characteristic fibre length                                                              |
| $A_f$          | Area of fibre                                                                            |
| $w$            | Effective matrix width between the fibres                                                |
| $G_m$          | Matrix shear modulus                                                                     |
| $h$            | Effective matrix thickness                                                               |
| $u_2$          | Velocity of transmission bar-specimen interface                                          |
| $F_1$          | Force observed by the specimen at incident bar – specimen interface                      |
| $F_2$          | Force observed by the specimen at specimen - transmission bar interface                  |
| $T_r$          | Time gap between starting point of incident and reflected pulse                          |
| $T_t$          | Time gap between starting point of incident and transmitted pulse                        |
| $C_s$          | Speed of sound inside the specimen                                                       |
| $f$            | Bin counts                                                                               |
| $x$            | Bin centres                                                                              |
| $X$            | Vector                                                                                   |
| $\omega$       | Damage variable                                                                          |
| $E_d$          | Degraded elastic stiffness                                                               |

|             |                                                  |
|-------------|--------------------------------------------------|
| $G_d$       | Degraded shear modulus                           |
| $E_{f22}$   | Transverse fibre modulus                         |
| $E_m$       | Matrix modulus                                   |
| $E_{l22}$   | Transverse composite modulus                     |
| $k_f$       | Fibre volume fraction in the composite           |
| $\bar{k}_f$ | Corrected fibre volume fraction in the composite |
| $k_v$       | Void volume fraction in the composite            |
| $S_{mc}$    | Matrix compressive strength                      |
| $S_{l22c}$  | Transverse compressive strength of the composite |
| $V_f$       | Fibre volume fraction in the composite           |
| $V_v$       | Void volume fraction in the composite            |

# **1 Introduction**

## **1.1 Carbon fibre reinforced plastic in aerospace applications**

Due to its improved properties, Carbon Fibre Reinforced Plastic (CFRP) has found extensive application in aerospace components such as aircraft wings, fuselages, empennage structures as well as satellite structures. CFRP has been considered for aircraft applications because it offers high specific stiffness and high specific strength, which are primary requirements for the structural performance of airworthy structures. CFRP forms around 52 percent of the structural weight of the Airbus A350 XWB and 50 percent of the Boeing 787 aircraft in addition to a broad range of structural applications [2-7]. CFRP has also been considered for applications in satellite structures as face sheet material for sandwich panels of heat pipes and for optical tables due to its high thermal performance, low coefficient of thermal expansion (CTE) and excellent dimensional stability against moisture and temperature [8]. One of the drawbacks of CFRP is the increased cost associated with material and manufacturing. With the advent of automated technologies for manufacturing and out of autoclave processes, CFRP can find further applications in aerospace, automobile, bridges, sports goods etc. CFRP structures used for aircraft applications also need to demonstrate its damage tolerant capability for impact events for the structural qualification of the part.

## **1.2 Dynamic impact requirements of CFRP in aircraft applications**

Aircraft, in particular, are prone to high velocity impact due to bird strike and space structures are prone to space debris impact, which can occur at hyper velocities. Aircraft structures need to meet the respective certification clauses, such as FAR 25.571 and FAR 25.631 [9], to prove their airworthiness for withstanding such impacts. CFRP, because of its constituent materials (fibre and resin), may show improved dynamic response compared to its static behaviour because of the strain rate dependent properties of the constituents. There are very few reported examples of work on the high strain rate behaviour of carbon fibres/fibre bundles [10, 11] and there is no consistency among researchers on the rate sensitivity of CFRP [12-20]. Depending on impact velocity, the associated strain rates vary across an impact zone and in zones further from the impact

region. The strain rate regimes of typical high velocity impact ( $50\text{-}500\text{ ms}^{-1}$ ), ballistic impact ( $500\text{-}5000\text{ ms}^{-1}$ ) and hypervelocity impact ( $> 5000\text{ ms}^{-1}$ ) range from  $10 - 10^3\text{ s}^{-1}$ ,  $10^3 - 10^6\text{ s}^{-1}$  and  $10^5 - 10^7\text{ s}^{-1}$  respectively [21-23]. Low velocity and crash events typically cover lower strain rate regimes ( $10^{-1} - 10\text{ s}^{-1}$ ), even though high strain rates ( $10^2 - 10^3\text{ s}^{-1}$ ) can be observed at the impact point for crash events as the speed of impact increases [21, 24]. For high velocity impact, away from the impact zone, strain rates can vary between  $10^{-1}$  to  $10\text{ s}^{-1}$  [25]. Such extreme loading conditions need careful design procedures to develop airworthy structures.

### **1.3 Approaches to design for impact**

The design approaches for impact applications followed by most teams are primarily based on empirical formulae for penetration velocities [26] or the use of quasi-static properties [27, 28] of the material with a considerable reserve factor to ensure safety in dynamic conditions and satisfy the certification requirements through testing. Such conservative design results in weight addition, higher fuel consumption and reduce the operational efficiency of the aircraft. Producing more efficient designs through experimental testing is time consuming and costly. In the last two decades, predictive modelling has gained considerable recognition [29-38], as there has been increased availability of high performance computers and cost efficient computational software. Predictive modelling plays an important role in the design as it helps in identifying the correct configuration, materials and processes and reduces experimentation, thereby saving considerable cost. One of the most important requirements in predictive modelling is obtaining reliable material input data (including high strain rate data) and the use of realistic material constitutive models. Some of the advanced material models used for impact simulations requires some material input parameters that are difficult to generate with standard test methods. This reinforces the need for development of new test methods or, alternatively, the use of a multi-scale modelling approach to predict these properties. However, the development of new test methods may not be feasible, due to practical difficulties e.g. the need to ensure the correct failure modes for a given loading scenario, or would be time consuming in terms of specimen preparation. Virtual testing [39], on the other hand, using high strain rate material data, can play a major role in realising an optimum design of composite parts for impact and crash resistance thereby limiting the need for experimental testing only to validation tests during the certification process.

## **1.4 Approaches adopted in this thesis**

A multi-scale modelling approach has previously been developed by Lidgett [1] to evaluate the high strain rate properties of composite materials. Multi-scale modelling focusses on the fundamental understanding of the material from a lower scale to a macroscopic scale. The multi-scale modelling approach followed by Lidgett [1] involved the prediction of lamina level properties from the constituent material properties (fibre and resin) at high strain rates using micro-scale modelling. The properties obtained for the lamina were used as input for the prediction of the properties for the laminate. The properties so generated can be used as input for the material models for the prediction of the overall behaviour of composite structures subjected to impact loadings. The material model used in Lidgett's micro-scale model requires extensive definition of parameters for the fibre, matrix and interface including the transverse isotropic properties of the fibre, failure criteria, strain rate effects and softening effects for the matrix. One of the important aspects in micromechanical models is the availability of the data of the constituents. In the study conducted by Lidgett [1], the properties used for his micro-scale models were the best data available at the time obtained primarily from literature sources. Hence, to further validate and develop the multi-scale modelling approach, it is required to generate constituent material data (fibres and resins) at high strain rates.

Two different approaches have been used for generation of the high strain rate data of the fibres and resins. The first one is the direct evaluation of properties through experimental testing and the other is an inverse modelling approach which uses the test data of composites to provide information on the constituent data, using the Lidgett [1] micro-scale models, through an iterative optimisation study. The experimental approach uses a specially developed split Hopkinson tensile bar (SHTB) for measuring the tensile properties of the fibre and a more standard split Hopkinson pressure bar (SHPB) [40] to determine the compressive properties of the matrix. The transverse and compressive properties of the fibres were estimated using the inverse modelling approach for which the composite characteristics, used for input, were obtained experimentally through high strain rate compression testing using the above mentioned standard SHPB.

In summary, this thesis describes work which develops different methodologies for the evaluation of high strain rate properties of composite material constituents. The issues and challenges in the methodologies are also discussed.

A critical review of the results obtained has been compared with information available in the literature.

## **1.5 Contents of the thesis**

In this thesis, the second chapter gives a description of the state-of-the-art in impact simulation through a detailed and focussed literature review. The literature review deals with applications where impact simulation is relevant, a critical overview of the state-of-the-art on material models available, and highlights the importance of multi-scale modelling work for the generation of high strain rate properties of composite materials. There is also a detailed literature review on the high strain rate data already available in the literature relating to composites and fibres. Finally, the rationale and objectives of the thesis are described in that Chapter. The third Chapter describes the development of a split Hopkinson bar for tensile testing of fibres. This includes detailed discussion of high strain rate testing methods available, the theory of split Hopkinson bar techniques, the development methodology and commissioning challenges. The fourth Chapter describes experimental work on the mechanical characterisation of composite materials and their constituents at different strain rates. The testing approach, results and challenges encountered during the testing are highlighted in this chapter. The fifth Chapter is a description of the inverse modelling method developed for estimation of high strain rate properties of composite material constituents. The sixth Chapter highlights the significance of statistical strength distribution in the results and gives a summary of these results obtained through Monte Carlo simulation from fibre distribution and inverse estimated data. Finally, the seventh Chapter concludes the thesis and describes the main findings of the various studies performed as part of this project and also gives suggestions for future work in each of the areas presented.

## **2 Impact simulation of composite structures – the need for high strain rate data**

Since composites have been extensively used for impact resistant and energy absorbing applications, the simulation of impact behaviour on composite structures has been the subject of much attention due to the need to reduce experimental testing, which is time consuming and expensive. There has been extensive research on impact simulation of composite structures in the past decade. One of the main issues in numerical simulation has been the use of the correct constitutive material models and the availability of relevant material data. The choice of constitutive model depends upon the type of problem attempted.

The aim of this Chapter is to first present a literature review of available high strain rate data for CFRP and GFRP and then identify the requirement of high strain rate data for impact simulation through a critical review on some of the published papers which have/have not used high strain rate data in their simulation. This Chapter also highlights the requirement of the need to generate different lamina properties through a review on the material model that have been used for impact simulation. This Chapter also discusses the advantages of using multi-scale modelling techniques for generation of these high strain rate properties. Since the multi-scale modelling approaches require constituent data, a literature review on the availability of high strain rate data of fibres is also presented. The main purpose of this Chapter is therefore to discuss the rationale and objective of the thesis through a literature review on the relevant areas and consequently highlight the importance of high strain rate data of composite material constituents and their relevance to virtual testing.

### **2.1 High strain rate data of composite materials**

The high strain rate behaviour of metals and alloys has been studied by many researchers in the past decades and has been well understood. The strain rate sensitivity of metals can be understood from the viewpoint of dislocation mechanics [41]. Unlike their metallic counterparts, the failure associated with composites is more complex because of the inherent multiphase structure which results in complex deformation processes [42]. Extensive work carried out by various researchers on the high strain rate characterisation of composites has been thoroughly summarised in a variety of review articles [42-44]. This section

gives a summary of the literature on the high strain rate data available for GFRP and CFRP materials only.

Various researchers have tried to evaluate the high strain rate sensitive behaviour of glass based composites. Barre et al. [24] evaluated the strain rate effect of glass reinforced phenolic and polyester resins using specially designed drop weight apparatus. Strain rates varying from  $10^{-4}$  to  $10 \text{ s}^{-1}$  were achieved during tests. The studies showed that elastic modulus was not influenced by strain rates up to a value of  $1 \text{ s}^{-1}$ , though a significant effect was observed in failure strength beyond strain rates of  $10^{-1} \text{ s}^{-1}$ . The increased strain rate sensitivity was observed for transverse and shear properties of composites which highlights the significance of viscoelasticity of the matrix systems used. However, the influence of strain rate depended upon the layup sequence and the reinforcement structure of the composites. The authors highlighted the need to understand the microstructure of the materials to correlate the role of various damage mechanisms against the dynamic behaviour of these materials. Sun et al. [45] investigated the dynamic compressive characteristics of multi-axial multi-layer warp knitted (MMWK) E-glass/epoxy composites using a split Hopkinson pressure bar. The through-thickness compressive stress strain curves were observed to be rate sensitive for strain rates between  $600\text{--}2100 \text{ s}^{-1}$ . The compressive stiffness and compressive stress were found to increase linearly with strain rate whereas failure strain was found to decrease with strain rate. Shokrieh and Omid [46] carried out tensile testing of unidirectional glass fibre composites at various strain rates from  $10^{-5}$  to  $160 \text{ s}^{-1}$ . The tests were carried out using a servo hydraulic testing machine with a specially designed fixture to hold the tensile specimen. The material properties such as the tensile strength, modulus, failure strain and the failure fracture energy were found to be strain rate sensitive beyond  $1 \text{ s}^{-1}$ . A summary of the results for high strain rate characterisation of glass based composites performed by various researchers is given in Table 2-1.

To summarise for GFRP, strength and stiffness in all the directions are found to be rate sensitive by most of the researchers and this can mostly be attributed to the visco-elastic behaviour of the matrix. It can also be seen that there is a threshold rate beyond which the GFRP materials are strain rate sensitive (above  $10^{-2} \text{ s}^{-1}$ ). Contradictory rate sensitive characteristics could be observed for the shear properties of GFRP conducted through two different methods by two different research groups ([47, 48]).

Table 2-1 Review of the high strain rate data available in the literature on GFRP

Note here in the observation column: 1 denotes fibre direction 2- denote transverse direction and 3- thickness direction. No direction mentioned if it is not unidirectional specimen such as cross ply, woven fabric etc. Trend:  $\uparrow$ -direct relation,  $\downarrow$ -inverse relation,  $\approx$  - no specific trend

| Material Type                                      | Strain Rate range ( $s^{-1}$ ) | Testing Equipment                               | Type of Loading              | Observation                                                  | Reference              |
|----------------------------------------------------|--------------------------------|-------------------------------------------------|------------------------------|--------------------------------------------------------------|------------------------|
| E-glass/ polypropylene woven fabric composite      | 0.001 - 100                    | Modified instrumented falling weight drop tower | Tension                      | $E_{11}\uparrow, \sigma_{11}\uparrow$                        | Brown et al.[49]       |
|                                                    |                                |                                                 | Compression                  | $E_{11}\uparrow, \sigma_{11}\uparrow$                        |                        |
| S2/8552 glass epoxy                                | 400-700                        | SHPB                                            | Off-axis compression         | $G_{12}\uparrow, \tau_{12}\uparrow$                          | Ninan et al. [47]      |
| Plain weave E-Glass Epoxy                          | 496-1000                       | TSHB                                            | Interlaminar shear behaviour | $G_{13}\uparrow, \tau_{13}\uparrow$                          | Naik et al.[50]        |
| Glass fibre reinforced phenolic composites         | 0.0001 - 10                    | Drop weight                                     | Tension                      | $E\uparrow, \sigma\uparrow$                                  | Barre et al. [24]      |
| Glass fibre reinforced polyester composites        | 0.0001 - 10                    | Drop weight                                     | Tension                      | $E\uparrow, \sigma\uparrow$                                  | Barre et al.[24]       |
| Unidirectional S2 Glass /8552 glass epoxy laminate | 0.001 - 600                    | SHPB                                            | Off-axis compression         | $\tau_{12}\uparrow, \gamma_{12}\downarrow$                   | Tsai and Sun[51]       |
| UD Glass epoxy composites                          | 0.001-100                      | High speed servo hydraulic testing apparatus    | Tension                      | $E_{11}\uparrow, \sigma_{11}\uparrow, \epsilon_{11}\uparrow$ | Shokrieh and Omid [46] |
| UD Glass epoxy composites                          | 0.001-100                      | High speed servo hydraulic testing apparatus    | Compression                  | $E_{11}\uparrow, \sigma_{11}\uparrow, \epsilon_{11}\approx$  | Shokrieh et al. [52]   |
| E Glass Ampreg 20 epoxy matrix                     | 0.001-450                      | SHPB                                            | Cross-ply compression        | $\epsilon\downarrow, \sigma\uparrow$                         | Ochola et al. [53]     |
| EGL/300/400/37                                     | 0-100                          | INSTRON VHS                                     | Tension                      | $\sigma\uparrow$                                             | Wang et al. [54]       |

|                                                                             |             |                                                                                             |                        |                                                                  |                        |
|-----------------------------------------------------------------------------|-------------|---------------------------------------------------------------------------------------------|------------------------|------------------------------------------------------------------|------------------------|
| S2/8552 glass epoxy                                                         | 0.0001-500  | Instron UTM & SHPB                                                                          | Transverse compression | $E_{22} \approx, \sigma_{22} \uparrow, \epsilon_{22} \downarrow$ | Tsai and Kuo [55]      |
| UD Glass epoxy composites                                                   | 0.001-100   | High speed servo hydraulic testing apparatus                                                | In-plane shear         | $G_{12} \downarrow, \tau_{12} \uparrow, \gamma_{12} \downarrow$  | Shokrieh and Omid [48] |
| Unidirectional E-glass fiber/vinylester                                     | 0.0001-600  | Instron UTM & SHPB                                                                          | Off-axis compression   | $E_{22} \uparrow, \sigma_{22} \uparrow, \epsilon_{22} \uparrow$  | Daniel et al. [16]     |
| Unidirectional glass fiber /epoxy composite materials                       | 1-100       | High-speed servo-hydraulic testing apparatus (Unique Fixture designed to hold the specimen) | Transverse tension     | $E_{22} \uparrow, \sigma_{22} \uparrow$                          | Shokrieh and Omid [56] |
|                                                                             |             |                                                                                             | Transverse compression | $E_{22} \uparrow, \sigma_{22} \uparrow$                          |                        |
| Pultruded polyester composite reinforced with unidirectional E-glass fibers | 0.0001-1000 | MTS 810 machine & Modified Hopkinson Bar                                                    | Tension                | $E_{11} \uparrow, \sigma_{11} \uparrow$                          | Asprone et al. [57]    |
| Woven-roving reinforced glass fibre/epoxy composite                         | 0.0001-870  | Instron UTM & SHTB                                                                          | Tension                | $E_{11} \uparrow, \sigma_{11} \uparrow, \epsilon_{11} \uparrow$  | Harding and Welsh [19] |
| Woven S2 Glass-Sc15 composite                                               | 550-3450    | SHPB                                                                                        | Compression            | $E \uparrow, \sigma \uparrow, \epsilon \approx$                  | Pan et al. [58]        |
| S-Glass/ epoxy                                                              | 0.001-27    | MTS Electro hydraulic closed loop system                                                    | Longitudinal tension   | $E_{11} \approx, \sigma_{11} \approx, \epsilon_{11} \approx$     | Daniel and Liber [59]  |
|                                                                             |             |                                                                                             | Transverse tension     | $E_{22} \approx, \sigma_{22} \approx, \epsilon_{22} \approx$     |                        |
|                                                                             |             |                                                                                             | In-plane shear         | $G_{12} \approx, \tau_{12} \approx, \gamma_{12} \approx$         |                        |

Various researchers have tried to evaluate the high strain rate sensitivity of carbon based composites. Gilat et al. [60] carried out tensile testing of the matrix and laminates with various fibre orientations at various strain rates from  $10^{-5}$  to  $650 \text{ s}^{-1}$  to understand rate effects upon carbon/epoxy composites. The characteristics of resin dominated configurations were found to be strain rate sensitive. Xuan and Yulong [61] reported the strain rate sensitivity of tensile strength of carbon fibre reinforced silicon carbide matrix (2D-C/SiC) composites. SEM evaluation of the failed samples revealed integrated fibre bundle pull out which demonstrates the increased interfacial bundle strength at high strain rates which in turn resulted in the rate sensitivity behaviour of the composite. Koerber et al. [12, 13] evaluated the high strain rate characteristics of unidirectional carbon-epoxy (IM7-8552) by carrying out longitudinal compression, transverse compression and in plane shear using SHPB apparatus. Strain rates corresponding to 63, 118 and  $350 \text{ s}^{-1}$  were achieved during testing. A dynamic compression fixture was developed for testing flat rectangular specimens. It was observed that the longitudinal compressive modulus was strain rate insensitive whereas the longitudinal compressive strength, transverse compressive modulus, transverse yield strength, transverse failure strength, in-plane shear modulus, in-plane shear yield strength and in-plane shear failure strength were found to increase linearly with strain rate. A summary of the results for high strain rate characterisation of carbon based composites performed by various researchers is given in Table 2-2.

To summarise for CFRP, conflicting rate sensitivity information can be seen from the table related to longitudinal characteristics of CFRP. Most of the studies (except for [62]) showed that the longitudinal tensile characteristics are rate insensitive. However, the longitudinal compressive characteristics showed rate sensitivity. This rate sensitivity could be attributed primarily to the rate sensitive deformation characteristics observed for end loading rather than the material characteristics as such. It was also observed that the strength and the stiffness estimated through the end loading method were significantly lower than the quasi-static properties estimated through standard test methods for the same materials. Another interpretation for the rate insensitivity observed in the longitudinal characteristics is that the rate of loading where data were generated was lower compared to the threshold strain rate. The transverse and shear characteristics of CFRP were mostly found to be strain rate sensitive and this can mainly be attributed to the rate sensitive characteristics of the matrix.

Table 2-2 Review of the high strain rate data available in the literature on CFRP (same note applies as in Table 2-1)

| Material Type                                   | Strain Rate range ( $s^{-1}$ ) | Testing Equipment                                  | Type of Loading                       | Observation                                                                                        | Reference                |
|-------------------------------------------------|--------------------------------|----------------------------------------------------|---------------------------------------|----------------------------------------------------------------------------------------------------|--------------------------|
| AS4/3501-6                                      | 400-700                        | SHPB                                               | Off-axis compression                  | $G_{12}\uparrow, \tau_{12}\uparrow$                                                                | Ninan et al. [47]        |
| Plain weave Carbon Epoxy                        | 496-1000                       | TSHB                                               | Interlaminar shear behaviour          | $G_{13}\uparrow, \tau_{13}\uparrow$                                                                | Naik et al. [50]         |
| IM7/977-2 Carbon epoxy matrix composite         | 0.00001 - 900                  | Instron UTM & SHPB                                 | Tension                               | $E_{22}\uparrow, \sigma_{22} \approx, G_{12}\uparrow, \tau_{12}\uparrow$                           | Gilat et al. [60]        |
| IM7-8552 Unidirectional Carbon Epoxy Prepreg    | 90-350                         | SHPB                                               | Transverse compression                | $E_{22}\uparrow, \sigma_{22}\uparrow$ (yield & Ultimate), $\epsilon_{22}\downarrow$                | Koerber et al. [13]      |
| IM7-8552 Unidirectional Carbon Epoxy Prepreg    | 63-118                         | SHPB                                               | Longitudinal compression              | $E_{11}\uparrow, \sigma_{11}\uparrow$                                                              | Koerber and Camanho [12] |
| AS4/3501-6 UD Epoxy Unidirectional carbon epoxy | 0.0001-400                     | Instron UTM & SHPB                                 | Off-axis Compression                  | $\tau_{12}\uparrow, G_{12}\uparrow$<br>$E_{22}\uparrow, \sigma_{22}\uparrow, \epsilon_{22}\approx$ | Daniel et al. [16]       |
| Graphite/ epoxy                                 | 0.001-27                       | MTS Electro hydraulic closed loop system           | Longitudinal tension                  | $E_{11}\approx, \sigma_{11}\approx, \epsilon_{11}\approx$                                          | Daniel and Liber [59]    |
|                                                 |                                |                                                    | Transverse tension                    | $E_{22}\approx, \sigma_{22}\approx, \epsilon_{22}\approx$                                          |                          |
|                                                 |                                |                                                    | In-plane shear                        | $G_{12}\approx, \tau_{12}\approx, \gamma_{12}\approx$                                              |                          |
| UTC 300/500 carbon Epoxy                        | 0.001-450                      | SHPB                                               | Cross-ply compression                 | $\epsilon\downarrow, \sigma \approx$                                                               | Ochola et al. [53]       |
| Woven Carbon epoxy composites                   | 500-1400                       | Modified SHPB                                      | Tension                               | $E\approx, \sigma\uparrow$                                                                         | Chiem and Liu [63, 64]   |
|                                                 | 1021-5410                      | TSHB                                               | Shear                                 | $G\approx, \sigma\uparrow$                                                                         |                          |
| CYCOM T700-300-300                              | 0-45                           | INSTRON VHS                                        | Tension                               | $\sigma\uparrow$                                                                                   | Wang et al. [54]         |
| IM7-8552 Unidirectional Carbon Epoxy Prepreg    | 0.0004-300                     | Standard electro mechanical Testing Machine & SHTB | Transverse tension & Off-axis tension | $E_{22}\uparrow, \sigma_{22}\uparrow$ (yield & Ultimate)                                           | Vogler et al. [18]       |

|                                                   |              |                                                        |                                               |                                                                |                        |
|---------------------------------------------------|--------------|--------------------------------------------------------|-----------------------------------------------|----------------------------------------------------------------|------------------------|
| UD Carbon Epoxy IM6G /3501-6                      | 0.001-1800   | Servo-hydraulic Testing Machine & SHPB                 | Transverse compression & Off-axis compression | $E_{22}\uparrow, \sigma_{22}\uparrow, \epsilon_{22}\approx,$   | Hsiao et al. [17]      |
|                                                   |              |                                                        | Shear                                         | $G_{12}\uparrow, \tau_{12}\uparrow, \gamma_{12}\downarrow$     |                        |
| UD Carbon T700/2521                               | 0.002-830    | Instron UTM & SHPB                                     | Longitudinal compression(0)                   | $E_{11}\uparrow, \sigma_{11} \approx, \epsilon_{11}\downarrow$ | Yokoyama et al. [15]   |
|                                                   |              |                                                        | Transverse compression(90)                    | $E_{22}\uparrow, \sigma_{22}\uparrow, \epsilon_{22}\downarrow$ |                        |
|                                                   |              |                                                        | Transverse compression(t)                     | $E_{33}\uparrow, \sigma_{33}\uparrow, \epsilon_{33}\downarrow$ |                        |
| AS4-PEEK                                          | 0.001-1000   | Instron UTM & SHPB                                     | Transverse compression                        | $E_{22}\uparrow, \sigma_{22}\uparrow, \epsilon_{22}\downarrow$ | Parry [65]             |
| AS4-Epoxy                                         |              |                                                        | Transverse compression                        | $E_{22}\uparrow, \sigma_{22}\uparrow, \epsilon_{22}\approx$    |                        |
| CFRP (vinyl ester)                                | 0.001-1000   | SHPB                                                   | Longitudinal compression                      | $E_{11}\approx, \sigma_{11}\uparrow, \epsilon_{11}\downarrow$  | Yuan et al. [66]       |
| CFRP_T1000 & duglycidylether of bisphenol A epoxy | 0.000033-600 | Shimadzu HITS-T10                                      | Tension                                       | $E_{11}\approx, \sigma_{11}\uparrow, \epsilon_{11}\approx$     | Naito [62]             |
| CFA graphite /epoxy composites                    | 0.0001-500   | Instron UTM & SHPB                                     | Transverse compression                        | $E_{22}\approx, \sigma_{22}\uparrow, \epsilon_{22}\downarrow$  | Tsai and Huo [55]      |
| HS160 REM UD Epoxy composites                     | 0.001-400    | Instron UTM & SHPB                                     | Quasi-isotropic laminate compression          | $\sigma\uparrow$                                               | Guedes et al. [67]     |
| UD Carbon EpoxyIM6G/3501-6                        | 0.001-200    | Servo-hydraulic testing machine & Drop Tower Apparatus | Longitudinal compression                      | $E_{11}\approx, \sigma_{11}\approx, \epsilon_{11}\approx$      | Hsiao and Daniel [14]  |
|                                                   |              |                                                        | Transverse compression                        | $E_{22}\uparrow, \sigma_{22}\uparrow, \epsilon_{22}\approx$    |                        |
| C/C composites (3K-PAN carbon felts)              | 0.0001-450   | Instron UTM & SHPB                                     | Cross Ply compression                         | $E\uparrow, \sigma\uparrow$                                    | Yuan et al. [68]       |
| Unidirectional HTA/6376C composite                | 0.001-1000   | Instron UTM & SHTB                                     | Transverse tension                            | $E_{22}\approx, \sigma_{22}\approx, \epsilon_{22}\approx$      | Melin and Asp [69]     |
| UD Carbon Epoxy (HYFIL-Torayca-130-S)             | 0.0005-450   | Instron UTM & SHTB                                     | Tension                                       | $E_{11}\approx, \sigma_{11}\approx, \epsilon_{11}\approx$      | Harding and Welsh [19] |
| IM7/8551-7 graphite/epoxy composite               | 0.001-2000   | MTS Servo Hydraulic system                             | Compression & off axis(balanced)              | $E_{11}\uparrow, \sigma_{11}\uparrow$                          | Jadhav[20]             |

The literature on the high strain rate characteristics of CFRP and GFRP show different level of responses of fibre reinforced composites. However, it can be summarised that the strength and stiffness of glass based composites are generally strain rate sensitive, whereas only the transverse and shear properties are considered rate sensitive for carbon based composites. The failure strain tends to decrease for increase in strain rate for both types of fibres for the majority of the literature reviewed. This may be because the matrix tends to be excessively brittle during high rates of loading [21]. Also, it can be observed that there is a threshold strain rate beyond which these materials become strain rate sensitive [42]. Polymer composite materials such as CFRP are known to be less rate sensitive below  $10 \text{ s}^{-1}$  whereas GFRP is typically rate insensitive below  $10^{-2} \text{ s}^{-1}$  [19, 24, 59, 70-72]. The viscoelastic characteristic of the matrix material and their influence on the interface strength can be regarded as the possible reason for the rate sensitive behaviour of composites.

## **2.2 The wide application of the impact simulation of composite structures**

This section will mainly focus only on articles related to high velocity and ballistic/hyper velocity impact simulations. Even though various articles on simulation of high velocity/ballistic and hyper velocity impact behaviour of composites have been published in the last two decades [31, 73-79], the attempt here is only to review relevant articles to highlight the requirement of high strain rate data for impact simulation.

### **2.2.1 Bird strike application**

Aircraft parts such as windshields, leading edges of wings, horizontal and vertical tail planes are prone to bird strike impact and are required to demonstrate safe landing when such impacts are encountered. These parts are required to meet the respective certification clauses to prove their airworthiness. Hence, different researchers have used impact simulation to develop these parts to save time and money as experimental testing is expensive.

McCarthy et al. [80, 81] have predicted the bird impact behaviour of fibre metal laminate (FML) leading edges (GLARE-GLass Aluminium REinforced laminates) using the bi-phase model, a material model based on continuum damage mechanics implemented in PAM-CRASH/SHOCK. Material Type 131 was used to define the behaviour of GLARE leading edges. The unidirectional composite global

ply model (Ply "Type 1") inside the Material Type 131 was used for definition of the strain rate effects for glass layers of FML. The smoothed particle hydrodynamics (SPH) method which is a gridless Lagrangian technique was used to model the bird [81]. Even though rate sensitive parameters of some of the material constants have been incorporated into the model, the extensive availability of rate sensitive data for all the required input properties was highlighted in the conclusions in both of their articles. Guida et al. [82] identified through FE simulation using MSC/Dytran a rib-less wing leading edge configuration consisting of sandwich construction made from FML, aluminium and honey comb which satisfied bird strike criteria, as specified in FAR [83, 84] through experimental tests. The "Orthotropic crushable material model", "Johnson-Cook model" and "Orthotropic failure material model" from MSC/Dytran material library were used for the materials definition for honeycomb, aluminium and fibre metal laminate respectively. The numerical prediction of the impact behaviour shows similarity with test with regard to global deformation characteristics. The localised deformation of the leading skin for experiments showed larger deformation along the span wise direction of the leading edge. Such a discrepancy could be related to the failure of the model to capture the actual deformation of the S glass layers of FML. The paper does not present much detail on the use of rate sensitive data of FMLs or for aluminium even though the Johnson-Cook model has the provision for the definition of strain rate characteristics. Since the FML used is glass-based, the material would tend to be rate sensitive for high velocity impact events as strain rates between 100-1000  $s^{-1}$  are observed in the vicinity during such impacts. Guida et al. [33] also performed a similar study of leading edge with sandwich construction of aluminium and honeycomb alone. Aluminium layers were modelled using the Cowper-Symonds law and rate sensitive parameters were defined in the model for simulation. The results showed good correlation of the simulation with the experiment with respect to global behaviour, final inter rib distance observed as a result of the inward movement of the deformed leading edge and relating to the non-penetration of the leading edge skin for the event. The ability of the numerical model to capture the deformation characteristics within 2% (defined in terms of the inter-rib distance) highlights the significance of correct rate sensitive parameters in the model.

The tensor skin concept demonstrated in papers [85, 86] is an interesting concept which could be employed in the design of various energy absorbing and impact resistant structures. The tensor skin concept replaces the brittle composite

by a polyethylene tensor skin with loops embedded inside a hybrid composite construction for skin elongation during transverse impact loads. These tensor skins do not serve any purpose during the normal operation, however they act as good absorbing members during transverse impact loading once the plies have failed, as the impact load results in unfolding of the tensor plies. This tensor skin acts like a membrane when loaded in tension. Strain rate effects were not incorporated in the material models in their simulations used for identification of the optimum configuration for bird strike resistance. This concept could be further extended for different energy absorbing structures using rate sensitive material models, as the mode of deformation especially for tensor skin involves large tensile deformation with large failure strains which for the class of the fibres considered are strain rate sensitive [87]. The tensor skin concept along with rate sensitive data can be very useful for development of energy absorbing structures through simulations.

Georgiadis et al. [28] have used a simulation based approach for the complete certification of a carbon fibre epoxy composite moveable trailing edge of the Boeing 787 aircraft with minimum validation tests performed at each scale. The SPH method was used for modelling the bird where the material characteristics were defined through an equation of state (EOS), whose parameters were determined separately using PAM-OPT through iterative optimisation of the numerical results with the respective experimental data of a bird strike event (inverse method). The bi-phase material model available in PAM-CRASH was used to define the composite material characteristics. The bird strike certification methodology was initially validated for a simplified model through series of tests and simulations. The deformation characteristics observed for low, medium and high velocity impacts showed that the model was sufficient to capture the deformation characteristics of the composite trailing edge subject to bird impact. Since there is no mention of the use of strain rate sensitive data or the use of a strain rate sensitive model, based on the results above, it can be inferred that the quasi-static behaviour of the CFRP is similar to its characteristics for strain rates corresponding to these impact events. The study, extended to a pre-production outboard flap, also showed similar results to the tests for four different cases of velocity.

Smojver and Ivancevic [88, 89] investigated the bird impact response of the inboard flap of a typical large transport aircraft through two different bird models (Lagrangian [88] and coupled Eulerian-Lagrangian (CEL) [89]) using the Abaqus

Explicit FE solver. The inboard flap consisted of a skin made of CFRP with the internal structural members made from different aluminium alloys. The leading edge and the trailing edge were made of sandwich constructions of CFRP-Nomex and aluminium (Al 2024)-Nomex combinations respectively. The post yield characteristics of aluminium alloy were modelled as rate dependent and those were implemented into the FE model through the VUMAT subroutine. CFRP was modelled as being rate insensitive to reduce the computational time. The numerical prediction showed damage was dependent on the direction of impact as well as the location. The bird strike simulation approach followed helped in the subsequent design of a damage tolerant, bird strike resistant flap structure which satisfied the bird strike certification requirements. No experimental validation of their results was presented. However, the numerical validation of a CFRP flat plate when subjected to impact at  $280 \text{ ms}^{-1}$  showed under-prediction of the damage characteristics of the plate by the numerical model used [90]. This shows that quasi-static properties alone cannot be used for the prediction of the impact characteristics for such high velocities. The threshold strain rate sensitivity for carbon could be between these velocity regimes ( $140\text{--}280\text{ms}^{-1}$ ) and hence could have such an influence in the impact characteristics. The under-prediction of the damage could also be attributed to the inverse sensitivity of the failure strain of CFRPs at high rates which has been observed by various researchers and is discussed in the literature survey on high strain rate characteristics of composite materials.

### **2.2.2 Ballistic/hyper velocity impact application**

Ballistic impact results in very high strain rates near the vicinity of the impact zone. Typical fibres that have been used for blast resistant applications are aramid (Kevlar, Twaron, Technora), polyethylene (Spectra, Dyneema), S2 Glass and polybenzoxazole (Zylon) [91, 92]. In the last decade, GLARE laminates have also been proposed for blast resistant applications due to the hybrid construction of the laminate, possibility of multiple failure mechanism and also due to the high strength of the fibres [93, 94]. The main requirement of blast resistant structures (soft body armours, helmets, blast resistant panels/containers in vehicle etc.) is to prevent the penetration of the projectile and dissipate the energy of the impact through various failure or deformation mechanisms of the material. The major energy absorbing mechanisms during ballistic impact of plain weave E-glass epoxy composites arise mainly from tensile failure of the primary yarns and deformation of the secondary yarns [95]. In the case of carbon composites such

as T300 epoxy, the deformation of the secondary yarns and shear plugging were the main energy absorbing mechanism during failure [95]. Shear plugging is a type of shear failure that occurs at the target material along the circumference of the projectile/punch as the induced shear plugging stress due to ballistic impact/punching exceeds the permissible shear plugging strength [96] of the target material [97]. Hoo Fatt et al. [98] discussed in detail the energy absorption mechanism occurring in GLARE panels through derivation of analytical expression for each mode of deformation.

In the case of perforation of fabrics or composite panels, the ballistic performance perforation is characterised in terms of a parameter called the Cunniff velocity [99] which is given by the relationship given in Eq. 2-1 (as taken from [100]). The higher the Cunniff velocity, the more appropriate is the material for the ballistic application. It may also be observed from Eq. 2-1 that the Cunniff velocity is a measure of the product of strain energy per unit volume (area under the stress strain curve) and the speed of sound in the material while taking into account the effect of density.

$$c^* = \left( \frac{\sigma_f \varepsilon_f}{2\rho_f} \sqrt{\frac{E_f}{\rho_f}} \right)^{1/3} \quad \text{Eq. 2-1}$$

The selection of materials for ballistic application is typically based on the Cunniff velocity parameter and comparison of various grades of fibre types based on the Cunniff velocity is shown in Fig 2-1. Hence most of the research on the topic of ballistic impact applications deals with materials likes Kevlar, Dyneema, Spectra fibres etc which all have high specific energy absorption.

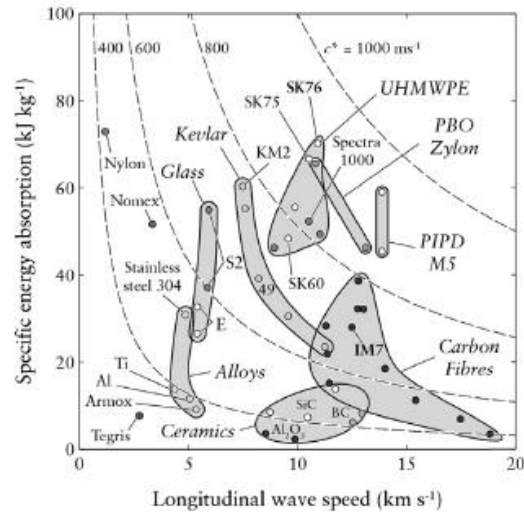


Fig 2-1 Materials typically used for ballistic application (contour lines indicate same Cunniff velocity number) [100]

Silva et al. [101] carried out ballistic impact simulation of thin composite laminated plates reinforced with Kevlar 29 performed using specially developed explicit finite element software called AUTODYN. An advanced material model implemented in AUTODYN to simulate the shock response of anisotropic materials was used for this purpose. Static and dynamic tests were performed for generation of the input data for simulation. The prediction relating to estimates for ballistic perforation velocity ( $V_{50}$ ), global damage and residual velocity were comparable to that obtained from experiments. Even though some of the high strain rate properties were used for the simulation, the authors highlight the importance of through thickness high strain rate properties for closer prediction through simulations. Duan et al. [92] performed simulation of the transverse impact of a cylinder on a single layer plain woven Kevlar fabric using LS-DYNA. Even though the model accounts for the frictional interaction between fabric, yarns and projectile, it does not account for the rate sensitivity of the fibre/yarn, as their study assumes Kevlar yarn to be a rate insensitive orthotropic material. The validation of the results obtained through simulation was not discussed in their study. Lim et al. [102] investigated the ballistic impact response of a Twaron fabric structure using a viscoelastic and strain rate dependent material model. Material Type 19 within DYNA3D was used in the study which offers the option for representation of the rate sensitive characteristics of the Young's modulus, yield stress, failure stress and tangent modulus in its isotropic elastic plastic model. The residual velocity and energy absorption observed for simulations and experiments closely match with each other. However, the authors also warn the readers of the errors that might have crept in due to the

simplification of the strain rate dependent model. Wicklein et al. [103] investigated the performance of CFRP subjected to hypervelocity impact using a material model developed based on an orthotropic continuum model using ANSYS AUTODYN. CFRP-Al honeycomb sandwich constructions are used in satellites and are required to demonstrate its performance for hypervelocity impact that might cause due to space debris impact. A material model for CFRP was developed and validated by comparison of numerical simulations to planar plate impact and hypervelocity impact experiments. The need for the generation of strain rate dependent parameters was emphasised as all the present data were generated from quasi-static experiments.

### **2.2.3 Conclusions from the literature review on impact simulation application**

The importance of using simulations for saving on the enormous cost of testing has been highlighted by various researchers. There has been considerable effort regarding the development of material models for prediction of impact characteristics. It can be seen from the literature review that various commercial FE packages are efficient in prediction of impact behaviour of composite materials through their own material models or through the option for definition of user defined material models through subroutines. A drawback of some of the work carried out was the use of inappropriate material models which cannot incorporate strain rate sensitive characteristics or make the blind assumption that the material is strain rate insensitive. The challenges in simulations are the use of the correct material model, boundary conditions, interaction of friction, use of actual experimental data and incorporation of the rate sensitive characteristics for accurate simulation of impact events of high strain rate regimes. Researchers have shown that the use of strain rate dependent data helped in close correlation of predictions of the impact behaviour of glass based composites to that observed from the experiments and also have highlighted the need for more comprehensive material input data relating to rate sensitivity for better predictions. However, the requirement for strain rate sensitive data is still a matter of debate regarding the simulation of impact behaviour of carbon based composite structures. Georgiadis et al. [28] have successfully predicted the damage behaviour of carbon composite structures with material models without rate sensitive parameters whereas the studies performed by Smojver and Ivancevic [90] under predicted the damage characteristics of the carbon composite plate subjected to high velocity impact using material models without rate sensitive parameters. Wicklein et al. [103]

emphasised the need for high strain rate properties for better prediction of the hyper velocity impact behaviour of carbon composite plates. The study performed by Georgiadis et al. [28] used velocities up to  $140 \text{ ms}^{-1}$  whereas the studies mentioned by Smojver and Ivancevic [90] go up to around  $280 \text{ ms}^{-1}$ .

Qi et al. [104] have illustrated the dependence of dynamic strength on strain rate for brittle materials with a simple curve with three defined regimes. They have identified that there is a regime 2 (Fig 2-2) for brittle materials where the rate sensitivity of the material is very high. The materials are comparatively less rate sensitive in regime 1 and regime 3. Yu et al. [105] has shown with experimental data that common engineering materials such as metals (aluminium 7075, stainless steel), limestone and concrete, silicon carbide and alumina, and polymers (resins, polycarbonates, polyethylene) do exhibit similar characteristics in strain rate dependency with regard to their strength.

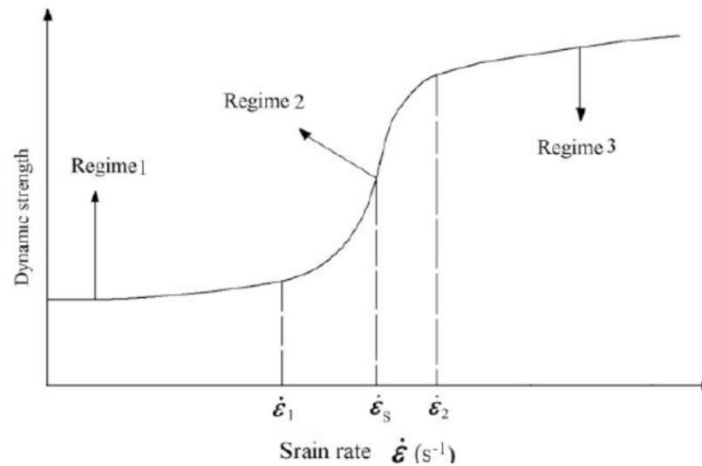


Fig 2-2 Dependence of strength on strain rates of engineering materials (as taken from [105], originally from [104])

Similarly, carbon composites may not be strain rate sensitive in the lower velocity impact regime. However, they could be rate sensitive in the higher velocity impact regimes. There is a promising scope for exploring the rate sensitive characteristics of carbon fibres and carbon based composites. As discussed in the previous section (section 2.1, page 5), there is contradictory information on the rate sensitivity of longitudinal characteristics of CFRP and there is non-availability of accurate high strain rate data of CFRP for longitudinal characteristics beyond  $500 \text{ s}^{-1}$ . Since CFRP materials are the material of choice in structural applications for aircraft and satellite structures, due to their high specific stiffness and strength, and the structures are required to show airworthiness for different cases

of impact application, it is highly desirable to generate information on the high strain rate behaviour of CFRP materials. The next section discusses some of the high strain rate material models available in the commercial finite element packages that have been used for impact simulation.

## **2.3 Material model definitions for impact simulation**

The use of material models for the prediction of failure initiation and damage propagation is very important for accurate prediction of the impact behaviour of composite materials. Even though extensive work has been done on the development of material models for impact applications, a complete validated methodology has not yet been achieved [106]. This is primarily attributed to the complex nature of the failure mechanisms in composites which are dependent on a range of parameters such as geometry, material anisotropy, layup sequence, material configurations, loading conditions, load history and failure modes [106]. A constitutive model definition for a material is required for describing the structural response during loading. This section discusses the material models that have been used by various researchers for impact simulation of composite materials. An assessment of the different failure theories available for composite materials for general application has been summarised in a variety of review articles [106, 107]. The most important aspect of the material models for higher velocity impact simulations is to incorporate the features of failure initiation, damage degradation, strain rate effect and inclusion of temperature effects, if any. The aspects relating to delamination propagation are less significant nearer to the impact zone from a simulation viewpoint for higher velocities, unlike low velocity events. For high velocity impact such as bird strike, energy absorption is the dominant requirement which is more dependent on the toughness and yield strength of the matrix, the ability of the structure to undergo shape changes and the fracture energy of the fibre materials. By comparison with fibre fracture, delamination is a low energy phenomenon which can be triggered due to stiffness mismatch when subjected to transverse impact loads. This section gives a brief overview of the material modelling techniques that have been used for the impact simulation of composite structures.

### **2.3.1 PAM CRASH Bi-phase model [108]**

The PAM CRASH bi-phase model assumes the composite to be a heterogeneous material with two phases, namely a fibre phase and matrix phase. The model requires the properties of the fibre and matrix to be defined to evaluate each

phase separately. The matrix phase is subjected to shear damage due to pure shear and volumetric damage due to the expansion or contraction of the matrix. In this model, the stiffness and strength of an element are calculated by superimposing the effects of each phase. Material Type 131 included in this model has the option to use separate material laws with a different failure law for each layer. This model uses the micromechanics approach to analyse failure of fibre and matrix separately for the fibre direction only. It assumes homogenised properties of the composites in all other direction (22, 33) including the shear (12, 23, and 13) directions. A series of coupon level tests on unidirectional and angle ply laminates is required for generation of all the parameters [80].

The unidirectional composite global ply model (Ply "Type 1") within Material Type 131 can be used for definition of the strain rate effects [80]. The strain rate effects incorporated into the CDM model are similar to the rate dependent models developed for metals. This model also has the option for incorporating anisotropic rate dependencies. The rate dependent model incorporates a threshold reference strain rate beyond which the property is rate dependent. The rate dependency is incorporated into the model based on the concept of viscous stress. This results in separate definition of viscosity functions for each element in the stiffness matrix [80]. The viscosity function defined for elastic moduli was used to take into account their rate effects of damage parameters. A separate viscosity function was introduced to account for the rate dependency of failure strains [80].

The bi-phase material model requires material input data of the constituents and the work described in this thesis aims to generate data for use directly in this material model.

### **2.3.2 LS DYNA MAT 162 [109]**

The MAT 162 material model in LS DYNA adopts the damage mechanics approach for simulating the softening behaviour of the composite after damage initiation. The quadratic failure criterion, as defined by Hashin's model [110], is used for definition of fibre failure. The matrix failure criterion considers two failure modes: one is perpendicular to the planes of layering (transverse matrix failure) and other is parallel to the layering planes (commonly known as delamination). This model has the capability of a definition of the strain rate effect which is based on a logarithmic law. The matrix softening behaviour after damage initiation is based on the damage mechanics approach proposed by Matzenmiller et al. [111].

This model requires a large volume of coupon level test results in order to characterise the parameters defined in the model.

### **2.3.3 Material model definition with Abaqus**

Smojver and Ivancevic [88] used a built-in progressive failure and damage model of Abaqus/Explicit for numerical prediction of bird impact response of an inboard flap structure. This built-in model uses Hashin's failure criteria which accounts for the fibre rupture in tension, fibre buckling and kinking in compression, matrix cracking during transverse tension and shear, matrix crushing under transverse compression and shear [88]. The degradation model is based on definition of damage parameters relating to fibre damage, matrix damage and shearing, all of which result in reduced values of material stiffness in the stiffness matrix, where the damage parameter varies from '0' for an undamaged material and '1' for fully degraded material. In the stiffness matrix, the ply level properties are degraded once the damage initiation criterion is met. This model however does not account for the rate effects of the material. However, Raimondo et al. [112] developed a phenomenologically based, strain rate dependent failure theory which was implemented in Abaqus/Explicit for one integration point solid elements and has been considered as suitable for numerical modelling of UD polymer composites especially CFRPs. In their model, a set of dynamic failure criteria was proposed which included a criterion for dynamic fibre compressive failure and a phenomenological strain rate dependent failure criterion for the definition of the tensile and compressive failure of the matrix. The model also proposes a strain rate dependent phenomenologically based hardening model which can be used in combination with the proposed failure criteria and replaces the conventional visco-plastic hardening material model definitions. This model follows a rate independent failure criterion for definition of fibre rupture during tensile loading. Lidgett [1] in his work used a failure model which was separately defined as a user subroutine for implementation with Abaqus/Explicit for analysis of micro-scale and meso-scale models. The micro-scale model which has definition of the fibre, matrix and interface characteristics assumes fibre to be rate independent and the matrix as rate dependent. The material model for meso-scale considers the yarn as rate dependent to consider for the effect of matrix. The details are further discussed in section 5.4 (page 131).

### **2.3.4 Conclusions from material model survey**

Most of the material models available in commercial software require lamina properties of the composites. The rate sensitivity of the properties is defined through different laws for each of the solvers. A numerical finite element approach for generation of the lamina properties from a micro-scale model can be useful for generation of the input data for these material models. This will significantly reduce the need for generation of too much experimental data from coupon level tests.

## **2.4 Multi-scale modelling of composite materials relating to high strain rate models**

All aircraft components need to show their functional capability in adverse environments to the requirements set by aviation regulators to prove their airworthiness. One of the approaches that can be used for reducing the number of tests involved in certification processes is the use of a multi-scale modelling approach for generation of material properties. Multi-scale modelling of composites involves study from micro-scale (fibre/matrix level to lamina level) to meso-scale (lamina to sub laminate level) and finally to macro-scale which is the laminate/structural scale. A multi-scale modelling platform can be used to simulate the characteristics of various composite structures from the information relating to fibres and matrices. The description of a model for micro-scale/ meso-scale/macro scale would be specific to the problem and will be based on the composite internal architecture. An interesting outlook on the necessity and challenges of multi-scale modelling of composites has been highlighted by Cox and Yang [39]. One of shortcomings of multi-scale modelling following the bottom up approach is that the mechanisms of failure that are modelled at the lower scale may not be very significant in larger scale testing. However, multi-scale modelling can be used as a means for conducting virtual mechanical testing and can thereby reduce the number of coupon tests and the costs involved in the design and testing cycle. This section gives an overview of the literature on multi-scale modelling and/or micro-scale modelling related to generation of high strain rate properties of composite materials.

Goldberg and Stouffer [113] modified the Ramaswamy-Stouffer [114] strain rate dependent elastic equation to model the deformation response of polymeric materials. The equation neglects the hardening effect observed for metals at low

strains. The equation is applicable only for small strain and the effect of temperature is neglected. The equations were introduced into the micromechanical models to analyse uniaxial composites at fibre orientation angles. Goldberg et al. [115] used the modified Ramaswamy-Stouffer equation for predicting the polymer composite material characteristics at strain rates of  $450 \text{ s}^{-1}$ . Tabiei et al. [116] also used the modified Ramaswamy-Stouffer strain rate dependent proposed by Goldberg and Stouffer [113] to model the deformation response of polymeric materials. LS Dyna was used to implement the modified constitutive equation through a user subroutine. The micromechanical model assumes the fibre to have a square geometric cross section in the representative volume cell unlike the conventional circular ones. The micro fibre criterion was incorporated to capture the various failure mechanisms including micro cracking, matrix crushing, fibre breakage, fibre buckling and kink band phenomena. The validation of the models was carried out for strain rates of 0.001, 0.1, and  $1 \text{ s}^{-1}$ .

Chen and Ghosh [117] used separate constitutive models for fibre and matrix in the development of a micromechanical model for understanding the deformation and damage process in fibre reinforced metal matrix composites. A unit cell comprising of a cylindrical fibre in square packed arrangement with the necessary periodic boundary condition was used for the high strain rate studies. A continuum damage model was used to describe the fibre behaviour. The matrix model uses a strain rate dependent elastic-viscoplastic constitutive model with damage evolution based on the Gurson [118]-Tvergaard [119]-Needleman [119, 120] model to simulate the behaviour of a porous ductile matrix whereas it uses a Johnson-Cook formula [121] for describing the behaviour of rate dependent matrix without porosity (voids). These models were implemented as a user material subroutine (VUMAT) in the Abaqus explicit FE code.

Lidgett et al. [1, 122, 123] used a multi-scale modelling approach for prediction of high strain rate properties of carbon and S glass composites using the constituent data at the micro-scale in order to predict the composite laminate characteristics at the meso-scale for strain rates of  $1\text{-}10^4 \text{ s}^{-1}$ . The micro-scale modelling carried out in the study involved the analysis of a hexagonally closed packed unit cell for eight different loading scenarios. The input data for material model used requires the elastic and strength properties of fibre, matrix and interface. The meso-scale models assumes the same matrix model but with only

a single strain rate scaling constant. The yarn model uses the properties from micro-scale and uses scaling constant differently for stiffness and strength.

Most of the studies mentioned above involved prediction of composite material characteristics at strain rates below  $500 \text{ s}^{-1}$  by using strain rate sensitive constitutive models for defining the matrix characteristics. The study conducted by Lidgett made was on multi-scale modelling for prediction of characteristics of composites at high strain rates ( $10^4 \text{ s}^{-1}$ ) which correspond to the high velocity and ballistic impact regimes. Even though an extensive study has been conducted at the micro and meso levels, the data used in the micro-scale models were obtained from published literature or data sheets for similar fibre materials/resins. Hence, there is scope to improve the prediction capability of Lidgett's model by using the properties and scaling parameters obtained from actual tensile and compression tests for the materials or through inverse methods. Also there is the scope to demonstrate the use of the multi-scale models for generation of design allowables at each scale through Monte Carlo simulations.

## **2.5 High strain rate data for carbon and glass fibres**

The experimental determination of high strain rate characteristics of fibres is always challenging and complex. The method of gripping the fibres and prevention of fibre breakage during the striker movement (in case of SHTB) or loading process are some of the factors that makes the determination of high strain rate properties of the fibre more complicated. There have been only a few articles on the measurement of high strain rate properties through experimental testing of single fibres. Most of these articles deal with high strain rate property determination of Kevlar fibres [124-126], PPTA (poly(p-phenylene terephthalamide)) [127-132], Dyneema [133, 134], wool fibres [135], spider silk [136] etc. This section gives an overview of the high strain rate data available in the public domain relating to carbon and glass fibres only. The section discusses the high strain rate data generated through fibre bundle testing for each of the fibre types.

Shokrieh et al. [137] used the experimental data on the longitudinal tensile properties of unidirectional glass epoxy composites [46] to generate the high strain rate data of E-glass fibres from the rules of mixtures method. The rate sensitivity observed for E-Glass fibres based on the estimated data is shown in Fig 2-3.

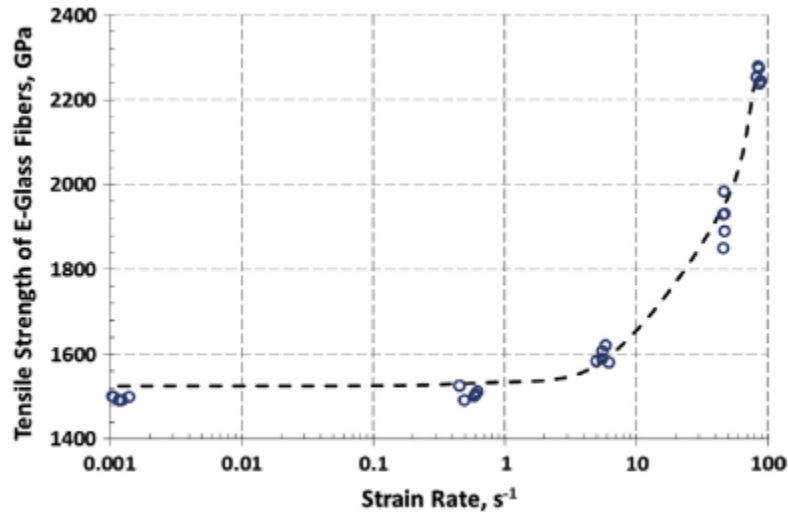


Fig 2-3 Rate sensitivity of tensile strength of E Glass fibres from estimated data [137]

Taniguchi et al. [138] evaluated the high strain rate tensile characteristics of glass fibres from experimental results available from fibre bundle testing. A 1K fibre bundle with average fibre diameter of  $17\mu m$  was considered for the testing. Studies have shown that the modulus of the fibre is rate independent, however strength showed strong dependency on strain rate. A strain rate of  $250 s^{-1}$  was achieved for dynamic characterisation using a split Hopkinson tensile bar (SHTB). Wang and Xia [139] investigated the statistical distribution of the tensile strength of E glass fibre bundles at high strain rates ( $90-1700 s^{-1}$ ) through a self-designed disc impact tensile test machine. Zhou et al. [10] experimentally evaluated the high strain rate tensile characteristics of T300 and M40J carbon fibre bundles at strain rates of  $0.001 s^{-1}$  to  $1300 s^{-1}$ . The results showed that the fibre bundle characteristics were strain rate insensitive as the strength, modulus and failure strain were found to have no dependency upon the rate of loading. Hill and Okoroafor [11] investigated the effect of interface friction by evaluating the strength characteristics of dry and lubricated fibre bundles. Fibres bundles of materials such as E glass, carbon and Kevlar 49 were tested.

It can be seen that, so far, no high strain rate data for single fibre testing of carbon or glass fibres are available in the public domain. The assessment of the strain rate sensitivity of the fibre strength from the fibre bundle strength data may not be the right approach. Hence, it would be desirable to develop a method for high strain rate tensile characterisation of fibres.

## 2.6 Rationale and objectives

### 2.6.1 Rationale

Based on the literature survey presented in the previous sections, the rationale of this work can be summarised as follows:

1. *Multi-scale modelling approaches require constituent material data.* A multi-scale modelling approach helps in reducing the number of tests as it can predict the overall structural behaviour of the laminate through information generated from lower scales (meso and micro). It is important to use the correct data in the micro-scale model for accurate prediction of the macro-scale properties/behaviour. Hence, there is a need to generate the high strain rate properties of composite material constituents which can be used as input for the micro-scale models to improve the predictive capability of the multi-scale models, in this case for the models developed by Lidgett [1].
2. *Extensive use of CFRP demands detailed evaluation of its impact characteristics.* CFRPs parts/components need to demonstrate their capability to withstand impact related events such as bird strike (in the case of aircraft) and space debris impact (in the case of satellite structures). There is a need to drive the design of such components through simulation. However, there are insufficient data available on the high strain rate characteristics of carbon based composites beyond  $500 \text{ s}^{-1}$ . Although there are published articles on the high strain rate tensile testing of single fibres, there are insufficient data on the high strain rate tensile characteristics of carbon fibres which could be used along with Lidgett's micro-scale model [1] for the generation of high strain rate characteristics of composite materials.
3. *The material model used within the micro-scale finite element model requires compressive and shear properties of fibre.* Generation of quasi-static or high strain rate data on transverse and compressive properties of fibres is impractical. Hence, there is scope for exploring the use of inverse methods in conjunction with these micro-scale model developed by Lidgett [1] for generation of these properties.
4. *Variability assessment and generation of design allowable for each lamina/laminate property.* The constituent data generated will have variability as a result of their processing methodology and also due to the

variable test conditions. For example, the strength of the fibres is expected to have a significant amount of scatter due to the presence of flaws along the length. The micro-scale model developed by Lidgett [1] is a deterministic model which accepts only single value for each material input. A Monte Carlo simulation with the multi-scale models could be useful to demonstrate how variability in the constituent parameters affects the macro level properties.

### **2.6.2 Objectives**

The general aim of this thesis is to develop methodologies for evaluation of high strain rate properties of composite material constituents. The activities undertaken as part of this work can be classified into four specific objectives as follows:

1. Development of a split Hopkinson tensile bar for single fibre testing (Chapter 3)

The purpose of this task is to develop a test facility for generation of high strain rate tensile properties of stiff single fibres such as carbon. The sub tasks involved with this work can be classified as:

- i. Identification of the design approach for SHTB through review of different methodologies for conducting high strain rate testing of materials
  - ii. Development of gripping methodology for single fibre testing
  - iii. Evaluation of wave propagation in SHTB through finite element analysis
  - iv. Fabrication and assembly of the designed rig
  - v. Resolution of the challenges involved during commissioning of rig
2. Mechanical Characterisation of composite material and its constituents for different strain rates (Chapter 4)

The purpose of these tasks is to generate the tensile fibre characteristics for input into the micro-scale model and generate data for use with inverse models. The sub tasks involved in this study is as follows:

- i. Longitudinal compression of UD composites at quasi-static and high strain rates
- ii. Transverse compression of UD composites at quasi-static and high strain rates
- iii. Compression of cross ply laminates at quasi-static and high strain rates

- iv. Tensile characterisation of T800, T700 and IM7 fibres at quasi-static and high strain rates
- 3. Development of an inverse modelling methodology for generation of high strain rate properties of composite material constituents from composite data (Chapter 5)

The main purpose is to develop a methodology for generation of compressive and transverse properties of fibre. The sub tasks involved in this study were as follows:

- i. Parametric studies for identification of parameters to which the measured characteristics are most sensitive in each case of loading
  - ii. Development of code for interfacing the finite element solver with the optimisation solver
  - iii. Inverse estimation of the most sensitive parameters and resolution of the challenges encountered
- 4. Monte Carlo Simulation for probabilistic prediction of strength of composite laminate at high strain rates (Chapter 6)

The main purpose of this section is to demonstrate the use of constituent material data with its variability with the multi-scale models to show the variability in the macro level properties. The main task involved in this study was to develop a Python code to generate the distribution data of composite properties through Monte Carlo simulations of the micromechanical models.

## **2.7 Conclusions**

The rationale of the thesis has been described through detailed discussions on the requirement of high strain rate data for impact simulation of composite structures, the significance of multi-scale modelling approaches for generation of the high strain rate data and need for high strain rate data of carbon fibres. The objectives were defined based on the gaps in the literature to identify methodologies to evaluate the high strain rate properties of composite material constituents and demonstrate its use in multi-scale for prediction of high strain rate characteristics of the composites.

## **3 Development of a split Hopkinson tensile bar for single fibre testing**

### **3.1 Introduction**

To use effectively the multi-scale modelling environment developed by Lidgett [1], micro-scale properties of the constituents are necessary. Carbon fibres have found extensive application in the primary components of modern aircraft due to their high specific modulus and high specific strength. Several aircraft structures such as the leading edges (wing, horizontal stabiliser and vertical stabiliser), wing spars and control surfaces such as ailerons, rudders and elevators are required to be cleared for bird strike impact to prove their airworthiness. The certification requirement for airworthiness requires a successful completion of flight for the airplane in the case of a bird strike with a relative velocity of 0.85 times the cruise velocity of the aircraft at 8000 ft or cruise velocity at sea level, whichever is more critical (FAR 25.571/ CS 25.571, FAR 25.631/CS 25.631) [9, 140]. The strain regimes for such a high velocity impact event are in the range of  $10^2 - 10^3 \text{ s}^{-1}$  near to the impact location. The tensile and flexural properties of the composites are controlled by the tensile properties of the fibre. As mentioned in the previous chapter, there is insufficient data on the high strain rate tensile properties of the carbon fibres or glass fibres for high strain rates. The high strain rate testing of materials is generally carried out using a split Hopkinson pressure bar which uses the one dimensional stress wave generated inside the bar to load the specimen. Depending on the magnitude of the stress wave and the size of the specimen, different strain rates up to  $10^4 \text{ s}^{-1}$  can be achieved using this technique. A split Hopkinson tensile bar has been designed and developed as part of this study for evaluation of the tensile characteristics of aerospace grade carbon fibres.

This chapter gives an overview of the literature related to various approaches for high strain rate testing, the theory relating to the split Hopkinson pressure bar (conventional and tensile version), detailed studies carried out as part of the SHTB rig development and challenges encountered during the commissioning of the rig. For consistency in referencing in this thesis, the compression version, tensile version and torsion version of split Hopkinson pressure bars will be referred as SHPB, SHTB and TSHB respectively.

## 3.2 Literature review

### 3.2.1 High strain rate testing methods

There are various dynamic test methods that have been employed for the evaluation of material characteristics such as Izod or Charpy tests, punch tests, drop weight, expanding ring tests, pressure bar, servo-hydraulic/pneumatic, flyer plate etc. The strain rate regimes and characteristic time involved for these test methods are as shown in Fig 3-1. Detailed discussions on the testing methodologies for high strain rates have been presented by various researchers [42, 43, 141, 142]. This subsection deals with dynamic test methods other than SHPBs which are used for strain rates greater than  $10^5 \text{ s}^{-1}$ .

#### 3.2.1.1 Taylor impact test method

The Taylor impact test method [141] basically uses the concept of measuring the dynamic yield stress of a cylindrical projectile based on the geometric measurements on it before and after its impact on a rigid target (Fig 3-2). Based on the information of density ( $\rho$ ) and velocity of impact ( $V$ ), the dynamic stress can be evaluated (Eq. 3-1).

$$\sigma = \frac{\rho V^2 (L - X)}{2(L - L_1) \ln\left(\frac{L}{X}\right)} \quad \text{Eq. 3-1}$$

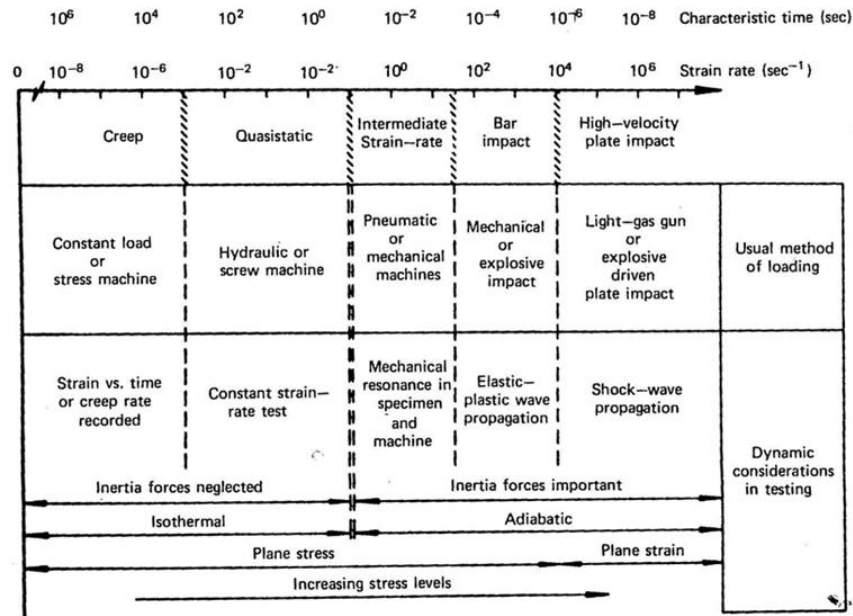


Fig 3-1 Strain rate regimes and characteristic time for different material testing techniques (as taken from [143], originally from [144] )

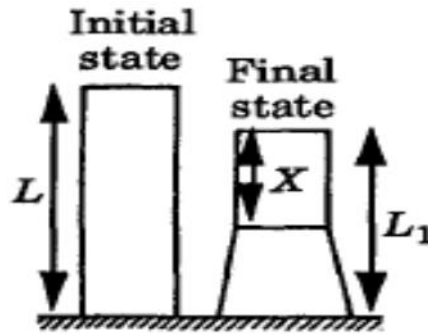
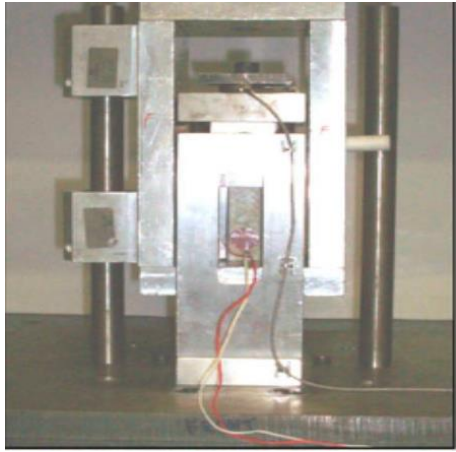


Fig 3-2 Initial and final states of Taylor impact specimen [141]

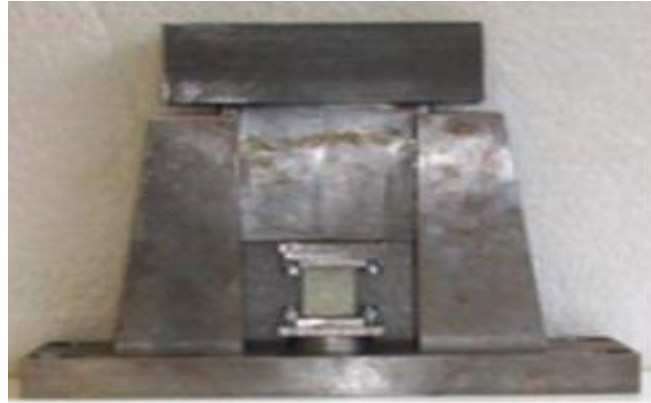
The technique was developed for metallic materials wherein the elastic strain was much lower compared to the plastic strain where an assumption of rigid plastic behaviour can be justified. The extension of this technique to polymers was carried out by Kukureka and Hutchings [145], by assuming the polymers to be a linear elastic perfectly plastic material. The length variation of polymer cylinders during an impact on a rigid anvil was analysed through elastic-plastic wave propagation theory to estimate the dynamic characteristics of the striking material. The yield stress and yield strain were evaluated from a separate expression proposed by Hutchings [146] from the dimensions of the specimen before and after impact, density of the specimen and the velocity of impact. Cylindrical specimens of 16mm diameter and 40 mm length were fired through a gas gun onto a massive rigid steel anvil. Five different unfilled polymers were evaluated using this technique. The strain rates achievable with this technique were quite similar to the strain rates that can be achieved through SHPB techniques.

### 3.2.1.2 Drop weight tests

A falling mass is allowed to impact on a specially designed fixture to generate uniaxial loading. The strain rates achieved are around  $1\text{-}100\text{ s}^{-1}$ . The strain rates achieved is mostly limited by the maximum height of the drop weight apparatus. To test at larger strain rates, one can give an initial velocity to the falling mass rather than allowing it to make a free fall. Brown et al. [49] used specially designed fixtures for high strain rate tensile and compression testing of E-glass/polypropylene woven fabric composites using a drop tower facility (Fig 3-3). The facility achieves higher velocities only by variation of the drop height.



a) Tension configuration



b) Compression configuration

Fig 3-3 Tensile and compression fixtures developed for high strain rate testing [49]

### 3.2.1.3 Expanding ring/explosive loading tests

This method uses the dynamic symmetric free expansion of a ring for evaluation of stress-strain characteristics of a specimen subjected to radial expansion (through fluid pressure [147], EMF [148] or explosive detonation [149]) during the test. Since rotational symmetry is preserved during expansion, tangential stress remains uniform and the radial stress would be negligible due to the use of thin ring specimen. This test can be used for testing at intermediate or high strain rates, e.g.  $10 - 10^3 \text{ s}^{-1}$ . Even though this method generates tensile loading, compressive loading has also been achieved with this technique using dynamic external pressure pulse which gets transferred to the ring specimen through liquid in a specially designed arrangement [150].

### 3.2.1.4 Servo-hydraulic/pneumatic tests

Hydraulic and pneumatic machines can be used to perform tests with great control and speed capability. However, the order of the strain rates achieved is of the order of  $10 - 10^2 \text{ s}^{-1}$ . One drawback with this method is the ringing effect which is significant with increasing impact speeds ( $5 - 20 \text{ ms}^{-1}$ ) [25]. The ringing effect, which is the excitation of the resonant frequency of the load cell, tends to affect the interpretation of the test data as the loading rates are increased [151]. This type of testing machine uses slack adaptors/innovative gripping jaws which grip the specimen only after the desired crosshead speed is achieved [54].

### 3.2.2 High strain rate testing using SHPB

#### 3.2.2.1 Theory of one dimensional longitudinal wave propagation in bars

When a body is subjected to a suddenly applied load, the deformation and stresses propagate through the material in the form of one or more stress disturbances which travel with a finite velocity from the point of application of load [152]. This velocity is a characteristic of the material and is independent of the method of application of these loads. Two types of waves are generated during such an impact. The first is a longitudinal wave (also known as dilatational) wherein the particle motion is parallel to the direction of propagation of wave. The second is a transverse/shear wave (also known as a distortional wave), wherein the particle motion is normal to the direction of propagation of wave. Since SHPBs are primarily slender bars with length at least twenty times the diameter, the theory based on the one-dimensional longitudinal wave propagation is applicable in such situations. The governing equation of motion for one dimensional wave propagation in bars is given by Eq. 3-2.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{Eq. 3-2}$$

where

$$c = \sqrt{\frac{E}{\rho}} \quad \text{Eq. 3-3}$$

The above equation assumes that the bar is homogeneous so that  $E$  and  $\rho$  do not vary with axial position. It is also assumed that plane, parallel cross sections remain plane and parallel and there exists a uniform axial stress distribution within the bar when subjected to impact loading along its axis. The above equation ignores the effect of transverse strain, lateral inertia, body forces and other dissipative forces such as internal friction etc. This equation holds true for propagating pulses having wavelength greater than 6 times the diameter of the bar [143]. The intensity of the stress wave propagated is given by Eq. 3-4.

$$\sigma = \rho c v \quad \text{Eq. 3-4}$$

where  $v$  is the particle velocity which is the rate at which the particle displacement occurs for a given loading ( $\frac{\partial u}{\partial t}$ ).

##### 3.2.2.1.1 Reflection and transmission of waves [152, 153]

The characteristics of wave reflection depend on the type of interacting boundary. The pulse reflected from a fixed boundary will have the same sign as that of the incident wave. i.e. a tensile pulse will be reflected as tensile pulse and a

compressive pulse will get reflected as a compressive pulse [152, 153]. However, in the case of a free boundary, the reflected pulse has the opposite stress characteristics from the incident pulse. i.e. a tensile pulse gets reflected as a compressive pulse and a compressive pulse gets reflected as a tensile pulse [152, 153].

When the stress pulse moves over an interface of discontinuous cross section or material, the wave will be partially reflected and partially transmitted. During wave propagation across such interfaces, two conditions must be satisfied at the interface. i.e the force acting on either side of the interface should be equal and there should be continuity of particle velocities at the interface. For a wave moving from zone A to zone B through an interface (Fig 3-4), the above conditions can be represented as in Eq. 3-5 and Eq. 3-6 .

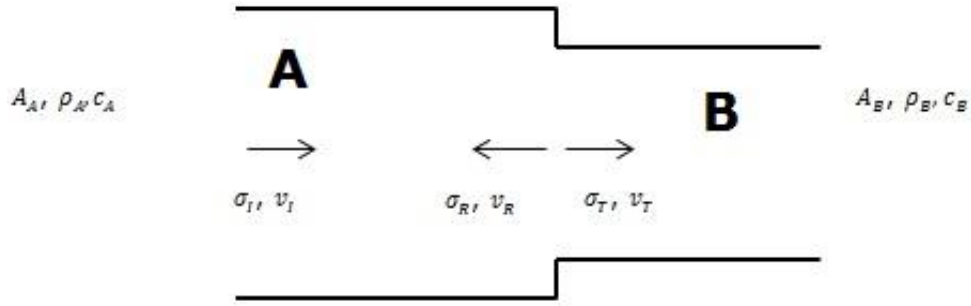


Fig 3-4 Wave reflection and transmission across interfaces

$$A_A(\sigma_I + \sigma_R) = A_B\sigma_T \quad \text{Eq. 3-5}$$

$$v_I - v_R = v_T \quad \text{Eq. 3-6}$$

Based on the above conditions, the stress wave transmitted ( $\sigma_T$ ) and stress reflected ( $\sigma_R$ ) at the interface can be represented as Eq. 3-7 and Eq. 3-8 respectively.

$$\sigma_T = \frac{2}{1+T} \frac{A_A}{A_B} \sigma_I \quad \text{Eq. 3-7}$$

$$\sigma_R = \frac{1-T}{1+T} \sigma_I \quad \text{Eq. 3-8}$$

$$\text{where,} \quad T = \frac{\rho_A c_A A_A}{\rho_B c_B A_B} \quad \text{Eq. 3-9}$$

These equations show that the reflected pulse characteristics can be tensile or compressive depending on the area and the impedance of the interacting zones. However, the transmitted pulse will always have the same characteristics as the incident pulse. It can also be observed from Eq. 3-7 that the stress amplification

and particle velocity amplification can be achieved by using multiple stepped bars.

### **3.2.2.2 History and development of SHPB**

The high strain rate characterisation of materials for tension, compression or torsion using SHPB is based on principles of one dimensional wave propagation theory. The SHPB concept evolved from the stress wave experiments performed on iron wires by John Hopkinson [154, 155]. His son Bertram Hopkinson [156] further extended the study to measure the pressure produced by the impact of a bullet or by the detonator at the end of cylindrical steel rod. The trapped momentum concept was used by Bertram Hopkinson in his pressure bar experiments to evaluate the applied stress and the pulse duration of the impact. Davies [157] then used electrical condenser unit along with an oscilloscope for measurement of the stress pulse in the pressure bar. Kolsky [158] introduced a two bar split Hopkinson bar system for indirect measurement of high strain rate characteristics of materials. The SHPB apparatus made use of a specimen in the shape of disc placed between the steel bars. The striking bar was triggered by a detonation process and the displacement of the free end of the transmitter bar was measured using a parallel plate condenser microphone. An additional condenser microphone fitted around the incident bar (between the detonator and specimen) was used for measurement of the incident pressure pulse. The dimensions of the bars were selected in such a way as to avoid dispersion of the pulses. The stress-strain characteristics were established based on the plot of stress vs. time obtained from the displacement measured by the parallel plate condenser microphones. Kolsky derived the equations for converting the experimental data obtained through testing into stress-strain characteristics of the material tested. The arrangement used by Kolsky for measurement is similar to that used by Davies [157]. Lindholm and Yeakley [159] used the SHPB concept for evaluation of tensile and compressive characteristics at high strain rates by introducing strain gauges for the measurement of the wave pulses in each bar. Poisson's ratio was deduced from the radial strain measurement which was attempted using a capacitance gauge in the form of coaxial ring. Most of present SHPB setups are similar to that described in this article.

Gama et al. [160] presented an extensive review of the historical development of the split Hopkinson bar and a detailed discussion on the technical aspects of the method, specimen dimensions, data processing and validity of the results obtained from SHPB experiments.

### **3.2.2.3 Operating principle of SHPB**

#### **3.2.2.3.1 Prerequisites**

1. To ensure that the stress wave propagation is one dimensional, the bars selected should be homogeneous and isotropic.
2. The length to diameter ratio of the bars should be greater than 20 to ensure a uniform axial stress distribution along the bars.
3. Techniques such as centreless grinding may be used to ensure that the bars are of uniform cross section throughout the entire length and the neutral axes of the bars are straight [160].
4. The striker velocity should be controlled in order that the stress pulse in the incident bar is within the elastic limit.
5. The striker-incident bar interface, incident bar- specimen interface and specimen-transmitter bar interface should remain plane at all times.
6. The specimen should achieve stress equilibrium after the initial ring up period during impact.
7. The specimen inertia and friction effect in specimen should be to a minimum to ensure that the specimen achieves stress equilibrium during testing.

#### **3.2.2.3.2 A typical SHPB**

A typical arrangement of a split Hopkinson pressure bar for high strain rate characterisation is shown in Fig 3-5. Even though all the configurations (compression, tension and torsion) of SHPB work on the same principle, the compression setup is used to explain the concept and the principle because of its simplicity. A typical SHPB setup consists two long cylindrical bars of uniform cross section acting as incident and transmission bar along with a striker bar which is used to generate the impact pulse on the incident bar. The specimen to be tested is sandwiched between bars. The striker bar is triggered using a stress generation system such as a gas gun apparatus. The bars have bearing supports to ensure proper alignment. The strain is measured using the strain gauges mounted on the bars and is amplified before being fed to a data acquisition system or an oscilloscope. The momentum trap is a bar which is introduced at the end of the transmission bar to mitigate the effect of the incident impact pulse. The momentum of the incident pulse gets trapped as it moves away from the contact interface due to the reflected tensile pulse reaching the interface.

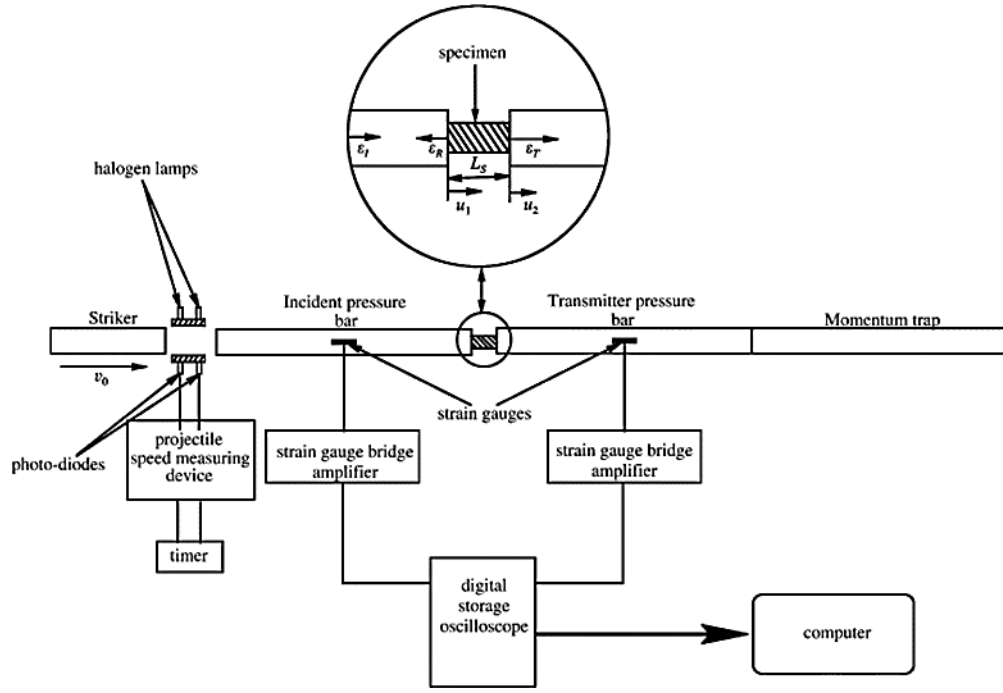


Fig 3-5 Schematic diagram of SHPB arrangement [161]

#### 3.2.2.4 Working principle

The impact of a striker bar with a velocity  $v_0$  on to an incident bar (with the same cross sectional area and impedance) will generate a rectangular compressive strain pulse of  $\frac{v_0}{2c_0}$  on the incident bar. The geometric length of the strain pulse generated is twice the length of the striker as the duration of the pulse is given by  $T = \frac{2L_0}{c_0}$ . This pulse propagates toward the incident bar-specimen interface, a strain gauge in the incident bar records the strain ( $\epsilon_I(t)$ ) and this pulse gets reflected from the incident bar specimen interface as a tensile pulse. The strain gauge in the incident bar records the reflected pulse ( $\epsilon_R(t)$ ) as it passes through its location (Fig 3-5). It is important to select the location of the strain gauges such that the incident pulse and reflected pulse signals do not interfere with each other. During the wave reflection from the incident bar specimen interface, a part of the incident wave propagates through the specimen. Since the specimen is very short compared to the bar length, it will cause multiple reflections of the compressive wave at the specimen-transmission bar interface and specimen-incident bar interface. These reflections occur until the stress reaches the dynamic strength of the material. During the complex reverberation phenomenon in the specimen between the incident bar-specimen interface and specimen-transmitter bar interface, a train of compressive waves will be transmitted through the specimen-transmission bar interface [160]. The strain gauge

mounted on the transmission bar records the transmitted pulse ( $\varepsilon_T(t)$ ) from the specimen side. The strain data from the incident and transmission bar are then used for the evaluation of the average stress, strain and strain rate observed by the specimen as given by Eq. 3-10, Eq. 3-11 and Eq. 3-12 respectively. The detailed derivation of the Eq. 3-10, Eq. 3-11 and Eq. 3-12 is given in Appendix 2 (page 194).

$$\sigma(t) = A_B E_B \varepsilon_T / A_S \quad \text{Eq. 3-10}$$

$$\varepsilon_s(t) = \left( \frac{2c_0}{L_s} \right) \int_0^{t_0} \varepsilon_R dt \quad \text{Eq. 3-11}$$

$$\dot{\varepsilon} = 2c_0 \varepsilon_R / L_s \quad \text{Eq. 3-12}$$

### 3.2.3 High strain rate tensile testing

Different methods of tensile testing have been used by various researchers in the past. Lindholm and Yeakley [159] used a top-hat shaped tensile specimen fitted between a solid cylindrical incident bar and a tubular transmission bar (Fig 3-6). The gauge section has four arms with the same length to width ratio. Specimens that showed failure in the mid-section were used for evaluation of the material characteristics.

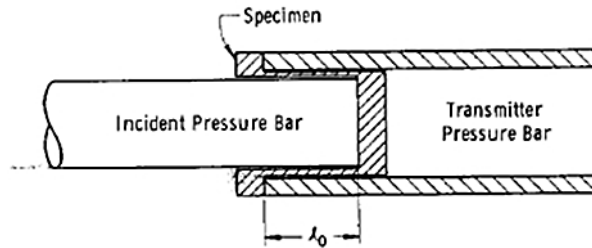


Fig 3-6 Hat shaped specimen used for high strain rate tensile testing [159]

Hauser [162] discusses an experimental arrangement for generation of a reflected tensile pulse based on an impact on transmitter bars with a ram, to evaluate the tensile characteristics of specimen using SHPB concept (Fig 3-7). This article does not give other details on the arrangement such as the supporting unit for the output bar, dimensions of each section etc other than the figure shown (Fig 3-7).

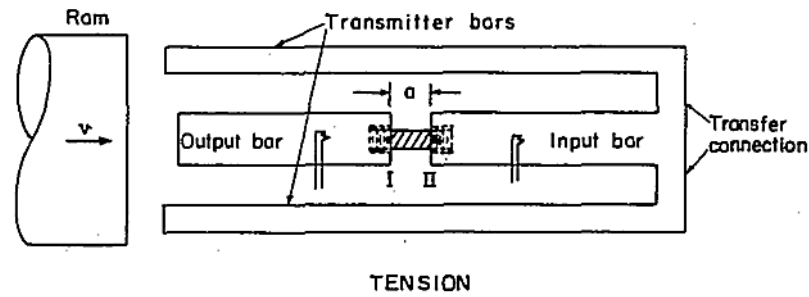


Fig 3-7 Tensile testing based on reflected pulse concept [162]

Theodore [163] introduced a hollow cylindrical shoulder piece over a threaded specimen in a compression split Hopkinson bar to generate a tensile loading on the threaded specimen (Fig 3-8).

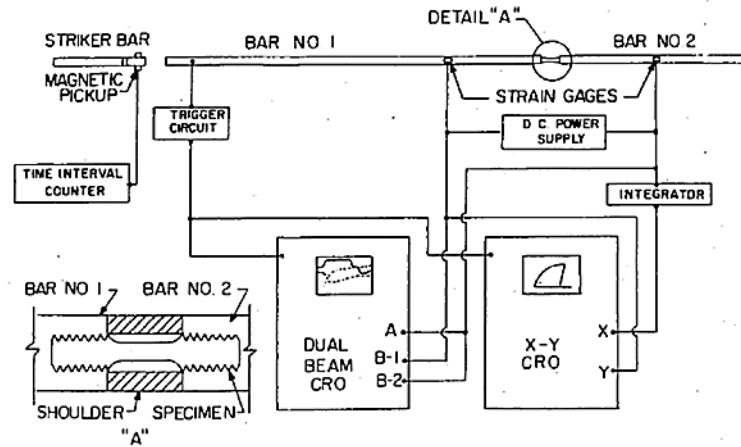


Fig 3-8 Introduction of hollow shoulder attachment along with the specimen (identified as "A") for tensile testing using conventional compression SHPB setup [163]

The principle behind this test approach is that the entire compressive pulse will pass through the hollow cylindrical shoulder piece and the reflected tensile pulse will then act as an incident pulse to load the threaded specimen as can be observed from the wave time diagram (Fig 3-9). Since the hollow cylindrical shoulder is not fastened to either of the bars through any joints, it cannot transmit any tensile pulse. This technique was used for high strain rate testing of various grades of aluminium, titanium and steel. Strain rates up to  $1000 \text{ s}^{-1}$  were observed for various materials using this technique.

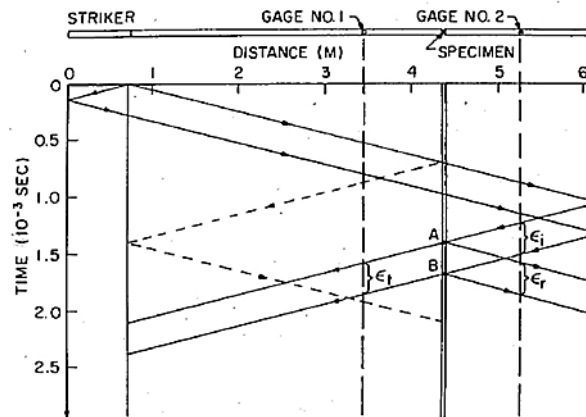


Fig 3-9 Lagrangian wave time diagram for the reflected pulse concept introduced by Theodore [163]

Chiem and Liu [63, 64] modified the incident bar of the conventional SHPB as a loading tube which, when subjected to impact from the striker, generates a compressive wave inside the tube, which on reflection, generates a tensile pulse and loads the specimen. Their concept is similar to the concept used by Hauser[162]. Staab and Gilat [164] demonstrated a method for direct tensile loading of specimen at high strain rates through the release of a stored load from a section of the input bar. A tensile load applied to an already clamped incident bar to store the load in a section of the incident bar. The release of the clamp from the incident bar through fracture of a pin, generates a dynamic tensile wave in the bar. Strain rates of up to  $2000 \text{ s}^{-1}$  were generated using the technique. The main drawback of this method is that it requires fracture of the pin for each test. Harding et al. [165] developed a test method of loading the specimen using the reflected tensile pulse generated based on compressive impact loading. The test configuration consists of two assemblies as shown in Fig 3-10 . One test assembly involves the uniform alloy bar inside a hollow weigh bar tube and other test assembly consist of specimen screwed to an inertia bar inside a hollow weigh bar tube. The purpose of the first assembly is to provide information of the input loading condition on the specimen. Two separate tests are carried out with the same velocity to get the realistic assessment of the input condition. Several significant issues would need to be addressed when this technique is applied to anisotropic materials such as composites. The shear failure in the grips and the difficulty in achieving stress equilibrium across the ends of specimen are some of the challenges that arise in tensile testing of composite materials. The main drawback of this approach is that the input strain data is extracted from a separate test performed at a different time. Harding and Welsh [19] modified the

earlier tensile version of the split Hopkinson bar [165] by attaching an instrumented input bar to the yoke so that it precedes the specimen and the inertia bar (Fig 3-11). The input bar is capable of sliding freely with respect to the inertia bar and also has two strain gauges to monitor the incident and reflected waves so that the strain determination could be made from a single test unlike the earlier technique. However, introduction of input bar restricts the maximum length of inertia bar and thereby the total duration of the dynamic test.

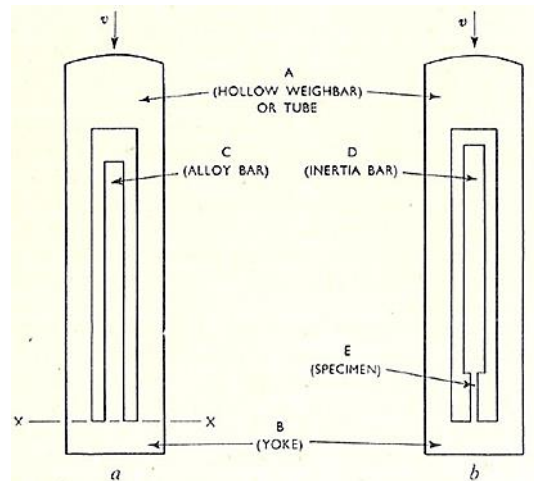


Fig 3-10 Elastic test assembly and specimen test assembly developed for tensile testing [165]

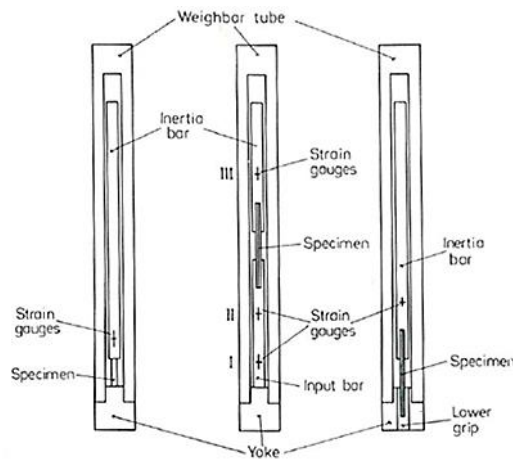


Fig 3-11 Modified version of tensile Hopkinson bar- Left: standard configuration, middle: configuration for CFRP and right: configuration for GFRP [19]

Li et al. [166] used a rotating disc attached with a cam to indirectly launch a hollow striker to impact the flange portion an incident bar through an impact block (Fig 3-12). The concept of loading the specimen using a direct tensile wave and through a combination of direct and reflected pulse was also elaborated through Lagrangian time diagrams. The different approaches of tensile loading

were achieved by changing the dimensions of striker and the flange of the incident bar. Gerlach et al. [167] used a long non symmetrical inverted U-section striker bar as a projectile instead of hollow striker to generate a clean and oscillation free rectangular pulse (Fig 3-13). The length of the striker bar was considerably larger compared to that of conventional hollow striker and hence the incident pulse was five times longer than for the conventional counterparts. This design allows the incident bar to be supported throughout the length. Strain rates up to  $1500\text{s}^{-1}$  were achieved during the testing. Guzman et al.[168] demonstrated high strain rate tensile testing using a split Hopkinson bar concept with a hollow incident bar and solid cylindrical striker. The striker moves inside the hollow incident bar and impacts on a pulse shaper on an impact cap fitted at the incident bar (Fig 3-14). The impact on the impact cap generates a compressive pulse on the impact cap and a tensile hollow incident bar tube. The apparatus was used for evaluation of dynamic tensile behaviour of Al 6061-T6511 and TRIP 800 for strain rates as high as  $1850\text{ s}^{-1}$ .

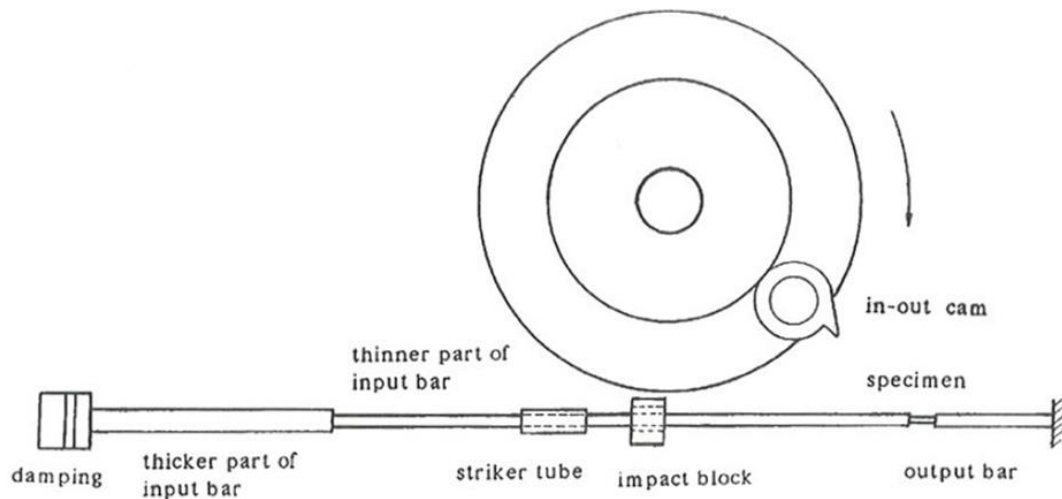


Fig 3-12 Direct Tension Kolsky bar setup based on hollow striker impact [166]

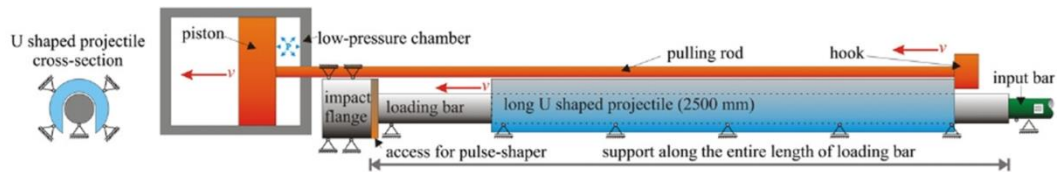


Fig 3-13 A novel tensile Hopkinson bar based on unsymmetrical striker bar [167]

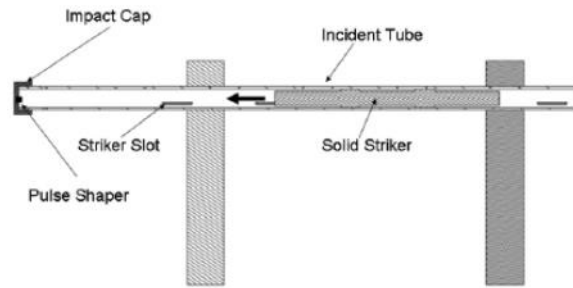


Fig 3-14 Cross sectional view of impact end of SHTB with hollow incident tube [168]

Mohr and Gray [169] used a specially designed specimen which resembles a letter 'M' to investigate the tensile characteristics of metal at high rates using a conventional split Hopkinson pressure bar (Fig 3-15). The issues in gripping relating to tensile testing were overcome by careful specimen design. The specimen dimensions were carefully optimised based on finite element analysis. There will be practical difficulties in realising such a concept through SHPB for single fibre/fibre bundle testing.

Kimberley and Paul [170] used a miniaturised Kolsky bar for tensile testing of thin metallic films. Ultra-high strain rates can be achieved through reduced length of the specimen. The setup uses a cylindrical launch tube and rectangular cross sectional bars as incident and transmission bars. Semi-conductor gauges were used for evaluation of strains from the rectangular bars. The setup demonstrated testing of aluminium foil samples of 100 microns thickness at strain rates up to  $10^4 \text{ s}^{-1}$ .

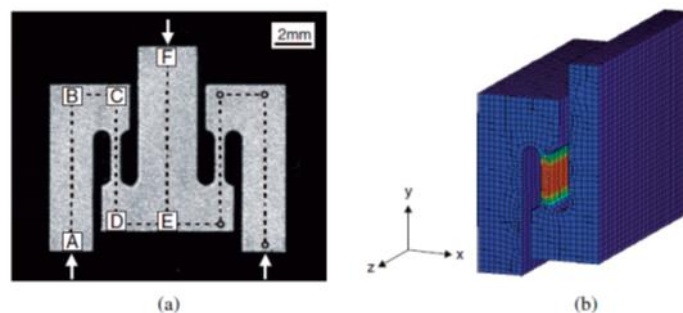


Fig 3-15 M shaped specimen(a) used for high strain rate tensile characterisation (dimensions optimised based using finite element analysis (b)) [169]

### 3.2.4 High strain rate single fibre testing

#### 3.2.4.1 Testing methods

Single fibre testing at high strain rates is challenging due to the difficulty in gripping. Lim et al. [124, 126] carried out single fibre tensile testing using a miniaturised tension Kolsky bar (Fig 3-16). The fibres were of 10  $\mu\text{m}$  diameter and were tested at strain rates of up to  $1500\text{ s}^{-1}$ . A non-contact laser technique was used to measure the fibre strain during testing. The effect of gauge length also was studied and the failure strains were found to be a function of the gauge length.

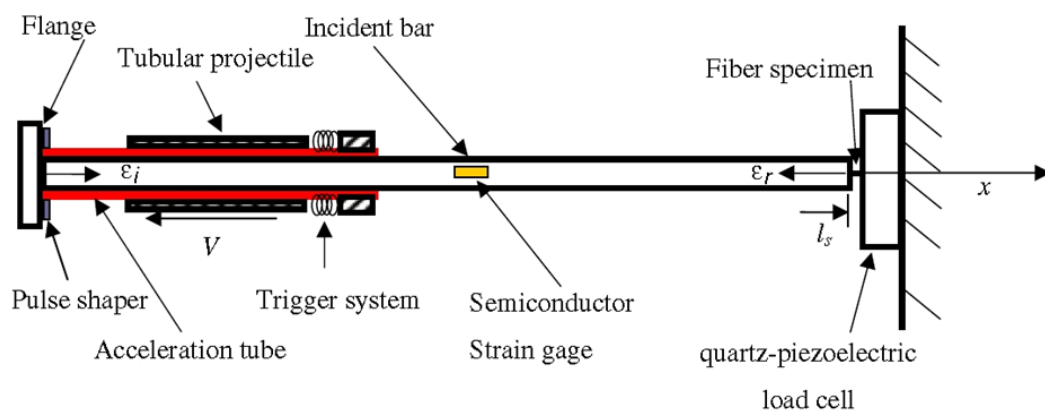


Fig 3-16 Schematic diagram of the setup used for single fibre testing [126]

Another approach for generating the properties of fibres is fibre bundle testing, however this also has several issues with respect to gripping, leading to premature failure. Tan et al. [171] adopted a special clamping mechanism, earlier used by Chocron et al. [172] for testing aramid composites, to hold the fibres during the tensile testing of aramid Twaron CT2040 1100dtex/ 1000f yarns at high strain rates in SHTB apparatus. The capstan grips were used for quasi-static testing. However, the supporting shaft of the capstan grips was found to break into two pieces after repeated dynamic tests. The grips shown in Fig 3-17 were successful in holding the fibre bundles for dynamic testing unlike the capstan grips. With a different approach, Bahnert et al. [173] used an innovative falling weight apparatus consisting of a drop weight and miniaturised tension arrangement (Fig 3-18) to evaluate the high strain rate tensile characteristics of the polyamide and poly (ethyleneterephthalate) yarns for strain rates up to  $200\text{ s}^{-1}$ . The freely hanging lower clamp to which yarns are attached will engage with the weight on its fall and applies the high strain rate tensile load to the yarns. This type of method can be employed for single fibre testing. However,

there will be challenges such as the weight of the lower clamp and ensuring repeatability of the failure modes observed when realising this method for fibre testing. In addition, to characterise the single fibres at higher strain rates larger than  $10^2 \text{ s}^{-1}$ , the falling weight apparatus needs to be propelled downwards with an initial velocity.

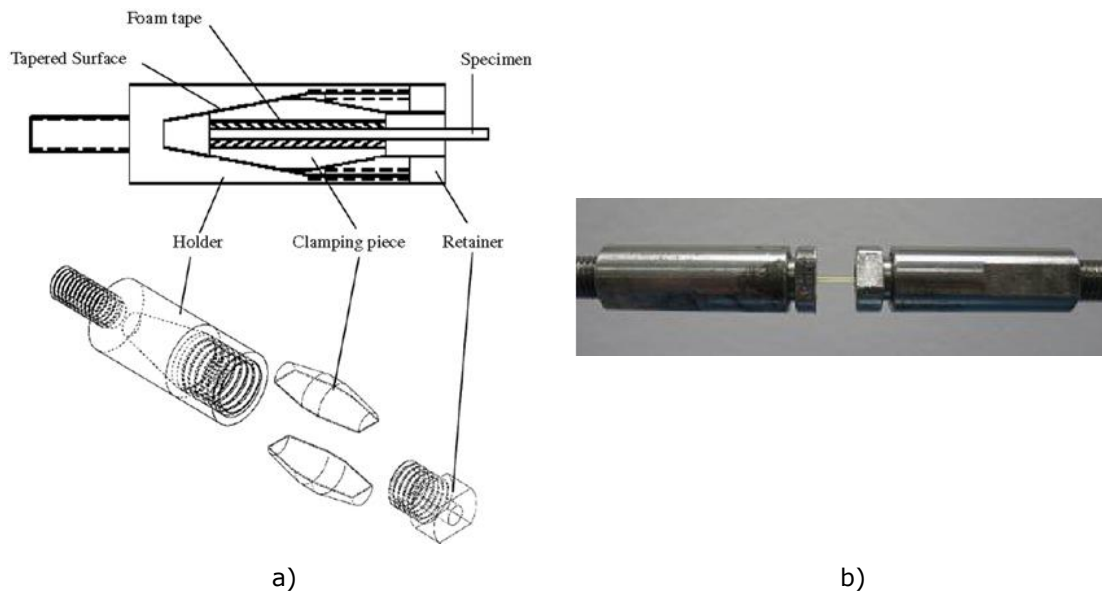


Fig 3-17 Clamping mechanism used for fibre bundle testing for high strain rates (a) clamping mechanism used (b) fibre bundle gripped using the clamping mechanism [171]

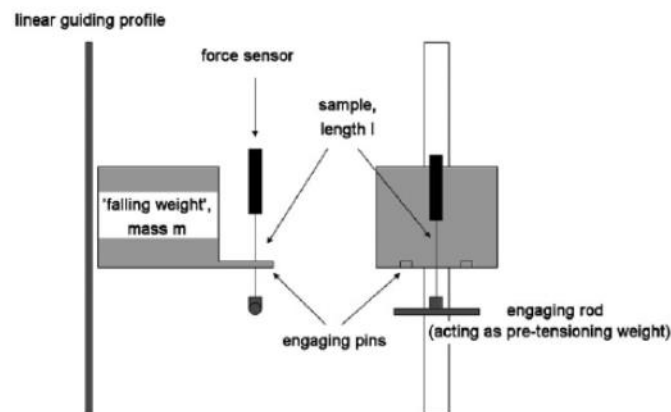


Fig 3-18 Falling weight apparatus used for fibre yarn testing [173]

### 3.2.4.2 Fibre/Fibre bundle gripping techniques

The challenge in single fibre testing is the method of gripping the fibre. Lim et al. [124, 126] used an adhesive bonding technique for attaching the fibres. The fibres are attached to the bar and the load cell through small flat plates to which it is bonded (Fig 3-19). Hudspeth et al. [134] bonded the fibre in a cardboard

before attaching to the bars through set screws which is more convenient compared to directly bonding the fibre to the bars.

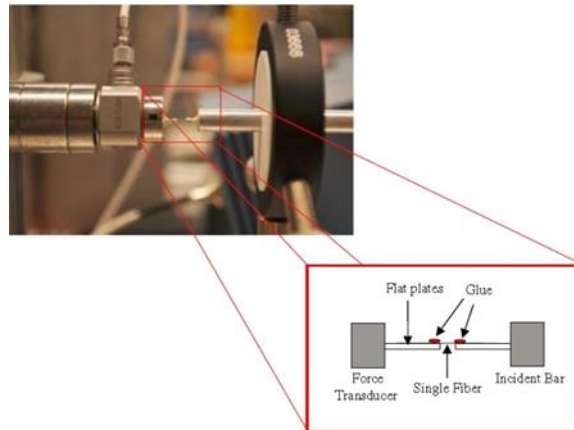


Fig 3-19 Gripping methodology followed for single fibre high strain rate tensile testing [126]

Kim et al. [131] studied the effect of gripping material and loading rates on tensile properties of single PPTA fibre using direct gripping methodology is shown in Fig 3-20. The influence of rate sensitive grip material such as PMMA and elastic material was compared. The initial works of Kim et al. on this study [128, 129, 132] highlighted the importance of high strain rate data of single fibres for ballistic performance simulation, but their work on high strain rate testing was not discussed. Two different approaches followed for gripping single PPTA (p-phenylene terephthalamide) fibres for quasi-static were described [130], one of which involved a direct grip and the other involved adhesively bonding the fibre to a tab. Statistical techniques were used to compare the distributions obtained for the mechanical properties like tensile moduli, failure strain and fibre strength for various fibre length for either methods. A direct fibre clamping methodology evaluated for quasi-static testing of Kevlar 29 fibres was also described [129].

It was decided to adopt the clamping methodology followed by Chocron et al. [172] and Tan et al. [171] and fibre bonding procedure followed by Hudspeth et al. [134] for developing the clamping mechanism for high strain rate single fibre tensile testing using SHTB. The fibre testing procedure followed by Lim et al. [124, 126], requires direct fibre bonding to the incident bar. The entire process would be tedious and time consuming and it would be challenging to achieve a consistent failure mode.

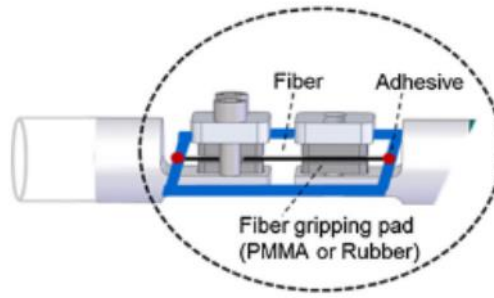


Fig 3-20 Direct fibre clamping procedure used for single PPTA fibre tensile testing [131]

### 3.2.5 General conclusions from literature review

There were three main approaches followed for high strain rate tensile testing is:

- 1) Direct tension concept by stored energy
- 2) Direct tension by impact
- 3) Reflected tensile pulse concept

The use of the reflected tension concept with the SHPB apparatus [40] available in The University of Nottingham was evaluated, but was not feasible with the present length of the bars (Appendix 3, page 196). Other approaches such as the expanding ring test or miniaturised desktop versions of SHPB/SHTB were not considered due to practical difficulties in realising single fibre testing and data measurement with the respective techniques. The direct tension concept by release of stored energy requires fracture of pin for each test and hence may not be ideal choice for single fibre testing. Hence, it was decided to use the direct tension by impact for development of SHTB. The detailed literature survey helped in identification of the testing methodology and clamping mechanism with respect to the development of a SHTB for single fibre testing.

## 3.3 Development of a SHTB for single fibre testing

### 3.3.1 General theoretical considerations

#### 3.3.1.1 Theory

The design approach for the SHTB developed as part of the present study is based on a direct tension concept described by Li et al. [166] using hollow striker impact. This involves a tensile Hopkinson bar wherein a tensile pulse is generated in the incident bar based on the impact of hollow striker on the flange of incident bar. The direct tension concept requires the length of the flange portion to be

larger compared to the striker length. This is important as it is necessary to ensure that the wave reflected from the far end of the flange portion does not interfere with the recording of the incident, reflected and transmitted waves. The theory of one dimensional wave propagation, its necessary conditions and assumptions are also followed for this design. The requirement of force equilibrium and the velocity continuity at the impact interface (Fig 3-21) between incident bar flange and the striker gives a series of equation which is different from Eq. 3-5 and Eq. 3-6, because of varying geometries. The conditions pertaining to the hollow bar tensile arrangement has been modified as in Eq. 3-13, Eq. 3-14 and Eq. 3-15 [166] respectively representing the force equilibrium, continuity of velocity condition and the opposite sign of the velocity pulses after impact of the striker at the flange.

$$A_0\sigma_0 + A_1\sigma_1 = A_2\sigma_2 \quad \text{Eq. 3-13}$$

$$V_0 - \frac{\sigma_0}{\rho_0 C_0} = \frac{\sigma_2}{\rho C} \quad \text{Eq. 3-14}$$

$$\frac{\sigma_2}{\rho C} = - \frac{\sigma_1}{\rho C} \quad \text{Eq. 3-15}$$

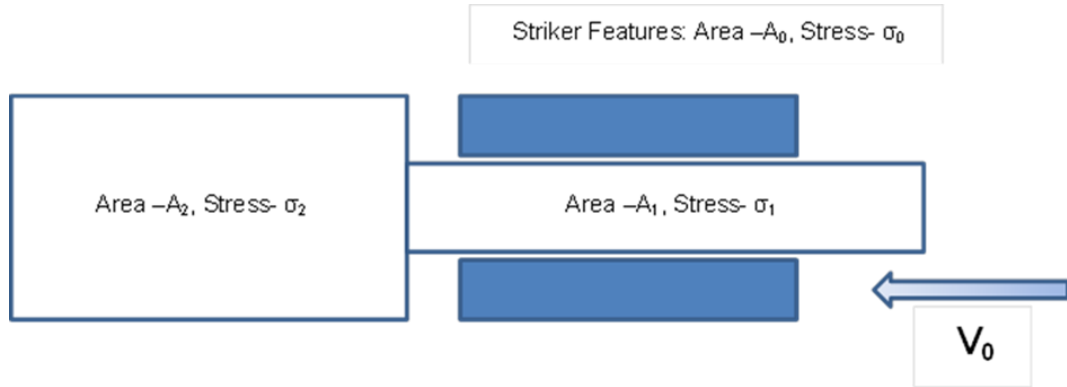


Fig 3-21 Impact section of a tensile Hopkinson bar showing hollow striker, flange and the initial zone of incident bar

Based on the above equations, the stresses across each member during impact are evaluated. In these calculations, compressive stress is assumed to be positive and tensile stress is regarded as negative. The formulae for stresses for each of the members shown in Fig 3-21 after the impact are as follows.

Striker stress is given by

$$\sigma_0 = \frac{A_1 + A_2}{A_0 + (A_1 + A_2) \frac{\rho C}{\rho_0 C_0}} \rho C V_0 \quad \text{Eq. 3-16}$$

Incident bar stress is given by

$$\sigma_1 = - \frac{A_0}{A_0 + (A_1 + A_2) \frac{\rho C}{\rho_0 C_0}} \rho C V_0 \quad \text{Eq. 3-17}$$

Incident bar flange stress is given by

$$\sigma_2 = \frac{A_0}{A_0 + (A_1 + A_2) \frac{\rho C}{\rho_0 C_0}} \rho C V_0 \quad \text{Eq. 3-18}$$

### 3.3.1.2 Incident bar particle velocity

The particle velocity generated in the incident bar, along with the gauge length of the specimen, controls the strain rate observed in the specimen. The particle velocity generated in the incident bar for a hollow striker configuration can be evaluated from Eq. 3-17. It can be noted that the particle velocity generated in the incident bar due to hollow striker impact is dependent on the dimensions of the incident bar, flange and striker and their impedances. Furthermore, it can be observed that particle velocity will always be less than half the striker velocity for a hollow striker configuration if the striker and the incident bar are of the same material (Fig 3-22). The area ratio of one in Fig 3-22 relates to a typical compression bar configuration where  $A_2$  is equal to  $A_0$  and  $A_1$  is equal to zero.

It can also be seen that using a denser material for the striker would help in achieving larger particle velocity inside the incident bar for the same impact velocity for the striker. Since a larger particle velocity results in larger stress in the striker and incident bar, the striker and incident bar material should have a yield stress larger than the critical stress levels observed for maximum striker velocity.

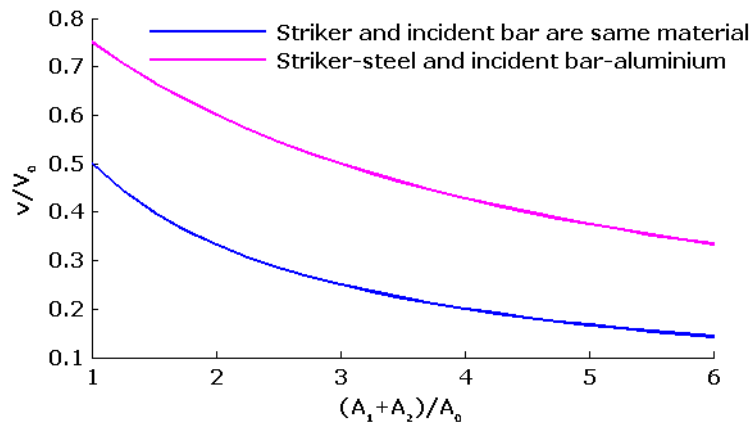


Fig 3-22 Incident bar velocity achieved for a hollow striker impact

### **3.3.1.3 Assessment of specimen loading condition for single fibre testing**

As the incident bar transmits the stress pulse generated due to an impact to the specimen, the specimen undergoes a complex reverberation of waves between the incident bar-specimen interface and specimen-transmission bar interface. The number of times the reverberation takes place before the specimen fails is a characteristic of the amplitude of the wave pulse, its pulse duration, the length of the specimen and the impedance of the bar and the specimen. The assessment of the boundary condition during high strain rate loading was found to be similar to single fibre testing under quasi-static conditions. The evaluation of the specimen failure during single fibre testing for different cases of boundary conditions is given in Appendix 4 (page 198).

### **3.3.2 Design objective and constraints**

The primary design objective of developing the SHTB apparatus was to perform tensile characterisation of fibres at strain rates up to  $10^4 \text{ s}^{-1}$ . These strain rates can be achieved by changing the striker velocity or by varying the gauge length. Since there are practical difficulties in dealing with small gauge length fibres, variable strain rates could be achieved through varying the striker velocity. The velocity of the incident bar is a critical parameter which controls whether the tensile pulse generated in the bar is capable of inducing failure in the fibres and fibre bundles and also capable of generating the desired strain rates. The conceptual design of SHTB was based on the assumption that 3mm fibre specimen subjected to an incident bar velocity of  $30\text{ms}^{-1}$  could achieve the above strain rate of  $10^4 \text{ s}^{-1}$ . The incident bar can achieve a maximum particle velocity of  $21\text{ms}^{-1}$  in the lowest diameter bar (12mm) for maximum allowed compression of the spring procured. The main design constraint was that the SHTB had to be accommodated within the length of the structural frame (4.4m).

### **3.3.3 Components and subsystems of SHTB**

The SHTB apparatus consists of the following components and subsystems:

- 1) Stepped incident bar with flange
- 2) Alignment system
- 3) Launch system
- 4) Load cell and data capturing system
- 5) Shock absorber unit and its support
- 6) Gripping mechanism

The stepped incident bar configuration consists of 3 steps and a flange zone as shown in Fig 3-23. Two configurations have been developed.

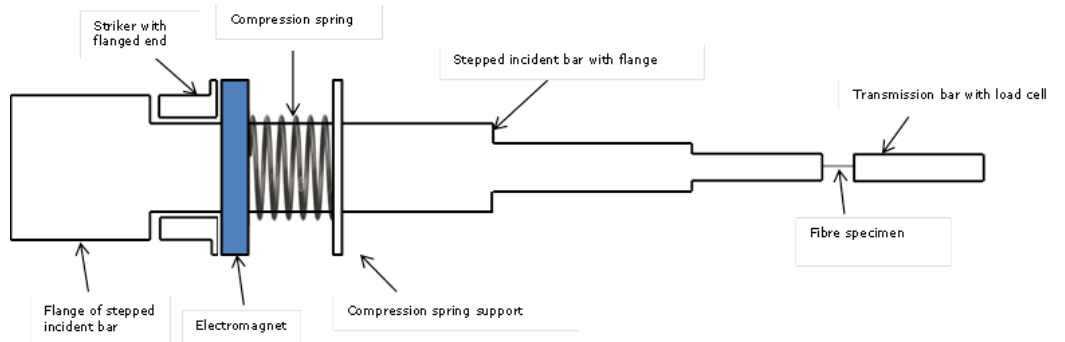


Fig 3-23 Schematic representation of main components of SHTB (not drawn to scale)

- Both stepped incident bar and striker made of hardened steel (EN 24T)
- Stepped incident bar and flange made of aluminium 7075-T6 and striker made of hardened steel (EN 24T)

The alignment system consists of nylatron bushes mounted on aluminium blocks. These aluminium blocks are held on aluminium extrusion members whose position can be adjusted laterally and across the frame. The extra aluminium extrusion members were procured to accommodate the SHTB along with the existing compression SHPB on the frame and to strengthen the frame structurally. The final configuration of the SHTB as assembled alongside the present rig (SHPB [40]) along with the high speed camera is shown in Fig 3-24.

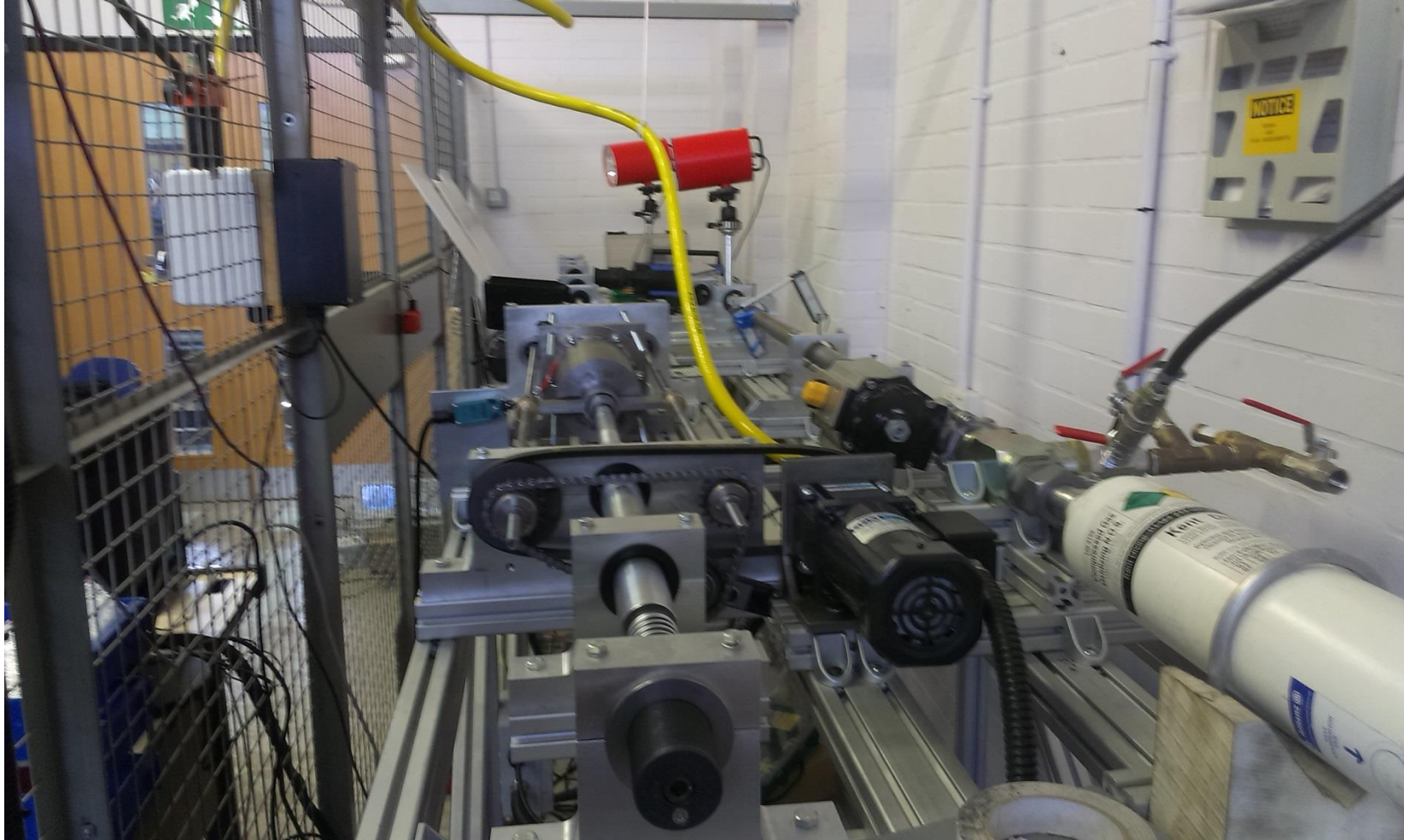


Fig 3-24 Final assembly of SHTB along with arrangement of high speed camera

### **3.3.3.1 Assessment of various striker launch mechanism options**

The main purpose of the launch system is to fire the striker at desired ranges of velocities with a great accuracy and repeatability. Various principles based on gas guns, mechanical springs, electromagnetic guns, linear induction motors and drives driven by induction motors could be used for launching the strikers and have been explored. Gas guns use compressed gases to propel the projectiles. The gas gun apparatus consists of a compressed gas reservoir, barrel tube/launchers and a solenoid valve. Gas gun systems are simple and very effective in powering strikers. The velocities can be varied by controlling the pressure of the working substance. The drawback in realising a gas gun based SHTB is that it requires large unsupported length of incident bar inside the gun. Some researchers have realised U shaped strikers to address this issue [174]. Electromagnetic guns propel projectiles to large velocities by converting electrical energy into useful mechanical work based on the Lorentz force equation. This is quite cumbersome and requires a large energy storage and delivery system. Another major disadvantage is that it is less efficient (around 2%) and is very costly. Linear induction motors works on the principle of conventional induction motors, and generates linear motion in a conductor based on the interaction of driving magnetic field and induced magnetic field. These linear induction motors are also cumbersome like the electromagnetic guns and a motor powerful enough to generate forces in the range of 300-400N is very large and heavy. Drives based on conventional induction motors could also be used to drive the hollow striker to the flange. However, taking into account the dimensions of the bar, it would consume a large space, and such a drive mechanism also would involve lot of mechanical maintenance issues.

### **3.3.3.2 Spring drive mechanism for launching striker**

In the developed SHTB, the striker is launched using a spring drive mechanism (Fig 3-25). The spring drive mechanism consists of two threaded shafts, an electromagnet, crosshead, striker, anti-buckling tube, tie rods, slider unit, supports (for threaded shafts, electromagnet and a compression spring), pulley-chain mechanism and a motor. The spring is compressed using the forward movement of the crosshead whose end is carried by on two threaded shafts which are driven simultaneously by a motor through a belt and chain transmission mechanism. The anti-buckling tube prevents significant buckling of the spring during compression. Once the striker crosshead unit reaches the energised electromagnet unit, the striker gets held to the electromagnet and keeps the

spring compressed. The introduction of tie rods between the electromagnet support and fixed support ensures that the forces are balanced and base supports does not experience bending moments, once the cross head is moved back. The energy stored by the compression of a spring is used for actuation of the striker whose movement is controlled by the demagnetisation of the electromagnet which holds the compressed spring and striker once the spring is compressed. The image in Fig 3-24 shows the spring in its compressed position restrained by the crosshead whereas the image in Fig 3-26 shows the SHTB before the spring is compressed. The detailed features of spring used and the electromagnet are given in Appendix 5 (page 201).

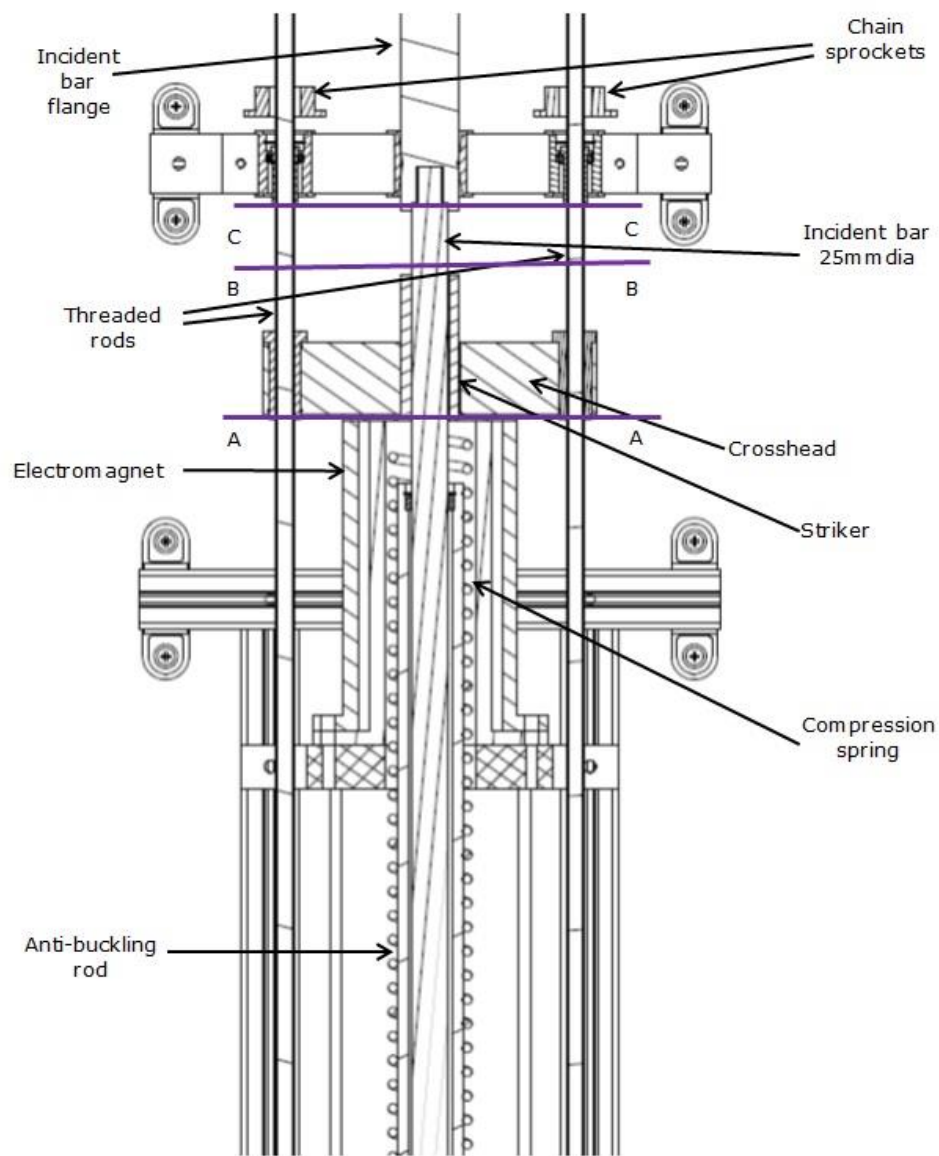


Fig 3-25 Layout of spring mechanism used for launching the striker in the developed SHTB

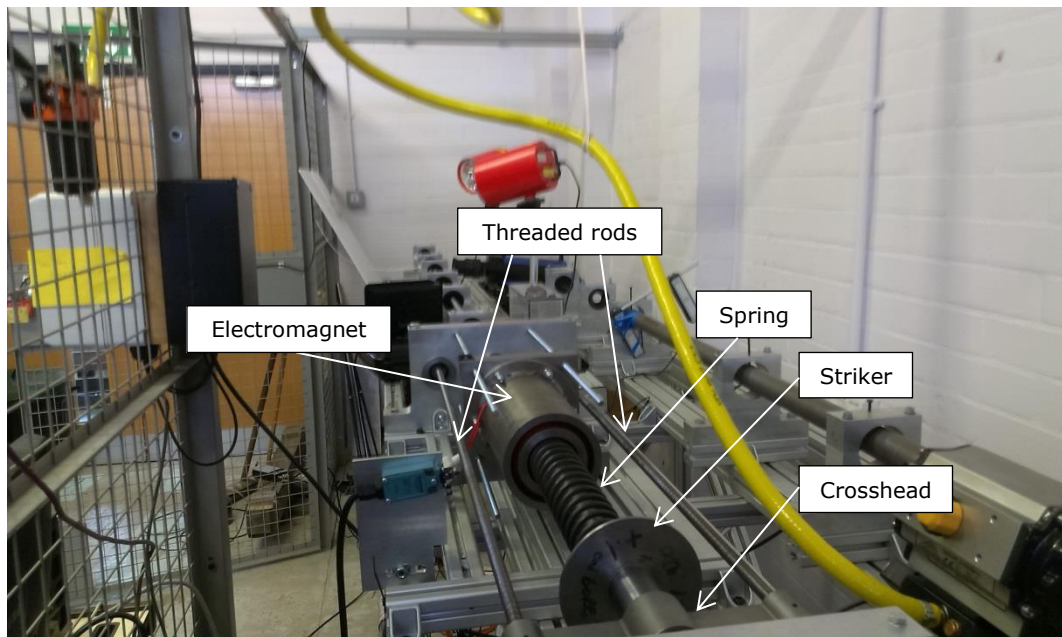


Fig 3-26 SHTB prior to compression of the spring

### 3.3.3.3 Clamping methodology for SHTB

The clamping mechanism used for clamping fibres and fibre bundles is similar to the clamping mechanism adopted by Chocron et al. [172] and Tan et al. [171] for fibre bundles (Fig 3-27). The clamping mechanism consists of a clamp cylinder, tapered clamp plates, and a clamp screw. The specimen (fibre/fibre bundle), sandwiched with softer material (paper/foam) is held between the clamp plates. The tightening of the clamp screw ensures sufficient clamping force is applied to the specimen so that the specimen pull-out is prevented when tensile load is applied.

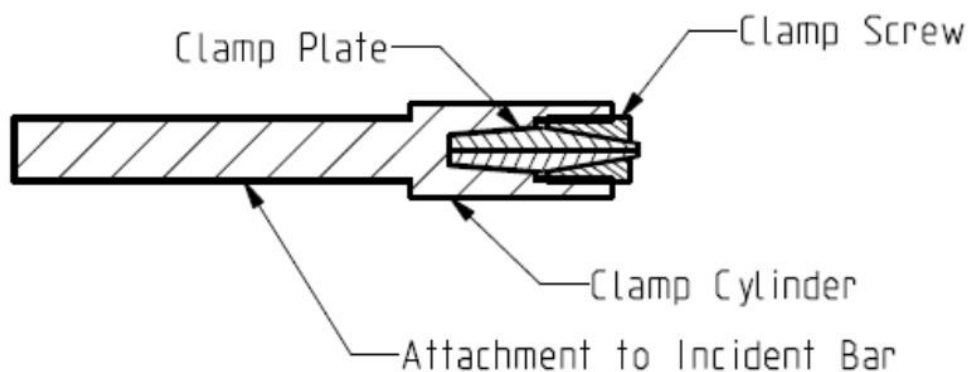


Fig 3-27 Clamping mechanism employed in SHTB

#### 3.3.3.4 Data capturing systems in SHTB

The data capture unit of the SHTB consist of strain gauges, a load cell, their respective signal conditioners and a data acquisition system. The strain gauges are mounted on the incident bar to record the incident wave and reflected wave. The transmitted signal from the single fibre would be very feeble and hence a load cell is used to evaluate the load while testing single fibre. A quartz piezoelectric load cell of type Kistler 9712B5 was procured along with its signal conditioning unit (as used by Lim et al.[124, 126]) for use during single fibre testing. In the case of fibre bundle testing (if required), the information from the strain gauge mounted on the transmission bar can be used to evaluate the stress characteristics of the specimen as in conventional SHPB/SHTB testing.

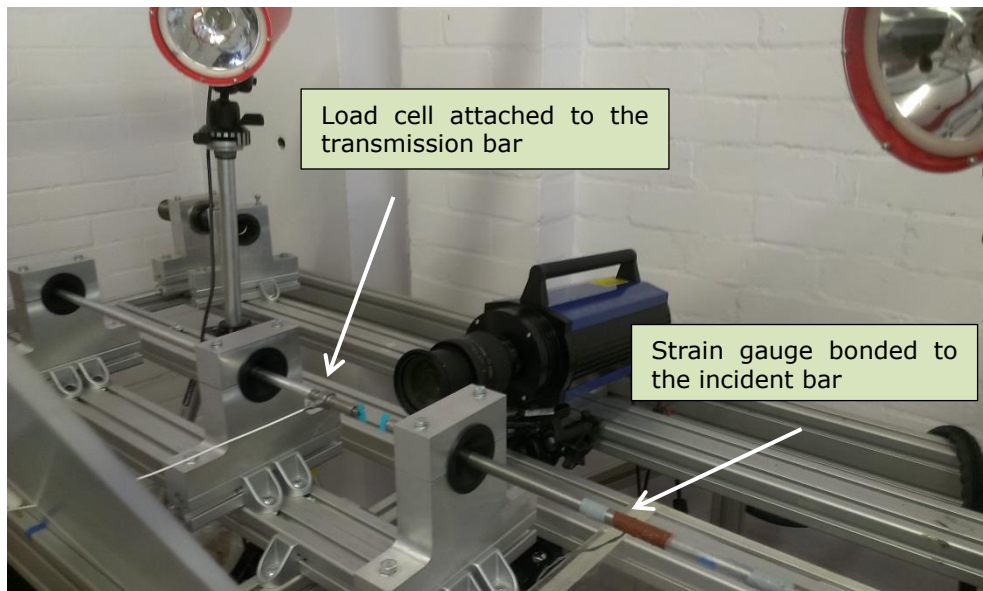


Fig 3-28 Data capturing systems used during high strain rate single fibre testing

#### 3.3.4 Characteristic wave diagram of SHTB

The characteristic wave time diagram for the SHTB configuration is shown in Fig 3-29. The characteristic wave time diagram gives the wave propagation information with respect to time on the y axis and the wave position along the bars on the x-axis. The wave time diagram shows the wave propagation that happens when striker impacts the flange portion of the incident bar. The impact time is regarded as the time datum,  $t=0$ . It can be observed that the strain gauge to monitor the incident and reflection wave signal can be mounted at any distance between 200mm - 388mm from the specimen end of the incident bar to prevent reflected pulse from the specimen from interfering with the reloading

tensile wave (as defined by the vertical magenta line in Fig 3-29) or the interference with main loading wave. The length of the flange is chosen to ensure the failure of the specimen due to the single tensile wave and also to avoid interference of reloading tensile wave with the reflected wave from the specimen end.

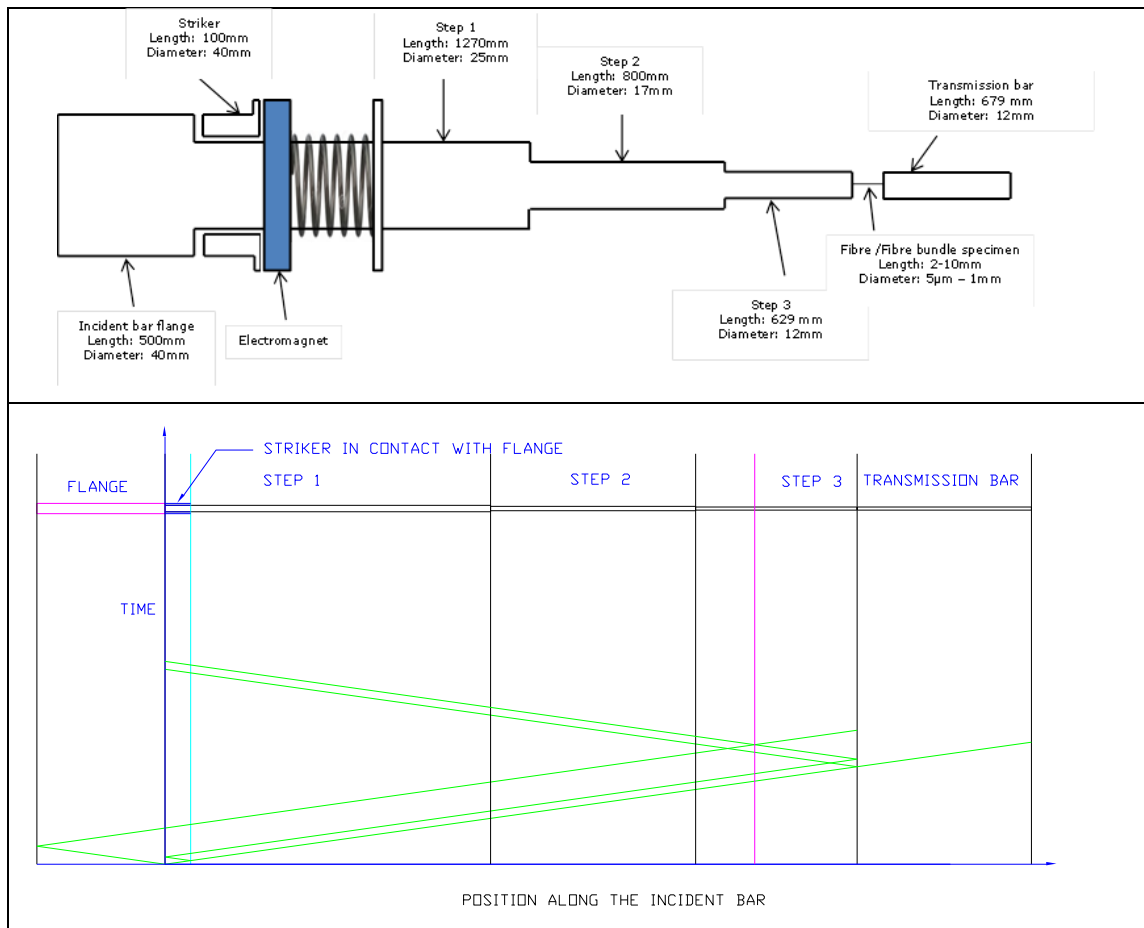


Fig 3-29 Dimensions of SHTB and its characteristic wave time diagram

### **3.3.5 Design evaluation through FE analysis**

#### **3.3.5.1 Analysis of wave propagation**

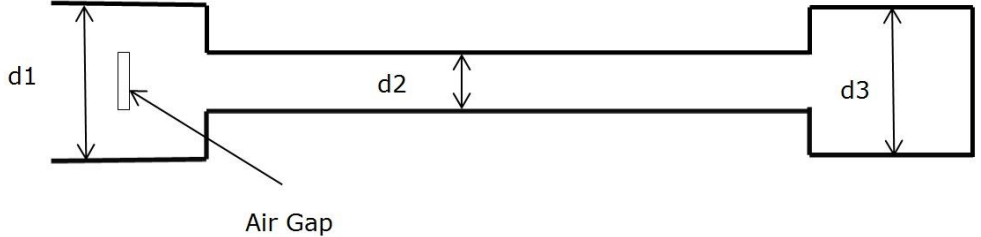
Various researchers have attempted to understand the wave propagation in SHPB using FE models for different purposes. Gerlach et al. [174] have analysed wave propagation for evaluating a novel pulse shaping concept using 3D finite element models. Kariem et al. [175] investigated various possible forms of misalignment that can occur in conventional split Hopkinson bar testing through 3D FE analysis using ANSYS-LS DYNA. The effect of neutral axis offset, uneven height of the incident and transmission bars, non-parallel impact faces, straightness of bars, convexity and concavity of the impact bar at the impact interfaces were studied through rigorous FE analysis. Li and Ramesh [176] used a half cylindrical model of the bars and the specimen to evaluate the parametric influences on the specimen strain characteristics using Abaqus Explicit. The interactions of the specimen and bars were defined through contact pair definitions at the threaded interfaces in each part. Ramirez et al. [177] investigated the input pulse characteristics on the dispersion observed during wave propagation using a 3D full finite element model of the bars. It was demonstrated that trapezoidal pulse with an increased rise time can be used for reducing wave dispersion in the bars.

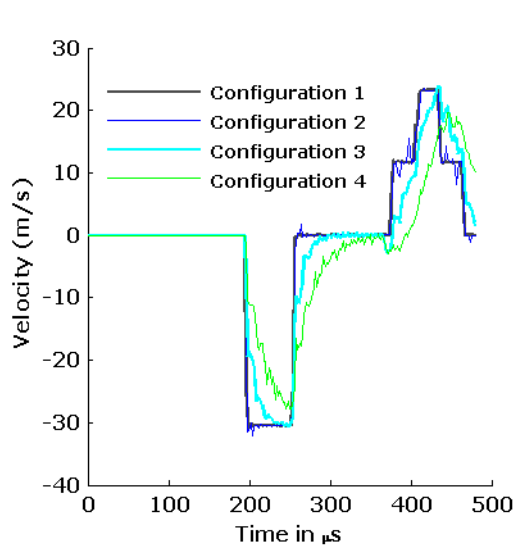
#### **3.3.5.2 Details of FE modelling**

In the present study, axisymmetric finite element models based on Abaqus/Explicit were used for finite element simulation of the wave propagation across the stepped configurations and around annular air gaps (due to complicated clamp configurations), as the conditions prevailing in SHTB can be assumed to have both geometric and loading axis symmetry. A uniform mesh size of 0.2mm and time intervals of 0.002 ms were defined to assess the wave propagation characteristics of various stepped bar configurations mentioned in Table 3-1 & Table 3-2. The velocity conditions that were imposed were based on the theoretical calculation of velocity at the given interface for typical wave propagation in a SHTB. The velocity pulse was defined for the entire pulse duration ( $2L_0/C_0$ ) and was input as an amplitude function in the Abaqus input file. The FE analysis of wave propagation shows velocity amplification at the step interfaces. The velocity at the free end is amplified to twice the velocity of the propagating wave (Fig 3-30a). An air gap to simulate the effect of the threaded joint between bars was introduced into the model (configuration 2 in Table 3-1).

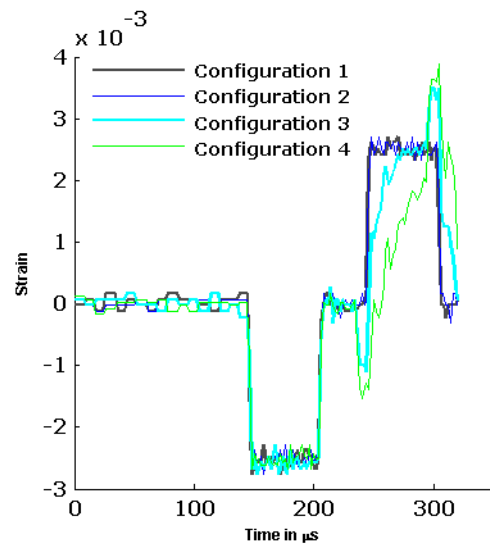
It could be observed from the velocity plot that such an introduction of an air gap had only minor distortional effects on the velocity curve. Also, its effect on strain observed at the mid node was minimal (Fig 3-30b).

Table 3-1 Different configurations of stepped bar to understand wave propagation across varying geometries explained through schematic representative section (not drawn to scale)

| Stepped Section                                                                    |        | Clamping Cylinder Section |        |         |
|------------------------------------------------------------------------------------|--------|---------------------------|--------|---------|
|  |        | d2                        | d3     |         |
| Configuration Type                                                                 | d1(mm) | d2(mm)                    | d3(mm) | Air Gap |
| Configuration 1                                                                    | 12     | 8                         | 8      | No      |
| Configuration 2                                                                    | 12     | 8                         | 8      | Yes     |
| Configuration 3                                                                    | 12     | 8                         | 12     | No      |
| Configuration 4                                                                    | 12     | 8                         | 18     | No      |



a) Velocity at end of the incident bar



b) Strain observed at mid-section of the incident bar

Fig 3-30 Effect of stepped bar and clamp cylinder sizes on wave characteristics

### 3.3.5.3 Effect of clamp diameter

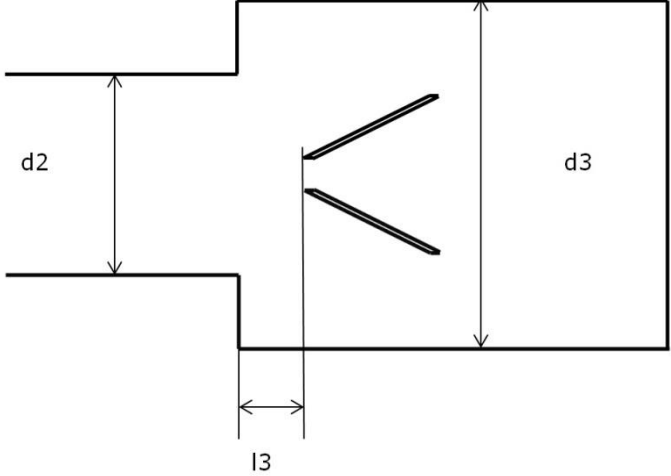
One of the design options considered in design of the SHTB rig was to use four steps to obtain greater velocity amplification. In one of the configurations evaluated for a 4 step SHTB configuration, the lowest diameter bar was 8mm.

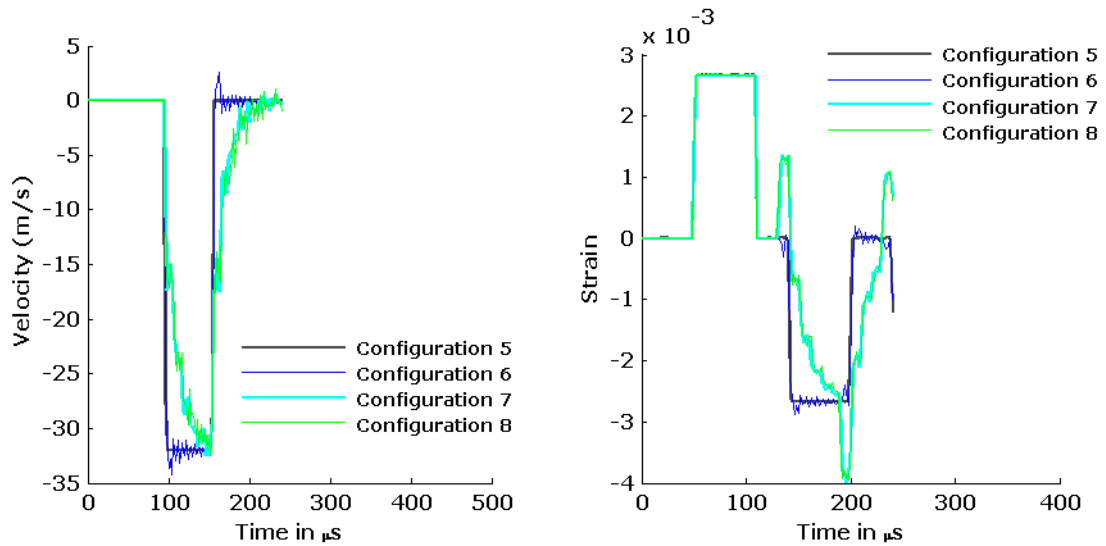
However, based on the feasible clamping mechanism configurations, it was observed that the clamping cylinder has to have a minimum diameter of 12mm. Hence, a study was performed to evaluate the wave propagation effects on a bar of 8mm diameter with a clamping cylinder of 12mm diameter (Configuration 3 on Table 3-1) and 18 mm diameter (Configuration 4 on Table 3-1) at its free end. With respect to Table 3-1,  $d_1$  denotes the diameter of stepped bar,  $d_2$  denotes diameter of smallest diameter bar and  $d_3$  denotes the diameter of clamping cylinder attached to the bar with diameter  $d_2$ . For both configurations, it could be observed that the velocity profile is no longer a trapezoidal pulse. The strain signal also shows significant distortion in the reflected wave. The undulation was primarily due to the series of wave reflections that happens between the clamped interface and the free end. This is undesirable as specimen strain information during SHTB testing needs to be gathered from the reflected signal (for fibre bundle and composite specimen testing). Another undesirable phenomenon that can be observed was that there was a pulse that was generated at the increased diameter clamp interface which results in further amplification of the stress wave due to superimposition effects. Hence, there would be the likelihood of the bar undergoing yield, if the initial velocity of propagation corresponds to stress level that is comparable to the yield strength of the bar. It is evident from the figures that the severity was higher when the diameter of the clamp cylinder was higher.

#### **3.3.5.4 Effect of annular gap**

During the clamping of the specimen, there will be a tapered annular air gap between the clamp cylinder and the clamp plates once the clamp screw is tightened. FE analyses were performed to evaluate the wave propagation across such annular air gap interfaces. Various configurations were studied as shown in Table 3-2. It can be observed that, even though there was an additional distortion because of annular air gap (Fig 3-31), it neither influenced the basic characteristic nor the magnitude of velocity or the strain (configurations 5&6 in Table 3-2). The effect of position of the annular air gap interface inside the clamp also was analysed (configuration 7&8 in Table 3-2) and was found not to have any significant influence upon their velocity and strain characteristics.

Table 3-2 Configurations to study the effect of annular air gap and explained through schematic representative section (not drawn to scale)

| Clamping Cylinder Section with annular air gap                                     |         |         |         |                 |
|------------------------------------------------------------------------------------|---------|---------|---------|-----------------|
|  |         |         |         |                 |
| Configuration Type                                                                 | d2 (mm) | d3 (mm) | l3 (mm) | Annular Air Gap |
| Configuration 5                                                                    | 8       | 8       | -       | No              |
| Configuration 6                                                                    | 8       | 8       | -       | Yes             |
| Configuration 7                                                                    | 8       | 14      | 5       | Yes             |
| Configuration 8                                                                    | 8       | 14      | 8       | Yes             |



a) Velocity at end of the incident bar

b) Strain observed at mid-section of the incident bar

Fig 3-31 Effect of annular air gaps on wave characteristics of SHTB

### 3.3.5.5 Conclusion from wave propagation studies

Since it was observed that increased diameter of clamp at the end of with incident bar can result in reflection of the shock pulse at the interfaces, it was

decided to set the diameter of the clamp cylinder to be the same as the bar diameter. Hence, a decision was made to discard the 8mm bar step and develop a SHTB setup with 3 steps of 25mm, 17mm and 12mm diameters. The clamp cylinder of 12mm with the features as mentioned in Fig 3-27 was attached to the ends of the 12mm bar. The transmission bar was also required to be 12mm diameter with provision for attaching the load cell and the clamp unit.

### **3.3.6 Adaptable features of the SHTB rig**

Even though the SHTB was developed for single fibre testing, the rig can also be used for fibre bundle testing and also for composite specimen testing.

- 1) *Use for fibre bundle testing:* If the mean diameter of the fibre bundle is smaller than 0.4 mm, then the present three step configuration of SHTB along with the clamp can be used directly for fibre bundle testing. If it is larger than 0.4 mm, then the similar clamp design can be scaled up for use with a 17mm bar. Accordingly, an additional 17mm bar will also be required to be used as the transmission bar.
- 2) *Use for composite testing:* The modification of the clamp mentioned above for the fibre bundle could be used directly for the tensile testing of composite specimen as well for lower strain rates. However, careful design of the specimen needs to be carried out to ensure that grip force is always higher than the maximum load that is transferred to the specimen.
- 3) *Change of actuation system for higher strain rates for composite specimen testing:* The gas gun unit of the compression version [40] developed for composite testing could be used with the present setup to increase further the strain rate capability of the system. A barrel (with the inner diameter and length of 40.5 mm and 1000mm respectively) for propelling the hollow striker the gas gun needs to be made so that higher strain rates may be achieved for composite specimen testing with the stepped bar. Since the barrel will be positioned over the 25mm bar section, the 25mm bar would be subjected to bending after repeated tests as it is unsupported throughout the length to facilitate striker movement.

### **3.3.7 General conclusions from rig design and development**

A SHTB based on direct tension through impact of a hollow striker on a flange was designed and developed to meet the requirement of single fibre tensile testing at high strain rates. The stepped bar concept was used to amplify the

velocity thereby reducing the energy requirement of the spring system used for actuation of the striker. The setup developed was able to be accommodated within the structural frame along with the already available SHPB.

### **3.4 Manufacturing, assembly and commissioning**

#### **3.4.1 Manufacturing and assembly issues**

Various issues encountered relating to manufacturing and assembly during the development of the rig are as follows:

1. *Fabrication of incident bar and support:* The fabrication of the bars and aluminium support along with the bushes (along with the units for electromagnet) was carried out by the Central Engineering Workshop, The University of Nottingham. Since the bars were fabricated to the required dimensions through turning process in lathe, bars were observed to have bending induced due to the turning operation. The bars corresponding to 17mm diameter needed to be replaced with centreless ground bars as they were observed to have severe bending due to the large amount of material removed during manufacture. Since the aluminium supports and nyatron bushes were given tight tolerances, there were difficulties in the smooth sliding of the bars inside the bushes. The tolerance of 0.1 mm was fixed for all the dimensions so that the bushes could be easily positioned inside the supports and the bars were able to slide freely inside the bushes. It would be advisable to have centreless ground bars for all the sections to improve the quality of the signal.
2. *Fabrication of clamp units:* The clamp unit developed was fabricated using turning and electro discharge machining (EDM) processes. The clamp screws were required to be split on both the incident and the transmission bar ends to avoid assembly issues for specimen during clamping.
3. *Stiffening of structural frames:* To resist the sideways moments generated in the structural frame due to impact event, the frames were further stiffened with cross braces.
4. *Positioning of support and power transmission system:* Due to the presence of the existing SHPB in the same frame, the electric motor for the belt drives was required to be accommodated within the space available between the pressure accumulator unit of SHPB and simple support of the developed SHTB.

### **3.4.2 Installation of electrical units of SHTB**

The electromagnet was wound within the Faculty. The control units for the operation of the crosshead and the electromagnet with the necessary safety features were also installed for safe use of the SHTB system. The details of the design features of electromagnet and the power transmission system is given in Appendix 5 (page 201). The electrical installations for the power transmission system and the electromagnet were carried out by the electrical engineering support team of The University of Nottingham.

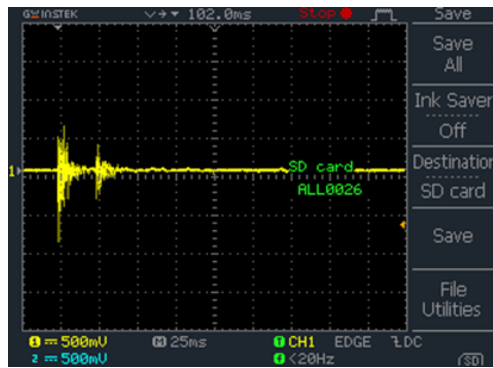
### **3.4.3 Challenges during commissioning of the rig**

Once the rig had been manufactured, assembled and the electrical installations were made, a number of practical issues were encountered during the commissioning process of the rig which is discussed below:

- 1) *Unsupported crosshead results in whirling of threaded shafts:* The crosshead motion during the return stroke caused excessive vibrations in the threaded shafts due to lack of support (as the crosshead reaches position BB from position AA (Fig 3-25)). There was no vibration during the forward phase as the crosshead moves along with the striker which ensured cross head alignment with the threaded shafts. There was also found to be an alignment issue of the entire power transmission system and careful re-alignment of the supports and crosshead was found to improve matters greatly.
- 2) *Crosshead pushes the flange during the return stroke:* The cross head pushes the flange due to undesirable contact with the flange at its edges during the return stroke (as the crosshead reaches position CC from position BB (Fig 3-25)). The ends of the flanges were chamfered and the interior faces and outer edges of crosshead were filed so that free movement of the cross head over the flange was possible.
- 3) *Simultaneous movement of incident bar along with crosshead:* During the forward motion of the crosshead towards the electromagnet, it was observed that the incident bar also got dragged forward along with the movement of the cross head. The movement of the bar once the specimen is attached can result in premature breaking of the fibre. Since the forward movement was due to the small drag force, it was decided to restrain one of the stepped interfaces of the incident bar to the nylatron bushes

attached to the adjacent supports before the crosshead movement is initiated. This was successful in preventing forward movement of the bar.

- 4) *Triggering Issues due to very serious electrical noise*: The high speed camera and flash light were set to be triggered using the voltage signal from the strain gauge mounted on the incident bar. However, it was observed that the camera and lights were triggered long before the impact had happened. It was identified that the electrical noise/voltage spikes resulted from the relay coils of the switches and as well as from the inductance of the electromagnet which, in turn, resulted in premature triggering of the camera and flashlights. The electrical spike observed from the electromagnet was very large and a freewheeling diode was connected in parallel to eliminate the large back-EMF which results from sudden switching-off of the electromagnet current by allowing the current to decay naturally over a short period. This helped in removing the electrical spike during the release of the striker from the electromagnet.
- 5) *Incident bar moves along with striker after the demagnetisation of the electromagnet*: The video images captured based on the trigger settings and time delay showed movement of the bar before the striker impacted on the flange. There were two periods of significant varying amplitude oscillations corresponding to two events which can be seen from the signal obtained from the strain gauge mounted on the incident bar (Fig 3-32a). Since the time gap between the two dominant oscillatory signals was around 25.6ms, it was understood that the incident bar moved along with the striker during the demagnetisation of the electromagnet as the time duration corresponds to the time taken by the striker to reach the flange after being released from the electromagnet. It was understood that the incident bar moved along with the striker during the demagnetisation of the electromagnet. This was also verified from the images obtained from the high speed camera relating to the starting time of the motion of the incident bar end (where the specimen is attached). This is mainly due to the magnetic interaction (through residual magnetism) of the steel incident bar with the steel striker. Hence, the steel incident bar was replaced with aluminium incident bar and the signal characteristics obtained thereafter were confined to oscillatory signal due to the striker impact on the flange alone (Fig 3-32b).



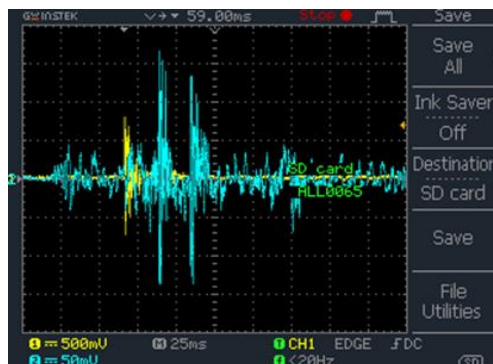
a) With steel incident bar



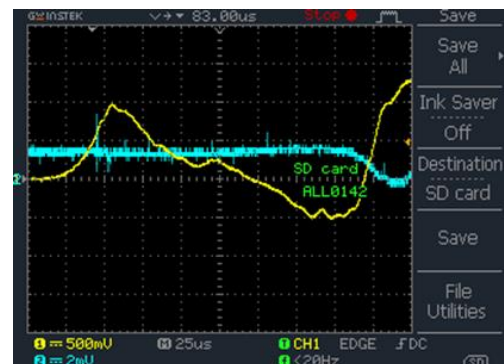
b) With aluminium incident bar

Fig 3-32 Oscillatory signals observed from the incident bar strain gauge

- 6) *Excessive noise in the load cell*: It was observed from the oscilloscope traces (Fig 3-33a) that the load cell (blue trace) picks up excessive noise long before the strain gauge (yellow trace) picks up movement of the incident bar. It could be observed that it would be difficult to distinguish the actual characteristics of the specimen from this noise as signal characteristics from load cell were comparable or lower in its amplitude compared to the noise signal observed. This noise was identified to be from the frame vibration due to the reaction of the supports during the demagnetisation process of the electromagnet/release of the striker. Hence a procedure as shown in Fig 3-34 and Fig 3-35 was followed to ensure that vibrations from the spring reaction force are not transmitted to the load cell via the frame, at least until the loading has reached the specimen (Fig 3-33b).



a) Before isolation



b) After isolation

Fig 3-33 Vibratory signals picked up by load cell (blue colour) during SHTB testing

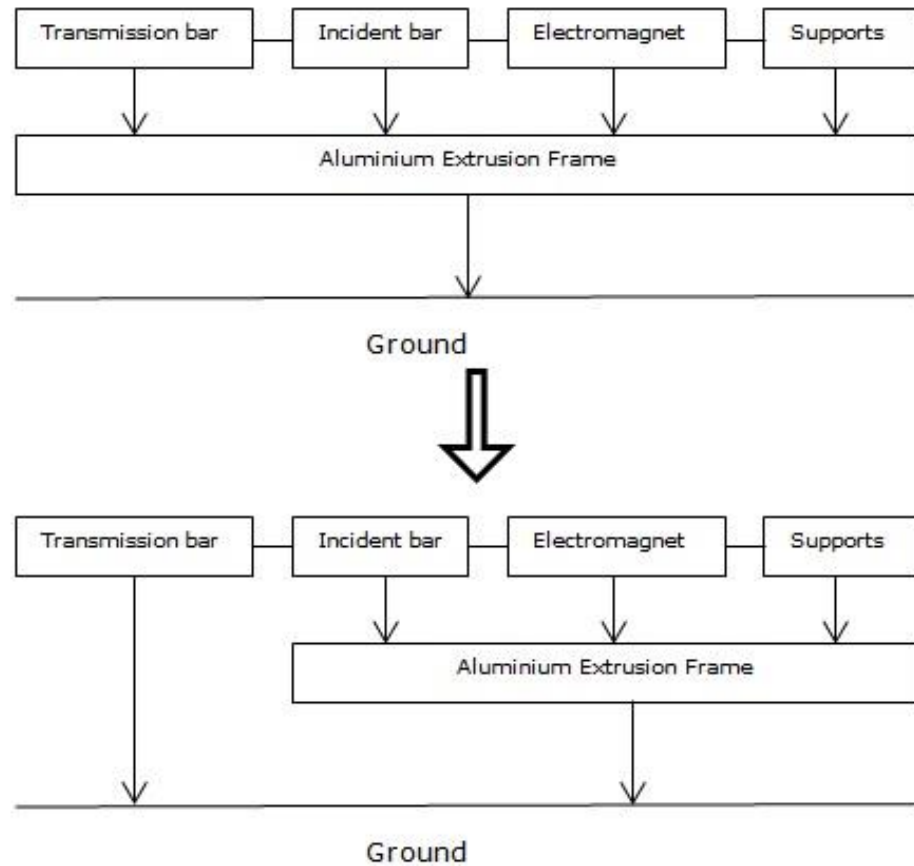


Fig 3-34 Schematic representation of the isolation procedure followed for transmission bar

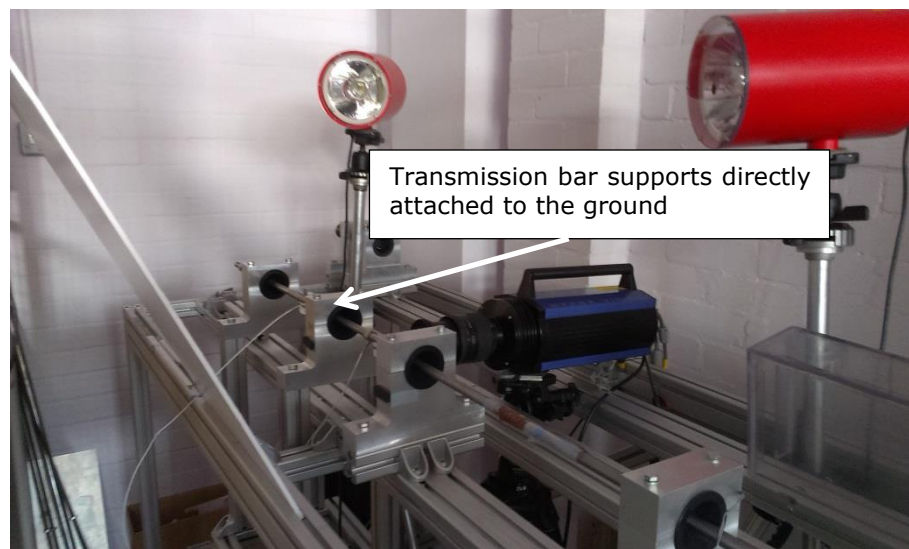


Fig 3-35 Isolation of transmission bar from the aluminium extrusion frame

- 7) *Fibre breakage during forward movement of the crosshead*: It was observed that the fibre tends to break due to excessive vibration transferred from the incident bar during the forward motion of the crosshead while compression of the spring. Hence to avoid the fibre

breakage, the standard operating procedure was modified so as to mount the specimen after the compression of the spring with the crosshead still holding the spring.

- 8) *Small movement of incident bar during the movement of the striker:* Due to the frictional interaction between the incident bar and the striker, fibre breakage occurred before the intended high strain rate loading. This was observed from the load cell signals captured during the test. Providing an intentional slackness prevented the fibre breakage before the high strain rate loading, however resulted in another issue of non-breakage of fibre during the first loading tensile pulse. Removing a bush from the flange end of the striker helped in reducing the frictional interaction between the striker and the incident bar. Small spikes in the strain gauge signal were anticipated as the striker slides along the incident bar as a result of removal of the bush. The series of tests performed showed no major difference in the magnitude of the incident bar strain signals compared to the test conditions when the striker was supported by two bushes, even though low magnitude spikes can be observed during the striker movement phase.

Lim et al. [126] had used a separate acceleration tube on which the striker slides after the release from electromagnet (Fig 3-16). The introduction of an acceleration tube over the incident bar should have been considered during the design phase or can be considered in future along with the modification of the striker dimensions to accommodate an acceleration tube over the incident bar.

- 9) *Distortion of the striker flange:* The flange of the striker was observed to have deformed to the shape of a dish (Fig 3-36). This deformation can be caused due to the inertia loading the flange experienced during each impact. As the impact occurs, the striker is immediately brought to rest whereas the flange of the striker would experience inertial loading as it is unsupported at the flange when the striker impacts on the flange of the incident bar. In due course of time, this resulted in the electromagnet being unable to hold the flange of the striker due to the excessive gap.

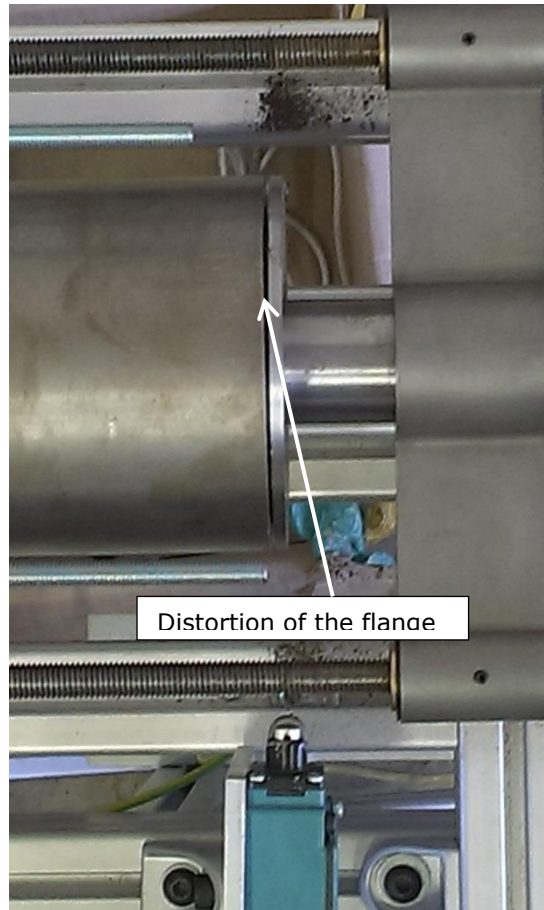


Fig 3-36 Distorted flange of the striker held by electromagnet

#### 3.4.4 SHTB configuration used for single fibre testing

A small modification to the SHTB configuration was required due to the non-availability of aluminium clamp sections. The high strain rate testing was performed using the SHTB with an incident bar consisting of bar sections made from different materials. Aluminium 7075-T6 bars were used for the flange (40mm diameter), step 1 (25mm diameter) and step 2 (17mm diameter) of the incident bar sections. Steel (En 24) was used for the 12mm and clamp sections of the incident bar and also for the 12mm transmission bar which holds the load cell and the steel clamp unit. This arrangement was adopted as the aluminium 7075-T6 clamp section was not available for use during the time of high strain rate testing with the Shimadzu HPV1 camera and this configuration was continued for consistency so as to get the same strain rate for other tests for which the camera was not available. Having interfaces with different impedances is undesirable as it would result in significant wave reflection at the interfaces. The modified

arrangement described above also affects the strain rate of the events, as the wave propagation through the aluminium-steel interface will result in larger wave reflections.

### 3.5 Results

#### 3.5.1 Typical data during SHTB

In order to be able to interpret the signals obtained from the SHTB tests, it is instructive to explore the exact sequence of events and the signals corresponding to each event. In particular, the time at which the strain gauge and the load cell sense the voltage signals corresponding to SHTB event is dependent on the initial condition and the constraints controlling each phenomenon during the actuation event. Typical data from the incident bar strain gauge and the load cell during the single fibre testing using the SHTB are shown in Fig 3-37.

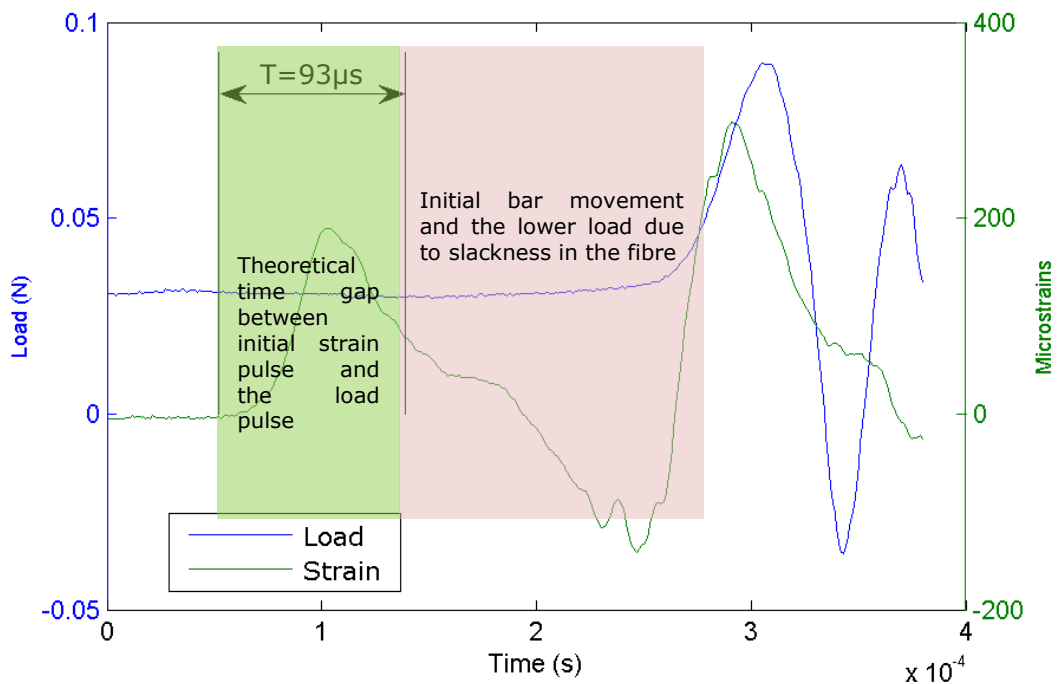


Fig 3-37 Typical raw data observed for SHTB testing

In order to interpret the data obtained from the incident bar strain gauge and the load cell which was mounted on the transmission bar, the times of events during the SHTB tests were assessed.

- The distance between the strain gauge and the free end of the bar is 340mm and it takes 69 microseconds to reach the end of the bar interface (clamp region included).
- The time taken for the wave to travel from the paper tab to the specimen is around 5 microseconds which is based on the assumption that the wave speed in the paper is  $2000\text{ms}^{-1}$  (160gsm paper of modulus 3.2 GPa).
- The time taken by the wave to travel inside the carbon fibre of 10mm gauge length will be less than 1 microsecond.
- The time taken by the wave to travel through the paper tab in the transmission bar side is 5 microseconds (assuming that the paper tab is of same length on both sides of the specimen).
- The time taken by the wave to travel through the EN 24 steel clamp unit in the transmission bar side is 6 microseconds.
- The time taken by the wave to travel further inside the load cell unit is 1 microsecond.
- The rise time of Kistler load cell is 6 microsecond

This means that the total theoretical time for the load cell to pick up the load signal from the time when the strain gauges observe the signal is around 93 microseconds ( $69+5+1+5+6+1+6$ ). A time gap of around 170-175 microseconds could be observed for all the specimens, which is approximately twice the time that was theoretically evaluated. This means that the initial pulse observed by the load cell relates to a phenomenon which is present for all the tests. The time delay due to slackness varies from test to test due to the varying amount of slackness induced in the samples. The initial tensile wave front is intended to load the specimen and cause the failure of the specimen within the duration of the stress pulse. This may happen only if the fibre is at optimum slackness (nearly zero slackness at the time when fibre is loaded).

It could be also observed from Fig 3-37 that there is a secondary tensile loading wave with time lag of 200 microseconds which has larger magnitudes than the initial tensile loading wave. This secondary wave corresponds to the compressive wave which was propagated from impact end of the flange towards the free end of the flange during the impact. The compressive wave then is reflected as a tensile wave from the free end of the flange and gets amplified at each stepped interface. This stress pulse is expected to be of higher magnitude compared to the initial tensile pulse due to additional amplification of the wave as it is transmitted from the 40mm to the 25mm section (Fig 3-29).

The fibre specimen which may not fail in the first tensile pulse due to larger slackness in the fibre has a possibility of fibre failure during the second tensile pulse which is at a higher strain rate compared to the first pulse. However, it would be difficult to infer load characteristics of the fibre as the load cell signal are significantly noisy due to ground vibration during the beginning phase of second pulse and thereafter for most cases.

### 3.5.2 FE analysis of SHTB single fibre testing

The SHTB arrangement developed consisted of a 12mm steel bar holding a single fibre through a clamp arrangement as shown in Fig 3-38. To understand the magnitude of the stress pulse transferred from the bar to the specimen and to the specimen to the transmission bar, an FE analysis of an equivalent axisymmetric model of the bar and the specimen was performed. The FE model consisted of step 3 of the incident bar along with the clamps and the transmission bar. A separate definition for the load cell was not incorporated in the FE model based on the assumption that the bars and the load cell were of the same material and diameter. The bars were assumed to be made of steel and the specimen was assumed to have a stiffness corresponding to carbon fibre (250 GPa).

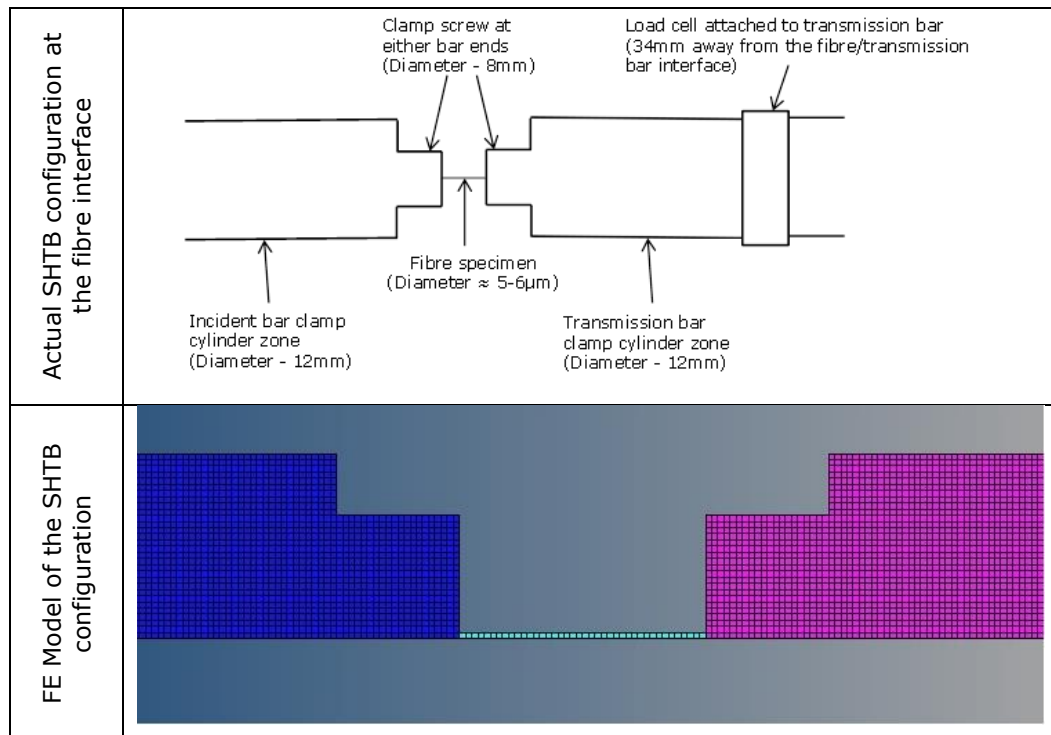


Fig 3-38 Configuration considered for FE simulation

To maintain the actual impedance mismatch between the bar and the specimen using a specimen of large diameter of 0.4 mm (compared to actual fibre diameter of 5  $\mu\text{m}$ ), the stiffness and the density of the specimen were both scaled down by a factor of  $1\text{e-}4$ . The wave speed across would be the same as in the actual case as scaling down was performed for both stiffness and density by an equal factor. The corrected stress was evaluated from the specimen stress obtained from the simulation to account for the actual diameter instead of that used for the simulation. This approach was followed for its simplicity.

**Stress equilibrium in the specimen:** It was observed that the specimen achieves stress equilibrium in the specimen within less than  $2\mu\text{s}$  irrespective of the nature of the loading pulse (step or ramp). The time required for the wave pulse to reverberate between the bar interfaces four times was also around  $2.48\mu\text{s}$  which validates the above observation achieved through simulation. This can be largely attributed to the higher wave speed of the carbon fibres due to the larger modulus and lower density. However, it is advisable to use a ramp pulse with a larger rise time as achieved during SHTB test conditions to minimise the fibre failure during the loading phase due to non-equilibrium of the stresses across the specimen and lower failure strain of the carbon fibre. Higher stresses are observed in the specimen as failure modelling was not considered in the model.

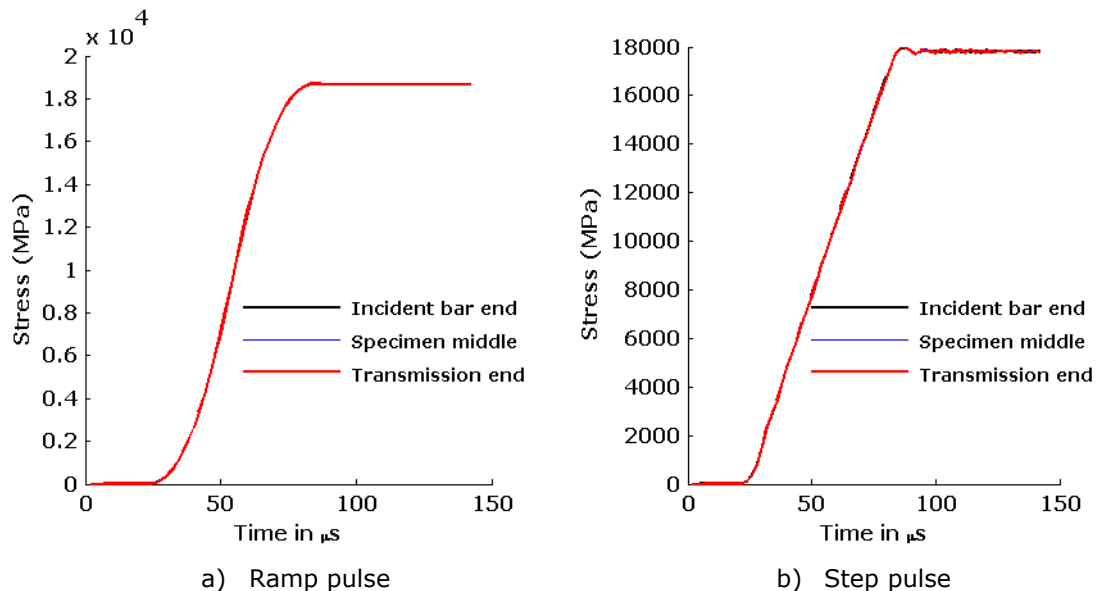


Fig 3-39 Stress equilibrium observed for the specimen for different loading pulse characteristics

**Effect of load cell position in the transmission bar:** The stress pulse transmitted to the transmission bar from the single fibre specimen during the

loading phase was used for evaluation of the load. It can be observed that the load monitored nearer to the specimen showed significant difference from that monitored away from the specimen. Significant reduction in the impedance from the actual was caused due to the scaling down of the modulus and the density by a factor of  $1e-4$ . This factor should have been  $1/6400$  than  $1e-4$  for a case for  $5\mu\text{m}$  diameter fibre. However, this study highlights the effect of the position of the load cell during single fibre testing. There is high possibility of large error in load measurement in the present study as load cell was 34mm away from the fibre/transmission bar interface. This study also shows that the location of the load cell is also a factor contributing to the longer response time of the load data in addition to the slackness that was introduced.

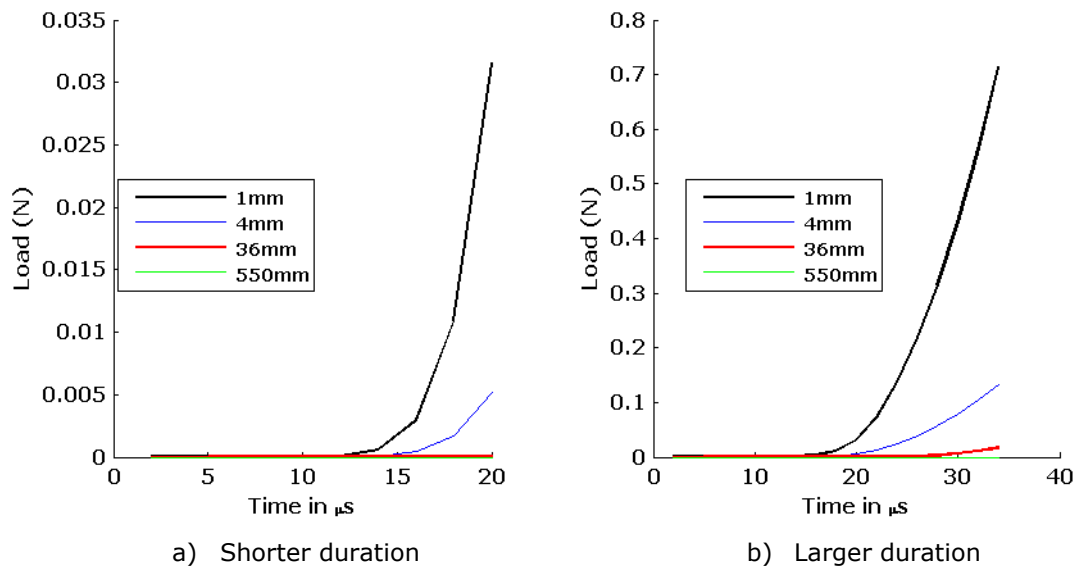


Fig 3-40 Load-time observed at different locations in the transmission bar during the fibre loading phase

### 3.6 Discussion and conclusions

A SHTB was designed and developed for single fibre testing. The developed SHTB can also be used for testing fibre bundles and composites. Most of the issues during the commissioning were addressed and solutions were found. Some of the persistent issues in the present SHTB configuration were the distortion of the flange of the striker and the movement of the bar during the sliding movement of the striker on the incident bar. The thickness of the flange could be increased to reduce the distortion of the flange that occurs during every test. The incident bar movement due to sliding of the striker was further addressed by removing the bush from the flanged end of the striker. This helped in reduction of the frictional contact between bar and the strikers. This issue could be resolved by use of an

acceleration tube. The tests conducted so far used a steel bar in the last step which resulted in lower strain rate for the specimens tested. Higher strain rates can be achieved through replacement of the steel bar with the aluminium bar for that step. The developed SHTB in general was successful in performing high strain rate testing of fibres and are suitable of high strain rate fibre data. However, the position of the load cell in the present study can induce significant errors in the load data. The next chapter discusses the results obtained from tensile testing of single fibres along with the test results from SHPB for various strain rates.

## **4 Mechanical characterisation of composite material and their constituents at different strain rates**

### **4.1 Introduction**

The material model definition in the micro-scale finite element model developed by Lidgett [1] requires strength and elastic properties of fibre, matrix and the interface to be given as input into the model. Hence, two different philosophies for generation of high strain rate properties of composite material constituents have been attempted in this study. The first of these is the direct evaluation of tensile properties of fibres using a universal testing machine and split Hopkinson tensile bar whose design and development has been discussed in the previous chapter. The other is the inverse estimation of high strain rate property of constituents from the test data of composites which will be discussed in detail in the next chapter. The inverse estimation uses the uni-directional composites data in the evaluation of an objective function for estimation of the constituent data through numerical optimisation of the objective function.

Hence, the first part of this Chapter discusses the results obtained from compression testing performed on unidirectional composite specimens for different strain rates which have been used with inverse modelling studies discussed in Chapter 5. The details of the test methodology followed and the results obtained are discussed in detail for longitudinal and transverse compressive testing of unidirectional composites. The second part of this chapter is about the methodology followed and the results obtained from single fibre testing performed on three different carbon fibres at different strain rates. The data obtained from single fibre testing were used for probabilistic strength prediction of composite laminates which are discussed in Chapter 6.

### **4.2 Compression characterisation of unidirectional (UD) composites**

The evaluation of the compressive characteristics of UD composites in the fibre direction is always a challenge as it is difficult to achieve a compressive mode of failure during testing. Premature failure due to global buckling of the composite is one of the major issues during compression testing. This global buckling,

popularly termed as Euler buckling, can be prevented by reducing the gauge length, thereby the aspect ratio of the specimen, or by employing an anti-buckling fixture. There are three different methods of compression testing for composite materials [178].

1. Direct end loading
2. Shear loading
3. Mixed loading of direct and shear

Direct end loading of the specimen (ASTM D 695 [179]) is not recommended for high strength composites as it can result in under prediction of the strength due to the end crushing effect. Shear loading of specimen is the most common method for compression testing wherein the compressive load is introduced into the gauge length of the specimen via the shearing action of the wedge grips loaded in compression [178]. The specimen is held between the tapered wedge grips which are accommodated in matching tapered sleeves which transfers the applied compressive load to the specimen through shear (ASTM D 3410 [180]). The combined mode of shear and end loading combines the features of both types and compared to the shear loaded test method, the clamping forces are less and this lowers stress concentration effects (ASTM D 6641 [181]) [178].

The split Hopkinson pressure bar (SHPB) employs the end loading method for compression of the specimens during testing, even though Hsiao and Daniel [14] had reported that the premature crushing failure during pure end loading compression testing results in underestimation of the longitudinal compressive strength. They also reported that the longitudinal compressive strength at different strain rates can only be compared if same method of loading is employed. Since the purpose of the present study is to evaluate the high strain rate characteristics, it was decided to have the same specimen dimensions and mode of testing for both quasi static and high strain rate testing. The end loading method followed in SHPB compression testing is through the use of a short flat specimen loaded on its flat surfaces [15, 182]. There is no specific anti-buckling fixture used as buckling is avoided by controlling the aspect ratio of the specimen.

#### **4.2.1 Material/specimen details used for compression testing**

Based on the availability of the material, the mechanical testing was performed on unidirectional laminated composites made from Hexply® IM7-8552 [183]. This carbon-epoxy prepreg has found extensive applications in the primary structural

components of military aircraft. The dimensions of the compression specimens were decided based on the consideration of the failure mode that is desired and ease of handling. Since there is an issue of buckling, the thickness of the laminate fabricated for compression testing was kept as 5mm (consisting of 40 plies). The aspect ratio of the compression specimen (ratio of length of specimen to the smallest cross sectional dimension) was kept under 2 (Table 4-1). The specimens of the required dimensions were cut using a diamond saw. Based on the wave propagation analysis of the stress build up that occurs inside the specimen, it was observed that the compression specimens fail in the initial loading pulse when subjected to an incident bar strain of greater than 800 microstrains. The different strain rates were achieved by varying striker velocity for a given specimen gauge length. Two laminate sequences were used:

- a. UD Laminate:  $[0]_{40}$
- b. Cross-ply Laminate:  $[0/90]_{10s}$

The cut prepreg plies were laid up according to the above laminate sequences and the laminates were cured in an autoclave (LBBC Quicklock Thermoclave) available in the Composites Group facilities in The University of Nottingham, based on the manufacturer's recommended cure cycle [183]. The size of the laminate fabricated was 100×100mm. The consolidation of the laminates was analysed using a void analysis of the specimens. The details of the procedure followed for preparation of the void analysis sample and method of evaluation of the void content are discussed in Appendix 6 (page 204). The void analysis showed a void content of around 2-9 % in the specimens analysed which is high compared to the acceptable void content (for autoclave cured parts in aerospace applications) of 1% for preregs [184, 185]. Higher content can adversely affect the compressive [186] and matrix dominant properties [185] of the composite laminates [186]. The higher void content observed is most likely to be due to artefacts arising from the methodology followed for preparation of the sample. The size of the laminate was smaller due to the limited availability of the prepreg materials.

## **4.2.2 Quasi-static compression testing**

### **4.2.2.1 Longitudinal compression of IM7-8552 UD composite specimens (0° direction)**

**Results and Observation:** The quasi-static tests were carried out on an INSTRON 50 kN servo-mechanical UTM at a constant displacement rate of

0.25mm/min. Table 4-1 shows the details of the longitudinal compression tests carried out on IM7 UD specimens. The schematic representation of the quasi-static compression testing performed with and without LVDT is shown in Fig 4-1.

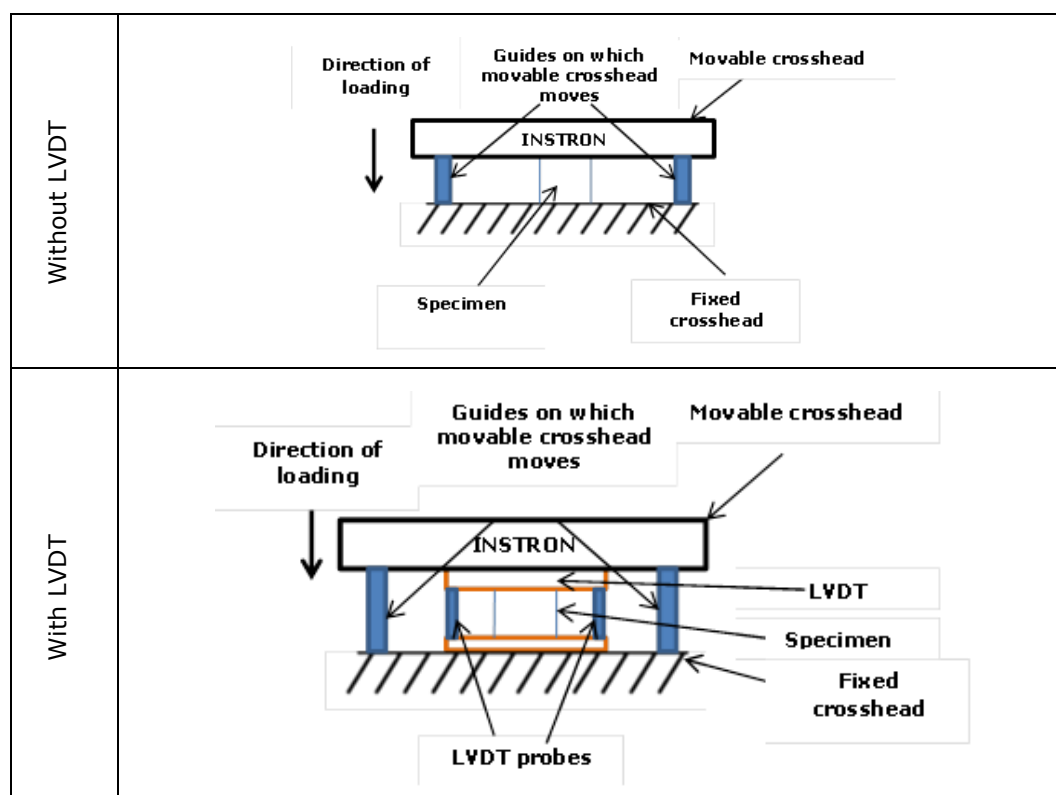


Fig 4-1 Schematic representation of quasi compression testing performed with and without LVDT

Since the crosshead components of the UTM are in series with the specimen, the crosshead displacement accounts for the elastic deflections of the frame, grips, load cell and the specimen ends in addition to the deformation experienced across the specimen gauge length. It was also observed that the use of LVDT sensors for strain measurement also under predict the stiffness. It was because the stiffness of the specimen is much higher compared to the loading unit of the LVDT and the loading frame would have bent during the loading process. It can be seen that the average longitudinal compressive strength obtained from unidirectional composites is around 60 percent lower than the data published in the data sheets and literature [183, 187]. The data given in the datasheets/literature were based on tests performed according to the ASTM D3410 standard [180]. The compressive stress vs crosshead displacement curves are shown in Appendix 7 (page 206). The longitudinal stress-strain curves are plotted only (Fig 4-2) for those specimens where strains have been evaluated from the measured displacement data obtained from the LVDT sensors mounted on the movable

cross head. The underestimated strength and stiffness can also be related to the mode of testing adopted.

Table 4-1 Details of specimens tested for quasi-static longitudinal compression

| Tests without LVDT |                      |                     |                         |                |         |
|--------------------|----------------------|---------------------|-------------------------|----------------|---------|
| Specimen ID        | Specimen length (mm) | Specimen width (mm) | Specimen thickness (mm) | Strength (MPa) | Average |
| IM7SL6SP1          | 6.28                 | 3.91                | 4.8                     | 890            | 876     |
| IM7SL6SP2          | 6.09                 | 3.92                | 4.68                    | 861            |         |
| IM7LL3SP1          | 3.56                 | 19.25               | 4.93                    | 685            | 629     |
| IM7LL3SP3          | 3.59                 | 19.14               | 4.77                    | 434            |         |
| IM7LL3SP4          | 3.57                 | 19.27               | 4.87                    | 769            |         |
| IM7LS3SP2          | 3.43                 | 8.1                 | 4.33                    | 873            | 804     |
| IM7LS3SP3          | 3.52                 | 9.33                | 4.55                    | 680            |         |
| IM7LS3SP4          | 3.56                 | 9.11                | 4.91                    | 766            |         |
| IM7LS3SP5          | 3.47                 | 9.07                | 4.49                    | 760            |         |
| IM7LS3SP6          | 3.44                 | 9.31                | 4.45                    | 941            |         |
| Tests with LVDT    |                      |                     |                         |                |         |
| IM7LL8SP1          | 8.18                 | 8.05                | 5.05                    | 507            | 501     |
| IM7LL8SP2          | 8.2                  | 8.14                | 4.98                    | 543            |         |
| IM7LL8SP3          | 8.26                 | 8.17                | 4.89                    | 453            |         |
| IM7LL3SP5          | 3.54                 | 9.08                | 4.54                    | 584            | 594     |
| IM7LL3SP6          | 3.56                 | 9.31                | 4.61                    | 518            |         |
| IM7LL3SP7          | 3.53                 | 9.18                | 4.98                    | 679            |         |

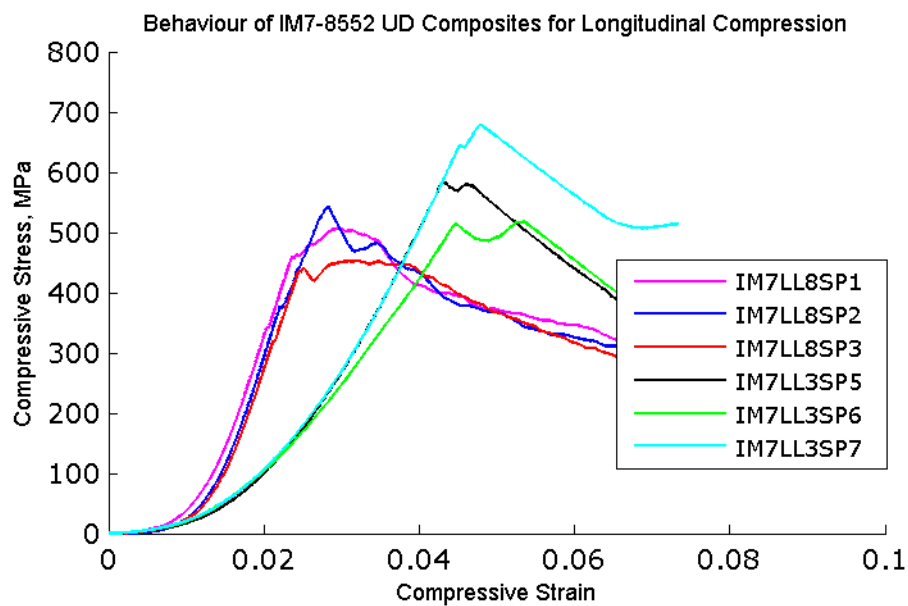


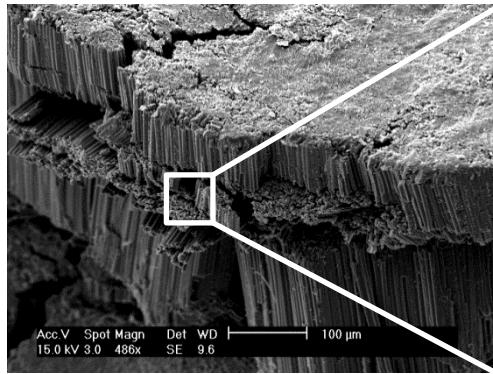
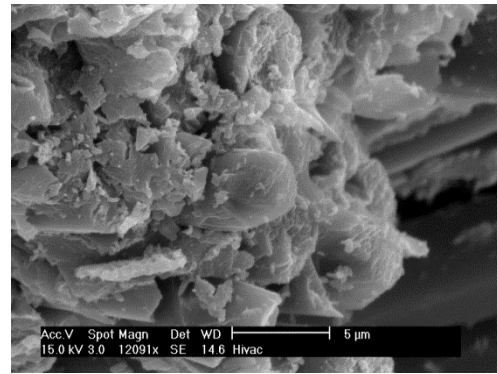
Fig 4-2 Quasi-static longitudinal compressive characteristics of IM7 UD preregs

Two different families of curves can be observed in the longitudinal stress-strain characteristics (Fig 4-2). Each group of curves can be correlated with the aspect ratio of the specimens (ratio of the length in the loading direction to the shortest transverse dimension of the specimen). Specimens with smaller aspect ratios have shown increased strength but lower stiffness compared to the specimens with higher aspect ratio for the cases where strains were evaluated. The higher strength for smaller aspect ratio specimens may be attributed to the load that is shared by the support probes of the LVDT. However, it could be observed that the wider specimens showed lower strength for shorter gauge length specimens. Also, the strength observed for the tests performed with the LVDT shows lower strength compared to that without LVDT sensors. This inconsistent behaviour can mostly be attributed to difference between the frictional interactions between the surfaces of the specimen for each test conditions. For cases where LVDTs were not mounted for strain measurement, the longitudinal strength of composite specimens was higher for specimens with larger aspect ratio (except for specimens with IDs - IM7LS3SP2 & IM7LS3SP6). This further emphasises the importance of the flatness of the interacting faces (both the specimen and the loading surfaces) for the end compression loading method. The end loading method causes the premature failure phenomenon inside the specimen which triggers the total collapse of the specimen on further loading. During initial loading, the stiff fibres in the composite specimen prevent the perfect contact of the end faces with the movable/fixed surface of the crosshead/LVDT unit.

More accurate measurement technologies such as small gauge length strain gauges or optical techniques should have been employed for measurement of strain. Generally, the shorter gauge length tends to show structural contribution rather than the actual material characteristics due to the shear stresses at the ends of the specimen which tend to restrict the lateral expansion of the specimen.

#### **Observations on failure mode:**

The failure modes analysed using the SEM images also showed fibre kinking near to the loading end as major cause for failure (Fig 4-3a). This results in localised matrix cracking and kink band formation near to the end faces which results in crushing failure of the composites as the specimens is further loaded. Further loading flattens the failed fibres as can be seen from the zoomed region of the SEM image (Fig 4-3b).



a) Kink band bones near to loaded end

b) Zoomed in zone

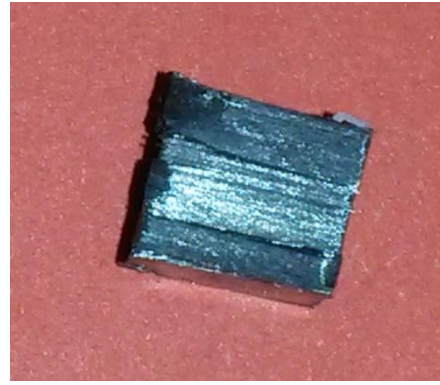
Fig 4-3 Compressive failure of UD composites subjected to longitudinal compression as observed through SEM

The method of achieving compression through end loading tends to cause localised fibre buckling near to the loading ends which can be related to the non-uniform loading of the ends. This results in the initiation of the matrix cracking in the vicinity of localised buckled fibre zones which in turn results in longitudinal splitting of the lamina. Single or multiple longitudinal splits may develop depending on the end loading conditions, matrix interfacial strength, void fraction and the mode of propagation of the matrix cracking etc.

The composite specimen tends to deform outwardly at either of its loading ends as crushing progresses which eventually makes the end similar to a broom in its appearance. This 'brooming' effect could be observed only at one end of the failed longitudinal compression specimen with multiple longitudinal splitting (Fig 4-4). The failure modes observed for compression testing of T700/2521 composite cubic specimens of size 8mm mentioned in Yokoyama et al. [15, 182] were also similar to the results obtained from the present study except that the multiple longitudinal splitting along with brooming were also observed in the present study. Steel end caps and rings were used in their study at each end to prevent premature kink band formation or longitudinal splitting near the ends [15, 182]. However, the strength observed in their study [15, 182] was quite similar to that obtained for IM7 in the present study which confirms that the introduction of steel caps and rings were not successful in controlling end localised kink band failure and splitting even though macroscopically it may appear so. Bing and Sun [188] have described longitudinal compressive strength evaluation to be a challenging task and have mentioned that end brooming or fibres splitting are the failure modes which are more dominant than the microbuckling during longitudinal compression testing which was also observed in this study.



a) 8×8×5mm specimen



b) 6×4×5 mm specimen

Fig 4-4 Failure mode of quasi-static longitudinal compression specimen

#### 4.2.2.2 Transverse Compression of IM7-8552 UD composite specimens (90° direction)

Specimens were also cut from the fabricated unidirectional laminates for transverse compression testing of IM7-8552 UD composite specimens. Specimens having two different aspect ratios were tested for transverse compression (Table 4-2).

Table 4-2 Details of specimens tested for quasi-static transverse compression

| Specimen ID    | Specimen length (mm) | Specimen width (mm) | Specimen thickness (mm) | Strength (MPa) |
|----------------|----------------------|---------------------|-------------------------|----------------|
| IM7TL6SP1      | 6.52                 | 4.94                | 4.94                    | 255            |
| IM7TL6SP2      | 6.48                 | 5.48                | 5.02                    | 267            |
| IM7TL8SP1      | 8.11                 | 8.18                | 4.94                    | 254            |
| IM7TL8SP2      | 8.11                 | 7.97                | 4.93                    | 261            |
| <b>Average</b> |                      |                     |                         | <b>259</b>     |

Stress-strain curves consistent behaviour for both the specimens where LVDT sensors were used (Fig 4-5). The strength measurements were consistent irrespective of the aspect ratio of the specimen. The low modulus of the composites in the loading direction ensured good contact between the specimen and the loading surfaces of the sensors/cross head of the UTM after the initial gradual loading phase. Hence, the variations observed for compressive strength and stiffness for cases with and without LVDT sensors were insignificant primarily due to the low modulus of the composite unlike the case for longitudinal compression. However, the transverse stiffness obtained was lower by 15-25 percent compared to the data available in the literature [187]. The transverse compression specimen showed transverse cracking of the composite along a line

at 50-60° to the direction of loading (Fig 4-6). The SEM images of the failure section shows the shear band formation at the matrix interface due to transverse shearing during the transverse compressive failure of UD composites (Fig 4-7).

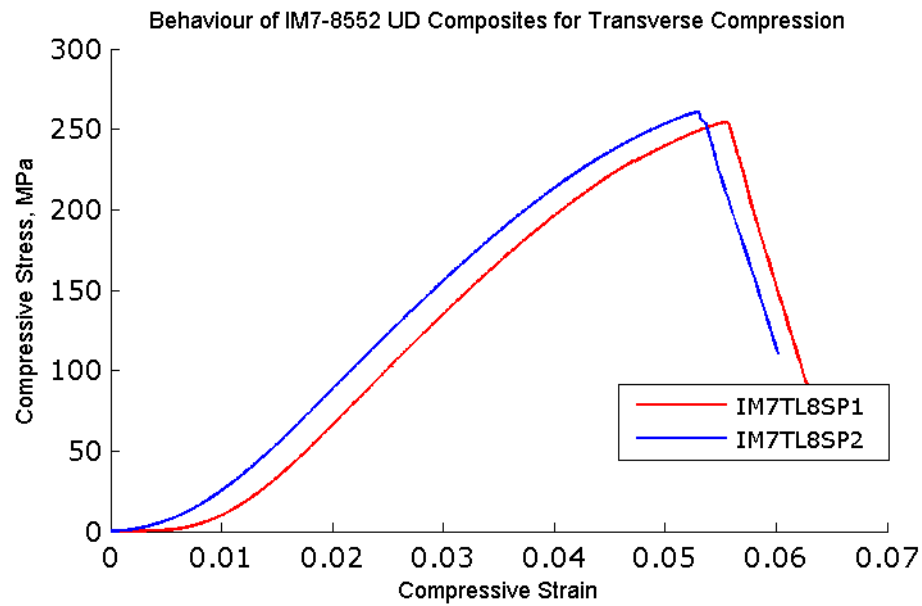
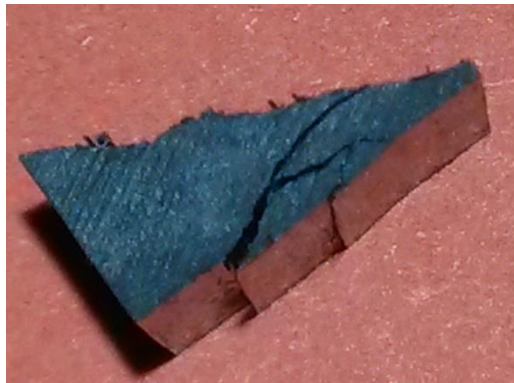


Fig 4-5 Quasi-static transverse compressive characteristics of IM7 UD composites

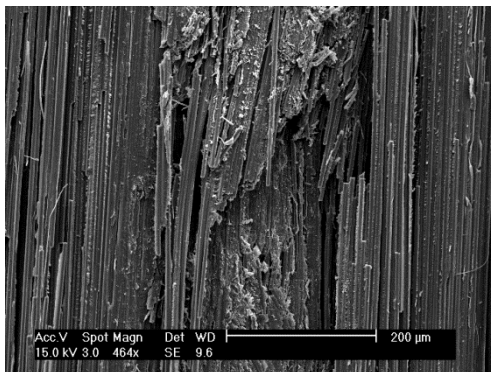


a) 8×8×5mm specimen

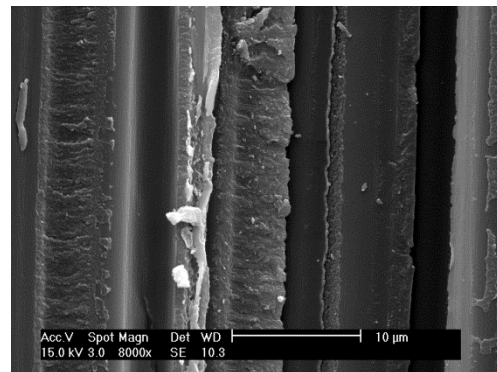


b) 6×4×5 mm specimen

Fig 4-6 Failure mode of quasi-static transverse compression specimen



a) Perpendicular to the section of transverse shear failure



b) Zoomed in zone

Fig 4-7 Transverse compressive failure of UD composites as observed through SEM

#### 4.2.2.3 Compression characteristics of cross ply laminates

The compressive strength of cross ply specimens was tested to indirectly evaluate the longitudinal compressive strength of unidirectional composites. The compressive strength of unidirectional laminate is evaluated by multiplication of the cross ply compressive strength with a back out factor typically evaluated from the stiffness properties of the unidirectional laminae [178]. This relationship for the back out factor is estimated using classical laminate theory [189, 190] (Eq. 4-1).

$$BF = \frac{\frac{1}{2} E_{11}(E_{11} + E_{22}) - (v_{12}E_{22})^2}{\frac{1}{4}(E_{11} + E_{22})^2 - (v_{12}E_{22})^2} \quad \text{Eq. 4-1}$$

Table 4-3 Details of cross ply specimens tested for quasi-static compression

| Specimen ID    | Specimen length (mm) | Specimen width (mm) | Specimen thickness (mm) | Strength (MPa) | Longitudinal compression strength (MPa) |
|----------------|----------------------|---------------------|-------------------------|----------------|-----------------------------------------|
| IM7CL6SP1      | 6.06                 | 4.05                | 5.2                     | 680            | 1267                                    |
| IM7CL6SP2      | 5.92                 | 4.46                | 4.75                    | 641            | 1194                                    |
| IM7CL6SP3      | 5.99                 | 4.45                | 4.75                    | 605            | 1127                                    |
| IM7CL6SP5      | 5.97                 | 4.02                | 4.91                    | 738            | 1375                                    |
| IM7CL6SP6      | 5.96                 | 4.05                | 4.99                    | 656            | 1222                                    |
| IM7CL6SP7      | 5.83                 | 4.05                | 5.13                    | 754            | 1405                                    |
| IM7CL6SP8      | 5.9                  | 3.78                | 5.17                    | 646            | 1203                                    |
| <b>Average</b> |                      |                     |                         | <b>674</b>     | <b>1256</b>                             |

The researchers who proposed this formula recommended the use of a  $[90/0]_{ns}$  cross ply laminate for better accuracy. However, for thick specimens, it does not make a significant difference in the value of the strength even if  $[0/90]_{ns}$  cross ply laminates are used. There are also other formulae for back out factors which have used to predict the longitudinal compressive strength from balanced angle ply laminates [191, 192]. The specimens were cut to approximately the same dimensions as used for the above cases. The quasi-static compressive characteristics of cross ply specimens made from IM7 prepregs are shown in Fig 4-8. The compressive strength of cross ply specimens showed comparable values with that obtained for longitudinal compression testing. This is primarily due to the fact that  $90^\circ$  plies helps in restraining localised lamina level buckling thereby resulting in improved compressive strength. The failures observed for a cross ply

laminate for quasi-static compression show a similar failure mode to longitudinal compression.

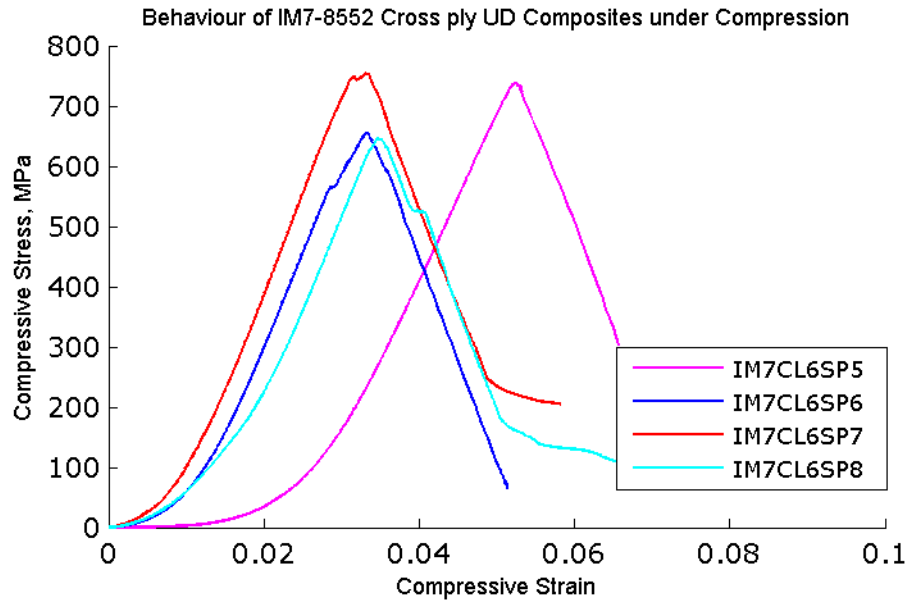
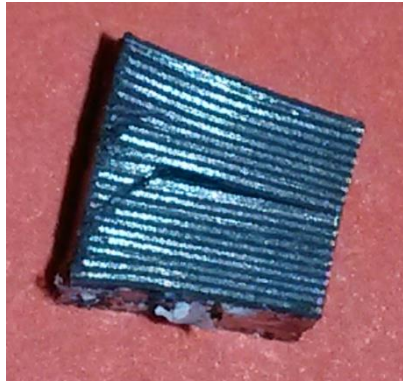
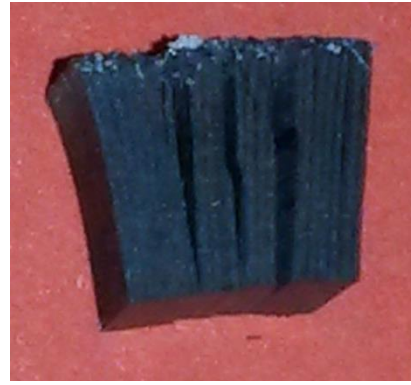


Fig 4-8 Quasi-static compressive characteristics of IM7 UD cross ply laminate

The estimated longitudinal compressive strength of  $0^\circ$  specimens from the cross ply compressive strength is shown Table 4-3. The  $E_{11}$  and  $E_{22}$  value based on the relationship defined for the back out factor in Eq. 4-1 for a composite lamina were taken based on data published for tensile characteristics [183]. Even though  $\nu_{12}$  was taken as 0.3 in the calculation, it was observed that the assumption of Poisson's ratio between 0.28-0.36 does not change the value of the back out factor even to two decimal places. The back estimated values of compressive strength are higher than the value obtained from testing of  $0^\circ$  specimens. The value obtained is still only 75-80 percent of the data published in the data sheet. The crushing effect in the ends was still prevalent in cross ply laminates which is the reason for the under estimation of the longitudinal compressive strength. The 'brooming' effect observed in  $0^\circ$  specimens was more severe for cross ply specimens (Fig 4-9).



a) 6×4×5 mm specimen



b) 6×4×5 mm specimen

Fig 4-9 Failure mode of quasi-static cross ply compression specimen

### 4.2.3 High strain rate compression testing

#### 4.2.3.1 Test apparatus and details

The high strain rate compression testing of UD composite specimens at strain rates between  $2000 \text{ s}^{-1}$  and  $6500 \text{ s}^{-1}$  was carried using the SHPB facility at the University of Nottingham. A general purpose lubricant (grease) was applied at the bar/specimen interfaces for all the SHPB tests to minimise the interfacial friction which can have adverse effects on the specimen characteristics. A schematic diagram of the SHPB apparatus and ancillary equipment is shown in Fig 4-10. Strain gauges of type FLA-1-11 from TML were mounted on the incident and transmission bar for capturing the strain information during the shock wave propagation in the bar due to the impact. The strain gauges were of gauge factor of 2.08-2.14 depending on the batch. Each of the strain gauges was connected to a signal conditioning amplifier (VISHAY 2310B) in a quarter bridge configuration using the inbuilt Wheatstone bridge circuitry.

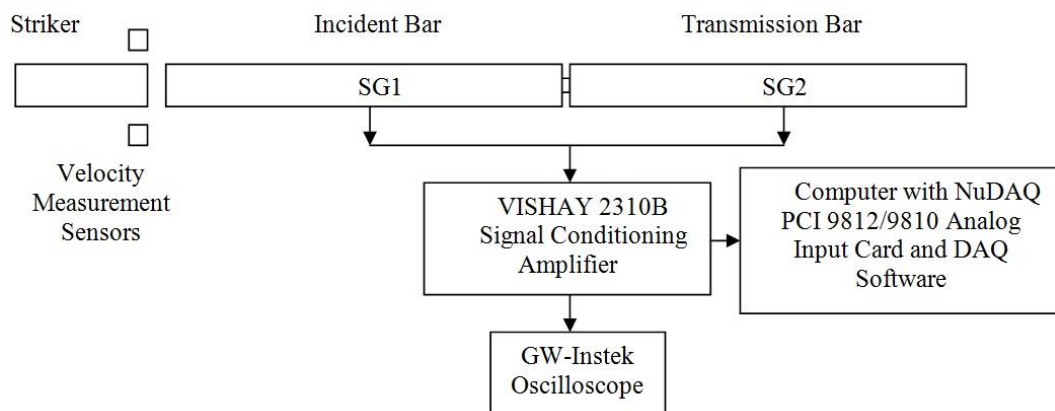


Fig 4-10 Schematic representation of SHPB arrangement used for high strain rate testing

The output voltage in each channel due to strain from the incident bar and the transmission bar (SG1 and SG2) was amplified inside the signal condition amplifier with gain setting of 100 (can be adjusted to any value from 1 to 11000) and was recorded using an ADLINK NuDAQ PCI 9812/9810 data acquisition card inside the computer. The event data is recorded based on the trigger settings in the data acquisition software (AD-Logger). The data recorded by the data acquisition were also simultaneously verified with a GW-Instek GDS-1022 oscilloscope. It was also observed that there is an additional gain of 5 is observed for the NuDAQ card due to its hardware and software settings as verified during calibration studies (Appendix 8, page 208). The velocity measurement sensors (DJB Microtech Ltd) consist of a light gate which passes a light beam through the path of movement of the striker. The TSA unit also from DJB Microtech Ltd measures the length of the time when the light beam has been cut. The light gate is positioned in a slot so that it measures the striker velocity just before the striker impacts the bar. The striker velocity measurements were used to check the input strain signal observed in the incident bar.

#### **4.2.3.2 Data processing of a SHPB event**

The stress, strain and strain rate observed by the specimen can be inferred from the reflected and transmitted pulse (Eq. 3-10, Eq. 3-11 and Eq. 3-12). An ideal signal generated during an SHPB event will consist of a rectangular pulse with strain amplitude ( $\frac{V_0}{2C_0}$ ) and pulse duration of  $\frac{2L_0}{C_0}$  based on wave theory for the cases where the bar and striker are made of the same material. However, it is observed in practice that there are distortions from the ideal signal due to the dispersions arising from resonance of the bar in addition to the other noise that can be present during the event. The effects of sabot impedance (sabot refers to additional material attached to the striker for alignment purposes, e.g. nylatron rings in this study) [193-195], alignment of the bars, parallelism of the interfaces of the bars, flatness/perpendicularity of the bar ends [175] etc. play a significant role in the characteristics of the incident pulse. Hence, the processing of the raw signal to obtain engineering material data has uncertainties or variabilities due to the following reasons:

- Inaccuracies in determination of the wave speed inside the bar
- Inaccuracies in identification of the datum point of the reflected and transmission bar signals.

- Validity of the data due to the non-uniform stress equilibrium conditions of the interfaces.
- Dispersion correction may also be needed for better data representation [160, 196].

The details of the calibration experiments carried out for evaluation of wave speed, the time shifting approach for identification of datum points and the methodology for stress equilibrium evaluation are given in Appendix 9 (page 213).

#### **4.2.3.3 Longitudinal compressive characteristics at high strain rates**

The methodology described above was used for extracting the dynamic stress-strain characteristics. Since the equation used for calculation of the specimen stress and strain from the strain data of the bars assumes stress equilibrium at each interface, the time required for the pulse to reach dynamic stress equilibrium was evaluated first as mentioned in the previous section. However, it should be noted that, for many of the specimens tested, it is quite unlikely that the specimen reached the state of dynamic stress equilibrium.

##### **4.2.3.3.1 General description of the longitudinal characteristics observed during SHPB testing**

Three distinct stress-strain curves were observed for longitudinal compression testing of IM7-8552 composite laminates at high strain rate (Fig 4-11). These curves were classified as 'Type A', 'Type B' and 'Type C'. 'Type A' characteristic is a case where gradual continuous failure of the specimen seems to occur due to lamina level buckling or localised fibre crushing, however the characteristic curve tends to follow a linear pattern even though minor deviations from the linear behaviour could be observed in its characteristics. The characteristic curve in 'Type B' has a specific region where stress remains constant for continuous increase of strain (shown as ellipsoid region in Fig 4-11). This characteristic curve has specific two defined zones of stiffness zones in its characteristic curves. However, a typical non-linear characteristic with gradually decreasing stiffness can be observed in a 'Type C' characteristic. This characteristics can assumed to occur in loading conditions when the specimen resisted the dynamic compressive load due to larger interaction of the rubbing surfaces which could be substantiated by steeper regions during initial loading. However, it disintegrated due to the poor quality of the specimen and structural loading conditions.

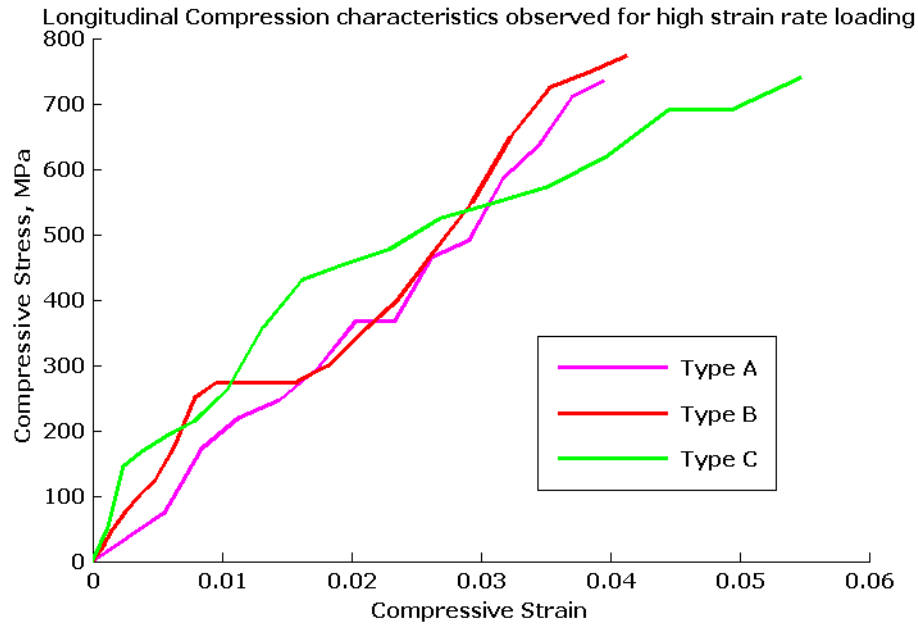


Fig 4-11 Different types of longitudinal compressive characteristics observed during SHPB testing

#### 4.2.3.3.2 Failure modes and their discussion

Some of the specimens were tested with the use of a high speed camera for capturing deformation behaviour. It could be observed that microbuckling induced matrix cracking and fibre crushing due to the non-uniform loading of the specimen triggers lamina level outward buckling of the composite. The non-uniform loading of the specimen may be due to the following reasons:

- *Concavity of the bars results in non-planar ends:* This happens over a period of time after a series of tests of very stiff specimens. This is largely due to the inertia effects at the unsupported area of the bar ends.
- *Non planar specimen ends:* Specimens were visually analysed for their flatness by positioning each on a surface table.

The non-uniform contact caused localised failure of the loaded fibres which triggered the eventual collapse of the specimen as it was subjected to further load. The strength evaluation is based on the total area of the specimen rather than the more realistic localised contact area, thus providing a likely explanation for the low stiffness and strength obtained for end loaded specimens. The value of stiffness tends to show inverse rate sensitivity which can be related to the rate sensitivity of localised crushing/buckling behaviour to the loading rate. Hence, this spurious rate sensitivity cannot be seen as the rate sensitivity of the material for longitudinal loading. The failure modes observed for high strain rate testing of

T700/2521 composite using the end loading method in a SHPB at strain rates around  $400\text{-}800\text{s}^{-1}$  [182] showed multiple longitudinal splitting rather than complete shattering of the specimen as seen in the present study. The difference in failure mode could be attributed primarily due to the increased strain rate in the present study.

The failure modes observed for high strain rate testing as observed through high speed videos reveals the lamina level outward buckling of the composite and crushing of the fibres at the ends due to localised buckling to be major reason for premature compressive failure observed for the composites at high strain rates (Fig 4-12). The data obtained from the corresponding bar signals for the respective test shown in Fig 4-12 were affected by noise which can be caused due to the minor misalignment of the bars during high strain rate testing.

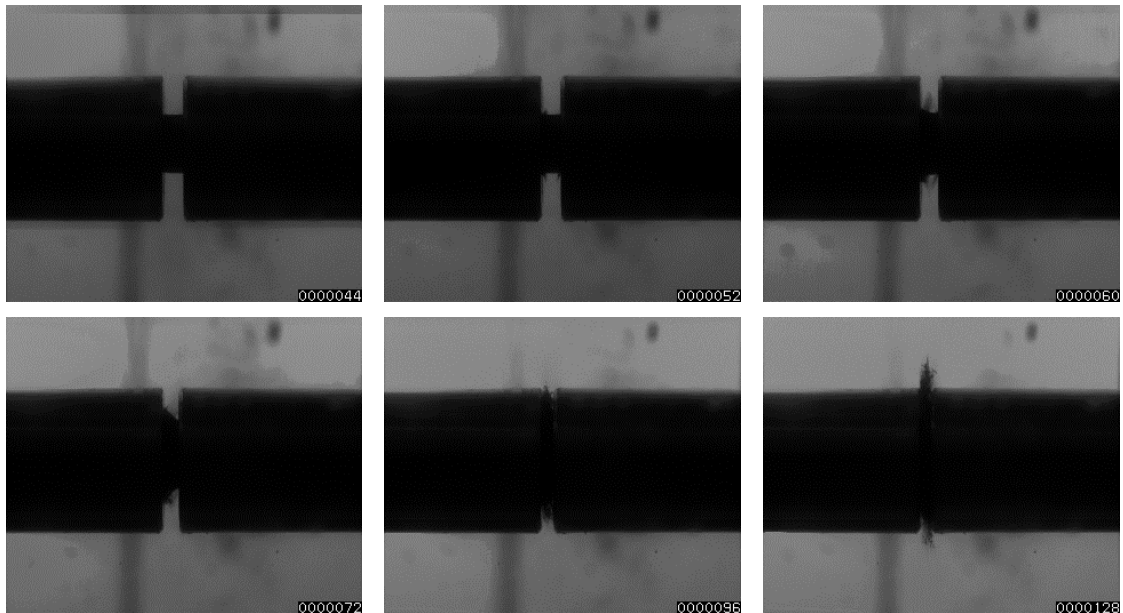


Fig 4-12 High speed images captured for longitudinal compression testing using SHPB

Since flash lights were used, the lighting provided was insufficient in the initial phase of loading due to the incorrect delay settings. The images that have been shown are for a shorter specimen of 3mm length. The images shown in Fig 4-12 were captured at 250,000 fps using a Shimadzu HPV 1 high speed camera available on loan through the EPSRC Instrument Pool (details of the arrangement are mentioned in section 4.3.4.1.1, page 114). Some of the tests with 6mm gauge length specimens (Table 4-4) were also captured using a different high speed camera but with a lower frame rate capability. Hence those images are not shown as they could not capture the deformation details observed during the

loading phase. The arrangement used for capturing images for compression testing using SHPB is shown in Fig 4-13 .



Fig 4-13 Arrangement used for capturing images during SHPB testing

The higher void content and voids present along the lamina interface might also have contributed lamina level buckling. The interlaminar shear strength should have been verified for the samples cut from the laminate. Since the laminate was smaller ( $100 \times 100$  mm) and thickness of the laminate was 5mm, it would mean that only a limited quantity of specimens would be available for compressive strength evaluation. Hence such a study was not undertaken before compression testing. In general, if the interlaminar shear strength of the composite laminate is lower than the acceptable value of 118 MPa [187, 197], then data obtained from compression testing would be unreliable. Since this study is a demonstration of development of a methodology, it was decided to do the compression testing with the available laminate. The consolidation level of laminates in general, is assessed through non-destructive techniques such as ultrasonic C-scan etc, if available.

#### **4.2.3.3.3 Results from stress-strain curves at different strain rates**

The longitudinal stress strain characteristics for strain rates 2800, 4500, 5000 and  $5900 \text{ s}^{-1}$  are shown in Fig 4-14, Fig 4-15, Fig 4-16 and Fig 4-17 respectively. Only two of the specimens had shown dynamic stress equilibrium for longitudinal compression testing. The coloured vertical lines shown in the characteristic curves define the region corresponding to the respective coloured specimen (as defined by the colour of the specimen curve in the legend) beyond which the dynamic equilibrium condition was satisfied. The specimen tends to deviate from dynamic stress equilibrium once the failure initiates. Hence, dynamic stress equilibrium

was assumed only until the point of maximum strength. The check followed for evaluation of the satisfaction of the dynamic equilibrium condition for each of the specimen is shown in Appendix 22 (page 248). The specimens for which vertical lines are not shown have not satisfied the dynamic equilibrium requirements. The plotted data shown in this section for each specific strain rate are the reprocessed data from the initial raw stress-strain characteristics. The methodology followed for replotting the stress-strain curves is given in Appendix 10 (page 218). Some of the characteristics observed are different from the defined type of characteristics such as 'SP4' for  $4500 \text{ s}^{-1}$ . There were also characteristics where the specimens showed combination of characteristics described above. The inconsistency in the failure characteristics appears to be primarily due to the poor consolidation of the laminate and also can be attributed to the non-planarity of the loading interfaces.

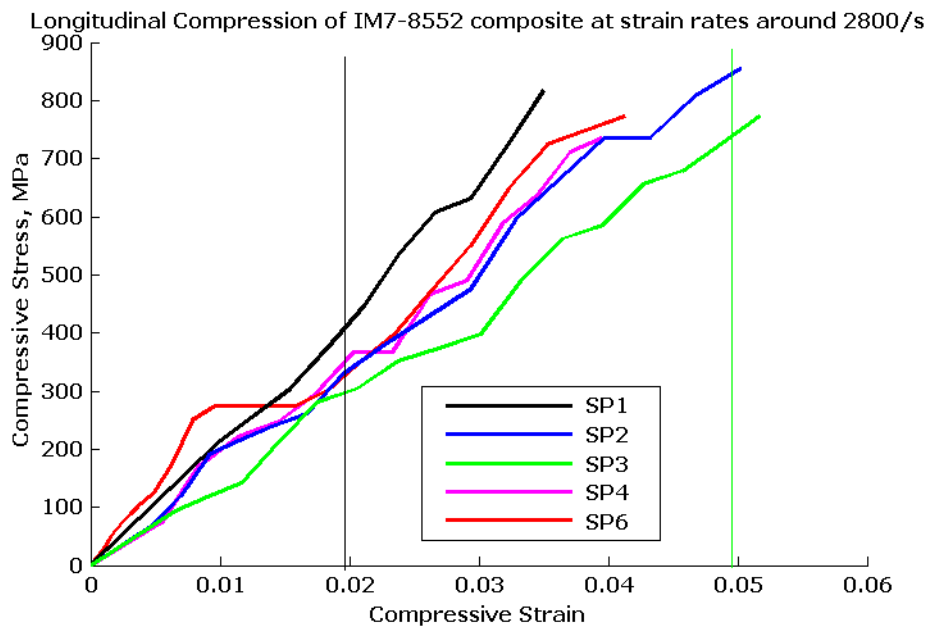


Fig 4-14 Longitudinal compressive characteristics of IM7-8552 composite laminates around  $2800 \text{ s}^{-1}$  showing the limits of stress equilibrium for specimens SP1 and SP3

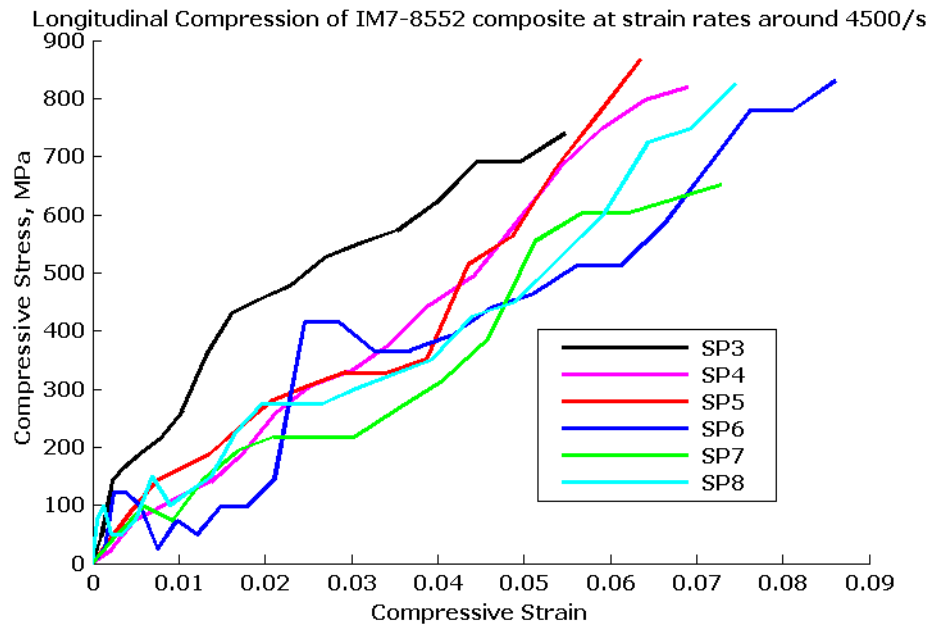


Fig 4-15 Longitudinal compressive characteristics of IM7-8552 composite laminates around  $4500 \text{ s}^{-1}$

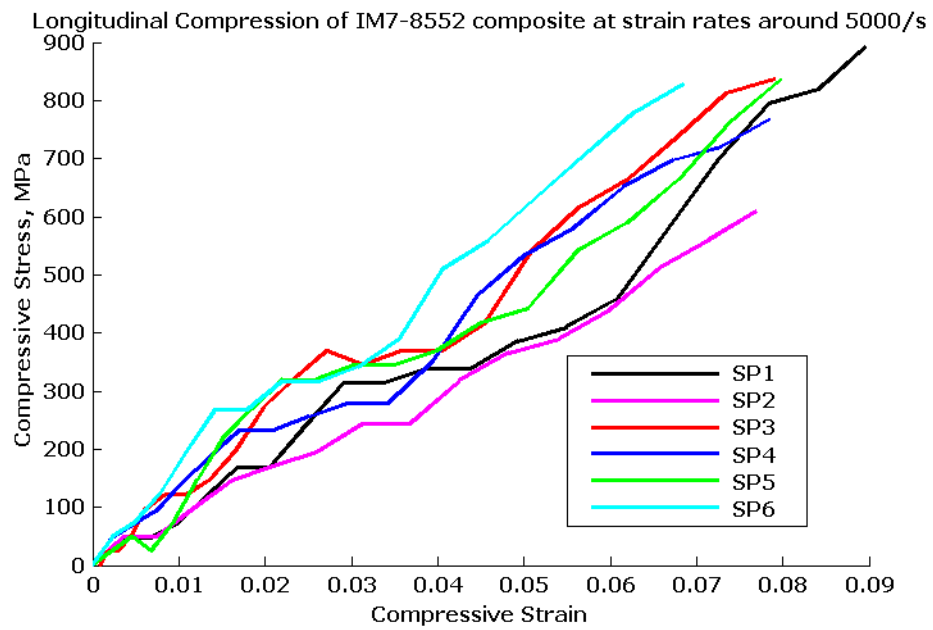


Fig 4-16 Longitudinal compressive characteristics of IM7-8552 composite laminates around  $5000 \text{ s}^{-1}$

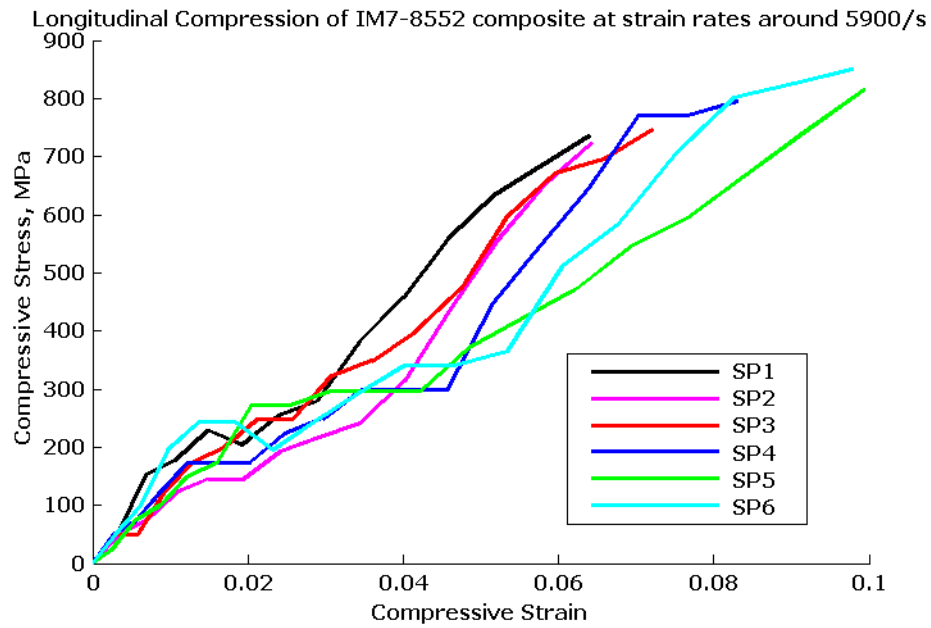


Fig 4-17 Longitudinal compressive characteristics of IM7-8552 composite laminates around  $5900 \text{ s}^{-1}$

The summarised details of all the specimens tested are shown in Table 4-4. The tests performed were grouped based on the average strain rate obtained (rounded off). It could be observed that, during the initial phase of loading, the specimens were subjected to a lower strain rate (corresponding to the rise phase of the pulse). The strain rate mentioned below is the strain rate obtained from the strain time plot for the specimens during the period corresponding up to the maximum loading from the slope of the strain-time plot. The stiffness and strength of the specimens tested at high strain rates were found to be 85-95 percent and 45-55 percent lower than the quasi-static compression values published in the literature respectively [187, 197]. The strength of the composites obtained was comparable to the quasi-static data discussed in the earlier sections, and hence do not show any influence of strain rate (Table 4-1 and Table 4-5). The stiffness obtained was much lower compared to quasi-static data. The lower stiffness was mainly due to the non-uniform loading of the longitudinal specimen.

Table 4-4 Details of specimens and its evaluated properties from longitudinal compression testing with SHPB

| Strain Rate ( $s^{-1}$ ) | Specimen ID | Dimensions (mm) |       |           | Evaluated Properties from the plots |                |                                |
|--------------------------|-------------|-----------------|-------|-----------|-------------------------------------|----------------|--------------------------------|
|                          |             | Length          | Width | Thickness | Stiffness (GPa)                     | Strength (MPa) | Actual Strain Rate( $s^{-1}$ ) |
| 2800                     | SP1         | 6.21            | 3.93  | 4.93      | 22.12                               | 816            | 2667                           |
|                          | SP2         | 6.22            | 3.87  | 4.92      | 17.35                               | 855            | 3233                           |
|                          | SP3         | 6.22            | 3.95  | 4.89      | 14.98                               | 842            | 2900                           |
|                          | SP4         | 6.35            | 3.84  | 4.81      | 18.11                               | 734            | 2674                           |
|                          | SP6         | 6.39            | 3.86  | 4.7       | 19.23                               | 772            | 2646                           |
| 4500                     | SP3         | 6.23            | 3.9   | 4.86      | 16.35                               | 740            | 4372                           |
|                          | SP4         | 6.21            | 3.94  | 4.9       | 12                                  | 820            | 4383                           |
|                          | SP5         | 6.21            | 3.94  | 4.9       | 12.3                                | 867            | 4466                           |
|                          | SP6         | 6.21            | 3.76  | 4.93      | 9.58                                | 829            | 4455                           |
|                          | SP7         | 6.18            | 3.91  | 4.79      | 9.31                                | 652            | 4909                           |
|                          | SP8         | 6.41            | 3.91  | 4.63      | 10.45                               | 824            | 4450                           |
| 5000                     | SP1         | 6.37            | 3.9   | 4.81      | 9.17                                | 891            | 5016                           |
|                          | SP2         | 6.07            | 3.97  | 4.67      | 7.63                                | 609            | 5003                           |
|                          | SP3         | 6.35            | 3.82  | 4.81      | 10.77                               | 836            | 4898                           |
|                          | SP4         | 6.2             | 3.94  | 4.94      | 10.12                               | 766            | 4930                           |
|                          | SP5         | 6.18            | 3.8   | 4.84      | 10.06                               | 836            | 5100                           |
|                          | SP6         | 6.33            | 3.87  | 4.8       | 12.35                               | 828            | 4786                           |
| 5900                     | SP1         | 6.4             | 3.85  | 4.63      | 11.7                                | 735            | 5299                           |
|                          | SP2         | 6.35            | 3.95  | 4.75      | 9.9                                 | 723            | 5369                           |
|                          | SP3         | 6.37            | 3.83  | 4.75      | 10.55                               | 746            | 5473                           |
|                          | SP4         | 6.37            | 3.83  | 4.75      | 9.51                                | 795            | 5550                           |
|                          | SP5         | 6.3             | 3.78  | 4.84      | 8.04                                | 816            | 6870                           |
|                          | SP6         | 6.2             | 3.79  | 4.91      | 8.87                                | 851            | 6850                           |

Table 4-5 Mean ( $\mu$ ) and standard deviation ( $\sigma$ ) of evaluated properties

| Strain Rate ( $s^{-1}$ ) | Stiffness (GPa) |          | Strength (MPa) |          |
|--------------------------|-----------------|----------|----------------|----------|
|                          | $\mu$           | $\sigma$ | $\mu$          | $\sigma$ |
| 2800                     | 18.36           | 2.62     | 804            | 50       |
| 4500                     | 11.67           | 2.6      | 789            | 79       |
| 5000                     | 10.02           | 1.58     | 794            | 99       |
| 5900                     | 9.76            | 1.28     | 778            | 51       |

#### 4.2.3.4 Transverse compressive characteristics at high strain rates

The replotted curves of transverse stress-strain characteristics for strain rates 2600, 4700, 5500 and 5700  $s^{-1}$  are shown in Fig 4-18, Fig 4-19, Fig 4-20 and Fig 4-21 respectively. The procedure followed for replotting is similar to that mentioned in Appendix 10 (page 218) for longitudinal characteristics except that the non-linear region also has been plotted in this case. The stress-strain characteristics initially processed from the bar strain data are shown in Appendix 12 (page 221). The details of the specimen tested and their properties evaluated have been tabulated in Table 4-6. The specimens that meet the dynamic stress equilibrium condition have been marked by specific vertical lines beyond which

the conditions were satisfied until failure similar to that shown for longitudinal compression characteristics. Even though the failure strain for transverse compression of the unidirectional composites are higher compared to longitudinal failure strain, the wave speed inside the UD specimen in the transverse direction is much slower than that in the longitudinal direction. Hence, only very few transverse compression specimens satisfied the dynamic stress equilibrium condition. It can be observed that the compressive strength is comparatively high compared to the quasi-static strength data. However, there is no significant difference in the strength data obtained for various strain rates. One of the limitations in the data is the low signal-to-noise ratio for the transmission bar signal. Better selection of specimen dimensions and bar materials could have resulted in improvement signal-noise ratio for these test conditions. The transverse stiffness showed increase in its stiffness compared to the quasi-static for  $2600\text{s}^{-1}$ , but show a decreasing trend for strain rates of 4700, 5500 and  $5700\text{s}^{-1}$ . It can be observed that there is large variability in the transverse stiffness data for these strain rates. This difference could be attributed to the higher void content present in the laminate and variability of consolidation that can exist in a small laminate. The stress-strain curves obtained for a strain rate of  $2600\text{s}^{-1}$  showed consistency in the strength and stiffness data. However, two specific ranges of stiffness could be observed for strain rates of  $4700\text{s}^{-1}$ . This inconsistency is more likely to be related to the quality of the laminate. Similar inconsistent characteristics curves could also be observed for strain rates of 5500 and  $5700\text{s}^{-1}$ . The average and standard deviation for the high strain rate transverse compression data is given in Table 4-7 . The failure mode after the tests showed complete disintegration of the specimen.

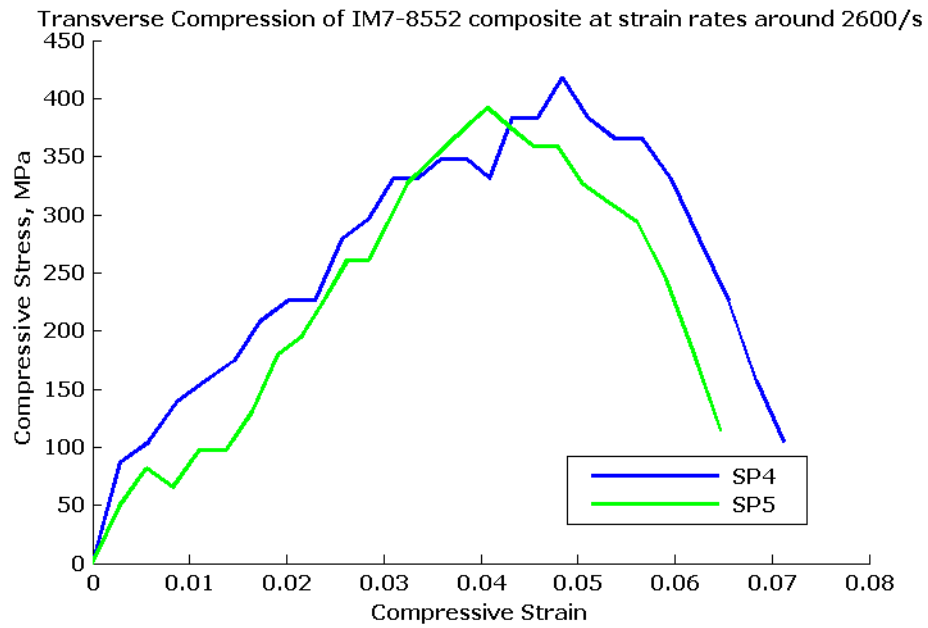


Fig 4-18 Transverse compressive characteristics of IM7-8552 composite laminates around  $2600 \text{ s}^{-1}$

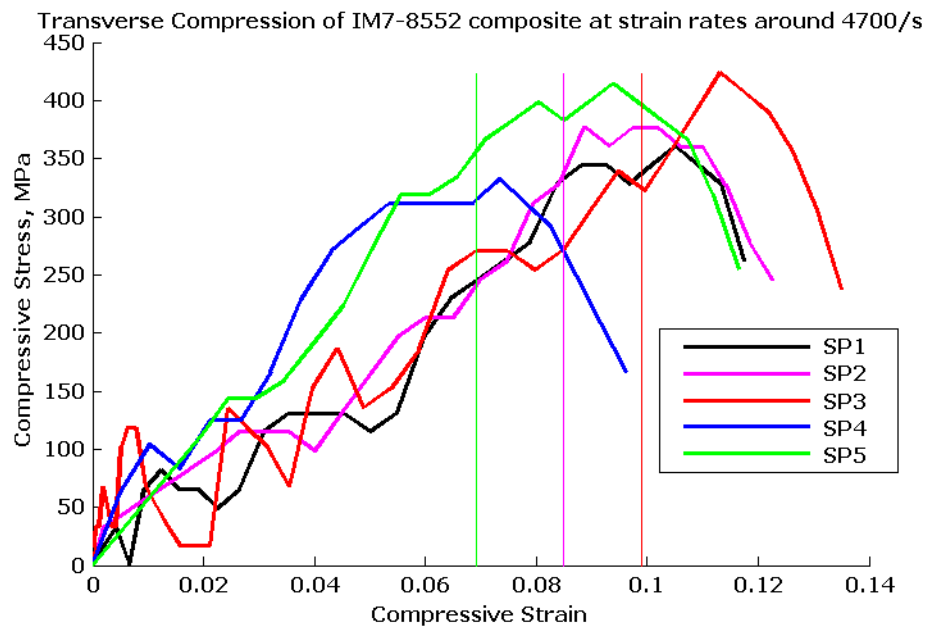


Fig 4-19 Transverse compressive characteristics of IM7-8552 composite laminates around  $4700 \text{ s}^{-1}$  showing the limits of stress equilibrium for specimens SP2, SP3 and SP5

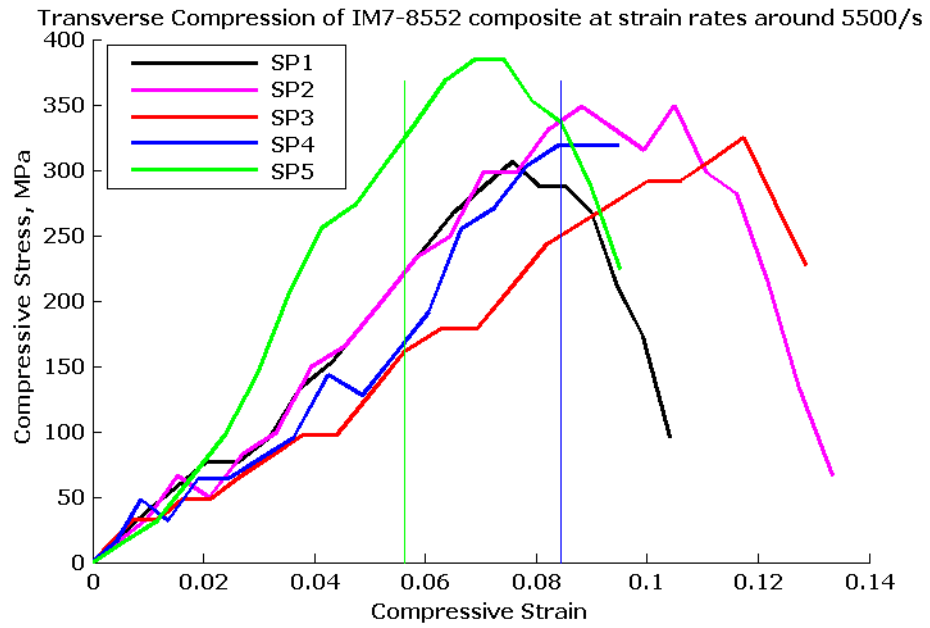


Fig 4-20 Transverse compressive characteristics of IM7-8552 composite laminates around  $5500 \text{ s}^{-1}$  showing the limits of stress equilibrium for specimens SP4 and SP5

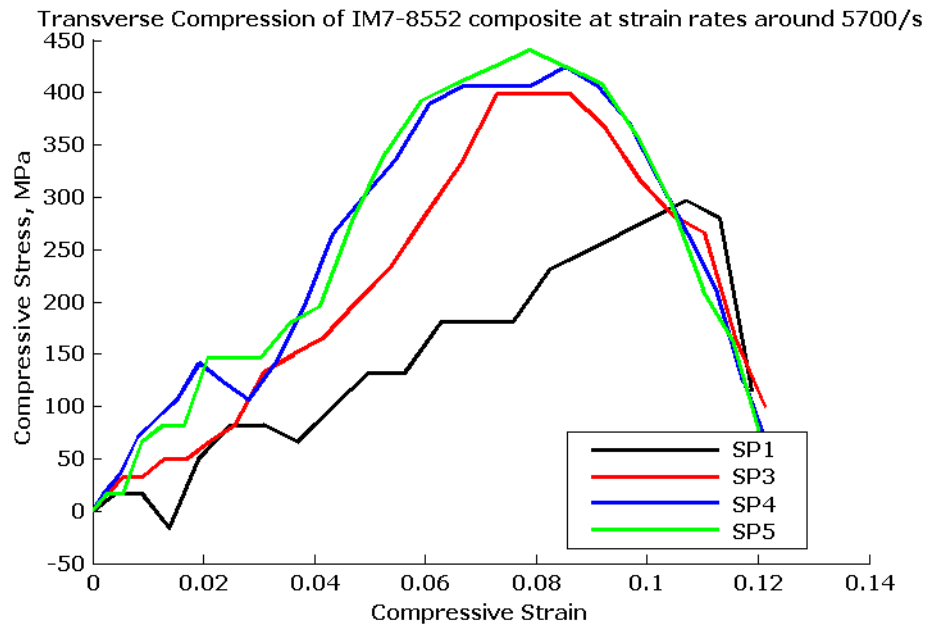


Fig 4-21 Transverse compressive characteristics of IM7-8552 composite laminates around  $5700 \text{ s}^{-1}$

Table 4-6 Details of specimens and its evaluated properties subjected to transverse compression testing using SHPB

| Strain Rate ( $s^{-1}$ ) | Specimen ID | Dimensions (mm) |       |           | Evaluated Properties from the plots |                |                                 |
|--------------------------|-------------|-----------------|-------|-----------|-------------------------------------|----------------|---------------------------------|
|                          |             | Length          | Width | Thickness | Stiffness (GPa)                     | Strength (MPa) | Actual Strain Rate ( $s^{-1}$ ) |
| 2600                     | SP4         | 6.24            | 5.18  | 5.01      | 9.05                                | 418            | 2687                            |
|                          | SP5         | 6.53            | 5.41  | 5.12      | 8.96                                | 391            | 2441                            |
| 4700                     | SP1         | 6.49            | 5.11  | 5.4       | 3.43                                | 360            | 4502                            |
|                          | SP2         | 6.46            | 5.41  | 5.1       | 3.75                                | 377            | 4490                            |
|                          | SP3         | 6.54            | 5.41  | 4.93      | 3.18                                | 424            | 4661                            |
|                          | SP4         | 6.05            | 4.34  | 5.01      | 5.56                                | 333            | 4972                            |
|                          | SP5         | 6.47            | 5.54  | 5.12      | 5.22                                | 414            | 4742                            |
| 5500                     | SP1         | 6.44            | 4.76  | 4.96      | 4.09                                | 306            | 5075                            |
|                          | SP2         | 6.23            | 5.34  | 5.1       | 4.14                                | 349            | 5801                            |
|                          | SP3         | 6.07            | 5.54  | 5.03      | 2.89                                | 325            | 5862                            |
|                          | SP4         | 6.11            | 5.56  | 5.1       | 3.62                                | 349            | 5526                            |
|                          | SP5         | 6.54            | 5.53  | 5.1       | 6.3                                 | 385            | 5449                            |
| 5700                     | SP1         | 6.54            | 5.45  | 5.04      | 2.85                                | 296            | 6157                            |
|                          | SP3         | 6.25            | 5.37  | 5.07      | 5.05                                | 399            | 5631                            |
|                          | SP4         | 6.5             | 5.11  | 5.01      | 5.86                                | 406            | 5323                            |
|                          | SP5         | 6.13            | 5.42  | 5.11      | 6.21                                | 441            | 5758                            |

Table 4-7 Mean ( $\mu$ ) and standard deviation ( $\sigma$ ) of evaluated transverse compressive properties for high strain rates

| Strain Rate ( $s^{-1}$ ) | Stiffness (GPa) |          | Strength (MPa) |          |
|--------------------------|-----------------|----------|----------------|----------|
|                          | $\mu$           | $\sigma$ | $\mu$          | $\sigma$ |
| 2800                     | 9.01            | 0.06     | 405            | 19       |
| 4500                     | 4.23            | 1.09     | 382            | 38       |
| 5000                     | 4.21            | 1.27     | 343            | 30       |
| 5900                     | 4.99            | 1.51     | 386            | 62       |

#### 4.2.3.5 Cross ply compressive characteristics at high strain rates

Cross ply specimens tested at high strain rates showed stress-strain curves and deformation characteristics similar to unidirectional specimens (Fig 4-22 and Fig 4-23). However two different families of curves could be observed in the stress-strain curves shown. This can be related to the inconsistency in the consolidation levels observed in the laminate. The data are only available for one strain rate as the stress-strain data for others could not be extracted due to noise in the transmission bar signal. The strength values obtained during the tests were comparable to the strength obtained for quasi-static testing for cross ply specimens. The stiffness values were higher compared to the quasi-static data obtained (Table 4-8). However, value of stiffness observed for the initial loading zones shows comparable ('SP3' and 'SP4') or even higher ('SP2' and 'SP6') stiffness values that would be expected for this kind of cross ply laminate of this

configuration using classical laminate theory. It should be noted that none of the specimens achieved a state of dynamic stress equilibrium during testing (Fig 4-22). The striker was fired with a pressure of 6.9 bar to achieve such higher strain rates during testing.

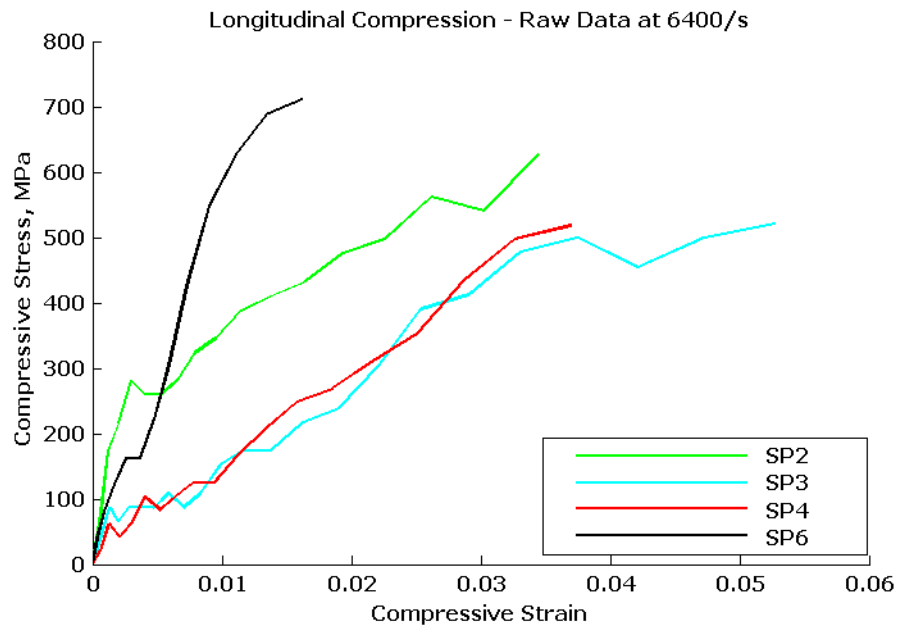


Fig 4-22 Cross ply compressive characteristics of IM7-8552 composite laminates around  $6400 \text{ s}^{-1}$

Table 4-8 Details of specimens and its evaluated properties subjected to cross ply compression testing using SHPB

| Strain Rate ( $\text{s}^{-1}$ ) | Specimen ID | Dimensions (mm) |       |           | Evaluated Properties |                |                                         |
|---------------------------------|-------------|-----------------|-------|-----------|----------------------|----------------|-----------------------------------------|
|                                 |             | Length          | Width | Thickness | Stiffness (GPa)      | Strength (MPa) | Longitudinal Compression Strength (MPa) |
| 6400                            | SP2         | 5.96            | 4.07  | 5.14      | 22.551               | 626            | 1166                                    |
|                                 | SP3         | 5.98            | 4.07  | 5.12      | 14.048               | 521            | 971                                     |
|                                 | SP4         | 5.93            | 4.22  | 5.17      | 14.723               | 518            | 965                                     |
|                                 | SP6         | 5.88            | 4.46  | 4.99      | 50.984               | 711            | 1324                                    |

The lamina level delamination buckling could be clearly observed in the images captured through high speed camera (Fig 4-23) for cross ply specimens tested at lower pressure (2.2 bar). These images were captured at frame rate of 250,000 fps with Shimadzu HPV1 camera available through loan from the EPSRC Instrument Pool. The transmission bar data relating to the test corresponding to Fig 4-23 were very noisy due to misalignment of the bar. Hence, only the high speed images corresponding to the observed damage phenomenon during the testing have been shown. Ochola et al. [53] have also used an end loading

approach to evaluate the high strain rate compressive strength of UT-C300/500 cross ply laminates. The strength was marginally higher compared to that obtained for quasi-static testing. The specimens completely disintegrated during high strain rate testing, and this was also observed in the project (Fig 4-23).

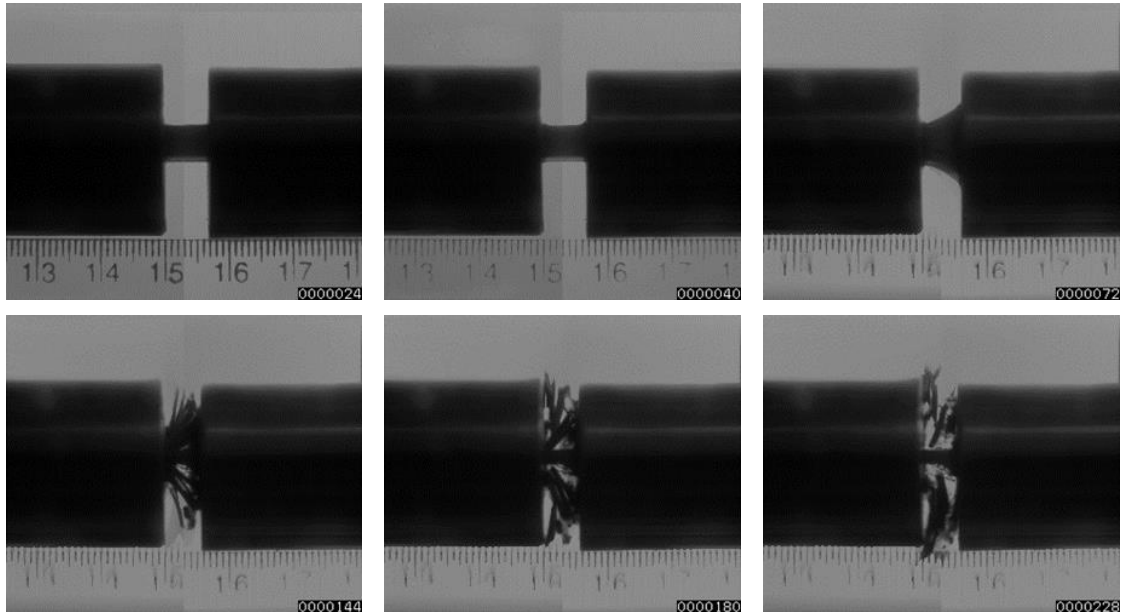


Fig 4-23 High speed images captured for cross ply laminate compression testing using SHPB

#### **4.2.4 Sources of error in high strain rate compression test data and suggestions for improvement**

*Importance of stress uniformity:* The equations used for evaluation of the stress and strain from the bar strain data follows an assumption that stress equilibrium exists at the ends of the bar [198]. A typical evaluation of stress equilibrium at specimen interfaces based on 1 wave and 2 wave comparisons for an SHPB test is shown in Fig 4-24. As mentioned previously, most of the specimens used for testing did not achieve dynamic stress equilibrium. The data obtained from SHPB tests could be improved by careful experimental design of the specimen dimensions and by employing pulse shapers to achieve uniform loading rate. Methods such as pulse shaping have been introduced to achieve stress uniformity in the specimen before the specimen fails [199, 200]. Pulse shaping techniques also tend to help in achieving uniform strain rate in the specimen during testing.

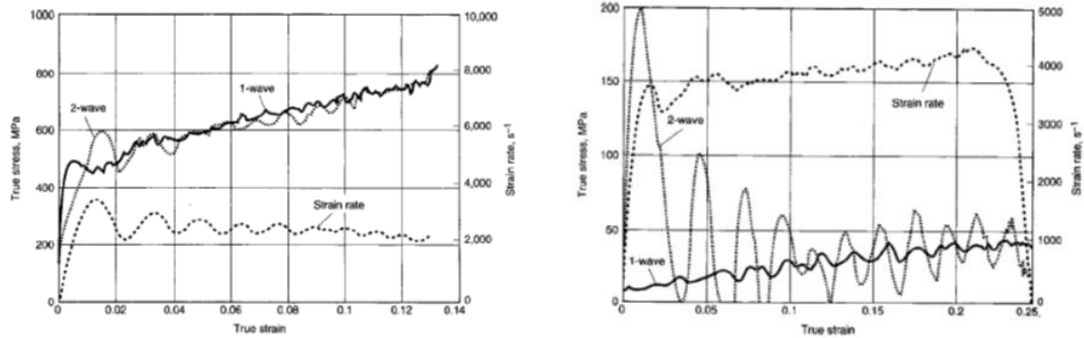


Fig 4-24 Stress equilibrium plots based on 1-wave and 2 wave analysis (as taken from [160])

**Specimen dimension effects:** Since the area of the specimen is small, it was observed that the stress pulse corresponding to the specimen loading was comparable to signal noise in the transmission bar which affected the accuracy of the measured data. Hence it is advisable to use a wider specimen, use a softer bar instead of steel bars or use semiconductor gauges instead of resistance type strain gauges for attaining a larger signal to noise ratio so as to improve the accuracy in the data.

**Inertial and frictional effects:** The interfacial friction tends to increase when the specimen size gets smaller. The friction at the specimen/bar interfaces can cause a non-uniaxial stress condition in the specimen and also can result in spuriously stiff results. The selection of the dimensions used in the study could have contributed to larger interfacial friction at the interfaces even though efforts were taken to reduce the friction through application of grease for all the tests. There will be inaccuracies in the measured stress due to longitudinal and radial inertial effects due to non-equilibrium of the stresses within the specimen. But having a shaped pulse with a larger rise time can help in achieving constant strain rate during loading can minimise the interfacial friction and thereby the inaccuracies due to radial and inertial effects.

**Wave velocity measurements:** The wave speed measurement is important as it influences the strain, strain rate and also influences the identification of datum points used for plotting the stress-strain characteristics. The bar wave velocity needs to be measured by performing series of calibration tests such that the proper incident and transmission bar signal without noise in the tail end of the incident pulse has been achieved. The noise in the tail end of the signals can be mostly attributed to the misalignment of bars and more calibration tests need to be performed with the realignment of the bars until this noise can be substantially eliminated. Calibration tests should be performed whenever the rig alignment is

disturbed before doing the actual tests on the specimens. The curves obtained during the calibration studies (Appendix 9, page 213) show significant noise and distortion in the tail end incident and transmission bar signals. The significant noise and distortion can be attributed to the misaligned bars which should have been corrected before attempting actual SHPB tests on specimens.

*Effect of distorted and bent bar:* Non-parallelism of impact interface due to the convexity or concavity of the impact surface on the incident bar can introduce severe distortion in the incident signal [175]. The non-parallelism can result in significant deviation in the magnitude of elastic strain, failure strain and stresses. Similarly, the bent bars showed large distortion in the incident signal in the author's earlier study and hence a new bar was procured which has been used for generation of the data described in this chapter. Bar straightness of 0.08mm per 305 mm is recommended and the bar surface should be perpendicular to the neutral axis within a tolerance of  $\pm 0.03^\circ$  [175].

Another more accurate method is to use the full field measurement data from a high speed camera for direct strain measurement of the specimen.

#### **4.2.5 Conclusion from UD compression testing**

The data obtained for longitudinal compression under predict the stiffness and strengths. Hence, it was decided not to use the data generated for longitudinal compression for the inverse modelling studies. The data obtained from transverse compression showed rate sensitivity in compressive strength. Different trends were observed for stiffness for different strain rates. The data can be used for inverse modelling as it was comparable with results available from literature for similar grade of carbon fibre composites. Cross ply data also showed similar results to longitudinal characteristics in the failure modes.

#### **4.3 Single fibre tensile testing**

The purpose of single fibre tensile testing is to understand the statistical variation of the tensile properties of carbon fibres at different strain rates and use the data for prediction of the probabilistic strength of composite laminates as described in Chapter 6.

#### **4.3.1 Specimen preparation and clamping procedure**

The procedure followed for single fibre testing was to bond the single fibre specimen onto a paper tab and use the clamp described in the section 3.3.3.3 (page 56) for clamping the paper tab to effectively transfer the force to the fibre during tensile testing. Paper tabs suitable for the developed clamp were cut using scissors for each fibre specimen for each test (Fig 4-25). The paper tab used is of 200 micron thickness with areal density of 160 gsm. The single fibre separated from the fibre bundle tow is bonded to paper tab with cyanoacrylate based adhesive (Elegant Touch- Brush on Nail Glue Clear). The fibre specimen is left overnight or for at least for two hours before any fibre testing is carried out (Fig 4-25). The specially developed clamp based on the clamping concept was used for clamping the fibre to the incident bar. The present clamp design requires careful handling of the fibre/paper tab while mounting the paper tab with bonded fibre on the clamps which is positioned on movable cross head (in the case of quasi-static testing) or incident bar (in the case of high strain rate testing) and the fixed cross head/load cell end. The paper tab with the bonded fibre is inserted into the gaps of the split clamp units. Once the position of the fibre and clamps are verified, the split clamp unit on either ends are fastened to respective clamp cylinder units to ensure that the paper tab is held with necessary clamp force. One of the major issues in single fibre testing was fibre failure during the clamping process. The clamping procedure was developed based on various trials and critical analysis on the cause of fibre failure during the clamping procedure for quasi-static testing. Once the paper tab is clamped, the tab is cut using a hot wire cutter.

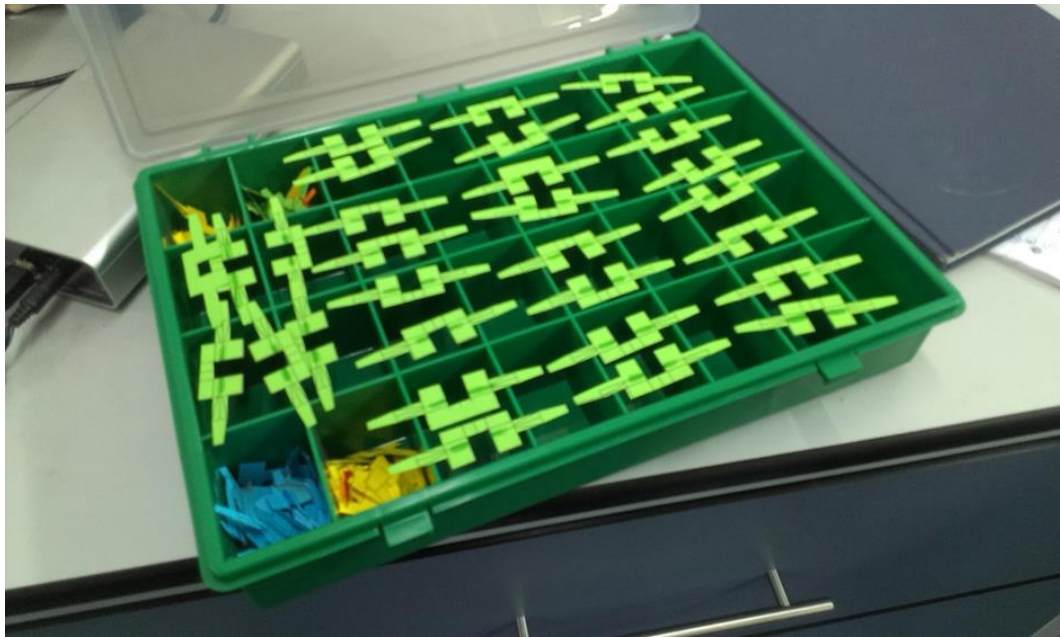


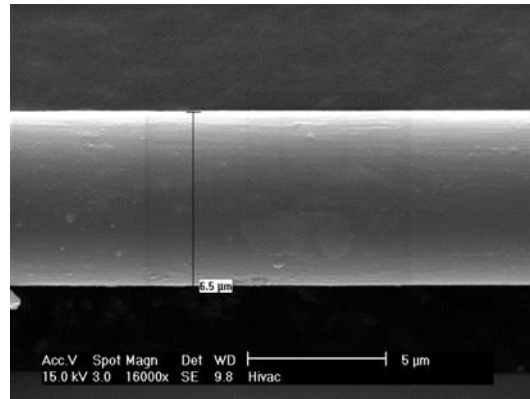
Fig 4-25 Fibre specimens glued on paper tabs prepared for single fibre testing

#### **4.3.2 Challenges in fibre diameter measurement**

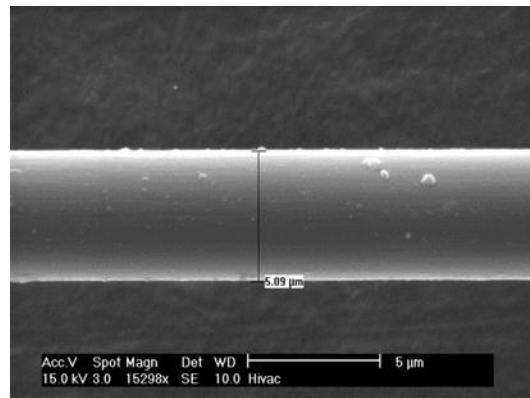
The diameter measurement using an optical microscope is limited by the diffraction effect which is a function of the wavelength of light [201, 202]. With an optical microscope, point objects would appear as fuzzy discs surrounded by diffraction rings at very high magnifications when using transmitted light. Therefore optical microscopy cannot be used for fibre diameter measurement as it cannot achieve an accuracy of 0.1 microns as wavelength of visible light is 550nm [202]. In this study, the diameter measurement of carbon fibres is carried out using laser diffraction technique or scanning electron microscopy. The fibre diameters were measured using the Dia-Stron LEX/LDS0200 system, which is based on the principle of laser diffraction [203]. This system is available in the Polymer Composites Group, The University of Nottingham. This technique performs accurate measurement (with diameter resolution of 0.01 microns) of very fine fibres such as carbon fibres [203]. However, the measurements carried out showed different values for different measurement attempts and orientations, hence demonstrating the inaccuracy of the procedure followed. The fibre mounted on paper tabs were positioned in the path of laser beam of the LDS0200 using clamps attached to height adjustable stands. The variations in the measurements were as high as 0.4-0.6 microns. Hence, the setup which was used with paper tab is inappropriate for this technique. The principle of laser diffraction technique requires that the fibre specimen is held in tension during the measurement process.

Around 3-5 specimens of each fibre type were also measured using scanning electron microscopy to verify the average diameter used for the study (Fig 4-26). The average diameter of fibre measurement for IM7 and T800 was comparable to that from the data sheet of the respective material from the suppliers [204, 205] and also with the data from Heil et al [206] who performed the study on the same batch of fibres. However, the measurements observed for T700 from this study as well as from the study performed by Heil was low compared to the data available in the data sheets [207].

T700 = 6.5  $\mu\text{m}$



IM7 = 5.09  $\mu\text{m}$



T800 = 5.2  $\mu\text{m}$

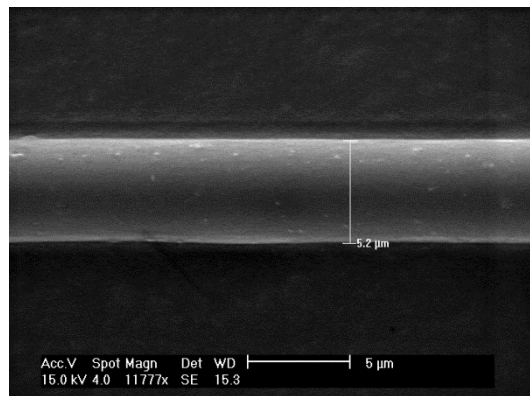


Fig 4-26 Fibre diameter measurements using SEM

### 4.3.3 Quasi-static single fibre testing

A quasi-static tensile test is performed on single filaments of carbon in order to find their axial tensile modulus, ultimate strength and failure strain, according to ASTM D3379-75 [4]. An Instron servo-mechanical UTM is used, with a 50 N load cell for carrying out this testing. Even though the load cell is of larger load rating compared to the requirement, it has a resolution of 0.00001N. The signal output from the load cell was also verified with a known calibration mass and the corresponding force was recorded consistently with a variation of only 0.0002 N. This variation is hence not expected to cause significant error in the test measurements for the single fibre test. The loading rate during testing was kept as 0.1 mm/min. For consistency, the cross-sectional area of a single fibre is calculated using the diameter specified in the respective data sheets. The clamp developed for high strain rate testing was used for quasi-static single fibre tests to assess its function and the issues involved in its design (Fig 4-27). The quasi-static single fibre testing was performed on three different grades of carbon fibres namely T700, T800 and IM7 so as to get the reference strength and stiffness value with respect to the clamp design for each of the above type for comparison with high strain rate. Around 25-30 samples were tested at each gauge length in order to evaluate the strength distribution.

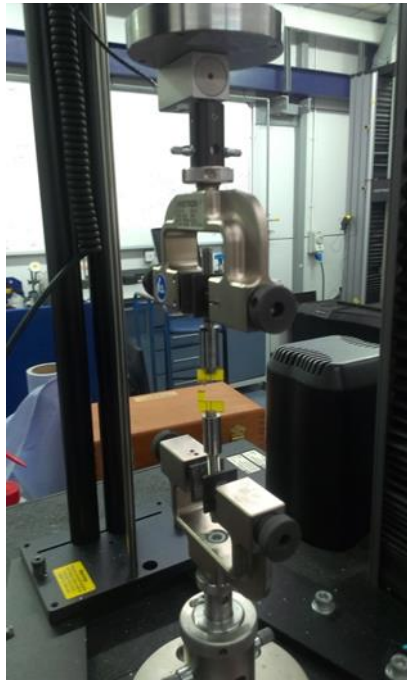


Fig 4-27 Quasi-static single fibre testing with specially designed clamps

The strength of each type of fibres was evaluated for different gauge length is shown in Table 4-9. The average strength of 5mm gauge length showed the highest strength compared to the larger gauge length for each of three fibre types. However, the average tensile strength of T700 and T800 fibres corresponding to the gauge length 10mm and 15mm shows similar range of strength values.

Table 4-9 Strength observed for different types of carbon fibres for quasi-static tensile testing as given by their mean ( $\mu$ ) and standard deviation ( $\sigma$ )

|                | Fibre Type | Gauge Length (mm)       |                         |                         |
|----------------|------------|-------------------------|-------------------------|-------------------------|
|                |            | 5                       | 10                      | 15                      |
| Strength (GPa) | IM7        | $\mu=4.79, \sigma=0.88$ | $\mu=4.45, \sigma=1.14$ | $\mu=4.05, \sigma=0.7$  |
|                | T800       | $\mu=5.81, \sigma=1.5$  | $\mu=5.34, \sigma=1.03$ | $\mu=5.42, \sigma=1.14$ |
|                | T700       | $\mu=3.66, \sigma=0.67$ | $\mu=3.29, \sigma=0.49$ | $\mu=3.29, \sigma=0.7$  |

To understand the cause of such a discrepancy in the distribution data, Weibull analysis was performed on the strength distribution data available for each carbon fibre type. The strength distributions for each gauge length for each of the three fibre types are shown in Appendix 14 (page 224). The strength distribution of the fibre types were assumed to follow a two parameter Weibull distribution where the probability of failure is defined by the equation below.

$$F(\sigma) = 1 - e^{-\left(\frac{\sigma}{\sigma_0}\right)^m} \quad \text{Eq. 4-2}$$

where,  $m$  is the Weibull modulus, commonly known as the shape parameter and  $\sigma_0$  is the characteristics strength, commonly known as the scale parameter of the Weibull distribution. The characteristic strength ( $\sigma_0$ ) is strength value,  $\sigma$ , at which  $F(\sigma)$  is equal to 0.632. To obtain the value of  $F(\sigma)$  for a given strength distribution, strength values obtained were arranged in ascending order,  $i=1,2,\dots, n$ , where  $n$  is the total of test specimens and  $i$  is the  $i$ th datum. Thereafter the ranked probabilities of failure were assigned to each datum value of strength based on Benard's approximation [208] median rank formula which is as given in Eq. 4-3.

$$F(\sigma) = \frac{(i - 0.3)}{(n + 0.4)} \quad \text{Eq. 4-3}$$

Weibull plots were generated for each of the fibre types for gauge length studied. For each gauge length, the best fit line was plotted, based on the method of least squares approximation. The slope of the fitted line gives the value of  $m$  for a given fibre type. The value of  $m$  is an indication of the distribution of flaws along

the fibre. Hence a material with homogeneous quality can be expected to have the same value of  $m$  irrespective of the gauge length [209]. This means that for a unimodal distribution of the flaw size, the average tensile strength of the fibre increases when gauge length is decreased and vice versa.

The Weibull plot are obtained by taking the double logarithm of Eq. 4-2 and plotting the  $\ln(\ln(1/(1 - F(\sigma))))$  versus  $\ln(\sigma)$ . The Weibull parameters ( $m$  and  $\sigma_0$ ) are obtained from the Weibull plot from the slope and the intercept of the fitted distribution data as mentioned in Appendix 21 (page 246). The Weibull plot for the test carried out on IM7 fibres shows variability in Weibull modulus (Fig 4-28). Carbon fibres generally have consistent Weibull modulus irrespective of the precursor material. The possibility of surface defects caused due to fibre handling could be the cause for variability observed for Weibull modulus. The 5mm and 15mm gauge lengths show the same Weibull modulus whereas 10mm shows large difference compared to the other two gauge lengths. The lower Weibull modulus indicates that there are fewer flaws along the length or flaws are randomly distributed along the length thereby resulting in larger variability in fibre strength distribution.

It can be observed that all the three types of fibres followed the two parameter Weibull distribution with the exception of the IM7 and T700 fibres (Fig 4-28, Fig 4-29 & Fig 4-30) at a gauge length of 15mm. This can be seen from p-value estimated from goodness of fit test performed using Minitab 16 (Table 4-10). The p-value for 'the goodness of fit test' should be more than 0.05 for an given distribution to be acceptable for statistical analysis [210]. A statistical distribution with highest p-value is the best distribution for the given data. The main reason for such a discrepancy could be multimodal failure of the fibre such as failure induced due to the clamping procedure followed or introduction of addition flaw sites due to mishandling while bonding the fibre etc.

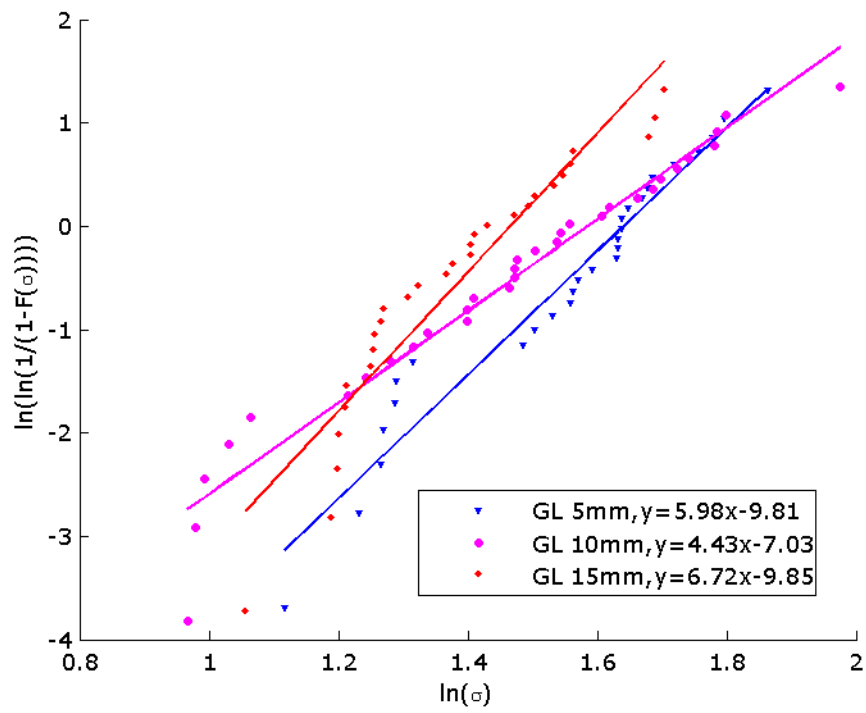


Fig 4-28 Weibull plot of quasi-static tensile distribution for different gauge length of IM7 carbon fibres

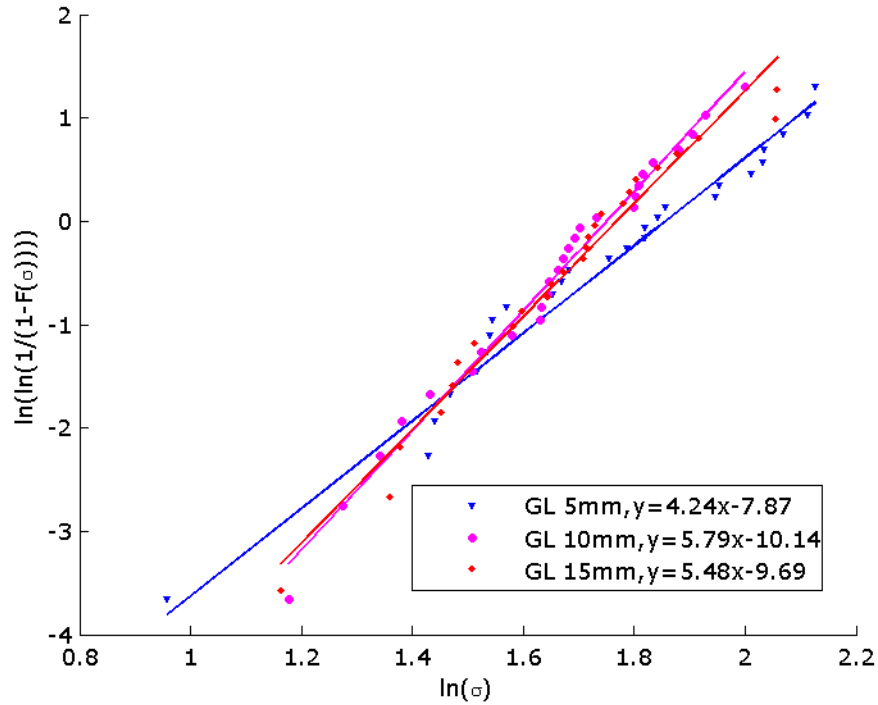


Fig 4-29 Weibull plot of quasi-static tensile distribution for different gauge length of T800 carbon fibres

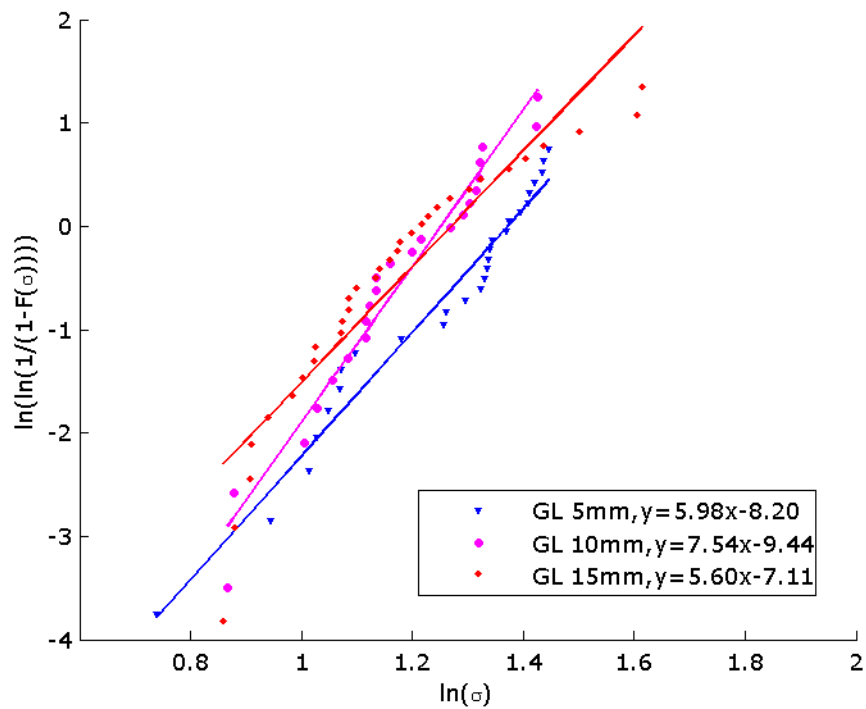


Fig 4-30 Weibull plot of quasi-static tensile distribution for different gauge length of T700 carbon fibres

Table 4-10 P-value estimated based goodness of fit analysis through Minitab

| Fibre Type | Gauge length (mm) | Weibull (p-value) | Best distribution     |         |
|------------|-------------------|-------------------|-----------------------|---------|
|            |                   |                   | Name                  | P-value |
| IM7        | 5                 | 0.233             | Weibull               | 0.233   |
|            | 10                | >0.25             | Normal                | 0.856   |
|            | 15                | 0.039             | Weibull (3-parameter) | >0.5    |
| T800       | 5                 | >0.25             | Lognormal             | 0.375   |
|            | 10                | >0.25             | Normal                | 0.851   |
|            | 15                | >0.25             | Lognormal             | 0.873   |
| T700       | 5                 | 0.208             | Weibull               | 0.028   |
|            | 10                | >0.25             | Normal                | 0.508   |
|            | 15                | <0.01             | Lognormal             | 0.529   |

#### 4.3.4 High strain rate single fibre testing

##### 4.3.4.1 Method of evaluation of strain

The conventional technique of measurement of specimen strain from the reflected strain data from the strain gauge mounted on the incident bar will not be accurate here for single fibre testing as there will be only minimal variation in the reflected wave characteristics as only small of energy is dissipated in the fibre.

[124, 126]. Hence, it would be quite difficult to distinguish the signature attributed to the specimen whose influence is comparable or smaller compared to other factors mentioned above. The direct measurement of strain through use of strain gauges mounted on the specimen or by painting of fibre surface with a pattern for direct image correlation is not possible for a single fibre. Similarly, it will be difficult to extract the specimen characteristics from the transmission bar strain data as the strain transmitted from the specimen to the transmission bar will be lower or comparable to the noise.

Several researchers [124, 126, 134] have used non-contact laser based setups for the dynamic axial strain measurement of the fibre. The principle followed was to make a focused laser beam with uniform intensity to pass through gap formed by bars/fixtures holding the fibre measurement so as to capture the movement of the incident bar which controls the amount of light detected by the photo detector placed on the other side of the incident bar opposite to laser source. The output from the photo detector is calibrated based on axial displacement which was used for evaluation of the displacement seen by the specimen. Kim et al. [131] have compared displacement of the incident bar measured using both DIC and non-contact laser techniques for different bar velocities for gauge lengths and both techniques agree well with its measurements.

The studies mentioned in the Appendix 4 (page 198) relating to evaluation of specimen failure concludes that prevailing condition for fibre during SHTB testing is quite similar to the condition of fibre testing during quasi-static loading with a universal testing machine, although the time scales involved are very much shorter. Hence, the incident bar movement alone as used by the above researchers can be used for evaluation of the strain rate/strain observed by the specimen, as the movement of the transmission bar can be assumed to be negligible. In the present study, strain measurements were evaluated from the bar movement through two different techniques as described in the following sub-sections.

#### **4.3.4.1.1 Fibre strain measurement from high speed images**

The technique measures the bar velocity through analysis of the images obtained through high speed camera. The series of images were processed using a digital image correlation code available in MATLAB under the BSD license [211, 212] to determine the bar displacement. The procedure mentioned in the documentation [211] was followed to determine the movement of the bar which was obtained

from the evaluation the spatially averaged pixel movement across the defined region of interest. The dimension corresponding to pixel movement was evaluated with respect to a reference dimension of clamp unit of the incident bar and its pixel coordinate information. The bar displacement in the defined region of interest were used for evaluation of strain and strain rate observed by the specimen.

A high speed camera (Shimadzu HPV 1) was available for use on loan through the EPSRC Instrument Pool for a period of 40 days which was used for commissioning of the SHTB rig as well as for initial fibre testing. The delay generator and flashlights were also supplied along with high speed camera. The analog voltage input from the strain gauge mounted on the SHTB is amplified through the signal conditioning amplifier and fed to the delay generator for triggering the camera and flash light. The delay timings and threshold voltage for the triggering the camera and flashlights were manually set in the delay generator supplied. A delay of 15 microseconds with a rise time of 50ns was set as a parameter in the delay generator which was used for triggering the high speed camera and the flash lights from the incident bar strain signal. The connections followed to trigger the high speed camera are shown in the block diagram in Fig 4-31.

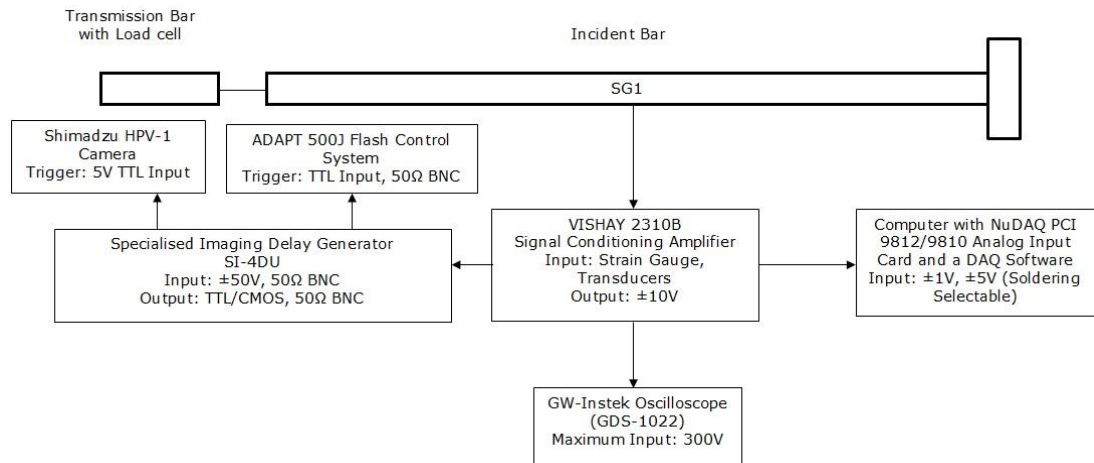


Fig 4-31 Block diagram showing connection between each units for triggering the camera

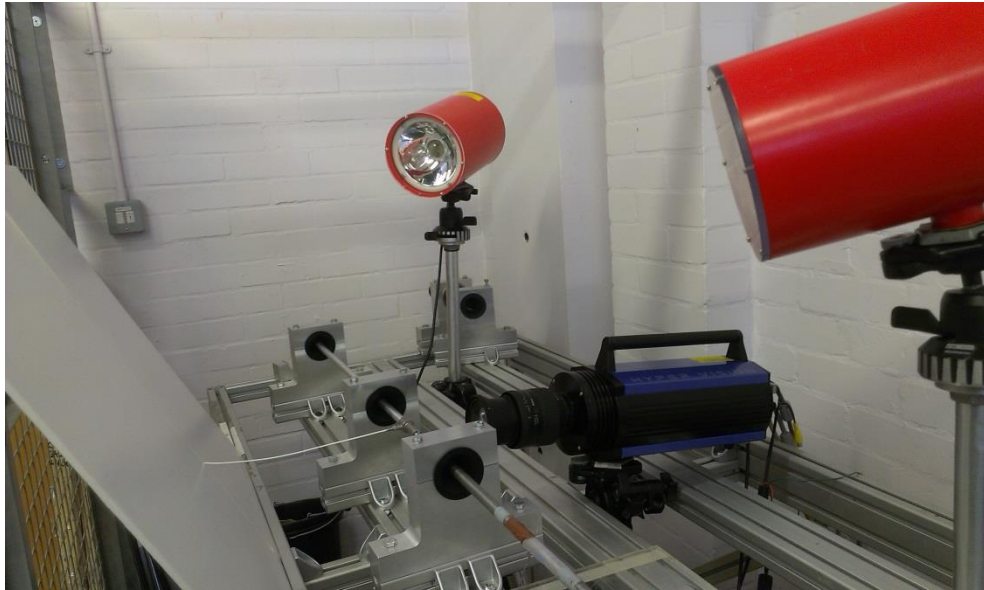


Fig 4-32 Arrangement used for capturing images during high strain rate single fibre testing

Since only 100 images can be stored using the high speed camera and SHPB events are typically in the range of 60-100 microseconds, the high speed camera was triggered along with a flash light using delay generators with different time delays for each unit so that the images of the event were captured every 4 microseconds (250,000 frames per second) through the camera in '.tif format' (TIFF). The images for each frame were stored using the software supplied with the camera. The trigger for the data acquisition card (for storing strain gauge and load cell data) and delay generator (for triggering the flash light and the camera) were based on the signal from the incident bar strain gauge. The images corresponding to the initial movement of the incident bar were identified by comparing the time of each event, the delay time and signals from the strain gauge and the load cell. The direct image measurement analysis performed on images captured from the high speed camera was used for direct strain measurement based on the movement of the clamps attached to the incident bar/movement of incident bar. The strain measurement using the direct measurement of images is quite dependent on the region of interest selected which is also related to the clarity of the images that is obtained. The clarity of the images captured during the event varied between the images as the light intensity varies during the event [213]. It was also observed that the first 5-8 images captured during an event were severely blurred and cannot be used for image processing for evaluation of bar movement. Hence, a shorter delay time was set on the delay generator (15  $\mu$ s instead of 60  $\mu$ s) to capture the event so that all the images of the actual event are available for processing. The images

obtained also showed split sections (Fig 4-33a and Fig 4-34a) which were primarily the limitation due to the CCD based sensors used in the camera which was confirmed by discussion with representatives from the camera manufacturer.

One of the challenges during high strain rate single fibre testing was that the taut fibre specimen tends to break at a lower strain rate before the high strain rate impact event occurs. This was primarily due to the fact that bar moves along with the striker due to the increased frictional contact at the base of the striker. One of the solutions was to provide slackness in the fibre to take into account the small movement before the actual impact occurs. Mostly, it was observed that the fibre either tends to fail before the main stress pulse corresponding to the impact is observed or the slackness was too large to cause the failure during the first loading pulse. Even though more than 30 fibre tests were performed for three grades of carbon fibres using the high speed camera (Shimadzu HPV1), it was observed that only one of the tests performed upon the T800 fibres showed proper fibre failure corresponding to the actual high strain rate loading.

#### **4.3.4.1.2 Fibre strain measurement from incident bar strain data**

It was described earlier that the condition during single fibre testing with SHTB can be correlated with the condition of quasi-static testing with UTM where one end is fixed and the other end is moving. Hence, to evaluate the incident bar movement at the instant of loading the fibre, the expression for the velocity at the incident bar-specimen interface (Appendix 2, page 194) has been used. The incident and reflected strain data from the strain gauge mounted on the incident bar were used to assess the bar velocity during the event which is given below (Eq. 4-4).

$$u_1 = c_0(-\varepsilon_I + \varepsilon_R) \quad \text{Eq. 4-4}$$

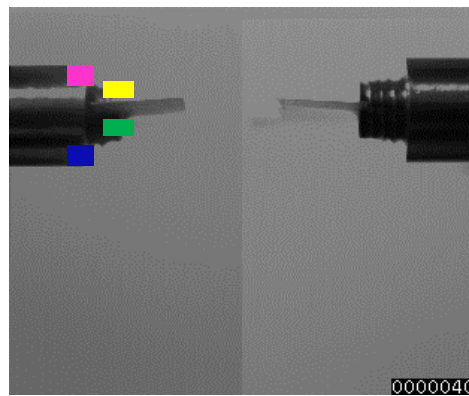
The strain rate observed by the specimen is evaluated from ratio of the incident bar velocity to the gauge length of the specimen as the transmission bar end is assumed to be stationary (Eq. 4-5).

$$\dot{\varepsilon} = \frac{u_1}{L_s} \quad \text{Eq. 4-5}$$

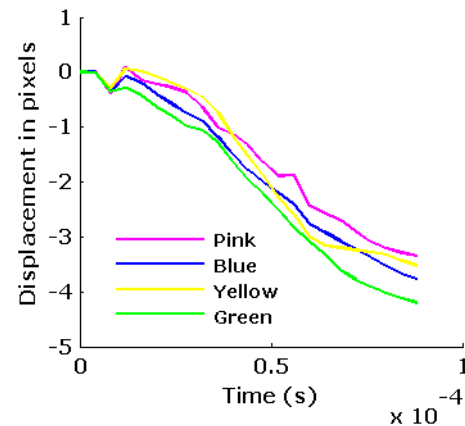
The deformation observed by the specimen is evaluated by integrating the bar velocity over the entire duration of the event. The strain observed by the fibre specimen during SHTB testing is then evaluated from the ratio of deformation to the gauge length or by integrating Eq. 4-5 for the entire duration of loading.

#### 4.3.4.1.3 Assessment of specimen end condition during high strain rate single fibre testing

The validity of using Eq. 4-5 for evaluation of specimen strain rate is based on the assumption that the transmission bar is assumed to be fixed during the loading process. The images obtained from a valid high strain rate single fibre testing performed on T800 fibre was considered for verification of the end conditions of the specimen. The movement of the bar ends were analysed for different windows at different locations of the bar ends so as to evaluate the bar displacement during the loading process. The detailed methodology followed for evaluation of the bar displacement based on the image analysis is given in Appendix 24 (page 261). At different region of interest (Fig 4-33a and Fig 4-34a), there seems to be consistency in the displacement data observed (Fig 4-33b and Fig 4-34b),. The difference in displacement (Fig 4-34b) observed for yellow window in the transmission bar (Fig 4-34a) can be due to the varying intensity of light across the images. It could be noted that the transmission bar tends to have a movement which is opposite in the direction of incident bar. The movement observed for the transmission bar could be due to the lateral movement of the support which supports the transmission bar. The movement of the incident bar is nearly 8-10 times larger than the movement observed for the transmission bar.

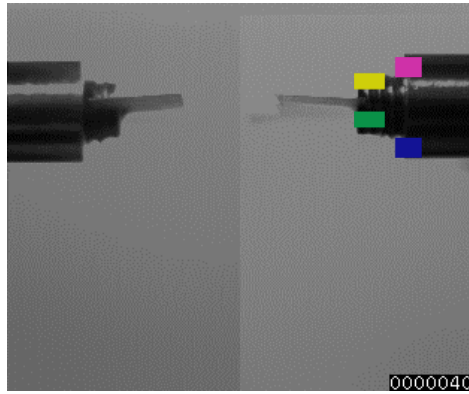


a) Region of interest defined in the incident bar at various ends

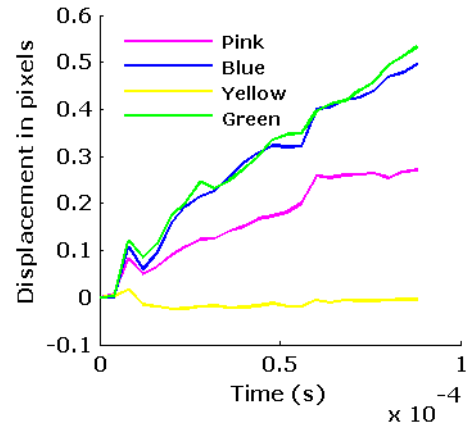


b) Displacement observed in pixels for respective colour windows

Fig 4-33 Incident bar displacement observed during high strain rate single fibre testing



a) Region of interest defined in the transmission bar at various ends



b) Displacement observed in pixels for respective colour windows

Fig 4-34 Transmission bar displacement observed during high strain rate single fibre testing

#### 4.3.4.1.4 Evaluation of the strain and comparison of the methods employed

**Comparison of the strain data analysed from image analysis:** The high strain rate data of T800 carbon fibre (specimen 1 shown in Fig 4-38) was only used for comparison of the two techniques mentioned above as it was the only case for which reliable high strain rate data for both the methods were available. The displacements in pixels were converted to displacements in mm by comparison of the pixel coordinates of known reference points in the image through MS paint. The average displacement in millimetres for all the windows was considered to evaluate the specimen strain. For the case where the evaluation of the specimen strain accounts for the movement of the transmission bar, the displacement corresponding to the yellow window (Fig 4-34) was neglected in the calculation of the average displacement of the transmission bar. The strain evaluated from the measured displacement from both the bars was higher compared to the strain evaluated from the measured displacement of the incident bar alone (Fig 4-35a). The movement observed for the transmission bar is significant as it resulted in 10-15 percent variation in the strain when compared to the case when the movement of the transmission bar was not accounted for. Hence, it would be more accurate to account for the transmission bar movement while evaluating specimen strain. The strain rates evaluated with the direct image measurement technique with and without transmission bar measurement were  $702 \text{ s}^{-1}$  and  $758 \text{ s}^{-1}$  respectively. The strain rate was evaluated from the slope fitted over the most linear region of the strain-time curve (Fig 4-35a).

**Comparison of strain measurement from incident bar data alone through different approaches:**

The strain evaluated from the bar displacement extracted from the measurements from the incident bar strain gauge data was lower compared to the strain evaluated from the incident bar displacement alone through image analysis (Fig 4-35b). The strain rates observed by the specimen measured through image analysis was 35 percent compared to that measured from bar strain data ( $518 \text{ s}^{-1}$ ). It could be observed that the constant strain rate was achieved during the high strain rate loading of the fibre specimen as could be observed in strain-time curves (Fig 4-35).

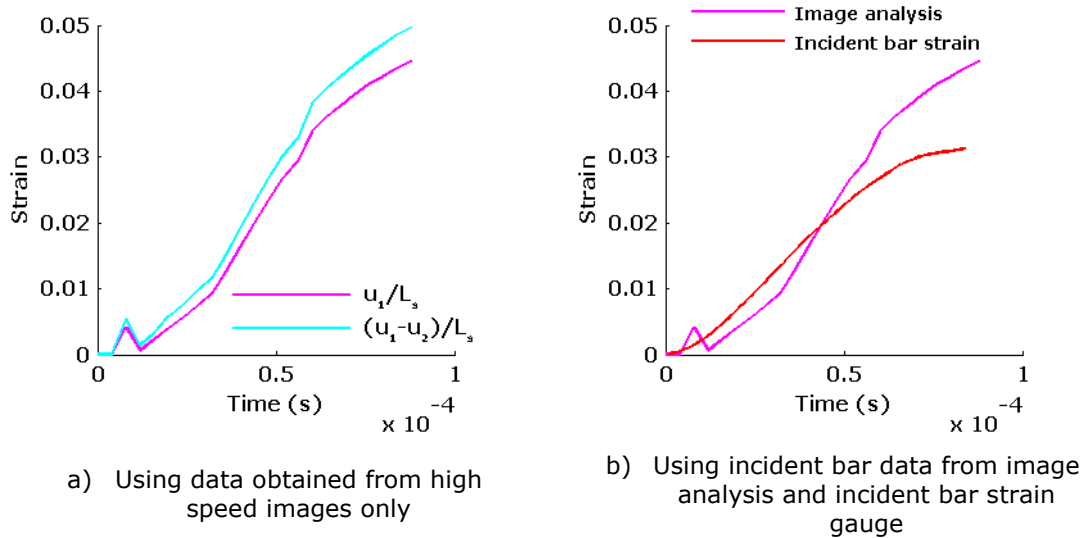


Fig 4-35 Strain comparison for data evaluated from different approaches

**Plot of stress-strain characteristics for the strain data obtained from different approaches:**

The load and incident bar strain data versus time obtained during single fibre testing is shown in Fig 4-36. The load cell data are captured directly to the oscilloscope (without amplification) from the power coupler of the piezoelectric load cell and the strain data shown are obtained after amplification via the signal conditioning amplifier (Gain=1000). Both the sets of data shown were also simultaneously captured through ADLINK DAQ card (Gain=5 (Appendix 8, page 208)) at a sampling rate of 1Ms/s so that the complete information of the test are available for data processing and evaluation.

The stress observed by the specimen was evaluated by dividing the load by the area evaluated from the mean assumed diameter of the specimen in the same way as for the quasi-static data. The stress-strain characteristics obtained for each methodology followed for strain measurement is as shown in Fig 4-37. The

percentage variation of failure strain measured through image analysis was around 42 percent higher compared to that evaluated through the bar strain data for the case where the displacement of the incident bar only was used (Fig 4-37b). This highlights the fact that the fibre strain measurement from the bar strain data or from image analysis may not be accurate. The bar strain data are significantly affected by the electrical noise or disturbances due to misalignment etc whereas the strain evaluation from image analysis is significantly influenced by the intensity of the light available during the capturing of the images. Hence, monitoring the bar displacement using another technique such as laser measurement could be useful to assess the accuracy of the above methods used.

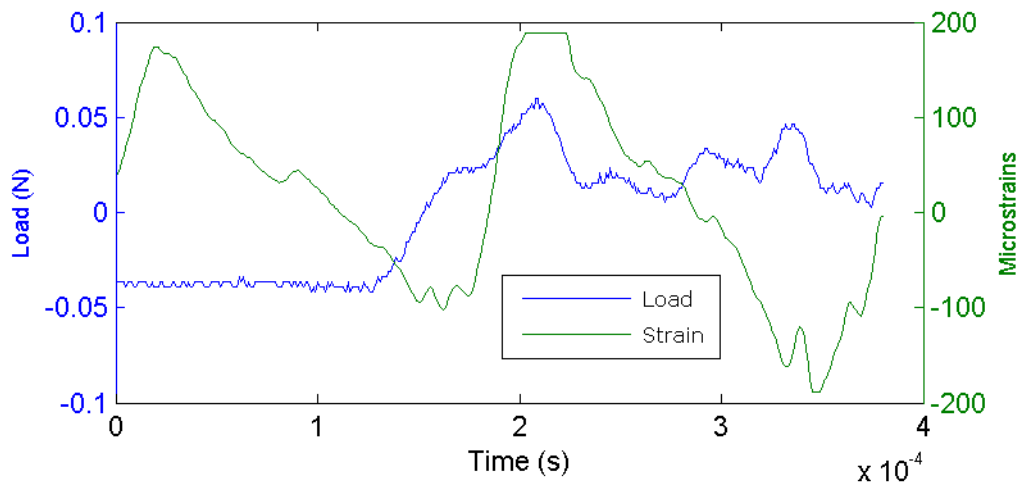
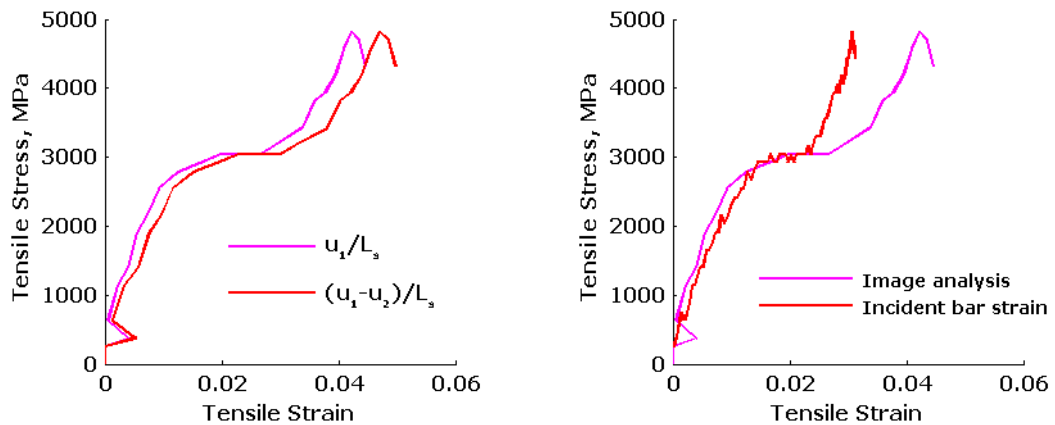


Fig 4-36 Load and strain data captured for high strain rate single fibre testing performed along with high speed camera



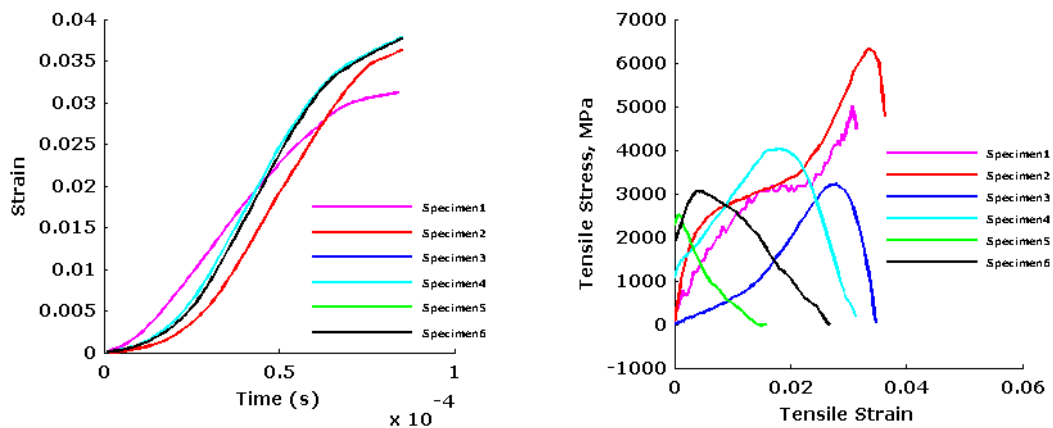
a) When the strain was evaluated from bar displacement data obtained from high speed images only (using incident and transmission displacement data)

b) When the strain was evaluated from incident bar data from image analysis and incident bar strain gauge

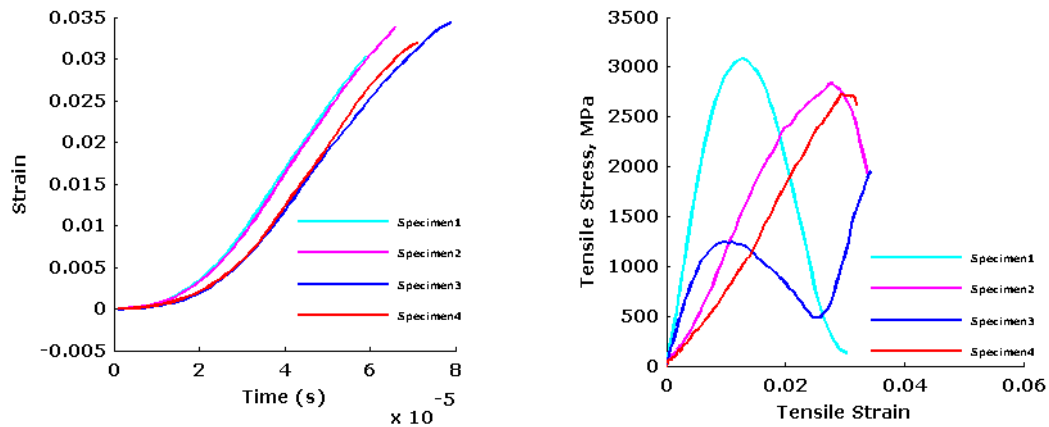
Fig 4-37 Stress-strain comparison for data evaluated from different approaches

#### 4.3.4.2 Results and discussion on other high strain rate single fibre test data

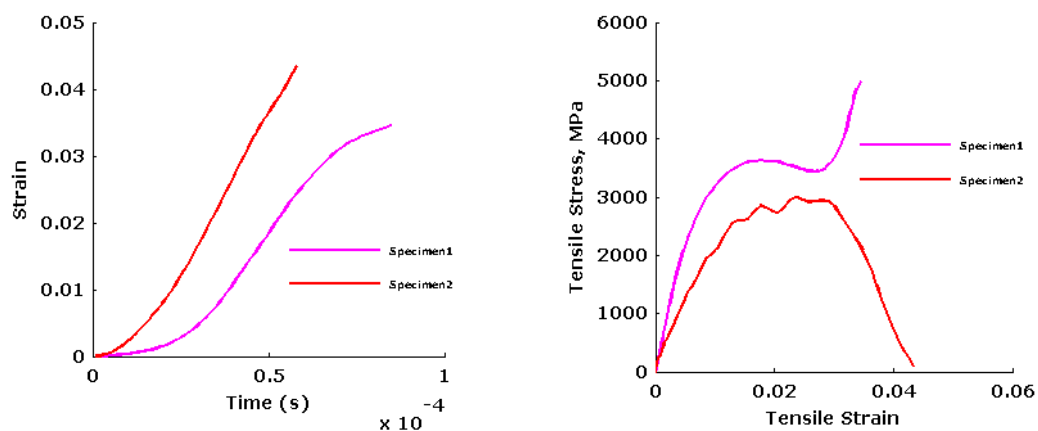
The respective load and strain data obtained for each specimen are shown in Appendix 23 (page 257). The mean diameter used for quasi-static studies for each of the three fibre types was used for evaluating the fibre stress observed during the high strain rate loading. The strain observed in the specimen were evaluated from the displacement of the bar through the strain data obtained from the incident bar strain gauge and the gauge length of the specimen used. The approach to use fibre strain measurement from the bar strain data was followed only due to the non-availability of the high speed camera. Significant variability could be observed in the strength and stiffness values obtained for T800 carbon fibres (Fig 4-38). The same trend is also observed for strength values of T700 and IM7 (Fig 4-39 & Fig 4-40). The data shown for stress could be inaccurate due to both the inaccuracies in the measurement of diameter as well as due to the fact that load cell is 34mm away from the fibre/transmission bar interface. It can be observed that constant strain rate was achieved during the high strain rate loading of all the fibre specimens. Strain rates around  $650\text{s}^{-1}$  were observed for almost all the specimens. The strain rate corresponding to Specimen 2 of T700 fibres (Fig 4-40)) was high, as it was considered to have failed during the second pulse. The failure strain shows increased rate sensitivity (more than 2%) for almost all the fibres [204, 205, 207]. The data obtained are insufficient to make a judgement on the rate sensitive characteristics.



a) Strain-time plots  
b) Stress-strain plots  
Fig 4-38 High strain rate characteristics of T800 carbon fibres



a) Strain-time plots  
b) Stress-strain plots  
Fig 4-39 High strain rate characteristics of IM7 carbon fibres



a) Strain-time plots  
b) Stress-strain plots  
Fig 4-40 High strain rate characteristics of T700 carbon fibres

The strain rates can be further increased without needing to change the spring compression by changing the 12mm section of the bar to Aluminium 7075-T6. Also, higher strain rate testing could be performed to obtain data for evaluation of the rate sensitive characteristics at higher strain rates. Further testing could not be taken up to assess the rate sensitivity of the fibres since the rig was moved to a new location in order to allow expansion of a teaching facility. Hence, only limited high strain rate data were generated as important time was lost due to the relocation of the rig and there was a substantial amount of time that was spent in addressing the commissioning issues related to the rig.

#### 4.3.4.3 Errors/Concerns associated with high strain rate single fibre testing

##### 1) Effect of impedance mismatch between bar and the specimen:

When the impedance mismatch is higher, there will be large reflection of the incident signal at the incident bar-specimen interface. Hence, only

direct measurement approaches of the strain is valid as it will be difficult to infer usual information relating to specimen from the reflected characteristics. The purpose of using the larger diameter rigid bar is to have provision for clamping the brittle fibre and also to provide the necessary boundary condition for loading the specimen at high rates. There can be premature failure in the loading end due to non-equilibrium of the stresses within the specimen. But having a shaped pulse with a larger rise time can help in achieving constant strain rate during loading can minimise the interfacial friction and thereby the inaccuracies due to radial and inertial effects. The single fibre specimens tend to have achieved constant strain rate after 15-20 $\mu$ s of the loading. It will also be a challenge to ensure uni-axial stress condition in the specimen during the loading due to the magnitude of the load that is taken by the specimen and the effect of other sources of errors in disturbing the uni-axial stress condition.

2) Effect of gauge length of the specimen

The gauge length of the specimen used in this study was 10mm. This can be considered to be long, if the incident loading pulse has a very short rise time which result in non-uniform high stress loading of the specimen at the loading end which can result in premature failure. However, the loading pulse achieved in this study has a larger rise time (35-45 $\mu$ s) compared to less than 5  $\mu$ s for typical rectangular pulses. The longer gauge length was only used for generating data for comparison with quasi-static data. The shorter gauge length could also be attempted with the clamping procedure developed.

3) Effect of load cell position and inaccuracy associated relating to it on load measurements: The fibre is attached to a clamp unit which is 34mm long to which the Kistler piezoelectric load cell was attached for measurement of the load. Since the stress pulse transmitted to the transmission bar is of low magnitude and maximum load a carbon fibre may take is in the range of 0.1 N, it would have been ideal if the fibre had been attached very close to the load cell unit as discussed in section 3.5.2 (page 73).

4) Method of fibre strain measurements: When the failure strain of the specimen is smaller, it will be difficult to accurately evaluate the strain observed by the specimen through image analysis or through the strain data from the incident bar strain gauges due to errors associated with the approaches. It would be useful to have other direct measurement

approaches such as non-contact laser technique for measuring the bar movement and thereby the strain observed by the specimen.

#### **4.4 Conclusions**

The work reported in this chapter was required to generate extensive data relating to high strain rate characteristics of composites and its constituents. However, it could be seen that there are shortcomings in the data generated for longitudinal compression because of premature failure of the composite caused primarily as a result of the method of loading employed. Sufficient high strain rate data were generated relating to transverse compression of unidirectional composites (IM7-8552) at high strain rates. The validity of the high strain rate data could be further improved by careful experimental design of the specimen so as to achieve dynamic stress equilibrium during the high strain rate loading of the specimen.

Sufficient data were generated for the quasi-static single fibre tensile testing for T700, T800 and IM7 fibres. The strength distribution observed for each of the three fibre types typically followed a Weibull distribution (except for one gauge length for T700 and IM7 fibres). The clamps developed for high strain rate testing were used to generate the quasi-static data and did not have any influence on the strength distribution characteristics.

The method of strain measurement for high strain rate single fibre testing from the displacement of the bar using the incident bar strain data is slightly different from that obtained through processing of images obtained from the high speed camera through image analysis. The stress evaluated has inaccuracies due to the inaccuracies in the load measurement as well as due to the assumption of a mean diameter for all the fibres. There is insufficiency in data to make an assessment on the rate sensitivity of the fibres.

The data obtained from transverse compression of unidirectional composite were used for the inverse estimation of the transverse properties of fibres in chapter 5 and the tensile data have directly been incorporated in the Monte Carlo simulation for prediction of the probabilistic strength of composite materials at different strain rates described in chapter 6.

## **5 Inverse modelling for estimation of high strain rate properties of composite constituents**

### **5.1 Introduction**

The micro-scale models developed by Lidgett [1] for prediction of lamina properties involved the use of user defined material models which required elastic and strength properties of the constituents to be input into the model for the prediction of the lamina characteristics. Even though there have been attempts by researchers to predict the transverse mechanical characteristics of single fibres at quasi-static rates [214], the experimental evaluation of shear and compressive properties of the fibres has so far been practically unachievable at high strain rates due to the smaller dimensions involved. This chapter describes the development of inverse modelling for estimation of the constituent properties of composite materials at high strain rates. The approach followed is to use the test data of UD composites at high strain rates to generate the respective high strain rate data of the constituent materials by inverse modelling using Lidgett's micro-scale finite element model as the forward problem. The procedure for inverse modelling was written in Python scripts which call an optimisation solver that interfaces with Abaqus/Explicit for solution of the forward problem. Data from high strain rate testing carried out for evaluation of the compressive properties of unidirectional composites described in the previous chapter, were used as inputs to the inverse problems described in this chapter. In some cases, where the data are missing or unreliable, data available from the literature have been used for demonstration of the concept.

This chapter gives an overview of inverse problems and reviews literature related to material parameter estimation using inverse methods involving numerical optimization and finite element modelling. It then describes the strategy followed for the inverse problem such as a parametric study to identify the most sensitive parameters, states the methodology for evaluation of the sensitive parameters and finally discusses the constituent parameter estimation from different mechanical characteristics of composites through inverse approach.

### **5.2 Inverse modelling: an overview**

A direct/forward problem in engineering involves the determination of the response of a structure/physical system from the model parameters such as

loads, geometry or configuration of the system, material properties, boundary conditions etc [215]. The model parameters relate to the response through the governing differential equations of the system involved [216]. There can be physical systems where not all the model parameters may always be available and such problems can become good candidates for inverse methodologies to be applied, if experimental or theoretical data are available with respect to overall characteristic behaviour [215]. For example, direct evaluation of properties through experimental testing may not be always possible for determination of all material constitutive parameters. In such cases, inverse methods are one of the common approaches that have been followed to evaluate the material constitutive parameters that cannot be directly evaluated. In an inverse problem, the objective would be to determine  $n$  model parameters from a set of  $m$  measurements, where  $m \geq n$  as described in Eq. 5-1 as given below [215].

$$F(\beta) = d \quad \text{Eq. 5-1}$$

where  $F$  represents the governing equations based on the physics of the problem, relating the  $n$  model parameters to the measured data  $d$  which consists of  $m$  measurements. An objective function ( $r_i$ ) which compares a predicted quantity from a numerical model with the test data would be generated for each function call for each data point. The optimisation solver then estimates the parameter through the quadratic minimization of the non-linear least squares problem (Eq. 5-2 and Eq. 5-3), which minimises the function  $S$  which is the sum of the squares of the residuals,  $r_i$ . In Eq. 5-3,  $y_i$  is the data measurements which is dependent on the input parameters,  $x_i$  and  $\beta$  is the column vector consisting of  $n$  model parameters that is to be estimated from the inverse problem [215, 217].

$$S = \sum_{i=1}^m r_i^2 \quad \text{Eq. 5-2}$$

$$\text{where,} \quad r_i = y_i - f(x_i, \beta) \quad \text{Eq. 5-3}$$

However, one of the major issues with respect to inverse problems is that they can be ill posed problems [215, 216]. Ill-posed problems either have no solutions in the desired class, more than one solution or are unstable [218]. The non-uniqueness of the solution can be due to various reasons such as noise in the measured data, round off errors in the inverse problem calculations or imperfections in the forward problem. The instability in the solution means that the solution is highly sensitive to small changes in the measured data. In other words, small errors in the measured data which are likely to happen in a real

world experiment can cause large errors in the parameter that needs to be estimated. In certain cases, these errors can be minimised or avoided by imposing constraints on the parameter to be estimated or selective definition of the objective function in the region where the parameter to be estimated is well behaved. The process of imposing constraints on the parameters to be estimated is also known as regularisation in the field of inverse modelling [215, 216].

### **5.3 Literature survey on inverse modelling for estimating parameters**

Even though inverse problems have found application in various fields, the literature survey presented here is primarily focussed on discussing the work related to estimation of material properties/parameters relating to relevant material constitutive models.

Kajberg et al. [219] estimated the parameters of a viscoplastic material model proposed by Perzyna [220] for simulation of a high strain rate event through inverse methods by repeated comparison of the FE model results with the deformation pattern obtained from microscopic high speed photography of a high strain rate testing event. The Simplex method, which is designed to find a minimum through direct searching, was considered for inverse modelling in their study. The effect of uncertainty in the measurements was also studied as part of the sensitivity analysis through introduction of uncertainties into the force-extension data generated from the experiments. The simulation of the experiment with estimated parameters showed close agreement with the experimental results. This methodology of using experimental data and a numerical FE model for evaluation of high strain rate properties of specimens is quite encouraging as the method was used for a non-homogeneous stress state condition of the specimen during testing. Chafra et al. [221] attempted numerical optimisation to determine the mechanical characteristics of the fibre including the transverse characteristics from the experimental data and numerical homogenisation model. The numerical homogenisation model used was focussed on determination of elastic properties and stiffness coefficients. The optimisation problem has been solved using the function '*lsqnonlin*' in Matlab which is an implementation of the Gauss-Newton algorithm. However, the illustration of the method followed for estimation of transverse properties of transversely isotropic fibres is not specifically mentioned in the paper. Anghileri et al. [222] used the inverse approach based on finite element updating for estimating parameters for

a constitutive material model used with PAM-CRASH for simulation of dynamic crushing characteristics of carbon woven fabric fibre reinforced composites. The constrained optimisation algorithm based on the sequential quadratic programming method was used for the estimation of the critical strain values and the damage parameters. A multi objective optimisation approach was required as single objective optimisation could generate only topology dependent optimised parameters which undermine the purpose of inverse modelling using simulation. The multi objective optimisation based on goal attainment methods [223] showed good consistency with the experimental data. The paper demonstrated the utility of the inverse approach for estimation of damage parameters of complex material models. Markiewicz et al. [224] demonstrated the use of an inverse approach to determine the elastic-plastic and viscoelastic model parameters of the Cowper-Symonds constitutive model from data obtained from quasi-static and dynamic axial crushing of thin walled tubes. The parameters were estimated using the mathematical optimisation solver Pam-Opt. The parameters thus obtained were used for simulating the dynamic three point bending impact test using Pam-Crash and it showed good correlation with actual experiment. Hwang et al. [225] used a hybrid genetic algorithm and finite element analysis of vibration testing to inverse evaluate the elastic constants of composite materials. Chamis et al. [226] used a micromechanics based optimisation subroutine to evaluate the stiffness and strength properties of the transversely anisotropic fibre and the isotropic matrix using in-plane test data. The constituent properties were evaluated by assuming a square cross section for the fibre and also ignoring the transverse strength contribution of the fibre. The estimation of the transverse fibre and shear properties showed a large variation from the given data which could be attributed to the discrepancies in the assumptions followed for the numerical model considered.

Sztefek and Olsson [227, 228] used an inverse method for evaluation of the reduction in the tensile and compressive stiffness of the impact damaged specimens in a composite laminates. Optically measured displacement fields (from DIC) from the uniaxial tensile and compression testing of the impact damaged specimen were fed to a numerical optimiser which compared the displacements with the FE data of the analysis and performed an iterative update of the material properties to match the predicted displacement with the actual test data. The objective of the inverse method was to minimise the difference between the measured and computed in plane components of the displacement fields. An optimisation algorithm which uses the steepest descent method was

used in the solver for identification of the parameters. Variations of the error function with respect to change in moduli and Poisson's ratio were determined for gradient calculations. The material parameters of the undamaged specimen were assumed to be known a priori so that the parameters corresponding to damage could be estimated. The effect of noise in the experimental data and its sensitivity to the estimated parameters also were assessed and found to be within the error range of 1.5 percent for both the parameters. The spatial variation of constitutive parameters across the impact damaged specimen was successfully evaluated with the inverse method developed. Even though the inverse procedure was written in Fortran 90, it required multiple calls of Abaqus using Python scripts for FE evaluation of the tensile/compressive characteristics and for post processing of the FE output data. Rahmani et al. [229] developed a hybrid optimization procedure for inverse identification of mechanical properties of composite materials wherein initial solution from a direct search algorithm such as Mesh Adaptive Direct Search (MADS) were used as the initial guess value for the gradient based search algorithms. This kind of hybrid optimising procedure resulted in better global searching and a high convergence rate to the desired solution. Also, regularization is achieved by imposing constraints to the minimisation problem so that predicted properties from a micromechanical model match with those obtained from experimental measurements. Peronnet et al. [230] presented his initial study, focused on identification of the local damage zones, using the displacement field measurements. The proposed concept uses the displacement field measurement for the evaluation of the degradation parameters and the estimation of the spatial variability of the elastic properties in the damaged zone. The initial study presented estimated the virgin properties of the orthotropic composites using the inverse modelling approach with the finite element model updating (FEMU) technique. The data from another numerical model were used to provide simulated experimental data for evaluation of the virgin properties of the composite laminates to avoid an "inverse crime" (i.e. use of the same forward model as the basis of the inverse problem and for generating test data for the inverse method).

It can be observed that various researchers have effectively used numerical optimisation procedures along with finite element techniques, popularly termed by some of the researchers as the finite element model updating (FEMU) technique, for inverse estimation of material properties/parameters defined in the constitutive models. Depending on the type of problem and flexibility of the solver for the users, different types of algorithm have been used by various researchers

for numerical optimisation. The inverse methodology has gained significant application along with DIC techniques for estimation of the damage degradation parameters and also in the assessment of spatial variation of elastic parameters as a result of damage. However, none of the published literature mentions transverse/compressive property estimation of composite fibres at high strain rates. Hence, the scope of using a micro-scale finite element model for inverse methodology would be a novel approach for estimating high strain rate transverse and compressive properties of fibres.

## 5.4 Description of micro-scale model used for inverse modelling

The micro-scale unit cell developed by Lidgett [1] consisted of fibres surrounded by interface embedded in a matrix material. The 3D solid elements were used to develop the finite element model of the hexagonally arranged unit cell (Fig 5-1).

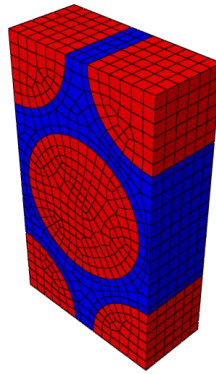


Fig 5-1 FE model of the hexagonal unit cell considered for inverse modelling

The same unit cell model was considered for all the loading cases to enable easier application of periodic boundary conditions based on translational symmetry. Periodic boundary conditions were applied to ensure that there is no separation or overlap between the adjacent unit cells. The details of the periodic boundary conditions used are given in Appendix 1 (page 192). The length, width and thickness of the unit cell were 17.259, 9.9645 and 4.47 microns respectively. The thickness of the unit cell was the same as the radius of the fibre. There were 2898 solid elements (type C3D8R elements) and 3667 nodes in a single hexagonal unit cell micro scale model.

For the material model, the fibre is assumed to be transversely isotropic and elastic until failure whereas the matrix is assumed to be isotropic and elastic until failure. The fibre is considered to be strain rate insensitive and the matrix was

considered to be strain rate sensitive in the material model used. Hence, a strain rate scaling constant required based on the logarithmic equation was included in the matrix material model to incorporate strain rate effects of its stiffness and strength (Eq. 5-4). A bi-linear scaling law was used as the matrix material exhibits two distinct regions of strain rate dependency.

$$A_{RT} = A_0 \left[ 1 + C_A \ln \left( \frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right) \right] \quad \text{Eq. 5-4}$$

The maximum stress criterion was used for element deletion for the fibre and for damage evolution in the case of the matrix. The damage evolution is based on the damage mechanics theory proposed by Matzenmiller [111] where a damage variable is evaluated which were then used to evaluate the degraded direct stiffness and shear stiffness parameters.

$$\omega = 1 - e^{(1-e_{max})/m} \quad \text{Eq. 5-5}$$

$$E_d = (1 - \omega)E \quad \text{Eq. 5-6}$$

$$G_d = (1 - \omega)G \quad \text{Eq. 5-7}$$

Where  $m$  is the strain softening parameter,  $e_{max}$  is the largest value of the maximum stress criteria and  $E, G$  are the respective direct stiffness and shear stiffness parameters after scaling.

The interface is assumed to have the same properties as the matrix. However, different strain rate scaling constants were used for the stiffness and interfacial shear strength. The failure of the interface is dictated by the shear stress along the interface in the longitudinal fibre direction which results in element deletion for further time increments. Dummy nodes which are linked to the faces, edges and corners of the unit cell through linear constraint equation were used to application of displacement. Based on the strain rate desired, the rate of loading applied to the dummy nodes was varied.

Table 5-1 Base reference values used in the micro scale model for inverse modelling

| Material Model | Property Type               | Material parameter                                                                    | Base values |
|----------------|-----------------------------|---------------------------------------------------------------------------------------|-------------|
| Fibre          | Elastic                     | Longitudinal Young's modulus (GPa)                                                    | 230         |
|                |                             | Transverse Young's modulus (GPa)                                                      | 15          |
|                |                             | Poisson's ratio in 1-3 direction                                                      | 0.25        |
|                |                             | Poisson's ratio in 2-3 direction                                                      | 0.0163043   |
|                |                             | Poisson's ratio in 1-2 direction                                                      | 0.2         |
|                |                             | Shear modulus in 1-3 direction (GPa)                                                  | 15          |
|                |                             | Shear modulus in 2-3 direction (GPa)                                                  | 15          |
|                |                             | Shear modulus in 1-2 direction (GPa)                                                  | 7           |
|                | Strength                    | Longitudinal tensile strength (GPa)                                                   | 4.9         |
|                |                             | Longitudinal compressive strength (GPa)                                               | 2.5         |
| Matrix         | Elastic                     | Young's modulus (GPa)                                                                 | 1.423       |
|                |                             | Poisson's ratio                                                                       | 0.34        |
|                |                             | Shear modulus (GPa)                                                                   | 0.53097     |
|                | Strength                    | Tensile strength (GPa)                                                                | 0.121       |
|                |                             | Compressive strength (GPa)                                                            | 0.131       |
|                |                             | Shear strength (GPa)                                                                  | 0.05        |
|                | Softening                   | Tensile strain softening parameter                                                    | 0.1         |
|                |                             | Compressive strain softening parameter                                                | 0.1         |
|                |                             | Shear strain softening parameter                                                      | 0.1         |
|                | Stiffness scaling constant  | Strain rate scaling constant for tensile stiffness at high rates                      | 0.1571      |
|                |                             | Strain rate scaling constant for tensile stiffness at low rates                       | 0.0401      |
|                |                             | Strain rate scaling constant for compressive stiffness at high rates                  | 0.1571      |
|                |                             | Strain rate scaling constant for compressive stiffness at low rates                   | 0.0401      |
|                |                             | Strain rate scaling constant for shear stiffness at high rates                        | 0.1571      |
|                |                             | Strain rate scaling constant for shear stiffness at low rates                         | 0.0401      |
|                | Stiffness scaling threshold | Tensile stiffness low rate / high rate threshold (/ms)                                | 0.19935     |
|                |                             | Compressive stiffness low rate / high rate threshold (/ms)                            | 0.19935     |
|                |                             | Shear stiffness low rate / high rate threshold (/ms)                                  | 0.19935     |
|                | Strength scaling constant   | Strain rate scaling constant for tensile strength at high rates                       | 0.1788      |
|                |                             | Strain rate scaling constant for tensile strength at low rates                        | 0.0329      |
|                |                             | Strain rate scaling constant for compressive strength at high rates                   | 0.1788      |
|                |                             | Strain rate scaling constant for compressive strength at low rates                    | 0.0329      |
|                |                             | Strain rate scaling constant for shear strength at high rates                         | 0.1788      |
|                |                             | Strain rate scaling constant for shear strength at low rates                          | 0.0329      |
|                | Strength scaling threshold  | Tensile strength low rate / high rate threshold (/ms)                                 | 0.08858     |
|                |                             | Compressive strength low rate / high rate threshold (/ms)                             | 0.08858     |
|                |                             | Shear strength low rate / high rate threshold (/ms)                                   | 0.08858     |
|                | Softening scaling constant  | Strain rate scaling constant for tensile strain softening parameter at high rates     | 0           |
|                |                             | Strain rate scaling constant for tensile strain softening parameter at low rates      | 0           |
|                |                             | Strain rate scaling constant for compressive strain softening parameter at high rates | 0           |
|                |                             | Strain rate scaling constant for compressive strain softening parameter at low rates  | 0           |
|                |                             | Strain rate scaling constant for shear strain softening parameter at high rates       | 0           |
|                |                             | Strain rate scaling constant for shear strain softening parameter at low rates        | 0           |
|                | Softening scaling threshold | Tensile strain softening parameter low rate / high rate threshold (/ms)               | 1           |
|                |                             | Compressive strain softening parameter low rate / high rate threshold (/ms)           | 1           |
|                |                             | Shear strain softening parameter low rate / high rate threshold (/ms)                 | 1           |
| Interface      | Elastic                     | Young's modulus (GPa)                                                                 | 1.423       |
|                |                             | Poisson's ratio                                                                       | 0.34        |
|                |                             | Shear modulus (GPa)                                                                   | 0.53097     |
|                | Strength                    | Interfacial shear strength (GPa)                                                      | 0.03703     |
|                |                             | Interfacial normal failure stress (GPa)                                               | 0.07406     |
|                | Strength scaling threshold  | Strain rate scaling constant for interfacial shear strength                           | 0.2647      |

## 5.5 Inverse modelling: strategy adopted

The strategy followed is to use the micro-scale model as a forward problem in the inverse modelling study by comparing the output from the micro-scale models with the respective test data and iteratively evaluate the parameters through numerical optimisation. The micro-scale model developed by Lidgett [1] has a separate user subroutine that requires elastic and strength properties of fibre, matrix and interface along with strain rate scaling constants to be given as input for solution. The detailed material definition requires around 52 material parameters to be given as input parameters for the user subroutine to describe

the material characteristics behaviour. Since inverse problems are ill posed problems, it is desirable to identify the parameters that strongly affect the composite characteristics for a given loading condition. Hence, a parametric study was conducted on the micro-scale model, to identify the sensitive parameters for each loading condition. From the parametric study, the sensitive parameters that can be experimentally evaluated were identified and the sensitive parameters that can be attempted for estimation through inverse modelling were identified.

### **5.5.1 Identification of sensitive parameters**

Parametric studies were carried out to evaluate how sensitive the material parameters are for each loading cases. This is done by comparing the composite stiffness and strength for each case of parametric variation in the material parameter. The approach followed for the parametric study is based on an example problem mentioned in the Abaqus documentation [231] for parametrisation. Accordingly, similar Python scripts were written to vary the parameters and sequentially run the parametric study. The parametric study was performed for all cases of loading (tension, compression and shear in the longitudinal and transverse directions). The dummy nodes in the micro-scale models used to apply appropriate boundary conditions were subjected to a rate of loading corresponding to a strain rate of  $10^4 \text{ s}^{-1}$ . The parametric study was performed without introduction of mass scaling as simulation time was reasonably fast compared to lower loading rates. The parameters that were considered for the study were mainly the elastic fibre properties, such as longitudinal fibre modulus, transverse fibre modulus, fibre shear modulus in all three planes ( $G_{12}$ ,  $G_{13}$ ,  $G_{23}$ ), fibre Poisson's ratio ( $\nu_{12}$ ,  $\nu_{23}$ ,  $\nu_{31}$ ), matrix modulus, matrix Poisson's ratio ( $\nu_m$ ), fibre tensile strength, fibre compressive strength, matrix tensile strength, matrix compressive strength, matrix shear strength and interfacial shear strength. The sensitivity of strain rate scaling constants on composite strength and stiffness were also evaluated during the study. Altogether, the sensitivities of 23 parameters were analysed for each load case as part of the parametric study. The parameters which influence the stiffness and strength of the composite significantly were identified for each load case as shown in Table 5-2. The parameters were varied by  $\pm 20$  percent on the base value (the value used by Lidgett in his models) to evaluate their influence upon the composite properties (stiffness and strength). The stress-strain characteristics for each value of the parameters for each load case are given in Appendix 19 (page 236). The longitudinal characteristics for tension and compression were found to be

dependent only on longitudinal fibre parameters. The elastic and strength properties of matrix tends to have a dominant effect on the transverse and shear characteristics of the composite. The transverse and shear moduli of the fibres also have shown to be sensitive to the transverse and shear properties of the composite respectively. The results obtained from the parametric study is in agreement with the approximate relationship defined based on micromechanics theory of composite materials for each case of loading [232, 233].

Table 5-2 Variation observed in composite stiffness and strength for  $\pm 20\%$  variation of sensitive parameters

| Loading Type                | Sensitive Parameters                    | Stiffness |       | Strength |       |
|-----------------------------|-----------------------------------------|-----------|-------|----------|-------|
|                             |                                         | 20%       | -20%  | 20%      | -20%  |
| Longitudinal Tension(Z)     | Longitudinal Fibre Modulus              | 19.8      | -19.8 | -5.5     | -3    |
|                             | Longitudinal Fibre Tensile Strength     | 0         | 0     | 20.9     | -21.1 |
| Longitudinal Compression(Z) | Longitudinal Fibre Modulus              | 19.8      | -19.8 | 2.6      | -8.3  |
|                             | Longitudinal Fibre Compression Strength | 0         | 0     | 14.5     | -28.8 |
| Transverse Tension(X)       | Matrix Modulus                          | 9.6       | -10.7 | -0.4     | 0.5   |
|                             | Transverse Fibre Modulus                | 8.9       | -11.1 | 0.6      | -0.7  |
|                             | Matrix Poisson's ratio                  | 6.9       | -3.8  | 18.1     | -7.7  |
|                             | Matrix Tension Strength                 | 0         | 0     | 19.6     | -19.8 |
| Transverse Tension(Y)       | Matrix Modulus                          | 8.8       | -11.7 | -2       | 2.3   |
|                             | Transverse Fibre Modulus                | 8.2       | -10.7 | 1.6      | -2.2  |
|                             | Matrix Poisson's ratio                  | 5.5       | -5.1  | 25.8     | -12   |
|                             | Matrix Tension Strength                 | 0         | 0     | 20.2     | -20.2 |
| Transverse Compression(X)   | Matrix Modulus                          | 9.5       | -10.7 | 1.2      | -1.5  |
|                             | Transverse Fibre Modulus                | 9.2       | -10.8 | -0.8     | 1.3   |
|                             | Matrix Poisson's ratio                  | 6.9       | -3.8  | 30.7     | -13.2 |
|                             | Matrix Compression Strength             | 0         | 0     | 19.7     | -19.8 |
| Transverse Compression(Y)   | Matrix Modulus                          | 8.7       | -11.9 | -1.2     | 1.2   |
|                             | Transverse Fibre Modulus                | 8.8       | -11.7 | 0.9      | -1.2  |
|                             | Matrix Poisson's ratio                  | 5.4       | -5.1  | 18.5     | -11   |
|                             | Matrix Compression Strength             | 0         | 0     | 20.1     | -20   |
| Transverse Shear (XY)       | Matrix Modulus                          | 7.9       | -11   | -3.5     | 3.4   |
|                             | Fibre Shear Modulus (G12)               | 13        | -9.2  | 1.1      | -0.9  |
|                             | Matrix Poisson's ratio                  | -1.8      | 2.8   | 14.1     | -8.2  |
|                             | Matrix Shear Strength                   | 0         | 0     | 20       | -20.9 |
| Longitudinal Shear (XZ)     | Matrix Modulus                          | 13.1      | -14.8 | -2.1     | -3    |
|                             | Fibre Shear Modulus (G13)               | 5.3       | -7    | -1.5     | -1    |
|                             | Matrix Poisson's ratio                  | -3.1      | 3.9   | -3.4     | -3.3  |
|                             | Matrix Shear Strength                   | 0         | 0     | 0        | 37.4  |
|                             | Interfacial Shear Strength              | 0         | 0     | 73.5     | -20   |

### 5.5.2 Methodology for evaluation of sensitive parameters

In Table 5-2, the parameters have been listed in the order of their sensitivity for each case of loading. For longitudinal tensile loading, strength and modulus could be evaluated directly from experimental testing as shown in the previous chapter. The longitudinal compressive fibre modulus and longitudinal compressive strength would be estimated through inverse modelling from the longitudinal compression data of a unidirectional composite available in the literature. The transverse fibre modulus and matrix compression strength were estimated from the transverse

compression data of composites. Even though the matrix modulus, matrix tensile and matrix compressive strengths would be obtained from high strain rate experimental testing of epoxy, the epoxy resin used here was different from the 8552 epoxy resin which forms the matrix medium in the composites tested described in chapter 4. The stiffness and strength of epoxies is known for its large variation among different grades due to the varying chemical toughening processes it is subjected to. Hence, matrix compressive strength was estimated through inverse modelling and the high strain rate data of the composites were then used for the estimation of the high strain rate scaling constant for matrix strength. The shear characteristics of UD composites available from the literature were used for estimation of shear modulus of the fibre and matrix shear and interfacial strength to understand the issues involved in their estimation. A table summarising the methodology followed for evaluation or estimation of the sensitive parameters is given in Table 5-3.

Table 5-3 Methodology for evaluation of sensitive parameters

| Methodology       | Parameters estimated /evaluated              |
|-------------------|----------------------------------------------|
| Inverse Modelling | Longitudinal compressive modulus (QS & HSR)  |
|                   | Longitudinal compressive strength (QS & HSR) |
|                   | Transverse fibre modulus (QS & HSR)          |
|                   | Matrix compressive strength (QS)             |
|                   | Matrix compressive strength scaling constant |
|                   | Longitudinal fibre shear modulus             |
|                   | Transverse fibre shear modulus               |
|                   | Matrix interfacial shear strength            |
|                   | Matrix shear strength                        |
| SHTB              | Fibre tensile modulus (QS & HSR)             |
|                   | Fibre tensile strength(QS & HSR)             |

### 5.5.3 Methodology for faster simulation with explicit micro-scale FE model

Since the inverse modelling is carried out via the finite element model updating technique, the optimisation solution procedure requires the explicit micro-scale finite element model developed by Lidgett [1] to be called a number of times until the optimisation solver converges to a particular solution. The number of times the optimisation solver calls Abaqus varies for each problem could be more than 50 or so, if it is a slowly converging inverse problem. Sometimes, due to tolerances being set too tightly, the solver may need to run an excessively large

number of times just to arrive at a single solution. The micro-scale finite element model is developed for implementation in Abaqus/Explicit along with the VUMAT user subroutine. The explicit procedure integrates through duration of the simulation event by using many small time increments. Since the minimum stable time increment related to minimum characteristic element length and dilatational wave speed is usually quite small, most problems require a large number of time increments to simulate a complete event. A typical explicit analysis of the micro-scale finite element model [1] for longitudinal tension loading at strain rates of  $1000 \text{ s}^{-1}$  and  $10000 \text{ s}^{-1}$  performed on a 64 bit Abaqus using Intel Core i3-2100 CPU, 3.10 Ghz processor with 32GB RAM, took around 443 minutes and 52 minutes respectively. Based on the above information, it is evident that it becomes computationally impractical to simulate a quasi-static event ( $0.0001 - 0.001 \text{ s}^{-1}$ ) in its natural time scale using explicit finite element procedures. Since the inverse modelling problem attempted requires estimation of parameters for different strain rates, standard approaches as described in Appendix 16 (page 228) was followed to achieve a meaningful and computationally efficient solution without affecting the validity of the mechanical characteristics obtained.

#### **5.5.4 Selection of optimisation solver**

The various optimisation solvers available from different packages use different algorithms[234] for solution to the non-linear least squares problems. There are four main algorithms that have been extensively used for solving non-linear least squares problems. They are as given below:

1. Gauss Newton method [235-237]
2. Gradient Descent method [238]
3. Levenberg-Markquardt method [239, 240]
4. Sequential Quadratic Programming Method [241]

Since Abaqus is typically scripted using the Python, it was decided to use a solver which could be easily interfaced with Abaqus and can be called from the Abaqus Python environment. Python is a general purpose fully-fledged programming language which has a powerful standard library, numerous special purpose extension modules and it interfaces well with external codes [242]. Hence, it was initially decided to use free software such as SciPy [242, 243] which can be used along with Python. SciPy [242, 243] is an open source computing environment which contains modules for various specific scientific computing tasks. It was decided to use 'scipy.optimize.leastsq' [244] which contains wrappers for the

MINPACK library in FORTRAN which has the Levenberg-Markquardt algorithm for minimising a non-linear least squares problem implemented in *lmdif* subroutine. The initial studies for the inverse modelling problem were carried out using the '*leastsq*' solver to estimate the material parameters. However, it was observed that even though the '*leastsq*' solver was successful in estimation of the modulus parameter, it was unsuccessful in estimation of the strength parameters. The details of the results obtained with the '*leastsq*' solver from SciPy are discussed in detail in reference [245]. Also, one of the issues in interfacing Abaqus with an unconstrained optimization solver is that the solver can give a value unacceptable as an input for Abaqus to solve which can result in abrupt termination of the optimization routine and incorrect parameter prediction. For example, a negative value for the modulus or a value greater than 0.5 for Poisson's ratio for an isotropic material can result in errors in the Abaqus analysis. Defining bounds or defining specific constraints (linear or non-linear) on the parameters can help in avoiding such issues during the optimisation process. However, the '*leastsq*' solver does not have the option of constrained optimisation or the option of defining bounds for the parameter to be estimated. It was observed that the '*leastsq*' requires more iterations (Fig 5-3) to converge to a solution and was found inappropriate for estimation of the strength parameter for the inverse modelling studies. Hence, an optimisation solver namely '*e04unc*' [246] from the NAG C library [247], which solves the optimisation problem based on the Sequential Quadratic Programming Method, has been explored and found suitable for these applications. It was successful in estimation of both modulus and strength parameters. This is discussed in detail in subsequent sections of this chapter. The challenges in interfacing Abaqus with the solver packages namely the '*leastsq*' solver from SciPy and the '*e04unc*' solver from NAG C library [248] have been described in detail in Appendix 17 (page 233).

### **5.5.5 Definition of objective function**

A typical stress strain characteristic from an actual experimental test has different features in its characteristic curve representing different phenomena during loading. These features may not be exactly replicated in the micromechanical model as they may not relate directly to the global material characteristics. In such cases, it is important to define the objective function appropriately to those points which exactly correlate with the data from the predicted numerical model. For an example, a typical stress strain characteristics is shown in Fig 5-2. There is a gradual loading zone in the initial part of the curve which is the load observed

on the load cell from the non-uniform loading of the specimen. The characteristic corresponding to this region may not completely relate to the material as slippage between the grips and the specimen is common phenomenon or this region may correspond to the localised deformation of the material at the loading surfaces. Hence, the points corresponding to the initial loading zone were not considered in the experimental data used for comparison in the objective function. The linear region of the curve was only considered in the definition of the objective function for the determination of the modulus (as shown by the green region), whereas strength related parameters were determined through comparison of the linear region and the non-linear region up to the maximum strength point (region corresponding to green and blue only). The degradation region was not considered to avoid the inaccuracy in estimation of the parameters.

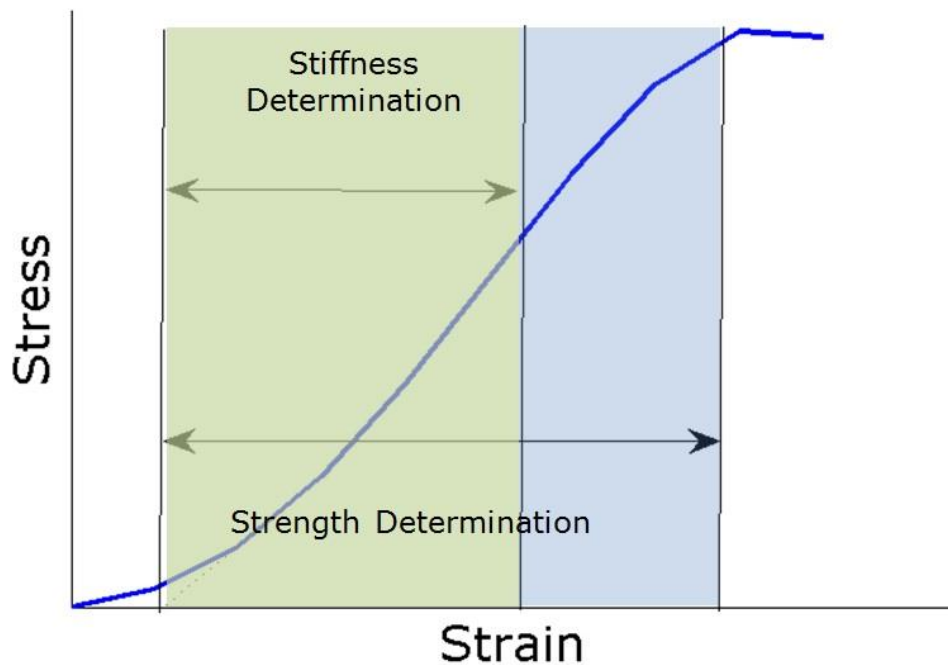


Fig 5-2 Methodology followed for inverse modelling for definition of objective function

For the estimation of stiffness/strength parameters, the stress levels predicted corresponding to each strain value were compared with the experimental data as mentioned by the relationship in Eq. 5-3. The solver 'e04unc' requires only the predicted value of stress to be defined in the objective function as can be seen from the screenshots shown in Appendix 18 (page 234). The evaluation of the residual vector and sum of the squares of the residuals as defined in Eq. 5-2 is done internally by the solver. The solver 'leastsq' requires definition of the

residual vector inside the code, however it calculates the sum of the squares of the residuals internally.

### **5.5.6 Definition of approximate Jacobian**

Optimisation solvers based on gradient based algorithms use the value of gradients/Jacobian/Hessian evaluated for each iteration for convergence to the solution. If the Jacobian is not defined, the solver evaluates an approximate Jacobian through finite difference method, evaluating the objective function repeatedly for small incremental variations of the parameters of the problem. This approach will further slowdown the computationally intensive optimisation study as each evaluation of the objective function requires a separate Abaqus analysis. Also, the limited precision of the results returned by Abaqus may result in stationary or spurious values of derivatives if the finite difference increments are set to be small. Hence, an approximation for the Jacobian can help save a considerable amount of computational time as it avoids the need to call Abaqus simply for finding derivatives of the objective function. The inverse model considered in the present study evaluates an approximate Jacobian for the parameters to be estimated using their relationships defined based on the micromechanics of composites [249-252] for respective loading cases.

The comparative study with and without the user defined Jacobian for estimation of the longitudinal fibre compressive modulus is shown in Fig 5-3. It can be seen that the user defined analytical Jacobian based on micromechanical relations helped in faster estimation than when the solver uses a Jacobian based on the finite difference approximation. Data corresponding to longitudinal compression for specimen 'SP1' (Table 5-4) for the quasi-static strain rate was considered for demonstration of the importance of user defined Jacobian. The micromechanical relationship was also used for evaluation of an analytical Jacobian for estimation of sensitive parameter such as longitudinal compressive strength, transverse fibre modulus, matrix compressive strength, matrix strength scaling constant etc.

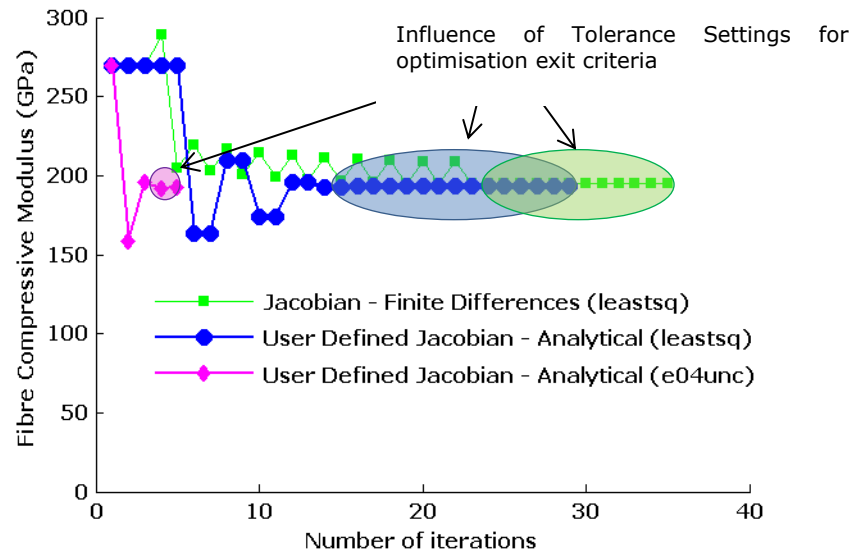


Fig 5-3 Influence of user defined Jacobian for estimation for inverse modelling

### 5.5.7 Methodology for code development

The main purpose of code development undertaken in this study is to create a platform for inverse modelling which helps to interface the optimisation solver with Abaqus for each iteration so as to help in successful estimation of parameters through optimisation. Instead of directly developing the code required for inverse modelling problem for estimation of parameters, involving the micro-scale finite element model, a methodical procedure was followed for the evolution of the code. Numerical solvers from the optimization module of Scipy and NAG C Library were identified based on their ability to estimate material parameters for a non-linear least squares problem and for a quasi-static linear problem. Python scripts were used to set up the problem to call the optimisation solver and also to interface with Abaqus FEA software for solving the forward problem. Based on the successful demonstration of the inverse modelling strategy for simple problems, Python scripts were then written to call the micro-scale explicit finite element model through the solvers for each inverse problem. Typical screenshots showing the details of Python scripts used for optimisation with the solvers '*leastsq*' and '*e04unc*' are shown in Appendix 18 (page 234). The procedure followed for inverse modelling in this study is summarised in the flowchart shown in Fig 5-4.

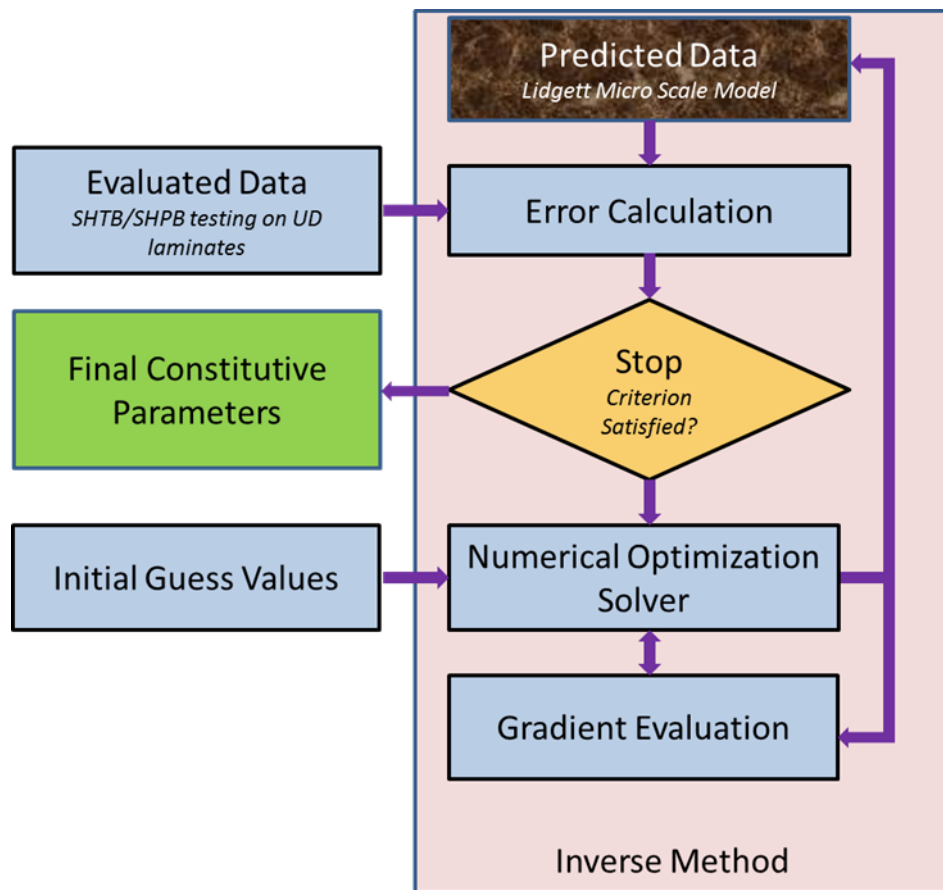


Fig 5-4 Flowchart describing the methodology followed for inverse modelling

The scripts written in Python to carry out the inverse problem have the following features:

- 1) Include modules required for execution of various operations such as calling the optimisation solvers and Abaqus, to perform math operations, to open and close files, to carry out efficient array operations etc.
- 2) Initialise the arrays for passing the input parameter, objective function value and the Jacobian in the format specified by the optimisation solver.
- 3) Call experimental data from the folder and assign into their variables for comparison in the objective function during the optimisation
- 4) Run Abaqus micro-scale finite element analysis and extract results from the output files (.odb) for evaluation in the objective function through function calls from the solver for each iteration. Some of the scripting methodology followed to extract results from odb file was taken from the scripts written by Lidgett [1] for post processing of output data from the micromechanical model.
- 5) Define the Jacobian in the format as required by the optimisation solver

- 6) Define the necessary parameters to be passed to the optimisation solver for carrying out the optimisation.
- 7) The function tolerance and solution tolerance are specified to cause exit from the optimisation solver once the solution converges.

## **5.6 Parameter estimation from longitudinal composite characteristics**

### **5.6.1 Approach**

Linear elastic behaviour was observed for the UD composites under longitudinal loading. This fibre dominant characteristic is expected for a typical aerospace composite of high volume fraction such as 0.65 which is the fibre volume fraction of the Lidgett's micro-scale model [1]. The parametric study to understand the sensitivity of the material parameters shows that the longitudinal composite stiffness is primarily controlled by variation in fibre stiffness in both tension and compression whereas the longitudinal composite strength is primarily controlled by fibre strength with marginal influence of the fibre stiffness (Table 5-2). Since the longitudinal fibre modulus and longitudinal fibre strength are separately coupled to the composite stiffness and strength respectively, it would be difficult to obtain these parameters through a two parameter inverse problem. Hence, the inverse problem was set up as two separate independent single parameter problems for estimation of the longitudinal fibre modulus and longitudinal fibre strength respectively.

For each inverse problem, the input data for the micro-scale model were mainly based on the values used by Lidgett (Table 5-1). The elastic properties of the resin were modified based on the data available for IM7-8552 so as to take into account the properties of the toughened epoxy resin. The estimation of longitudinal fibre modulus was carried out by defining an objective function which compares the linear region in the predicted and experimental curve. The estimated longitudinal fibre modulus was then input into the model directly for the estimation of the longitudinal fibre strength. A curve fitted to the experimental data was used in the objective function for estimation of the fibre compressive strength. This was done to prevent the optimisation solver locating a local minimum in the experimental curve during the optimisation process which can result in false estimation. The option of constrained optimisation available in the e04unc solver was utilised where specific bounds were set for the parameters

and this approach was observed to be successful for estimation of fibre compressive strength. The detailed methodology particular to the inverse problem for longitudinal parameter estimation is given in Appendix 20 (page 243). Since the data presented in the sections 4.2.2.1 (page 79) and 4.2.3.3 (page 90) for longitudinal compression testing of IM7-8552 is an under prediction of the actual strength and stiffness, due to the type of loading method employed, this section describes estimation of longitudinal fibre properties from experimental data presented in the literature relating to high strain rate properties of IM7-8552 unidirectional composites [12] (Fig 5-5).

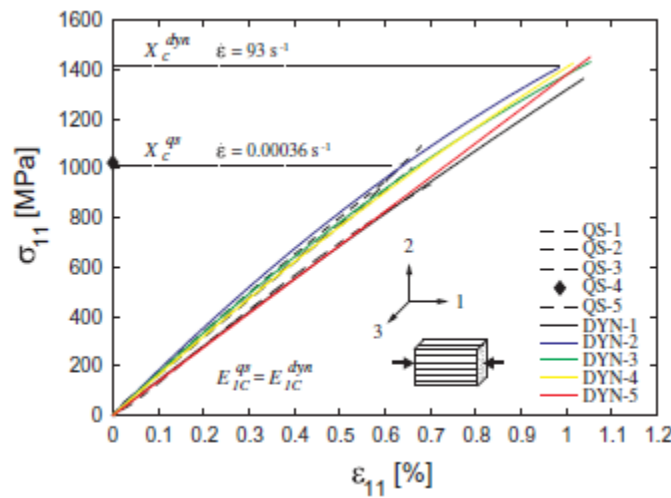


Fig 5-5 Longitudinal characteristics of UD composites [12] used for inverse modelling

## 5.6.2 Results and discussion

The inverse estimated longitudinal compressive stiffness and strength showed consistency irrespective of the initial guess values (Table 5-4). Small variations could be observed in the estimated values for different guesses which can be related to the tolerance settings defined as part of exit criteria for the optimisation solver. The average of the estimated values from all the specimens for quasi-static longitudinal fibre stiffness and strength are 207 GPa and 1.62 GPa and for dynamic ( $93 \text{ s}^{-1}$ ) longitudinal fibre stiffness and strength are 210 GPa and 2.4 GPa respectively. The longitudinal fibre compressive strength tends to show rate sensitivity whereas only little rate sensitivity could be observed for the longitudinal fibre stiffness. It could be observed that estimated parameters directly evaluated from the analytical Chamis formula [226, 251] using the same laminates and constituent properties showed significant difference from those estimated using inverse modelling.

Table 5-4 Inverse estimated longitudinal parameters for different specimens at different strain rates and its comparison with estimated values using Chamis formula

| Rate of loading                          | Specimen ID | Fibre compressive modulus (GPa) |                | Fibre compressive strength (GPa) |                |
|------------------------------------------|-------------|---------------------------------|----------------|----------------------------------|----------------|
|                                          |             | Inverse modelling               | Chamis formula | Inverse modelling                | Chamis formula |
| Quasi-Static                             | SP1         | 192.6                           | 211.01         | 1.52                             | 1.43           |
|                                          | SP2         | 209                             | 237.88         | 1.64                             | 1.53           |
|                                          | SP3         | 220                             | 260.5          | 1.69                             | 1.67           |
| High strain rate ( $93 \text{ s}^{-1}$ ) | SP1         | 181.8                           | 192.58         | 2.24                             | 2.08           |
|                                          | SP2         | 191.7                           | 205.3          | 2.35                             | 2.21           |
|                                          | SP3         | 217.4                           | 250.2          | 2.37                             | 2.19           |
|                                          | SP4         | 220                             | 256.13         | 2.66                             | 2.18           |
|                                          | SP5         | 239.8                           | 282.95         | 2.61                             | 2.15           |

**Influence of noise on inverse estimation:** The sensitivity of the estimated parameters with noisy data was also investigated and has shown only a marginal variation in the estimated parameters (Fig 5-6). A random variation of up to  $\pm 5/\pm 10$  percent from the initial reference value was introduced into each item of data to produce noise in the data.

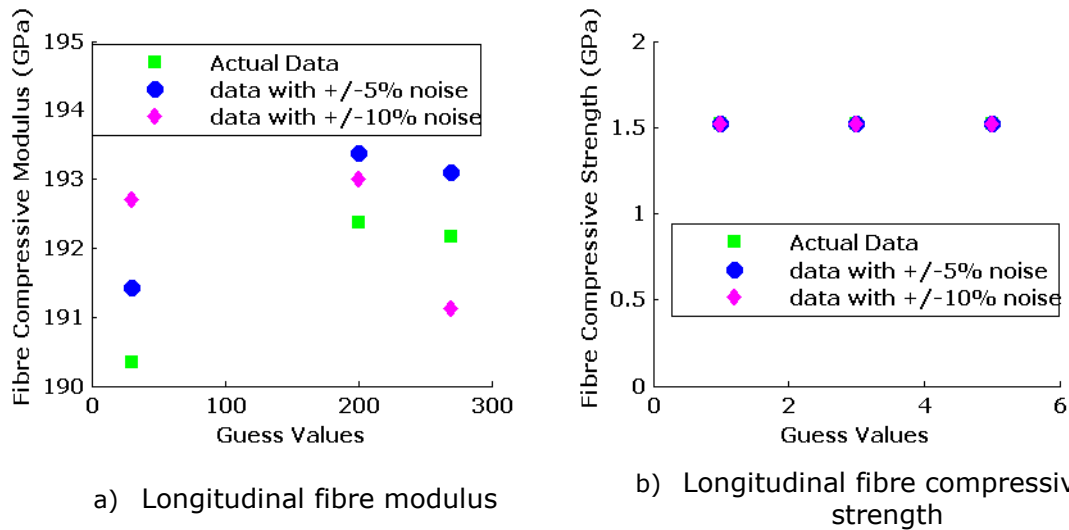


Fig 5-6 Influence of random noise on the estimation of longitudinal fibre parameters at quasi-static strain rate

**Effect of matrix properties:** The matrix Poisson's ratio and the matrix scaling constants for stiffness and strength which were used in this study were derived [1] from the published strain rate data for the Epon epoxy resin [13] rather than 8552 resin data. However, it should be noted that neither the elastic modulus of the matrix (defined as  $E_m$  in the legend) nor the respective strain rate scaling constant (defined as  $CECH$  in the legend) had an influence on the estimation of longitudinal fibre compressive modulus for dynamic strain rate (Fig 5-7). Neither

the compressive strength properties of the matrix (defined as  $S_{cm}$  in the legend) nor its strain rate scaling constant (defined as  $CSCH$  in the legend) had an effect on the estimation of the longitudinal fibre compressive strength at dynamic strain rates as expected (Fig 5-8). It can be seen that the estimation of fibre compressive strength was not dependent on the matrix strength parameters as the estimated values were the same for all the different values of matrix strength and its scaling constant.

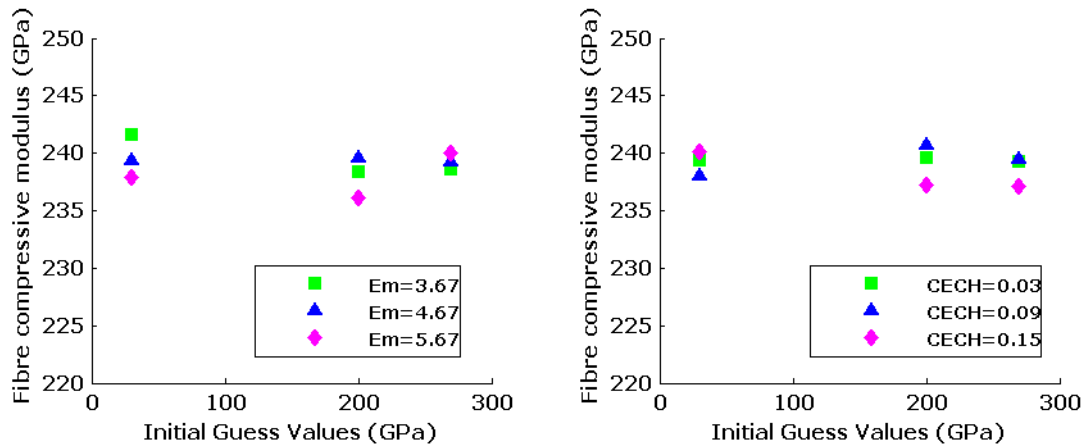


Fig 5-7 Influence of elastic properties of matrix on estimation of longitudinal fibre compressive modulus at high strain rate ( $93 \text{ s}^{-1}$ )

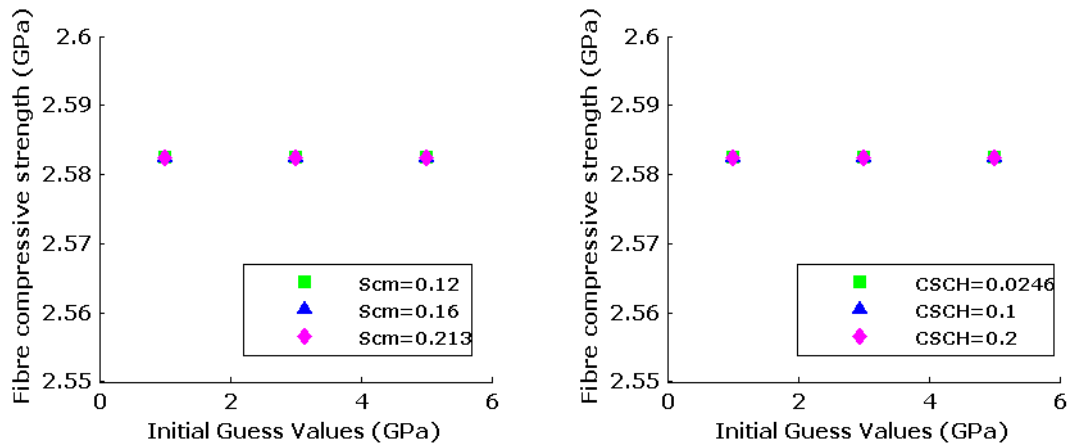


Fig 5-8 Influence of strength properties of matrix on estimation of longitudinal fibre compressive strength at high strain rate ( $93 \text{ s}^{-1}$ )

The non-influence of the matrix parameters on the estimation of the longitudinal fibre parameters can be attributed to the failure of the micro-scale finite element model to capture the actual failure modes as discussed in section 6.2.3 (page 159) during the compressive failure of unidirectional composites. It should be noted that the micro-scale finite element model considered for this study only takes into account the composite failure, primarily fibre compressive failure. The rate sensitivity observed in the compressive failure of UD composites can be

primarily be attributed the rate sensitivity of the matrix which restrains the fibres from micro-buckling.

Even though the material model in the micromechanical finite element model does not incorporate the rate sensitive fibre behaviour in its material definition, rate sensitive characteristics of the fibre can be extracted from the rate sensitive test data through the inverse modelling approach. The comparison of the estimated fibre stiffness and strength shows spurious strain rate sensitivity for both longitudinal fibre stiffness and longitudinal fibre strength. Moreover, it should be noted that the accuracy of the estimated results is very much dependent on the accuracy of the micromechanical finite element used, the test data used and the effectiveness of the test method in measuring actual compressive stiffness and strength of the composite. This is particularly the case for compressive strength, where low values have been reported [16] due to delamination and other unintended failure mechanisms such as end crushing, longitudinal splitting etc. Hence, the failure of the composite due to delamination, transverse shear or microbuckling, which results in under prediction of the strength, is not accounted for in the model. In such cases, the estimated strength values for the fibre will tend to be an underestimate and the data need to be used with care in design and simulations.

**Prediction of the stress-strain characteristics with estimated data:** The estimated longitudinal fibre data were then used in the micro-scale model to evaluate the composite characteristics and results shows close prediction from the micro-scale model with the experimental data for both quasi-static and high strain rate (Fig 5-9). It can be seen that the characteristic curve (SP2 and SP3) predicted through simulation tends to deviate from the linear characteristics for larger fibre modulus specimens even though close correlation could be observed in the initial loading phase with the experimental data. This may be attributed to the large ratio between the constituent moduli which may result in gradual failure of the composite at the interphases. For high strain rate, the experimental and the simulation for almost all the specimen showed close correlation with each other except for specimen 'SP1' (Fig 5-10). In general, this close prediction validates the micromechanical material model and also the inverse methodology approach for predicting parameters.

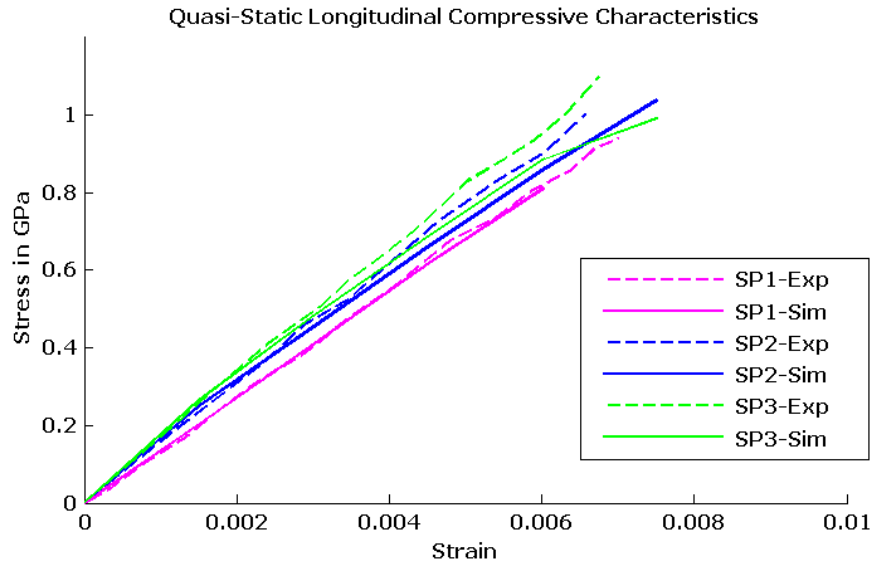


Fig 5-9 Comparison of experimental data [12] for quasi-static longitudinal compression with micromechanical prediction based on inverse estimated fibre data

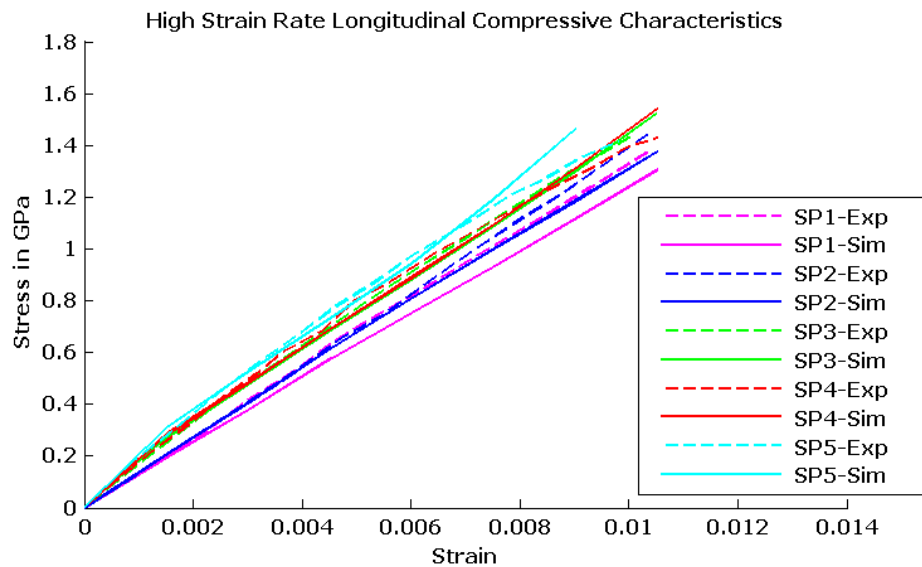


Fig 5-10 Comparison of experimental data [12] for high strain rate longitudinal compression with micromechanical prediction based on inverse estimated fibre data

## 5.7 Parameter estimation from transverse composite characteristics

A non-linear characteristic could be observed when the uni-directional composites are subjected to transverse loading. The experimental stress-strain curve for transverse compression shows a linear elastic region followed by gradual yielding and then failure of the composite (Fig 5-12). From the parametric study (Table 5-2), it can be observed that the transverse compressive stiffness of the

unidirectional composites is influenced by matrix modulus, transverse fibre modulus and the matrix Poisson's ratio. On the other hand, its transverse compressive strength is influenced primarily by matrix compressive strength and matrix Poisson's ratio.

The experimental data presented in the sections 4.2.2.2 (page 84) and 4.2.3.4 (page 97) relating to the transverse compression of Hexply® IM7-8552 UD composites was used for inverse estimation of the transverse modulus of the fibre and compressive strength properties of the matrix.

### **5.7.1 Parameter estimation from quasi-static data**

When two or more parameters are strongly coupled with the output data in the physical model under consideration, it would not be mathematically possible to estimate two parameters from a single linear equation. Since the transverse compressive modulus of UD composite laminates are influenced by more than one parameter, it is important to assign a value to one of the material parameters so as to predict the other parameter through an inverse approach. Since matrix materials are generally isotropic, the matrix data can be experimentally generated much more easily than the transverse fibre data. Hence, it was decided to assign the matrix modulus based on the value of tensile modulus from the data sheet of 8552 epoxy resin [253] for inverse estimation of the transverse fibre modulus. Similar to estimation of parameters from longitudinal compressive characteristics, the inverse problem was therefore set up to predict the transverse fibre modulus and matrix compressive strength from the quasi-static transverse compression data of UD composites as two independent single parameter problems. The approximate relationship for transverse modulus of composites, as formulated by Chamis [251], was used for definition of the Jacobian. Different values were estimated for transverse compression parameters through inverse modelling and directly through the Chamis formula using the same laminate and constituent properties (Table 5-5). The direct evaluation of the constituent parameters through Chamis formula [226, 251] without the micro-scale model was carried out using the composite modulus and strength evaluated from the test data that was considered in the inverse modelling studies. The relationship given by Chamis was rewritten as in Eq. 5-8 and Eq. 5-10 for evaluation of the transverse fibre modulus and matrix compression strength respectively. The matrix properties and volume fraction were assumed to be the same as that used for estimation of parameters using inverse modelling with micro-scale model.

$$E_{f22} = \frac{E_m}{\left[1 - \frac{1}{\sqrt{\bar{k}_f}} \left(1 - \frac{E_m}{E_{l22}}\right)\right]} \quad \text{Eq. 5-8}$$

where

$$\bar{k}_f = (1 - k_v)k_f \quad \text{Eq. 5-9}$$

$$S_{mc} = \frac{S_{l22c}}{\left[ \left\{1 - (\sqrt{V_f} - V_f) \left(1 - \frac{E_m}{E_{f22}}\right)\right\} * \left\{1 - \sqrt{\frac{4V_v}{(\pi(1 - V_f))}}\right\} \right]} \quad \text{Eq. 5-10}$$

Table 5-5 Inverse estimated transverse parameters for different specimens for quasi-static loading condition and its comparison with estimated values using Chamis formula

| Parameters estimated              | Specimen ID | Inverse modelling | Chamis formula |
|-----------------------------------|-------------|-------------------|----------------|
| Transverse fibre modulus (GPa)    | SP1         | 6.9               | 7.28           |
|                                   | SP2         | 6.61              | 7.18           |
| Matrix compressive strength (GPa) | SP1         | 0.212             | 0.27           |
|                                   | SP2         | 0.21              | 0.28           |

Even though the estimation of transverse fibre modulus and the matrix compressive strength was consistent irrespective for initial guess values, the estimated values were quite dependent on the matrix modulus (defined as  $E_m$  in the legend) and matrix Poisson's ratio (defined as  $\nu$  in the legend) respectively (Fig 5-11). The average quasi-static transverse fibre modulus and matrix compressive strength estimated using the inverse modelling approach are 6.76 GPa and 212 MPa respectively (for  $E_m=4.67$  GPa and  $\nu=0.34$ ).

The estimated values used for prediction of the quasi-static transverse compressive characteristics of the carbon epoxy composites showed good agreement with the experimental characteristic curves obtained from the testing (Fig 5-12). The characteristic region corresponding to linear region and point of degradation of the experimental curves for both the specimens have been predicted accurately by the model.

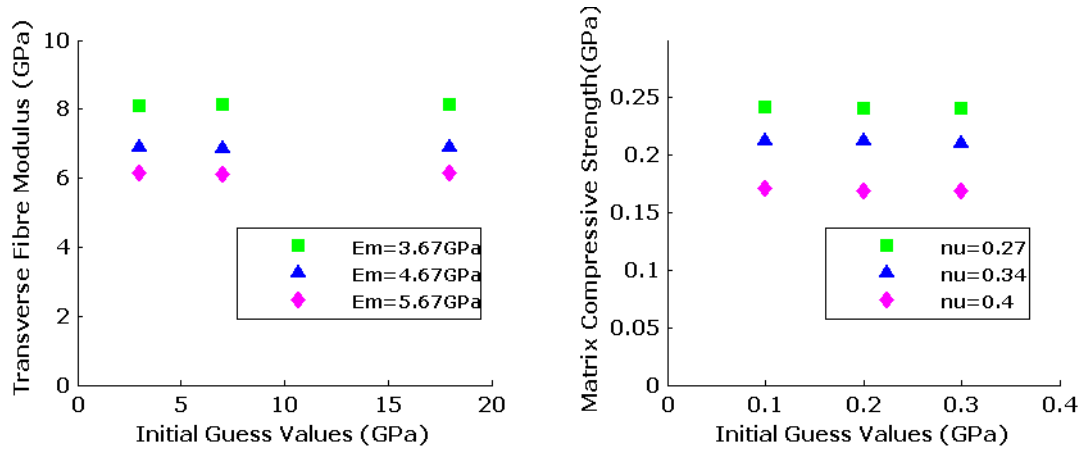


Fig 5-11 Influence of properties of matrix on estimation of transverse fibre modulus and matrix compressive strength from quasi-static transverse compressive characteristics

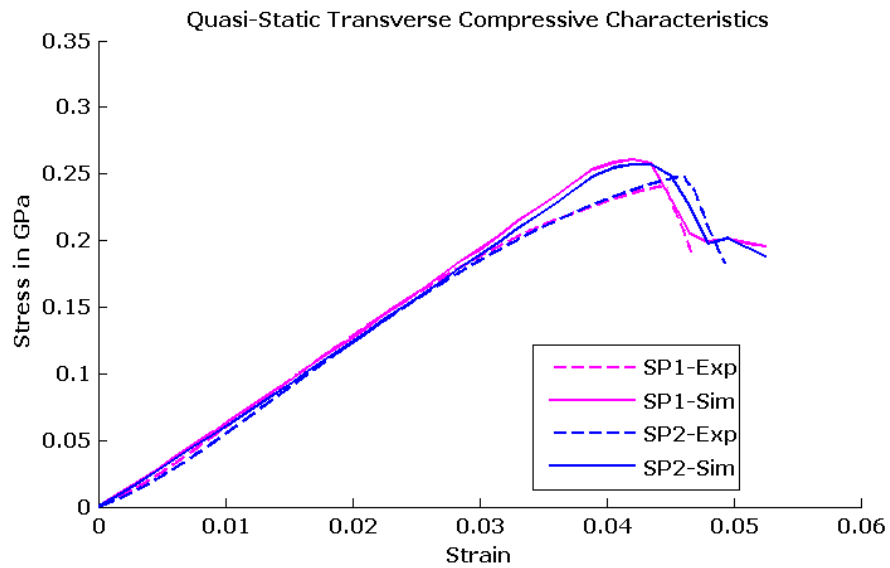


Fig 5-12 Comparison of experimental data for quasi-static transverse compression with micromechanical prediction based on inverse estimated constituent data

### 5.7.2 Parameter estimation from high strain rate data

The material model defined by the micro-scale user subroutine defines the elastic and strength properties of the matrix through the quasi-static compressive strength and the strain rate scaling constants to take into account the strain rate effects for each loading type (tension, compression and shear). The inverse estimation approach was used with high strain rate composite transverse compression test data for evaluation of rate sensitivity of transverse fibre modulus and the matrix compressive strength scaling constant. The method of estimation of the transverse modulus is similar to the method adopted for its estimation with quasi-static data. Changes were made for the strength and stiffness scaling constants corresponding to the strain rate of the test data and

modified to account for the increased loading rate effect as mentioned in Appendix 16 (page 228). Once the transverse fibre modulus had been estimated from the high strain rate data, the estimated data were entered into the model for the estimation of the matrix compressive strength scaling constant from the high strain rate characteristics. The approximate relationship for transverse compressive stiffness and strength of composites as formulated by Chamis [251] along with relationship mentioned in Eq. 5-4 was used for definition of the Jacobian. The values of the estimated transverse compression parameters at high strain rates were also different from those estimated through micromechanical relations defined by Chamis [226, 251] (Table 5-6). For evaluation of the matrix scaling constant using the Chamis formula, the matrix compressive strength at high strain rate was estimated from the Chamis formula (Eq. 5-10) using the high strain rate transverse compression strength of the composite which were then compared with the quasi-static matrix compressive strength for evaluation via the relationship defined by Eq. 5-4. The dynamic transverse fibre modulus and matrix compressive strength scaling constant estimated using the inverse modelling approach are 6.74 GPa and 0.045 respectively. The high strain rate transverse fibre modulus and the matrix compressive strength were also dependent on the matrix properties such as matrix stiffness scaling constant ((defined as CECH in the legend) and Poisson's ratio (defined as nu in the legend) (Fig 5-13). The estimated strength scaling constant is lower compared to the scaling constants derived for other epoxy resins tested and published in the literature [254].

Table 5-6 Inverse estimated transverse parameters for different specimens at high strain rate loading condition ( $2600 \text{ s}^{-1}$ ) and its comparison with estimated values using Chamis formula

| Parameters estimated                         | Specimen ID | Inverse modelling | Chamis formula |
|----------------------------------------------|-------------|-------------------|----------------|
| Transverse fibre modulus (GPa)               | SP1         | 6.9               | 7.28           |
|                                              | SP2         | 6.61              | 7.18           |
| Matrix compressive strength scaling constant | SP1         | 0.051             | 0.061          |
|                                              | SP2         | 0.041             | 0.065          |

The matrix materials are generally visco-elastic and their characteristics have a dominant effect on the transverse behaviour of the unidirectional composite laminates as can be seen from the parametric study and from the micromechanical relations developed by Chamis [226, 251]. Hence, the actual

properties of the matrix should only be considered for inverse modelling for estimation of transverse parameters from the transverse compression data of the composite. The estimated values were then used for prediction of the high strain rate transverse compressive characteristics of the carbon epoxy composites showing good agreement with the experimental characteristic curves obtained from the testing (Fig 5-14). The non-linear behaviour observed within the raw data of transverse compressive characteristics was not considered.

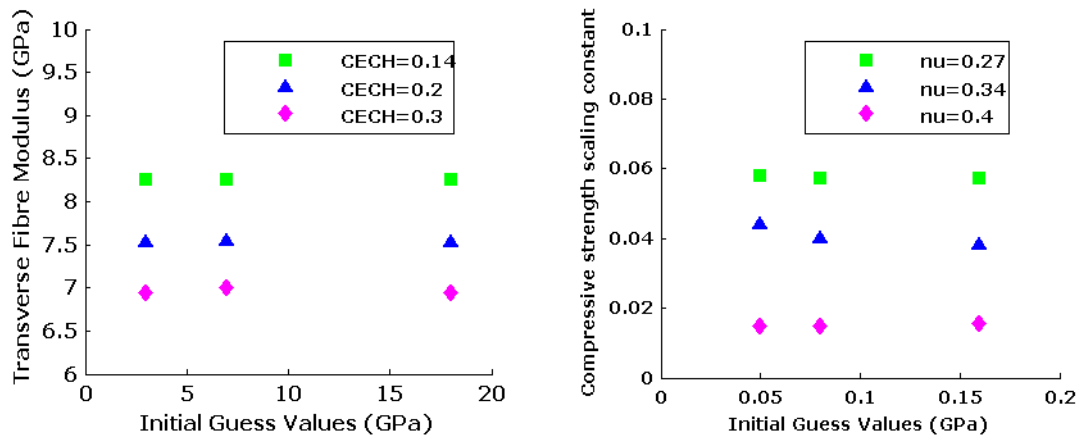


Fig 5-13 Influence of properties of matrix on estimation of transverse fibre modulus and matrix compressive strength at high strain rate transverse compressive characteristics

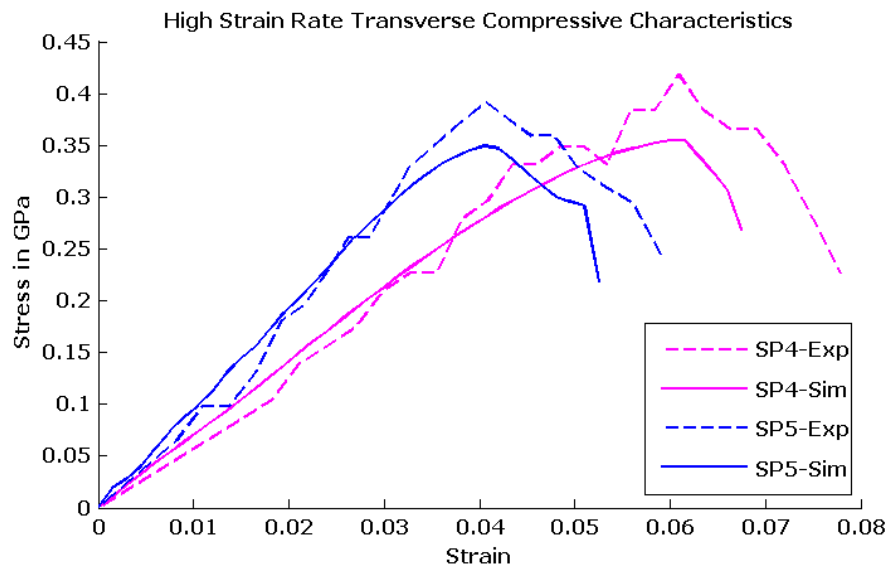


Fig 5-14 Comparison of experimental data for transverse compression with micromechanical prediction based on inverse estimated constituent data for high strain rate ( $2600 \text{ s}^{-1}$ )

## 5.8 Parameter estimation from shear characteristics of composite laminates

Different methodologies have been adopted by various researchers for experimental evaluation of the shear characteristics of composites. Different standards are available for evaluation of quasi-static shear characteristics/properties of composites [255-258]. Torsional split Hopkinson bars (TSHB) have been used for evaluation of shear characteristics at high strain rates [50, 259, 260]. Direct characterisation of the shear properties similar to the ASTM 5379 (Iosipescu method) [256] for high strain rate testing using TSHB is quite complex. Some researchers have used off-axis compression to evaluate simultaneously the transverse compressive and shear characteristics of UD composites indirectly [13, 16, 17, 47, 261]. The global in-plane shearing effect induced on the laminate from off-axis compression testing results in longitudinal shear loading of the UD composite. Since the longitudinal shear modulus is a function of the matrix modulus and the longitudinal shear modulus of the fibre ( $G_{13}$ ), it is important to assign a value to one of these to estimate the other parameter (fibre shear modulus) through inverse modelling. Similarly, it was important assign values for matrix shear strength and Poisson's ratio to determine the matrix interfacial shear strength. In this study, the experimental data presented for the in-plane shear characteristics [262] was used as a reference test data for estimation of fibre shear modulus and interfacial shear strength of the matrix. The approach followed for parameter estimation described for longitudinal and transverse compressive characteristics was extended for parameter estimation of shear characteristics. It was observed that the model gave non-unique solution for the inverse problem for estimation of shear modulus; however the inverse methodology was successful in estimation of the interfacial shear strength (Table 5-7).

Table 5-7 Estimated interfacial shear strength for different guess value

| Matrix interfacial shear strength (GPa) |             |               |
|-----------------------------------------|-------------|---------------|
| Guess1=0.03                             | Guess2=0.05 | Guess 3 = 0.1 |
| 0.0203                                  | 0.0193      | 0.0196        |

The non-unique solution for fibre shear modulus was primarily because of the poor coupling of shear moduli with the longitudinal shear characteristics of the

composites. The matrix modulus has much more dominant influence on the longitudinal shear characteristics as can be seen from Table 5-2.

## **5.9 Conclusions**

Inverse modelling was undertaken to generate the high strain rate transverse, compression and shear properties of fibres and matrices where they could not be measured directly. The study shows that the methodology is successful in estimation of parameters which had strong coupling with the output data such as longitudinal compressive properties and transverse modulus of the fibre. The methodology was also successful in estimation of all the major strength properties of fibre and matrices. A parametric study was useful in identification of the strongly coupled and weakly coupled parameters. The failure in estimation of shear moduli demonstrates the ill-posedness of the inverse problems. The predicted characteristics from the properties estimated shows close correlation with experimental data which validates the inverse modelling approach and material model definition used in the micro-scale finite element model. The parameters evaluated directly using the Chamis formula from the same composite properties disagrees with that estimated using the inverse modelling approach. The next chapter discusses the use of the data generated from both testing and through inverse modelling for the probabilistic prediction of strength of UD composite laminates.

## **6 Application of constituent data generated for prediction of strength of composites**

### **6.1 Introduction**

Significant variability is observed in the strength of composites compared to their stiffness. The variability associated with strength can be observed in all types of composites irrespective of the kind of reinforcement or the matrix medium. This can be attributed to the variability of strength that is associated with its constituents and the mode of interaction of these constituents under loading at the microstructural level [263]. The longitudinal tensile strength of a UD composite is governed by the strength of its fibres. The variability associated with the composite strength also arises from the fact that the strength of a fibre cannot be described by a single value. Longitudinal compressive behaviour is influenced by the properties of the constituents and their respective failure modes. The longitudinal modulus and strength of the fibre influences composite failure in the fibre crushing mode whereas shear modulus and strength properties of the matrix plays an important role in other failure modes such as delamination buckling. The transverse compressive strength of the composite is influenced by the matrix modulus, transverse fibre modulus, matrix compressive strength and matrix Poisson's ratio as can be seen from the micromechanical relations and also from the parametric study described in the previous chapter. The shear strength of composites is influenced by matrix shear strength, interfacial shear strength and elastic properties of the matrix along with the respective shear modulus of the fibres. When life safety is a critical concern, the critical design objective may be to restrict the probabilities of failure to be less than  $10^{-6}$  to achieve higher reliability for the parts developed [263]. In such situations, the number of tests required to demonstrate these values will be extremely large. Hence, in such scenarios, an approach for stochastic prediction of the strength based on the micromechanical behaviour would provide strength estimates that could be used in the design of these structures.

This chapter discusses work related to Monte Carlo simulation of the micromechanical models for establishing the strength distribution of the composites from the estimated distribution of the constituents obtained from tests and inverse modelling.

## **6.2 Literature review**

### **6.2.1 Tensile failure of composites**

The tensile failure of composites is a complex process which is governed by the accumulation of various microstructural damages [264]. Unlike the homogeneous situation, wherein the failure is caused by pre-existent defects which cause the eventual failure [264], the failure of composites is caused by accumulation of fibre fractures at the failure sites that might have evolved during loading due to weak fibres or could be a pre-existent defect site which triggers further fibre fractures as the material is loaded [265]. The fracture of a particular fibre in a composite is determined by the flaws it possesses along its length which could have been present in the precursor material or may have been introduced during the processing stage [265]. When a composite consisting of unidirectional fibres is subjected to applied tension in the fibre direction, uniform strain is induced inside the composite. At some critical strain, a fibre failure would result at the location of the most severe flaw [265]. Unlike the case of a dry fibre bundle, a fibre breakage at a flaw tends to cause the fibre ends to retract but they are restrained by the matrix and hence the load carrying capability of the fibre is potentially restored through shear interaction at the fibre/matrix interface [265]. A broken fibre thereafter is only unloaded over a short distance on either side of the fibre break commonly termed the ineffective length. The load that was taken by the fibre over the ineffective length gets redistributed to the remaining composite. In the case of a dry fibre bundle, the contribution of the entire fibre strand is eliminated once a fibre fails at a flaw location and the load is then equally shared by the surviving fibres (equal load sharing (ELS)) [265]. Daniels [266] has developed an asymptotic fibre bundle strength distribution model for evaluation of dry bundle strength considering the ELS theory for non-failed fibres. However, in composites, the presence of the matrix as binder material causes localisation of the redistribution of the load to the neighbouring fibres alone which is commonly termed as 'Local Load Sharing' (LLS) [267]. The mode of stress transfer is influenced by the elastic properties of the constituents and their interfaces. Due to overload, fibres nearer to the vicinity of the fibre break have more probability of failure over the same region as the ineffective length as the load is further increased and reaches to the critical strength of the short fibre. This short fibre length is known as the positively affected length of the overloaded fibres. A failure can occur at all points of imperfections along the fibres in the whole composite as the fibres are loaded. The phenomenon of overstressing

would take place at all the broken fibre sites in the composite. The eventual failure of a composite occurs when a critical number of fibre failures has occurred at a particular critical flaw site in an analogous manner to the Griffith crack criterion [268] resulting in the failure of other neighbouring fibres across the cross section without further increase in strain [269, 270]. In other words, it is the statistical accumulation of flaws within a composite material that results in the eventual composite failure. The strength of the composite is expected to have a significant amount of scatter based on the presence of flaws along the length of the fibres and the strength distribution of the fibres at the positively affected length [265]. Hence, variability in the fibre strength which is a function of its length would dictate the strength of a composite. Due to this, there will be large variability in the composite tensile strength for different specimens.

Various researchers have approached the prediction of the probabilistic strength of a composite with different models. These range from two and three dimensional finite element models to a shear lag model [271]. There has been further refinement to the shear lag methods with techniques such as Break Influence Superposition [272, 273] and Green's function technique [274, 275], where the size of the numerical problem is related to the number of breaks in the composite in contrast to the FEM where the size of the problem is the total volume of the composite.

The present study uses the micromechanical models of the composite for probabilistic prediction of the tensile strength of the composite through Monte Carlo simulation with the evaluated fibre data. The micromechanical models based on homogenisation approaches have been used by some researchers for prediction of the strength of composites [276]. The homogenisation assumes a geometric arrangement of the composite and the response of the micromechanical is evaluated for respective loading conditions. Even though these models cannot be used for direct simulation of the statistical process of failure of composite, the variability associated with the properties of the interacting material and their interfaces on the strength can be assessed with these simplified homogenised models.

### **6.2.2 Description of the strength of fibres**

The strength of fibres varies as a function of the length. A longer fibre has a higher probability of encountering a severe flaw along its fibre length. Hence, the

strength of a longer fibre is lower than a shorter fibre. The number of flaws in a fibre of length  $L$  which can fail at or below stress  $\sigma$  is given by [277]

$$N(\sigma, L) = \frac{L}{L_0} \left( \frac{\sigma}{\sigma_0} \right)^\rho \quad \text{Eq. 6-1}$$

The strength distribution of the fibre is traditionally assumed to follow a 2 parameter Weibull distribution [277] wherein the probability of fibre failure of a fibre of length  $L$ , subjected to stress of  $\sigma$  is given by Eq. 6-2.

$$P_{f(\sigma, L)} = 1 - \exp \left[ - \left( \frac{L}{L_0} \right) \left( \frac{\sigma}{\sigma_0} \right)^\rho \right] \quad \text{Eq. 6-2}$$

where  $L_0$  is the reference gauge length of the fibre whose strength distribution is used for evaluation of the scale parameter ( $\sigma_0$ ) and shape parameter ( $\rho$ ). However, it has been observed that the above equation is not accurate for describing the strengths of various carbon, graphite and glass fibres [277]. Eq. 6-2 was slightly modified with an additional parameter  $\alpha$  to take care of the inaccuracies [278, 279] and the probability of failure of fibre at a given stress  $\sigma$ , modified as:

$$P_{f(\sigma, L)} = 1 - \exp \left[ - \left( \frac{L}{L_0} \right)^\alpha \left( \frac{\sigma}{\sigma_0} \right)^\rho \right] \quad \text{Eq. 6-3}$$

Based on Eq. 6-3, the strength of the fibre for any given length could be estimated from

$$\sigma_L = \sigma_0 \left( \frac{L_0}{L} \right)^{1/\rho'} \quad \text{Eq. 6-4}$$

$$\rho' = \frac{\rho}{\alpha} \quad \text{Eq. 6-5}$$

Eq. 6-3 and Eq. 6-4 are also known as "Power Law Accelerated Weibull" model which was found to account for variability in fibre diameter. This Weibull model was used as it was mentioned in the study performed earlier by Beyerlein and Phoenix [280] that for study relating to composite considered as chain of bundles, the traditional Weibull distribution would tend to overestimate the relevant fibre strength and thereby the composite strength. .

### 6.2.3 Longitudinal compressive failure of composites

The compressive strength of composites is generally 50-60 percent of the evaluated tensile strength [66, 281]. Different researchers have described about

different models for compressive strength prediction based on the failure modes. Microbuckling (elastic/plastic), fibre crushing, splitting of the matrix, delamination buckling and shear band formation are the main competing failure modes during compression testing of laminates [282, 283]. Elastic microbuckling occurs in a composite with elastic matrix wherein there exist two possible buckling modes: a mode where the matrix undergoes strains transverse to the fibres and other where the matrix shears parallel to the fibres [284]. The lowest compressive strength observed in the elastic buckling mode is equal to the shear modulus of the matrix [283, 284]. However, in reality, when the fibres do have pre-existing fibre waviness or misalignment and the matrix exhibits a non-linear behaviour in its characteristics, the strength of the composites observed are about one-fourth of the shear modulus due to imperfection sensitive plastic microbuckling [283]. Fibre misalignment is also known to induce large scatter in fibre compressive strength. One of the processes associated with microbuckling of the fibre is kinking which results in kink band formation [285]. The kinking phenomenon is associated with the phenomenon of fibre breakage at two points as a result of fibre rotation due to microbuckling which tends to be initiated as the matrix is subjected to shear loading [286]. Another cause of compressive failure is fibre crushing phenomenon which occurs when the uniaxial strain in the composite reaches the crushing strain of the fibres [283]. The compressive strength due to fibre crushing could be evaluated by the rule of mixtures formula [282, 283]. Microbuckling and longitudinal splitting mode of failure are also associated along with the fibre crushing failure modes. Matrix splitting is commonly associated with those composite where the stiffness of the matrix exceeds that of the fibres [283]. Delamination buckling is related to phenomenon of outward buckling of the composite of the composites which occurs as result of lamina level splitting [283]. The lamina level splitting might have arisen due to impact damage, out of plane loading due to fibre waviness and manufacturing defect etc. Shear band formation occurs in composites with low volume fraction of the fibres [283].

The micro-scale model developed by Lidgett [1] predicts the composite crushing failure based on the defined compressive properties of the fibre. Hence, the study uses information relating to variability of the stiffness of the fibre and the compressive strength of the matrix only for assessing the compressive strength of the composite.

### **6.3 Monte Carlo simulations**

Statistical predictions of behaviour of the composite materials have been attempted by various researchers in the past. In this study, Monte Carlo simulations have been performed to evaluate the bundle strength distributions based on simple ELS models and also to evaluate the composite strength distribution based on the micromechanical models developed by Lidgett [1] from the fibre strength distribution data and inverse estimated data. A Monte Carlo Simulation refers to the practice of evaluating the outcome of a process or an experiment by randomly assigning a value based on the distribution pattern of the underlying random variables which dictates the outcome of the process or an experiment [287]. The procedure is composed of  $n$  experiments, where  $n$  is assumed to be large enough to provide a good assessment of the statistical characteristics of the experimental/process outcomes. The strength distribution of UD composites was then estimated based on the data available from Monte Carlo simulations.

### **6.4 Prediction of dry bundle tensile strength distribution assuming ELS model of load sharing**

#### **6.4.1 Numerical model development and validation**

The main assumption that is followed in the Equal Load Sharing (ELS) chain of bundles models is that the failed fibres do not support any load as failure progresses and the surviving fibres share the applied load equally. A fibre bundle of 100 fibres of length corresponding to the positively affected length in a composite (analogous to a failure zone in a composite) was analysed by incrementally loading the small bundle. The stress levels of every fibre in the bundle for each increment of load were compared with their respective strengths. The strengths of the fibres ( $S_{(1)}, S_{(2)}, \dots, S_{(n-1)}, S_{(n)}$ ) are independent and are randomly generated based on a pseudo random number generator particular to the assumed distribution type (Weibull distribution) for a typical simulation. The experimental data available in the literature [288] regarding T800 fibres have been used in this study for prediction of the bundle strength. The fibre strength is observed to follow a Weibull distribution in that study [288]. The properties/parameters obtained from that study relating to fibre and its distribution based on Eq. 6-3 is as follows: Modulus of fibre: 294 GPa;  $\alpha = 0.7$ ,  $\sigma_0 = 4.73$  GPa and  $p = 4.2$ . The diameter of the fibre is taken as 8.46 microns which

was obtained from Lidgett's micro-scale model [1]. The value used by Lidgett for matrix properties was used for evaluation of the length of the fibre considered which was used for estimation of the strength of each link with the aid of a pseudo-random number generator.. Also, the cross sectional area of the fibre, stiffness of the fibre and length were assumed to be same for all fibres. The bundle strength is then evaluated as the total load divided by the product of the number of fibres and the mean area of the fibres. A Monte Carlo simulation for 100 such cases was carried out to evaluate the bundle strength distribution. The predicted bundle strength based on the code developed was compared with bundle strength prediction obtained from analytical formula (Eq. 6-6) valid for the case described above [263].

$$\sigma_{bundle} = \max \left\{ S_{(1)}, \frac{n-1}{n} S_{(2)}, \dots, \frac{2}{n} S_{(n-1)}, \frac{1}{n} S_{(n)} \right\} \quad \text{Eq. 6-6}$$

The strength of the individual fibres were ranked in an ascending order before being evaluated according to Eq. 6-6 (where  $(S_{(1)} < S_{(2)} < \dots < S_{(n-1)} < S_{(n)})$  ). The comparison of the bundle strength predicted using the above formula and the iterative loading procedure based on the code that was written shows similar results and thus validates the code developed (Fig 6-1).

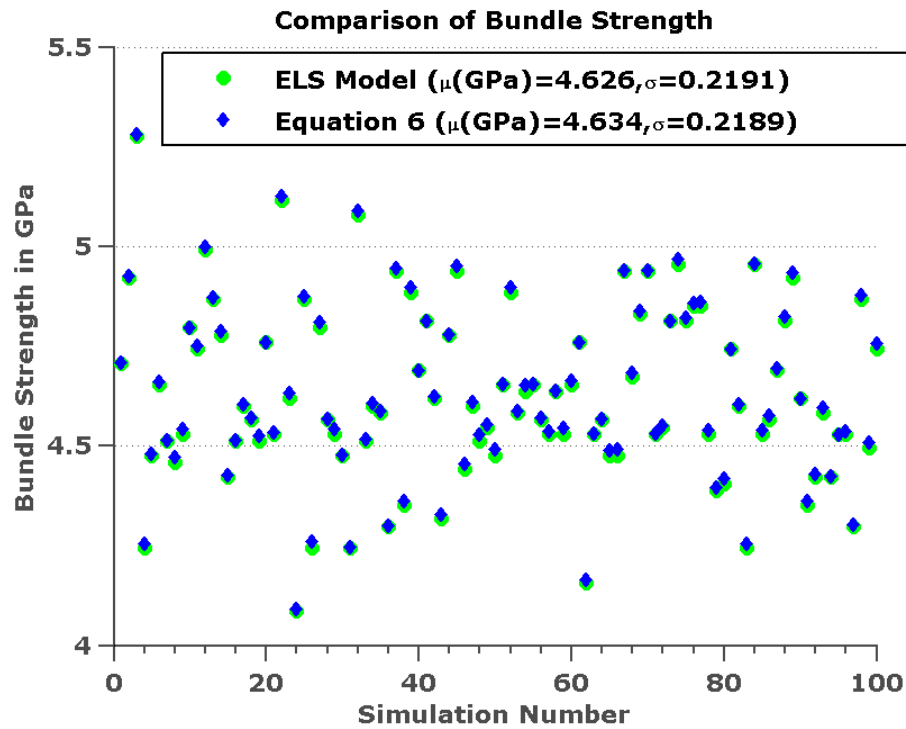


Fig 6-1 Comparison of bundle strength prediction based on numerical ELS model and results based on Eq. 6-6

The mean and the standard deviation of model and based on the equation are as given in the legend. It can also be observed that the bundle strength predicted using the ELS model was lower than that predicted using Eq. 6-6 for all simulations. This was mainly due to the fact that the bundle strength estimation from the code developed was dependent on the value of the load increments used.

The bundle strength distribution obtained is shown in Fig 6-2. When the number of simulations carried out was 100, it can be observed that the bundle strength distribution for the given parameters used for the fibre distribution tends not to follow any specific distribution pattern. However, when the number of simulation was 1000, it can be clearly observed that the bundle strength follows a normal distribution pattern. Hence, it is quite important to have a large database of strengths to make the right assessment of the distribution type. When the tow size of the bundle was increased to 24000, it was observed that the mean bundle strength decreased from 4.6 to 4.5. However, the scatter observed in the bundle strength was also lower as indicated by a lower standard deviation. The decrease in the mean bundle strength for increased tow sizes ( $n$  in Eq. 6-7) can be understood based on the relationship mentioned for the asymptotic mean bundle strength [263].

$$\mu_n^* = \mu^* \left\{ 1 + 0.996n^{-2/3} (e^{2/\rho} / \rho)^{1/3} \right\} \quad \text{Eq. 6-7}$$

where  $\mu^*$  is the asymptotic mean bundle strength

The tow sizes of fibre bundles used in composites are of 12K or 24K for most of the fibre types and their average strength prediction tends to be higher if the data is generated with the lower tow size bundle. It could also be noted that for larger tow size, the mean distribution strength is comparatively the same when the number of simulations were varied from 100 to 1000. Since the data plotted form a probability density function, it should be noticed that the distribution will be influenced by the bin size used. The methodology followed for evaluating the probability density function and for plotting the histogram and the respective distribution is described in Appendix 21 (page 246). The composite strength comprising of these tows can be approximated as [232, 289]

$$\bar{\sigma} = V_f \mu \quad \text{Eq. 6-8}$$

where,  $V_f$  is the fibre volume fraction and  $\mu$  is the mean bundle strength obtained from the distribution.

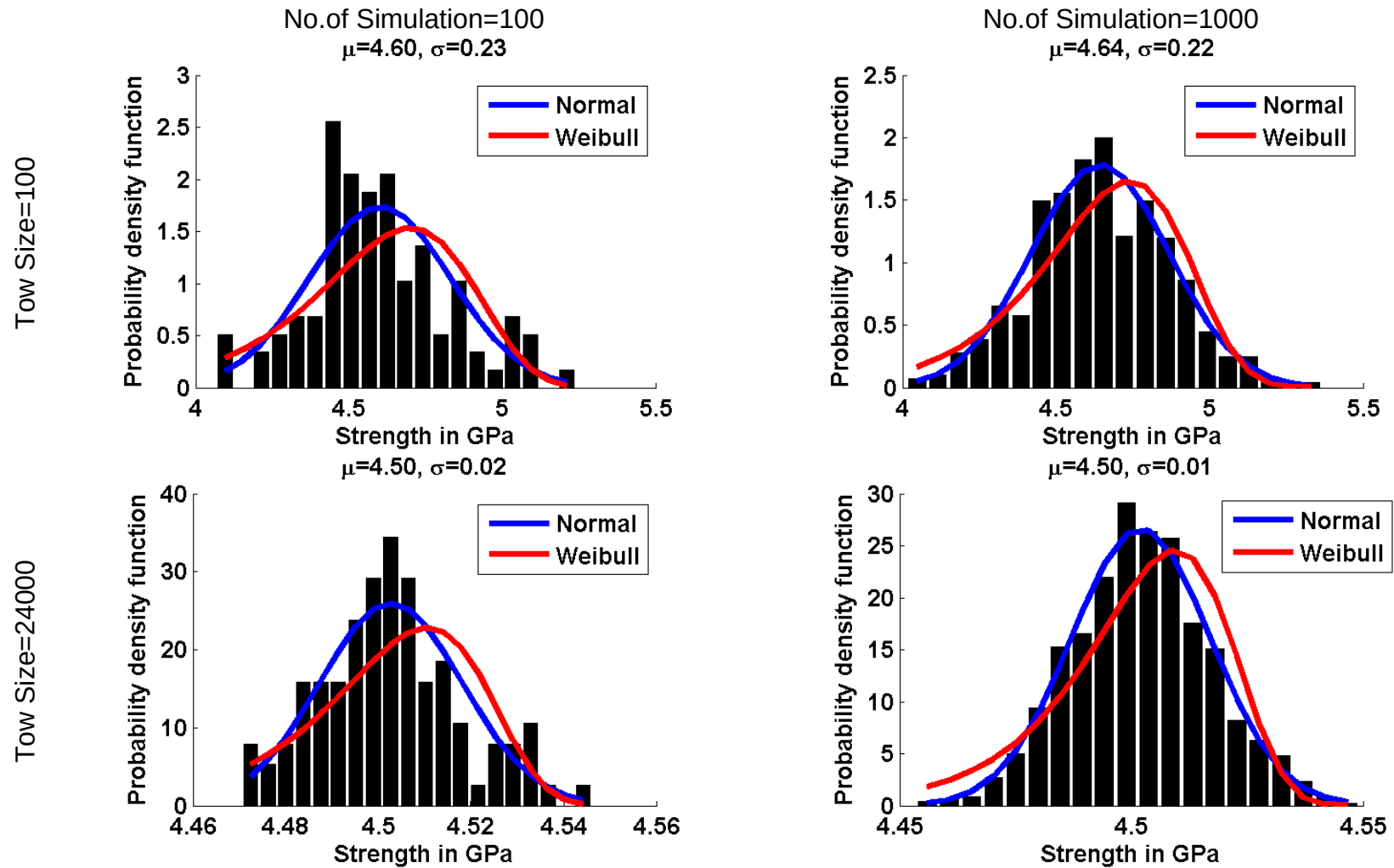


Fig 6-2 Bundle strength distribution observed when variability in strength alone was considered

### 6.4.2 Effect of fibre strength and stiffness variability

The above studies were carried out by assuming a constant value of fibre modulus for all the fibres in a tow. However, in reality it is observed that modulus of the fibre follows a normal distribution. The variability in the fibre strength and the parameters used is same as defined for the case described in the previous section. This effect was incorporated into the model to evaluate the bundle strength and the changes it causes in the distribution pattern. The mean modulus and its standard deviation were 294GPa and 40GPa respectively [288]. A tow size of 3000 was assumed, as there was no significant difference in the predicted mean bundle strengths for tow sizes greater than 3000 (Fig 6-3). It can also be observed that the variability of stiffness did not have much influence on the bundle strength as the bundle size increased (Fig 6-4).

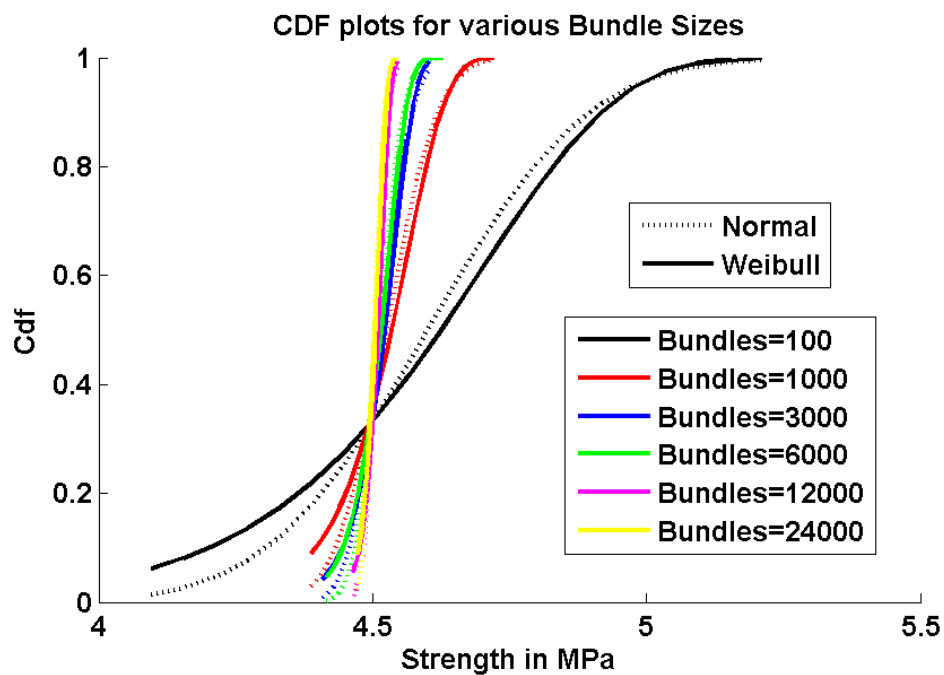


Fig 6-3 Cumulative probability of failure of dry fibre bundle for various bundle sizes (ELS model)

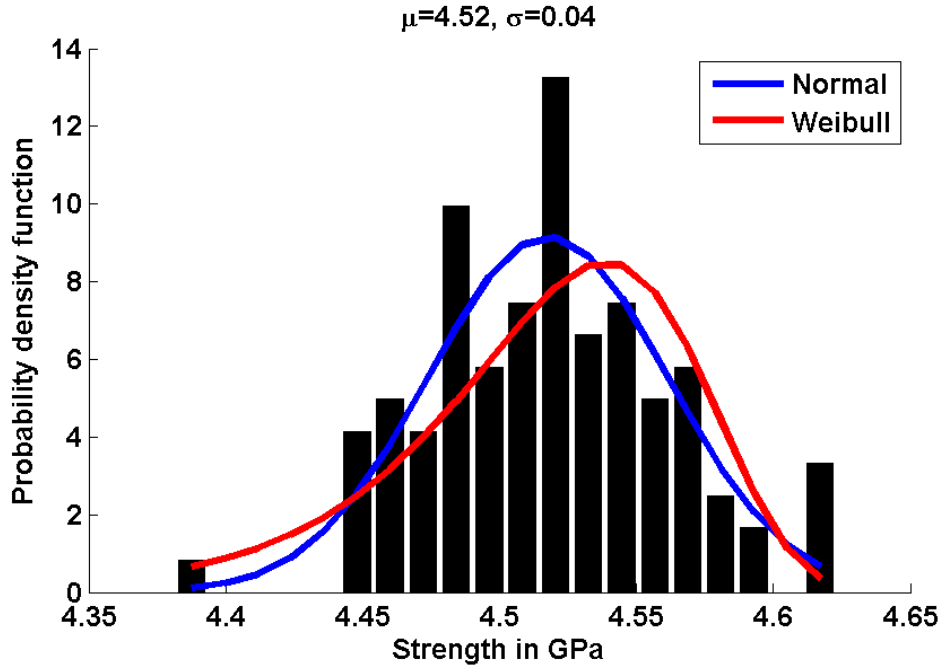


Fig 6-4 Influence of variability of both fibre stiffness and strength on bundle strength distribution

### 6.4.3 Composite strength prediction using dry fibre bundle distribution

The strength of a dry bundle consisting of multiple links was analysed. The number of links in the bundle was determined by the ratio of gauge length to the characteristic fibre length. The characteristic fibre length ( $\delta_e$ ) also known as ineffective length (described earlier) is approximated by a relation as mentioned in [263]:

$$\delta_e = \sqrt{E_f A_f w / (G_m h)} \quad \text{Eq. 6-9}$$

The gauge length of the specimen for calculation of number of links was assumed to be 10mm. In a typical dry fibre bundle, when a fibre breaks, it no longer carries any load. The load taken by the bundle is shared between the remaining surviving bundles following the principle referred to as Equal Load Sharing rule (ELS). Since the model assumes that a fibre no longer carries a load once it breaks at any link, it was observed that the mean bundle strength obtained is much lower than the strength expected for the composite (around 2.8-3 GPa) made of the same fibre bundles (Fig 6-5). Hence, such an approach cannot be considered for evaluation of the composite strength, as load transfer happens due to shear interaction between fibre and fibre/matrix interface in composites/wet bundles when a fibre link has failed.

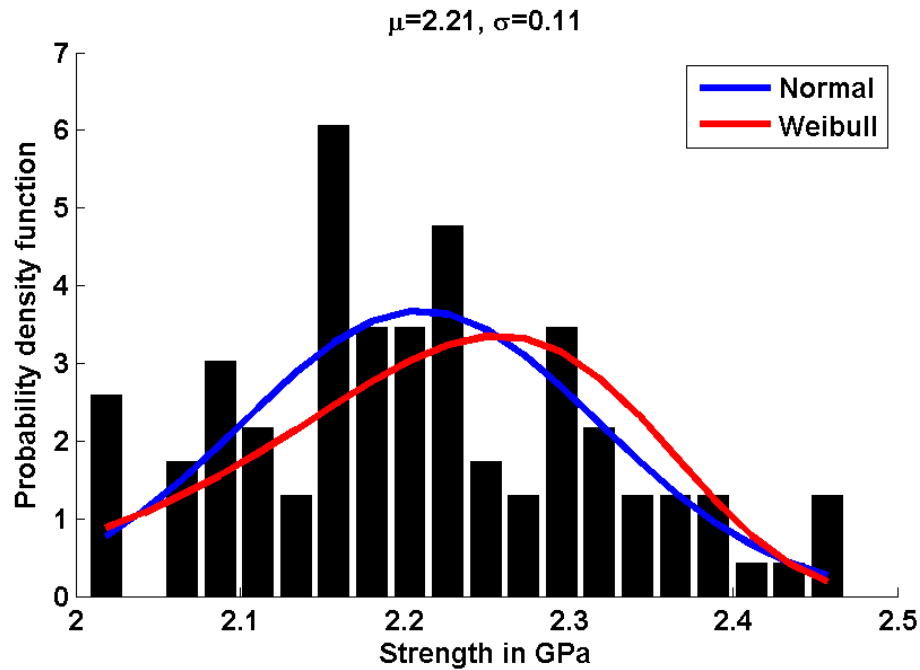


Fig 6-5 Bundle strength distribution observed for a multiple link fibre bundle for ELS model

## 6.5 Use of a micro-scale FE model for Monte Carlo simulation

The micro-scale model developed by Lidgett [1] uses the explicit formulation available in Abaqus/Explicit for its solution to analysis problems. These models are capable of generating the high strain rate characteristics of the composite lamina when the constituents' data are available. Since Monte Carlo simulations were used for the generation of the distribution data, it would involve running the simulation for around 100 or more times to generate the statistical distributions. It is impractical to model the quasi-static simulation in its natural time period to obtain the quasi-static strength distribution data. Hence, approaches for faster simulation as described in Appendix 16 (page 228) were followed to achieve a meaningful and computationally efficient solution without significantly changing the predicted mechanical characteristics.

## 6.6 Longitudinal tensile strength distribution of composites from fibre strength data obtained from single fibre testing

Since the longitudinal tensile characteristics of the composites are dependent only on the stiffness and the tensile strength of the fibres, the variability associated

with these parameters only was assessed for determination of the tensile strength distribution of the composites. An accurate model for strength of composites would require accurate characterisation of the fibres. However, the data available from tensile testing of fibres show large deviation of the strength and stiffness especially for high strain rate data. The influence of such variability on the composite distribution is investigated in this study. The data available from the experimental quasi-static studies and high strain rate tensile testing were used to generate the composite strength distribution data for respective strain rates through Monte Carlo simulations. In this study, the stiffness variations in the fibres were assumed to follow a normal distribution whereas its strength was assumed to follow a Weibull distribution. Various case studies were performed to assess the variability of stiffness and strength as given in Table 6-1. Case 1 uses the respective distribution parameters for stiffness and strength from the actual data obtained during quasi-static single fibre testing of IM7 fibres (section 4.3.3, page 109). As for case 1, the stiffness and strength measurement is based on the average diameter of the fibre for all the fibres tested. The distribution parameters of strength and stiffness due to the variability in diameter were reassessed through a pseudo random number generator for diameter. The highest values of standard deviation of stiffness distribution were considered for case 2 and the lowest value of Weibull modulus of strength distribution was considered as case 3. Case 4 is based on using the variability parameter for stiffness from case 2 and variability parameter for strength from case 3.

Table 6-1 Different Monte Carlo simulations for various cases to assess the quasi-static longitudinal tensile strength distribution

| Case Studies | No of Simulations | Stiffness (GPa) |          | Strength (GPa) |            |          |
|--------------|-------------------|-----------------|----------|----------------|------------|----------|
|              |                   | $\mu$           | $\sigma$ | $\rho$         | $\sigma_0$ | $\alpha$ |
| 1            | 100               | 290             | 42       | 4.43           | 4.83       | 0.7      |
| 2            | 100               | 291             | 87       | 4.43           | 4.83       | 0.7      |
| 3            | 100               | 290             | 42       | 3.37           | 4.724      | 0.7      |
| 4            | 100               | 291             | 87       | 3.37           | 4.724      | 0.7      |
| 5            | 1000              | 291             | 87       | 3.37           | 4.724      | 0.7      |

It could be observed from cumulative probability of failure plots that the larger scatter for stiffness as in case 2 did not significantly influence strength of the composites (Fig 6-6). However, it could be observed that the scatter in stiffness reduced the respective B-basis value compared to Case 1. B-basis and A-basis material properties are the 10<sup>th</sup> and 1<sup>st</sup> percentiles of a material property distribution [290]. Similar observations can also be made with respect to Case 3

and Case 4. B-basis values estimation for Case 5 is comparatively larger compared to case 3 and Case 4 which shows influence of the number of simulations. This means that the estimation of the composite strength distribution data from fibre data with lesser number of simulations will result in lower estimates for B-basis values which would result in a conservative design. This will indirectly affect the overall weight and operational efficiency especially if this value is used for aircraft structures.

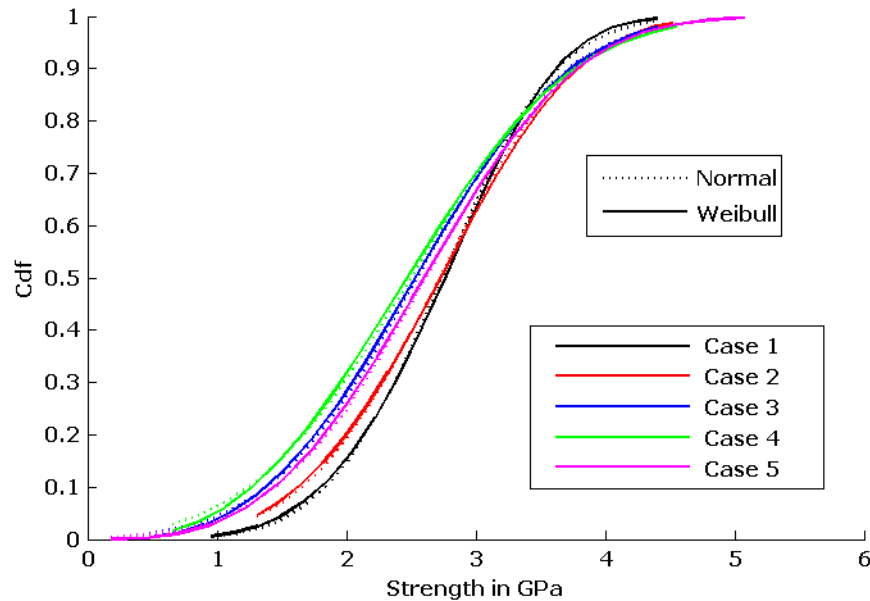


Fig 6-6 Cumulative probability of failure of unidirectional composites for quasi-static tensile loading for different case studies (Table 6-1)

It could be observed from section 4.3.4.2 (page 122) that there was a limitation in the data available for the fibre testing performed both due to the relocation of the rig and substantial amount of time that spent on commissioning the rig and eliminating problems. Hence, the standard deviation for stiffness was comparatively higher. The strength of IM7 fibres did not show significant scatter and hence a high value can be observed for Weibull modulus (Case 1 in Table 6-2). The high strain rate data obtained from tensile testing of IM7 fibres were used to generate the reference set of parameters for strength estimation of the composites (Case 1 in Table 6-2). The approach for generation of parameters for other cases was similar to that followed for quasi-static studies. From the cdf plots, it could be observed that the tensile strength data for larger number of simulations (1000) predicts the b-basis value ( $P_f=0.1$ ) higher compared to the cases when the number of simulations was 100. However, the higher b-basis values predictions are due to the fact that bounds were set for stiffness

distribution to ensure that the stiffness value is between 80 and 500 GPa in addition to the lower value for Weibull modulus(Fig 6-7).

Table 6-2 Different Monte Carlo simulations for various cases to assess the high strain rate longitudinal tensile strength distribution

| Case Studies | No of Simulations | Stiffness (GPa) |          | Strength (GPa) |            |          |
|--------------|-------------------|-----------------|----------|----------------|------------|----------|
|              |                   | $\mu$           | $\sigma$ | $\rho$         | $\sigma_0$ | $\alpha$ |
| 1            | 100               | 194             | 111      | 7.45           | 3.221      | 0.7      |
| 2            | 100               | 261             | 195      | 7.45           | 3.221      | 0.7      |
| 3            | 100               | 194             | 111      | 3.14           | 3.285      | 0.7      |
| 4            | 100               | 261             | 195      | 3.14           | 3.285      | 0.7      |
| 5            | 1000              | 261             | 195      | 3.14           | 3.285      | 0.7      |

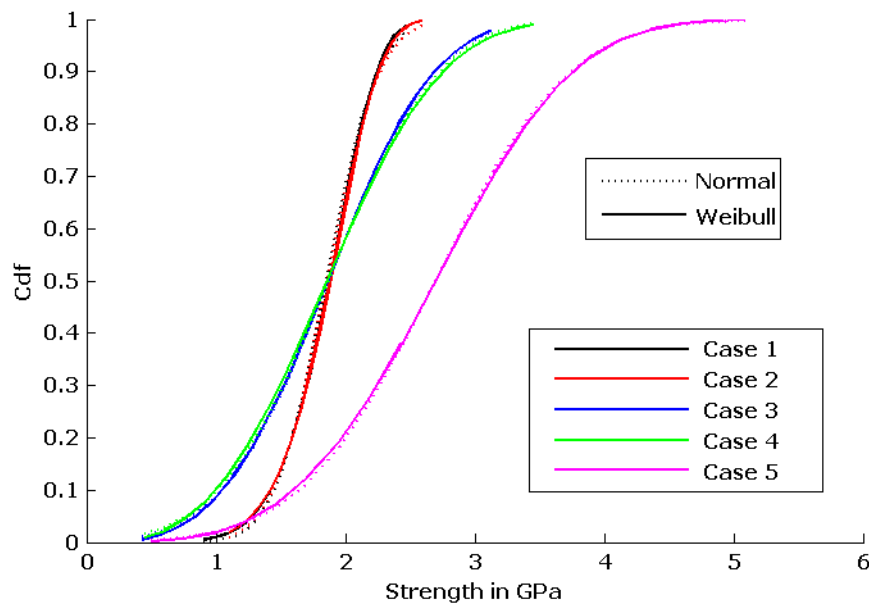


Fig 6-7 Cumulative probability of failure of unidirectional composites for high strain rate tensile loading for different case studies (Table 6-2)

## 6.7 Longitudinal compressive strength distribution of composite

The inverse estimated data of the fibres relating to longitudinal compression were used for the prediction of the compressive strength of unidirectional composites. Since the micromechanical model only captures the fibre compressive/crushing failure of the composite, variability in the longitudinal stiffness and compressive strength were only considered for prediction of the compressive strength distribution. The variability in the estimated compressive stiffness and strength is lower compared to that observed for compressive stiffness and strength generally obtained for compression testing. Hence, the case studies performed also

involved cases where larger variations of compressive stiffness and the strength were considered (Table 6-3). The predicted strength distribution is significantly influenced by the Weibull modulus of the fibre strength distribution (Fig 6-8). The predicted strength from the estimated data and its variability is comparable to the data published in the literature for IM7-8552 composites [12].

Table 6-3 Different Monte Carlo simulations for various cases to assess the quasi-static longitudinal compressive strength distribution

| Case Studies | No of Simulations | Stiffness (GPa) |          | Strength (GPa) |            |
|--------------|-------------------|-----------------|----------|----------------|------------|
|              |                   | $\mu$           | $\sigma$ | $\rho$         | $\sigma_0$ |
| 1            | 100               | 207             | 14       | 17.44          | 1.652      |
| 2            | 100               | 207             | 40       | 17.44          | 1.652      |
| 2            | 100               | 207             | 40       | 6              | 1.652      |
| 4            | 1000              | 207             | 40       | 6              | 1.652      |

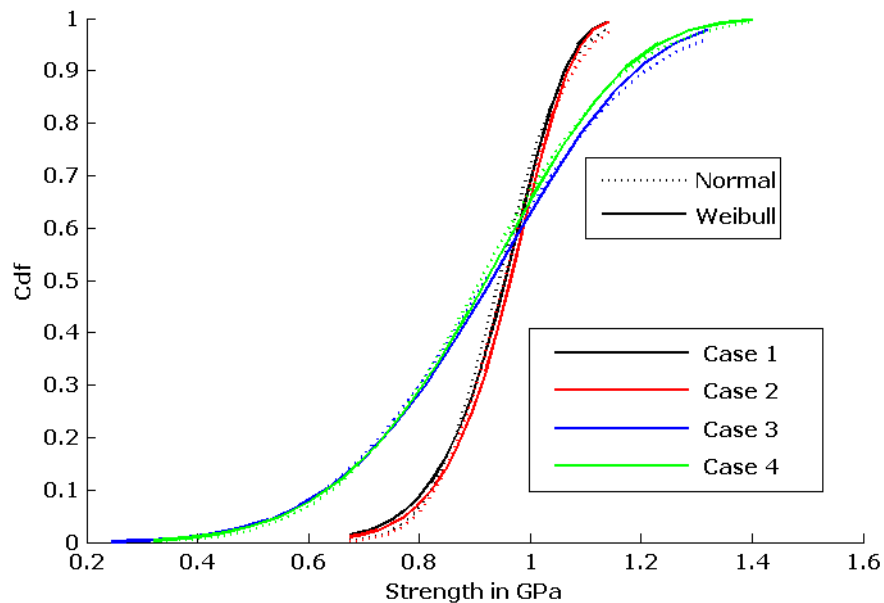


Fig 6-8 Cumulative probability of failure of unidirectional composites for quasi-static compressive loading for different case studies (Table 6-3)

Similarly, the data estimated from the high strain rate longitudinal compression characteristics through inverse modelling were also used for prediction of the compressive strength distribution of the composites through Monte Carlo studies (Table 6-4). The b-basis value predicted is higher when a larger database (Case 2) is used for plotting the distributions (Fig 6-9).

Table 6-4 Different Monte Carlo simulations for various cases to assess the high strain rate longitudinal compressive strength distribution

| Case Studies | No of Simulations | Stiffness (GPa) |          | Strength (GPa) |            |
|--------------|-------------------|-----------------|----------|----------------|------------|
|              |                   | $\mu$           | $\sigma$ | $\rho$         | $\sigma_0$ |
| 1            | 100               | 210             | 23       | 13.46          | 2.514      |
| 2            | 1000              | 210             | 23       | 13.46          | 2.514      |

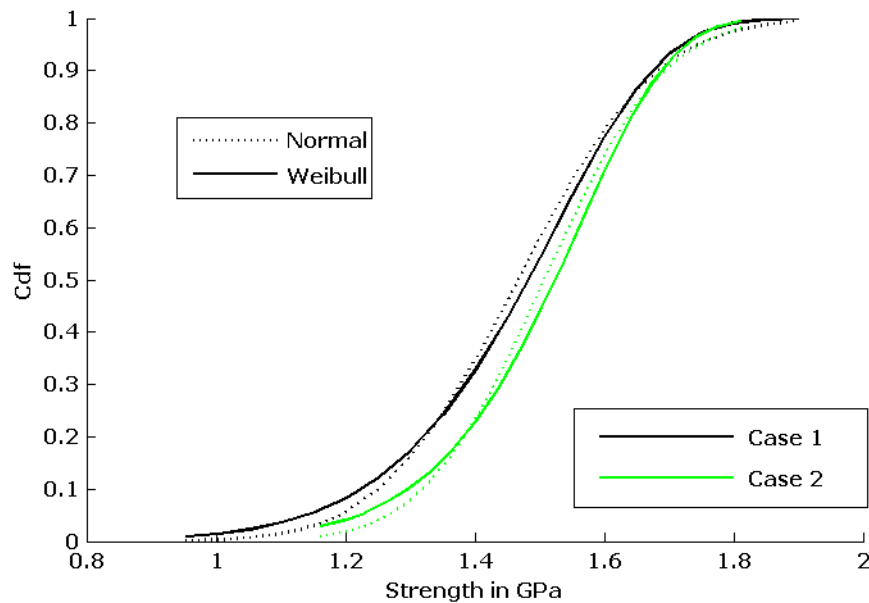


Fig 6-9 Cumulative probability of failure of unidirectional composites for high strain rate compressive loading for different case studies (Table 6-4)

## 6.8 Transverse compressive strength distribution of composites

Even though the inverse modelling procedure followed was successful in its estimation of the respective constituent's properties, it was observed that the estimation of the constituent properties from transverse compressive characteristics was mainly influenced by the matrix properties. The laminate used for transverse compression testing of UD carbon composites was observed to have slightly higher void content. The void content would degrade the matrix modulus quite significantly. Hence, these variabilities were introduced into the matrix properties along with the variability of transverse fibre modulus for prediction of transverse compressive strength distribution. All the sensitive parameters (Table 6-5) were assumed to follow a normal distribution. The same variables were used for quasi-static and high strain rate except that additional variability with respect to stiffness and strength scaling constants were introduced

for high strain rate characteristics strength distribution prediction. It can be observed from Fig 6-10 that the mean strength predicted ( $\approx 230$  MPa) for quasi-static case is comparable to the mean strength obtained for experimental quasi-static transverse compression studies (260 MPa). However, the strength distribution observed for the high strain rate predictions showed larger scatter compared to the distribution data for with experimental high strain rate transverse compressive characteristics (300-420MPa). This was mainly due to the higher values used for mean scaling constant for strength. The estimated matrix strength scaling constant through inverse modelling was in the range of 0.03-0.05, whereas a value of 0.1 with a variation of 0.04 has been used in this study.

Table 6-5 Variability introduced for constituent properties for prediction of transverse compressive characteristics

| Strain Rates | No of Simulations | Matrix Modulus (GPa) |          | Matrix Poisson's Ratio (GPa) |          | Matrix Compression Strength (GPa) |          | Transverse Fibre Modulus (GPa) |          | Matrix Modulus Scaling Constant (GPa) |          | Matrix Strength Scaling Constant (GPa) |          |
|--------------|-------------------|----------------------|----------|------------------------------|----------|-----------------------------------|----------|--------------------------------|----------|---------------------------------------|----------|----------------------------------------|----------|
|              |                   | $\mu$                | $\sigma$ | $\mu$                        | $\sigma$ | $\mu$                             | $\sigma$ | $\mu$                          | $\sigma$ | $\mu$                                 | $\sigma$ | $\mu$                                  | $\sigma$ |
| QS           | 100               | 4.67                 | 1.2      | 0.34                         | 0.02     | 0.181                             | 0.04     | 6.76                           | 2        | 0                                     | 0        | 0                                      | 0        |
| 2000         | 100               | 4.67                 | 1.2      | 0.34                         | 0.02     | 0.181                             | 0.04     | 6.76                           | 2        | 0.1                                   | 0.04     | 0.1                                    | 0.04     |

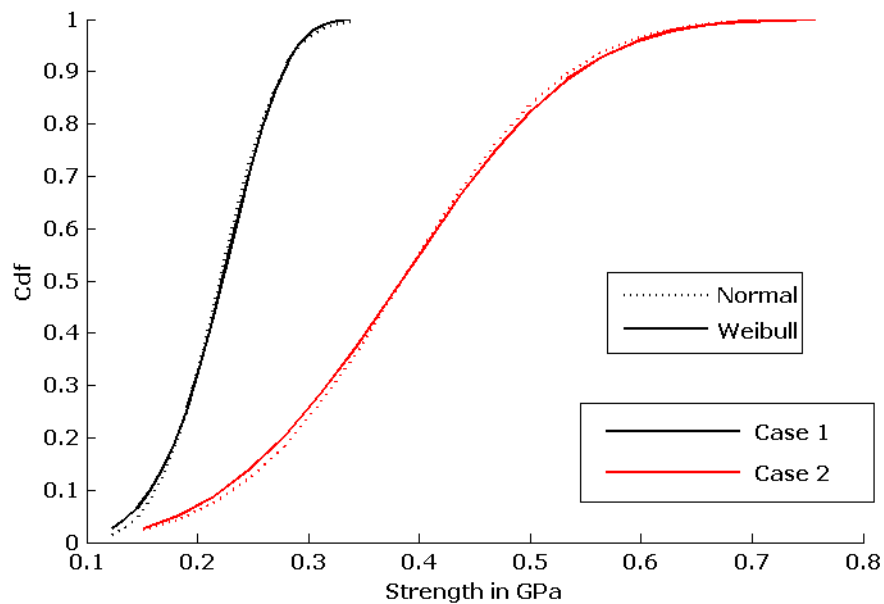


Fig 6-10 Cumulative probability of failure of unidirectional composites for compressive loading for different strain rates (Table 6-5)

## 6.9 Conclusions

Monte Carlo simulations of the micro-scale model for the respective loading conditions can be used to generate more realistic assessment of b-basis values

(design allowables) than depending on smaller experimental data pool. However, the accuracy of the data generated through inverse modelling and tensile data needs to be verified. Since there was lot of variability in the longitudinal tensile fibre data, it would be useful to generate more data on the constituent level for potential better predictions of the composite strength. The mean strength predicted for quasi-static UD transverse compression was comparable with the quasi-static experimental data. However, larger scatter was found for strength distribution predicted for high strain rate transverse compressive characteristics compared to the test data.

## 7 Conclusions

An attempt has been made to develop methodologies for the generation of high strain rate properties of composite material constituents for use with multi-scale models. This study was required as very little data is available on the high strain rate properties of carbon and glass fibres. Two different methodologies were explored that can successfully estimate the high strain rate properties of the constituent materials. The first method was direct evaluation of the high strain rate tensile properties using a specially developed SHTB and the second method was an inverse method which was used for of generation of high strain rate compressive and transverse properties of the fibres and the matrix properties. A stepped SHTB was developed with an innovative clamping methodology for tensile testing of single fibres. The developed SHTB was designed such that it can also be used for high strain rate tensile testing of fibre bundles and composite materials. The approach for inverse modelling used the test data of UD composites at high strain rates to estimate the respective high strain rate data of the constituent materials by using a micro-scale finite element model as the forward problem through a numerical optimisation process.

Only a limited set of data was generated from the SHTB testing due to the time involved in investigating the technical issues and also due to relocation of the rig during the last phase of testing. However, the technical issues in the operation of the SHTB were identified successfully and to a large extent resolved. The inverse modelling approach developed was successful in estimation of the longitudinal compressive and transverse compressive properties of fibres. The matrix compressive strength and the scaling constant were also successfully estimated from the inverse modelling approach. However, the methodology was unsuccessful in estimation of the shear moduli of the fibre as it was observed to have comparatively poor coupling with the output data. A probabilistic prediction of the strength properties of the composites was also successfully undertaken using the micro-scale models with the data evaluated/estimated. The variability associated with the data of several specimens was considered for prediction of strength distribution of composites for different loading cases.

This approach of using the constituent's properties can also be extended for development of high strain rate properties of composite materials with different architectures using appropriate meso/macro scale models.

The detailed conclusion with respect to each main activity conducted as part of this study is discussed in subsequent sections.

## **7.1 Development of SHTB for single fibre testing and the data generated with SHTB**

A SHTB based on the concept of generation of direct tension by hollow striker impact was designed and developed for single fibre testing. The SHTB developed successfully generated the required tensile pulse for loading the fibre specimen. The tensile wave generated, based on the impact of the hollow striker on the flange, using a three stepped incident bar which helped in amplification of the tensile wave generated during impact. The electromagnet designed and developed as part of launch system successfully held the compressed spring and released the striker on demagnetisation. The successful commissioning of the SHTB demonstrated the concept of a spring based system along with a stepped incident bar for carrying out high strain rate tensile testing. The SHTB developed in the demonstrated configuration itself is capable of achieving  $4500\text{s}^{-1}$  for a 3mm gauge length specimen at full compression of the spring. This can be further increased to around  $6000\text{s}^{-1}$  when the 12mm diameter steel section bars are replaced with 12mm aluminium bars along with aluminium clamp sections. The demonstrated SHTB configuration was successful in generating high strain rate data for three classes of fibres at a strain rate of  $675\text{ s}^{-1}$ .

One of the major limitations of the developed rig is that the flange of the striker deforms to a shape of a dish due to inertia loading effects caused during the impact of the striker on the flange. This will be a persistent problem due to the shape of the striker. It can be avoided by using a hollow striker without the flange. This may force the system to be converted to a gas gun driven system. Increasing the thickness of the flange would help increase the duration of usage after which the striker needs to be replaced to carry out further testing. However, these changes will have an effect on the strain rate capability of the system. Another limitation observed during the commissioning and the testing phase was the movement of the incident bar as the striker is released by the electromagnet. Removing the bush from the flange end of the striker to a large extent resolved the problem. An acceleration tube concentric to the incident bar on which the striker is allowed to slide could be used to avoid the frictional interaction of the striker directly with the incident bar.

The limitation of the data generated is that the fibre diameter used for calculation is the diameter provided by the manufacturer in their data sheet. Even though sample diameter measurements were performed using SEM, a method to measure the diameter of the fibre specimen at the rig location prior to testing would give more accuracy in the evaluated strength data. The position of the load cell needs to be closer to the specimen for improving the accuracy of the load data generated. This may require modification of the clamp in the transmission bar

The following suggestions are made for improvement of the rig:

- All the bars can be replaced with centreless ground bars to improve the quality of the incident bar signals obtained.
- The launch system can be replaced with air guns to further increase the strain rate capability of the system.

## **7.2 High strain rate testing of composite materials**

The quasi-static and high strain rate longitudinal compressive and transverse compressive characteristics of unidirectional composites were evaluated through the end loading method. It was observed that the end loading method cannot be used for accurate prediction of longitudinal characteristics as it results in premature failure of the composites triggered from the ends, resulting in under prediction of the actual compressive strength. However, transverse compressive characteristics were estimated successfully with end loading whose values were comparable to that mentioned in the literature for a similar group of fibres. It was also observed that high strain rate transverse compressive strength was around 25-80% higher compared to the quasi-static characteristics. The failure modes observed through high speed camera showed that lamina level buckling, resulting from non-uniform loading, is the main reason for under prediction of the longitudinal composite strength.

The main limitation of the high strain rate data generated was due to the inaccuracies involved in identifying the datum point of the reflected and transmitted waveform. Another limitation is that the laminate showed a high void fraction which can affect the compressive strength of the material.

Solutions to following issues can improve the quality of the result obtained from testing:

- Careful experimental design of the specimen can help in ensuring more specimens achieve state of dynamic stress equilibrium during the high strain rate loading phase prior to failure.
- Identification and removal of the source of the disturbance observed at the tail end of the incident bar pulse can help in achieving more accurate results. This noise can be mostly attributed to the misalignment of bars and more calibration tests need to be performed with the realignment of the bars until noise can be eliminated significantly from the tail end of the pulses. Calibration tests should be performed on every occasion as and when the rig alignment is disturbed before doing the actual tests on the specimens.
- Correction of the dispersion observed in the raw signals can help in generation of less noisy stress-strain characteristics of the specimen. However, this might affect the ability to capture useful information regarding specimen failure.

### **7.3 Inverse estimation of compressive and transverse properties of fibres**

A methodology was developed for inverse identification of the high strain rate constituent properties of composite materials using a numerical optimisation approach. The micromechanical model developed as part of the previous study by Lidgett was used as the forward problem in the inverse modelling study considered here. The parameters attempted for estimation through inverse modelling were identified based on a parametric study using the micromechanical finite element model with different load cases. The inverse modelling problem was developed with Python scripts to set the problem to call the optimisation solver and also to interface with Abaqus FEA software for solving the forward problem. Two different numerical optimisation solvers from packages were evaluated as part of the study. The numerical solver 'e04unc' from the NAG C library was successful in estimation of both the stiffness and strength parameters through the inverse method developed. The strength and stiffness parameters of longitudinal characteristics were successfully identified with the inverse methodology developed. Even though the inverse methodology was successful in identification of the respective stiffness and strength parameters for quasi-static and high strain rate, it was observed that the estimated parameter is dependent on the assumed values of matrix properties.

However, the approach developed could not estimate the shear properties of the fibre from the shear characteristics available from the literature, as it had poor coupling with the output data. Also, it was difficult to estimate the strength and stiffness parameters from a single inverse estimation study with the present approach of defining the objective function.

## **7.4 Probabilistic strength prediction**

The data available from the testing along with its variability were assessed to predict the probabilistic strength of composites under longitudinal and transverse loading. Python scripts were written to perform the Monte Carlo simulation with the micromechanical model using the necessary variable parameters. Monte Carlo simulations of the micro-scale model with the constituent data can be used as a numerical tool to generate more realistic assessment of b-basis values (design allowables) instead of having to rely on a small experimental data pool. One of the major limitations was that some of the case studies were performed with assumed variability and distribution types which need to be further verified to reflect real property variations.

## **7.5 Future studies**

The following issues still remain to be investigated:

- 1) *Applicability of such an approach for complex composite architectures.*  
One of the promising aspects of composites is the possibility of various failure mechanisms during a dynamic event which can help in increased energy absorption. It is known that composites with complex architectures are ideal for energy absorption. Meso and macro level models for these complex architectures could be developed and the data from micro-level studies could be used to predict the strength and stiffness of such composite structures. This approach can reduce the large amount of testing that is required in evaluation of the suitability of new architectures for energy absorbing applications.
- 2) *Further studies on single fibre testing at high strain rates.* Only a limited amount of data was generated through this study. More data at high strain rates can give a better picture of the statistical variation of fibre properties which will be useful information for tailoring the composites for impact applications. There is concern regarding the accuracy of the data generated due to the position of the load cell and the method used for

evaluation of the strain. These concerns should be addressed for further studies on high strain rate single fibre testing.

- 3) *Reliable method of strain measurement during high rate testing.* One of the drawbacks of the present technique is the inconsistency of displacement data measurement from high speed images. The inconsistency was attributed to the uneven brightness, poor quality of initial images and decreasing quality of the image at each frame rates. The techniques using laser measurement could be explored for development of more reliable data measurement systems from SHTB.
- 4) *Generation of high strain rate shear characteristics for further evaluation of the sensitive parameters through inverse modelling.* The data that was used in the inverse estimation of the shear properties of the fibres were mostly the data that was available in the literature for different materials from than those used here. This data may not be appropriate for estimation through the inverse modelling approach. Methods for evaluation of high strain rate interlaminar shear properties can be explored which will generate useful information for modelling impact behaviour of laminated composites.
- 5) *Development of a test fixture for high strain rate loading which can induce compressive loading through shear without premature failure of composites.* The data generated for longitudinal characteristics of UD composites was much lower than the data available in the manufacturer's data sheets for both quasi-static and high strain rate loading. This was mainly due to the method employed for compressive loading, which resulted in premature failure triggered by the failure of the fibre and the matrix from the end loading faces. Hence, a fixture suited for SHPB application with a design concept similar to the fixture described in the quasi-static standard for compression testing ASTM D 3410 (through shear loading). This could be achieved by extracting and developing some of the features of the clamping mechanism used in the present study. However, slippage between the loading jaw and tab or tab and the specimen will be an issue at high loading rates.
- 6) *Modification of the material model to incorporate the rate sensitivity of fibres.* The material model developed by Lidgett only considers the rate sensitivity of the matrix. Development of a rate sensitive model for the fibres will allow for inverse evaluation of the rate sensitive characteristics of the fibre.

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## Appendix 1 Lidgett's [1] work on multi-scale modelling for generation of high strain rate properties

### A 1.1 Micro-scale model

The micro-scale unit cell comprises of fibre with interface embedded in matrix material. The interfacial layer is very thin and assumed to be one tenth of the fibre radius. A hexagonal close packed arrangement has been used for the unit cell model considered in the study (Fig A. 1).

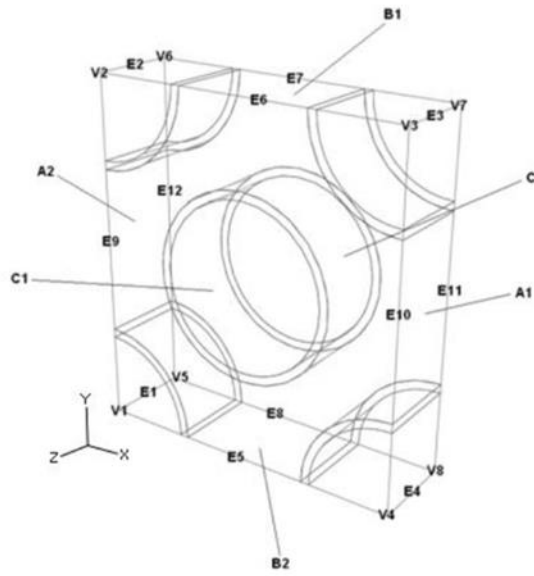


Fig A. 1 Hexagonal closed packed unit cell used for micro-scale models and the definition of each face, edges and corners [1]

To ensure that each unit cell has same deformation mode with no separation and no overlap, periodic boundary condition were applied to the faces, edges and corners (Fig A.2), though linear constraint equations. The linear constraint equation is developed from Eq A. 1, Eq A. 2 and Eq A. 3.

$$u' - u = 2ai\epsilon_x + 2bj\epsilon_{xy} + 2ck\epsilon_{xz} \quad \text{Eq A. 1}$$

$$v' - v = 2bj\epsilon_y + 2ck\epsilon_{yz} \quad \text{Eq A. 2}$$

$$w' - w = 2ck\epsilon_z \quad \text{Eq A. 3}$$

where 2a, 2b, 2c represent the width, height and depth of the unit cell respectively. The detailed derivation for the boundary conditions can be found in work done by Li and Wongsto [291].

| Faces                                                                                                                                                                                                                                                                                                                                                                                                                                  | Edges 1–4                                                                                                                                                                                                                                                                                                                                                                              | Edges 5–8                                                                                                                                                                                                                                                                                                        | Edges 9–12                                                                                                                                                                                                                                                                                                |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $i=1, j=0, k=0$<br>$u_{A2}-u_{A1}=2ac_x$<br>$v_{A2}-v_{A1}=0$<br>$w_{A2}-w_{A1}=0$<br>$i=0, j=1, k=0$<br>$u_{B2}-u_{B1}=2bc_{xy}$<br>$v_{B2}-v_{B1}=2bc_y$<br>$w_{B2}-w_{B1}=0$<br>$i=0, j=0, k=1$<br>$u_{C2}-u_{C1}=2cc_{xz}$<br>$v_{C2}-v_{C1}=2cc_{yz}$<br>$w_{C2}-w_{C1}=2cc_z$                                                                                                                                                    | $i=0, j=1, k=0$<br>$u_{E2}-u_{E1}=2bc_{xy}$<br>$v_{E2}-v_{E1}=2bc_y$<br>$w_{E2}-w_{E1}=0$<br>$i=1, j=1, k=0$<br>$u_{E3}-u_{E1}=2ac_x+2bc_{xy}$<br>$v_{E3}-v_{E1}=2bc_y$<br>$w_{E3}-w_{E1}=0$<br>$i=1, j=0, k=0$<br>$u_{E4}-u_{E1}=2ac_x$<br>$v_{E4}-v_{E1}=0$<br>$w_{E4}-w_{E1}=0$                                                                                                     | $i=0, j=1, k=0$<br>$u_{E6}-u_{E5}=2bc_{xy}$<br>$v_{E6}-v_{E5}=2bc_y$<br>$w_{E6}-w_{E5}=0$<br>$i=0, j=1, k=1$<br>$u_{E7}-u_{E5}=2bc_{xy}+2cc_{xz}$<br>$v_{E7}-v_{E5}=2bc_y+2cc_{yz}$<br>$w_{E7}-w_{E5}=2cc_z$<br>$i=0, j=0, k=1$<br>$u_{E8}-u_{E5}=2cc_{xz}$<br>$v_{E8}-v_{E5}=2cc_{yz}$<br>$w_{E8}-w_{E5}=2cc_z$ | $i=1, j=0, k=0$<br>$u_{E10}-u_{E9}=2ac_x$<br>$v_{E10}-v_{E9}=0$<br>$w_{E10}-w_{E9}=0$<br>$i=1, j=0, k=1$<br>$u_{E11}-u_{E9}=2ac_x+2cc_{xz}$<br>$v_{E11}-v_{E9}=2cc_{yz}$<br>$w_{E11}-w_{E9}=2cc_z$<br>$i=0, j=0, k=1$<br>$u_{E12}-u_{E9}=2cc_{xz}$<br>$v_{E12}-v_{E9}=2cc_{yz}$<br>$w_{E12}-w_{E9}=2cc_z$ |
| <b>Corners</b>                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                           |
| $i=0, j=1 \text{ and } k=0$<br>$u_{V2}-u_{V1}=2bc_{xy}$<br>$v_{V2}-v_{V1}=2bc_y$<br>$w_{V2}-w_{V1}=0$<br>$i=1, j=1 \text{ and } k=0$<br>$u_{V3}-u_{V1}=2ac_x+2bc_{xy}$<br>$v_{V3}-v_{V1}=2bc_y$<br>$w_{V3}-w_{V1}=0$<br>$i=1, j=0 \text{ and } k=0$<br>$u_{V4}-u_{V1}=2ac_x$<br>$v_{V4}-v_{V1}=0$<br>$w_{V4}-w_{V1}=0$<br>$i=0, j=0 \text{ and } k=1$<br>$u_{V5}-u_{V1}=2cc_{xz}$<br>$v_{V5}-v_{V1}=2cc_{yz}$<br>$w_{V5}-w_{V1}=2cc_z$ | $i=0, j=1 \text{ and } k=1$<br>$u_{V6}-u_{V1}=2bc_{xy}+2cc_{xz}$<br>$v_{V6}-v_{V1}=2bc_y+2cc_{yz}$<br>$w_{V6}-w_{V1}=2cc_z$<br>$i=1, j=1 \text{ and } k=1$<br>$u_{V7}-u_{V1}=2ac_x+2bc_{xy}+2cc_{xz}$<br>$v_{V7}-v_{V1}=2bc_y+2cc_{yz}$<br>$w_{V7}-w_{V1}=2cc_z$<br>$i=1, j=0 \text{ and } k=1$<br>$u_{V8}-u_{V1}=2ac_x+2cc_{xz}$<br>$v_{V8}-v_{V1}=2cc_{yz}$<br>$w_{V8}-w_{V1}=2cc_z$ |                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                           |

Fig A.2 Equations for boundary conditions enforced at the faces, edges and corners of the unit cell [1]

## A 1.2 Meso-scale model

The meso scale modelling is carried out to predict the mechanical characteristics of a composite lamina for various loading cases. The meso scale unit cells consists of yarns arranged in the form of a 0°/90°/90°/0° NCF fabric architecture embedded in a matrix. The details of yarn measurements of yarn width and yarn height can be found in [1]. The material model for the matrix is the same as that mentioned for the micro-scale model. The yarns characteristics are modelled similarly to that of fibre except that it has strain rate scaling defined for stiffness and strength. The periodic boundary conditions applied to the meso-scale unit cell are the same as those followed for the micro-scale model (Fig A.2). The information obtained from the micro-scale models would be used as an input to the meso scale models.

## Appendix 2 Derivation for stress-strain equation for SHPB

The properties of the bar and the specimen's dimensions are required for data analysis. Hence, it is necessary to do a proper calibration test and evaluate the actual properties such as wave speed ( $C$ ) of the bars. Each of the strain signals obtained in the form of voltage signal needs to be converted to a strain signal based on its conversion factors. The conversion factors are related to the type of circuit used, gauge factor, bridge excitation and the amplification factor used by the amplifier. The measured quantities ( $\varepsilon_I(t)$ ,  $\varepsilon_R(t)$  and  $\varepsilon_T(t)$ ) from the SHPB test can be used for evaluation of the average stress and strain of the specimen. This derivation followed here assumes that the material and cross section of incident bar and transmission bar are the same.

The solution of Eq. 3-2 using D'Alembert's method applied to the incident bar is

$$u(x, t) = f(x - c_0 t) + g(x + c_0 t) \quad \text{Eq A. 4}$$

The solution of Eq. 3-2 using D'Alembert's method applied to the transmission bar is

$$u(x, t) = h(x - c_0 t) \quad \text{Eq A. 5}$$

In Eq A. 4 and Eq A. 5,  $f$ ,  $g$  and  $h$  represents independent arbitrary functions. Also, it can be noted that  $x - c_0 t$  is forward travelling wave whereas  $x + c_0 t$  is a backward moving wave. Hence, to represent both the incident and reflected wave, the particle displacement in the incident bar is represented as the sum of functions representing the forward and backward moving waves as seen in Eq A. 4. However for the transmission bar, the particle displacement of the bar is only represented by a forward moving wave alone (Eq A. 5).

Differentiating Eq A. 4 with respect to time gives the deformation rate at the incident bar – specimen interface as:

$$\dot{u}_1 = c_0(-\varepsilon_I + \varepsilon_R) \quad \text{Eq A. 6}$$

Differentiating Eq A. 5 with respect to time gives the deformation rate at the specimen - transmission bar interface as:

$$\dot{u}_2 = -c_0 \varepsilon_T \quad \text{Eq A. 7}$$

Hence, the average strain rate across the specimen is given by:

$$\dot{\varepsilon} = \frac{\dot{u}_1 - \dot{u}_2}{L_s} \quad \text{Eq A. 8}$$

Substituting Eq A. 6, Eq A. 7 in Eq A. 8,

$$\dot{\varepsilon} = \frac{c_0(-\varepsilon_I + \varepsilon_R + \varepsilon_T)}{L_s} \quad \text{Eq A. 9}$$

The force observed by the specimen at incident bar – specimen interface is given by:

$$F_1 = A_B E_B (\varepsilon_I + \varepsilon_R) \quad \text{Eq A. 10}$$

The force observed by the specimen at specimen - transmission bar interface is given by:

$$F_2 = A_B E_B \varepsilon_T \quad \text{Eq A. 11}$$

The specimen is expected to achieve stress equilibrium after the initial ringing period based on the assumption, hence based on this, condition becomes

$$F_1 = F_2 \quad \text{Eq A. 12}$$

Based on Eq A. 12, we get a condition as,

$$\varepsilon_T = \varepsilon_I + \varepsilon_R \quad \text{Eq A. 13}$$

Accordingly Eq A. 9 can be re-written as,

$$\dot{\varepsilon} = 2c_0 \varepsilon_R / L_s \quad \text{Eq A. 14}$$

Based on the condition given by Eq A. 13, average stress experienced by the specimen based on Eq A. 10 and Eq A. 11 becomes,

$$\sigma(t) = A_B E_B \varepsilon_T / A_s \quad \text{Eq A. 15}$$

The engineering strain observed by the specimen can be obtained integrating Eq A. 14 with respect to time (from 0 to  $t_0$ ),

$$\varepsilon_s(t) = \left( \frac{2c_0}{L_s} \right) \int_0^{t_0} \varepsilon_R dt \quad \text{Eq A. 16}$$

### **Appendix 3      Reflected tensile pulse concept**

The concept of using the reflected pulse from a conventional compression SHPB already developed to generate the tensile pulse for loading the fibre specimen was also evaluated as part of the study. The setup for this is based on the methodology described by Theodore [163]. The test setup consisting of two identical bars with slotted end was considered in this feasibility study for evaluation of reflected tensile pulse concept. The ends were assumed to be slotted to have provision for mounting the single fibre/fibre bundles. Two semi-circular discs made of the same material as that of the bars held between the ends of the bars placed on either side of the specimen helps to transfer the compressive pulse generated by the impact of striker (Fig A. 3). The semi-circular disc was assumed to have the length/thickness same as that of the gauge length of the specimen material and assumed to be held between the two bars through frictional force. The transmitted compressive pulse from the incident bar was assumed to load only the semi-circular discs due to higher structural stiffness compared to the specimen. The transmitted compressive pulse reflected as a tensile pulse from its transmission bar free end will load the specimen according to the reflected pulse concept. The semi-circular disc between the bars cannot take any tensile load as it is held only friction and will slide down under the influence of gravity. The characteristics wave time diagram for the reflected tensile concept is shown Fig A. 4. With reference to the figure, Red line shows the compressive wave and green line shows the tensile stress wave and blue line shows that there is both compressive wave and tensile wave at the same time hence shows the superimposed strains on the bar. The compressive wave needs to be separated from the signal to actually get the transmitted tensile wave. That is with the present compression setup it would be difficult to separate the two pulses as the bars are of same length. But the option of using different length for incident bar and the transmission bar is a possible solution for separating the two pulses.

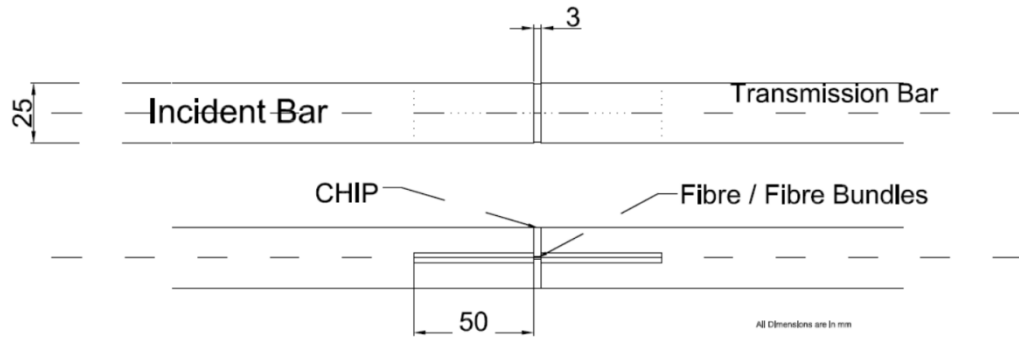


Fig A. 3 Front view and plan of the modifications incorporated in the incident and transmission bar at the specimen interface

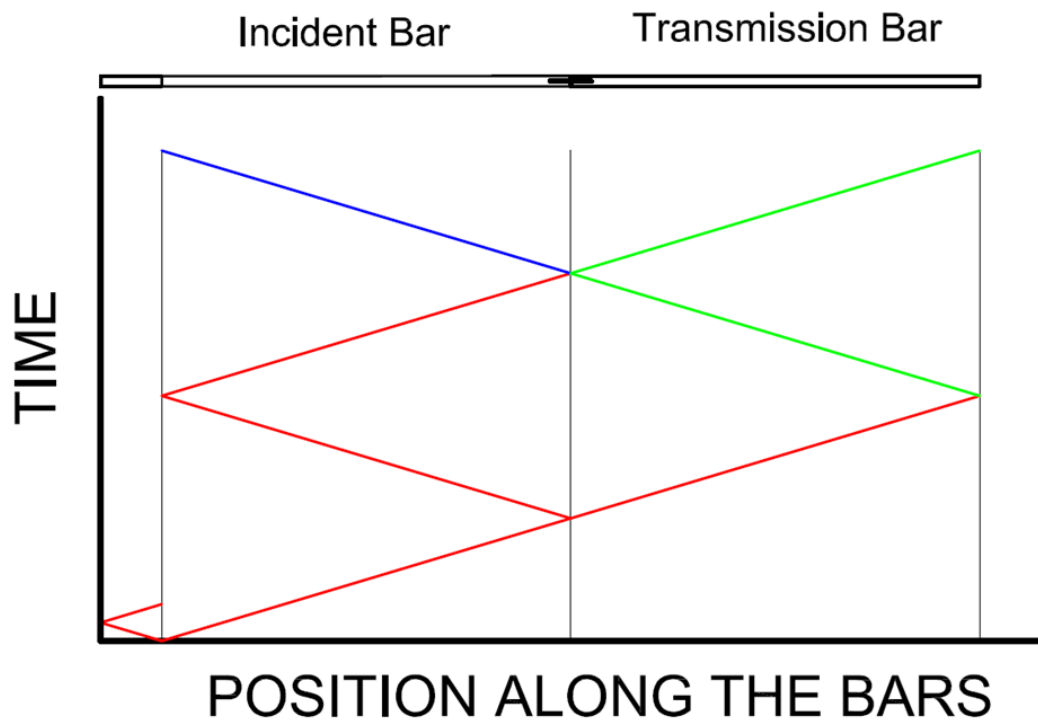


Fig A. 4 Characteristic wave time diagram for reflected tensile concept for present SHPB setup

## Appendix 4 Evaluation of fibre failure in SHTB

To understand the stress build up phenomenon, a specimen subjected to a perfectly rectangular pulse from the incident bar-specimen interface AA travelling towards specimen-transmission bar interface BB (Fig A. 5a) is considered for evaluation.

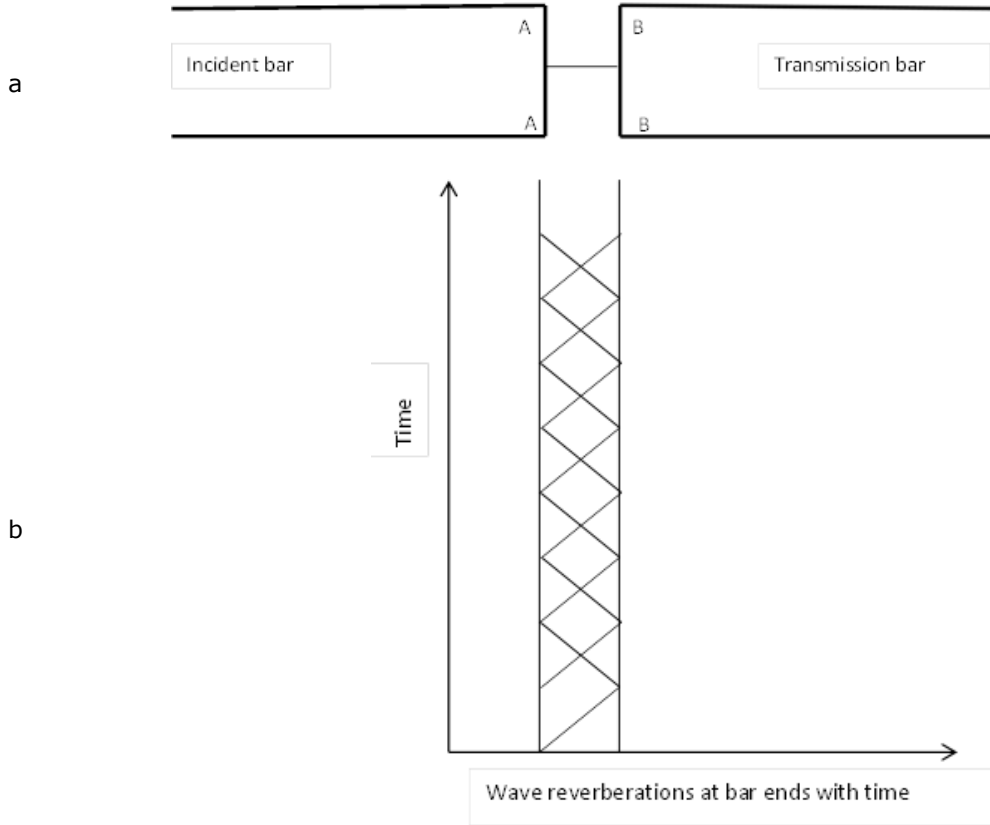


Fig A. 5 Wave reverberations at bar end with incremental incident pulse with time

A perfect rectangular pulse can be assumed to be obtained in an actual test, if there is perfect contact between the striker and the bar and the pulse propagates without dispersion. This ensures that specimen is under a state of one dimensional stress. The rectangular pulse has a pulse duration which is given by  $T = \frac{2L_0}{C_0}$ . Now since it is a rectangular pulse, it can be assumed to combination of  $N \left( = \frac{T}{\Delta t} \right)$  rectangular pulses, of pulse duration,  $\Delta t$ . The transit time required for an incident wave to travel through the specimen is given by  $\Delta t = L_s / C_s$ . So, the specimen gets subjected to a loading due to incremental incident stress pulse as well as due to reverberations across the interfaces (Fig A. 5b). The reverberations within the interfaces (reflection of wave across a fixed end) and the incident loading pulse are of the same characteristics.

A specific case of a single carbon fibre specimen being subjected to an incident strain pulse of 200 microstrains is taken for analysing the stress build up. In this situation, two different conditions of specimen loading were evaluated. The first condition will be referred to as Theory 1 and will consider both ends to be free to move. This condition represents a situation which is comparable to the loading observed by the specimen in the SHPB condition. The second condition is named as Theory 2 which considers that the right end (BB) condition is fixed and the left end (AA) is free to move.

Since the difference in the magnitude of velocity from each theory is marginal (Fig A. 6), it could be concluded that the prevailing condition for SHPB (Theory 1) is quite similar to a condition of specimen being loaded through displacement control in a UTM (Theory 2). The stress observed by the specimen for both the theories showed only a marginal difference (Fig A. 7). The stress build up can be understood by considering the force equilibrium evaluation at both the interfaces at each incremental time. The average specimen stress evaluated over a period of time based on the evaluation of the forces shows a gradual build-up of stresses in the specimen for both the theories (Fig A. 7a). Once the stress builds up reaches the dynamic failure limit of the specimen (not modelled in the plot), the fracture of the specimen occurs. The failure is induced during the loading process, if the initial stress pulse is of significant magnitude such that it can induce fracture within the duration of the pulse. Since it depends on  $\Delta t$  and impedances, the failure process and the initial critical strain pulse required to induce failure will vary with the specimen material type. Fig A. 7a shows that within the pulse duration period, the fibre has seen the stress levels corresponding to failure (4800 MPa). Closer examination of the stress build-up for both the theories shows a similar trend for stresses at the interfaces AA and BB (Fig A. 7b). This means that the specimen loading condition can be considered to be similar to that in the quasi-static loading without much significance of the wave effects. This representation is for the ideal case; however, in actual situations, the loading pulse could be a ramp, with dispersion etc. Also, this representation assumes that the fibre is directly attached to the bars. Attachments for specimen mounting can induce wave dispersion due to various factors such as size, shape and method of attachment followed.

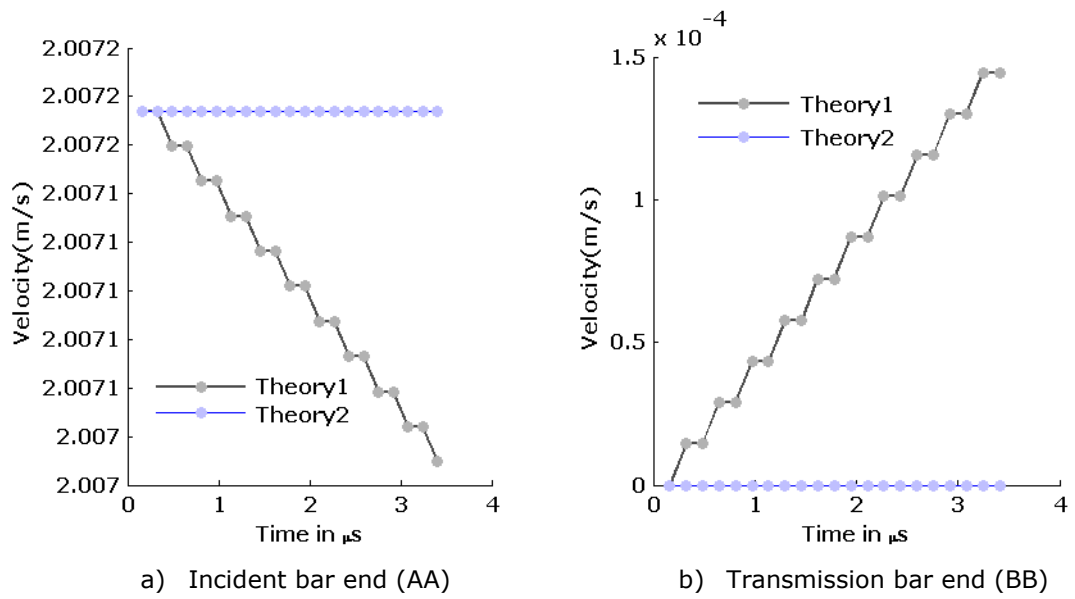


Fig A. 6 Velocity condition on each bar ends for both theories

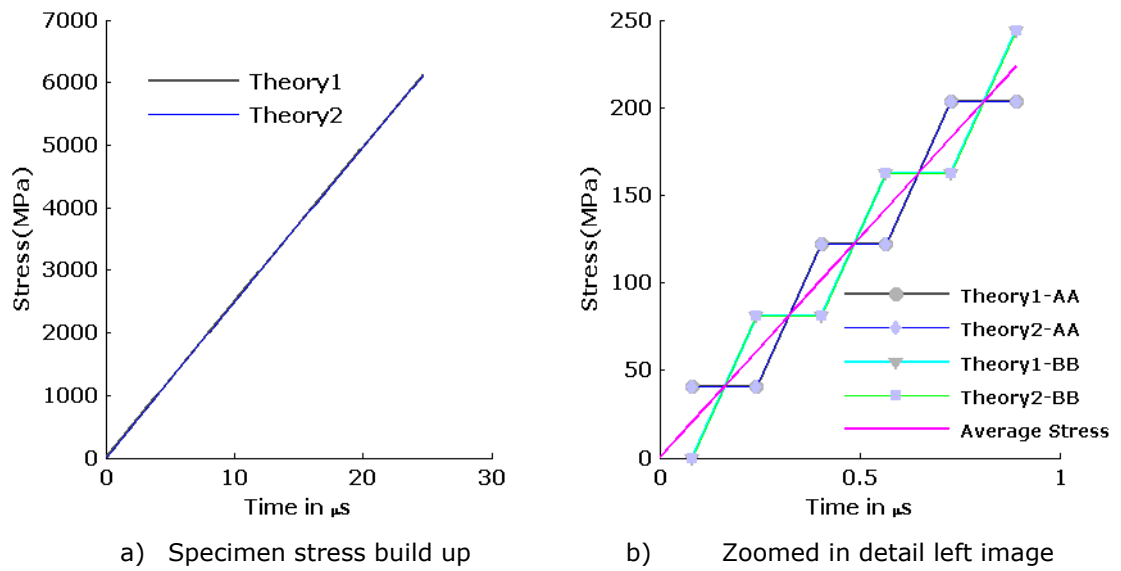


Fig A. 7 Stress build up in specimen for both the theories

## Appendix 5 Design of sub systems of SHTB

### A 5.1 Spring based actuators and its design

The design of spring is based on the assumption that spring characteristics obey Hooke's law ( $F = kx$ ). The force required in the design is based on the average force generated in the spring. The total energy required to be stored in the spring is evaluated based on the total moving mass (striker mass (including flange) and the effective mass of spring) and maximum desired striker velocity. The effective mass of the spring is taken to be  $1/3^{\text{rd}}$  of the total spring mass. The wire diameter and the mean diameter are selected such that the design meets the energy and force requirements as well as the allowable stress levels in the spring. The spring configuration which was identified from the quotation which nearly meets the initial design requirements is given in Table A. 1.

Table A. 1 Final spring configuration

| Final Spring Configuration           |                   |
|--------------------------------------|-------------------|
| Wire Diameter (mm)                   | 6.4               |
| Mean Coil Diameter(mm)               | 51.2              |
| Spring Rate (N/mm)                   | 3.54              |
| Free Length (mm)                     | 700               |
| Solid Length (mm)                    | 236.8             |
| Solid Load (N)                       | 1639.8            |
| Spring Index                         | 8                 |
| Active Coils                         | 35                |
| Rigidity Modulus(N/mm <sup>2</sup> ) | 79300             |
| Density of Wire(kg/m <sup>3</sup> )  | 7830              |
| End Type                             | Closed and Ground |
| Helix Angle(degrees)                 | 6.96              |
| Outer Diameter(mm)                   | 57.6              |
| Inner Diameter(mm)                   | 44.8              |
| Weight (kg)                          | 1.5097            |
| Natural Frequency (rpm)              | 1498.9            |
| Wire Length(mm)                      | 5993.5            |
| Solid Stress(N/mm <sup>2</sup> )     | 955.39            |

### A 5.2 Design of an electromagnetic release

The purpose of an electromagnetic release is to hold the compressed spring and striker assembly and release the striker based on deactivation of electromagnet.

The electromagnetic release basically consists of inner annular tube, current carrying coils, outer tube, hollow disc cap and the flanged hollow striker assembly. The current carrying coil is wound on the inner annular tube. The inner annular tube is a hollow cylinder enclosing the spring and the incident bar. The length of the inner annular tube is taken such that it is 10 percent more than the solid length of the spring. The outer tube also has a hollow cylindrical cross section and encloses the current carrying coil and the inner annular tube. The hollow disc cap connects the inner annular tube and the outer tube. The inner annular tube and outer tube holds the flanged hollow striker assembly across its cross section and completes the magnetic circuit for the electromagnet. The plate thickness of the hollow striker flange is kept as 4mm to withstand the magnetic force acting on it. The force to be held by an electromagnet is based on the maximum force that is generated by the spring. The cross sectional area of the inner annular tube and the outer tube is varied so that it gives a magnetic flux density which is well below the saturation point in the magnetisation curve of the selected core material. The details of the electromagnetic release are given in Table A. 2.

### **A 5.3 Screw mechanism for compressing the spring**

The screw mechanism used for the compression of the spring consists of the following components.

- 1) Motor with belt driven pulleys
- 2) Chain drive system
- 3) Crosshead
- 4) Threaded rods
- 5) One double row ball bearing at fixed supported end for each threaded rod
- 6) One single row ball bearing at simply supported end for each threaded rod

The crosshead moves along the screw mechanism and applies the compressive force on the spring. The construction of the crosshead is such that the centre hollow circular unit has diameter greater than the striker's outer diameter and hence can move over the striker so that the compressive can be transferred to the spring through the armature plate directly. A chain drive is used to simultaneously transfer the torque applied at the pulley by the motor to both the threaded rods. Based on the movement of the crosshead, the linear motion is seen by the flanged hollow striker assembly at the incident bar and thus the spring is compressed.

Table A. 2 Details of electromagnetic release used in the SHTB

| Electromagnetic Release |           |    |        |                                 |        |                    |        |                         |                |        |
|-------------------------|-----------|----|--------|---------------------------------|--------|--------------------|--------|-------------------------|----------------|--------|
| Construction            |           |    |        | Reluctance of each Path(/henry) |        | Main Calculations  |        | Based on wire selection |                |        |
| Length                  |           |    | 260.48 | Ist Length                      | 111032 | B^2                | 0.73   | Circular mils           |                | 666.13 |
| Inner Annular Tube      | Inner Dia | mm | 63     | 2nd Length                      | 15734  | B                  | 0.85   | Turns/Layer             |                | 209.72 |
|                         | Outer Dia | mm | 84     | 3rd Length                      | 81628  | Total Flux         | 0.0021 |                         | <b>Rounded</b> | 210    |
|                         | Mass      | kg | 4.08   | Air Gap1                        | 164110 | Total Ampere Turns | 1096   | Layers                  |                | 8.06   |
|                         | Area      | m2 | 0.0025 | Armature                        | 39333  | I(amps)            | 0.65   |                         | <b>Rounded</b> | 10     |
| Outer Tube              | Inner Dia | mm | 101    | Airgap2                         | 120650 | No. of Turns       | 1690   | Total Length            | mm             | 485082 |
|                         | Outer Dia | mm | 120    |                                 |        | AWG no.            | 21     |                         | m              | 486    |
|                         | Mass      | kg | 5.54   |                                 |        |                    |        |                         | ft             | 1592   |
|                         | Area      | m2 | 0.0033 |                                 |        |                    |        | Wire Mass               | kg             | 2.25   |
| Total                   | Mass      | kg | 11.06  |                                 |        |                    |        | Resistance              | ohms           | 174    |
|                         |           |    |        |                                 |        |                    |        | Inductance              | Henry          | 5.37   |

## **Appendix 6      Void content and volume fraction studies**

The samples for void content analysis were cut from the cured laminates of size 20mm\*20mm using the diamond cutting machine. The cut specimens were then positioned inside a cylindrical mould for development of the holding pattern for polishing and grinding the void analysis samples. The mixture polyester resin with accelerator (0.5%) and catalyst (1%) were poured into the mould and were allowed cure in room temperature. Then, one of the circular faces was grounded to make it flat and was used as reference during polishing of opposite face. The face used for void analysis was polished with eight different grades of emery paper (240, 400, 600, 800, 1200, 2500 and 4000). The sample was further polished with alumina powder for obtaining the surface finish required for imaging through digital optical microscope. The predefined procedure prescribed for void analysis using the optical microscope was followed for obtaining images for void analysis. The images of the cross section obtained from microscope were analysed using ImageJ software through image processing of the void areas and analysis of the void information. The void analysis showed different void content (2-9%) based on the assessment of the void signatures in the section for the same void analysis sample (Fig A. 8). There is also subjective judgement involved in this method of analysis. The higher void content of the sample was mainly due to the poor consolidation of the laminate at the edges and also can be attributed to the artefacts involved in the preparation of the sample. In general, the quality or the consolidation tends to be poorer at the edges of the laminate. Since the laminate was of small size, only two samples were cut from the edges which may not be the actual representation of the void content in the middle regions of the laminate. The same approach was used for volume fraction studies on the composite laminates fabricated. The volume fraction of the laminate varied from 61-68 % (Fig A. 9).

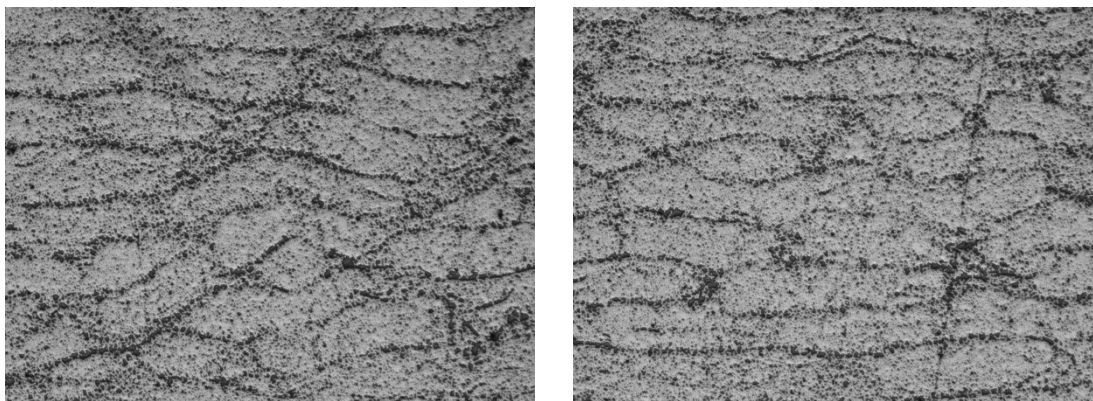
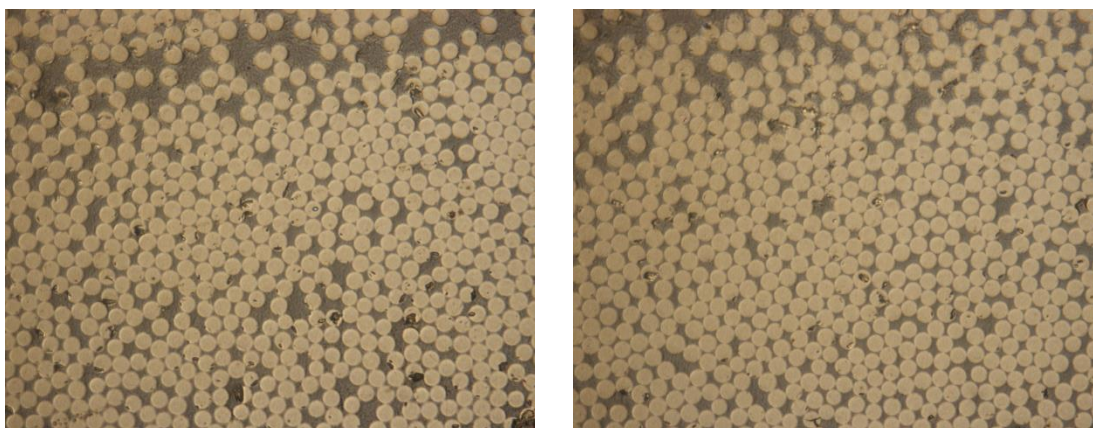


Fig A. 8 Micrographs captured on avoid analysis sample of UD CFRP laminate



a) Volume fraction = 65.47%

b) Volume fraction = 69.38%

Fig A. 9 Volume fraction of IM7-8552 UD CFRP laminate

## Appendix 7 Quasi-static compressive characteristics of unidirectional composites for different specimen aspect ratio

### A 7.1 Longitudinal compressive characteristics

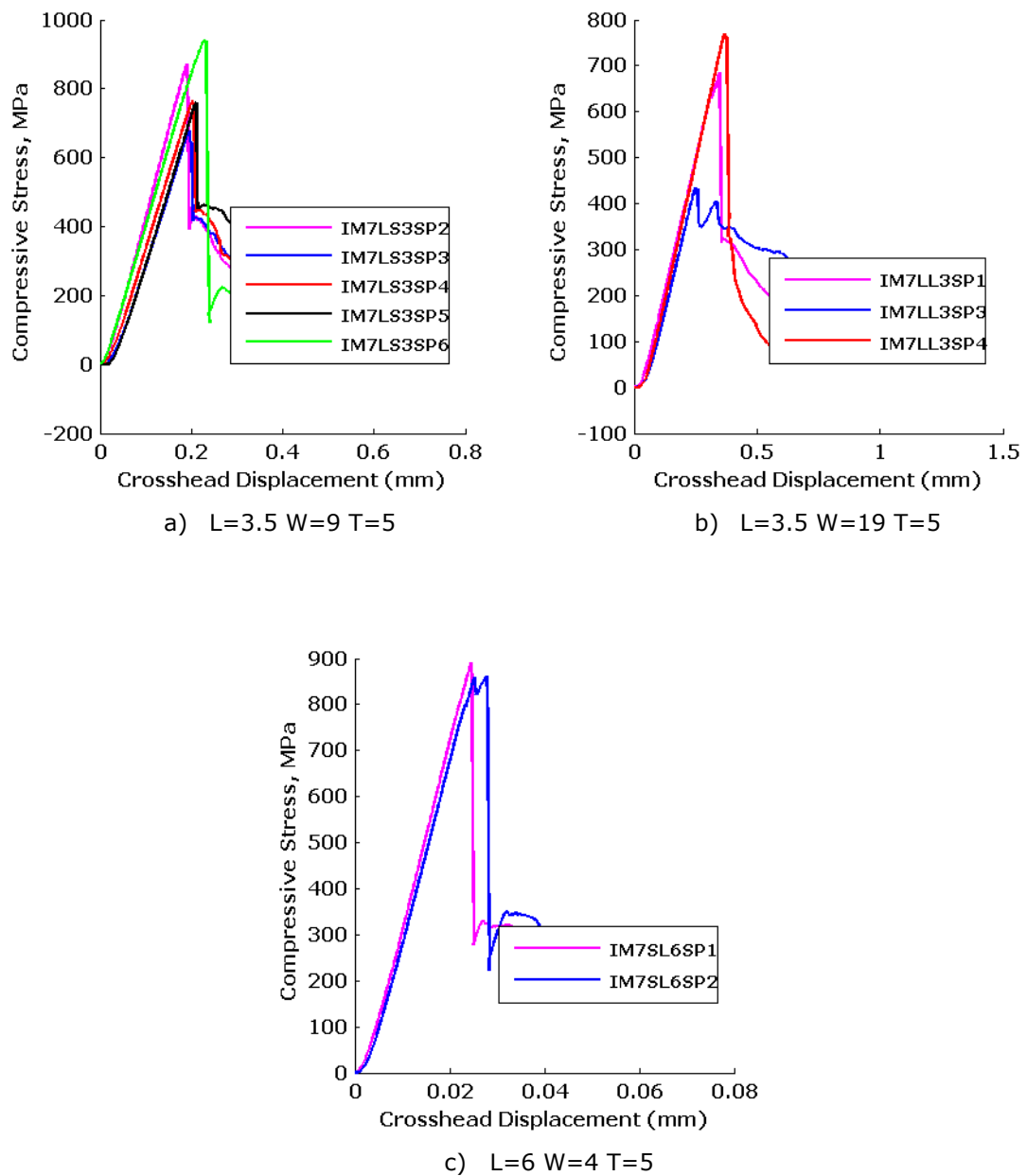
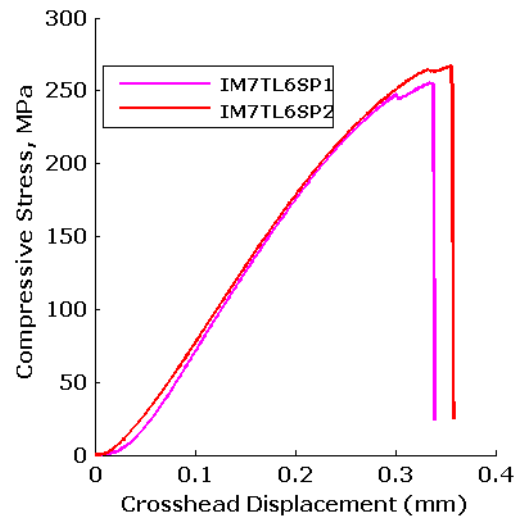


Fig A. 10 Stress-cross head displacement curves of UD prepregs composites subjected to longitudinal loading for different specimen aspect ratios

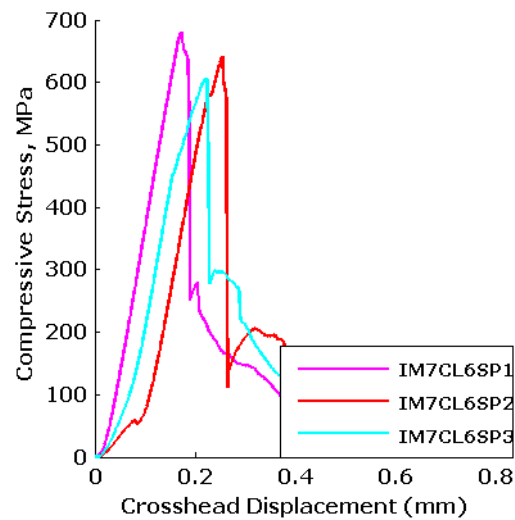
## A 7.2 Transverse compressive characteristics



a) L=6 W=5 T=5

Fig A. 11 Stress-cross head displacement curves of UD prepregs composites subjected to transverse compressive loading

## A 7.3 Compressive characteristics of cross ply laminates



a) L=6 W=4 T=5

Fig A. 12 Stress-cross head displacement curves of UD prepregs cross ply composite laminates subjected to compressive loading

## Appendix 8 Calibration of VISHAY 2310B signal conditioning amplifier and NuDAQ PCI-9812/9810 data acquisition card

### A 8.1 Evaluation of gain for NuDAQ PCI-9812/9810 data acquisition card based on signal from a function generator

An output from signal generator (GW Instek) in the form of sinusoidal voltage signal was monitored simultaneously using an oscilloscope (GW Instek) and to one of the channels of the NuDAQ PCI-9812/9810 data acquisition card. Likewise, signals observed for each channel in the data acquisition card was recorded for the same input waveform features from the function generator using the DAQ software AD Logger. Fig A. 13 shows that there is very little observable difference between the gain settings of the NuDAQ channels.

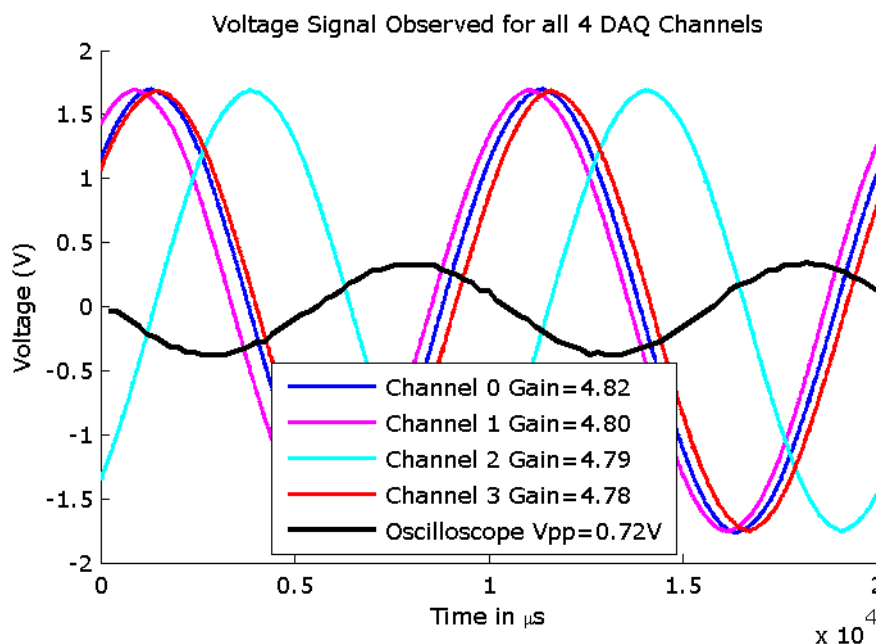


Fig A. 13 Gain observed for NuDAQ channels for same signal generator input

However, there is a gain of 4.8 compared to the signal waveform observed in the oscilloscope. Since the recording of the data for each channel in the DAQ card was performed at different times, it could be observed that the each of the waveform recorded were at different phases with respect to each other. The gain was evaluated by comparison of peak to voltage for each channel with respect to peak to peak voltage of the waveform recorded from the oscilloscope.

## A 8.2 Evaluation of gain for VISHAY 2310B signal conditioning amplifier channels based on inbuilt shunt calibration method

A 2 wire 120 ohm resistance strain gauge was shunted with inbuilt shunt resistance available for each channel of the signal conditioning amplifier for evaluation of its gain settings. A simulated tensile strain of 1000  $\mu$  strains was expected for the inbuilt shunt resistance available in the amplifier channels. The equivalent voltage reading for the simulated strain was appropriately calculated for the gain setting and the gage factor was then used for comparison with the voltage shift observed in the oscilloscope. Fig A. 14 shows that the variation in voltage reading observed for the Gain 1 settings for all the channels of the signal conditioning is within 2 percent compared to predicted voltage based on the simulated strain.

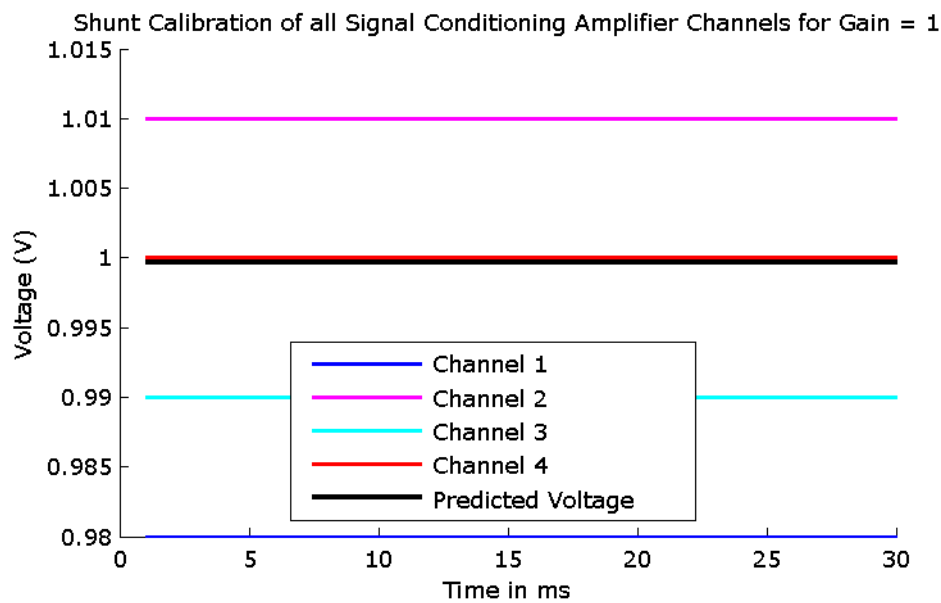


Fig A. 14 Shunt calibration of all channels of signal conditioning amplifier

For other gain settings, it can be observed that in most of the cases error is within  $\pm 2$  percent with random exceptions for certain gain settings (Fig A. 15). Similar voltage shifts were observed with external shunt calibration with the same strain gauge with a large value of shunt resistance (30K ohms) except that it simulated compressive strain compared to the inbuilt shunt calibration method where tensile strains were simulated in each channels.

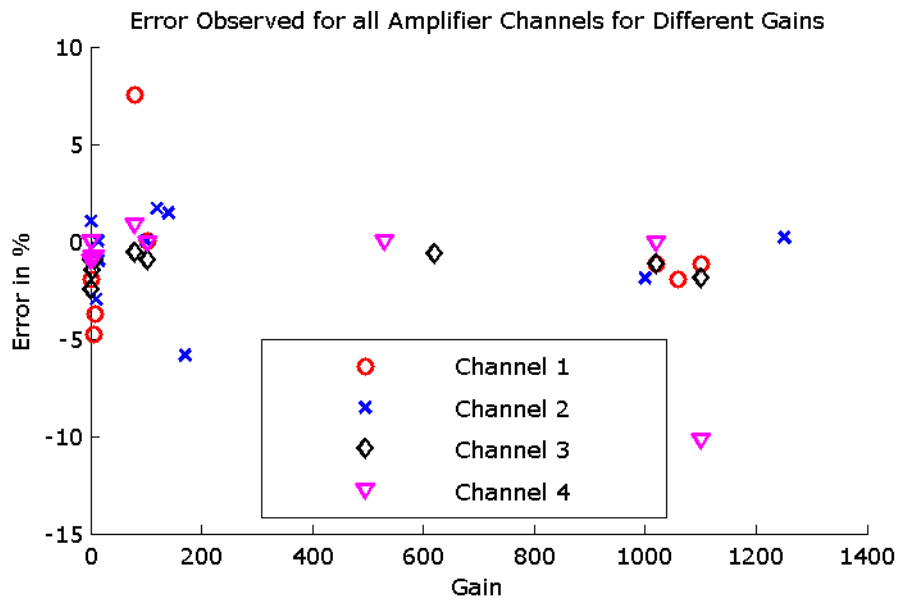


Fig A. 15 Error observed in voltage observed to predicted voltage in each channel for various gain settings

### A 8.3 Evaluation of output impedance settings of signal conditioning amplifier unit

It was observed that there is a voltage drop when the NuDAQ card (Input impedance= $50\Omega$ ) is connected to the signal generator (Input impedance= $50\Omega$ ) due to comparable impedance between the two systems (Fig A. 16). From Fig A. 16, it could be observed that voltage drop is nearly half of the supply voltage was measured without NuDAQ being connected. Hence, to assess the impedance settings of the signal conditioning amplifier, SHPB tests (with bars attached together with no specimens) were performed with and without the NUDAQ system connected to the signal conditioning amplifier. During each test the signal from the signal conditioning amplifier was monitored using the oscilloscope. The incident bar signals for the same input conditions shows only around 2.5 percent of variation in the input characteristics signal for the maximum amplitude observed (Fig A. 17). This variation could be due to the varying input conditions for these tests performed. This means that the impedance of the signal conditioning amplifier was much larger compared to the NuDAQ.

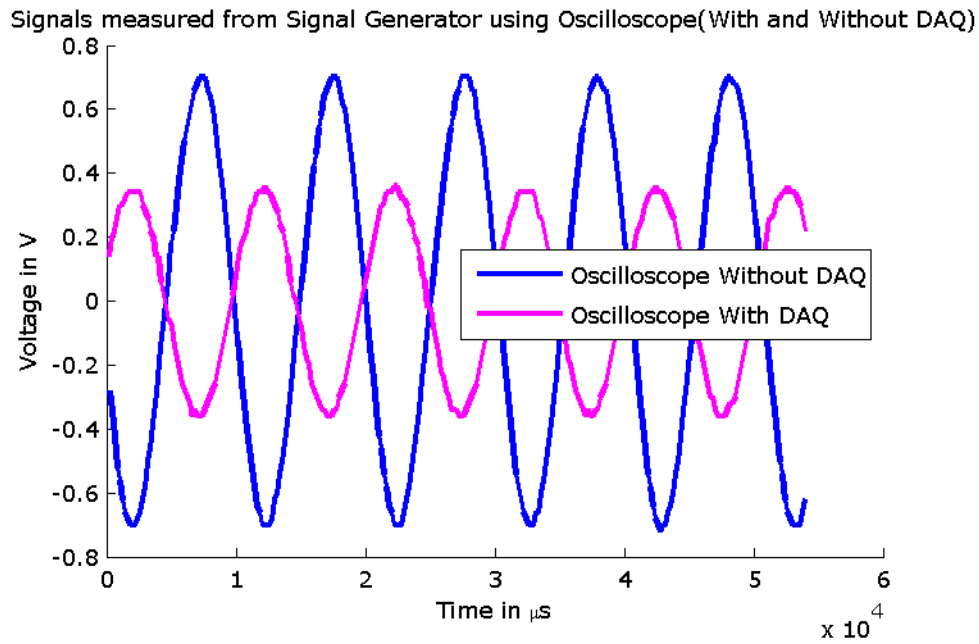


Fig A. 16 Voltage observed when signal generator was connected to NuDAQ

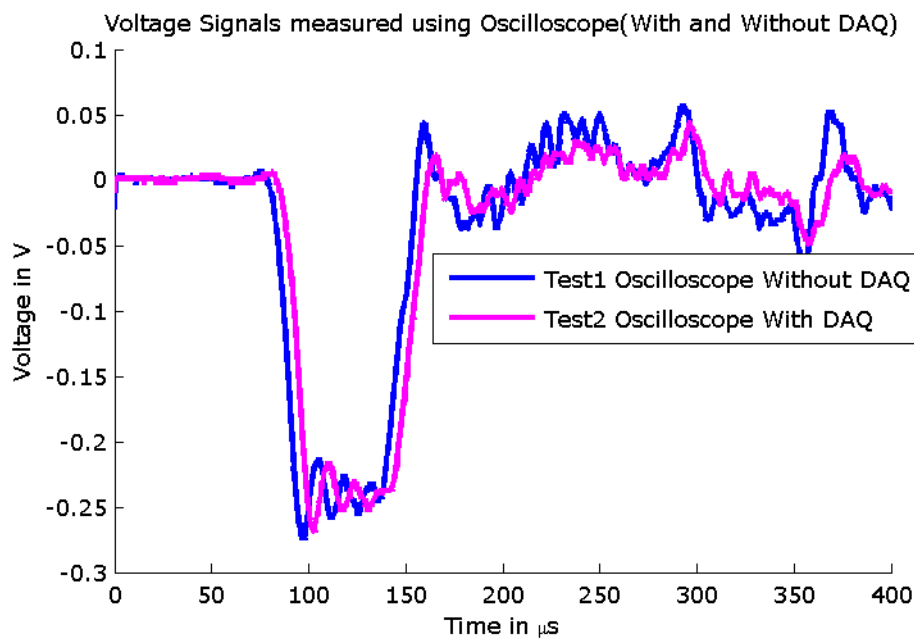


Fig A. 17 Voltage signal observed through signal conditioning amplifier for actual shpb tests with and without NuDAQ connected

## A 8.4 Exact evaluation of actual gain between oscilloscope and NUDAQ data

The gain setting of NuDAQ was found to be 4.8 for all the channels, when signal waveform from the signal generator was compared signal waveform observed in

the oscilloscope and NuDAQ (Fig A. 13). Since the recording of the data for each channel in the DAQ card was performed at different times, there is possibility of error as peak to peak voltages of different wave waveforms have been compared. The actual gain of NuDAQ was evaluated from the calibration tests performed using SHPB described in the previous section (Fig A. 18). The comparison of the peak to peak signals revealed a gain of 4.9988( $\approx 5$ ) between the oscilloscope and NuDAQ raw data. The data with corrected value was plotted and compared with oscilloscope data as shown in Fig A. 18. It was later observed that the hardware of NuDAQ was configured for  $\pm 1V$ , whereas the software was configured for  $\pm 5V$  voltage settings, which explains the reason for the gain factor of 4.9988( $\approx 5$ ).

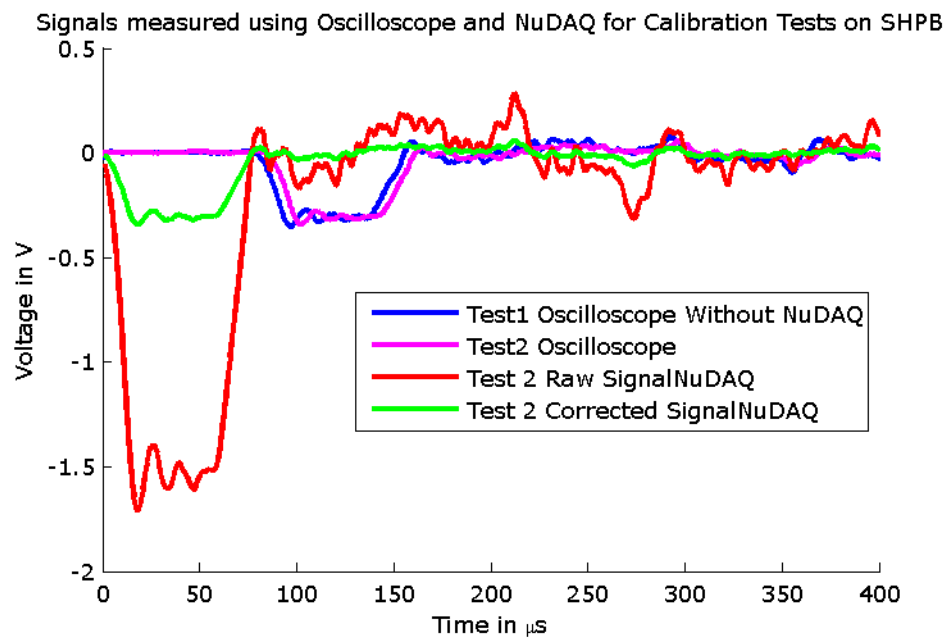


Fig A. 18 Corrected signal based on calibration factor compared with actual raw data NuDAQ and raw oscilloscope data

## Appendix 9      Approaches followed for processing SHPB data

The stress strain relationship used for indirect measurement is based on the assumption of stress equilibrium in either of the specimen interfaces. Also, the strain relationship is a function of the wave speed of the bars and hence it is important to evaluate accurate wave speed of the bars before processing the data. This section describes some of the approaches relevant to processing the data from SHPB.

### A 9.1 Determination of wave speed from calibration experiments

The purpose of the calibration test is to evaluate the wave speed in both the bars. The following calibration experiments were carried out using two test conditions without the specimen.

- Bars separated experiment
- Bars in contact experiment

A typical signal obtained for bars separated approach is shown in Fig A. 19.

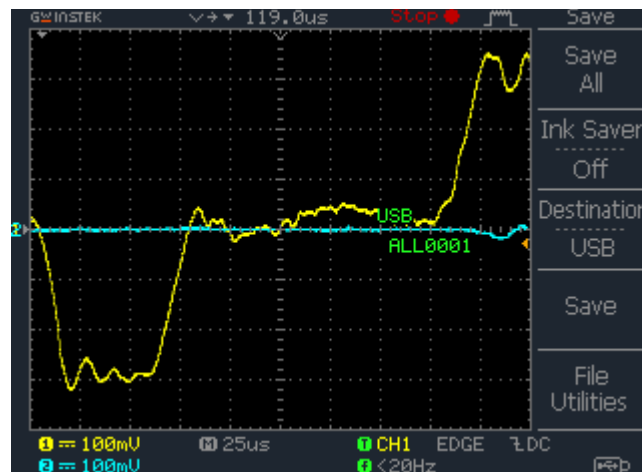


Fig A. 19 Raw signal as captured through oscilloscope from bars separated calibration study

For the bars separated approach, there will be not be any transmission bar signal other than the noise or signal picked up because of base vibration. It could be observed from the incident bar signal that the time duration between the incident and reflected signal is in the range of 200-205 microseconds. Hence, the

calculated wave speed is in the range of  $4850\text{--}5000\text{ms}^{-1}$  depending on the exact identification of the datum of the reflected signal.

A typical strain signal observed during a 'bars in contact' calibration test is as shown in Fig A. 20. As the bars are in contact, there should be no reflected signal from the interface with the incident bar data. Also, in theory, the transmission signal from the incident bar should be exactly the same as the incident signal as two bars of same material are in contact. However, in practice there will be a small reflected pulse and a marginal variation in the transmission bar characteristic due to the presence of air gap at the contacting surfaces. The wave speed in the transmission bar was evaluated from the ratio of twice the length of bar to the time interval between the corresponding points in the consecutive transmission signals (comparison of consecutive tensile pulse or compressive pulses). The wave speed thus evaluated was found to be lower ( $4600\text{ ms}^{-1}$ ) than the wave speed evaluated from the standard formula ( $\sqrt{E/\rho}$ ) which was  $4975\text{ms}^{-1}$ .

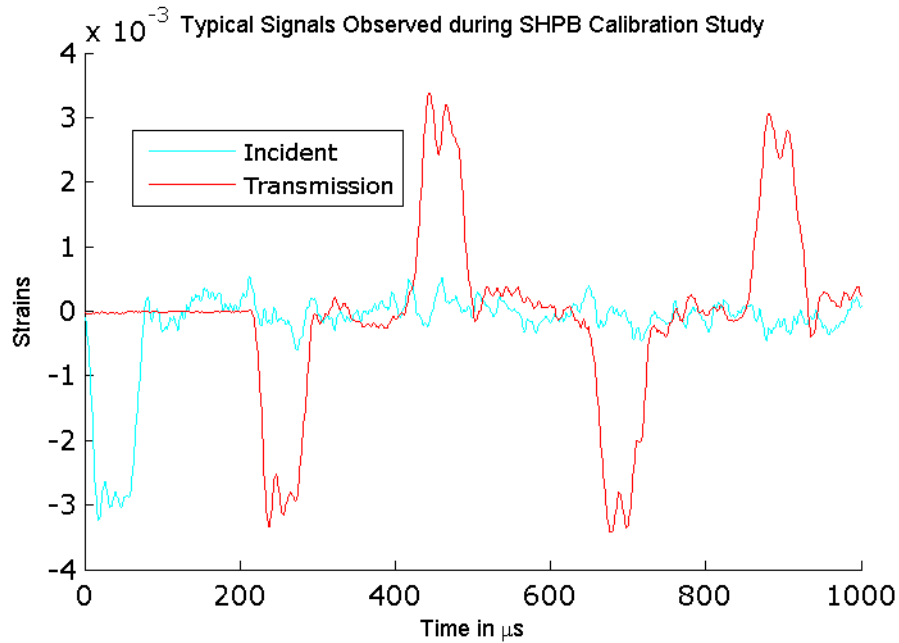


Fig A. 20 Raw signal converted to strains from 'bars in contact' calibration study

This difference in wave speed compared to theoretical value can be due to the difficulty in identifying the corresponding points from the two consecutive pulses due to the distortion in the signal as a result of superposition of the actual pulse with noise. No two rectangular pulses look alike in their characteristics and this could be the reason for the error. There are other methods which discuss on the measurement of wave speed such as the method proposed by Lifshitz and Leber

[196] which employs an iterative dispersion correction method to determine the wave speed in the bars.

The value of **4975ms<sup>-1</sup>** was used as the wave speed in the bars for evaluation of dynamic stress-strain relationship of the materials tested.

## **A 9.2 Time shifting method for identification of the datum points**

The identification of the datum point of each signal is important for evaluation of the time taken for the bar interfaces to reach stress equilibrium during specimen loading. The identification of the datum point of the reflected and the transmitted pulse is complicated because of the following reasons:

- 1) The tail ends of the incident signals are noisy and distorted.
- 2) Noise exists in the transmission bar signal because of the vibration experienced by the supports. This vibration is present even before the actual impact pulse reaches the strain gauge.

If  $L_0$  and  $L_1$  are the distances of the strain gauges mounted on incident and transmission bar from the respective specimen end interfaces and  $L_s$  is the length of the specimen, then the starting point for reflected wave from the starting point of incident bar is given by

$$T_r = \frac{2L_0}{C} \quad \text{Eq A. 17}$$

Similarly, the starting point of the transmitted signal from the instant the incident bar senses the incident pulse from the striker is given by

$$T_t = \frac{L_0 + L_1}{C} + \frac{L_s}{C_s} \quad \text{Eq A. 18}$$

where,  $C$  and  $C_s$  are the wave speeds in the incident/transmission bar (same material) and specimen respectively. The identification of the datum point for the transmission signal is based on an assumed value of wave speed of the specimen. A time shifting approach is therefore adopted in which the time when the incident wave reaches the specimen is taken as the origin for comparing the different signals. In order to achieve this, a backward time shift of the incident signal and a forward time shift of the reflected signal by a defined time increment were carried out (Fig A. 21). This time interval is the same for both the incident and reflected pulses as the strain gauge is at a fixed position from the specimen interface in the incident bar ( $T_r/2$ ). The forward time shift of the exact time of

transmission signal is obtained by subtracting the wave travel time inside the incident bar from  $T_t$  ( $T_t - \frac{L_0}{c}$ ). Hence, the approach followed in this study is to assume a wave speed for the specimen based on its equivalent in-plane modulus and density for evaluation of the backward time shift of the transmitted pulse. The data are then used for evaluation of the time taken to achieve stress equilibrium. The mandatory condition for stress equilibrium of the interfaces is given below

$$\varepsilon_T = \varepsilon_I + \varepsilon_R$$

Eq A. 19

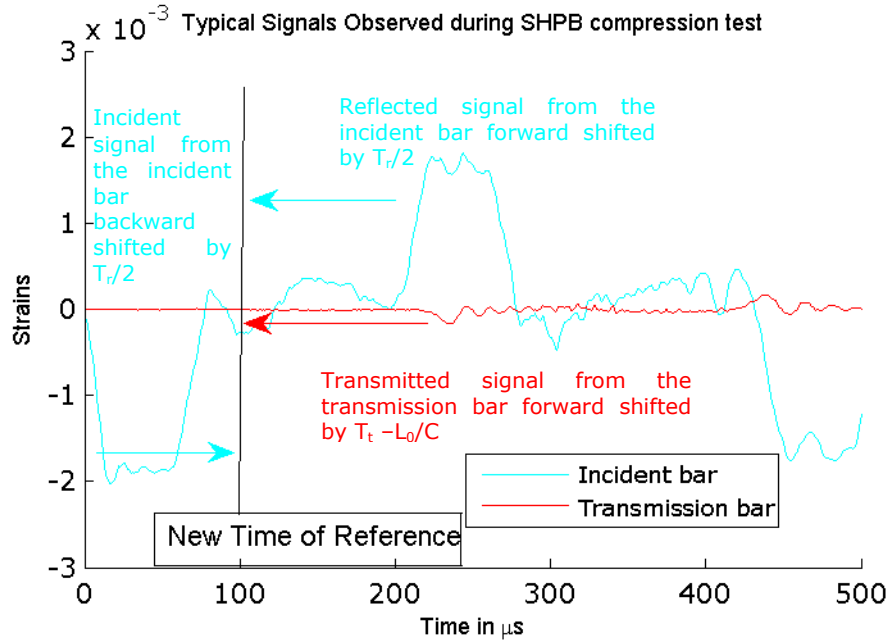


Fig A. 21 Time shifting approach followed for data processing of SHPB results

### A 9.3 Approach for checking stress equilibrium evaluation

The signals plotted on the new reference time based on time shifting approach (incident bar end as  $t=0$ ) are shown in Fig A. 22a. The incident and the reflected signal from the incident bar strain gauge were shifted to the new reference time which denotes the time at which the incident wave first reaches the incident bar-specimen interface. Similarly, the transmitted signal in Fig A. 22a obtained from forward shifting the transmission bar strain data corresponding to the time interval mentioned above. The data were used for evaluation of the state of stress equilibrium in the specimen by comparison of the 2-wave and 1 wave analysis (Fig A. 22b and Eq A. 19). The ' $V_i + V_r$ ' denotes the resultant strain pulse at the incident bar end which is based on the 2-wave analysis which is sum of the incident and reflected signal as given in right hand side of Eq A. 19 and ' $V_t$ '

denotes the strain pulse at the transmission bar end which is based on 1-wave analysis which is the left hand side of Eq A. 19. It can be observed that both the specimen ends were not in stress equilibrium during the initial phase of loading. However, during the load pick up region (shown circled), both the interfaces tends to show similar magnitude for the strain pulses which indicates that stress equilibrium exists at the specimen ends during this loading phase. Hence, such data obtained during testing can be used for processing the dynamic stress-strain characteristics of the material tested. One of the common approaches for achieving stress equilibrium during dynamic testing is to use pulse shapers [174, 199, 200] which helps in gradual loading of the specimen (larger rise time in the incident pulse), hence the specimen achieves stress equilibrium after initial 3-4 reverberation of the wave pulses.

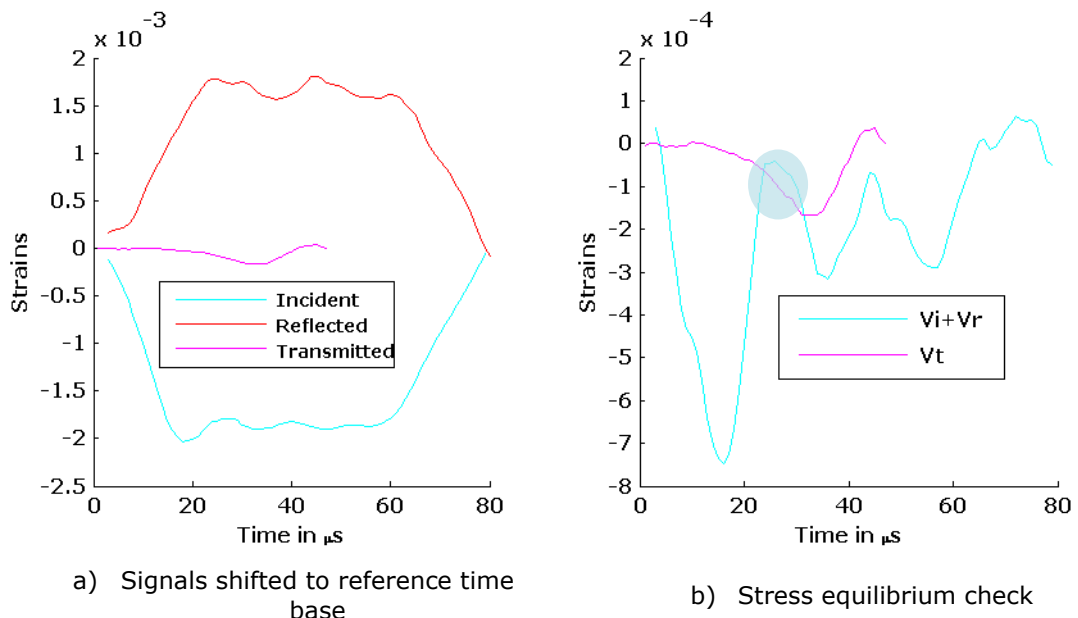


Fig A. 22 Processing of SHPB raw data for evaluation of stress equilibrium

## **Appendix 10    Curve replotting approach**

Since the plots obtained for each specimen was observed to be noisy, a modification of the curve was done for better comparison of its characteristic regions. The procedure followed for replotting the curve from the raw data is as given below:

- 1) Identify the steepest linear region in the curve.
- 2) Identify a line that fits the steepest linear region of the curve and its point of intersection with the x-axis.
- 3) Locate the nearest point in the strain data with respect to x coordinate of the intersection point.
- 4) Subtract all the strain data obtained with respect to the nearest data point identified in step 3.
- 5) Select the data corresponding to all non-negative values of strain data (both strain and stress)
- 6) Change the first value of stress as zero
- 7) Remove the data points up to the lowest data point (excluding the modified first point (0, 0)) that fit the steepest linear region plotted earlier.
- 8) Fit a linear curve that fits all the resulting points up to the maximum strength only.

## Appendix 11 Longitudinal compressive characteristics of IM7-8552 UD composites at high strain rates

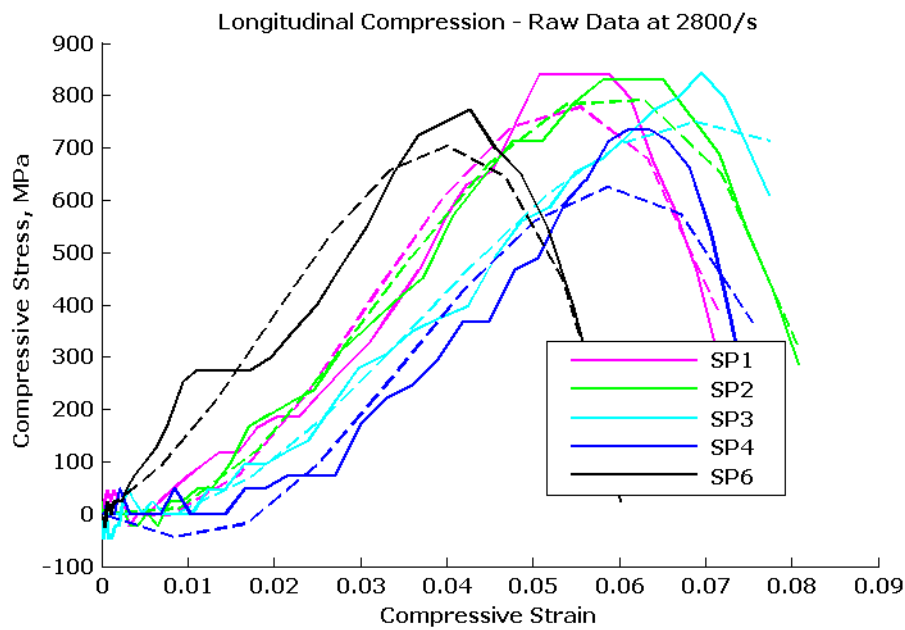


Fig A. 23 Longitudinal compressive characteristics of IM7-8552 composite laminates around  $2800 \text{ s}^{-1}$

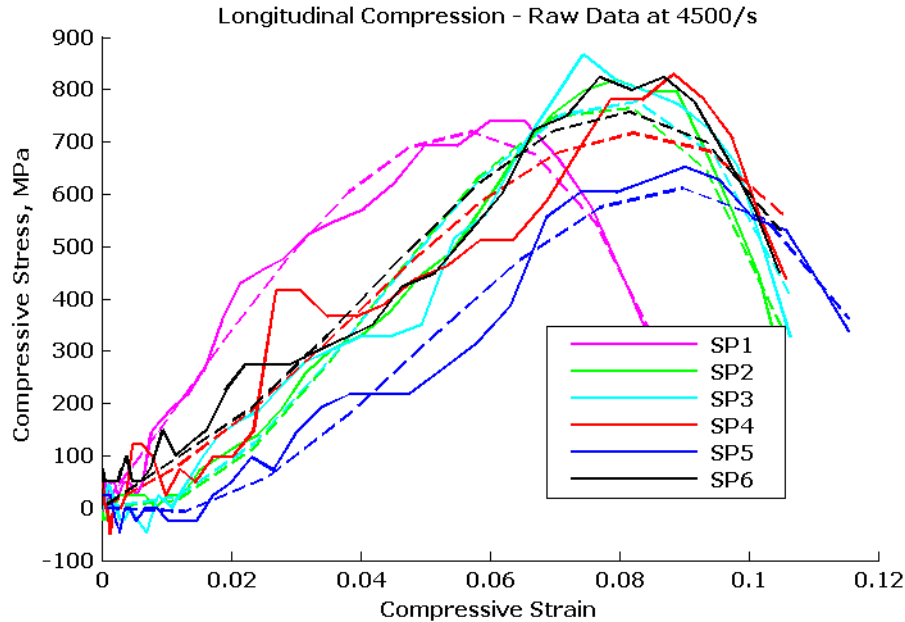


Fig A. 24 Longitudinal compressive characteristics of IM7-8552 composite laminates around  $4500 \text{ s}^{-1}$

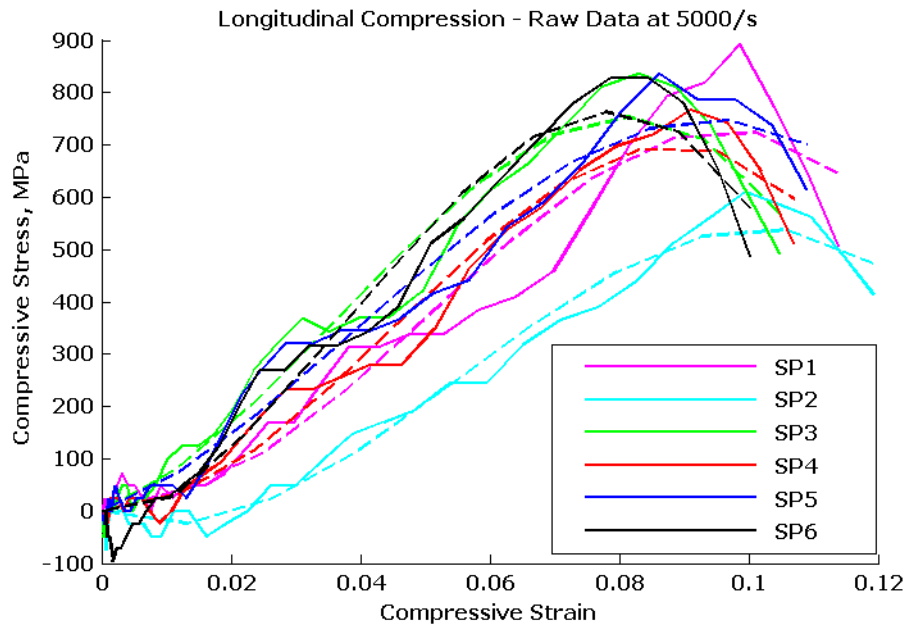


Fig A. 25 Longitudinal compressive characteristics of IM7-8552 composite laminates around 5000 s<sup>-1</sup>

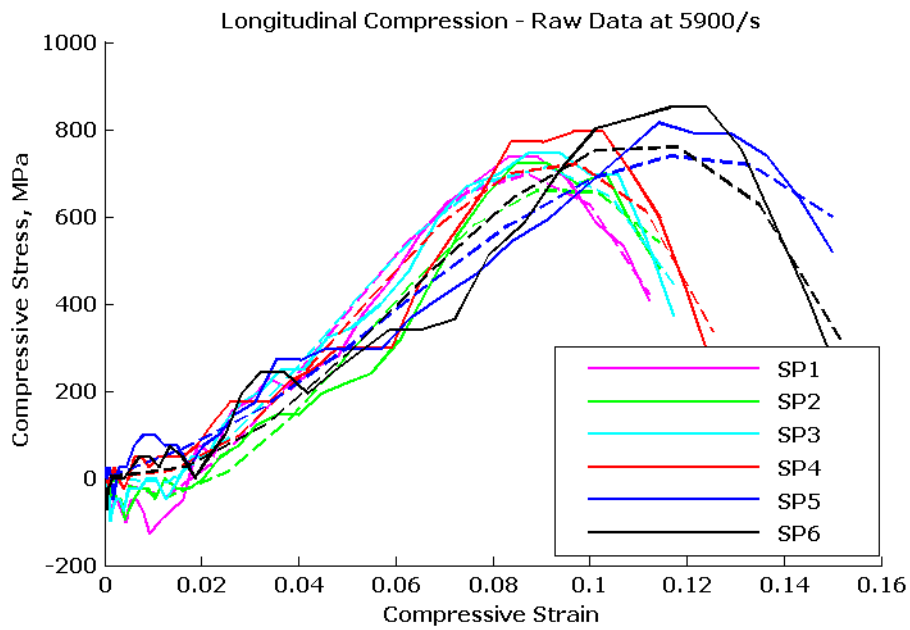


Fig A. 26 Longitudinal compressive characteristics of IM7-8552 composite laminates around 5900 s<sup>-1</sup>

## Appendix 12 Transverse compressive characteristics of IM7-8552 UD composites at high strain rates

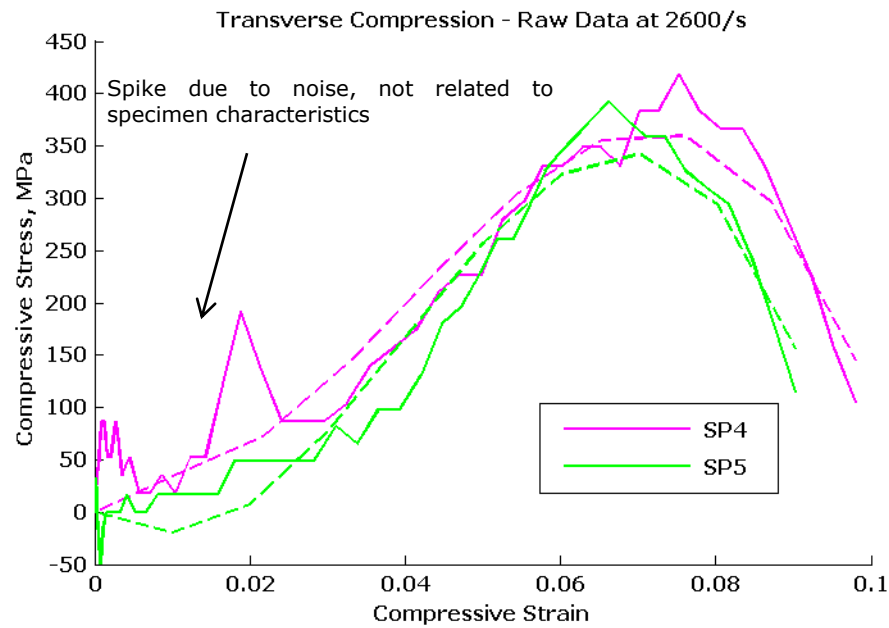


Fig A. 27 Transverse compressive characteristics of IM7 -8552 composites around  $2600 \text{ s}^{-1}$

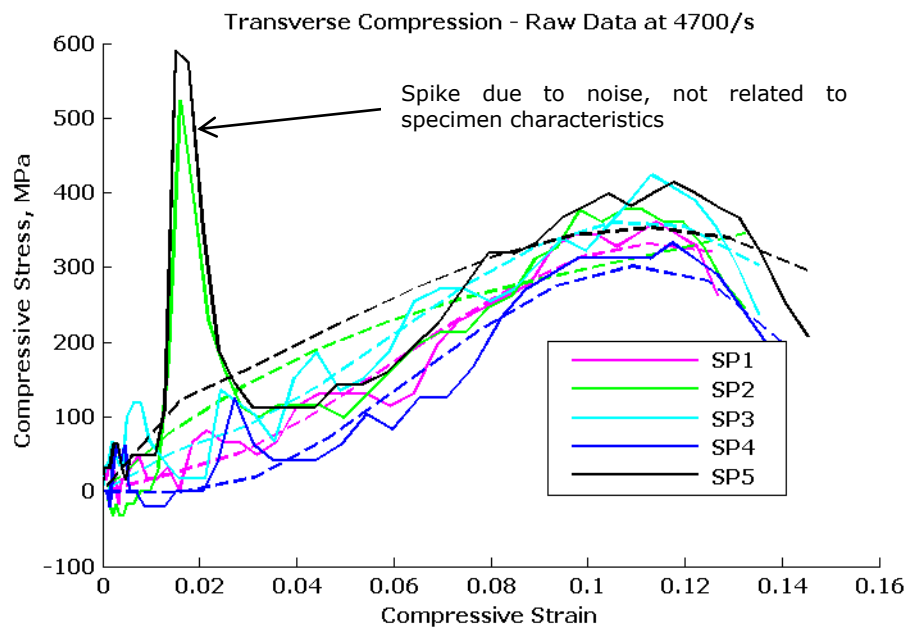


Fig A. 28 Transverse compressive characteristics of IM7 -8552 composites around  $4700 \text{ s}^{-1}$

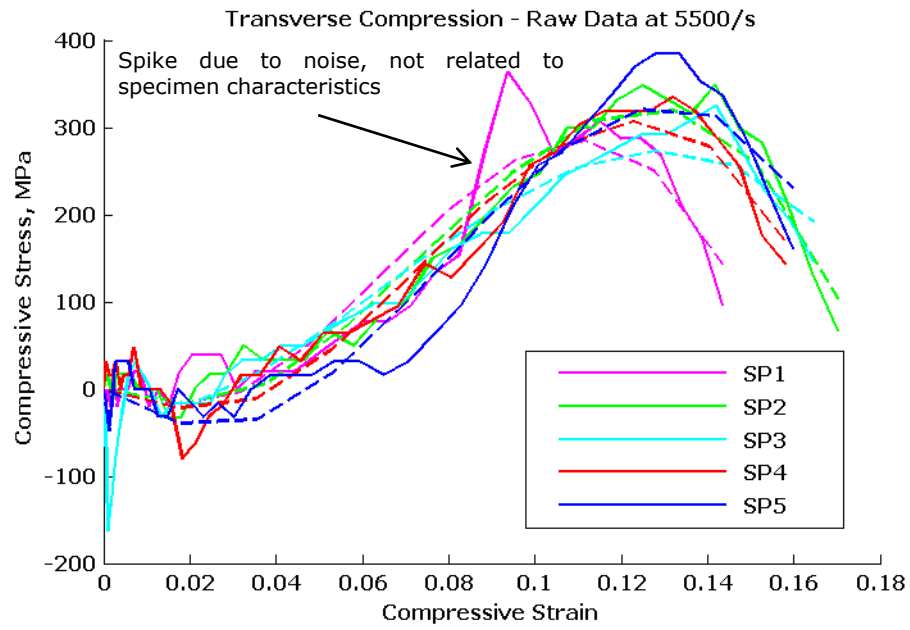


Fig A. 29 Transverse compressive characteristics of IM7 -8552 composites around 5500 s<sup>-1</sup>

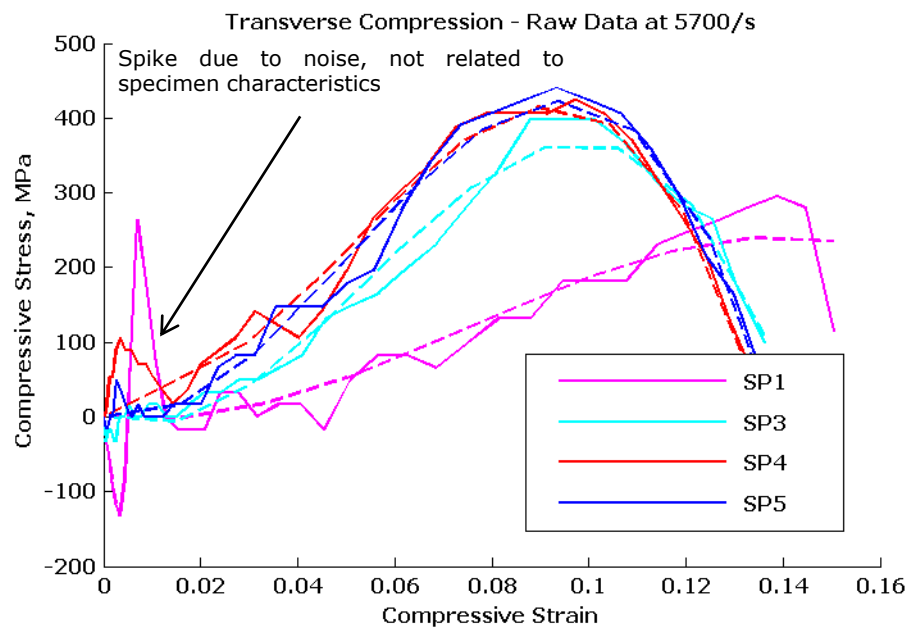


Fig A. 30 Transverse compressive characteristics of IM7 -8552 composites around 5700 s<sup>-1</sup>

## Appendix 13 Cross ply compressive characteristics of IM7-8552 UD composites at high strain rates

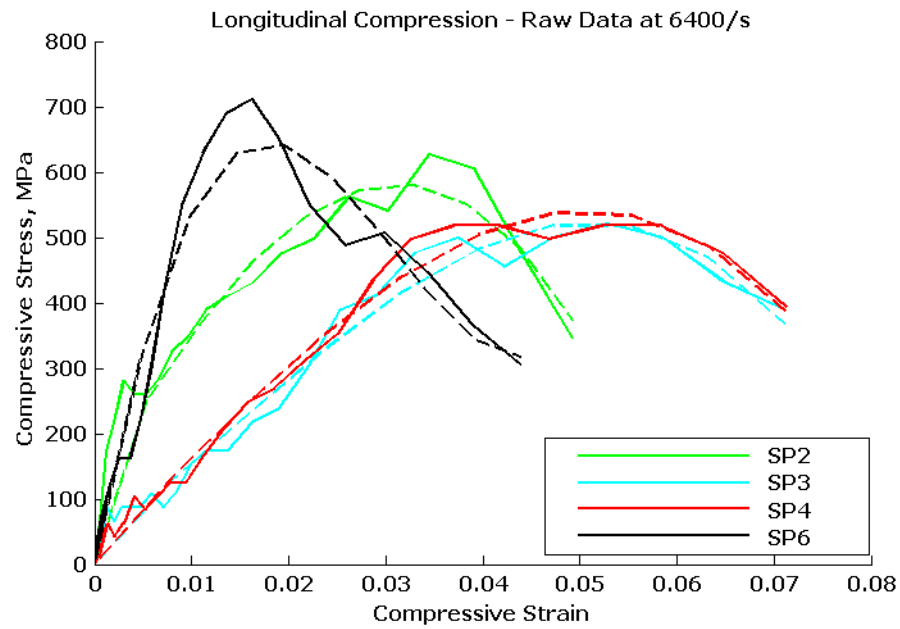


Fig A. 31 Compressive characteristics of IM7 -8552 of cross ply laminated composites around 6400 s<sup>-1</sup>

## Appendix 14 Quasi-static Single Fibre Test Data of Carbon Fibres

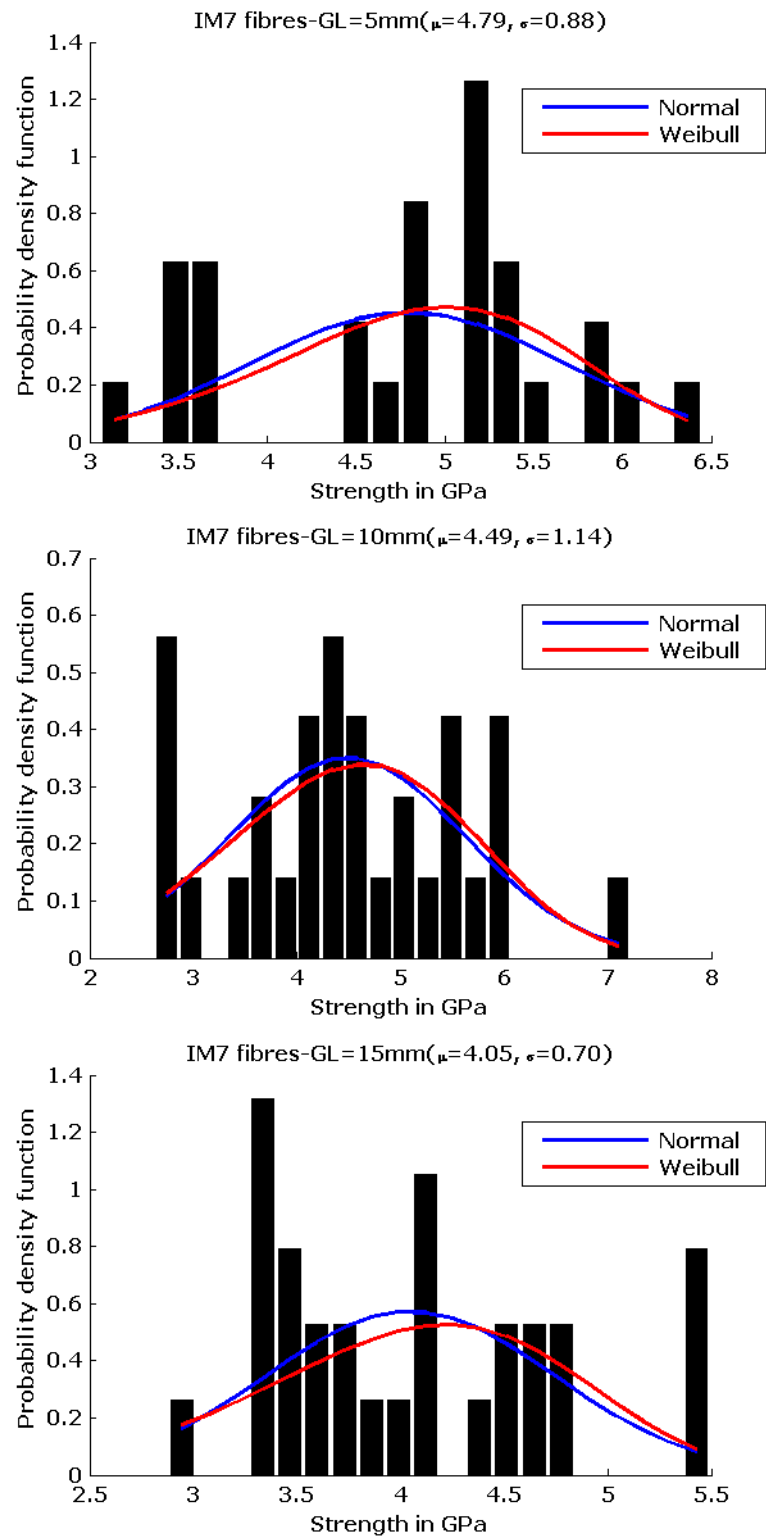


Fig A. 32 Pdf of quasi-static strength distribution of IM7 fibres for different gauge length

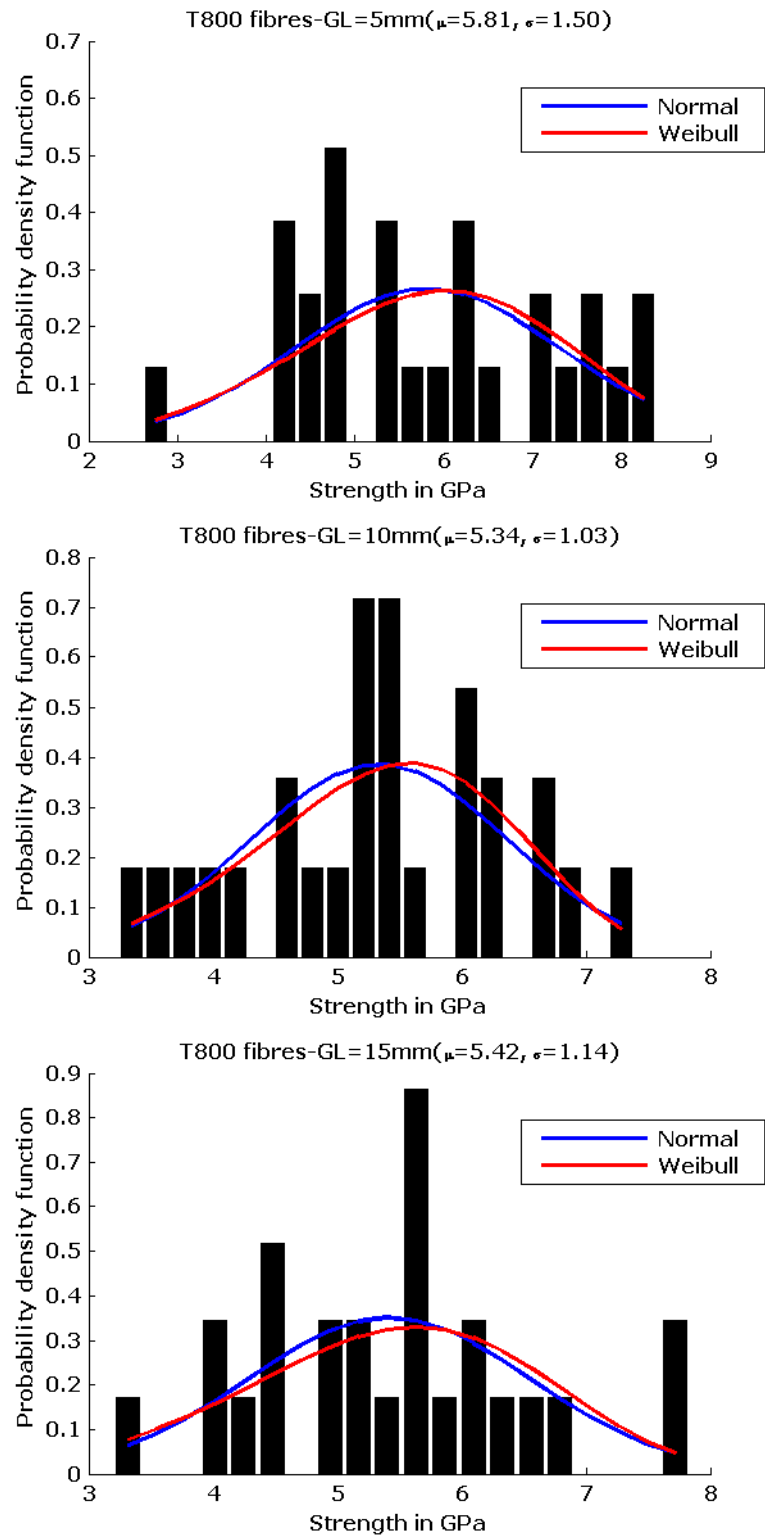


Fig A. 33 Pdf of quasi-static strength distribution of T800 fibres for different gauge length

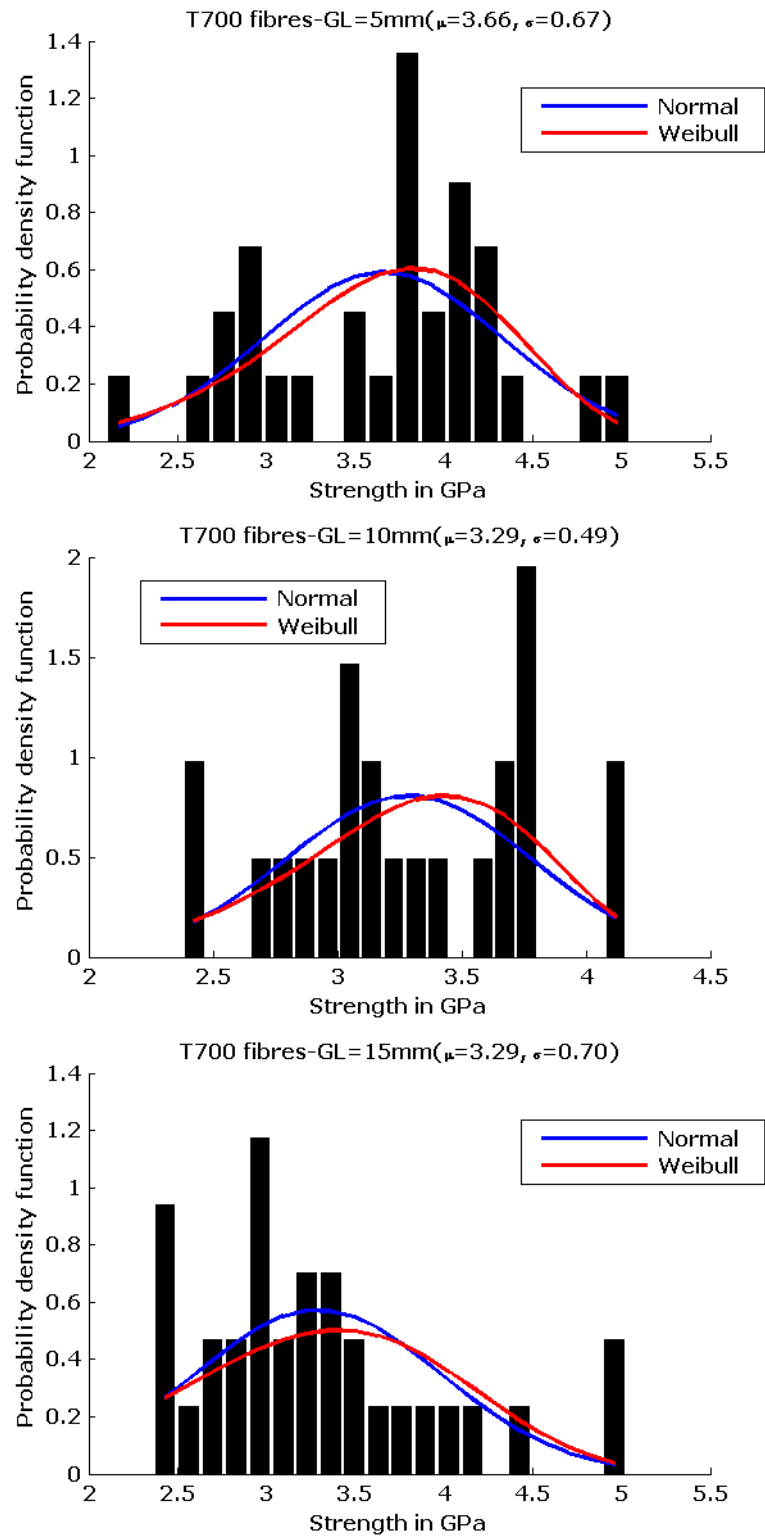


Fig A. 34 Pdf of quasi-static strength distribution of T700 fibres for different gauge length

## **Appendix 15 Procedure followed for stiffness measurement**

Once the compliance was evaluated, the fibre specimens of 10mm gauge length which showed linear load-cross head displacement characteristics without slippage was only considered for stiffness evaluation. The stiffness evaluation was carried out based on 'Method B' described in BS ISO standard 11566 [292] for carbon fibre tensile testing. The procedure followed for evaluation of stiffness from the load-deflection characteristics is as described below:

- 1) Identify the datum point in the load deflection curve when the specimen gets loaded.
- 2) Subtract the cross head displacement prior to this point from all the cross head displacement data.
- 3) Use the tensile strength and tensile modulus given in the data sheet of respective fibres for estimation of the range of the force and cross head displacement will be evaluated.
- 4) Since the gauge length is 10 mm, identify the slope of the load-crosshead displacement curve in the region where the cross head displacement is between 0.01 and 0.06mm.
- 5) Use the formula specified in 'Method B' for evaluation of the stiffness of the fibre specimen.

## **Appendix 16 Approaches for faster simulation in explicit analysis**

Two approaches that have been followed to complete the simulation in reasonable time with Abaqus explicit procedure are

- 1) Increasing the loading rate
- 2) Using mass scaling options by scaling the density

The increase in loading rate can change the equilibrium condition of the system as it changes from static to dynamic, where inertia forces becomes dominant. Hence when loading rates were increased, the objective was to model the simulation event in the shortest time period in which inertial forces remain insignificant. Mass scaling also would help in reducing the time for simulation by increasing the stable time increment. This has been done by scaling the density of each material. The effect of loading rate and mass scaling were studied for typical longitudinal compression problem defined using a micro-scale finite model to evaluate its influence in the mechanical characteristics. It can be observed from Fig A. 35 that there is a threshold mass scaling (given by MS in legend) for each loading rate/strain rate (given by SR in legend) beyond which a change in the characteristic behaviour due to inertia effects can be distinguished. The density used for each material for simulation is the product of the relative density of the material and MS as specified in Fig A. 35 where the inertia effect is insignificant. The dotted lines in each colour (representing each strain rate) shows the influence of inertia on the characteristic behaviour which can be clearly be seen to be different from the mechanical characteristics described by the line for each strain rate.

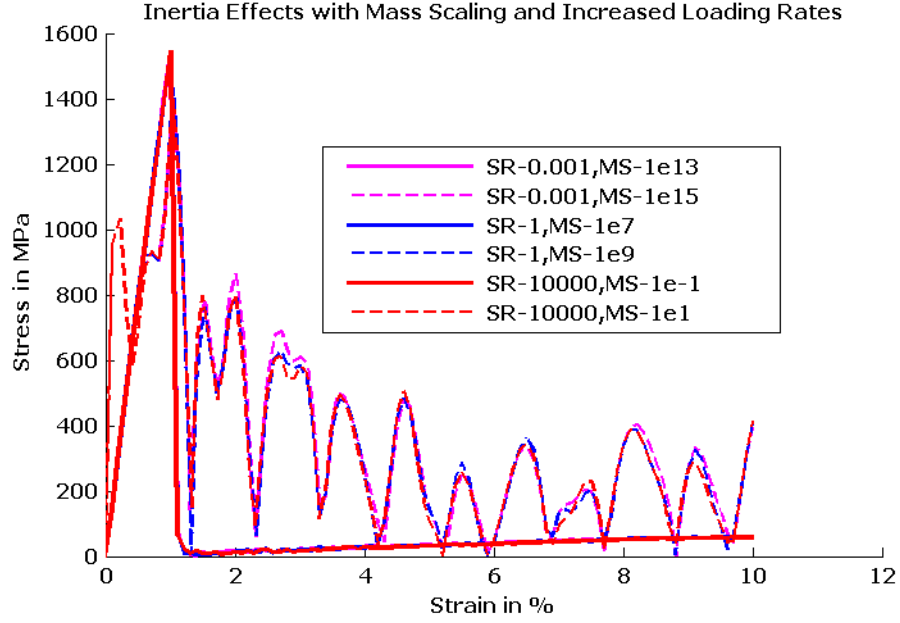


Fig A. 35 Effect of inertia on characteristics curves due to mass scaling and increased loading rates

Another issue is that the material strain rate will also increase according to the increase in loading rate. This means that material characteristics will be different, if the material is strain rate sensitive. In the micro-scale model, the matrix behaviour is modelled as strain rate sensitive where its stiffness and strength is assumed to be based on Eq. 5-4. Here the material response changes based on the value of  $C_A$  which has different values based on a threshold strain rate. For the material model, Lidgett had used a bi-linear strain rate scaling law which uses different values for low strain rate and high strain rate with reference to a particular threshold strain rate obtained from the characteristic curve. The scaling constant of zero (Eq. 5-4) was used in the analysis for characteristics curves plotted in Fig A. 35. Based on the values used by Lidgett ( $C_A=0.0401$  and  $0.1571$  at low strain rates and at high strain rates for stiffness and  $C_A=0.0329$  and  $0.1788$  at low strain rates and at high strain rates for strength), the matrix material is expected to show rate-sensitive behaviour according to its scaling constant for different loading rates. Since the longitudinal characteristics are primarily fibre dependent, the longitudinal characteristics curves is not significantly affected by the value of matrix stiffness and strength scaling constant, even though small changes could be seen in the values of strength and stiffness (Fig A. 36). In the legend shown in Fig A. 36, 'CA-0' denotes that all the matrix scaling constant have been put to zero value to simulate the quasi-static

characteristics and 'CA-Lidgett' denotes that the scaling constant as used by Lidgett in his studies.

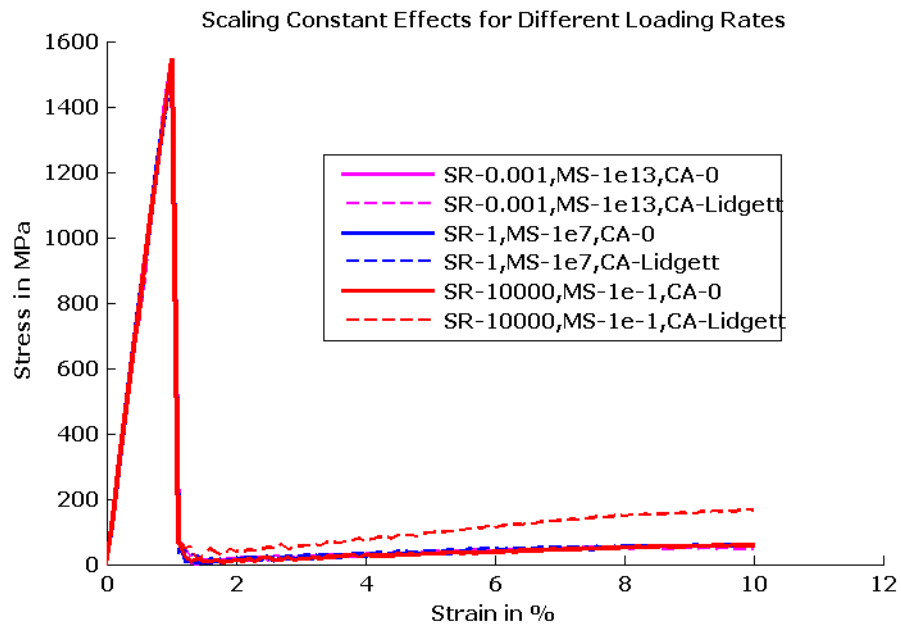


Fig A. 36 Influence of matrix scaling constant in the longitudinal composite compressive characteristics

However, the matrix scaling constant does seem to affect the transverse compressive characteristics as can be seen from the transverse characteristic curve predicted for different values of scaling constant (Fig A. 37). It can be observed that there is only a small difference between the SR-0.001 and SR-1 as values used for strain rate scaling constant was smaller value as it is below the threshold strain rate whereas larger values were used for the strain rate constant for stiffness and strength for the rate of loading corresponding to SR-10000. Similar effects can be expected for shear characteristics as well, as shear is also primarily a matrix dominant characteristic.

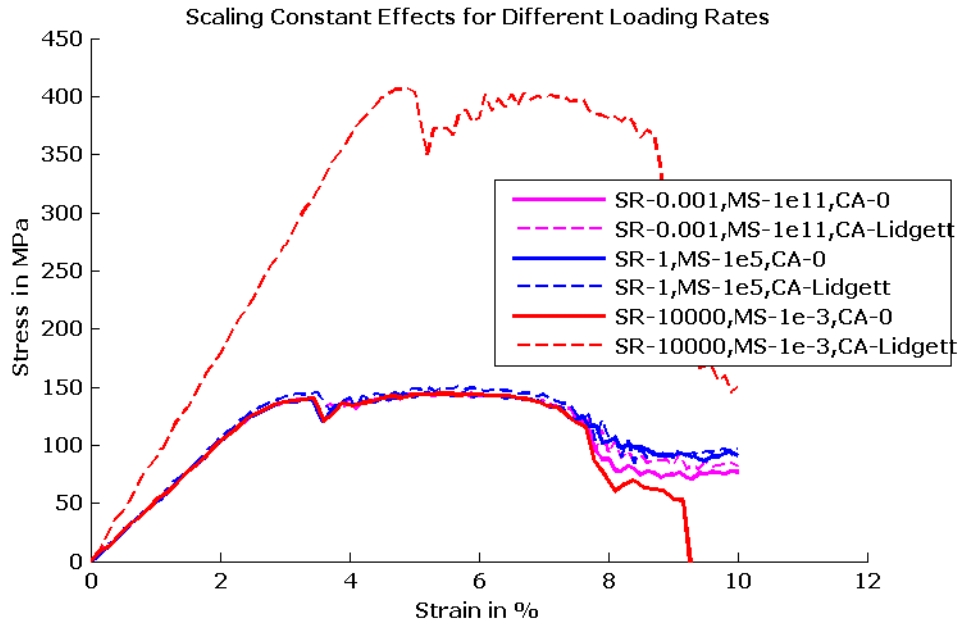


Fig A. 37 Influence of matrix scaling constant in the transverse composite compressive characteristics

Another issue that needs to be addressed was the value of strain rate scaling constant used when the loading rates are changed. When the loading rates change, the material strain rates also would change. Hence, it is important to modify the strain rate scaling constant in Eq. 5-4 (page 132) so as to nullify the effect of artificial increase in loading rate which is done for enhancing the speed of simulation. The strain rate constant that need to be appropriately used for different strain rates when the simulation is done with increased loading rate is as given in Table A. 3.

Table A. 3 Strain scaling constant used for modified loading rate simulations

| Strain rates ( $s^{-1}$ ) | Simulation strain rate ( $s^{-1}$ ) | Strain Rate scaling constant used in Lidgett microscale model | Modified strain rate scaling constant (corresponding to simulation strain rate) |
|---------------------------|-------------------------------------|---------------------------------------------------------------|---------------------------------------------------------------------------------|
| 0.0001                    | 10000                               | 0.0401                                                        | 0                                                                               |
| 0.0001                    | 10000                               | 0.0329                                                        | 0                                                                               |
| 0.0001                    | 10000                               | 0.1571                                                        | 0                                                                               |
| 0.0001                    | 10000                               | 0.1788                                                        | 0                                                                               |
| 0.0001                    | 10000                               | 0.2647                                                        | 0                                                                               |
| 93                        | 10000                               | 0.0401                                                        | 0.0299                                                                          |
| 93                        | 10000                               | 0.0329                                                        | 0.0245                                                                          |
| 93                        | 10000                               | 0.1571                                                        | 0.0299                                                                          |
| 93                        | 10000                               | 0.1788                                                        | 0.0245                                                                          |
| 93                        | 10000                               | 0.2647                                                        | 0.1975                                                                          |
| 2000                      | 10000                               | 0.0401                                                        | 0.0401                                                                          |

|      |       |        |        |
|------|-------|--------|--------|
| 2000 | 10000 | 0.0329 | 0.0329 |
| 2000 | 10000 | 0.1571 | 0.1434 |
| 2000 | 10000 | 0.1788 | 0.1632 |
| 2000 | 10000 | 0.2647 | 0.2416 |
| 7    | 10000 | 0.0401 | 0.0243 |
| 7    | 10000 | 0.0329 | 0.0199 |
| 7    | 10000 | 0.1571 | 0.0251 |
| 7    | 10000 | 0.1788 | 0.0206 |
| 7    | 10000 | 0.2647 | 0.1603 |
| 450  | 10000 | 0.0401 | 0.0401 |
| 450  | 10000 | 0.0329 | 0.0329 |
| 450  | 10000 | 0.1571 | 0.1307 |
| 450  | 10000 | 0.1788 | 0.1487 |
| 450  | 10000 | 0.2647 | 0.2201 |
| 3700 | 10000 | 0.0401 | 0.0401 |
| 3700 | 10000 | 0.0329 | 0.0329 |
| 3700 | 10000 | 0.1571 | 0.1486 |
| 3700 | 10000 | 0.1788 | 0.1691 |
| 3700 | 10000 | 0.2647 | 0.2504 |
| 4300 | 10000 | 0.0401 | 0.0401 |
| 4300 | 10000 | 0.0329 | 0.0329 |
| 4300 | 10000 | 0.1571 | 0.1499 |
| 4300 | 10000 | 0.1788 | 0.1706 |
| 4300 | 10000 | 0.2647 | 0.2526 |
| 5000 | 10000 | 0.0401 | 0.0401 |
| 5000 | 10000 | 0.0329 | 0.0329 |
| 5000 | 10000 | 0.1571 | 0.1512 |
| 5000 | 10000 | 0.1788 | 0.1721 |
| 5000 | 10000 | 0.2647 | 0.2547 |
| 650  | 10000 | 0.0401 | 0.0257 |
| 650  | 10000 | 0.0329 | 0.0185 |
| 650  | 10000 | 0.1571 | 0.1338 |
| 650  | 10000 | 0.1788 | 0.1523 |
| 650  | 10000 | 0.2647 | 0.2254 |

## **Appendix 17 Challenges/Issues in using Abaqus with Optimisation Solvers**

The methodology followed in the code was to call the optimisation solver within Abaqus Python. To achieve this, solvers should be able to communicate with the solver library within the Abaqus Python environment.

### **A 17.1 SciPy**

SciPy has to be first installed in Python directory. Then the installed SciPy files should be copied from the site packages of Python directory to the site packages in the Abaqus directory. It needs to be ensured that Python version in Abaqus is the same as the Python version installed outside Abaqus. SciPy requires Numpy to be installed in the Python directory before it is installed. NumPy also known Numerical Python is required for efficient array operations. The installed NumPy version in Python directory has to be the same as the preinstalled NumPy inside Abaqus. Scipy version (<8.0) is compatible with the NumPy version (1.4.1) preinstalled inside Abaqus. Since the Abaqus installed was 64 bit version, it was necessary to install 64 bit version of NumPy and SciPy inside the Python directory to ensure the optimisation runs without any error. There were delays in the installation process as 64 bit version of compatible SciPy package weren't available in the open domain. An optimisation run involving micro-scale FE model performed on a 32 bit Abaqus using Intel Pentium 4 CPU, 3Ghz processor with 2GB RAM took more than 8 days (including queue time) to complete a single optimisation run whereas the same optimisation run on a 64 bit Abaqus using Intel Xeon E5 2670 2.60 GHz processor with 32GB RAM took only 22 hours.

### **A 17.2 NAG C Library**

The python bindings available from the website were downloaded and installed to call the NAG Library from Python (nag4py). The licence key supplied by NAG for the NAG C Library, CLL6A23DHL is required to use the bindings. The nag4py bindings can only work inside Python 2.7 or greater versions. Hence, this cannot be used inside Abaqus version which uses Python 2.6. The release of Abaqus 6.14.1 is based on Python 2.7 and has been used for the inverse modelling studies. The NAG C Library need to be loaded into the directory where the python scripts calls the NAG C library through nag4py.

## Appendix 18 Screen shots of Python scripts used for optimisation

### A 18.1 SciPy (Leastsq)

Optimisation solver command used for calling the function (without Jacobian)

```
Mode = optimize.leastsq(fun,Modulus,full_output=1,ftol=1.49012e-10, xtol=1.49012e-10,gtol=0.0, epsfcn=0.005)
print Mode
```

Optimisation Solver command used for calling the function (with Jacobian)

```
Mode = optimize.leastsq(fun,Modulus, Dfun=dfunc,col_deriv=1,full_output=1,ftol=1.49012e-10, xtol=1.49012e-10,gtol=0.0)
print Mode
```

Inclusion of modules inside the file

```
import sys,os
#from xlrd import open_workbook
import math
from numpy import *
import numpy as np
from scipy import optimize
import subprocess
from odbAccess import *
from abaqusConstants import *
from odbMaterial import *
from odbSection import *
import shutil
```

Setting up parameter to be estimated as parametric study problem

```
def fun(Modulus):
    # Setting the first parameter of guess matrix to g11
    g11 = Modulus[0]
    print "g11:", g11

    uni = parStudy(par=('Efl'),directory=ON, verbose=ON)
    # DEFINE THE NAMES OF THE INPUT DECKS
    inputFiles=('c_z_-10000_micro_Efl')

    words = inputFiles.split('_')

    print words

    # define the parameters
    uni.define('DISCRETE', par='Efl', domain=(g11))
```

Definition of the objective function

```
for i in range (m):
    StressInt[i]=polyval(kk,Epsilon_Act[i])
    Err = Err + [(Sigma[i]- StressInt[i])]
```

Removing analysis folders

```
myodb.close()

shutil.rmtree(path='/panfs/panasas01.panfs.cluster/eaaxscs1/AbaqusTestRuns4/C_2_-10000_MICRO_EFL_P1_C1')
return Err
```

## A 18.2 NAG C Library (e04unc)

Optimisation Solver command used for calling the function

```
xx=nag_opt_nlin_lsq(m, n, nolin, nonlin, a, tda, bl, bu, y, c_objfun, c_confun, x, objf, f, fjac, tdfjac, options, comm, fail)
print("nag_opt_nlin_lsq (e04unc) Python Example Script Results.")
if fail.code == NE_NOERROR:
    print("Final objective value is %.7e." % objf)
```

Inclusion of modules inside the file

```
def main():
    # include the modules relevant to E04UNC from Nagpy
    from nag4py.e04 import nag_opt_free, nag_opt_init, nag_opt_nlin_lsq
    from nag4py.util import (Nag_Comm, NAG_E04UNC_CONFUN, NAG_E04UNC_OBJFUN, Pointer,
                             Nag_E04_Opt, Nag_FALSE, Nag_NoCheck, Nag_NoPrint,
                             NE_NOERROR, noisy_fail)
    # include the modules relevant to for arrays from Numpy
    from numpy import array, empty, ones
```

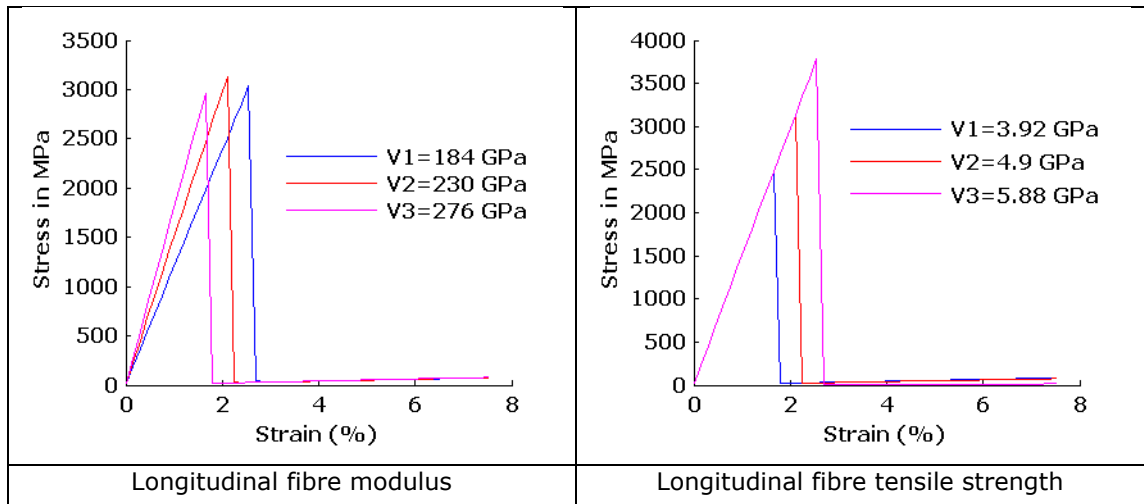
Definition of the objective and Jacobian function

```
for i in range(m):
    if comm[0].needf in [i+1, 0]:
        print "funcall:",i
        StressInt[i]=polyval(kk,Epsilon_Act[i])
        f[i]=StressInt[i]
        print "f:", f

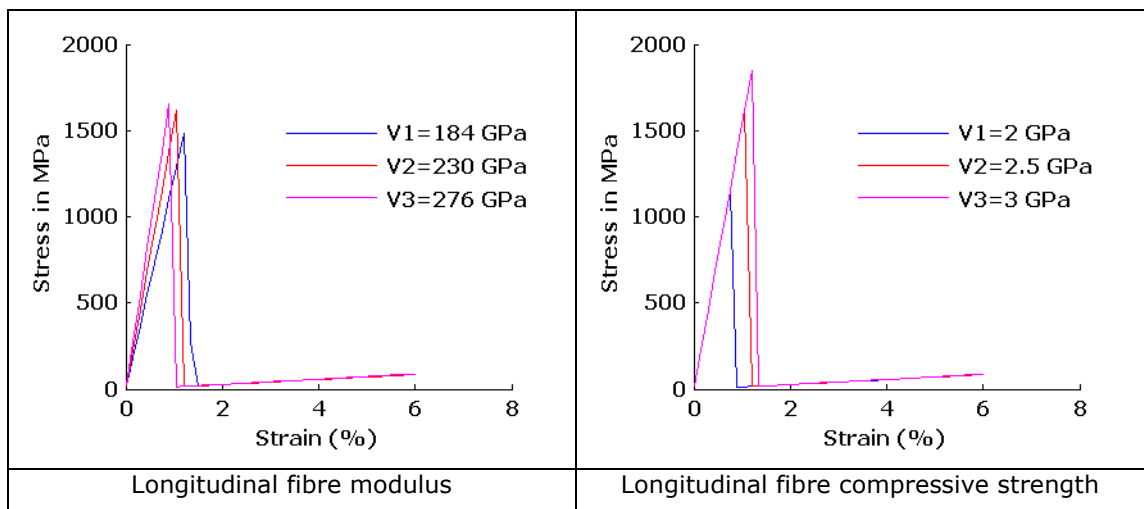
    if comm[0].flag == 2:
        print "jacobiancall:",i
        k0=i*tdfjac
        #k1=i*tdfjac+1
        fjac[k0]=Epsilon_Act[i]*0.63
```

## Appendix 19 Characteristics curves obtained for each sensitive parameters for different case of loading

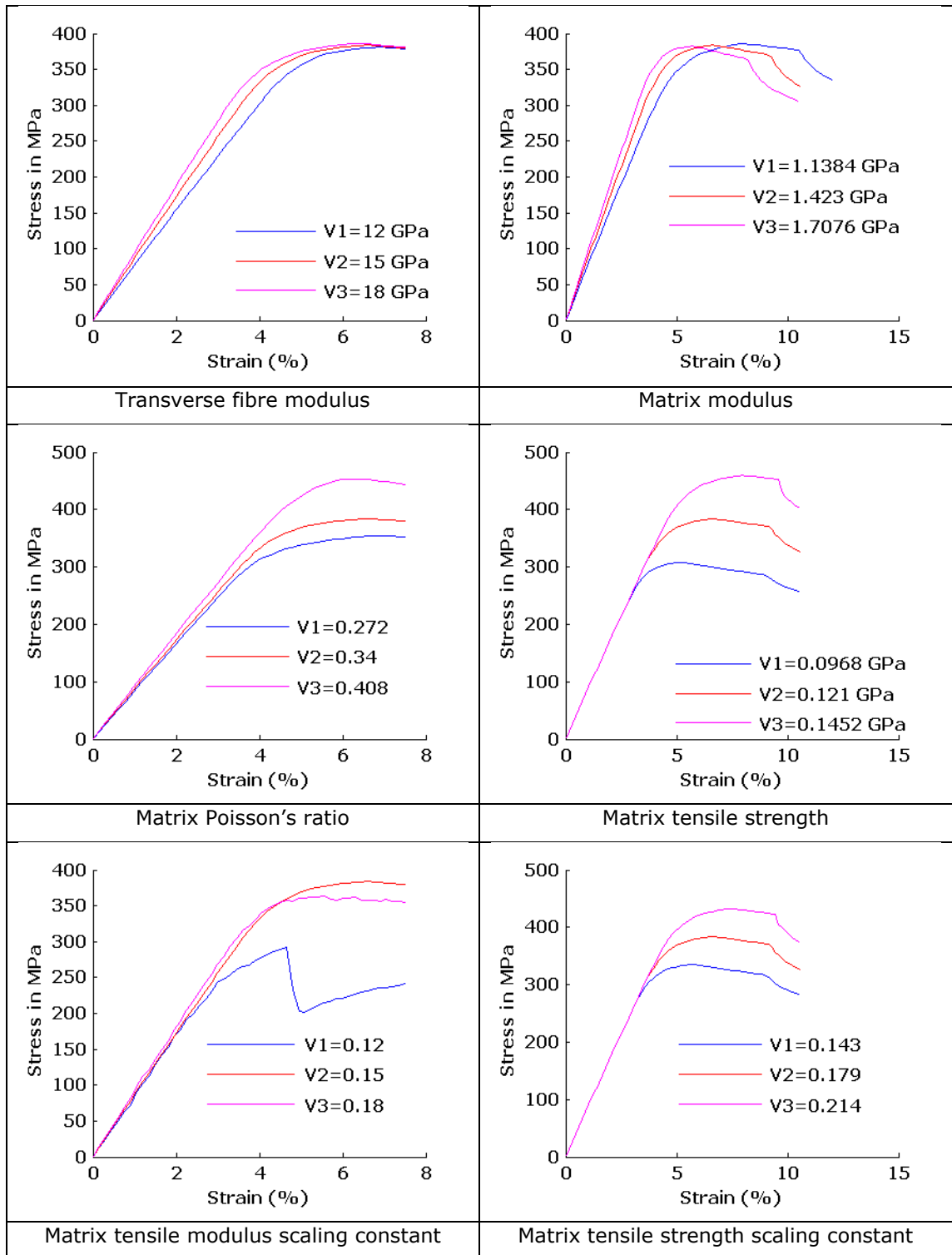
### A 19.1 Longitudinal Tension (Z)



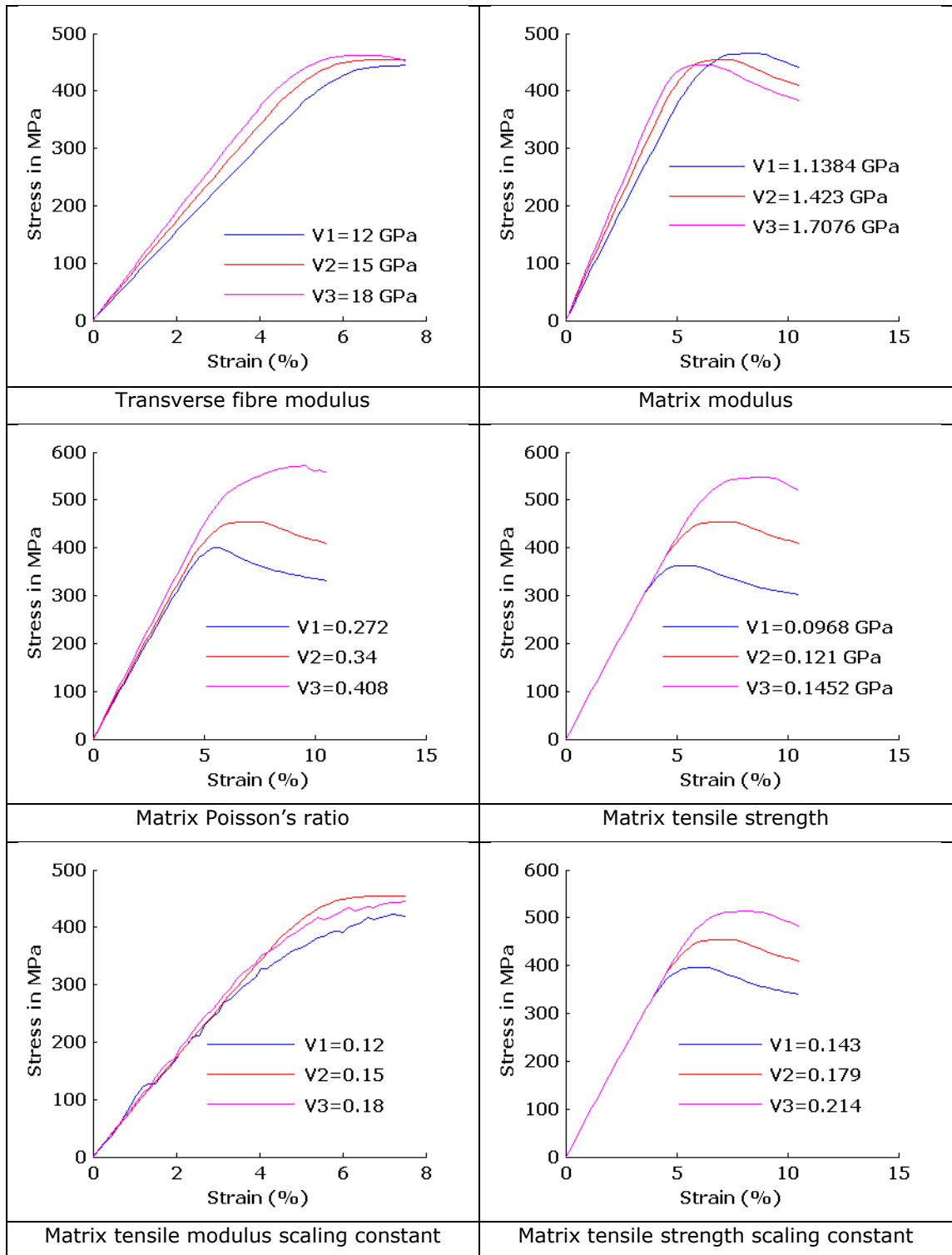
### A 19.2 Longitudinal Compression (Z)



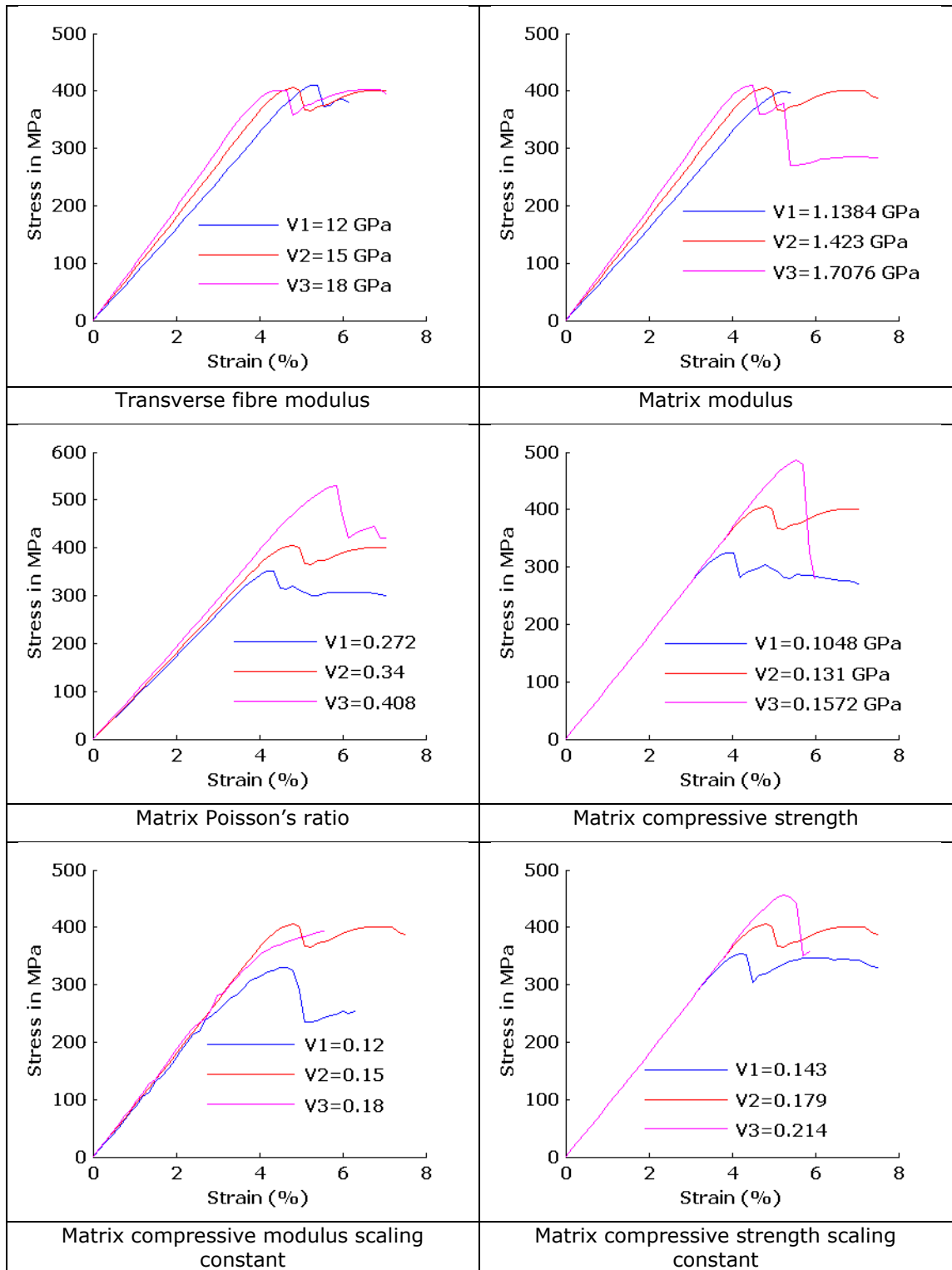
## A 19.3 Transverse Tension (X)



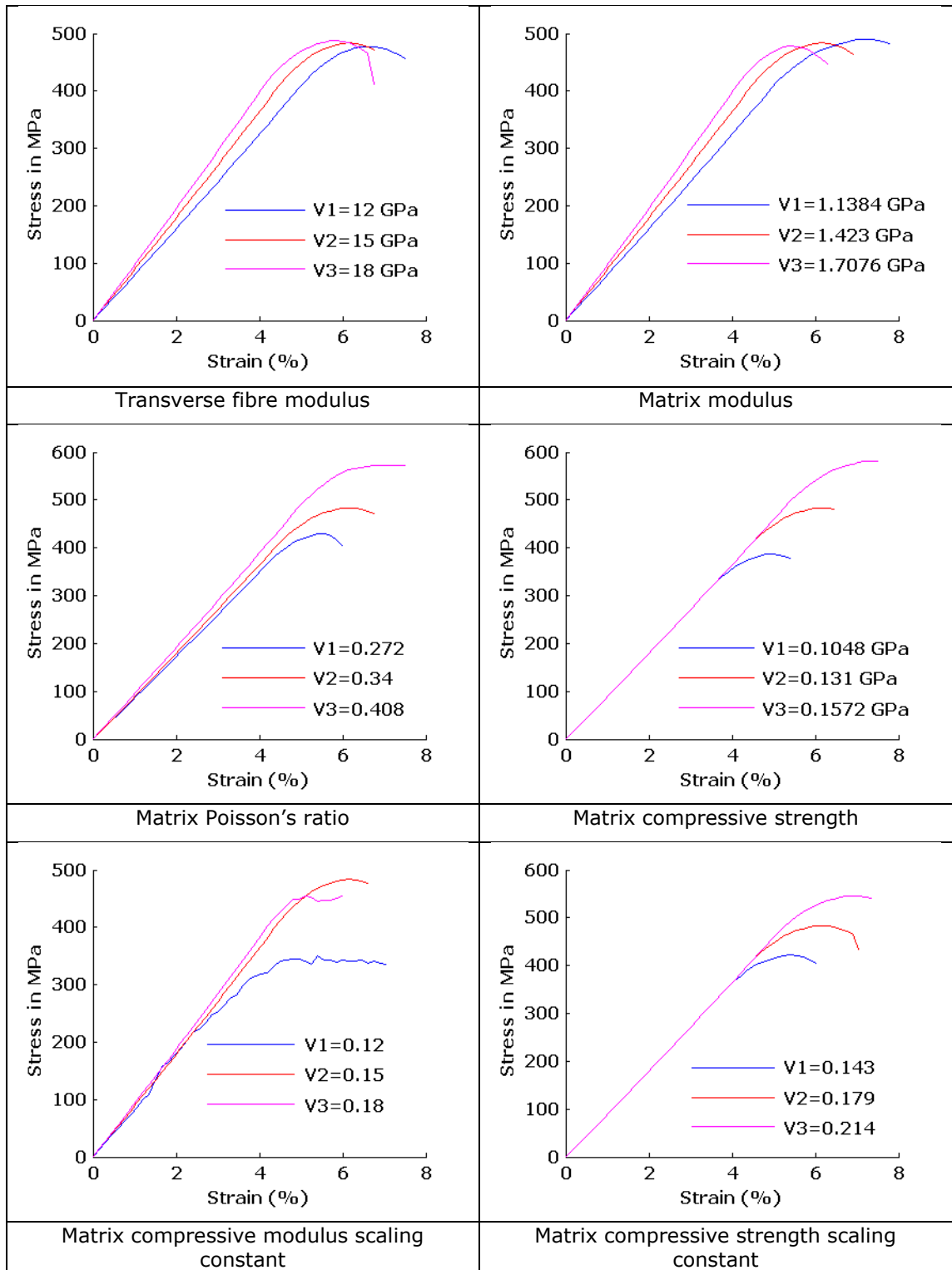
## A 19.4 Transverse Tension (Y)



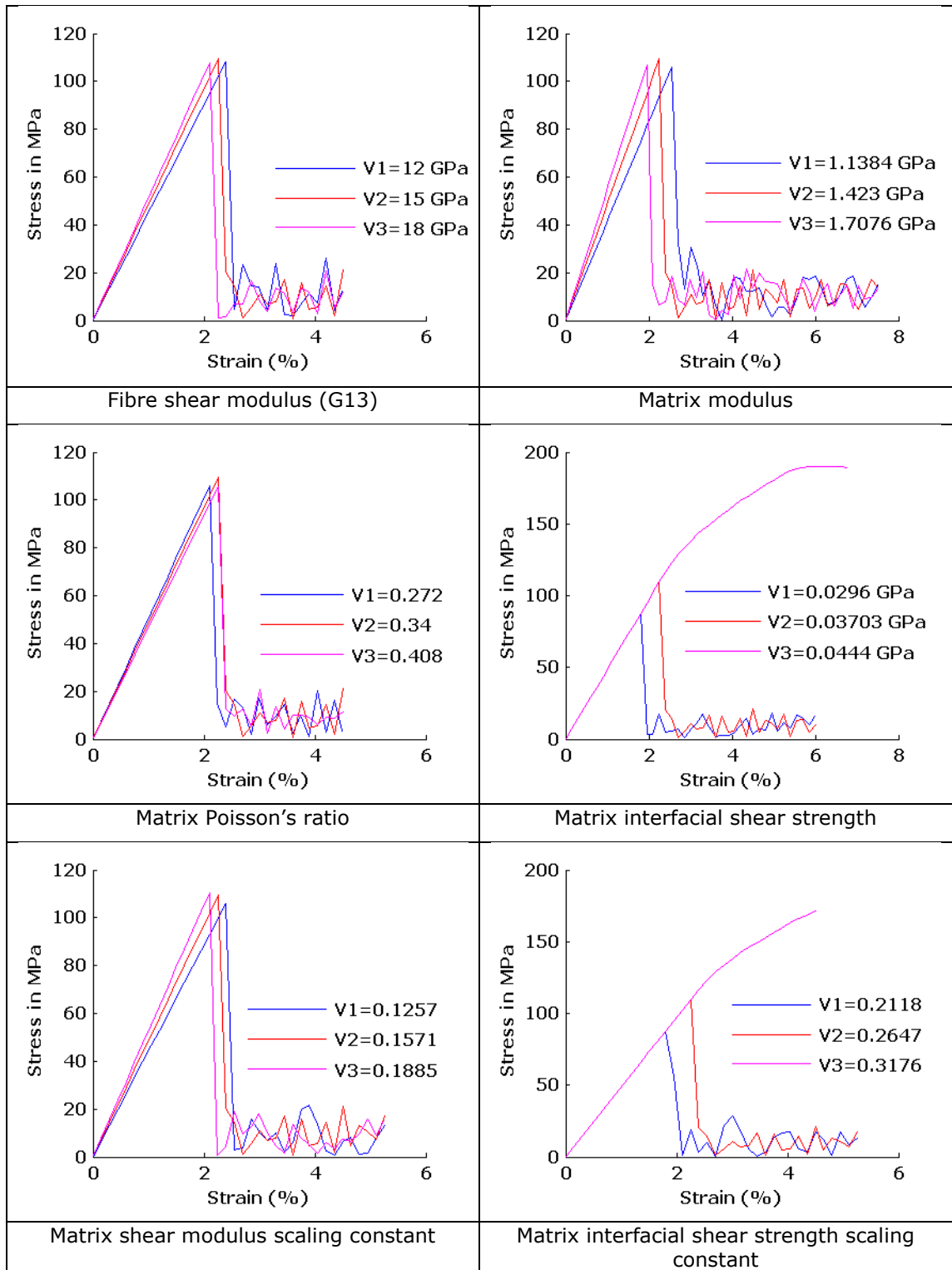
## A 19.5 Transverse Compression (X)



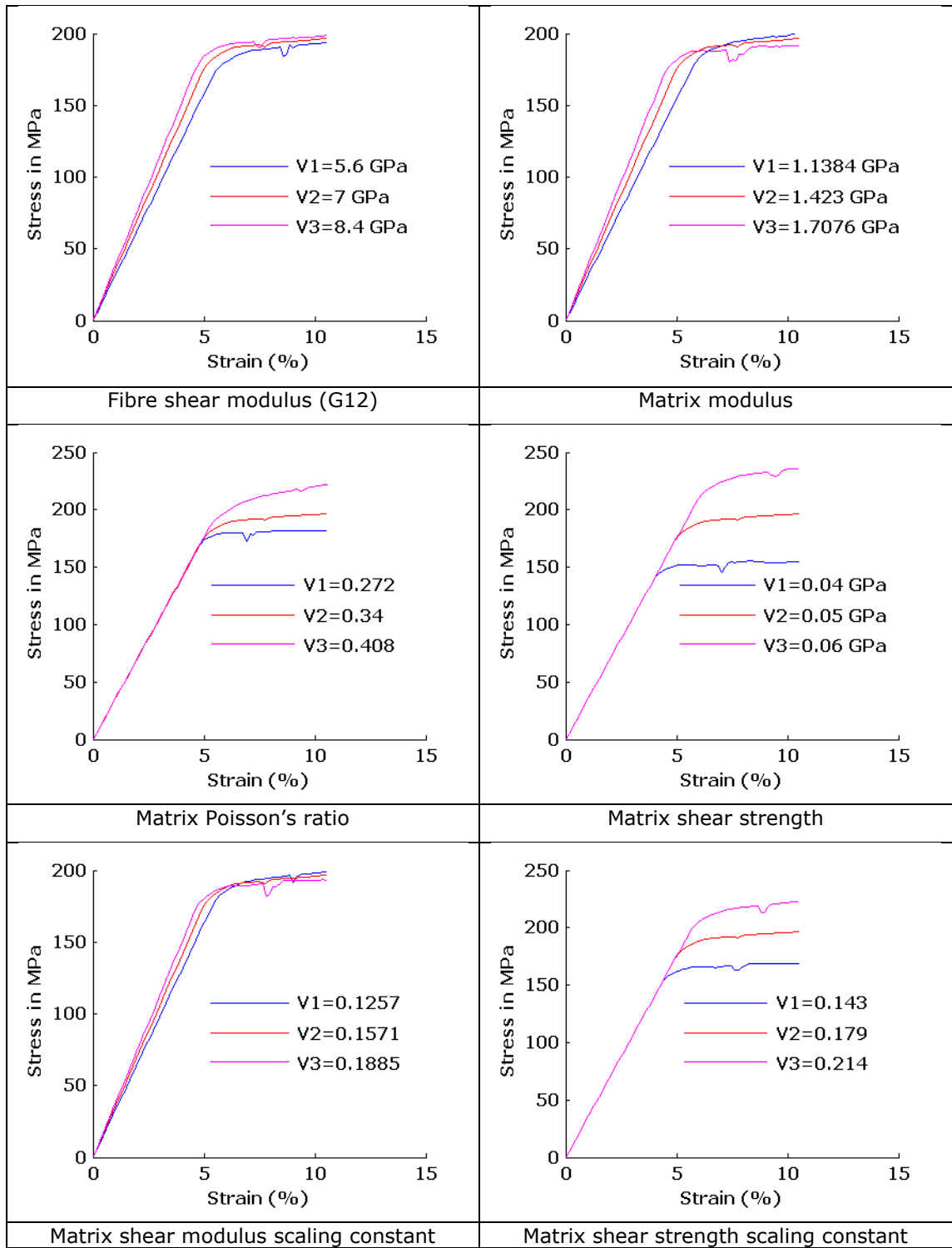
## A 19.6 Transverse Compression (Y)



## A 19.7 Longitudinal Shear (XZ)



## A 19.8 Transverse Shear (XY)



## **Appendix 20 Methodology followed for estimation of each parameters**

### **A 20.1 Modulus (QS & HSR- longitudinal fibre compressive/transverse fibre compressive/ longitudinal fibre shear/ transverse fibre shear)**

- 1) Use the data set mentioned in Lidgett's thesis with strain rate scaling constants modified to accommodate for increasing loading rates.
- 2) Set the longitudinal fibre Modulus/transverse modulus as parameter to be changed.
- 3) Scale the density for mass scaling corresponding to loading rate of 10,000/s ( $t=0.01$ )
- 4) Matrix modulus and matrix shear modulus also were changed to 4.67 and 1.743 respectively from the Lidgett's data to correlate to the modulus of 8552 as published in manufacturer's data sheet
- 5) Replace the strain rate scaling constant as given in Table A. 3 for the input files.
- 6) Prepare input file (.inp) to accommodate for the changes (eg. longitudinal compressive fibre modulus estimation).
- 7) Identify the curve that will be used as test data (either experimental data from the test programme conducted as part of this study or available from open literatures)
- 8) Check the test data for any unusual characteristic points.
- 9) Identify the best fit curve for the experimental data.
- 10) Compare the best fit curve with the predicted characteristic obtained through parametric study
- 11) Prepare python scripts for calling optimisation solvers (SciPy / NAG).
- 12) Estimate the Jacobian based on RoM micromechanical relations.
- 13) Define the objective function for the data points that will be considered.
- 14) Set the precision value(NAG (f.prec) and SciPy (x tol and ftol))
- 15) In case, there is a large discrepancy between estimated parameters for different guess value, look for the output file and plot the estimated characteristics curves for different values.

```

*Amplitude, name=Amp-1, time=TOTAL TIME
| 0., 0., 0.015, 1.5
**
** MATERIALS
**
*Material, name=FIBRE
*Density
1.8e-01,
*Depvar, delete=7
45,
*User Material, constants=11
15., 15., <Efl>, 0.2, 0.0163043, 0.25, 7., 15.
| 15., 4.9, 2.5
*Material, name=INTERFACE
*Density
1.301e-01,
*Depvar, delete=7
45,
*User Material, constants=15
4.67, 0.34, 1.743, 0.07406, 0.03703, 0.0000, 0.0000, 0.19935
0.0000, 0.0000, 0.19935, 0.0000, 0.0000, 0.19935, 0.0000
*Material, name=MATRIX
*Density
1.301e-01,
*Depvar, delete=7
45,
*User Material, constants=38
4.67, 0.34, 1.743, 0.121, 0.127, 0.05, 0.1, 0.1
| 0.1, 0.1, 1.301e-01, 0.0000, 0.0000, 0.19935, 0.0000, 0.0000
0.19935, 0.0000, 0.0000, 0.19935, 0.0000, 0.0000, 0.08858, 0.0000
0.0000, 0.08858, 0.0000, 0.0000, 0.08858, 0., 0., 1.
| 0., 0., 1., 0., 0., 1.
**
**
** STEP: TS_1
**
*Step, name=TS_1
Time Step 1
*Dynamic, Explicit
, 0.015
*Bulk Viscosity
0.06, 1.2

```

Fig A. 38 Screenshot of input file relating to material definition

## A 20.2 Strength (QS & HSR- longitudinal fibre compressive /matrix compressive strength/ matrix compressive strength scaling constant/ matrix shear strength/ interfacial matrix shear strength)

1. Perform strength estimation only after the respective modulus estimation is evaluated
2. Change the fibre modulus in the input file (longitudinal/transverse) to the estimated fibre modulus. Also accordingly change the value for v23 as well (not necessary, as it does not influencing longitudinal characteristics significantly)

3. Set the longitudinal fibre Strength as parameter to be changed
4. Replace the strain rate scaling constant as given in Table A. 3 for the input files.
5. Prepare input file (.inp) to accommodate for the changes.
6. Prepare separate python scripts for calling SciPy and NAG solvers to accommodate for the changes in the estimation parameter definitions.
7. Estimate the Jacobian based on Chamis micromechanical relations
8. Define the objective function for the data points that will be considered.
9. Set the precision value (NAG (f.prec) and SciPy(x tol and ftol)) and check for its sensitivity on the results.
10. In the case where there is a large discrepancy between estimated parameters for different guess value, look for the output file and plot the estimated characteristics curves for different values.

## Appendix 21 Methodology followed for generation of pdf/cdf plots

### A 21.1 Strength distribution bar chart

- Use *hist/histogram* function to generate the distribution pattern of a vector set(X).

$$[f, x] = \text{hist}(X, \text{bin sizes}) \quad \text{Eq A. 20}$$

- Using the information of bin counts (f) and bin centres(x), evaluate the difference between each bin centres (dx).
- Plot the bar chart as given in Eq A. 21 to generate the probability density function of the distribution.

$$\text{bar}(x, f / \text{sum}(f * dx)) \quad \text{Eq A. 21}$$

### A 21.2 Fitting of approximate curves of standard distribution patterns

#### A 21.2.1 Normal distribution

- Evaluate the mean and standard deviation and use those values in the *makedist* function to generate the coefficients for the normal distribution curve
- Then use the coefficients to generate the pdf value for all values of x in Eq A. 20 using pdf function
- Using plot command plot the curves along with the bar chart (Using *fitdist* can also be used to generate the coefficients for a given distribution without separately evaluating mean and standard deviation). Since method of evaluation of mean and standard deviation is the same, both the method (using *fitdist* or *makedist* gives the same fitted distribution for the probability distribution functions for a given distribution data).

#### A 21.2.2 Weibull distribution

- To obtain the scale parameter ( $\sigma_0$ ) and shape parameter ( $\beta$ ) as given in Eq A. 22 to be used in the *makedist* function, the approach given below was followed. The probability of failure under a stress ( $\sigma$ ) for a 2 parameter weibull distribution is given by

$$F(\sigma) = 1 - e^{-\left(\frac{\sigma}{\sigma_0}\right)^\beta} \quad \text{Eq A. 22}$$

Taking the double log of above equation and representing the equation as straight line would help in estimation of the parameters.

$$\ln\left(\ln\left(\frac{1}{1-F(\sigma)}\right)\right) = \beta \ln(\sigma) - \beta \ln(\sigma_0) \quad \text{Eq A. 23}$$

When  $\ln\left(\ln\left(\frac{1}{1-F(\sigma)}\right)\right)$  is plotted vs  $\ln(\sigma)$ , the slope represents the Weibull modulus of the distribution and  $\sigma_0$  can be evaluated from intercept b as given in equation below. The slope and intercept were obtained from the parameters polyfit option for fitting a linear straight line. Polyfit estimates the parameter based on the least square best fit.

$$\sigma_0 = e^{-\left(\frac{b}{\beta}\right)} \quad \text{Eq A. 24}$$

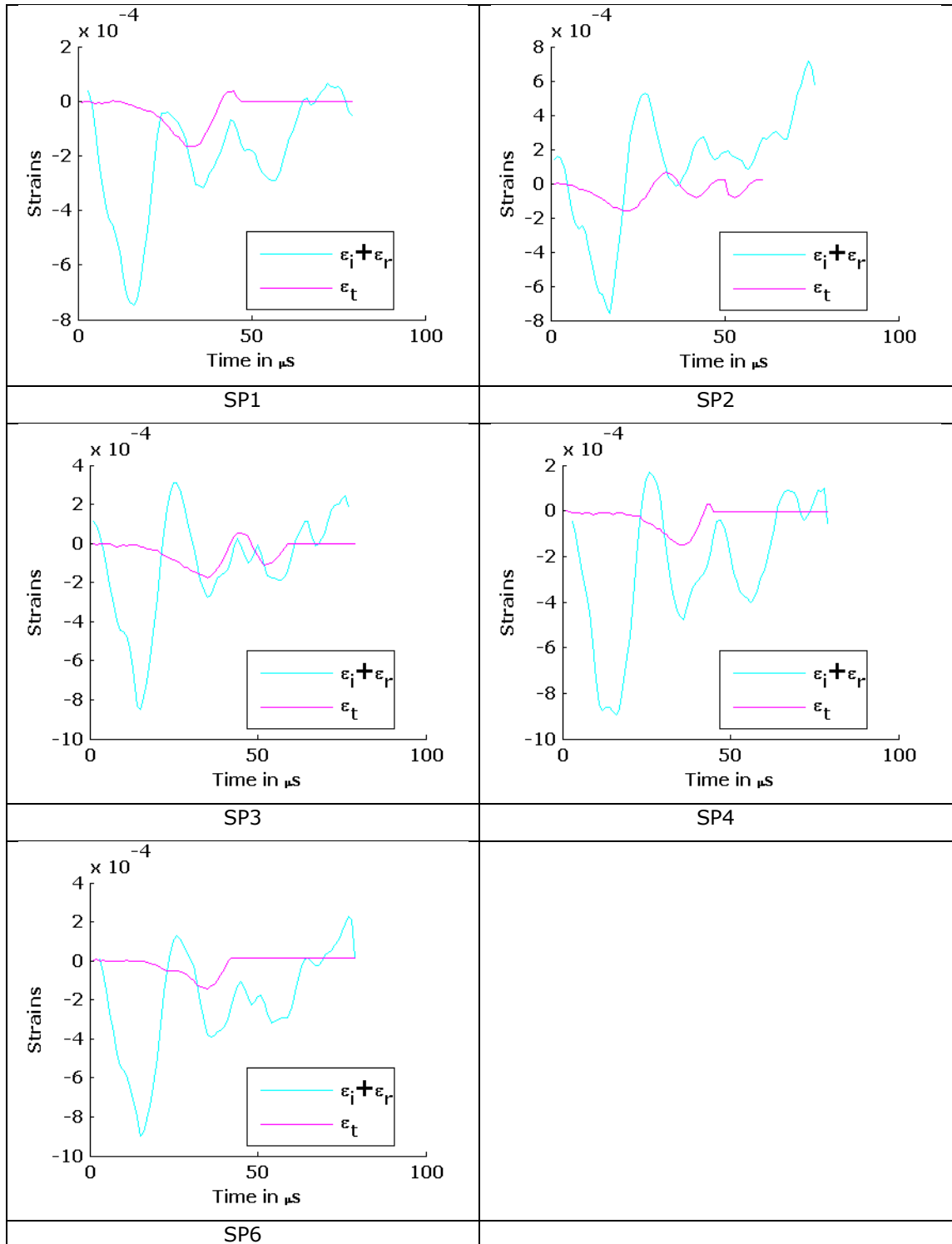
- Then use the coefficients to generate the pdf value for all values of x in Eq using pdf function
- Using plot command plot the curves along with the bar chart (Using *fitdist* can also be used to generate the coefficients for a given distribution without separately evaluating the Weibull parameter). Since the method of estimation of  $\sigma_0$  and  $\beta$  is different for the two methods (using *makedist* with data from *polyfit* and using data fitted data from *fitdist*), difference in the fitted distribution could be observed for both the methods. The *polyfit* estimates the parameters from the method of least square error approximation whereas *fitdist* uses parameters generated through maximum likelihood estimates. Hence, it was decided to use the *fitdist* for generation of the cdf and pdf fitted distribution plots.

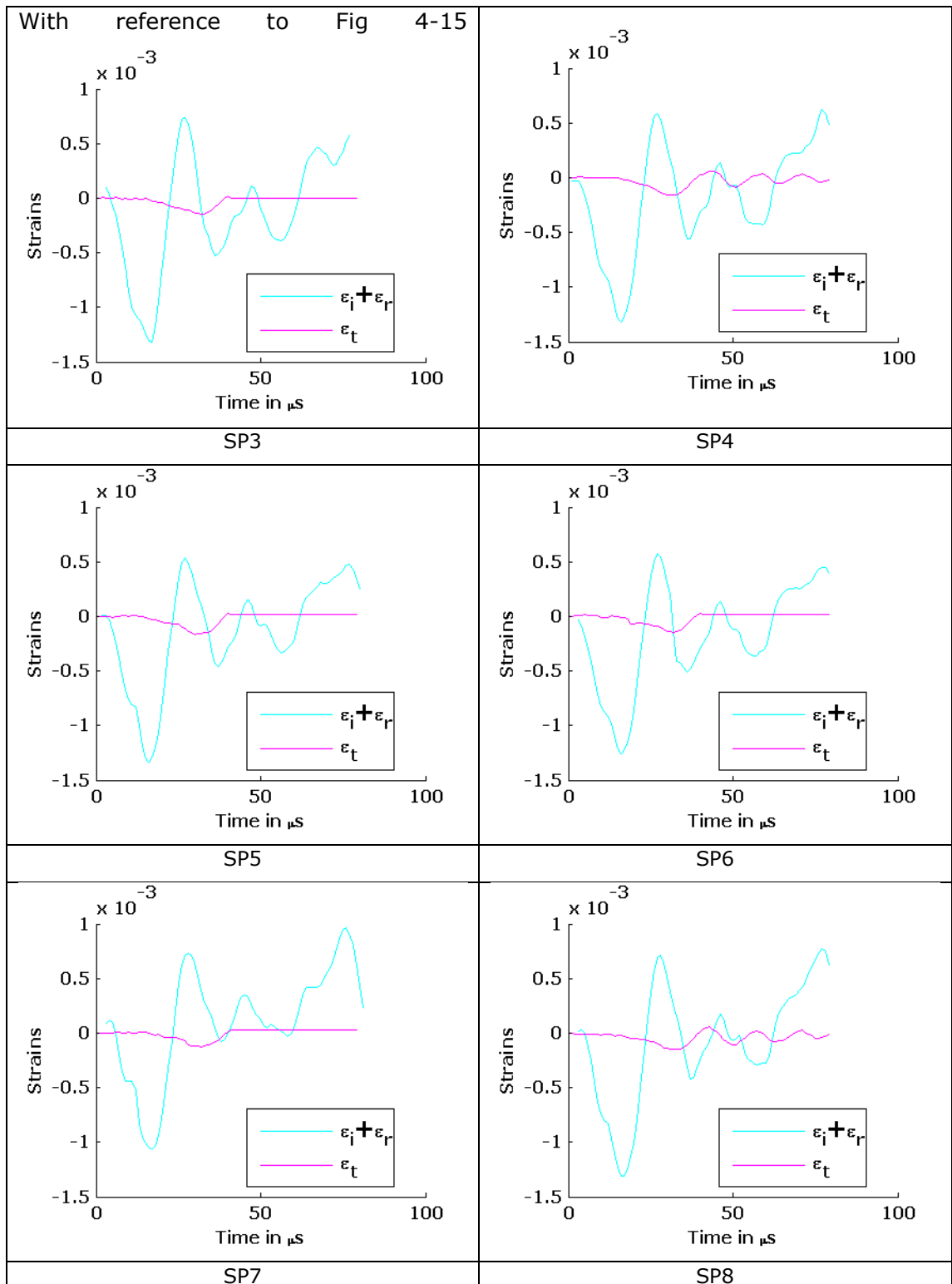
### **A 21.3 Plotting cumulative distribution functions based on the assumed fitted distributions**

The above methods can be used for plotting cdf for both distributions except that instead of *pdf* function, *cdf* function needs to be used.

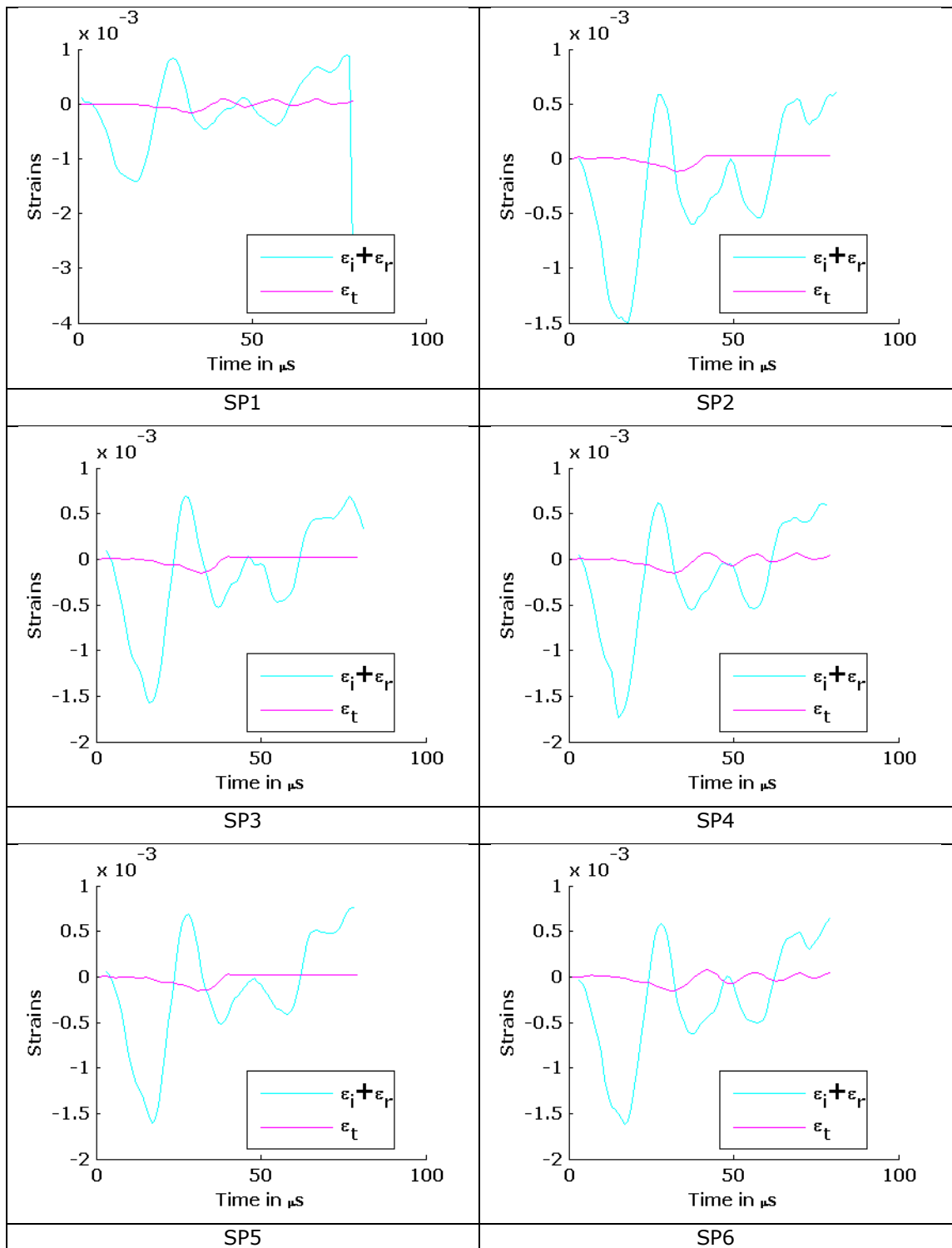
## Appendix 22 Dynamic stress equilibrium curves of SHPB testing

With reference to Fig 4-14

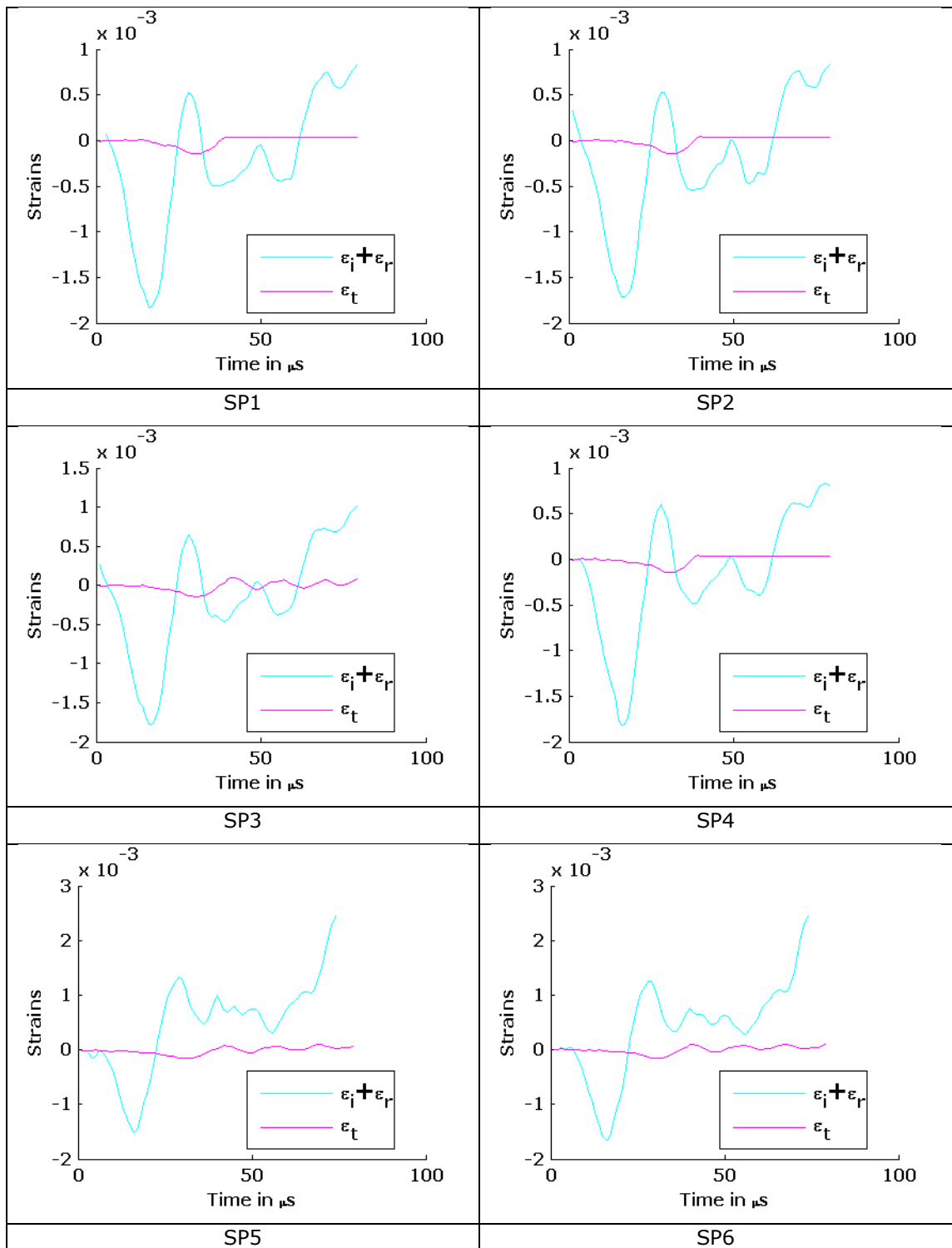




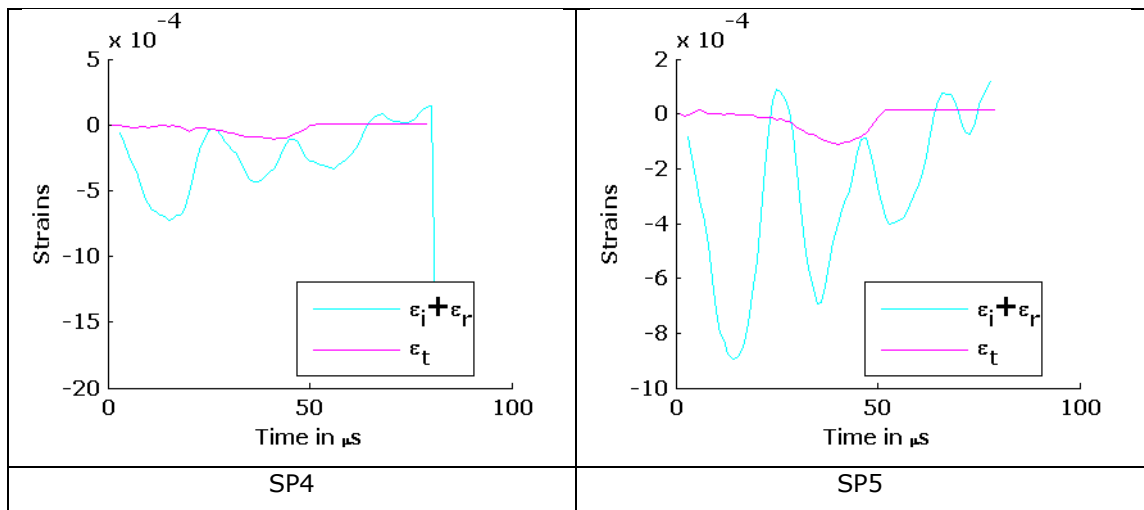
With reference to Fig 4-16



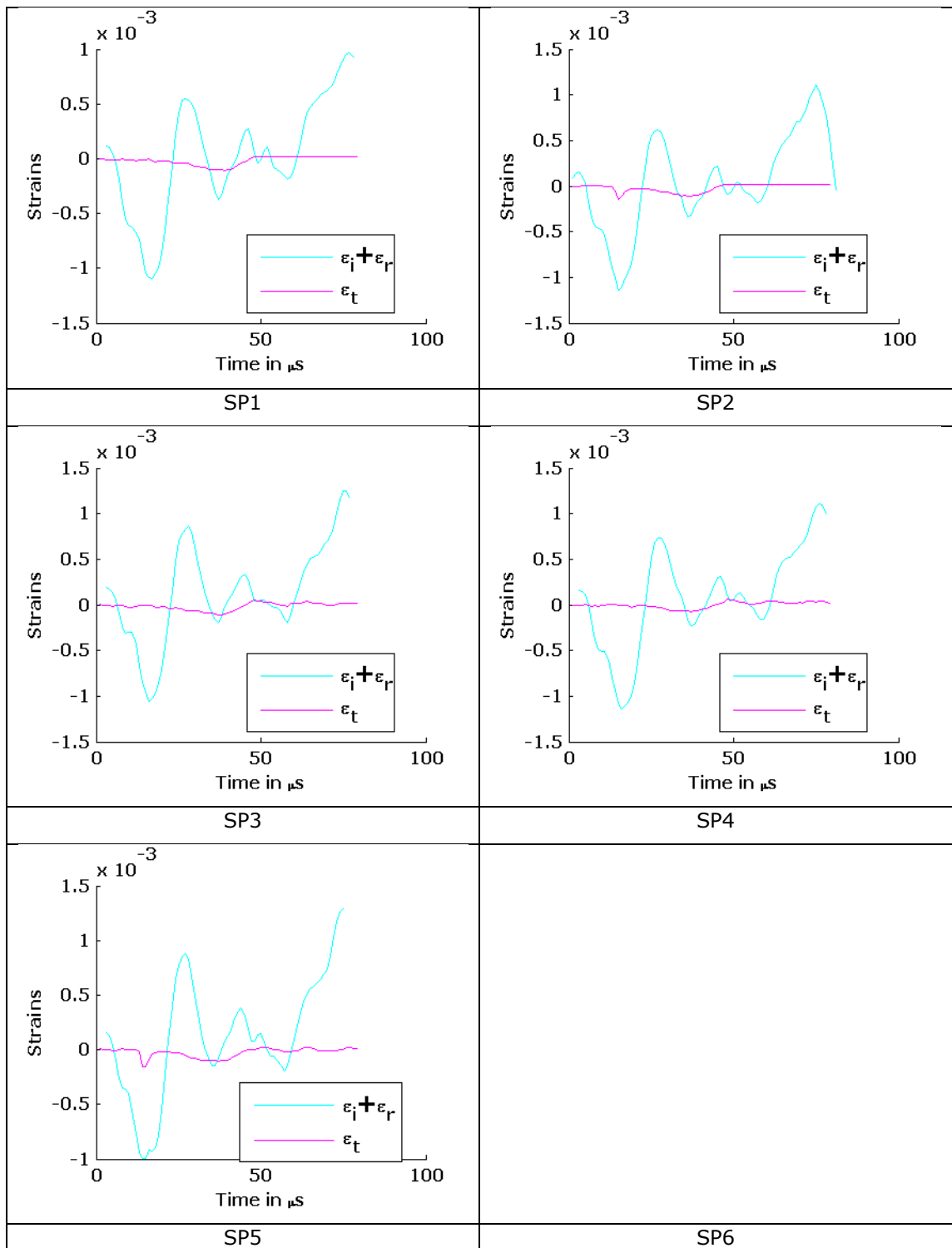
With reference to Fig 4-17



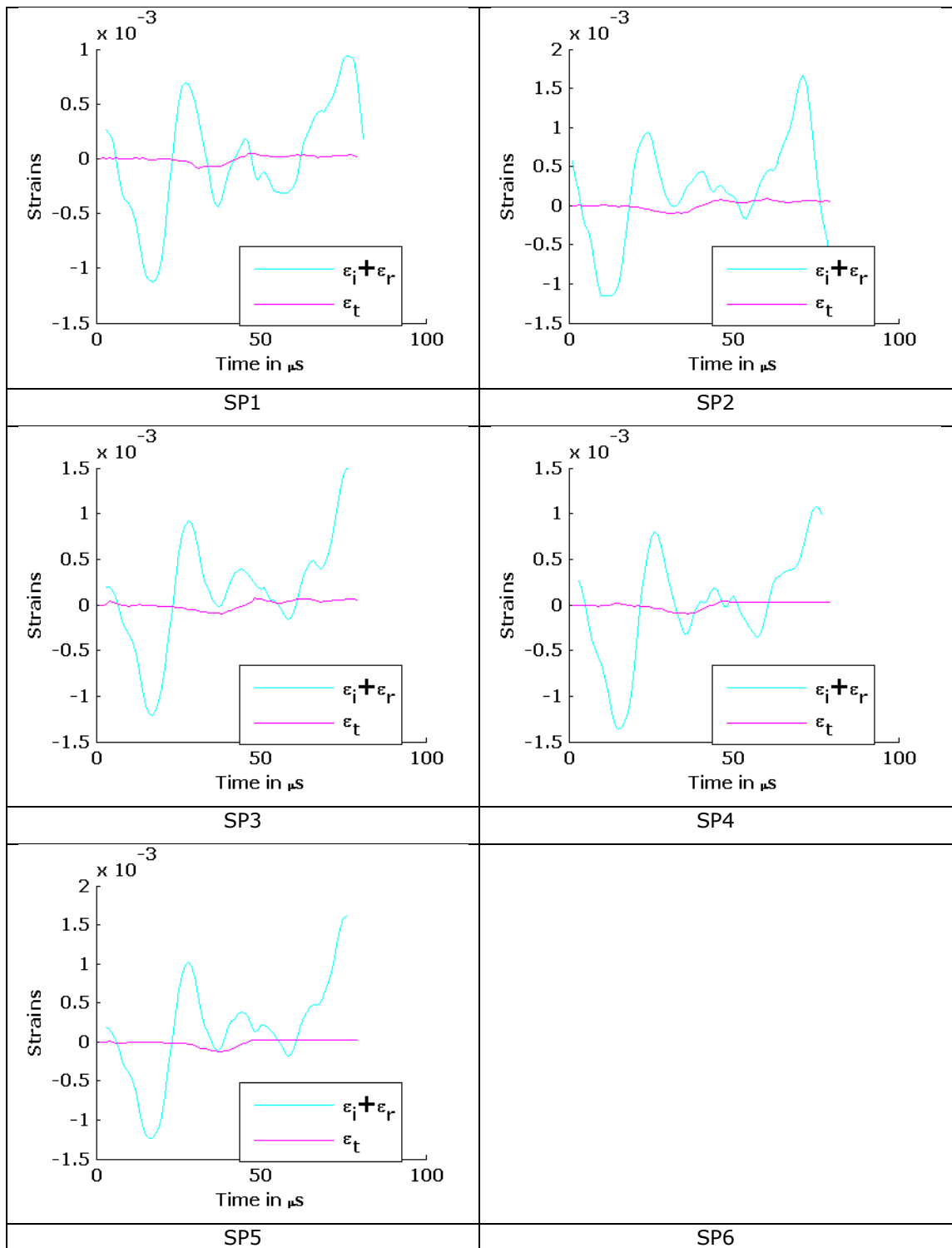
With reference to Fig 4-18



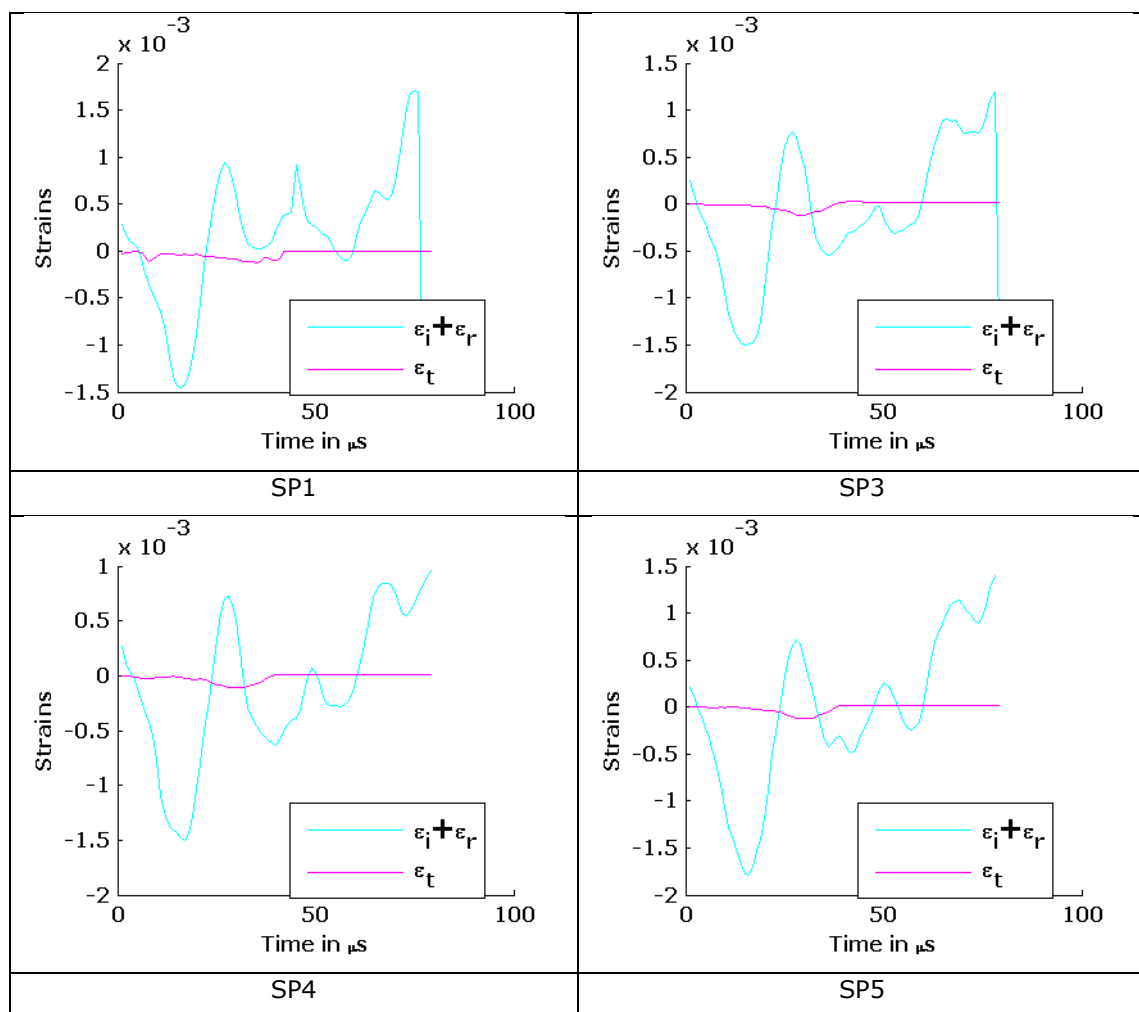
With reference to Fig 4-19



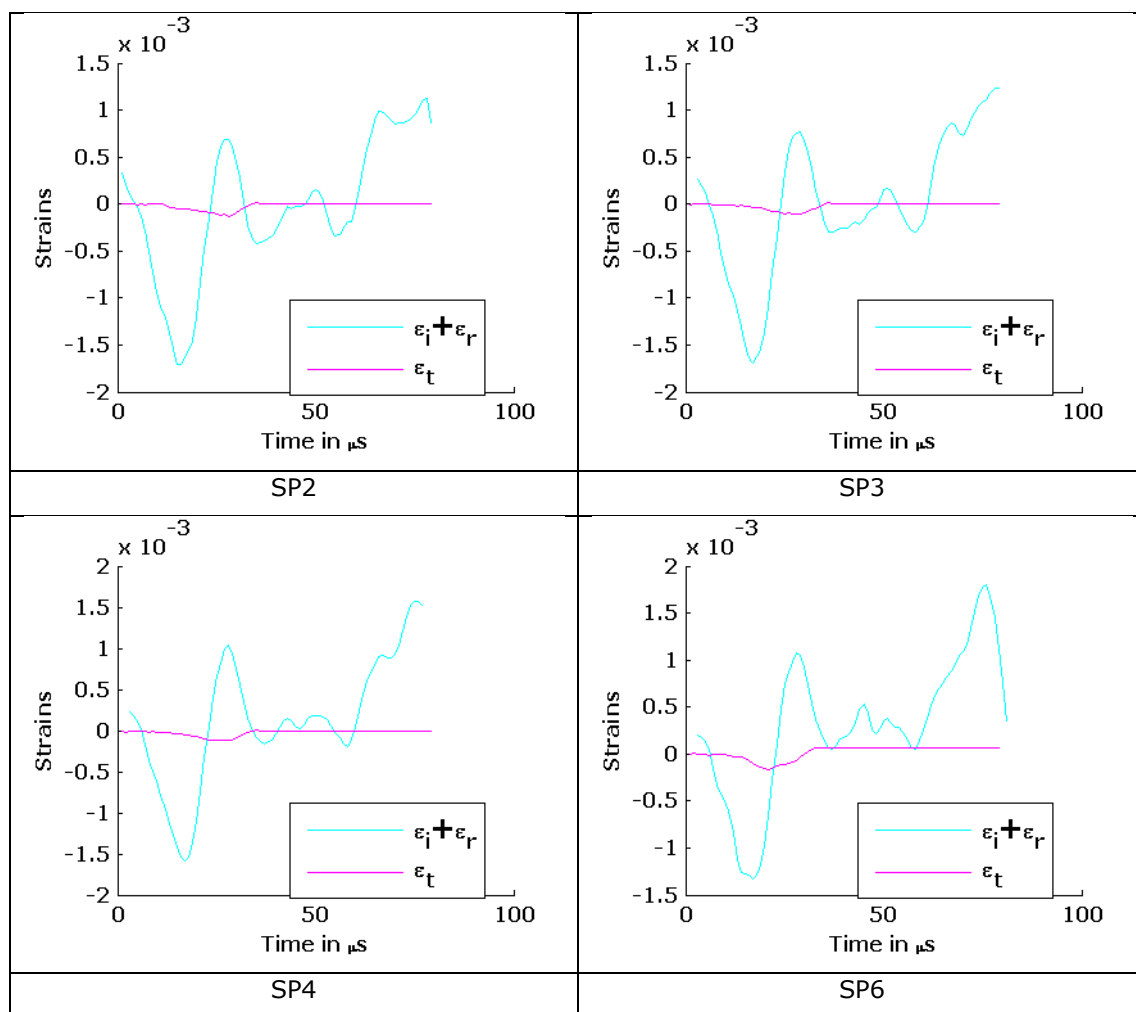
With reference to Fig 4-20



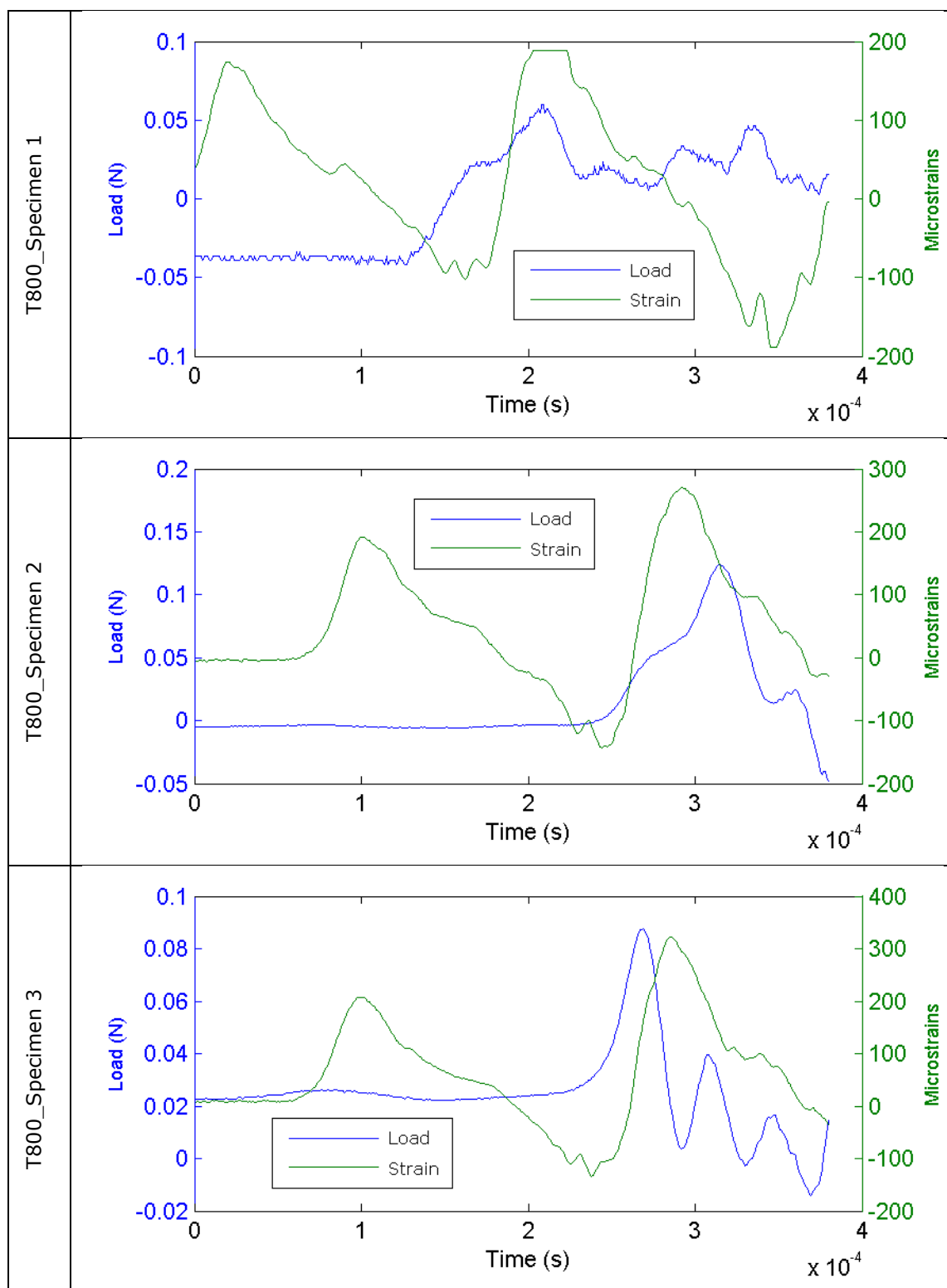
With reference to Fig 4-21

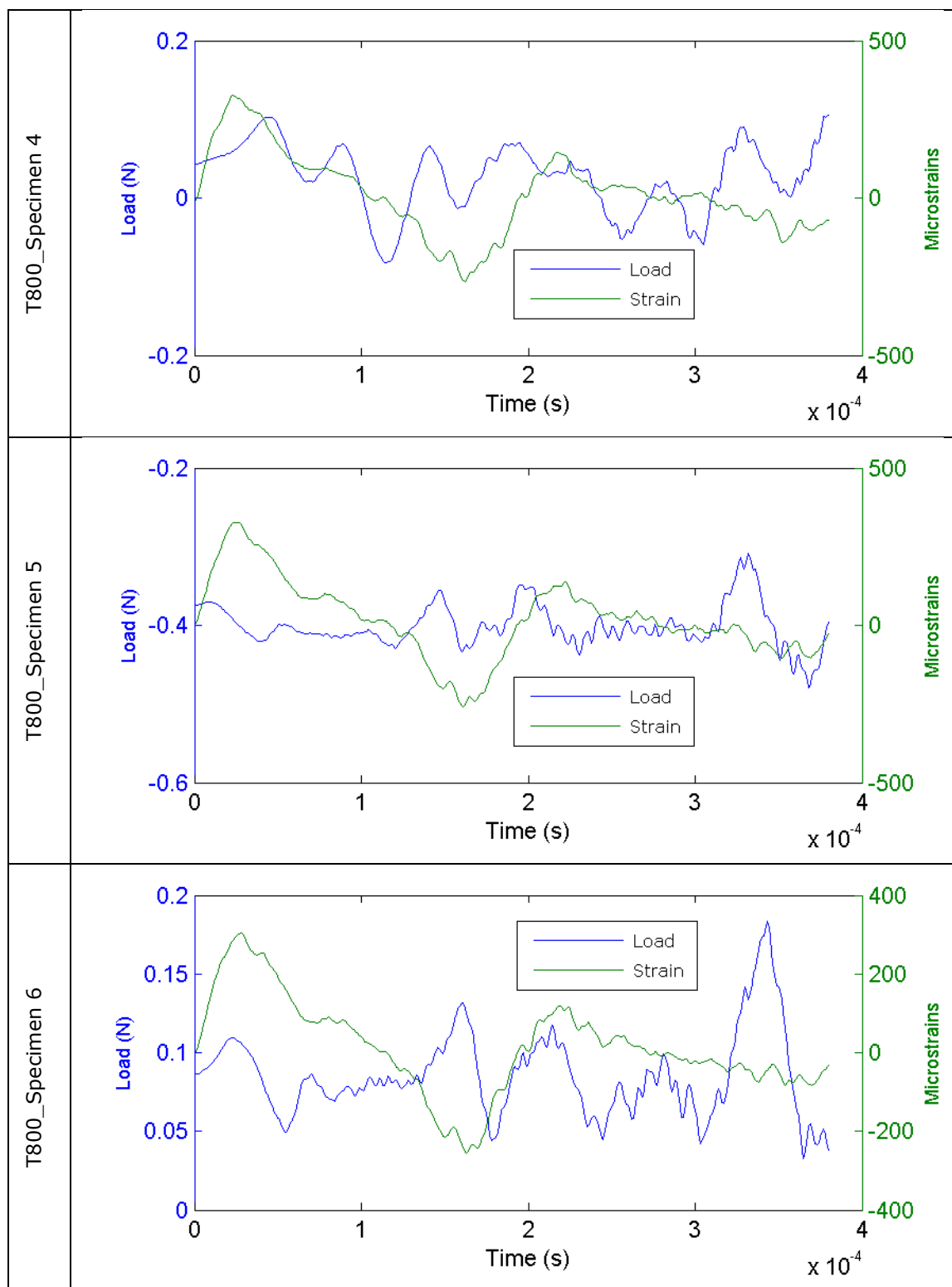


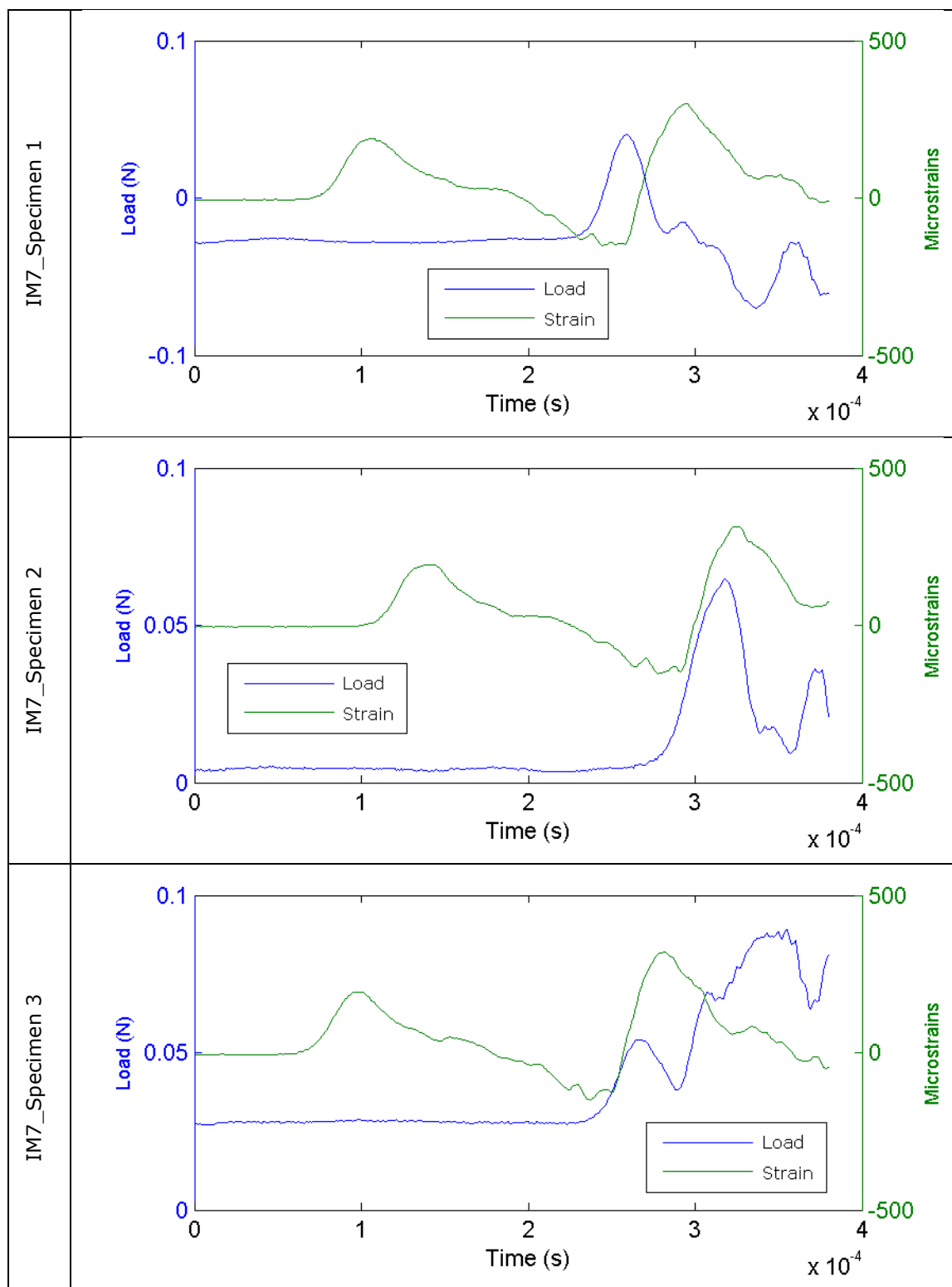
With reference to Fig 4-22

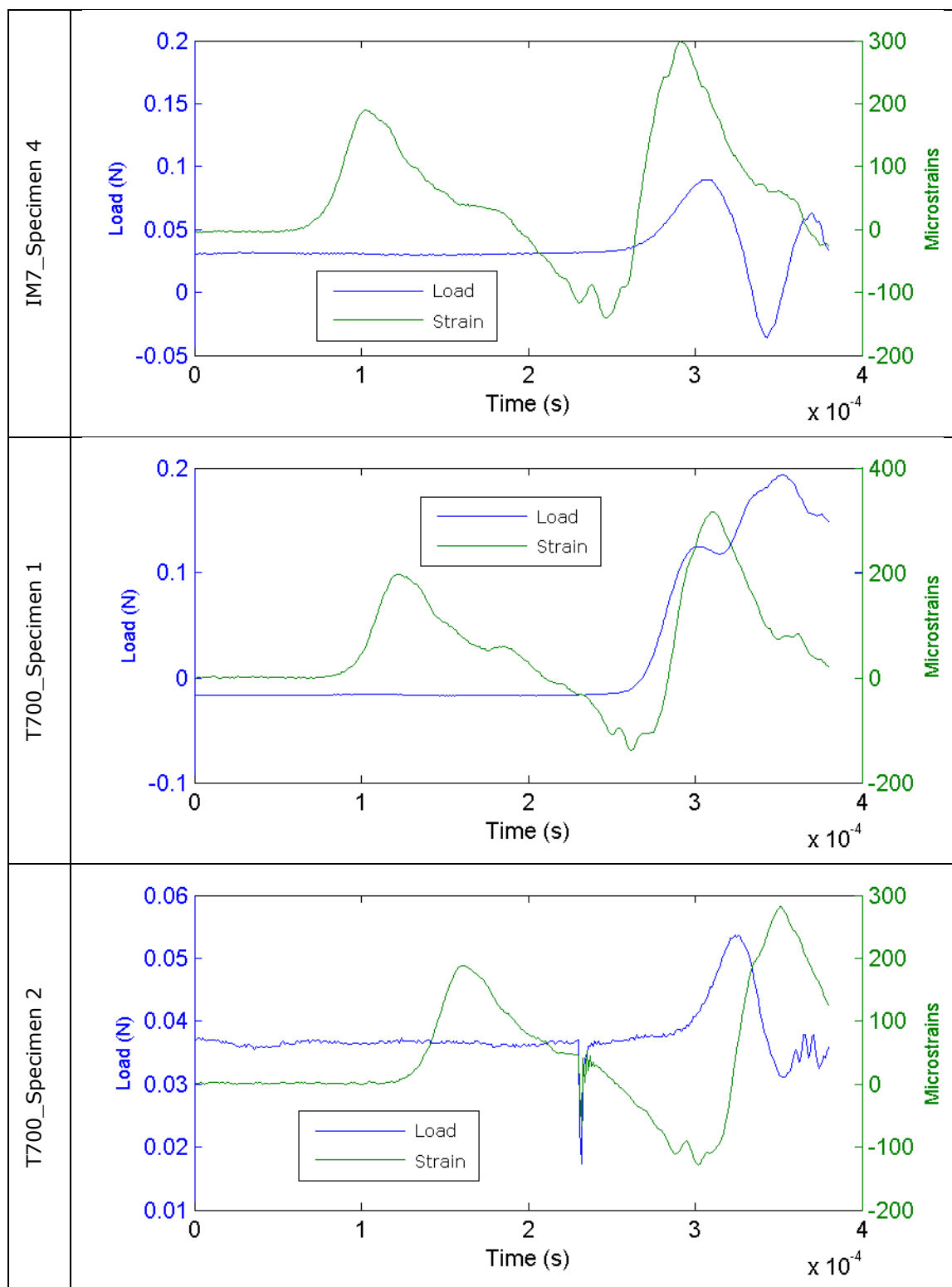


## Appendix 23 Raw data for high strain rate single fibre testing



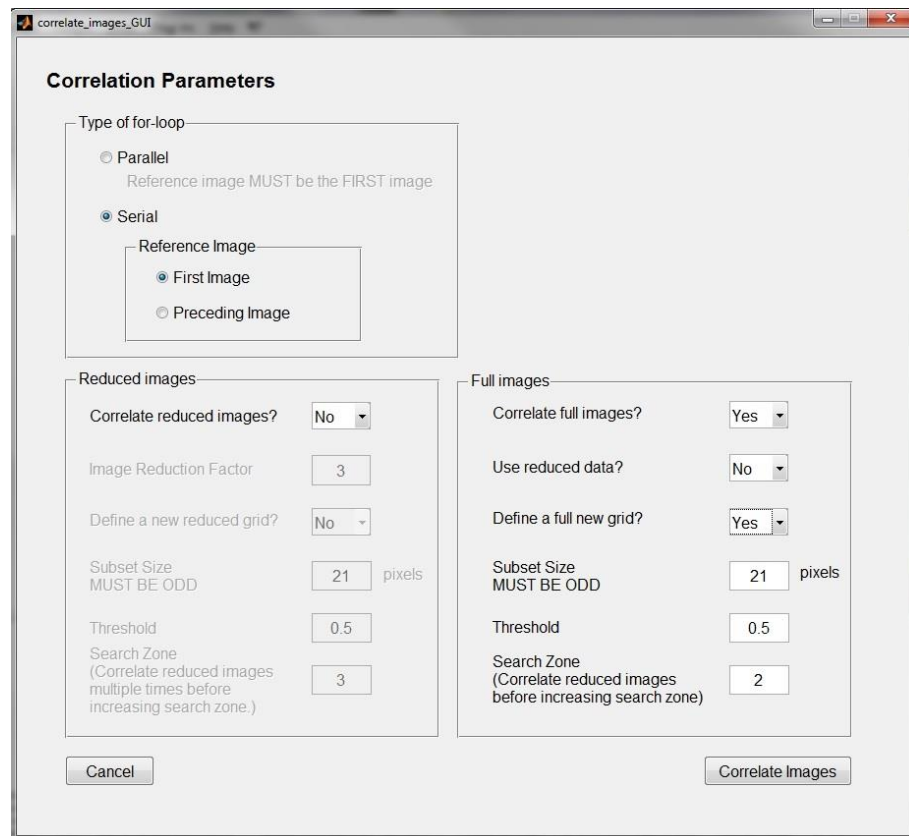




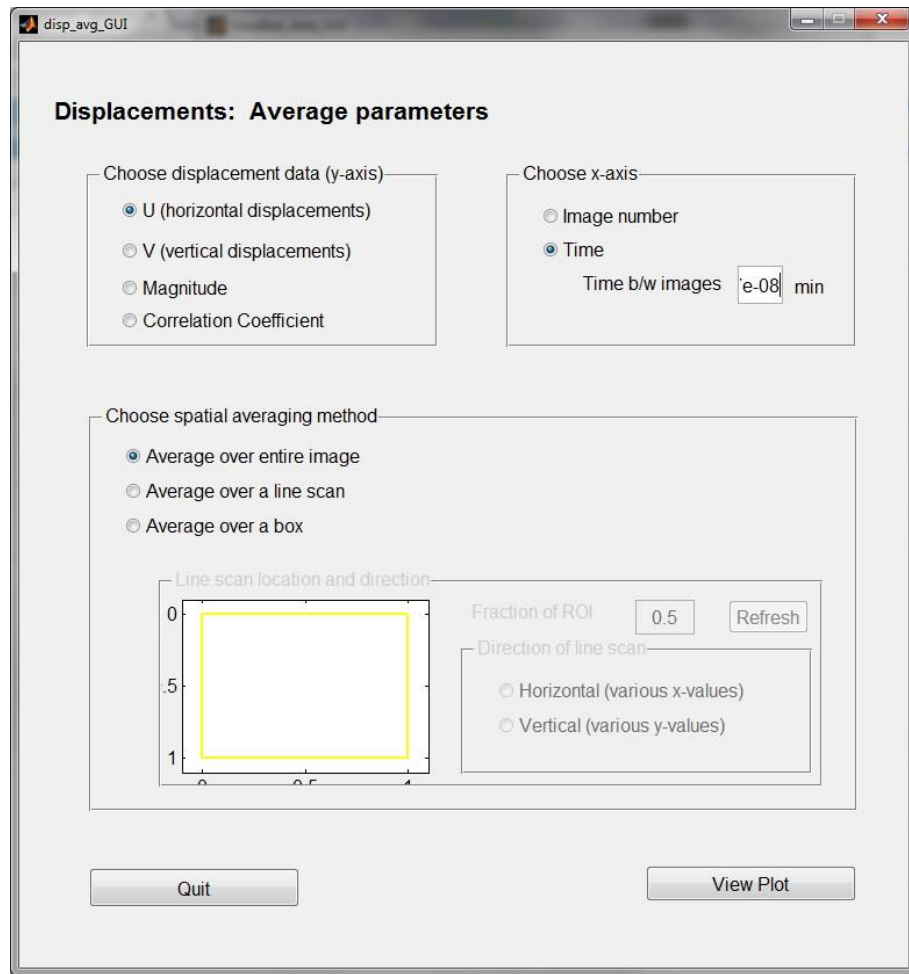


## Appendix 24 Methodology followed for evaluation of strain from high speed images

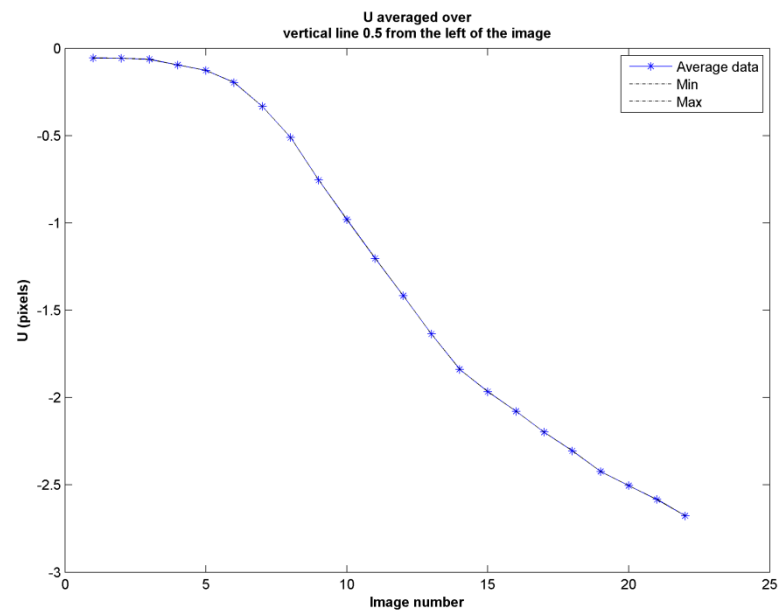
1. Use the digital correlation code as available in MATLAB and follow the procedure mentioned in the documentation.
2. Put all the images in the same folder.
3. Run the command- image\_setup\_GUI.
4. Run the command-correlate\_images\_GUI and following settings were made.



5. Then identify the region of interest from the reference image
6. Select the region of interest such it contains a contrasting zone
7. Define the step size as 1 pixel
8. Run the command- visualize\_data\_GUI
9. Choose full data and select spatial average method
10. The following selections shown in the screen shot below were made to plot the displacement data.



11. This generates the displacement for the region of interest in terms of pixels.



12. The pixel displacement is converted to equivalent displacement in millimetres through comparison of the known length of the clamp and their pixel coordinates. The pixel coordinate information was evaluated through MS Paint. It was observed that 3mm clamp length corresponds to 25 pixel coordinates. Based on this information, the displacement obtained was evaluated.

13. The bar movements evaluated for each time interval were then used to evaluate the strain during the loading process by dividing by the gauge length of the specimen.