

Thermoelastic Damping and Support Loss in MEMS Ring Resonators

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Abstract

Micro Electro-Mechanical Systems (MEMS) rate sensors are inertial devices based on a resonating structure to measure and detect rate of angular rotation. For applications requiring high performance angular rate measurements it is important to be able to design MEMS rate sensors with high quality factors (Q-factor or Q). However, the device performance is affected by physical damping mechanisms which influence the overall quality factor of the device. High performances from a damping perspective can be achieved by identifying the dominant damping mechanism and considering different ways to reduce damping. For vacuum encapsulated devices, thermoelastic damping (TED) and support loss are the most important damping mechanism in MEMS resonators. This thesis focuses on understanding and quantifying the effects of thermoelastic damping and support loss on the damping performance of a supported ring resonator.

An extensive review of different methods of modelling TED and support loss is presented. It is concluded that analytical models are not applicable to complex structures to predict damping due to their restrictive assumptions, and numerical approaches are required for accurate prediction. In this thesis a finite element based model using a fully coupled thermo-elastic approach is developed to quantify the thermoelastic dissipation in supported ring resonators and a detailed parameter study is conducted to understand the influence of ring geometry, support legs, micro-machined slots around the ring circumference, and material properties. The results show that damping in the support legs can have significant influence on TED in the resonator, and the optimum leg geometry can be identified to achieve high-Q. It is also ob-

served that the addition of slots improves Q for resonators having higher energy loss. However, for high- Q rings slots have a detrimental effect. The results are considered for designing high- Q ring resonators with reduced levels of thermoelastic damping, and devices fabricated based on high TED Q designs are tested experimentally to compare the performance with predicted values.

To analyse support loss behaviour in supported rings, a finite element model is developed using Perfectly Matched Layer (PML) technique to accurately predict the dissipation of energy through the elastic wave propagation from the resonator into its substrate. The model is validated by identifying key modelling issues, and strategies have been developed to effectively calculate PML parameters are discussed. The model is used to analyse support loss in supported ring resonators, and it is concluded that support loss is negligible compared to TED. Further investigations consider the effect of key design parameters on support loss, including a quantitative evaluation of the influence of an unbalanced vibration on the support loss by considering systematic design variation and manufacturing imperfections in the supported ring resonators. It is concluded that mass imperfection in the resonator and cycling asymmetry of the central supporting structure not only induce frequency splitting between degenerate pair of modes but also increase support loss due to the unbalance of the mode.

It is expected that the results and contributions of this work will aid the development of high performance MEMS angular rate sensors by reducing the damping behaviour.

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Nomenclature

α	Coefficient of thermal expansion (1/K)
β	End value of PML absorbing function
β_n	Mode related constant of cantilever beam
χ_n	Mode shape factor of cantilever beam
Δf	Frequency split (Hz)
ΔT	Change in temperature from reference (K)
ΔW	Energy lost per cycle of vibration of a resonator (J)
ΔW_u	Energy lost per cycle of flexural vibration (J)
ΔW_w	Energy lost per cycle of longitudinal vibration (J)
δ	Thickness of adsorbate layer (m)
Δ_M	Relaxation strength
δ_{ij}	Kronecker delta
Δ_i	Energy dissipated from i 'th part of a system (J)
Δ_{legs}	Energy dissipated from support legs (J)
Δ_{ring}	Energy dissipated from ring (J)
η	Loss factor
η_{other}	Loss factor for other damping sources
$\eta_{overall}$	Overall loss factor of a resonator
η_{TED}	Thermoelastic loss factor
λ	Eigenvalue
Λ, μ	Lamé constants
λ_l	Longitudinal wavelength (m)
λ_s	Shear wavelength (m)

λ_{wave}	Wavelength (m)
ν	Poisson's ratio
Ω	Rate of rotation about the rotational axis (rad/s)
ω_0, ω_n	Natural frequency (rad/s)
ω_d	Damped natural frequency (rad/s)
$\Omega_{(Brownian)}$	Brownian noise floor ($^{\circ}/\text{sec}/\sqrt{\text{Hz}}$)
$\Omega_{(Electronic)}$	Electronic noise floor ($^{\circ}/\text{sec}/\sqrt{\text{Hz}}$)
$\Omega_{(min)}$	Minimum rotation rate ($^{\circ}/\text{sec}/\sqrt{\text{Hz}}$)
ω_{die}	Natural frequency of die (rad/s)
ω_{res}	Natural frequency of resonator (rad/s)
ρ	Material density (kg/m^3)
$\sigma(s)$	Absorbing function
σ_{ij}	Stress tensor ($i, j = x, y, z$)
τ	Thermal relaxation time (s)
τ_{legs}	Thermal relaxation time of support legs (s)
τ_{ring}	Thermal relaxation time of a ring (s)
$\theta_{slot-gap}$	Angular gap between neighbouring slots ($^{\circ}$)
θ_{slot}	Angular extent of a slot ($^{\circ}$)
$\tilde{x}$	Stretched coordinate
Υ	Thermoelastic constant
ε_{ij}	Strain tensor ($i, j = x, y, z$)
ζ	Damping ratio
A	Cross-sectional area of ring (m^2)
A_g	Angular gain
a_r	Axial thickness of ring (m)
b	Dimension of beam in the direction of flexing (m)
b_r	Radial thickness of ring (m)
C	Damping coefficient (kg/s)
c	Wave speed (m/s)

c_1	Damping coefficient for pin (kg/s)
c_2	Damping coefficient for die (kg/s)
c_l	Longitudinal wave speed (m/s)
C_p	Specific heat capacity at constant pressure (J/kg K)
c_P	Damping coefficient for 1θ rigid body motion (kg/s)
c_Q	Damping coefficient for 2θ motion (kg/s)
c_R	Damping coefficient for 3θ motion (kg/s)
C_s	Sense capacitance (F)
c_s	Shear/Transverse wave speed (m/s)
C_v	Specific heat capacity at constant volume (J/m ³ K)
$C_{parasitic}$	Parasitic capacitance (F)
d	Diameter of ring (m)
d_0	Capacitive gap (m)
E	Young's modulus (N/m ²)
e	Strain dilatation
E_0	Young's modulus at reference temperature (N/m ²)
E_c	Complex Young's modulus (N/m ²)
E_d	Dissipation part of Young's modulus (N/m ²)
E_R	Relaxed value of Young's modulus (N/m ²)
E_R	unrelaxed value of Young's modulus (N/m ²)
$f(x)$	Stretching function
f, f_0	Damped natural frequency (Hz)
F_N	Normal force (N)
F_S	Shear force (N)
F_x	Driving force (N)
H	Thickness of cantilever beam (m)
i	Complex number
I_i	Mechanical energy flux (W/m ²)
I_z	Area moment of inertia about axial axis z (m ⁴)

J_0	Bessel function of the first kind of zero order
J_1	Bessel function of the first kind of first order
K	Stiffness (N/m)
k	Thermal conductivity (W/m K)
k_1	Stiffness coefficient for pin (N/m)
k_2	Stiffness coefficient for die (N/m)
k_B	Boltzmann constant, 1.38×10^{-23} (J/K)
k_P	Stiffness coefficient for 1θ rigid body motion (N/m)
k_Q	Stiffness coefficient for 2θ motion (N/m)
k_R	Stiffness coefficient for 3θ motion (N/m)
k_w	Wave number (1/m)
L	Length of cantilever beam (m)
L	Truncation distance of a semi-infinite rod (m)
L_{offset}	Distance of hub offset from the centre (m)
L_{PML}	PML length (m)
M	Mass (kg)
m, n	Mode number
M_0	Molecular mass of a gas (kg/kmol)
m_d, m_{die}	Mass of die (kg)
m_p	Mass of pin (kg)
m_{res}	Mass of resonator (kg)
m_{ring}, m_r	Mass of ring (kg)
N_{el}	Number of elements
P	Pressure of the gas (Pa)
p	Order of polynomial
Q	Quality factor
$Q_{coupled}$	Quality factor die-resonator coupled system
Q_{die}	Quality factor of die
q_{drive}	Drive amplitude (m)

Q_{gas}	Quality factor associated to gas damping
Q_i	Quality factor of i 'th part of a system
Q_{legs}	Quality factor of support legs
Q_{min}	Minimum value of quality factor
Q_{other}	Quality factor associated to other damping sources
$Q_{overall}$	Overall quality factor
Q_{res}	Quality factor of resonator
Q_{ring}	Quality factor of a ring
$Q_{support}$	Quality factor associated to support loss
Q_{TED}	Quality factor associated to thermoelastic damping
Q_{total}	Total quality factor
r	Radius of ring (m)
R_0	Universal gas constant, 8.314 (J/mol K)
r_i	Inner radius of ring (m)
r_o	Outer radius of ring (m)
r_{arc}	Support arc radius (m)
r_{disk}	Disk radius (m)
r_{hub}	Hub radius (m)
r_{PML}	PML radius (m)
r_{post}	Support Post radius (m)
r_{sub}	Substrate radius (m)
T	Temperature (K)
T_0	Reference/absolute temperature (K)
t_{disk}	Disk thickness (m)
t_{post}	Support Post thickness (m)
$u(x, t)$	Axial displacement of rod
v_j^*	Complex conjugate of the velocity vector
V_p	Polarisation voltage (V)
V_i	Energy stored in the i 'th part of a system (J)

V_{legs}	Energy stored in support legs (J)
V_{ring}	Energy stored in ring (J)
W	Total stored energy in a resonator (J)
W	Width of cantilever beam (m)
w_{gap}	Leg gap (m)
$W_{leg-nominal}$	Nominal leg width (m)
w_{leg}	Leg width (m)
W_{PML}	PML width (m)
w_{slot}	Slot width (m)
W_{sub}	Substrate width (m)
Z	Acoustic Impedance (kg/m ² s)
[K]	Stiffness matrix
[M]	Mass matrix

Abbreviations

CVD	Chemical Vapor Deposition
FE	Finite Element
FEA	Finite Element Analysis
FEM	Finite Element Method
HRG	Hemispherical Resonator Gyroscope
HSR	Hemispherical Shell Resonator
MEMS	Micro Electro-Mechanical Systems
PML	Perfectly Matched Layer
SCS	Single Crystal Silicon
SFD	Squeeze Film Damping
TCE	Temperature coefficient of Young's modulus
TE	Thermoelastic
TED	Thermoelastic Damping

Chapter 1

Introduction and Literature Review

1.1 General Introduction

Micro Electro-Mechanical Systems (MEMS) is a promising technology for industrial and consumer devices, combining silicon based microelectronics with micromachining technologies [1]. Since the early 1980's, MEMS engineering has rapidly expanded. Low cost and large volume production are the distinct advantages of MEMS devices over conventional, macro-scale counterparts. With the advancement of MEMS technology, a wide range of devices are currently being used in various areas of science and engineering. Major shares of the annual MEMS production are held by applications in ink-jet printer heads, mirror devices for projectors, and various sensors including accelerometers and angular rate sensors [2].

Rate sensors are one of the most important MEMS inertial sensors after accelerometers, and are used to detect and measure the rate of angular rotation. The most popular MEMS rate sensors are classed as vibratory gyroscopes which use simple mass, shells or rings as the vibrating element (commonly referred as the resonator). Due to their small size, light weight, low energy consumption and low fabrication costs, there has been a great deal of interest in a wide range of applications from automotive to aerospace and consumer electronics. The performance of these devices is governed by how close the frequencies are and by energy dissipation due to damping of the vibration. Damping is quantified in terms of the quality factor (Q-factor)

and a high Q-factor (or high-Q) in rate sensors enables higher rate resolution, better bias stability, and lower power consumption [3] to be achieved. As the dissipation of energy in the resonator increases, lower Q-factors degrade sensor performance. Research into the design of resonators having high-Q or low levels of damping mainly focuses on identifying the dominant source of energy dissipation. The current work is motivated by the need to develop high performance angular rate sensors with high-Q resonators.

To design high-Q resonators, all dissipation mechanisms that contribute need to be considered. The most important sources of energy dissipation are gas damping, thermoelastic damping (TED), surface loss, and support loss [4]. However, vacuum encapsulation of the device eliminates the contribution from gas damping and surface losses become significant when the surface to volume ratio increases as devices are further miniaturised, so thermoelastic damping and support loss become the major bottlenecks governing resonator performance. The ability to accurately model and predict damping effects is required to aid the development of high-Q resonators. Previous research work on thermoelastic damping and support loss has focused on analytical models for simple resonator geometries, and are not sufficient to accurately predict damping behaviour in complex structures due to the restrictive assumptions made. The work in this thesis is motivated by the need to gain an improved understanding of energy dissipation mechanisms in resonators, and uses numerical models to quantify and predict damping behaviour in complex resonating structures, to develop high-Q resonators.

1.2 MEMS Vibratory Gyroscope

Vibratory gyroscopes are inertial devices used to measure the angular motion of an object. Depending on whether the angular velocity (rate of rotation or simply rate) or orientation is being measured, all vibratory gyroscopes are classified in two categories [5]: rate gyroscopes or *rate sensors* measure the rate of rotation, and whole angle

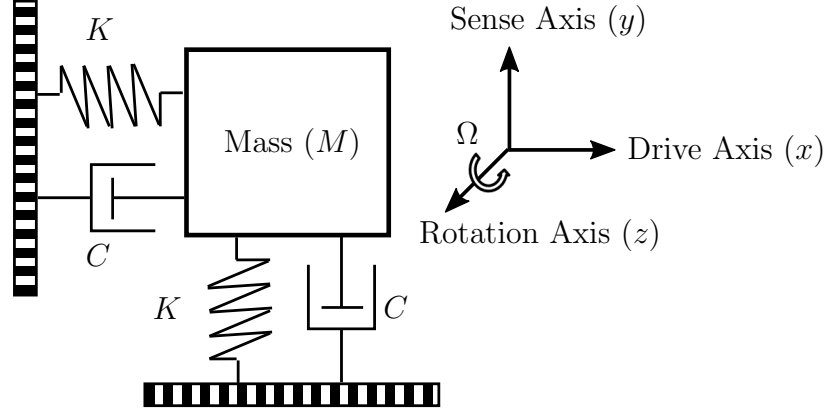


Figure 1.1: Simplified 2DOF model of a vibratory gyroscope.

gyroscopes or *rate integrating* gyroscopes measure the angular position or orientation directly. Most MEMS vibratory gyroscopes are of the rate measuring type [5]. Unlike the conventional gyroscopes which use a spinning wheel, a vibrating element (also called the resonator) is used in MEMS vibratory gyroscopes. The resonators can be simple proof-masses, tuning-forks or axisymmetric structures such as rings or shells. The principle of an ideal vibratory gyroscope, operating in the rate mode, is discussed in the following section.

1.2.1 Principle of Operation

All vibratory gyroscopes depend on the phenomenon of Coriolis effect. This effect is experienced by an object when undergoing motion in a rotating non-inertial reference frame [6].

The operation of an ideal vibratory gyroscope can be described by an inertia element with spring suspensions and dampers having two degrees of freedom (DOF). A simplified model for a vibratory gyroscope is shown in Figure 1.1.

A mass M is attached to a fixed frame by two springs of stiffness K and two dampers with damping coefficients C . An external force F_x is applied to the mass (M) to vibrate at a frequency ω_0 along the drive axis x using an actuation mechanism (e.g. electrostatic or electromagnetic). This mode is called the drive or the primary mode. When the mass experiences a rotation Ω about the rotational axis z , the Coriolis force

causes the mass to vibrate in a direction orthogonal to the drive (x) and rotational axes (z) i.e. the y axis. This mode is called the sense or the secondary mode. The Coriolis force couples the primary/drive mode of vibration to the secondary/sense mode, and the change in movement of the mass is used as a measure of the applied rate Ω . Generally, rate sensors are designed to detect small angular velocities ($\Omega \ll \omega_0$), and often the constant rate (Ω) condition is assumed i.e. $\dot{\Omega} = 0$ [5].

Considering a constant rate input and ignoring Ω^2 terms, the dynamics of the rate sensor based on this model can be expressed using the following equations of motion [5]

$$M\ddot{x} + C\dot{x} + Kx + 2M\Omega\dot{y} = F_x \quad (1.1)$$

$$M\ddot{y} + C\dot{y} + Ky - 2M\Omega\dot{x} = 0 \quad (1.2)$$

Eqs. (1.1) and (1.2) assumes that both drive mode and sense mode have the same resonance frequency ($\omega_0 = \sqrt{K/M}$) and damping coefficient (C), i.e. the rate sensor is operated in a matched-mode condition. It is worth mentioning that a split-mode condition is also used in some rate sensors, where the frequencies of the drive and sense mode are different from each other. However, most rate sensors are designed such that the matched-mode condition is observed [5]. Although due to imperfections during fabrication, the two modes are never perfectly matched and frequency split occurs. $2M\Omega\dot{x}$ is the Coriolis coupling term from the x to y axis. However, the Coriolis coupling term $2M\Omega\dot{y}$ from y to x axis is always three orders of magnitude smaller [7], hence can be neglected in Eq. (1.1). In the system considered, a constant amplitude $x(t) = X_D \sin \omega_0 t$ is maintained with a constant amplitude drive force F_x at the drive frequency ω_0 i.e. $F_x = F_D \sin \omega_0 t$. Substitution of the constant drive amplitude ($x(t) = X_D \sin \omega_0 t$) in Eq. (1.2) gives the Coriolis induced amplitude of

the response in the sense direction y [5]

$$|y(t)| = \frac{2X_D\Omega Q}{\omega_0} \quad (1.3)$$

where $Q = \frac{\omega_0 M}{C}$ is the quality factor.

Eq. (1.3) shows that amplitude of the sense mode is proportional to the angular rate input. Thus the amplitude of the response in the y direction provides a measure of the applied angular velocity Ω . It is worth noticing that the sense mode displacement is amplified by Q , and a higher Q will allow transfer of more energy from the drive to sense mode [8].

Using a simple vibrating structure for rate sensors comprising a suspended mass has significant disadvantages. For example, a false rate output can occur due to coupling of the drive and sense motion even when there is no applied rate. Ideally, there should be no response in the sense direction at zero rate input, but false rate output is observed mainly due to asymmetries in stiffness and damping associated with the suspension and mounting [9]. In addition, they are susceptible to external linear vibration, in particular in the sense direction, which can cause the mass to vibrate with the resulting response interpreted as input angular rate. To avoid such problems, more sophisticated axisymmetric structures, such as cylindrical shells and circular rings are used. A brief discussion of ring based rate sensors, which form the main focus of this thesis, is presented in the following section.

1.2.2 Ring Based Rate Sensor

Ring resonator based rate sensors measure the angular velocity of a body about the polar axis of the ring, based on the Coriolis effect. For a perfect ring, two orthogonal vibration modes with the same shape and equal natural frequencies are used as the primary and secondary modes. The modes most commonly observed in ring resonators are the extensional mode (0θ), rigid-body modes (1θ) and flexural modes (2θ , 3θ etc.). For a perfect ring the modes occur in degenerate pairs (except

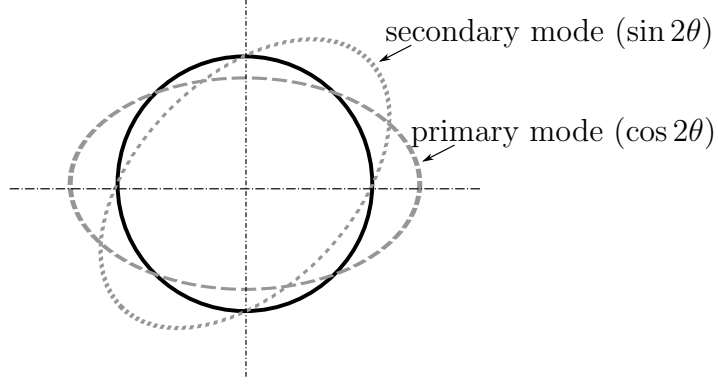


Figure 1.2: Degenerate pair of 2θ in-plane modes of a ring resonator. Dashed Line: primary mode ($\cos 2\theta$) and dotted line: secondary mode ($\sin 2\theta$).

the 0θ mode) at a mutual angle of $\pi/2n$, where n is the mode number. For angular rate sensors, it is the 2θ in-plane mode generally utilised as a means for detecting rate [9, 10]. The degenerate pair of 2θ in-plane modes are shown in Figure 1.2. The secondary mode has the shape and frequency the same as the primary mode, but the mode shape is rotated by 45° about the primary mode. When the ring vibrates (in-plane) in its primary mode, any rotation about its polar axis results in energy transfer to the secondary mode due to the Coriolis force. The magnitude of the response of the secondary mode is directly proportional to the angular rate to be measured. The balanced symmetric structure of the rings are one of the important features of the ring based rate sensors, which makes them less sensitive to spurious vibrations [10]. Also, the two identical modes with equal frequencies provide higher sensitivity due to the secondary mode response being amplified by the Q-factor, as seen in Eq. (1.3). Imperfections (due to mass or stiffness asymmetries) during fabrication of rings are inevitable. These imperfections fix the position of the modes relative to the ring and yield small frequency splits [11]. Different techniques are available to correct this frequency mismatch in practical devices such as mass trimming via laser ablation [12] or electrostatic trimming based on balancing/tuning electrodes located around the ring structure [10].

In practical rate sensors (see Figure 1.3), ring resonators are supported by flexible legs and symmetry considerations require at least eight support legs/springs to be

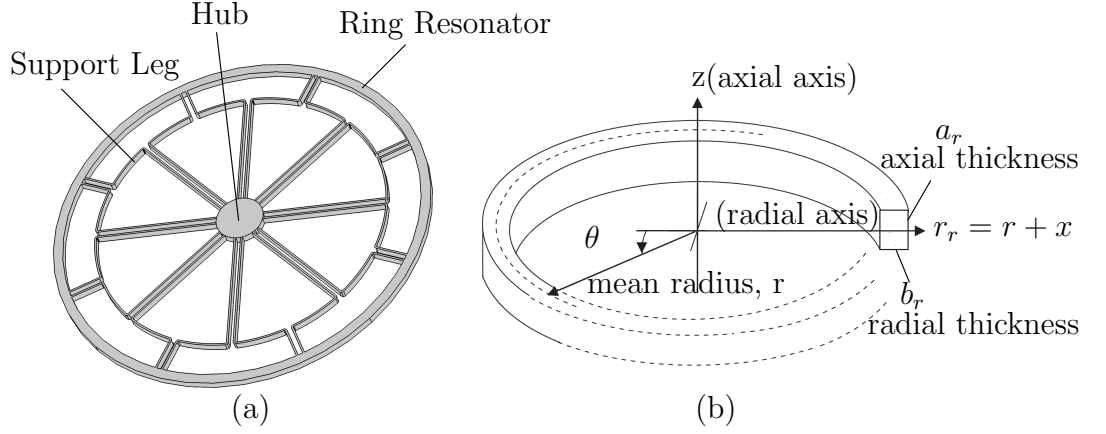


Figure 1.3: (a) A ring structure supported to a central hub by supporting legs, and (b) schematic of a thin ring with its global polar coordinate system (r_r, θ, z) [16].

used to ensure the device is balanced and the degenerate pair of flexural modes do not split [10]. It is necessary to include the contribution of the support legs when modelling the dynamic behaviour of ring based rate sensors. Ong [13] presented a model of the resonator that used point masses and three discrete springs (radial, tangential and torsional) to model each support leg. This lumped parameter model was used to determine the governing equation of motion and natural frequencies for the complete sensor. Wong [14] developed a model to predict the thermoelastic damping in ring resonators, which neglected the influence of the support legs. Chouvioun *et al.* [15] provided a detailed analytical model of support loss from a ring resonator with leg supports positioned outside the ring. The model used a ray tracing method to predict the natural frequencies and mode shapes of the ring and support beam structure, but no explicit expression for the natural frequency of the supported ring is available. This thesis focuses on modelling and understanding the damping behaviour of supported ring resonators taking proper account of the effect of support legs.

1.3 Quality Factor (Q-factor or Q)

The quality factor (Q-factor or Q) is a measure of performance of a MEMS resonator. Due to the presence of damping, a fraction of the total energy of a resonator is

converted into other form of (irrecoverable) energy [17]. The Q-factor is a measure of cyclic energy loss in a resonator and is defined as the ratio of the total energy stored in the resonator to the energy lost per cycle of vibration, i.e.

$$Q = 2\pi \frac{\text{Total energy stored}}{\text{Energy lost per cycle of vibration}} \quad (1.4)$$

It is clear from Eq. (1.4) that a higher Q-factor indicates a lower amount of energy dissipation from the resonator.

The Q-factor for a resonator can be evaluated using different methods. The details of these methods can be found in the work by Gorman [17] and Koyama [18]. However, the two most commonly used methods in evaluating the Q-factor from numerical simulations and experimental measurements are discussed in the following section.

1.3.1 Eigenvalue Approach

The eigenvalue evaluation of the Q-factor can be explained simply by considering a single degree of freedom lumped system with mass M , stiffness K and damping coefficient C . The equation of motion, under free vibration conditions can be expressed as

$$M\ddot{x} + C\dot{x} + Kx = 0 \quad (1.5)$$

The form of the solution to Eq. (1.5) depends on the amount of damping. It is generally assumed that MEMS resonators are weakly damped. If λ is the eigenvalue of the system defined by Eq. (1.5), then for a lightly damped system it can be shown that

$$\lambda = -\omega_0\zeta \pm i\omega_0\sqrt{1 - \zeta^2} \quad (1.6)$$

where ζ is the damping ratio ($\ll 1$ for light damping), which characterises the amount of damping in the system and ω_0 is the undamped natural frequency of the resonating

mode. Eq. (1.6) can be simplified further as,

$$\lambda = -\omega_0\zeta \pm i\omega_0 \approx -\frac{\omega_0}{2Q} \pm i\omega_0 \quad (1.7)$$

Here, the damped natural frequency, ω_d and quality factor, Q are given by

$$\omega_d = |Im(\lambda)| \approx \omega_0 \quad (1.8)$$

$$Q = \frac{1}{2} \frac{|Im(\lambda)|}{|Re(\lambda)|} \quad (1.9)$$

It is evident from Eq. (1.8) that a highly under-damped system has a natural frequency approximately equal to its undamped natural frequency. In many cases it is convenient to evaluate the Q-factor in terms of eigenfrequency (f), where $\omega_d = 2\pi f = \lambda i$. Eq. (1.9) can be rearranged as

$$Q = \frac{1}{2} \frac{|Re(f)|}{|Im(f)|} \quad (1.10)$$

Eq. (1.10) is used extensively in this thesis to evaluate Q-factor in ring resonators numerically.

1.3.2 Resonant Response Approach

For sufficiently small values of damping, the Q-factor can be experimentally measured from the frequency response curve and defined as:

$$Q = \frac{f_0}{\Delta f} \quad (1.11)$$

where f_0 is the damped resonant frequency corresponding to the maximum amplitude of the frequency response curve, and Δf (also referred as Bandwidth) is the difference between frequencies f_1 and f_2 at an amplitude of vibration 3 dB lower than the maximum amplitude. This approach is often called half-power bandwidth method, and is illustrated in Figure 1.4. The Q-factor is used to characterise the sharpness of the resonant peak. Systems with a high-Q have a sharp, large magni-

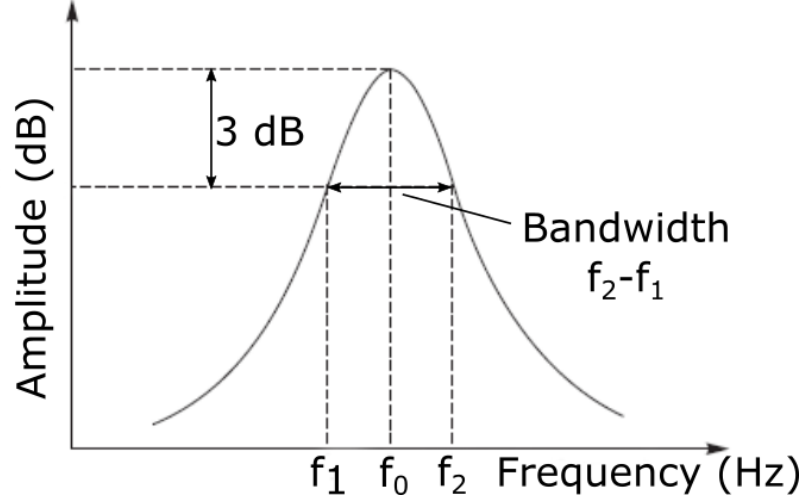


Figure 1.4: Frequency response curve showing half power method to determine Q-factor.

tude and well-defined resonance, and equivalently low levels of energy loss [19].

1.3.3 Advantages of High-Q

It is clear from the previous section that an increase in damping reduces the Q-factor, and that increasing the damping in the resonator subsequently reduces the amplitude but also widens the frequency range of the mode. For the resonator to vibrate continuously at large amplitude at a desired frequency, damping needs to be as low as possible and for this reason high-Q resonators are desirable. It was mentioned in Section 1.2.1 that high-Q increase the sensitivity (i.e. sense mode deflection) of the resonator in matched-mode operation. The advantages of high-Q resonators for vibrating gyroscope angular rate sensors are discussed in this section.

Resolution: The minimum change in input that changes the output of a resonator is referred as resolution, expressed in units of $^{\circ}/\text{sec}/\sqrt{\text{Hz}}$ [20]. The minimum detectable rotation rate (Ω_{min}) is limited by the noise floor. The overall resolution of a resonator is determined by two uncorrelated noise components: Brownian noise and

electronic noise. The minimum rotation rate is given by [10]

$$\Omega_{(min)} = \sqrt{\Omega_{(Brownian)}^2 + \Omega_{(Electronic)}^2} \quad (1.12)$$

where $\Omega_{(Brownian)}$ is the Brownian noise floor and $\Omega_{(Electronic)}$ is the electronic noise floor.

Brownian motion of the resonator occurs due to molecular collision with the surrounding fluid [21]. The expression for Brownian noise in a matched-mode operation of a resonator is obtained from the displacement caused by Brownian motion and the displacement induced by Coriolis acceleration, as given by

$$\Omega_{(Brownian)} \propto \frac{1}{q_{drive}} \frac{4k_B T_0}{\omega_0 M Q} \sqrt{BW} \quad (1.13)$$

where q_{drive} is the drive amplitude, M , ω_0 and Q are the mass, natural frequency and Q-factor of the resonating mode, k_B is the Boltzmann constant (1.38×10^{-23} J/K), T_0 is the absolute temperature, and BW is the bandwidth (Hz).

The electronic noise floor is dependent upon the noise of the interface circuitry [22], and the minimum detectable electronic signal for matched-mode operation of a capacitive rate sensor can be expressed by [10]

$$\Omega_{(Electronic)} \propto \frac{1}{q_{drive}} \frac{d_0}{V_p C_s A_g} \frac{\omega_0}{Q} V_n \sqrt{BW} \quad (1.14)$$

where d_0 , V_p , C_s and A_g are the capacitive gap, polarisation voltage, sense capacitance and angular gain, respectively.

It is evident from Eqs. (1.13) and (1.14) that a resonator should have higher Q-factor to reduce the noise level, thus increasing the resolution of the sensor. It should be noted that the fundamental limiting noise component of the mechanical structure is due to the Brownian motion. However, for high-Q resonators operating in vacuum, resolution is not limited by Brownian motion of the structure; it is limited by the electronic noise [8]. Another noticeable factor is the bandwidth (BW) of the resonator which is proportional to ω_0/Q . A reduced BW due to high-Q will also

increase the resolution.

Scale Factor: Scale factor is defined as the amount of change in the output signal per unit change of rotation rate and is expressed in $V/^\circ/s$ [8]. For a capacitively sensed rate sensor, the expected scale factor is given by [22]

$$\text{Scale Factor} = \frac{2V_p C_s q_{drive} Q}{(C_s + C_{parasitic}) d_0 \omega_0} \quad (1.15)$$

where V_p , C_s , $C_{parasitic}$ and d_0 are respectively polarisation voltage, the sense mode capacitance, the parasitic capacitance and the capacitive gap at sense electrodes, respectively.

From Eq. (1.15), a high-Q contributes to a high scale factor for the sensor.

Zero Rate Output (ZRO): It has been mentioned in Section 1.2.1 that any structural and damping asymmetries in the resonator results in a false output in the absence of an input rate. The measure of this offset to the output is the zero rate output. It can occur due to imperfection in the location of the sense and drive electrodes as well as electrical coupling between the electrodes. By electrically and mechanically decoupling the sense and drive modes, and by minimizing the fabrication process errors, ZRO can be significantly reduced [23]. However, a higher Q-factor can further reduce ZRO due to the increased sensitivity of the resonator [8], thus improving the accuracy of the sensor.

One of the other benefits of having a high-Q resonator is the low power consumption to maintain vibration, due to low energy dissipation per cycle of vibration. Although high-Q is desired for achieving high performance, the Q-factor is limited by the level of damping present in the resonator. The following section focuses on damping mechanisms that affect sensor performance.

1.4 Damping Mechanisms

To obtain high performance MEMS resonators, having a high-Q is essential. There are several mechanisms of energy dissipation in MEMS. Each of the energy loss mechanisms can be described quantitatively in terms of a corresponding Q-factor Q_i , and the total Q-factor Q can be expressed in terms of the individual Q-factor as [24]:

$$\frac{1}{Q} = \frac{1}{Q_1} + \frac{1}{Q_2} + \cdots = \sum_i \frac{1}{Q_i} \quad (1.16)$$

Eq. (1.16) shows that Q can not exceed the smallest Q_i .

It is important to identify the dominant energy dissipation mechanism occurring in MEMS resonators, and loss mechanisms can be classified into two categories: intrinsic and extrinsic. Extrinsic damping mechanisms are related to the operating environment of the resonator. Gas damping and support losses are the most important extrinsic losses that needed to be considered in a MEMS resonator. Intrinsic damping mechanisms depend primarily on the material and geometric properties of the structures [24]. Thermoelastic damping, material losses (e.g. losses due to dislocation and grain boundary relaxation) and surface losses are key intrinsic sources of damping.

Since sources of energy loss combine together directly, the overall quality factor of a MEMS resonator can be expressed as

$$\frac{1}{Q_{overall}} = \frac{1}{Q_{gas}} + \frac{1}{Q_{TED}} + \frac{1}{Q_{support}} + \frac{1}{Q_{surface}} + \frac{1}{Q_{other}} \quad (1.17)$$

Q_{gas} is the quality factor associated with gas damping, Q_{TED} is the effect of thermoelastic damping, $Q_{support}$ is the quality factor for energy loss through the supports, $Q_{surface}$ corresponds to surface loss, and Q_{other} represents the effect of other damping such as material damping and electrical damping.

A review of the key sources of extrinsic and intrinsic damping in MEMS devices is presented in the following section.

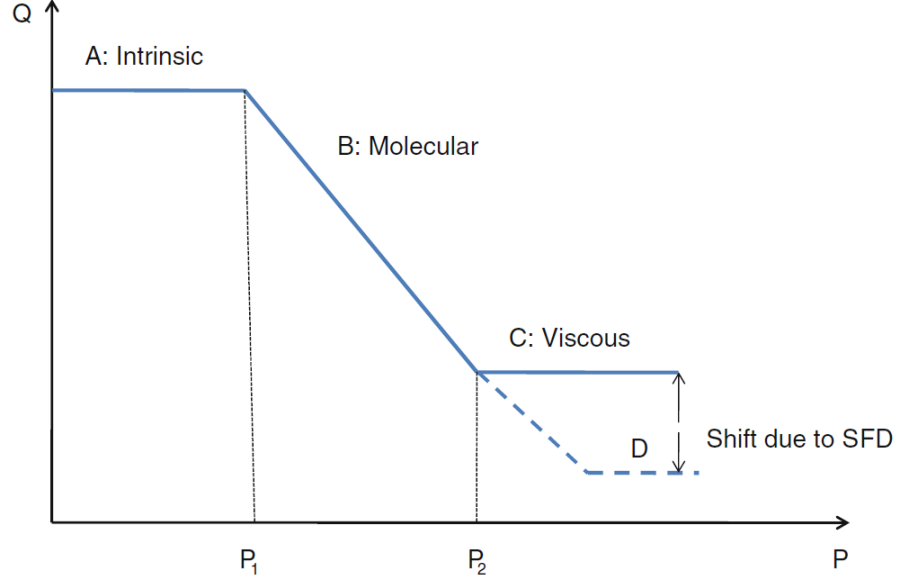


Figure 1.5: Schematic illustrating Newell's classification of Q-factor on pressure [24].

1.4.1 Gas Damping

For MEMS devices operating at atmospheric pressure or low vacuum, gas damping is generally the dominant loss mechanism [4]. Fluid damping or air damping are also often used interchangeably to express gas damping. It strongly depends on the pressure of the surrounding medium. Newell [25] investigated the effect of gas damping on miniaturizing cantilever resonators, and identified three different regimes for the dependence of Q-factor on pressure as shown in Figure 1.5:

- Intrinsic damping region: In this region the pressure is close to vacuum and gas damping is negligible. In this region other sources of damping e.g. TED are expected to be dominant.
- Molecular region: In the molecular region the interaction of air/gas molecules with the vibrating resonator causes damping. Damping occurs due to momentum exchange during the collision of molecules with the resonating structure, and air/gas molecules behave as kinetic particles.
- Viscous region: In this region the air/gas acts as a viscous fluid.

In MEMS, two types of viscous damping effects occur due to fluid surrounding

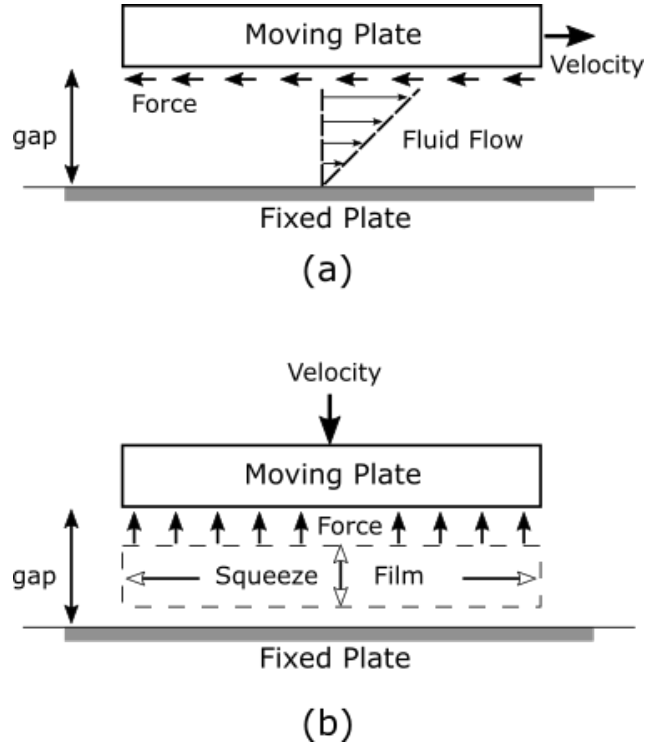


Figure 1.6: Schematic illustrating (a) slide film damping and (b) squeeze film damping.

the moving structure [4]:

- Slide film damping: This is also known as shear or Couette flow damping. The damping occurs when two parallel plates are in relative tangential motion [26], as shown in Figure 1.6a. The relative motion of the parallel plates induce a fluid flow that exerts a force opposing the relative motion. There is no pressure change in the gap between plates, but a velocity gradient occurs across the gap.
- Squeeze film damping (SQFD or SFD): Squeeze film damping occurs when two parallel plates are in relative perpendicular motion [26], as shown in Figure 1.6b. The air/gas trapped inside the gap is compressed and begins to squeeze out the sides of the gap due to increase in pressure. The energy is lost due to the work performed by the plate to move the air/gas out of the gap. However, when the plates separate, the pressure drops between the plates and the air/gas is drawn into the gap.

In many MEMS devices, viscous damping is dominated by squeeze film damping,

and the energy loss is much higher than the slide film damping due to the dependencies on the gap [4]. Most notably, in devices which use electrostatic actuation and capacitive sensing, narrow air gaps often result. The dynamic behaviour of these devices can be significantly affected by squeeze film damping and dominate the overall damping [27–30]. The damping of devices is inversely proportional to the gap size, therefore smaller gap between electrodes and the vibrating structure results in greater damping force. Other factors affecting SFD are: geometry of the structure, resonant frequency, packaging pressure, thickness of the fluid film, boundary conditions [31]. Accurate models and approaches for predicting air damping are very important to the design of high-performance MEMS devices.

Energy losses due to SFD are complex to calculate for practical MEMS devices. However, much research effort has focused on developing models to predict the damping level. Some of the common models for SFD in MEMS are discussed here. The most popular approach to model SFD is based on solving Reynolds equation. This approach is used to predict damping in the viscous region of Figure 1.5, and assumes the gas to be a continuum. One of the earliest works focused on SFD in MEMS devices is by Starr [32], in which Reynolds equation is simplified by assuming it to be incompressible. This assumption reduces Reynolds equation to Poisson’s equation, which can be solved either analytically for simple shapes or numerically for complicated shapes. Starr [32] derived an exact expression for the damping force of a circular disk and an approximate expression for the damping force of a rectangular plate. Hosaka *et al.* [28] considered the expression of Starr [32] to model squeeze film damping on a resonating beam, and derived an approximate closed form equation for Q-factor in cantilevers. Yang *et al.* [33] used linearised Reynolds equation to simulate dynamic behavior of a clamped-clamped micro-beam. They used finite element simulation to extract the free mode shape of the micro-beam, and used this as a harmonic deflection in the linearized Reynolds equation. They solved the equation using the finite difference method to extract effective damping and spring coefficients to be used in a mass-spring-damper model. The Q-factor and resonant frequencies

were then calculated from this model. Nayfeh and Younis [34] used perturbation techniques to solve the compressible Reynolds equation for micro-beams and micro-plates actuated by electrostatic forces. They used the solution into the structural equation and solved the resulting equation using a finite-element method, and found good agreement between predicted and measured Q-factors. Another approach to model SFD problems in MEMS devices is to solve Navier-Stokes equations using the finite element method [31].

Modelling gas damping in the molecular region is more challenging because the continuum theory is not applicable in this region. This region is also known as the rarefied gas regime, and statistical thermodynamics is utilised to study the mechanics of rarefied gas [24]. The effect of collisions of the vibrating structure on individual molecules of gas needs to be investigated to understand the damping behavior in this region. Gas damping in a low vacuum is usually estimated using the free molecular model proposed by Christian [35]. He developed a free molecular theory to model gas damping of a vane that is free to swing in a low vacuum and used the Maxwell-Boltzmann velocity distribution function for gases. Newell [25] applied Christian's model to predict the gas damping in a vibrating beam considering the change in momentum of the gas molecules, and expressed the Q-factor as:

$$Q = \left(\frac{\pi}{2}\right)^{3/2} \rho H f \sqrt{\frac{R_0 T_0}{M_0}} \frac{1}{P} \quad (1.18)$$

where ρ , H , and f , are the density, thickness and resonant frequency (Hz) of the vibrating beam, respectively. P is the pressure, R_0 is the universal gas constant, T_0 is the absolute temperature, and M_0 is the molecular mass of the gas.

It was found that the theoretical prediction of the quality factor is almost one order of magnitude higher than the experimental results [36]. Despite being quantitatively inaccurate, it is still useful for a qualitative prediction of gas damping and how it relates to important design parameters as well as the operating pressure. Kadar *et al.* [37] modified Christian's model with new molecular speed distribution function (the Maxwellian-Stream distribution) to achieve better agreement with experiments.

This reduced the Q-factor in Eq. (1.18) by a factor of π . Kadar's model was further modified by Li *et al.* [38] to take account of the velocity of the contact surface. This modification provides improved accuracy compared to the experimental results [36]. Bao *et al.* [39] showed more accurate prediction of Q-factor, using an energy-transfer model to study the effect of a moving structure on changing the kinetic energy of the gas molecules. The Q-factor equation is similar to Eq. (1.18), but modified by a correction factor.

In summary, modelling techniques for gas damping are available for the two regimes: viscous regime and molecular damping regime. In the viscous regime, the continuum theory is applicable and the rarefaction of gases are negligible. The damping behaviour is modelled analytically or numerically by solving Reynolds equation for compressible or incompressible fluids. In the low vacuum region, i.e. molecular damping regime, free molecular theories are applied to capture the momentum transfer mechanism of gas particles interacting with vibrating structure. These models have been derived based on beam type resonators, and to date gas damping in ring resonator based MEMS devices has not been addressed in the literature. All of the molecular damping models available in literature have predicted a qualitatively similar behavior and a drop of in quality factor with pressure that is proportional to $1/P$.

Since gas damping is extrinsic, it can be reduced by operating the device in a vacuum in order to enter the intrinsic damping region.

1.4.2 Thermoelastic Damping

In this section Thermoelastic damping will be introduced and the main contributions of different TED models available in the research literature are summarised.

Thermoealstic damping (TED) is an inevitable damping mechanism, and exists in all solids with non-zero thermal expansion [40]. The damping originates from the alternating strain field in an oscillating structure, inducing temperature gradients due to an adiabatic volume change [41]. The energy is lost due to relaxation of

temperature gradients through an irreversible heat flow. Following section reviews the principle of TED in detail.

Principle of Thermoelastic Damping The purpose of this section is to review the principle of thermoelastic damping in materials of vibrating structure. Further details can be found in e.g. [42, 43].

Thermoelastic Damping (TED) occurs in any structure made from a thermoelastic solid that is subjected to a cyclic stress. It is an internal source of energy dissipation which occurs due to the coupling between thermal and mechanical domain within a solid. In a solid with non-zero thermal conductivity, an increase in temperature induces expansion, and a decrease in temperature produces contraction. Similarly, compression increases the temperature while expansion lowers the temperature. When a solid vibrates, the induced motion causes it to be in a non-equilibrium state with an excess of energy (kinetic and potential). If the solid is isolated from its environment, the solid would vibrate indefinitely because energy is not dissipated. The local temperature gradient, generated by coupling between strain and temperature fields, is dissipated by the irreversible heat flow to return the solid to its equilibrium state. This is an irrecoverable energy loss because the mechanical energy is converted to thermal energy which generates a thermal gradient. This dissipated thermal energy causes thermoelastic relaxation of the solid. From the thermodynamic point of view, thermoelastic damping occurs when the thermal energy is lost to the entropy which actually increases the entropy of the system.

Thermoelastic damping is one of the dominant damping sources in MEMS resonators. It is significant when the period of vibration and the thermoelastic relaxation time have a similar order. The resonators (e.g. beams, ring, hemispherical shell etc.) used in MEMS angular rate sensors generally vibrate in flexural modes, so when a resonator vibrates, cyclic stresses occur. These cyclic stresses typically occur in the form of tensile and compressive stresses within different regions of the resonator. The region under compression is heated and the stretched region is cooled (see Figure 1.7). A temperature gradient is generated across these two regions and an irreversible heat

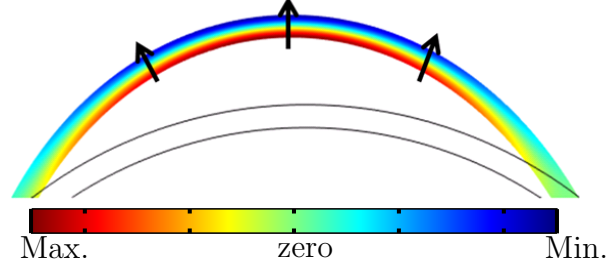


Figure 1.7: Temperature distribution in a ring section under flexure. Red and blue colours indicates higher and lower temperatures, respectively. Black arrows indicate the direction of heat transfer.

flow occurs from warmer to colder region to dissipate energy.

Thermoelastic Damping Models: In the 1930s Zener developed a detailed theory of thermoelastic damping for thin rectangular beams under flexural mode vibrations [44, 45], and later demonstrated the presence of thermoelastic damping experimentally [46]. Roszhart [47] experimentally verified that TED is the dominant damping mechanism in micro-scale silicon beam resonators, and more recently Duwel *et al.* [48] illustrated the importance of TED in MEMS gyroscopes made from different semiconductor materials. Zener [44, 45] derived an approximate equation for the Q-factor for homogeneous, isotropic and uniform beams undergoing flexural mode vibrations, based on 1D theory of anelastic solids. In the elastic analysis of solids it is generally assumed that the elastic strain is a function of stress only. However, there is a time dependence of the elasticity which is known as anelasticity. Anelastic solids show a temporal shift between an applied stress and the resulting strain. For periodically applied stress, this results in a phase shift between the stress and strain in the steady state.

The main approximations, in Zener's model, were the consideration of Euler-Bernoulli beam bending and using the first transverse thermal mode, neglecting all other thermal modes, to represent the thermal field. Zener's approximate solution for Q-factor exhibits a Lorentzian behaviour. A detailed discussion on Zener's theory of thermoelastic damping is presented in Chapter 2.

As Zener's theory is based on 1D anelasticity it cannot be used to describe the

thermoelastic damping behaviour in structures of arbitrary form. For this reason there have been some interest analyzing TED considering coupled equations of thermoelastic dissipation. Alblas [49] and Chadwick [50] provided rigorous foundation for solving the governing equations of thermoelasticity in 3D. Alblas [49] developed a method for solving thermoelastic loss in 3D problems based on the linear theory of elasticity where the conversion of mechanical energy into heat was considered. He derived detailed and explicit solutions for the thermoelastic damping in vibrating beams [51]. However, more thorough analysis was conducted by Chadwick [50], providing exact solutions to the coupled thermoelastic equations in terms of complex resonant frequency of a vibrating structure. The Q-factor of TED in any arbitrary geometry can be predicted using the complex frequency expressions.

In the context of MEMS resonators, Kinra and Milligan [52] and Lifshitz and Roukes (L-R) [53] developed exact, closed-form expressions for TED. Kinra and Milligan [52] derived the coupled isotropic thermoelastic equations and provided a solution for uni-dimensional structures, including a discontinuous beam. L-R developed the exact solution to the coupled heat equation of linear thermoelasticity for a flexural mode of a beam. The analytical solution by L-R uses a similar approach as Zener when considering 1D heat conduction. However, the approximation of the thermal field to only the first thermal mode is avoided in the analysis. L-R's solution technique is widely used to estimate TED in micro and nano scale resonators. The exact expression for TED developed by L-R is discussed in Chapter 2.

Extension of Thermoelastic Damping Models: The analytical models of Zener and Lifshitz-Roukes (L-R) only considered simple beam like structures. However, Zener's classical theory and L-R's solution technique have been extended to MEMS resonators of various geometries. The extension of Zener's model involves identifying the mode through which the energy is dissipated, and quantifying the thermal path lengths that are involved in the relaxation process. Alternatively, the equations of thermoelasticity are resolved as in L-R's model when extending for different geometries.

Wong *et al.* [16, 54] applied Zener's theory and L-R's approach to silicon rings and obtained theoretical expressions for Q-factors. The extension of these two methods resulted in similar expressions to those originally derived for beams. A wide range of ring sizes relevant to MEMS resonators were investigated, and the values of Q-factor predicted by the two approaches were in very good agreement for lower order modes (low frequency), as well as with experimental results. A similar closed form expression for Q in a ring resonator based on the L-R's technique was derived by Hao *et al.* [3]. The extended model for a ring resonator based on Zener's theory is extensively used throughout this thesis to understand thermoelastic damping behaviour in these resonators. It is worth highlighting that the rings analysed in these models do not take account of the effect of supporting legs.

Choi *et al.* [55] investigated thermoelastic damping effect in the inextensional hemispherical shells based on the extended Zener's theory of rings by [16]. The Q-factor is calculated using Rayleigh energy method. However, the validity of the model is doubtful as thermal expansion is only assumed to occur in the circumferential direction of the hemispherical shell. To address this issue, Sorenson *et al.* [56] conducted a rigorous analysis on hemispherical shells to understand the TED behaviour both analytically and numerically. A closed form expression for thermal path for bulk TED in a hemispherical shell is derived by relaxing the inextensional assumption in Rayleigh's shell theory. Unlike rings [16] or the hemispherical shell model by Choi *et al.* [55], the thermal path is not dictated by thermal gradients through the shell's thickness. An expression for the thermal path is developed and used in Zener's theory to obtain a closed form expression for Q-factors in hemispherical shells. An investigation of thermoelastic damping in hemispherical shell resonators (HSR) is presented in Appendix A. It is observed that a HSR having similar dimensions to a ring resonator exhibits a higher Q-factor due to the differences in thermal path between these two resonators.

Nayfeh and Younis [34] presented an analytical expression for Q-factor for thermoelastic damping in rectangular micro-plates. The decoupling of the thermal equa-

tion from the structural equation of a plate is achieved by solving the heat equation for the heat flow across the micro-plates. A perturbation method is then used to obtain an analytical expression for the Q-factor. Hao [57] used a thermal-energy approach instead of commonly used complex frequency method, to analytically investigate the contour mode vibration of micro and nano circular thin-plate resonators. It has been inferred by Hao [57] that TED becomes a significant source of dissipation for circular thin-plate resonators at the nano-scale.

Houston *et al.* [58, 59] and Photiadis *et al.* [60] developed a model for MEMS oscillators based on Zener's model. It is assumed that resonant mode of the oscillator always contains some flexural component, and the damping is caused by the flexural motion. A flexural modal participation factor (MPF), defined as the fraction of potential elastic energy stored in flexure, is included in the results obtained from Zener's theory (e.g. for flexural beam) to quantify the thermoelastic damping. The MPF can be calculated by integrating the curvature tensor of the vibration mode over the volume of the oscillator, provided the displacement field of the mode is known [61]. However, finite element analysis or experimental techniques must be used to determine the mode shape.

The 1D heat conduction equation in L-R's method is refined by Prabhakar and Vengallatore [62] with the consideration of 2D heat conduction for beam resonators. However, it is found that effect of 2D heat conduction on thermoelastic damping compared to 1D heat conduction is negligible for thin flexural structures for a wide range of frequencies.

Zener's theory is sufficiently accurate for many practical purposes, particularly in the case of transverse thermal mode during flexural vibration of homogeneous structures. However, some extended analytical models have been reviewed in the aforementioned section. But it is not straightforward to obtain closed form solutions for complex cases (e.g. complex geometry, anisotropic material etc.) beyond those simple structures already considered in the research literature. The limitations of Zener's theory and L-R's approach and differences in these models are discussed in

Chapter 2. A numerical method is required to investigate TED in more complex structures.

Finite Element Based Models to Predict TED: Numerical methods based on the finite element analysis (FEA) have been developed to directly solve the governing equations of thermoelasticity. In the literature, three different numerical approaches i.e. a fully coupled approach [63], a weakly coupled approach [63] and a thermal energy method [64] exist. Duwel *et al.* [63] formulated fully coupled and weakly coupled thermomechanical approaches based on the FE model developed by Gorman [17]. The fully coupled approach has been used to investigate TED in flexural, torsional and longitudinal modes of beam resonators [63], and flexural modes of hemispherical shell resonators [65–68]. The fundamentals of these approaches are discussed in the following section.

In the weakly coupled approach [63], the equations are solved by neglecting thermal expansion, thereby uncoupling the equations. The uncoupled mechanical and thermal equations are solved separately based on eigenvalue solutions. These modal solutions are coupled using an analytic formula to calculate Q_{TED} [41, 63]. The thermal energy method, described in [64], is based on the TED principle that dissipation of vibration energy is permanently converted into thermal energy. This enables TED to be calculated from the generation of thermal energy per cycle of vibration. Similar to the weakly coupled approach this method also neglects thermal expansion. The uncoupled mechanical equations and the transient heat conduction equation are solved separately using numerical simulations. The deformation resulting from uncoupled elastic modal simulation is used as the internal heat source for the transient heat analysis. The dissipated heat per cycle (obtained from temperature variation in the resonator) and, the stored mechanical energy are then used to evaluate the based on eigenvalue solutions. These modal solutions are coupled using an analytic formula to calculate Q_{TED} . The fully coupled approach solves the set of coupled thermoelastic equations by considering the complete bidirectional coupling between the mechanical and thermal equations. Using the finite element method, a complex

thermo-mechanical modal analysis is carried out to extract the complex eigenvalue associated with the mode of vibration [41]. The complex eigenvalue contains the mechanical vibration frequency and the rate of decay for an unforced vibration due to TED. The ratio of these two quantities is used to evaluate the Q-factor for any geometry under TED (i.e. using Eq.(1.9)) .

In recent years, researchers have also implemented the finite element method to investigate TED in specific resonator geometries. Yi and Matin [69] developed a method to solve TED problems in MEMS resonators using a combination of the finite element method and eigenvalue formulation. In this method the linear eigenvalue equation is obtained by applying a perturbation of temperature and displacements in the governing equations of thermoelasticity. The resulting equations are solved using the finite element method to obtain an approximate solution which contains information about resonant frequency and thermoealstic Q-factor. The model is validated for a beam resonator, and is used to solve TED in contour-mode vibrations of rings, disks and plates [70, 71]. While solving problems like circular disk and ring geometries, the order of the problem and computational time is reduced by using a Fourier reduction method. However, this reduction method is only applicable to symmetric geometries. The results obtained for the ring were compared with the results by Wong *et al.* [54], and were found to be in close agreement.

Lepage and Golinval [72] presented a 2D finite element model (based on Hamilton's vibrational principle) for thermoelastic coupling in an oscillating clamped-clamped beam and, a vibrating bar. The results obtained from this model were found to be in good agreement with the results predicted by Zener's theory and L-R's technique, which could allow complex structures to be investigated using this formulation. Serra and Bonaldi [73] applied a finite element formulation based on a weak form of the boundary value problem for fully coupled thermoelasticity to flexural beams. Thermoelastic damping is calculated from the irreversible flow of entropy due to the thermal fluxes that have originated from the volumetric strain variations. This finite element framework is applicable to the evaluation of ther-

moelastic damping in multi-layered structures made by bonding silicon wafers and laminated composites of layered thin films, used in the manufacture of some beam resonators [74].

To achieve higher Q-factors, the introduction of trench, slots or channels have been considered specifically in micro beam resonators [75–78]. Candler *et al.* [75, 76] introduced slots into flexural beam resonators to alter their mechanical and thermal coupling. A comprehensive study was performed using the fully coupled approach to investigate the effect of slots on TED, and showed that slots have significant impact on damping by either increasing or decreasing the Q-factor depending on slot location. A detailed analysis on how slots affect TED behaviour in resonators is presented in Chapter 2. Furthermore, FE models to predict TED in flexural beam resonators containing structural discontinuities in the form of slots and channels is developed by Prabhakar and Vengallatore [77]. The results showed that trenching in structures can shift resonant frequencies, while slots in the beam can enhance Q-factor. More recent work by Guo *et al.* [78] implemented a finite element method to investigate effects of slots on TED in clamped-clamped and cantilever beam resonators. The Q-factors and resonant frequency are obtained as a function of various geometric parameters of the slots i.e. location, size, number. The results showed increases in Q-factor when slots are placed in the centre and at the clamped ends of the resonators. However, slot located in the clamped end is found to reduce the resonant frequency significantly.

In summary, analytical and finite element models are available for predicting thermoelastic damping in MEMS resonators and were reviewed. Apart from the general approaches to solve thermoelastic dissipation for arbitrary structures, most FE models have only been applied to specific geometries, especially simple beam-like geometries. A limited number of studies on ring resonators have been performed in the literature to understand TED behaviour. However, these models are not capable of taking into account support legs or slots which can modify the TED behaviour, and it is upto the present date still explicit calculation of TED for complex cases of support legs and slots in rings have not been treated in the literature.

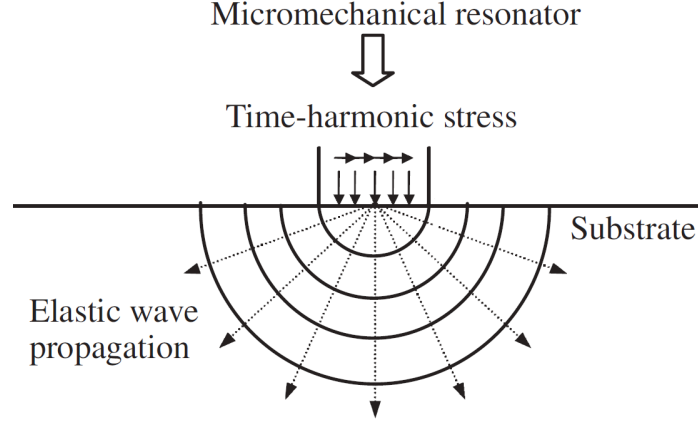


Figure 1.8: Physical mechanism of support loss in a MEMS resonator [79].

1.4.3 Support Loss

Support loss, also known as anchor loss/attachment loss/clamping loss, is the dissipation of mechanical energy via the coupling to the support structure. In MEMS devices, resonators are attached to substrates which can be several magnitudes larger. As the resonator vibrates harmonic stresses are applied across the attachment region to the substrate. These stresses act as excitation sources that excite elastic waves which propagate into the substrate. The vibrational energy dissipated through elastic wave propagation in the substrate is referred to as support loss [79]. The physical mechanism of support loss is illustrated in Figure 1.8. The time-harmonic stress source can be normal or shear stress, depending on the resonant mode. Support loss has been observed to be prominent in resonant MEMS devices at high frequencies and depends on the design and mode of vibration of the device [80–82]. Innovative designs such as isolating the vibrating element from the mount using a mechanical isolation mechanism can remove the coupling between the resonator and its support, reducing energy loss by reducing support motion [83]. It can also be minimized by designing a symmetric resonator so that it vibrates in a balanced way without moving its center of mass [84].

There exist three approaches to model support loss behaviour in MEMS resonators: analytical, semi-analytical and numerical approaches. Models based on

these approaches are discussed in the following sections.

1.4.3.1 Analytical Approach

In 1968, Jimbo and Itao [85] derived an expression for the energy loss from a cantilever vibrator into a semi-infinite elastic medium, by comparing the vibration energy of the cantilever with the energy associated with strain induced in the elastic medium by the shear force and bending moment at the root of the cantilever. Their result has been applied to a number of later studies related to MEMS applications [28, 86, 87] to estimate support loss. Cross and Lifshitz [88] considered elastic wave transmission across the junction between two plates of differing widths but the same thickness, and calculated the associated energy loss. They suggested that thin-plane elasticity theory provides a useful and tractable approximation to the three dimensional geometry. However, the support loss was not derived from the viewpoint of the resonant modes of a beam resonator and hence no explicit expressions were provided.

Hao *et al.* [89] derived an analytical model for calculating support loss in MEMS cantilever and clamped-clamped beam resonators exhibiting in-plane flexural vibrations. By assuming the resonator and the substrate have the same out-of-plane thickness, the response of the resonating beam was determined using Euler-Bernoulli beam theory. The support of cantilever and clamped-clamped beam resonator is assumed and modelled as a semi-infinite and an infinite thin plate, respectively, so that the support behaviour can be described by the 2D elastic wave theory. The stresses induced by the resonator excite elastic waves in the substrate, and the displacement field in the substrate is obtained analytically by modelling its behaviour using 2D linear elastic theory, subject to the assumption of fixed boundary conditions at the attached region. The Q-factor is computed by assuming that the work imposed on the substrate by the support/resonator is equal to the energy loss. The work done is calculated by integrating the product of stress at the attachment point with the displacement, over one period of vibration. A closed form expression for Q-factor is obtained from the ratio of total energy of the resonator to the energy lost per cycle.

One of the limitations of this approach is that it assumes the wavelengths of propagating waves to be much larger than the thickness of the resonator, therefore assumes the propagating waves are 2D. This approach has been extended to model support loss in disk [90, 91] and ring [92] resonators, and verified with experimental results. Hao and Ayazi used separation and coupling between disk, support beam and substrate using the same approach as [89] to determine support loss in side supported [90] and centre supported [90, 91] disk resonators. In this case, the resonant frequencies, mode shapes, and vibration energy of the disk resonator was determined using the circular thin-plate theory. The support beam was modelled as a cantilever and longitudinal rod for side supported and centre supported disk resonators, respectively. The substrate was assumed to be a semi-infinite medium, and its behaviour described using 3D elastic theory. The displacement in the substrate under harmonic stress was used to derive closed form expression for Q-factor, the same as Hao *et al.* [89]. The results show the existence of a strong dependence of Q-factor on the material properties for both disk resonators. Also, Q-factor for the side supported disk resonator depends only on support dimension, whereas the centre supported disk depends on both disk and support beam dimensions. The analytical model developed for a ring resonator is reviewed in Section 4.2. Hao and Xu [79] presented a general approach to quantitatively evaluate support loss in MEMS resonators. They developed expressions for two different resonator-substrate configurations with different number of clamped regions under various time-harmonic stress sources: resonator on top and within the substrate. The support loss was calculated in terms of the time-harmonic stress from the resonator and its corresponding vibration displacement on substrate. The contribution of their work is mainly the derivation of vibration displacement on semi-infinite or infinite substrates using 2D and 3D elastic wave theories in a much more accessible way. The support loss expressions offer insight on design improvements for reducing support loss, by considering the relation of support loss with key design parameters of a resonator as well as the material properties of the substrate.

The analytical closed form expressions are useful due to their simplicity, but

the major disadvantage is that they are only valid under the assumptions that are made and available for restricted geometries. Therefore if the geometry is complex such that the mechanism of damping is not known, the analytical approach is not applicable.

1.4.3.2 Semi-analytical Approach

A semi-analytical approach was proposed by Park and Park [93, 94] for predicting the propagation of waves from a vibrating beam to the substrate. The resonator system is partitioned into modular subsystems consisting of the substrate, resonator and the electrostatic actuators. Each of these are modelled independently, and then combined together under proper interface matching conditions to obtain the overall system response.

The response of the resonator is calculated using the finite element method, and the response matrix relating the force and displacement at the anchor nodes, as well as the excitation nodes is constructed. An electromechanical model between the actuator nodes on the substrate and the corresponding excitation nodes on the resonator is used to construct another response matrix. Assuming the substrate to be an infinite half space, the response matrix relating force and displacements corresponding to points on the substrate is constructed. These three response matrices are combined to construct the response matrix of the entire structure. This matrix is used to derive stiffness and damping matrices, and hence calculate the Q-factor.

The disadvantage of this method is that a spatial inverse Fourier transform is required to evaluate entries of the response matrix for the substrate, which can be computationally costly and intensive. Since the substrate is not modelled as semi-infinite, this can be an important advantage when the substrate cannot be considered semi-infinite such as the case when the propagating wavelength in the substrate is large.

1.4.3.3 Numerical Approach

The numerical approach generally implies solving the elastic wave equations using discretisation techniques such as the finite element method. To model support loss behaviour in MEMS resonators, it is generally assumed that the elastic waves radiated into the substrate are lost and do not return to the resonator. Hence, the substrates are considered to be semi-infinite or infinite domains. In numerical simulations, to model the non-returning wave behaviour in the substrate, the semi-infinite or infinite domain must be truncated finitely with appropriate boundary conditions (BC) to mimic the semi-infinite or infinite domain behaviour [18]. If the boundary of the substrate is truncated with standard boundary conditions, i.e. Dirichlet or Neumann boundary conditions, unacceptable artifacts from the boundary reflections occur due to a fast decay of wave equations [95]. Therefore, numerical boundary conditions are required to absorb these waves without reflections.

Different numerical boundary conditions are available for simulating of wave dissipation: boundary dampers, absorbing boundary conditions, infinite elements, boundary integrals, and absorbing layers (e.g. perfectly matched layers) [96]. These boundary conditions truncate the simulation domain with an artificial boundary that absorbs outgoing waves. However, all have certain failings when it comes to solving the resonator problem [97].

The simplest method is to apply damping elements at the boundary of the truncated domain [98]. Viscous elements are used which have higher damping coefficients than the material of the truncated domain. The propagating waves attenuate due to the higher damping of these elements. However, the main drawback with this method is that a sudden change in the material impedance induces spurious reflections at the boundary. Also, this method is only effective for cases when the incoming wave is almost normal to the boundary.

High order Absorbing Boundary Conditions (ABCs) and the Perfectly Matched Layer (PML) are commonly applied to solve wave problems in semi-infinite and infinite domains using FE [99]. However, PML has become the method of choice in

MEMS applications [18, 96, 100–103], since it appears to be the most flexible and applicable to general boundary conditions and geometries. The concept of PML was first introduced by Berenger [104] for electromagnetic waves in the finite difference time domain (FDTD) method. This numerical absorbing boundary consists of a layer of artificial material placed around the computational domain, and designed to absorb all propagating waves at any angle of incidence without reflection. This is done by matching the mechanical impedance between the PML and the computational domain, which eliminates spurious reflections from the artificial interface. Basu and Chopra [105] formulated PMLs for elastic waves based on complex coordinate stretching method. This coordinate transformation in the PML region causes waves to exponentially decay in this direction. The physical mechanism of PMLs is discussed in Section 3.2.

In the context of MEMS resonators, Bindel and Govindjee [96] first applied the PML based finite element method to quantify support loss in a radially driven centre-supported disk resonator. They provided an alternative interpretation to simplify implementation of PML in a finite element code, to that of Basu and Chopra’s work [105]. Later on, they developed a computer tool called *HiQLab* which has the capability to simulate support loss based on the finite element method [106]. More recently, Frangi *et al.* [101, 102] has provided extensive benchmarking against available analytical estimates for 2D and 3D cantilever beams by employing a custom built FE code to solve a generalised complex symmetric eigenvalue problem. They also simulated support loss in piezoelectric MEMS resonators with in-plane longitudinal mode vibrations, and found excellent quantitative agreement with experimental results [100]. The work of Bindel and Govindjee [96] and Frangi *et al.* [101, 102] simulated support loss behaviour using custom built FE codes, making it difficult to adopt and, require longer development time for generic resonators. However, the advantage of PML is that it allows an easy implementation in commercial FE packages such as COMSOL [107]. The support loss work presented in this thesis utilises the PML capability in COMSOL, and extensive validation against published work [18, 96, 102] is provided

in Chapter 3. Furthermore, difficulties in obtaining PML parameters to accurately model support loss behavior in resonators, as mentioned in Reference [92], are also addressed.

Steeneken *et al.* [108] proposed a Matched Layer (ML) method to calculate the Q-factor associated with support loss using COMSOL. This is similar to PML except that it only matches perfectly for normal incident waves, making it less effective than PML. The method was applied to a MEMS disk resonator, and found to give reasonable agreement (<5%) with Q-factor obtained using PML based model.

Another numerical boundary condition uses the concept of infinite elements. In this method elements with a special shape function for the geometry considered are placed at the infinite boundary, in conjunction with standard finite elements. The details of this method are provided by Chouvion *et al.* [92] and used to study the fundamental in-plane mode of a simple cantilever beam resonator in commercial FE package ABAQUS [109].

1.4.4 Surface Loss

Surface losses become important as the ratio of surface area to volume increases for a resonator. They are mostly caused by surface stress, which are modified by adsorbates on the surface or surface defects [110]. The surface layer will not store vibrational energy, however it dissipates energy leading to damping. The surface loss is modelled by considering the complex Young's modulus in the surface layer: $E_c = E + iE_d$, where E_d is the dissipation part of Young's modulus [86]. Yang *et al.* [110] derived the following expression for Q-factor related to surface loss for a cantilever beam resonator:

$$Q = \frac{WH}{2\delta(3W + H)} \frac{E}{E_d} \quad (1.19)$$

where W and H are the width and thickness of the cantilever, δ is the thickness of adsorbate layer, E is the Young's modulus, and E_d is the dissipation value of Young's modulus of the surface layer. In Eq. (1.19), Q is proportional to thickness

when $H \ll W$.

Yang *et al.* [110] found that surface loss is dominant in sub-micron cantilever beams and the Q-factor was largely a function of surface damping, particularly as the ratio of the surface area to volume increased. However, the Q-factor of structures dominated by surface losses can be modified through heat or surface treatments. The dimensions of the ring resonators considered in this thesis are not in the sub-micron level, so surface loss effect can be neglected.

1.5 Research Aims and Objectives

The aim of this research work is to investigate the effects of dominant damping mechanisms, particularly thermoelastic damping and support loss, and develop improved understanding of their influence on the damping performance of a supported ring resonator. As high-Q are needed to enhance the performance of vibrating ring based angular rate sensors, the research work described in this thesis focuses on understanding and quantifying damping in supported ring resonators and designing supported ring resonators having high Q values. The research work can be divided into two main tasks:

Thermoelastic Damping (TED): This involves a fundamental study of thermoelastic damping in ring resonators taking into account the effects of the support leg structure. The main aims of the work are to:

- Model, analyse and quantify thermoelastic damping behaviour in supported ring resonators to develop improved understanding of this damping mechanism.
- Design supported ring resonators having low levels of thermoelastic damping to aid the development of high performance rate sensors.

The following methodology is used to achieve the aims of this work:

- Develop a robust FE model based on the fully coupled thermoelastic approach to accurately predict thermoelastic dissipation.

- Analyse TED behaviour in supported ring resonators using the developed FE model to understand and explain the influence of TED in the ring and support legs.
- Investigate the influence of key geometric and material properties to improve the design of supported ring resonators for achieving high-Q.
- Design high-Q resonators by optimising key geometric parameters, and perform experimental testing on proposed high-Q designs to compare predicted TED with measured results.

Support Loss: This involves a detailed study of support loss in supported ring resonators. The main aims of the work are to:

- Model, analyse and quantify support losses in supported ring resonators to understand the damping behaviour.
- Develop understanding on the key design parameters that influence support loss in supported ring resonators, including systematic design variations and manufacturing imperfections.

The methodology used is:

- Develop a robust FE model based on Perfectly Matched Layer (PML) to accurately predict support losses in MEMS resonators.
- Analyse support loss behaviour in supported ring resonators to understand and explain the influence of design parameters and balanced vibration on support loss, considering the effects of systematic design variations and manufacturing imperfections.

1.6 Thesis Outline

The thesis is organised in six chapters. The contents of the remaining chapters are summarised below.

- **Chapter 2** investigates the thermoelastic damping in MEMS resonators. The physical process of thermoelastic dissipation, and Zener’s classical theory is reviewed. The focus of the chapter is design and modelling of thermoelastic damping behaviour in ring resonators considering the effect of support legs, slots, and different materials. A finite element method is used to predict the damping of the flexural mode, employing a fully coupled approach in the commercial FE package COMSOL. In addition, several high-Q designs are proposed by selecting ring and support leg geometries that have reduced levels of TED.
- **Chapter 3** presents a methodology to evaluate support loss in MEMS resonators, which uses Perfectly Matched Layers (PML). Firstly, the general physics behind PMLs is discussed. The implementation of PMLs in finite element model is explained and, key parameters affecting the PML accuracy are identified. An extensive validation of the model is performed using beam and disk resonators, against analytical, numerical and experimental results, where applicable. Based on the validation and verification of the model, different modelling strategies are proposed for accurate prediction of support loss in MEMS resonators.
- **Chapter 4** provides an application of the support loss model presented in Chapter 3 to a supported ring resonator. A nominal ring based device is introduced, and support loss behaviour of different resonant modes are analysed using the PML based model. This chapter also focuses on the influence of key design parameters and manufacturing variations in a supported ring resonator on support loss. Furthermore, the contribution of support loss to the high-Q rings designed in Chapter 2 to have low levels of TED is investigated.
- **Chapter 5** focuses on the devices manufactured based on the high-Q ring resonators designed in Chapter 2. Initially the predicted TED behaviour in different resonant modes of the nominal device (introduced in Chapter 4) is compared with measured values, which provides an understanding of the damping effect on different modes. The experimental test setup and procedures used

for the measurements of frequency and Q-factor of the high-Q devices are explained. Furthermore, the measured values are compared against theoretically predicted values obtained using the FE model developed in Chapter 2. Finally, the mismatch between theoretical and experimental results are analysed in the context of TED, die coupling, support loss and gas damping in rate sensors.

- **Chapter 6** summarises the key findings and conclusions of this research project, and suggests suitable future work in this area of research.

Chapter 2

Thermoelastic Damping in MEMS Resonators

2.1 Introduction

Thermoelastic damping (TED) is an intrinsic source of energy dissipation which occurs due to coupling between thermal and mechanical state within a structure. For MEMS resonators such as beams [47, 48, 53] and rings [10], TED is the most important damping mechanism and in the design of MEMS angular rate sensors it is desirable to have ring resonators that have low levels of Q_{TED} to improve sensor performance. Previous studies predicting Q_{TED} in micro-rings [3, 16, 54] are restricted to simple geometries and are not capable of taking account of the support structure, deliberate modifications like slots, geometric asymmetry or anisotropic materials. To overcome these restraints and design high-Q resonators a more general method is needed to predict TED.

In this work the finite element method is used to analyse the damping in supported ring resonators and parameter studies are performed to gain understanding into the influence of dimensional changes to the ring and support legs and the effect of slots around the ring. As TED depends on the thermal properties of materials, careful consideration is also given to the effect of using different materials. The work presented in this chapter is useful for designing high-Q resonators to improve the performance of angular rate sensors from a thermoelastic damping perspective.

Section 2.2 briefly introduces analytical results for predicting TED in simple res-

onators, including Zener’s theory and outlines the limitations of these models.

Section 2.3 presents the background to using the finite element method to predict TED in resonators. The governing equations of thermoelasticity are based on constitutive stress-strain relations, equations of motion and the heat diffusion equation. A description of the finite element model and its validation against analytical and experimental results is provided.

Section 2.4 considers TED in unsupported rings and numerical results are presented to illustrate the effect of ring geometry on frequency and Q_{TED} . Zener’s theory is used to explain this damping behaviour and investigate the effect of using different materials (including diamond, quartz and single crystal silicon).

Section 2.5 investigates the impact of supporting legs on the frequency and Q_{TED} for the resonator. To explain the observed behaviour a semi-analytical approach is used that expresses the Q_{TED} for the resonator in terms of the Q_{TED} for the ring and Q_{TED} for the support legs. Numerical studies are also used to investigate the effect of using different materials.

Section 2.6 investigates the impact of incorporating slots around the ring circumference on the overall Q_{TED} , and an explanation for the complex coupling between mechanical and thermal modes caused by the slots is provided.

Section 2.7 proposes supported ring resonators designs that have high Q_{TED} . Chapter 5 describes the experimental testing and performance (frequency and Q-factor) for these designs.

Section 2.8 summarises the research work and presents the main conclusions for the chapter.

2.2 Analytical Methods for Predicting TED in Simple Resonators

2.2.1 Introduction

Analytical expressions for Q-factor associated with the thermoelastic damping in flexural mode resonators are well established, see Chapter 1 for details. Although

these expressions are based on some restrictive assumptions they provide relatively accurate predictions for simple geometries like beam, rings, etc. In this section the main results for the two analytical models developed by Zener and Lifshitz-Roukes are reviewed, to provide insight into the damping behaviour of simple resonators.

2.2.2 Zener's Theory

Zener [44, 45] was the first to develop the theory of thermoelastic damping for thin rectangular beams under flexural vibration. Zener develop the following equation for the Q-factor due to thermoelastic effect:

$$Q_{TED}^{-1} = \Delta_M \frac{\omega_0 \tau}{1 + (\omega_0 \tau)^2} \quad (2.1)$$

where ω_0 is the fundamental undamped natural frequency. The effective relaxation time is

$$\tau = \frac{C_v b^2}{k \pi^2} \quad (2.2)$$

and the relaxation strength is

$$\Delta_M = \frac{E \alpha^2 T_0}{C_v} \quad (2.3)$$

In Eqs. (2.2) and (2.3), $C_v (= \rho C_p)$ is the specific heat capacity at constant volume, T_0 is the reference temperature, α is the linear coefficient of thermal expansion, E is the Young's modulus, k is the thermal conductivity, and b is the dimension in the direction of the flexing.

For a ring vibrating in its in-plane flexure mode at undamped natural frequency of ω_0 , Wong *et al.* [16] used Eq. (2.1) to calculate Q_{TED} and showed that

$$\tau = \frac{C_v b_r^2}{k \pi^2} \quad (2.4)$$

b_r is the radial thickness of the ring.

The relaxation time (τ), which is the characteristic time required to equalize the temperature, is one of the key parameters in Zener's approximate expression. The

relaxation strength of a thermoelastic solid is represented by the difference between unrelaxed (E_U) and relaxed values (E_R) of the Young's modulus [44]:

$$\Delta_M = \frac{E_U - E_R}{\sqrt{E_U E_R}} \quad (2.5)$$

The magnitudes of E_R and E_U are measures of two equilibrium states for the same applied stress, and the values are governed by the material properties.

Based on Zener's theory there are three different regimes that influence the behaviour of Q_{TED} : $\tau \approx \omega_0^{-1}$ ($\omega_0 \tau \approx 1$), $\tau \gg \omega_0^{-1}$ ($\omega_0 \tau \gg 1$) and $\tau \ll \omega_0^{-1}$ ($\omega_0 \tau \ll 1$).

Low Frequency: When $\tau \ll \omega_0^{-1}$, i.e. the low frequency range, the vibration is considered isothermal and a very small amount of energy dissipates from the solid. At this low frequency, only a small temperature gradient is formed. This is due to the fact that the temperature gradient is dependent on strain rate. In the low frequency range, the strain rate is reduced, consequently the temperature gradient reduces, leading to small levels of energy dissipation.

High Frequency: In the high frequency range, i.e. $\tau \gg \omega_0^{-1}$, very small levels of energy is dissipated similar to the low frequency range, and the vibration is adiabatic. Due to the high frequency, the temperature gradient has a shorter time period to relax back to the equilibrium state, leading to very low energy dissipation. During this short time period the temperature gradient vanishes before heat transport occurs.

Maximum Energy Dissipation (Debye Peak Region): When $\tau \approx \omega_0^{-1}$, maximum energy dissipation occurs, i.e. Q_{TED} is at a minimum. In this case, the relaxation time and flexural period of the system are of the same order of magnitude, allowing the temperature gradient to relax, through heat transport, to restore thermal equilibrium. This maximum dissipation occurs at the Debye peak (see Figure 2.1) and is defined by $Q_{TED} = Q_{min} = \frac{2}{\Delta_M}$.

These three regimes will be used later to develop an understanding of damping behaviour in more complex resonators.

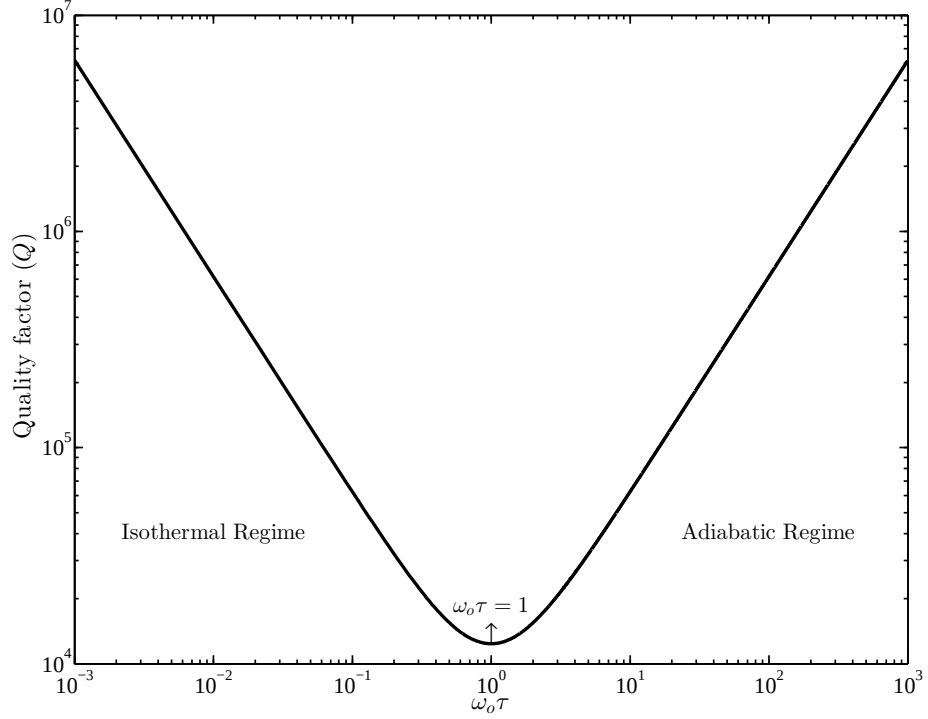


Figure 2.1: Q_{TED} as a function of resonant frequency. The Debye peak is observed when $\tau \approx \omega_0^{-1}$.

2.2.3 Lifshitz-Roukes (L-R) Approach

Lifshitz and Roukes developed the following expressions for a beam in flexure based on the same principle as used in Zener's expression. The main differences are that the transverse temperature profile has been modelled accurately, and considers the effect of thermoelastic damping on the natural frequencies using expressions of complex frequencies [53]. L-R showed that the Q-factor can be expressed as:

$$Q_{TED} = \frac{E\alpha^2 T_0}{C_v} \left(\frac{6}{\xi^2} \frac{6}{\xi^3} \frac{\sin \xi + \sinh \xi}{\cos \xi + \cosh \xi} \right), \text{ with } \xi = b \sqrt{\frac{\omega_0}{2\chi}} \quad (2.6)$$

where b is the dimension of the beam in the direction of the flexing and χ is the thermal diffusivity of the material.

Wong *et al.* [54] and Hao and Ayazi [3] derived similar expressions as Eq. (2.6) for Q_{TED} in a ring resonator vibrating in its in-plane flexural modes based on L-R's approach, where they showed $b = b_r$ and ω_0 in Eq. (2.6) are the radial thickness and undamped natural frequency of ring, respectively.

An expression similar to Eq. (2.6) for TED in ring resonators derived by Wong *et al.* [54] will be used for comparison with the results obtained using Zener's expression and numerical simulations. Furthermore, the analytical expressions will be used to validate the numerical model developed in this chapter.

2.2.4 Limitations of Analytical Approach

With the advancement of MEMS technologies, designers are now considering resonators with complex geometries. Although Zener's closed form expression provides an accurate approximation of Q_{TED} for simple beam and ring-like structures, it incorporates some simplifying assumptions. One of the assumptions is that heat conduction only occurs in the direction of flexing and neglects heat conduction along the longitudinal axis. Although it has been later proved that heat conduction in the longitudinal direction is negligible for flexural vibrations (see Chapter 1). Another assumption is that the temperature profile is approximated by a sinusoidal function only. Zener showed that for a beam vibrating in its transverse mode, 98.6% of the relaxation occurs within the first transverse thermal mode [53] and hence, a single relaxation time is considered for the calculation of Q_{TED} . This assumption induces a small error in the prediction of Q_{TED} .

Lifshitz-Roukes' approach provides an exact solution by considering more accurate transverse temperature profile compared to Zener's approximation. Zener's formula overestimates Q_{TED}^{-1} by 2% compared to L-R's model for low frequencies (i.e. isothermal side of the Debye curve), and underestimates by a maximum of 20% for high frequencies (adiabatic side). Both these models assume that surfaces are thermally insulated, which is a valid assumption for resonators operating in vacuum. However, these models do not account for the effects of different structural boundary conditions, or different mode shapes of flexural vibration on TED [62]. Both analytical models are not applicable to resonators with complex geometries to evaluate Q_{TED} , not only because of the simplifying assumptions but also because these models are only valid for isotropic materials. Hence, evaluation of any frequency and/or

Q-factor shifts resulting from anisotropic materials is difficult to accomplish using analytical models. On this basis, a numerical approach is required to investigate TED in complex resonators.

2.3 Numerical Method for Predicting TED in Complex Resonators

2.3.1 Introduction

As mentioned in the previous section, it is necessary to use numerical methods to predict TED for complex geometries. These approaches discretize the coupled equations of thermoelasticity and use the finite element method for their solution [18]. Referring back to Chapter 1, three different numerical approaches are available in the literature to solve these equations: a fully coupled approach, a weakly coupled approach and a thermal energy method [63, 64]. The fully coupled approach is more rigorous because of the full coupling between thermal and mechanical domains being considered, which enables a precise evaluation of Q_{TED} . The fully coupled approach is used in this work.

The remainder of this section describes the methodology used to evaluate TED numerically, including the basic principles of deriving coupled equations of thermoelasticity and constructing the numerical model in the commercial FE package COMSOL [111]. The effect of number of elements, i.e. meshing, on the accuracy of the solution is presented. Furthermore, the finite element model is validated for a simple ring resonator for which analytical and experimental results are available in the literature.

2.3.2 Methodology

2.3.2.1 Governing Equations of Thermoelasticity

In every solid there exists a fundamental thermoelastic coupling between the thermal and mechanical states (e.g. between stress and temperature fields), where the thermal

expansion coefficient acts as the coupling constant [40]. The governing equations of thermoelasticity are derived from the equations of motion of solids and a heat diffusion equation.

The equations of motion for a three dimensional (3D) elastic solid (neglecting external body force) in a Cartesian coordinate system can be expressed as [112]

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2} \quad (2.7)$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2} \quad (2.8)$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2} \quad (2.9)$$

where u, v, w are displacements, and $\frac{\partial^2 u}{\partial t^2}, \frac{\partial^2 v}{\partial t^2}, \frac{\partial^2 w}{\partial t^2}$ are the acceleration of an infinitesimal element of the body, in the x, y and z directions. ρ is the material density.

The constitutive equations of thermoelasticity for a homogeneous isotropic material are given by [112]

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + (\Lambda e - \Upsilon T)\delta_{ij} \quad (2.10)$$

where σ_{ij} and ε_{ij} are the stress and strain tensors ($i, j = x, y, z$), respectively. The thermoelastic constant is, $\Upsilon = \alpha(3\Lambda + 2\mu)$ and the strain dilatation is given by $e = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$. Also, T is the temperature change from the reference temperature (T_0) and α is the thermal expansion coefficient. Here, one has the *Lamé* constants, $\Lambda = \frac{E}{(1+\nu)(1-2\nu)}$ and $\mu = \frac{E}{2(1+\nu)}$. ν is the Poisson's ratio of material. The Kronecker delta is defined by

$$\delta_{ij} = \begin{cases} 1 & \text{when } i = j \\ 0 & \text{when } i \neq j. \end{cases}$$

Combining Eqs. (2.7) to (2.10) and the linear strain displacement equations gives [63]

$$\rho \frac{\partial^2 u}{\partial t^2} = \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + (\Lambda + \mu) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} \right) - \Upsilon \frac{\partial T}{\partial x} \quad (2.11)$$

$$\rho \frac{\partial^2 v}{\partial t^2} = \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + (\Lambda + \mu) \left(\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z} \right) - \Upsilon \frac{\partial T}{\partial y} \quad (2.12)$$

$$\rho \frac{\partial^2 w}{\partial t^2} = \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + (\Lambda + \mu) \left(\frac{\partial^2 u}{\partial z \partial x} + \frac{\partial^2 v}{\partial z \partial y} + \frac{\partial^2 w}{\partial z^2} \right) - \Upsilon \frac{\partial T}{\partial z} \quad (2.13)$$

The heat conduction equation in 3D for thermoelastic damping is given by [63]

$$\rho C_p \frac{\partial T}{\partial t} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) - T_0 \Upsilon \frac{\partial e}{\partial t} \quad (2.14)$$

where C_p is the specific heat at constant pressure, and k is the thermal conductivity.

In MEMS, the temperature fluctuations from the equilibrium temperature are considered small, allowing Eq. (2.14) to be linearised about the reference temperature T_0 .

Eqs. (2.11) to (2.14) are known as the coupled equations of thermoelasticity. The coupling between structural, Eqs. (2.11) to (2.13), and the thermal, Eq. (2.14) is via the thermal expansion coefficient (α) and the thermoelastic constant (Υ). The third term on the right hand side of Eqs. (2.11) to (2.13) correspond to thermal expansion and the second term on the right of Eq. (2.14) represents strain induced heat. These equations can be solved relatively easily using the finite element (FE) method.

2.3.2.2 Finite Element Modelling

Model Setup

Eqs. (2.7) to (2.9) are integrated into the finite element package COMSOL [111]. Coupling with the heat diffusion equation, thermal expansion is implemented in these equations based on the constitutive relation (Eq. (2.10)). This gives a set of equations similar to Eqs. (2.11) to (2.13). The heat source term from diffusion Eq. (2.14) is specified in the heat conduction equation available in the software for the

bi-directional coupling based on Eqs. (2.11) to (2.13). For the work discussed in this thesis, simulations are performed using two-dimensional fully coupled thermo-mechanical damped eigenfrequency analysis. The two-dimensional plane stress assumption ($\sigma_{zz} = \sigma_{zx} = \sigma_{zy} = 0$) is used to reduce the computational effort. Two displacement variables (u, v) and a temperature variable (T) are solved for each node of the discretised resonator geometry, which includes the phase differences between them. The damped eigenfrequency analysis results in complex eigenvalues (λ) from which the damped natural frequencies and Q-factors are evaluated using Eq. (1.9).

An important aspect of the finite element modelling is the scaling of variables and equations. If the dependent variables (displacements and temperature in this case) in the model have widely different magnitudes, the solver in the COMSOL can experience problems with the ill-conditioned matrix [111]. To avoid such problems COMSOL internally rescales the variables so that a well-scaled system is analysed.

Boundary Conditions

Two types of boundary conditions (BCs) are applied to the equations: structural BCs and thermal BCs. The boundaries of the resonator can be fixed i.e. zero displacement, and this fixed constraint is used later in this chapter to model the attachment of the support legs to the central hub. The resonator is assumed to vibrate in a vacuum, so there will be no convective heat transfer with the surroundings. Also, the radiation effects are considered negligible as temperature change remains low. The adiabatic (thermal insulation) boundary condition is applied to all external boundaries within the model i.e. $\mathbf{n} \cdot k \nabla T = 0$, where $\mathbf{n}$ is the normal vector at the boundary. Physically, this implies that there is no heat flux across the external boundaries.

Mesh Generation

The models are meshed using quadratic triangular elements generated by the built-in meshing technique in COMSOL. The mesh density and element size are controlled by a set of meshing parameters. Throughout the mesh the minimum element size does not exceed the minimum thickness within the geometry, e.g. radial thickness

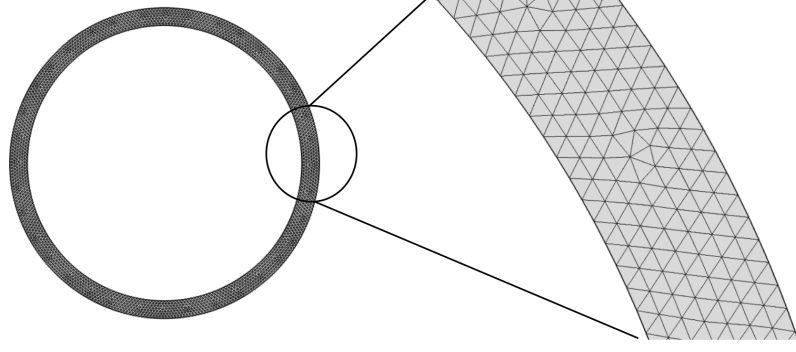


Figure 2.2: Meshed ring resonator using triangular elements

of a ring, or support leg width. Also, the maximum element growth rate, which determines the maximum rate at which the element size can grow from a region with small elements to a region with larger elements [111], is kept within 1.1 to 1.2. This ensures the element size can grow by at most 10-20% (approximately) from one element to another. Depending on the ring and support dimensions, the total number of elements used in later simulations varies between ~ 70000 and ~ 160000 .

In the following sections, mesh convergence and validation studies are performed to determine the accuracy of the finite element model developed in this section.

2.3.3 Convergence Study

A study is performed to determine the effect of the number of elements on the convergence of the Q-factor prediction using the finite element analysis. The results shown in Figure 2.3 are obtained for a 3 mm radius and 120 μm thick ring (unsupported) resonator. It is clear that the Q_{TED} increase asymptotically as the number of meshing elements (mesh density) is increased. Also, the Q-factor values for the degenerate pair of modes i.e. primary and secondary mode become almost identical as number of elements increase. This is expected as the simulation does not consider any imperfections. However, the negligible differences could be due to small amount of asymmetry induced in the meshing, relative to the modes concerned. For Q_{TED} of this ring resonator, the finite element model requires the number of elements to be ≥ 10000 to give converged results. Based on this converged results, the FE model is

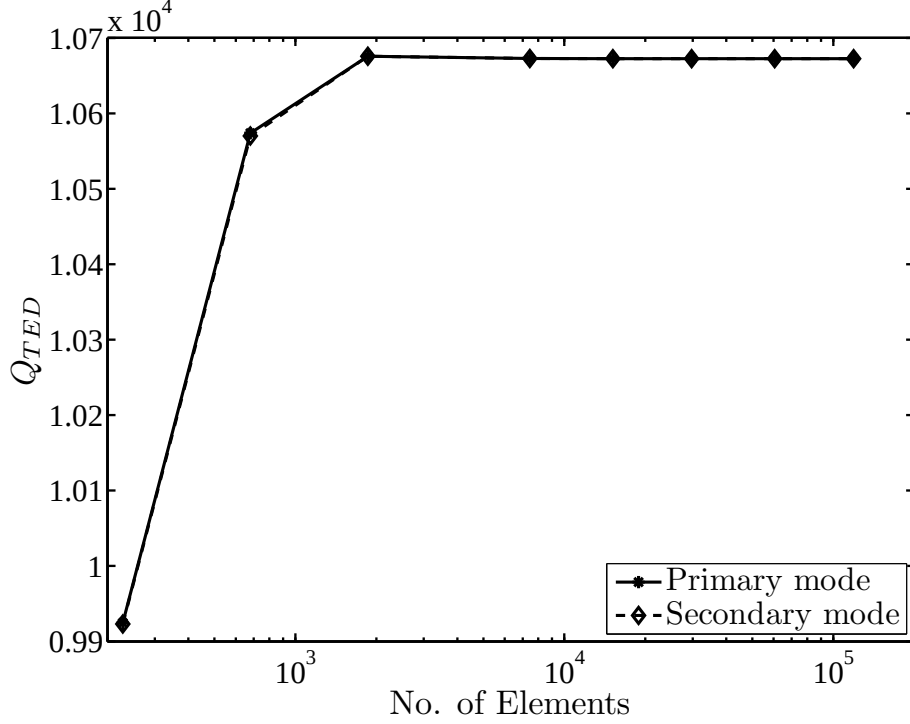


Figure 2.3: Mesh convergence study showing relationship between Q-factor and the number of elements for a simple ring resonator vibrating in 2θ in-plane mode.

validated using 3 mm- 120 μ m unsupported ring in the following section.

2.3.4 Model Validation

The purpose of this section is to validate the FE model developed for a simple ring vibrating in its 2θ in-plane mode. Numerical results are compared with the analytical results using Eqs. (2.1) and (2.6) presented in Section 2.2. These results depend on the natural frequency and it is well known that the natural frequency for an unsupported ring vibrating in its in-plane $n\theta$ mode is [113]

$$\omega_0 = \frac{n(n^2 - 1)}{r^2 \sqrt{n^2 + 1}} \sqrt{\frac{EI_z}{\rho A}} \quad (2.15)$$

where n is the mode number (for 2θ in-plane modes, $n=2$), r is the radius of the ring, I_z is the area moment of inertia about the axial direction, A is the cross-sectional area of the ring, and ρ is the material density.

The natural frequencies and Q-factors presented in Table 2.1 are for the ring

resonator (unsupported) of 3 mm radius and 120 μm radial thickness, considered in Section 2.3.3. The Table compares the results obtained using numerical simulation against analytical and experimental values obtained using Eqs. (2.1), (2.6), (2.15) and Ref. [16], for the 2θ in-plane mode.

It is worth mentioning that performing eigenfrequency analysis to solve the problem will provide multiple eigenfrequencies and the shape of eigenmodes, if it is not solved for the desired modal frequency. Therefore, to avoid this problem, analytical frequency is used as the initial guess around which the eigenfrequency analysis is performed to obtain the desired eigenmode and frequency.

Table 2.1: Results of a simulation for a ring vibrating in its in-plane 2θ mode

Natural Frequency (kHz)		Q_{TED}			Overall, Q (Exp.[16])
Analytical	Simulated	Analytical (Zener) Eq. (1)	Analytical (L-R) Eq. (2)	Simulated	
14.04	14.03	10569	10670	10672	10500

Comparing the natural frequencies, it can be seen that they are in good agreement and the analytical and simulated values are almost identical. The analytical frequency is calculated based on the expression for the natural frequency of an undamped ring vibrating in its flexural 2θ (in-plane) mode. In contrast the simulated value is the damped natural frequency, so a small difference could be expected. The simulated Q_{TED} is in excellent agreement with the analytical and experimental values, with $<1\%$ difference between the simulated and analytical values. Given that the analytical results for TED do not account for TED in the support legs or any other sources of damping, it can be deduced from the experimental result that for the case considered TED in the ring is the dominant damping mechanism and the influence of the support is negligible.

2.3.5 Summary

In this section, the methodology used for numerically predicting TED in MEMS resonators was presented. The governing equations of thermoelasticity depend on the equations of motions and the heat diffusion equation, and in the FE model

used full coupling between the mechanical and thermal domains within the structure are included. The discretization of these equations using a finite element model in COMSOL has been discussed. In order to validate the finite element model, a simple ring resonator has been investigated. The results show excellent agreement with analytical (based on Zener's theory and L-R's approach) and experimental results. The comparison with experimentally obtained Q-factor suggests that TED is the dominant mechanism in ring resonators operating in the 2θ in-plane mode. This conclusion will be discussed again later in this chapter.

2.4 Thermoelastic Damping in Unsupported Rings

2.4.1 Introduction

This section presents a parametric method for determining ring configurations having high-Q and discusses the damping behaviour in these rings based on Zener's theory. A further investigation is conducted to consider the impact of material selection on resonator damping based on a simplification Zener's equation.

2.4.2 Effect of Ring Geometry on Frequency and TED

This section presents a parameter study investigating the influence of ring geometry on the natural frequencies and Q_{TED} for the flexural 2θ in-plane modes of an unsupported ring.

Figures 2.4 and 2.5 show how the damped natural frequency and Q_{TED} vary as the diameter (d) and radial thickness (b_r) are varied. These results have been obtained using the finite element model, but similar results are obtained using Eqs. (2.1), (2.6) and (2.15).

Eq. (2.15) indicates that the undamped natural frequency of an unsupported ring

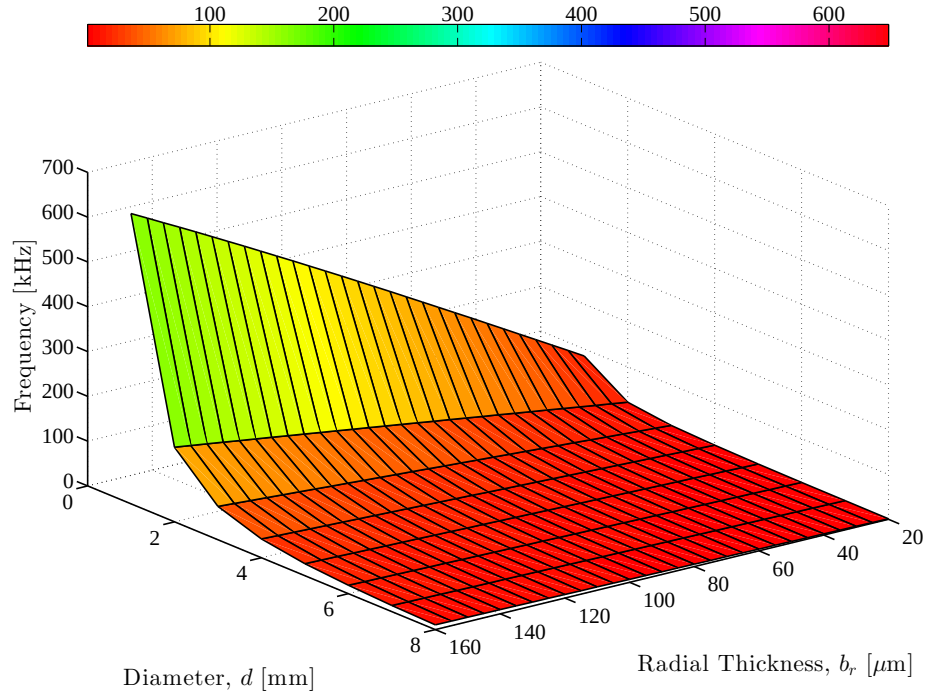


Figure 2.4: Frequency (damped) as a function of diameter and radial thickness.

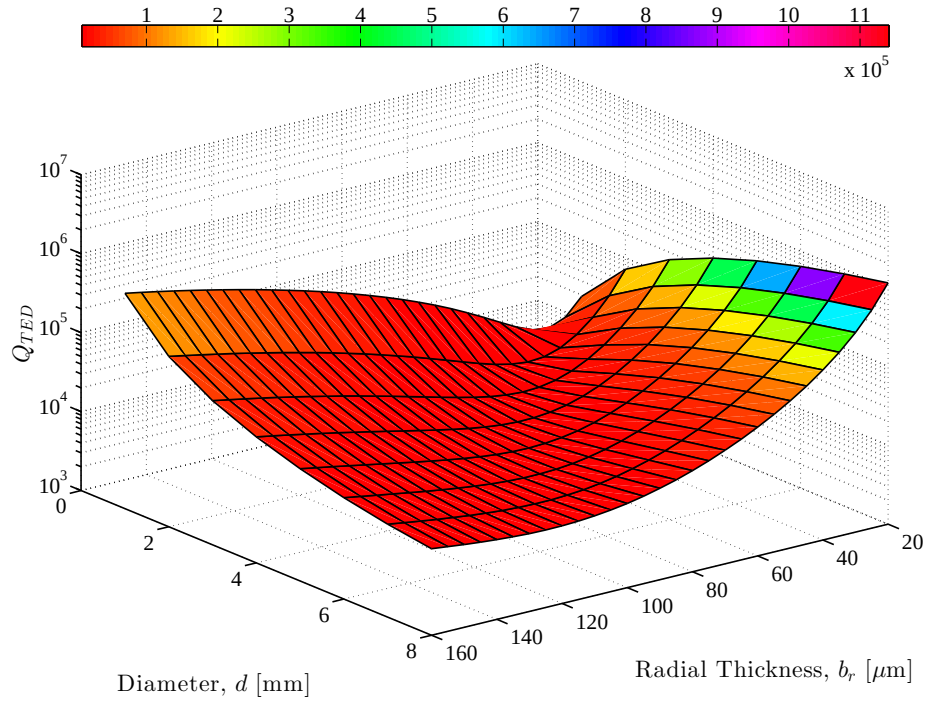


Figure 2.5: Q_{TED} as a function of diameter and radial thickness.

varies as follows

$$\omega_0 \propto \frac{1}{r^2}; \text{ for fixed } b_r \quad (2.16)$$

$$\omega_0 \propto b_r; \text{ for fixed } r \quad (2.17)$$

These relationships can be observed in Figure 2.4, and indicate that high frequencies occur for small diameter thick rings, and low frequencies occur for large diameter thin rings, as expected.

In Figure 2.5, it can be seen that for a given fixed diameter, Q_{TED} decreases as b_r decreases until it reaches a minimum value before it starts increasing again. The critical value b_r when the minimum Q_{TED} occurs is different for each diameter. For example, for $d = 8 \text{ mm}$ the critical value for b_r is $120 \mu\text{m}$, whilst for $d = 3 \text{ mm}$ the critical value is $60 \mu\text{m}$. For large values of b_r , Q_{TED} decreases as d increases. In contrast, for small values of b_r , Q_{TED} decreases as d decreases. This study suggests that high Q_{TED} values can be achieved either with large diameter thin rings or with small diameter thick rings.

Although high Q_{TED} values can be obtained using small diameter thick rings, these configurations are not suitable for practical MEMS rate sensor applications because higher frequency applications are preferred to avoid unwanted couplings, and typically the frequency range is 10 to 30 (kHz) [114] based on published values. For this reason a trade-off is often required between Q_{TED} and frequency when selecting the basic ring dimensions to achieve high Q . The general approach would be to consider a fixed diameter ring and then perform a parametric analysis to decide the radial thickness. In this section, two different ring diameters ($d = 4$ and 8 mm) are considered for the purpose of comparison.

To highlight the trade-off between frequency and Q-factor, Figure 2.6 shows the frequency and Q_{TED} plotted as a function of radial thickness for $d = 4 \text{ mm}$. The plot includes results obtained using numerical simulations as well as analytical results, and they are all in excellent agreement. It is evident that the numerically simulated

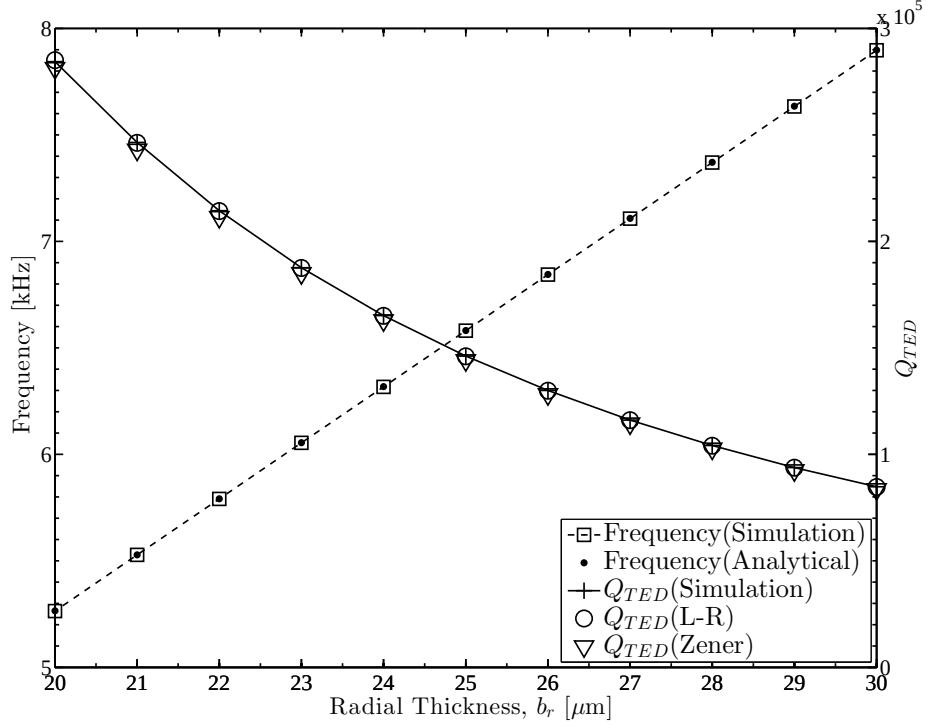


Figure 2.6: Frequency (damped) and Q_{TED} as a function of radial thickness for 4 mm diameter ring.

Q_{TED} values are in excellent agreement with those calculated using Eq. (2.6). The Q_{TED} values predicted using Zener's expressions are slightly underestimated, and this is expected because Lifshitz and Roukes derived an exact expression compared to Zener's approximate expression.

2.4.3 Understanding TED Behaviour in Unsupported Rings-Based on Ring Geometry

In the previous section it was shown that high Q_{TED} values can be achieved either with large diameter thin rings or with small diameter thick rings, and low Q_{TED} values are obtained for critical b_r thickness values.

To understand this behaviour, three different cases of high and low Q_{TED} are considered; these are: (a) 2 mm diameter ring with 160 μm radial thickness, (b) 4 mm diameter ring with 20 μm radial thickness and (c) 8 mm diameter ring with 120 μm radial thickness. The frequencies and Q_{TED} values, obtained from numerical simulation, for the three cases are summarised in Table 2.2. Figure 2.7 shows the

Table 2.2: Frequency and Q-factor for three different cases of unsupported rings

Ring	Frequency (kHz)	Q_{TED}
2 mm-160 μm	166.758	1.44×10^5
4 mm-20 μm	5.264	2.85×10^5
8 mm-120 μm	7.895	9624

Q_{TED} for these rings on the Debye peak curve (see Figure 2.1), and are obtained using Eqs. (2.1) and (2.15). The vertical and horizontal dashed lines show the damped natural frequency and corresponding Q_{TED} values respectively for the three rings considered. It is evident that the 4 mm-20 μm ring operates in the isothermal region, i.e. a small temperature gradient is formed during its vibration, and this leads to very low energy dissipation, high Q_{TED} . In contrast, the 8 mm-120 μm ring operates in the region close to the Debye peak where maximum dissipation occurs. In this case, the flexural period and relaxation time have the same order of magnitude, resulting in a low Q_{TED} value. However, the 2 mm-160 μm ring operates in the adiabatic region due to its high frequency. Since the thermal gradient vanishes due to the shorter time period, it exhibits a high Q_{TED} value. In summary, the small diameter thick rings and large diameter thin rings have high Q_{TED} due to operating in the adiabatic and isothermal regions, respectively. For thicker rings, one should be careful about the practical limitations that exist for ring geometries, i.e. $0.5b_r < \frac{d}{2}$.

2.4.4 Effect of Material Properties on Frequency and TED

Here, the aim is to identify key material properties that influence the damping behaviour of rings. If the ring dimensions remain fixed, Eq. (2.15) and Eq. (2.2) indicate that the frequency and thermal time constant vary as follows

$$\omega_0 \propto \sqrt{\frac{E}{\rho}}; \quad (2.18)$$

$$\tau \propto \frac{\rho C_p}{k} \quad (2.19)$$

For a ring operating in **Debye peak region** i.e. $\omega\tau \approx 1$, Eq. (2.1) can be simplified

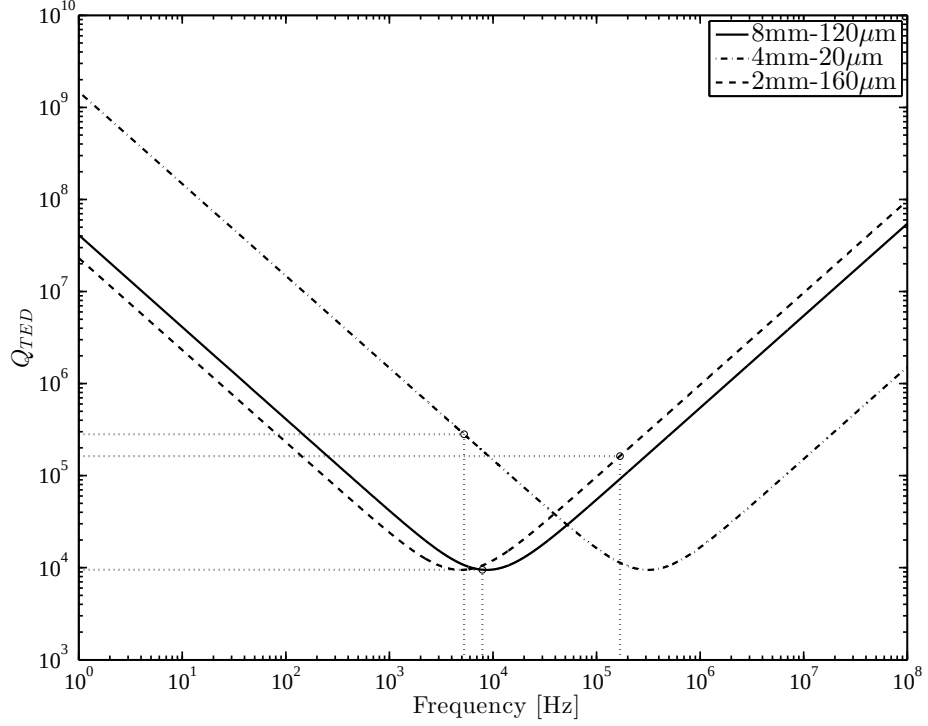


Figure 2.7: Debye peak curve for 2 mm-160 μm , 4 mm-20 μm and 8 mm-120 μm rings.

to

$$Q_{TED} = \frac{2\rho C_p}{E\alpha^2 T_0}; \quad (2.20)$$

It is evident from Eq. (2.20) that for a ring operating in Debye peak regime, Q_{TED} increases if the density and specific heat capacity of the material increases; Q_{TED} increases for materials with a low Young's modulus and thermal expansion coefficient. Eq. (2.20) represents the minimum Q_{TED} value for a ring.

For a ring operating in the **isothermal region** i.e. $\omega_0\tau \ll 1$, Eq. (2.1) simplifies to

$$Q_{TED} = \frac{\rho C_p}{\omega_0\tau E\alpha^2 T_0} \quad (2.21)$$

Combining Eqs. (2.18) (2.19) and (2.21), it can be shown that

$$Q_{TED} \propto \frac{\sqrt{\rho}k}{E^{3/2}\alpha^2} \quad (2.22)$$

For a ring operating in the **adiabatic region** i.e. $\omega_0\tau \gg 1$, Eq. (2.1) can be

simplified to

$$Q_{TED} = \frac{\rho C_p \omega_0 \tau}{E \alpha^2 T_0} \quad (2.23)$$

Combining Eqs. (2.18) (2.19) and (2.23), it can be shown that

$$Q_{TED} \propto \frac{\rho^{3/2} C_p^2}{\sqrt{E} k \alpha^2}; \quad (2.24)$$

The main distinction between Eqs. (2.22) and (2.24) is the relation between Q_{TED} and thermal conductivity, k . For rings operating in the adiabatic regime, Q_{TED} is higher for low k materials whereas in the isothermal regime Q_{TED} is higher for high k materials.

2.4.5 Understanding TED Behaviour in Unsupported Rings-Based on Material Properties

This section investigates the influence of material properties on natural frequency and Q_{TED} for the flexural 2θ in-plane modes of an unsupported ring.

Predominantly silicon is the preferred choice of material in most MEMS devices mainly due to its availability and ease of processing. For high performance applications (111)-oriented single crystal silicon (SCS) is used due to its uniform and homogeneous material properties over the (111) surface [115]. Due to difficulties in microfabrication technology, few other materials can be considered for use in MEMS devices. However developments in processing techniques make it possible to use non-metals like Ge, ceramics like diamond, SiC, quartz (SiO_2), etc. [116]. To investigate the effects of material properties on resonator performance, quartz and diamond are considered along with SCS-(111). The mechanical and thermal properties of these materials are summarised in Table 2.3, where the diamond is assumed to be prepared by chemical vapor deposition (CVD). The thermal conductivity of CVD diamond is represented using the following equation at $T=300K$ [117]:

$$k = (2.833 \times 10^6)/T^{1.245} \quad (2.25)$$

For any ring dimension, the frequency will be much higher for diamond compared to quartz and SCS-(111). This is because, from Eq. (2.18) the frequencies of rings are directly proportional to the Young's modulus. Since the Young's modulus for diamond is almost 10 times higher than the other materials, rings made of diamond will exhibit much higher frequencies.

Table 2.3: Thermo-mechanical properties of single crystal silicon(SCS), quartz and diamond. The values in brackets indicate the relative magnitudes with respect to the SCS-(111). The properties are obtained from Maluf and Williams [118], The CVD Diamond Booklet [119] and, Shinde and Goela [117]

Material	Young's Modulus, E (GPa)	Density, ρ (kg/m ³)	Poisson's Ratio, ν	Specific Heat, C_P (J/kg.K)	Coefficient of Thermal Expansion, α (1/K) $\times 10^{-6}$	Thermal Conductivity, k (W/m.K)	Q_{min}
SCS-(111)	170 (1)	2330 (1)	0.22 (1)	700 (1)	2.60 (1)	130 (1)	9472 (1)
Quartz (fused silica)	107 (0.63)	2650 (1.14)	0.16 (0.72)	787 (1.12)	0.55 (0.21)	1.4 (0.01)	4.29×10^5 (45.40)
Diamond (CVD)	1050 (6.18)	3500 (1.5)	0.1 (0.45)	520 (0.74)	1 (0.38)	2340 (18)	11556 (1.22)

Figures 2.5, 2.8 and 2.9 show how Q_{TED} varies as a function of ring diameter and radial thickness for SCS-(111), quartz and diamond, respectively. It was observed previously that, high Q_{TED} values can be achieved either with small diameter thick rings or large diameter thin rings. Within the selected range of diameter and radial thickness, SCS-(111) exhibits both types of behaviour. However, from Figures 2.8 and 2.9 it is evident that, quartz shows higher Q_{TED} values for small diameter thin rings whereas diamond exhibit high-Q values for large diameter thin rings.

The high-Q rings usually operate either in the isothermal or adiabatic regimes of the Debye curve, as mentioned earlier. To understand damping behaviour in high-Q rings for different materials, the Debye curves for a large diameter thin ring, i.e. 8 mm-20 μ m ring and a small diameter thick ring i.e. 2 mm-160 μ m, are compared for SCS-(111), quartz and diamond in Figures 2.10 and 2.11, respectively. For the large diameter thin ring (see Figure 2.10), it can be seen that diamond operates in the isothermal regime and has higher Q_{TED} . The ring made of quartz has the lowest Q_{TED} and operates close to the Debye region. SCS-(111) on the other hand, also operates in the isothermal regime. Referring back to Eq. (2.22), rings operating in the isothermal regime will have higher Q_{TED} for a material that has higher k value.

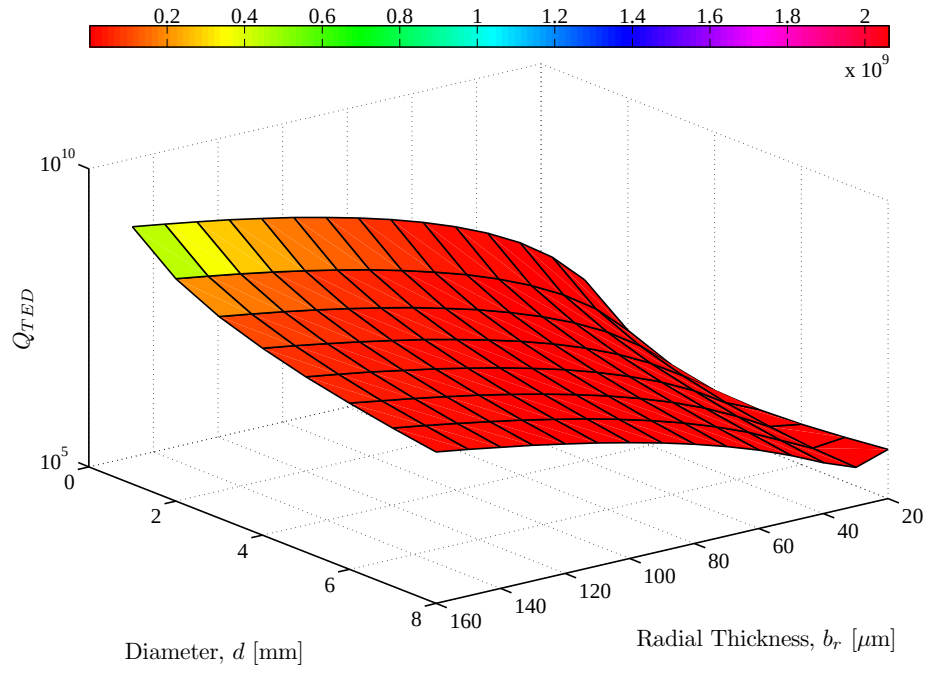


Figure 2.8: Q_{TED} as function of diameter and radial thickness (quartz).

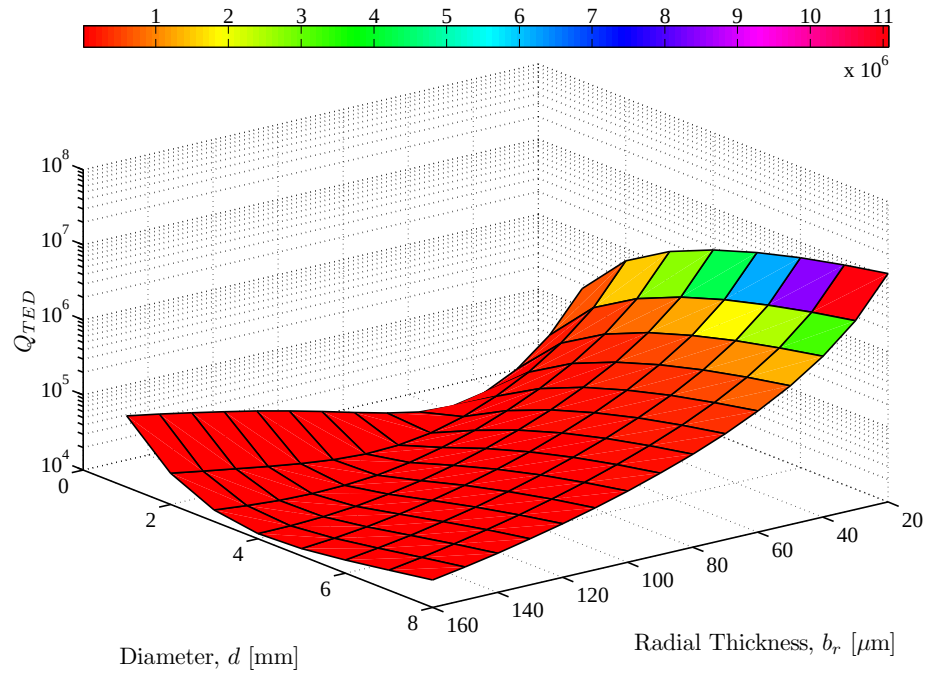


Figure 2.9: Q_{TED} as function of diameter and radial thickness (diamond).

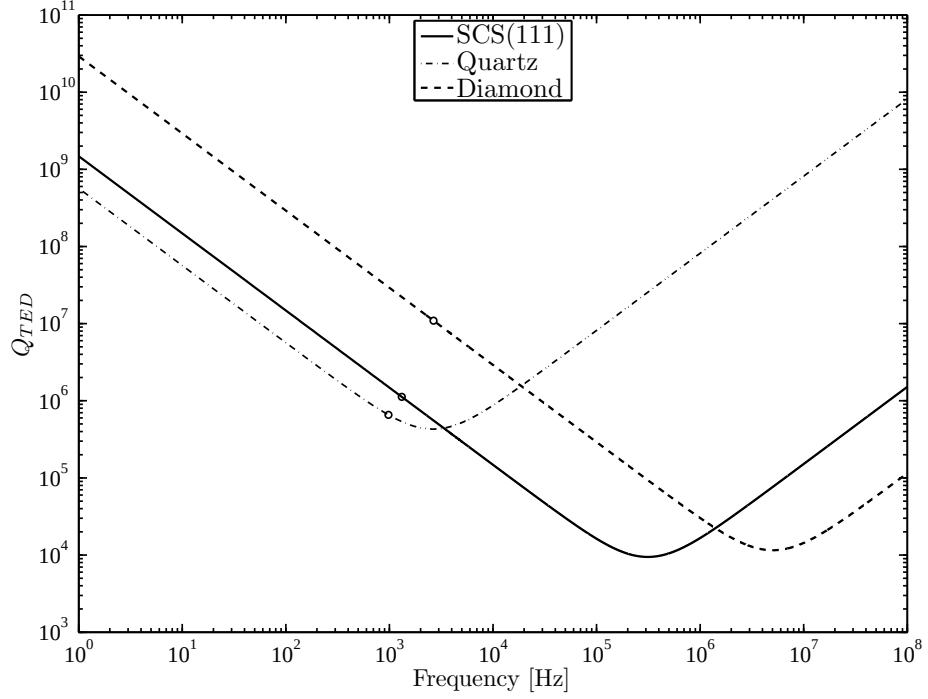


Figure 2.10: Debye peak curve for 8 mm-20 μm ring with different materials.

This explains why large diameter thin rings made of diamond exhibit higher Q_{TED} than SCS-(111) and quartz.

For small diameter thick rings (see Figure 2.11), it can be seen that the quartz ring operates in the adiabatic regime and has a higher Q_{TED} compared to SCS-(111) and diamond. The diamond ring has the lowest Q_{TED} and operates close to the Debye region. SCS-(111), on the other hand, also operates in the adiabatic regime. From Eq. (2.24), it is evident that rings operating in the adiabatic region have higher Q_{TED} for materials with low k .

2.4.6 Summary

The damping behaviour of unsupported rings has been presented, and is the first step to designing high-Q ring resonators. Predominantly silicon is the preferred material for MEMS fabrication and the results suggest that, irrespective of frequency, unsupported ring resonators with high-Q can be achieved with either small diameter thick rings or large diameter thin rings. Large diameter thin rings have lower frequen-

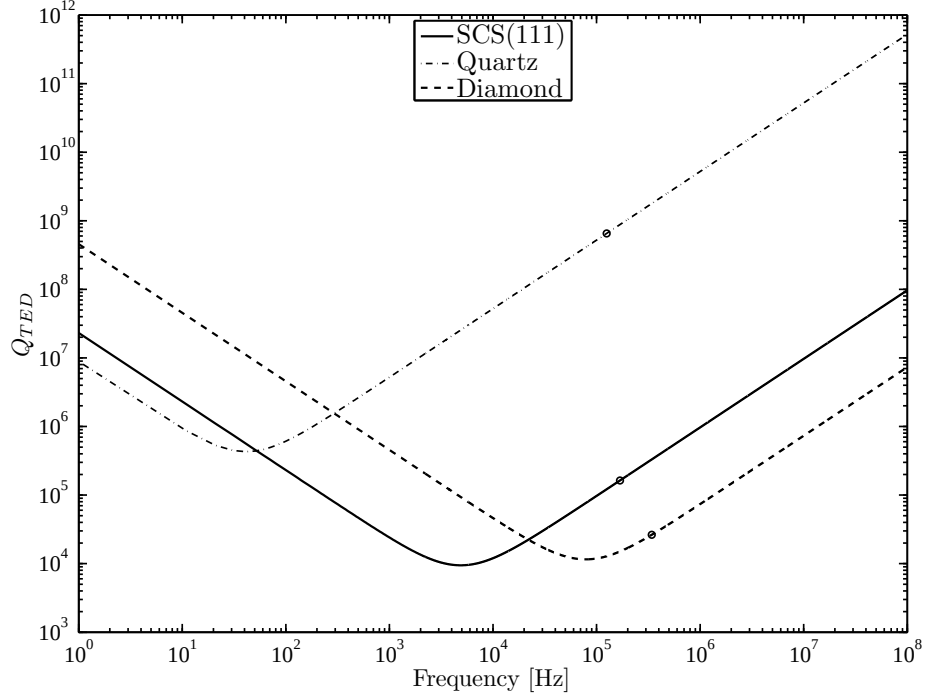


Figure 2.11: Debye peak curve for 2 mm-160 μ m ring with different materials.

cies than small diameter thick ring, and based on Zener's theory the small diameter thick rings operate in the adiabatic region of the Debye peak curve whereas the large diameter thin rings operate in the isothermal regime. A further simplification of Zener's theory in terms of material properties indicates that, for rings operating in the isothermal and adiabatic regions, Q_{TED} increases for material having high and low thermal conductivity (k), respectively.

2.5 Thermoelastic Damping in Supported Rings

2.5.1 Introduction

This section considers thermoelastic damping in supported ring resonators. In MEMS angular rate sensors, the ring is attached to a rigid frame or central anchor/hub by support legs. Different types of support legs are used, but the most common consist of slender beams. In previous sections the effects of support legs were neglected. In what follows it will be shown that thermoelastic damping in legs can significantly

influence Q_{TED} . To predict and analyse damping in supported ring resonators, the FE model described in Section 2.3 is used. The aim of the section is to investigate the contribution of leg damping on the damping of the whole resonator structure.

The impact of adding support legs and modifying leg width on the frequency and Q-factor is investigated with the aid of the fully coupled thermoelastic model. A semi-analytical model is developed based on Zener's theory and the stored energies obtained using a finite element model to understand the influence of support legs on damping behaviour. By considering the ring and legs separately, Zener's expression is used to quantify Q_{TED} for each of them separately, while the stored energy in the rings and legs are evaluated from an undamped eigen-frequency analysis. The damping behaviour of supported rings is illustrated by modifying the coupled equation of thermoelasticity to understand the significance of leg damping on the overall damping of the resonator. Furthermore, the semi-analytical model is used to analyse the effect of different materials on TED in supported rings.

2.5.2 Supported Ring Resonator Design

The design of support legs is important to achieve desired sensor performance. The legs are designed in such a way that the 2θ in-plane mode shapes of the supported ring are not significantly different to that of an unsupported or free ring. In practice, each legs supporting the ring structure will apply a local perturbation to the mass and stiffness of the ring at the point of attachment [120]. Therefore, the number and location of the support legs must be such that the dynamic symmetry between the two 2θ in-plane modes are maintained. In order to maintain symmetry relative to the 2θ modes to ensure the natural frequencies do not split, legs should be placed symmetrically around the ring. This is preferably achieved using eight legs [121]. The natural frequencies of the support legs should not be close to the natural frequencies of the ring to prevent coupling between ring and supports. Three different support leg configurations are considered here, as shown in Figure 2.12. Figure 2.12(a) shows support legs consisting of 8 pairs of 'z' shaped support legs connected to a circular

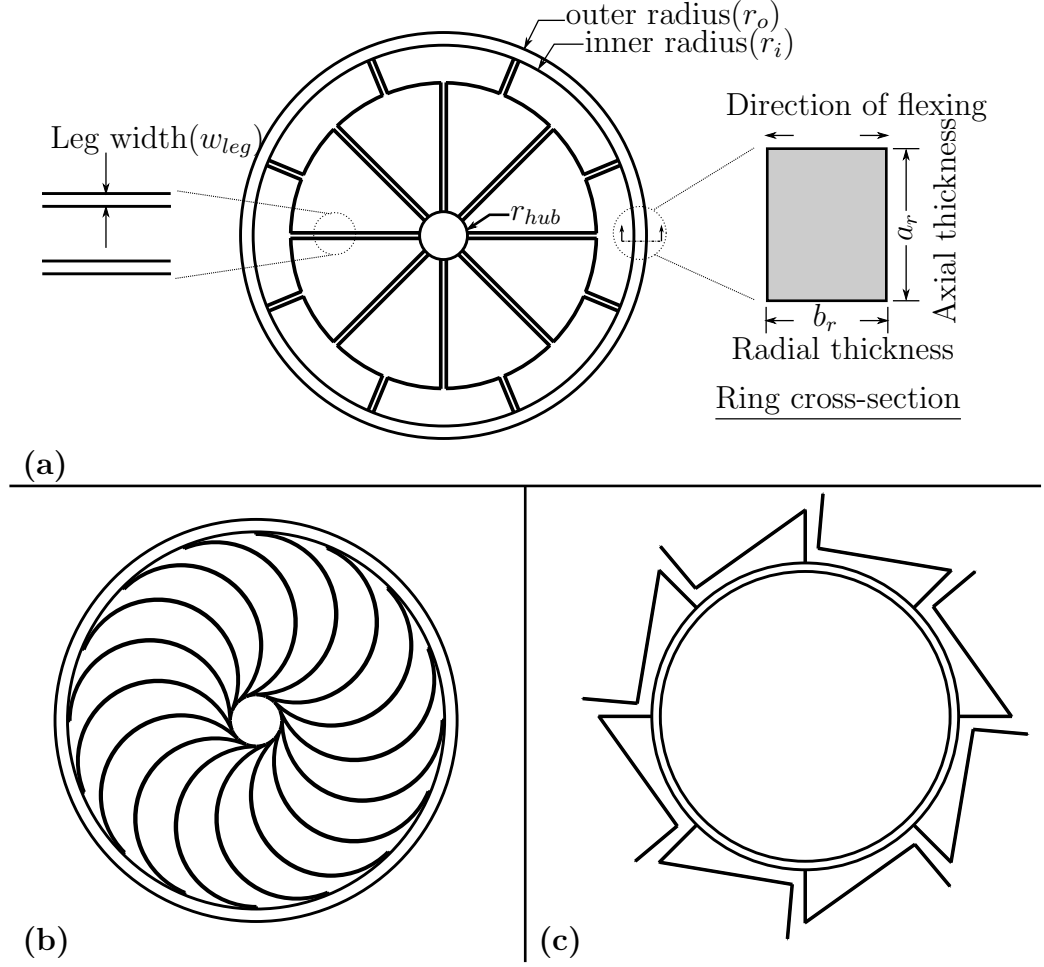


Figure 2.12: The ring resonator with different support leg configurations: (a) pair of 'z' shaped legs, (b) semi-circular legs, and (c) outer support legs.

central hub, this configuration is considered in most studies later. Figure 2.12(b) shows the support leg configuration considered by Ayazi and Najafi [10] and consists of 16 semi-circular support legs connected to a central hub. Figure 2.12(c) shows a configuration with 8 support legs connected to the outside of the ring, with the “free ends” attached to a substrate.

2.5.3 Effect of Support Legs on Frequency and TED

Figure 2.6 shows how the natural frequency (damped) and Q_{TED} vary as a function of radial thickness for $d = 4\text{ mm}$ unsupported ring. To investigate the effect of the support legs on frequency and Q_{TED} , 4 mm diameter rings with radial thicknesses in the range $20 - 25\text{ }\mu\text{m}$ are calculated using the FE method with supports of leg width

Table 2.4: Natural frequency (damped) and Q-factor of 4 mm diameter rings with and without support legs and hub

Radial Thickness, b_r (μm)	Unsupported Ring		Supported Ring	
	Frequency (kHz)	Q_{TED}	Frequency (kHz)	Q_{TED}
20	5.264	2.850×10^5	6.111(13.86%)	$4.823 \times 10^5(40.90\%)$
21	5.528	2.462×10^5	6.247(11.50%)	$4.002 \times 10^5(38.46\%)$
22	5.791	2.142×10^5	6.394(9.43%)	$3.359 \times 10^5(36.22\%)$
23	6.054	1.875×10^5	6.551(7.58%)	$2.848 \times 10^5(34.16\%)$
24	6.317	1.650×10^5	6.716(5.94%)	$2.437 \times 10^5(32.26\%)$
25	6.580	1.460×10^5	6.888(4.47%)	$2.102 \times 10^5(30.56\%)$

Table 2.5: Frequency and Q-factor of a 8 mm diameter rings with and without support legs and hub

Radial Thickness b_r (μm)	Unsupported Ring		Supported Ring	
	Frequency (kHz)	Q_{TED}	Frequency (kHz)	Q_{TED}
20	1.316	1.140×10^6	2.331(43.54%)	$3.432 \times 10^6(66.77\%)$
21	1.382	9.852×10^5	2.333(40.76%)	$2.791 \times 10^6(64.70\%)$
22	1.447	8.652×10^5	2.337(38.08%)	$2.293 \times 10^6(62.26\%)$
23	1.513	7.572×10^5	2.344(35.45%)	$1.900 \times 10^6(60.15\%)$
24	1.579	6.664×10^5	2.354(32.92%)	$1.589 \times 10^6(58.06\%)$
25	1.645	5.896×10^5	2.367(30.50%)	$1.340 \times 10^6(56.0\%)$

$w_{leg}=6\mu\text{m}$ connected to the resonator. As was seen in Figure 2.5, this range of b_r values ensure high Q_{TED} values for unsupported rings. In this study, the leg configuration shown in Figure 2.12(a) is used to support the ring. Table 2.4 summarises the numerically simulated results for the frequency and Q_{TED} and compares results for supported and unsupported rings. The values in brackets show the percentage change in frequency and Q_{TED} compared to the unsupported rings. The results suggest both the frequency and Q_{TED} increase when the support is added to the ring structure. Although the support legs provide additional modal mass and stiffness to the ring, the frequency increase suggests that the dominant effect is to increase the modal stiffness. Assuming that the support legs are not a source of damping, it can be deduced that the frequency increase will also cause the Q-factor to increase. This argument is only valid if the thermoelastic mode shape is unchanged when support legs are attached.

Similar characteristics are observed when support legs are added to 8 mm diameter ring, see Table 2.5. Although, the large diameter thin rings give higher Q_{TED} values, the frequencies are quite low in comparison.

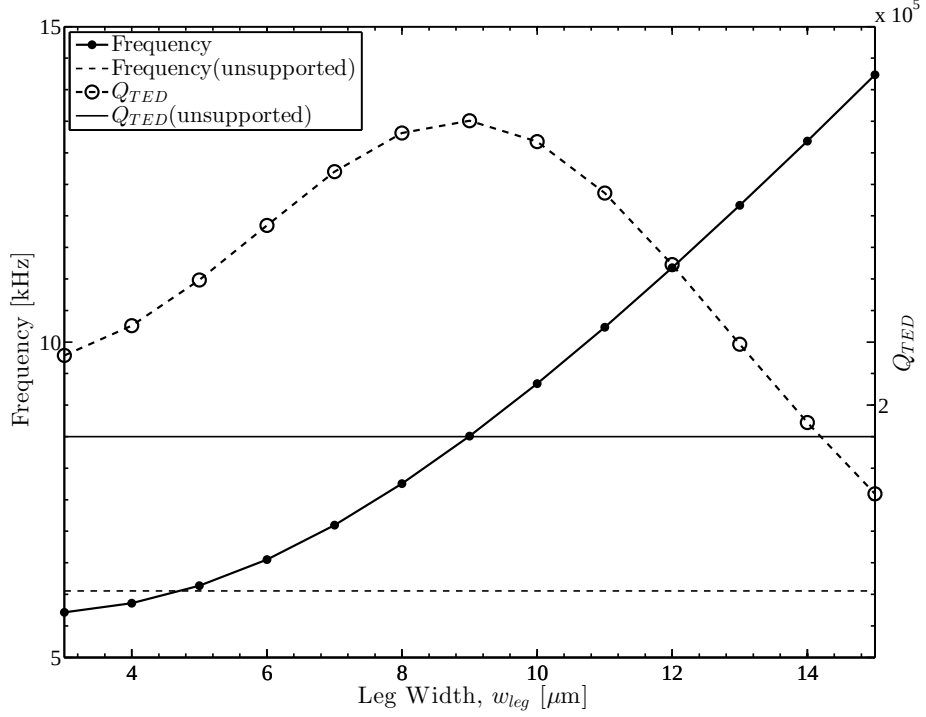


Figure 2.13: Effect of leg width on frequency and Q_{TED} for 4 mm-23 μm rings.

2.5.4 Effect of Leg Width on Frequency and TED

Adding support legs was shown in the previous section to increase the frequency and Q_{TED} of the ring resonator, at least for the example considered. In this section, the influence of leg width on Q_{TED} is investigated. This is achieved by considering two rings, with different diameters and thicknesses, and by varying the leg width w_{leg} . In addition the different leg configurations introduced earlier in Figure 2.12 are considered and for each of these cases the leg width is assumed constant and takes the same value in all sections of the leg. Figure 2.13 shows the frequency and Q_{TED} values for a ring of 4 mm diameter and 23 μm radial thickness with z-shaped support legs (Figure 2.12(a)) having leg widths in the range 3 – 15 μm . The frequency increases non-linearly for leg widths 3 μm to 9 μm , and for larger values it increases approximately linearly. For thin legs, e.g. 3 μm , the frequency (damped) of the supported ring is lower than the unsupported ring. This is due to the dominating effect of modal mass. But, the higher Q-factor compared to the unsupported ring may be attributed due to the increase in stored energy for the supported ring. As

expected the natural frequency (damped) increases as the leg width increases due to increase in leg stiffness. However, the Q_{TED} value shows a different trend. The Q_{TED} increases gradually up to a peak value before decreasing with further increases in w_{leg} . Although the frequency increases, Q_{TED} decreases significantly after reaching its peak value. This suggests that the support legs can have a significant influence on the overall damping, which is in contrast to the example considered in Section 2.3.4 where Q_{TED} in the ring was observed to be dominant, based on a comparison with experiment. It is interesting to note that Q_{TED} reaches its peak at the w_{leg} -value that coincides with the transition in frequency from increasing non-linearly to being in the linear range, see Figure 2.13. It is also worth noting that Q_{TED} of the supported ring can decrease compared to unsupported ring due to increased damping in the thicker legs.

Similar characteristics are observed for a large diameter thick ring with supports. Figure 2.14 shows the frequency and Q_{TED} values for a ring of 8 mm diameter and 120 μm width for z-shaped legs having leg widths in the range 20 μm to 60 μm . Similar to Figure 2.13, Q_{TED} increases to a peak value before decreasing with further increases in w_{leg} . These results indicate that there are limits to the maximum value of Q_{TED} that can be achieved when support legs are included.

To further investigate this behaviour, semi-circular (see Figure 2.12(b)) support legs and a central hub are implemented for a ring of 4 mm diameter and 23 μm radial thickness. In Figure 2.15, frequency and Q_{TED} are plotted as a function of w_{leg} . The results show similar characteristics to those in Figures 2.13 and 2.14. The Q_{TED} reaches a peak value before decreasing with w_{leg} , and the frequency also increases approximately linearly. This trend is also apparent for 8 mm-120 μm ring with semi-circular supports, see Figure 2.16. Also, when supports outside of the rings are considered, (Figure 2.12(c)), similar behaviour for Q_{TED} and frequency is observed, see Figure 2.17. It can be inferred from the analyses that regardless of the support leg design, minimum damping occurs at an optimal leg width.

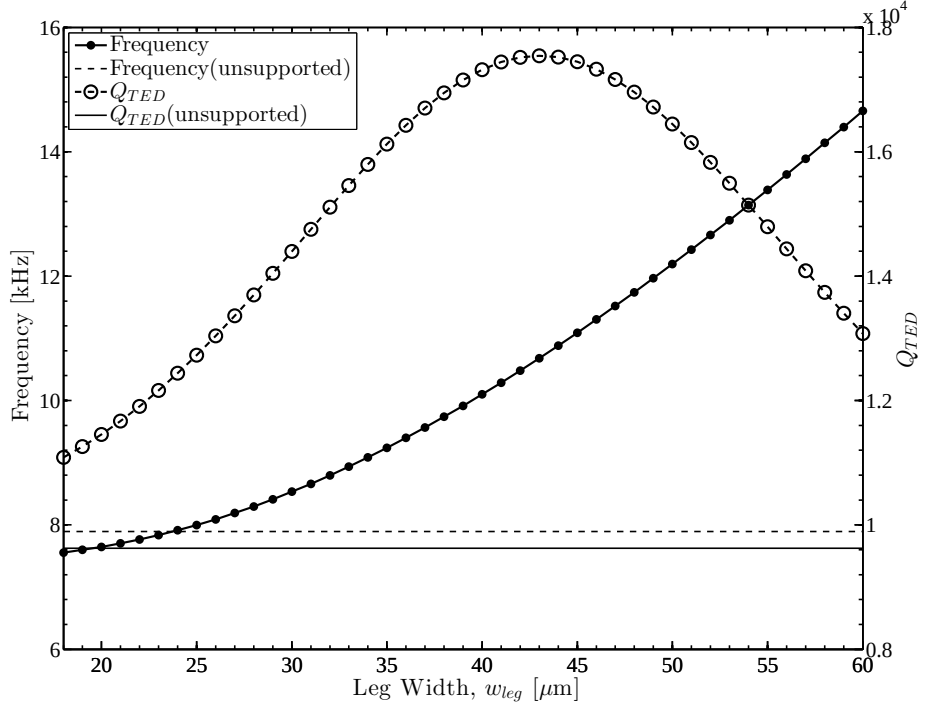


Figure 2.14: Effect of leg width on frequency and Q_{TED} for 8 mm-120 μm rings.

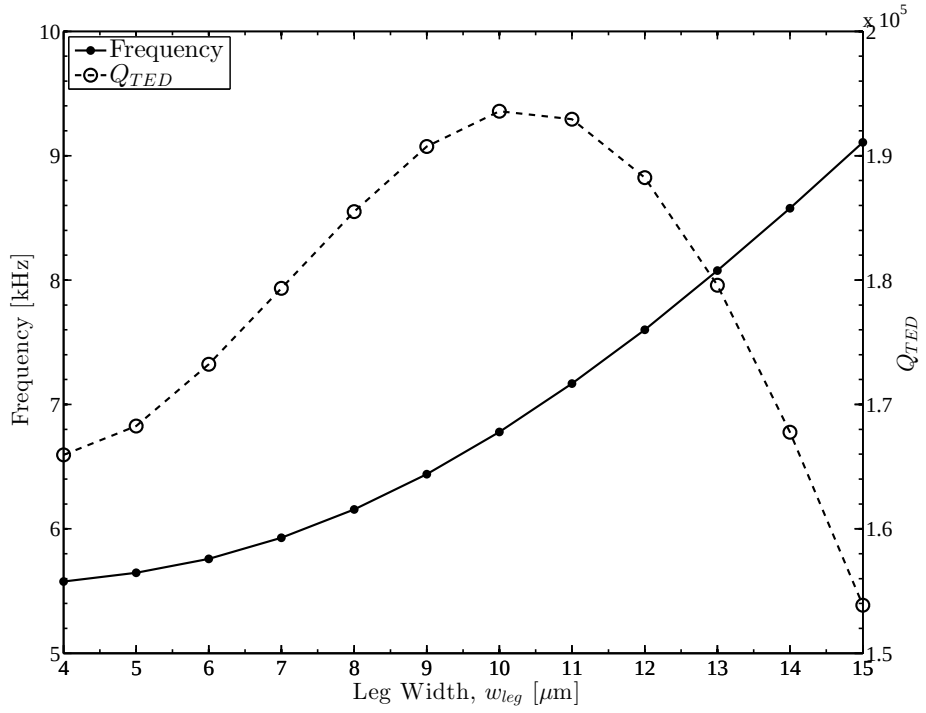


Figure 2.15: Effect of leg width (semi-circular support) on frequency and Q_{TED} for 4 mm-23 μm rings.

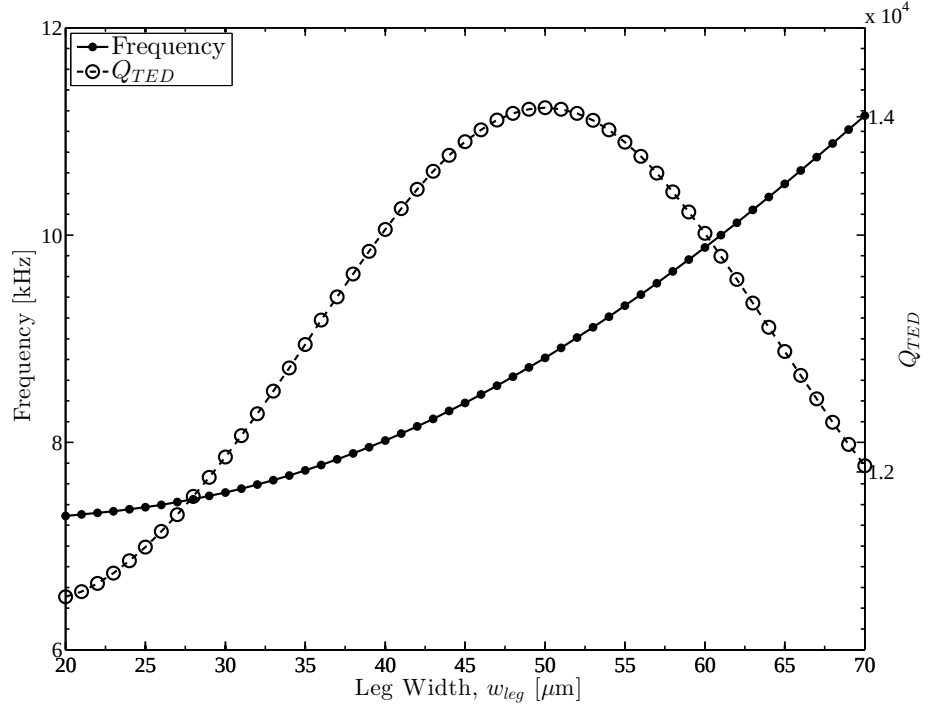


Figure 2.16: Effect of leg width (semi-circular support) on frequency and Q_{TED} for 8 mm-120 μm rings.

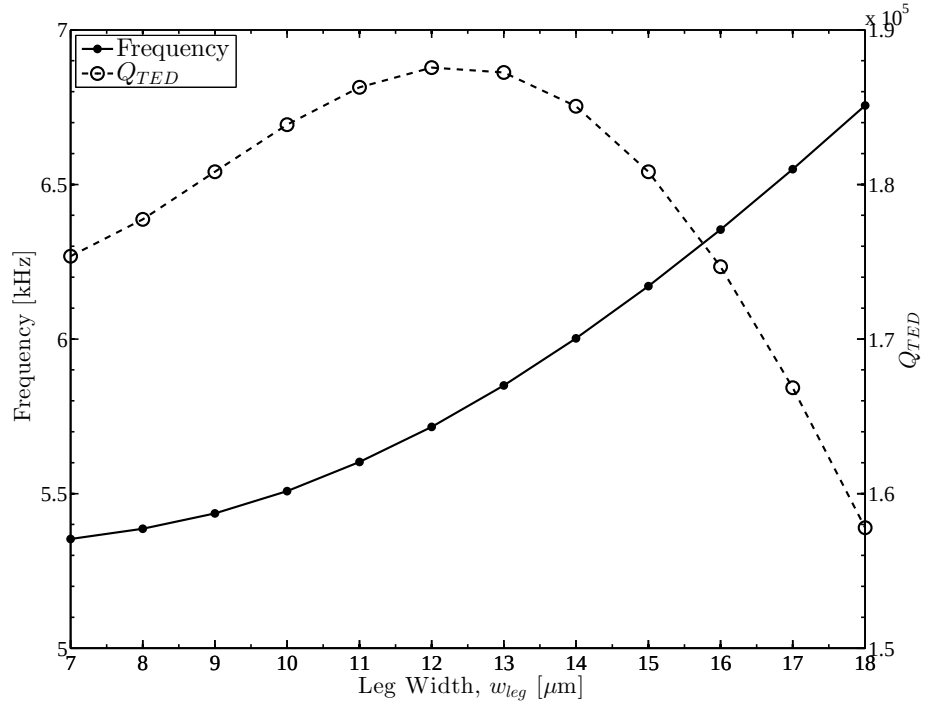


Figure 2.17: Effect of leg width (outer supports) on frequency and Q_{TED} for 4 mm-23 μm rings.

2.5.5 Semi-Analytical Modelling for Supported Rings

Results presented in the previous section indicate that damping in the legs can have significant effect. In this section a semi-analytical model for predicting Q_{TED} in a supported ring resonator is presented. It is based on analysing the strain energy stored in different parts of the system and uses Zener's equation (Eq. (2.1)) to calculate damping for the supported ring. The model is referred to as "semi-analytical" because the stored energy is calculated using results from finite element modelling. The purpose of the model is to explain the observed damping behaviour in supported rings.

As mentioned in Section 1.3, the Q-factor is defined as the ratio of the energy stored and energy dissipated per cycle within a resonator and Eq. (1.4) can be expressed as

$$\frac{1}{Q_{total}} = \frac{1}{2\pi} \frac{\text{energy dissipated per cycle}}{\text{energy stored in the system}} \quad (2.26)$$

Here, Q_{total} represents the Q-factor associated with TED in the complete resonator including the ring and supports. For a system composed of N parts Eq. (2.26) can be expressed as:

$$\frac{1}{Q_{total}} = \frac{1}{2\pi} \frac{\sum_{i=1}^N \Delta_i}{\sum_{i=1}^N V_i} \quad (2.27)$$

where, Δ_i and V_i are the energy dissipated from and, energy stored in the i 'th part of the system, respectively.

It can be deduced from the definition of Q-factor that the Q-factor for the i 'th part of the system can be expressed as

$$\frac{1}{Q_i} = \frac{1}{2\pi} \frac{\Delta_i}{V_i} \quad (2.28)$$

where Q_i is the Q_{TED} for the i 'th part of the system. Combining Eqs. (2.27) and (2.28) gives the following result

$$\frac{1}{Q_{total}} = \frac{\sum_{i=1}^N \frac{V_i}{Q_i}}{\sum_{i=1}^N V_i} \quad (2.29)$$

This equation express Q-factor for the complete resonator in terms of the Q-factors for different parts and the energy stored.

The energies stored in different parts of the system V_i can be calculated from FE model of the undamped system. The Q-factors can be calculated using Zener's theory, where the thermal time constants for different parts of the system are evaluated using Eq. (2.2) or Eq. (2.4) and the undamped natural frequency of the whole system required for Q-factor calculation is obtained from FE modelling. It is worth mentioning that Eq. (2.1), used for calculating Q-factor for different parts, uses the undamped natural frequency for the whole system and not individual parts.

If Δ_{ring} , V_{ring} , Δ_{legs} and V_{legs} are the energy dissipated from and stored energy in the ring and support legs of a supported ring, respectively, it can be deduced from Eq. (2.28) that

$$\Delta_{ring} = 2\pi \frac{V_{ring}}{Q_{ring}} \quad (2.30)$$

$$\Delta_{legs} = 2\pi \frac{V_{legs}}{Q_{legs}} \quad (2.31)$$

The values of Q_{ring} and Q_{legs} can be calculated from Eq. (2.1). The thermal time constants are determined from ring radial thickness (b_r) and leg width (w_{leg}) in Eq. (2.4), respectively. Combining Eqs. (2.29), (2.30) and (2.31) gives

$$\frac{1}{Q_{total}} = \frac{1}{Q_{ring}} \frac{V_{ring}}{V_{ring} + V_{legs}} + \frac{1}{Q_{legs}} \frac{V_{legs}}{V_{ring} + V_{legs}} \quad (2.32)$$

If there is no dissipation in the legs i.e. $Q_{legs} \rightarrow \infty$, Eq. (2.32) becomes

$$Q_{total} = Q_{ring} \left(1 + \frac{V_{legs}}{V_{ring}}\right) \quad (2.33)$$

Eq. (2.32) can be used to interpret how Q_{TED} for the supporting ring changes, considering individual effects of ring and support legs. Since adding support legs influence TED, the total damping of the resonator can be approximated using Eq. (2.32) to understand the damping behaviour due to changes in support leg width.

Table 2.6: Stored energy calculated from FE for ring and legs

w_{leg} (μm)	Frequency (kHz) (undamped)	Stored Energy/length (J/m) $\times 10^5$		
		Total ($V_{ring} + V_{legs}$)	Ring only (V_{ring})	Legs Only (V_{legs})
3	5.713	7.26	6.56 (90.37%)	0.69 (9.63%)
4	5.858	6.90	5.75 (83.30%)	1.15 (16.70%)
5	6.134	6.34	4.63 (72.97%)	1.71 (27.02%)
6	6.550	6.02	3.70 (61.42%)	2.32 (38.57%)
7	7.095	5.95	3.00 (50.30%)	2.96 (49.70%)
8	7.753	6.02	2.44 (40.58%)	3.58 (59.42%)
9	8.507	6.24	2.04 (32.60%)	4.21 (67.40%)
10	9.338	6.57	1.73 (26.29%)	4.84 (73.70%)
11	10.232	6.95	1.49 (21.42%)	5.46 (78.58%)
12	11.178	7.33	1.30 (17.70%)	6.03 (82.30%)
13	12.165	7.88	1.17 (14.87%)	6.71 (85.12%)
14	13.187	8.29	1.06 (12.75%)	7.23 (87.25%)
15	14.236	8.73	0.97 (11.15%)	7.75 (88.84%)

2.5.5.1 Model Validation

The model developed in the previous section is validated against results obtained using the numerical approach described in Section 2.3.2. For the purpose of validation, 4 mm-23 μm rings with different leg widths are considered. The initial step is to calculate the stored energy within the ring and support legs for the resonator. The results are presented in Table 2.6 and were evaluated from an undamped eigenfrequency analysis of the structure. Since the ring and leg have the same thickness, the stored energy is calculated from the stored energy per unit length (J/m) along the axial direction. There is no stored energy in the hub as it is assumed not to undergo any deformation. It is evident from Table 2.6 that the total stored energy of the supported ring resonator achieves a minimum for $w_{leg}=7\mu\text{m}$. In contrast, the stored energy in the ring decreases as the leg width increases, whilst the stored energy in the legs increase. Both the ring and legs have similar amount of stored energy at the minimum point. The percentage of total stored energy in ring and legs are shown in brackets.

The natural frequency (undamped) of the supported ring, obtained using the FE model, is used in Eq. (2.1) to calculate Q_{ring} and Q_{leg} . Table 2.7 shows calculated

Table 2.7: Comparison between results obtained from the semi-analytical model and the simulation

$w_{leg}(\mu\text{m})$	Frequency (kHz) (Undamped)	Q_{ring}	Q_{legs}	Q_{total} Eq. (2.32)	Q_{TED} (simulation)	% Difference
3	5.713	196100	11520000	216610	219644	1.38
4	5.858	191250	6319400	228202	231457	1.40
5	6.134	182660	3862500	246001	249590	1.43
6	6.550	171070	2511900	267077	271222	1.52
7	7.095	157950	1703700	287659	292468	1.64
8	7.753	144570	1193700	302589	307843	1.70
9	8.507	131790	859600	306968	312730	1.84
10	9.338	120090	634330	298371	304417	1.98
11	10.232	109630	478460	278067	284070	2.11
12	11.178	100390	368040	250054	255726	2.21
13	12.165	92284	288180	219017	224161	2.29
14	13.187	85171	229260	188590	193129	2.35
15	14.236	78935	185040	160913	164841	2.38

values for Q_{total} using Eq. (2.32) and those obtained in Section 2.5.4. The results are in good agreement with $<2.5\%$ error. Q_{TED} values obtained using the semi-analytical model and fully-coupled FE model are plotted in Figure 2.18. Since Q_{ring} and Q_{leg} are calculated using Zener's equation, the differences observed in Table 2.7 is likely to be due to the approximations in Zener's theory. Q-factors are calculated based on the assumption that only one of the thermal modes contribute to the damping. In contrast, the fully coupled approach considers contribution of all thermal modes. If the dominant thermal mode has strong coupling with one of the thermal modes which have a lower level of damping, Q_{TED} for the resonator is likely to have higher value than the Q-factor obtained using Zener's equation. At the interface between the leg and ring, it is expected that there will be transport of heat from ring to legs and vice versa. Since in the semi-analytical model the effect of the interface is not taken into account, this may also cause some difference between the results obtained using the model and fully coupled simulation. The above model can be used to approximately predict the damping behaviour of a supported ring resonator and explain damping behaviour for different leg widths, if strain energies can be calculated.

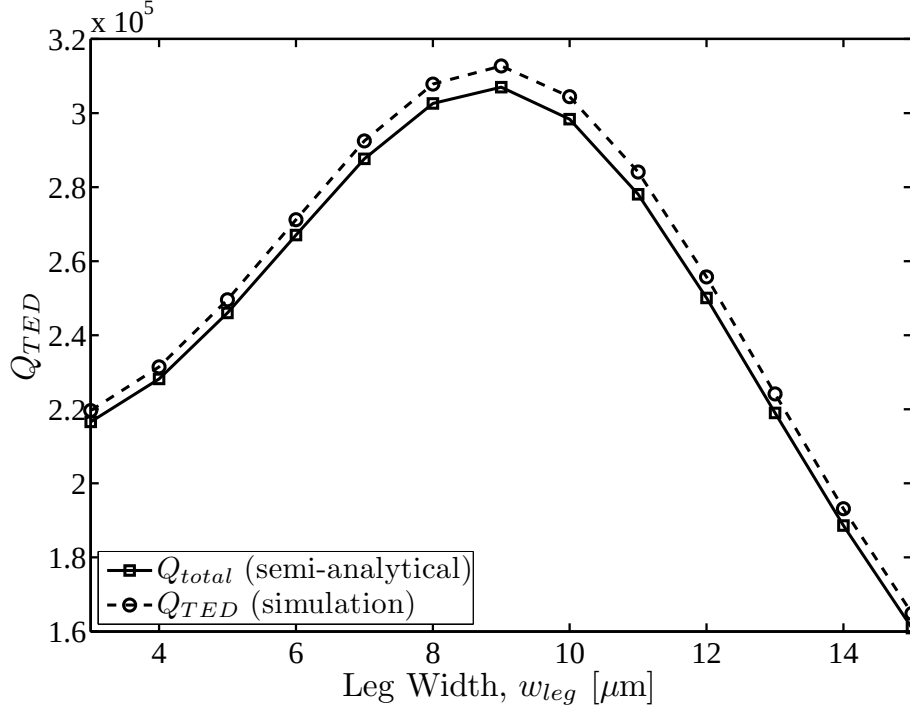


Figure 2.18: Validation of the semi-analytical model using 4 mm-23 μm ring

2.5.6 Understanding TED Behaviour in Supported Rings

In Section 2.5.4 it was shown that as the support leg width increases, Q_{TED} decreases after reaching a maximum even though the frequency increases monotonically. This behaviour suggests that as w_{leg} increases, the damping in the legs become significant.

This is investigated by re-considering the 4 mm-23 μm ring with the support legs constrained so they have no TED - this is achieved by modifying the coupling equation for the support legs, which negates the thermal expansion and strain induced heat only in the support legs. The results obtained for Q_{TED} when TED is limited to the ring only are compared with previous results including the legs as damping sources in Figure 2.19. For the case when TED is limited to the ring, Q_{TED} increases monotonically as w_{leg} increases. As in Section 2.5.4, this behaviour can be explained by noting that Q_{TED} increases as frequency increases provided that the thermoelastic mode shape is unchanged. When w_{leg} is small, the Q_{TED} values with and without leg damping are similar to each other, suggesting that leg damping is insignificant for small w_{leg} values. However, increasing w_{leg} introduces a significant amount of

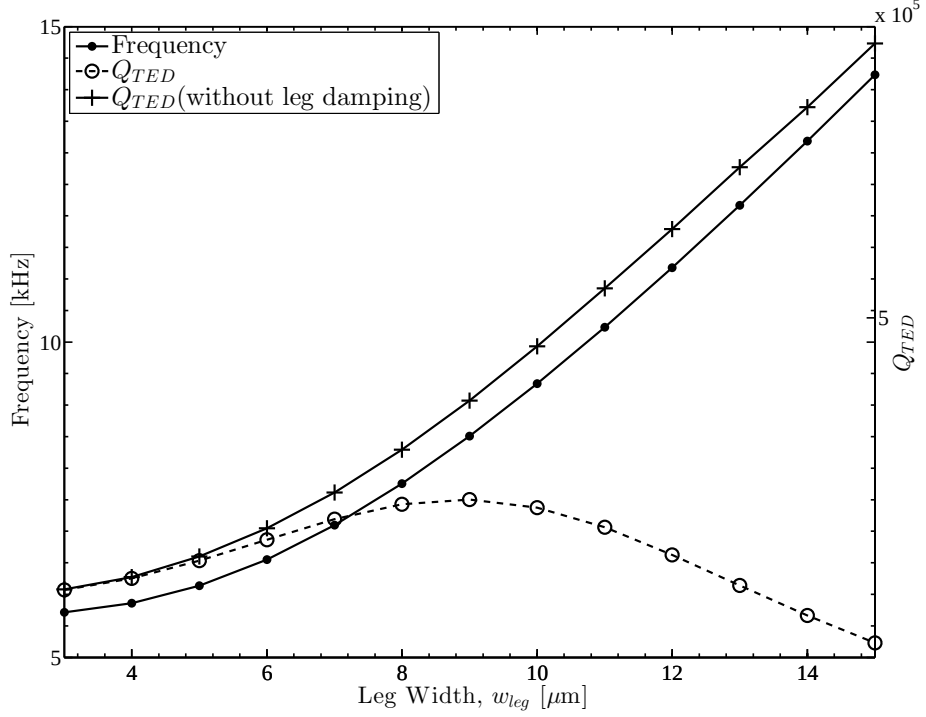


Figure 2.19: Influence of thermoelastic damping in support legs on Q_{TED} of a 4 mm-23 μm ring.

TED in the support legs which acts to reduce Q_{TED} . For example, for a leg width of 14 μm , Q_{TED} is reduced by $\approx 72\%$ due to the presence of leg damping.

2.5.7 Damping in Different Leg Sections

To better understand the damping behaviour, it is helpful to study how each part of the support leg contributes to the damping for supported rings with high damping. In this section only the ‘z’-shaped legs in Figure 2.12(a) are considered for brevity, and the damping in each section is achieved by dividing each ‘z’-shaped support leg into three sections, as shown in Figure 2.20, and by evaluating the TED in each section. The results presented in this section are obtained from FE simulation.

Figure 2.21 shows results for the (high damping) 4 mm-23 μm ring case with leg width $w_{leg} = 14 \mu\text{m}$. It is worth mentioning that the results are interpreted in terms of loss factor (η) which is the inverse of quality factor. The first column shows the overall loss factor (η_{TED} or $\frac{1}{Q_{TED}}$) for the complete ring and support structure. The loss factor for the ring only is represented by $\frac{1}{Q_{Ring}}$ and $\frac{1}{Q_{P1}}$, $\frac{1}{Q_{P2}}$ & $\frac{1}{Q_{P3}}$ are the

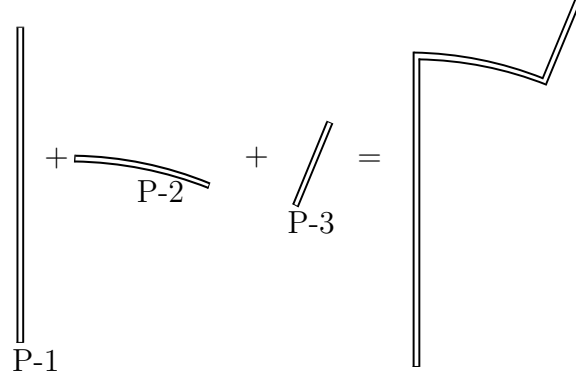


Figure 2.20: Different parts of support leg.

individual loss factors for the P-1, P-2 and P-3 sections, respectively. The overall loss factor ($\frac{1}{Q_{TED}}$) is the sum of all individual loss factors, such that:

$$\eta_{TED} = \frac{1}{Q_{TED}} = \frac{1}{Q_{Ring}} + \frac{1}{Q_{P1}} + \frac{1}{Q_{P2}} + \frac{1}{Q_{P3}} \quad (2.34)$$

As expected the results indicate that leg damping is significant for the case considered. The results also indicate that damping in P-3 is higher than damping for the ring only. The TED in the ring only case accounts for $\approx 27.64\%$ of the overall loss factor. Of the three different leg sections, P-3 has the highest damping and P-2 has the lowest energy dissipation. This is likely to be because this section (P-3) experiences higher levels of deformation, and consequently has a higher strain rate compared to the other sections.

2.5.8 Effect of Material Properties on Frequency and TED

The influence of material properties on the damping behaviour of a supported ring is investigated in this section. Similar to Section 2.5.4, a 4 mm-23 μm ring having leg widths in the range 3–15 μm is considered. The results are plotted in Figure 2.22 for both quartz and diamond. As observed in Figure 2.13 for SCS-(111), the frequency increases linearly. However the diamond ring has higher frequencies due to having a higher specific stiffness ($\frac{E}{\rho}$). Q_{TED} for diamond shows similar trends as those observed for SCS-(111) in Figure 2.13 i.e. reaches a peak as leg width increases

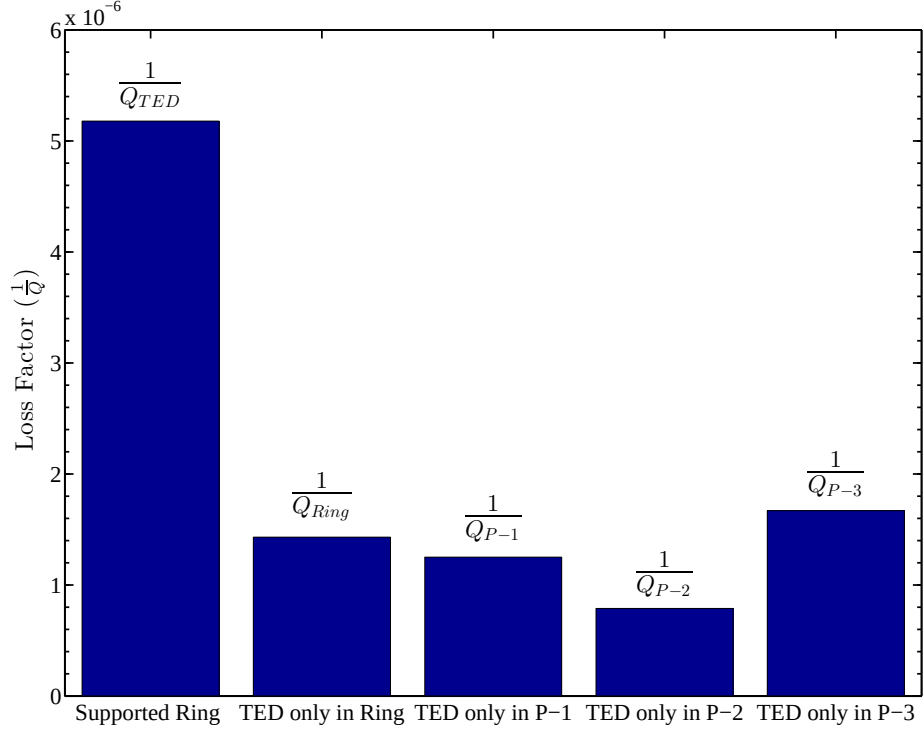


Figure 2.21: Loss factor ($\frac{1}{Q}$) in different leg sections of 4 mm-23 μm ring with $w_{leg} = 14 \mu\text{m}$.

before decreasing with leg width. This is mainly because leg damping dominates for leg widths greater than that for the peak value. Q_{TED} for quartz shows a different trend. In this case, Q_{TED} increases to a peak as leg width increases and then achieves a minimum value before increasing again. A similar characteristics is observed for 8 mm-120 μm rings in Figure 2.23. However, the peak is reached within the leg width range 10 – 14 μm . It is important to mention that, for this case, leg widths smaller than 10 μm induce coupling between the ring and legs for the 2θ in-plane mode (see Section 4.3.5.1 for more details). For the same ring configuration, diamond shows higher Q_{TED} values.

This damping behaviour for supported quartz rings indicates that the local damping in support legs has significant impact. The next section investigates this phenomenon with the aid of the model developed in Section 2.5.5.

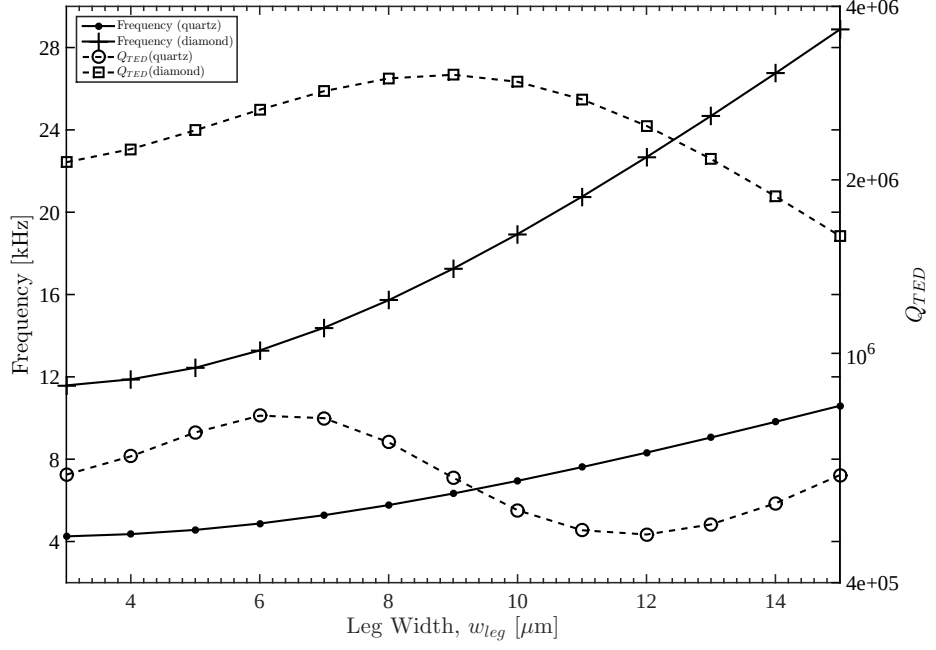


Figure 2.22: Frequency and Q_{TED} as a function of leg width for 4 mm-23 μm rings using quartz and diamond.

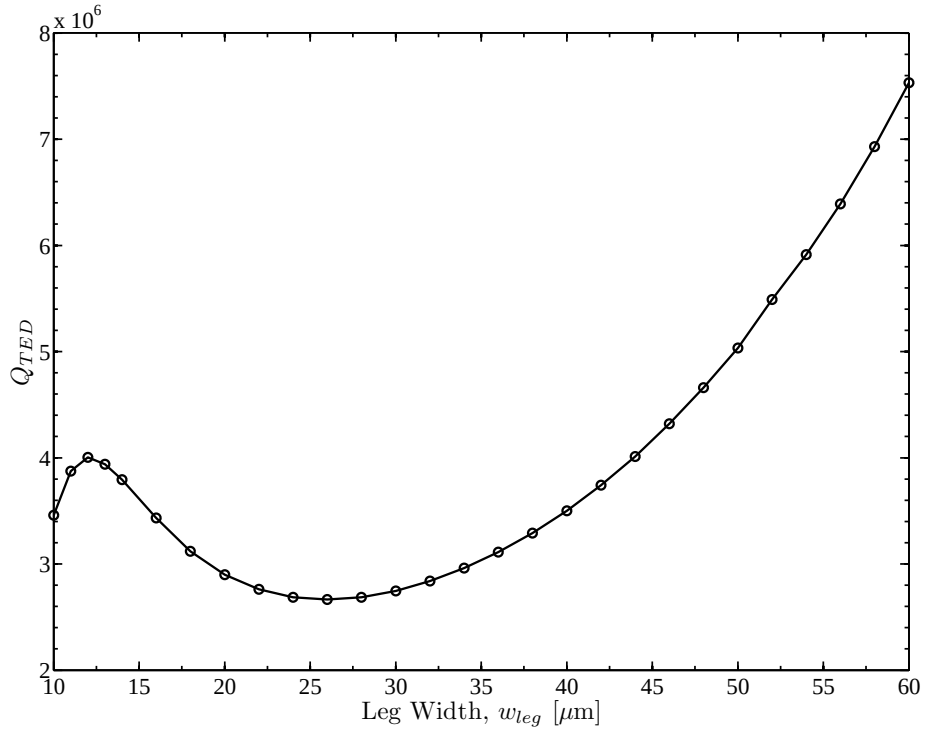


Figure 2.23: Q_{TED} as a function of leg width for 8 mm-120 μm quartz rings.

2.5.8.1 Leg Damping Behaviour for Quartz

The semi-analytical model described in Section 2.5.5 is used to predict the TED behaviour for a 4 mm-23 μm quartz ring considered in the previous section. Referring to Eq. (2.32), the total Q_{TED} in the supported ring resonator depends on local Q -factors for the ring and the legs i.e. Q_{ring} and Q_{leg} , respectively, and the energy distribution in the ring and legs. The distribution of total stored energy in the ring and legs are illustrated in Table 2.8. The energy distribution in the ring and legs is similar to that observed for a 4 mm-23 μm SCS-(111) ring in Table 2.6. Figure 2.24 shows Q_{TED} , Q_{ring} and Q_{legs} for a 4 mm-23 μm quartz ring. Similar to the silicon ring in Table 2.7, Q_{ring} increase monotonically with leg width. However, for quartz, Q_{legs} decrease with leg width until it reaches a minimum indicating the local Debye curve before increasing, whereas Q_{legs} decreases with leg width for SCS-(111) as shown in Table 2.7. The contribution of ring on Q_{total} is dominant until it reaches a peak because Q_{ring} increases whereas the ratio $V_{ring}/(V_{ring}+V_{legs})$ decreases with leg width. But, after the peak leg damping starts to dominate because Q_{ring} decrease whereas $V_{legs}/(V_{ring}+V_{legs})$ increases with leg width thus reducing the Q_{total} . For the quartz ring considered here, Q_{total} follows the similar trend as Q_{legs} after the transition point where Q_{ring} and Q_{legs} is almost equal. The minimum Q_{legs} observed in the local Debye curve for legs is related to the thermal relaxation time and is discussed in the following section. It is worth mentioning that low k material will have higher relaxation rate (inverse of τ) for same leg width compared to higher k material e.g. SCS-(111).

In Figure 2.25, the magnitude of $\omega_0\tau_{ring}$ and $\omega_0\tau_{legs}$ parameters are compared to illustrate the local damping regime (isothermal/adiabatic/Debye peak) for the ring and legs. The dashed red line, indicates the $\omega_0\tau \approx 1$ condition for which maximum damping would occur. It is apparent from Figure 2.25 that $\omega_0\tau_{ring}$ is well above the critical value of 1 and indicates that the ring damping is in the adiabatic regime for the local ring Debye curve. On the other hand, $\omega_0\tau_{legs}$ is well below 1. As the leg width tend to increase, the $\omega_0\tau_{legs}$ is approximately close to 1, hence more damping

Table 2.8: Stored energy for ring and legs in a 4 mm-23 μm quartz ring resonator

w_{leg}	Frequency (kHz) (undamped)	Stored Energy/length (J/m) $\times 10^5$		
		Total ($V_{ring} + V_{legs}$)	Ring only (V_{ring})	Legs only (V_{legs})
3	4.250	6.51	5.89 (90.37%)	0.62 (9.63%)
4	4.358	6.66	5.55 (83.29%)	1.12 (16.71%)
5	4.564	6.55	4.78 (72.96%)	1.77 (27.03%)
6	4.879	6.57	4.03 (61.4%)	2.53 (38.59%)
7	5.279	6.69	3.36 (50.28%)	3.33 (49.72%)
8	5.769	7.04	2.86 (40.56%)	4.19 (59.44%)
9	6.330	7.51	2.44 (32.57%)	5.06 (67.43%)
10	6.948	7.99	2.10 (26.26%)	5.89 (73.74%)
11	7.614	8.63	1.85 (21.39%)	6.78 (78.61%)
12	8.318	9.19	1.62 (17.66%)	7.56 (82.33%)
13	9.053	9.92	1.47 (14.84%)	8.45 (85.15%)
14	9.813	10.5	1.33 (12.71%)	9.16 (87.29%)
15	10.59	11.1	1.24 (11.12%)	9.91 (88.88%)

occurs in legs. This is the point where Q_{leg} reaches minimum value. With further increase in leg width, $\omega_0\tau_{legs}$ gradually becomes higher than 1 i.e. shifting to the adiabatic regime. For the quartz ring, the transition of damping from the isothermal regime to the adiabatic regime in the local Debye peak occurs due to the low k value, as a consequence Q_{total} exhibits the trend shown in Figure 2.24.

2.5.8.2 Effect of Thermal Conductivity values on TED

In the previous section it was found that the low k value of quartz is the primary reason for the minimum value of Q observed in the supported ring resonator. To show the influence of thermal conductivity a range of values are considered for quartz and the results are presented in Figure 2.26. It is important to note that apart from the k value, other thermo-mechanical properties remain the same. The stored energy in the ring (V_{ring}) and legs (V_{legs}) will be the same as before as the mechanical properties considered are unchanged. It is evident from Figure 2.26 that as k increases, the overall damping follows a similar trend as SCS and diamond (see Figure 2.13 and 2.22). For higher k values, the local Debye peak for the rings and legs does not occur within the range, as no minimum Q value is observed.

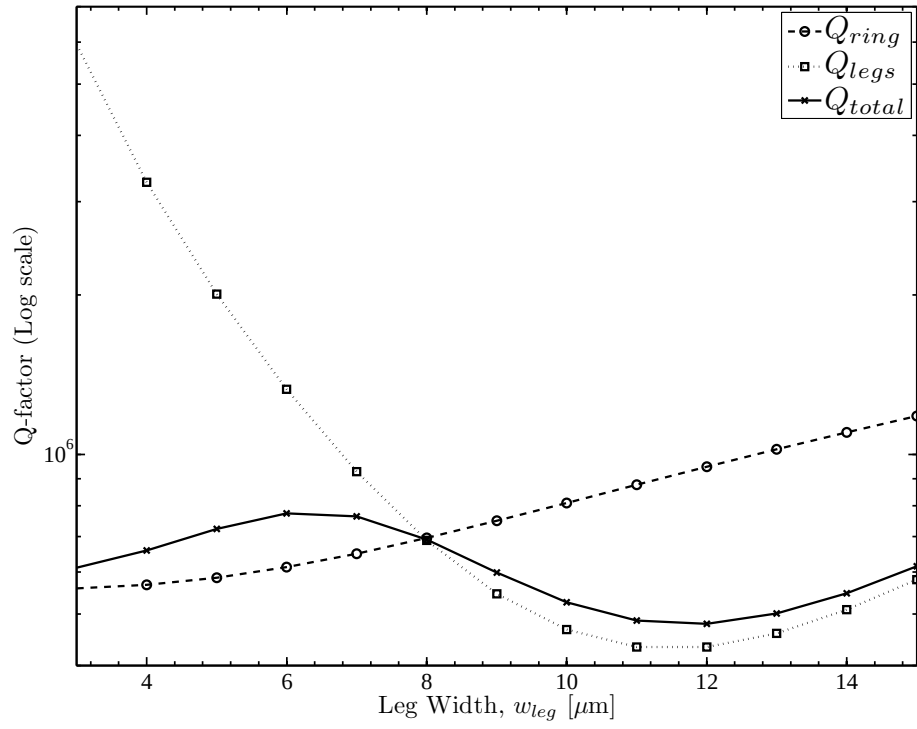


Figure 2.24: Q_{total} , Q_{ring} and Q_{legs} as a function of leg width for 4 mm-23 μm quartz rings.

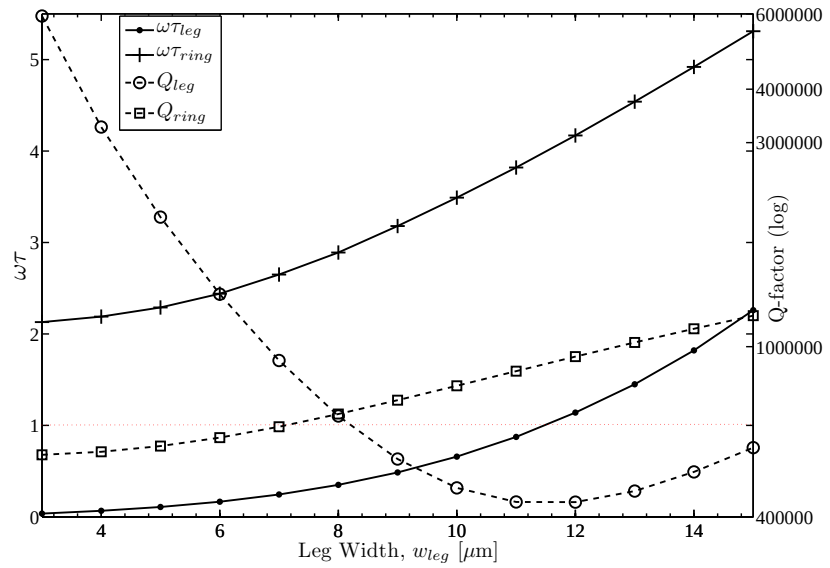


Figure 2.25: Comparison of local $\omega\tau$ parameter as a function of leg width for 4 mm-23 μm quartz rings.

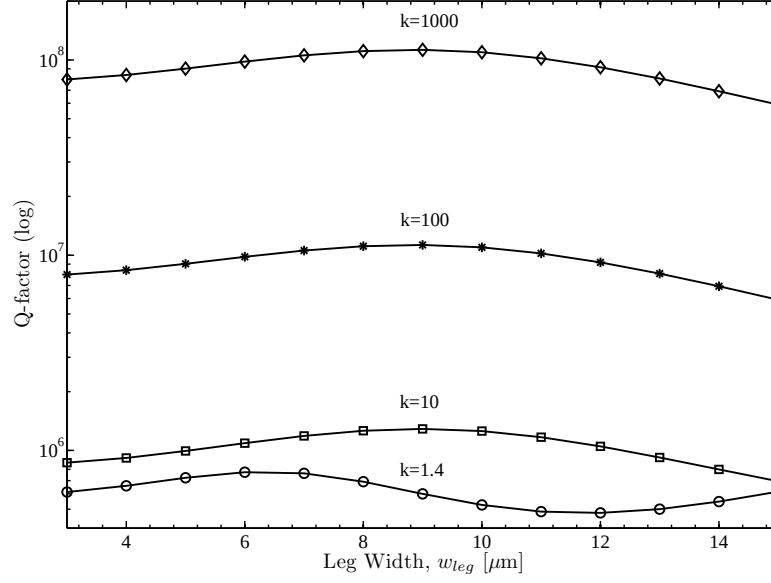


Figure 2.26: Effect of thermal conductivity value (k) on the overall damping of a 4 mm-23 μm supported ring.

2.5.9 Summary

Adding support legs to the ring can increase or decrease the frequency compared to unsupported rings, depending on the contribution of legs on the modal mass and stiffness of the whole structure. The frequency of a supported ring resonator always increases as the leg width increases. However, a strong relationship between Q_{TED} and the leg width is identified. The Q_{TED} of the supported ring reaches a peak before decreasing with leg width. Therefore, a maximum Q_{TED} value can be achieved for an ‘optimum’ leg width and the damping in the support legs dominates the overall damping after the maximum Q_{TED} . It is also shown that analysing the damping in different sections of the support legs can be useful to distinguish their contribution on the overall Q_{TED} of the resonator structure. A semi-analytical model is introduced to explain the damping behaviour for supported ring resonators. The model uses Zener’s approach to calculate the Q-factor for the energy dissipation within the complete resonator in terms of the ring and legs, and the strain energies for the ring and legs, evaluated from a FE model. For low thermal conductivity materials, e.g. quartz, the local Debye curve affects the overall damping behaviour of the resonator as the

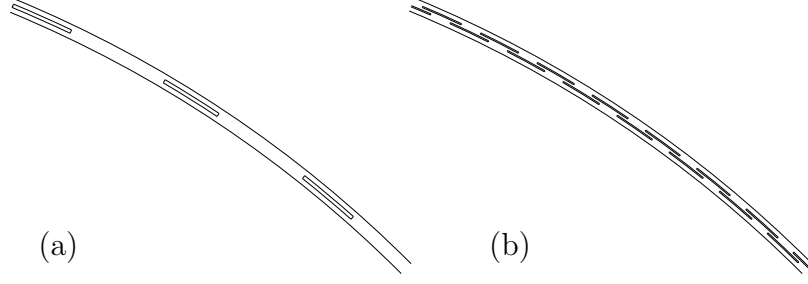


Figure 2.27: (a) Single and (b) double layer slots equally positioned from the neutral plane.

leg width changes. For high thermal conductivity materials, e.g. diamond, similar damping behaviour is observed to silicon for supported rings.

2.6 Thermoelastic Damping in Slotted Rings

2.6.1 Introduction

Candler *et al.* [75, 76] and Guo *et al.* [78] have shown that Q_{TED} of a beam resonator is influenced significantly by incorporating micro-machined slots in the beam. This section considers the influence of slots on various rings and provides an explanation for the damping behaviour of slotted rings.

2.6.2 Effect of Different Slot Configurations

To investigate the effect of slots, two different configurations of slots are considered, as shown in Figure 2.27. Figure 2.27(a) shows a ring with single layer of slots arranged around the ring circumference on the neutral plane. Each slot has a width w_{slot} of $6\text{ }\mu\text{m}$ while the slot angular extent θ_{slot} is 3° and the angular gap between neighbouring slots $\theta_{slot-gap}$ is 5° . Figure 2.27(b) shows a ring with a double layer of slots. In this case the slots are positioned at equal distance ($2\text{ }\mu\text{m}$) from the neutral plane and each slot layer has $w_{slot} = 2\text{ }\mu\text{m}$, $\theta_{slot} = 2^\circ$ and $\theta_{slot-gap} = 1^\circ$. The meshing of slotted ring is important to achieve accurate results, and is discussed in Appendix B. From the previous results obtained, it is known that a ring with 8 mm diameter and $120\text{ }\mu\text{m}$

radial thickness has relatively low Q_{TED} value due to operating near the Debye peak region. The two slot configurations are used on this ring, and results obtained using numerical simulations for unslotted and slotted cases are shown in Table 2.9. It is clear that the presence of micro-machined slots increases Q_{TED} in this case. Slots

Table 2.9: Frequency and Q_{TED} of 8 mm-120 μm ring with single and double layer slots

Unslotted Ring		Slots Included			
		Single Layer		Double Layer	
Frequency (kHz)	Q_{TED}	Frequency (kHz)	Q_{TED}	Frequency (kHz)	Q_{TED}
7.895	9624	7.959	13347	7.455	61162

work by disrupting the heat flow due to temperature gradients formed in different parts of the ring, causing Q_{TED} to increase, more significantly for the double layer slots. Of course, Q_{TED} of slotted rings is influenced by various factors like: number of slots, location of slots, slot sizes and gaps, but this is not the purpose of this work and these factors are not directly discussed here.

To investigate the effect of slots on high-Q ring resonators, 4 mm diameter rings with radial thicknesses in the range 20 – 25 μm are selected based on results shown in Figure 2.5. These rings operate in the isothermal region of the Debye curve, and so have high-Q values. Table 2.10 shows numerically simulated results for the frequency and Q_{TED} for rings with single and double layer slots, and no support legs. For the rings considered including slots, Q_{TED} decreases significantly, indicating that the damping has been increased - this is in contrast to the results in Table 2.9 where the damping was reduced. Table 2.11 shows similar behaviour to Table 2.10 but for a 2 mm diameter ring with a radial thicknesses in the range 140 – 160 μm , operating in the adiabatic region. Again it is found that including slots causes the damping to increase, however, for the selected slots the decrease in Q_{TED} is not as large as for the isothermal ring considered earlier.

Table 2.10: Frequency and Q_{TED} of 4 mm ring with single and double layer slots

Radial Thickness b_r (μm)	Unslotted Ring		Slots Included			
			Single Layer		Double Layer	
	Freq. (kHz)	Q_{TED}	Freq. (kHz)	Q_{TED}	Freq. (kHz)	Q_{TED}
20	5.264	2.850×10^5	5.531	3.086×10^4	5.503	1.308×10^4
21	5.528	2.462×10^5	5.797	2.977×10^4	5.773	1.260×10^4
22	5.791	2.142×10^5	6.063	2.879×10^4	6.041	1.228×10^4
23	6.054	1.875×10^5	6.327	2.789×10^4	6.309	1.210×10^4
24	6.317	1.650×10^5	6.592	2.708×10^4	6.575	1.202×10^4
25	6.580	1.460×10^5	6.857	2.633×10^4	6.840	1.204×10^4

 Table 2.11: Frequency and Q_{TED} of 2 mm ring with single and double layer slots

Radial Thickness b_r (μm)	Unslotted Ring		Slots Included			
			Single Layer		Double Layer	
	Freq. (kHz)	Q_{TED}	Freq. (kHz)	Q_{TED}	Freq. (kHz)	Q_{TED}
140	146.26	9.966×10^4	146.94	9.957×10^4	144.31	9.586×10^4
145	151.40	1.095×10^5	152.05	1.090×10^5	149.22	1.031×10^5
150	156.52	1.197×10^5	157.15	1.191×10^5	154.11	1.105×10^5
155	161.64	1.314×10^5	162.22	1.294×10^5	158.98	1.178×10^5
160	166.75	1.439×10^5	167.30	1.405×10^5	163.84	1.252×10^5

2.6.3 Understanding TED Behaviour in Slotted Rings

Candler *et al.* [75] mention in their work that the addition of slots can decrease the Q-factor, depending on the specific geometry and frequency of a thermoelastically limited beam resonator. This behaviour has also been observed by Duwel *et al.* [63] and Candler *et al.* [76] for flexural beam resonators. In the previous section it was found that Q_{TED} reduces in some cases when slots are added. Due to the complexity added by the slots to the geometry of the ring, Zener's theory based on single thermal mode is not sufficient to explain this behaviour. Candler *et al.* [76] proposed a rigorous explanation based on coupling of thermal and mechanical modes. The same explanation can be applied to the case of slotted high-Q rings. Due to thermoelastic coupling, several thermal modes can couple with the mechanical mode of interest. For an unslotted ring, it is a valid approximation (based on Zener's theory) that a single thermal mode (usually the first mode) is strongly coupled to the mechanical mode, and Q_{TED} is dominated by energy dissipation from that mode. However, the addition of slots causes multiple thermal modes to be coupled to the mechanical mode. The initial thermal mode of an unslotted high-Q ring resonator is fully coupled to the mechanical mode and has very low energy dissipation if it is operating in the

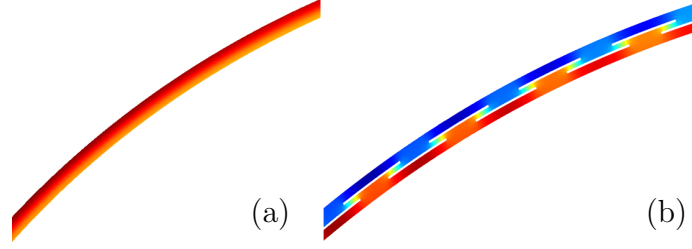


Figure 2.28: Temperature distribution in (a) unslotted and (b) slotted high-Q rings. Red and blue indicates higher and lower temperatures, respectively.

isothermal or adiabatic regime. Adding slots to the ring couples the mechanical mode to higher frequency thermal modes. This reduces the coupling effect with the initial thermal mode. The partially coupled higher thermal modes also have energy dissipation. This dissipation is significantly higher than the energy dissipated by the initial thermal mode which becomes partially coupled when slots are added, leading to a reduction in Q_{TED} . Figure 2.28 shows the change in temperature distribution due to the addition of slots into a ring operating in the isothermal regime. A similar explanation is valid for the case where Q_{TED} of a ring resonator, operating near the Debye peak, increases by the addition of slots. In this case, the initial thermal mode has higher energy dissipation, and incorporating slots reduces the coupling of this mode with the mechanical mode. Also, the energy dissipation from the partially coupled higher frequency thermal modes is such that it does not decrease Q_{TED} . In summary, the addition of slots introduces a complex coupling of thermal modes and the mechanical mode of interest which can increase or decrease Q_{TED} depending on whether the ring is operating near or away (isothermal/adiabatic regime) from the Debye peak.

2.6.4 Summary

The impact of slots on the ring resonator was studied and the results obtained suggest that slots can improve Q_{TED} for a ring having low Q, i.e. operating near the Debye peak, but slots do not improve Q_{TED} for high-Q rings operating in the isothermal or adiabatic regimes. This is mainly due to the complex interaction between the

thermal and mechanical domains.

2.7 Design of High-Q Ring Resonators

This section discusses the design of ring resonators with high Q_{TED} based on the results and conclusions presented in the previous sections. Since 2θ in-plane modes are utilised to detect the angular rate of rotation, the frequency and Q-factor for this mode are considered as the design targets. Based on existing architectures the following design specifications were set:

- Quality factor due to TED, $Q_{TED} \approx 3 \times 10^5$
- 2θ in-plane frequency range: 6 kHz-14 kHz

From the manufacturing perspective, the ring diameter needs to be kept within the limits of the available manufacturing process within the sponsoring company i.e. dry reactive etching (DRIE) method, and with this in mind only ring diameters in the range 3-8 μm are considered.

Referring to Section 2.4, high Q_{TED} can be achieved using small diameter thick rings or large diameter thin rings. However, based on the imposed frequency constraints and the results in Table 2.4, 4 mm diameter unsupported rings having radial thickness in the range 20-23 μm have high Q_{TED} i.e. $\sim 300,000$. The results in Table 2.4 show that adding support legs increase frequency and Q_{TED} for these ring configurations, and leg widths in the range 6-10 μm satisfy the frequency and Q_{TED} design target for a 4 mm-23 μm ring, as shown in Figure 2.13. Based on these results, ring resonators having ring and support dimensions shown in Table 2.12 are selected as having potential to achieve high-Q. Figure 2.29 shows the support leg design used for these resonators.

The support loss and TED behaviour for different modes of the high-Q resonators are discussed in Chapters 4 and 5, respectively. Also, experimental testing of the devices fabricated using these high-Q design are reported in Chapter 5.

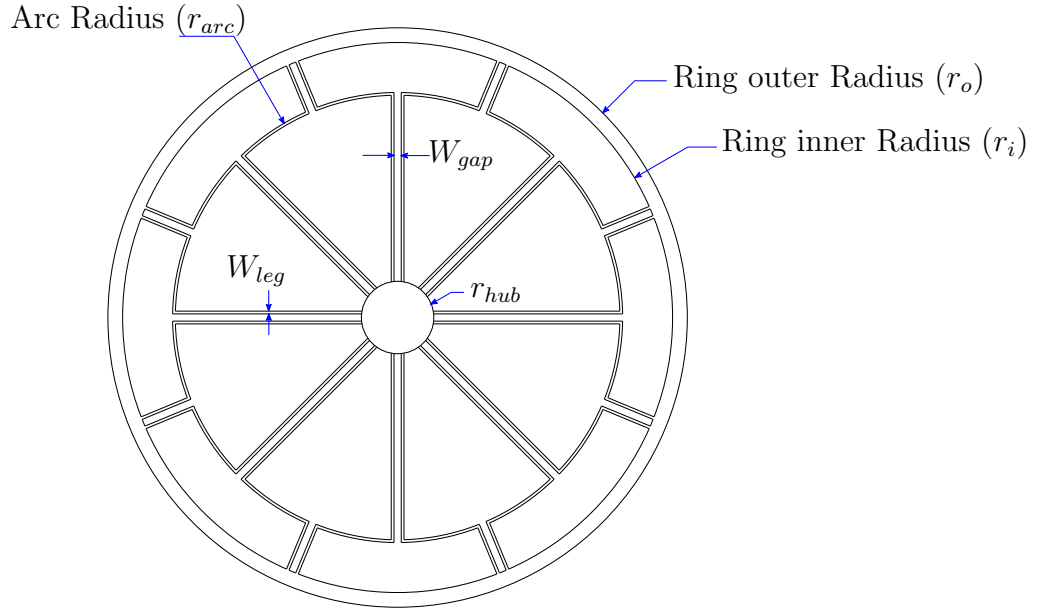


Figure 2.29: Design of support leg for the high-Q resonators.

Table 2.12: Ring and support dimensions for the selected high-Q resonators

Diameter, d (mm)	Radial Thickness, b_r (μm)	Leg Width, W_{leg} (μm)	Hub Radius, r_{hub} (μm)	Leg Gap, W_{gap} (μm)	Arc Radius, r_{arc} (μm)	Axial Thickness, a_r (μm)
4	22	6	250	50	1.6	150
4	23	6				
4	23	8				
4	23	10				

2.8 Summary and Conclusions

In this chapter, numerical finite element based simulations and available analytical solutions have been used to predict thermoelastic damping (TED) in ring resonators. The FE model is developed based on a fully coupled thermoelastic approach to accurately predict the thermoelastic energy dissipation of resonators. The key purpose of the chapter is to present a parameter study to investigate the influence of bulk geometry changes in the ring and support structure and ring slot enhancements as well as the effects of material properties on TED, providing explanations for the observed behaviour where possible. In summary, the following conclusions can be drawn:

- The TED behaviour of unsupported rings is adequately explained by Zener's theory, and as such it is possible to achieve high-Q rings by operating in either the isothermal or adiabatic regions. In practice this can be achieved by using either large diameter thin rings or small diameter thick rings. However, large diameter thin rings have lower frequencies because of their low stiffness than small diameter thick rings, and this means that there is a trade-off between frequency and Q_{TED} that needs to be addressed when designing high-Q resonators for rate sensor applications.
- Adding support legs to high-Q rings tends to increase Q-factors compared to unsupported rings, but depending on the support leg design and ring size it could decrease also. However, as the width of the supporting legs increases, the Q-factor increases to a maximum before decreasing, suggesting that there is an optimum leg width at which maximum Q_{TED} is achieved. This behaviour is attributed to TED in the support legs which is dominant compared to the damping in the ring.
- A semi-analytical model based on strain energy is introduced to investigate the TED behaviour in supported rings. The model uses FEA to calculate strain energy and Zener's equation to calculate Q_{TED} in the ring and support legs separately. The model predicts similar TED behaviour for supported ring

resonators as obtained using the fully coupled FE model. The model provides insight on strain energy distribution in the ring and support legs, which also influences Q_{TED} .

- For materials that have low thermal conductivity (k), high-Q is generally achieved with small diameter thick rings as they operate in the adiabatic region. However, high thermal conductivity materials have higher Q_{TED} for large thin rings. Also, if the material has low k , the damping behaviour is greatly influenced by the local Debye peak for leg damping, which has been explained using the semi-analytical model. As a consequence, a minimum Q_{TED} is observed after the peak as leg width increases.
- The effect of incorporating slots around the ring circumference was also investigated for high-Q unsupported rings, and it was found that slots can decrease Q_{TED} (increase damping) by reducing the coupling between thermal modes with low energy dissipation and the prevailing mechanical mode.
- Several high-Q ring resonator designs with low levels of thermoelastic damping are proposed based on the imposed design constraints. These ring designs are fabricated and experimentally tested in Chapter 5.

Chapter 3

Modelling of Support Loss in MEMS Resonators

3.1 Introduction

Support loss is one of the important extrinsic source of energy dissipation in the context of MEMS resonators. The coupling between the resonator and the substrate to which it is attached induce support loss, resulting in the the generation of elastic waves which carry energy from the resonator to the substrate during vibration. It is important to understand the physical mechanism of support loss, which could restrict the maximum achievable Q-factor. As mentioned in Chapter 1, analytical models to quantify support loss in simple structures like beam and disk resonators are available in the research literature [88, 89, 122]. However, these models are based on restrictive assumptions (e.g. assuming wavelengths of propagating waves much larger than the thickness of the resonator translates propagating waves to be 2D), and require knowledge of forces/stresses exerted at the support area, which can be difficult to evaluate for complex resonators. For this reason, energy radiation into the substrate by elastic wave propagation is often predicted using numerical approaches [4, 92] e.g. Finite Element method (FE).

Because substrates are orders of magnitude bigger than resonators, the substrate is modelled as a semi-infinite/infinite domain. However, in practice the model needs to be truncated at a computationally manageable size, with appropriate boundary conditions (BC) applied. If the boundary of the substrate is truncated with a stan-

dard BC (e.g. Neumann BC or Dirichlet BC), unwanted reflections from the truncation of the domain at the boundary due to interaction with propagating waves “pollute” the solution. Since the substrate is considered as a semi-infinite/infinite domain and requires the waves to be absorbed, numerical boundary conditions, e.g. infinite elements, absorbing layers etc., are employed to mimic the semi-infinite/infinite extent of domains, which has been discussed in Chapter 1.

In this chapter, the support loss is modelled using a non-physical absorbing layer, known as the Perfectly Matched Layer (PML) [105]. Although, the PML has been implemented for different MEMS resonators [96, 100, 102] using FE, these custom-built codes are difficult to adopt for generic resonators. In this work a commercial FE package, COMSOL [107], is used to predict support loss using PMLs.

The purpose of this chapter is to provide insight and understanding into PMLs and to validate the PML-based model, which proved difficult in previous studies [19, 92]. In addition, insight into support loss behaviour for different resonators is provided. For the purposes of illustration, a simple cantilever beam and disk resonator are analysed in this chapter. The methodology and understanding developed here will be useful for analysing support loss in high-Q ring resonators in Chapter 4.

In Section 3.2, the physics behind PMLs is illustrated with the aid of a semi-infinite elastic rod model. The limitations of PMLs are also outlined and a description is provided of the finite element method used to predict support loss in resonators.

Section 3.3 presents a validation of the finite element model for a cantilever beam vibrating in its first bending mode. Several factors affect the efficiency of PMLs and these are discussed in the context of achieving converged results. Simulated results are compared with analytical results from the literature, and support loss in beam resonators is discussed briefly.

A practical, high-frequency MEMS resonator in the form of a disk resonator is analysed in Section 3.4. This study is based on work of Bindel and Govindjee [96] and Koyama [18], and provides further verification of the PML model described in Section 3.2. Support loss depends on the design of the resonator, and a parameter study

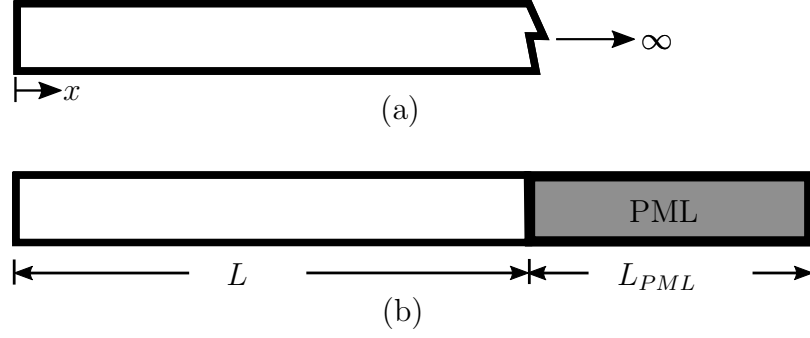


Figure 3.1: (a) Semi-infinite elastic rod, and (b) Perfectly matched layer (PML), adjacent to the bounded domain for the semi-infinite rod.

is presented based on varying the resonator and support dimensions, and material properties. The results are compared with experimental results, and findings show the importance of design parameters in order to minimise support loss.

The highlights from the chapter are presented with concluding remarks in Section 3.6.

3.2 Methodology

3.2.1 Physical Mechanism of Perfectly Matched Layers (PMLs)

Since this work focuses on the application of PMLs to support loss in MEMS resonators, this section explains the physical mechanism used by PMLs to truncate a boundary condition for solving the elastic wave equation numerically. This is illustrated in the following case for 1D wave propagation in a semi-infinite elastic rod.

Figure 3.1(a) shows a schematic of a semi-infinite rod modelled both physically and using a PML. The wave propagating from the origin in the positive x direction is expressed by the 1D wave equation (scalar)[123]:

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0 \quad (3.1)$$

Where, $u(x, t)$ is the axial displacement of the rod and c is the wave speed at which it propagates.

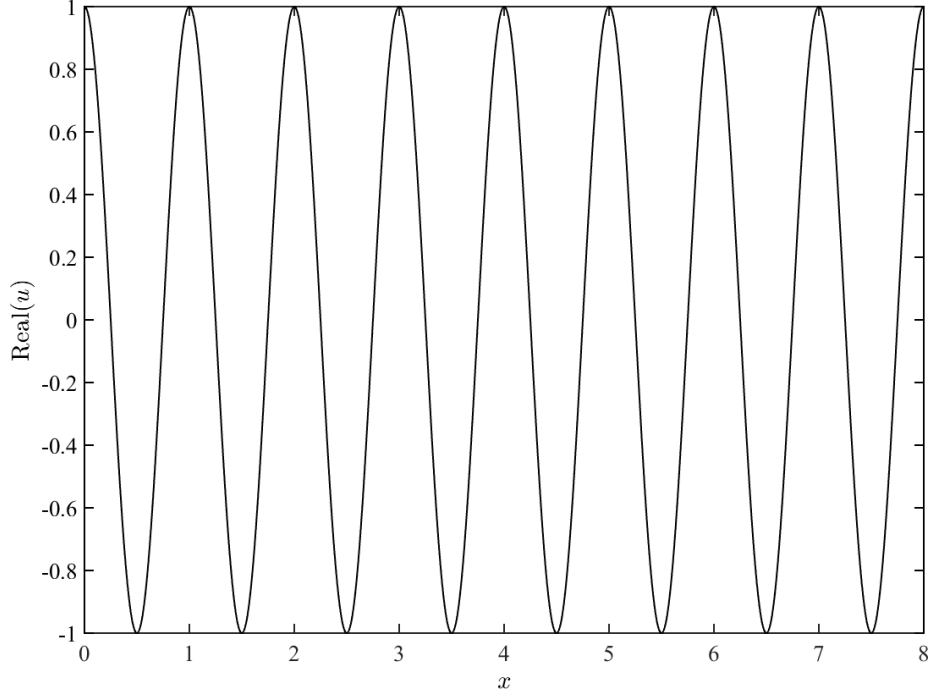


Figure 3.2: Real part of steady-state harmonic solution in semi-infinite elastic rod.

For an initial displacement of u_0 , the steady-state harmonic solution to Eq.(3.1) has the form:

$$u(x, t) = u_0 e^{-i(k_w x - \omega t)} \quad (3.2)$$

Where, ω is the angular frequency and k_w is the wave number, i.e. $(\frac{\omega}{c})$.

The real part of Eq.(3.2) represents the wave, as given below:

$$u(x, t) = u_0 \cos(k_w x - \omega t) \quad (3.3)$$

Eq.(3.3) is plotted in Figure (3.2) for arbitrary values of k_w, ω and x . As depicted in the figure, the wave will propagate along the semi-infinite rod until it is truncated at a finite distance. If Eq. (3.1) is solved numerically, the unbounded elastic rod must be truncated using appropriate boundary conditions to reduce the computational effort. However, truncating the computational domain using for example a rigid boundary produces wave reflections into the computation space producing errors in the computed results. To overcome this difficulty, the bounded domain is augmented by incorporating a numerical absorbing boundary or layer. A PML is a

non-physical damping layer positioned adjacent to the truncated domain, which absorbs all propagating wave energy without producing any spurious reflections. The PML presented in this work uses a complex stretching of coordinate axes to introduce damping to the waves.

Development of a PML for Eqs.(3.1) to (3.3) using complex coordinate stretching is achieved in three conceptual steps [95]: i) analytic continuation, ii) coordinate transformation, and iii) truncation of the computational domain. Each of these steps are considered next.

Analytic continuation The PML is obtained by analytic continuation of the equations in the complex plane. This transforms the propagating waves into exponentially decaying waves inside the PML region without generating any reflections from the interface between the PML and the truncated domain. In the PML region, the x coordinate is replaced by a stretched coordinate $\tilde{x}$:

$$\tilde{x} = x - if(x) \quad (3.4)$$

where, $f(x)$ is called the stretching function which determines the rate at which the real x coordinate is stretched along the imaginary axis. The stretching function is generally expressed as a function of frequency (ω) and an absorbing function $\sigma(x)$, given by Eq.(3.5).

$$f(x) = \frac{1}{\omega} \int^x \sigma(x) dx \quad (3.5)$$

Within the physical domain the absorbing function is zero, but within the PML the absorbing function is a positive real number or a function of the coordinate variable.

Applying Eq.(3.5) to (3.2) it can be shown that the steady-state harmonic solution inside the PML region is:

$$u(x, t) = u_0 e^{-i(k_w x - \omega t)} e^{\frac{\int^x \sigma(x) dx}{\omega}} \quad (3.6)$$

The real part of Eq.(3.6) governs the wave propagation inside the PML, i.e.:

$$u(x, t) = u_0 \cos(k_w x - \omega t) e^{\frac{\int^x \sigma(x) dx}{\omega}} \quad (3.7)$$

If the semi-infinite elastic rod is truncated at a distance $x=L$, and a PML of length L_{PML} is introduced adjacent to the physical domain (see Figure 3.1(b)), the behaviour of the absorbing function is defined as:

$$\sigma(x) = 0 \quad \text{for } x \leq L \quad (3.8)$$

$$\sigma(x) > 0 \quad \text{for } L < x \leq L + L_{PML} \quad (3.9)$$

When the propagating wave enters the PML region, the real coordinate is stretched into the complex plane, i.e. the absorbing function has a finite value. From Figure 3.3, it is evident that inside the PML region, the propagating wave decays exponentially and the rate of decay depends on the value of the absorbing function ($\sigma(x)=1$ used in this figure). Importantly, the wave equation and its solution remain unchanged in the bounded region of the elastic rod.

Coordinate Transformation In the physical and PML domains, a coordinate transformation is performed to express the complex x coordinate as a function of real x coordinate. The transformation used is as follows:

$$\frac{\partial}{\partial \tilde{x}} = \frac{1}{1 + \frac{i\sigma(x)}{\omega}} \frac{\partial}{\partial x} \quad (3.10)$$

When extending to three dimensional (3D) elastic solids the PML can be implemented in the y and z directions using the following transformations:

$$\frac{\partial}{\partial \tilde{y}} = \frac{1}{1 + \frac{i\sigma(y)}{\omega}} \frac{\partial}{\partial y} \quad (3.11)$$

$$\frac{\partial}{\partial \tilde{z}} = \frac{1}{1 + \frac{i\sigma(z)}{\omega}} \frac{\partial}{\partial z} \quad (3.12)$$

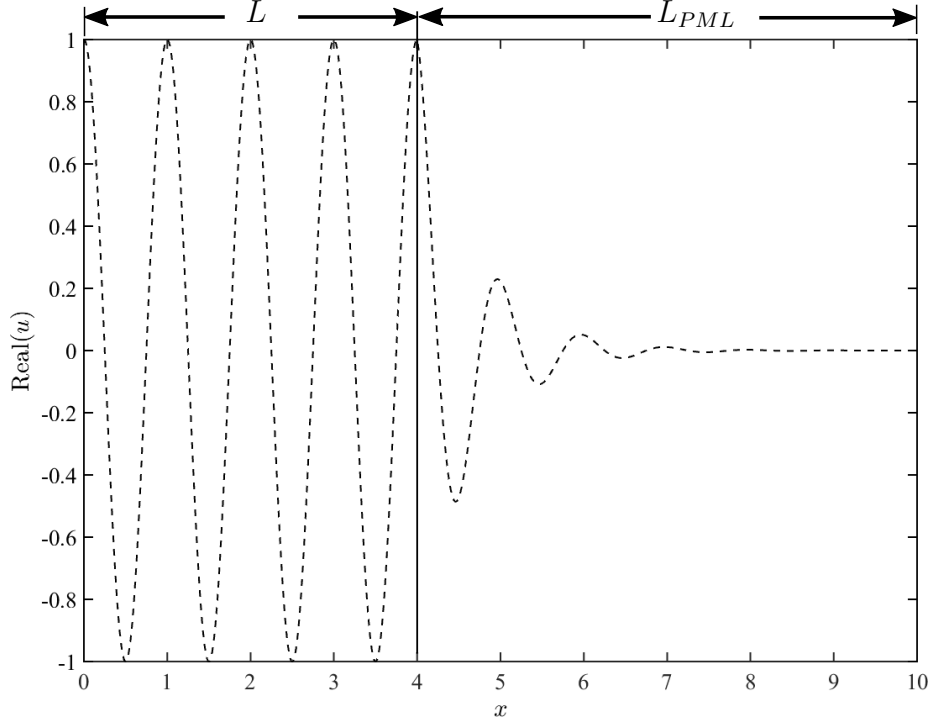


Figure 3.3: Damping of the propagating wave inside the PML region.

Truncation of Computational Domain As the final step of the PML implementation, the truncation inside the PML domain is required for solving the wave equation using numerical techniques. Since the solution decays inside the PML, the boundary condition will not affect the solution significantly if it is truncated at a sufficient distance. Generally, a rigid boundary condition is applied at the outer surface. This will produce some reflected waves when the decayed waves reach the rigid boundary. However, they are attenuated before they reach the interface between the physical domain and the PML.

3.2.1.1 Absorbing Function (σ)

In the previous section an absorbing function was introduced to cause the propagating wave to decay within the PML region. In Figure 3.3, a constant absorbing profile ($\sigma(x)=1$) was used. Figure 3.4 provides a similar plot using a range of constant absorbing function value i.e. $\sigma(x) = 0.5$ to 2.0 . It can be seen that an increase in the magnitude of the absorbing function produces a faster rate of decay inside the PML, indicating that the value of σ has a significant impact on the rate of decay.

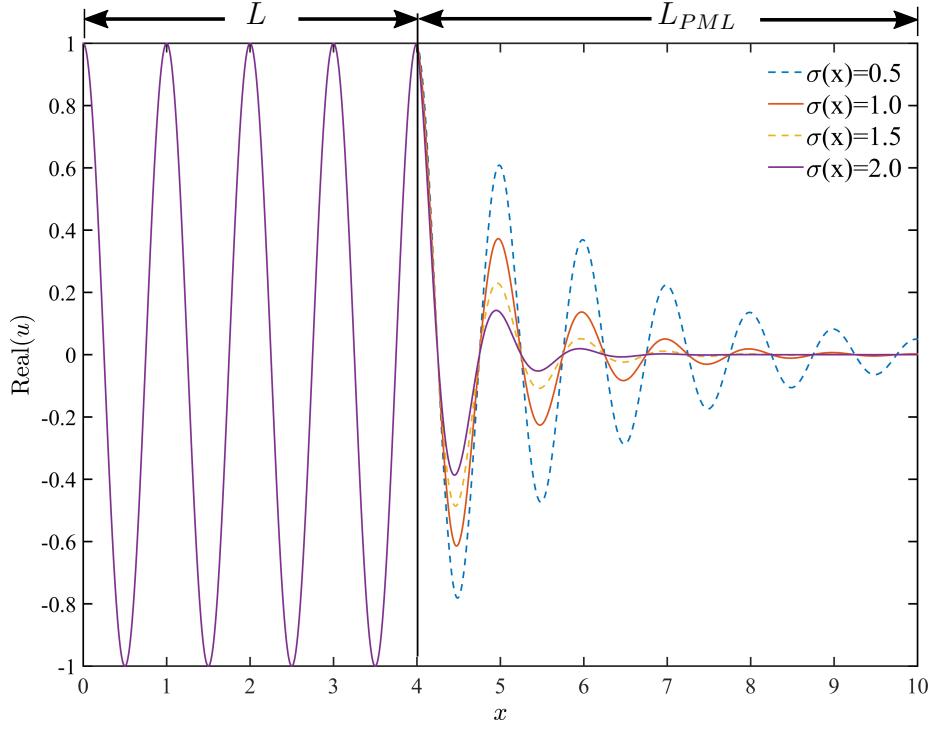


Figure 3.4: Damping of the propagating wave inside the PML region for different absorbing function values.

Increasing the PML length would also decrease the decay produced by an absorbing function. For example, Figure 3.5 shows how the wave is attenuated as the length of the PML is increased by a factor of 3 for $\sigma(x) = 0.5$.

As observed earlier, the value of absorbing function can be tuned to control the rate of decay. Depending on the type of PML used, the absorbing function can be expressed in several forms [99]. For example, assuming σ is positive and monotonically increasing for monotonically damped wave propagation inside the PML [18, 102]:

$$\sigma(s) = \beta \gamma(s) \left(\frac{s - L}{L_{PML}} \right), s \in [L, L + L_{PML}] \quad (3.13)$$

where, $\gamma(s)$ is a normalized function defined on the unit interval $[0, 1]$ with $\gamma(1) = 1$. Generally, $\gamma(s)$ is represented using a polynomial profile i.e. $\gamma(s) = s^p$, β is the end value of the PML absorbing function, and p is the order of the polynomial.

Eq.(3.13) is the standard form of absorbing function commonly used in the literature for MEMS applications, and a similar form of absorbing function is used in

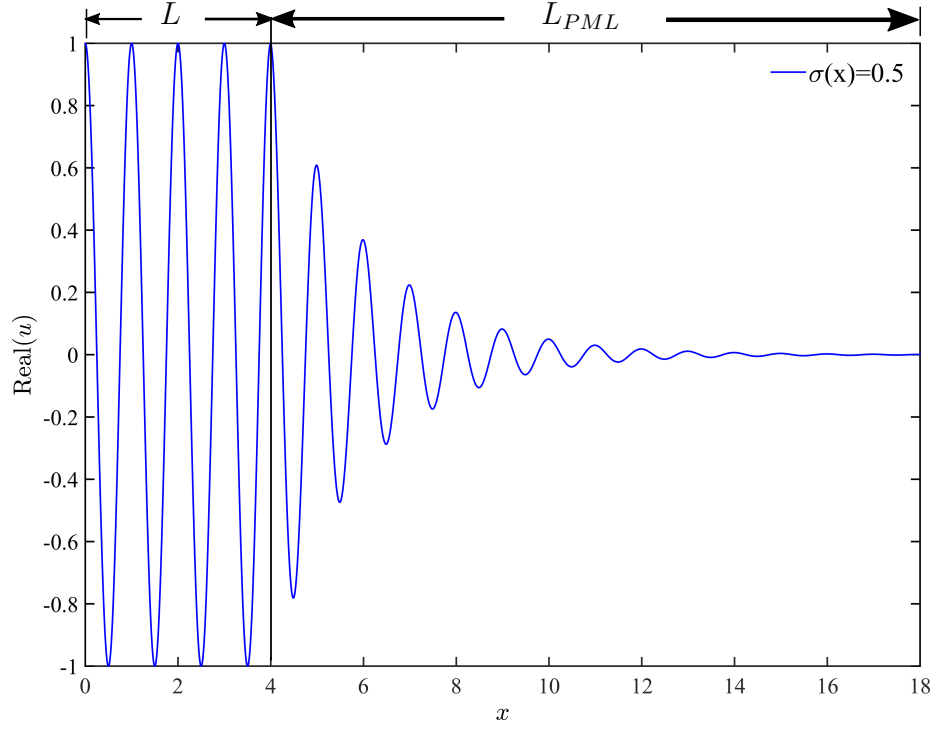


Figure 3.5: Damping of the propagating wave inside the PML region with increased PML length.

COMSOL later.

3.2.1.2 Limitations of PML

Theoretically, PMLs produce zero reflections at the interface between the physical domain and the PML for any propagating waves. This is because the exact wave equations are solved for a continuous problem. In this case the only reflections that arise are from the finite termination of PML, and can be reduced to zero by selecting a large β value [96].

When solving the wave equations using a numerical discretisation method like the finite element method, the equations become approximate. The discretisation process induces reflections at the interface, leading to less accurate results. A detailed study on the influence of end and interface reflections can be found in Koyama [18] and Bindel *et al.* [96]. The wave reflection can be made arbitrarily small by proper selection of the absorbing function, provided discretisation is obtained with sufficient resolution (mesh elements) [95]. Furthermore, the impedance between the physical

domain and PML domain must be matched. Physically this implies that the material properties inside the PMLs should be the same as that of the physical domain to which PMLs are applied.

3.2.2 Finite Element Modelling

In this section several aspects of finite element modeling to quantitatively evaluate support loss are presented. A brief review of the elastic wave equations solved using FE method is provided, along with the fundamental assumptions for modelling support loss using PMLs. The implementation of PMLs in the commercial package COMSOL is also discussed.

3.2.2.1 Governing Equation

The equations of motion for a three dimensional (3D) elastic solid (neglecting external body forces) in a Cartesian coordinate system are given by Eqs. (2.7) to (2.9). Using Hooke's Law and constitutive relations in these equations, yields the following elastic wave equations in Cartesian coordinates [124]:

$$c_l^2 \frac{\partial^2 u}{\partial x^2} + c_s^2 \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + (c_l^2 - c_s^2) \left(\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z} \right) = \frac{\partial^2 u}{\partial t^2} \quad (3.14)$$

$$c_l^2 \frac{\partial^2 v}{\partial y^2} + c_s^2 \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} \right) + (c_l^2 - c_s^2) \left(\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial^2 w}{\partial y \partial z} \right) = \frac{\partial^2 v}{\partial t^2} \quad (3.15)$$

$$c_l^2 \frac{\partial^2 w}{\partial z^2} + c_s^2 \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + (c_l^2 - c_s^2) \left(\frac{\partial^2 u}{\partial z \partial x} + \frac{\partial^2 v}{\partial z \partial y} \right) = \frac{\partial^2 w}{\partial t^2} \quad (3.16)$$

Where, $c_l = \sqrt{\frac{\Lambda + 2\mu}{\rho}}$ and $c_s = \sqrt{\frac{\mu}{\rho}}$ are propagation velocities for longitudinal and shear (transverse) waves in an elastic medium of density, ρ and Young's modulus, E . Here, one has the *Lamé* constants, $\Lambda = \frac{E}{(1+\nu)(1-2\nu)}$ and $\mu = \frac{E}{2(1+\nu)}$. ν is the Poisson's ratio of the material. It worth mentioning that the longitudinal (λ_l) and shear/transverse (λ_s) wavelengths are expressed as $\frac{c_l}{f}$ and $\frac{c_s}{f}$, respectively, where f is the frequency.

Eqs. (3.14) to (3.16) can be expressed in vector notation as:

$$(c_l^2 - c_s^2)\nabla(\nabla \cdot \mathbf{u}) + c_s^2\nabla^2\mathbf{u} = \ddot{\mathbf{u}} \quad (3.17)$$

Where, $\mathbf{u}$ is the displacement vector.

This equation is integrated into COMSOL and solved using eigenfrequency analysis to extract the modal parameters of a resonator.

3.2.2.2 Modelling Assumptions

In FE modelling, the resonator and the substrate to which it is attached is treated as the physical domain. During each cycle of vibration, an amount of energy flows from the resonator to the substrate through elastic wave propagation. Generally, the substrate is considered to be much larger (typically several orders of magnitude) in size than the resonator i.e. semi-infinite domain. It is assumed that all propagating wave energy is absorbed within the semi-infinite substrate and is considered to be lost.

3.2.2.3 PML Implementation in COMSOL

The careful selection of the absorbing function is essential to the accurate and efficient performance of the PMLs to calculate support loss. In COMSOL, the absorbing function (σ) is defined as a function of a dimensionless coordinate ξ , which varies linearly from 0 to 1 over the PML layer [125]. For a PML absorbing waves propagating in the direction of unit vector $\mathbf{n}_\xi$, ξ is defined as:

$$\xi = \frac{\mathbf{n}_\xi \cdot \mathbf{x} - x_0}{W_{PML}} \quad (3.18)$$

Where, $\mathbf{x}$ is the coordinate vector $\{x, y, z\}^T$, W_{PML} is the PML width and x_0 is a reference distance defined such that $\xi = 0$ when $\mathbf{x}$ lies on the boundary between the PML and the physical domain. This is depicted in Figure 3.6 for a 1D PML along x axis. $\mathbf{n}_\xi$ and ξ are computed automatically.

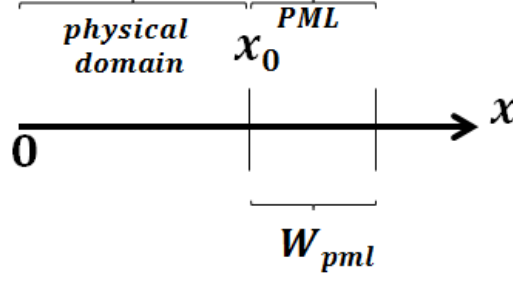


Figure 3.6: Schematic of a 1D PML.

The polynomial absorbing function used in COMSOL is similar to Eq. (3.13). However, the real and imaginary parts are stretched separately. The imaginary part, which is responsible for wave attenuation is of primary concern. The absorbing function is defined as [125]:

$$\sigma(\xi) = S\lambda_{wave}\xi^p(i-1) \quad (3.19)$$

where, λ_{wave} is the wavelength, p is the polynomial order, and S is the scaling factor. It is worth mentioning that the scaling factor is similar to the end value of the absorbing function (β), defined in Eq.3.5. So, S in Eq. (3.19) is replaced with β to ensure consistency with notation throughout the thesis.

$$\sigma(\xi) = \beta\lambda_{wave}\xi^p(i-1) \quad (3.20)$$

Finally, the wave equations are reformulated inside the PML using the following stretched coordinate vector, and a similar form of stretching is used in all directions

$$\tilde{\mathbf{x}} = \mathbf{x}_0 + \mathbf{n}_\xi\sigma(\xi) \quad (3.21)$$

PML Geometry It is possible to model PMLs as rectangular (Cartesian), cylindrical or spherical geometries in COMSOL. The selection of PML geometry depends on the geometry of the physical domain around which PMLs are to be applied. A comparison between models using different PML geometries is discussed later in this

chapter.

Treatment of PML Corners Modifications are required during PML implementation if the physical domain is bounded by rectangular PMLs. The complex coordinate stretching occurs in such a way that the waves are attenuated in the direction perpendicular to the interface between the PML and the physical domain [126]. However, the treatment of corners requires the superposition of the intersecting PMLs. In Figure 3.7, the physical domain is surrounded by PML regions which are separated into different parts. Inside the PML region in x and y directions, either $\sigma(x)$ or $\sigma(y)$ is zero and the wave is attenuated in one direction. In the corners both absorbing functions ($\sigma(x)$ and $\sigma(y)$) are positive and act together, because of the two intersecting PMLs. In 3D, depending on whether the PMLs are attached to a surface, edge, or a point in the physical domain, they are stretched in one, two or three directions, respectively [125]. It is important to define distinct domains for the corners in the FE analysis, otherwise erroneous results will be obtained. This modification of corners can be avoided by using a cylindrical PML.

3.2.2.4 Model Setup

This section describes the general procedures to setup a model in COMSOL to quantify support loss in different resonators. Depending on the nature of the problem, the resonator is modelled in either 2D or 3D. In many cases it is assumed that the the out-of-plane dimensions of the resonator and the substrate are similar and there are no stresses acting in the out-of-plane direction, hence the two-dimensional plane stress assumption ($\sigma_{zz} = \sigma_{zx} = \sigma_{zy} = 0$) can be used to model the resonator and substrate (for example the beam resonator considered in Figure 3.8). It also reduces the computational effort since two displacement variables (u, v) are solved for each node of the discretised resonator geometry. However, in 3D, the out-of-plane displacement (w) is also solved, increasing the total number of degrees of freedom and computational effort. Furthermore, an axisymmetric resonator vibrating with an axisymmetric mode shape can be modelled as a 2D axisymmetric problem. An

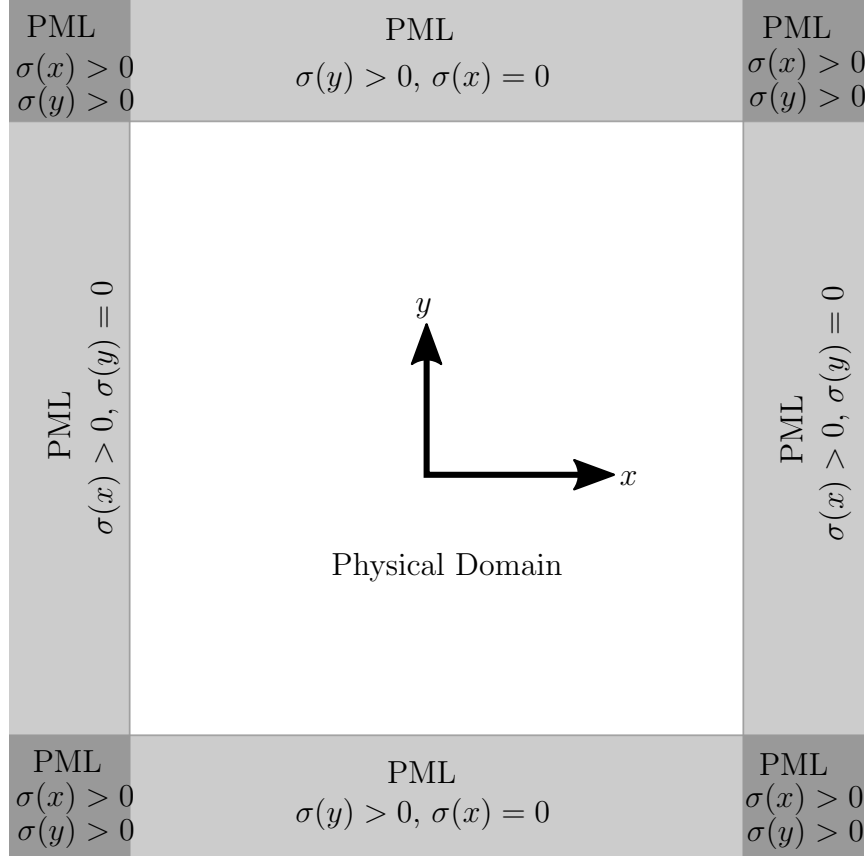


Figure 3.7: The behaviour of absorbing function inside the PML regions including corners in 2D.

axisymmetric model of a disk resonator has been considered later in this chapter.

Boundary Condition A rigid boundary condition, namely a fixed constraint ($\mathbf{u} = 0$), is applied at the outer edges (for 2D) and outer surfaces (for 3D) of the PML domain. This truncates the computational domain, and is the final step of PML implementation mentioned in Section 3.2.1.

Mesh Generation In COMSOL, 2D models are meshed using quadratic triangular elements generated by the built-in meshing routine. This technique has been proved to be effective in previous studies (see Section 2.3). For 3D models, the resonator and substrate are meshed using quadratic tetrahedral elements, and swept meshes are used for the PML domain.

Calculation of Q-factor The implementation of PML leads to the following complex-valued eigenvalue problem:

$$Det([K] - \lambda^2[M]) = 0 \quad (3.22)$$

Where, λ is the eigenvalue of the system, and $[M]$ and $[K]$ are the mass and stiffness matrices, respectively.

The Q-factor associated with support loss ($Q_{support}$) is extracted as follows [103]:

$$Q_{support} = \frac{|Im(\lambda)|}{2|Re(\lambda)|} \quad (3.23)$$

3.3 Model Validation

3.3.1 Introduction

Chouvion [19, 92] reported difficulties obtaining converged results when modelling support loss using PMLs. The work studied a simple cantilever beam, but the results obtained did not agree precisely with the exact Q-factor values. It also showed that selecting appropriate parameter values was a time-consuming process. The main parameters that influence the accuracy of the Q-factor calculation using PMLs are:

- the dimension of the PML, specifically PML width (W_{PML})
- the dimension of the substrate used in the FE model, specifically substrate width (W_{sub})
- the number of mesh elements (N_{el}) in the PML region
- the end value of the absorbing function (β)
- the polynomial order (p) of the absorbing function

The purpose of this section is to validate the model developed earlier. The simple case of cantilever beam vibrating in its first bending mode is considered here. The

beam model is created based on the work of Frangi *et al.*[102], for bench-marking against published values. The numerical results are also compared with analytical results (discussed in the following section). The effect of some of the parameters mentioned earlier, is investigated further. Finally, the damping behaviour in beam resonators is presented to develop physical understanding.

3.3.2 Analytical Models

Analytical models for a cantilever beam are discussed briefly in this section, and used for the purpose of comparison with numerical results later.

Natural Frequency of a Cantilever Beam It is well known that the natural frequency of the n^{th} resonant mode of a cantilever beam (clamped-free) is [127]:

$$\omega_n = \frac{\pi^2 \beta_n^2 H}{L^2} \sqrt{\frac{E}{12\rho}} \quad (3.24)$$

Where, L is the beam length, H is the beam height, β_n is a mode related constant, E and ρ are the Young's modulus and density of the material, respectively. For the first bending mode ($n = 1$), $\beta_n=0.597$. Eq. (3.24) was derived based on Euler-Bernoulli beam theory.

Support Loss Models Hao *et al.* [89] derived a closed form expression for support loss of a cantilever beam. The support was modelled as a semi-infinite thin plate using 2D elastic wave theory and it was shown that the Q-factor for support loss is expressed as:

$$Q_{support} = \left[\frac{0.24(1-\nu)}{(1+\nu)\Psi} \right] \frac{1}{\beta_n^2 \chi_n^2} \left[\frac{L}{H} \right]^3 \quad (3.25)$$

Where, $\Psi = \int_0^\infty \frac{\sqrt{\zeta^2 - (\frac{c_l}{c_s})^2}}{\{2\zeta^2 - (\frac{c_l}{c_s})^2\}^2 - 4\zeta^2 \sqrt{\zeta^2 - (\frac{c_l}{c_s})^2} \sqrt{\zeta^2 - 1}} d\zeta$, and χ_n is the mode shape factor which is -0.734 for $n = 1$. The value of Ψ is calculated numerically, more details are provided in Hao *et al.* [89].

It is evident from (3.25) that $Q_{support}$ is dependent on the dimensions of the beam

i.e. the cube of the ratio of beam length to the beam height.

The above model is valid in 2D when the plane stress assumption is used. As mentioned by Judge *et al.* [122], the effect of bending moment is not included in that model and it is also not affected by the out-of-plane beam width (W). To overcome these deficiencies, Judge *et al.* [122] derived a closed form expression for $Q_{support}$ of a beam vibrating in its first bending mode, assuming the support can be modelled as a semi-infinite elastic media:

$$Q_{support} = A_\nu \frac{L}{W} \left(\frac{L}{H} \right)^4 \quad (3.26)$$

Where, A_ν is a weak function of the Poisson's ratio (ν), and $A_\nu = 3.175$ for $\nu = 1/3$, $A_\nu = 3.23$ for $\nu = 0.3$. W is the width of the beam.

Eq. (3.26) is used later in this chapter to compare numerical results obtained from a 3D model of the cantilever beam.

3.3.3 Convergence Analysis

3.3.3.1 Model Setup: Cantilever Beam Resonator

Figure 3.8 shows a schematic of the 2D base beam model used for validation purposes, together with various parameters. A cantilever beam is attached to a finite substrate that is surrounded by PMLs (corners are modelled using distinct domains). The dimensions and material properties of the beam are the same as those reported in Frangi *et al.* [102]. The dimensions of the beam: $L=10\mu\text{m}$ and $H=1\mu\text{m}$, and the materials properties: $E=130\text{ GPa}$, $\rho=2330\text{ kg/m}^3$ and $\nu=0.3$ are used in the FE simulation. The analytical estimates for frequency and $Q_{support}$ are 12 MHz and 2081 using (3.24) and (3.25), respectively. Since the shear/transverse wave length (λ_s) of the first bending mode is around $390\mu\text{m}$, it is not a feasible option to select the geometry of the PML close to λ_s due to increased computational cost. For the base model, $W_{PML}=10\mu\text{m}$ and $W_{sub}=5\mu\text{m}$ is selected, obtained from Frangi *et al.* [102]. The simulation is conducted for a range of β values, to identify the optimum β value

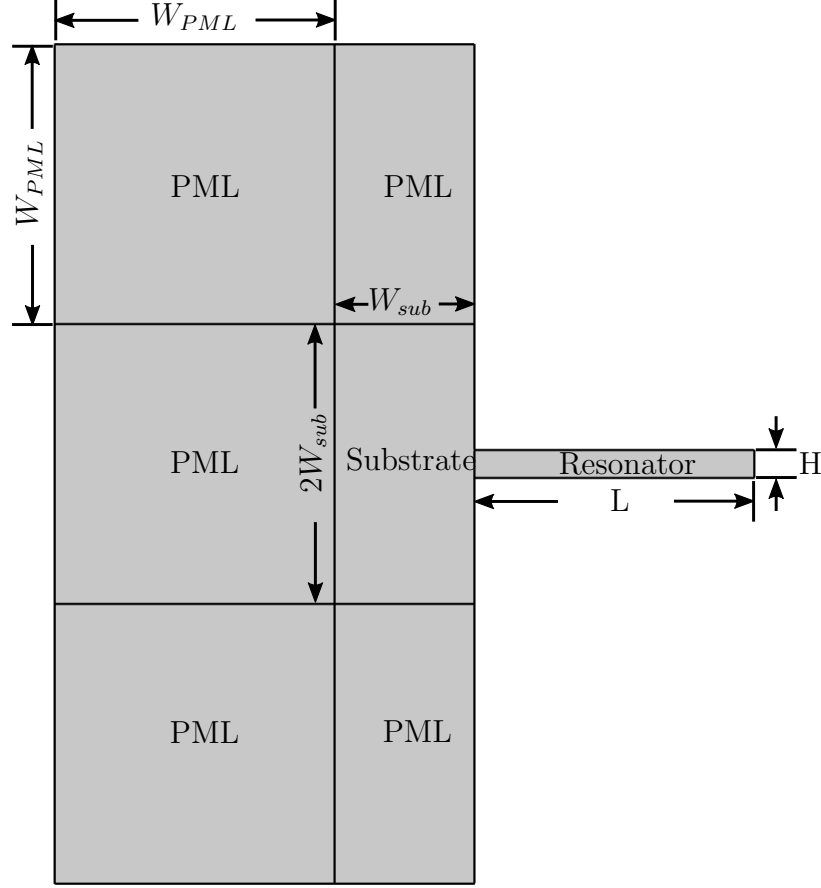


Figure 3.8: 2D Cantilever Beam: geometry of the substrate and the PML.

around which $Q_{support}$ is stable. Also, a linear absorbing function is selected i.e. $p=1$ in Eq. 3.20 for simulation.

3.3.3.2 Convergence of Frequency

The convergence of frequency with increased number of elements (N_{el}) in the model is shown in Figure 3.9. The frequency of the beam vibrating in its first bending mode converges to a value 11.6 MHz. The difference between analytical (12 MHz) and simulated frequency is approximately 3%. Although both values are in good agreement, the difference is mainly due to the boundary condition assumption used in analytical expression. Eq. (3.24) is for a cantilever beam clamped at the root. In this study, the beam is not fixed at the root, but is attached to a substrate with finite stiffness.

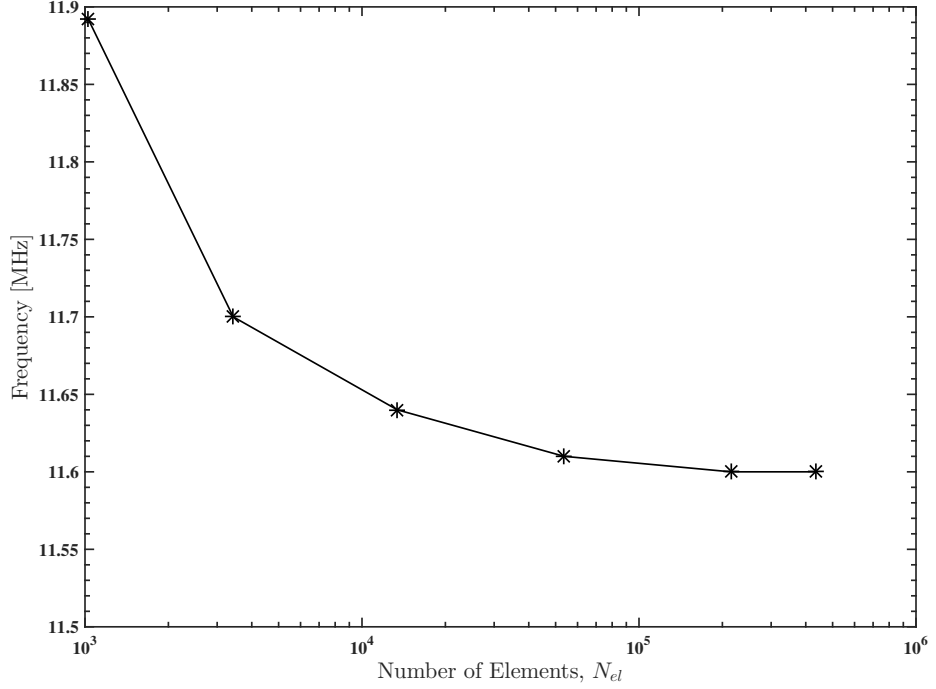


Figure 3.9: 2D Cantilever Beam: convergence of frequency as a function of N_{el} .

3.3.3.3 Convergence of $Q_{Support}$

This section illustrates convergence of $Q_{support}$, considering the effect of various parameters.

Effect of Number of Elements (N_{el}) and β The sensitivity of $Q_{support}$ to the total number of mesh elements (N_{el}) in the model and β values is shown in Figure 3.10. For a coarse mesh, the $Q_{support}$ does not converge as β is increased. As the mesh resolution increases, the solution approaches the exact wave equation. Hence, $Q_{support}$ starts to converge with β . However, large β values can have detrimental effect on the result, as spurious reflections in the PMLs start polluting the solution. For large β value, the waves entering the PML decay rapidly, and using a coarser mesh to resolve this decay induces spurious reflections [96]. This is evident in Figure 3.10, though N_{el} increases. A trade-off between N_{el} and β is essential for an effective PML. In this study, $Q_{support}$ is stable at around 1657. A β value within the range 30-40 and a mesh with N_{el} between 1×10^5 and 2×10^5 is sufficient for convergence. Furthermore, the results are compared with published values from Frangi *et al.* [102]

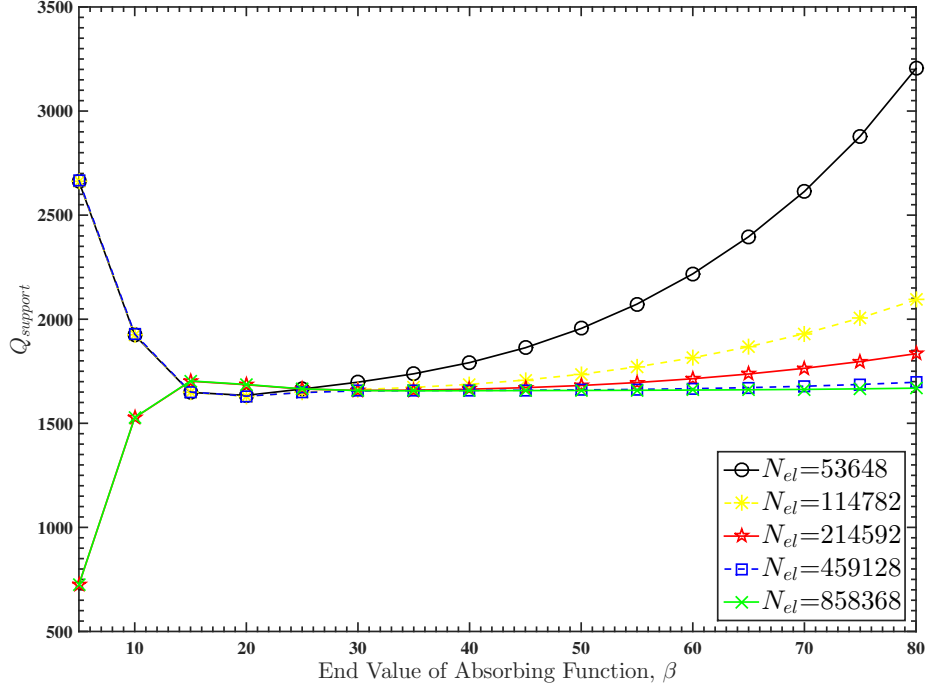


Figure 3.10: 2D Cantilever Beam: convergence of $Q_{support}$ as a function of N_{el} and β .

Table 3.1: Effect of substrate width (W_{sub}) on $Q_{support}$ of a cantilever beam

W_{sub} (μm)	5	10	15	20	25
$Q_{support}$	1657	1656	1655	1656	1656

in Figure 3.11. The $Q_{support}$ values from this study are in excellent agreement with the converged value of ≈ 1650 and $\beta=20$ to 30 , mentioned in Frangi *et al.* [102].

The model has been validated successfully and converged values for Q-factor are obtained. A good resolution mesh and optimum β values is required to achieve stable $Q_{support}$ values.

Effect of Substrate Width (W_{sub}) The influence of substrate width on the convergence of $Q_{support}$ is investigated for a range of W_{sub} values from $10\ \mu\text{m}$ - $25\ \mu\text{m}$ ($5\ \mu\text{m}$ is the base model), using $\beta=30$. Theoretical results indicate the substrate width should not affect the solution as the absorbing function is zero in every domain except in the PMLs. This is confirmed in Table 3.1, with small variation caused by numerical errors.

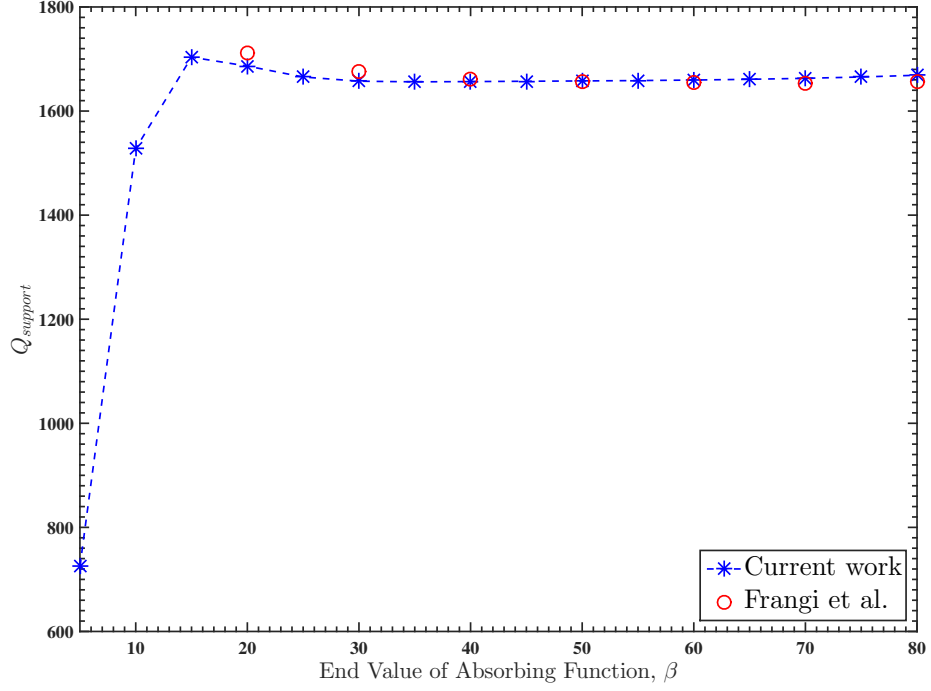


Figure 3.11: 2D Cantilever Beam: comparison of $Q_{support}$ with published results from Frangi *et al.*[102].

Effect of PML Width (W_{PML}) In Section 3.2.1 it was illustrated that the PML length can have a significant impact on the damping of the propagating waves. Based on theoretical observation, the effect of PML width (W_{PML}) on $Q_{support}$ is further investigated. Here, W_{PML} is varied from $5\text{ }\mu\text{m}$ to $50\text{ }\mu\text{m}$ using $\beta=30$, while other parameters are kept constant. However, N_{el} is fixed within 4.5×10^5 to 4.6×10^5 , depending on the dimensions in the model. The results presented in Table 3.2 show very little change in $Q_{support}$ for $W_{PML}=20\text{ }\mu\text{m}$ to $30\text{ }\mu\text{m}$ compared to the base case of $W_{PML}=10\text{ }\mu\text{m}$. A further increase in W_{PML} causes deviation from the converged value most likely due to discretisation error, i.e. as N_{el} is fixed, sufficient mesh resolution might not have been achieved with an increased W_{PML} . On the other hand, $Q_{support}$ is slightly underestimated for $W_{PML}=5\text{ }\mu\text{m}$. This may be due to the fact that $\beta=30$ is not the optimal value for $W_{PML}=5\text{ }\mu\text{m}$. These results show the sensitivity of Q-factor to the PML size, and a correct combination of W_{PML} and β value(s) is required to achieve accurate results.

Table 3.2: Effect of PML width (W_{PML}) on $Q_{support}$ of a cantilever beam

W_{PML} (μm)	5	10	20	30	40	50
$Q_{support}$	1651	1657	1656	1659	1666	1681

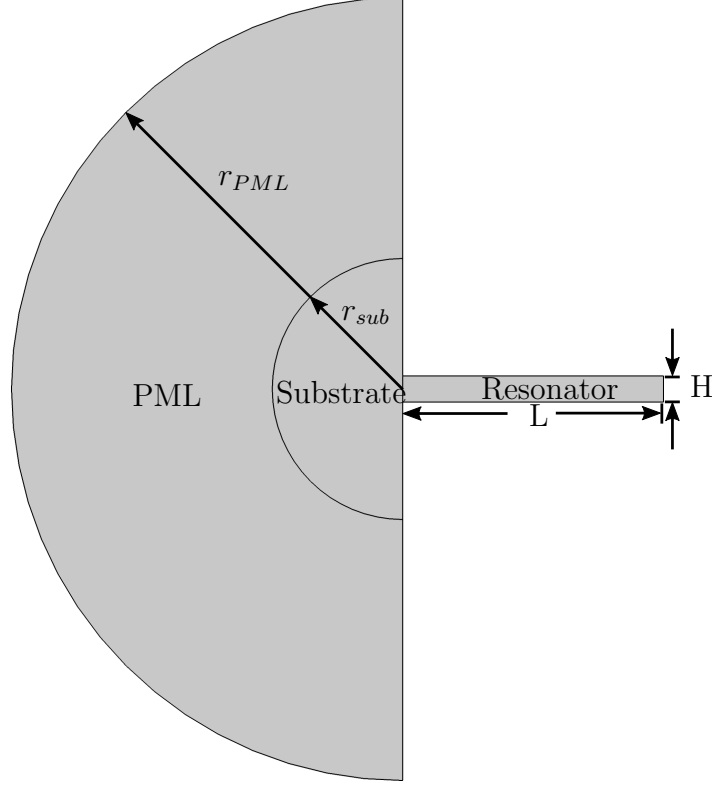


Figure 3.12: Circular PML (cylindrical coordinate) applied to the 2D cantilever beam.

Effect of PML Geometry For this investigation, the beam resonator is considered to be attached to a semi-circular substrate of radius, r_{sub} . Figure 3.12 shows how the semi-circular PML (r_{PML}) is applied around the substrate. The circular PML stretches the coordinate in radial and axial directions, i.e. a cylindrical coordinate system. All the parameters are kept the same as the base case, with $r_{sub}=5\mu\text{m}$ and $r_{PML}=10\mu\text{m}$. Results are obtained using $N_{el} = \approx 8 \times 10^5$ and compared with the results in Figure 3.10 (i.e. $N_{el}=858368$). For a circular PML, $Q_{support}$ converges for a similar range of β i.e. $\beta=30$ to 40 , as shown in Figure 3.13. The results are similar to those obtained using rectangular PMLs. However, no special treatment of corners is required using circular PMLs which is an advantage over rectangular PMLs.

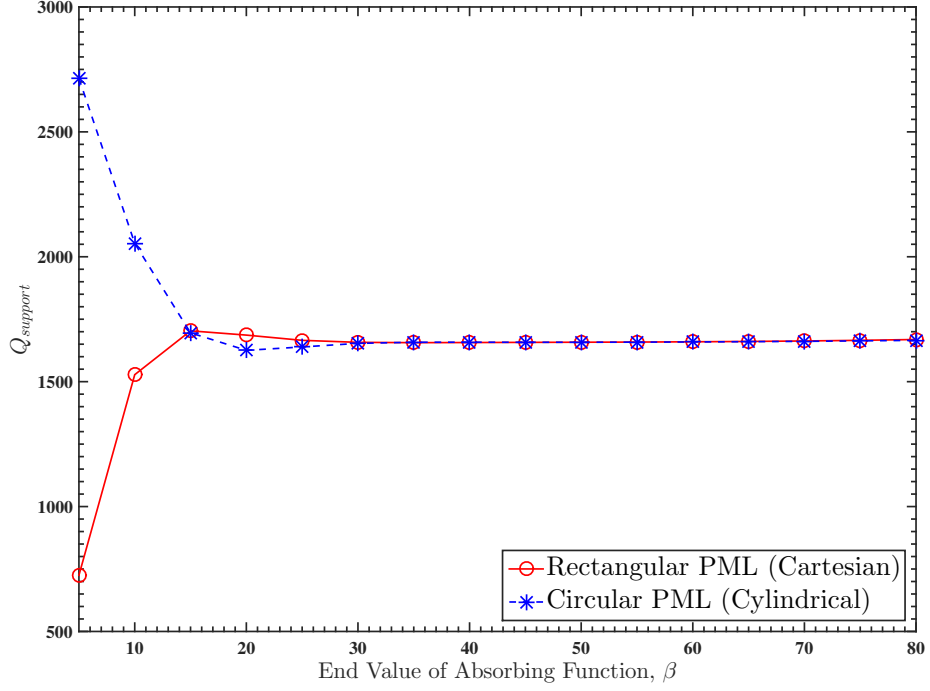


Figure 3.13: 2D Cantilever Beam: comparison of $Q_{support}$ values using rectangular and circular PML geometries.

Simulation of a 3D model A 3D model of the beam resonator is considered here with dimensions: length of beam, $L=10\ \mu\text{m}$, height of beam $H=1\ \mu\text{m}$, and width of beam $W=1\ \mu\text{m}$. A spherical PML was used around the substrate, which does not require any treatment of corners and simplifies modelling of the geometry. Furthermore, to reduce computational effort, symmetric boundary condition are considered. The 3D model is shown in Figure 3.14, where the substrate and the resonator is meshed with tetrahedral elements and a swept mesh is used in the PML region. In this section, the effect of the out-of-plane dimension, W , of the beam on $Q_{support}$ is investigated. It is important to validate the model for 3D geometries, which can be employed for resonators where propagating waves travel in the out of plane direction. For this study, $r_{PML}=20\ \mu\text{m}$ and $r_{sub}=10\ \mu\text{m}$ was used. The results are compared with an analytical estimate of $Q_{support}=317500$, obtained using (3.26).

In Figure 3.15, the convergence of $Q_{support}$ is shown for different β values with increased number of elements, N_{el} . It is evident that $Q_{support}$ is stable at $\beta=15$ with an increase in N_{el} . The best resolved simulation yielded a $Q_{support}$ value of

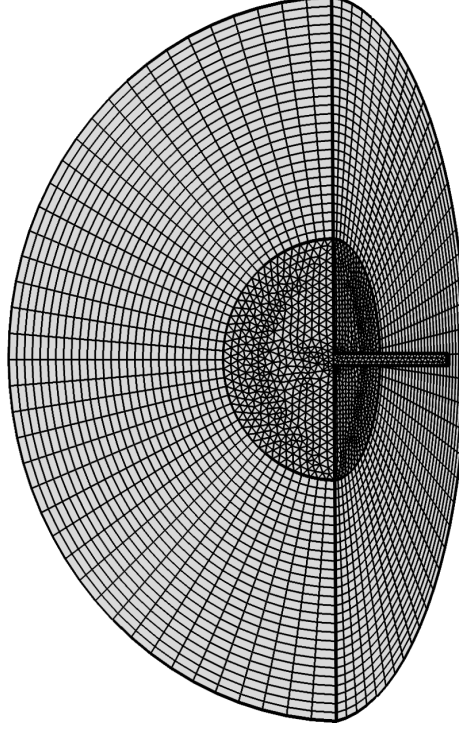


Figure 3.14: 3D cantilever beam - swept mesh used to discretise spherical PML.

317111 which is in excellent agreement with the analytical estimate, and simulated $Q_{support}=316104$ from Frangi *et al.* [102]. However, the computational effort further restricts the simulation using finer meshes. It is worth noting that simulations have been performed using a quadratic absorbing function i.e. $p = 2$. The linear absorbing function i.e. $p = 1$ was found to induce reflections and give erroneous results. For a cubic function ($p = 3$), $Q_{support}$ has a similar value and does not change significantly compared to that obtained using the quadratic function. The polynomial absorbing function allows the waves to decay smoothly in this case. A comparison of different polynomial absorbing functions is presented in Figure 3.16.

3.3.3.4 Damping Behaviour in a Cantilever Beam Resonator

The support loss mechanism in a cantilever beam is briefly discussed in this section. Support loss is the radiation of energy from the resonator to the support, hence, an excitation source is required for the waves to propagate. When a cantilever beam vibrates, it exerts both shear force and bending moment at the attachment region.

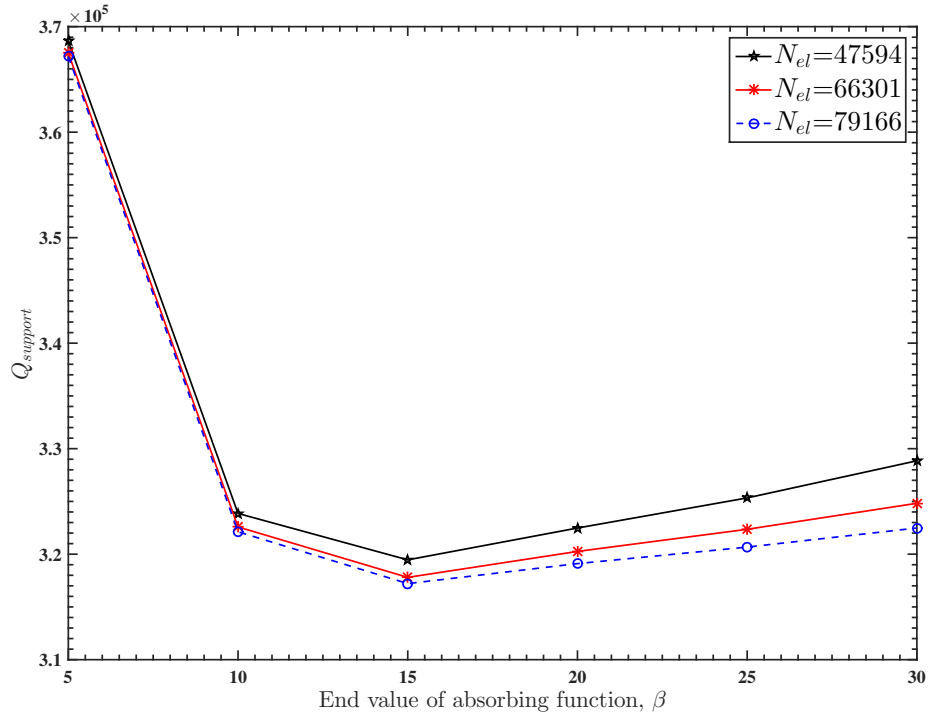


Figure 3.15: 3D Cantilever Beam: convergence of $Q_{support}$ as function of β for different N_{el} .

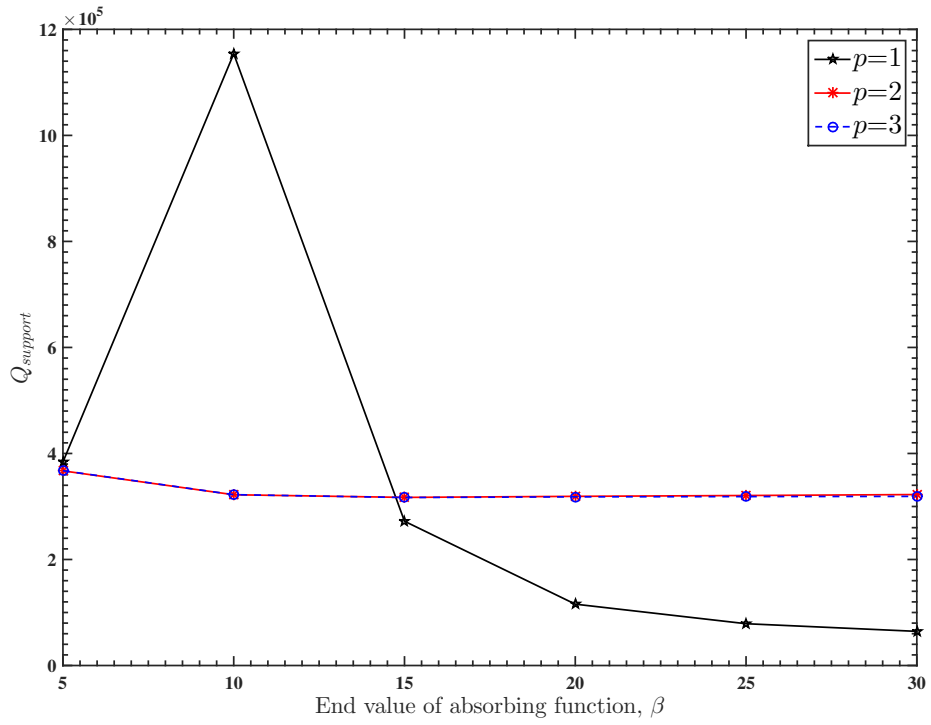


Figure 3.16: 3D Cantilever Beam: convergence of $Q_{support}$ for different polynomial order of the absorbing function.

This shear force and bending moment excites the elastic waves which propagate into the support. As a consequence, some of the resonator energy is lost to the support structure leading to $Q_{support}$. It has been proved theoretically that the effect of the bending moment on support loss is negligible and that shear stress at the attachment region is the dominant excitation source for a vibrating cantilever beam [88]. Figure 3.17(a) shows a plot of the first bending mode, whereas Figure 3.17(b) to (d) illustrates the wave propagation inside the support and PMLs, obtained from COMSOL using the 2D base beam model with rectangular PMLs in Section 3.3.3.1. The maximum range of total displacement (in μm) of the color bars is scaled down gradually to show the propagating waves responsible for the energy loss (see Figure 3.17(b)-(d)). It is apparent from the figure how waves are absorbed by the PMLs.

The support loss in a cantilever beam has been predicted using a PML-based model. The $Q_{support}$ value obtained from simulation is 1657, which is 20% lower than the analytical estimate which is 2081. The deviation between numerical and analytical results is expected due to the restrictive assumptions used in deriving the analytical equations. In Eq. (3.25), the beam and the support structure have been modelled separately, while the natural frequency expression for the beam assumes a clamped boundary condition. As mentioned earlier, the beam is not clamped, rather it is attached to an elastic structure with finite stiffness. The decoupling assumptions adopted in the analytical model accounts for this large variation. However, a FE analysis using beam elements and decoupling assumptions yields a $Q_{support} = 2106$, without PMLs. This is within 1.2% of the analytical estimate.

Due to dependency on the geometry of the resonator structure, the support loss is often termed “structural damping”. It is evident from Eq. (3.25) that damping is directly related to the frequency of the beam i.e. $Q_{support} \propto \frac{1}{f_n^3}$, and is plotted in Figure 3.18 for different aspect ratios ($\frac{L}{H}$) of the beam. It is worth mentioning that aspect ratio of a beam is inversely related to the frequency i.e. $f_n \propto \frac{H}{L^2}$, from Eq. 3.24. In this study, $L=500\text{ }\mu\text{m}$ has been considered with H varied. A similar trend is observed from simulated results, for $L=500\text{ }\mu\text{m}$ and $H=3\text{ }\mu\text{m}$ to $6\text{ }\mu\text{m}$. The

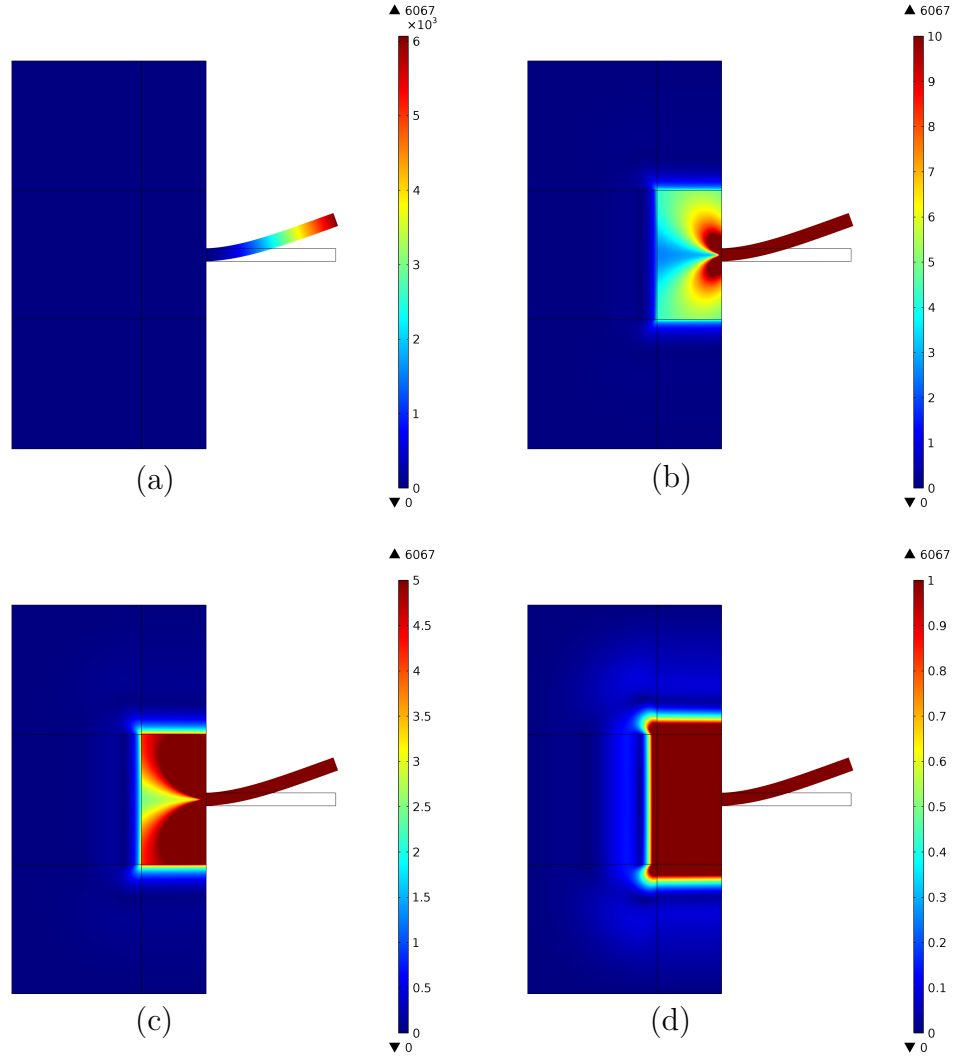


Figure 3.17: (a) First bending mode of the cantilever beam, (b)-(d) total displacement of the resonator and support, showing propagation of wave in the support and PMLs. The color bar indicates total displacement in μm , and has been scaled down gradually from (b) to (d) emphasizing wave propagation.

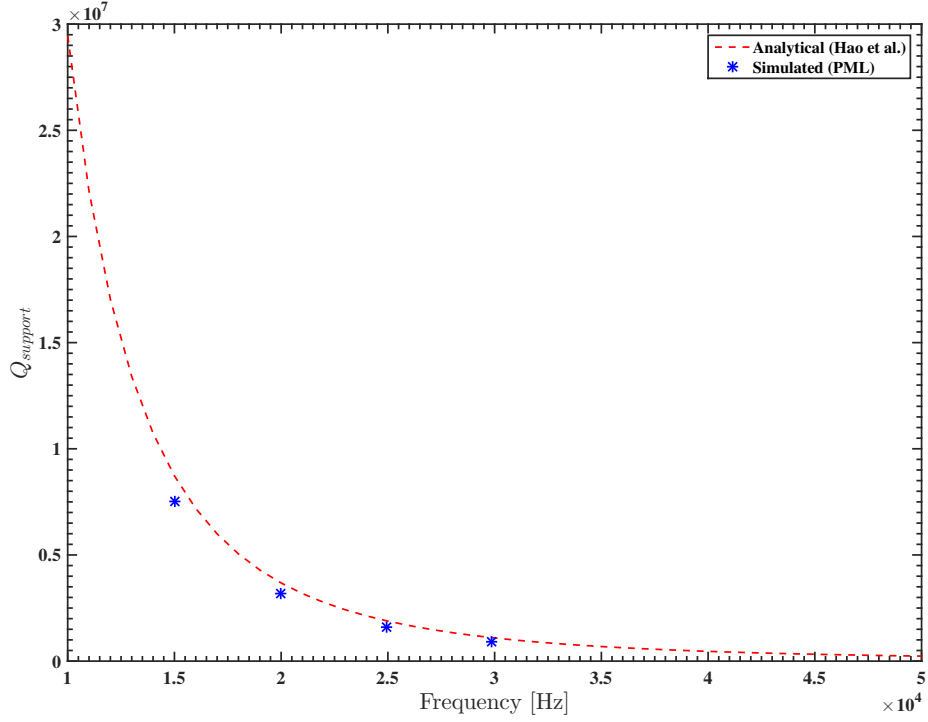


Figure 3.18: $Q_{support}$ as a function of frequency for a cantilever beam.

decrease in $Q_{support}$ is mainly due to an increase in shear stress, with frequency, at the attachment region. A FE analysis for various beam aspect ratios shows a linear relationship between shear stress (attachment region) and frequency for a cantilever beam. However, for a 3D geometry, the $Q_{support}$ is also dependent on the out-of-plane dimension, namely beam width (W).

3.3.4 Summary

In summary, the methodology behind the implementation of PMLs as an absorbing layer that behaves as an infinite domain is presented. Theoretically, PMLs are reflectionless and absorb all propagating waves inside it. In practice, discretisation of the PML domain causes spurious reflection at the interface with the physical domain, providing erroneous results. The magnitude of the reflection depends on the discretisation of the problem, and the selection of PML parameters. The PML-based FE model has been validated successfully by considering a cantilever beam resonator and comparison have been made against analytical and published results from the

research literature. There are several factors that affect convergence of solution. Proper selection of PML width, end value of absorbing function, and mesh resolution are critical for achieving efficient PMLs. The correct combination of PML width and the end value of absorbing function is required to prevent end termination reflections due to insufficient wave absorption, and the interface reflection at the PML and physical domain interface. Generally, the PML width should be chosen close to the longest wave length for proper attenuation of waves. However, the biggest challenge in quantifying support loss using FE model is the computational effort. For waves with longer wave lengths, it is not feasible to select a PML width which is several magnitudes larger in size compared to the resonator size, such as the cantilever beam considered in the previous sections. In that case, a modelling strategy needs to be applied to find the correct combination of PML parameters without increasing the computational effort. Based on the validation of the FE model, the following strategy is recommended for solving support loss problems in resonators with longer wave lengths i.e. low frequency:

- For longer wave lengths, selecting a PML width close to that is not computationally feasible. Therefore, the PML width should be selected rationally to a manageable size. For the selected PML width, a parameter study of the end value of absorbing function is required to find the optimal value for which the Q-factor is stable. However, sufficient mesh density also needs to be considered for performing the simulations. Thus, the optimal combination of parameters would yield accurate results with the least computational effort.

3.4 Case Study: Disk Resonator

3.4.1 Introduction

In the previous sections, a PML-based FE model has been validated successfully for the simple case of cantilever beam. A different class of MEMS resonator often used in practical applications is a disk resonator, which is investigated in this

section. Due to higher frequencies and Q-factors in radial contour modes (MHz to GHz), disk resonators have been studied extensively in the literature [80–82, 91]. The first implementation of PMLs in MEMS application was to model support loss in disk resonators [96], that was further analysed by Koyama [18]. In most cases, comparison of numerical [18] and experimental [82] results suggest that support loss is the dominant damping mechanism in high-frequency resonators. The rationale for conducting this study is to investigate support loss behaviour in high frequency disk resonators, and provide further verification of PML based models against the work of Bindel and Govindjee [96], and Koyama [106]. The effect of several design parameters, like resonator and support dimensions, material mismatch between the resonator and support, are also illustrated to gain insight on damping, and compared against experimental results where applicable.

3.4.2 Convergence Analysis

3.4.2.1 Model Setup: Disk Resonator

The contour modes (radial extension and compression) in disk resonators are axisymmetric [103]. To reduce computational effort, simulations for a disk resonator are performed by reducing the problem to a 2D axisymmetric model, as depicted in Figure 3.19. The disk is attached to the substrate, supported on a post. The values of material and geometric parameters are presented in Table 3.3, and are identical to the disk resonator analysed by Bindel and Govindjee [96] and Koyama [18], to allow benchmarking against their results. The analytical estimate of natural frequency for the 2^{nd} radial mode (the disk expands and contracts in the radial direction only) of a disk resonator is approximately 719 MHz, using the following equation of the natural frequency of n^{th} disk contour mode [128].

$$f_n = \frac{1}{2\pi} \frac{\lambda_n}{r_{disk}} \sqrt{\frac{E}{\rho(1 - \nu^2)}} \quad (3.27)$$

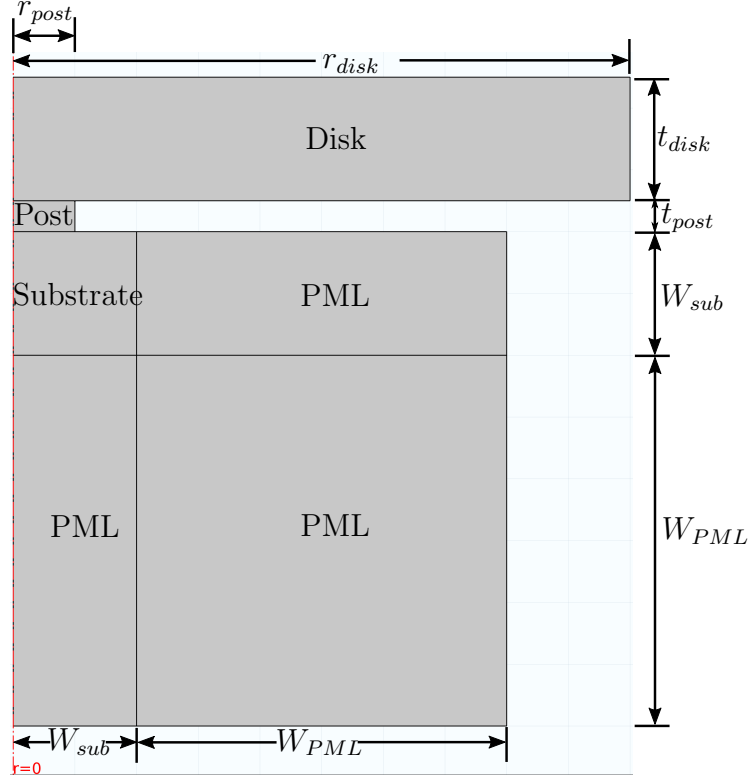


Figure 3.19: Axisymmetric model of the disk resonator (cross-section).

Where, λ_n is the eigenvalue for the n^{th} mode, and is calculated using: $\lambda_n \frac{J_0(\lambda_n)}{J_1(\lambda_n)} = 1 - \nu$. Here, J_0 and J_1 are Bessel functions of the first kind of zero and first order, respectively. For $n = 2$, $\lambda_n = 5.3740$.

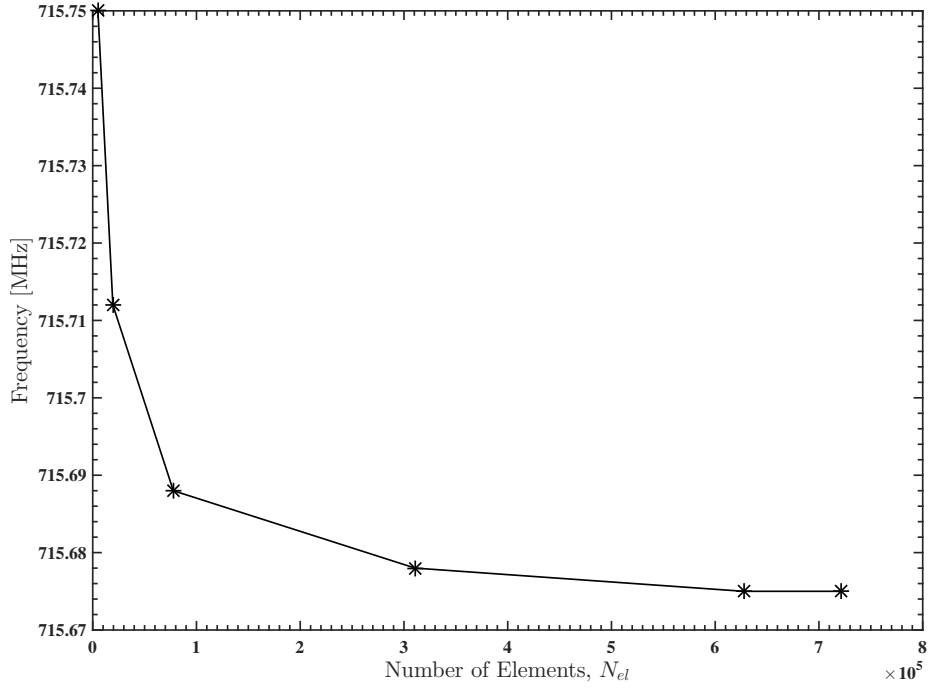
This translates into a shear wavelength of (λ_s) $6.9 \mu\text{m}$. Choosing a value of W_{PML} close to this wave length is manageable for calculating support losses by using the finite element method. The value $W_{PML} = 7 \mu\text{m}$ is used throughout this study, along with a linear absorbing function profile ($p = 1$) and $\beta = 1$. It is noteworthy that the axisymmetric cylindrical PML behaves identically to the plane 2D Cartesian PML [125]. This is why the corners have been distinctly modelled. The model uses quadratic triangular elements.

3.4.2.2 Convergence of Frequency and $Q_{support}$

Firstly, the natural frequency of the 2^{nd} radial mode of the disk is considered. The computed frequencies reported in [96] and [18] are 715.70 MHz and 715.60 MHz,

Table 3.3: Geometrical and material parameters used in modelling a disk resonator

Parameter	Value
Young's Modulus, E (GPa)	150
Material Density, ρ (kg/m ³)	2330
Poisson's Ratio, ν	0.3
Disk Radius, r_{disk} (μm)	10
Disk Thickness, t_{disk} (μm)	2
Support Post Radius, r_{post} (μm)	1
Support Post Thickness, t_{post} (μm)	0.5
Radius/Width of Substrate, W_{sub} (μm)	2


 Figure 3.20: Disk resonator - convergence of frequency as a function of number of elements (N_{el}).

respectively. In comparison to that, the current model results in a frequency of 715.67 MHz, which is in excellent agreement with the aforementioned values. However, there is a small difference with the analytical frequency (719 MHz) due to the effect of the support post not being considered in Eq. (3.27). A smooth convergence is achieved with increased numbers of elements (N_{el}), as shown in Figure 3.20.

The convergence of $Q_{support}$ with increasing number of elements is shown in Figure 3.21, and the results converges at 6025 for N_{el} between 6.2×10^5 to 7.2×10^5 . Since the exact wave equation is approached with sufficient mesh resolution, PMLs

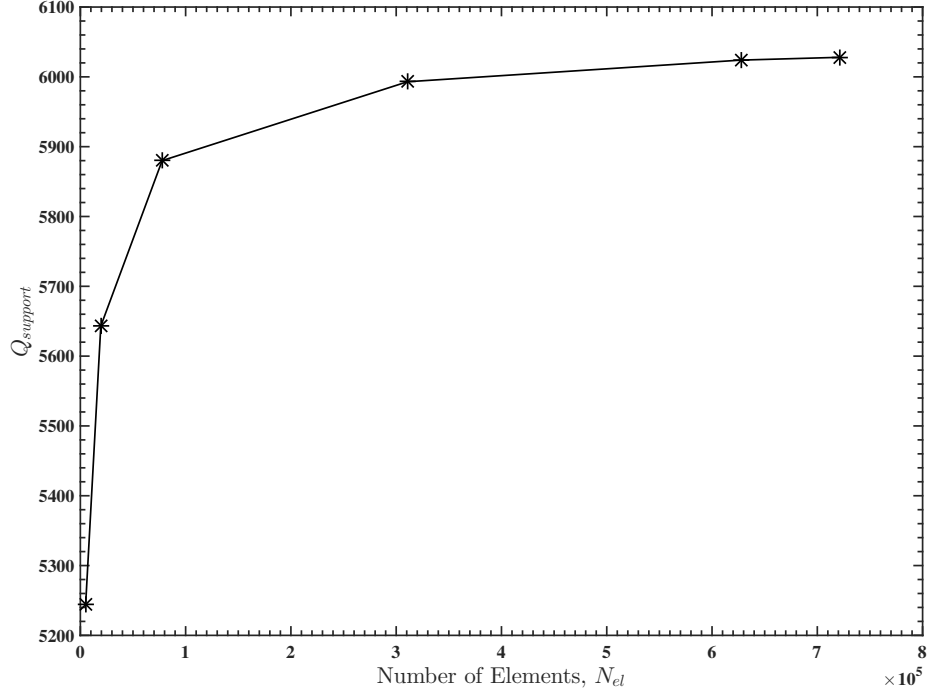


Figure 3.21: Disk resonator - convergence of $Q_{support}$ as a function of number of elements (N_{el}).

behave almost perfectly to absorb energy fluxes into the substrate from the disk. The predicted $Q_{support}=6250$ from [96] is overestimated due to a lack of fine mesh resolution and failing to resolve small fluxes reaching the support base. On the other hand, the best resolved simulation by Koyama [18] using a 2D axisymmetric model was $Q_{support} \approx 5900$. Although all these results are in reasonably good agreement, the current analysis provides a better converged $Q_{support}$ value.

3.4.2.3 Damping Behaviour in a Disk Resonator

The support loss mechanism in a disk resonator is explained by Hao *et al.* [91] and Bindel *et al.* [96], and the damping behaviour is discussed qualitatively in this section. As the disk vibrates, the support post follows the vibration. The post exerts force on the attachment region in the substrate resulting in stress waves being excited into the substrate. These waves carry energy away from the resonator, inducing support loss. Unlike beam resonators where shear stress acts as the dominant excitation source, the normal stress at the post/substrate interface also generates elastic waves into the

substrate.

The 2nd radial mode shape is shown in Figure 3.22. The mode is not purely radial, but is coupled by vertical motion of the disk and post (see Figure 3.23). As mentioned in Hao *et al.* [91] and Bindel *et al.* [96], the coupling of radial motion with bending motion is caused by Poisson's effect of the material and the post motion excites stress waves into the substrate, incurring support loss.

The energy 'leaking' from the resonator to substrate can be investigated by computing the mechanical energy flux, using the following equation [125]:

$$I_i = \frac{1}{2} \text{Re}(-\sigma_{i,j} v_j^*) \quad (3.28)$$

Where, $\sigma_{i,j}$ is the stress tensor, and v_j^* is the complex conjugate of the velocity vector. The Figure 3.24 shows the mechanical energy flux vector field in the disk resonator. The inset shows a similar behaviour to that reported by Bindel and Govindjee [96], where the energy cycle pattern is broken near the post and energy travels down the post into the substrate. However, the flux would cycle around the disk resonator if the post motion is decoupled.

3.4.3 Parameter Study

Unlike some damping mechanisms, support loss depends on the design of the resonator and so can be minimised by optimising design parameters. The design parameters that affect disk resonator performance are addressed in this section. The material properties are similar to Table 3.3, but $\rho=2300 \text{ kg/m}^3$ and $\nu=0.226$ is used for the following simulations to compare with the published results.

3.4.3.1 Effect of Resonator Dimension

In this section, the effect of disk radius (r_{disk}) on the $Q_{support}$ is investigated. The simulation is performed on a disk resonator of radius (r_{disk}) 10 μm and 18 μm , with post radius (r_{post}) of 0.8 μm . The results are shown in Table 3.4, and excellent

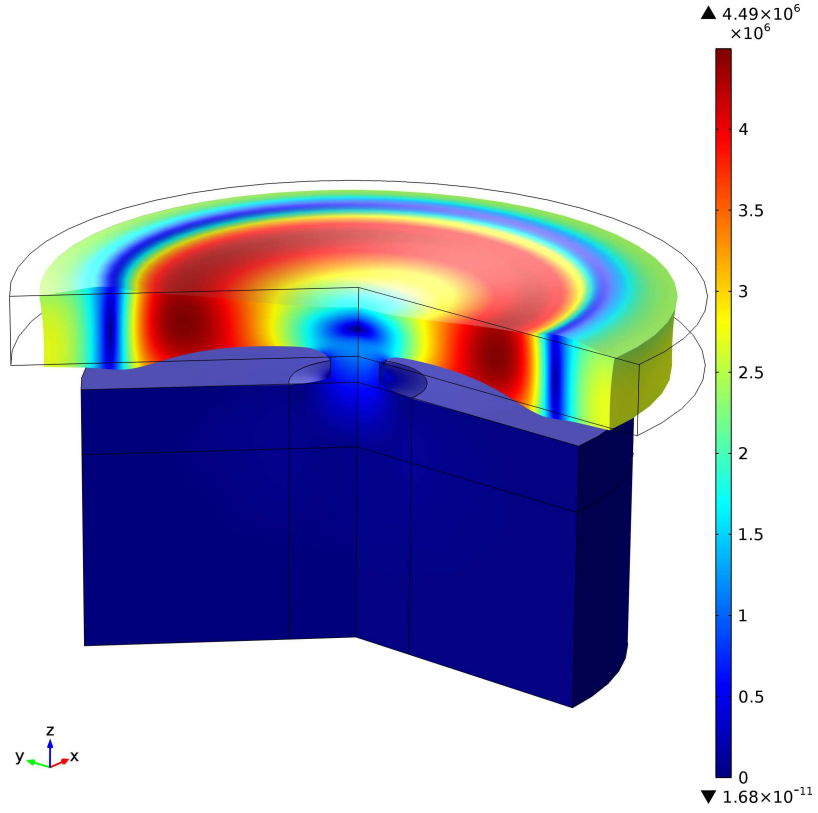


Figure 3.22: Disk resonator - 2nd radial contour mode. The colorbar represents the total displacement in μm .

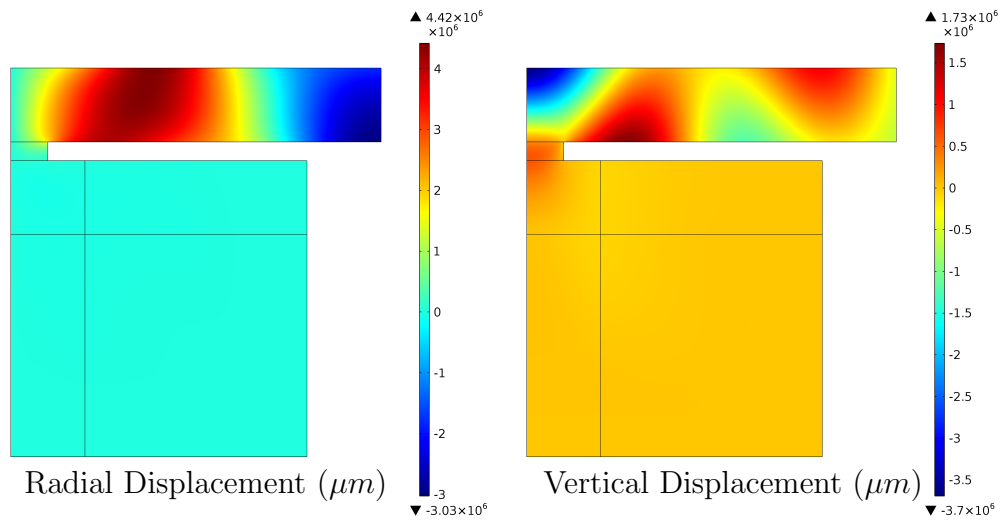


Figure 3.23: Radial and vertical displacement in 2nd radial mode of disk resonator.

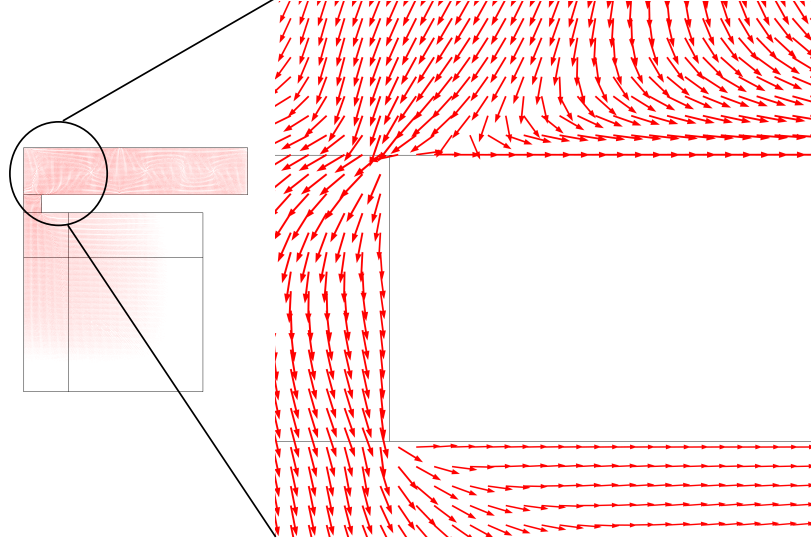


Figure 3.24: Mechanical energy flux vector field in the disk resonator.

Table 3.4: Frequency and $Q_{support}$ as a function of disk radius (r_{disk})

Disk radius, r_{disk} (μm)	10		18	
	Current work	Koyama[18]	Current work	Koyama[18]
Frequency (MHz)	706.22	706.5	393.57	393.6
$Q_{support}$	21642	21320	18631	18420

agreement is achieved compared with Koyama [18]. As r_{disk} increases, the frequency of the mode decreases due to the increased modal mass. $Q_{support}$ increases with r_{disk} , leading to less energy dissipation to the substrate. Qualitatively this can be explained by the fact that at low frequency, coupling between post motion and disk motion is small, therefore the stress exerted on the support region to excite wave propagation is much smaller compared to a higher frequency disk resonator.

3.4.3.2 Effect of Support Dimension

In this study, the post radius (r_{post}) is varied from $0.8\mu\text{m}$ to $1\mu\text{m}$ for a disk of $18\mu\text{m}$ radius. The results of the simulation are shown in Table 3.5 and compared with simulated and experimental results from Koyama [18] and Wang *et al.* [81], respectively. The experimental values for $Q_{support}$ are lower than the simulated values. This may be due to other energy loss mechanisms having a significant influence on the overall measured Q. Also, a simplified geometry was used in simulations compared

Table 3.5: Frequency and $Q_{support}$ as a function of disk radius (r_{post})

Post Radius, r_{post} (μm)	1			0.9			0.8		
	Current Work	Ref. [18]	Experiment [81]	Current Work	Ref. [18]	Experiment [81]	Current Work	Ref. [18]	Experiment [81]
$Q_{support}$	8607	8482	5551	12487	12330	11740	18631	18420	14660
Frequency (MHz)	393.68	393.7	405.2	393.62	393.6	404.6	393.57	393.6	404.1

to the experimentally fabricated devices. The change in post radius has negligible impact on frequency values as observed in the simulations and experiments. But, increasing r_{post} induces more energy dissipation through the support which lowers the $Q_{support}$ as seen from the results. A small change in r_{post} has significant influence on $Q_{support}$. It is evident that $Q_{support}$ is very sensitive to support post geometry and a careful design is required to achieve high Q.

3.4.3.3 Effect of Material Mismatch

It has been observed experimentally that the use of dissimilar materials for a disk resonator and its support structure can increase Q substantially [82]. Using an identical material for support and disk creates a perfect impedance ($Z = \rho\sqrt{\frac{E}{\rho}}$) match, therefore waves propagate without any reflections allowing maximum energy transfer from disk to support. For example, a disk with radius 11 μm and post radius 1 μm fabricated using polysilicon material has a Q-factor of 4500 [82]. The impedance mismatch between the disk resonator and post allows some waves to be reflected back into the resonator - this can be quantified in terms of the reflection coefficient which is the ratio of the reflected to incident waves amplitude. A result of this is that the Q-factor would be much higher, and for the aforementioned disk resonator a Q-factor of 27500 is achieved when dissimilar materials are used for disk and post [82].

In this section, a disk resonator vibrating in its 1st radial contour mode is investigated. Although perfect impedance is required between the PML and substrate, using different materials for resonator, support and substrate does not restrict PMLs to evaluate $Q_{support}$ in these more complex devices. For this study, a diamond disk (r_{disk}) is attached to a silicon substrate by a polysilicon post/stem (see Figure 3.25).

Table 3.6: Effect of material mismatch on $Q_{support}$ of a disk resonator

Disk Configuration	Similar Material [current work]		Material Mismatch				
			Current Work		Matched Layer [108]	Experiment [82]	
	Frequency (MHz)	$Q_{support}$	Frequency (MHz)	$Q_{support}$	$Q_{support}$	Frequency (MHz)	Overall Q
$r_{disk} = 12 \mu\text{m}$ $r_{post} = 0.8 \mu\text{m}$	217.94	8726	449.18	35930	34000	455.8	24117
$r_{disk} = 11 \mu\text{m}$ $r_{post} = 1 \mu\text{m}$	237.90	5071	486.80	12477	12000	495.64	27500
$r_{disk} = 10 \mu\text{m}$ $r_{post} = 0.8 \mu\text{m}$	261.48	8669	537.15	21935	21900	545.9	17458

The simulation is performed using a 2D axisymmetric model of the disk resonator with a cylindrical PML (effectively a spherical PML in 3D). The results obtained from the simulation are shown in Table 3.6, and compared with simulated and experimental results by Steeneken *et al.* [108] and Wang *et al.* [82], respectively. It is evident that impedance mismatch between diamond disk and polysilicon post increases the Q-factor. The results are in good agreement with experiment. However, the experimental Q-value is higher than the simulated results, for $r_{disk} = 11 \mu\text{m}$ and $r_{post} = 1 \mu\text{m}$, by approximately 120%. The reason for is not clear, but manufacturing variation can influence the experimental results. The comparison with experimental measurements also suggests that support loss is dominant in disk resonators. The propagation of waves in a disk resonator is shown in Figure 3.26, where the waves are absorbed well by the PML.

The impedance mismatch between the resonator and support can be utilised in other classes of resonator to reduce damping where support loss is dominant.

3.5 Modelling Strategy

It has been identified that the PML width (W_{PML}) and the end value of absorbing function (β) are the two most important parameters to accurately model support loss using PML. Two different modelling strategies have been used implicitly in Sections 3.3 and 3.4, respectively. This section summarises the two modelling strategies to be used for quantitative evaluation of $Q_{support}$ in MEMS resonators.

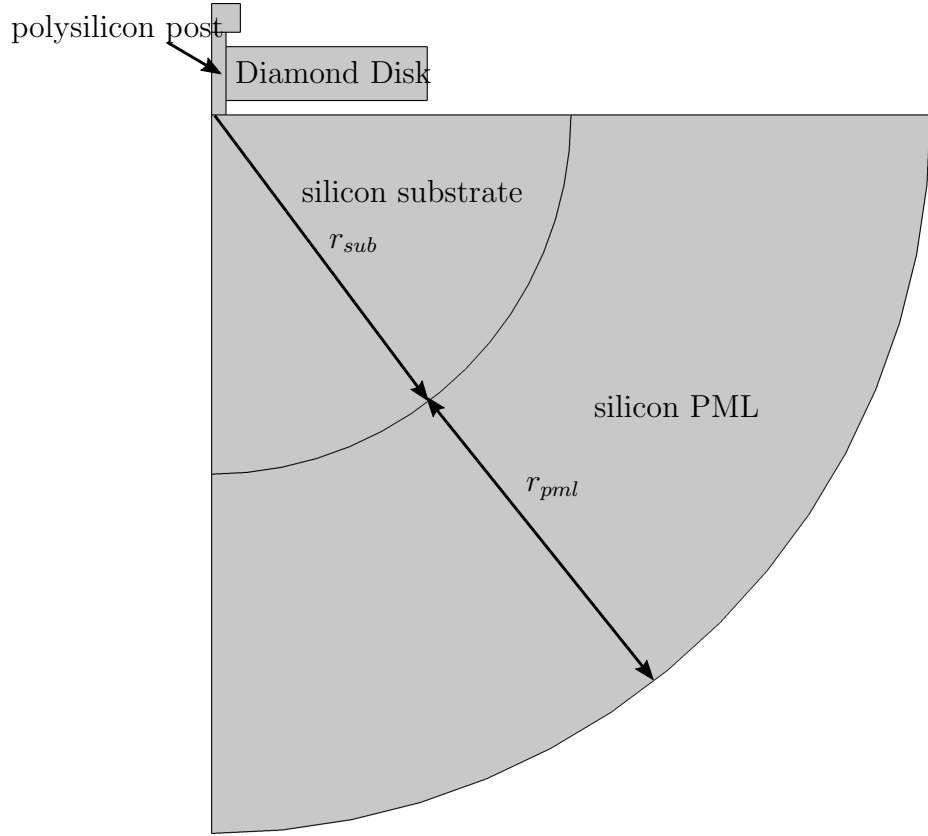


Figure 3.25: Geometry of the disk resonator with impedance mismatch between disk, post and substrate.

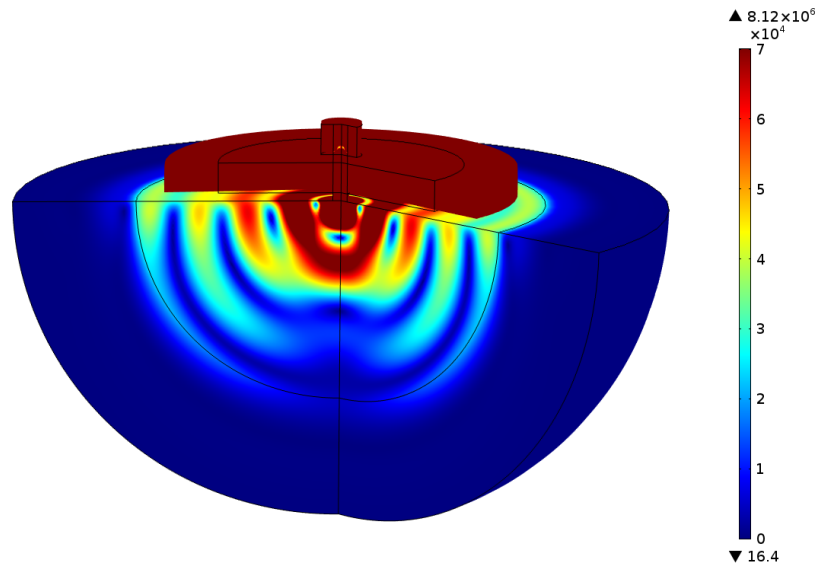


Figure 3.26: The total displacement in a disk resonator, showing propagation of wave in the substrate and PML. The color bar indicates total displacement in μm , and has been scaled down to emphasize the wave propagation.

- The size of the PML needs to be in same order of the wavelength, for appropriate attenuation of waves propagating in PML. Therefore, the first step is to calculate the wavelength using the undamped natural frequency of the resonator i.e. $\lambda_s = \frac{c_s}{f}$. For low frequency resonators selecting a PML width (W_{PML}) close the λ_s is not computationally feasible. For example, if the wavelength is few meters then it is impossible to model a structure of that magnitude while having a resonator in the size of few micro-meters. The modelling strategy can be categorised for low frequency and high frequency applications.
- **Low Frequency:** For low frequency resonators with longer wavelengths, W_{PML} should be selected to a computationally manageable size considering the size of the resonator. As a reference, W_{PML} could be selected as 1/4, 1/10, 1/100th etc. of the λ_s . The next step is to obtain the optimal value of β for which PML is perfectly matched, and a parameter study for a range of β using different number of mesh elements (N_{el}) is required to achieve convergence. The convergence in this case is defined as the β value(s) and number of elements (N_{el}) for which stable $Q_{support}$ value is obtained. For sufficient mesh density the convergence is expected at the optimal β value(s). The optimal β value(s) for the corresponding W_{PML} would absorb the waves appropriately, and divergence of $Q_{support}$ can be expected at any other β values higher than optimal value(s) during the parameter study. In most cases, linear absorbing function i.e. $p = 1$ works fine. However, in such cases where convergence issues occur $p=2$ or 3 should be used to absorb the waves without reflections.

It is worth mentioning that the optimisation of absorbing function for a given number of mesh elements is often cumbersome in terms of the computational cost. For example, a parametric study with eight β values and 110422 quadratic tetrahedral mesh elements (i.e. 815634 degrees of freedom to be solved for), require approximately 11 hours 30 minutes to perform the simulation. However, once the mesh parameters and optimal β value(s) are determined for a specific resonator geometry, the model can be used to conduct further studies

on resonator parameters efficiently.

- **High Frequency:** The high frequency resonators are relatively easier to model due to the shorter wavelengths, typically similar magnitude as the resonator dimensions. In this case W_{PML} should be selected close to λ_s , and a convergence study should be performed in terms of N_{el} . A linear absorbing function i.e. $p = 1$ and $\beta=1$ will provide appropriate attenuation of waves with adequate mesh elements.

3.6 Summary and Conclusions

Support loss occurs due to the transfer of energy from the resonator to the substrate, where energy is radiated by propagating waves. It is often difficult to model the damping behaviour using analytical models due to restrictive assumptions and proper evaluation of the propagating wave excitation source. For this reason, numerical techniques are considered. In this chapter, support loss modelling was achieved using Perfectly Matched Layers (PMLs) in a finite element analysis. PMLs are used as an absorbing boundary layer to mimic the semi-infinite/infinite behaviour of substrates (several order of magnitude larger than the resonator) without causing reflections of any of the propagating waves. A brief introduction to PMLs was presented based on the stretching of coordinates in the complex plane inside the PML region to attenuate waves. Theoretically, PMLs are reflectionless, but discretisation during FE modelling introduces numerical reflections which are generally resolved by using sufficient mesh density. The two most important parameters that influence accuracy and effectiveness of PMLs are the size of the PML region and the end value of the absorbing function. If the optimal combination of these two parameters are not selected, the results obtained from the simulations may be inaccurate. The effect of these parameters were investigated, and the following modelling strategies have been developed in order to quantify support loss in any class of resonator:

- Generally, high frequency devices have shorter wavelength. Therefore, selecting

a PML dimension close to the wavelength is computationally feasible. In such resonators, PML dimensions close to the transverse/shear wavelength provides accurate predictions of support loss with a linear absorbing function profile.

- Longer wave lengths (typically several order of magnitude larger than the resonator size) occur in low frequency resonators and make it difficult to select a PML size based on wavelength. For these resonators, the PML dimensions should be selected so that they have similar size to the resonator to reduce the computational effort. In addition, a parametric study of the end value of the absorbing function is required to determine the optimal value(s) to ensure a converged value of Q-factor is obtained. Adequate discretisation of the PML region ensures desired accuracy is achieved.

The objective of this chapter was to validate a PML-based FE model using COMSOL to predict support loss behavior in MEMS resonators. The support loss model is successfully validated, overcoming previous modelling issues, for cantilever beam and disk resonators. The results are in good agreement with analytical estimates, and published results from the research literature. Furthermore, the parameter study on resonator and support dimensions and material properties provided insight into ways to reduce support loss. It is observed that the Q-factor is very sensitive to support dimensions, a small change can significantly increase/decrease the level of damping in the resonator. Finally, the methodology presented in this chapter is used in the next chapter to analyse support loss in supported ring resonators.

Chapter 4

Support Loss in Ring Resonators

4.1 Introduction

In Chapter 3, the method of Perfectly Matched Layers (PMLs) was introduced and discussed as a means for quantifying support loss in MEMS resonators, where the PML acts to absorb waves propagating within the substrate. The current chapter considers support losses from ring resonators used in MEMS angular rate sensors. The ring resonator is supported by spokes/legs which connect it to the substrate and package. These devices have a more complex structure than the simple cantilever beam and disk resonators considered in the Chapter 3. Here, the purpose is to analyse and explain the support loss behaviour and consider the influence of design modifications and manufacturing variations.

Section 4.2 reviews an analytical approach for quantifying support loss for supported ring resonators with legs located on the outside of the ring that are attached to a substrate lying in the same plane as the resonator. The results obtained using this model are compared with results obtained using the FE model incorporating PMLs described in the previous chapter, and the key differences are discussed. Section 4.3 presents results for the support losses obtained for the resonator used in a nominal device. This section also investigates the influence of key design parameters, i.e. support and hub dimensions, and manufacturing variations, i.e. mass imperfection and systematic variation of hub location, on support loss for different modes of vibration. Section 4.4 analyses support losses for rings with high thermoelastic

damping Q-factor, and Section 4.5 provides a summary and concluding statements.

4.2 Analytical Approach: Review

As mentioned in Chapter 3, analytical models are available in the literature for simple structures, like beams, disks etc. Chouvion [19, 92] describe an analytical method used to calculate support loss i.e. $Q_{support}$ for supported ring resonators used in an inductive MEMS angular rate sensors with leg supports located outside the ring [14]. The method used is based on the work by Hao *et al.* [89] and Miller and Pursey [129], and a summary of the approach is presented below.

Figure 4.1 shows a schematic diagram of the supported ring resonators analysed by Chouvion [19, 92]. When the resonator vibrates, energy propagates from the ring through the supports into the substrate, causing energy dissipation. The stresses induced at the interface between the support and substrate cause energy to propagate in the substrate, leading to energy dissipation (see Chapter 1). For a cantilever beam resonator, the interface is subjected to shear stresses only [89]. However, the legs supporting the ring resonators induce both shear and normal stresses at the interface with the substrate, and it has been shown that the bending moment contributes little to support loss and can be neglected [89, 122]. However, the effect of bending moment can be important, for example, in supported disk resonators where the mode is not purely radial, but contains a bending motion which induces a vertical motion in the support [96]. For the special case when the bending moment is zero and the stress is uniformly distributed over the interface, it is possible to derive analytical expressions for the resulting displacement of the substrate [129]. The modelling can be simplified further if the resonator thickness is much smaller than the transverse/shear wavelength of the propagating elastic waves. In this situation the substrate can be modelled as a two-dimensional thin plate undergoing plane stress. These modelling assumptions are used by Chouvion [19] and Hao *et al.* [89] to determine analytical results for support loss.

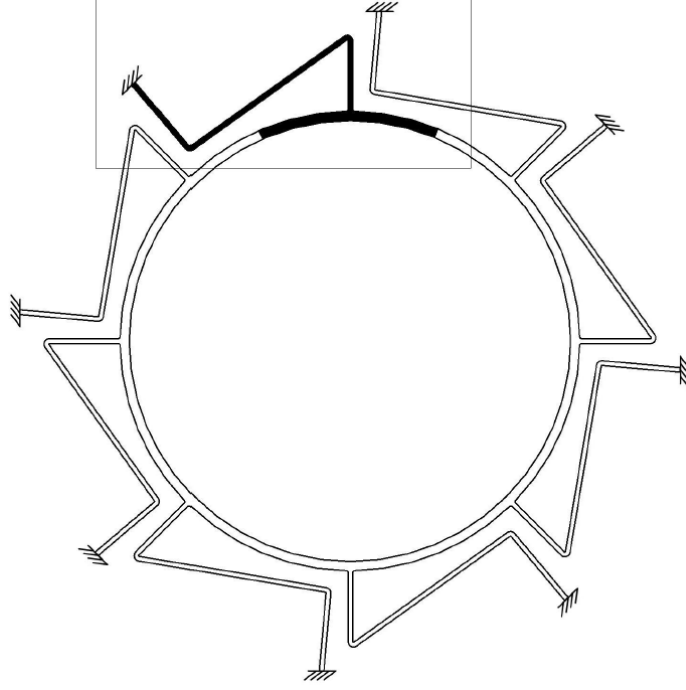


Figure 4.1: Schematic of a supported ring resonator, where the free ends of the support legs are considered to be clamped to the substrate [19].

The Q-factor for a resonator can be expressed as:

$$Q = 2\pi \frac{W}{\Delta W} \quad (4.1)$$

where W is the total energy stored in the resonator and ΔW is the energy lost per cycle.

The energy lost per cycle due to longitudinal (w) and flexural (u) vibrations are determined by:

$$\Delta W_w = \frac{4(1+\nu)}{Eh(1-\nu)} |F_N|^2 \text{Im}(\Psi_w) \quad (4.2)$$

$$\Delta W_u = \frac{4(1+\nu)}{Eh(1-\nu)} |F_S|^2 \text{Im}(\Psi_u) \quad (4.3)$$

where E is the Young's modulus, ν is the Poisson's ratio of the material and h is the thickness of the resonator and substrate (assuming they both lie in the same plane and have equal thickness). F_N and F_S are the normal and shear forces acting at the support/substrate interface, respectively. $\text{Im}(\Psi_w)$ and $\text{Im}(\Psi_u)$ represents the

contribution of mean longitudinal and flexural displacements to ΔW_w and ΔW_u , and are calculated numerically using the following equations:

$$\Psi_w = \int_0^\infty \frac{\sqrt{\zeta^2 - 1}}{(2\zeta^2 - r^2)^2 - 4\zeta^2 \sqrt{\zeta^2 - 1} \sqrt{\zeta^2 - r^2}} d\zeta \quad (4.4)$$

$$\Psi_u = \int_0^\infty \frac{\sqrt{\zeta^2 - r^2}}{(2\zeta^2 - r^2)^2 - 4\zeta^2 \sqrt{\zeta^2 - 1} \sqrt{\zeta^2 - r^2}} d\zeta \quad (4.5)$$

For example, $\text{Im}(\Psi_w) = 0.22153$ and $\text{Im}(\Psi_u) = 0.33503$ assuming $\nu = 0.28$. Here, r is the ratio of longitudinal ¹ (c_l) and transverse (c_s) wave speeds, i.e. $\frac{c_l}{c_s}$, and $\zeta = \xi \frac{c_l}{\omega}$ (ξ is a variable of the Fourier transform used in the derivation of the equations). The derivation of Eqs. (4.2) to (4.5) can be found in Chouvion [19].

Approximate expressions for the normal and shear forces and total energy stored (W) are determined by assuming the leg supports are clamped at their ends, and making use of the mode shape and frequency of the resonator. Chouvion [19, 92] achieved this using a novel ray tracing method, but the FE method could be used instead. Quantities ΔW_w and ΔW_u are calculated using Eqs. (4.2) to (4.5), based on the normal and shear forces at the clamped ends, and then combined together to determine the total energy lost (ΔW). The stored energy (W) is calculated from the modes shapes of the resonator. Finally, Q_{support} is calculated using Eq. (4.1).

This approach was used in Chouvion *et al.* [92] to predict support losses for the 2θ in-plane mode of a nominal ring resonator with support legs located outside the ring. In what follows, the FE model developed in the previous chapter is used to calculate the support losses for the same structure. The model is developed using the 2D plane stress assumption and rectangular PMLs are applied adjacent to the outer boundaries of the substrate. The PML arrangement used is shown in Figure 4.2, where it is worth noticing the distinct PML domains at the corners. Results are obtained using two different PML widths $W_{\text{PML}} = 4\mu\text{m}$ and $6\mu\text{m}$. In addition, following the modelling strategy outlined in Section 3.5, it is necessary to determine

¹ $c_l = \sqrt{\frac{\Lambda + 2\mu}{\rho}}$ and $c_s = \sqrt{\frac{\mu}{\rho}}$

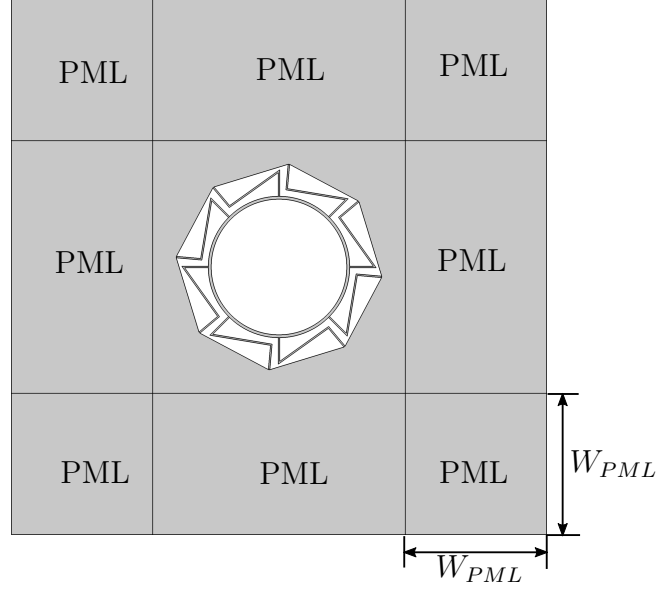


Figure 4.2: 2D model of the ring resonator with outer legs, surrounded by rectangular PMLs.

Table 4.1: Comparison of $Q_{support}$ obtained using FE and analytical model for 2θ in-plane mode

Model	Frequency (kHz)	$Q_{support}$	Q_{TED}	$Q_{overall}$	Measured Q [14]
Analytical [92]	14.233	147600	11383	10567	9455
FE	14.185	97460		10192	

the optimum β value(s) for the PMLs to accurately calculate support loss. This is described below. For further details on β , which is the end value of absorbing function, see Section 3.2.

Figure 4.3 shows a plot of calculated $Q_{support}$ for different values of β , for two different W_{PML} values considered in this study. Results are not presented for $\beta < 50$ because the $Q_{support}$ values obtained are highly unstable due to internal numerical reflections. For this reason Figure 4.3 is plotted for the range $\beta = 50$ to 400. It is evident from the results that $Q_{support}$ achieves a converged value in the range $\beta = 280$ to 300. As mentioned in Chapter 3, the term converged is used to define $Q_{support}$ values which are stable at the optimum β value(s).

Table 4.1 compares results obtained using the analytical approach [92], the FE method described in Chapter 3 and Wong's [14] TED predictions and experimental measurements, for the 2θ in-plane mode. It can be seen that the predicted $Q_{support}$

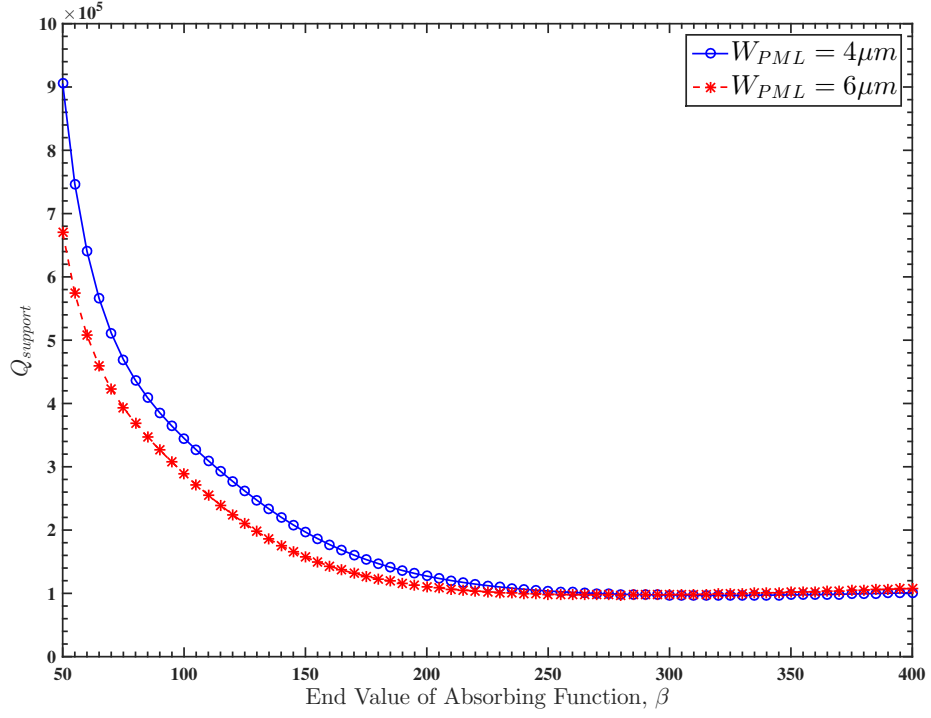


Figure 4.3: Convergence of $Q_{support}$ (2θ in-plane mode) as a function of β for different W_{PML} .

values are all much higher than the TED prediction and measured results, indicating that support loss is not the dominant source of damping for this device. The FE method predicts higher levels of support loss compared to the analytical method. The main reasons for the difference are: i) the leg/substrate decoupling assumptions used to derive the analytical results; and ii) the assumption that the support legs are rigidly clamped to the substrate; which are both used in the derivation of the analytical results. In practice the support legs are attached to a flexible (non-rigid) substrate having finite stiffness, which is considered in the FE model but not in the approximate method.

The results indicate that the analytical model is capable of predicting the relative magnitude of support loss, but its accuracy is limited by the assumptions made. However it is important to note that the analytical model is only applicable when the resonator and substrate lie in the same plane. The following sections consider ring resonators that are supported centrally and do not lie in the same plane as the substrate. For these resonators it is not valid to apply the 2D plane stress assumption

when quantifying the support loss, and the FE model described in Chapter 3 must be applied instead.

4.3 Support Loss in a Nominal Device

The modelling and analysis of support losses in the nominal device are considered in this section. The damping behaviour of different modes of the resonator is analysed, and parameter studies are performed to understand the effects of design parameters and manufacturing variations on support loss.

4.3.1 Description of the Device

The nominal device used in this work is a ring based angular rate sensor which utilises electrostatics to excite the drive mode and to detect the sense mode. The general lay-out of the device is introduced and discussed in this section.

Figure 4.4 shows a schematic of the sensor including the supported resonator, insulators and base layer. The ring resonator is mounted to a central hub that is rigidly connected (anodically bonded) to two insulator (glass) plates above and below the resonator. The insulator layers, central hub and supported ring resonator form one body, called the *die* [130]. The bond layer connects the die to the base, and the die, base and lid together form one body referred to as the *package* [130]. The base of the device where the lower glass plate is bonded is referred to as the substrate in this chapter. The resonator for the nominal device consists of a ring having 7.797 mm diameter with 203 μm radial thickness and 38 μm thick legs. The ring is designed to have low levels of TED by incorporating a double layer of slots at equal distances from the neutral axis around the ring circumference.

Figure 4.5 shows results obtained from a Finite Element (FE) study of the structural mode shapes for the supported ring resonator used in the nominal device. In the FE model the central hub is assumed to be rigidly fixed, so no account is taken of the flexible insulator plates, die bond, base or lid. The results indicate that the 10

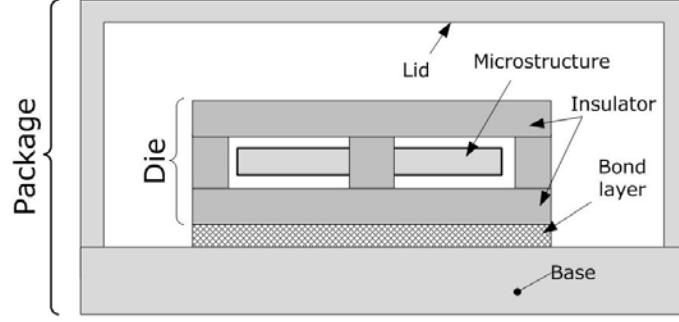


Figure 4.4: Schematic drawing of the whole sensor with the resonator (microstructure) enclosed in a sealed cavity formed by two insulator layers [130].

in-plane mode, 1θ out-of-plane (rocking) mode and bounce mode involve rigid body motions of the ring on flexible leg supports, whilst the 2θ in-plane, 3θ in-plane and 2θ out-of-plane modes involve flexural motion of the ring and leg supports.

4.3.2 Model Setup

As described in Figure 4.4 and Section 4.3.1, the ring resonator in the nominal device is connected to glass plates above and below the resonator via a central hub. Considering intermediate layers of glass plate and die bond makes it difficult to model. The mechanical properties of die bonds are not well understood, and requires the consideration of hyper-elastic material during modelling. To simplify the modelling in this study the hub is considered to be attached to a semi-infinite silicon substrate below the ring resonator, as shown in Figure 4.6. The purpose of this model is to represent the support loss behaviour in this device. However, an investigation using glass substrate is presented later in this chapter. A hemispherical PML is used to model support loss and avoids the need for any special corner treatment required for rectangular PMLs. The model size is reduced by a factor of 2 by imposing symmetric boundary conditions, which reduces computational effort. The resonator and the substrate assembly is meshed with tetrahedral elements, and swept meshes are used to discretise the PML region, as shown in Figure 4.6(b). It is worth mentioning that the slots incorporated in the ring are not been considered in the modelling, since the slots are used to reduce TED and have no effect on support loss.

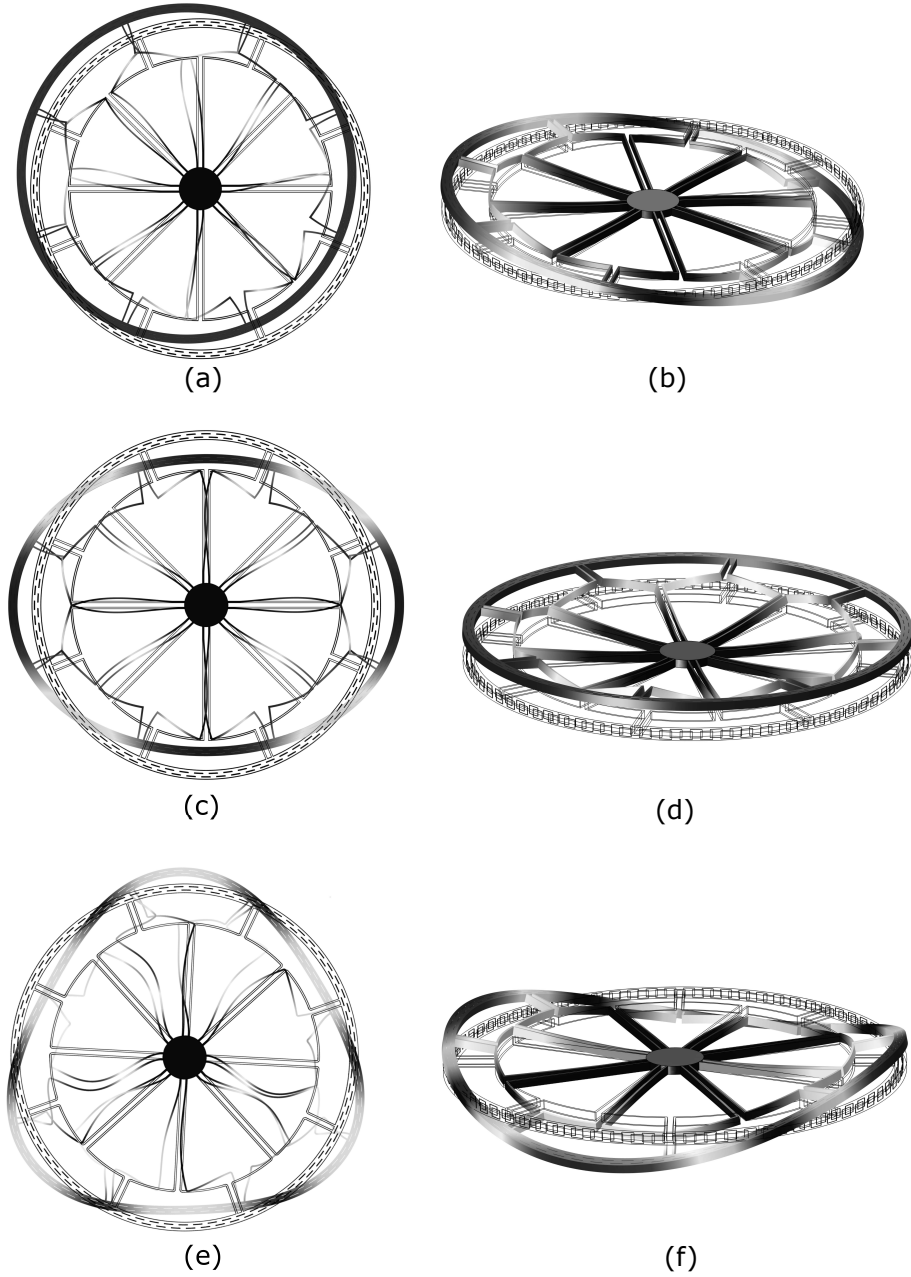


Figure 4.5: Analysed mode shapes for the nominal device: (a) 1θ in-plane mode; (b) 1θ out-of-plane (rocking) mode; (c) 2θ in-plane mode; (d) bounce mode; (e) 3θ in-plane mode, and (f) 2θ out-of-plane mode.

Table 4.2: r_{PML} values considered to quantify support loss in different modes of the nominal device

Mode	r_{PML} (mm)	r_{sub} (mm)
1 θ in-plane	10	2
1 θ out-of-plane	10	
Bounce	10	
2 θ out-of-plane	6	
2 θ in-plane	4	
3 θ in-plane	4	

For the resonator considered, the frequency of the 2 θ in-plane mode is ≈ 14 kHz, which translates to shear wavelength ² (λ_s) ~ 0.39 m, based on the following materials properties: $E=170$ GPa, $\rho=2330$ kg/m³ and $\nu=0.22$. Following the modelling strategy discussed in Section 3.5, a PML radius $r_{PML}=4$ mm ($\approx \lambda_s/100$) was considered to reduce the computational effort. FE predictions were conducted for a range of number of elements (N_{el}) values, to identify the optimum β value(s) that provide converged $Q_{support}$ values for the selected r_{PML} value. A linear absorbing function i.e. $p = 1$ was also considered. A similar approach was used for the other modes shown in Figure 4.5. The 1 θ in-plane mode has the lowest frequency ≈ 4.2 kHz ($\lambda_s \sim 0.13$ m) and a $r_{PML}=10$ mm is selected for this mode. A summary of r_{PML} values for different modes of the nominal device used in this chapter is presented in Table 4.2. In the next section, the optimal β value(s) for each of these modes are determined based on a convergence study for $Q_{support}$.

4.3.3 Convergence Study to Calculate $Q_{support}$

This section presents a convergence study to calculate $Q_{support}$ for different modes of vibration in the nominal device.

Figure 4.7 plots $Q_{support}$ for 2 θ in-plane mode against β values for different numbers of elements, N_{el} . For $\beta < 20$, $Q_{support}$ values are highly unstable due to numerical reflections, and for this reason $Q_{support}$ is plotted for β in the range 20 to 100.

The results indicate the Q-factor converges to 1.82×10^{11} for $\beta=45$ and 50 and N_{el} between 1.14×10^5 and 1.35×10^5 . In this case, the convergence refers to the stability

² $\lambda_s = \frac{c_s}{f}$, where f is the frequency

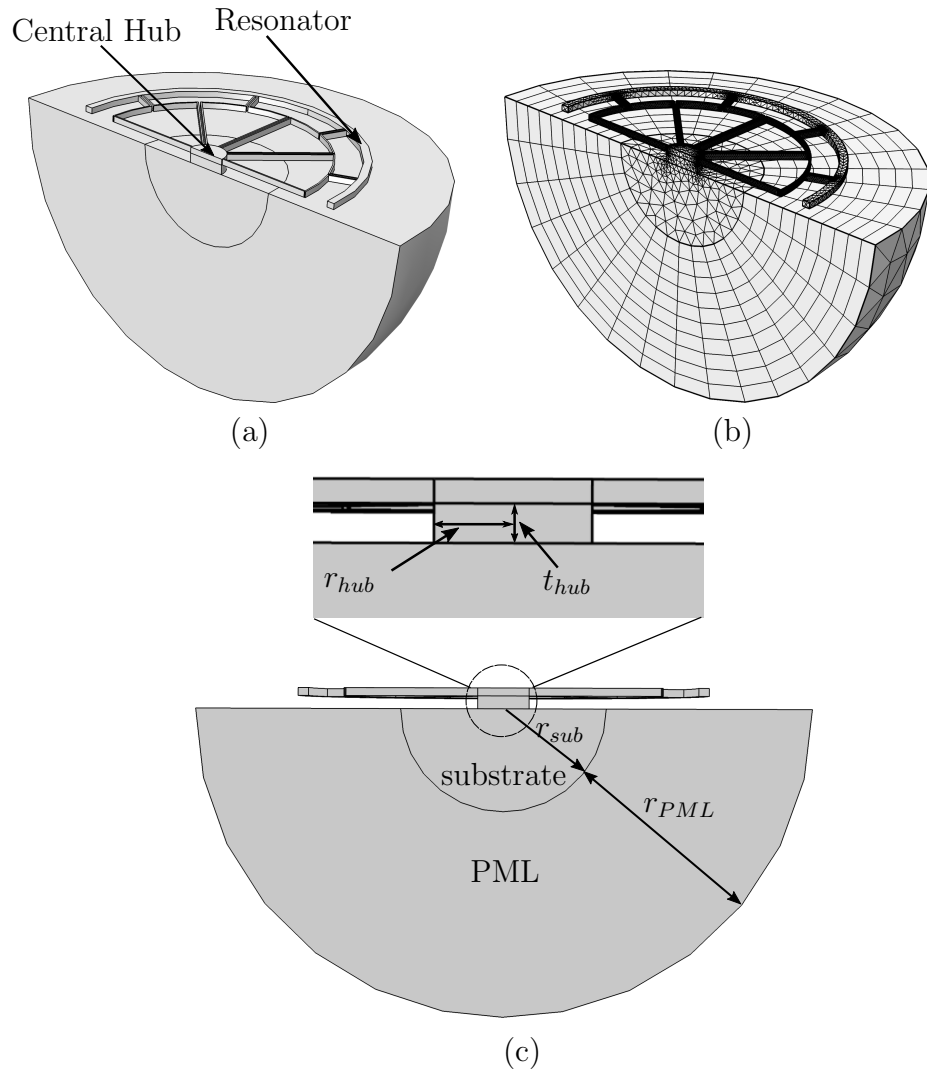


Figure 4.6: (a) Half-symmetry model of the nominal device with PML and substrate, (b) meshed geometry, and (c) cross-sectional view of the model.

of $Q_{support}$ value for a combination of β value(s) and increasing number of elements. As seen in Figure 4.7, the $Q_{support}$ is stable or reaches minimum value at $\beta=45$ and 50 with the increase in N_{el} before increasing again. This β value(s) refers to the optimal value, and can be used for parametric studies efficiently with the sufficient mesh elements. Larger β values induce reflections that pollute the solution, causing $Q_{support}$ to increase as discussed in Chapter 3. The effect of PML size has also been investigated and it was found that $r_{PML}=6$ mm also yielded $Q_{support} \approx 1.82 \times 10^{11}$ for β in the range 40 to 45-these results are not shown in Figure 4.7. Based on these results the value of β used in later sections is $\beta=45$ for the 2θ in-plane modes. It is also worth noting that, in general, the computational resources limit the maximum number of mesh elements to $\sim 1.35 \times 10^5$ for the results shown in this chapter. Figure 4.8(a) shows a plot of the 2θ in-plane mode, whereas Figure 4.8(b) to (d) illustrates the wave propagation from the central hub to the substrate and PML. The maximum range of total displacement (in μm) is shown in the color bars and is scaled down gradually to show the propagating waves responsible for energy dissipation, as they are absorbed inside the PML.

To optimize the β values for all modal frequencies, it is necessary to consider convergence curves similar to Figure 4.7 for all modes. Figure 4.9 shows the convergence curve for the 3θ in-plane mode, where it can be seen that $Q_{support}$ is stable in the range $\beta=30$ to 35. Similarly, Figure 4.10 shows the convergence curve for the leg dominated bounce mode and 1θ in-plane and out-of-plane modes, and Figure 4.11 shows the convergence curve for 2θ out-of-plane mode. The optimal β value(s) based on the r_{PML} values mentioned in Table 4.2 for each of these modes are summarised in Table 4.3.

For the purpose of comparison, $Q_{support}$ values for the different modes are summarised in Table 4.4. It is evident that the ring dominated modes have much lower levels of damping compared to the leg dominated modes. Also, it is worth mentioning that the out-of-plane leg modes have relatively low Q . An analysis of TED for the 2θ in-plane mode (without slots) showed a $Q_{TED}=25456$ which is much lower compared

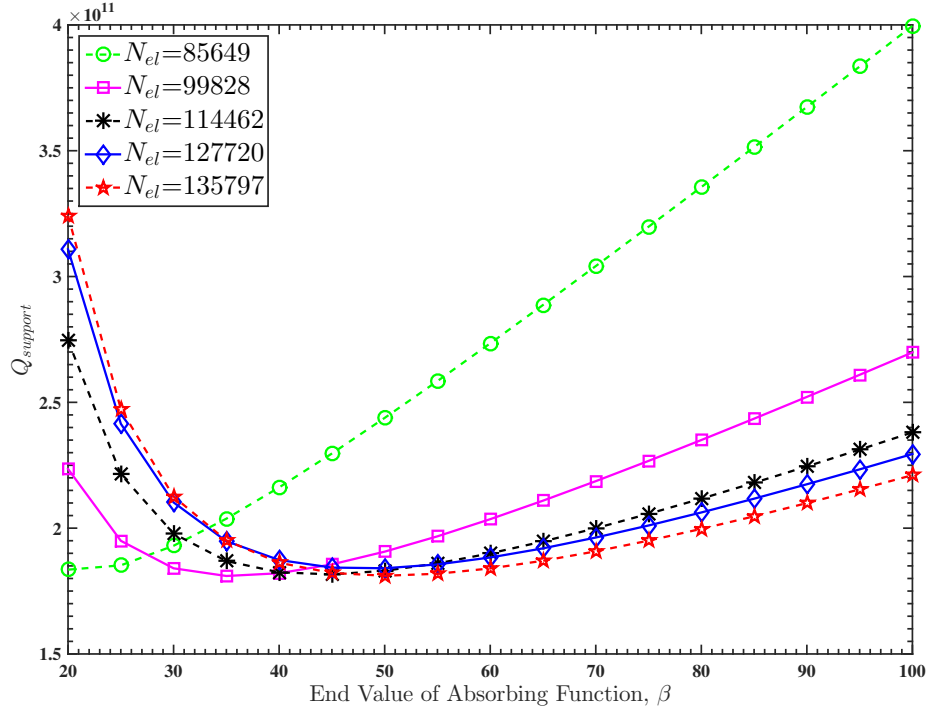


Figure 4.7: Convergence of $Q_{support}$ for 2θ in-plane mode as a function of N_{el} and β (nominal device).

Table 4.3: Optimal β value(s) for different modes of the nominal device

Mode	Optimal β value(s)
1θ in-plane	45-50
1θ out-of-plane	35
Bounce	45-50
2θ out-of-plane	50
2θ in-plane	45-50
3θ in-plane	30-35

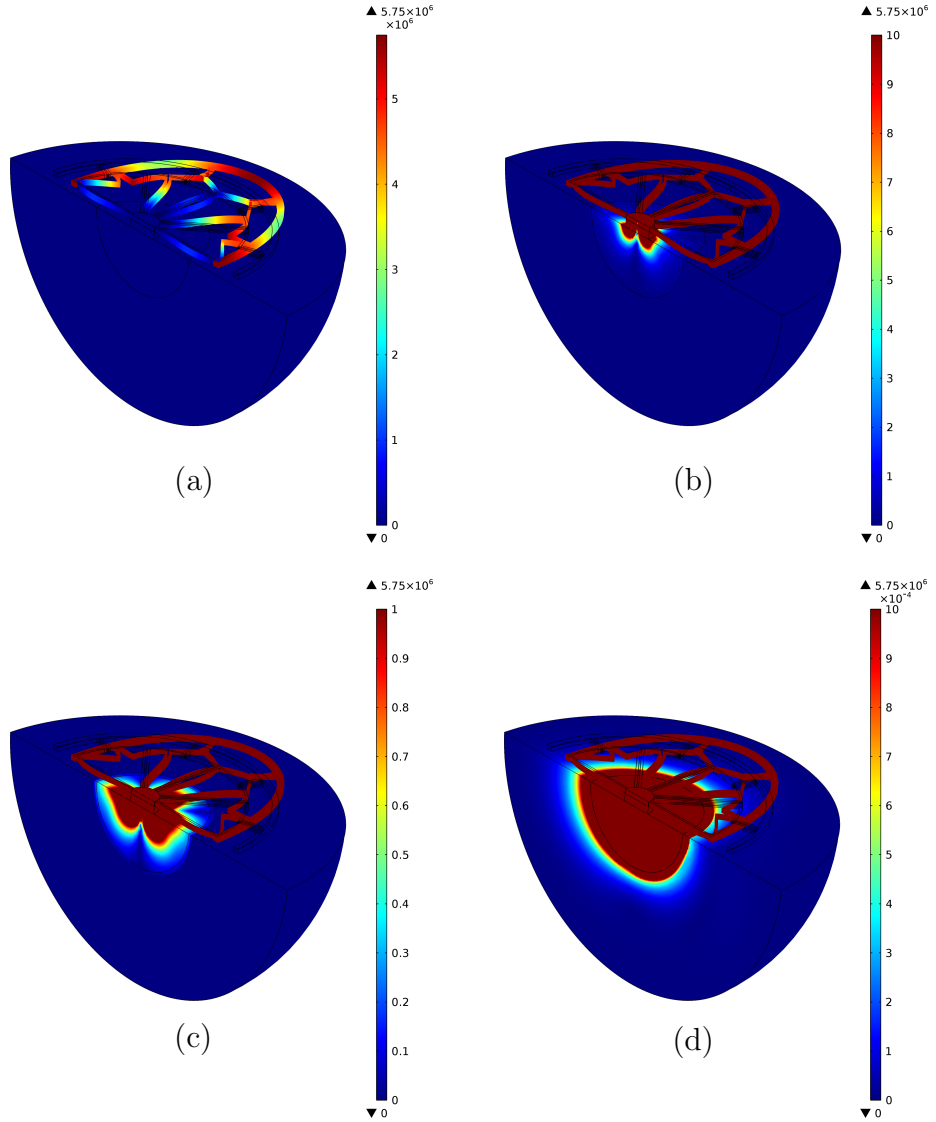


Figure 4.8: (a) 2θ in-plane mode of the ring resonator, (b)-(d) total displacement of the resonator, hub and substrate, showing propagation of wave in the substrate and PMLs. The color bars are in μm , and have been scaled down gradually from (b) to (d) emphasizing wave propagation from central hub to substrate.

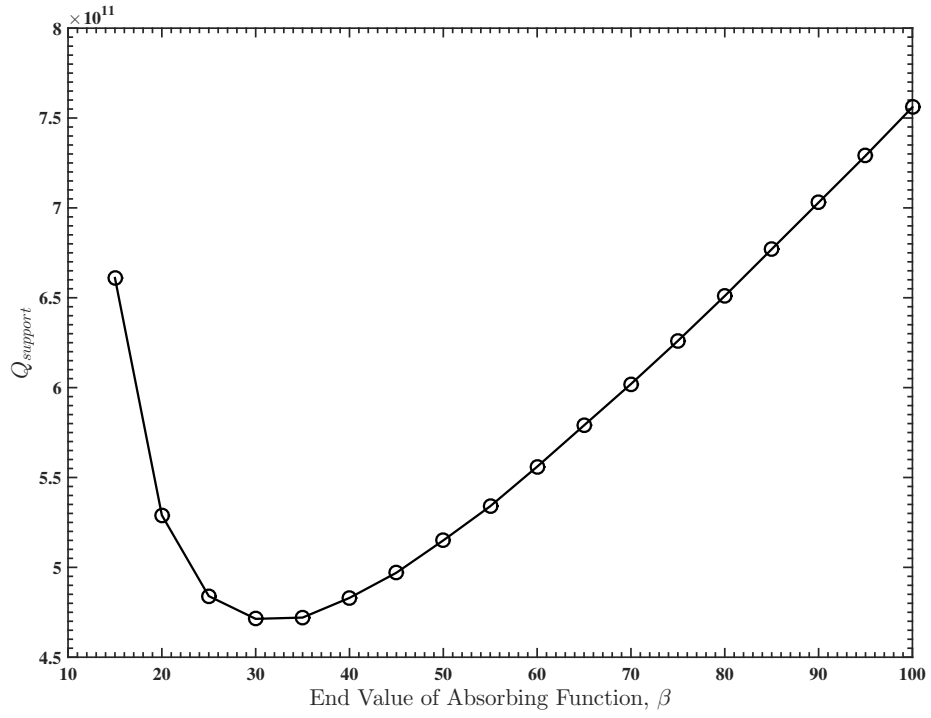


Figure 4.9: Convergence of $Q_{support}$ for 3θ in-plane mode as a function of β (nominal device).

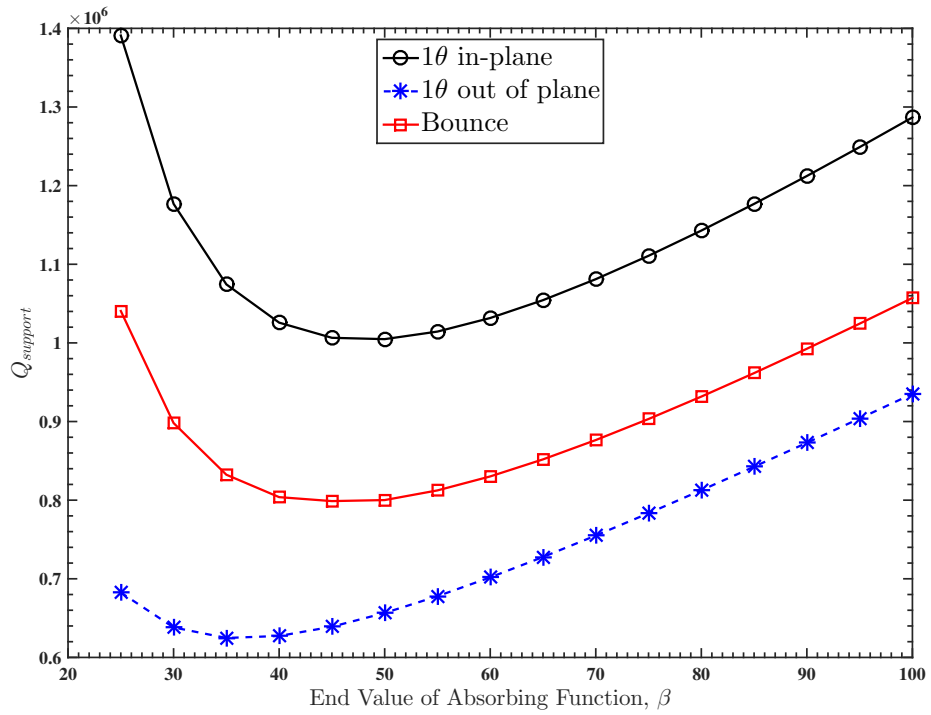


Figure 4.10: Convergence of $Q_{support}$ for 1θ in-plane, 1θ out-of-plane mode, and bounce mode as a function of β (nominal device).

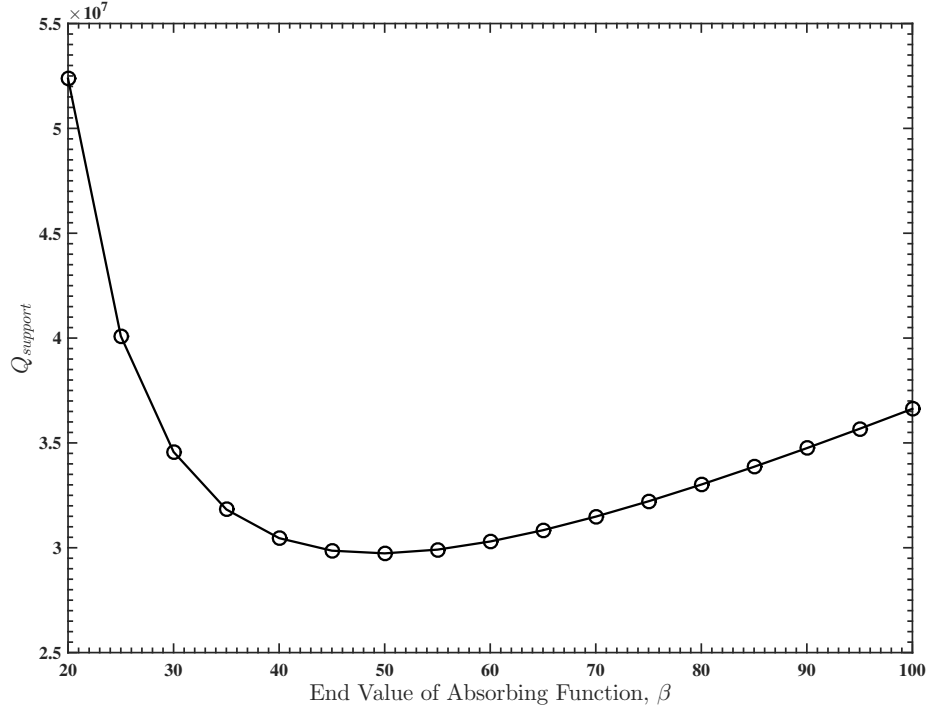


Figure 4.11: Convergence of $Q_{support}$ for 2θ out-of-plane mode as a function of β (nominal device).

Table 4.4: Frequency and $Q_{support}$ values for different modes of the ring resonator in nominal device

Mode	Frequency (kHz)	$Q_{support}$
1 θ out-of-plane	4.228	6.25×10^5
1 θ in-plane	4.879	1.01×10^6
Bounce	5.151	8.04×10^5
2 θ out-of-plane	11.091	2.97×10^7
2 θ in-plane	13.972	1.82×10^{11}
3 θ in-plane	33.080	4.72×10^{11}

to the calculated $Q_{support}=1.82 \times 10^{11}$ for this mode. This indicates that the amount of support loss dissipation is extremely small for the 2θ in-plane mode. A further analysis on TED in other modes is discussed in Chapter 5.

4.3.4 Support Loss Behaviour in a Supported Ring Resonator

This section investigates the dominant factors that influence the support loss in supported ring resonators. In particular, the stresses exerted across the clamped/attachment region on the substrate and the dynamic balance of the resonating mode are consid-

ered.

The 3D model used to quantify the stresses across the clamped/attachment region in this section is shown in Figure 4.12. As illustrated in Figure 4.12, the supported ring is attached to a central hub with eight pairs of attachment points around the circular hub, referred to as the leg/hub interface. The central hub is attached to a region on the substrate referred as the hub/substrate interface. However, it is worth mentioning that support loss is quantified using the symmetric model described in Section 4.3.2 and illustrated in Figure 4.6.

The vibrating ring resonator causes harmonic stresses to act across the leg/hub interface generating elastic waves to propagate in the substrate. The energy dissipated through elastic wave propagation can be formulated as the integral of the product of the stress and vibration displacement over the attachment interface per cycle [79]. Based on this formulation and the work of Miller and Pursey [129], Hao *et al.* [79] derived mathematical expressions for calculating support loss in a MEMS resonator considering different cases: i) location of the resonator (within or on top of a substrate), ii) stress source (normal or shear), and iii) number of stress sources (single or double). For example, for a resonator located on the top of a substrate with rectangular attachment area (see Figure 4.13), the energy loss due to vibration of the resonator caused by normal and shear stresses are expressed in [79] as

$$\Delta W = \frac{4}{\pi^2} d^2 c^2 \sigma_{zz}^2 \omega \left\{ \frac{\gamma}{2\rho C_s^3} \Pi_6(\gamma) \right\} \quad (4.6)$$

$$\Delta W = \frac{4}{\pi^2} d^2 c^2 \tau_{zy}^2 \omega \left\{ \frac{1}{2\rho C_s^3 \gamma} \Pi_7(\gamma) \right\} \quad (4.7)$$

where σ_{zz}^2 and τ_{zy}^2 are the uniform normal and shear stress, $2d$ and $2c$ are the length and width of the rectangular attachment area, ω is the frequency of the resonator and γ is the ratio of longitudinal (c_l) and transverse (c_s) wave speeds, i.e. $\frac{c_l}{c_s}$. Π_6 and Π_7 are integral constants, and their values for different materials are given in Hao *et al.* [79].

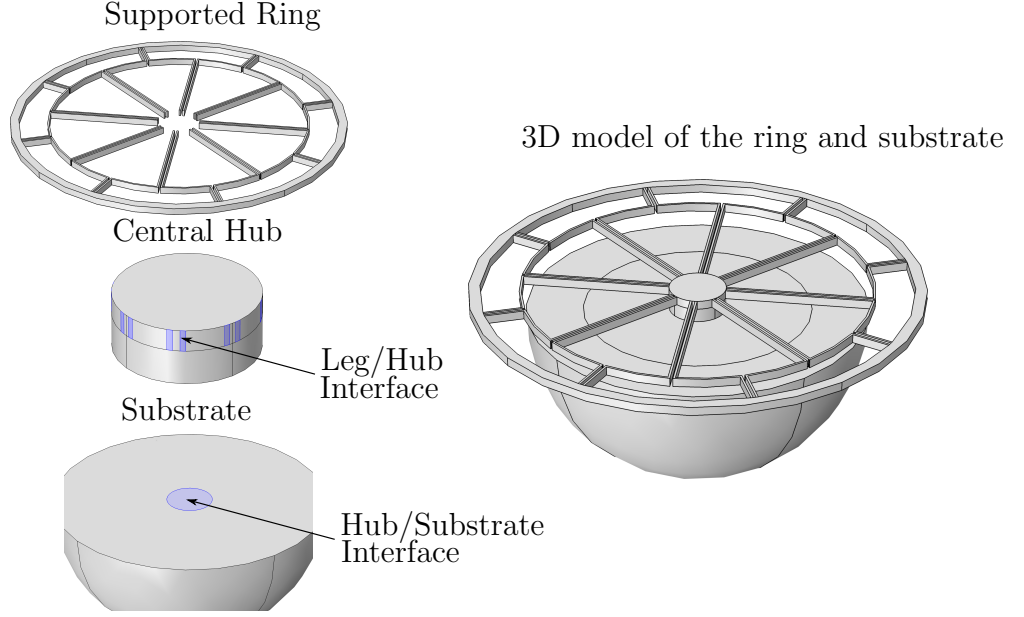


Figure 4.12: The interface between the support legs/hub, and hub/substrate in the model, highlighted in blue.

Using Eqs. (4.6) and (4.7) for energy dissipated, $Q_{support}$ can be calculated from Eq. (4.1). It is evident from these equations that the energy loss is directly related to the stresses at the attachment region, and is proportional to the stress squared. It can also be seen that support loss is linearly proportional to the frequency of the resonator and the square of the attachment area. For a ring resonator, the support legs flex during vibration and exert stresses at each of the attachment points around the hub. The resultant effect of these stresses is elastic wave propagation through the interface with the hub into the substrate. Ultimately the amount of energy radiated away by the propagating wave is dependent on the level of stress exerted at the hub/substrate interface.

Stresses at the Interface: It is clear from Table 4.4 that the flexural modes of a ring resonator have much higher Q values. Additional insight into the observed levels of damping for different modes is gained by evaluating the stresses at the interface between hub and substrate. For this purpose, averaged von Mises stresses across the interface for different modes are compared. The von Mises stress takes account of all stress components across the hub/substrate interface, and therefore provides insight

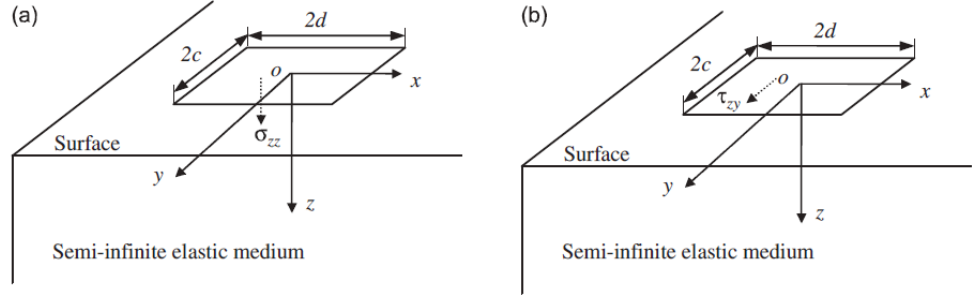


Figure 4.13: (a) Single normal stress source and (b) single shear stress source applied on top of a semi-infinite elastic medium (substrate) [79].

into the relative stress levels for comparison.

Figure 4.14 shows results for the von Mises stresses at the hub/substrate interface for different modes. The stresses have been normalised with respect to the result for the 2θ in-plane mode. The stress levels at the interface are highest for the 2θ out-of-plane mode. Similarly, the bounce mode and 1θ out-of-plane mode also exert relatively high stress levels. In contrast the 2θ in-plane mode has the lowest interface stress level. Interestingly, the 3θ in-plane mode has a higher stress level compared to 2θ in-plane mode, but $Q_{support}$ is higher for this mode. The reason why the interface stress levels are higher for 3θ in-plane mode is the dynamic balance of the modes, which is explained later in this section. Similar behaviour can be seen for the bounce mode and 1θ out-of-plane mode.

Depending on the resonant mode, the stress source at the interface can be either shear stress or normal stress. Therefore, it is important to identify which stress source provides the dominant contribution to support loss for each of the modes considered in this section. The levels of normal and shear stresses at the interface for each mode are compared in Figure 4.15. As illustrated in the figure, normal stress is dominant for the bounce mode and the 2θ out-of-plane mode. In contrast, the shear stresses are dominant for both 1θ modes and, the 2θ and 3θ in-plane modes. This is due to shear forces exerted at the interface, by the flexing of the ring and support occurring in a plane parallel to the substrate's plane. As a consequence, the motion of the hub is considered to be in-plane.

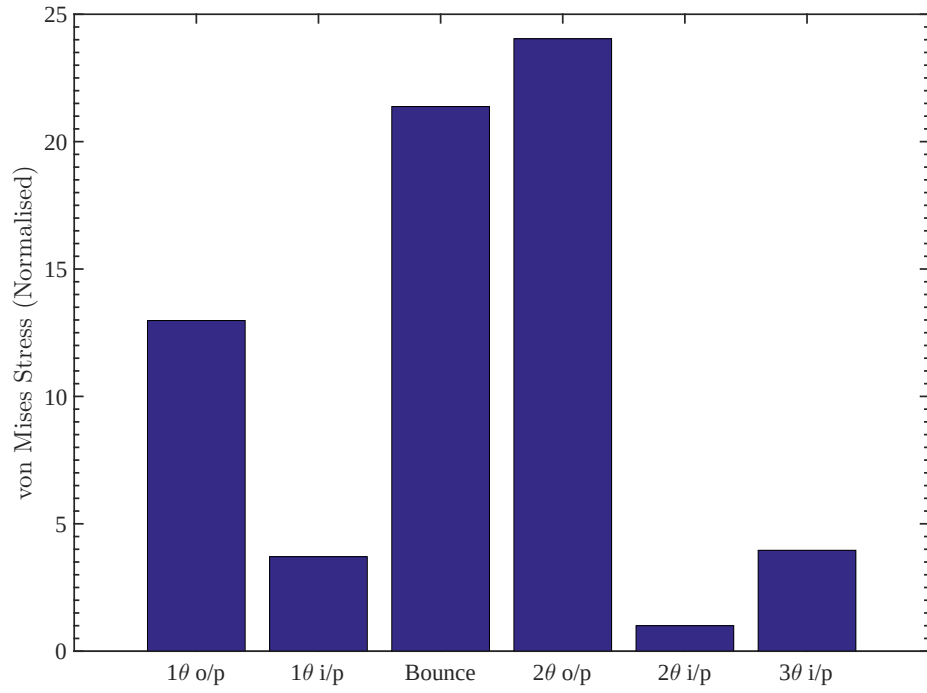


Figure 4.14: The level of von Mises stress at the interface between hub and substrate for different modes of the nominal device. (i/p: in-plane, and o/p: out-of-plane)

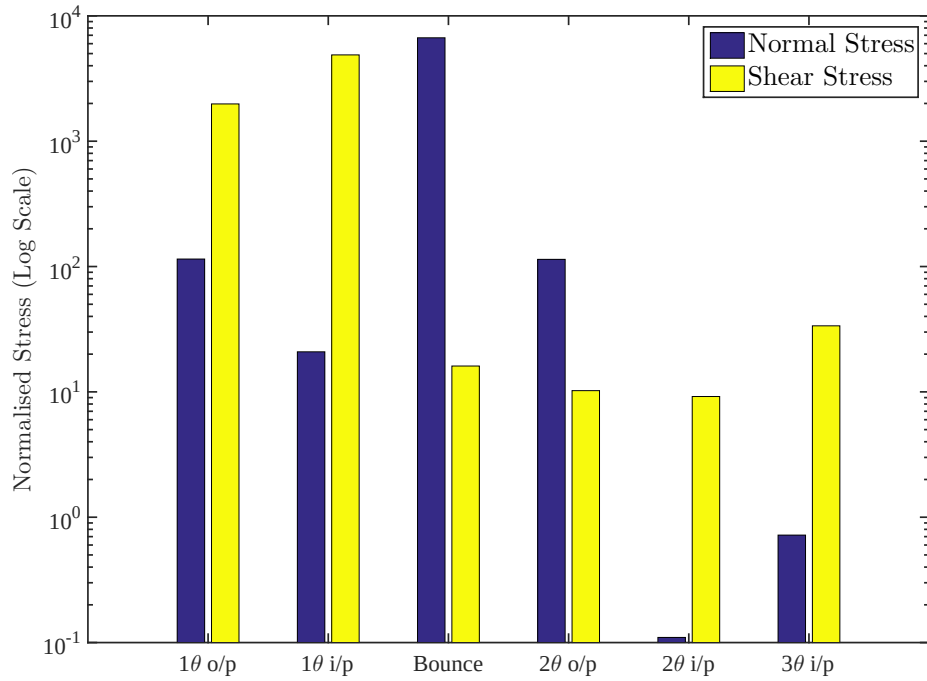


Figure 4.15: Comparison of the contribution from normal and shear stresses for each mode of the nominal device.

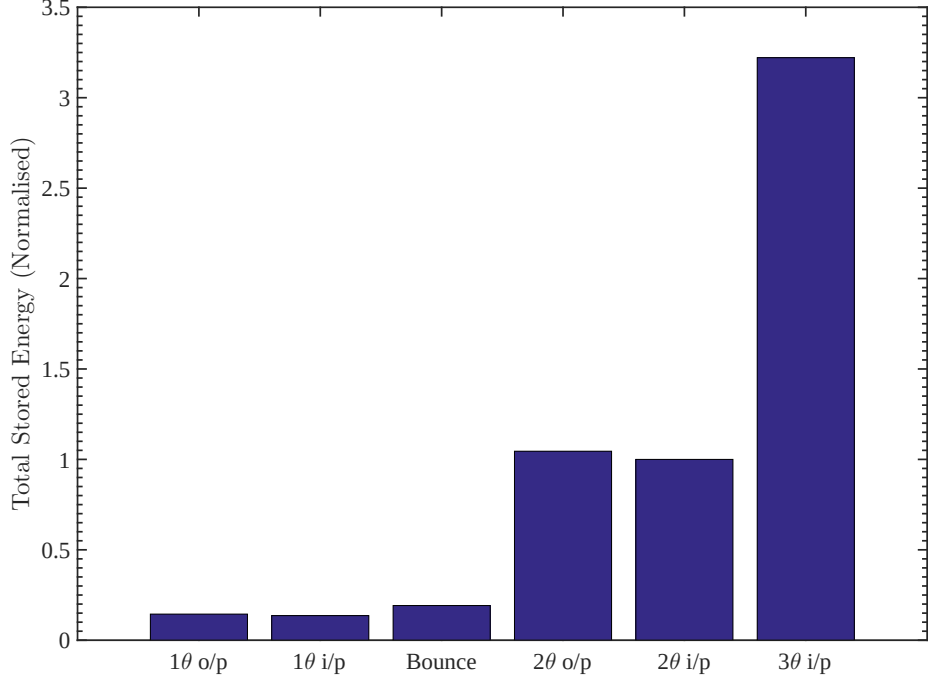


Figure 4.16: Comparison of total stored energy in different modes of a nominal device.

The observed damping behaviour can be explained using Eq. (4.1), where $Q_{support}$ depends on the energy dissipation (ΔW and total energy stored in the resonator W). It is evident from the equation that for same level of energy dissipation, higher levels of Q-factor can be achieved for a mode with a larger amount of stored energy. The normalised total stored energy in the resonator for different modes is illustrated in Figure 4.16. It can be seen that the amount of energy stored is higher for the 3θ in-plane mode than the 2θ in-plane mode, this fact is likely to cause the higher $Q_{support}$ value obtained for 3θ in-plane mode. In contrast, for the leg dominated modes (1θ in-plane, bounce and 1θ out-of-plane) the levels of energy stored are much lower than for the 2θ in-plane mode. This is because the ring does not deform for these modes, so energy is only stored in the legs. In essence, the leg dominated modes exhibit low $Q_{support}$ values (as observed in Table 4.4) due to higher stresses at the interface and lower levels of stored energy. For ring-dominated modes, the highest levels of damping are observed for the 2θ out-of-plane mode due to the higher stresses generated at the hub/substrate interface.

Dynamic Balance: Stemme [84] discussed and explained the importance of balanced vibration in different resonator types e.g. double- and triple-beam, and bridge structures used in tuning fork resonators. For a resonator to be perfectly balanced it must have a mass centre that is motionless as the resonator oscillates, i.e. the resultant force acting at the mass centre must be zero throughout the oscillation period. Beeby *et al.* [131] investigated the damping performance of tuning fork resonators computationally by analysing the stress levels at the fixed supports. It was inferred that high stresses indicate an out-of-balance mode or structure with high energy loss and mechanical coupling with the supports. As such, unbalanced resonators are expected to generate energy losses at the mounts/supports, which degrade the Q-factor. For this reason, the selected mode of operation is important as it needs to ensure the resultant force is zero to eliminate energy dissipated due to support loss.

A perfect unsupported ring has inherent cyclic symmetry and a fixed centre of mass when the ring vibrates in the 2θ in-plane mode, i.e. a balanced mode. A characteristic of a balanced mode is that the nodal points occur in pairs with each member of a pair being located directly opposite the other; this applies to anti-nodal points as well. The interaction with ring supports is minimised by selecting the number of support legs to maintain the cyclic symmetry of a particular mode, so the frequencies do not split [10]. These attributes are evident in the design of the supported ring resonator used in the nominal device, where there are effectively 8 uniformly spaced support locations. It is likely that this will contribute to achieving low levels of support damping for the 2θ in-plane mode.

Higher stress levels indicate the presence of an out-of-balance mode. From Figure 4.14 it is evident that the 2θ in-plane mode has the lowest levels of stress at the interface for the modes considered, indicating a high level of balance. In contrast, the bounce mode and 2θ out-of-plane mode have high levels of stress, confirming they are unbalanced. Figure 4.14 indicates that the out-of-plane modes (both ring and leg dominated modes) exert much higher stresses at the interface and are therefore highly unbalanced. However, it should be re-emphasized that $Q_{support}$ also depends

on the total energy stored. The higher stresses for the 3θ in-plane mode compared to the 2θ in-plane mode indicates that the 3θ in-plane mode is an unbalanced mode. However, due to the larger amount of energy stored, the 3θ in-plane mode exhibits a slightly higher $Q_{support}$.

4.3.5 Design Parameter Study

This section focuses on considering the influence of key design parameters of a supported ring resonator on support loss. The insights gained into their effect will help engineers design resonators with reduced levels of support damping. As ring based angular rate sensors excite and sense the 2θ in-plane mode to perform measurements, the work described in this section considers support loss damping for this mode - unless otherwise stated.

4.3.5.1 Effect of Support Leg Width (W_{leg})

The influence of support leg width (W_{leg}) on frequency and $Q_{support}$ for 2θ in-plane modes was investigated. For this purpose, the support leg width (W_{leg}) of the nominal device was modified and the other ring dimensions remain unchanged.

Effect on Frequency: Figure 4.17 shows the frequency as a function of ratio of support leg width to nominal leg width ($W_{leg-nominal}$) i.e. $W_{leg}/W_{leg-nominal}$, where $W_{leg-nominal} = 38\mu\text{m}$. The results indicate the frequency increases monotonically with leg width for ratios greater than 0.39. However, for smaller leg widths, i.e. $W_{leg}/W_{leg-nominal} = 0.26$ to 0.39, the opposite trend is observed. To investigate the underlying reasons behind this behaviour the mode shapes are considered. The 2θ in-plane mode shapes for different $W_{leg}/W_{leg-nominal}$ ratios are illustrated in Figure 4.18. It is evident from these results that the mode contains a significant leg component for smaller leg thicknesses. In these cases the leg deformation is significant and the ring does not deform into a perfect $\cos 2\theta$ shape. However, the coupling between the leg mode and ring mode is weaker for $W_{leg}/W_{leg-nominal} = 0.39$. For ratios greater than

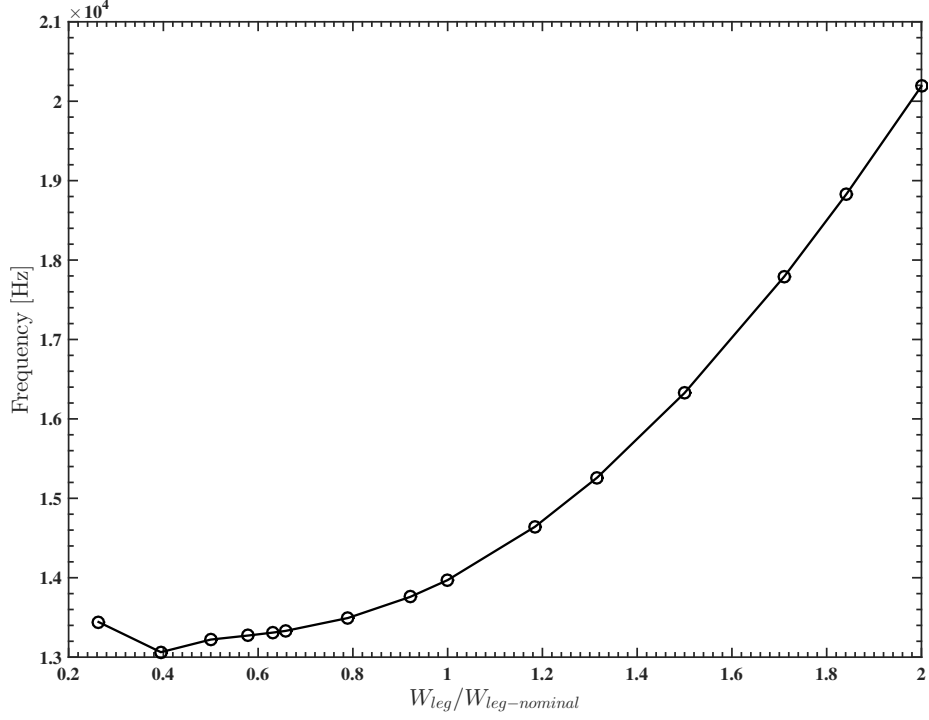


Figure 4.17: Frequency of 2θ in-plane mode as a function of $W_{leg}/W_{leg-nominal}$.

0.5, the leg deformation reduces.

Effect on Q-factor: Figure 4.19 shows the $Q_{support}$ values as a function of $W_{leg}/W_{leg-nominal}$. It is evident that $Q_{support}$ decreases for $W_{leg}/W_{leg-nominal} > 0.5$, but opposite trend is observed for $W_{leg}/W_{leg-nominal} = 0.26$ to 0.39 .

To help explain this behaviour the level of stresses at the hub/substrate interface for different ratios of leg widths are plotted in Figure 4.20. It can be seen that the stresses increase monotonically with $W_{leg}/W_{leg-nominal}$ indicating that thicker legs exert larger stresses during the vibrating period. Recalling that $Q_{support}$ also depends on the total energy stored in the resonator (see Eq. (4.1)), the total energy stored is shown in Figure 4.21 where it increases monotonically with leg width. The percentage of total stored energy in the ring, and support legs are illustrated in Figure 4.22. As leg width increases, the stored energy in the ring decreases as the energy stored increases in the support legs. However, the opposite trend is observed for thin legs, as previously observed for $Q_{support}$ and frequency. The percentage of stored energy in the legs are relatively higher for ratios less than 0.5, indicating the leg deformation is significant.

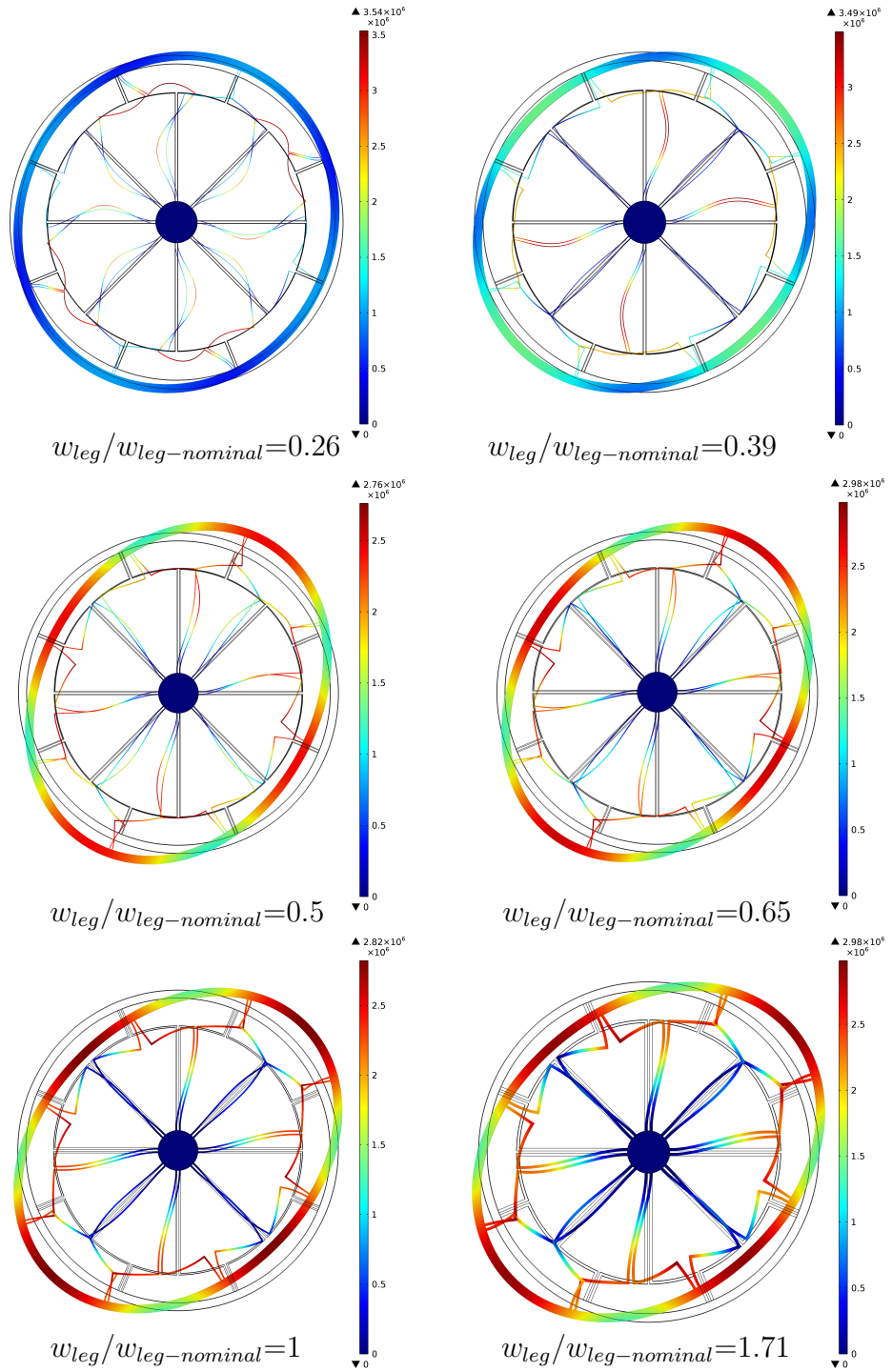


Figure 4.18: 2θ in-plane modeshapes for different $W_{leg}/W_{leg-nominal}$ ratios. The color bar indicates the total displacements in μm .

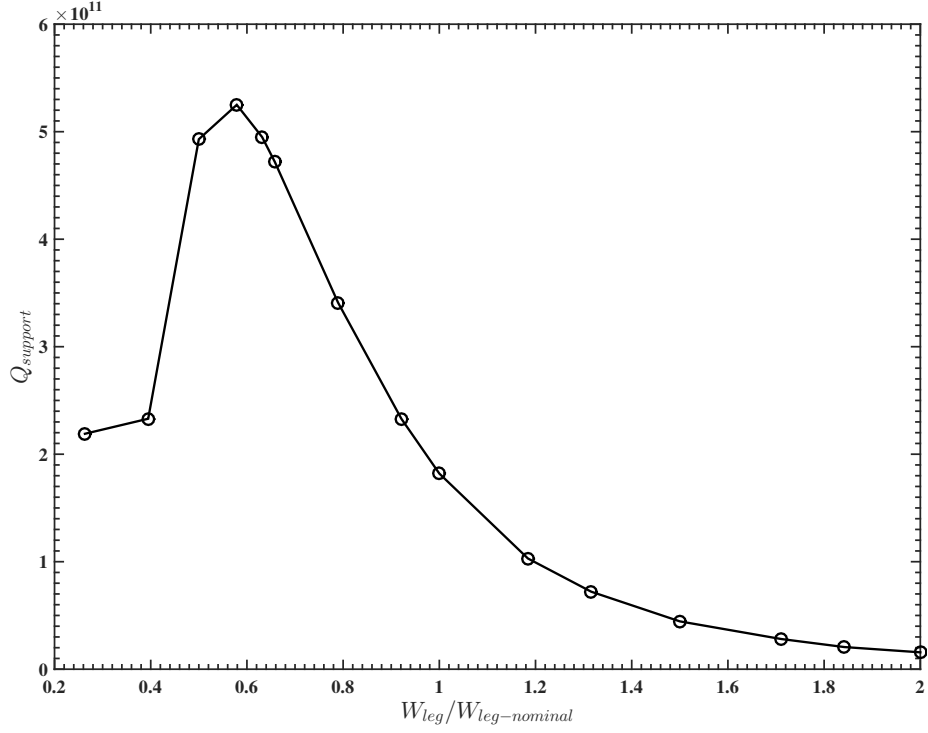


Figure 4.19: $Q_{support}$ as function of $W_{leg}/W_{leg-nominal}$ for the nominal device.

In summary, for the 2θ in-plane, reducing the support leg width can lead to reduced support loss due to lower stresses at the hub/substrate interface. However, very thin legs can lead to lower $Q_{support}$ due to coupling between the leg and ring.

4.3.5.2 Effect of Hub Radius (r_{hub})

In this study, the effect of hub radius (r_{hub}) on support loss was investigated. Since the central hub works as the link between the substrate and the supported ring, it is important to understand the behaviour of support damping considering the effect of the hub. The nominal value for r_{hub} is $500\text{ }\mu\text{m}$, and the $Q_{support}$ is calculated for $r_{hub} = 250\text{ }\mu\text{m}$, $500\text{ }\mu\text{m}$ and $750\text{ }\mu\text{m}$ in Table 4.5. The results show that $Q_{support}$ decreases with r_{hub} .

Eqs. (4.6) and (4.7) indicate support loss is proportional to the square of the attachment region, provided the stress level and energy stored is constant. It is also worth mentioning that the frequency of the resonant mode is not significantly affected by a small change in hub radius. This would also ensure similar amount of

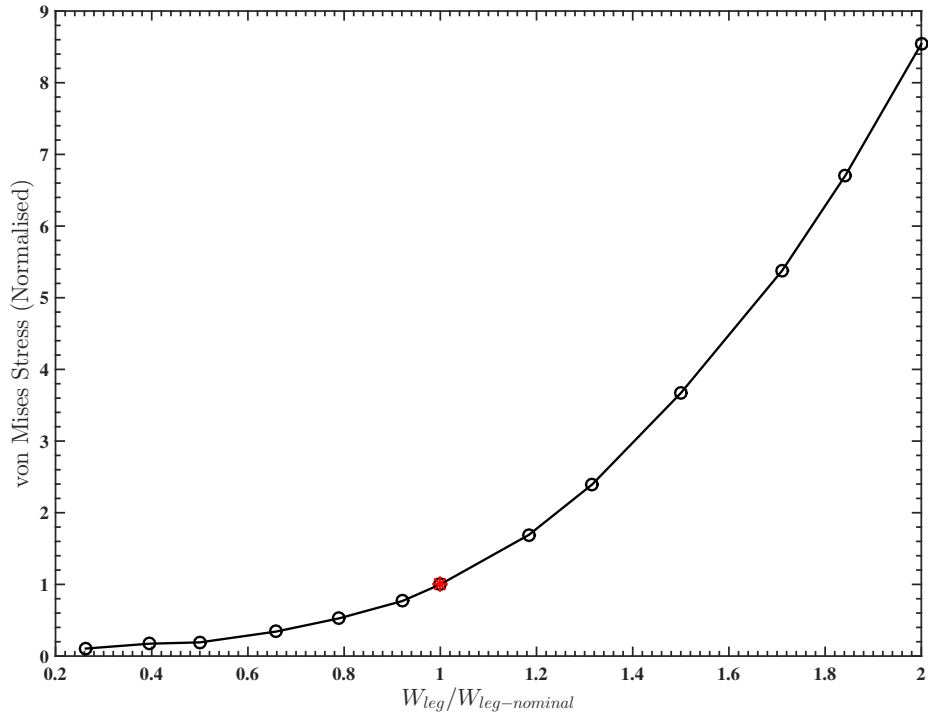


Figure 4.20: von Mises stresses (normalised) at the interface as function of $W_{leg}/W_{leg-nominal}$ for the nominal device. * denotes the nominal leg width.

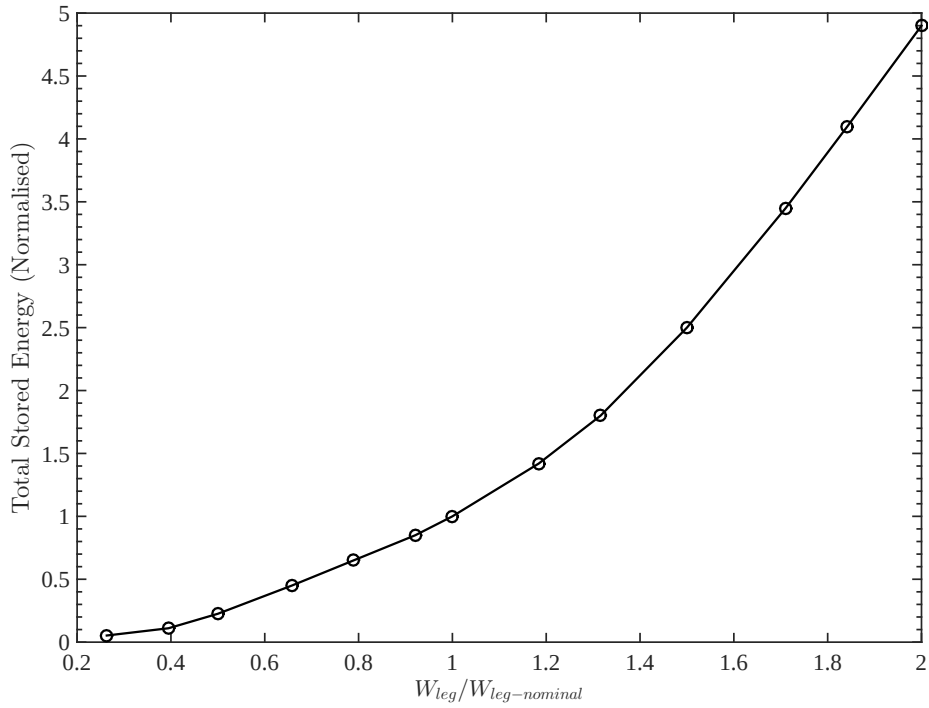


Figure 4.21: Total stored energy (normalised) in the resonator as a function of $W_{leg}/W_{leg-nominal}$ for the nominal device.

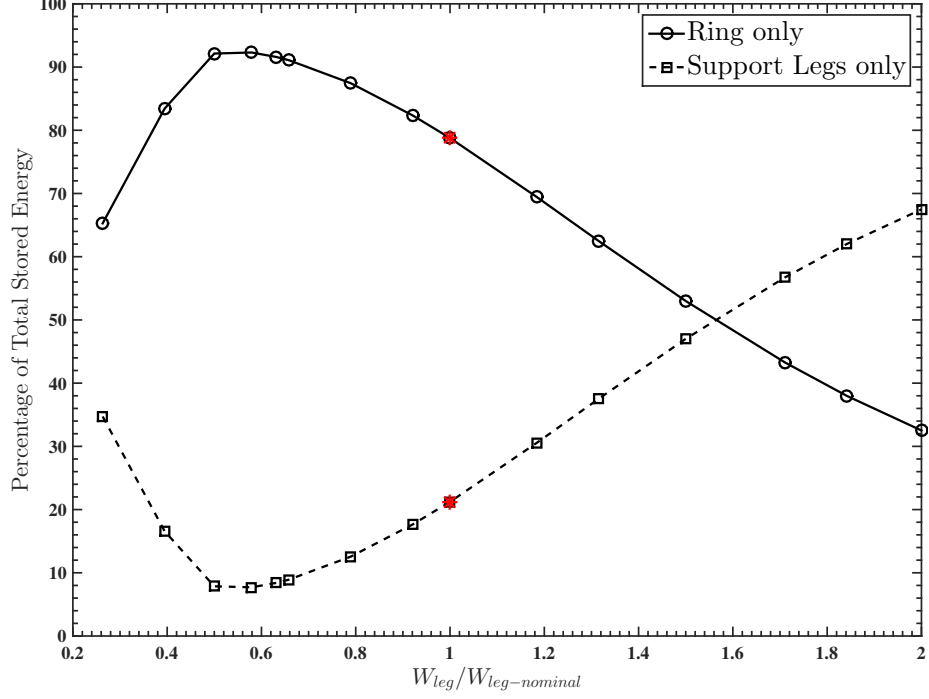


Figure 4.22: Percentage of total stored energy in the ring, and support legs. Red * denotes the nominal value.

Table 4.5: Effect of hub radius (r_{hub}) on the $Q_{support}$ of 2θ in-plane mode

r_{hub} (μm)	250	500	750
$Q_{support}$	5.02×10^{12}	1.82×10^{11}	3.60×10^9

stored energy in the resonator, and the $Q_{support}$ value using Eq. (4.1) is dominated by ΔW . The effect of attachment region is evident from the results presented in Table 4.5, where $Q_{support}$ decreases due to an increase in hub/substrate interface (a similar behaviour was observed for disk resonators in Chapter 3). On this basis support loss can be minimised by considering a smaller hub.

4.3.5.3 Effect of Substrate Material

In Section 4.3.2 the model considered a ring resonator attached to a silicon substrate via the central hub, where the glass plates attached to the hub were modelled as a silicon substrate. In this study the effect of the substrate material was investigated. It was assumed that the hub was attached to a semi-infinite glass substrate with material properties: $E=64$ GPa, $\nu=0.2$ and $\rho=2330$ kg/m³. For a glass substrate, the proposed PML model predicts $Q_{support}=1.15 \times 10^{11}$, which is $\sim 36\%$ lower than

the predicted value of 1.82×10^{11} using a silicon substrate. It is worth noting that $r_{PML}=4\text{ mm}$ is used in this study, and the optimal β -values are within the range 50 to 60.

The lower Q value observed for the glass substrate can be explained using Eqs. (4.6) and (4.7). In these equations terms $\frac{\gamma}{C_T^3}$ and $\frac{1}{\gamma C_T^3}$ can be used to show that support loss is proportional to $\frac{1}{E^{\frac{3}{2}}}$ for both stress types. This indicates reduced support loss for substrate materials with a higher Young's modulus, E . Since E for glass is much lower than E for si-(111), a higher level of damping is expected using a glass substrate. However, the impedance mismatch at the hub/substrate interface will cause some of the energy to be reflected back into the resonator.

4.3.6 Effect of Manufacturing Variations on Support Loss

The symmetry of the resonator relative to the vibration mode is broken by manufacturing and/or material imperfections. This section considers the effects of ring mass imperfection and systematic assembly variations based on hub offset on the support loss behaviour for 2θ and 3θ in-plane modes.

4.3.6.1 Effect of Mass Imperfection

The presence of mass imperfections in the resonator give rise to a dynamic imbalance resulting in motion of the centre of mass as the resonator vibrates. This dynamic imbalance causes reaction forces to act at the supports and causes support loss. The effect of ring mass imperfection on the frequency and support loss $Q_{support}$ for the 2θ and 3θ in-plane modes is investigated. Mass imperfection is introduced to an otherwise perfect ring resonator as shown in Figure 4.23, where the mass imperfection is distributed around a quarter section of the ring on the edge of the ring. The magnitudes of the mass imperfections are considered to be 0.5%, 0.75% and 1% of the total mass of the ring element of the resonator (m_{ring}) i.e. mass of a 7.797 mm diameter and 203 μm thick ring.

Table 4.6 shows calculated frequency and $Q_{support}$ values for the different mass

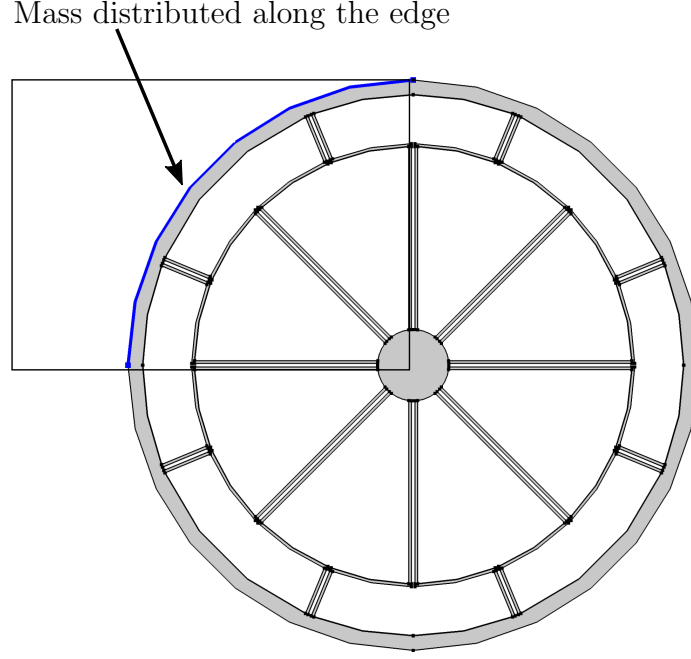


Figure 4.23: Mass imperfection distributed around a section on the edge of the ring.

imperfections. The frequencies of both the 2θ and 3θ in-plane modes decrease as the mass imperfection increases compared to the perfect resonator, see Table 4.4. This is expected and is caused by an increase in modal mass. It is also worth noticing that the frequency split (Δf) between the pairs of modes is higher for the 3θ in-plane modes than for the 2θ in-plane modes. $Q_{support}$ decreases as the mass imperfection increases for both in-plane modes, and can be explained by noting that the imperfection mass breaks the structural symmetry, which causes the structure to become unbalanced and the support loss to increase. The reduction in $Q_{support}$ is much higher for the 3θ in-plane modes compared to the 2θ in-plane modes. As discussed previously, the relatively high interface stresses for the 3θ in-plane mode indicates an unbalanced mode compared to the 2θ in-plane mode. The dynamic balance of the mode is likely to be affected by mass imperfection and explains why the reduction in $Q_{support}$ is higher for the 3θ in-plane mode.

4.3.6.2 Effect of Hub Offset

Now, the effect of offsetting the central hub of the resonator on the frequency and support loss is investigated in this section. Figure 4.24 shows the offsetting of the

Table 4.6: Effect of mass imperfection on support loss behaviour of 2θ and 3θ in-plane modes

% of m_{ring}	Mode	Frequency (Hz)		$Q_{support}$	
0.5 (8.97 μg)	2θ in-plane	13955.013	13955.230	1.37×10^{11}	1.22×10^{11}
		$\Delta f = 0.21$ Hz		$\Delta Q = 1.5 \times 10^{10}$	
	3θ in-plane	33056.035	33041.585	1.80×10^{11}	1.70×10^{11}
		$\Delta f = 14.44$ Hz		$\Delta Q = 1 \times 10^{10}$	
0.75 (13.46 μg)	2θ in-plane	13941.991	13942.240	1.03×10^{11}	8.30×10^{10}
		$\Delta f = 0.25$ Hz		$\Delta Q = 2 \times 10^{10}$	
	3θ in-plane	33040.991	33019.626	1.13×10^{11}	1.07×10^{11}
		$\Delta f = 21.36$ Hz		$\Delta Q = 0.6 \times 10^{10}$	
1 (17.95 μg)	2θ in-plane	13929.055	13929.360	7.71×10^{10}	5.68×10^{10}
		$\Delta f = 0.30$ Hz		$\Delta Q = 2.03 \times 10^{10}$	
	3θ in-plane	33026.013	32997.770	7.29×10^{10}	6.89×10^{10}
		$\Delta f = 27.96$ Hz		$\Delta Q = 0.4 \times 10^{10}$	

 Table 4.7: Effect of offsetting the central hub on $Q_{support}$ for 2θ and 3θ in-plane modes

L_{offset} (μm)	Mode	Frequency (Hz)		$Q_{support}$	
10	2θ in-plane	13968.90	13969.48	7.77×10^9	7.65×10^9
		$\Delta f = 0.58$ Hz		$\Delta Q = 1.2 \times 10^8$	
	3θ in-plane	32960.98	33009.01	8.16×10^9	1.05×10^9
		$\Delta f = 48.03$ Hz		$\Delta Q = 7.11 \times 10^9$	
20	2θ in-plane	13960.40	13961.09	1.96×10^9	1.95×10^9
		$\Delta f = 0.69$ Hz		$\Delta Q = 1 \times 10^7$	
	3θ in-plane	32548.90	32751.13	1.43×10^9	9.56×10^8
		$\Delta f = 202.23$ Hz		$\Delta Q = 1.2 \times 10^8$	

hub by a distance L_{offset} from the original centre. It is worth noting that the length of the support legs reduces in the direction of the offset, and increase in the opposite direction.

Table 4.7 shows the results obtained for the frequency and $Q_{support}$, which clearly show that increased levels of structural asymmetry increase support loss. Following earlier discussions, the increase in support loss is caused by increased stress levels at the leg/hub interface. A 10 μm offset of the central hub reduces $Q_{support}$ by two orders of magnitude and introduces a Q-factor split for both modes. The splitting of both Q-factors and frequencies are higher for the 3θ in-plane mode compared to the 2θ in-plane mode. This is likely to be because the 3θ in-plane mode is dynamically unbalanced.

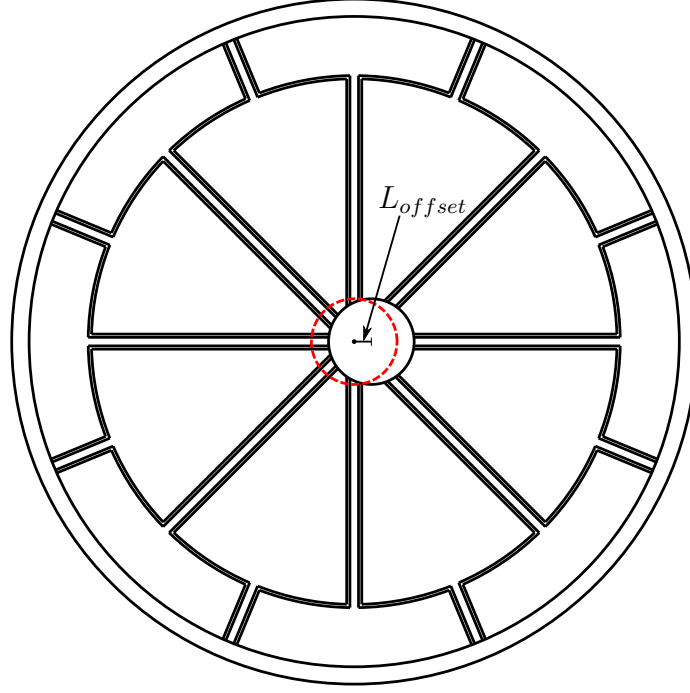


Figure 4.24: Structural asymmetry due to offset of central hub. The dotted circle shows the original position of the hub.

4.3.7 Summary

Support loss for the nominal device has been analysed. The results for different modes suggest that support loss is negligible for this device, with the highest Q -factors being obtained for the 2θ and 3θ in-plane modes. The Q value depends on two main factors: i) the energy dissipated at the interface; and ii) the total energy stored in the resonator. The energy dissipated depends on the magnitude of the stresses at the attachment region, e.g. hub/substrate interface, whilst the total energy stored depends on whether the mode is leg or ring dominated. The damping for leg-dominated modes, e.g. bounce mode, is higher than that for the ring-dominated modes, because leg-dominated modes have higher stresses at the leg/hub interface and lower levels of stored energy. On the other hand, the dynamically balanced 2θ in-plane mode ensures that the level of damping in this mode is significantly lower than other modes.

The relation of support loss to key design parameters of the ring resonator and the substrate was investigated for the 2θ in-plane mode. It has been inferred that a ring

resonator with thin legs exhibit low levels of damping due to lower levels of stresses exerted at the hub/substrate interface. However, the level of damping can be further reduced by considering smaller attachment areas by reducing the hub radius and using a substrate material with a large elastic modulus. This is because support loss is proportional to the square of the attachment area, and inversely proportional to elastic modulus of the substrate material i.e. $E^{\frac{3}{2}}$. Further investigations on the effect of manufacturing variations such as mass imperfection and central hub offset showed a significant reduction in Q-factor. Any form of imperfection or systematic variation of geometry breaks the cyclic symmetry of the structure and introduces unbalance to the mode of vibration.

4.4 Support Loss in High-Q Ring Resonators

In Chapter 2, thermoelastic damping behaviour of supported ring resonators was investigated and several ring configurations and designs considered. It was shown that supported ring resonators can be designed to exhibit very high-Q. For example a 4 mm-23 μ m ring with 8 μ m support leg thickness was designed to have a $Q_{TED} \approx 3 \times 10^5$. The purpose of this section is to analyse the support loss behaviour for high-Q ring resonators designed to have low levels of TED, and determine if the support loss is significant for these resonators. Primarily, the 4 mm-23 μ m ring with 8 μ m thick legs will be investigated.

Table 4.8 shows the $Q_{support}$ values for different resonant modes of the 4 mm-23 μ m ring with 8 μ m support legs. The damping behaviour of this resonator is similar to the nominal device analysed in the previous section (see Table 4.4). It is evident that the leg-dominated modes have much lower Q values compared to the ring-dominated modes. The coupling between the substrate and supports is stronger for these modes. However, Q-values are higher compared to the nominal device. Also, the higher $Q_{support}$ value for the 2θ in-plane mode suggests that support loss is negligible for this ring.

Table 4.8: Support loss in different modes of a 4 mm-23 μm ring with 8 μm support legs.

Mode	Frequency (kHz)	Q_{support}
1 θ in-plane	4.893	8.31×10^6
1 θ out-of-plane	6.425	9.52×10^6
Bounce	7.959	4.15×10^6
2 θ in-plane	8.008	5.13×10^{12}
2 θ out-of-plane	16.515	2.20×10^{10}
3 θ in-plane	15.302	1.59×10^{13}

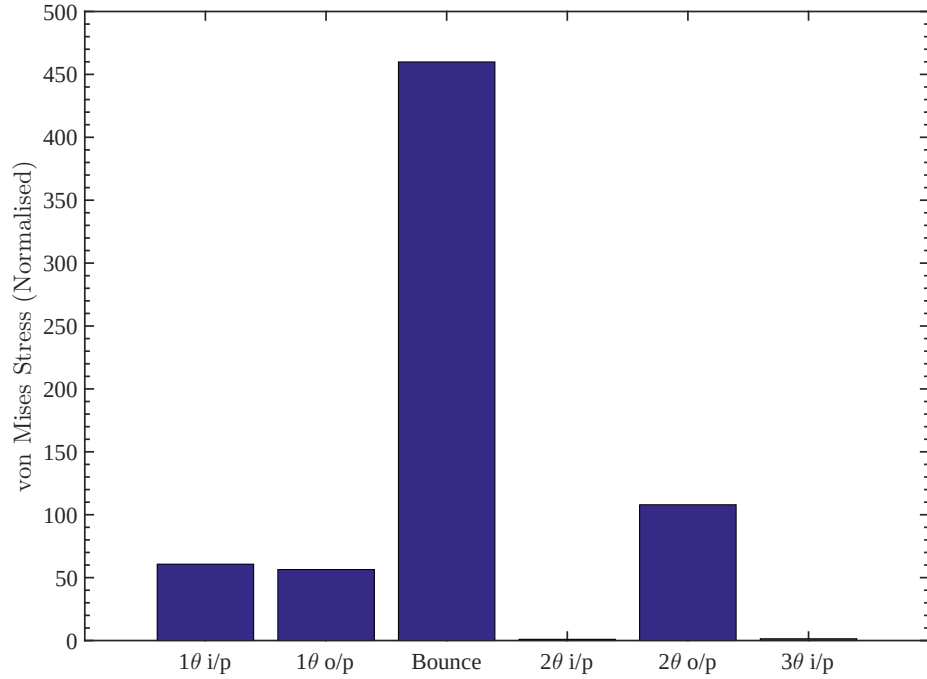
Figure 4.25: The level of von Mises stress at the interface between hub and substrate for different modes of the 4 mm-23 μm ring with 8 μm legs.

Figure 4.25 shows the von Mises stresses at the hub/substrate interface for the 4 mm-23 μm ring with 8 μm thick legs. The high interface stresses for the bounce mode are caused by out of balance and result in the highest level of damping. The stresses for the ring modes i.e 2 θ and 3 θ are much lower, which translates into higher Q -values. Similar to the nominal device, the 3 θ mode has a higher Q_{support} due to having a larger amount of stored energy. The total amount of stored energy for different modes are shown in Figure 4.26.

A parameter study is performed to find the effect of various leg width (W_{leg}) on the Q_{support} of the 2 θ in-plane mode. The ring dimensions are unchanged, and the

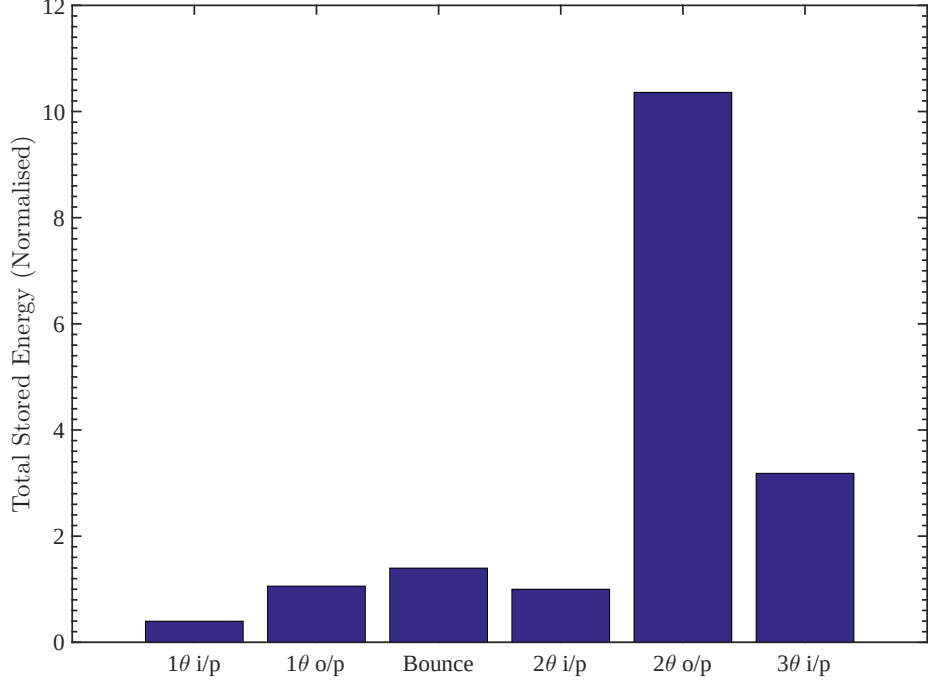


Figure 4.26: Comparison of total stored energy in different modes of the 4 mm-23 μm ring with 8 μm legs.

results are plotted in Figure 4.27. It is seen that the Q_{support} is higher for thin legs and the variation with W_{leg} is almost exponential. A similar behaviour was observed for the nominal device in Section 4.3.5.1.

In conclusion, the results presented in this section indicate that the high-Q rings exhibit similar behaviour to the nominal device and have exceptionally low levels of support, particularly for the 2θ in-plane mode. Furthermore, the higher Q_{support} values compared to Q_{TED} shows contribution of support loss is not as significant as TED.

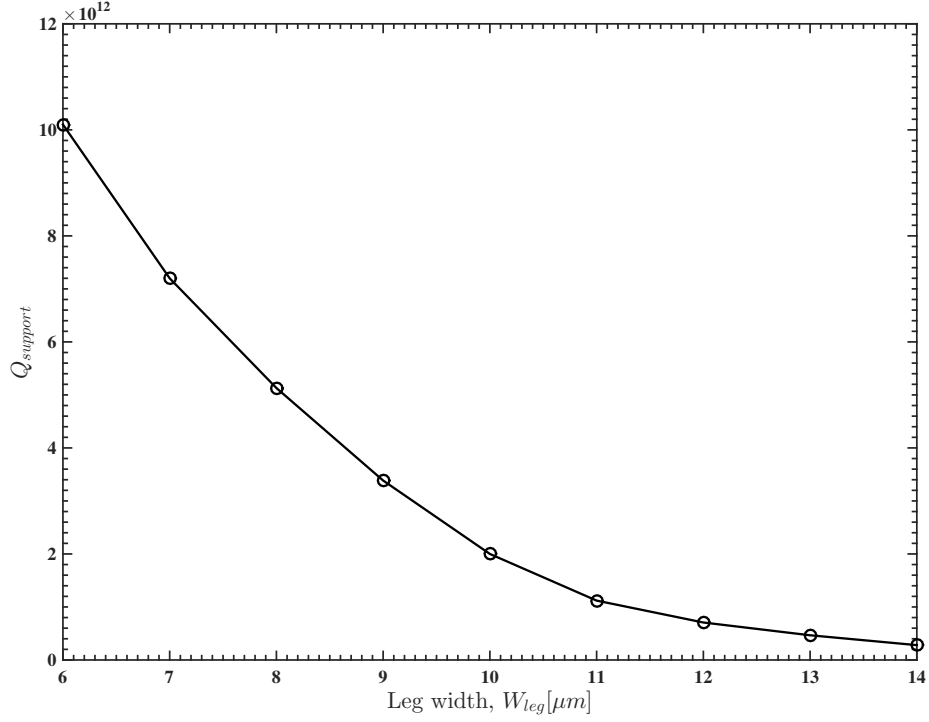


Figure 4.27: $Q_{support}$ as function of W_{leg} for 4 mm-23 μm ring.

4.5 Summary and Conclusions

The aim of this chapter was to analyse support losses in supported ring resonators. The modelling strategy discussed in Chapter 3 was used to quantify support loss. An analytical approach to predict support loss in ring resonator with support legs located outside the ring was reviewed. Comparisons of the FE model with an analytical model for this case showed better agreement, but the higher Q-value suggested that support loss is not the dominant damping mechanism in that device.

The results presented in this chapter mainly focused on a nominal device but the support loss behaviour in the high-Q ring resonators discussed in Chapter 2 were also investigated. The resonators analysed in this chapter showed negligible support loss. In summary, the following conclusions can be drawn:

- The leg-dominated modes show relatively high levels of damping mainly due to higher stresses at the interface between hub and substrate. The higher levels of interface stress translates to higher amount of vibration displacement in the substrate, and indicates the mode is unbalanced.

- For ring modes such as the 2θ in-plane mode $Q_{support}$ is much higher indicating that support loss is negligible in ring resonators. This is mainly due to lower levels of interface stress compared to leg-dominated modes. This damping behaviour has been observed for both the nominal device and high-Q ring resonators.
- 2θ in-plane mode is a balanced mode because the nodal points as well as anti-nodal points occur in pairs, which translates into low interface stress. In contrast, the 3θ in-plane mode is considered to be unbalanced with odd numbers of nodal and anti-nodal points. However, 3θ in-plane modes show relatively higher $Q_{support}$ than 2θ in-plane modes. This is because Q-factor also depends on the stored energy in the resonator, and 3θ in-plane mode exhibits higher amount of stored energy than 2θ in-plane mode.
- The mode of vibration becomes unbalanced in the presence of imperfections in the resonator. The structural asymmetry due to offsetting the central hub showed significant reduction in Q-factor. This confirms that as the cyclic symmetry is broken, the vibration becomes unbalanced. Similarly, mass imperfections not only induce frequency splitting between pair of modes but also increases support loss as the centre of mass moves.
- The parameter study investigating the influence of design parameters show that thinner legs have low levels of damping due to less coupling with the substrate resulting in low interface stress. However, coupling can occur between leg and ring for very thin legs significantly reducing the Q-factor. On this basis, support leg width needs to be selected carefully. Also, a smaller hub and stiffer substrate material can improve the Q-factor significantly, since support loss is directly proportional to the interface area and shows an inverse relationship with the elastic modulus of the substrate material.

Chapter 5

Experimental Study of High-Q Ring Resonators

5.1 Introduction

To achieve tactical and inertial grade performance levels (i.e. resolution and bias stability better than 1-10 deg/hr [10]) it is necessary to have sensors with high-Q. Ring resonators, used in angular rate sensors, have been designed in Chapter 2 to have low damping by selecting ring and support leg geometries that have reduced levels of Thermoelastic Damping (TED). This chapter describes the experimental work carried out to measure the performance of these high-Q rate sensors and compares them against theoretical predictions.

Section 5.2 presents results from thermoelastic damping predictions for different modes of the nominal device (see Chapter 4) and compares them with measured values by Sieberer [130].

Section 5.3 introduces the ring resonators proposed in Section 2.7 to have low levels of TED and presents predicted frequencies and Q_{TED} . These devices are referred to as “high-Q devices” throughout the thesis.

Section 5.4 describes the experimental testing procedure used to measure the frequencies and Q factors for these high-Q devices - which are bespoke, non-commercial devices for validating design, and discusses the differences between the experimental and predicted results.

Section 5.5 discusses potential causes for the differences between prediction and

Table 5.1: Predicted and experimentally measured frequencies and thermoelastic Q-factors for the nominal device.

Mode	1θ in-plane	1θ out-of-plane	Bounce	2θ in-plane	2θ out-of-plane	3θ in-plane
Frequency (kHz)	<i>Simulated</i>					
	5.223	4.347	5.290	14.222	10.196	33.272
	<i>Measured</i>					
	5.272	4.140	5.230	14.252	10.196	33.428
	5.280	4.142	-	14.253	-	33.443
	$\Delta f=8\text{Hz}$	$\Delta f=2\text{Hz}$	-	$\Delta f=1\text{Hz}$	-	$\Delta f=15\text{Hz}$
Q_{TED}	<i>Simulated</i>					
	87636	19975	17525	37608	15865	30982
	<i>Measured</i>					
	1150	2700	6070	31000	-	29530

measurement and investigates the mismatch. The section includes considerations of different energy dissipation mechanisms, such as die coupling, support loss and gas damping. In addition the effects of operating temperature and geometric variations on thermoelastic damping are analysed.

Section 5.6 presents a summary and concluding remarks for the chapter.

5.2 Thermoelastic Damping in a Nominal Device

A detailed description of the nominal device was presented in Chapter 4, along with an analysis of support losses for this device. This section considers the presence of thermoelastic damping (TED) in the ring resonator used in the nominal device, and compares predicted and measured frequencies and Q-factors for different modes.

Figure 5.1 shows a schematic diagram of the ring resonator used in the nominal device. To reduce the levels of TED a double layer of slots was incorporated an equal distance from the neutral axis around the ring circumference. A detailed discussion about the influence of slots was provided in Chapter 2, and for this device slots reduce the damping.

The FE model described in Chapter 2 is used to predict the thermoelastic damping for the same modes analysed in Chapter 4 for support loss. Table 5.1 shows the predicted frequencies and Q_{TED} together with experimentally measured results.

Experimental results for the nominal device are reported in Sieberer [130], where

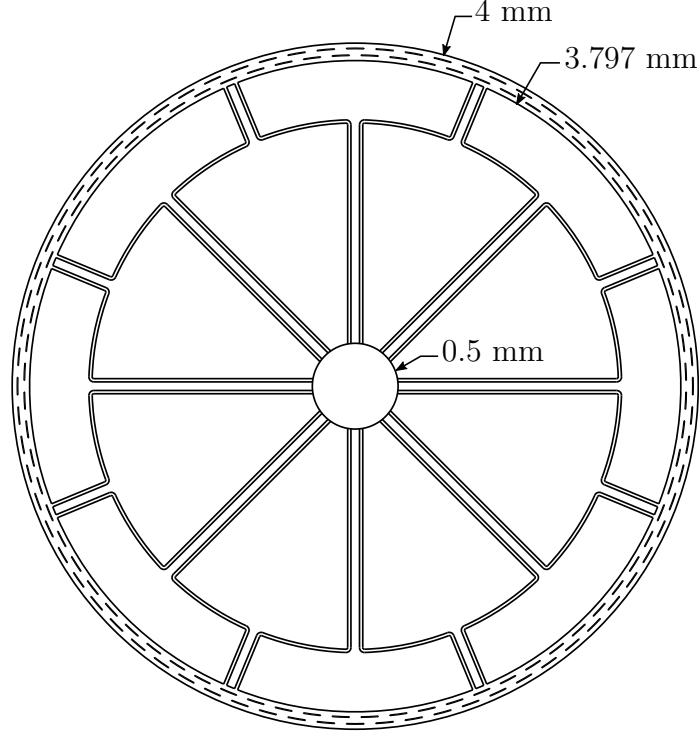


Figure 5.1: Ring resonator for the nominal device with slots around the circumference.

three devices (#0047, #0451, #0358) having similar dimensions were tested. The experimental results presented in Table 5.1 correspond to device #0047. The only exception to this is for the 1θ out-of-plane mode, which corresponds to device #0451. This is because in device #0047 the 1θ out-of-plane mode was not observed. In contrast, the bounce mode was not observed in device #0451. The experimentally measured Q-factors take account of all physical sources of damping including thermoelastic damping, support loss, electronic damping etc. As the device operates in a vacuum, gas damping is expected to be negligible. Details on how these experimental measurements were obtained are available in Sieberer [130] and are similar to those discussed in Section 5.4 later.

Frequency: The measured frequencies are all in good agreement with FE predictions with small frequency splits (Δf) indicating that the manufacturing imperfections are small. The 1θ in-plane, 1θ out-of-plane, 2θ in-plane, 2θ out-of-plane and 3θ in-plane modes occur in pairs (equal frequency and degenerate mode shape for a

perfect device). However, the bounce mode is unique and does not occur in a pair.

Q-factor: The predicted Q_{TED} are higher for the 1θ in-plane, 2θ in-plane, and 3θ in-plane modes. The 1θ in-plane mode has the highest Q-factor of the modes considered because there is no TED in the ring (because the ring does not deform) and the selected leg widths ensure the leg supports have low levels of TED. However, it can be seen that the 1θ in-plane mode has a significantly higher level of measured damping. The origins of the high levels of measured damping were investigated by Sieberer [130] and it was concluded that it was caused by significant coupling between the die motion and the 1θ in-plane response. It was found that die bond stiffness has a significant effect on the damping, and die bonds having low stiffness and high damping can induce high levels of damping in the 1θ in-plane response. No such affect is expected for the 2θ in-plane and 3θ in-plane modes because these modes are ring dominated and do not couple to the die motion. Further details about die coupling are provided in Section 5.5.2.

The results in Table 5.1 show good agreement for the 2θ and 3θ in-plane mode frequencies and Q_{TED} when comparing predictions with experimental measurements. It can be seen that the experimentally measured Q values for the 2θ and 3θ in-plane modes are both lower than the simulated Q_{TED} values indicating the presence of additional sources of energy dissipation (Q_{other}) to TED.

For the 2θ in-plane mode it can be shown that additional damping mechanisms having a combined $Q_{other} \sim 175000$ are needed to ensure the overall Q (inverse of sum of $\frac{1}{Q_{TED}}$ and $\frac{1}{Q_{other}}$) matches the experimentally measured Q. The high value of Q_{other} indicates that energy dissipation via other damping sources is small compared to TED, and TED is the dominant damping mechanism for these modes. In addition, the inferred value for Q_{other} indicates there is an upper limit to the value of Q that might be achieved if thermoelastic damping was eliminated completely.

The measured damping and TED predictions are significantly different for the 1θ out-of-plane mode and bounce mode. These differences have not been investigated but it is anticipated that coupling to out-of-plane die motion is the most likely cause

similar to the explanation provided earlier for the 1θ in-plane mode.

5.3 High-Q Devices

For the nominal device it was found that TED is the dominant damping mechanism and that the Q-factors are relatively low. To achieve improved sensor performance, it is desirable to significantly increase the Q-factor for the 2θ in-plane mode. This section re-introduces the high-Q devices proposed in Section 2.7 which are designed to have low levels of TED.

The ring and support legs for these “high-Q” devices are chosen with the target of achieving $Q_{TED} \approx 300,000$ and frequencies in the range 6 kHz-14 kHz for the 2θ in-plane mode, see in Section 2.7.

The following ring configurations were selected for the high-Q devices:

- Device-1: 4 mm ring with 23 μm radial thickness and 8 μm thick support legs
- Device-2: 4 mm ring with 23 μm radial thickness and 6 μm thick support legs
- Device-3: 4 mm ring with 22 μm radial thickness and 6 μm thick support legs

These rings are designed to provide devices with high thermoelastic Q-factors based on reduced ring radial and support leg thicknesses (see Chapter 2), and were fabricated by the sponsoring company for the purpose of experimental testing. The devices were fabricated from SCS-(111) and patterned and etched using the Deep Reactive-Ion Etching (DRIE) process. The general layout of these devices are similar to the nominal device, and a schematic diagram of the general layout is shown in Figure 5.2. The ring layer manufactured from the silicon wafer is encapsulated by glass layers on either side of the central hub, formed by anodic bonding. The ring layer and glass layers constitute the die. The ring layer is surrounded by electrodes internally and externally, which are also anodically bonded to the glass layers. The die is hermetically sealed and the resonator cavity on the inside is highly evacuated

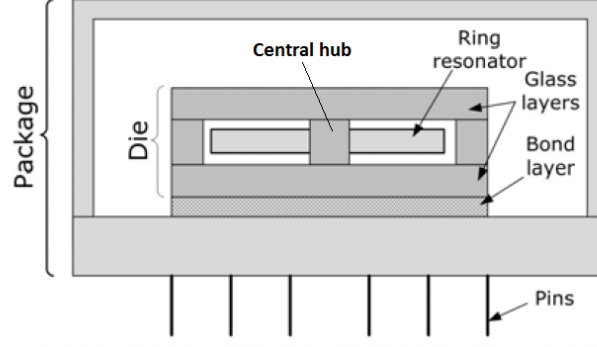


Figure 5.2: General layout of the high-Q devices [130].

to provide a low-pressure atmosphere for the ring resonator, as mentioned in Sieberer [130]. The pins provide connections to external instrumentation and power supply.

Table 5.2 show FE predicted frequencies and thermoelastic Q-factors for the high-Q devices. Comparing these results with Table 5.1 indicates that high-Q ring designs have much higher Q_{TED} compared to the nominal device and that the frequencies are quite different, with the 2θ in-plane frequencies being much lower. Similar to the nominal device the Q-factors for the 1θ in-plane modes are the largest indicating that TED in leg supports is very small for the high-Q devices. This is further indicated by the higher Q_{TED} values for the leg dominated bounce mode and the 1θ out-of-plane mode. The highest level of damping is predicted for the 3θ in-plane mode.

In the next section, measurements of frequency and Q-factor are conducted for the high-Q devices and comparisons made with theoretical predictions.

5.4 Experimental Testing of High-Q Devices

5.4.1 Test Setup

A schematic diagram of the test setup is shown in Figure 5.3, which consists of a gain phase analyser (HP 4194A), oscilloscope, power supply unit (PSU), digital multi-meters (DMM), and the MEMS test board. The rate sensor is mounted on the test board, which has external sockets for connecting to other devices in the setup,

Table 5.2: Predicted frequency and Q_{TED} for different modes of Device-1 (4 mm-23 μ m ring and 8 μ m leg, Device-2 (4 mm-23 μ m ring and 6 μ m leg), Device-3 (4 mm-22 μ m ring and 6 μ m leg) and Nominal Device.

Device	Mode	1θ in-plane	1θ out-of-plane	Bounce	2θ in-plane	2θ out-of-plane	3θ in-plane
1	Frequency (kHz)	4.893	6.425	7.959	8.008	16.515	15.302
	Q_{TED}	1.83×10^6	1.67×10^5	1.53×10^5	3.024×10^5	1.12×10^5	92350
2	Frequency (kHz)	3.301	4.352	5.440	6.694	15.808	14.772
	Q_{TED}	5.21×10^6	3.17×10^5	2.78×10^5	2.78×10^5	1.137×10^5	95130
3	Frequency (kHz)	3.355	4.430	5.532	6.544	15.314	14.186
	Q_{TED}	4.97×10^6	3.13×10^5	2.75×10^5	3.25×10^5	1.18×10^5	1.08×10^5
Nominal Device	Frequency (kHz)	5.223	4.347	5.290	14.222	10.196	33.272
	Q_{TED}	87636	19975	17525	37608	15865	30982

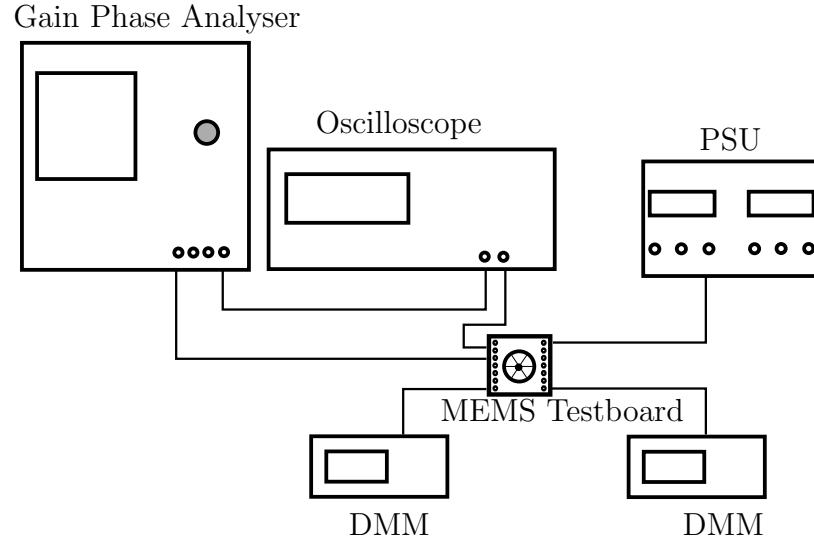


Figure 5.3: A schematic diagram showing the experimental setup.

and the gain phase analyser is used to generate a sinewave signal to excite the device. The sensor is operated in open loop when measuring the resonant frequencies and Q-factors.

5.4.2 Test Method

The resonator is actuated by electrostatic forces and the radial ring motion is detected capacitively. Figure 5.4 shows a schematic diagram of the supported ring including the arrangement of primary and secondary drive and pick-off (sense) electrodes used

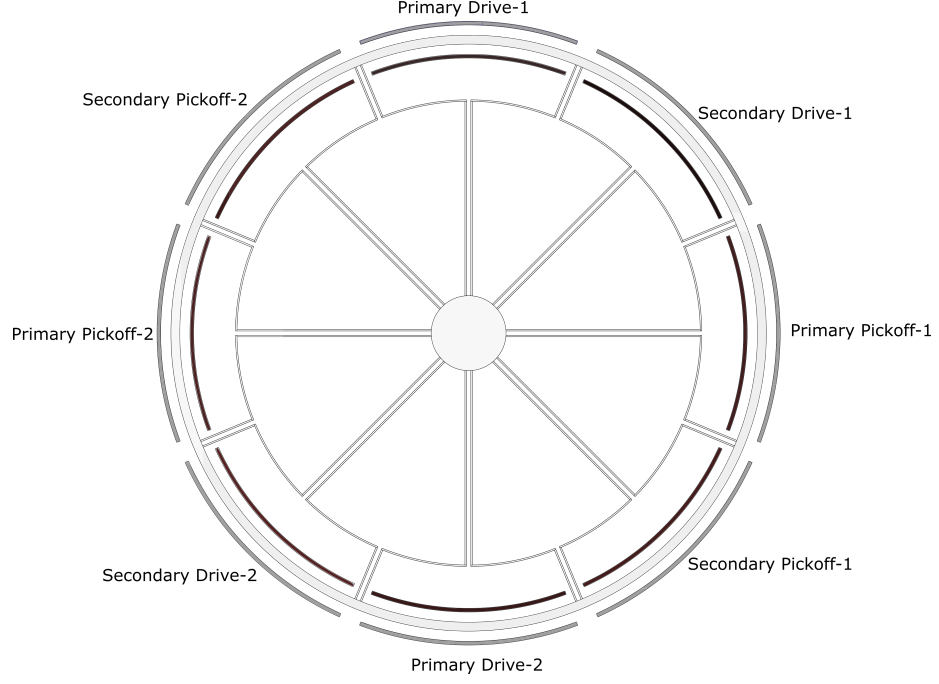


Figure 5.4: General arrangement of electrodes for a ring based gyro. The outer electrodes are used for driving and sensing purpose. The inner electrodes are used for balancing/trimming frequency splits.

to excite and measure the dynamic response of the ring resonator.

Table 5.2 indicate the modes to be considered are expected to have frequencies in the range 3 kHz to 17 kHz. Initially a coarse frequency sweep was conducted from 3 kHz to 18 kHz to identify response (gain) peaks in the pick-off measurements using the signal analyser. Refined frequency sweeps over a frequency range near the gain peaks were then performed to identify and measure the frequency and Q-factor for each mode. Frequency measurements were determined based on the 90 degree phase shift, whilst the Q-factor was calculated using the half-power method [132].

For the 2θ in-plane modes the frequencies and Q factors for both the primary and secondary modes were measured. The primary mode is characterised by driving the device using the primary drive and analysing the primary pick-off measurements for the mode exhibiting high gain. In a similar way, the secondary mode is characterised by driving the device using the secondary drive and analysing the secondary pick-off measurements for the high gain mode. Using this method it was possible to determine both mode frequencies in a pair.

For the other modes considered it is necessary to carefully consider the influence of the electrodes and adapt the device to drive these modes. This is because the drive and pick-off electrodes shown in Figure 5.4 are specifically designed to drive and sense the 2θ in-plane modes. The main complexity relates to driving and sensing the translational modes such as the 1θ in-plane modes which are leg dominated modes and do not involve ring deformation. To excite modes other than the 2θ in-plane modes, one of the drive and pick-off connections for either the primary or secondary was isolated. This was achieved by removing the wires for a set of drive and pick-offs. In the experiment, one of the secondary drive and secondary pick-off electrodes was isolated. A consequence of this is that it was only possible to measure the frequencies and Q factors for one mode in a pair. Other disadvantages of this arrangement are that the force magnitude is smaller than that obtained for the 2θ in-plane modes, and modes with nodal points close to the active drive axis will only experience low levels of excitation, and so only exhibit low levels of response.

It was difficult to excite the out-of-plane modes (e.g. bounce mode) using the existing electrode arrangements, as the device is designed to drive and sense 2θ in-plane motions. One way the out-of-plane modes could be excited would be to apply an excitation normal to the plane of the ring [130]. Unfortunately, there was insufficient time to perform these tests, which took place at the sponsoring company. Given that the excitations applied act in-plane, it is expected that the modes identified will be in-plane modes of vibration.

5.4.3 Results and Discussion

In this section, the experimental measurements for the frequencies and Q factors are discussed and compared to the predicted values presented in Section 5.3. For testing purposes, the following two devices were considered:

- Device-1 (#0411): 4 mm-23 μ m ring with 8 μ m leg thickness
- Device-2 (#0311): 4 mm-23 μ m ring with 6 μ m leg thickness

Table 5.3: Measured frequency and Q-factor for initial testing of Device-1

	Primary Mode	Secondary Mode	Splitting
Frequency (Hz)	8851.568	8856.480	$\Delta f=4.912$
Q-factor	32542	34594	$\Delta Q=2052$

A functional device similar to Device-3 (see Section 5.3) was not available for testing.

5.4.3.1 Device-1 Testing: 2θ in-plane modes

Initial testing on Device-1 was conducted to evaluate the frequencies and Q-factors for the pair of 2θ in-plane modes. Based on predictions in Table 5.2, these modes are characterised by a pair of frequencies at ~ 8 kHz. The frequencies were identified easily and the results for the frequencies and Q factors for the primary and secondary modes are shown in Table 5.3. When the device was driven using the primary drive the highest gain (higher peak) response was found to coincide with the low-frequency mode. When the device was driven using the secondary drive the highest gain (higher peak) response was found to coincide with the high-frequency mode. In practice manufacturing imperfections break the symmetry of the resonator and fix the orientation of the modes, and the associated mode angle dictates whether the primary mode is the low-frequency or the high-frequency mode.

A frequency split (Δf) between the pair of modes is expected and is mainly caused by 4θ variations in the distribution of mass and stiffness around the circumference of the ring. These arise due to manufacturing imperfections and material variations in both the ring and support legs. A damping split (ΔQ) is also expected due to the presence of a frequency split and can also be induced by 4θ variations in the distribution of damping around the ring circumference.

The predicted frequency and Q-factor results presented in Table 5.2 are significantly different to the measured results presented in Table 5.3. The experimental frequencies are much higher than the simulated frequencies indicating the presence of systematic variations in dimensions and material properties of the fabricated device compared to the simulated results. Also the experimentally measured Q factors

Table 5.4: Measured frequency and Q-factor for other modes of Device-1

	Mode-1 (1θ i/p)		Mode-2 (2θ i/p)		Mode-3 (3θ i/p)
Frequency (Hz)	6047.162	6066.868	8859.506	8853.132	15017.920
Δf (Hz)	19.706		6.374		-
Q-factor	~ 15920 -15990	-	~ 33100 -33600	-	~ 35200 -36100

are significantly lower (approximately 10 times) than the predicted Q_{TED} value of $\sim 3 \times 10^5$.

5.4.3.2 Device-1 Testing: Other modes

Table 5.4 reports the measured frequencies and Q factors obtained from frequency sweeps, where the Q-factors for each pair of modes are based on the mode with the highest gain peak when driven. Five distinct gain peaks are observed, with two frequency pairs in the ~ 6 kHz and ~ 8.8 kHz regions and a single peak observed in the ~ 15 kHz region. Based on earlier discussions, the frequencies in the ~ 8.8 kHz region correspond to the 2θ in-plane modes. The frequencies presented in Table 5.4 were obtained by averaging the frequencies over three sets of experimental measurements. Also, the range of Q-factor values obtained for the three sets of measurements is shown, as it is difficult to obtain a distinct Q-factor value mainly due to drift in the device i.e. the output of the device is subject to a slowly changing random function over a period of time [20]. It is worth noting that the modes in the ~ 6 kHz and ~ 15 kHz regions exhibited relatively low gain peaks compared to the 2θ in-plane modes discussed earlier, for all cases considered. This is due to aforementioned limitations in the method used to drive these modes, which is not well suited to modes other than 2θ in-plane modes.

Identification of Modes: The measured Mode-1 frequencies in Table 5.4 are close to the 1θ out-of-plane frequencies appearing in Table 5.2. However it is unlikely that the peaks correspond to out-of-plane modes because they are difficult to excite using the arrangement of drive electrodes in the device. For this reason it is expected that

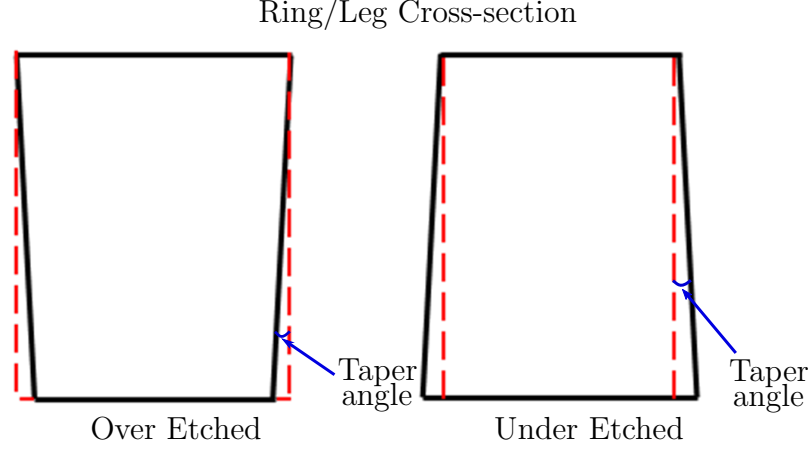


Figure 5.5: Tapered ring/leg cross-section, where dashed line indicates perfect etch.

Mode-1 corresponds to the 1θ in-plane modes. The measured Mode-1 frequencies are higher than the simulated results, and as the 1θ in-plane modes only involve leg deformations it is suggested that the increase in frequency is due to under-etching of the support legs. The under-etching occurs during the etching process when not enough material is removed from the wafer, and the cross-section of the ring and/or support becomes trapezoidal. Figure 5.5 shows a cross-section through the ring/leg when it is over-etched and under-etched. An analysis of etch variations is presented in Section 5.5.1.1. The results in Table 5.4 indicate that the frequency split Δf between the 1θ in-plane modes is ~ 20 Hz compared to ~ 6 Hz for 2θ in-plane modes. This indicates the presence of ring and leg support imperfection, because the 1θ in-plane modes are leg dominated modes and the 2θ in-plane modes are ring dominated modes.

The measured Mode-2 frequencies in Table 5.4 are similar to those in Table 5.3. The small differences observed are likely to be due to the isolation (i.e. removing wired connections) of one of the secondary drive and secondary pick-off electrodes (see Figure 5.4) required to excite the rigid body modes. Also, change in ambient conditions could affect the results, since the experiments were conducted on different days. Based on comparisons between measured and simulated frequencies, and the fact that these frequencies occur in a pair, Mode 2 corresponds to the 2θ in-plane

modes.

Only one frequency was measured for Mode-3. As the frequency is in reasonable agreement with predicted results for the 3θ in-plane mode in Table 5.2, it is suggested that this mode corresponds to one of the 3θ in-plane modes. It might be suggested that Mode-3 is one of the 2θ out-of-plane modes. However, as the electrode arrangement is designed to excite in-plane modes, it is more likely that Mode-3 is one of the 3θ in-plane modes.

In all cases it can be seen that the measured Q-factors in Table 5.4 are significantly lower than the predicted values presented in Table 5.2, indicating a much higher level of measured damping. The largest difference is for Mode 1 where the experimentally measured results are ~ 120 times lower than the prediction ($\sim 2 \times 10^6$). Similar behavior was observed for the nominal device (Table 5.1) and as discussed in Section 5.2, this is caused by die motion coupling to 1θ in-plane motion. Similarly it is feasible that die coupling accounts for the high levels of damping observed in Mode-1 for Device-1. The differences between predicted and measured Q factor values are smaller for Mode-2 and Mode-3 (than for Mode-1) and the measured results are ~ 10 and ~ 3 times lower than the predicted values ($\sim 3 \times 10^5$ and $\sim 1 \times 10^5$, respectively).

5.4.3.3 Device-2 Testing

Initial testing on this device involved exciting the primary drive and measuring the response at the primary pick-off. This indicated a single peak (composite mode) at 6863.038 Hz with $Q \approx 24598$. However, measurements performed by exciting the secondary drive and measuring at the secondary pick-off indicated two closely spaced peaks, as indicated in Table 7. The fact that the frequencies have a small frequency split and are similar to those in Table 5.2 indicates that these frequencies correspond to the 2θ in-plane modes.

Table 5.5 reports the frequencies and Q-factors obtained from frequency sweeps, where the Q-factors for each pair of modes are based on the mode with the highest gain peak when driven. A frequency sweep across the 3 kHz to 18 kHz range indicates

Table 5.5: Measured frequency and Q-factor for different modes of Device-2

	Mode-1 (1θ i/p)		Mode-2 (2θ i/p)		Mode-3 (3θ i/p)	Mode-4 (2θ o/p)	
Frequency (Hz)	3997.297	4019.980	6862.626	6863.501	13262.860	13378.748	13423.956
Δf (Hz)	22.683		0.875		-	45.208	
Q-factor	~ 11700 - 11800	-	~ 22750 - 22800	-	~ 36000 - 37000	~ 31500 - 33200	-

seven distinct gain peaks, with pairs of modes in the ~ 4 kHz, ~ 6.8 kHz and ~ 13.4 kHz regions, and a single peak observed at ~ 13.2 kHz region. Based on above discussions, the frequencies in the ~ 6.8 kHz region are expected to correspond to the 2θ in-plane modes.

Identification of Modes: The measured Mode-1 frequencies in Table 5.5 are quite close to the 1θ out-of-plane frequencies appearing in Table 5.2, but are more likely to correspond to the 1θ in-plane modes (see earlier discussion for Device 1). The measured frequencies are larger than the predicted values (as for Device-1) suggesting the support legs have been under-etched. The presence of a large Δf confirms the presence of leg imperfections, where the ~ 20 Hz split is similar to that obtained for Device-1.

As observed in initial testing for this device, Mode-2 is expected to be the 2θ in-plane mode. For this device, the simulated frequencies are close to the experimentally measured frequencies. The predicted frequencies for Device-2 are $\sim 2.5\%$ lower than the experimentally measured values, whilst for Device-1 the prediction was $\sim 10\%$ lower. As discussed for Device-1, the observed 2θ in-plane frequency split is caused by 4θ variations around the ring circumference. The frequency split for Device-2 is ~ 1 Hz, which is much smaller than ~ 5 Hz achieved for Device-1, indicating a lower level of manufacturing imperfection for Device-2.

Based on the results in Table 5.5, the measurements for Mode-3 and Mode-4 correspond to either 2θ out-of-plane or 3θ in-plane modes. It is evident from Table-3 that the 2θ out-of-plane mode has higher predicted frequencies than the 3θ in-plane mode, suggesting Mode-3 is the 3θ in-plane mode and Mode-4 is the 2θ out-of-plane mode.

Similar to Device-1, only one frequency was measured for Mode-3. The reason only one mode was detected is likely to be because the response level was too small to detect the other mode. This might be because the electrode arrangement is not well suited to exciting the mode concerned (i.e it is a 2θ out-of-plane mode) and/or there is a nodal point close to the drive or pick-off. Noting the similarities to measurements for Device-1, it is suggested that Mode-3 is the 3θ in-plane mode and Mode-4 is the 2θ out-of-plane mode. For the 2θ out-of-plane mode the ring and legs deform out-of-plane, and any 4θ variations around the ring induce frequency split Δf . The presence of imperfection is evident in the experimental results, where $\Delta f = \sim 45$ Hz is observed.

The measured Q-factors are all significantly lower than the thermoelastic predictions, indicating much higher levels of measured damping than predicted TED. The findings are similar to those observed for Device-1 with the largest difference for Mode-1, which is likely to be due to coupling to the die motion. Smaller but significant differences are present for Mode-2 and Mode-3. These differences are unlikely to be caused by die coupling - this point is discussed in detail in Section 5.5.2.

5.4.3.4 Summary

For the proposed high-Q devices comparisons have been made between measured and predicted frequencies and Q factors.

It was found that the measured frequencies are reasonably close to the predicted values, but all measured frequencies are slightly higher. These differences are likely to be caused by systematic etch variations (e.g. under-etch) in the ring and/or support legs. A larger frequency split is observed for the 1θ in-plane mode, which is attributed to support leg imperfections.

The measured Q-factors for the devices did not achieve the design target, i.e. Q-factor $\sim 300,000$. All measured Q-factors are significantly lower than the predicted Q_{TED} for the modes considered, indicating the presence of other damping mechanisms. For the 1θ in-plane mode, coupling between die motion and 1θ in-plane motion is

expected to reduce the overall Q-factor similar to that observed for the nominal device by Sieberer [130]. An investigation into possible causes of increased damping in the high-Q devices is presented in the next section.

5.5 Potential Root Causes

The measured levels of damping in high-Q devices are significantly higher than TED predictions for all modes considered. Potential root causes for these high levels of observed damping are discussed in this section.

5.5.1 Thermoelastic Damping

5.5.1.1 Influence of Geometric Etch Variations on Frequencies and TED

As mentioned in Section 5.4, some of the observed differences between predicted and experimentally measured frequencies are likely to be caused by etch imperfections for the ring and leg supports. It is anticipated that etch variations will also affect the TED and this might help explain the observed high levels of damping.

A study is performed to determine how taper angle affects the frequencies and TED Q factors for the supported ring. A zero taper angle means that the ring and support leg have a rectangular cross-section (perfect etch), whilst a positive taper angle means they are trapezoidal in shape with the upper dimension smaller than the lower dimension (see Figure 5.5).

The predicted frequencies (obtained from FEA) for 4 mm-23 μ m ring with 8 μ m leg (i.e. Device-1) is plotted in Figure 5.6, when a small taper angle ($<2^\circ$) is introduced to the ring and support legs. The frequencies are seen to increase approximately linearly as taper angle increases. This can be explained easily for the case of an unsupported ring by noting that under-etching produces a larger mean radial thickness than the “perfect” zero angle taper case. It has been seen from Eq. (2.15) that the frequency is proportional to the ring radial thickness for an unsupported ring with rectangular cross-section. However, for small taper angles the mean radial thickness

varies linearly. Therefore, on this basis, it is expected that the frequency will increase if the ring is under-etched.

Figure 5.7 shows predicted Q_{TED} values for 4 mm-23 μm ring with 8 μm leg for different taper angles. These results indicate that the Q-factors reduce as taper angle increases. This behaviour can be explained for an unsupported ring based on Zener's theory. Zener's equation (also mentioned in Chapter 2) for the Q-factor due to thermoelastic damping in a ring resonator is:

$$Q_{TED}^{-1} = \Delta_M \frac{\omega_0 \tau}{1 + (\omega_0 \tau)^2} \quad (5.1)$$

where ω_0 is the fundamental undamped natural frequency of the ring. The effective relaxation time is

$$\tau = \frac{C_v b_r^2}{k \pi^2} \quad (5.2)$$

and the relaxation strength is

$$\Delta_M = \frac{E \alpha^2 T_0}{C_v} \quad (5.3)$$

In Eqs.(5.2) and (5.3), $C_v (= \rho C_p)$ is the specific heat capacity at constant volume, T_0 is the reference temperature, α is the linear coefficient of thermal expansion, E is the Young's modulus, k is the thermal conductivity, and b_r is the the radial thickness of the ring, respectively.

Referring to Eq. (5.1), if a ring is operating in the isothermal regime (i.e. $\omega \tau \ll 1$), an increase in $\omega \tau$ will cause the resonator to operate closer to the Debye peak, causing Q_{TED} to decrease. Alternatively for a ring operating around the Debye peak or in the adiabatic regime (i.e. $\omega \tau \geq 1$), an increase in $\omega \tau$ will cause the resonator to operate away from the Debye peak, causing Q_{TED} to increase. When an unsupported ring is under-etched, the mean ring radial thickness increases causing both ω and τ to increase. For the resonator considered in Figure 5.7, the zero taper case operates in the isothermal regime, so Q_{TED} decreases. This behaviour is similarly valid for supported rings also.

Although etch tapering is observed to increase the level of damping, the increase

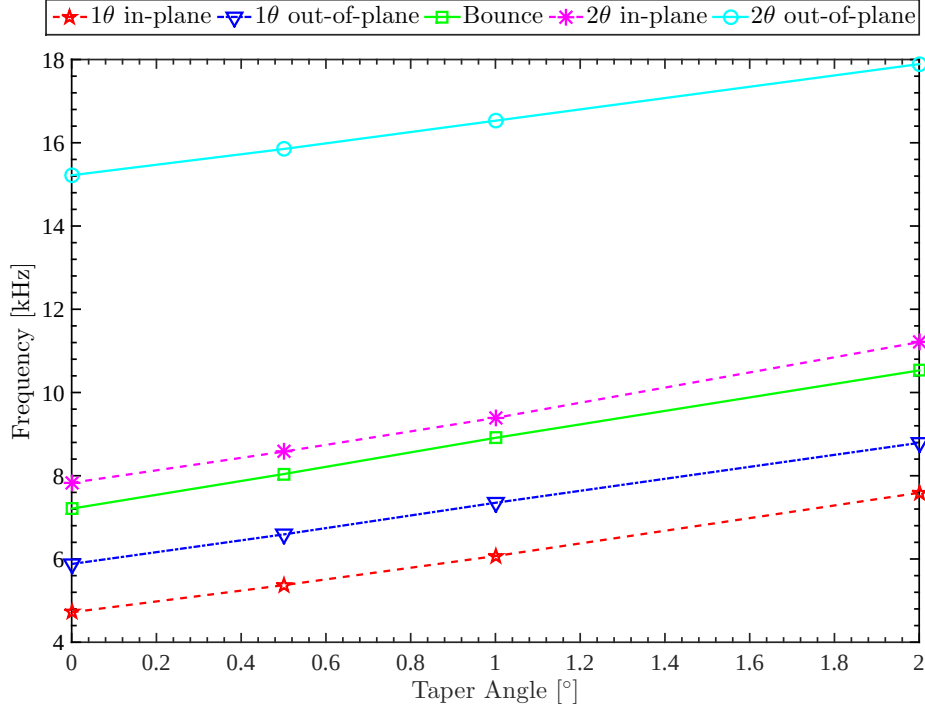


Figure 5.6: Frequency for different modes in an under etched supported ring (4 mm-23 μm ring and 8 μm leg).

is small in comparison to the increases observed in Section 5.4 and so does not explain the high levels of damping. This indicates taper angle variations are not sufficient alone to explain the measured results.

Systematic variations in etch width, leading to larger ring radial thickness and leg widths are expected to contribute to the higher frequencies observed in experiments. In Section 5.4, the largest discrepancy between predicted and measured frequencies was observed for the 4 mm-23 μm ring with 8 μm leg thickness. Table 5.6 shows FE results for the frequencies and Q_{TED} for an identical ring but with slightly wider leg thickness (i.e. 4 mm-23 μm ring with 9 μm leg thickness). Comparing the predicted and measured results in Table 5.6, it can be seen that the 2θ in-plane frequencies are in good agreement ($<0.5\%$ difference) and the 1θ in-plane frequency is also in closer agreement. However, agreement with the experimental measurement for the 3θ in-plane frequency is worse.

The Q_{TED} results in Table 5.6 are all similar in magnitude to those predicted for 8 μm leg thickness, and are much higher than the measured values. This can

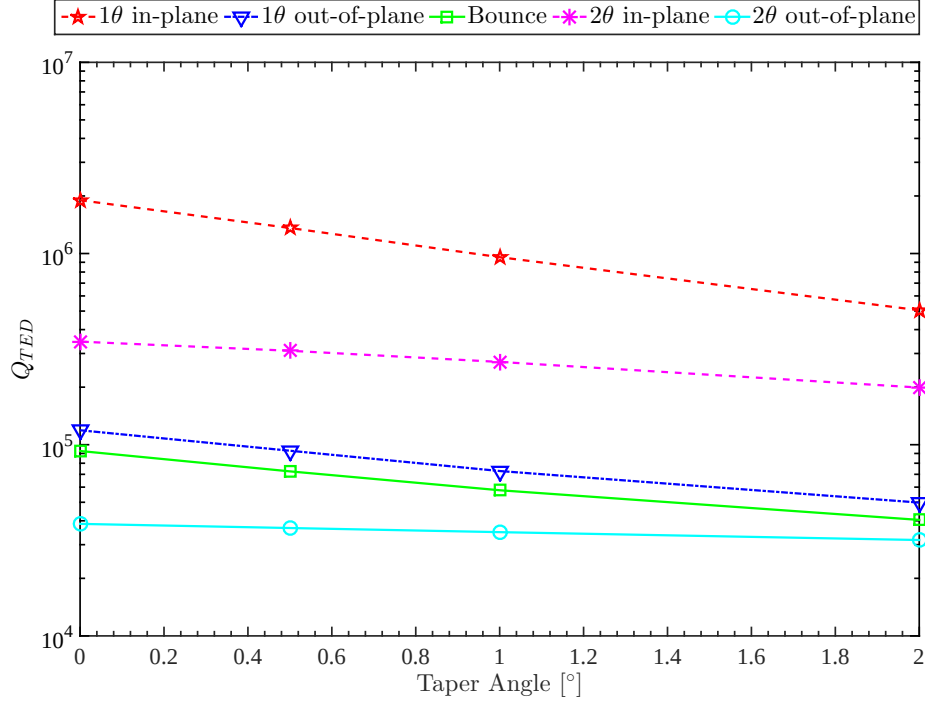


Figure 5.7: Q_{TED} for different modes in an under etched supported ring (4 mm-23 μ m ring and 8 μ m leg).

be explained based on Zener's theory (Eq. 5.1) by noting that a 1 μ m change in leg width changes the relaxation time constant by $\sim 20\%$ and so a large change in damping is unlikely.

In summary, the results presented in this section show that the etch variations can have a significant influence on frequency, but are unlikely to cause a sufficiently large change in TED to account for the high levels of measured damping observed for the experimental values for high-Q devices in this work.

Table 5.6: Predicted frequency and Q_{TED} for different modes of 4 mm-23 μ m ring with 9 μ m leg thickness.

Mode	1 θ in-plane	1 θ out-of-plane	Bounce	2 θ in-plane	2 θ out-of-plane	3 θ in-plane
<i>Prediction (9 μm leg)</i>						
Frequency (kHz)	5.735	7.492	9.219	8.821	17.069	15.693
Q_{TED}	1.24×10^6	1.35×10^5	1.26×10^5	2.944×10^5	1.106×10^5	94608
<i>Measured (8 μm leg)</i>						
Frequency (kHz)	6.047 6.066	-	-	8.859 8.853	-	15.017
Q_{TED}	~ 15920 -15990	-	-	~ 33100 -33600	-	~ 35200 -36100

5.5.1.2 Effect of Temperature on TED

Thermoelastic damping is an intrinsic damping mechanism that is strongly dependent on material properties. As thermo-mechanical properties of materials change with temperature, thermoelastic damping behavior is expected to be affected. For silicon, thermal properties change significantly over the normal operating temperature range (250K-350K). Over this range, the thermal expansion coefficient (α) reduces by 70% [133]; the specific heat capacity at constant pressure (C_p) increases by $\sim 17\%$, and the thermal conductivity (k) decreases by approximately 33% [134, 135].

The effect of temperature on frequency and Q_{TED} is investigated based on Zener's theory. In addition results from FE modelling and experimental testing as temperature is varied are compared to investigate if TED is the major cause for the observed high levels of damping. This study considers the 4 mm-23 μm ring with 8 μm leg thickness (i.e. Device-2).

Effect of Temperature on Frequency: Young's modulus (E) governs the elasticity of a material and is dependent on temperature. An increase in temperature causes the material to expand and the distance between atoms to increase. This causes the bond strength between atoms to decrease and can result in a complex non-linear behavior [136]. For small variations, the change in Young's modulus can be considered to be linear and Young's modulus can be expressed as a linear function of temperature [136] i.e.:

$$E = E_0(1 + TCE \cdot \Delta T) \quad (5.4)$$

where E_0 is the Young's modulus at the reference temperature T_0 , ΔT is the change in temperature about the reference temperature, and TCE is the temperature coefficient of Young's modulus. For silicon [135], TCE is approximately $-63.7 \text{ ppm}/^\circ\text{C}$ at normal operating temperatures.

In general, the frequency of a resonator changes with temperature due to changes in thermal expansion and changes in mechanical properties. The contribution of thermal

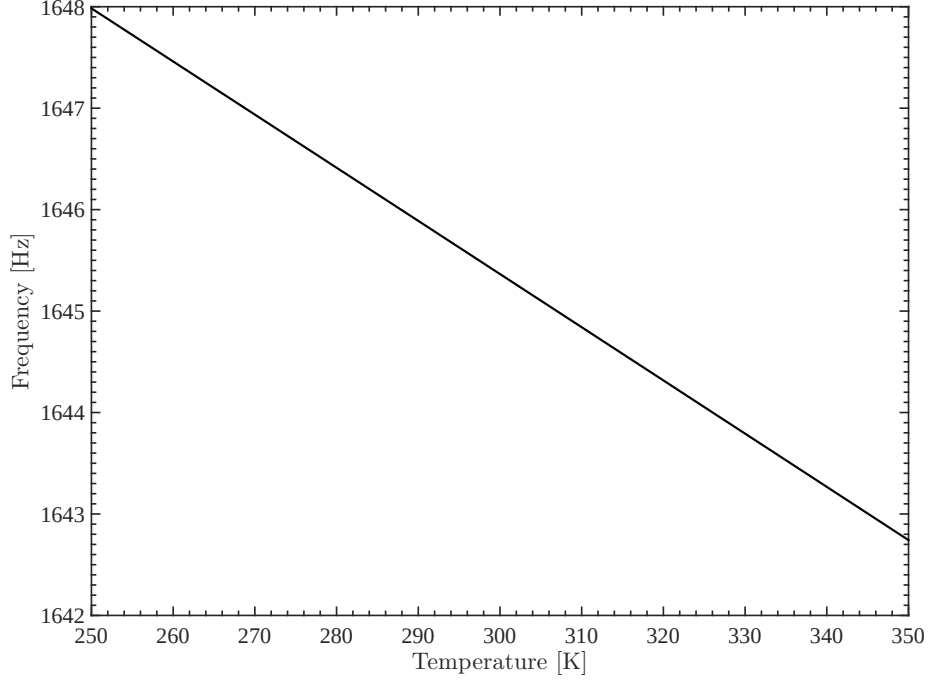


Figure 5.8: Frequency as a function of temperature (analytical).

expansion to the change in dimension (e.g. ring radius r and ring thickness b_r) and density (ρ) is attributed to the $\alpha\Delta T$ factor. However the effect of thermal expansion on frequency is governed by $\alpha\Delta T/2$. Noting that α is approximately 1.3 ppm/ $^{\circ}\text{C}$ [136] it can be deduced that the change in frequency due to thermal expansion is insignificant and can be neglected.

The influence of temperature variations in E on the resonant (in-plane) frequency of an unsupported ring can be evaluated based on Eq. (5.4). Figure 5.8 shows how the 2θ in-plane frequency for an unsupported ring having the same ring dimensions as Device-2 (i.e. 4 mm-23 μm) changes with temperature. It is clear from the figure that the frequency decreases approximately linearly as temperature increases, and it can be shown easily that the rate of change of frequency is $\approx -0.003\%/^{\circ}\text{C}$.

Effect of Temperature on Q_{TED} : The influence of variations in temperature on the thermoelastic Q-factor for an unsupported ring can be evaluated using Zener's equation. Referring to Eq. 5.3, the relaxation strength (Δ_M) depends on material properties and is independent of resonator dimensions. As temperature increases

the relaxation strength increases (approximately) linearly with temperature, mainly because temperature T_0 increases, and the value of Δ_M influences the magnitude of Q_{TED} . Although the resonant frequency is independent of thermal properties, the thermal time constant (τ) depends on both ring dimensions and thermal material properties. For silicon, regardless of resonator dimensions, τ increases with temperature. This is because the physical value of C_p is higher than k , which increases with temperature. As mentioned previously, $\omega\tau$ determines where the resonator operates on the Debye curve i.e. isothermal regime if $\omega\tau \ll 1$, adiabatic regime if $\omega\tau \gg 1$ or Debye peak if $\omega\tau \approx 1$. For a ring resonator (assuming no change in frequency) operating in the isothermal regime, a temperature increase shifts the operating point towards the Debye peak. Similarly, if it operates in the adiabatic regime it shifts away from the Debye peak. This is illustrated in Figure 5.9 for a ring undergoing a temperature change from $25^\circ C$ to $70^\circ C$, which is within the normal operating temperature range for the nominal device i.e. $-20^\circ C$ to $80^\circ C$. This figure shows Debye curves for both temperatures. The ring considered operates near the Debye peak (green circle) at $25^\circ C$. However, an increase in τ (not considering the change in Δ_M) with temperature to $70^\circ C$ shifts the point to the right (blue circle). Q_{TED} shifts downwards due to the increase in Δ_M with temperature (red circle) until it lies on the Debye curve for $70^\circ C$. Regardless of the operating region, a decrease in Q_{TED} is observed with increasing temperature. On this basis Q_{TED} is expected to reduce with an increase in temperature, as shown in Figure 5.10.

Comparison Between Predicted TED and Measured Damping: Device-

2 Experimental testing was conducted to investigate the effect of temperature on the frequencies and damping behavior of Device-2 by taking measurements inside a temperature controlled chamber. Initial measurements were taken at base (ambient) temperature of $22^\circ C$ (295K) for all modes reported in Table 5.5. Additional measurements were taken at $50^\circ C$ (323K) and $70^\circ C$ (343K), and a settling time of 5 minutes was used to ensure the temperature had settled to the desired value before measurements were taken. The experimental results obtained are shown in Table 5.7, and

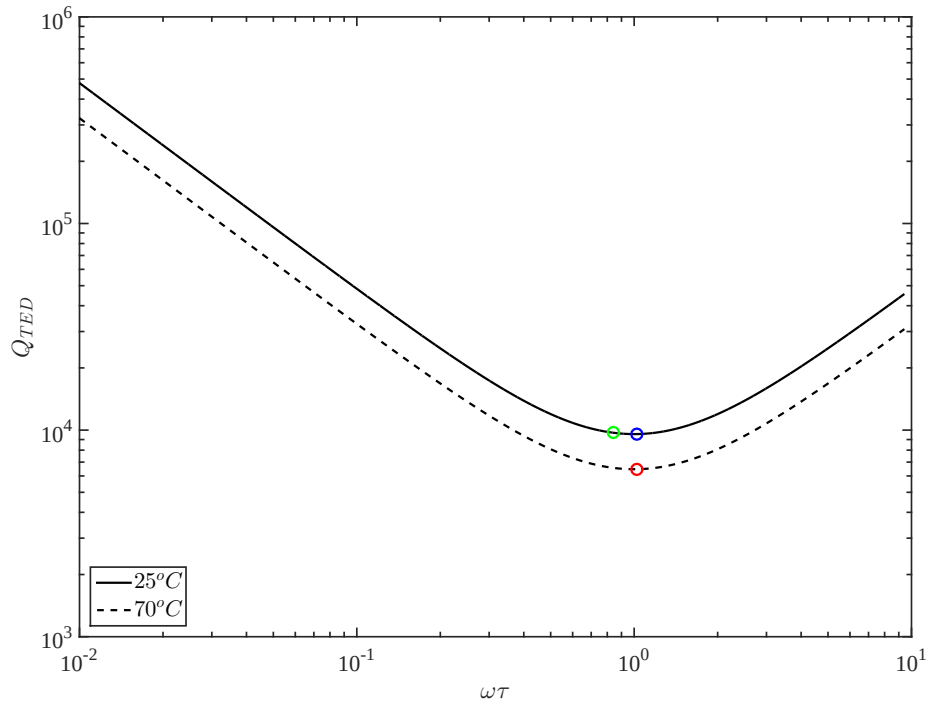


Figure 5.9: Frequency as a function of temperature (analytical).

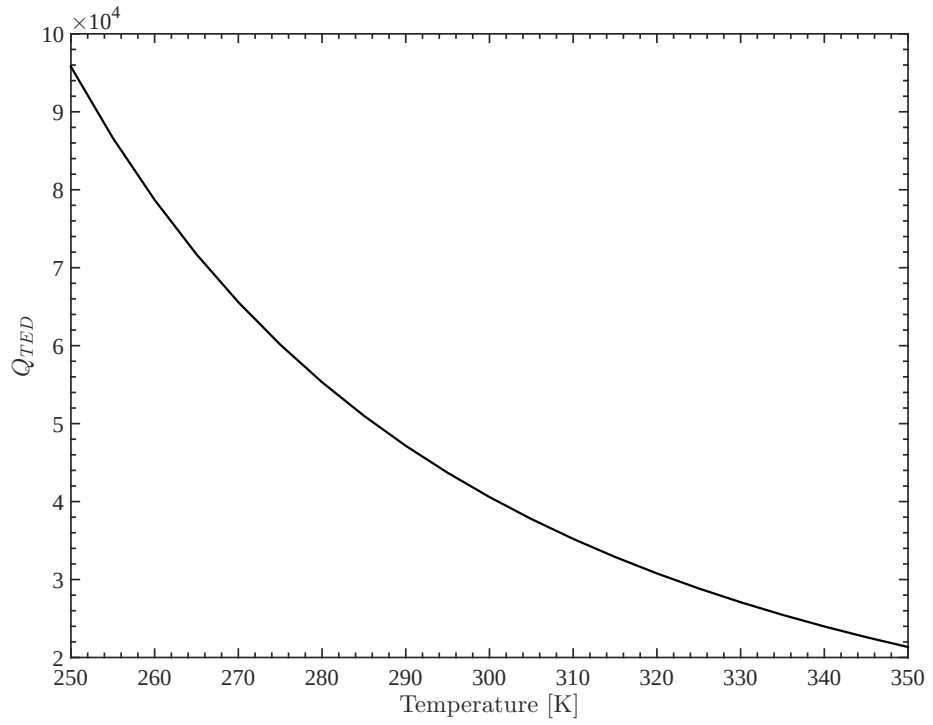

 Figure 5.10: Q_{TED} as a function of temperature (analytical).

Table 5.7: Temperature effect on Q_{TED} and frequency for Device-2 (measurement)

Temperature (°C)		1θ in-plane		2θ in-plane		3θ in-plane	2θ out-of-plane	
22	F (Hz)	3997.479	4020.101	6862.915	6863.867	13263.040	13379.175	13423.968
	Q	~13810- 13910		~22810- 22910	-	~34870- 35420	~32500- 33000	-
50	F (Hz)	3993.766	4016.791	6856.590	6857.549	13251.686	13367.548	13412.256
	Q	~12050- 12250	-	~20850- 21650	-	~29900- 30650	~27100- 27650	-
70	F (Hz)	3991.250	4014.570	6852.249	6853.889	13243.584	13259.252	13403.888
	Q	~10600- 11200	-	~16800- 17250	-	~27490- 28950	~24550- 24850	-

the results indicate that all frequencies and Q-factors decrease as the temperature increases.

Figures 5.11 and 5.12 show the predicted frequencies and Q_{TED} for the 2θ in-plane mode based on FE analysis for a 4 mm-23 μm ring with 9 μm support legs, together with measurements. These results both indicate how the frequencies and Q-factors decrease as temperature increases. In Figure 5.11, the change in frequency is $<0.1\%$ when the temperature is increased from 22°C (295K) to 50°C (323K). As this change is small, the effect of incorporating E as a function of temperature in the FE model is small and Q_{TED} only changes by $\sim 0.3\%$ compared to using the base E value. For this reason, the plot of Q_{TED} shown in Figure 5.12 is determined using the base E value (i.e. 170 GPa).

Frequencies and predicted TED values with measured values over temperature are presented in Table 5.8. In contrast to the results shown in Section 5.2 for the nominal device it is clear that TED is not the dominant damping source for the temperatures considered.

To understand how damping changes with temperature it is convenient to convert Q-factors to loss factors, η (i.e. reciprocal of the Q-factor). The predicted TED loss factors (η_{TED}), overall loss factor ($\eta_{overall}$) and loss factor accounting for all other losses (η_{other}) are included in Table 5.8 and plotted in Figure 5.13, where $\eta_{overall}$ is the sum of the two loss factors η_{TED} and η_{other} , and the percentages indicated in Table

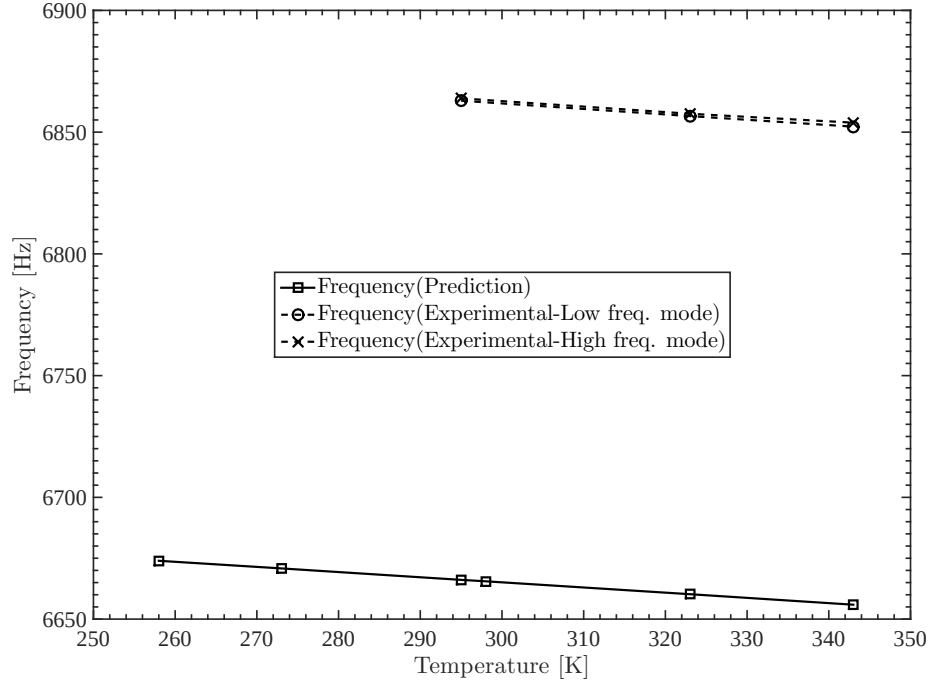


Figure 5.11: Dependence of Frequency on temperature for 2θ in-plane mode [Device-2].

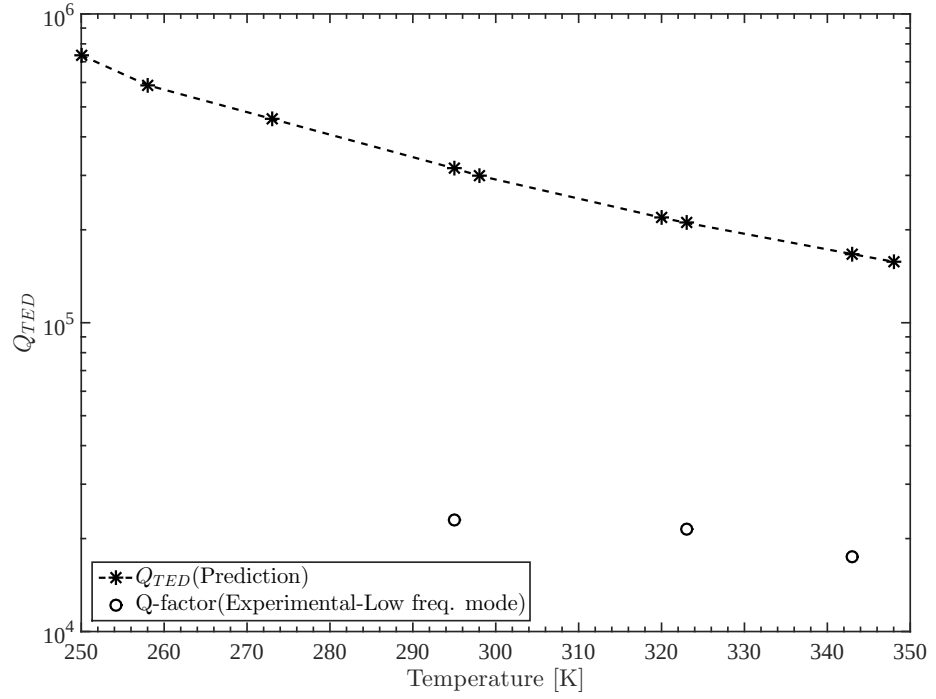


Figure 5.12: Dependence of Q_{TED} on temperature for 2θ in-plane mode [Device-2] (Y-axis in log scale).

Table 5.8: Effect of TED and other damping mechanisms on the overall damping based on predicted and measured results as a function of temperature

Temperature ($^{\circ}C$)	22	50	70
Q_{TED} (Prediction)	3.16×10^5	2.1×10^5	1.66×10^5
η_{TED} (Prediction)	3.16×10^{-6} (7%)	4.8×10^{-6} (10%)	6.02×10^{-6} (11%)
$Q_{overall}$ (Experiment)	~ 23000	~ 21500	~ 17500
$\eta_{overall}$ (Experiment)	$\sim 4.3 \times 10^{-5}$	$\sim 4.7 \times 10^{-5}$	$\sim 5.7 \times 10^{-5}$
Q_{other}	~ 24800	~ 23950	~ 19500
η_{other}	$\sim 4.0 \times 10^{-5}$ (93%)	$\sim 4.2 \times 10^{-5}$ (90%)	$\sim 5.1 \times 10^{-5}$ (89%)
Frequency (Hz) (Prediction)	6666.11	6660.23	6655.91

5.8 represent the contribution to the total loss factor. As the temperature increases the loss factors increase. From $22^{\circ}C$ to $50^{\circ}C$ it can be seen that the overall increase in damping is dominated by the increase in TED. However, from $50^{\circ}C$ to $70^{\circ}C$ the increase in damping is shared between TED and the other damping losses. The results suggest that the other damping mechanisms are less sensitive to temperature than TED, especially at low temperatures. Furthermore, the damping results shown in Figure 5.12 indicate that the measured Q-factors follow a different trend to TED predictions with temperature, providing further evidence to confirm that the high levels of damping observed experimentally are not due to TED.

5.5.2 Die Coupling

For the nominal device discussed in Section 5.2 it was noted that the high levels of damping for the 1θ in-plane mode are caused by the presence of coupling between the die motion and 1θ in-plane motion [130]. This sub-section provides an overview of die coupling and explains why die coupling is not expected to influence the 2θ and 3θ in-plane modes.

Figure 5.14 shows a schematic diagram of the ring resonator and macro structure including die, die bond and pins connecting the package to the substrate. In the figure c_1 , c_2 , c_P , c_Q and c_R are the viscous damping coefficients for the pin, die, 1θ rigid body motion, 2θ motion and 3θ motion, respectively, and k_1 , k_2 , k_P , k_Q and k_R

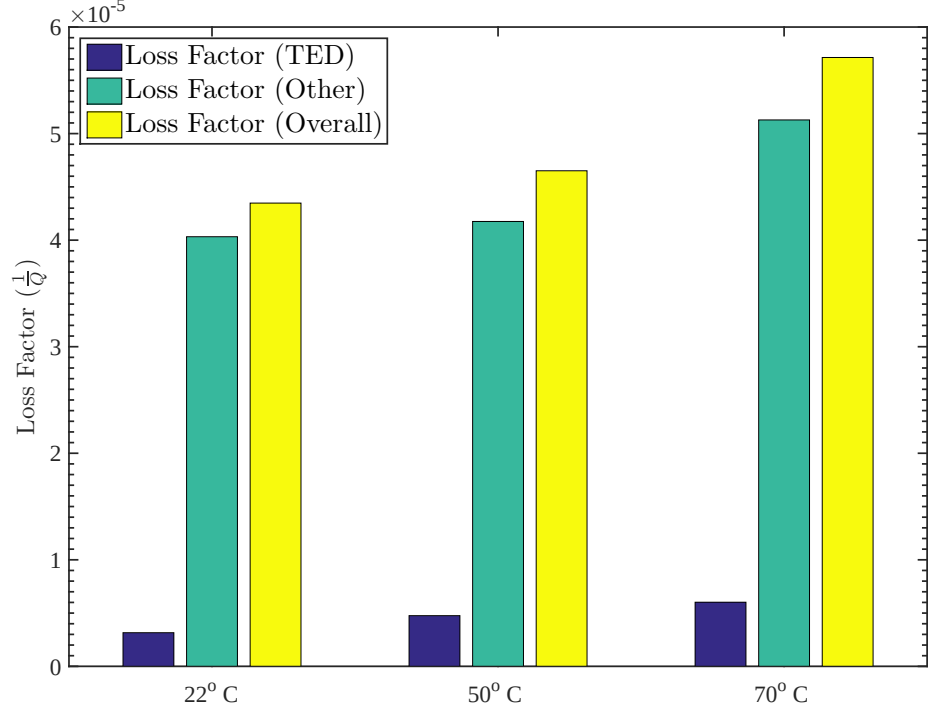


Figure 5.13: Loss factors for the Device-2.

are the stiffness coefficients for the pin, die, 1θ rigid body motion, 2θ motion and 3θ motion, respectively.

Sieberger [130] derived the equations of motion governing the generalised coordinates described in Table 5.9. To simplify the analysis, only rigid body motions in one direction were considered, so only one of the pairs of flexural coordinates and only one of the rigid body motions were included [130]. This assumes the anti-nodes of the flexural coordinates are aligned with one rigid body coordinate and is only applicable

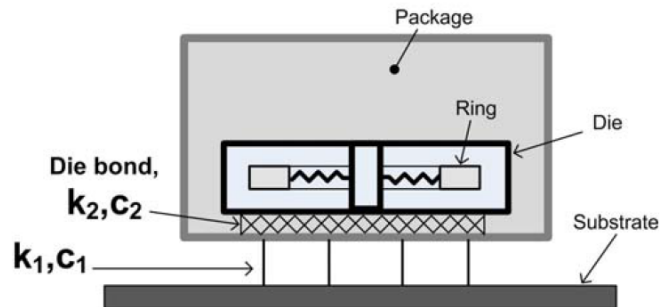


Figure 5.14: A rate sensor, modelled as three connected bodies: package, die and ring. The package is mounted on a fixed base [130].

Table 5.9: Generalised coordinates of package, die and ring

Body	Coordinate	Absolute/Relative
Package	x_p	Absolute
Die	x_d	Absolute
Ring 1θ	P_a	Absolute
Ring 2θ	Q	Relative to P_a
Ring 3θ	R	Relative to P_a

when the sensors is axisymmetric, so no imperfection breaks the symmetry.

The equations of motion for the system are:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}_s \quad (5.5)$$

where, the vector of generalised coordinates $\mathbf{q} = [x_p, x_d, P_a, Q, R]^T$ and the mass, spring and damping matrices for the mechanical system can be expressed as:

$$\mathbf{M} = \text{diag} \left[m_p, m_d, m_r, \frac{5m_r}{8}, \frac{10m_r}{18} \right] \quad (5.6)$$

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_1 + k_P & -k_P & 0 & 0 \\ 0 & -k_P & k_P & 0 & 0 \\ 0 & 0 & 0 & k_Q & 0 \\ 0 & 0 & 0 & 0 & k_R \end{bmatrix} \quad (5.7)$$

$$\mathbf{C} = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & 0 & 0 \\ -c_2 & c_1 + c_P & -c_P & 0 & 0 \\ 0 & -c_P & c_P & 0 & 0 \\ 0 & 0 & 0 & c_Q & 0 \\ 0 & 0 & 0 & 0 & c_R \end{bmatrix} \quad (5.8)$$

where m_p, m_d, m_r are the package, die and ring masses, respectively.

It is important to note that the model does not consider the presence of imperfections in the ring, supports or surrounding macro-structure. However, the presence

of non-zero off-diagonal terms in the stiffness and damping matrices indicates that the 1θ in-plane motion (P_a) is directly coupled to the die motion (x_d). In contrast, the flexural ring modes i.e. 2θ and 3θ modes having stiffness (k_Q, k_R) and damping (c_Q, c_R) are not coupled to other coordinates (i.e. no non-zero off-diagonal terms). Numerical simulations performed by Sieberer [130] showed there is no modal participation from any other modes in the 2θ and 3θ flexural modes since they decoupled from the rigid-body motion. On this basis the high levels of damping observed for the 2θ and 3θ in-plane modes in Section 5.4 are not expected to be caused by die coupling.

The above model was used to estimate the Q-factor for different modes of the nominal device [130]. The modal parameters were estimated from experimental results and it was concluded that die coupling was responsible for the 1θ in-plane motion having damping $Q=2396$. Since the ring flexural modes are decoupled from the rigid body motion, no change in Q-factor is expected and none was observed. This Q value has a similar magnitude to the experimentally measured Q for the 1θ mode in Table 5.1. It is worth noting that the Q value for the 1θ mode is strongly dependent on the die bond stiffness [130]. The influence of the die and die bond on the in-plane rigid body motion of the resonator can be explained using a two degree of freedom (2-DOF) model of the die-resonator. Appendix C describes an analytical model for analysing the coupled die-resonator to determine the following approximate expressions for the coupled natural frequency ($\omega_{coupled}$) and Q-factor ($Q_{coupled}$)

$$\omega_{coupled} = \omega_{res} \frac{1 - \frac{\omega_{res}^2}{\omega_{die}^2} \left(2 - \frac{3m_{res}}{2m_{die}}\right) + \frac{\omega_{res}^4}{\omega_{die}^4} \left(1 - \frac{3m_{res}}{2m_{die}} + \frac{m_{res}^2}{2m_{die}^2}\right)}{1 - \frac{\omega_{res}^2}{\omega_{die}^2} \left(1 - \frac{m_{res}}{m_{die}}\right)^2} \quad (5.9)$$

$$Q_{coupled} \approx Q_{res} \frac{1 - \frac{\omega_{res}^2}{\omega_{die}^2} \left(2 - \frac{3m_{res}}{2m_{die}}\right) + \frac{\omega_{res}^4}{\omega_{die}^4} \left(1 - \frac{3m_{res}}{2m_{die}} + \frac{m_{res}^2}{2m_{die}^2}\right)}{1 - \frac{2\omega_{res}^2}{\omega_{die}^2} + \frac{m_{res}\omega_{res}^3 Q_{res}}{m_{die}\omega_{die}^3 Q_{die}} + \frac{\omega_{res}^4}{\omega_{die}^4} \left(1 - \frac{m_{res}}{m_{die}} + \frac{m_{res}^2}{m_{die}^2}\right)} \quad (5.10)$$

where ω_{res} , Q_{res} , m_{res} and ω_{die} , Q_{die} , m_{die} are the natural frequency, Q-factor and mass of the resonator and the die, respectively. The die is assumed to be rigid, therefore the stiffness and damping of the die is attributed from the die bond. Eqs.

(5.9) and (5.10) are derived based on the assumption that the die and die bond has small effect on the natural frequency of the resonator. This has been observed in Section 5.2, where predicted frequency (neglecting die) is close to the measured frequency. It is also assumed the coupling is small, and the Q-factor of uncoupled resonator is much higher than Q-factor for die.

Eq. (5.9) approximates the damped natural frequency ($\omega_{coupled}$) and is independent of damping i.e. Q_{res} and Q_{die} , because the resonator is assumed to be weakly damped. Eq. (5.10) approximates the Q-factor for the coupled system in terms of Q_{res} and Q_{die} . For $Q_{coupled}$ to be approximately equal to Q_{res} (i.e. the die and die bond have no influence on the resonator damping) it is shown in Appendix C that the following condition must be satisfied:

$$Q_{die} \gg \frac{\frac{m_{res}\omega_{res}^3 Q_{res}}{m_{die}\omega_{die}^3}}{1 - \frac{2\omega_{res}^2}{\omega_{die}^2} + \frac{\omega_{res}^4}{\omega_{die}^4} \left(1 - \frac{m_{res}}{m_{die}} + \frac{m_{res}^2}{m_{die}^2}\right)} \quad (5.11)$$

The parameters used by Sieberer [130] to determine the frequency and Q-factor, for 1 θ in-plane mode using his model are: $m_{die}=0.295$ g, $m_{res}=0.295$ g, $\omega_{die}=50\,265.6$ rad/sec (i.e. 8000 Hz), $\omega_{die}=33\,093.6$ rad/sec (i.e. 5267 Hz), $Q_{res}=90000$ (considering TED only) and $Q_{die}=8$. Using these values in Eqs. (5.9) and (5.10) give $\omega_{coupled}=32\,923.9$ rad/sec (i.e. 5240 Hz) and $Q_{coupled}=2578$. These results indicate that die coupling reduces the Q-factor significantly but the coupled frequency does not change significantly compared to the resonator frequency, which was also observed by Sieberer [130]. Using Eq. (5.11) it can be shown that Q_{die} needs to be much greater than 271.7 to reduce the effect of coupling on $Q_{coupled}$.

In summary, for the 1 θ in-plane mode the high level of damping is caused by the die motion coupling to the 1 θ in-plane motion and this can be explained using the model developed by Sieberer [130]. The model presented in Appendix C is capable of predicting the coupled die and 1 θ rigid body motion of a ring resonator, and can also be used to study the effect of die motion on the out-of-plane bounce mode.

In practice, manufacturing imperfection induces small amounts of coupling between the different ring motions and the die motion. The resulting influence of die

damping on the other coordinates depends on the frequency separation and the coupling effect reduces as frequency separation increases. Noting that the frequency separation between the 2θ (~ 14.2 kHz) and 3θ in-plane (~ 33.3 kHz) modes and the die mode (~ 8.2 kHz) is large, it is expected that the contribution of die damping to these modes will be very small.

An observation that supports this assertion is based on the fact that the measured Q-factors for all modes (except the 1θ in-plane mode) have (roughly) similar values. If die coupling is the dominant cause for increased damping, then the high frequency modes would be influenced to a lesser extent than the low frequency modes, but this trend is not observed.

For these reasons it is unlikely that coupling to the die is the dominant cause for the high levels of damping observed in the 2θ and 3θ in-plane modes for the high-Q devices.

5.5.3 Support Loss

Methods to predict the support loss were discussed in Chapter 3 and applied to supported ring resonators in Chapter 4. The main conclusion drawn was that the levels of support loss are much lower than those predicted for TED. This point is reinforced by comparing the predicted support losses with measured Q factors for the high-Q devices. As discussed in Chapter 4 this can be attributed to the symmetric design of the resonator relative to the mode of vibration, which minimises the dissipation of energy through the supports by dynamically balancing the resonator. In principle this ensures no elastic waves are excited at the central attachment hub when the ring vibrates in the 2θ in-plane mode, but this is not the case for the other modes. Manufacturing and materials imperfections in the ring and support legs destroy the perfect balance of the 2θ in-plane mode, leading to energy dissipation, whilst the balance of others modes is also affected. However, the results presented in Section 4.3.6 indicate that the expected level of out of balance does not induce sufficiently high levels of stress at the hub interface to explain the observed high levels of damping. Further

details about ring support loss can be found in Chapter 4.

5.5.4 Gas Damping

As mentioned in Chapter 1, for devices operating in atmospheric pressure, gas damping is generally the dominant source of energy dissipation. For MEMS ring based angular rate sensors, the space surrounding the resonator is normally evacuated to reduce/eliminate the influence of gas damping. However, it is feasible that only a partial evacuation is achieved for the high-Q devices. In this section a qualitative discussion and description of the effects of gas damping on the damping of ring resonators is presented.

Although the high-Q devices are evacuated so the resonator operates in a vacuum, the high levels of measured damping (see Tables 5.4 and 5.5) indicate the possibility of gas damping. As it was not practically possible to measure the air/gas pressure, it was not possible to predict the regime in which the device operates under experimental test conditions. If the gas pressure in the device is sufficiently large that viscous damping is present, then the damping is expected to increase significantly. As the ring vibrates, radial motion of the ring changes causing air/gas trapped in the gap between the outer surface of the ring and electrodes to be squeezed out and sucked back in again. The flow of gas/air in and out of the gap generates viscous drag which creates a reactive force that opposes the ring motion, increasing energy dissipation. MEMS devices using capacitive actuation and sensing often have narrow air gaps, and as a result squeeze film damping has a significant effect on the dynamic behaviour [30].

To illustrate the influence of gas damping, experimentally measured Q-factor values for the 2θ in-plane mode for a partially slotted device with nominal dimensions are considered as the gas pressure varies [137]. Figure 5.15 shows the arrangement of slots, where the ring resonator contains slots at 4 uniformly spaced locations around the ring. The primary mode is aligned midway between the slotted regions, whilst the secondary mode is aligned with the slots. The results for the secondary mode are

useful as they show the influence of slots on Q , and exhibits the damping behaviour similar to a fully slotted device.

Figure 5.16 shows how the Q -factor varies as a function of pressure. It can be seen that the Q -factor for the secondary mode of the partially slotted device (~ 32000) at low pressure is similar to the measured Q -factor (~ 37600) for a fully slotted nominal device, see Table 5.1. The device was tested over a pressure range of 0.001-1 Torr ($\sim 0.0013 \times$ atmosphere) and a significant drop in Q observed as gas pressure surrounding the resonator increases. For the pressure range from 0.001 to 0.01 Torr the resonator can be considered to be operating in the intrinsic damping regime, where the Q remains relatively constant and is dominated by TED. For higher pressures, Q reduces because the resonator operates in the molecular region and interactions take place with gas/air molecules. Squeeze film damping is unlikely to occur in this low pressure range, where air particles behave as kinetic particles. For a pressure ~ 1 Torr, Q is ~ 5000 . At this pressure gas damping and other non-TED sources contribute (i.e. Q_{other}) ~ 5800 compared to $Q_{TED} \sim 37600$. The high levels of gas damping caused by a relatively small change in pressure emphasises the detrimental effect that gas damping can have on the overall Q value.

It is evident from the above discussion that gas damping can significantly reduce the Q -factor with a relatively small change in operating pressure inside the nominal device. For the high- Q devices tested here it is argued that a small amount of air/gas is responsible for the observed high levels of damping. Further work is required to confirm if air/gas is indeed present inside the device. This would require a study to be performed to quantify the influence of gas damping on the Q factor, and would involve analysing the interaction of fluid flow with the moving ring. Similar analyses are available for simple structures and trivial boundary conditions [138]. For complex devices with structural elements that move relative to each other and within close proximity of stationary objects, computational modelling is required to properly consider the fluid-structure interactions and reliably evaluate the gas damping. Analytical and computational models are unavailable for ring resonators,

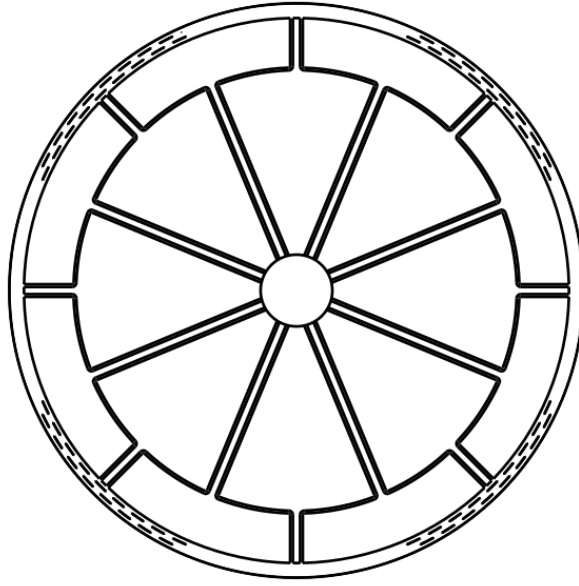


Figure 5.15: Partially slotted ring device with nominal dimensions.

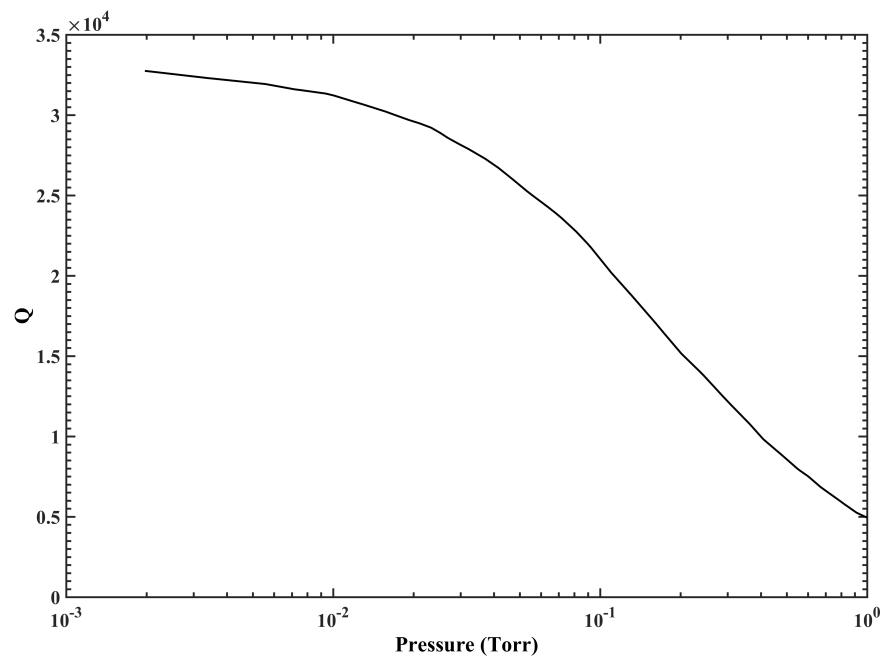


Figure 5.16: Measurement of Q as a function of pressure for a partially slotted nominal device [137].

and it was not within the scope of the research project to model gas damping in these devices. This is suggested as an area for future research.

5.6 Conclusion

The main focus of the Chapter was to compare measured frequencies and Q-factors for high-Q devices, designed to have low levels of thermoelastic damping, against FE predicted results. In addition some results were presented for the nominal device for the purposes of comparison and interpretation.

The following conclusions were drawn:

- The measured frequencies were found to be higher than those predicted by the FE method, and this was attributed to the presence of manufacturing imperfections (e.g. under-etch) in the high-Q devices causing inaccuracies in ring and support width dimensions during the fabrication stage.
- The measured Q-factors for all modes were found to be significantly lower than those predicted using the thermoelastic damping model developed in Chapter 2. In particular, the design target of $Q \sim 300000$ for the 2θ in-plane mode was not achieved, for all devices tested.
- The highest level of measured damping occurred in the 1θ in-plane mode. This is in contrast to the predicted results, where the highest thermoelastic Q-factor was achieved for this mode. The nominal device also possesses high levels of measured damping in this mode, and previous work based on the nominal device study indicates that increased levels of damping (beyond those arising from TED) occur due to coupling between the ring rigid body motion and die motion, particularly when the die bond is highly damped. It is expected that die bond coupling is responsible for the observed high levels of damping in the 1θ in-plane mode for the high-Q devices tested.
- The ring resonators are designed to have flexural ring modes (e.g. 2θ in-plane

mode) which are uncoupled from the rigid body motion, die motion and out-of-plane modes. On this basis, it is unlikely that die bond coupling is the origin of the increased levels of observed damping.

- Further modelling and testing was conducted to analyse the effect of temperature on TED predictions and fabricated devices. Reductions in frequency and Q were observed in both predicted and measured results. The frequency decreases because the Young's modulus decreases with temperature. The Q -factor related to TED has strong dependence on temperature, which decreases with increase in temperature. The observed reduction in Q is relatively small compared to predicted values, suggesting TED is not the dominant cause for the observed high levels of damping in high- Q devices.
- The effect of geometric etch imperfections were investigated. These are likely to increase frequency, but have little influence on Q -factor based on TED considerations.
- Based on discussion in Chapter 4, support losses have negligible influence on damping for the high- Q devices.
- For the nominal device it was observed that gas damping causes a significant reduction in Q -factor when the space around the resonator is partially evacuated. It is inferred that similar behaviour is expected for the high- Q devices and hypothesized that gas damping is responsible for the observed high levels of damping for the high- Q devices. Further work is required to investigate this hypothesis.

Chapter 6

Conclusions and Future Work

6.1 Introduction

Thermoelastic damping (TED) and support loss are important sources of damping in MEMS resonators in the absence of gas damping in vacuum encapsulated devices. The aim of this thesis was to investigate the effects of thermoelastic damping and support loss, and develop improved understanding of their influence on the performance of supported ring resonators used in MEMS angular rate sensors. The research work is divided into two main parts: a fundamental study of thermoelastic damping in ring resonators considering the effect of supporting structure and analysis of support loss in supported ring resonators. For both TED and support loss, robust models were developed using the finite element method to quantify the damping in ring resonators. Since a high-Q is the primary demand, this research work is expected to aid the development of high performance MEMS rate sensors by reducing the damping.

This chapter summarises the main conclusions and contributions drawn from each chapter and discusses the relevant future work in this area of research.

6.2 Conclusions and Contributions

The main conclusions of this research work are summarised, and the major contributions are highlighted as follows:

- Existing analytical models are limited to simple structures due to their restric-

tive assumptions. A robust FE model has been developed considering fully coupled thermoelastic equations to predict TED in complex structures. The model solves mechanical and thermal fields of a structure simultaneously using COMSOL, and has been validated successfully against analytical estimates and experimental results in terms of Q-factor and frequency. The model is used to investigate the effects of key design parameters and material properties in resonators.

- The TED behaviour of unsupported rings was explained by Zener's theory, providing in-depth understanding of the damping mechanism. It is possible to achieve high-Q by selecting either large diameter thin rings or small diameter thick rings, which operate in the isothermal and adiabatic regions away from the Debye peak, respectively. However, a trade-off between frequency and Q_{TED} needs to be considered based on the fact that large diameter thin rings demonstrate low stiffness and consequently low frequency compared to small thick rings. The effect of incorporating slots around the unsupported ring circumference was also investigated using the fully coupled FE model. It is concluded that slots can increase the damping in a high-Q ring significantly, thus lowers the Q-factor. In this case, slots reduce the coupling between the thermal modes having low energy dissipation and the prevailing mechanical mode.
- One of the primary aims of the research work was to investigate TED behaviour in supported ring resonators used in angular rate sensors to achieve higher performance, since TED behaviour in supported rings has not been studied sufficiently in the past. It was observed that adding support legs to high-Q rings tends to increase Q-factors compared to unsupported rings, but depending on the support leg design and ring size it could decrease also. The results indicate that TED in support legs can significantly affect the overall Q-factor of the resonator. To further advance this study, TED in support legs was quantified, and it was found that as the width of support legs increases the

Q-factor increases to a maximum before decreasing. The damping behaviour of ring and support legs were separately investigated using the FE model and a semi-analytical model. The strain energy based semi-analytical model approximates the TED behaviour in supported ring resonators using Zener's equation to calculate Q_{TED} in ring and support separately. It was found using both models that damping in support legs dominate overall damping as the leg width increases compared to the damping in the ring. It is concluded that an optimum leg width exists at which maximum Q_{TED} is achieved, and this leg width needs to be considered to design a high-Q ring resonator.

- Most MEMS devices are fabricated from silicon, but other materials can be used. Other materials provide an opportunity for enhancing the performance of the device. To achieve this goal it is important to gain understanding on the effect of material properties on TED behaviour, particularly thermal properties which dictate TED. On this basis, the influence of materials with relatively low and high thermal conductivity were investigated. It was found that high-Q is generally achieved with unsupported small diameter thick rings for materials with low thermal conductivity, since they operate in the isothermal region. In contrast, high thermal conductivity materials have high-Q for large thin rings. The damping behaviour of supported rings are greatly influenced by the local Debye peak associated with leg damping, for materials with low thermal conductivity. As a consequence, a minimum Q_{TED} is observed as leg width increases.
- Support loss occurs due to the energy coupling between the resonator and the substrate, where the energy is radiated by propagating waves into the substrate. While analytical approximations are available for simple structures based on restrictive assumptions using classical wave theory, numerical approaches based on finite element method provide a robust means of predicting support loss in complex resonating structures. A FE model was developed based on Perfectly Matched Layer (PML) technique using COMSOL to quantify support loss in

MEMS resonators. PMLs mimic the behaviour of substrate as semi-infinite or infinite domain, and absorbs propagating waves to quantify support loss. The model was successfully validated, overcoming previous modelling issues with convergence, for cantilever beam and disk resonators and achieved good agreement with analytical estimates and published results. It was shown that the two most important parameters that influence accuracy and effectiveness of PMLs are the size of the PML region and end value of the absorbing function. Modelling strategies have been developed to select the optimal combination of these two parameters to quantify support loss in MEMS resonators, and depending on the nature of the problem one of the strategies should be employed.

- Following the modelling strategies developed in Chapter 3, the FE model was used to analyse support loss behaviour in supported ring resonators. Although the 2θ in-plane mode is the operating mode of the ring resonators, other in-plane and out-of-plane modes were also investigated. It is concluded that support loss is negligible in ring resonators compared to Q_{TED} due to much higher $Q_{support}$ for the 2θ in-plane mode. This is due to the balanced 2θ in-plane mode providing less coupling between the resonant mode and the supporting hub, which translates to lower stress at the interface between hub and substrate. However, leg-dominated modes showed lower $Q_{support}$ because the modes are unbalanced.
- Qualitative discussions on the influence of out-of-balance modes on support loss are available in the research literature. In this work the influence of unbalanced modes on support loss is analysed quantitatively and investigates the influence of systematic design variations and manufacturing variations. Introducing mass imperfections not only induces frequency splitting between pair of modes but also increases support loss because out-of-balance is introduced into the 2θ in-plane mode. Similarly, structural asymmetry due to offsetting the central supporting structure showed significant reduction in Q-factor as the cyclic asymmetry causes unbalance. Since fabrication imperfections are

inevitable the 2θ in-plane will always possess a degree of imbalance, but the energy dissipated through the support structure is much lower compared to the dominant TE effects.

- Based on the conclusions from a TED analysis of supported ring resonators, high-Q ring resonators were designed to have low levels of TED within specified design constraints. Some of these high-Q designs were fabricated, and experimental testing of the devices conducted to measure frequency and Q-factor for different modes. The measured frequencies were found to be higher than those predicted by the FE method, and this was attributed to the presence of manufacturing imperfections in the form of under-etch. However, measured Q-factors for all modes were found to be significantly lower than the theoretical Q_{TED} predictions, and the design target for the 2θ in-plane was not achieved for any of the devices tested. Theoretical predictions using the fully coupled thermoelastic damping model showed the lowest level of damping in the 1θ in-plane mode. In contrast, the highest level of measured damping occurred in the 1θ in-plane mode due to the presence of die coupling. However, a qualitative analysis showed that die coupling is unlikely to affect the damping of flexural ring-modes such as 2θ in-plane mode, because the ring modes are uncoupled from rigid body motion and die motion. Further modelling and testing conducted to analyse the effects of temperature on TED concluded that TED is not the dominant cause for the observed high levels of damping in high-Q devices. Since support loss is negligible in supported ring resonators, it was hypothesised that gas damping is responsible for the low-Q observed in these devices. But, further research is required to validate this hypothesis.

6.3 Future Work

This thesis has focused on the improvement of thermoelastic and support loss behaviour of MEMS ring resonators, but there still remains need for further research

for improving the overall performance of rate sensors. Based on the obtained results and conclusions, some topics that should be further researched are explained:

- **Non-Linearity:** For large vibration amplitudes, it is well known that unsupported rings exhibit softening behaviour due to geometric non-linearity, where the restoring force becomes a non-linear function of its displacement. Serandour [139] investigated the non-linear flexural vibration of supported ring resonators, and showed that support legs modify the behaviour and provide either softening or hardening behaviour. The linear theory is valid in capacitive rate sensors since ring displacements are usually small. In these devices non-linearity occurs from electrostatic actuation force, and has been found to change the nature of TED significantly by De and Aluru [140]. However, inductive rate sensors are affected by the geometric non-linearity due to amplitude of motion. The FE model developed in Chapter 2 to predict TED behaviour considers geometric linearity since deformations are assumed to be small, but the geometric non-linearity can be introduced in the model and investigate its effect on TED.
- **Support Loss Model:** Although it has been concluded from the analysis presented in Chapter 4 that support loss is negligible in supported ring resonators, the PML based FE model considered the resonator to be directly attached to the substrate neglecting the effects of die and die bonds. For more accurate predictions the existing model can be extended to include the die bond layer. However, it is not a straightforward approach since the behaviour of die bonds requires to be modelled as hyperelastic material, and research on die bond characteristics is currently being conducted [141].
- **Gas Damping:** In Chapter 5 it has been hypothesised that high levels of damping observed is due to the presence of gas damping. But, experimental results and analytical/numerical models are required to validate the hypothesis. Any residual gas molecules in the system e.g. gas released from anodic bonds would affect the damping, and experimental testing needs to be conducted to

identify the presence of gas molecules using residual gas analyser. Furthermore, it will be useful to measure Q-factor as a function of pressure using a vacuum chamber to derive a correlation between the Q-factor and the surrounding gas pressure for high-Q devices. Modelling gas damping is more challenging since the gas pressure determines whether the energy is dissipated due to molecular damping or squeeze film damping (SFD). A fluid-structure interaction model of the system could be used to capture the effect of SFD, shear damping and molecular damping, but appropriate flow properties and boundary conditions are crucial for reliable estimation [138]. However, SFD will be more important than molecular damping if the thickness of gas film is smaller than one third of the width of the structure [30]. Therefore, for most MEMS resonators it is the most important gas damping mechanism due to the narrow capacitive gap between the resonator and driving and sensing electrodes. In the high-Q devices SFD can be assumed to be dominant due to the low frequency and smaller gaps. A finite element model in COMSOL using thin film damping can be developed to investigate the damping resulting from the squeezing of a thin film of gas between two surfaces.

- **Noise:** A higher Q-factor increases the resolution of MEMS resonators by reducing the noise level, as discussed in Section 1.3.3. The reduction of intrinsic noise level is important for achieving tactical and inertial grade performance in angular rate sensors. The fundamental limiting noise component of a resonator is due to the Brownian motion, and it becomes more significant as the size of the resonator decreases. Ayazi *et al.* [10] and Leland [142] provides analytical expression for the effect of Brownian noise on angular rate sensors, where noise level is a function of Q-factor, resonance frequency and mass. During this research work, a preliminary analysis on the effect of frequency and Q_{TED} as a function of radius and radial thickness of unsupported ring resonators on Brownian noise was conducted. The expression derived by Ayazi *et al.* [10] (see Eq. (1.13)) was used for the purpose of analysis. It was seen that the noise

level decreases with an increase in radius, whereas it reaches a peak value before decreasing with radial thickness. It was also observed that Q_{TED} dominates until noise reaches a peak and after that frequency dominates the noise level. The analysis was limited due to the scope of this research project. A rigorous analysis is required to understand the fundamental mechanism of Brownian noise on the sensor performance.

References

- [1] Prime Faraday Technology Watch, An introduction to MEMS (2002).
- [2] M. Tanaka, An industrial and applied review of new MEMS devices features, *Microelectronic Engineering* 84 (58) (2007) 1341–1344. doi:10.1016/j.mee.2007.01.232.
- [3] Z. Hao, F. Ayazi, Thermoelastic damping in flexural-mode ring gyroscopes, in: *ASME 2005 International Mechanical Engineering Congress and Exposition*, 2005, pp. 335–343. doi:10.1115/IMECE2005-79965.
- [4] M. Weinberg, R. Candler, S. Chandorkar, J. Varsanik, T. Kenny, A. Duwel, Energy loss in MEMS resonators and the impact on inertial and RF devices, in: *International Solid-State Sensors, Actuators and Microsystems Conference, TRANSDUCERS*, 2009, pp. 688–695. doi:10.1109/SENSOR.2009.5285418.
- [5] A. M. Shkel, Type I and type II micromachined vibratory gyroscopes, in: *Position, Location, and Navigation Symposium*, 2006 IEEE/ION, 2006, pp. 586–593. doi:10.1109/PLANS.2006.1650648.
- [6] C. Acar, A. Shkel, *MEMS Vibratory Gyroscopes: Structural Approaches to Improve Robustness*, Springer US, 2008. doi:10.1007/b117574.
- [7] P. Shao, *Microscale hemispherical shell resonating gyroscopes*, Ph.D. thesis, Georgia Institute of Technology, Atlanta, Georgia (2014).
- [8] N. Yazdi, F. Ayazi, K. Najafi, Micromachined inertial sensors, *Proceedings of the IEEE* 86 (8) (1998) 1640–1659. doi:10.1109/5.704269.

-
- [9] C. H. J. Fox, Vibrating cylinder rate gyro: theory of operation and error analysis, in: Proceedings of the DGON Symposium on Gyro Technology, Stuttgart, 1988.
- [10] F. Ayazi, K. Najafi, A HARPSS polysilicon vibrating ring gyroscope, *Journal of Microelectromechanical Systems* 10 (2) (2001) 169–179. doi:10.1109/84.925732.
- [11] S. McWilliam, J. Ong, C. Fox, On the statistics of natural frequency splitting for rings with random mass imperfections, *Journal of Sound and Vibration* 279 (12) (2005) 453–470. doi:10.1016/j.jsv.2003.11.034.
- [12] C. Fox, A simple theory for the analysis and correction of frequency splitting in slightly imperfect rings, *Journal of Sound and Vibration* 142 (2) (1990) 227–243. doi:10.1016/0022-460X(90)90554-D.
- [13] J. Ong, Uncertainty effect on the dynamics of vibrating structure rate sensors, Ph.D. thesis, The University of Nottingham (2003).
- [14] S. J. Wong, Thermoelastic damping in MEMS ring resonators, Ph.D. thesis, The University of Nottingham, Nottingham (2005).
- [15] B. Chouvion, C. Fox, S. McWilliam, A. Popov, In-plane free vibration analysis of combined ring-beam structural systems by wave propagation, *Journal of Sound and Vibration* 329 (24) (2010) 5087–5104. doi:10.1016/j.jsv.2010.05.023.
- [16] S. J. Wong, C. H. J. Fox, S. McWilliam, C. P. Fell, R. Eley, A preliminary investigation of thermo-elastic damping in silicon rings, *Journal of Micromechanics and Microengineering* 14 (9) (2004) S108–S113. doi:10.1088/0960-1317/14/9/019.
- [17] J. Gorman, Finite element model of thermoelastic damping in MEMS, Master's thesis, Massachusetts Institute of Technology (2002).
-

-
- [18] T. Koyama, Efficient evaluation of damping in resonant MEMS, Ph.D. thesis, University of California, Berkeley (2008).
- [19] B. Chouvion, Vibration transmission and support loss in MEMS resonators, Ph.D. thesis, The University of Nottingham, Nottingham (2010).
- [20] S. Beeby, G. Ensell, M. Kraft, N. White, MEMS Mechanical Sensors, Artech House, London, 2004.
- [21] T. B. Gabrielson, Mechanical-thermal noise in micromachined acoustic and vibration sensors, IEEE Transactions on Electron Devices 40 (5) (1993) 903–909. doi:10.1109/16.210197.
- [22] H. Johari, Micromachined capacitive silicon bulk acoustic wave gyroscopes, Ph.D. thesis, Georgia Institute of Technology (2008).
- [23] W. Geiger, B. Folkmer, J. Merz, H. Sandmaier, W. Lang, A new silicon rate gyroscope, in: IEEE Micro Electro Mechanical Systems Workshop (MEMS'98), Heidelberg, 1998, pp. 615–620.
- [24] M. I. Younis, MEMS Linear and Nonlinear Statics and Dynamics, Springer, New York, 2011.
- [25] W. Newell, Miniaturization of tuning forks, Science 161 (3848) (1968) 1320–1326. doi:10.1126/science.161.3848.1320.
- [26] W. Wang, J. Jia, J. Li, Slide film damping in microelectromechanical system devices, Proceedings of the Institution of Mechanical Engineers, Part N: Journal of Nanoengineering and Nanosystems 227 (4) (2013) 162–170. doi:10.1177/1740349913486097.
- [27] F. R. Blom, S. Bouwstra, M. Elwenspoek, J. H. J. Fluitman, Dependence of the quality factor of micromachined silicon beam resonators on pressure and geometry, Journal of Vacuum Science & Technology B 10 (1) (1992) 19–26. doi:10.1116/1.586300.
-

-
- [28] H. Hosaka, K. Itao, S. Kuroda, Damping characteristics of beam-shaped micro-oscillators, *Sensors and Actuators A: Physical* 49 (1) (1995) 87–95. doi:10.1016/0924-4247(95)01003-J.
- [29] S. Middelhoek, K. Cammann, R. Puers, T. Corman, P. Enoksson, G. Stemme, Proceedings of eurosensors x gas damping of electrostatically excited resonators, *Sensors and Actuators A: Physical* 61 (1) (1997) 249–255. doi:10.1016/S0924-4247(97)80270-1.
- [30] M. Bao, H. Yang, Squeeze film air damping in MEMS, *Sensors and Actuators A: Physical* 136 (1) (2007) 3–27. doi:10.1016/j.sna.2007.01.008.
- [31] M. Gologanu, C. G. Bostan, V. Avramescu, O. Buiu, Damping effects in MEMS resonators, in: *CAS 2012 (International Semiconductor Conference)*, Vol. 1, 2012, pp. 67–76.
- [32] J. B. Starr, Squeeze-film damping in solid-state accelerometers, in: *Solid-State Sensor and Actuator Workshop. 4th Technical Digest.*, IEEE, 1990, pp. 44–47. doi:10.1109/SOLSEN.1990.109817.
- [33] Y.-J. Yang, M. A. Gretillat, S. D. Senturia, Effect of air damping on the dynamics of nonuniform deformations of microstructures, in: *International Conference on Solid State Sensors and Actuators. TRANSDUCERS '97, Chicago.*, Vol. 2, 1997, pp. 1093–1096 vol.2. doi:10.1109/SENSOR.1997.635390.
- [34] A. H. Nayfeh, M. I. Younis, Modeling and simulations of thermoelastic damping in microplates, *Journal of Micromechanics and Microengineering* 14 (12) (2004) 1711. doi:10.1088/0960-1317/14/12/016.
- [35] R. Christian, The theory of oscillating-vane vacuum gauges, *Vacuum* 16 (4) (1966) 175–178. doi:10.1016/0042-207X(66)91162-6.
- [36] J. Zook, D. Burns, H. Guckel, J. Sniegowski, R. Engelstad, Z. Feng, Characteristics of polysilicon resonant microbeams, *Sensors and Actuators A: Physical* 35 (1) (1992) 51–59. doi:10.1016/0924-4247(92)87007-4.
-

-
- [37] Z. Kádár, W. Kindt, A. Bossche, J. Mollinger, Quality factor of torsional resonators in the low-pressure region, *Sensors and Actuators A: Physical* 53 (1) (1996) 299–303. doi:10.1016/0924-4247(96)80153-1.
- [38] B. Li, H. Wu, C. Zhu, J. Liu, The theoretical analysis on damping characteristics of resonant microbeam in vacuum, *Sensors and Actuators A: Physical* 77 (3) (1999) 191–194. doi:10.1016/S0924-4247(99)00072-2.
- [39] M. Bao, H. Yang, H. Yin, Y. Sun, Energy transfer model for squeeze-film air damping in low vacuum, *Journal of Micromechanics and Microengineering* 12 (3) (2002) 341–346.
- [40] M. Blanter, I. Golovin, H. Neuhäuser, H. Sinning, *Internal Friction in Metallic Materials: A Handbook*, Springer Series in Materials Science, Springer, Berlin, 2007.
- [41] T. Künzig, M. Niessner, G. Wachutka, G. Schrag, H. Hammer, The effect of thermoelastic damping on the total Q-factor of state-of-the-art MEMS gyroscopes with complex beam-like suspensions, *Procedia Engineering* 5 (2009) 1296–1299. doi:10.1016/j.proeng.2010.09.351.
- [42] A. Kovalenko, *Thermoelasticity: basic theory and applications*, Wolters-Noordhoff, Groningen, 1969.
- [43] W. Nowacki, *Thermoelasticity*, 2nd Edition, Pergamon Press, Warszawa, 1986.
- [44] C. Zener, Internal friction in solids I. theory of internal friction in reeds, *Physical Review* 52 (3) (1937) 230–235. doi:10.1103/PhysRev.52.230.
- [45] C. Zener, Internal friction in solids II. general theory of thermoelastic internal friction, *Physical Review* 53 (1) (1938) 90–99. doi:10.1103/PhysRev.53.90.
- [46] C. Zener, W. Otis, R. Nuckolls, Internal friction in solids III. experimental demonstration of thermoelastic internal friction, *Physical Review* 53 (1) (1938) 100–101. doi:10.1103/PhysRev.53.100.
-

-
- [47] T. Roszhart, The effect of thermoelastic internal friction on the Q of micromachined silicon resonators, in: Solid-State Sensor and Actuator Workshop, 1990. 4th Technical Digest., IEEE, 1990, pp. 13–16. doi:10.1109/SOLSEN.1990.109810.
- [48] A. Duwel, J. Gorman, M. Weinstein, J. Borenstein, P. Ward, Experimental study of thermoelastic damping in MEMS gyros, *Sensors and Actuators A: Physical* 103 (12) (2003) 70 – 75. doi:10.1016/S0924-4247(02)00318-7.
- [49] J. Alblas, On the general theory of thermo-elastic friction, *Applied Scientific Research* 10 (1) (1961) 349–362. doi:10.1007/BF00411929.
- [50] P. Chadwick, Thermal damping of a vibrating elastic body, *Mathematika* 9 (1) (1962) 38–48. doi:10.1112/S0025579300003077.
- [51] J. Alblas, A note on the theory of thermoelastic damping, *Journal of Thermal Stresses* 4 (3-4) (1981) 333–355. doi:10.1080/01495738108909973.
- [52] V. Kinra, K. Milligan, A second-law analysis of thermoelastic damping, *Journal of Applied Mechanics* 61 (1) (1994) 71–76. doi:10.1115/1.2901424.
- [53] R. Lifshitz, M. L. Roukes, Thermoelastic damping in micro- and nanomechanical systems, *Physical Review B* 61 (8) (2000) 5600–5609. doi:10.1103/PhysRevB.61.5600.
- [54] S. J. Wong, C. H. J. Fox, S. McWilliam, Thermoelastic damping of the in-plane vibration of thin silicon rings, *Journal of Sound and Vibration* 293 (1-2) (2006) 266–285. doi:10.1016/j.jsv.2005.09.037.
- [55] A. Choi, Y. Na, J. Kim, Thermoelastic damping of inextensional hemispherical shell, *World Academy of Science, Engineering and Technology* 32 (2009) 177–182.
- [56] L. Sorenson, P. Shao, F. Ayazi, Bulk and surface thermoelastic dissipation in
-

- micro-hemispherical shell resonators, *Journal of Microelectromechanical Systems* 24 (2) (2014) 486–502. doi:10.1109/JMEMS.2014.2333528.
- [57] Z. Hao, Thermoelastic damping in the contour-mode vibrations of micro- and nano-electromechanical circular thin-plate resonators, *Journal of Sound and Vibration* 313 (12) (2008) 77–96. doi:10.1016/j.jsv.2007.11.035.
- [58] B. H. Houston, D. M. Photiadis, M. H. Marcus, J. A. Bucaro, X. Liu, J. F. Vignola, Thermoelastic loss in microscale oscillators, *Applied Physics Letters* 80 (7). doi:10.1063/1.1449534.
- [59] B. Houston, D. Photiadis, J. Vignola, M. Marcus, X. Liu, D. Czaplewski, L. Sekaric, J. Butler, P. Pehrsson, J. Bucaro, Loss due to transverse thermoelastic currents in microscale resonators, *Materials Science and Engineering: A* 370 (12) (2004) 407 – 411, 13th International Conference on Internal Friction and Ultrasonic Attenuation in Solids. doi:10.1016/j.msea.2003.05.007.
- [60] D. M. Photiadis, B. H. Houston, X. Liu, J. A. Bucaro, M. H. Marcus, Thermoelastic loss observed in a high Q mechanical oscillator, *Physica B: Condensed Matter* 316-317 (2002) 408–410. doi:10.1016/S0921-4526(02)00528-8.
- [61] A. N. Norris, D. M. Photiadis, Thermoelastic relaxation in elastic structures, with applications to thin plates, *The Quarterly Journal of Mechanics and Applied Mathematics* 58 (1) (2005) 143–163. doi:10.1093/qjmath/hbi002.
- [62] S. Prabhakar, S. Vengallatore, Theory of thermoelastic damping in micromechanical resonators with two-dimensional heat conduction, *Journal of Microelectromechanical Systems* 17 (2) (2008) 494–502. doi:10.1109/jmems.2008.916316.
- [63] A. Duwel, R. Candler, T. Kenny, M. Varghese, Engineering MEMS resonators with low thermoelastic damping, *Journal of Microelectromechanical Systems* 15 (6) (2006) 1437–1445. doi:10.1109/JMEMS.2006.883573.

-
- [64] Z. Hao, Y. Xu, S. Durgam, A thermal-energy method for calculating thermoelastic damping in micromechanical resonators, *Journal of Sound and Vibration* 322 (4-5) (2009) 870–882. doi:10.1016/j.jsv.2008.12.005.
- [65] D. Senkal, C. Raum, A. Trusov, A. Shkel, Titania silicate / fused quartz glass-blowing for 3-D fabrication of low internal loss wineglass micro-structures, in: *Solid-State Sensors, Actuators, and Microsystems Workshop*, Hilton Head Island, South Carolina, 2012.
- [66] L. Sorenson, X. Gao, F. Ayazi, 3-D micromachined hemispherical shell resonators with integrated capacitive transducers, in: *IEEE 25th International Conference on Micro Electro Mechanical Systems (MEMS)*, 2012, 2012, pp. 168–171. doi:10.1109/memsys.2012.6170120.
- [67] P. Shao, C. L. Mayberry, X. Gao, V. Tavassoli, F. Ayazi, A polysilicon micro-hemispherical resonating gyroscope, *Journal of Microelectromechanical Systems* 23 (4) (2014) 762–764. doi:10.1109/JMEMS.2014.2327107.
- [68] P. Shao, V. Tavassoli, C. L. Mayberry, F. Ayazi, A 3D-HARPSS polysilicon microhemispherical shell resonating gyroscope: Design, fabrication, and characterization, *IEEE Sensors Journal* 15 (9) (2015) 4974–4985. doi:10.1109/JSEN.2015.2431857.
- [69] Y.-B. Yi, M. A. Matin, Eigenvalue solution of thermoelastic damping in beam resonators using a finite element analysis, *Journal of Vibration and Acoustics* 129 (4) (2007) 478–483. doi:10.1115/1.2748472.
- [70] Y.-B. Yi, Geometric effects on thermoelastic damping in MEMS resonators, *Journal of Sound and Vibration* 309 (3-5) (2008) 588–599. doi:10.1016/j.jsv.2007.07.055.
- [71] Y.-B. Yi, Finite element analysis of thermoelastic damping in contour-mode vibrations of micro- and nanoscale ring, disk, and elliptical plate resonators,
-

- Journal of Vibration and Acoustics 132 (4) (2010) 041015–7. doi:10.1115/1.4001506.
- [72] S. Lepage, J.-C. Golinval, Finite element modeling of the thermoelastic damping in micro-electromechanical systems, in: Third international conference on Advanced Computational Methods in Engineering, Ghent, 2005.
- [73] E. Serra, M. Bonaldi, A finite element formulation for thermoelastic damping analysis, International Journal for Numerical Methods in Engineering 78 (6) (2009) 671–691. doi:10.1002/nme.2502.
- [74] S. Prabhakar, S. Vengallatore, Thermoelastic damping in bilayered micromechanical beam resonators, Journal of Micromechanics and Microengineering 17 (3) (2007) 532–538. doi:10.1088/0960-1317/17/3/016.
- [75] R. Candler, M. Hopcroft, C. Low, S. Chandorkar, B. Kim, M. Varghese, A. Duwel, T. Kenny, Impact of slot location on thermoelastic dissipation in micromechanical resonators, in: Solid-State Sensors, Actuators and Microsystems, 2005. Digest of Technical Papers. TRANSDUCERS '05. The 13th International Conference on, Vol. 1, 2005, pp. 597–600. doi:10.1109/SENSOR.2005.1496488.
- [76] R. Candler, A. Duwel, M. Varghese, S. A. Chandorkar, M. Hopcroft, W.-T. Park, B. Kim, G. Yama, A. Partridge, M. Lutz, T. Kenny, Impact of geometry on thermoelastic dissipation in micromechanical resonant beams, Journal of Microelectromechanical Systems 15 (4) (2006) 927–934. doi:10.1109/JMEMS.2006.879374.
- [77] S. Prabhakar, S. Vengallatore, Thermoelastic damping in hollow and slotted microresonators, Journal of Microelectromechanical Systems 18 (3) (2009) 725–735. doi:10.1109/jmems.2009.2016287.
- [78] X. Guo, Y.-B. Yi, S. Pourkamali, A finite element analysis of thermoelastic

- damping in vented MEMS beam resonators, *International Journal of Mechanical Sciences* 74 (2013) 73–82. doi:10.1016/j.ijmecsci.2013.04.013.
- [79] Z. Hao, Y. Xu, Vibration displacement on substrate due to time-harmonic stress sources from a micromechanical resonator, *Journal of Sound and Vibration* 322 (1-1) (2009) 196–215. doi:10.1016/j.jsv.2008.11.004.
- [80] J. Wang, Z. Ren, C.-C. Nguyen, Self-aligned 1.14-ghz vibrating radial-mode disk resonators, in: *TRANSDUCERS, Solid-State Sensors, Actuators and Microsystems, 12th International Conference on*, 2003, Vol. 2, 2003, pp. 947–950 vol.2. doi:10.1109/SENSOR.2003.1216923.
- [81] J. Wang, Z. Ren, C.-C. Nguyen, 1.156-ghz self-aligned vibrating micromechanical disk resonator, *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control* 51 (12) (2004) 1607–1628. doi:10.1109/TUFFC.2004.1386679.
- [82] J. Wang, J. Butler, T. Feygelson, C.-C. Nguyen, 1.51-ghz nanocrystalline diamond micromechanical disk resonator with material-mismatched isolating support, in: *17th IEEE International Conference on Micro Electro Mechanical Systems (MEMS)*, 2004, pp. 641–644. doi:10.1109/MEMS.2004.1290666.
- [83] H. A. Tilmans, M. Elwenspoek, J. H. Fluitman, Micro resonant force gauges, *Sensors and Actuators A: Physical* 30 (1) (1992) 35–53. doi:http://dx.doi.org/10.1016/0924-4247(92)80194-8.
- [84] G. Stemme, Resonant silicon sensors, *Journal of Micromechanics and Micro-engineering* 1 (2) (1991) 113–125. doi:10.1088/0960-1317/1/2/004.
- [85] Y. Jimbo, K. Itao, Energy loss of cantilever vibrator, *Journal of the Horological Institute of Japan* 47 (1968) 1–15.
- [86] K. Y. Yasumura, T. D. Stowe, E. M. Chow, T. Pfafman, T. W. Kenny, B. C. Stipe, D. Rugar, Quality factors in micron- and submicron-thick cantilevers, *Journal of Microelectromechanical Systems* 9 (1) (2000) 117–125. doi:10.1109/84.825786.

-
- [87] A. B. Hutchinson, P. A. Truitt, K. C. Schwab, L. Sekaric, J. M. Parpia, H. G. Craighead, J. E. Butler, Dissipation in nanocrystalline-diamond nanomechanical resonators, *Applied Physics Letters* 84 (6) (2004) 972–974. doi:10.1063/1.1646213.
- [88] M. Cross, R. Lifshitz, Elastic wave transmission at an abrupt junction in a thin plate with application to heat transport and vibrations in mesoscopic systems, *Physical Review B* 64 (2001) 1–22. doi:10.1103/PhysRevB.64.085324.
- [89] Z. Hao, A. Erbil, F. Ayazi, An analytical model for support loss in micromachined beam resonators with in-plane flexural vibrations, *Sensors and Actuators A: Physical* 109 (12) (2003) 156 – 164. doi:10.1016/j.sna.2003.09.037.
- [90] Z. Hao, F. Ayazi, Support loss in micromechanical disk resonators, in: 18th IEEE International Conference on Micro Electro Mechanical Systems., 2005, pp. 137–141. doi:10.1109/MEMSYS.2005.1453886.
- [91] Z. Hao, F. Ayazi, Support loss in the radial bulk-mode vibrations of center-supported micromechanical disk resonators, *Sensors and Actuators A: Physical* 134 (2) (2007) 582–593. doi:10.1016/j.sna.2006.05.020.
- [92] B. Chouvion, S. McWilliam, C. H. J. Fox, A. A. Popov, Review and comparison of different support loss models for micro-electro-mechanical systems resonators undergoing in-plane vibration, *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* 226 (1) (2012) 283–295. doi:10.1177/0954406211413351.
- [93] P. Yong-Hwa, K. Park, High-fidelity modeling of MEMS resonators. part i. anchor loss mechanisms through substrate, *Journal of Microelectromechanical Systems* 13 (2) (2004) 238–247. doi:10.1109/jmems.2004.825300.
- [94] P. Yong-Hwa, K. Park, High-fidelity modeling of MEMS resonators. part ii. coupled beam-substrate dynamics and validation, *Journal of Microelectromechanical Systems* 13 (2) (2004) 248–257. doi:10.1109/jmems.2004.825298.
-

-
- [95] S. Johnson, Notes on perfectly matched layers(PML), Available at `math.mit.edu/~stevenj/18.369/pml.pdf`(12/4/2013).
- [96] D. S. Bindel, S. Govindjee, Elastic PMLs for resonator anchor loss simulation, *International Journal for Numerical Methods in Engineering* 64 (6) (2005) 789–818. doi:10.1002/nme.1394.
- [97] D. S. Bindel, E. Quevy, T. Koyama, S. Govindjee, J. W. Demmel, R. T. Howe, Anchor loss simulation in resonators, in: 18th IEEE International Conference on Micro Electro Mechanical Systems. MEMS 2005., 2005, pp. 133–136. doi:10.1109/MEMSYS.2005.1453885.
- [98] M. Pandey, Analysis of entrainment and clamping loss in an optically actuated mems, Ph.D. thesis, Cornell University, New York (2008).
- [99] D. Rabinovich, D. Givoli, E. Becache, Comparison of high-order absorbing boundary conditions and perfectly matched layers in the frequency domain, *International Journal for Numerical Methods in Biomedical Engineering* 26 (2010) 1351–1369. doi:10.1002/cnm.1394.
- [100] A. Frangi, M. Cremonesi, A. Jaakkola, T. Pensala, Analysis of anchor and interface losses in piezoelectric MEMS resonators, *Sensors and Actuators A: Physical* 190 (2013) 127–135. doi:10.1016/j.sna.2012.10.022.
- [101] A. Frangi, A. Bugada, M. Martello, P. T. Savadkoobi, Numerical modelling of anchor losses in MEMS resonators, in: 13th International Conference on Thermal, Mechanical and Multi-Physics Simulation and Experiments in Microelectronics and Microsystems (EuroSimE), 2012, pp. 1/8–8/8. doi:10.1109/ESimE.2012.6191719.
- [102] A. Frangi, A. Bugada, M. Martello, P. T. Savadkoobi, Validation of PML-based models for the evaluation of anchor dissipation in MEMS resonators, *European Journal of Mechanics - A/Solids* 37 (2009) 256–265. doi:10.1016/j.euromechsol.2012.06.008.
-

-
- [103] D. Paci, S. Stoffels, H. Tilmans, Simulations of the anchor losses in MEM disk resonators, in: Proceedings of NSTI Nanotechnology Conference, 2006, pp. 558–561.
- [104] J.-P. Berenger, A perfectly matched layer for the absorption of electromagnetic waves, *Journal of Computational Physics* 114 (2) (1994) 185–200. doi:10.1006/jcph.1994.1159.
- [105] U. Basu, A. Chopra, Perfectly matched layers for time-harmonic elastodynamics of unbounded domain: theory and finite-element implementation, *Computer Methods in Applied Mechanics and Engineering* 192 (2003) 1337–1375.
- [106] T. Koyama, D. S. Bindel, H. Wei, E. P. Quevy, S. Govindjee, J. W. Demmel, R. T. Howe, Simulation tools for damping in high frequency resonators, in: *Sensors, 2005 IEEE*, 2005. doi:10.1109/icsens.2005.1597708.
- [107] COMSOL Multiphysics 4.3a is a product of COMSOL Ltd.
URL <http://www.uk.comsol.com>
- [108] P. Steeneken, J. Ruigrok, S. Kang, J. van Beek, J. Bontemps, J. Koning, Parameter extraction and support-loss in MEMS resonators, in: *COMSOL Users Conference*, Grenoble, 2007.
- [109] ABAQUS is a product of Dassault Systèmes.
- [110] J. Yang, T. Ono, M. Esashi, Energy dissipation in submicrometer thick single-crystal silicon cantilevers, *Journal of Microelectromechanical Systems* 11 (6) (2002) 775–783. doi:10.1109/JMEMS.2002.805208.
- [111] COMSOL Multiphysics 4.2a is a product of COMSOL Ltd.
URL <http://www.uk.comsol.com>
- [112] N. Noda, R. Hetnarski, Y. Tanigawa, *Thermal Stresses*, 2nd Edition, Taylor & Francis, New York, 2003.
-

-
- [113] R. Blevins, *Formulas for Natural Frequency and Mode Shape*, Krieger Publishing Company, Florida, 1979.
- [114] J. Liewald, B. Kuhlmann, T. Balslink, Y. Manoli, A 100 kHz vibratory MEMS rate gyroscope with experimental verification of system model's frequency scaling, in: *IEEE 25th International Conference on Micro Electro Mechanical Systems (MEMS)*, 2012, pp. 492–495. doi:10.1109/MEMSYS.2012.6170245.
- [115] G. He, K. Najafi, A single-crystal silicon vibrating ring gyroscope, in: *The Fifteenth IEEE International Conference on Micro Electro Mechanical Systems*, 2002, pp. 718–721. doi:10.1109/MEMSYS.2002.984371.
- [116] M. Madou, *Fundamentals of microfabrication: the science of miniaturization*, CRC Press, Boca Raton, 2002.
- [117] S. Shinde, J. Goela (Eds.), *High Thermal Conductivity Materials*, 2nd Edition, Springer, New York, 2006.
- [118] N. Maluf, K. Williams, *Introduction to microelectromechanical systems engineering*, Artech House, Massachusetts, 2004.
- [119] The CVD diamond booklet, Online booklet, Diamond Materials GmbH, Freiburg (2008).
URL <http://www.diamond-materials.com/downloads/>
- [120] C. Fell, R. Eley, C. Fox, Vibratory gyroscopic rate sensor, US Patent App. 10/475,003 (Jun. 24 2004).
URL <https://www.google.com/patents/US20040118204>
- [121] C. Fell, Rate sensor, US Patent 5,915,276 (Jun. 22 1999).
URL <https://www.google.com/patents/US5915276>
- [122] J. A. Judge, D. M. Photiadis, J. F. Vignola, B. H. Houston, J. Jarzynski, Attachment loss of micromechanical and nanomechanical resonators in the limits
-

- of thick and thin support structures, *Journal of Applied Physics* 101 (1) (2007) 013521–11. doi:10.1063/1.2401271.
- [123] A. Bedford, D. Drumheller, *Introduction to Elastic Wave Propagation*, Wiley, Chichester, 1996.
- [124] L. Landau, E. Lifshitz, *Theory of Elasticity*, Pergamon Press, London, 1959.
- [125] COMSOL Multiphysics Reference Guide 4.4 (2013).
- [126] M. Zampolli, A. Tesei, F. B. Jensen, N. Malm, J. B. Blottman III, A computationally efficient finite element model with perfectly matched layers applied to scattering from axially symmetric objects, *Journal of Acoustical Society of America* 122 (3) (2007) 1472–1485. doi:10.1121/1.2764471.
- [127] P. Morse, *Vibration and Sound*, McGraw-Hill, New York, 1948.
- [128] M. Onoe, Contour vibrations of isotropic circular plates, *The Journal of the Acoustical Society of America* 28 (1956) 1158–1162. doi:10.1121/1.1908579.
- [129] G. F. Miller, H. Pursey, The field and radiation impedance of mechanical radiators on the free surface of a semi-infinite isotropic solid, *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 223 (1155) (1954) 521–541. doi:10.1098/rspa.1954.0134.
- [130] S. Sieberer, In-plane shock response of capacitive MEMS ring rate sensors, Ph.D. thesis, The University of Nottingham, Nottingham (2014).
- [131] S. P. Beeby, M. J. Tudor, Modelling and optimization of micromachined silicon resonators, *Journal of Micromechanics and Microengineering* 5 (2) (1995) 103–105. doi:10.1088/0960-1317/5/2/011.
- [132] R. E. D. Bishop, G. M. L. Gladwell, An investigation into the theory of resonance testing, *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences* 255 (1055) (1963) 241–280. doi:10.1098/rsta.1963.0004.

-
- [133] B. Kim, M. A. Hopcroft, R. N. Candler, C. M. Jha, M. Agarwal, R. Melamud, S. A. Chandorkar, G. Yama, T. W. Kenny, Temperature dependence of quality factor in mems resonators, *Journal of Microelectromechanical Systems* 17 (3) (2008) 755–766. doi:10.1109/JMEMS.2008.924253.
- [134] P. Flubacher, A. Leadbetter, J. Morrison, The heat capacity of pure silicon and germanium and properties of their vibrational frequency spectra, *Philosophical Magazine* 4 (39) (1959) 273–294. doi:10.1080/14786435908233340.
- [135] C. J. Glassbrenner, G. A. Slack, Thermal conductivity of silicon and germanium from 3k to the melting point, *Phys. Rev.* 134 (4A) (1964) A1058–A1069. doi:10.1103/PhysRev.134.A1058.
- [136] A. Hopcroft, Temperature-stabilized silicon resonators for frequency references, Ph.D. thesis, Stanford University, Stanford, USA (2007).
- [137] A. Kazer, Partially Slotted Devices. UTAS (2014).
- [138] S. Gorelick, J. R. Dekker, M. Leivo¹, U. Kantoj^{rvi}, Air damping of oscillating MEMS structures: Modeling and comparison with experiment, in: COMSOL Users Conference, Rotterdam, 2013.
- [139] G. Sérandour, Non-linear dynamics of vibratory rate sensors, Ph.D. thesis, The University of Nottingham, Nottingham (2005).
- [140] S. K. De, N. R. Aluru, Theory of thermoelastic damping in electrostatically actuated microstructures, *Phys. Rev. B* 74 (2006) 144305. doi:10.1103/PhysRevB.74.144305.
- [141] P. Kendall, Analysis of die bond charactersite for mems inertial sensors, The University of Nottingham.
- [142] R. P. Leland, Mechanical-thermal noise in mems gyroscopes, *IEEE Sensors Journal* 5 (3) (2005) 493–500. doi:10.1109/JSEN.2005.844538.
-

- [143] D. M. Rozelle, The hemispherical resonator gyro: From wineglass to the planets, in: 19th AAS/AIAA Space Flight Mechanics Meeting, 2009, pp. 1157–1178.

Appendix A

Thermoelastic Damping in Hemispherical Shell Resonator

A.1 Introduction

Macro-scale hemispherical shell resonators (HSR) based on fused quartz (fused silica) with Q -factors over 10×10^6 motivated its development for MEMS applications [143]. With the latest advances in micro-fabrication technology several hemispherical shell resonators have been reported [65–68] based on different materials e.g. silicon, pyrex, fused quartz, titania silicate (TSG). Senkal *et al.* [65] used finite element analysis to calculate Q_{TED} of a thin hemispherical shell for different materials, and TSG showed the highest Q_{TED} i.e. 1×10^{10} due to its low thermal expansion coefficient. Sorenson *et al.* [66] achieved Q_{TED} of 10.3×10^6 for a polysilicon hemispherical shell. The 3D curved structure of hemispherical shell has less coupling between the vibration mode and the support structure compared to other planar axial symmetric structures such as disks and rings reducing the support loss [7]. It seems a HSR intrinsically have high Q_{TED} i.e. low thermoelastic damping and it is of particular interest to investigate the fundamental differences in damping behaviour between HSR and ring.

This appendix investigates the TED behaviour of hemispherical shell resonators using fully coupled finite element method (see Chapter 2) and compares the frequency and Q_{TED} values with those of ring resonators.

Table A.1: Frequency and Q_{TED} values for HSR and ring resonators				
Configuration	Frequency (kHz)		Q_{TED}	
	Ring	HSR	Ring	HSR
4 mm-23 μm	6.054	10.340 ($\sim 71\%$)	1.875×10^5	8.385×10^5 ($\sim 345\%$)
8 mm-120 μm	7.895	13.108 ($\sim 66\%$)	9624	77207 ($\sim 700\%$)

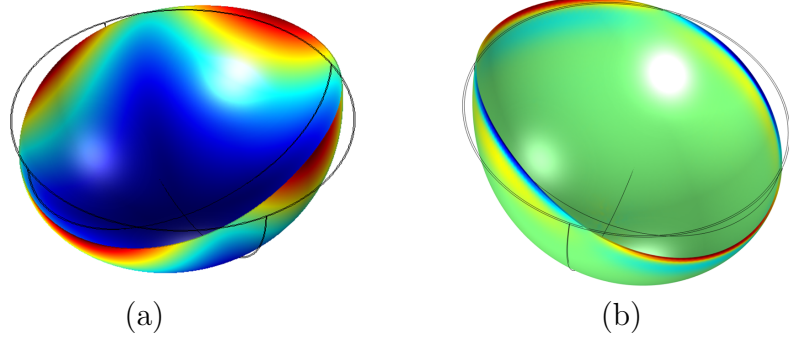


Figure A.1: (a) 2θ in-plane mode shape and (b) temperature distribution of a HSR. Red and blue colour indicates higher and lower total displacement in figure (a) and temperature in figure (b).

A.2 Thermoelastic Damping Behaviour in HSR

To investigate the the TED behaviour in hemispherical shell resonators, a 3D hemispherical shell geometry is considered and solved using fully coupled approach in COMSOL. The material is considered to be (111)-single crystal silicon (SCS). Two configurations of hemispherical shells are considered, and the frequency and Q_{TED} values are shown in Table A.1: 4 mm diameter with 23 μm shell thickness and 8 mm diameter with 120 μm shell thickness. However, frequency and Q_{TED} values for unsupported ring resonators with similar configurations are also shown in Table A.1, and will be used later in this section for comparison. The 2θ in-plane mode is considered in this investigation, and the mode shape and the temperature distribution in a hemispherical shell is illustrated in Figure A.1. It is worth mentioning that a degenerate pair of mode occur during the flexural vibration of hemispherical shell at a mutual angle of $\pi/2m$ where m is the mode number, similar to a ring resonator (see Section 1.2.2).

The results in Table A.1 show that the large diameter thick shells have much lower Q-factor compared to small diameter thin shells. In contrast, frequency is higher for

large diameter thick rings. These behaviour will be discussed in the following section.

Frequency: The natural frequency for a hemispherical shell vibrating in its m th in-plane flexural mode is approximated by [56]

$$\omega_m = \sqrt{\frac{2}{3}} m(m^2 - 1) \sqrt{K_m} \sqrt{\frac{E}{2(1 + \nu)\rho}} \frac{h}{R^2} \quad (\text{A.1})$$

Here K_m is a mode-dependent constant determined by

$$K_m = \frac{\int_0^{\phi_f} \tan^{2m}(\frac{\phi}{2}) / \sin^3 \phi d\phi}{\int_0^{\phi_f} (m^2 + 1 + \sin^2 \phi + 2m \cos \phi) \sin \phi \tan^{2m}(\frac{\phi}{2}) d\phi} \quad (\text{A.2})$$

where E , ν , ρ are the Young's modulus, Poisson's ratio and mass density of the material, respectively. h is the shell thickness and R is the mid-shell radius of the HSR. ϕ_f indicates the extent of the polar angle, which terminates at the rim of the shell. For a true hemisphere $\phi_f = \frac{\pi}{2}$.

Eq. (A.1) indicates that the undamped natural frequency of a vibrating HSR is proportional to the thickness of the shell, and inversely proportional to the square of the shell radius i.e. $\omega_m \propto \frac{h}{R^2}$. It is evident in Table A.1 where 8 mm-120 μm shell showed higher frequency due to higher $\frac{h}{R^2}$ ratio than 4 mm-23 μm shell. Using Eq. (A.1) undamped natural frequency of 8 mm-120 μm shell is 13.963 kHz and 10.705 kHz for 4 mm-23 μm shell; both are in reasonable agreement with simulated values in Table A.1 with <7% difference.

Q-factor: The damping behaviour of the HRG is not well understood using the Zener's equation (see Eq. (2.1)). Sorenson *et al.* [56] conducted a rigorous analysis on hemispherical shells to understand the TED behaviour both analytically and numerically. By relaxing the inextensional assumption in Rayleigh's shell theory, the thermal path for bulk TED in hemispherical shell is expressed in closed-form. The thermal path (d), which is the average path length between hottest and coolest

regions of the shell, is defined as

$$d = \left(\frac{4}{5}\right)\pi\sqrt{Rh} \quad (\text{A.3})$$

Sorenson *et al.* [56] used Zener's expression to derive closed form expression of bulk TED in HRGs. Using Zener's equation the thermal relaxation time (τ) of a HSR is given by

$$\tau = \frac{\rho C_P d^2}{\pi^2 k} \quad (\text{A.4})$$

where k and C_p are the thermal conductivity and the specific heat capacity of the material, respectively.

It is assumed in [56] that for thin micro hemispherical shells the vibration is isothermal i.e. $\omega\tau \ll 1$, and Q_{TED} is approximated using Eq. (A.4) into Zener's equation as

$$Q_{TED} = \left(\frac{5}{4}\right)^2 \frac{\sqrt{3}}{m(m^2 - 1)\sqrt{K_m}} \frac{k\sqrt{(1 + \nu)\rho}}{sE^{3/2}\alpha^2 T_0} \frac{R}{h^2} \quad (\text{A.5})$$

where α is the coefficient of thermal expansion of the material, and T_0 is the reference temperature. The factor s is included to account for the difference between the relaxation strength before and after averaging of the temperature due to diffusion both laterally and across the thickness. An exact value for s is required for estimating Q_{TED} and could be determined by the use of Green's functions to calculate the exact temperature profile for a given maximum displacement after accounting for diffusion assuming as input the heat source distributions determined when $k = 0$.

Eq. (A.5) cannot be used to quantify Q_{TED} without actual value for s , but it indicates relationship between key geometric parameters and material properties with Q_{TED} . It is evident that $Q_{TED} \propto \frac{R}{h^2}$, and the results Table A.1 further verifies this relationship as 4 mm-23 μm shell showed higher Q_{TED} due to higher $\frac{R}{h^2}$ ratio than 8 mm-120 μm shell.

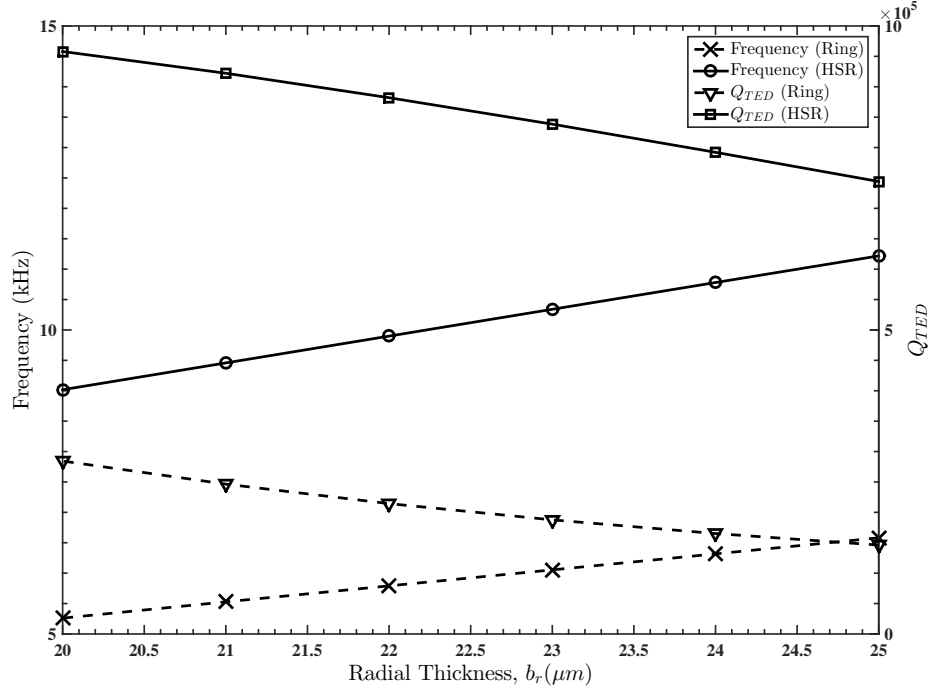


Figure A.2: Comparison of frequency and Q_{TED} as a function of radial/shell thickness for HSR and ring resonator.

Comparison with Ring Resonator: From Table A.1 it is clear that for a ring and hemispherical shell of same radius and shell/radial thickness, HSR show higher frequency and Q_{TED} . A parameter study on ring and shell thickness in the range $20\ \mu\text{m}$ - $25\ \mu\text{m}$ with $4\ \text{mm}$ diameter is shown in Figure A.2. For both ring and shell, a linear decrease in Q_{TED} is observed with thickness, whereas frequency increases linearly.

Similar to HSR, the undamped natural frequency (ω_n) for a ring is proportional to the radial thickness (b_r) and inversely proportional to the square of the ring radius (r) i.e. $\omega_0 \propto \frac{b_r}{r^2}$, where n is the mode order (see Eq. 2.15)). However, the higher frequency in HSR is due to the additional stiffness along the curved surface (i.e. polar direction ϕ) during vibration compared to the planar motion of ring.

The difference in Q_{TED} for ring and shell can be explained by the thermal behaviour. In The thermal path of a ring depends on the radial thickness (b_r) as the temperature predominantly diffuse in the radial direction, and thermal relaxation

time of a ring is given by

$$\tau = \frac{\rho C_p b_r^2}{k\pi^2} \quad (\text{A.6})$$

In contrast, from Eq. (A.3), thermal path of a shell depends on both shell thickness and radius. However, the thermal path is dictated by oscillating deviations of the neutral surface away from the shell's mid surface and not thermal gradients through shell's thickness as in the case of ring [56]. The main heat source arise from lateral thermal gradients over the surface of the shell. Because of the difference in thermal path, a shell has a longer relaxation time compared to ring for same radius and thickness. On the other hand, a shell has relatively shorter period of vibration than ring, hence thermal gradient has shorter time period to relax back to equilibrium state compared to a ring. This leads to small levels of energy dissipation in HSR.

A.3 Summary

This appendix investigates the TED behaviour in HSR. It is seen that higher frequency in shell is obtained for higher ratio of shell thickness to the square of shell radius i.e. $\frac{h}{R^2}$. However, higher Q_{TED} is achieved for higher $\frac{R}{h^2}$ ratio. Therefore, a trade-off between shell thickness and radius is needed to achieve the desired performance. For ring and shell of similar dimensions, HSR always have higher frequency and Q_{TED} than ring resonator due to the higher stiffness and difference in thermal behaviour, respectively.

Appendix B

Convergence Study for Slotted Ring Resonators

It is important that a sufficient number of mesh elements are placed around the slots to have good convergence. A study is performed in this appendix to determine the effect of number elements (N_{el}) on the convergence of the Q-factor for slotted ring resonators using the TED model developed in Chapter 2.

The slotted rings are meshed using quadratic triangular elements generated by the built-in meshing technique in COMSOL. To ensure sufficient number of elements around the slotted region, the minimum element size is considered not to exceed the half of the slot width i.e. $w_{slot}/2$. The mesh is further refined to achieve a converged solution for Q-factor. Also, the maximum element growth rate is kept to 1.1, which increases the element size not more than 10% from one element to other. Figure B.1 shows a section of the single slotted ring meshed with triangular elements for two different mesh resolution. The results shown in Figure B.2 are obtained for a 4 mm-23 μ m ring resonator. It is evident that Q_{TED} increase asymptotically as the number of elements is increased. It is apparent that N_{el} within 80000 to 100000 provide a converged value, and Q_{TED} does not change significantly for $N_{el} \sim 150000$. For the results in Section 2.6, total N_{el} is kept within ~ 80000 to ~ 100000 for a 4 mm ring with various radial thickness.

A similar approach is also used for double slotted ring resonators. Figure B.3 show meshed section of the double slotted ring. The results in Figure B.4 show that convergence is achieved with $N_{el} \sim 130000$ to ~ 150000 , which is used in Section 2.6

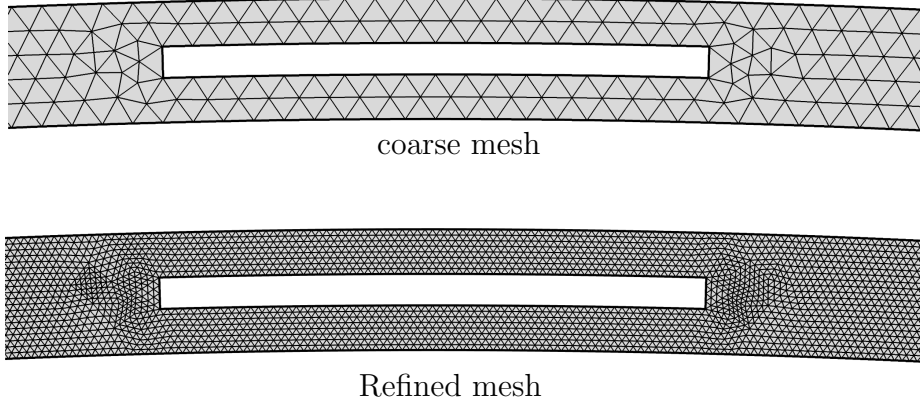


Figure B.1: Meshed single slotted ring resonator.

for the 4 mm ring.

The meshing approach described in this appendix provides sufficient mesh quality to achieve convergence, and this approach is applied for all the cases considered in Section 2.6.

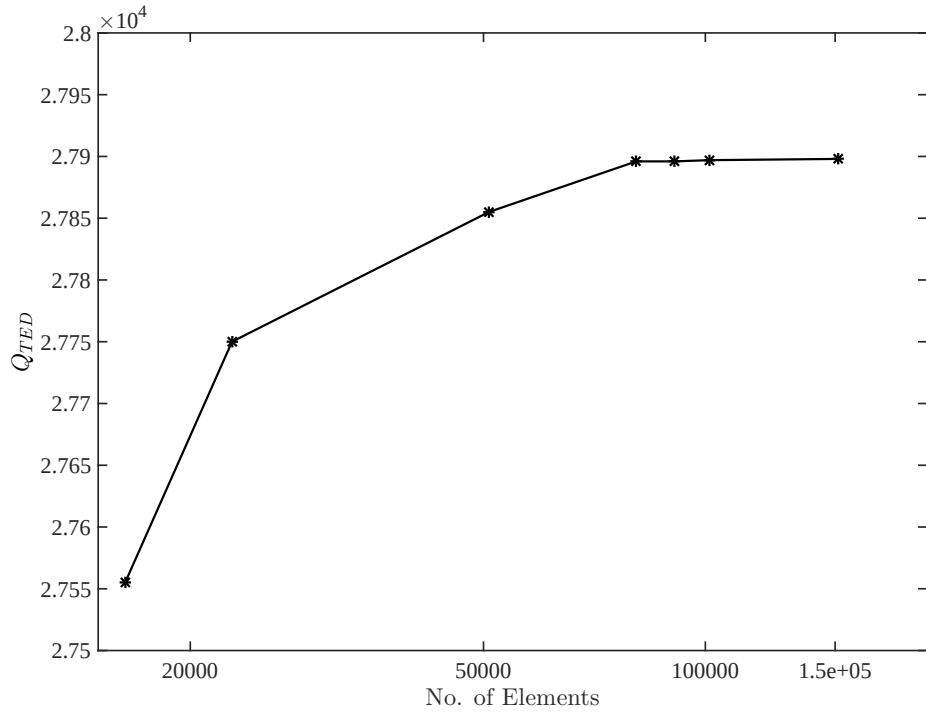
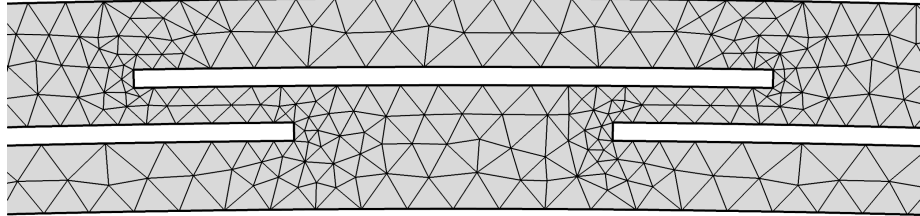
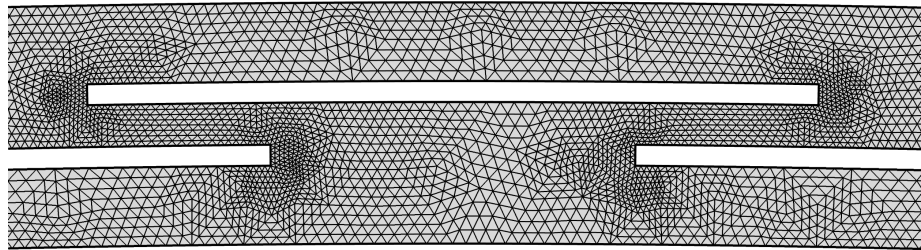


Figure B.2: Mesh convergence study showing relationship between Q-factor and the number of elements for a single slotted 4mm-23 μ m ring resonator in 2θ in-plane mode.



Coarse mesh



Refined mesh

Figure B.3: Meshed double slotted ring resonator.

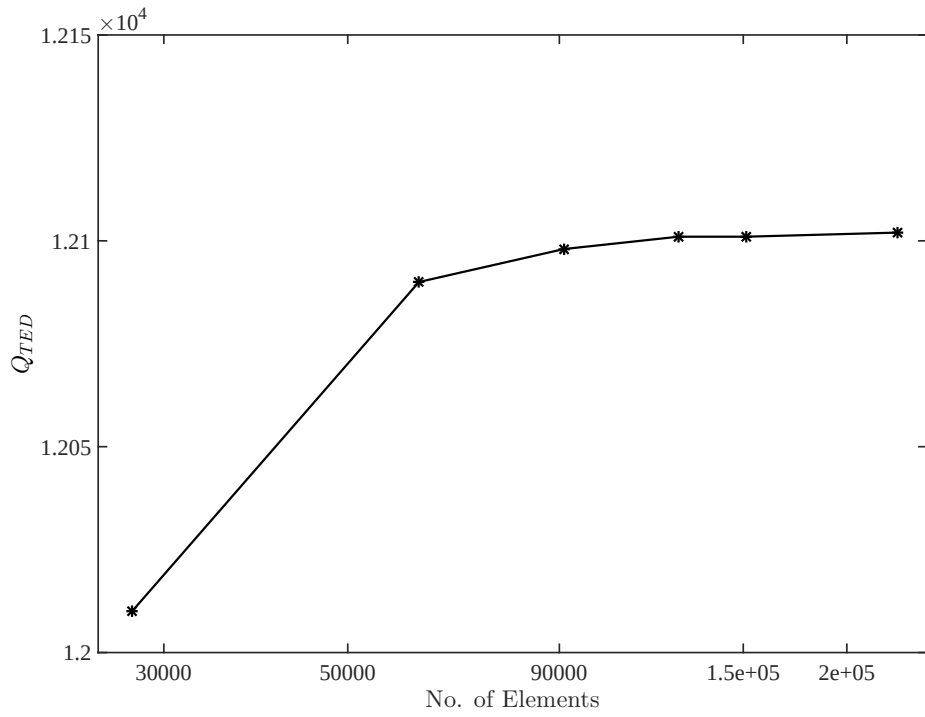


Figure B.4: Mesh convergence study showing relationship between Q-factor and the number of elements for a double slotted 4 mm-23 μ m ring resonator in 2θ in-plane mode.

Appendix C

Analysing the Influence of Die on a Coupled Resonator

In this appendix the influence of die (including the effects of die bond) on the frequency and Q-factor of the coupled die-resonator system is analysed analytically. A simplified two degree of freedom (2-DOF) lumped parameter model is used to derive and determine the coupled system's frequency and Q-factor. The model presented here only considers the effect of die and 1θ rigid body motion compared to Sieberer's model [130] where the effects of package motion and flexural 2θ and 3θ motions were considered. Although it is concluded that flexural ring motions are not coupled to rigid body motions.

C.1 Uncoupled Resonator System

Before considering the coupled system, the uncoupled resonator vibrating in single direction can be modelled using a 1-DOF system shown in Figure C.1.

The equation of motion of the system under free vibration, shown in Figure C.1 is given by

$$m_{res}\ddot{x}_{res} + c_{res}\dot{x}_{res} + k_{res}x_{res} = 0 \quad (C.1)$$

where m_{res} , c_{res} , and k_{res} are the mass, damping coefficient and stiffness of the resonator, respectively. x_{res} is the displacement of the resonator during vibration.

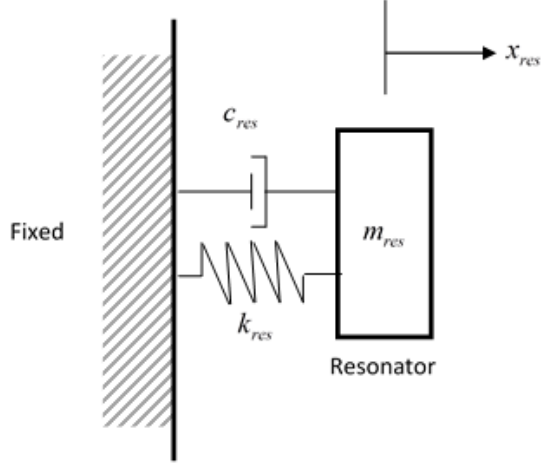


Figure C.1: 1-DOF model of the uncoupled resonator.

Eq. (C.1) can be simplified as

$$\ddot{x}_{res} + \frac{\omega_{res}}{Q_{res}}\dot{x}_{res} + \omega_{res}^2 x_{res} = 0 \quad (C.2)$$

where ω_{res} is the undamped natural frequency of the resonator and $Q_{res} = \frac{\sqrt{m_{res}k_{res}}}{c_{res}}$ is the quality factor of the resonator.

To investigate the free vibration characteristics of the resonator it is assumed that the solution to Eq. (C.2) can be expressed as

$$x_{res} = X_{res}e^{s_0 t} \quad (C.3)$$

Substitution of Eq. (C.3 into C.2) gives roots of the characteristic equation by

$$s_0^2 + \frac{\omega_{res}}{Q_{res}}s_0 + \omega_{res}^2 = 0 \quad (C.4)$$

where s_0 is the eigenvalue governing the free vibration. The real part of equation gives the decay rate or the damping of the resonator, and imaginary part governs the damped frequency.

$$s_0 = -\frac{\omega_{res}}{2Q_{res}} + i\omega_{res}\sqrt{1 - \frac{1}{4Q_{res}^2}} \quad (C.5)$$

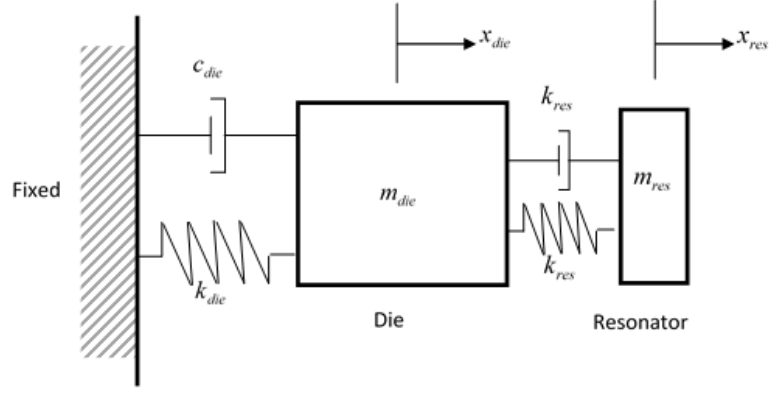


Figure C.2: A 2-DOF die-resonator coupled system.

C.2 Die-Resonator Coupled System

The resonator is now considered to be coupled to the die by the die bond as shown in Figure C.2, and it is assumed that both die and resonator are vibrating. Ideally, the frequency and Q-factor of the coupled system should be identical to the resonator i.e. die bond is rigid. This model will provide an insight on the effect of die vibration on the coupled system.

The equation of motion of this coupled 2-DOF system can be expressed in matrix form as

$$\begin{pmatrix} m_{res} & 0 \\ 0 & m_{die} \end{pmatrix} \begin{pmatrix} \ddot{x}_{res} \\ \ddot{x}_{die} \end{pmatrix} + \begin{pmatrix} c_{res} & -c_{res} \\ -c_{res} & c_{res} + c_{die} \end{pmatrix} \begin{pmatrix} \dot{x}_{res} \\ \dot{x}_{die} \end{pmatrix} + \begin{pmatrix} k_{res} & -k_{res} \\ -k_{res} & k_{res} + k_{die} \end{pmatrix} \begin{pmatrix} x_{res} \\ x_{die} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (C.6)$$

where m_{die} , k_{die} and c_{die} are the mass of the die, stiffness of the die bond (assuming die is rigid) and damping coefficient of the die bond (assuming die is rigid) . x_{die} is the displacement of the die.

Noting that the inverse of the stiffness matrix is

$$\begin{pmatrix} \frac{1}{k_{res}} + \frac{1}{k_{die}} & \frac{1}{k_{die}} \\ \frac{1}{k_{die}} & \frac{1}{k_{die}} \end{pmatrix} \quad (C.7)$$

and pre-multiplying Eq. (C.6) by (C.7) gives

$$\begin{pmatrix} \frac{1}{\omega_{res}^2} + \frac{m_{res}}{m_{die}\omega_{die}^2} & \frac{1}{\omega_{die}^2} \\ \frac{m_{res}}{m_{die}\omega_{die}^2} & \frac{1}{\omega_{die}^2} \end{pmatrix} \begin{pmatrix} \ddot{x}_{res} \\ \ddot{x}_{die} \end{pmatrix} + \begin{pmatrix} \frac{1}{Q_{res}\omega_{res}} & -\frac{1}{Q_{res}\omega_{res}} + \frac{1}{Q_{die}\omega_{die}} \\ 0 & \frac{1}{Q_{die}\omega_{die}} \end{pmatrix} \begin{pmatrix} \dot{x}_{res} \\ \dot{x}_{die} \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{res} \\ x_{die} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (C.8)$$

where $c_{res} = \frac{\sqrt{m_{res}k_{res}}}{Q_{res}}$, $c_{die} = \frac{\sqrt{m_{die}k_{die}}}{Q_{die}}$, $\omega_{res} = \sqrt{\frac{k_{res}}{m_{res}}}$ and $\omega_{die} = \sqrt{\frac{k_{die}}{m_{die}}}$

Here ω_{die} and Q_{die} are the undamped natural frequency and Q-factor for the uncoupled the die, respectively.

To investigate the free vibration characteristics, the solutions to Eq (C.8) are assumed to be

$$x_{res} = X_{res}e^{st} \quad (C.9)$$

$$x_{die} = X_{die}e^{st} \quad (C.10)$$

where the real and imaginary parts of s govern the frequency and damping of the coupled system, respectively.

Substituting Eq. (C.9) and (C.10) into Eq. (C.8) and writing the result in matrix form, the characteristic equation for the system can be obtained from

$$\begin{pmatrix} \frac{s^2}{\omega_{res}^2} + \frac{s^2 m_{res}}{m_{die}\omega_{die}^2} + \frac{s}{Q_{res}\omega_{res}} + 1 & \frac{s^2}{\omega_{die}^2} - \frac{s}{Q_{res}\omega_{res}} + \frac{s}{Q_{die}\omega_{die}} \\ \frac{s^2 m_{res}}{m_{die}\omega_{die}^2} & \frac{s^2}{\omega_{die}^2} + \frac{s}{Q_{die}\omega_{die}} + 1 \end{pmatrix} \begin{pmatrix} X_{res} \\ X_{die} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (C.11)$$

The roots s that satisfy Eq. (C.11), govern the natural frequencies and Q-factors for the coupled system. To determine the roots the determinant of the matrix appearing on the left hand side of Eq. (C.11) is set equal to zero, and the polynomial eigenvalue problem solved. The aim here is to determine closed form expressions for Q-factor ($Q_{coupled}$) and damped natural frequency ($\omega_{coupled}$) of the coupled system. For this purpose, the following assumptions are used

- The die and die bond have a small effect on the natural frequency of the resonator. This is based on the observation that the predicted frequency (neglecting the die) is close to the measured frequency in the Chapter 5.
- Coupling between die and resonator is small
- Q-factor for uncoupled resonator is much larger than Q-factor for die

C.3 Closed form Expression for $\omega_{coupled}$ and $Q_{coupled}$

Following steps are followed to derive closed form expressions for $\omega_{coupled}$ and $Q_{coupled}$.

- s in Eq. (C.11) is expanded as $s = s_0 + s_1$, where s_0 corresponds to the root of the uncoupled resonator system, given by Eq. (C.5) and s_1 is assumed to be small. Using this expansion in Eq. (C.11) gives

$$\begin{pmatrix} s_0^2 \frac{m_{res}}{k_{die}} + 2s_0 s_1 \left(\frac{1}{\omega_{res}^2} + \frac{m_{res}}{k_{die}} \right) + \frac{s_1}{Q_{res}\omega_{res}} + s_1^2 \left(\frac{1}{\omega_{res}^2} + \frac{m_{res}}{k_{die}} \right) \\ (s_0^2 + 2s_0 s_1 + s_1^2) \frac{m_{res}}{k_{die}} \\ \frac{s_0^2}{\omega_{die}^2} + s_0 \left(\frac{2s_1}{\omega_{die}^2} - \left(\frac{1}{Q_{res}\omega_{res}} - \frac{1}{Q_{die}\omega_{die}} \right) \right) - s_1 \left(\frac{1}{Q_{res}\omega_{res}} - \frac{1}{Q_{die}\omega_{die}} \right) + \frac{s_1^2}{\omega_{die}^2} \\ \frac{(s_0^2 + 2s_0 s_1 + s_1^2)}{\omega_{die}^2} + \frac{s_0 + s_1}{Q_{die}\omega_{die}} + 1 \end{pmatrix} \begin{pmatrix} X_{res} \\ X_{die} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (C.12)$$

- Non-trivial solutions to Eq. (C.12) are obtained by setting the determinant of the matrix to zero. Neglecting second and higher order terms in s_1 , the characteristics equation can be expressed as:

$$as_1 + b = 0 \quad (\text{C.13})$$

where

$$a = s_0^3 \frac{2}{\omega_{res}^2 \omega_{die}^2} + s_0^2 \left(\frac{3m_{res}}{Q_{res} \omega_{res} m_{die} \omega_{die}^2} + \frac{2}{\omega_{res}^2 Q_{die} \omega_{die}} + \frac{1}{Q_{res} \omega_{res} \omega_{die}^2} \right) + s_0 \left(\frac{2}{\omega_{res}^2} + \frac{2m_{res}}{m_{die} \omega_{die}^2} + \frac{1}{Q_{res} \omega_{res} Q_{die} \omega_{die}} \right) + \frac{1}{Q_{res} \omega_{res}} \quad (\text{C.14})$$

$$b = s_0^2 \frac{m_{res}}{m_{die} \omega_{die}^2} \left(1 + \frac{s_0}{Q_{res} \omega_{res}} \right) \quad (\text{C.15})$$

- Since the resonator is assumed to be lightly damped, $\frac{1}{Q_{res}}$ is considered small and higher order terms in $\frac{1}{Q_{res}}$ are neglected. Based on these conditions, the following approximations are used to simplify a and b in Eqs. (C.14) and (C.15).

$$s_0 \approx \omega_{res} \left(-\frac{1}{2Q_{res}} + i \right) \quad (\text{C.16})$$

$$s_0^2 \approx \omega_{res}^2 \left(-1 - i \frac{1}{Q_{res}} \right) \quad (\text{C.17})$$

$$s_0^3 \approx \omega_{res}^3 \left(\frac{3}{2Q_{res}} - i \right) \quad (\text{C.18})$$

These approximations can be used to simplify Eqs. (C.14) and (C.15).

- Based on these results the approximate root s for the characteristic Eq. (C.13) is given by

$$s = s_0 - \frac{a^* b}{|a|^2} \quad (\text{C.19})$$

where $*$ denotes the complex conjugate. The approximate expressions for $|a|^2$ and a^*b are determined from Eqs. (C.14) to (C.18). The second and higher order terms in $\frac{1}{Q_{res}}$ are neglected. The root s can then be expressed by

$$s = -\frac{\omega_{res}}{2Q_{res}} + i\omega_{res} - \frac{1}{\frac{4}{\omega_{res}^2}(1 - \frac{\omega_{res}^2}{\omega_{die}^2}(1 - \frac{m_{res}}{m_{die}}))^2} \left(2\frac{m_{res}\omega_{res}^2}{m_{die}\omega_{die}^2} \left(-\frac{2}{Q_{res}\omega_{res}} + \frac{1}{Q_{die}\omega_{die}} + \frac{\omega_{res}}{Q_{res}\omega_{die}^2} \right) + i\frac{2m_{res}\omega_{res}^2}{m_{die}\omega_{die}^2} \left(\frac{1}{\omega_{res}} + \frac{2}{Q_{res}Q_{die}\omega_{die}} - \frac{\omega_{res}}{\omega_{die}^2} \left(1 - \frac{m_{res}}{m_{die}} \right) \right) \right) \quad (C.20)$$

- Using Eq. (C.20) the natural frequency $\omega_{coupled}$ and Q-factor $Q_{coupled}$ for the coupled system can be expressed as

$$\omega_{coupled} = \omega_{res} \frac{1 - \frac{\omega_{res}^2}{\omega_{die}^2}(2 - \frac{3m_{res}}{2m_{die}}) + \frac{\omega_{res}^4}{\omega_{die}^4}(1 - \frac{3m_{res}}{2m_{die}} + \frac{m_{res}^2}{2m_{die}^2})}{1 - \frac{\omega_{res}^2}{\omega_{die}^2}(1 - \frac{m_{res}}{m_{die}})^2} \quad (C.21)$$

$$Q_{coupled} \approx Q_{res} \frac{1 - \frac{\omega_{res}^2}{\omega_{die}^2}(2 - \frac{3m_{res}}{2m_{die}}) + \frac{\omega_{res}^4}{\omega_{die}^4}(1 - \frac{3m_{res}}{2m_{die}} + \frac{m_{res}^2}{2m_{die}^2})}{1 - \frac{2\omega_{res}^2}{\omega_{die}^2} + \frac{m_{res}\omega_{res}^3 Q_{res}}{m_{die}\omega_{die}^3 Q_{die}} + \frac{\omega_{res}^4}{\omega_{die}^4}(1 - \frac{m_{res}}{m_{die}} + \frac{m_{res}^2}{m_{die}^2})} \quad (C.22)$$

It can be seen from Eq. (C.21) that damped natural frequency ($\omega_{coupled}$) is independent of damping i.e. Q_{res} and Q_{die} . This is because the resonator is weakly damped. Also, from equation Eq. (C.22) it can be observed that $Q_{coupled}$ depends on both Q_{res} and Q_{die} . Ideally (i.e. rigid die bond) the $Q_{coupled}$ should be equal or close to Q_{res} , which means there is no damping in the die. Eq. (C.22) can be used to determine the condition when $Q_{coupled}$ is approximately equal to Q_{res} . It can be shown that $Q_{coupled}$ will be approximately equal to Q_{res} when

$$Q_{die} \gg \frac{\frac{m_{res}\omega_{res}^3 Q_{res}}{m_{die}\omega_{die}^3}}{1 - \frac{2\omega_{res}^2}{\omega_{die}^2} + \frac{\omega_{res}^4}{\omega_{die}^4}(1 - \frac{m_{res}}{m_{die}} + \frac{m_{res}^2}{m_{die}^2})} \quad (C.23)$$

In Chapter 5 it has been mentioned that high levels of damping observed in 1θ in-plane mode of a ring resonator is due to the coupled die motion. Eqs. (C.21) and (C.22) are used in the Chapter 5 to explain the influence of die and die bond

on the 1θ in-plane mode of the nominal device, and Eq. (C.23) is used to predict the value of Q_{die} needed to reduce the effect of coupling. It is also worth mentioning that this model is also applicable to bounce mode, where both the resonator and die have out-of-plane motions.