Policy Interactions, Uncertainty, and Credit Cycles in Financial Dynamic Stochastic General Equilibrium Models



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Abstract

This dissertation joins a vibrant conversation in macroeconomics about the role of financial frictions in business cycles spurred on by the recent financial crisis. Our proposed study contributes to this lively debate in a fundamental way by putting forward two DSGE models. Firstly, we consider a DSGE model which accounts for the financial sector and assumes a distorted steady state. Unlike in studies where the welfare of distinct groups i.e. households and entrepreneurs are not tracked, in our proposed model, the welfares of savers and borrowers are tractable. To the best of our knowledge, this model is the first to consider welfare-based mandates to the central bank and the macroprudential institution that targets related inefficiencies. In addition, a key innovation of this model is the use of such welfare-based mandate in a game-theoretical framework. Secondly, we introduce a DSGE model, which in addition to financial frictions, is augmented with stochastic volatility and nominal rigidities. This model is then used to assess the effectiveness of conventional, unconventional monetary policies and macroprudential policies in mitigating the effects of disturbances in the presence of risk shocks. To the best of our knowledge, this second model is the first to analyse the effectiveness of unconventional monetary policy and macroprudential policies with stochastic volatility. Furthermore, another key novelty of our research is the analysis of the interactions between three policy options, namely conventional, unconventional monetary and macroprudential policies.

Aknowlegement

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Contents

1 Introduction and Related Literature **7**

1.1 Introduction 7

1.2 Literature review 10

2 Interactions between monetary and macroprudential polices using welfare based mandates **19**

2.1 Introduction. 19

2.2 Objectives of Macroprudential policy 23

2.3 Related literature 26

2.4 Benchmark Model 31

2.4.1 Savers 31

2.4.2 Borrowers 32

2.4.3 Final goods firms 33

2.4.4 Intermediate goods firms 33

2.4.5 Market clearing conditions 36

2.4.6 Policy rules 36

2.5 Calibration and impulse responses 38

2.5.1 Calibration 38

2.5.2 Technology shock 39

2.5.3 Monetary policy shock 39

2.5.4 Housing demand shock 42

2.6 Efficient Equilibrium and Optimal policy 43

2.6.1 Loss function 45

2.6.2 Optimal Policy Mix 50

2.7 Conclusion 59

2.8 Appendix 61

3 Uncertainty in a model of financial frictions and policy interactions 71
3.1 Introduction . . . 71
3.2 Related Literature . . . 75
3.2.1 Uncertainty . . . 75
3.2.2 Policies . . . 84
3.3 Model . . . 93
3.3.1 Households . . . 94
3.3.2 Wholesale firms . . . 96
3.3.3 Retail Firms . . . 98
3.3.4 Firm default and financial settlement . . . 100
3.3.5 Aggregation . . . 103
3.3.6 Banks . . . 105
3.3.7 Steady State Economy . . . 108
3.3.8 Full de-trended Model . . . 109
3.4 Calibration/Estimation . . . 112
3.5 Results . . . 132
3.5.1 Liquidity policy as an unconventional monetary policy tool and Macroprudential policy . . . 136
3.5.2 Volatility and policies . . . 142
3.5.3 Sensitivity Analyses . . . 146
3.5.4 Optimal Policy and Welfare analysis . . . 154
3.6 Conclusion . . . 160
3.7 Appendix . . . 162

List of Figures

2.1 Impulse responses to the technology shock 40
2.2 Impulse responses to the expansionary monetary policy shock 41
2.3 Impulse responses to the housing demand shock. 42
2.4 Actual Welfare and Negative Loss Function 49
2.5 Best responses of the central bank 52
2.6 best responses of the macroprudential institution 53
2.7 Nash Equilibrium 54
2.8 Best responses of the central bank 56
2.9 Best responses of the macroprudential authority 57

3.1 Lending to deposit ratio 106
3.2 Determinacy regions 107
3.3 Priors and Posteriors 130
3.4 Priors and Posteriors 131
3.5 Uncertainty shock, Sticky vs Flexible prices 134
3.6 Credit shock, Sticky vs Flexible prices 135
3.7 Uncertainty shock 137
3.8 Credit shock 138
3.9 Monetary policy shock 140
3.10 Marginal cost shock 141
3.11 Volatility and the Liquidity Policy 142
3.12 LTV rule and volatility 143
3.13 Volatility responses to an increase in the policy rate 144
3.14 Volatility responses to a negative credit shock 145
3.15 Banks leverage and the credit shock 146
3.16 Banks leverage and the uncertainty shock 147
3.17 Firms' leverage and the credit shock 148
3.18 Firms' leverage and the uncertainty shock 149
3.19 Uncertainty, Term Premium and Firm's leverage 150

3.20 Uncertainty, Term Premium and Banks' leverage 151
3.21 Monetary Policy and Term Premium 152
3.22 Uncertainty shock and EZ parameter 153
3.23 Nash game and cooperative institutional design 157
3.24 Nash game and cooperative institutional design 158
3.25 Firm profit and the relative price 165

List of Tables

2.1 Macro - versus microprudential perspectives 24
2.2 Model parameters 38
2.3 Omptimized Parameters 51
2.4 Omptimized Parameters 55
2.5 Omptimized Parameters 58

3.1 Fixed/calibrated parameters 127
3.2 Estimated parameters 129
3.3 Maximum likelihood estimation of the stochastic volatility 129
3.4 Welfare optimal policies 154
3.5 Coordination vs Nash - Taylor rule and LTV 156
3.6 Coordination vs Nash - Reserve Requirement and LTV 157

Chapter 1

Introduction and Related Literature

1.1 Introduction

In recent decades, macroeconomic attention shifted to account for financial frictions explicitly in business cycle models (e.g., Moore & Kiyotaki (1997), Gertler et al. (2011), Iacoviello (2005), Bernanke et al. (1999)). These modeling developments have contributed to our knowledge concerning the role played by the financial market in the propagation of policy and external shocks into macroeconomic dynamics. They have also assisted us to better understand the consequences of financial instability for the macroeconomy.

Yet, there remain large gaps in our knowledge in relation to key macroeconomic interactions and transmissions mechanisms in the presence of financial frictions. For instance, the interaction between macroprudential and monetary policies under financial frictions and nominal rigidities is not well understood. There are different views on the extent of this interaction, the efficacy of independent policies for the financial and price stability, whether the monetary policy can contain financial instability, if and to what extent the monetary policy needs to include financial stability concerns, e.t.c. Smets et al. (2014). The same applies to the role played by uncertainty in the amplification of shocks under a general equilibrium setting and in the presence of financial frictions. Furthermore, there is no clear consensus as to the optimal policy design to ensure monetary and financial stability. Besides conventional policies, the transmission mechanism of unconventional monetary policy such as the use of reserve requirement policies remains far from clear. Filling these gaps is a key motivation of the present thesis.

The present thesis is divided into three chapters. The first (current) chapter of the thesis presents a common literature review which aims to identify gaps in our comprehension concerning policy interactions (studied in both models of the present thesis), and the role of uncertainty (the second model of the thesis) in financial DSGE models. In the second chapter, we consider a DSGE model which accounts for the financial sector. Specifically, the model we propose assumes a distorted steady state (the steady state that assumes an efficient risk sharing but allows for the presense of steady state markup) and unlike in studies where the welfare of distinct groups i.e. households and entrepreneurs are not tracked, we propose a model where the welfare of savers and borrowers are tractable. One of the main focuses of the thesis is on welfare implications of macroprudential and unconventional monetary policies. As Svensson (2012) argues, there is a risk of "confusion between monetary policy and financial-stability policy" which is also referred to as macroprudential policy in the present, both from theoretical and practical perspective. Indeed, some have argued that the design of monetary policy needs to be extended to incorporate financial stability objectives and that financial-stability and monetary policies ought to be consolidated and carried out jointly (Eichengreen et al. (2011), Eichengreen (2011)). We espouse (Svensson 2012) view that such proposals may be unsuitable since they dismiss the fact that monetary and financial stability policies are separate policies with distinct objectives. Thus, in addition to discussing the interaction between monetary and financial stability policies, our literature review also discusses in separate sections the objectives and policy instruments needed for the conduct of macroprudential policy. To the best of our knowledge, the model we introduce in our first chapter is the first to consider welfare based mandates to the central bank and the macroprudential institution that targets related inefficiencies. In addition, a key innovation of this study is the use of such welfare based mandate in a game theoretical framework.

In our third chapter, we introduce a DSGE model, which in addition to financial frictions, is augmented with stochastic volatility and nominal rigidities. Specifically, the proposed model build on Pesaran & Xu (2016) which features firm default. These features allow for a time variation in firms' default probability away from the steady state where its distribution is linked to underlying volatility in the productivity process and wider economic developments such as movements in the nominal side of the economy. In this chapter, we shed light on the effects of uncertainty on macroeconomic variables using a DSGE model. In addition, we also assess the efficiency of conventional, unconventional, and macroprudential policies in moderating the effects of uncertainty shocks. Specifically, our proposed DSGE model is different from many of the existing

ones in that it accounts for financial frictions, it allows uncertainty to be modelled as a time-varying shock process and, it uses the Epstein-Zin preferences which disentangles the simple link between the coefficient of relative risk aversion and the intertemporal elasticity of substitution in consumption. To the best of our knowledge, the model proposed in our second chapter is the first to examine the effectiveness of unconventional monetary policy and macroprudential policies in the presence of stochastic volatility. Furthermore, another key innovation of our research is the analysis of the interactions between three policy options, namely conventional, unconventional and macroprudential policies.

In sum, the rest of this thesis is structured as follows. We start with a common literature review for the thesis described above. The role of this section is to highlight what has been done in the research areas relevant to this thesis and identify potential gaps in the literature. We then offer two chapters dealing with the issues discussed above. Our second chapter is divided into 5 sections. Section 2.1 introduces the chapter. Section 2.2 is a literature review which revolved around key findings on financial frictions and the interplay between monetary and macroprudential policies. Section 2.3 introduces our DSGE model with financial frictions, section 2.4 presents our key findings and we conclude in section 2.5. A technical appendix section which shows the key model derivations is also included for this chapter. Our third chapter is structured as follows. We start with an introduction to the chapter in section 3.1. In section 3.2, we review the existing literature on uncertainty and its impact on the macroeconomy. This literature review also looks at existing findings on the interplay between monetary and macroprudential policy in the presence of uncertainty. In section 3.3, we introduce a DSGE model, which in addition to financial frictions, is augmented with stochastic volatility. Specifically, the proposed model build on Pesaran & Xu (2016) which features firm default. Key results from this model are presented in section 3.4. We conclude in section 3.5.

1.2 Literature review

The importance of financial market frictions in dynamics of business cycles has been drawing the interest of both academics and policymakers alike, notably following the recent financial crisis. This resulted in an increasing number of theoretical frameworks that to a large extent build on two main perspectives to account for financial frictions in macroeconomic models. The first class of models chiefly builds on the seminal work by Bernanke et al. (1999) (BGG hereafter) that features price frictions in an agency cost framework. BGG shows that endogenous developments in the financial sector can considerably amplify and propagate exogenous innovations to the macroeconomy which is referred as the financial accelerator mechanism. Specifically, the driving factor of the financial accelerator relies on the negative correlations between the firms' net worth and the external premium charged by financial intermediaries. The second class of models build on Moore & Kiyotaki (1997) which is extended by Iacoviello (2005) where financial frictions operate through collateral constraints.

One of the earliest extension to the BGG (1999) model is introduced by Christiano et al. (2004) in the study of the Great Depression. To account for the driving factors of economic fluctuations, these authors also introduced features such as risk perceptions, capacity utilization, wage-setting rigidities, and labour demand shocks. Furthermore, the model is augmented with a banking sector which is sufficently elaborate to analyze the interactions between several nominal aggregates such as currency, demand deposits, bank reserves and real economic activity. For the analyzed period the augmented model is able to identify a rise in preferences towards holding money and a considerable decline in savings. Aoki et al. (2004) augment the BGG (1999) model with housing investment. In this model, capital producers are replaced by homeowners, who offer investment projects to provide residential services to households.

Christensen & Dib (2008) augment the model of BGG (1999) with two key properties. First, in their set up, the debt contracts are expressed in the nominal interest rate. This practically captures the feature of debt contracts in emerging markets. This means that the model can generate debt-deflation responses that widely cited in studies of the Great Depression. Next, the monetary policy is described by an extended Taylor rule in which the central bank regulates the short-term nominal interest rate in response to changes in inflation, output, and money-growth. This is significant addition given the fact that the quantitative relevance of the financial accelerator will, generally rely on the response of monetary policy. As BGG (1999) points out, the significance of the financial accelerator in magnifying and propagating business cycles fluctuations in output and investment is decreasing with the extent of output targeting in the

monetary policy. Christensen & Dib (2008) using maximum likelihood ratio test find that adding the financial accelerator mechanism into the model improves the fit of the observed data properties.

In general, there is some evidence that the inclusion of an external finance premium leads to a better match of the economic data features, for example, the size and persistence of fluctuations (Brázdik et al. 2012). Though, the comparison reveals that the extent of the financial accelerator mechanism largely depends on the underlying modelling assumptions (Brázdik et al. 2012). However, DSGE models that rely on external finance premium largely dismiss the immediate role of expectations regarding future economic developments at the present level of premium as the external finance premium is only derived based on the current value of the capital producers' net worth (Brázdik et al. 2012). Owing to this feature, models that include financial frictions, generally, have only a restricted capacity to identify the rise in default instances in times of stress (Brázdik et al. 2012). The model introduced in the third chapter of the thesis overcomes the constant rate of bankruptcy restriction by introducing two additional features. In the model the bankruptcy rate is endogenously determined by the expected level of uncertainty in the economy and by price setting constraints of ex-post heterogeneous firms.

A different perspective to model the accelerator mechanism of external finance premium is to impose an upper bound on the number of available external funds (Brázdik et al. 2012). The constraint derived from the balance sheet is justified by the debt-worthiness of the debtor, whilst the accessibility and the value of the borrowers assets facilitate the allocation of credits because these assets can be used to secure the loans (Brázdik et al. 2012). Where a collateral constraint is used, lenders can get back the debt only if loans are completely secured in the event of the insolvency of an investment project (Brázdik et al. 2012). Naturally, loans can be secured by the debtors durable assets, for example, land, houses, or long-term capital. Such durable assets are seized by the creditor in the case of liquidation of the investment project with the nominal value of the assets applied to determine the extent of the funds the debtor can receive (Brázdik et al. 2012). To this end, while our first chapter assumes that borrowers use their assets (houses) as collateral, the second chapter allows for financial intermediaries to incur losses. Specifically, the external finance premium in the model is determined by two constraints, and developments in the level of uncertainty and prices. The first constraint is on firms' equity and the second constraint is on banks' leverage.

Overall, at the firm level, as Brunnermeier & Sannikov (2014) point out, the micro-

foundations of financial frictions reside in studies on the capital structure with an agency and informational frictions on the one hand, and on financial intermediation with bank runs on the other. At the aggregate level, the research is focused on the impacts of prices and values of collateral, examining financial rigidities in a general equilibrium framework.

At the firm level, research by Townsend (1979), Bolton & Scharfstein (1990), and DeMarzo & Sannikov (2006) show how deviations from the Modigliani Miller theorem generate restrictions on the capacity of borrowing of agents, also limitations on risk sharing (Brunnermeier & Sannikov 2014). Sannikov (2012) looks at a survey of implications of financial frictions of capital structure and find that the distribution of among agents is important for how productive resources will be allocated in the economy. Similarly, Scheinkman & Weiss (1986) finds that the distribution wealth between different agents is critical for the aggregate macroeconomic activity. Whereas, Diamond (1984), and later Holmstrom & Tirole (1997) argue that the monitoring function performed by banks is critical because they direct funds from savers to borrowers. In this regard, we also allow for heterogeneity in our models. While the first model in thesis abstracts from financial intermediaries, the second model allows for the simple banking sector. Specifically, our first chapter assumes heterogenous households while the second chapter considers ex-post heterogenous firms that are subject to idiosyncratic shocks. The first chapter of the present also highlights the importance of wealth distribution between different types of households. Particularly, the aggregate welfare measure of the economy depends on the consumption gap between patient and impatient households. In the second chapter, the distribution of idiosyncratic productivity process directly determines the economywide default rate and thus the interest rate spread in the economy. The banking sector in this model, yet, is simple and there is no friction between banks and households. Therefore we also abstract from bank runs. In Diamond & Dybvig (1983) and Allen & Gale (2007), however, banks are allowed to run. He & Xiong (2012) move runs to nonfinancial firms, and Shleifer & Vishny (2010) highlights stability of banking sector together with sentiments of investors (Brunnermeier & Sannikov 2014).

At the aggregate level, several papers build upon the notion that unfavorable movements in prices deteriorate the net worth of borrowers and thereby their borrowing constraints (Brunnermeier & Sannikov 2014). Shleifer & Vishny (1992) make a case for the significance of the liquidation price of the capital which is evaluated at the time when actual buyers are more likely to be restrained. Shleifer & Vishny (1997) argue that the risk of insolvency limits the fund managers capacity for trading against

mispricing. Similarly, Geanakoplos et al. (1997) find that the distinctions of marginal buyers are important for prices of assets. Brunnermeier & Pedersen (2009) emphasized margin constraints that are determined by uncertainty while Rampini & Viswanathan (2010) argue that most firms with higher productivity are located in the proximity of their debt capacity and therefore are affected more in times of distress.

In addition to the above, other studies that investigate financial accelerator in infinite-horizon macro setup, such as Christiano et al. (2010), Carlstrom et al. (2010), Gertler & Karadi (2011), Gertler et al. (2011), and Curdia & Woodford (2010), Gerali et al. (2010) address questions including but not limited to the impact of financial frictions on monetary policy transmission and on optimal monetary policy. Specifically, some researchers have sought to examine whether models building on BGG provided justication for the monetary policy to "lean against the wind" (e.g., Gilchrist & Leahy (2002)); that is, to raise interest when asset prices increase (Duncan & Nolan 2017). For the majority of these studies, the consensus appeared to be that such a policy added little to the efficacy of monetary stabilization policy (Duncan & Nolan 2017). Concerning monetary policy, BGG model predicts that a surprise increase in the nominal interest rate reduces the demand for capital and thereby leads to a fall in its price (Nguyen 2015). The drop in the capital's value deteriorates the net worth entrepreneurs and therefore induces a higher external premium. This further reduces output and investment Nguyen (2015).

Along with financial frictions, another prevalent topic in the recent literature has been assessing quantitative importance of uncertainty shocks in business cycle dynamics. In his influential paper, Bloom (2009) attributes the fall in investment and output to a jump in uncertainty. According to this author, hikes in uncertainty as a reaction either to significant economic or political shocks results in firms postponing their hiring and investment which leads to a reduction in productivity growth, output, and employment. Furthermore, Alexopoulos et al. (2009) and Bloom et al. (2012) argue that a jump in the uncertainty causes a rapid decline and sluggish return to the steady-state in output.

Following an uncertainty shock, the presence of nominal rigidities is critical to restore the observed comovement in the data. More to the point, in models with sticky prices, an increase in uncertainty reduces firms' marginal costs of production through precautionary labour supply. Slowly-adjusting prices together with lower marginal cost increase firms' markups over marginal costs which lead to reduced demand for consumption and investment goods; output and employment decline when investment and consumption both fall as they are demand-determined in these models (Basu

and Bundick,2012). Leduc and Liu (2015) explain that both labour market frictions and nominal rigidities are fundamental as amplifying factors of uncertainty shocks that decrease aggregate demand and raise firms' markups. In the presence of search and match rigidities, the fall in aggregate demand lowers the value of new matching. Therefore, firms announce fewer job vacancies which push the employment down. Household's incomes decline further as more of them fail in finding a maching job match. This leads to a further reduction in aggregate demand and amplifies the impact of uncertainty shocks.

Although there is extensive literature and as well as results on the impacts of uncertainty shocks, recently, not many of them have looked at their effects in the presence of credit frictions (Bonciani & Van Roye 2016). Gilchrist et al. (2014) demonstrate, by giving empirical and theoretical evidence, that time-varying volatility by interacting with credit market rigidities amplifies business cycles. Using a conventional bond-contracting set-up, these authors find that a rise in uncertainty shocks is advantageous to equity holders whereas it is costly for bondholders since these shocks raise the cost of capital and consequentially decreases investment (Bonciani & Van Roye 2016). Furthermore, they find that decreasing credit supply obstructs efficient reallocation of capital which causes a further reduction in TFP.

Christiano et al. (2014) use a DSGE model with the financial accelerator feature initially introduced by BGG and also estimate the model for the U.S. data. They show that uncertainty shocks, defined as variations in the cross-sectional volatility of idiosyncratic productivity process, play a significant role in explaining the U.S. business fluctuations (Bonciani & Van Roye 2016). Christiano et al. (2014) solely examine idiosyncratic volatility shocks, whereas Balke et al. (2017) also analyze the impact of macroeconomic volatility shocks with financial market rigidities. In a model with agency costs, these authors find that financial frictions magnify the contractionary effects of uncertainty under nominal rigidities (Christiano et al. 2014). In addition, Cesa-Bianchi & Corugedo (2014) find that financial frictions expand the adverse effect of uncertainty shocks on investment, output, and consumption by using a modified variant of the financial accelerator model of Faia & Monacelli (2007). Also, they show that micro or idiosyncratic volatility shocks appears to be more significant quantitatively than a macro or aggregate volatility shocks (Bonciani & Van Roye 2016). Our second chapter contributes to this line of debate. Particularly, we find that the macroeconomic or systemic uncertainty leads to a sizeable and persistent decline in economic activity through the price of lending. The contraction in the economy is also present when prices are flexible and nominal rigidities enhances the recession.

In addition to integrating nominal rigidities, volatility, and financial frictions into a united framework, the present study is one of the few that sets out to assess the potency of monetary and macroprudential instruments as well as unconventional monetary policies (i.e., reserve requirements or liquidity provisioning) [1] under general equilibrium conditions. There is, in fact, growing literature incorporating macroprudential policy in economic models (Shao and Khashanah, 2016). Some papers focus on the pros and cons of adopting Basel III, these include Angelini et al. (2014), Bean et al. (2010) and Roger & Vlček (2011)). There are also papers advocating the use of macroprudential instruments - LTV ratios - to improve social welfare (see Gruss & Sgherri (2009), Lambertini et al. (2011) and Funke & Paetz (2012)). Other authors argue that it is necessary to improve the interaction of macroprudential and monetary policy for a sound financial system (see Borio & Shim (2007) and Angelini et al. (2014)). The present study is more closely related to this later strand of research. To echo Angelini et al. (2014), coordination between macroprudential and monetary policy, which may enhance their efficiency in times of downturns, requires additional analysis. The present study seeks to provide such analysis. Certainly, the subject of whether the monetary policy rule needs to be extended to include financial stability concerns, or whether there should be a separated policy tool for the financial stability is not new. In fact, this question is equal to asking whether Tinbergens Rule holds in the design of monetary and macroprudential policies (Carrillo et al. 2017). In other words, are two policy tools needed for two policy objectives? The relevance of this question resides in the fact that quantitative examination of a New Keynesian DSGE model with the credit friction and uncertainty shocks indicates that violations of Tinbergens Rule and strategic interaction between policymaking institutions may impair the effectiveness of transmission for both policies significantly(Carrillo et al. 2017).

Following the financial crisis, there is commonly shared belief evolving around a view that assigns financial stability targets to financial policies, where a task for monetary policy are held for exceptional occasions (IMF, 2015). Whereas monetary policy continues responsible for overall macroeconomic stability, in the majority of cases via an inflation targeting framework (Chen & Columba 2016). In relation to macroprudential tools, the current wisdom is that if implemented promptly, these can efficiently target most vulnerabilities, despite their impacts, notably on welfare, require additional study (Claessens 2014). Filling this gap is a key motivation for the present

[1] In the present, we interchangeably use terms of reserve requirement and liquidity policy to refer the unconventional monetary policy in the second model of the thesis. Although, liquidity policy would be a more appropriate label to reflect the fact that lending to deposit ratio in our sample period is above unity.

study. Indeed, macroprudential policies are still unproven, and as Bernanke (2015) argues risks to financial soundness cannot be entirely dismissed from the attention of decisions behind the monetary policy. For example, it may be the case that risks to financial stability cannot be adequately addressed solely by financial policies, but considering their macroeconomic consequence, a leaning against the wind policy by the monetary authority to address these vulnerabilities require a thorough cost-benefit analysis (Svensson 2016).

The literature offers a number of important studies exploring the interplay between monetary and macroprudential policy under a general equilibrium setting. Among them, Rubio and Carrasco-Gallego(2013) analyze the impact of the interaction between macroprudential and monetary policies on the economy, they show that incorporating the macroprudential policy largely dampens the impact of booms on the economy by limiting the credit on housing. Chen & Columba (2016) show that demand-side macroprudential policies are more efficient in moderating household debt ratios than the traditional monetary policy, and such policies are also less costly for the welfare of agents. Furthermore, these authors argue that a tighter macroprudential stance improves welfare by suppressing the volatility of consumption in response to shocks, notably when several combinations of macroprudential measures are used jointly. Silvo (2016) finds that to achieve or replicate the first-best outcome a social planner needs two distinct choice variables under financial and nominal rigidities. According to this author, having two separate policy variables, namely, a monetary and a financial one, renders the constrained optimum efficient. Furthermore, Silvo (2016) finds that when the Ramsey planner only sets the inflation rate, he cannot fully stabilize the economy, and also cannot eliminate a policy trade-off between stabilizing inflation and output gap. Angelini et al. (2013) results suggest that the active use of capital requirements by a macroprudential authority has limited benefit on macroeconomic stability during normal times or when the fluctuations in the economy are predominantly driven by supply shocks. Moreover, the absence of cooperation between a macroprudential body and the central bank may produce conflicting policies and hence excessive volatility in both policy instruments (interest rate and capital requirements). Therefore, one obvious implication is no improvement in the volatility of the key macroeconomic variables (output and the loans-to-output ratio) (Angelini et al. 2013). Suh (2012) results show that countercyclically implemented macroprudential policy can stabilize the credit-driven cycles. In this study, however, the LTV ratio that reacts to housing price, being a market-specific measure that targets the price of a specific asset, creates regulatory arbitrage which reallocates resources from household sector to business

sector. Suh (2012) Suh (2012) finds that the optimal policy differentiates objectives of macroprudential from monetary policy, with the reasoning being that these measures are used to stabilize separate target variables. Finally, Afanasyeva & Karasulu (2013) find that prolonged periods of accommodative monetary policy produce inflationary pressures and lower the efficiency of the macroprudential policy (loan-to-value ratio) which leads to further credit growth, thus increasing vulnerabilities.

Two reasons are often cited as a rationale for using the LTV as the preferred macroprudential instrument in many of the papers mentioned above. First, Cerutti et al. (2015) and Lopez et al. (2015) show that among the macroprudential measures available, both emerging and advanced countries favour loan-to-value related instruments. Second, Basel III reform conceives a countercyclical capital buffer, together with a rich list of other policy tools (Angelini et al. 2014). It is therefore meaningful to assess the macroeconomic effect of LTV ratio so that more discretion is allowed in its application for macroprudential goals (Shao and Khashanah, 2016). Specifically, in these studies, it is assumed that the macroprudential body targets a loss based mandate that take as arguments variances of the credit-to-output ratio, output, and of bank profits (Shao and Khashanah, 2016). Similarly, in the present study, we use the LTV as the preferred macroprudential instrument. Specifically, two definitions of LTV are used in our analysis. Our first definition relates to how much households are able to secure against their collateral, which in this case are houses. Our second definition relates to the existing firms' equity. This allows us to explore two different channels of financial frictions. The first chapter focuses on the financial friction arising as a result of borrowing constraint of impatient households. The second chapter apart from considering an exogenously given borrowing constraint for heterogeneous firms also includes the bank lending channel.

Similar to assessing the potency of the interplay between monetary and macroprudential policy in the presence of nominal rigidities, volatility, and financial frictions, very little evidence exists on the transmission mechanism of unconventional monetary policy. This is notwithstanding the fact that central banks in a number of emerging economies practice reserve requirements as an additional policy tool. Furthermore, among policymakers, reserve requirements are under consideration both as a financial stability measure, particularly, when dealing with the volatility of capital flows, and also as an unconventional monetary policy instrument to achieve inflation stability, notably when interest rate policy is restrained by the effective lower bound or objectives related to the exchange rate. In fact, financial regulation measures has traditionally been referred as a financial repression policies to characterize the tight regulation of the

financial sector in developing countries. Therefore, pros and cons of such polices has been a matter of long-standing debate. For instance, **?** analysing the sample of Latin American countries find that financial repression had a negative impact on economic growth. On the other hand, **?** referring to financial liberalization reforms from early 1980's in those countries raises concerns of financial instability as a consequence of these deregulations. More to the point, **?** using the data for 73 banks in Greece, Thailand, Mexico, Malaysia, and Taiwan empirically show that the probability of banking crises increases following the financial liberalization. These debates came to fore once again following the recent global financial crisis where financial regulation policies are more amiably labelled as macroprudential policies. However, despite often having overlapping policy tools, some make distinctions between these two definitions where financial repression are generally discussed in the context of governments channelling resources on preferential basis, specifically to finance the government debt through negative real interest rates, or to increase its' tax base. For example, citereinhart2012return point out financial intermediation helped to significantly liquidate accumulated governmen-t debts during the Second World War with over-regulated financial markets of the Bretton Woods system. Macroprudential polices, however, are often discussed in the context of reducing exposures and riskiness of financial endeavours to achieve overall financial stability. Therefore, despite the use of both policy labels interchangeably in the literature, there is some distinction between their objectives. However, this does not preclude the possibility of using financial policies for both purposes at the same time.

Chapter 2

Interactions between monetary and macroprudential polices using welfare based mandates

2.1 Introduction.

The recent financial crisis has brought to the fore the necessity of going beyond a solely micro approach to financial supervision and regulation (Galati & Moessner 2013). The urgency to consider alternative macroeconomic models which cast aside the underlying assumptions of the Modigliani & Miller (1958) theorem, has never been so strong. This is largely due to the fact that the presence of important transaction costs in the credit markets which led to the expanding debt stocks and leverage, particularly in the housing sector, have been recognized by many as the key trigger of the financial crisis.

In the midst of the crisis, one particularly worrying development has been the ineffectiveness of existing policies, e.g., monetary policy, to prevent the contagion to the real economy both within and across international borders. As the IMF (2012) points out, price stability does not guarantee macroeconomic stability. Whilst the level of financial innovation and financial deregulation has played a major role in magnifying both the boom and the unwinding of financial imbalances, it is difficult to overlook the fact that there has been a fundamental gap in our knowledge in understanding the system-wide risk.

The above explains why macroprudential policy has recently been drawing considerable attention among policymakers and researchers. Among the many interesting and important new avenues of research which have so far been identified, our research will pay a particular attention to the relationship between macroprudential policy and

monetary policy. We believe that identifying the right policy tools/instruments and understanding the welfare implications of this interaction is of paramount importance for policymakers.

According to the IMF (2012), experience and knowledge on the effectiveness of macroprudential policies and, especially, interactions with the monetary policy are still limited at this juncture. Though there is some evidence of macroprudential measures in the form of countercyclical capital buffers [1] responding to the credit growth, countercyclical provisioning [2], loan-to-value ratio (LTV), or direct regulation on lending to specific sectors being implemented by central banks in certain countries (Rubio 2014). Furthermore, there is rapidly growing literature that includes a macroprudential policy in the DSGE modeling context to examine the relative efficiency of using macroprudential policies together with monetary policies in addressing housing booms (Christensen et al. 2011).

For example, Rubio & Carrasco-Gallego (2014) analyse the implications of macroprudential and monetary policies for financial stability, welfare, and business cycles. These authors consider a macroprudential rule on the LTV within a DSGE (Dynamic Stochastic General Equilibrium) framework. They find that augmenting the variability of output and inflation with the variability of borrowing improves the stability of the system. Tayler & Zilberman (2014) find that following a credit shock, the countercyclical regulation of the bank capital is more efficient than a conventional monetary policy to improve financial, price and as well as overall economic stability. Christensen et al. (2011) add a countercyclical regulatory LTV to a DSGE model with a housing sector where housing serves as a collateral. These authors study shocks that simulate a housing boom, including an innovation that raises the borrowing threshold of constrained households. They find that in the presence of a shock that affects the value of collateral by increasing the borrowing capacity of constrained households, a countercyclical LTV policy is more efficient to stabilize the economy than monetary policy.

One of the key weaknesses of many of the DSGE models that the literature offer revolves around the derivation of the welfare loss function which is often done in an ad-hoc fashion. In cases where a theoretical loss function is derived, often the focus of the research is not on the interaction between macroprudential and monetary policies. For example Andrés et al. (2013) analyze optimal monetary policy in a model that features

[1] BASEL III defines the countercyclical capital buffer as a capital requirement for banks that respond to the macro-finance environments in which they operate.

[2] Countercyclical provisioning essentially refers bank setting aside funds from their surplus of profits in g̈oodẗimes against the probability of increased non-performing loans in downturns.

two forms of financial rigidities; first, a monopolistically competitive banking sector that endogenously creates lending spreads, and second, constraints in collateral. These authors show that welfare maximization requires stabilization of four goals: prices, output gap, the consumption gap between patient and inpatient households, and the housing gap which reflects the distortion in the allocation of the collateralizable assets between two agents. However, the authors do not consider the welfare implications of the interaction between macroprudential and monetary policies. In addition to the derivation of a theoretical welfare loss function, for some of the DSGE models proposed, the complexity of the welfare analysis often means that not all the relevant welfares in the model are tracked. For example, De Paoli & Paustian (2013) assume that in equilibrium entrepreneurs hold their consumptions because the return to internal funds exceeds their subjective discount factor. This assumption considerably facilitates the welfare analysis, because it is not required any more to track the welfare of both agents (entrepreneurs and households).

We contribute to the literature by analysing the interactions between monetary and macroprudential policy whilst deriving a welfare loss function as in Andrés et al. (2013). The model we propose assumes a partially distorted steady state where we impose efficient risk sharing between patient and impatient households in the steady state. However, the model still allows for the inefficiency related to the markup in the steady state. Furthermore, unlike in studies where the welfare of distinct groups i.e. households and entrepreneurs are not tracked, we propose a model where the welfares of savers and borrowers are tractable.

As Svensson (2012) argues: "there is a risk of conceptual and practical confusion between monetary policy and financial-stability policy" also referred to as macroprudential policy in the present. Indeed, some have argued that the objectives of monetary policy should be extended to financial stability targets and that monetary policy and macroprudential policy ought to be combined and implemented jointly (Eichengreen 2011, (Eichengreen et al. 2011)). We adopt Svensson (2012) view that such statements are unsuitable because they dismiss the fact that monetary and macroprudential policies are distinct policies with different objectives. Thus, in addition to discussing the interaction between monetary and macroprudential policy, our literature review also discusses in separate sections the objectives and policy instruments needed for the conduct of macroprudential policy.

The remainder of the paper is organised as follows. Section 2 discusses the objectives of macroprudential policy, Section 3 discusses macroprudential policy tools and Section 4 presents a discussion on the interaction between monetary and macroprudential

policies following a brief historical background of the various modelling strategies.

2.2 Objectives of Macroprudential policy

Caruana (2010) describes the objective of financial or macroprudential policy as "to reduce systemic risk by explicitly addressing the inter-linkages between, and common exposures of, all financial institutions, and the procyclicality of the financial system". According to Perotti & Suarez (2011) macroprudential policy aims to prevent individual bank strategies that produce a system-wide risk and put a negative externality on the entire financial system. For Rubio & Carrasco-Gallego (2014) the role of the macropruedntial regulation would be to avoid the transmission of shocks originating from the financial sector to the broader economy. Indeed, whilst the literature on monetary policy has witnessed a certain concurrence in opinions on its policy objective over the past decades, the same does not hold true for macroprudential policy as evidenced by the contrasting views of these authors.

One of the major reasons for this divergence in views is that there is no prevailing definition of financial stability (Galati & Moessner 2013). Though, as Galati & Moessner (2013) argue the different opinions can be categorized into two perspectives. The first perspective describes financial stability as a resilience of the financial system to withstand external disturbances (e.g.Allen & Wood (2006), Padoa-Schioppa (2003)). Whereas the second perspective, highlights the endogenous character of financial stability where it is described as resilience to innovations emerging within the financial market (e.g., Houben et al. 2004) and the fragility of financial system following normal-sized innovations instead of large innovations (Borio 2003).

Furthermore, it is also worth noting that whilst some authors put more emphasis on the micro nature of macroprudential policy others highlight its macroeconomic dimensions. Indeed, a useful starting point of this interesting debate as Crockett (2000) points out is explaining the macroprudential policy referring to the distinction between the macro and the microprudential viewpoints to regulation. To this end, Borio (2003) proposed the following description of the different characteristics of the two approaches (2.1):

Borio's characterisation is not only useful in helping develop a better understanding of the nature of macroprudential policy, but the above is also helpful when thinking about the policy instruments of macroprudential policy. These are discussed next.

Policy Instruments of Macroprudential policy.

As Galati & Moessner (2013) points out, there are significant differences between macroprudential policies which are described as prudential instruments designed with a

Table 2.1: Macro - versus microprudential perspectives

	Macroprudential	Microprudential
Proximate Objective	Limit financial system-wide distress	Limit distress of individual institutions
Ultimate Objective	Avoid macroeconomic costs linked to financial instability	Consumer (investor/depositor) protection
Characterisation of Risk	"endogenous" (dependent on collective behaviour)	"exogenous" (independent of individual agents' behaviour)
Correlations and common exposures across institutions	Important	Irrelevant
Calibration of prudential controls	In terms of system-wide risk; top-down	In terms of risks of ; individual institutions bottom-up

Source: Borio (2003).

macro perspective and the rest of macroeconomic policies that can also strengthen the stability of the financial system, e.g., fiscal policy (please see Hannoun 2010). Whilst the policy rate and standard repos are widely acknowledged as the primary instruments of monetary policy, in relation to macroprudential policy, there is a lack of consensus as to what the primary instrument is/are. As Galati & Moessner (2013) argue, the research on particular macroprudential policies can be grouped in different and in some places overlying ways (please see BIS 2008 for a more comprehensive discussion).

Several important distinctions exist between the various macroprudential instruments. One such distinction as Galati & Moessner (2013) point out is the distinction between policies that are designed to address the time-series aspect of the stability in the financial system (e.g., the procyclicality in the financial stability), and those that address the cross-sectional nature (e.g., distribution of the risk at a given point in time inside the financial system or relative contributions to system-wide risk of individual institutions). Whilst the former reflects the evolution of risk over time, in other words, the procyclical characteristics of risk, the latter emphasizes the distribution of risk inside the financial system at a given point in time (Galati & Moessner 2013). To address the procyclicality and their origins among many tools highlighted in the literature are practices related to the evaluations of collateral and loan-to-value ratios that can be mitigated through maximum loan-to-value ratios, the provisions of loan

losses, the haircut-setting, differentiating practices in the financing of securities, and over-the-counter derivatives transactions (Galati & Moessner 2013).

Another important distinction is between rules and discretion in parameterizing the instruments of prudential policies (Borio & Shim 2007). Whereas, the historical wisdom of monetary policymaking coupled with evidence from the academic literature has backed simple rules for their accountability, transparency, and effectiveness in the context of monetary policy design (Galati and Moessner, 2011). Also, as demonstrated in the literature on time-consistency that, despite being the first-best for agents' welfare fully optimal policymaking are not time consistent (Kydland & Prescott 1977). Therefore, in rational expectations models rules can achieve at least a second-best outcome (Galati & Moessner 2013). Furthermore, there is a consensus that the monetary policy to be efficient, it needs to be flexible, and the discretion is also required in extraordinary times (Galati & Moessner 2013).

There are some precedents in relation to the use of some of the macroprudential policy instruments mentioned above. Indeed, as McCauley (2009) argues, central banks in emerging economies have been regularly practicing macroprudential policies, despite not labeling it by this name. For example, the Reserve Bank of India decided to increase the Basel I weights on mortgages in 2005 (RBI, 2005). A detailed summary of practices of macro-prudential policies in emerging economies was provided by Borio & Shim (2007) and Board (2010). Agénor et al. (2012) analyzed the cyclical implications of capital requirement policies for banking in emerging economies, to understand the cyclical consequences of Basel regulations in the mitigating and amplifying the financial crisis (cited in Galati & Moessner 2013).

2.3 Related literature

As discussed in the introductory part of the present, the canonical Macroeconomic models largely relies on the frictionless financial markets assumption (Beau et al. 2012). According to the Modigliani & Miller (1958) paradigm, these models assume that the structure of agents' balance sheets does not affect their optimal spending decision. Hence, the canonical macro models often struggle to account for the interlinkages between developments in financial markets and the real macroeconomic activity in periods of distress in the financial sector (Beau et al., 2012).

Bernanke et al. (1999) (1999, BGG henceforth) seminal paper represent a key departure from macroeconomic models based on the Modigliani-Miller theorem. In the BGG model, borrowers are faced by an external finance premium, which emerges by the differences in cost of internally and externally raised funds (Beau et al, 2012). The premium on externally raised funds inversely relates to the net worth of borrowers which is pro-cyclical because of the pro-cyclicality in asset prices and as well as profits. Because the external finance premium is countercyclical, this helps to enhance the fluctuations in borrowing and thereby in investment and as well as in aggregate demand. Therefore the external finance premium transmits disturbances to the rest of the economy which magnifies economic cycles (Beau et al. 2012).

Gilchrist et al. (2009) assesses the importance of financial frictions in economic fluctuations in an estimated DSGE model with the financial friction which connects developments in the balance sheet to the real macroeconomic activity by changes in the external finance premium. They find strong evidence for the existence of a significant financial accelerator mechanism in business cycle fluctuations of the U.S. for the period of 1990–2008 period where increases in the external finance premium generate large and persistent reductions in investment and output. These authors' approach is closely related to the work of Christiano et al. (2009), De Graeve (2008), Christensen and Dib (2008), and Queijo von Heideken (2009), who show that accounting for the financial accelerator mechanism as in BGG improves the fit of the DSGE models of the macroeconomic data considerably (Gilchrist et al. 2009).

Moore & Kiyotaki (1997) proposed an alternative framework, which is based on borrowers' balance-sheets. An overriding assumption in this framework is that creditors cannot compel borrowers to pay their debt; this, in turn, means that durable assets, such as land or machinery, serve both as factors of production as well as the collateral to obtain loans at the same time (Beau et al. 2012). The latter has a bearing on borrowers' credit lines which ultimately change asset prices. The resulting dynamic interplay between borrowing constraint and prices of assets serve as transmission device

through which the impacts of financial disturbances persist and, magnified and spill over to the rest of economy (Beau et al. 2012).

Liu et al. (2010) claims that the reduced impact of credit limits arises because the mechanism to account for the observed persistent and co-movements between house prices and productive investment is often dismissed from the modeling. These authors introduce this mechanism by combining two key features into their DSGE model; first, they incorporate shocks into the demand for collateral assets, second, they also allow productive agents to have credit constraints. They find a positive co-movement between house prices and productive investment. Liu et al. (2010) results show that the amplification mechanism in Moore & Kiyotaki (1997) is empirically important.

Whilst the above models (except the BGG model) all assume that the borrowing constraints are always binding, which means that there is no default in equilibrium, some alternative frameworks have been proposed to this effect. For example Carlstrom & Fuerst (1997) propose a DSGE model with endogenous agency costs that potentially change business-cycle dynamics. In this setting, the risk of default exists in equilibrium. They find that their agency-cost model matches the empirical observation on output growth showing positive autocorrelation at shorter horizons. Furthermore, the hump-shaped output behaviour predicted by their model is explained as the result of households delaying their investment decisions until agency costs are considerably reduced. The latter results suggest that regulatory policies are a viable option.

Faia & Monacelli (2007) examine optimal implementable interest rate rules in two model economies with price and financial market rigidities as in Carlstrom & Fuerst (1997). They model monetary policies as interest rate rule that maximize welfare. These authors findings suggest that strict inflation targeting is a robust policy option. According to Faia & Monacelli (2007) this result is explained by the fact that financial frictions act towards suppressing the cyclical properties of the inflation relative to its level without financial frictions. Therefore none of the economies produces a trade-off between inflation and credit markup stabilization in spite of generating different dynamic in inflation and investment (Faia & Monacelli 2007). Another important implication of their results is that responding to asset prices does not yield any welfare benefits on its own in the monetary policy.

Iacoviello (2005) uses a financial accelerator model in the BGG spirit with collateral constraints linked to values of real estate as in Moore & Kiyotaki (1997). Furthermore, an additional friction is introduced to the model which is debt contracts being expressed in nominal terms. This permits the consideration of distributional implications of nominal frictions as in Fisher (1933). Two findings from this research are worth

mentioning here; first, collateral effects improve the model's match of the positive reaction in real spending to a shock in house prices, and second, nominal debt enables to correctly match the sluggish response of real spendings to a surprise inflation. Iacoviello also shows that the debt-deflation channel magnifies demand shocks but helps to stabilize supply shocks; this, in turns, improves the trade-off between inflation and output stabilization for the monetary authority. In the proposed model, reaction to asset prices does not bring considerable welfare improvements. Though, a limitation of this model is that it dismisses buffer-stock behaviour since the key to the model's results is that the quantity of aggregate uncertainty is modest compared to the degree of borrowers' impatience.

Indeed, most of the models we have mentioned so far only study the demand side of financial rigidities (e.g., balance sheets of borrowers). There is some evidence that supply-side determinants are likely to have a more significant role in the transmission mechanism. Among the studies that look at banks' balance sheet as a way of accounting for supply-side factors, the study by Christiano et al. (2007) is worth mentioning. These authors introduce a banking sector into a DSGE model that also contains a debt-deflation mechanism. They find that financial frictions as in BGG play an important quantitative role as the main driver of business cycle fluctuations, both in the U.S. and the euro area. More specifically, these authors find that owing to the larger inertia in the ECB's policy rule - the ECB's policies had more stabilizing efficiency than did those of the Fed. According to Christiano et al. (2007), the latter largely helped the ECB to keep the inflation close to its target level of price stability and, thereby, moderating effects of recessions. This study also finds that other factors that can potentially explain the different economic outcomes in the Euro Area and the U.S. were differences in innovations and distinctions in the degree of price and wage rigidities.

Meh & Moran (2010) develop a DSGE model which features a banking sector where the bank capital endogenously derived by solving an asymmetric information problem between banks and their borrowers. In this model, the bank's capital position, in other words, its ability to draw loanable funds plays a crucial role by affecting the business cycle through the bank capital transmission channel. Moreover, Meh & Moran (2010) use their model to examine the economy's responses to various shocks, including, technology, monetary policy shocks, and financial shocks that originate within the financial sector. They show that the bank capital channel expands the impacts of shocks on inflation, investment, and output. In addition, they also show that financial shocks that exogenously reduce the bank capital cause considerable drops in output and investment. A key finding of this study is that bank capital strengthens the economy's

capacity to resist shocks and, thereby affecting the implementation of monetary policy.

As Beau et al. (2012) point out, only a few models specifically consider macroprudential policies in a wider sense. For example, Kannan et al., (2009) show that stronger monetary response to accelerator mechanisms that drives up either the credit growth or house prices can improve macroeconomic stability. In addition, they also find that implementing a macroprudential policy explicitly designed to address credit cycles would also present stabilization benefits for shocks emerging from the financial sector or shocks housing demand. However, their study reveals that when the economy faces productivity shocks the optimal macroprudential rule is not to intervene. A key conclusion of the study relates to the importance of understanding causes of house price booms for the design of monetary and macroprudential policies. Their findings are in line with Cecchetti et al. (2000)..

Quint & Rabanal (2013) study the optimal mixture of monetary and macroprudential policies for a two-country model estimated for the Euro area. In their model, these authors include real, nominal and financial rigidities, so that both monetary and macroprudential policy can have a scope. They find that the introducing a macroprudential rule may benefit in reducing the overall volatility of the economy, enhance welfare, and partly substitute for the inadequacy of monetary policies at the national level. Their finding also reveals that macroprudential regulations always improve the welfare measure of savers, whilst their influences on the welfare of borrowers depend to a large extent on the origin of the shock that the economy faces. For example, following a technology shock, they find that macroprudential policy may cause welfare losses for borrowers by raising the countercyclical behaviour of lending spreads.

Rubio & Carrasco-Gallego (2014) consider a DSGE model with housing and borrowing constraints. In these authors' model, a macroprudential policy defined as an LTV ratio, which is set accroding to deviations in output and house prices, interacts with the conventional interest rate rule. Among other key results, they find that monetary and macroprudential policies may be conflicting with each other the economy faces supply-side shocks. Furthermore, they also find that the introducing the macroprudential measure improves welfare. Another key finding of these authors is that the optimal LTV policy should respond more strongly to house prices than to the output gap, in sum, they find that both policies acting jointly improves the stability of the economy. In a more recent study, Tayler & Zilberman (2014) looks at the role of bank capital regulation as macroprudential policy and monetary policy in a DSGE framework with endogenous credit frictions and a borrowing cost channel. These authors point out various channels of transmission of how the credit risk, losses of commercial

banks, the conduct of monetary policy, and bank capital requirements can affect the macroeconomic activity. Tayler & Zilberman (2014) find these channels to produce large financial accelerator effects, thereby providing motivations for macroprudential policy. One of their key findings is that for credit shocks, the countercyclical bank capital regulation is found to be more efficient than monetary policy in improving financial, inflation and overall economic stability. In contrast, following a supply shock, macroprudential regulation in conjunction with substantial feedback to inflation in the monetary policy gives the highest welfare gains. The findings highlight the significance of the Basel III regulatory accords and challenge the merit of traditional Taylor rules during episodes of financial distress.

As we made the case earlier, a key weakness of many of the DSGE models mentioned here resides in the fact that the derivation of the loss function which is often done in an ad-hoc fashion. In cases where a theoretical loss function is derived, often the focus of the research is not on the interaction between monetary and macroprudential policies. Thus, we contribute to the literature by analysing the interactions between monetary and macroprudential policy whilst deriving a welfare loss function as in Andrés et al. (2013). The model we propose assumes a distorted steady state. Furthermore, unlike in studies where the welfare of distinct groups, for example., households and entrepreneurs are not tracked, we propose a model where the welfares of savers and borrowers are tractable (i.e., the aggregate welfare in this economy will depend on the distribution of resources and risks between model's agents).

In the next section, we discuss our proposed model.

2.4 Benchmark Model

The model economy is based on Iacoviello (2005) and Rubio and Carrasco-Gallego (2013). The model subsumes savers, borrowers, final goods firms, intermediate goods producers, monetary and macroprudential authorities. Each type of households works and consumes both final goods and housing. Savers are infinitely lived, and borrowers face a probability δ of dying at the end of each period as in Andrés et al. (2013). At the beginning of each period an old borrower is replaced by a newly born of the same type who owns assets and liabilities of the former. A positive probability of death makes the borrowers relatively less patient in discounting future streams of utility than savers. This produces credit flows form savers to borrowers in equilibrium and in its neighbourhood. The production sector is divided into two parts, firms that produce differentiated intermediate goods, and firms that convert these intermediate goods into a final good.

2.4.1 Savers

Savers choose consumption of final goods (C_t^s), housing (H_t^s) and labour hours (L_t^s) to maximize following utility function:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t^s + j_t \ln H_t^s - \frac{(L_t^s)^\eta}{\eta} \right] \tag{2.1}$$

where $\beta \in (0,1)$ is savers' discount factor, $1/(\eta - 1)$ is the elasticity of labour with $\eta > 1$, $j_t = e^{u_t} j$ is an exogenously time-varying stochastic weight on utility from housing with mean j and $u_t \sim (0, \sigma_u^2)$ .This is is a shock to the marginal utility of housing, and acts as exogenous disturbances to house prices with AR(1) time path.

Subject to:

$$C_t^s + q_t \left(H_t^s - H_{t-1}^s \right) + b_t \leq w_t^s L_t^s + \frac{R_{t-1} b_{t-1}}{\pi_t} + F_t \tag{2.2}$$

where b_t is bank deposits with gross returns R_t, q_t is the price of houses in consumption units, and w_t^s is real wages, F_t is lump sum transfers from firms' profits. The first order conditions for the optimization are as follows:

$$\frac{1}{C_t^s} = \beta E_t \frac{R_t}{C_{t+1}^s \pi_{t+1}} \tag{2.3}$$

$$w_t^s = C_t^s \left(L_t^s \right)^{\eta - 1} \tag{2.4}$$

$$\frac{j_t}{H_t^s} = \frac{1}{C_t^s} q_t - \beta E_t \frac{1}{C_{t+1}^s} q_{t+1} \tag{2.5}$$

Equations (2) and (4) are Euler equations for consumption and housing. Euler condition for housing equates the marginal utility of house consumption to its cost in consumption units of final goods Equation (3) is the labour supply condition.

2.4.2 Borrowers

Borrowers solve the following problem:

$$\max E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \left[\ln C_t^b + j_t \ln H_t^b - \frac{\left(L_t^b\right)^\eta}{\eta} \right] \tag{2.6}$$

where $\tilde{\beta} \equiv \beta (1 - \delta)$ is borrowers discount factor that takes into account their survival probability.

subject the to budget and collateral constraints:

$$C_t^b + q_t \left(H_t^b - H_{t-1}^b \right) + \frac{R_{t-1} b_{t-1}}{\pi_t} \leq w_t^b L_t^b + b_t; \tag{2.7}$$

and

$$b_t \leq \frac{k_t}{R_t} E_t \pi_{t+1} q_{t+1} H_t^b \tag{2.8}$$

where k_t is the loan to value ratio in the borrowing constraint of the impatient household. The first order condition to the borrowers' problem are as follows:

$$\frac{1}{C_t^b} = \widetilde{\beta} E_t \frac{1}{C_{t+1}^b} \frac{R_t}{\pi_{t+1}} + \lambda_t R_t \tag{2.9}$$

$$w_t^b = C_t^b \left(L_t^b \right)^{\eta - 1} \tag{2.10}$$

$$\frac{j_t}{H_t^b} = \frac{1}{C_t^b} q_t - \widetilde{\beta} E_t \frac{1}{C_{t+1}^b} q_{t+1} - \lambda_t k_t E_t \left(\pi_{t+1} q_{t+1} \right) \tag{2.11}$$

where λ_t is the Lagrangian multiplier on the borrowing constraint and is positive in the steady state:

$$\lambda = \frac{\beta - \widetilde{\beta}}{C^b} > 0 \tag{2.12}$$

Therefore, the borrowing constraint always binds in equilibrium.

2.4.3 Final goods firms

There is a continuum of identical final goods' firms that behaves competitively.The representative final goods firm purchases intermediate goods and aggregates according to the following technology:

$$Y_t = \left[Y_t(j)^{\frac{\varepsilon - 1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon - 1}} \tag{2.13}$$

and sells the final good under flexible prices. $\varepsilon > 1$ is the elasticity of substitution between intermediate goods. The integral is raised to power $\varepsilon/(\varepsilon - 1)$ to make the production function display constant returns to scale. In each period the final goods' firm maximizes profits:

$$profits_t = P_t Y_t - \int_0^1 P_t(j) Y_t(j) \, dj \tag{2.14}$$

subject to the above production function. Thus, the problem of the final goods firm becomes:

$$\max_{Y_t(j)} P_t \left[\int_0^1 Y_t(j)^{\frac{\varepsilon - 1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon - 1}} - \int_0^1 P_t(j) Y_t(j) \, dj$$

resulting in the following demand function for the intermediate good j:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} Y_t \tag{2.15}$$

The price index is given by following CES aggregator:

$$P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{\varepsilon - 1}} \tag{2.16}$$

2.4.4 Intermediate goods firms

The intermediate goods sector is made by a continuum of monopolistically competitive firms owned by savers, indexed by $j \in (0.1)$. Each firm faces a downward slopping demand for its product $Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\epsilon} Y_t$. The production technology for intermediate goods firm j is

$$Y_t(j) = A_t \left(L_t^s\right)^{\alpha}(j) \left(L_t^s\right)^{1-\alpha}(j) \tag{2.17}$$

Borrowers' and savers' labour are not perfect substitutes in the model. This apart from making the derivation of steady-states analytically tractable, can economically explained by assuming that savers are the managers of firms and therefore earn higher wages (Rubio & Carrasco-Gallego 2014).

All firms are subject to the common technology shock A_t whose stochastic properties are specified as below:

$$\log(A_t) = \rho_A \log(A_{t-1}) + \varepsilon_t^A \tag{2.18}$$

The steady state value of the technology is normalized to 1.

Market for both types of the labour is perfectly competitive, so each firm takes wages as given.

$$w_t^s = \frac{1}{X_t} \alpha \frac{Y_t}{L_t^s} \tag{2.19}$$

$$w_t^b = \frac{1}{X_t} (1-\alpha) \frac{Y_t}{L_t^b} \tag{2.20}$$

X_t is the markup or the inverse of the marginal cost.

In each period a representative firm sets its price optimally with probability $1-\theta$, which is independent from the previous period. Assuming the law of large numbers the average price level will be a CES aggregate of all prices in the economy will take the following form:[3]

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{2.21}$$

Price setters maximize the expected discounted sum of all future profits which is made conditional on the chosen price and weighted by the probability that this price will apply to future periods/ They solve following program:

$$\max_{P_{jt}} \sum_{k=0}^{\infty} (\theta\beta)^k E_t \left[\Lambda_{t,k} \left(\frac{P(j)_t}{P_{t+k}} - Z_{t+k}\right) Y_{jt+k}\right] \tag{2.22}$$

[3]The maximum attainable value for P_t is the solution to the equation $1 = \theta P_t^{\varepsilon-1} \Rightarrow P_t < \theta^{\frac{1}{1-\varepsilon}}$.Thus, NK models can never generate hyper-inflations. Equation (2.21) also imposes a lower bound on optimal reset price: $P_t^* > \left(\frac{1}{1-\theta}\right)^{\frac{1}{1-\varepsilon}}$.

where θ^k is the probability that the price fixed in period t will continue into period $t+k$, and $\Lambda_{t,k} \equiv C_t^s / C_{t+k}^s$,

subject to the production technology:

$$Y_{t+k}\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} = A_{t+k}\left(L_{t+k}^s\right)^{\alpha}(j)\left(L_{t+k}^s\right)^{1-\alpha}(j)$$

The solution to this problem is as follows:

$$P_{jt}^* = X\frac{\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left[\Lambda_{t,k}Y_{j,t+k}^* P_{t+k}^{-1} Z_{t+k}^n\right]}{\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left[\Lambda_{t,k}Y_{j,t+k}^* P_{t+k}^{-1}\right]} = (\frac{1}{Z})\sum_{k=0}^{\infty}\phi_{t+k}Z_{t+k}^n \tag{2.23}$$

where Z_{t+k}^n is the nominal marginal cost, and

$$\phi_{t+k} \equiv \frac{(\theta\beta)^k E_t\left[\Lambda_{t,k}Y_{j,t+k}^* P_{t+k}^{-1}\right]}{\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left[\Lambda_{t,k}Y_{j,t+k}^* P_{t+k}^{-1}\right]} \tag{2.24}$$

When $\theta > 0$ optimal price depends on expected future values of aggregate variables and future nominal marginal costs. The aggregate price level becomes:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(X\sum_{k=0}^{\infty}\phi_{t+k}Z_{t+k}^n\right)^{\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{2.25}$$

Log linearizing this equation gives the familiar New-Keynesian Phillips curve:

$$\hat{\pi} = \beta\hat{\pi}_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}\hat{z}_t \tag{2.26}$$

However, for the welfare analyses which we discuss in following chapters the model needs to be set-up in its non-linear form as a set of difference equations. In the first order conditions for Calvo contract we are confronted with the expected discounted sums of the following general form:

$$\Upsilon_t = E_t\left[\sum_{k=0}^{t}\beta^k\Theta_{t,t+k}Y_{j,t+k}\right] \tag{2.27}$$

where $\Theta_{t,t+k}$ has the property of $\Theta_{t,t+k} = \Theta_{t,t+1}\Theta_{t+1,t+1}$ and Using the recursive nature of the expression we can rewrite the Calvo first order condition as (see Appendix 3.1):

$$\Upsilon_t = Y_j^* + \beta E_t\left[\Theta_{t,t+1}\Upsilon_{t+1}\right] \tag{2.28}$$

This gives equations for the dynamics of inflation:

$$\frac{P_t^*}{P_t} = \frac{\lambda_t^1}{\lambda_t^2} \tag{2.29}$$

$$\lambda_t^2 - \theta\beta E_t\left[\pi_{t+1}^{\varepsilon-1}\lambda_{t+1}^2\right] = Y_t U_{c,t}^s$$

$$\lambda_t^1 - \theta\beta E_t\left[\pi_{t+1}^{\varepsilon}\lambda_{t+1}^1\right] = \frac{1}{1-\frac{1}{\varepsilon}} Y_t U_{c,t}^s \dot{Z}_t \tag{2.30}$$

$$1 = \theta\pi_t^{\varepsilon-1} + (1-\theta)\left(\frac{\lambda_t^1}{\lambda_t^2}\right)^{1-\varepsilon} \tag{2.31}$$

As before, by the virtue of the law of large numbers, we get the following difference equation for the price dispersion that relates it to the inflation (see Appendix 2.2):

$$\Delta_t = \theta\pi_t^{\varepsilon}\Delta_{t-1} + (1-\theta)\left(\frac{\lambda_t^1}{\lambda_t^2}\right)^{-\varepsilon} \tag{2.32}$$

2.4.5 Market clearing conditions

Market clearing in final goods market:

$$\frac{Y_t}{\Delta_t} = C_t^s + C_t^b \tag{2.33}$$

Up to the first order and around zero inflation rate the price dispersion is unity and can be ignored. However, for high order approximations we need to keep track of this variable.

The total measure of housing in the economy is fixed to one and the equilibrium condition is given by:

$$H_t^s + H_t^b = 1 \tag{2.34}$$

2.4.6 Policy rules

In fully commitment rules or in discretionary policy all state variables have to enter into the policy maker's decision rule. However, we assume that the policymaker can pre-commit not to reoptimize over time, while restricting the set of variables to which the policy responds, using optimal simple rules. According to Levin et al. (2003) optimal simple precommitment rules have several advantages. First of all, they provide a compact representation of policy maker's decision rule and perform relatively well to

fully optimal precommitment policy. Levin et al. (2003) show that Taylor-type rules are approximately 85% as efficient as fully optimal precommitment rules. Moreover, the policy rule is not effective if it is forced to react to information that is not available at the time when policy decisions are made Levin et al. (2003). Whereas, Taylor-type optimal simple rules overcome this by restricting variables to which the policymaker can respond Levin et al. (2003). The nominal interest rate is given by the following Taylor rule that responds to the inflation and output gap:

$$r_t = r_{t-1}^{\rho} \left[\pi_t^{(1+\varphi_\pi)} \left(\frac{Y_t}{Y_{t-1}} \right)^{(1+\varphi_y)} (\bar{r}) \right]^{(1-\rho)} M_t \tag{2.35}$$

where M_t is a monetary shock process

$$M_t = \exp(\sigma_M * \epsilon_t^M) \tag{2.36}$$

For the macroprudential policy we assume following Taylor-type rule:

$$k_t = k \left[\frac{b_t}{b_{t-1}} \right]^{\varphi_k} \tag{2.37}$$

which states that the macroprudential institution countercyclically responds to movements in the total borrowing (b_t) in the economy.

2.5 Calibration and impulse responses

2.5.1 Calibration

Our model is calibrated in line with the parameters values listed in table (2.2) below. These values are on based our key macroeconomic and microeconomic stylized facts.

Table 2.2: Model parameters

Parameter values		
β^s	0.99	Savers discount factor
β^b	0.98	Borrowers discount factor: $\delta = 0.01$
η	2	Parameter for labour elasticity
X	1.2	Steady state mark-up
θ	0.75	Calvo parameter
κ	0.08583	$(1-\theta)(1-\theta\beta^s)/(\theta)$
φ^r	0.8	Interest rate smoothing parameter in Taylor Rule
$1+\varphi^\pi$	1.5	Inflation elasticity in Taylor Rule
φ^r	0.5	Output elasticity in Taylor Rule
α	0.64	Labour share of savers
j	0.1	Weight of Housing in the utility function
k^{ss}	0.9	Steady state, collateral constraint
μ^{mp}	0/0.45	Parameter for the macroprudential rule
ρ^a	0.9	Persistence of the technology shock
ρ^h	0.95	Persistence of the housing shock
r^{ss}	$1/\beta^s$	steady state inflation/interest rate

Both the savers and borrowers discount factors are set to 0.99 and 0.98, respectively which determine their degree of patience. The former is set to 0.99 so that the annual interest rate is 4% in the steady state whilst the latter reflects Lawrance (1991) estimates. The labour elasticity parameter is set to 2 which in turns implies a value of labour supply elasticity of 1. In addition, the labour income share for savers is set to 0.64, in line with Iacoviello (2005) estimates. Because the ratio of housing wealth to GDP is approximated to be 1.40 in the steady state based on US data, a parameter value of 0.1 is assumed for the steady state weight of housing in the utility function.

Three shocks are considered in our model, these are; monetary policy shock, technology shock, and housing demand shock. As in the standard literature, technology is assumed to be normally distributed and to follow an autoregressive process with a 0.9 persistence. The weight of housing on the utility function is assumed to be equal to its value in the steady state in addition to a shock which also follows an autoregressive process with 0.95 persistence.

2.5.2 Technology shock

A 1% shock to technology results in an increase in both output and consumption. Furthermore, given the fixed supply of housing in the economy, the shock also leads to an increase in borrowing which in turns causes borrowers to demand more housing. The latter is compensated by a decrease in the demand for housing product by the savers. The higher demand for housing product by borrowers causes house prices to increase which in turn increases borrowers' consumption further, given the collateral constraint. Unambiguously, as the value of the required collateral increases, the resulting wealth effects cause the economy to expand. The effects of the shock on borrowing are dampened owing to the automatic increase in the LTV rule which makes the collateral constraint tighter. As a result of the tighter collateral constraint, the increase in consumption and housing demand is moderate. Thus, we subsequently observe a moderate response of both output and inflation when the LTV rule is active (Fig. 2.1). In sum, our results show that the use of macroprudential policy along with monetary policy could help achieve price stabilisation.

2.5.3 Monetary policy shock

Next we examine the effects of monetary policy by allowing for a one standard deviation increase in interest rate (Fig. 2.2). Our results reveal that the effects of monetary policy are amplified in the absence of macroprudential rules. More to the point, an active macroprudential rule such as a decrease in the LTV ratio restricts borrowing by making credit availability tighter. In our example, an expansionary monetary policy (i.e. decrease in interest rate) in the absence of macroprudential rule encourages borrowing which subsequently leads to more consumption with output and house prices both increasing. Therefore the LTV rule helps to dampen asset prices bubbles and output booms that both might require greater tightening of monetary policy subsequently if left unchecked.

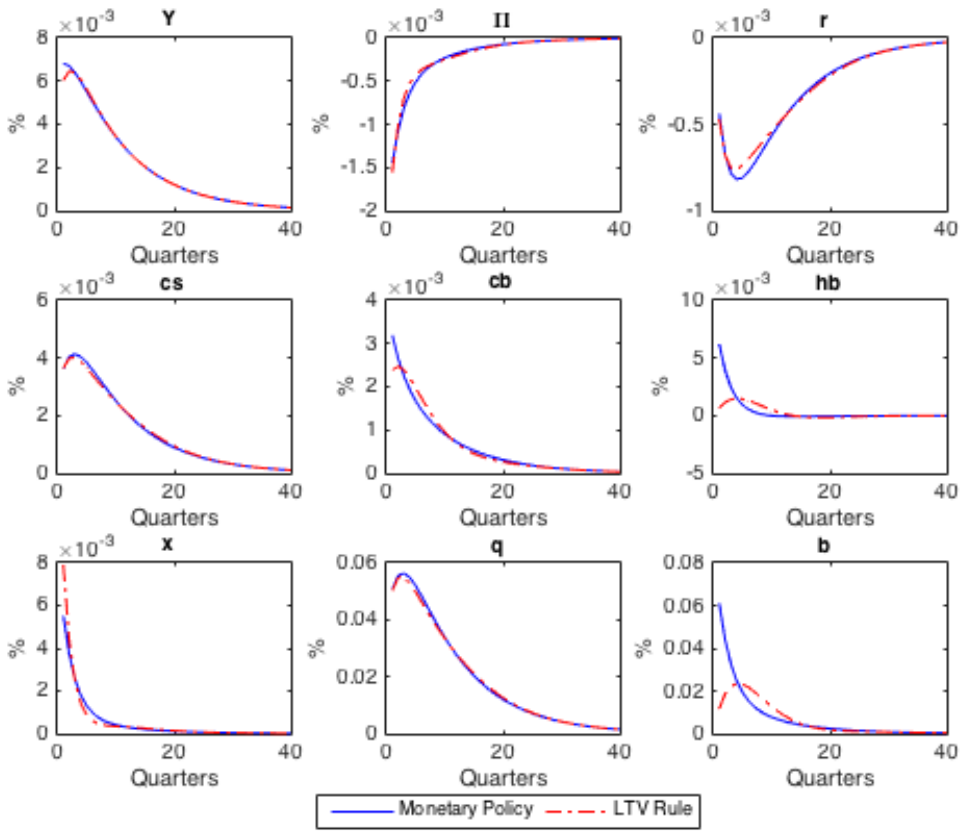


Figure 2.1: Impulse responses to the technology shock

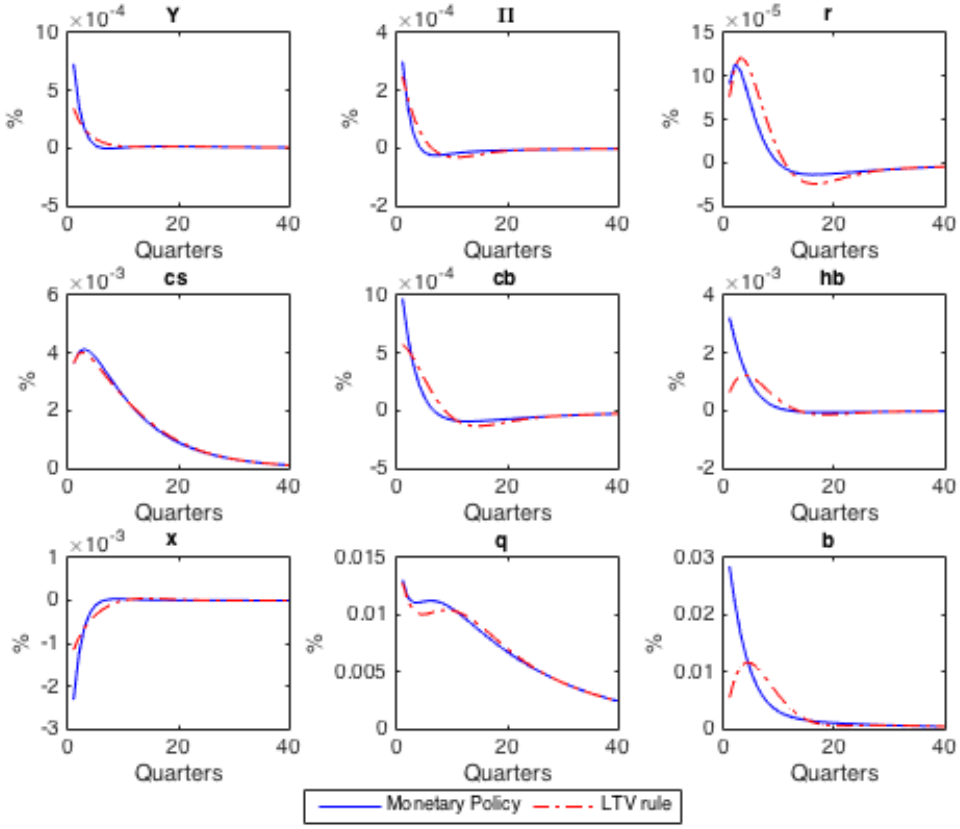


Figure 2.2: Impulse responses to the expansionary monetary policy shock

2.5.4 Housing demand shock

Our results show a moderate response from consumption, housing, borrowing and house prices following a 1% house demand shock. These results are largely dampened by the LTV policy. As stated in the previous section, the increase in house prices leads to a wealth effects in that borrowers can now afford to borrow more against their housing collateral. As more is consumed in a way of housing products and other consumption goods, overall output increases. In light of the latter, the macroprudential rule is activated, the LTV ratio decreases, and this subsequently restricts credit in the economy. In the presence of macroprudential rule, as we have seen in the previous case, we only observe moderate increases in both consumption and housing demand.

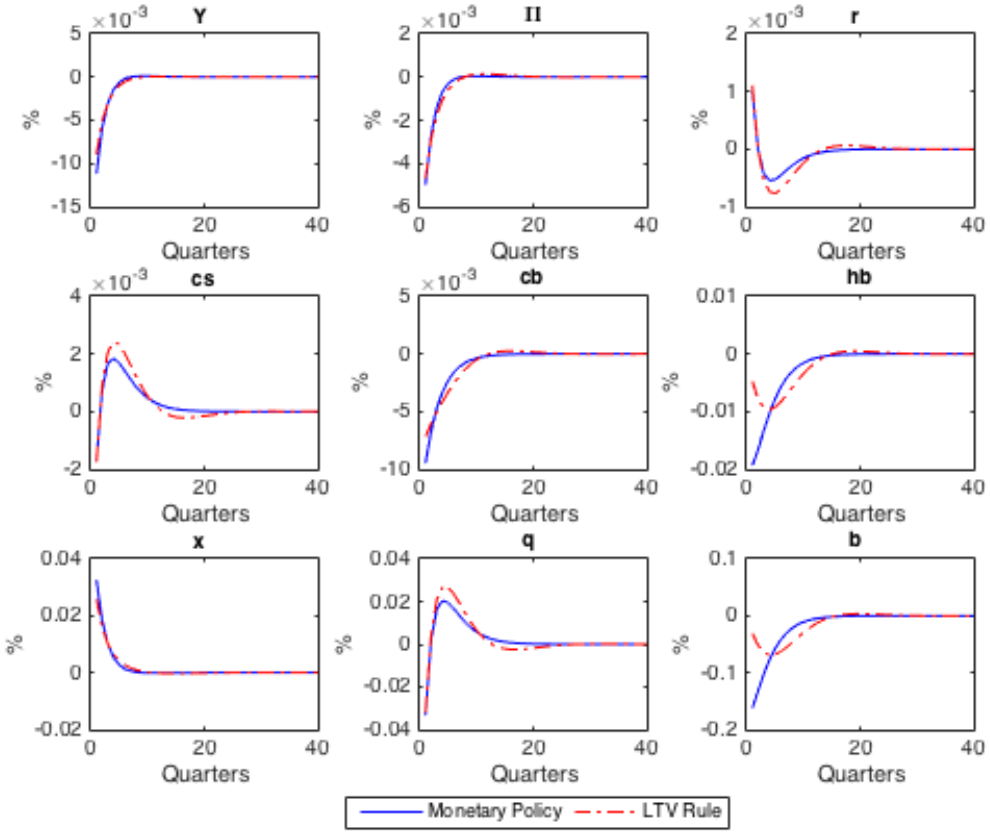


Figure 2.3: Impulse responses to the housing demand shock.

2.6 Efficient Equilibrium and Optimal policy

The model developed above features three types of distortions which often are present in financial NK models. We will now discuss in turn the implications of each of them separately. The first source of sub-optimality is the presence of market power in the intermediate goods sector. This distortion is independent of the presence of sticky prices, and it remains valid even under the flexible prices assumption. This is due to the fact that the demand for each firm's product is imperfectly elastic which leads to pricing above-marginal cost policies. The presence of markup distortions leads to an inefficiently low level of employment and thus output:

$$-\frac{U_{L,t}^{s,b}}{U_{c,t}^{s,b}} = w_t^{s,b} = \frac{1}{X} MPL_{s,b} < MPL_{s,b} \tag{2.38}$$

The second source of inefficiency relates to the assumption on the frequency of price adjustment which means that the average markup of the economy will change over time in response to shocks and will be different from its constant frictionless level.

The third inefficiency is related to the presence of financial frictions. Equation (2.9) implies that borrowers fail to optimally smooth their consumption path. Equation (2.11) implies that, because of borrowing constraint, borrowers tend to acquire inefficiently high level of housing as a measure of insurance against shocks in the economy. Therefore it will result in an inefficient risk sharing between agents of the model. The efficient allocation associated with the model economy can be pinned down by solving the problem of a benevolent social planner that maximize the aggregate welfare, given preferences and evolution of the technology. We weight individual utility streams by respective labour shares of individual households. In the benchmark model it can be shown that the labour share parameter incorporates the steady-state consumption to output ratio and labour mass of each household.

$$\alpha = \frac{C_s}{Y} L_s^{\eta} X \tag{2.39}$$

$$1 - \alpha = \frac{C_b}{Y} L_b^{\eta} X \tag{2.40}$$

We also assume that the social planner maximizes utility of existent and future generations of borrowers. With this assumption we can consider the common discount factor for both types of households (for proof please see Andres et. al. (2013)):

At each period, the optimal allocation needs to maximize following welfare criterion::

$$W = \alpha U_t^s + (1-\alpha)\, U_t^b = \alpha \left[\log C_t^s + j \log H_t^s - \frac{(L_t^s)^\eta}{\eta} \right] + (1-\alpha) \left[\log C_t^b + j \log H_t^b - \frac{(L_t^s)^b}{\eta} \right] \tag{2.41}$$

subject to the aggregate resource constraint,

$$C_t^s + C_t^b = A_t \left(L^s\right)^\alpha \left(L^b\right)^{1-\alpha} \tag{2.42}$$

and the equilibrium condition in housing market

$$H_t^s + H_t^b = 1 \tag{2.43}$$

First order conditions to this problem:

$$(1-\alpha)\, C_t^s = \alpha C_t^b \tag{2.44}$$

$$(1-\alpha)\, H_t^s = \alpha H_t^b \tag{2.45}$$

$$\frac{L_t^s}{L_t^b} = 1 \tag{2.46}$$

First order conditions imply that aggregate welfare maximizing requires consumption and housing ratios need to be equal to the ratio of respective production participation of the each household. Furthermore, both patient and impatient households will provide the same hours of labour in equilibrium.

2.6.1 Loss function

Once appropriately chosen subsidies and taxes are in place, the equilibrium allocation of a decentralized economy will correspond to the model's efficient allocation. To obtain the welfare theoretical loss function, we use the assumption of the efficient risk-sharing steady state. This is similar to the efficient steady-state assumption that Woodford (2003) used to derive optimal welfare based mandates for the central bank where the markup inefficiency is eliminated in the steady state by introducing equilibrium tax subsidies in a simple NK model. The definition of the efficient steady-state in the present framework differs from the definition that commonly referred in the literature. Explicitly, we allow for price distortions in the efficient steady-state, and our definition of efficiency relates to efficient risk-sharing in equilibrium between patient and inpatient households. The steady-state tax subsidies on markup give a purely quadratic objective function. This is important in fully optimal policies because properties of algorithms with quadratic objective and linear constraints are well-known. However, our main focus is on optimal simple rules and therefore we maintain linear terms in the objective function because optimization in simple rules does not require fully quadratic objective function. The efficient risk sharing assumption however, allows to tractably disentangle utility consequences of each distortions (price and financial), as well as to separate effects of individual policies (monetary and macroprudential) on the welfare. Therefore, we can decompose the overall welfare losses to its' individual constituents that approximate related inefficiencies. This allows to assign welfare based mandates to the macroprudential and monetary authorities that will address utility costs of inefficiencies related to their objectives. Regarding the efficient risk sharing steady-state, we assume that there is a fiscal authority that passively taxes resources from patient households and transfers them to borrowers so that the steady-state distribution of consumption is consistent with efficient steady-state assumption. This condition guarantees efficient risk sharing between savers and borrowers in the steady-state. Second, we assume that the housing stock of savers is taxed to equate the distribution of real estates between borrowers and savers. Government income and spending are assumed to be exogenous to the model. We make the following modifications to the baseline model:

$$C_t^s + q_t \left(1 + \tau^h\right) \left(H_t^s - H_{t-1}^s\right) + b_t \leq w_t^s L_t^s + \frac{R_{t-1} b_{t-1}}{\pi_t} + F_t - \Lambda^c \tag{2.47}$$

$$C_t^b + q_t \left(H_t^b - H_{t-1}^b\right) + \frac{R_{t-1} b_{t-1}}{\pi_t} \leq w_t^b L_t^b + b_t + \Lambda^c \tag{2.48}$$

Where equations (2.47) and (2.48) refer to savers' and borrowers' budget constraints, respectively. We can solve for Λ^c and τ^h that are consistent with efficient equilibrium assumption as follows:

Borrowers' budget constraint in the steady steady state could be written as:

$$\frac{\Lambda^c}{C^b} = 1 + (R-1)\frac{b}{C^b} - \frac{w^b L^b}{C^b} \tag{2.49}$$

Equations (2.10) and (2.20) give following expression:

$$\frac{w^b L^b}{C^b} = (1-\alpha)\frac{Y}{XC^b} \tag{2.50}$$

Imposing the efficient equilibrium assumption we have:

$$\frac{w^b L^b}{C^b} = \frac{1}{X} \tag{2.51}$$

Equations (2.8), (2.9), (2.10) solve for borrowing to consumptions ratio:

$$\frac{b}{C^b} = \left[\frac{R}{kj}\left[\left(1-\frac{k}{R}\right) - \widetilde{\beta}\,(1-k)\right]\right]^{-1} \tag{2.52}$$

Borrowers efficient steady state labour supply can be found from equation (2.10):

$$L_b = \left[\frac{w^b L^b}{C^b}\right]^{\frac{1}{\eta}} = \left[\frac{1}{X}\right]^{\frac{1}{\eta}} \tag{2.53}$$

Implying the following expression for the efficient steady state output:

$$Y^{EF} = \bar{A}\left[\frac{1}{X}\right]^{\frac{1}{\eta}} \tag{2.54}$$

The efficient steady state transfers becomes:

$$\Lambda^c = (1-\alpha)\,Y^{EF}\left[1 + \left(\frac{1-\beta}{\beta}\right)\left[\frac{R}{kj}\left[\left(1-\frac{k}{R}\right) - \widetilde{\beta}\,(1-k)\right]\right]^{-1} - \frac{1}{X}\right] \tag{2.55}$$

From equation (2.11) and equation (2.12) we can obtain the expression for the steady state house prices:

$$q = \frac{C^b j}{\left[1 - \widetilde{\beta} - \left(\beta - \widetilde{\beta}\right)k\right]H^b} \tag{2.56}$$

Savers optimality condition for housing in the steady state takes the following form:

$$\frac{jC^s}{H^s q} = (1-\beta)\left(1+\tau^h\right) \tag{2.57}$$

which together with equation (2.56) solves for the efficient steady state housing tax:

$$\tau^h = \frac{\left[1-\widetilde{\beta}-\left(\beta-\widetilde{\beta}\right)k\right]}{(1-\beta)} - 1 \tag{2.58}$$

A second order approximation to households welfare.

The main motivation of this paper is to derive the welfare theoretical mandates for the two policy-makers that our model features. Hence, the scope of the present precludes the provision of any positive or real-life policy prescriptions which, ultimately, is beyond the reach of most theoretical models, no matter their level of complexity. Thus, we will switch off all shocks other than the shock to the technology. To obtain a welfare theoretical loss function we assume that the efficient steady state transfers and taxes are in place. However, the concept of efficiency does not apply to markup distortions and only requires efficient risk sharing between patient and impatient households. Once we adopt the efficient steady state assumption (efficient risk sharing) welfare costs related to markup distortions, price adjustment distortions, and financial distortions that contribute to overall welfare loses are readily identified and isolated from each other.

Before presenting the loss function as a formal second order welfare ranking for alternative policies, we need to introduce a few concepts using as a reference framework the basic New Keynesian model. In the basic New Keynesian model the efficient output refers to the level of output under flexible prices with constant markup. Exogenous shocks to the economy create variations in markup and prices which lead to deviations in actual output and its efficient level. When prices are flexible, the efficient level of output becomes a function of the technology only, as in RBC models. Moreover, the output gap defined as the difference between output and its efficient counterpart is proportional to deviations in markup. Because of financial frictions in the model, however, we need to introduce a new definition for the efficient level of output. We define the efficient output as a level of output that would be, both, under flexible prices and efficient risk sharing between patient and impatient households:

$$Y^{EF} = \left[\frac{1}{X}\right]^{\frac{1}{\eta}} A_t \tag{2.59}$$

where the efficient level of output is the function of the technology only. We also need to make clear distinctions between two types of output gaps in the model economy.

The first concept of the output gap refers to the deviation of "efficient risk sharing output" from the output under both efficient risk sharing and flexible prices. This notion, essentially, applies to the output gap that would normally arise in financial frictionless economy. The second concept of the output gap represents the deviations in output from its "efficient risk sharing" counterpart. The first concept output gap in log deviations is proportional to log deviations in markup:

$$\hat{y}^{GAP1} \equiv \hat{y}_t^{ef} - \hat{y}_t^{ef_risk} = \frac{\hat{x}_t}{\eta} \tag{2.60}$$

The second form of the output gap emerges as a result of financial frictions in the economy:

$$\hat{y}^{GAP2} \equiv \hat{y}_t^{ef_risk} - \hat{y}_t = a_t + \frac{\hat{x}_t}{\eta} - \hat{y}_t$$

Under the efficient risk sharing assumption in the steady state, our model implies the following loss function approximated around distorted steady state (steady state with markup) :

$$\sum_{t=0}^{\infty} \beta^t \frac{W_t - W}{W_c C} \simeq -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t L + O^3 + t.i.p \tag{2.61}$$

$$L = \left[\begin{array}{c} 2\left[\hat{y}_t^{ef_risk} - \hat{y}\right] + \left(1 - \frac{1}{X}\right)\frac{2}{\eta}\hat{x}_t + \frac{1}{X}\frac{1}{\eta}\hat{x}_t^2 + \left(1 - \frac{1}{X}\frac{1}{\eta}\right)\frac{\varepsilon}{\kappa}\pi_t^2 + \\ +\alpha\left(1-\alpha\right)\left[c_t^s - c_t^b\right]^2 + j\left[\left(\frac{1 - 2\alpha + 2\alpha^2}{\alpha^2}\right)\left(\hat{h}_t^b\right)^2\right] \end{array} \right] \tag{2.62}$$

This loss function features four policy objectives where optimal policy should stabilize output gaps (the output gap due to the inefficient risk sharing, the output gap due to the price dispersion), inflation, consumption gap, and movements in borrowers collateral. The first two objectives are already present in NK models without financial distortions. When prices are sticky, and not all firms can adjust prices, inflation leads to inefficient dispersion of relative prices and production among individual firms. Because, the final goods firm is using a CES aggregation technology, employing all intermediate goods at the same amount in production is better than shifting from relatively more expensive goods to cheaper alternatives. Thus, inflation creates a reduction in aggregate utility. Whereas, the consumption gap and borrowers housing stock enter to the loss function because of the financial friction that our model features. The consumption gap is due to the inefficient risk sharing between savers and borrowers and it is a result of the collateral constraint of impatient households.

To check whether our approximation gives a correct welfare ranking we plot unconditional means of actual welfare and the negative loss function for set of policy vectors:

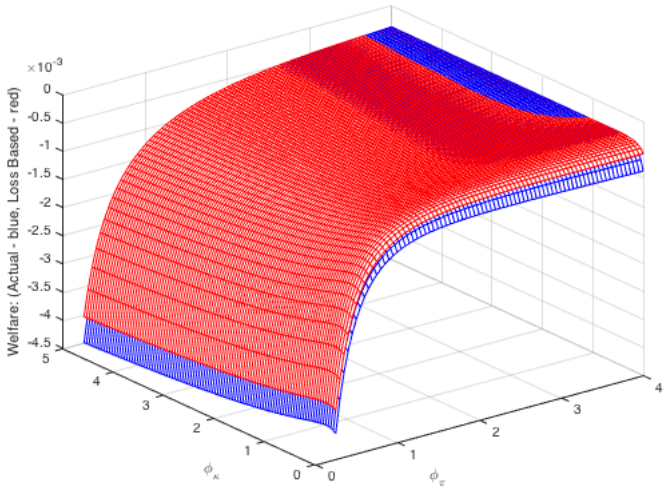


Figure 2.4: Actual Welfare and Negative Loss Function

Fig. 2.4 shows that the actual welfare that relies on the dynamics of the entire model's variables and our approximated loss function that relies on the dynamics of only few model's variables moves together for all combinations of policy vectors. Therefore it confirms that the loss function indeed does a decent job to mimic the actual welfare.

2.6.2 Optimal Policy Mix

The main focus of our paper is on welfare gains from cooperation. To analyse these gains we consider cooperative and non-cooperative equilibria. When two policymakers cooperate they jointly set their policy tools to minimize the aggregate loss function given by equation (2.61). However, under the non-cooperative setting we need to specify separate mandates for each institution. Coordination issues could be removed by assigning the same set of mandates for both authorities. In this case when the solution to the problem minimises the loss function of each player the outcome will be Pareto optimal and cooperation will overlap with non-cooperation equilibria, exactly because of the reason that both authorities would have no incentives to deviate from cooperation. In reality, however, due to accountability concerns each institution is often required to carry out more narrowly specified objectives.

For this reason, it is legitimate to think that each institution is assigned mandates to deal with particular inefficiencies in the economy, ignoring the rest. For instance, it might be natural to assume that the central bank is assigned to eliminate inefficiencies related to inflation and output gap, whereas the macroprudential authority is only concerned of inefficiencies arising from credit constraint. Accordingly, we can assume the following separation of mandates:

$$L^{CB} = \left(1 - \frac{1}{X}\right)\frac{2}{\eta}\hat{x}_t + \frac{1}{X}\frac{1}{\eta}\hat{x}_t^2 + \left(1 - \frac{1}{X}\frac{1}{\eta}\right)\frac{\varepsilon}{\kappa}\pi_t^2 \tag{2.63}$$

$$L^{MP} = 2\left[\hat{y}_t^{ef_risk} - \hat{y}\right] + \alpha\left(1-\alpha\right)\left[c_t^s - c_t^b\right]^2 + j\left[\left(\frac{1-2\alpha+2\alpha^2}{\alpha^2}\right)\left(\hat{h}_t^b\right)^2\right] \tag{2.64}$$

where L^{CB}, and L^{MP} refers to the loss functions of the central bank and the macropudential institution, respectively. This delegation of mandates seems plausible, at least for the reason that in traditional literature without financial frictions NK models concludes with similar objectives for the central bank, namely, stabilising output gap and inflation.

Our joint optimal policy problem picks optimized simple rule where the vector of instruments react to an observed subset of macroeconomics variables

$$\log i_t = \Gamma \log \Lambda_t \tag{2.65}$$

where $i_t \equiv [i_{1,t}, i_{2,t}, ..., i_{n,t}]'$ denotes the vector of n instruments. The matrix Γ selects a subset of variables Λ_t from which to feedback. The optimized simple rules then solves following maximization problem at $t = 0$ for given initial values of Z_0..

Table 2.3: Omptimized Parameters

	$1+\phi^{\pi}$	ϕ^{y}	ϕ^{b}	L^{CB}	L^{MP}	L	CE
[1]	5	1.1786	-	-	-	-0.01105	-
[2]	5	0	5	-0.00201701	-0.00180115	-0.00381817	0.2613
[3]	5	0	5	-0.00201701	-0.00180115	-0.00381817	0.2613
[1] Monetary policy only, [2] Coordinatied, [3] Non-coordinated							

$$\min_{i,\Gamma} \Omega_0(Z_0, i, \Gamma) \tag{2.66}$$

where the inter-temporal welfare loss at time t is defined in Bellman form:

$$\Omega_t = L_t + \beta E_t \left(\Omega_{t+1}\right) \tag{2.67}$$

We examine optimized welfare rules over ϕ_π, ϕ_y and ϕ_b where interest rate smoothing parameter is fixed at its' calibrated value.

We model non-cooperation as a Nash equilibrium to the non-coordinated game between the macroprudential institution and the Central Bank. In this set up the Central Bank optimizes over its own policy parameters taking the macroprudential policy as given. Similarly, macroprudential policy is set up taking into account the behaviour of the Central Bank. The intersection of two best responses will give the Nash equilibrium of the game. To solve the problem we use the grid search method together with the above described procedure. All policy parameters are constrained between 0 and 5.

Welfare comparisons of different policies are interpreted in terms of consumption equivalents as follows. Given particular equilibrium for c_t, h_t and l_t we compute the increase in the single period utility by 1% increase in consumption, keeping h_t and l_t constant. For 2 welfare outcomes (W_1 and W_2) we define the consumption equivalent as:

$$ce_t = \frac{W_1 - W_2}{W(1.01C, H, L) - W(C, H, L)} \tag{2.68}$$

Table (2.3) summarizes results of simulations for the technology shock.

A number of interesting points arise. First, including a macroprudential rule increases the welfare by around 0.26% consumption equivalents. Second, reacting to the output is not optimal when macroprudential regulation is in place. Furthermore, once

we separate mandates based on welfare theoretical loss functions the non-coordinated game picks the same set of optimal parameters as in the coordination case. Thus our findings suggest no welfare losses associated with non-coordinated policy making for the technology shock. In both situations it is optimal to react to the inflation and borrowing most aggressively.

To check the robustness of our results we also look at the Nash equilibrium with optimal simple rules that are extended to include smoothing parameters in both policy rules (ρ_b, ρ_r) for the optimization. The best response of the central bank in this set-up is a three-dimensional object, $\mathbb{R}_{\phi_b} \times \mathbb{R}_{\rho_b} \times \mathbb{R}_r$, where $\mathbb{R}_{\phi_b}$ are number of policy choices of the macroprudential institution for the lending growth (ϕ_b), $\mathbb{R}_{\phi_\rho}$ are number of policy choices of the macroprudential institution of the smoothing parameter, $\mathbb{R}_r$ are the number of parameters that the central bank optimizes. All parameter choices are defined in discretized intervals as in the previous example. Thus, the central bank's optimal responses are characterized $\mathbb{R}_{\phi_b} \times \mathbb{R}_{\phi_\rho}$ matrices in $\mathbb{R}_{\phi_\rho}$ "pages". Below we plot the central bank's best responses.

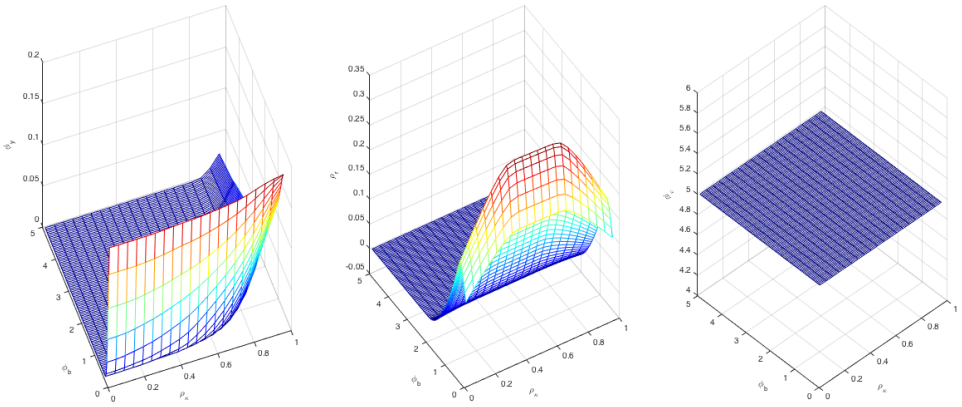


Figure 2.5: Best responses of the central bank

The left graph in fig. 2.5 corresponds to the output reaction in the interest rate rule. As it can be seen from this graph, the output response of the central bank is decreasing with the lending growth parameter of the macroprudential rule where without prudential smoothing it is not optimal for the central bank to respond to the output. Whereas, stronger smoothing of the macroprudential policy slightly increases the optimal output targeting for the central bank. Therefore, while LTV smoothing choices generate moderate optimal responses $(\phi^*_y = 0.2)$ together with no credit targeting, for high smoothing and credit parameters optimal output targeting drops to 0.02. The graph in the middle describes the optimal interest rate smoothing in response

to the macroprudential policy choices. As the graph shows optimal interest rate smoothing decreases with the lending response of the macroprudential rule. When this response is greater than 2 it is optimal not to smooth the interest rate policy. Whereas, smaller and larger LTV smoothing choices require weaker interest rate smoothing. For medium LTV smoothing, optimal interest rate smoothing responses are slightly above 0.3. Finally, the graph on the right side shows optimal inflation targeting of the central bank and it is independent of the policy choices of the macroprudential institution. Thus, it is optimal to respond to the inflation most aggressively regardless of the LTV policy choices.

Next, we analyze optimal responses of the macroprudential policymaker to the policy choices of the central bank. The best response of this policymaker is a four-dimensional vector, $\mathbb{R}_{\rho_r} \times \mathbb{R}_{\phi_y} \times \mathbb{R}_{\phi_\pi} \times \mathbb{R}_{\kappa}$, where the latter is the number of the choice parameters of the macroprudential institution. However, because it is always optimal for the central bank to choose the highest inflation targeting parameter irrespective of the policy parameters in the LTV rule we can reduce the dimensionality of the problem by fixing ${\phi^r}_\pi$ to its optimal value from the perspective of the central bank. Below we plot best responses of the financial regulator.

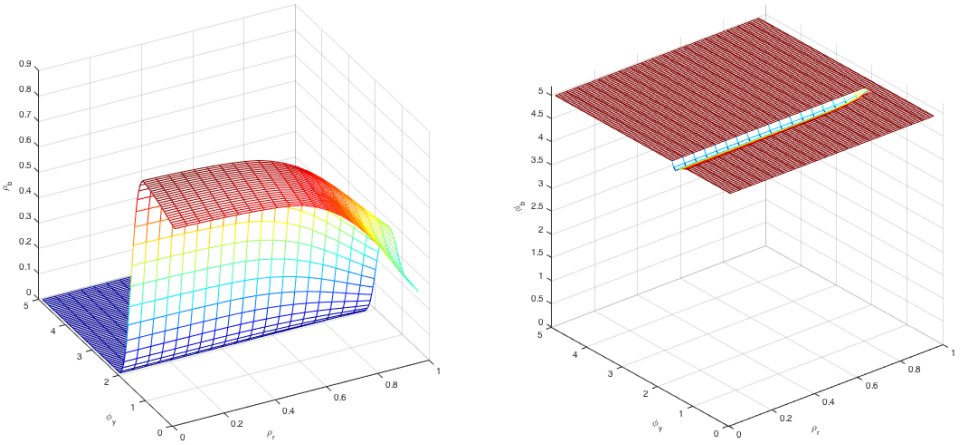


Figure 2.6: best responses of the macroprudential institution

The graph on the left presents optimal smoothing responses in the LTV rule against the interest rate smoothing and the output targeting choices of the monetary authority. First, for lower values of the output targeting it is optimal to choose high smoothing parameters in the LTV rule from the macroprudential perspective. However, as the output targeting gets higher LTV smoothing decreases and converges to zero. For the higher interest rate smoothing parameter the LTV smoothing also decreases but stays

above zero. The graph on the right presents optimal reactions to the lending growth. Similar to the inflation targeting in the interest rate rule, responses to the lending growth are independent of the policy choices of the other policymaker. This observation significantly reduces the complexity of the problem to find the Nash equilibrium of this game. First, as the optimal interest rate smoothing of the central bank is zero when the $\phi_b = 5$, we can eliminate all policy combinations with a positive interest rate smoothing parameter. Thereby, we can represent the solution in the two-dimensional space where all of the strategic interactions operate between the output targeting in the interest rate rule and the smoothing parameter in the LTV rule.

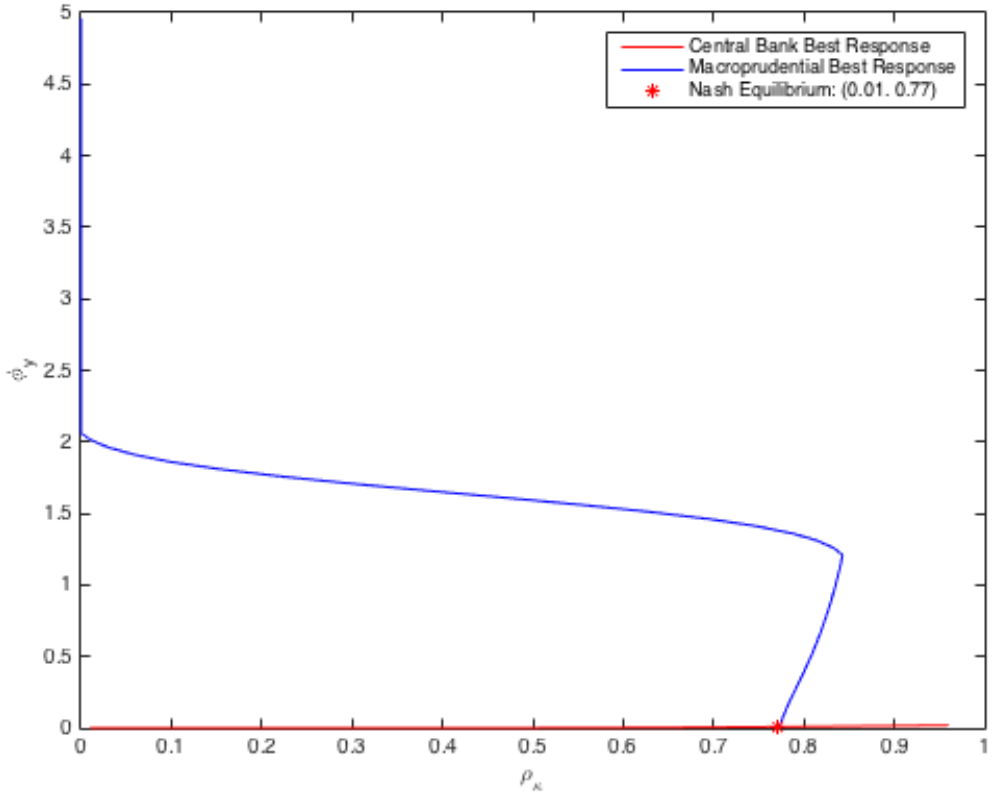


Figure 2.7: Nash Equilibrium

As it is shown on the fig. 2.7 the Nash equilibrium involves high LTV smoothing ($\rho_\kappa = 0.77$) and minimal feedback to the output gap ($\phi_y = 0.01$) by the central bank. However, it still involves most aggressive feedbacks to inflation and lending growth. In table 2.5 we summarize results of this exercise where we compare welfare outcomes relative to the optimal monetary policy (first row). Coordination implies a higher response to the overall output gap in the interest rate rule and also a slightly smoother LTV policy. Nonetheless, non-coordination results in negligible overall welfare losses

Table 2.4: Omptimized Parameters

	$1+\phi^{\pi}$	ϕ^{y}	ρ^{r}	ϕ^{b}	ρ^{κ}	L^{CB}	L^{MP}	L	CE
[1]	5	0.9390	0	-	-	-	-	-0.008222	-
[2]	5	0.3745	0	5	0.8589	-0.0016	-0.0012	-0.0029	0.2197
[3]	5	0 .01	0	5	0.77	-0.0004	-0.0036	-0.0041	-0.01
[1] Monetary policy only, [2] Coordinatied, [3] Non-coordinated									

(0.01% consumption equivalents) compared to the optimal monetary policy (the first row). Next we look payoffs of individual institutions. Although, non-coordination involves higher losses in the macroprudenial regulator's payoff, the central bank is better of not to cooperate with the macroprudential policy maker.

As a next experiment, we allow the macroprudential authority to optimally choose and react to the output gap along with the smoothing and lending parameters to all combinations of the central bank's policy choices. Similarly, the central bank chooses its' optimal reaction parameters in response to the combination of three policy parameters of the prudential policymaker. We begin our analyses by describing the best responses of the central bank. The central banks best response is a four-dimensional object, $\mathbb{R}_{\phi^{\kappa}{}_{y}} \times \mathbb{R}_{\phi_b} \times \mathbb{R}_{\rho_\kappa} \times \mathbb{R}_r$. I.e., they are $\mathbb{R}_{\phi_b} \times \mathbb{R}_{\rho_\kappa}$ matrix in $\mathbb{R}_{\phi^{\kappa}{}_{y}}$ books, and $\mathbb{R}_r$ pages. The latter refers to the number of parameters to optimize in the interest rate rule which is three in our case. For each of the central bank's choice parameter we have separate pages in $\mathbb{R}_{\phi^{\kappa}{}_{y}}$ number of books.[4] For the presentation purposes we only plot best responses for equally distanced "rough" points of the output parameter of the prudential authority ($\phi^{\kappa}{}_{y} = [0,1,2,3,4,5]$). As it is shown in fig. 2.8 when we allow the prudential policymaker to respond to the output gap the central banks best rest response for the interest rate smoothing converges to zero quickly for values of the $\phi^{\kappa}{}_{y} \geq 1$. The central banks best response for the output is zero for values of the $\phi^{\kappa}{}_{y} \geq 4$. Again central banks best response for the inflation is independent of the policy choices of the prudential policymaker.

We next describe best responses of the macroprudential authority. Using the above observation concerning the inflation parameter in the interest rule we can reduce the dimensionality of macropudential best responses by fixing this parameter at its optimal level from the perspective of the central bank. Fig. 2.9 plots results. The figure on the left is the macroprudential best response to the output gap. The optimal

[4]for each discrete choice of the output targeting by the macroprudential policymaker.

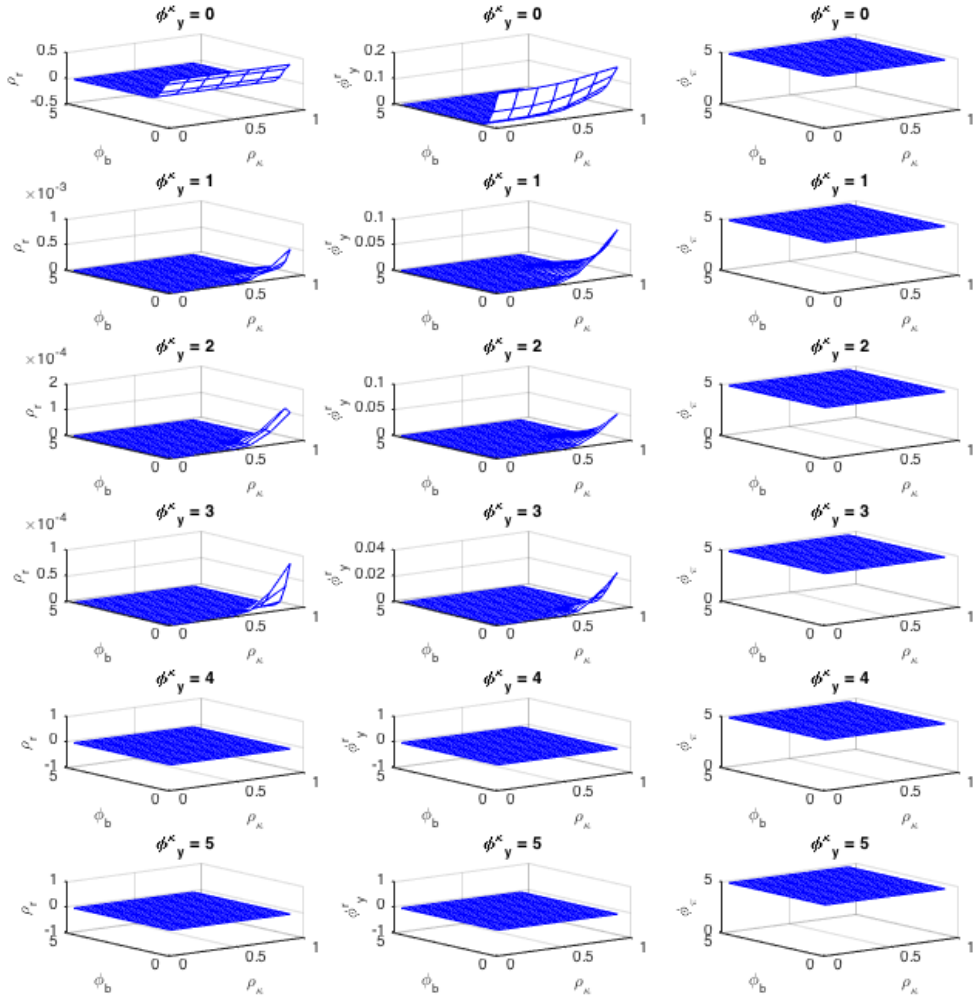


Figure 2.8: Best responses of the central bank

output targeting in the prudential rule approximately ranges between 4.7 and 5 and is decreasing with the output parameter of the central bank. However, the interest rate smoothing does not change its' value. The optimal LTV smoothing (figure in the center) varies approximately in a range of 0.9 and 0.99. This parameter is increasing with the output targeting parameter in the interest rate rule and has an inverse "U" shape in the domain of the interest rate smoothing. Finally, prudential response to the lending is about at its' maximum level of 5 irrespective of the monetary policy stance. This observation allows trivially pin down the equilibrium levels of the interest rate smoothing (ρ_r) and the interest rate response to the output gap (${\phi^r}_y$) from the figure 2.8. As it can be seen from the left and the center columns, both parameters are zero when $\phi_b = 5$. Therefore, the equilibrium prudential responses to the output gap and LTV smoothing can easily be determined from the figure 2.9 at $\phi_y = 0$ and $\rho_r = 0$.

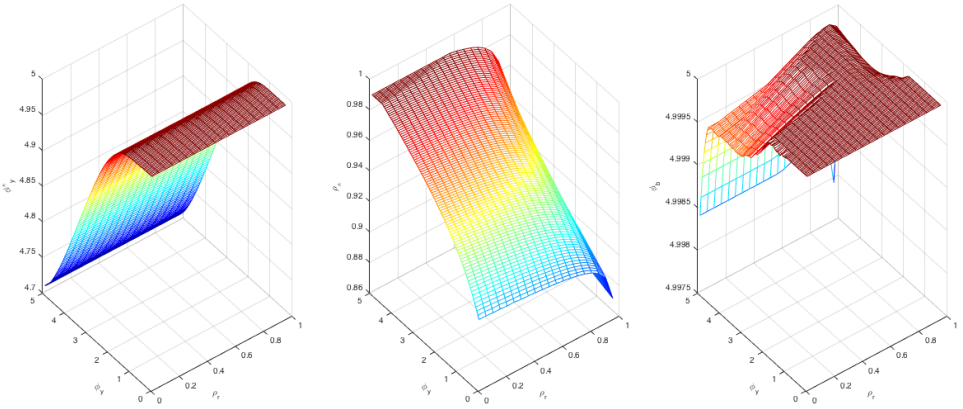


Figure 2.9: Best responses of the macroprudential authority

Table 2.5: Omptimized Parameters

	$1+\phi^{\pi}$	ϕ^{y}	ρ^{r}	ϕ^{b}	ρ^{κ}	${\phi^{\kappa}}_{y}$	L^{CB}	L^{MP}	L	CE
[1]	5	0.9390	0	-	-	-	-	-	-0.008222	-
[2]	5	0.3350	0	5	0.9327	5	-0.0012;	-0.0014	-0.0026	0.2397
[3]	5	0	0	5	0.9061	5	-0.007	-0.0029	-0.0036	0.03
[1] Monetary policy only, [2] Coordinatied, [3] Non-coordinated										

Both, coordination and non-cooperation, pick the same level of output targeting in the LTV rule. LTV smoothing parameters also have similar values (0.93 for the coordination, 0.91 for the Nash game). However, coordination prefers a moderate output targeting in the interest rate rule (0.335). Again, comparing individual losses we can conclude that the central bank is better off not to cooperate with the macroprudential institution. Differently from the previous example, non-coordination does not result in welfare losses compared to the optimal monetary policy by improving the aggregate welfare by 0.03% in consumption equivalents.

2.7 Conclusion

In this paper, we present a DSGE model and analyse the interactions between monetary and macroprudential policies whilst deriving a welfare loss function as in Andres et al (2013). Unlike the loss function that features in the canonical NK models, the loss function we propose, apart from inflation, subsumes the "consumption gap", two types of output gap, and movements in borrowers' collateral.

In the model economy the consumption gap arises from the inefficient risk sharing between households where the presence of financial frictions pushes the distribution of consumption from its' efficient level along the dynamic path. The output gap, however, is decomposed into two parts. Owing to the inability of some firms to reset their prices, the first part of the output gap measures the deviation of efficient risk sharing output from the output with the efficient risk sharing and constant mark-up. The second part of the output gap emerges as a result of financial frictions in the economy and measures the deviation of output from its efficient risk sharing level. A welfare-based loss function allows us to assign theoretically founded mandates for both policy institutions. To analyze the optimal policy mix we assume that each policy maker is delegated with mandates to deal with particular type of inefficiencies, ceteris paribus. Our results suggest that macroprudential policy (LTV rule) is welfare enhancing. Moreover, when we assign welfare based theoretical mandates to policy institutions, non-coordination does not bear any welfare losses in the case of the technology shock.

Among our key results is the finding that macroprudential tools lessen the effects of bubbles in the economy by controlling the credit market through the LTV ratio. Furthermore, our study also reveals that the effects of monetary policy are amplified in the absence of macroprudential rules. More to the point, an expansionary monetary policy (i.e. decrease in interest rate), in the absence of macroprudential rule, encourages borrowing which subsequently leads to more consumption with output and house prices both increasing.

Although the primary aim of this study is not to provide policy recommendations, from a normative standpoint, our results unambiguously suggests that macroprudential and monetary policies acting together can potentially enhance the stability of the economic system.

In terms of future avenues of research, the model proposed here can be easily extended to look at a "two country model" with a single monetary authority and separate financial regulators. As we have done in the present, policy coordination issues among national financial regulatory institutions and the central bank can then be explored. An equally interesting extension would be to look at sequential equilibria

of the optimal policy making game.

2.8 Appendix

Appendix 1

Steady states:

$$R = 1/\beta \tag{2.69}$$

$$\frac{jC^s}{H^s q} = (1-\beta) \tag{2.70}$$

$$1 + \frac{b}{C^b}\left(\frac{1-\beta}{\beta}\right) = \frac{w^b L^b}{C^b} \tag{2.71}$$

$$\frac{C^b}{b} = \frac{R}{kj}\left[\left(1-\frac{k}{R}\right) - \widetilde{\beta}\,(1-k)\right] \tag{2.72}$$

$$\frac{w^b L^b}{C^b} = 1 + \frac{jk\,(1-\beta)}{\left[1-\beta k - \widetilde{\beta}\,(1-k)\right]} \tag{2.73}$$

$$\lambda = \frac{1}{C^b}\left(\beta - \widetilde{\beta}\right) \tag{2.74}$$

$$L^b = \left[\frac{w^b L^b}{C^b}\right]\frac{1}{\eta} \tag{2.75}$$

$$\frac{Y}{C_b} = \frac{X}{(1-\alpha)}\frac{w^b L^b}{C_b} \tag{2.76}$$

$$\frac{C_s}{C_b} = -1 + \frac{Y}{C_b} \tag{2.77}$$

$$\frac{w^s L^s}{C^s} = \frac{\alpha}{X}\frac{Y}{C^s} \tag{2.78}$$

$$L^s = \left[\frac{w^s L^s}{C^s}\right]\frac{1}{\eta} \tag{2.79}$$

$$Y = A\,(L^s)^{\alpha}\left(L^b\right)^{1-\alpha}$$

$$C_s = Y - C_b \tag{2.80}$$

$$b = \frac{C^b \beta}{1-\beta}\left(\frac{w^b L^b}{C^b} - 1\right) \tag{2.81}$$

$$H^s = \left[\frac{(1-\beta)}{\left(1-\frac{k}{R}\right) - \widetilde{\beta}\,(1-k)}\frac{C^b}{C^s} + 1\right]^{-1} \tag{2.82}$$

$$H^b = 1 - H^s \tag{2.83}$$

$$q = \frac{jC^s}{H^s(1-\beta)} \tag{2.84}$$

Appendix 2

Non Linear model:

$$\frac{1}{C_t^s} = \beta E_t \frac{R_t}{C_{t+1}^s \pi_{t+1}} \tag{2.85}$$

$$w_t^s = C_t^s \left(L_t^s\right)^{\eta-1} \tag{2.86}$$

$$\frac{j_t}{H_t^s} = \frac{1}{C_t^s} q_t - \beta E_t \frac{1}{C_{t+1}^s} q_{t+1} \tag{2.87}$$

$$C_t^b + q_t\left(H_t^b - H_{t-1}^b\right) + \frac{R_{t-1} b_{t-1}}{\pi_t} = w_t^b L_t^b + b_t; \tag{2.88}$$

$$b_t = \frac{k_t}{R_t} E_t \pi_{t+1} q_{t+1} H_t^b \tag{2.89}$$

$$\frac{1}{C_t^b} = \widetilde{\beta} E_t \frac{1}{C_{t+1}^b} \frac{R_t}{\pi_{t+1}} + \lambda_t R_t \tag{2.90}$$

$$w_t^b = C_t^b \left(L_t^b\right)^{\eta-1} \tag{2.91}$$

$$\frac{j_t}{H_t^b} = \frac{1}{C_t^b} q_t - \widetilde{\beta} E_t \frac{1}{C_{t+1}^b} q_{t+1} - \lambda_t k_t E_t\left(\pi_{t+1} q_{t+1}\right) \tag{2.92}$$

$$Y_t = A_t \left(L_t^s\right)^{\alpha} \left(L_t^s\right)^{1-\alpha} \tag{2.93}$$

$$w_t^s = \frac{1}{X_t} \alpha \frac{Y_t}{L_t^s} \tag{2.94}$$

$$w_t^b = \frac{1}{X_t}(1-\alpha)\frac{Y_t}{L_t^b} \tag{2.95}$$

$$\frac{P_t^*}{P_t} = \frac{\lambda_t^1}{\lambda_t^2} \tag{2.96}$$

$$\lambda_t^2 - \theta\beta E_t\left[\pi_{t+1}^{\varepsilon-1}\lambda_{t+1}^2\right] = Y_t\frac{1}{C_t^s} \tag{2.97}$$

$$\lambda_t^1 - \theta\beta E_t\left[\pi_{t+1}^{\varepsilon}\lambda_{t+1}^1\right] = \frac{1}{1-\frac{1}{\varepsilon}}Y_t\frac{1}{C_t^s}\frac{1}{X_t} \tag{2.98}$$

$$1 = \theta\pi_t^{\varepsilon-1} + (1-\theta)\left(\frac{\lambda_t^1}{\lambda_t^2}\right)^{1-\varepsilon} \tag{2.99}$$

$$\Delta_t = \theta\pi_t^{\varepsilon}\Delta_{t-1} + (1-\theta)\left(\frac{\lambda_t^1}{\lambda_t^2}\right)^{-\varepsilon} \tag{2.100}$$

$$\frac{Y_t}{\Delta_t} = C_t^s + C_t^b \tag{2.101}$$

$$H_t^s + H_t^b = 1 \tag{2.102}$$

$$r_t = r_{t-1}^{\rho}\left[\pi_t^{(1+\varphi_\pi)}\left(\frac{Y_t}{Y_{t-1}}\right)^{(1+\varphi_y)}(\bar{r})\right]^{(1-\rho)}M_t \tag{2.103}$$

$$k_t = k\left[\frac{b_t}{b_{t-1}}\right]^{\varphi_k} \tag{2.104}$$

$$M_t = \exp(\sigma_M * \epsilon_t^M) \tag{2.105}$$

$$A_t = A^{(1-\rho_A)} * A_{t-1}^{\rho_A} * \exp(\sigma_A * \epsilon_t^A)$$

$$J_t = J^{(1-\rho_J)} * J_{t-1}^{\rho_J} * \exp(\sigma_J * \epsilon_t^J) \tag{2.106}$$

Appendix 3

3.1 F.O.C. for the Calvo contract:

At each period a representative firm sets its price optimally with probability $1-\theta$.. Each period this probability is independent from the previous period. Assuming the law of large numbers the average price level will be a CES aggregate of all prices in the economy and will take the following form:

$$\Upsilon_t = E_t\left[\sum_{k=0}^{t}\beta^k\Theta_{t,t+k}Y_{j,t+k}\right]$$

$$= \Theta_{t,t}Y_t + E_t\left[\sum_{k=1}^{\infty}\beta^k_{t,t+k}\Theta Y_{t+k}\right]$$

$$= Y_t + E_t\left[\sum_{k'=0}^{\infty}\beta^{k'+1}_{t,t+k'+1}\Theta Y_{t+k'+1}\right]$$

$$= Y_t + \beta E_t\left[\sum_{k'=0}^{\infty}\beta^{k'}_{t,t+1\Theta t+1,t+k'+1}\Theta Y_{t+k'+1}\right]$$

$$= Y_t + \beta E_t\left[\Theta_{t,t+1}\Upsilon_{t+k}\right]$$

3.2 Non-linear price dynamics:

In each period, θ share of firm will keep their old prices, and $(1-\theta)$ will update their prices to P_t^* By the law large numbers, we can assume that the distribution of prices among the firms that do not update their prices is the same as the overall distribution in previous period $(t-1)$. Then we can write:

$$\Delta_{t+1} = \theta\sum_{\text{no change, } j}\left(\frac{P_t(j)}{P_{t+1}}\right)^{-\varepsilon} + (1-\theta)\left(\frac{\lambda^1_{t+1}}{\lambda^2_{t+1}}\right)^{-\varepsilon}$$

$$= \theta\left(\frac{P_t}{P_{t+1}}\right)^{-\varepsilon}\sum_{\text{no change, } j}\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} + (1-\theta)\left(\frac{\lambda^1_{t+1}}{\lambda^2_{t+1}}\right)^{-\varepsilon}$$

$$= \theta\left(\frac{P_t}{P_{t+1}}\right)^{-\varepsilon}\sum_{j}\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} + (1-\theta)\left(\frac{\lambda^1_{t+1}}{\lambda^2_{t+1}}\right)^{-\varepsilon}$$

$$= \theta\pi^{\varepsilon}_{t+1}\Delta_t + (1-\theta)\left(\frac{\lambda^1_{t+1}}{\lambda^2_{t+1}}\right)^{-\varepsilon}$$

Appendix 4

Loss function:

$$\sum_{t=0}^{\infty}\beta^t\frac{W_t-W}{W_cC}\simeq-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\begin{array}{c}2\left[\hat{y}_t^{ef_risk}-\hat{y}\right]+\left(1-\frac{1}{X}\right)\frac{2}{\eta}\hat{x}_t+\frac{1}{X}\frac{1}{\eta}\hat{x}_t^2+\left(1-\frac{1}{X}\frac{1}{\eta}\right)\frac{\varepsilon}{\kappa}\pi_t^2+\\+\alpha\left(1-\alpha\right)\left[c_t^s-c_t^b\right]^2+j\left[\left(\frac{1-2\alpha+2\alpha^2}{\alpha^2}\right)\left(\hat{h}_t^b\right)^2\right]\\+O^3+t.i.p.\end{array}\right] \tag{2.107}$$

Derivation of the loss function (distorted case):

We use following second order approximation of relative deviations in terms of log deviations:

$$\frac{Z_t-Z}{Z}\simeq\hat{z}_t+\frac{1}{2}\hat{z}_t^2 \tag{2.108}$$

where $\hat{z}_t\equiv z_t-z$ is the log deviation from the steady state for any generic z_t. Applying this to equation (2.41) allows for following second order approximation of the aggregate welfare criterion:

$$W_t-W\simeq\alpha\left(\hat{c}_t^s+J\hat{h}_t^s+U_L^sL^s\left[\hat{l}_t^s+\frac{\eta}{2}\left(\hat{l}_t^s\right)^2\right]\right)+(1-\alpha)\left(\hat{c}_t^b+J\hat{h}_t^b+U_L^bL^b\left[\hat{l}_t^b+\frac{\eta}{2}\left(\hat{l}_t^b\right)^2\right]\right)+\\+O^3+t.i.p. \tag{2.109}$$

where O^3 are terms higher than order two, and $t.i.p.$ denotes terms independent of the policy.

The efficient steady state assumption implies:

$$U_L^sL^s=U_L^bL^b=-\frac{1}{X} \tag{2.110}$$

and we can write equation(2.109)

$$W_t-W\simeq\alpha\left(\hat{c}_t^s+j\hat{h}_t^s-\frac{1}{X}\left[\hat{l}_t^s+\frac{\eta}{2}\left(\hat{l}_t^s\right)^2\right]\right)+(1-\alpha)\left(\hat{c}_t^b+j\hat{h}_t^b-\frac{1}{X}\left[\hat{l}_t^b+\frac{\eta}{2}\left(\hat{l}_t^b\right)^2\right]\right)+O^3+t.i.p. \tag{2.111}$$

The aggregate resource constraint in goods market is given by:

$$\frac{Y_t}{\Delta_t}=C_t^s+C_t^b \tag{2.112}$$

The second order approximation to this:

$$\hat{y}_t + \frac{1}{2}\hat{y}_t^2 - \Delta_t = \frac{C^s}{Y}\left(\hat{c}_t^s + \frac{1}{2}\left(\hat{c}_t^s\right)^2\right) + \frac{C^b}{Y}\left(\hat{c}_t^b + \frac{1}{2}\left(\hat{c}_t^b\right)^2\right) + O^3 \tag{2.113}$$

where Δ_t- is already a second order term, and terms involving its' products and powers are dropped.

If we raise to the power two of both sides of the above equation we get next steps:

$$\left(\hat{y}_t + \frac{1}{2}\hat{y}_t^2 - \Delta_t\right)^2 = \left[\frac{C^s}{Y}\left(\hat{c}_t^s + \frac{1}{2}\left(\hat{c}_t^s\right)^2\right) + \frac{C^b}{Y}\left(\hat{c}_t^b + \frac{1}{2}\left(\hat{c}_t^b\right)^2\right)\right]^2 + O^3 \Rightarrow$$

$$\left(\hat{y}_t^2 + \hat{y}_t\hat{y}_t^2 + \frac{1}{4}\hat{y}_t^4\right) - 2\left(\hat{y}_t + \frac{1}{2}\hat{y}_t^2\right)\Delta_t + \Delta_t^2 = \left[\frac{C^s}{Y}\left(\hat{c}_t^s + \frac{1}{2}\left(\hat{c}_t^s\right)^2\right) + \frac{C^b}{Y}\left(\hat{c}_t^b + \frac{1}{2}\left(\hat{c}_t^b\right)^2\right)\right]^2 + O^3 \tag{2.114}$$

Where we dropped all third and higher order terms. Since Δ_t is already a second order term its product with all other first order terms will be in third order and can be dropped. Thus we can simplify it further:

$$\hat{y}_t^2 = \left(\frac{C^s}{Y}\right)^2\left(\hat{c}_t^s\right)^2 + \left(\frac{C^b}{Y}\right)^2\left(\hat{c}_t^b\right)^2 + 2\frac{C^s}{Y}\frac{C^b}{Y}\hat{c}_t^s\hat{c}_t^b + O^3 \tag{2.115}$$

We use this expression to substitute for $\hat{y}_t^2$ in the second order approximation of the aggregate resource constraint:

$$\hat{y}_t = \frac{C^s}{Y}\hat{c}_t^s + \frac{C^b}{Y}\hat{c}_t^b + \Delta_t + \frac{1}{2}\left[\frac{C^s}{Y}\left(1 - \frac{C^s}{Y}\right)\left(\hat{c}_t^s\right)^2 + \frac{C^b}{Y}\left(1 - \frac{C^b}{Y}\right)\left(\hat{c}_t^b\right)^2 - 2\frac{C^s}{Y}\frac{C^b}{Y}\hat{c}_t^s\hat{c}_t^b\right] + O^3 \tag{2.116}$$

We now, use the efficient equilibrium steady state $\frac{C_{ss}^s}{Y_{ss}} = \alpha; \frac{C_{ss}^b}{Y_{ss}} = (1-\alpha)$ and get following expression:

$$\alpha\hat{c}_t^s + (1-\alpha)\hat{c}_t^b = \hat{y}_t - \Delta_t - \frac{\alpha(1-\alpha)}{2}\left[c_t^s - c_t^b\right]^2 + O^3$$

Thus the welfare function can be written as:

$$\begin{aligned} W_t - W \simeq \hat{y}_t - \Delta_t - \frac{\alpha(1-\alpha)}{2}\left[c_t^s - c_t^b\right]^2 + \alpha\left(j\hat{h}_t^s - \frac{1}{X}\left[\hat{l}_t^s + \frac{\eta}{2}\left(\hat{l}_t^s\right)^2\right]\right) + \\ + (1-\alpha)\left(j\hat{h}_t^b - \frac{1}{X}\left[\hat{l}_t^b + \frac{\eta}{2}\left(\hat{l}_t^b\right)^2\right]\right) + O^3 + t.i.p. \end{aligned} \tag{2.117}$$

Equations (2.10), (2.4), (2.33) imply following expression:

$$\frac{X_t}{\Delta_t} = \alpha \left(L_t^s\right)^{-\eta} + (1-\alpha)\left(L_t^b\right)^{-\eta} \tag{2.118}$$

We take the second order approximation to this equation:

$$\hat{x}_t + \frac{1}{2}\hat{x}_t^2 - \Delta_t = \frac{-\eta a}{(L^s)^\eta X}\left[\hat{l}_t^s - \frac{\eta}{2}\left(\hat{l}_t^s\right)^2\right] + \frac{-\eta(1-a)}{(L^b)^\eta X}\left[\hat{l}_t^b - \frac{\eta}{2}\left(\hat{l}_t^b\right)^2\right] \tag{2.119}$$

Using the efficiency of the steady we can simplify the above equation as following:

$$\hat{x}_t + \frac{1}{2}\hat{x}_t^2 - \Delta_t = -a\left[\hat{l}_t^s - \frac{\eta}{2}\left(\hat{l}_t^s\right)^2\right] - \eta(1-a)\left[\hat{l}_t^b - \frac{\eta}{2}\left(\hat{l}_t^b\right)^2\right] \tag{2.120}$$

Rearranging this gives:

$$\alpha\left[\frac{\eta}{2}\left(\hat{l}_t^s\right)^2\right] + (1-a)\left[\frac{\eta}{2}\left(\hat{l}_t^b\right)^2\right] = \frac{1}{\eta}\left(\hat{x}_t + \frac{1}{2}\hat{x}_t^2 - \Delta_t\right) + \hat{y}_t - a_t \tag{2.121}$$

We now replace the LHS of equation (2.121) in the welfare function:

$$\begin{aligned} W_t - W \simeq \hat{y}_t - 2\frac{\hat{y}_t - a_t}{X} - \frac{1}{X}\frac{1}{\eta}\hat{x}_t - \frac{1}{X}\frac{1}{\eta}\frac{1}{2}\hat{x}_t^2 - \left(1 - \frac{1}{X}\frac{1}{\eta}\right)\Delta_t - \\ - \frac{\alpha(1-\alpha)}{2}\left[c_t^s - c_t^b\right]^2 + \alpha j \hat{h}_t^s + j(1-\alpha)\hat{h}_t^b + O^3 + t.i.p. \end{aligned} \tag{2.122}$$

$$\begin{aligned} W_t - W \simeq \hat{y}_t - \frac{\hat{y}_t - a_t}{X} - \frac{1}{X}\left[\hat{y}_t - a_t + \frac{1}{\eta}\hat{x}_t\right] - \frac{1}{X}\frac{1}{\eta}\frac{1}{2}\hat{x}_t^2 - \left(1 - \frac{1}{X}\frac{1}{\eta}\right)\Delta_t - \\ - \frac{\alpha(1-\alpha)}{2}\left[c_t^s - c_t^b\right]^2 + \alpha j \hat{h}_t^s + j(1-\alpha)\hat{h}_t^b + O^3 + t.i.p. \end{aligned} \tag{2.123}$$

Using the following relation

$$\hat{y} - \hat{y}_t^{ef_risk} = a_t + \alpha\hat{l}_t^s + (1-\alpha)\hat{l}_t^b - \left(a_t - \frac{1}{\eta}\hat{x}_t\right) = \hat{y}_t - a_t + \frac{1}{\eta}\hat{x}_t$$

we can rewrite the welfare criterion as

$$\begin{aligned} W_t - W \simeq \hat{y}_t - \frac{\hat{y}_t - a_t}{X} - \frac{1}{X}\left[\hat{y} - \hat{y}_t^{ef_risk}\right] - \frac{1}{X}\frac{1}{\eta}\frac{1}{2}\hat{x}_t^2 - \left(1 - \frac{1}{X}\frac{1}{\eta}\right)\Delta_t - \\ - \frac{\alpha(1-\alpha)}{2}\left[c_t^s - c_t^b\right]^2 + \alpha j \hat{h}_t^s + j(1-\alpha)\hat{h}_t^b + O^3 + t.i.p. \end{aligned} \tag{2.124}$$

$$W_t - W \simeq \left(1-\frac{1}{X}\right)\left(y_t - a_t + \frac{1}{\eta}\hat{x}_t - \frac{1}{\eta}\hat{x}_t\right) - \frac{1}{X}\left[\hat{y} - \hat{y}_t^{ef_risk}\right] - \frac{1}{X}\frac{1}{\eta}\frac{1}{2}\hat{x}_{tt}^2 -$$
$$- \left(1-\frac{1}{X}\frac{1}{\eta}\right)\Delta - \frac{\alpha\left(1-\alpha\right)}{2}\left[c_t^s - c_t^b\right]^2 + \alpha j\hat{h}_t^s + j\left(1-\alpha\right)\hat{h}_t^b + O^3 + t.i.p. \tag{2.125}$$

$$W_t - W \simeq \left(1-\frac{1}{X}\right)\left(\hat{y} - \hat{y}_t^{ef_risk} - \frac{1}{\eta}\hat{x}_t\right) - \frac{1}{X}\left[\hat{y} - \hat{y}_t^{ef_risk}\right] - \frac{1}{X}\frac{1}{\eta}\frac{1}{2}\hat{x}_t^2 -$$
$$- \left(1-\frac{1}{X}\frac{1}{\eta}\right)\Delta_t - \frac{\alpha\left(1-\alpha\right)}{2}\left[c_t^s - c_t^b\right]^2 + \alpha j\hat{h}_t^s + j\left(1-\alpha\right)\hat{h}_t^b + O^3 + t.i.p. \tag{2.126}$$

$$W_t - W \simeq \left(1-\frac{1}{X}\right)\left(\hat{y} - \hat{y}_t^{ef_risk}\right) - \frac{1}{X}\left[\hat{y} - \hat{y}_t^{ef_risk}\right] - \left(1-\frac{1}{X}\right)\frac{1}{\eta}\hat{x}_t - \frac{1}{X}\frac{1}{\eta}\frac{1}{2}\hat{x}_t^2 -$$
$$- \left(1-\frac{1}{X}\frac{1}{\eta}\right)\Delta_t - \frac{\alpha\left(1-\alpha\right)}{2}\left[c_t^s - c_t^b\right]^2 + \alpha j\hat{h}_t^s + j\left(1-\alpha\right)\hat{h}_t^b + O^3 + t.i.p. \tag{2.127}$$

Using equation (2.60) and the definition of efficient output we can rewrite the output gap in terms of mark up:

$$W_t - W \simeq -\left[\hat{y}_t^{ef_risk} - \hat{y}\right] - \left(1-\frac{1}{X}\right)\frac{1}{\eta}\hat{x}_t - \frac{1}{X}\frac{1}{\eta}\frac{1}{2}\hat{x}_t^2 - \left(1-\frac{1}{X}\frac{1}{\eta}\right)\Delta_t -$$
$$- \frac{\alpha\left(1-\alpha\right)}{2}\left[c_t^s - c_t^b\right]^2 + \alpha j\hat{h}_t^s + j\left(1-\alpha\right)\hat{h}_t^b + O^3 + t.i.p. \tag{2.128}$$

The second order approximation to the aggregate resource constraint in the housing market gives us following expression:

$$H_{ss}^s\left(\hat{h}_t^s + \frac{\left(\hat{h}_t^s\right)^2}{2}\right) + H_{ss}^b\left(\hat{h}_t^b + \frac{\left(\hat{h}_t^b\right)^2}{2}\right) = O^3 \tag{2.129}$$

We now replace the corresponding efficient steady state values for H_{ss}^s,and H_{ss}^b:

$$a\left(\hat{h}_t^s + \frac{\left(\hat{h}_t^s\right)^2}{2}\right) + \left(1-\alpha\right)\left(\hat{h}_t^b + \frac{\left(\hat{h}_t^b\right)^2}{2}\right) = O^3 \Rightarrow$$
$$a\hat{h}_t^s + \left(1-\alpha\right)\hat{h}_t^b = O^3 - \frac{\left(\hat{h}_t^s\right)^2}{2} - \frac{\left(\hat{h}_t^b\right)^2}{2} \tag{2.130}$$

Using the first order log-linear approximated relationship between $\hat{h}_t^b$ and $\hat{h}_t^s$:

$$\hat{h}_t^s = -\frac{H^b}{H^s}(\hat{h}_t^b) \tag{2.131}$$

we can obtain following equation:

$$a\hat{h}_t^s + (1-\alpha)\,\hat{h}_t^b = O^3 - \frac{1}{2}\left[\left(\frac{1-2\alpha+2\alpha^2}{\alpha^2}\right)\left(\hat{h}_t^b\right)^2\right] \tag{2.132}$$

$$W_t - W \simeq -\left[\hat{y}_t^{ef_risk} - \hat{y}\right] - \left(1-\frac{1}{X}\right)\frac{1}{\eta}\hat{x}_t - \frac{1}{X}\frac{1}{\eta}\frac{1}{2}\hat{x}_t^2 - \left(1-\frac{1}{X}\frac{1}{\eta}\right)\Delta_t - \tag{2.133}$$

$$-\frac{\alpha\,(1-\alpha)}{2}\left[c_t^s - c_t^b\right]^2 - \frac{1}{2}j\left[\left(\frac{1-2\alpha+2\alpha^2}{\alpha^2}\right)\left(\hat{h}_t^b\right)^2\right] + O^3 + t.i.p. \tag{2.134}$$

It is possible to show that (see Woodford,2003):

$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{1}{2}\frac{\varepsilon\theta}{(1-\theta)\,(1-\theta\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 \tag{2.135}$$

Using equation (2.135) finally we can write the aggregated welfare as:

$$\sum_{t=0}^{\infty}\beta^t\frac{W_t - W}{W_c C} = -\frac{1}{2}\left[\sum_{t=0}^{\infty}\beta^t L\right] + O^3 + t.i.p \tag{2.136}$$

where

$$L \equiv \left(\begin{array}{c} 2\left[\hat{y}_t^{ef_risk} - \hat{y}\right] + \left(1-\frac{1}{X}\right)\frac{2}{\eta}\hat{x}_t + \frac{1}{X}\frac{1}{\eta}\hat{x}_t^2 + \left(1-\frac{1}{X}\frac{1}{\eta}\right)\frac{\varepsilon}{\kappa}\pi_t^2 + \\ +\alpha\,(1-\alpha)\left[c_t^s - c_t^b\right]^2 + j\left[\left(\frac{1-2\alpha+2\alpha^2}{\alpha^2}\right)\left(\hat{h}_t^b\right)^2\right] \end{array}\right) \tag{2.137}$$

Chapter 3

Uncertainty in a model of financial frictions and policy interactions

3.1 Introduction

Imperfect financial markets have been pointed as a major reason for the Great Recession and also for the following sluggish recovery. An increasing number of recent studies (such as Christiano et al. (2010), Carlstrom et al. (2010), Gertler & Karadi (2011), Curdia & Woodford (2010), Gerali et al. (2010), Iacoviello (2005), and others) have suggested that credit market frictions are important for explaining economic fluctuations. In these studies, credit market frictions expand the impacts of shocks on the macroeconomy and, in many examples, shocks emerging from the financial market are themselves primary causes of economic fluctuations. There also has been rapidly growing attention in the effects of uncertainty shocks on the economy, notably, following the seminal contribution by Bloom (2009). In micro-finance literature uncertain production outcomes or some other broad definition of uncertainty over productivity often either creates or contributes to agency costs. Nevertheless, there has been considerably scarce research on the transmission of uncertainty or volatility shocks by financial market frictions in general equilibrium setup.

One of the aims of the present chapter is to investigate the role of uncertainty together with financial market frictions in accounting for business cycle fluctuations. The present study offers a unified framework which allows analyzing interactions among uncertainty, nominal rigidities, and financial frictions. The present builds on a model by Pesaran & Xu (2016) with a firm default and show that financial frictions can generate additional channels through which uncertainty may impact economic outcomes. In our model, uncertainty operates through the bank lending channel and the interaction

between nominal rigidities and financial frictions that cause a feedback loop which further worsens financial conditions. We also study the role of unconventional monetary and macroprudential policies in the model. In particular, our focus is on interactions among different policies.

The present reconciles increasing empirical consent on sizeable adverse effects of uncertainty shocks which general equilibrium models largely failed to replicate. For example, Born & Pfeifer (2014), and Bachmann & Bayer (2013) analyzing the quantitative impact of uncertainty shocks, show that the effects on output and aggregate output are most likely insufficient to be regarded as the primary driving factor of business cycle fluctuations. They suggest that price rigidities are necessary in order to expand the impacts of uncertainty shocks. Similarly, Basu and Bundick (2012, 2017) also find that an elevated level of uncertainty always has a negative impact on aggregate demand under nominal frictions. In a model of sticky prices, an increase in uncertainty reduces firms' marginal costs of production through precautionary labor supply. Slowly-adjusting prices together with lower marginal cost increase firms' markups over marginal costs which lead to reduced demand for consumption and investment goods; output and employment decline when both consumption and investment decrease as they are demand-determined in these models (Basu and Bundick, 2012, 2017). Thus, adding nominal rigidities restores observed comovement in the data while in smaller magnitude (i.e. not quantitatively sizeable to be a source of buisness cycle fluctuations). Leduc & Liu (2016) show that both labour market frictions and price rigidities are fundamental to amplify uncertainty shocks that reduce aggregate demand, and rising firms' markups. When the model is augmented with search and matching rigidities, the fall in aggregate demand lowers the value of new matches, and hence firms announce fewer jobs which push the employment down. Household's incomes decline further because fewer searching workers find a matching job. This causes an even greater fall in aggregate demand and amplifies the negative impact of uncertainty shocks.

The main motivations and contributions of the present research are briefly summarized as follows. First, this research contributes to the recently expanding debate on effects of uncertainty shocks and its transmission mechanism to the economy, especially, under credit frictions. We aim to shed light on the interactions of uncertainty, credit and nominal frictions in a model with heterogeneous firms which face a default risk. Moreover, as it will be discussed in the literature review section, apart from independent effects of uncertainty shocks, there exist important gaps in our knowledge on the role of uncertainty in propagating shocks to the economy, particularly, credit and monetary policy shocks which the present study intends to address. Second, this research

attempts answering important policy questions in addressing economic stability and welfare concerns in the presence of uncertainty and risk shocks, under expected and recursive preferences, in a setup that accounts for further non-linearities by solving, estimating, and simulating the model around the ergodic mean using underlying third order pruned approximation of Meyer-Gohde (2014) [1]. Third, we look at the effects of macroprudential and unconventional liquidity policies on model's dynamics, stability, and welfare of agents in this setup. Fourth, this research attempts to answer important questions related interactions among monetary, macroprudential, and liquidity policies on model's stability with conflicting policy targets, which urge for consideration with rapidly growing usage of macroprudential and unconventional policies in the new era of policymaking.

One of the aims of the present is to investigate the independent role of credit frictions and interactions between the financial friction with nominal rigidities in transmitting effects of uncertainty shocks. We use stochastic volatility to describe the variance of the common technology process. We assume that the variance of the level shock evolves as an autoregressive process. Thus, shock processes have two separate innovations: one that affects the level of the shock and the second that affects the variance of the shock. Our model introduces a new channel through which the volatility shocks can affect economic outcomes. Specifically, we include stochastic volatility in the technology process in Pesaran & Xu (2016) model, along with other modifications. In our model uncertainty increases a firm's default probability, thus widening the interest rate spread in the economy. The model can generate substantial contractions in the economy under flexible prices alone. We also add nominal frictions into the model. This enhances the contractionary effects of uncertainty shocks. In the present model with a firm default, as we show, the default probability is also a function of the marginal cost and economy-wide price dispersion. Therefore, it adds further variations in the probability distribution of the default threshold. This causes an additional feedback loop from the nominal side to the severity of the financial conditions. We also examine the relation between financial frictions and uncertainty. Namely, in the model, the borrowing constraint is one of the major determinants of the default probability. To capture independent effects of time-varying volatility on the policy rules, we take a third-order perturbed approximation to our model using as in Fernandez-Villaverde and Rubio-Ramirez (2010) and Fernandez-Villaverde et al. (2009). We parametrize the non-linear model using likelihood inferences (the mixture of, both, Bayesian and

[1] We use the Dynare code for the non-deterministic steady-state approximation by Meyer-Gohde (2014) which is slightly modified by T.Holden.

maximum likelihood estimations) based on the approximation around the ergodic mean as in Meyer-Gohde (2014). Despite an exogenously given volatility shock, the model also accounts for low volatility endogenously with increased banks' leverage during the "great moderation." Therefore, the drop in exogenously given volatility process possibly allowed financial intermediaries to increase their leverage and this strengthened the reduction in the second moments of systems' variables.

Our DSGE model also has the similar main features as several other finance models, namely, an elevated level of uncertainty increases the term premium by raising the size of risk (please refer to Cox et al. (1985), Engle et al. (1987), e.t.c). Most finance literature finds substantial term premium that has a considerable time variation (e.g., Campbell & Shiller (1991), Cochrane & Piazzesi (2005)). From a practical standpoint, many monetary authorities use the yield curve to evaluate expectations of monetary policy and inflation. Therefore understanding movements in term premium appears to be necessary for the conduct of monetary policy. However, general equilibrium-macro models often fail to account not only for the dynamics of asset prices but also their associated risk premia. We find that stochastic volatility shocks can produce sizeable variation in the term premium without compromising the model's ability to replicate main macroeconomic moments. To obtain both macroeconomic and financial coherence, as in recent papers by Andreasen (2009), Rudebusch & Swanson (2012), Guvenen (2009), Tristani et al. (2010) among others, we use non-time-separable preferences in the structural specification of agents. This allows increasing the risk aversion of the agents without compromising the model's ability to replicate key macroeconomic empirics where a higher level of the risk aversion is often used in the finance literature.

The present chapter is structured as follows. First, we present a review of relevant literature on impacts of uncertainty and identify gaps in our knowledge regarding policy issues in presence stochastic volatility and concerning interactions among different policy tools. The next section presents the model. Section 3 describes the estimation of the present non-linear model. Section 4 presents our main findings.

3.2 Related Literature

This section reviews the literature on uncertainty. Specifically, we present a comprehensive review on how uncertainty has been measured by different authors, its impacts on the economy and what the literature has to say about the interaction between conventional, macro-prudential and unconventional monetary policies in a model which is subject to uncertainty shocks. To this end, existing evidence on such interactions and findings related to reserve requirement are presented in the second section of this literature review.

3.2.1 Uncertainty

The year 2017 marks the 40th anniversary of the publication of Harvard economist John Kenneth Galbraiths book The Age of Uncertainty. 40 years ago the world was still reeling from the Nixon shock aftermath, which saw the Bretton Woods monetary system replaced by a regime based on freely floating flat currencies. Around the same time, proponents of the idea that inflation and recession were mutually exclusive were being rudely awakened by the reality of an unprecedented global stagflation. These events and the uncertainty that ensued led to a decline in the U.S. economy between the 1970s and 1980s.

Starting with the 2007 financial crisis and more recently Brexit, the nature of the uncertainty today is markedly different from the one in the 1970s. For instance, an interconnected world means that what happens in China or Brazil, does not stay in China or Brazil. On the contrary, the important role played by these countries in sustaining global growth means that any developments there will have implications for the world. In other words, any uncertainty resulting from major policy shifts in either the U.S., U.K., China or Brazil, can quickly have far reaching consequences for the world economy.

In light of the above, in an environment of effective lower bound policy rate, the role of monetary policy in offsetting the adverse impacts of uncertainty shocks on the economy in normal times has never been more critical.

Regarding both the role and conduct of monetary policy, the recent financial crisis has taught us that a policy which focuses solely on price stability cannot be expected to deliver macroeconomic stability. Albeit it is now evident that the achievement of financial stability is a requisite for the pursuit of macroeconomic stability, there is currently no consensus around a new policy measure.

Ultimately, the above unambiguously begs the following question, how monetary

policy and macro-prudential policy should be set together in order to deliver macroeconomic stability. And perhaps, more importantly, how effective are these policies in dealing with uncertainty shocks. In the interim, key findings from early research on this subject is the fact that there are important sources of conflict between the two policies especially in instances where they are not coordinated.

Albeit from a technical standpoint the present chapter makes numerous contributions to the existing literature on DSGE modeling, our study has two main aims. First, to shed some light on the impacts of uncertainty on macroeconomic variables using a general equilibrium framework. Second, to assess the effectiveness of monetary and macro-prudential policies in mitigating the effects of uncertainty shocks. Specifically, our proposed DSGE model is different from many of the existing ones in that it accounts for financial frictions, it allows uncertainty to be modeled as a time-varying shock process and, it uses the Epstein-Zin preferences which separate the simple connection between the intertemporal elasticity of substitution and coefficient of relative risk aversion.

The main challenge in the empirical examination of uncertainty and its relation to the macroeconomy is the lack of commonly agreed measure for the uncertainty. The existing literature relies mostly on the number of measures of uncertainty. For instance, one widely used proxy is the realized volatility of stock market returns. The second common measure is the cross-sectional distribution of firms' productivity, profits, or stocks. More interesting measures are the cross-sectional distribution of individual forecasts and the number of the frequency of the coverage of uncertainty-related keywords in news articles (e.g., Baker et al. (2016)).

In his seminal work, Bloom (2009) proxies uncertainty by the volatility of the stock market and finds that it is strongly countercyclical with real economic activity. Bloom (2009)'s estimates imply that innovation in his measure of uncertainty effects employment and output which develops in following next six months. An elevated level of volatility first depresses the real economic activity and then boosts it where investment and output overshoots of their long-run levels. This may be in line with the implications of models where uncertainty is considered as a main contributing factor of business cycles fluctuations, however still missing their persistence. Bloom et al. (2012) also analyzed the relationship between uncertainty and real economic activity where uncertainty is measured by cross-sectional dispersions in total factor productivity, firm-level and industry-level earnings, and also by the divergence in forecasters' predictions.

Although an obvious advantage of most of these proxies reside in the fact that they

are readily observable, their fitness as adequate measures of uncertainty relies to a greater extent on how closely they are correlated with the actual stochastic process describing uncertainty (Jurado et al. 2015). This, may not always be the case. For instance, volatility in the stock market can vary over time despite having any change in the underlying uncertainty around economic fundamentals. This is particularly true during leverage changes, or when movements sentiments and the risk aversion are important in driving asset market fluctuations (Jurado et al. 2015). Being more promising proxy for the uncertainty, the cross-sectional distribution of individual stock market returns also has its own drawbacks. For example, when there is significant heterogeneity in common risk factor loadings, the cross-sectional dispersion in individual stock returns can change regardless of changes in underlying uncertainty. Furthermore, cross-sectional distribution in firm-level productivity, profits or sales can change along cycles due to to the heterogeneity in the cyclicality of individual firms business activities (Jurado et al. 2015).

While these measures of stock market volatility and proxies related to different kinds of dispersions may or may not be closely related to true underlying uncertainty on economic fundamentals, it is evident that they represent a sensible starting point and important cases to understand. For instance, a commonly used proxy for the uncertainty that is closely associated with financial market volatility is the VIX index, which is likely to have significant components that are driven by factors related to the time-varying risk-aversion instead of actual economic uncertainty (Bekaert et al. 2013).

Regarding other proxies for uncertainty used in the literature, D'Amico et al. (2008) is one of earlier works to study various measures related to analyst ambiguity and discrepancies in the Survey of Professional Forecasters. While analysts forecasts being appealing in their own right, there are many recognized disadvantages in applying them as a proxy for uncertainty. The most obvious one is that subjective expectations are often available for only a restricted number of series (Giordani & Söderlind 2003). Second, it remains unclear whether the responses obtained from these surveys correctly reflect the conditional expectations of the entire economy. The sampled respondents usually are practitioner analysts', and it should not be surprising that systematic biases may often present in their forecasts; either because they may dismiss relevant forecasting information, or some forecasters may have other strategic considerations to bias their predictions (Laster et al. 1999). Moreover, discrepancies in survey forecasts could just be an indication of differences in opinions rather than pertaining to the underlying uncertainty (e.g., (Qu et al. 2003) (Mankiw et al. 2003)). Fourth, as Lahiri & Sheng (2010) point out, even if these forecasts were impartial, the discrepancy

in analysts point forecasts (when aggregated across analysts) does not necessarily translate into forecast error uncertainty, especially, when the variance of accumulated aggregate innovations is different from zero over the forecast horizon. These authors referring to the Survey of Professional Forecasters empirically show that the variance of the aggregate-accumulated shocks may lead to a divergence between uncertainty and cross-sectional distribution in point forecasts when the forecast horizon is too long or in periods of important economic changes.

Needless to say that developing a consistent measure of uncertainty is a critical step towards understanding its impact on the economy as a whole. Indeed, many authors are aware of the limitations of current measures of uncertainty, as such, often use more than one proxy in their research. For example, Bachmann et al. (2013) acknowledge many of the issues mentioned above and show precautions to address them by using further measures for uncertainty, for instance, an ex-post measure of forecast error variance from the expectations' surveys. A related approach is followed by Scotti (2016) who bases his analyses on series for which real-time data became available.

Another crucial, yet unsettled question in the empirical study of uncertainty relates to the persistence of these shocks. For instance, in models studied by Bloom et al. (2012) negative cycles are triggered by a rise in uncertainty, which in turns leads to a decline in productivity growth. However, Schaal et al. (2011), Leduc & Liu (2016), Lin (2014) along with other studies that have considered models in which uncertainty is allowed to play a major role have argued that commonly used empirical measures of the uncertainty, for example, the cross-sectional distribution in firms sales growth, do not exhibit sufficient persistence to account for persistent levels of unemployment that have accompanied most recessions, notably, slow recovery following the 2007-2009 recession.

Indeed, the literature offers some evidence that in fact both, the particular proxy used to measure uncertainty as well as the methodology used may be crucial in explaining the sometimes conflicting findings discussed above. For example, in their study, Jurado et al. (2015) introduce novel time series measures for the uncertainty and find that their estimates exhibit considerable independent variations from other commonly used uncertainty measures. The main finding of their study is that their estimates indicate far fewer, yet quantitatively more prominent episodes of uncertainty than what is usually obtained from all other commonly used uncertainty proxies they considered (Jurado et al. 2015). Furthermore, these authors' estimates for macroeconomic uncertainty display larger persistence than, for instance, those inferred from stock market volatility measures. In sum, their findings suggest that much of movements in commonly used

uncertainty proxies, such as measures of cross-sectional dispersion and stock market volatility, have in fact poor association with broad-based variations in macroeconomic economic uncertainty. The significance of the latter is that a considerable share of the variation in commonly referred uncertainty proxies may not be driven by economic uncertainty. From a methodological standpoint, Carriero et al. (2015) show that failure to account for measurement errors can also result in erroneously assigning a smaller role for uncertainty to account for the business cycle. Specifically, these authors by using a simulation exercise demonstrate that whenever the uncertainty proxy differs from the actual underlying measure, estimated impulse responses from VARs that include the uncertainty proxy will exhibit a downward bias (Carriero et al. 2015). However, they also find that structural VARs which instead use the measure of uncertainty as an external instrument suffer less from such bias (i.e. this methodological approach find quantitavely more important and long-lasting effects of uncertainty). Furthermore, when these authors' proxy SVAR approach is applied to the empirical work of Bloom (2009), the implied impact of the uncertainty shocks become more persistent and considerable.

At this stage, it is worth to point out that the most of the definitions of uncertainty mentioned above have been exclusively applied in the empirical analysis. In the context of DSGE modeling, the literature has proposed three distinct alternatives to account for changes in the volatility of shocks: Markov regime switching, GARCH, and stochastic volatility models. In the present analysis, we use the latter method as in most of the related studies on volatility and macroeconomics in general equilbrium context (for example, Born & Pfeifer (2014), Gilchrist et al. (2014), Cesa-Bianchi & Corugedo (2014), Fernández-Villaverde & Rubio-Ramírez (2010), Arellano et al. (2010), Justiniano & Primiceri (2008)). We now discuss each of these approaches in turn.

In the GARCH models, a single innovation that drives the dynamics of both the volatility and the level shock. The significance of this resides in that when there is a big innovation in the level shock; there also will be high volatility in the following period. Thus, it becomes impossible to isolate a level shock from a volatility shock, and periods of higher volatilities are induced only by higher level shocks. Although this constraint may be less important when working with time series in reduced-form models, it introduces constraints in structural models (Fernández-Villaverde & Rubio-Ramírez 2010). Specifically, the interconnection of volatility and level innovations prevents the use of GARCH specification to evaluate the effects of volatility shocks separately from the effects of level innovations in DSGE models.(Fernández-Villaverde & Rubio-Ramírez 2010). Although in time series empirics, the GARCH specification is a

preferred alternative over the stochastic volatility, as they are much simpler to estimate, this simplicity gain does not prevail in DSGE models as with either stochastic volatility or GARCH, the resulting model needs to be solved non-linearly (Fernández-Villaverde & Rubio-Ramírez 2010). Furthermore, having two separate shocks in stochastic volatility model gives an extra degree of freedom, unlike in the GARCH specification.

An alternative approach to GARCH and stochastic volatility models to describe the time-varying volatility is Markov regime switching specification. A key distinction between the Markov process and the previous two approaches revolves around the size and as well as the duration of the change. One can describe both stochastic volatility and GARCH models as a continuously evolving processes with innovations in each period (Fernández-Villaverde et al. 2011). Whereas, Markov regime switching processes emerge in a discrete way, where sudden hikes in uncertainty are interrupted by the episodes of calm (Fernández-Villaverde et al. 2011).

As Fernández-Villaverde et al. (2011) argue, there is no clear benefit of using GARCH models instead of stochastic volatility. First, the GARCH specification has one less degree of freedom which precludes one to neatly separate volatility from level shocks. It also gives a poorer match to the data, and, in the DSGE models, it is not any more straightforward to handle. However, the choice between stochastic volatility and Markov processes is more intricate. In the real world, movements in the volatility of, for instance, the technology process are likely to be the combination of discrete and continuous episodes (Fernández-Villaverde et al. 2011). Although there are some events describing the technological change which are easier to be understood as discrete shifts (for instance, when a new patent law is approved), some other developments (e.g., advances in our knowledge of natural laws) are easier to interpret as continuous and gradual changes (Fernández-Villaverde et al. 2011). Thus, it seems that the choice over one or another specification is an empirical issue.

From a technical standpoint, one advantage of stochastic volatility is that, under certain specifications, the standard deviation of the model innovation can have any value. Thus, the policy rules of the agents in the model can be differentiated with respect to volatility innovations which gives a computational advantage of using standard perturbation methods to compute the equilibrium dynamics of the model very efficiently (Fernández-Villaverde et al. 2011). This convenience does not hold for Markov regime switching specification because the standard deviation of the model takes values only from a finite set (Fernández-Villaverde et al. 2011). Though, it is worth to point out that stochastic volatility model has a few disadvantages of its own. A key one is that, when the actual process evolves as a discrete jump, stochastic volatility

will predict a rise in volatility before the real changes in the volatility materialize (Fernández-Villaverde et al. 2011). This is because the likelihood prefers smaller and gradual changes overtime over huge changes in a single period; therefore it will show a sequence of smaller innovations before and following the actual movements in volatility for an abnormally large innovation that reflects the jump (Fernández-Villaverde et al. 2011).

Having clearly outlined the various proxies or measures of uncertainty in the existing literature, one important question remains, namely what is the impact these indicators on the economy? Indeed, the primary aim of developing a consistent measure of uncertainty is to understand its impact on the economy, most importantly, understand various channels how it will be transmitted to the rest of economy. In fact, the link between volatility shocks and key economic variables has received a lot of attention in academic research. Studies from Bernanke (1983), Hartman (1976) and Kimball (1990) to name but a few, have provided ample evidence on how higher volatility can affect macroeconomic activity through precautionary savings, irreversible investments, or marginal revenues. Bloom (2009) by simulating the impact of a large temporary volatility innovation shows that it leads to a rapid decline andrebound which is followed by an overshoot in output, investment, hiring, and productivity growth. This author finds that investment and hiring rates decline dramatically in the four to six months following the uncertainty shock since high uncertainty raises the real-option value of "wait and see" and therefore firms need to scale back their hiring and investment plans. When uncertainty lessens, the firms' activity rapidly bounces back because firms need to meet their supressed demand for capital and labour (Bloom 2009). In addition to the latter, Bloom also finds that the aggregate productivity growth declines significantly following the shock owing to the reduction in investment and hiring. This, in turn, reduces the efficiency of how fast resources can be reallocated from firms with lower productivity to higher ones. This acts as the main source of productivity growth in the model, as well as, in the real economic activity (Bloom 2009). Overall, the real option value channel implies that in the medium run increased uncertainty will cause a "volatility overshoot" (Bloom 2009). This is because, according to Bloom (2009), most of the firms are located in the proximity of their investment and hiring thresholds, above which they invest or hire and below which they enter into a region of inaction. Thus, while small positive shocks will create an investment and hiring response, small and negative shocks will produce no response. This author concludes that investment hiring decisions feature local convexity in business conditions, namely in demand and in productivity, in which an elevated uncertainty around business conditions growth

leads to a rise in capital and labour in the medium-term. Indeed, these volatility effects cause a rapid slowdown which is followed by a quick bounce-back in economic activity, which have been shown to be entirely consistent with the empirical evidence based on financial market-related measures of uncertainty. This is entirely different from the much more prolonged and persistent recession that generally occurs following negative first-moment productivity and/or demand shocks that are widely modeled within the DSGE literature (Bloom 2009). In the present study, we abstract from the option value channel by removing the capital accumulation processes and rigidities attached to it. This allows isolating effects of financial frictions in the transmission of uncertainty shock beside keeping the model dynamics simple. Therefore, we can check if financial frictions have independent effects of transmitting uncertainty shocks. We do not find evidence for the overshooting in model variables ((i.e., rapid rebound and increase in economic activity after short-lived recession) for the estimated set of parameters and calibrated rule for the monetary policy in our general equilibrium framework.

In fact, although there are some agreements around the adverse consequences of uncertainty shocks on models' economic variables in a partial equilibrium framework, there are substantial differences in findings regarding the effects of such shocks in general equilibrium set-up, (Bloom 2009). Using a dynamic stochastic general equilibrium (DSGE) with heterogeneous firms, Bachmann & Bayer (2013) conclude that uncertainty shocks are unpromising to explain business cycle fluctuations as a major driver as opposed to standard aggregate productivity shocks. Under the assumption of sticky prices and the zero lower bound constraint of the central bank, Basu & Bundick (2017) find uncertainty to be an important source of fluctuation in a general equilibrium setting. The contrast in findings of these two studies encapsulate rather well the current conundrum when trying to assess the impact of uncertainty in a general equilibrium model with the results being sensitive to the chosen modeling assumptions.

It is important to note that although the majority of the studies exploring the impact of uncertainty on the economy find that uncertainty is counter-cyclical, these studies differ and their estimation of both the duration and severity of the ensuing recession. For example, the class of models developed by Bernanke (1983), Dixit & Pindyck (1994), Bloom (2009), and Bloom et al. (2012) predict that high uncertainty generates a fairly rapid bust-boom cycles (Bachmann et al. 2013). The second class of models, which stresses that the connection of uncertainty shocks and real economic activity transmitted through credit frictions, mainly argue in favour of a persistent decline in economic activity. For example, Gilchrist et al. (2014) finds that a rise in firm riskiness increases bond premia and, thereby, the cost of capital which generates a more

prolonged reduction in investment and output. In addition, Christiano et al. (2014) develop a DSGE model that accounts for credit frictions in which uncertainty shock also generate a slowly building and sizable drop in output. Narita (2011) introduces a model where firms with higher agency and information problems are more prone to breakups in times of high uncertainty which results in surviving units taking projects that involve less risk. This also leads to a persistent and slowly building reduction in economic activity (Bachmann et al. 2013). Finally, for Fernández-Villaverde et al. (2011), volatility innovations in the monetary policy rule in small open economies in the presence of high investment adjustment costs also result in more persistent output declines (Bachmann et al. 2013). Indeed, our findings are more in line with this second class of models.

Beside the slow down in economic activity described above, the impact of uncertainty shocks on the economy seems to differ across different geographical regions. Specifically, Bachmann et al. (2013) argue that the effect of uncertainty shock on economic activities differ greatly depending on the country. These authors find that a surprise increase in the survey related indicators of uncertainty is followed by a nonsizeable reduction in employment and production in both the United States and Germany. Specifically, their findings reveal that for the German data, production declines but rebounds rapdily following an increase in uncertainty, in a way which is line with the predictions of the wait and see (i.e. firms postpone their hiring and invesntment decision until uncertainty is resolved) dynamics. Specifically, their findings reveal that for the German production data, drops and rebounds appear quite promptly after a rise in uncertainty, in a way which is more consistent with the predictions of the wait and see channel. Furthermore, these authors argue that the economy's conditional response to a surprise increase in volatility in the U.S. is qualitatively not similar to the one in Germany. This is because, in the U.S., these authors find the response of output to innovation in uncertainty to be slow building, and more persistent with the negative response of output reaches to its peak about two years after the realization of the shock. They also fail to find satisfactory evidence for the rebound effect for the United States. In sum, findings of Bachmann et al. (2013) indicates that, for the United States, some of the channels suggested in the literature, namely, financial frictions, the option value (the option value of waiting is higher than the value of investment of hiring) when coupled with an endogenous growth mechanism, or the "by-product"[2] hypothesis may all be appealing explanations of the observed empirical link between the volatility and economic activity.

[2]By-product hypothesis refers to the elevated uncertainty of being a consequence or by-product of recessions instead of a driving factor

(Bachmann et al. 2013).

3.2.2 Policies

As stated in the introductory part of the previous section, one of the key contributions of the present research is to explore on the one hand what is the implication of uncertainty on optimal policies and on the other, the interaction between conventional, macro-prudential and unconventional monetary policies in a model which is subject to uncertainty shocks.

From the outset, the pursuit of financial stability is a requisite for the achievement of macroeconomic stability, and there is currently lack of consensus around a new policy measure. Specifically, according to Mishkin (2011), monetary policy ought to lean against only credit cycles. For Viñals et al. (2012), rather than being passive to credit and indebtedness bubbles, monetary policy rules should react to these financial variables, but only for the price stability purpose. Curdia & Woodford (2010) propose a Taylor Rule that is extended to respond to contemporaneous credit spreads, their model apart from decreasing the distortions introduced by a financial shock also enhances economy's response to rest of disturbances.

Although many economists would now tend to accept that price and financial stability are central objectives to achieve the macroeconomic stability as an ultimate goal, there may be important side effects from monetary policy on macro-prudential targets, e.g., leverage or credit, and also from macro-prudential policies on targets of the monetary policy, e.g., inflation or output gap Beau et al. (2012). According to Beau et al. (2012), it is possible for a potential macro-prudential policy to change the propagation of monetary policy as such policy also functions through the similar channels of the monetary policy, e.g., balance sheet and lending channels. This is to say that under different economic conditions, the consequences of both policies on financial and price stability can be independent, complementary or conflicting (Beau et al. 2012). For Beau et al. (2012) and others such as Galati & Moessner (2013), conflicting outcomes on targets of interest will largely rely on the source and propagation of demand and supply imbalances across the real economy and financial sector and on whether financial imbalances are relevant in the framework of monetary policy.

From a macroeconomic modelling standpoint, the above debate points to the inadequacy of DSGE models that are strong at matching macroeconomic data but with a weak performance on the financial side. This is to say that many of the existing structural macroeconomic models generate risk premia that are too small with little volatility compared with non-structural measures. For some authors, the latter

is indicative of a clear model misspecification, for asset pricing and macroeconomic behaviours are inextricably linked (Cochrane et al. (2005), Rudebusch & Swanson (2008)).

Still, recent studies by Andreasen (2009), Rudebusch & Swanson (2012), Guvenen (2009), Tristani et al. (2010) to name but a few - to ensure both macroeconomic and financial coherence - have made use of non-time-separable preferences, namely Epstein Zin preferences, in the structural specification of agents utility, within New Keynesian frameworks. This is because the Epstein-Zin preferences provide an additional degree of freedom that allows to increase risk aversion and hence generate substantial time-varying risk premia (Rudebusch & Swanson 2012).

The literature offers a number of relevant examples where Epstein-Zin preferences were successfully used to ensure macroeconomic and financial coherence. For instance, Rudebusch and Swanson et al. (2009) used the Epstein-Zin preferences to replicare both macroeconomic and financial coherence, including a 106 basis points term premium average for the U.S. ten-year nominal bond within a New Keynesian modelling framework. In addition, Andreasen (2009) provides an estimation of a DSGE macro-model using Epstein-Zin preferences and is able to generate term premium with the mean value of 69 basis points.

In light of the above, there is no doubt that the Epstein-Zin preferences are a key component to make the link between the macroeconomy and the financial side, as they improve the fit with financial data without significantly deteriorating the already well fitted macroeconomic dynamic (Pariès & Loublier 2010). It is important to note that these promising features also hold for the structural models where monetary policy is committed to a Taylor-type rule to minimize an ad hoc loss function (Pariès & Loublier 2010).

Moreover, Epstein-Zin preferences may bear important quantitative implications for not only asset pricing but also for macroeconomic allocation in the context of welfare-based monetary policy. A study by Levin et al. (2008), points out that models relying on Epstein-Zin preferences may yield some microeconomic dissonance: to a first order approximation, the underlying microeconomic difference in the agents preferences may affect the macroeconomic allocations under Ramsey policy. Thus, if Epstein-Zin preferences are to gain widespread use within DSGE models for example, recent papers like van Binsbergen et al. (2008), Caldara et al. (2012) provide a methodology to solve and estimate DSGE with a non-standard preferences an analysis of the non-linear properties through their impact on the welfare and the optimal policy is a relevant research avenue.

On the subject of non-linear properties of DSGE models, it is worth noting that what is estimated in the most structural macroeconomic models is the reduced form derived from the solution of linearized equilibrium conditions of these models. More to the point, although this approach has some clear advantages in that it is simple and can be used to compare models against other empirical approaches such as VARs, it may omit important dynamics at play. This is because the world is inherently non-linear and understanding the implication of the many nonlinear characteristics embedded in DSGE models may help towards developing ever more coherent models.

As Amisano & Tristani (2010) argue, compared to linear models, nonlinear models are better when modeling the macroeconomy when subject to large deviations from the steady state. Indeed, as they point out, since 1970, the average inflation in European countries has varied between a maximum and minimum of 14.56 and 0.69 percent, respectively. Furthermore, a linear model is inadequate when explaining large deviations and it is likely to produce biased estimates (Amisano & Tristani 2010).

Furthermore, as Amisano & Tristani (2010) point out, it is possible that the dynamic characteristics of the inflation to be governed by its proximity to the steady state. For example, small departures could be manifested with a smaller degree of amplitude and persistence in responses of inflation to innovations. On the other hand, dynamic features such as the persistence and amplitude can increase for a larger deviation from the steady-state, where the inflation shocks could more quickly become embedded in expectations (Amisano & Tristani 2010). Such dynamic properties of the economy can better be detected by higher-order, nonlinear solutions (Amisano & Tristani 2010).

So far, our discussion has revolved around how to best develop a DSGE model with the aim to assess the effectiveness of monetary and macro-prudential policies in mitigating the effects of uncertainty shocks. Although the latter is the central aim of our research, it is important to recognise the fact that monetary policy shocks in turn, under certain circumstances, can exacerbate macroeconomic volatility. For example, Mumtaz & Theodoridis (2015) find that a surprise increase of 150 basis points in the policy rate raises volatility of output and inflation about 10 % relative to their steady-state standard deviations. According to these authors; the volatility impact of the shock is embedded in by agents' concerns about the inadequacy of the central bank to reverse such deviations from the policy rule which is further amplified by the presence positive steady-state inflation. Understanding these second-order effects is another motivation of the present research in the presence of firm default. We find that banks leverage plays important role in responses of second moments.

Indeed, the current literature on DSGE modeling does not offer a satisfactory

account of the roots for movements in aggregate volatility over time. Therefore In the majority of the DSGE models the literature has to offer, stochastic volatility is assumed to be an exogenous process. In selected more sophisticated models, for example, models that account for shifts in policies, some components of the movements in volatility can be endogenized. However, again, the same concerns apply to answering questions about unexplained changes in policy (Fernández-Villaverde & Rubio-Ramírez 2010). This is to say that macroeconomics, typically, even does not offer a satisfactory theory of why we observe shocks, either in preferences, technological process, or any other. Most of the effort has been made just simply examining the outcomes of a given exogenous innovation with little regard to their roots (Fernández-Villaverde & Rubio-Ramírez 2010). The present study also does not deviate from this approach by assuming an exogenous volatility innovations. Despite the model's ability of endogenously generating movements in the volatility responses for different levels of the leverage, our model also agnostic in explaining changes in banks' leverages over time.

Our discussion so far has mainly revolved around conventional monetary policy; we now turn to unconventional policies, namely reserved requirement. Reserve requirements are defined as the minimum percentages or amounts of liabilities that commercial banks are required to preserve as cash or deposits with their central banks (O'Brien 2007). Since these liabilities cannot be used for the provision of private credit, or buying securities, this in turns means that when reserve requirements increase it would unambiguously decrease the money multiplier. That is to say that for any given monetary base, the broad money will reduce with an increase in reserve requirements (Glocker & Towbin 2012).

The effects of reserve requirement shocks are poorly understood despite the application of reserve requirement as a policy tool in many emerging economies. Whilst there is some research on partial equilibrium effects of reserve requirement instruments, not much has been done in investigating dynamics of macroeconomic variables jointly. Gelos (2009) examining a sample of Latin American countries finds that banking spreads increase with higher reserve requirement. Related studies for Brazil which has actively been actively using such interventions, examined the impact of reserve requirement changes on banking spreads (Takeda et al. (2005), Cardoso (2003)), and on banks stock returns (De Carvalho & Azevedo 2008) (cited in Glocker & Towbin (2012)). Similar studies for number of developing countries include Herrera et al. (2010), Treisman (2004), Betancourt and Vargas Herrera (2008), Robitaille et al. (2011) (cited in Glocker & Towbin (2012)). Most of the research rather focuses on the link between reserve requirements and the credit market and often dismisses its impact on

aggregate macroeconomic activity and other external variables. For example, Loungani & Rush (1995) study the relation between reserve requirement policies with output and investment for the United States using single equation set-up, however, dismiss possible contemporary interactions between the interest rate and reserve requirement and also do not examine the impacts on external variables (cited in Glocker & Towbin (2012)).

Our study fills the existing gap in the literature from a theoretical standpoint by specifically exploring how reserve requirement policy interacts with bank lending behaviour. Before providing a detailed explanation of our contribution, it is important to; first give a more thorough description of the central bank use of reserve requirement and second, describe how the reserve requirement systems differ across geographies.

As argued above, in the reserve requirement, central banks have a potent monetary policy too. For example, if the monetary authority targets quantities while keeping the monetary base unchanged, the impact of a rise in reserve requirements is expected to be similar to a conventional monetary policy contraction with the level of interest rates increasing in reserve requirements (Glocker & Towbin 2012). To meet the reserve requirements without cutting the credit, banks have to draw in more deposits, which increases deposit rates. Therefore, the higher marginal costs of funding, subsequently, will also raise lending rates as well as drive up the general level of interest rates (Glocker & Towbin 2012).

On the contrary, if the monetary authority targets a particular interest rate by setting the price of money, a hike in reserve requirements or liquidity will have different consequences. To address a possible deviation of the interest rate from its target level, the monetary authority will need to raise the monetary base and to accommodate the contractionary effects of the increase in reserve requirements (Glocker & Towbin 2012). In simplistic models, it has been argued that changes in reserve requirement are likely to be neutral when the monetary authority targets interest rates (Horrigan 1988). One should anticipate actual effects of the reserve requirement policy if they are numerated under the market rate and also when there is no numeration attached to them (Glocker & Towbin (2012) and Reinhart & Reinhart (1999)). In such instance, higher reserve requirements, in effect, work as a tax on deposits of banks. Therefore spreads between deposit and lending rates would rise because the cost financial intermediation increases (Glocker & Towbin 2012). When the monetary authority regulates the interbank rate, one should anticipate lending rates to grow and deposit rates to decline (Glocker & Towbin 2012). When the monetary policy targets the interbank rate, one should anticipate an increase in the rate of lending and a decrease in the deposit rate (Glocker

& Towbin 2012). Therefore it has been suggested that within the interest rate targeting set-up, changes in reserve requirement are likely to be neutral on the general interest rates levels but can possibly change interest rate spreads (Glocker & Towbin 2012).

However, the above discussed reasoning mostly assumes a limited role for the bank lending channel in the economy. As Kashyap & Stein (2000) and Bernanke & Blinder (1992) point out because reserve requirements usually are relevant only for commercial depositary institutions, the consequences of changes in reserve requirements on the aggregate economy rely upon the role of bank lending and especially the bank lending channel of monetary policy. This can particularly be the case in the presence of credit frictions in the economy.

According to Glocker & Towbin (2012), for changes in reserve requirements or in liquidity provisioning to have actual outcomes, two propositions have to be satisfied. The first requirement is that deposits cannot easily be replaced as a means of funding. Should this not be the case, banks could easily offset higher costs of deposit funding by other means of financing, for example, by wholesale funding (Glocker & Towbin 2012).Wholesale funding can have many forms,but usually are carried out by means of raising brokered deposits (deposits made by third party deposit brokers in larger denominations which are then divided and sold to private customers), foreign deposits (deposits sold in foreign branches), or federal funds (borrowing from the Fed. at overnight rate to meet liquidity requirements on demand). Analysing the relations between reserve requirement changes and liability structure of banks for Brazil, Robitaille et al. (2011) finds that banks respond to increases in reserve requirements by substituting funding through deposits by issuing more deposits' certificates (deposits that cannot be withdrawn until fixed time period). The second requirement applies to firms where bank credit cannot frictionlessly be substituted with other means of financing. For instance, if this not to be the case and the bank credit could easily be replaced, an increase in reserve requirement would decrease the bank lending which would be offset by an expansion in other forms of liabilities, such as capital market funding (equity backed securities). This will leave investment decisions and assets of the private sector unchanged (Glocker & Towbin 2012). Motoki & Funchal (2009) and Zonenschain (1997) present a survey on the role of bank funding for firms in Brazil and their findings suggest that bank funding is important (cited in (Glocker & Towbin 2012)).

In emerging markets, a number of financial institutions implement reserve requirements as an additional policy tool, in addition to conventional monetary policy on the interest rate. Indeed, there are certain incentives behind monetary authorities

regulating reserve requirements over economic cycles. First of all, emerging economies are often cautious about interest rate increases as a response to credit growths that mainly financed by inflows of capital as they recognize that when interest rates are high, it can draw in even larger capital inflows and thereby appreciate the domestic currency. Thus, reserve requirements are considered as a mean of tightening internal financial conditions (see Reinhart & Reinhart (1999), Terrier et al. (2011)). As reserves requirements often are remunerated under market rates, a hike in reserve requirements serves as an implicit tax on the banks and raises the wedge between lending and deposit rates. When the spread is higher, this renders lending to domestic banks unattractive for foreign investors and increases the cost of borrowing for the domestic sector. The latter means that increases in reserve requirements may attain a tightening in domestic credit conditions, and avoid bringing capital inflows and appreciating the domestic currency.

In the context of above discussion, the central bank of Turkey uses the interest rate policy as a primary tool to stabilize the inflation while considers the reserve requirement policy as a primary instrument for financial stability objectives (Basi 2010). In this regard, the reserve requirement and reserve remuneration are explicitly applied as a macroprudential device. Brazil, Croatia, Peru, Columbia, and Russia among other countries that regulate reserve requirements for a related purpose as Turkey (Lim et al. (2011), cited in Glocker & Towbin (2012)). To the best of our knowledge, there is not extensive empirical and theoretical examination of the connection between reserve requirement policies and domestic credit and also external variables, particularly, of theoretical implications of this policy in models parameterized for advanced economies.

Next, as Glocker & Towbin (2012) points out, in theory, the reserve requirement policy can also be adopted as a tool to stabilize prices. Since a contractionary reserve requirement policy reduces the money multiplier, it follows that if the central bank maintains the stable monetary base, an increase in reserve requirement would reduce broad money and lead to a rise in interest rate, which in turn would lower inflation. Indeed, moderate levels of liquidity intervention can decrease the volatility of inflation. In fact, in many economies currently, there exists an interest rate or exchange rate objective where the monetary base responds endogenously to this targets. Within this setup, the impacts of an increase in reserve requirement on inflation are hence not entirely apparent from a theoretical standpoint Glocker & Towbin (2012). Nevertheless, market observers recognize that a number of economies, for instance, China, exercise reserve requirement management as a substitute for conventional interest rate policy to accommodate inflationary pressures Glocker & Towbin (2012). There is an

argument that reserve requirement increases may help both stabilize the economy and prices. However, there are those who argue the opposite, in that expansionary reserve requirement policies to address the capital outflows can act as a way of simulating the economy without generating inflationary pressure (e.g., Montoro & Moreno (2011)). Although our set-up is a closed economy model and is parametrized to match the U.S. data and thereby abstracts from the capital in/outflow concerns, we find that liquidity policy has a greater convergence pressure on inflation to the equilibrium for all shocks except the monetary policy shock where the liquidity policy greatly assists to stabilize inflation. Our main motivation is to examine the scope how such policies will affect the economy in developed markets.

To recap, we summarize the main motivations of the present research as follows. First, the aim is to shed light on important questions regarding effects of uncertainty shocks under financial frictions with heterogeneous firms that face the risk of default. Apart from considering individual effects of volatility shocks we also consider how the presence of stochastic volatility and the default risk affect the transmission of monetary, financial and other shocks. Therefore, the present study contributes to the above discussed and recently rising debate on effects of volatility shocks, especially the role of credit frictions on its transmission to the economy. In addition, we also analyse relevant policy questions to address the stability of model's variables and optimal policymaking in the presence of risk under expected and recursive preferences, and in the model that is solved, estimated, and simulated around the ergodic mean. We also look at implications of an unconventional monetary policy (liquidity or reserve requirement policy) and macroprudential for the model's dynamics, stability, and welfare. As discussed in this literature review effects of liquidity policies in a general equilibrium framework lacks thorough analyses which is another main motivation of the present study. Finally, we consider policy interactions in the collaborative and non-cooperative framework and with conflicting targets. Analyzing interactions among three policy choices in the presence of risk is another gap in the literature that the present study intends to address.

Our study considers interactions among three policy instruments (Traditional interest rate rule, LTV, Reserve Requirement policies). In the present, we show that when the central bank engages in active reserve requirement policy, there exists a Pareto optimal set of Nash strategies where institutional design is irrelevant (non-coordination does not result in worse outcomes than cooperation). This arises when the inflation parameter in the Taylor rule is fixed to its upper bound, with the central bank optimization solely based on reserve requirement policy as its best response.

Furthermore, the present study also shows that, in a traditional setting (Taylor rule versus LTV), complementing the monetary policy with fixed reserve requirement policy can substantially reduce losses in non-cooperative set-up, although this would also result in a sub-optimal Nash equilibrium.

3.3 Model

The model closely follows Pesaran & Xu (2016) with four key differences. First, we introduce stochastic volatility in the shock process. This allows for a variation in firm's default rate which is time-invariant in the original model. The change in firms' default probability is endogenously determined since the stochastic volatility alters the probability distribution of the level shock. Thus, we also track cumulative distributions of shocks dynamically that endogenously determine time evolution of the firm default rate. This simple extension to the model endogenously alters the intensity of financial rigidities. Thereby, the model differs from other related models by endogenously linking volatility and tightening in financial conditions.

Second, we introduce price stickiness to analyse interactions between uncertainty, nominal rigidities, and financial frictions. Nominal rigidities also add a time variation in firms' default rate through the marginal cost and the price dispersion. In consequence, policies and wider economic developments directly affect the default rate in our model. This leads to additional sequence of events how the monetary policy will be transmitted into the economy. Furthermore, price stickiness potentially can lead to inflationary pressure following a positive credit shock and/or macroprudential policy according to the prevailing belief. Therefore transmission of macroprudential policy or credit shocks will differ from the economy with flexible prices. We do not find any supportive evidence for presumed inflationary pressure in our estimated model with firm default following the credit shock. Price stickiness also introduces strategic conflict between objectives of the central bank and macro-prudential authority. Above all, extending the model with price stickiness has more fundamental consequences on the transmission of shocks. Nominal rigidities by affecting the default probability of firms also endogenously influence financial conditions. Consequently, this causes a feedback loop from the nominal side of the economy to the financial side. Thereby, a decrease in the marginal productivity of firms because of price stickiness increases the level of financial friction. Overall, stochastic volatility together with nominal rigidities influence financial conditions throughout the dynamic path but not only in the steady state. Another important feature of the model is that, because contracts for wages and capital purchases are signed before the production begins, firms are unable to perfectly adjust their inputs to changing prices and overall economic circumstances. Therefore, this timing convention will have similar effects to adjustment costs even without such rigidities attached to the capital accumulation. Whereas, with flexible prices, real prices of inputs remain unaffected and, thus, uncertainty will have a limited effect on economic outcomes.

Third, we adopt Epstein-Zin preferences in household's problem. This allows for an

explicit consideration of attitudes towards risk in the economy without fundamentally distorting dynamics of macroeconomic variables for higher risk-aversion coefficients. Furthermore, up to the second order approximation with new preferences, model dynamics are identical to the expected utility case. Finally, we include two types of additional policy rules into the model to complement the monetary policy. These are reserve requirement policy and macro prudential policy as an LTV rule .

We adopt third-order perturbation to solve policy rules as opposed to the log-linear approach implemented by Pesaran & Xu (2016). Although the model is risk sensitive for the log-linear solution alone (due to the variable default rate), the third order approximation allows internalising effects of uncertainty beyond the default channel as standard deviations of shocks enter to policy rules. We also approximate the model around the ergodic mean. Because of non-linearities in the model, the composition of the balance sheet of banks and lending rate (therefore the steady-state financial friction) is different around the non-deterministic point. This further affects the transmission of shocks in the model subject to the risk.

3.3.1 Households

There is a continuum of infinitely lived identical households with the following temporal utility function over streams of consumption and leisure as in Pesaran & Xu (2016).

$$U(C_t, N_t) = \frac{1}{1-\gamma}\left[\left(C_t - \frac{\chi_0}{1+\chi}N_t^{1+\chi}\right)^{(1-\gamma)} - 1\right] \tag{3.1}$$

where γ is the inverse of elasticity of substitution, χ is inverse of Frish elasticity of the labour supply, and χ_0 is the scaling parameter. This utility specification first introduced by Greenwood et al. (1988) which isolates the wealth channel from preferences. As a result, labour supply and wages co-move together.This makes it easy to solve for the labour supply for any given wage. However, we need to reparametrize the technology to render the model consistent with the balanced growth path, so it has a well-defined steady state along the trend growth. Following Epstein & Zin (1989) and Rudebusch and Swanson (2009) we assume following dynamics for the value function of the household.

$$V_t = U\left(C_t, N_t\right) + \beta\left[E_t\left(V_{t+1}\right)^{(1-\gamma_{EZ})}\right]\frac{1}{1-\gamma_{EZ}} \tag{3.2}$$

The parameter γ_{EZ} gives an additional degree of freedom to increase the risk aversion without affecting the intertemporal elasticity of substitution. That is, when $U(C_t, N_t)$ is positive higher values of γ_{EZ} implies higher risk aversion. The opposite is true when

$U(C_t, N_t)$ is negative (Svanson, 2010). Furthermore, when γ_{EZ} is equal to zero the value function reduces to the function implied by expected utility preferences. For negative values of the temporal utility the literature assumes the following sign convention for the value function.

$$V_t = U\left(C_t, N_t\right) - \beta \left[E_t \left(-V_{t+1}\right)^{(1-\gamma_{EZ})}\right] \frac{1}{1-\gamma_{EZ}} \tag{3.3}$$

Households maximize (3.3) subject to flow of funds that is given by following constraint:

$$P_t C_t = R_{d,t} D_t - D_{t+1} + P_t W_t N_t - P_T S_t + \Pi_{t,h} \tag{3.4}$$

where D_t is deposit holdings of household in banks at the beginning of the period, S_t equity investments in firms capital, R_d returns from deposits, $\Pi_{t,h}$ lump-sum transfers from firms profit at the end of the period at the realization of shocks. We also abstract from capital accumulation and attached rigidities, such as investment adjustment costs and capacity utilization. This allows to analyze independent effects of financial frictions in transmission of uncertainty shocks. Furthermore, it is common in banking models with financial frictions not to include capital accumulation. This assumption along with making the model more tractable also allows isolating effects of financial frictions and highlight interactions among financial frictions, nominal rigidities, and volatility in economic outcomes of the model. Household optimization yields following first order conditions

$$\Xi_{t+1} \frac{R_{d,t+1}}{\pi_t} = 1 \tag{3.5}$$

$$W_t = \chi_0 N_t^{\chi} \tag{3.6}$$

where Ξ_{t+1} is the stochastic discount factor of the household,

$$\Xi_{t+1} = \beta E_t \left[\frac{C_{t+1} - \frac{\chi_0}{1+\chi} N_{t+1}^{1+\chi}}{C_t - \frac{\chi_0}{1+\chi} N_t^{1+\chi}}\right]^{-\gamma} \left(\frac{-V_{t+1}^{1-\gamma}}{E_t V_{t+1}^{1-\gamma}}\right)^{-\gamma_{EZ}} \tag{3.7}$$

Where γ_{EZ} is the Epstein-Zin parameter and reflects the attitude of households towards the timing of the resolution of the risk. Thus, while γ is the risk aversion of the household across different states of the nature at the given point int time, γ_{EZ} describes the risk aversion of the household in the time dimension.

3.3.2 Wholesale firms

The production side of the economy is consists of ex-ante identical firms where production for each firm i is given by following technology:

$$Y_{i,t}^{W} = A_{i,t}^{\varphi} N_{i,t}^{1-\alpha} K_{i,t}^{\alpha} \tag{3.8}$$

where $Y_{i,t}^{W}$ is the output, $N_{i,t}$ and $K_{i,t}$ are labour and capital inputs for the firm i in period t. The technology variable, $A_{i,t}^{\varphi}$, combines a common business cycle component, Z_t, and an idiosyncratic factor, $\Lambda_{i,t}$, which are independently distributed.

$$A_{i,t} = Z_t \Lambda_{i,t} \tag{3.9}$$

where $\Lambda_{i,t}$ is the idiosyncratic productivity shock of the firm i. It is assumed that the common technology, Z_t, follows a random walk growth

$$Z_t = Z_{t-1} \exp\left(\mu + u_t\right) \tag{3.10}$$

with first-order autoregressive technology shock, u_t:

$$u_t = \rho_u u_{t-1} + \sigma_{u,t} \varepsilon_t \tag{3.11}$$

where $\sigma_{u,t}$ accounts for the stochastic volatility and follows an AR(1) process:

$$log(\sigma_{u,t}) = \rho_\sigma log(\sigma_{u,t-1}) + \varepsilon_{\sigma,t} \tag{3.12}$$

It is further assumed that firms choose capital and labour inputs before the aggregate technology shock is realised, at the beginning of the period. The capital is financed by households' equity investments $S_{i,t}$, and bank loans $L_{i,t}$:

$$K_{i,t} = L_{i,t} + S_{i,t} \tag{3.13}$$

The leverage ratio, $\upsilon_i \equiv \frac{K_{i,t}}{S_{i,t}}$, is exogenously given.Therefore, the share of capital financed by bank loans is given by

$$L_{i,t} = \left(1 - \frac{1}{\upsilon_i}\right) K_{i,t} \tag{3.14}$$

Households' profits or losses are revealed at the end of the period after shocks arrive. Banks cannot observe idiosyncratic shocks. Hence the same amount of loans are allocated for all firms which act as a measure of insurance in effect.

Firms' decide on levels of inputs, $K_{i,t}$, $N_{i,t}$ to maximize the discounted sum of profits

$$\max_{\{K_{i,t+j};N_{i,t+j};j=0,1,2,...\infty\}} E\left[\sum_{j=0}^{\infty}\Xi_{t+s}\Pi^{f}_{i,t+j}|\Omega_{i,t}\right] \tag{3.15}$$

where Ξ_{t+s} is the stochastic discount factor of the representative household (households own firms),$\Omega_{i,t}$ is the information set available for the firm i at the beginning of the period, and $\Pi_{i,t}$ is firm's profit that is given by

$$\Pi^{f}_{i,t} = Y^{W}_{i,t} - R_{k,t}K_{i,t} - W_t N_{i,t} \tag{3.16}$$

$R_{k,t}$ and $W_{i,t}$ are prices of capital and labour. Firm i's first order conditions for factor inputs equate their marginal products to their respective prices.

$$E\left[\frac{\alpha}{MP_t}\left(Z_t\Lambda_{i,t}\right)^{\varphi}\left(\frac{N_{i,t}}{K_{i,t}}\right)^{1-\alpha}|\Omega_{i,t}\right] = R_{kt} \tag{3.17}$$

$$E\left[\frac{(1-\alpha)}{MP_t}\left(Z_t\Lambda_{i,t}\right)^{\varphi}\left(\frac{K_{i,t}}{N_{i,t}}\right)^{\alpha}|\Omega_{i,t}\right] = W_t \tag{3.18}$$

Where MP_t is the marginal product. When common and idiosyncratic shocks are independently distributed we have

$$E\left[\left(Z_t\Lambda_{i,t}\right)^{\varphi}|\Omega_{i,t}\right] = E\left(e^{\varphi z_t}|\Omega_{i,t}\right)E\left(e^{\varphi\lambda_{i,t}}|\Omega_{i,t}\right) \tag{3.19}$$

where $z_t = \ln Z_t$ and $\lambda_{i,t} = \ln\Lambda_{i,t}$. Further $E\left(e^{\varphi z_t}|\Omega_{i,t}\right)$ and $E\left(e^{\varphi\lambda_{i,t}}|\Omega_{i,t}\right)$ can be replaced by their respective moment generating functions

$$M_{\lambda}(\varphi) \equiv E\left(e^{\varphi\lambda_{i,t}}|\Omega_{i,t}\right) \tag{3.20}$$

$$M_{z}(\varphi) \equiv E\left(e^{\varphi z_t}|\Omega_{i,t}\right) \tag{3.21}$$

Given that all firm are ex-ante identical and make choices before technology shocks arrive we have

$$\frac{K_{i,t}}{N_{i,t}} = \left[\frac{\alpha M_{\lambda}(\varphi)M_{z}(\varphi)}{MP_{t-1}R_{kt,}}\right]^{\frac{1}{1-\alpha}} e^{\frac{\varphi\left(z_{t-1}+\mu+\rho u_{t-1}\right)}{1-\alpha}}, \qquad \forall i \tag{3.22}$$

Because hiring and investment is done before the productivity shocks surface, firms choose inputs based on marginal productivity at the beginning of the period. In equilibrium $K_{i,t} = K_t$, $N_{i,t} = N_t$, $L_{i,t} = L_t$ for all firms. Using the ratio of firms' first order conditions, the optimal level of capital then is given by:

$$K_t = \frac{\alpha\chi_0}{1-\alpha}\frac{N_t^{1+\chi}}{R_{k,t}} \tag{3.23}$$

Households' optimal labour supply together with equations (3.22 , 3.23) gives optimal labour hours:

$$N_t = \left[\frac{(1-\alpha)}{\alpha\chi_0}\left[\alpha M_\lambda(\varphi) M_z(\varphi) MC_{t-1}\right]^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\chi}} (R_{kt})^{\frac{-\alpha}{(1-\alpha)\chi}} e^{\frac{\varphi(z_{t-1}+\mu+\rho u_{t-1})}{\chi(1-\alpha)}}$$

The economy contains a perfectly competitive industry producing wholesale goods. In period t, the representative firm rents K_{t-1} units of capital from banks, at a real rental rate of $r_{K,t}$ per unit, and hires H_t units of labour from households at a real wage of W_t per unit.

3.3.3 Retail Firms

The retail sector using homogeneous wholesale output produce differentiated goods for consumption in a monopolistic market. The retail sector allows to introduce nominal rigidities into the model.Households' total consumption is given by Dixit-Stiglitz (1977) aggregator:

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\zeta-1}{\zeta}} dj\right]^{\frac{\zeta}{\zeta-1}} \tag{3.24}$$

where each firm j produces a differentiated good $\{C_t(j)\}$, and ζ is the elasticity of substitution among these goods. The consumer chooses each product j with the price $P_t(j)$ to minimize the total expenditure:

$$\int_0^1 P_t(i)C_t(i)di \tag{3.25}$$

This implies following consumption demand for each differentiated good:

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\zeta} C_t \tag{3.26}$$

with P_t being the aggregate price index:

$$P_t = \left[\int_0^1 P_t(j)^{1-\zeta} di\right]^{\frac{1}{1-\zeta}} \tag{3.27}$$

When investment spending is also aggregated with same the elasticity of substitution the demand for output will be given by

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\zeta} Y_t \tag{3.28}$$

Price setting for each differentiated good follows Calvo (1983) contracts. At each time point, only $(1-\xi)$ fraction of firms can revise their prices, and the rest of retail firms have to fix prices of their goods as in previous period. We also assume, that at each period defaulted firms are replaced by the same number of new firms, so that proportion of re-optimizing firms remains constant in the economy. This allows for a closed-form solution for the steady-state with positive inflation. Therefore, given the real marginal cost and the probability of not re-optimizing, each firm maximizes it's discounted profit to solve for the optimal price at time t.

$$E_t \sum_{j=0}^{\infty} \xi^k \Xi_{t,t+k} Y_{t+k}(i) \left[P_t^0(j) - P_{t+k} MC_{t+k}\right] \tag{3.29}$$

subject to 3.28. Here $\Lambda_{t,t+j}$-is the stochastic discount factor of households. The solution should satisfy the following F.O.C.:

$$E_t \sum_{k=0}^{\infty} \xi^k \Xi_{t,t+k} Y_{t+k}(j) \left[P_t^*(i) - \frac{1}{1-(1/\xi)} P_{t+k} MC_{t+k}\right] = 0 \tag{3.30}$$

Thus, using the law of large numbers the evolution of price index can be written as:

$$P_{t+1}^{1-\xi} = \xi P_{t+1}^{*1-\xi} + (1-\xi) P_t^{1-\xi} \tag{3.31}$$

Following the F.O.C. and the evolution of price index, the nominal side of the economy, then, can be written by set of difference equations in nonlinear form:

$$\frac{P_t^*}{P_t} = \frac{\Omega_t}{\widetilde{\Omega}_t} \tag{3.32}$$

$$\widetilde{\Omega}_{t+1} - \beta\xi E_t \left[\Pi_{t+1}^{\xi-1} \widetilde{\Omega}_{t+1}\right] = Y_t V_{c,t} \tag{3.33}$$

$$\Omega_t - \beta\xi E_t \left[\Pi_{t+1\Omega_t t+1}^{\xi}\right] = \frac{1}{1-\frac{1}{1-\xi}} Y_t V_{c,t} MC_t e^{\varepsilon_{MC,t}} \tag{3.34}$$

$$\xi \Pi_t^{\xi-1} + (1-\xi)\left(\frac{\Omega_t}{\widetilde{\Omega}_t}\right)^{1-\xi} = 1 \tag{3.35}$$

$$\Delta_t = \xi \Pi^{\zeta} \Delta_{t-1} + (1-\xi)\left(\frac{\Omega_t}{\Omega_{t-1}}\right)^{-\zeta} \tag{3.36}$$

where Ω_t and $\widetilde{\Omega}_t$ are auxiliary variables to define the system, MC_t is the marginal cost, Π_t is the inflation, and Δ_t is the price dispersion. The mark up shock ($\varepsilon_{MC,t}$) in equation (3.34) multiplies the marginal cost.

3.3.4 Firm default and financial settlement

Firm i defaults under certain threshold of technology shocks when firm's returns after wage payments drop below its' callable liabilities $R_{l,t}L_t$. As all firms are ex-ante identical, the set up does not involve firm-specific collateral or monitoring by banks. Banks diversify loans across all firms or idiosyncratic shocks as a measure of insurance. Therefore, firm i defaults only and only if

$$\frac{P_t^i}{P_t} Y_{i,t} - W_t N_{i,t} < R_{k,t} L_{i,t} \tag{3.37}$$

where P^i is the price of goods for the firm i relative to the price in this economy and $Y_{i,t} = P_t Y_{i,t}^W$. To define the default probability let $\vartheta_{i,t} = \lambda_{it} + \varepsilon_t$, $\vartheta_{i,t} \sim iid\,(0, \sigma_{\vartheta}^2)$ where $\sigma_{\vartheta}^2 = \sigma_{\lambda}^2 + \sigma_{\epsilon}^2$. Then, the moment generating function for $\vartheta_{i,t}$ is given by:

$$M_{\vartheta}(\varphi) = E\left(e^{\varphi \vartheta_{i,t}} | \Omega_{i,t}\right) = M_{\lambda} M_{\epsilon} \tag{3.38}$$

Using first order conditions for capital and wages (equations 3.17, 3.18) together with the borrowing constraint of firms, the default condition can be expressed as

$$\begin{aligned} \frac{P_t^i}{P_t} e^{\varphi \vartheta_{i,t}} N_t^{1-\alpha} K_t^{\alpha} - M_{\vartheta,t}(\varphi)(1-\alpha) MC_{t-1} \left(\frac{K_{i,t}}{N_{i,t}}\right)^{\alpha} N_{i,t} < \\ \alpha M_{\vartheta,t}(\varphi) MC_{t-1} \left(\frac{K_{i,t}}{N_{i,t}}\right)^{\alpha-1} \left(1 - \frac{1}{\nu_i}\right) K_{i,t} \end{aligned} \tag{3.39}$$

Dividing both side of the inequality by $M_{\vartheta}(\varphi) N_{i,t} MC_t$ gives:

$$\begin{aligned} \frac{P_t^i}{P_t} e^{\varphi \vartheta_{i,t}} \left(\frac{K_{i,t}}{N_{i,t}}\right)^{\alpha} \left[M_{\vartheta,t}(\varphi) MC_{t-1}\right]^{-1} - (1-\alpha)\left(\frac{K_{i,t}}{N_{i,t}}\right)^{\alpha} < \\ \alpha \left(\frac{K_{i,t}}{N_{i,t}}\right)^{\alpha} \left(1 - \frac{1}{\nu}\right) \end{aligned} \tag{3.40}$$

Which simplifies to

$$\frac{P_t^i}{P_t} e^{\varphi \vartheta_{i,t}} \left[M_{\vartheta,t}(\varphi) MC_{t-1}\right]^{-1} - (1-\alpha) < \alpha \left(1 - \frac{1}{\nu}\right) \tag{3.41}$$

and this is further simplified as

$$\frac{P_t^i}{P_t} e^{\varphi \vartheta_{i,t}} \left[M_{\vartheta,t}(\varphi)\, MC_{t-1} \right]^{-1} < \left(1 - \frac{\alpha}{\nu} \right) \tag{3.42}$$

Noting that the RHS of the equation is always positive, the default can occur only if $\upsilon_i > \alpha$, regardless of productivity shocks. The the economy-wide price level (P_t) in NK models, however, is undetermined. Even so the inequality (3.42) can be transformed that allows to write the default condition in terms of economy-wide price dispersion. This done by raising both sides of the inequality (3.42) to the Calvo price resetting probability, ξ. For any positive ξ, then, this should also hold:

$$e^{\varphi \vartheta_{i,t} \xi} < \left(1 - \frac{\alpha}{\nu} \right)^{\xi} \left(\frac{P_t^i}{P_t} \right)^{-\xi} \left[M_{\vartheta,t}(\varphi)\, MC_{t-1} \right]^{\xi} \tag{3.43}$$

To find the threshold level of the shock ($\vartheta_{i,t}$) that causes default we take the natural log of boss sides of the equation (3.43)

$$\vartheta_{i,t} = \frac{\left[\ln\left(1 - \frac{\alpha}{\nu} \right) + \ln\left(M_{\vartheta,t}(\varphi) \right) + \ln(MC_{t-1}) \right] + \frac{1}{\xi} \ln\left[\left(\frac{P_{i,t}}{P_t} \right)^{-\xi} \right]}{\varphi} \tag{3.44}$$

where

$$\frac{1}{m} \sum_{i=1}^{m} \ln\left[\left(\frac{P_{i,t}}{P} \right)^{-\xi} \right] = \Delta_t \tag{3.45}$$

as m $\to \infty$. Therefore, when the default threshold is integrated over all firms the mean default threshold technology converges to

$$\vartheta_t = \frac{\ln\left(1 - \frac{\alpha}{\nu} \right) + \ln M_{\vartheta,t}(\varphi) + \ln MC_{t-1} + \frac{1}{\xi} \ln(\Delta_t)}{\varphi} \equiv \mu_1 \tag{3.46}$$

Therefore the firm defaults if aggregate technology shock (idiosyncratic and common) falls below μ_1. Given the moment generating function,

$$M_{\vartheta,t} = \exp\left(\frac{\varphi^2 \sigma_{\vartheta,t}^2}{2} \right) \tag{3.47}$$

we can find the probability of default by normalizing the default threshold by the standard deviation of the aggregate technology shock

$$\kappa = P\left(\vartheta,t < \mu_1 \right) = \Phi\left(\frac{\ln\left(1 - \frac{\alpha}{\nu} \right) + \ln(MC_{t-1}) + \frac{1}{\xi} \ln(\Delta_t)}{\varphi \sigma_{\vartheta,t}} + \frac{\varphi \sigma_{\vartheta,t}}{2} \right) \tag{3.48}$$

where $\Phi()$ is the cumulative distribution function of a standard normal. We also allow for variations in $\sigma_{\vartheta,t}$. Thus the default probability becomes time variant as opposed to the fixed default probability in Pesaran & Xu (2016). The aggregate default probability also depends on nominal rigidities in the economy (marginal cost and price dispersion). For the positive steady-state inflation and non-linear approximation $Delta_t$ is bigger than unity, thereby it increases the default probability. The monopolistic competition in the intermediate goods sector (a positive markup), however, reduces the default probability in the deterministic steady state ($MC < 1$, $\ln(MC) < 0$). All the same, an increase in the marginal cost also increases the default condition dynamically. The default probability also depends on firm's leverage ratio, ν. Higher leveraged firms increase the default probability.

We now distinguish two possible outcomes. First, the firm does not default, with probability of $(1-\kappa)$, banks receive the principal with interest and households receive profits from firms.

In the second scenario, when the firm defaults, depending on the realization of technology shocks several sub-cases arise. If the revenue after wage payments $(Y_{i,t} - W_t N_{i,t})$ is positive, banks seizes the revenue of the defaulted firm and households receive zero transfers. When the revenue after wage repayment is negative banks does not get any payment and households cover rest of costs. This happens when

$$M_{\vartheta,t}^{-1}(\varphi)\, e^{\varphi\vartheta_{i,t}} < 1-\alpha$$

The threshold realized technology for having a negative after wage payments revenue, then, is given by

$$\vartheta_{i,t} = \frac{\ln(1-\alpha) + M_{\vartheta,t}(\varphi) + \ln(MC_{t-1}) + \frac{1}{\xi}\ln(\Delta_t)}{\varphi} \equiv \mu_2$$

The corresponding probability for this can be written as a function of the cumulative distribution of a standard normal

$$\tau = P(\vartheta,t < \mu_w) = \Phi\left(\frac{\ln(1-\alpha) + \ln(MC_{t-1}) + \frac{1}{\xi}\ln(\Delta_t)}{\varphi\sigma_{\vartheta,t}} + \frac{\varphi\sigma_{\vartheta,t}}{2}\right)$$

Thus with probability τ, households receive a negative transfer from firms $\left(\Pi_t^h = Y_{i,t} - W_t N_{i.t} < 0\right)$ and banks' payment is zero $\left(\Pi_t^b = 0\right)$. Next section describes aggregate outcomes in the economy.

3.3.5 Aggregation

Aggregate outcomes in this economy is same as in Pesaran & Xu (2016). Using the law of large numbers the aggregate output of the wholesale firms is given as,

$$Y_t^W = \frac{1}{m}\sum_{i=1}^{m} Y_{i,t} = \left(\frac{1}{m}\sum_{i=1}^{m} e^{\lambda_{i,t}\varphi}\right) A_t^{\varphi} N_t^{1-\alpha} K_t^{\alpha} \tag{3.49}$$

Given that $e^{\lambda_{i,t}\varphi}$ is identically and independently distributed then by the law of large number we have,

$$\frac{\sum_{i=1}^{m} e^{\lambda_{i,t}\varphi}}{m} \quad \overrightarrow{p} \quad E\left[e^{\lambda_{i,t}\varphi}\right] = M_{\lambda} \tag{3.50}$$

where $E\left[e^{\lambda_{i,t}\varphi}\right]$ is the expected cross-sectional productivity. Therefore, the aggregate output of the wholesale firms can be written as

$$Y_t^W = M_{\lambda} A_t^{\varphi} N_t^{1-\alpha} K_t^{\alpha} \tag{3.51}$$

Since households and banks receive transfers from firms after the realization of technological innovations, we also need to aggregate these transfers. Let, $\Pi_{h,t}$ and $\Pi_{b,t}$ denote average transfers to households and banks, respectively. Using that the Walras law (when n markets clear then $n-1$ markets also clear) we only consider the mean profit of banks. The average profit of banks is given as

$$\Pi_{t,b} = \frac{\sum_{i=1}^{m} d_{i,t} Max(0, Y_{i,t}^W - W_t N_{i,t}) + \sum_{i=1}^{m}(1-d_{i,t}) L_{i,t} R_{k,t}}{m} \tag{3.52}$$

which simply means that banks earn any remaining output after wage payments or nothing in case of default ,and full lent money with the interest from firms which do not default. The max operator in above equation can be replaced with the following expression by letting $1(\cdot)$ to denote an indicator function,

$$1(\mu_2 < \vartheta_{i,t} < \mu_1)[Y_{i,t} - W_t N_{i,t}] + 1(\vartheta_{i,t} < \mu_2)0 \tag{3.53}$$

Since $d_{i,t}$ can also be expressed as an indicator function $d_{i,t} = 1(\vartheta_{i,t} < \mu_1)$

$$d_{i,t} \times 1(\mu_2 < \vartheta_{i,t} < \mu_1)[Y_{i,t} - W_t N_{i,t}] = 1(\mu_2 < \vartheta_{i,t} < \mu_1)[Y_{i,t} - W_t N_{i,t}] \tag{3.54}$$

Therefore this can be written as

$$1(\vartheta_{i,t} < \mu_1)\left[Y_{i,t} - W_t N_t\right] - 1(\vartheta_{i,t} < \mu_2)\left[Y_{i,t} - W_t N_t\right] \tag{3.55}$$

Decomposing the aggregate technology to idiosyncratic and common components ($\vartheta = \lambda_{i,t} + u_t$) allows to find average output of defaulted firms with $\vartheta_{i,t} < \mu_1$ and $\vartheta_{i,t} < \mu_2$. Therefore,

$$\frac{\sum_{i=1}^{m} e^{\lambda_{i,t}\varphi} 1(\lambda_{i,t} < \mu_1 - u_t)}{m} \quad \overrightarrow{p} \quad \int_{-\infty}^{\mu_1 - u_t} e^{\varphi\lambda} f_\lambda(x)dx \tag{3.56}$$

where $f(x)$ is the density of idiosyncratic technology. Normalizing this by the standard deviation of the idiosyncratic technology allows for a closed form solution for this integral,

$$\int_{-\infty}^{\mu_1 - u_t} e^{\varphi\lambda} f_\lambda(x)dx = M_\lambda \iota_1(u_t) \tag{3.57}$$

where

$$\iota_1 = \Phi\left(\frac{\mu_1 - u_t - \sigma_\lambda^2\varphi}{\sigma_\lambda}\right) \tag{3.58}$$

and M_λ is given by equation (3.20). Therefore the mean output of defaulted firms is given by $\iota_1(u_t)Y_t^W$. Similarly the average output of the defaulted firms that receive a negative profit after wage payments is given by $\iota_2(u_t)Y_t^W$,where ι_2 is derived using same steps and given as,

$$\iota_2 = \Phi\left(\frac{\mu_2 - u_t - \sigma_\lambda^2\varphi}{\sigma_\lambda}\right) \tag{3.59}$$

Finally, using the law of large numbers gives expected values of indicator functions,

$$\frac{\sum_{i=1}^{m} 1(\vartheta < \mu 1)}{m} \quad \overrightarrow{p} \quad \kappa \tag{3.60}$$

$$\frac{\sum_{i=1}^{m} 1(\vartheta < \mu 2)}{m} \quad \overrightarrow{p} \quad \tau \tag{3.61}$$

Therefore banks profits aggregate to:

$$\Pi_{B,t} = (1-\kappa)R_{k,t}L_t - (\kappa - \tau)W_t N_t + \big[\iota_1(u_t) - \iota_2(u_t)\big]Y_t^W \tag{3.62}$$

3.3.6 Banks

Banks raise deposits from households and channel it to firms. We assume retail banks operate in a perfectly competitive market and there is no friction between households and financial intermediaries. Therefore, the deposit rate is same as the policy rate set by the central bank. Following, Pesaran & Xu (2016) we assume that banks may need to keep minimum deposit reserves $(1-\theta)$ with the central bank

$$L_t = \theta_t D_t, \tag{3.63}$$

where θ is exogenously given. Pesaran & Xu (2016) proposes three calibrations for this parameter; 0.95, 1, and 1.05. However, even the highest calibrated value for the parameter θ generates spread larger than observed in data, in Pesaran & Xu (2016) (5%, quarterly). The value of θ bigger than unity ($\theta > 1$) is also possible when banks are allowed to issue securities that are not backed by deposits. Alternatively, it may also be possible due to deposits in foreign branches of domestic banks. Such deposits are not subject to the deposit insurance or reserve requirements. Loose restrictions on foreign deposits are often justified to compete with offshore banking cnetres and thus Federal Reserve does not apply reserve requirements to foreign liabilities of the US commercial banks. Another possibility is extra liquidity coming with countries with large liquidity surpluses. Historically, lending to deposit ratio in the U.S. fluctuated around unity (Pesaran & Xu, 2016). However, for our sample period, it was above one starting from ends of 70's and substantially increased till 2001 (Figure 3.1.) [3]. With the onset of the financial crisis, the ratio rapidly dropped in 2007. Over the period what is referred as a "Great Moderation" banks leverage ratio in the economy was substantially high despite a relatively low level of inflation.

Although there is a minimum reserve requirement in the US, it applies only to depository institutions with higher net transactions and has not been actively used as a policy instrument. Nevertheless, we assume that the central bank can influence the supply of credit into the economy through the variable θ_t which serves as an additional policy tool. For example, during expansions, the central bank can cut the supply of credit by increasing deposit reserve requirement $(1-\theta_t)$ to reduce the inflationary pressure on economy (Pesaran & Xu, 2016). Or when the lending risk is high, the central bank can also increase the deposit requirement ratio to curb potential bank losses. Alternatively, θ can be thought as a macro-prudential instrument through which financial regulator can target the level of loans channeled into the economy, to mitigate

[3]For the data source refer to the estimation section

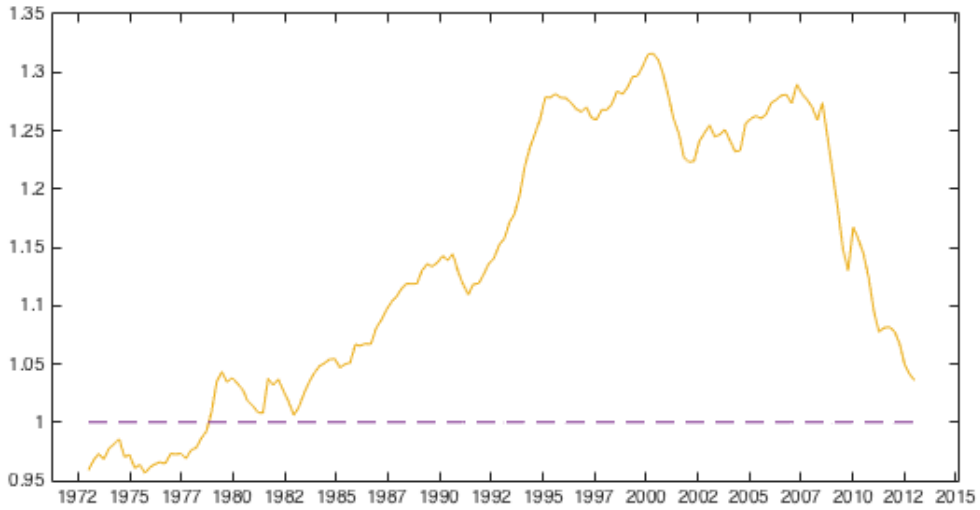


Figure 3.1: Lending to deposit ratio

effects of credit cycles. We also allow for exogenously driven credit shocks that affect the leverage of banks.

$$\theta_t = \rho_\theta \theta_{t-1} + \varepsilon_{\theta,t} \tag{3.64}$$

Banks to be solvent the end of period asset position of banks needs to be greater or equal to bank's liabilities. Perfectly competitive banking sector will earn zero profits, thereby, the above condition binds,

$$R_{d,t} D_t = \Pi_{t,b} \tag{3.65}$$

Perfectly competitive banking sector also equates the deposit rate and the rate of interest set by the central bank.

$$R_{d,t} = R_{n,t} \tag{3.66}$$

Finally, we close the model by a Taylor rule. The central bank sets the nominal interest rate in response to current inflation and the output gap. We also allow for some degree of inertia in this rule. Below we plot determinacy regions of our model under such policy. All free parameters are fixed to their calibrated values (3.1: On the left graph we fix the interest rate smoothing parameter to 0.5. On the right graph we fix the output parameter to 0.2).

$$R_{n,t} = {R_{n,t-1}}^{\rho_r} \left[\left(\frac{Y_t}{\overline{Y}} \right)^{\rho_y} \left(\frac{\Pi_t}{\overline{\Pi}} \right)^{\rho_\pi} \right]^{1-\rho_r} e^{\varepsilon_{M,t}}$$

where $\varepsilon_{M,t}$ is the monetary policy shock. As figure (3.2) shows, higher output targeting pushes the model either to indeterminacy or instability when combined with low inflation targeting. Whereas the inertia in the monetary policy pushes the inflation targeting higher for the model to have a determinate solution.

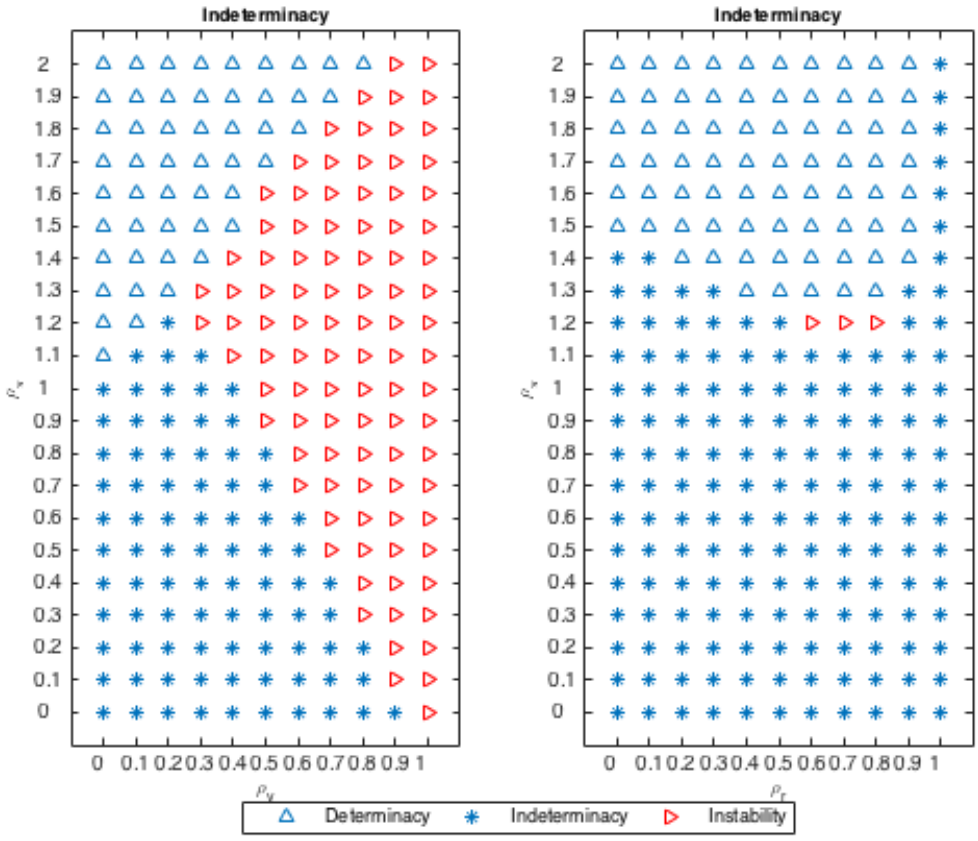


Figure 3.2: Determinacy regions

3.3.7 Steady State Economy

As the model assumes a unit root technological growth with a deterministic growth rate (μ),we scale endogenous variables ($C_t, Y_t, N_t, K_t, L_t, D_t, S_t$) with the technology, A_{t-1} such that the transformed variables are stationary along the balanced growth path. Thus endogenous variables in efficiency units are given as: $\dot{C}_t = \frac{C_t}{A_{t-1}}, \dot{Y}_t = \frac{Y_t}{A_{t-1}}, \dot{N}_t = \frac{N_t}{A_{t-1}}, \dot{K}_t = \frac{K_t}{A_{t-1}}, \dot{L}_t = \frac{L_t}{A_{t-1}}, \dot{D}_t = \frac{D_t}{A_{t-1}}, \dot{S}_t = \frac{S_t}{A_{t-1}}$. Whereas, wages are scaled by $A_{t-1}^{\frac{\chi}{1+\chi}}$, $\dot{W}_t = \frac{N_t}{A_{t-1}^{\frac{\chi}{1+\chi}}}$, and hours is scaled by $A_{t-1}^{\frac{1}{1+\chi}}$, $\dot{N} = \frac{N_t}{A_{t-1}^{\frac{1}{1+\chi}}}$. Therefore all variables in efficiency units are ergodic, despite the unit root technology process and properties of the log normal distribution can be used to write the steady state of shocks as a function of their mean and standard deviations.

Pesaran & Xu (2016) also assumes that interest rates in this economy ($R_{d,t}$, $R_{k,t}$) are bounded in probability ($R_{d,t} = O_p(1), R_{k,t} = O_p(1)$). This ensures the existence of the steady state of the economy and thus stationariness along the balanced growth path. They derive the condition for the existence of the steady state (to have stochastically bounded interest rates) of the economy defined by F.O.C. of households and stochastic processes which restricts the exponent of the technology process $A_{i,}$

$$\psi = \frac{(1-\alpha)\chi}{1+\chi} \tag{3.67}$$

The full derivation of the steady state is given in Appendix 2.1.

3.3.8 Full de-trended Model

$$V_t = \big(1-\beta(1+\mu)^{1-\gamma}\big)\left(\frac{1}{1-\gamma}\right)\left[\left(C_t - \frac{\chi_0}{1+\chi}N_t^{1+\chi}\right)^{1-\gamma} - 1\right] - \\ -\beta g_{t+1}^{1-\gamma}\left[-\frac{V_{E,t}}{V_{norm}^{1-\gamma_{EZ}}}\right]^{\frac{1}{1-\gamma_{EZ}}} \tag{3.68}$$

$$V_{E,t} = -\big[-V_{norm}V_{t+1}\big]^{1-\gamma_{EZ}} \tag{3.69}$$

$$V_{norm} = -\frac{1}{\bar{V}} \tag{3.70}$$

$$\Xi_{t+1} = \beta\big(g_{t+1}\big)^{-\gamma}\left[\frac{C_{t+1} - \frac{\chi_0}{1+\chi}N_{t+1}^{1+\chi}}{C_t - \frac{\chi_0}{1+\chi}N_t^{1+\chi}}\right]^{-\gamma_{EZ}}\left[\frac{-\big(-V_{t+1}V_{norm}\big)}{V_{E,t}}\right]^{-\gamma_{EZ}} \tag{3.71}$$

$$\Xi_{t+1}R_{f,t+1} = 1 \tag{3.72}$$

$$R_{f,t} = \frac{R_{d,t}}{\Pi_{t-1}} \tag{3.73}$$

$$K_t = \chi_0\frac{\alpha}{1-\alpha}\frac{N^{1+\chi}}{R_{k,t}} \tag{3.74}$$

$$N_t = \left[\frac{1-\alpha}{\alpha\chi_0}\big(\alpha M_\lambda M_\epsilon MC_{t-1}\big)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\chi}} exp\left[\frac{\mu(1-\rho_u)}{1+\chi}\right]R_{k,t}^{\frac{-\alpha}{(1-\alpha)\chi}}g_{t-1}^{\frac{\rho_u}{1+\chi}} \tag{3.75}$$

$$g_t = (1+\mu)e^{u_t} \tag{3.76}$$

$$u_t = \rho_u u_{t-1} + \sigma_u\varepsilon_{u,t} \tag{3.77}$$

$$\sigma_{u,t} = \rho_{\sigma,u}\sigma_{u,t-1} + \varepsilon_{\sigma,t} \tag{3.78}$$

$$x_{\kappa,t} = \frac{\ln\left(1-\frac{\alpha}{\upsilon}\right) + \ln\left(MC_{t-1}\right) + \frac{1}{\xi}\ln(\Delta_t)}{\varphi\sigma_{\vartheta,t}} + \frac{1}{2}\varphi\sigma_{\vartheta,t} \tag{3.79}$$

$$\kappa_t = \Phi\big(x_{\kappa,t}\big) \tag{3.80}$$

$$x_{\tau,t} = \frac{\ln\left(1-\alpha\right) + \ln\left(MC_{t-1}\right) + \frac{1}{\xi}\ln(\Delta_t)}{\varphi\sigma_{\vartheta,t}} + \frac{1}{2}\varphi\sigma_{\vartheta,t} \tag{3.81}$$

$$\tau_t = \Phi\big(x_{\tau,t}\big) \tag{3.82}$$

$$\sigma_{\vartheta,t} = \sqrt{\sigma_\lambda^2 + \sigma_{u,t}^2} \tag{3.83}$$

$$M_\lambda = e^{\frac{\varphi^2\sigma_\lambda^2}{2}} \tag{3.84}$$

$$M_{u,t} = e^{\frac{\varphi^2\sigma_{u,t}^2}{2}} \tag{3.85}$$

$$L_t = (1 - \frac{1}{\nu})K_t \tag{3.86}$$

$$K_t = L_t + S_t \tag{3.87}$$

$$L_t = \theta D_t \tag{3.88}$$

$$Y_t^W = M_\lambda g_t^\varphi K_t^\alpha N_t^{1-\alpha} \tag{3.89}$$

$$R_{d,t}D_t = \big(1 - \kappa_t\big)R_{k,t} - \chi_0\big(\kappa_t - \tau_t\big)N_t^{1+\chi} \tag{3.90}$$

$$Y_t = \left(1 - \frac{1}{\zeta}\right)\frac{Y_t^W}{\Delta_t} \tag{3.91}$$

$$Y_t - C_t = S_t + D_{t+1}\frac{g_{t+1}}{\Pi_{t+1}} \tag{3.92}$$

$$\frac{P_t^*}{P_t} = \frac{\Omega_t}{\widetilde{\Omega}_t} \tag{3.93}$$

$$\widetilde{\Omega}_{t+1} - \Xi_{t+1}\xi E_t\left[\Pi_{t+1}^{\xi-1}\widetilde{\Omega}_{t+1}\right] = Y_t V_{c,t} \tag{3.94}$$

$$\Omega_t - \Xi_{t+1}\xi E_t\left[\Pi_{t+1\Omega_t t+1}^{\xi}\right] = \frac{1}{1 - \frac{1}{1-\xi}}Y_t V_{c,t} MC_t e^{\varepsilon_{MC,t}} \tag{3.95}$$

$$\xi\Pi_t^{\xi-1} + (1-\xi)\left(\frac{\Omega_t}{\widetilde{\Omega}_t}\right)^{1-\xi} = 1 \tag{3.96}$$

$$\Delta_t = \xi\Pi^\zeta\Delta_{t-1} + (1-\xi)\left(\frac{\Omega_t}{\Omega_{t-1}}\right)^{-\zeta} \tag{3.97}$$

$$R_{n,t} = {R_{n,t-1}}^{\rho_r} \left[\left(\frac{Y_t}{\bar{\bar{Y}}} \right)^{\rho_y} \left(\frac{\Pi_t}{\bar{\bar{\Pi}}} \right)^{\rho_\pi} \right]^{1-\rho_r} e_{M_t} \tag{3.98}$$

$$R_{d,t} = R_{n,t} \tag{3.99}$$

3.4 Calibration/Estimation

Existing standard techniques for estimating linear models are ill-suited in estimating the present non-linear DSGE model. The reason for this is manifold. First, the techniques for estimating linear models only work because linear transformations of Gaussian random variables are themselves Gaussian. However, once one moves away from a linear Gaussian setup, the tricks that enabled estimation of linear models no longer work, and one is often left with no choice but to perform simulations of the model of interest for each point in the parameter space (Deak S (2014)). Second, non-linear estimation is made particularly difficult because in non-linear models, generally, one cannot analytically solve for the distribution of the time-path of unobserved states, given the data Deak S (2014). Without knowing where the states were in the past, one potentially has to not only integrate over the space of all parameters but also over the space of all possible paths for the state variables Deak S (2014). Third, since our DSGE model has many state variables, this integral will be high-dimensional, and thus also very expensive to evaluate numerically. An obvious implication of all that precede is that is that one can no longer capture the effects of shock realizations subject to risk.

The shape of likelihood in non-linear DSGE models is unknown and often approximated by methods with poor convergence properties. In this section, we combine several most recent techniques that allow estimating the present non-linear model efficiently. First, we exploit the structure of the third order pruned solution to write the model that is linear in an augmented state as in Kollmann (2013). Second, to gain further accuracy, we approximate the likelihood around the ergodic mean. The estimation strategy is largely based on Deak S (2014), and on the toolkit, by T.Holden [4] and performed in Dynare. To explain our estimation strategy we give background information on filtering and discuss potential obstacles to approximate the likelihood of the model and how we circumvent around these problems. For this purpose, let us consider a general set-up for a non-linear DSGE model where the solution takes the form of a following law of motion:

[4]The toolkit includes routines which originally proposed by Lan & Meyer-Gohde (2013) and Meyer-Gohde (2014) for nonlinear moving average representation of the state-space model and approximation around non-deterministic steady-state with some modifications by T. Holden. The toolkit of T. Holden also includes functions to transform the non-linear system to linear augmented state-space, *gaussian_approximation.m*, and makes few changes in Dynare internal files to declare new options ($non_central_approximation$,*gaussian_approximation.m*). More information and the link to the toolkit can be found on author's web page: $https://www.tholden.org$

$$x_t = f(x_{t-1}, \epsilon_{t-1}) \tag{3.100}$$

We observe $y_t = g(x_t)$. Then for a given, x_{t-1} observables are related to model shocks as $y_t = g(f(x_{t-1}, \epsilon_t))$. A necessary condition to have a solution for the latter non-linear system is $dim(y_t) >= dim(\epsilon_t)$. Provided an existence of a solution, we can link model's shocks to observables as $y_{t-1} = h_{t-1}(\epsilon_t)$, where $h_{t-1}(e) = g(f(x_{t-1}, e)$. However, it is usually expensive to invert h. Many macro-economics models, in practice, often feature complex non-linearities for which analytical inverses do not exist, and many of equations of the model contain expectations, necessitating computationally expensive numerical methods. Nonetheless, if we can evaluate underlying shock processes then it is straightforward also to evaluate the likelihood of the model given the parameters and the data, $f(y_t|\theta, x_t)$. However, we are interested in the unconditional likelihood, $f(y_t|\theta)$, to be passed to our estimation routine:

$$f(y_t|\theta) = \int f(y_t|\theta, x_t) f(x_t|\theta) dx_t \tag{3.101}$$

Therefore, to find the unconditional likelihood, we also need to know the distribution of initial states, given the parameters of the model. When shocks are Gaussian, for some states it may be possible to derive their stationary distributions which largely simplifies integration. However, when states are not linear in shocks, the derivation of their stationary distribution is not feasible without substitutional approximation to select equations of the model. Other obstacles are to infer shocks using forward-looking equations and to infer future dated shocks (Deak S 2014).

Filtering avoids the necessity of recovering underlying, true shocks processes to estimate the states of the model. It also makes it feasible to approximate the likelihood in models with more shocks than observables. To see how filtering works, let x_t be a stochastic Markov process, where the time evolution of x_t is fully described by the transition density $f(x_{t+1}|x_t)$. Our information set is given by $Y_t = [y_t, y_{t-1}, ...]$ where observables, y_t, are also entirely determined by some observation density $f(y_t, x_t)$. Filtering algorithms use this information to evaluate the predictive density of the future state with predict and update steps. Thus given the Markov property of x_t and by the law of conditional probability, $f(x_{t+1}|Y_t)$, can be written in terms of the transition density and the density of the current state:

$$
\begin{aligned}
f(x_{t+1}|Y_t) &= \int f(x_{t+1}, x_t|Y_t)dx_t \\
&= \int f(x_{t+1}, x_t|Y_t)f(x_t|Y_t)dx_t \\
&= \int f(x_{t+1}|x_t)f(x_t|Y_t)dx_t
\end{aligned}
\tag{3.102}
$$

Next, given $f(x_{t+1}|Y_t)$ and new information set (Y_{t+1}) we update the estimate of the next period's state using Bayes' rule:

$$
\begin{aligned}
f(x_{t+1}|Y_{t+1}) &= \frac{f(x_{t+1}, y_{t+1}|Y_t)}{f(x_{t+1}|Y_t)} \\
&= \frac{f(y_{t+1}|x_{t+1}, Y_t)f(x_{t+1}, Y_t)}{f(y_{t+1}|Y_t)} \\
&= \frac{f(y_{t+1}|x_{t+1})f(x_{t+1}, Y_t)}{f(y_{t+1}|Y_t)} \\
&\propto f(y_{t+1}|x_{t+1})f(x_{t+1}, Y_t)
\end{aligned}
\tag{3.103}
$$

In nonlinear models, there is no closed form solution to state transition density (3.102). Besides, for DSGE models with many state variables, this will be a high dimensional integral. One can not only sample from, but even evaluate it. Given a suitable approximation for distributions, Monte Carlo sampling is often very efficient to evaluate high dimensional integrals such as one in the predicting step. Fernandez-Villaverde & Rubio-Ramirez (2005, 2007) use a particle filter to approximate the log-likelihood function and the state transition state in a nonlinear DSGE model with potentially non-gaussian shocks through repeated use of importance sampling and resampling. Following their work, the most common choice for distributional approximation in non-linear filtering framework has been the particle filter algorithm. Particle filter picks up an approximation for a distribution under which Monte Carlo is most natural. Simply, the particle filter approximates the distribution of the state at a given time by a vector of points. For example, the distribution of x_t is approximated by n particles $x_{1,t}, x_{2,t}, ..., x_{n,t}$ and their associated weights $\omega_{1,t}, \omega_{2,t}, ..., \omega_{n,t}$:

$$
\begin{aligned}
f(x_{t+1}|Y_{t+1}) &\propto f(y_{t+1}|x_{t+1}) \int f(x_{t+1}|x_t)f(x_t|Y_t)dx_t \\
&\approx f(y_{t+1}|x_{t+1}) \sum_{i=1}^{n} f(x_{t+1}|x_{i,t})\omega_{i,t}
\end{aligned}
\tag{3.104}
$$

Perfect sampling from $f(x_{t+1}|y_{t+1,x_t})$ cannot directly be applied in the particle filter. To sample from this distribution, particle filter utilizes importance sampling where an

unknown distribution is replaced by a proposal distribution, $\pi(x_{t+1}|Y_{t+1}, x_t)$, from which one can sample. The transition prior, $f(x_{t+1}|x_t)$ is often used as a proposal distribution (An (2005), Strid et al. (2006), Chang et al. (2007), and An & Schorfheide (2007)). First, sampling from it is easy and in effect it is just equivalent to simulating the model. Second, it simplifies the calculation of weights in the next step. Importance sampling algorithm first draws observations, $x_{i,t}$, from a proposal distribution, $\pi(x_{t+1}|Y_{t+1}, x_t)$, and then constructs weights for each point. However, this proposal distribution is suboptimal, because it does not utilize new information.

$$\omega_{i,t+1} = \omega_{i,t} \frac{f(y_{t+1}|x_{i,t+1}) f(x_{i,t+1}|x_{i,t})}{\pi(x_{t+1}|Y_{t+1}, x_t)} \tag{3.105}$$

Then all weights are normalised to construct a proper density:

$$\tilde{\omega}_{i,t+1} = \frac{\omega_{i,t+1}}{\sum_{j=1}^{n} \omega_{j,t+1}} \tag{3.106}$$

When the transition prior $f(x_{i,t+1}|x_t)$, replaces the proposal distribution, $\pi(x_{t+1}|Y_{t+1}, x_t)$, equation (3.105) simplifies to

$$\omega_{i,t+1} = \omega_{i,t} f(y_{t+1}|x_{i,t+1}) \tag{3.107}$$

In effect, after carrying forward each particle by the same law of motion, each of them will have a related probability of observing a new y_{t+1}. Then these probabilities (weights) are normalized by the sum of all weights and transformed into the density.The sequential importance sampling also requires recalculating weights when a new observation is available. Without resampling the standard deviation of the importance weights increase over time and after few iterations all but only one of normalized weights practically converge to zero. Re-sampling circumvents this problem, by drawing new particles from the previous ones in proportion to their weights. For instance, particles with higher importance weights are passed over for the next sampling.

However, in practice, the particle filter only provides an accurate fit to the distribution with infeasibly many particles. Andreasen (2008) finds persistent inaccuracy using the particle filter in models with many state variables even with reasonably many particles. This is because using the transition distribution as a proposal distribution in importance sampling is suboptimal. Therefore, a great deal of inaccuracy may still remain even with increasing number of particles (Andreasen, 2010). The standard particle filter is suboptimal because the state transition as a proposal distribution ($\pi(x_{t+1}|Y_{t+1}, x_t) \approx f(x_{t+1}|x_t)$) does not use information from new observations. This is particularly an important drawback when new observations contain useful information

about the state vector which the proposal distribution omits. It is, specifically important, when small changes in the state drive large variations in observables or when there is a higher signal to noise ratio in the data. Omitting new observations from the proposal distribution in importance sampling also makes the particle filter easily affected by outliers, because the posterior distribution, in that case, will be centered in the tail of state transition distribution (Andreasen, 2010). Therefore, in the presence of state outliers, the support overlap between the posterior and proposal distribution will be poor, resulting in uneven distribution of importance sample weights. Because many particles will end up with zero weights, one needs to increase further the number of particles for sufficiently accurate approximation of the $(\pi(x_{t+1}|Y_{t+1}, x_t)$. Although there are some improvements on standard particle filter to bring new information to the proposal distribution, these alternatives often add more computational cost to approximate the proposal distribution. For example, Doucet et al (2000) use Extended Kalman filter (EKF) within the particle filter where they iterate each particle through the EKF to generate a new probability distribution so that the mean and covariance matrix of this distribution include information from new observations.

$$\pi(x_{t+1}|Y_{t+1}, x_t) = N(\hat{x}_{t+1}^{EKF(i)}, \hat{P}_{xx}^{EKF(i)}(t+1)), \quad \text{for all } i = 1...N \tag{3.108}$$

where $\hat{x}_{t+1}^{EKF(i)}$ is the posterior mean and, $\hat{P}_{xx}^{EKF(i)}(t+1))$ is the posterior covariance matrix of the particle i. This, in turn, moves particles to areas of higher probability in sampling. Although Doucet et al. (2000) finds that this gives more accurate estimates for states than the standard particle filter, the mean and the covariance matrix needs to be calculated for each particle at each time and for a large number of them in total, thus rendering the inference almost infeasible within a reasonable time. Another equally critical issue is that approximations of the mean and covariance matrix in EKF are only accurate up to the first and second order accuracy, respectively (Wan & Van Der Merwe (2000); Andreasen et al. (2010)). Therefore approximations ignore the probability distributions of the state vector. Wan & Van Der Merwe (2000), Van Der Merwe (2004) addresses this by using Central Difference Kalman filter (CDKF) which is an extension of the standard Kalman filter non-linear state-space systems where the non-linear moments are approximated to the at least second order of accuracy by a deterministic sampling based on multivariate Stirling interpolations (Andreasen et al. 2010)).

$$\pi(x_{t+1}|Y_{t+1}, x_t) = N(\hat{x}_{t+1}^{CDF(i)}, \hat{P}_{xx}^{CDF(i)}(t+1)), \quad \text{for all } i = 1...N \tag{3.109}$$

where, again, $\hat{x}_{t+1}^{CDF(i)}$ is the posterior mean and, $\hat{P}_{xx}^{CDF(i)}(t+1))$ is the posterior covariance matrix of the particle i. Merwe et al. (2010) find that CDKF is more accurate than EKF when used in importance sampling. Nonetheless, the CDKF still inherits computational expense of the EKF.

Finally, Andreasen et al. (2010) proposes an optimized particle filter (OPF) where particles for all endogenous states are iterated forward through the transition equation with uniform weights, as in standard particle filter to maintain the singularity of the transition function in the proposal distribution. Whereas, proposal draws on exogenous state variables use information from CDKF. In principle, the filter generates a Gaussian proposal distribution by using the posterior mean and the square root of the covariance matrix for each particle in the exogenous state.

$$x_{ex,t+1}^{(i)} = \hat{x}_{ex,t+1}^{CDKF} + \gamma_{t+1}\hat{S}_{ex}^{CDKF}(t+1)\epsilon_{t+1}^{(i)} \quad \text{for all } i = 1...N \tag{3.110}$$

where the parameter γ_{t+1} allows to account for deviations from normality by increasing the variance of the proposal distribution. This parameter is chosen by minimizing the variance of weights in the importance sampling. Andreasen et al. (2010) finds substantial efficiency gains (number of particles needed to achieve a similar level accuracy in OPF as opposed to the number of particles in standard particle filter) using OPF over the standard particle filter. These efficiency gains are, especially, high in models with one or two shocks. Nonetheless, efficiency gains start to drop when the model has more than two shocks. Therefore, the accuracy and feasibility of the OPF in medium size DSGE models is not clear. This is because, despite OPF requiring smaller number of particles (at least when the exogenous state vector is small), a numerical optimization is necessary at each iteration for the exogenous state vector to choose the parameter γ_{t+1} which, in effect, may need a similar computational time.

As an alternative to particle filter, Kollmann (2013) uses a simple Kalman filter to a pruned solution of Kim et al. (2008) of the second order model. The state equation in the pruned system is linear in the latent state equation. This augmented state equation contain terms only solved to first and second order accuracy, and products of first-order accurate terms. Kolmann (2013) shows that a simple Kalman filter on this latent state delivers more accurate estimates than the particle filter (even with 100,000 and 500,000 particles), especially, when the model has high curvature and when shocks are sizeable. Kalman filter replaces predict and update steps by simple matrix algebra. Moreover, second or higher order models often have extraneous steady states (not inherent in the original model) and some of these mark transitions to explosive sample path. Pruning eliminates explosive path by omitting higher order effects than the

considered approximation when the system is iterated forward. Thus pruning is often necessary not only for estimating, but also for just solving and simulating non-linear DSGE models in presence of large shocks. Andreasen et al. (2016) extends the pruning method to the 3rd order model. The third order Taylor expansion to the state equation is given as follows

$$\begin{aligned} x_{t+1}^{(3)} = h_x x_t^{(3)} + \frac{1}{2}H_{xx}(x_t^{(3)} \otimes x_t^{(3)}) + \frac{1}{6}H_{xxx}((x_t^{(3)} \otimes x_t^{(3)} \otimes x_t^{(3)}) + \\ + \frac{1}{2}h_{\sigma\sigma}\sigma^2 + \frac{3}{6}h_{\sigma\sigma x}\sigma^2 x_t^{(3)} + \frac{1}{6}h_{\sigma\sigma\sigma}\sigma^3 + \sigma\eta\epsilon_{t+1} \end{aligned} \tag{3.111}$$

where $H_{xx}$is $n_x \times n_x^2$ matrix derivatives of $h(x_t, \sigma)$ w.r.t. (x_t, x_t), H_{xx} is $n_x \times n_x^3$ matrix of derivatives of $h(x_t, \sigma)$ w.r.t. (x_t, x_t, x_t), $h_{xx\sigma}$ is $n_x \times n_x$ matrix of derivative of $h(x_t, \sigma)$ w.r.t. (x_t, x_t, σ) and $h_{\sigma\sigma\sigma}$ is $n_x \times 1$ matrix of derivatives $h(x_t, \sigma)$ w.r.t. (σ, σ, σ). Pruning decomposes $x_t^{(3)}$ to the first, second and third order effects, as $x_t^{(3)} = (x_t^I + x_t^{II} + x_t^{III})$. Then equation (3.111) can be written as:

$$\begin{aligned} x_{t+1}^{(3)} = h_x(x_t^I + x_t^{II} + x_t^{III}) + \frac{1}{2}H_{xx}((x_t^I + x_t^{II} + x_t^{III}) \otimes (x_t^I + x_t^{II} + x_t^{III})) + \\ \frac{1}{6}H_{xxx}((x_t^I + x_t^{II} + x_t^{III}) \otimes (x_t^I + x_t^{II} + x_t^{III}) \otimes (x_t^I + x_t^{II} + x_t^{III})) + \\ + \frac{1}{2}h_{\sigma\sigma}\sigma^2 + \frac{3}{6}h_{\sigma\sigma x}\sigma^2(x_t^I + x_t^{II} + x_t^{III}) + \frac{1}{6}h_{\sigma\sigma\sigma}\sigma^3 + \sigma\eta\epsilon_{t+1} \end{aligned} \tag{3.112}$$

To get third order accurate pruned state all terms higher than third order ($x_t^I \otimes x_t^{III}, x_t^{II} \otimes x_t^{III}, x_t^{II} \otimes x_t^{II}, ..., \frac{3}{6}h_{\sigma\sigma x}\sigma^2 x_t^{II}, \frac{3}{6}h_{\sigma\sigma x}\sigma^2 x_t^{III}$) are omitted from this equation. Thus, the third order accurate representation of the state equation is given by,

$$x_{t+1}^{III} = h_x x_t^{III} + H_{xx}(x_t^I \otimes x_t^{II}) + \frac{1}{6}H_{xxx}(x_t^I \otimes x_t^I \otimes x_t^I) + \frac{3}{6}h_{\sigma\sigma x}\sigma^2 x_t^I + \frac{1}{6}h_{\sigma\sigma\sigma}\sigma^3 \tag{3.113}$$

where

$$x_{t+1}^I = h_x(x_t^I) + \sigma\eta\epsilon_{t+1} \tag{3.114}$$

and

$$x_{t+1}^{II} = h_x x^{II} + \frac{1}{2}\big(H_{xx}(x^I \otimes x^I) + h_{\sigma\sigma}\sigma^2\big) \tag{3.115}$$

and likewise, a pruned approximation for the observation equation is expressed by keeping terms only up to the third order,

$$\begin{aligned} y_{t+1}^{III} = g_x(x_t^I + x_t^{II} + x_t^{III}) + \frac{1}{2}G_{xx}((x_t^I \otimes x_t^I) + 2(x_t^I \otimes x_t^{II})) + \frac{1}{6}G_{xxx}(x_t^I \otimes x_t^I \otimes x_t^I) \\ + \frac{1}{2}g_{\sigma\sigma}\sigma^2 + \frac{3}{6}g_{\sigma\sigma x}\sigma^2 x_t^I + \frac{1}{6}g_{\sigma\sigma\sigma}\sigma^3 \end{aligned} \tag{3.116}$$

To apply the Kalman filter, finally, we can rewrite the pruned approximation of the state equation where it is linear in an augmented state,

$$z_{t+1}^{(3)} = c^{(3)} + A^{(3)} z_t^{(3)} + B^{(3)} \xi_{t+1}, \quad \xi \sim (0, \Sigma) \tag{3.117}$$

The augmented state $z_t^{(3)}$ is given as,

$$z_t^{(3)} = \begin{bmatrix} (x_t^I)^T & (x_t^{II})^T & (x_t^I \otimes x_t^I)^T & (x_t^{III})^T & (x_t^I \otimes x_t^{II})^T & (x_t^I \otimes x_t^I \otimes x_t^I)^T \end{bmatrix}^T \tag{3.118}$$

and for the observation equation we have,

$$y_t^{(3)} = C z_t^{(3)} + d^{(3)} + \Omega \tag{3.119}$$

which is also linear in states. Ω denotes measurement errors. Coefficients of the transformed state-space system are given as,

$$A^{(3)} = \begin{bmatrix} h_x & 0 & 0 & 0 & 0 & 0 \\ 0 & h_x & \frac{1}{2} H_{xx} & 0 & 0 & 0 \\ 0 & 0 & h_x \otimes h_x & 0 & 0 & 0 \\ \frac{3}{6} H_{\sigma\sigma} \sigma^2 & 0 & 0 & h_x & H_{xx} & \frac{1}{6} H_{xxx} \\ h_x \otimes \frac{1}{2} h_{\sigma\sigma} \sigma^2 & 0 & 0 & 0 & h_x \otimes h_x & h_x \frac{1}{2} H_{xx} \\ 0 & 0 & 0 & 0 & 0 & h_x \otimes h_x \otimes h_x \end{bmatrix}$$

$$B^{(3)} = \begin{bmatrix} B_1^{(3)} & B_2^{(3)} & B_3^{(3)} \end{bmatrix}$$

where

$$B_1^{(3)} = \begin{bmatrix} \sigma\eta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \sigma n \otimes \sigma\eta & \sigma\eta \otimes h_x & h_x \otimes \sigma\eta \\ 0 & 0 & 0 & 0 \\ \sigma\eta \frac{1}{2} h_{\sigma\sigma} \sigma^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_2^{(3)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \sigma\eta \otimes h_x & \sigma\eta \otimes \frac{1}{2} H_{xx} & 0 & 0 \\ 0 & \sigma\eta \otimes h_x \otimes h_x & h_x \otimes h_x \otimes \sigma\eta & h_x \otimes \sigma\eta \otimes h_x \end{bmatrix}$$

$$B_3^{(3)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ h_x \otimes \sigma\eta \otimes \sigma\eta & \sigma\eta \otimes h_x \otimes \sigma\eta & \sigma\eta \otimes \sigma\eta \otimes h_x & \sigma\eta \otimes \sigma\eta \otimes \sigma\eta \end{bmatrix}$$

$$\xi_{t+1}^{(3)} = \begin{bmatrix} \epsilon_{t+1} \\ \epsilon_{t+1} \otimes \epsilon_{t+1} - vec(I_{n_e}) \\ \epsilon_{t+1} x_t^{(1)} \\ x_t^{(1)} \epsilon_{t+1} \\ \epsilon_{t+1} x_t^{(2)} \\ \epsilon_{t+1} x_t^{(1)} x_t^{(1)} \\ x_t^{(1)} x_t^{(1)} \epsilon_{t+1} \\ x_t^{(1)} \epsilon_{t+1} x_t^{(1)} \\ x_t^{(1)} \epsilon_{t+1} \epsilon_{t+1} \\ \epsilon_{t+1} x_t^{(1)} \epsilon_{t+1} \\ \epsilon_{t+1} \epsilon_{t+1} x_t^{(1)} \\ (\epsilon_{t+1} \otimes \epsilon_{t+1} \otimes) - E(\epsilon_{t+1} \otimes \epsilon_{t+1} \otimes \epsilon_{t+1}) \end{bmatrix}$$

$$c^{(3)} = \begin{bmatrix} 0_{n_x \times 1} \\ \frac{1}{2} h_{\sigma\sigma} \sigma^2 \\ (\sigma\eta \otimes \sigma\eta) vec(I_{n_e}) \\ \frac{1}{6} h_{\sigma\sigma\sigma} \sigma^3 \\ 0_{n_x^2 \times 1} \\ (\sigma\eta \otimes \sigma\eta \otimes \sigma\eta) - E(\epsilon_{t+1} \otimes \epsilon_{t+1} \otimes \epsilon_{t+1}) \end{bmatrix}$$

$$C^{(3)} = \begin{bmatrix} g_x + \frac{3}{6} g_{\sigma\sigma x} \sigma^2 & g_x & \frac{1}{2} G_{xx} & g_x & G_{xx} & \frac{1}{6} G_{xxx} \end{bmatrix}$$

$$d^{(3)} = \begin{bmatrix} \frac{1}{2} g_{\sigma\sigma} \sigma^2 & \frac{1}{6} g_{\sigma\sigma\sigma} \sigma^3 \end{bmatrix}$$

Andreasen et al. (2016) show that the third order pruned system has also finite unconditional moments when the first order approximation is stable. Given pseudo-linear state space system (equations 3.121 & 3.118) Kalman filter replaces prediction (equation 3.102) by,

$$z_{t|t-1}^{(3)} = A z_{t-1} + c \tag{3.120}$$

to obtain a priori estimate of the state, and a priori estimate for the covariance matrix of the state,

$$P_{t|t-1} = A^{(3)} P_{t-1} A^{(3)T} + B^{(3)} \Sigma {B^{(3)}}^T \tag{3.121}$$

To update the state (equation, 3.103), first, a priori state is projected to calculate predicted measurement and its covariance:

$$y_{z_{t|t-1}} = C \tilde{z}^{(3)}_{t|t-1} + d^{(3)}, \quad \Sigma_{t|t-1} = C^{(3)} P_{t|t-1} C^{(3)T} + \Omega \tag{3.122}$$

Then the state is updated by a Kalman Gain, which is calculated as a function of discrepancy between actual observed and predicted measurements,

$$K = P_{t|t-1} C^{(3)T} (\Omega + C^{(3)} P_{t|t-1} C^{(3)T})^{-1} \tag{3.123}$$

Finally, the posterior estimate of the state and its covariance matrix (equation 3.103) is expressed as,

$$\tilde{z}^{(3)}_t = z^{(3)}_{t|t-1} + K(y^{(3)}_t - y^{(3)}_{z_t|t-1}) \tag{3.124}$$

$$\tilde{P}_t = P_{t|t-1} - K C^{(3)} P_{t|t-1} \tag{3.125}$$

Given the distribution of state (3.124, 3.125) for each t, then it is straightforward to calculate the log-likelihood of the model given parameters:

$$L(\theta, y_t) = c - \frac{1}{2} \sum_{t=1}^{T} \{ ln(det(P_{t|t-1})) + (y^{(3)}_t - y^{(3)}_{z_t|t-1})^T P^{-1}_{t|t-1} (y^{(3)}_t - y^{(3)}_{z_t|t-1}) \} \tag{3.126}$$

Therefore, exploiting the structure of the pruned approximation to the third order model allows to evaluate the likelihood efficiently without expensive numerical integration algorithms, however, with a loss of some theoretical inviolability.[5] Therefore, to gain further accuracy improvements, we take an approximation to the model around a point other than its non-stochastic steady state. Specifically, by approximating around the models mean, we ensure that the mean of our approximation will be comparable to the mean of the true model (Deak S 2014). By default, standard perturbation methods construct local approximations by taking a Taylor expansion in the variance of the shock process around the point at which the variance is equal to zero (Deak S 2014). Thus, first order approximation will have a mean equal to the deterministic

[5]The model is still not linear in underlying shocks while the Kalman filter assumes that the state space is linear in shocks.

steady-state of the original model. Nevertheless, if we can find a way of evaluating the models mean, and its Jacobian at this point, then there is no reason why we cannot take an approximation around that point instead. The difficulty is in finding the mean of the model, and its Jacobian at that point, since determining this point is impossible without knowing the full policy functions of the models agents (Deak S 2014).

Meyer-Gohde (2014) introduces a new algorithm that uses an underlying third-order pruned approximation to the model to derive a first-order approximation around either the mean or the risky steady state. Our modeling approach addresses the above shortcomings by adopting the risk-corrected approximations introduced in Meyer-Gohde (2014) with the aim of correctly capturing the effects of shock realizations in our model subject to risk. As Meyer-Gohde (2014) points out, linear approximations around the nonstochastic steady state cannot address effects of the risk adequately in many contexts, such as for welfare analyzes, for models with recursive preferences, or when precautionary considerations have greater scope in the decision making of agents. Meyer-Gohde (2014) finds that standard Kalman filter provides competing or exceeding accuracy with particle filters - both with state-space or moving average representations of the policy function, using risk corrected approximation around either the risky steady state and the ergodic mean. Besides, Kalman filter is not affected by the sampling variation present in particle filters under weak identification (Meyer-Gohde 2014).

Following Meyer-Gohde (2014) also means that we can propose two different approximations, one is to approximate the model around it's mean and another to approximate the model around the stochastic steady state. In general, competing methods can provide only the former. Furthermore, the approximation method proposed by the Meyer-Gohde (2014) is one of the first methods to work solely with derivative information obtained from conventional perturbation algorithm and given moments of the exogenous process - no re-evaluation of derivatives or recalculation of decision rules are needed to build the approximation Meyer-Gohde (2014). The equations used in the estimation are linear in the unknown coefficients of the approximation which allows getting around the fixed point approximations or other recursive filters with poor convergence properties Meyer-Gohde (2014). These two conveniences are achieved by working implicitly with the unknown decision or policy rules of the model rather than it's equilibrium conditions Meyer-Gohde (2014).

citemeyer2014risky defines a "risky" linear approximation as an approximation that while preserving linearity in states, it is corrected to the desired order for risk. The desired order of risk corresponds to the targeted moments of the distribution of the exogenous innovations in equation (1) described below. The correction for the risk is

achieved by taking nonlinear taylor expansion to the policy function in σ, which is the scaling index of the risk. For example, first order expansion in σ corrects for the first moment of the risk, the second order expansion in σ corrects for the second moment of the risk, and so on.

Specifically, Meyer-Gohde (2014) risk-corrected approximation can be summarized as follows. Meyer-Gohde (2014), first, expresses the model as a set of equilibrium conditions of the following form of discrete-time rational expectations models given as:

$$E\big[f(y_{t+1}, y_t, y_{t-1}, \sigma\epsilon_t)\big] = 0 \tag{3.127}$$

Where is f is a $\mathbb{R}_{ny} \times \mathbb{R}_{ny} \times \mathbb{R}_{ny} \times \mathbb{R}_{n\epsilon}$ dimensional vector-valued function that collects the equilibrium conditions which describe the model; y_t is the vector of endogenous and exogenous variables; and ϵ_t is the vector of exogenous shocks, where ny and $n\epsilon$ are positive integers . The auxiliary parameter σ scales the risk in the model, i.e., the stochastic model (3.127) is where $\sigma = 1$ and the deterministic model corresponds to $\sigma = 0$. The solution to the system can be summarized by a set of policy functions,

$$y_t = g\big(y_{t-1}, \sigma\epsilon_t, \sigma_t\big) \tag{3.128}$$

The stochastic steady state of this system is a fixed point given as,

$$\bar{y}^{stoch} = g\big(\bar{y}^{stoch}, 0, 1\big) \tag{3.129}$$

which corresponds to a point in the absence of shocks, but while expecting future shocks with a predefined probability distribution. However, in the presence of risk, $\bar{y}^{stoch}$ is defined only implicitly in state-space policy function(3.129) and solving for it requires to calculate an integral over the probability distribution of all innovation. To solve for the risky fixed point, Meyer-Gohde (2014) instead, considers a nonlinear moving average representation of the policy function which is given by Lan & Meyer-Gohde (2013). This allows recovering the ergodic mean and the stochastic steady state of the model. Inverting or recursively substituting the policy function, maps the state-space to a nonlinear moving average form. Then approximated to an arbitrary M^{th} order around the deterministic steady state, the non-linear moving average policy function is given as,

$$y_t \approx \sum_{m=0}^{M} \frac{1}{m!} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \dots \sum_{i_m=0}^{\infty} \left[\sum_{n=0}^{M-m} \frac{1}{n!} y_{\sigma^n i_1 i_2 \dots i_m} \sigma^n \right] \big(\epsilon_{t-i_1} \otimes \epsilon_{t-i_2} \otimes \dots \otimes \epsilon_{t-i_m}\big) \tag{3.130}$$

where $y_{\sigma^n i_1 i_2 \dots i_m} \sigma^n$ is the n_{th} degree derivative of y_t with respect to m^{th} fold Kronekcker products of shocks realized at time $t - i_1, t - i_2, ..., t - i_m$ and with respect to

the risk parameter, σ. Then, given that all history of shocks converge to zero equation (3.130) approximately reduces to the stochastic steady state of the model:

$$\bar{y}^{stoch} \approx \sum_{n=0}^{M} \frac{1}{n!} y_{\sigma^n} \tag{3.131}$$

For the estimation, however, as Meyer-Gohde (2014) argues, the ergodic mean of the model is more useful to build an approximation around, because this point corresponds to high probability density in empirical applications. The ergodic mean is defined as an unconditional expectation of the policy function (3.4) in presence of, both, shocks and uncertainty:

$$\bar{y}^{mean} = E(y_t) = E\big(g(y_{t-1}, \epsilon_t, 1)\big) \tag{3.132}$$

which also requires, integration over endogenous variables through unknown policy function (Meyer-Gohde 2014). Whereas, an m^{th} order approximation of the nonlinear moving average counterpart of the policy function gives more straight expression for the ergodic mean:

$$\bar{y}^{mean} = E(y_t) \approx \sum_{m=0}^{M} \frac{1}{m!} \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \dots \sum_{i_m=0}^{\infty} \left[\sum_{n=0}^{M-m} \frac{1}{n!} y_{\sigma^n i_1 i_2 \dots i_m} \sigma^n \right] \big(\epsilon_{t-i_1} \otimes \epsilon_{t-i_2} \otimes \dots \otimes \epsilon_{t-i_m} \big) \tag{3.133}$$

As equation (3.131) shows, for the local approximation of the stochastic steady state, derivatives of the policy function evaluated at the non-stochastic steady state is sufficient through implicit differentiation of the policy function. Whereas, for the ergodic mean we only need derivatives of the policy function at the non-stochastic steady state and known moments of exogenous innovations. Therefore, performing a conventional perturbation around the deterministic steady state to the order of $m+1$ gives m^{th} order of approximation for these points. The resulting derivatives at the deterministic steady state of the third order pruned approximation (Andreasen, 2016) discussed above, can be used to approximate the stochastic steady state as follows,[6]

$$\bar{y}^{stoch} \approx \bar{y} + \frac{1}{2} \big[I - g_y \big]^{-1} g_{\sigma\sigma} \tag{3.134}$$

The second order σ approximation for the ergodic mean is given as,

$$\bar{y}^{mean} \approx \bar{y} + \frac{1}{2} \big[I_{n_y} - g_y \big]^{-1} \left[g_{\sigma\sigma} + \left(g_{\epsilon\epsilon} + \big(I_{n_y \times n_y} - g_y \otimes g_y \big)^{-1} g_\epsilon \otimes g_\epsilon \right) E\big(\epsilon_t \otimes \epsilon_t \big) \right] \tag{3.135}$$

[6]The replication of derivations are given in Appendix

in terms of derivatives of the third order pruned solution. We find that approximating the model (and likelihood) around mean gives improved identification of the model's parameters subject to the risk, as Meyer-Gohde(2015) argues. Therefore, we calculate our likelihood function around the ergodic mean.

Following the approximation method described above, the next step would be to estimate the model. To estimate our model, we propose using a Bayesian estimation approach. Compared to other econometric estimation techniques, the latter has a number key advantages. First, as opposed to the GMM estimation that builds on equilibrium relationships (Euler equation, labour supply etc.) the Bayesian inference is instead a system-based approach and matches the solved DSGE model to a vector of aggregate time series (An & Schorfheide 2007). Moreover, the Bayesian inference is based on the likelihood function of the DSGE model instead of, for example, the differences between in impulse responses of the DSGE and VAR models (An & Schorfheide 2007). Third, prior distributions can be combined to utilize additional information into the parameter estimation. This helps for the estimation to avoid the likelihood peaking at the extraneous points and thereby helps identification of parameters. Whereas, the plain maximum likelihood estimation can pick up extraneous estimates for the parameters of interest (An and Schorfheide, 2006). This describes instances wherein estimates of structural parameters inferred with maximum likelihood are at odds with the conventional wisdom (e.g., the estimate of discount factor needs to be in line with observed average interest rate).

In a Bayesian modeling framework, the likelihood function is re-weighted by a prior distribution. This can help to combine information that is not necessarily present in the sample used for the estimation. Since priors scan be revised at later stages, the change from the prior to the posterior distribution can be a sign of the conflict between different sources of information (An and Schorfheide, 2006). Thus, should the likelihood function peak at a value that is conflicting with the information in proposed the prior density, the marginal data density of the DSGE model, will be lower than in VAR, for example. Therefore in a posterior odds comparison, the DSGE model will be penalized for not being able to accommodate different origins of information with a unique set of parameters (An and Schorfheide, 2006).

In addition to the dilemma of absurd parameter estimates discussed above, another equally thorny issue is that researchers are often confronted with when estimating DSGE model in general and non-linear DSGE models in particular, is the identification problem. As Canova (2007) points out for meaningful inference and identiability, the likelihood function must have a unique minimum and have sufficient curvature in all

respective dimensions of the parameter space. It is extremely complex to easily recognize identification problems in large DSGE models. Therefore, an immediate benefit of using a Bayesian estimation approach is that a comparison of priors and posteriors can often give helpful insights about the degree to which data bear information about the estimated parameters versus priors (An and Schorfheide, 2006). Therefore, Bayesian MLE can better deal with identification problems, although it is not able to entirely avoid them (An and Schorfheide, 2006). Finally, another main advantage of using a Bayesian approach is that posteriors provide a measure of uncertainty around the parameters of interest.

Deak S (2014) points out that Gaussian approximation to estimate the stochastic volatility may yield inaccurate inference. Therefore we estimate the stochastic volatility parameter separately using the non-linear version of the Kalman filter (CDKF). Because the procedure is relatively slow for the Bayesian estimation, we suffice with the maximum likelihood estimation. We fix some of the model parameters to which there are comparably tight priors in the literature except the Epstein -Zin parameter. We find that estimating this parameter weakens the identification of the rest of model parameters. Implications of the Epstein-Zin parameter for the term-premium strongly depends on the central bank's stance on inflation, and small changes to this parameter, together with other parameters (for example, borrowing constraint) implies very large jumps in the term premium. Second, fitting the term premium which is largely volatile may compromise matching of the rest of parameters. Principally, however, main macroeconomics variables in observations may not have enough curvature for the identification of this parameter. Since by design recursive preferences help to fit dynamics of the term premium without considerably altering dynamics of macroeconomic variables (which are already matched well matched with expected utility preferences), Therefore, to identify this parameter it's necessary to include observations on the term premium in estimation. However, the filtering with simple Kalman filter is potentially less accurate to estimate states when the term premium is included in observations. Nevertheless, while fixing the Epstein-Zin parameter in estimation, in following analyses, we look at the implications of the model under various assumptions of the risk aversion.

All fixed parameters are given in Table (3.1). The rest of parameters are fixed to their commonly cited values in the literature. Particularly, the inflation parameter is often estimated between 1.5 and 2, thus we chose the mid point (1.75). The output targeting is often estimated between 0 and 0.5. However, most of estimates lies in the lower end of this range. For example Nguyen (2015) estimates it about 0.069. We also choose elasticity of substitution ($\eta = 1$) higher than in Pesaran & Xu (2016), (0.7)

because this parameter can be underestimated in the presence of financial frictions Rubio & Carrasco-Gallego (2014).

Table 3.1: Fixed/calibrated parameters

Parameters	Values	Source
β, Discount factor	0.996	Pesaran & Xu (2016)
γ, Inverse of IES	1	Pesaran & Xu (2016)
η, Inverse elasticity of subst. of labour	1	Rubio & Carrasco-Gallego (2014)
α, Capital share	0.35	Pesaran, 2016
ζ, Elasticity of subst. differentiated goods	7	14% Markup
χ, Calvo parameter	0.725	
ρ_r, Taylor rule persistence	0.5	
ρ_π, Taylor rule inflation	1.75	
ρ_y, Taylor rule output gap	0.2	
γ_{EZ}, Epstein-Zin parameter	10	

The rest of model's parameters are estimated for the quarterly US data from 1973Q1 to 2013Q2 on consumption, output, lending rate, deposit rate, deposits, lending, and CPI. We use the same data source as Pesaran & Xu (2016) from Federal Reserve. For the deposit rate, we use the mean of "U.S. CD Secondary Market 1 month - Middle rate", "U.S. CD Secondary Market 3 month - Middle rate" and "U.S. CD Secondary Market 6 month - Middle rate". Following Pesaran & Xu (2016) we also use "U.S. Bank prime loan -Middle Rate" for the loan rate. This series is the rate charged by top 25 insured chartered U.S. commercial banks. The prime rate is used by banks to charge for short-term business loans. For the inflation, we use the CPI series "Consumer price index for all urban consumers: All items." The series is indexed at the years 1982-84 and seasonally adjusted. For the consumption, we use "U.S. Real Personal Consumption Expenditures." The source of output series is the Federal Reserve Bank of St Louis (GDPC96) and is in 2005 billion dollars. For the lending we use "TOTBKCR

Bank Credit of All Commercial Banks, Billions of U.S. Dollars, Quarterly, Seasonally Adjusted." For deposits we use "DPSACBM027SBOG Deposits, All Commercial Banks, Billions of U.S. Dollars, Quarterly, Seasonally Adjusted." Data sources for both series

All trending variables are normalized by the trend growth rate of population. We also difference trending observables and keep mean growth rates. In contrast to HP filtering which disregards sample means, differencing preserves mean growth rates which may contain useful information for the estimation. For example, if the average inflation rate were positive, demeaning will result in the same number of observations with both signs. Keeping information about means allows estimated model parameters to account for this, utilizing the information content of sample averages. Regarding the choice of priors, we allow for lending to deposit ratio to be grater than unity as suggested by the data. Subtracting 0.2 allows using the "beta" distribution for this parameter while letting lending to be bigger than deposits (i.e., $\theta > 1$). We also restrict the parameter ν to unity from below that gives a positive leverage and use "gamma" distribution. Priors for the rest of parameters are standard as in literature.

Our estimate indicates considerable time variation in the technology process (Table 3.3), yet still is less than found, for example, in Fernández-Villaverde, Guerrón-Quintana & Rubio-Ramírez (2015) (0.7720), but higher than in Nguyen (2015) (0.168). Diebold et al. (2016), Clark (2011) also find much larger estimates than our's for the standard deviation of the stochastic volatility process. Therefore, our estimate lies in midrange found in the literature. The estimate for the persistence of the technological growth (0.2711) is also lower than in Pesaran & Xu (2016) (0.439) but is close to estimates of Justiniano & Primiceri (2008). Justiniano & Primiceri (2008) finds the posterior mean of this parameter 0.32 in a model with stochastic volatility, and 0.26 when the central bank reacts to the output ($\rho_{r,y} = 0.23$). We estimate the standard deviation of the micro uncertainty (0.457) slightly higher than Pesaran & Xu (2016) (0.439). Our estimate of the standard deviation of the technology growth (0.0172) is also marginally higher than estimates reported in Pesaran & Xu 2016 (0.011). Overall, most estimates are largely similar to the parametrization by Pesaran & Xu (2016) with minor differences. However, the likelihood peaks at considerably higher values for the firms' leverage ratio. This is because a positive steady state markup also decreases the steady-state default probability and therefore firms leverage needs to be larger to generate similar default probability.

Table 3.2: Estimated parameters

Estimated Parameters	Prior	Posterior	i
	Mean & Distribution	Mode	90% interval
Π, s.s.Inflation	Inverse Gamma(1.001, 1)	1.0023	[1.0016 - 1.0033]
μ, s.s. growth rate	Inverse Gamma(0.004, 1)	0.0036	[0.0034 - 0.0041]
θ-0.2, $(1-\theta)$: deposit			
requirement ratio:	BETA(0.85, 0.125)	0.8950	[0.9023 - 0.9994]
υ-1, $(1-\frac{1}{\upsilon})$: borrowing constraint	Inverse Gamma(1.5, 1)	1.6681	[1.4576 - 1.7762
Persistence parameters:			
ρ_e, common technology	BETA (0.7, 0.125)	0.2711	[0.2548 - 0.2871]
ρ_σ, technology uncertainty	BETA(0.7, 0.125)	0.8465	[0.8304 - 0.9024]
ρ_θ, credit shock	BETA(0.7, 0.125)	0.7683	[0.7670 - 0.0.8301]
Standard deviation of shocks:			
σ_l, micro uncertainty	Inverse Gamma(0.5, 1)	0.4573	[0.4561 - 0.4595]
σ_e, common uncertainty	Inverse Gamma(0.1, 1)	0.0172	[0.0143- 0.0216]
ε_m, monetary policy	Inverse Gamma(0.002, 2)	0.0034	[0.0035 - 0.0042]
ε_θ, credit shock	Inverse Gamma(0.002, 2)	0.00088	[0.0004 - 0.0007]
ε_{mc}, marginal cost	Inverse Gamma(0.002, 2)	0.00064	[0.0100 - 0.0129]
Observation errors			
ε_C, Consumption	Inverse Gamma(0.001, 1)	0.0052	[0.0045 - 0.0059
ε_Y, Output	Inverse Gamma(0.001, 1)	0.0064	[0.0057 - 0.0074]
ε_Π, Inflation	Inverse Gamma(0.001, 1)	0.0011	[0.00095 - 0.0012]
ε_L, Lending	Inverse Gamma(0.001, 1)	0.0096	[0.0085 - 0.0109]
ε_D, Deposit	Inverse Gamma(0.001, 1)	0.0091	[0.008- 0.0083]
ε_{R_k}, Lending rate	Inverse Gamma(0.001, 1)	0.00028	[0.0002 - 0.0004]
ε_{R_d}, Deposit rate	Inverse Gamma(0.001, 1)	0.00027	[0.00023 - 0.0007]

Table 3.3: Maximum likelihood estimation of the stochastic volatility

Estimated Parameters	Mean	Standard deviation
Standard deviation		
of the stochastic volatility $\sigma_{\sigma,\epsilon}$	0.4150	2.35

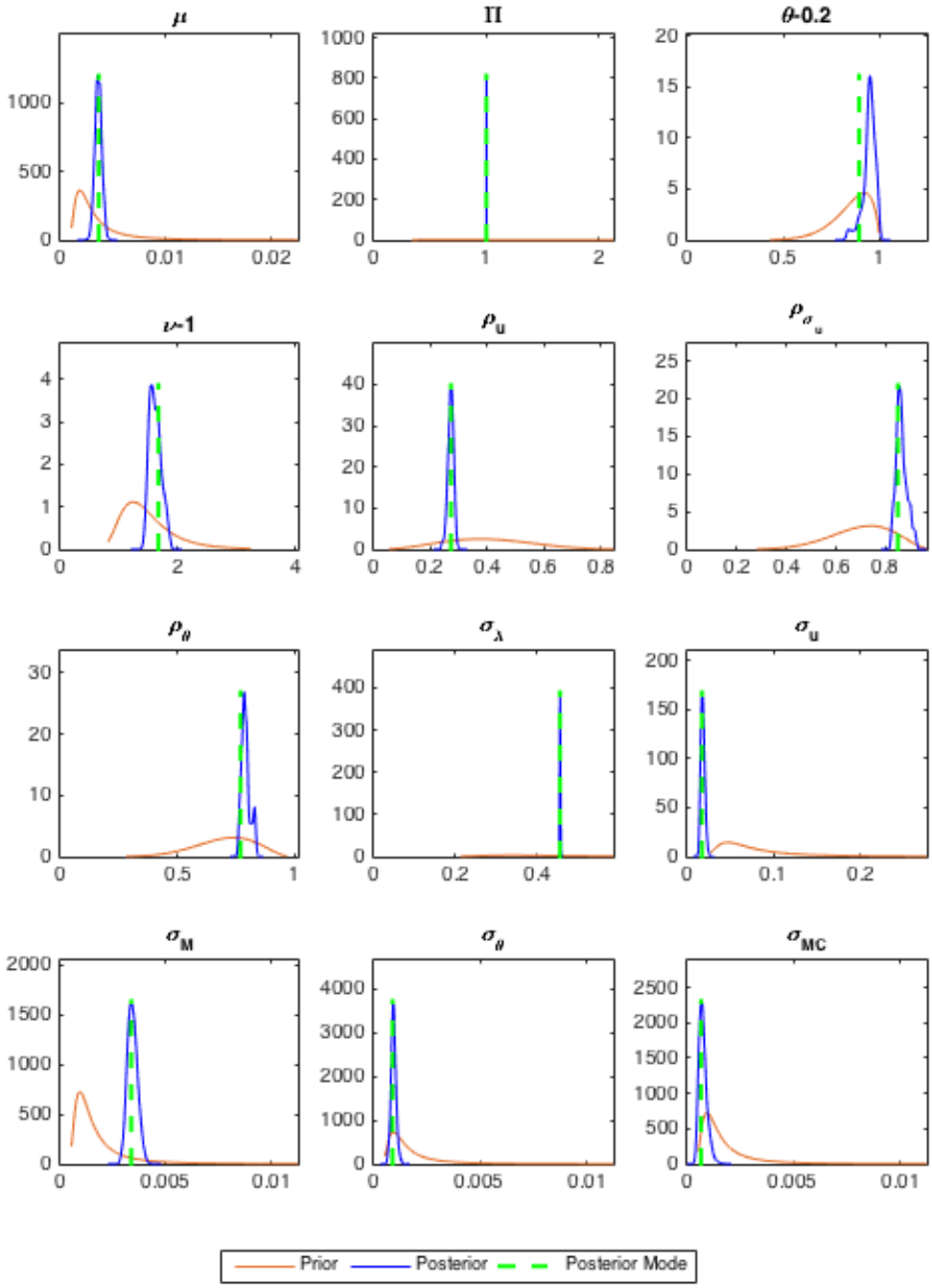


Figure 3.3: Priors and Posteriors

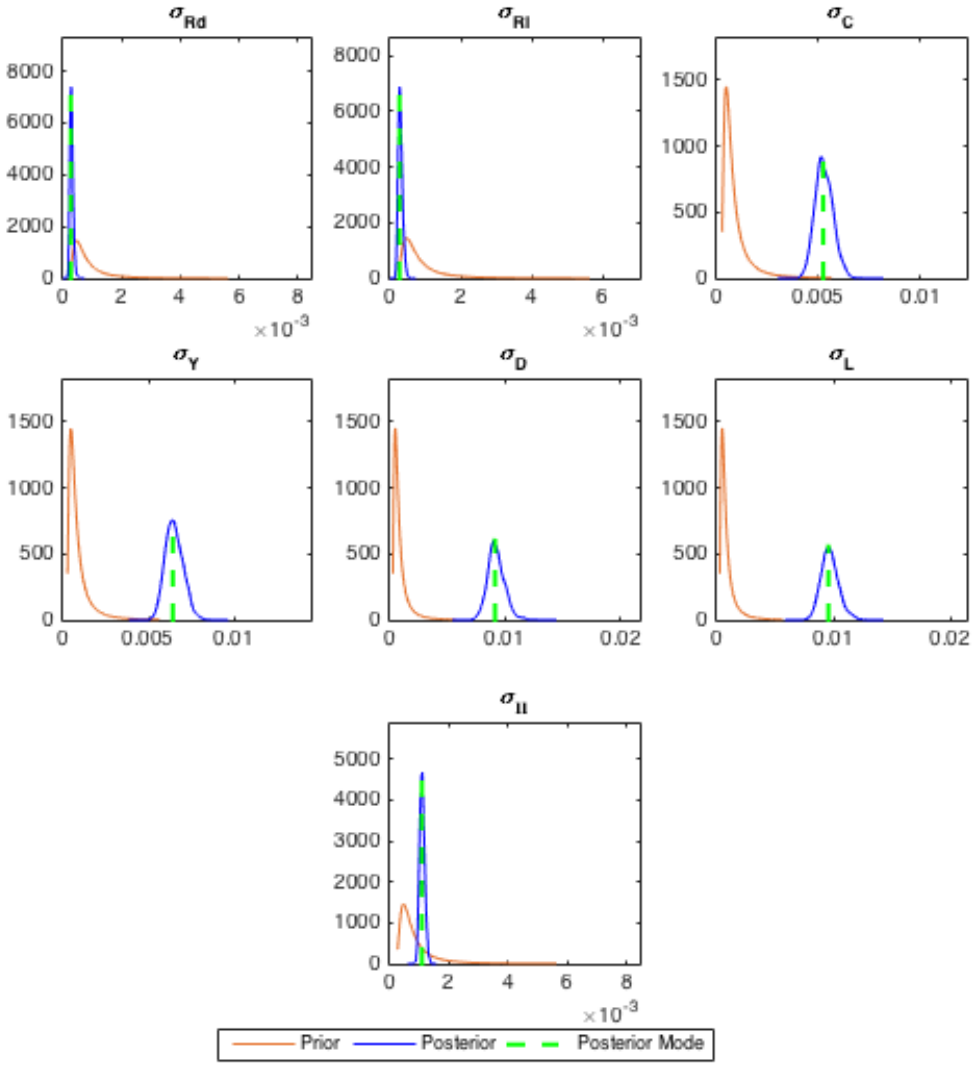


Figure 3.4: Priors and Posteriors

3.5 Results

We plot impulse responses (figure 3.5)[7] following uncertainty shock under sticky and flexible prices under expected utility and Epstein-Zin preferences. [8] Although the uncertainty shock can create substantial contractions in the economy with flexible prices alone, nominal rigidities enhances the recession. With flexible prices, the increase in lending rate is partially matched with a substantial decrease in the deposit rate. The uncertainty shock widens the distribution of productivity and therefore increases the default probability. This, in turn, increases the spread. Thus because of the stochastic volatility, the intensity of the financial friction is no longer constant along the dynamic path of this economy where increased volatility in the production technology compounds the financial accelerator. Another enhancing mechanism of the uncertainty shock arises because of non-linearities retained in the approximation of the model. Specifically, we solve and plot impulse response functions around the ergodic mean. The balance sheet of banks and the price of lending are different when agents expect the presence of shocks from the deterministic steady state. Therefore a model subject to the risk implies a higher level of financial friction or financial conditions to worsen further.

Under sticky prices, the lending rate increases more on impact compared to the economy with flexible prices. However, the deposit rate does not drop following the uncertainty shock. In the face of higher inflation, the central bank cannot considerably reduce the nominal interest rate. This is, particularly, enhanced by agents seeking higher interest in the inflationary economy. Therefore, overall the spread widens to a greater extent than in the economy with flexible prices. The sticky price model shifts much of the variation from the deposit rate to the lending rate. In sticky price economy, the default rate increases about the same as in the flexible prices economy on the impact of the uncertainty shock. Markup considerably reduces demand and, thereby the total lending into the economy contracts further. Firms' markup also slightly reduces the default probability. This highlights an additional channel through which nominal rigidities contribute to the transmission of the uncertainty shock. In the model marginal cost enters into the default condition of firms. Thus nominal rigidities interacts with financial rigidities, and there is a feedback loop from the nominal side of

[7] All impulse responses are calculated around the ergodic mean and show quarterly changes following the shock

[8] For flexible prices, we assume monopolistic competition but no constraint on optimal price resetting. This allows to get similar default rate with sticky prices in the steady state for identical values of micro and macro uncertainties and financial constraints

the economy to the financial market. The variation in the markup (or the marginal cost) through the price rigidities affects the financial conditions and, thereby, the intensity of the financial friction itself fluctuates over the business cycle. Despite having a matching increase in default probability, convex price adjustment implies firms choose the lower quantity of production with relatively higher prices, and thus inflation increases both interest rates further which also pushes the cost of production up. Therefore, in the model nominal rigidities allow for an additional transmission channel of the uncertainty shock apart from the demand channel of nominal frictions highlighted in the previous literature. This result arises because of the interaction among financial frictions, nominal rigidities, and the uncertainty. Regarding the role of nominal rigidities, Calvo contracts imply asymmetric profits over pricing decisions. As Fernández-Villaverde, Guerrón-Quintana, Kuester & Rubio-Ramírez (2015) shows a lower relative price implies higher losses compared to the higher price than competitors in this set-up. Figure (3.25) in the Appendix plots steady state profits and the relative price of the firm for different levels of the mark up in the steady state. Because lower relative prices imply higher loss than the same amount of increase in the relative price, firms choose higher prices in the face of elevated uncertainty.

In figure (3.6) we consider positive credit shocks in NK and flexible prices models.

In sticky prices model, inflation and, thereby the default rate considerably decreases because of further feedback effect from nominal rigidities following the credit shock. Whereas, the default rate remains unchanged with flexible prices model. In the flexible prices model, the marginal cost is constant. However, a sudden increase in available credit lowers the cost of capital in the nominal frictions model. This increases the response of the economy to the credit shock. Thus, including nominal rigidities makes firms default responsive to developments in the financial sector and, overall, to wider economic outcomes. A sudden increase in the credit also substantially decreases the lending rate. Therefore, the presence of a financial friction creates a larger drop in the cost of capital relative to the deposit rate (which is also a policy rate in the model). This together with convex price adjustment costs results in firms choosing a higher volume of production with substantially lower prices. Thus, overall the inflation considerably drops.

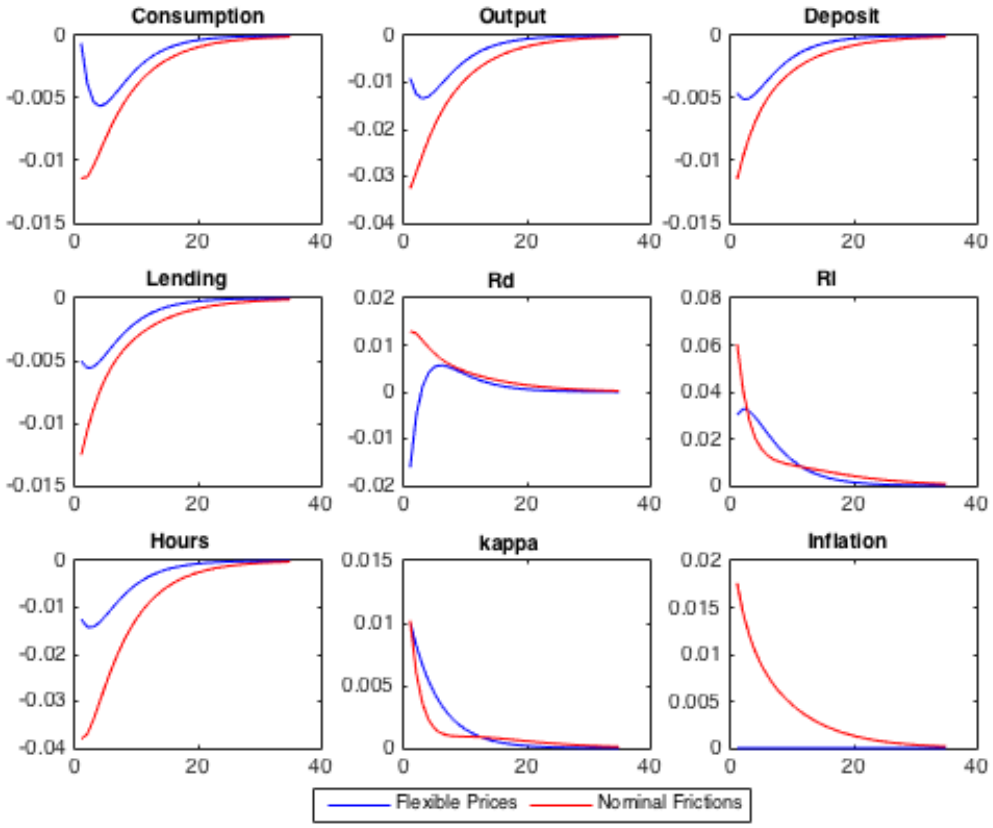


Figure 3.5: Uncertainty shock, Sticky vs Flexible prices

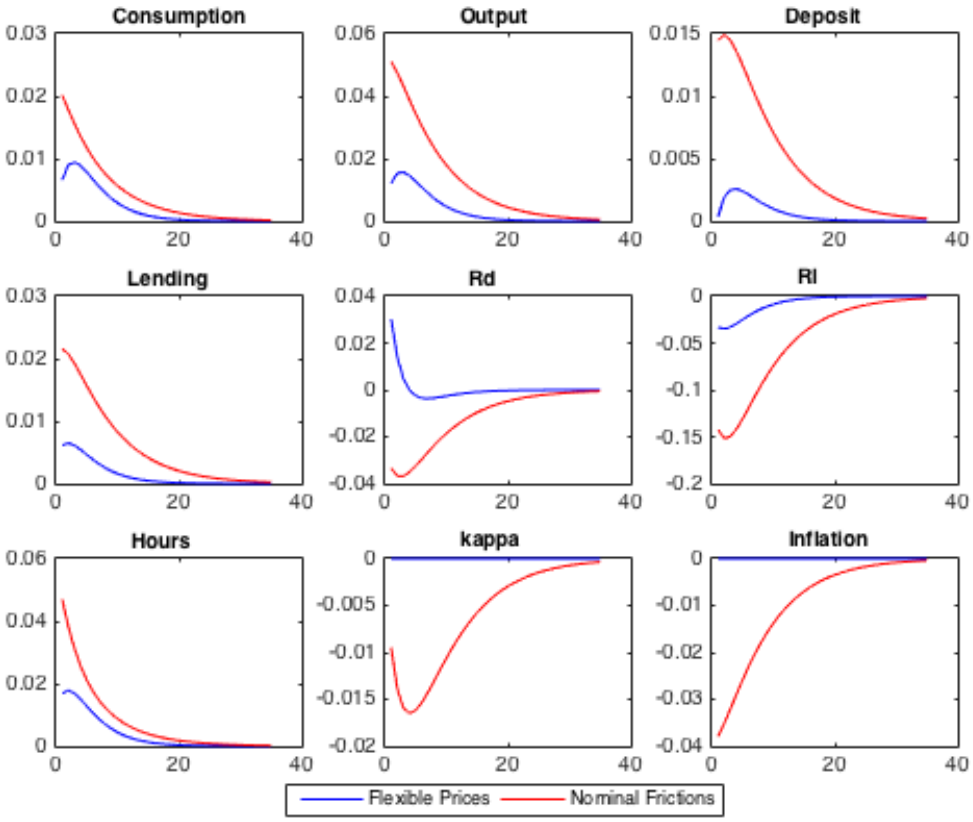


Figure 3.6: Credit shock, Sticky vs Flexible prices

3.5.1 Liquidity policy as an unconventional monetary policy tool and Macroprudential policy

For the brevity of description, we analyse the transmission of shocks with liquidity policy and macroprudential policies together. First, we consider a liquidity rule that adjusts the leverage ratio of banks in response to the amount of the credit in the economy. That is when the total lending is too high the financial regulator limits the volume of lending channeled into the economy to minimize bank losses. Alternatively, when the lending is too low, the central bank can offer an additional source of liquidity by cutting the leverage ratio. To be consistent with Taylor rule describing the conventional monetary policy, we also allow for persistences in the augmented policies as well ($\bar{\rho} = \rho_r = 0.5$). We fix the reaction parameters, ρ_ν, ρ_θ, to 0.5 in both polices. Parameters for the monetary policy are fixed to their calibrated values as in Table 3.1.

$$\theta_t = \left(\theta_{t-1}\right)^{\bar{\rho}} \left(\bar{\theta} \left(\frac{L_t}{\bar{L}} \right)^{-\rho_\theta} \right)^{1-\bar{\rho}} \tag{3.136}$$

As a macroprudential tool we consider a policy that regulates firms leverage and reacts to the lending as well. This policy is similar to the LTV rule considered in Rubio & Carrasco-Gallego (2014), and in the first chapter of the present where the collateral requirement dynamically changes in response to movements in borrowers' collateral. In this model, instead of collateral we use firms equity requirement to react movements in the total lending.

$$\nu_t = \left(\nu_{t-1}\right)^{\bar{\rho}} \left(\bar{\nu} \left(\frac{L_t}{\bar{L}} \right)^{-\rho_\nu} \right)^{1-\bar{\rho}} \tag{3.137}$$

Uncertainty shock. As shown in figure (3.7), estimated effects of uncertainty shock are very persistent and sizeable. It notably increases the default probability. As a result, the lending rate grows which shrinks the lending into the economy. An uncertainty shock leads to a persistent recession with notably high default probability and the lending rate, where the lending rate increases more than four times more than the deposit rate. While higher policy reduces the demand, the disproportional increase in the lending rate makes production very costly. Therefore, because of convexities in the price setting, firms lower their output further and set higher prices. Active liquidity policy considerably moderates transmission of the uncertainty shock. Whereas, the LTV rule only results in negligible moderation in consumption, lending and hours. Second, with the liquidity policy, lower lending rate reduces the cost of production, so firms set lower prices which keep the inflation relatively low compared to the economy without active liquidity policy. Therefore, relatively higher lending relocated into the economy under the liquidity policy, does not increase inflation. Quite the opposite, inflation

is much lower as the liquidity policy considerably decreases the cost of production following the uncertainty shock. Thus, firms faced with lower uncertainty and relatively lower cost of production keep prices lower. Overall, the liquidity policy substantially mitigates the contractionary effect of the uncertainty shock for all variables. ***Credit***

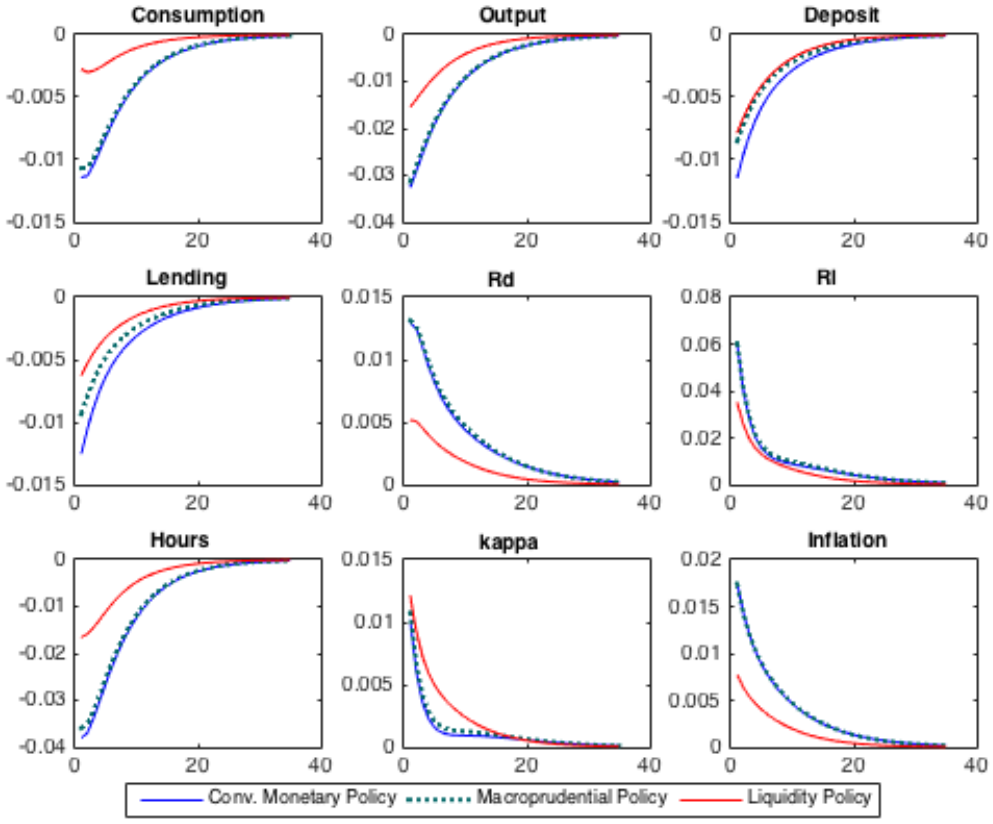


Figure 3.7: Uncertainty shock

shock. Figure (3.8) plots orthogonal impulse responses to a positive credit shock. Not surprisingly, but still in contrast to the prevailing belief, an expansionary credit shock, with more lending available to firms, does not put inflationary pressure on the economy. The availability of more lending in the economy lowers both interest rates. The credit shock noticeably decreases the default probability in our model. This together with lower interest on loans substantially lowers the cost of production and inflation falls. This is mainly due to the price setting behaviour of firms. With the availability of more lending and substantially lower interest rate for the capital, firms optimally choose lower prices with a higher volume of production. Therefore, an expansionary credit shock can lead to large increase in real activity with a substantially lower inflation rate. This makes inflation targeting and conventional monetary policy, not a very effective tool to respond to financial shocks. In the face of lower inflation, the policy rate (which is

also a deposit rate in our economy) falls. Lower savings rate together with lower prices raises the demand for output. The presence of a financial friction, and thereby, the distinction between deposit and lending rates leads to uneven responses in these rates. Consequently, the lending rate drops more than four times than the policy rate which reinforces the cost channel. With a liquidity policy rule, the lending rate drops less than in an economy with only monetary policy due to the decrease in the total lending. As a result, the default probability is slightly higher with an active liquidity policy rule. Overall, an active credit policy in the form of liquidity policy substantially mitigates the response of the economy to the credit shock for all variables. The LTV (green dotted line) rule also noticeable stabilizes lending and deposits. However, this policy puts relatively weaker convergence pressure on consumption, output, and employment.

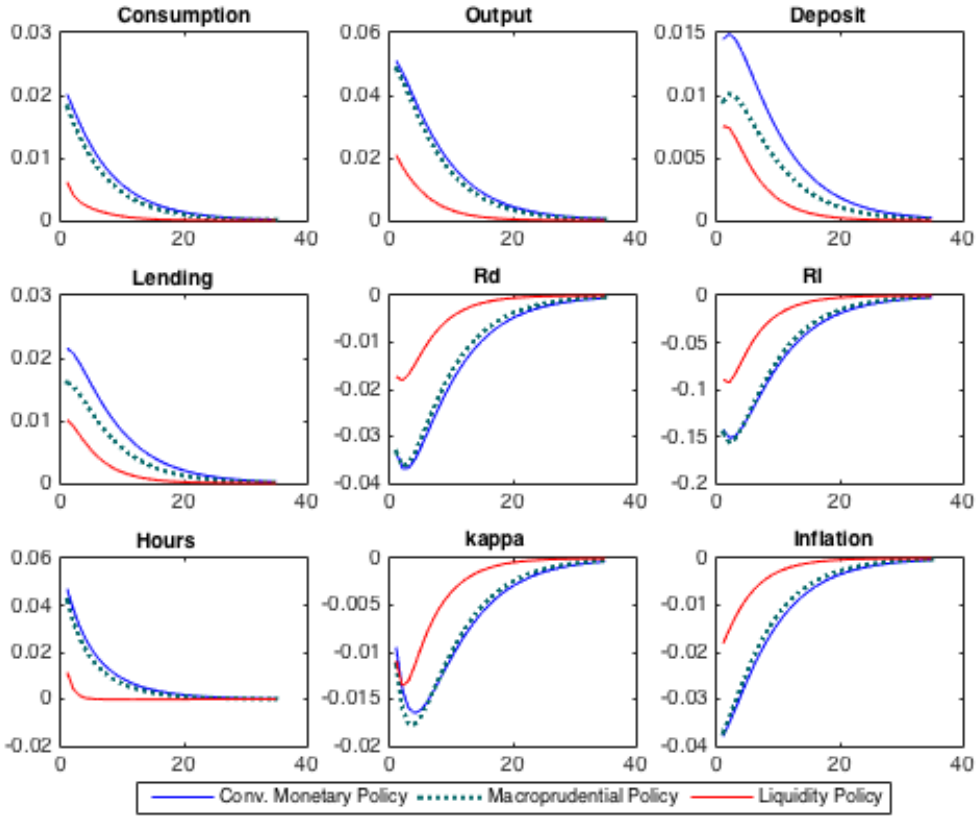


Figure 3.8: Credit shock

Monetary policy shock. Figure (3.9) plots impulse responses for a contractionary monetary policy shock. Estimated effects of the monetary policy shock are notably higher than effects of the uncertainty shock. Increasing the policy rate slightly decreases the default probability. The decrease in inflation is also trivial. However, reduced demand causes a sizeable reduction in output, consumption, and employment. Lower

demand for the labour decreases earnings of households, and thereby, their deposit holdings in banks shrinks. This together with lower demand for the capital compounds the fall in the lending. As a result, the lending rate expands in relation to the policy rate. An active liquidity policy, however, reduces contractions in output, whereas lending and deposits grow with the liquidity intervention. This substantially lowers the lending rate. With a cheaper lending rate households increase their equity holdings in firms. Therefore, consumption drops slightly more with a liquidity policy. The lower cost of the capital notably decreases the default probability, and this pushes prices down due to convex price adjustments. Availability of more loans in the economy, and thus, higher profit transfers from firms together with lower prices, decreases the labour supply following the liquidity intervention. Overall, the liquidity policy decreases the fall in output but expands the reduction in the default probability, inflation, and employment following an unexpected increase in the policy rate. However, the LTV rule, leads to identical responses in consumption, output, and employment, despite largely decreasing the inflation, lending rate, and the default probability compared to the economy with the only conventional monetary policy rule. Overall, the present economy differs from standard NK models in the transmission of the contractionary monetary policy because of the insufficient drop in prices. The distinction of interest rates creates disproportional changes in their levels, and higher cost of production encourages firms to decrease the quantity of output with relatively higher prices.

Marginal cost shock. Figure (3.10) plots orthogonal impulse to a cost push shock. An unexpected increase in the marginal cost of production raises the default probability. As a result, the lending rate increases. This through the imperfect financial markets raises the market price for the capital, and therefore the cost of production further. Thus, firms set higher prices, and inflation in the economy rises. In perspective of higher inflation, monetary policy raises the interest rate through the Taylor rule, however, to the lesser extent than the lending rate. In the meanwhile, the liquidity policy reinforces monetary policy to bring inflation back to its' target rate. Availability of more lending following a liquidity intervention substantially reduces the lending rate. Therefore, much of the mitigation is channeled through a lower lending rate. The liquidity policy also decreases the default probability. In total, the fall in output and inflation largely reduced, whereas, consumption and employment have an opposite sign under liquidity intervention. For the liquidity policy, much of the equilibrium convergence is maintained by a significant reduction in the lending rate. The LTV rule, however, does not reduce the lending rate. Loosening in firms leverage mitigates the drop in lending, thus with more lending available firms increase their input hirings. As

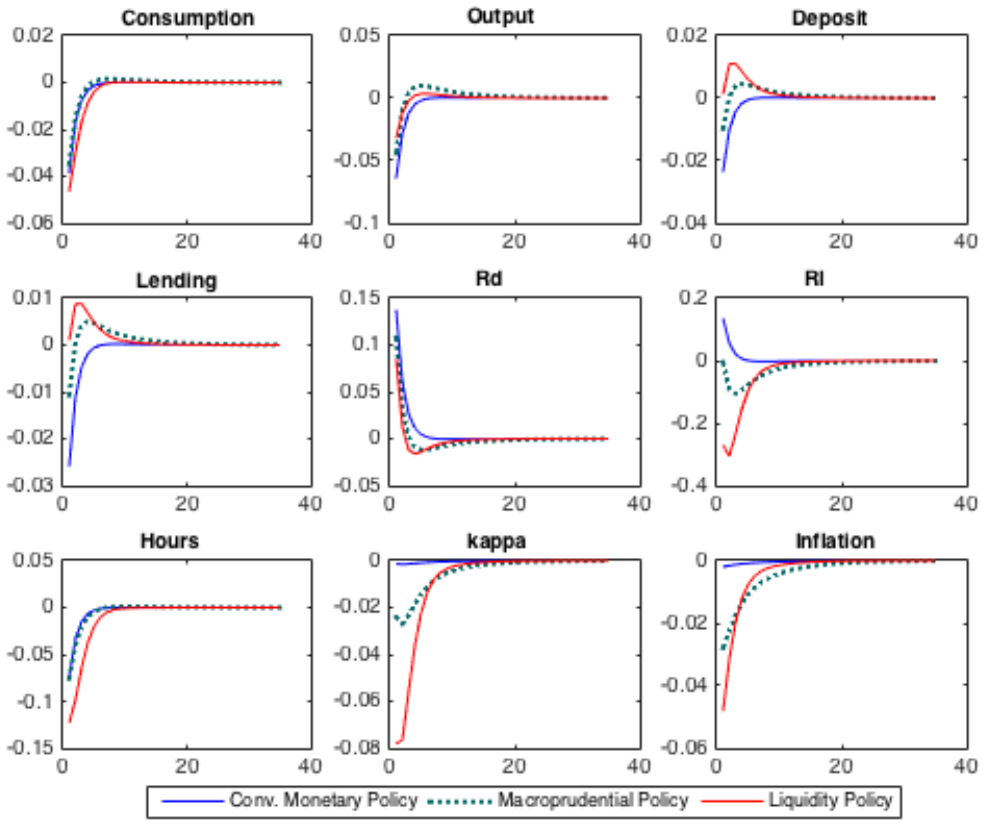


Figure 3.9: Monetary policy shock

a result, employment increases which reduce the drop in consumption by more earnings available for households. The LTV rule also slightly stabilizes the output and inflation. However, this much less compared to the liquidity policy.

Overall, stabilization benefits of the liquidity policy are much higher than the LTV rule, specifically for uncertainty, credit and cost-pushed shocks. For the monetary policy shock, the liquidity policy enhances responses of inflation and default rate and leads to slightly decreased consumption together with lower employment. The LTV rule is more efficient to stabilize financial variables such as lending and deposits, however, also results in slight moderations for the rest system.

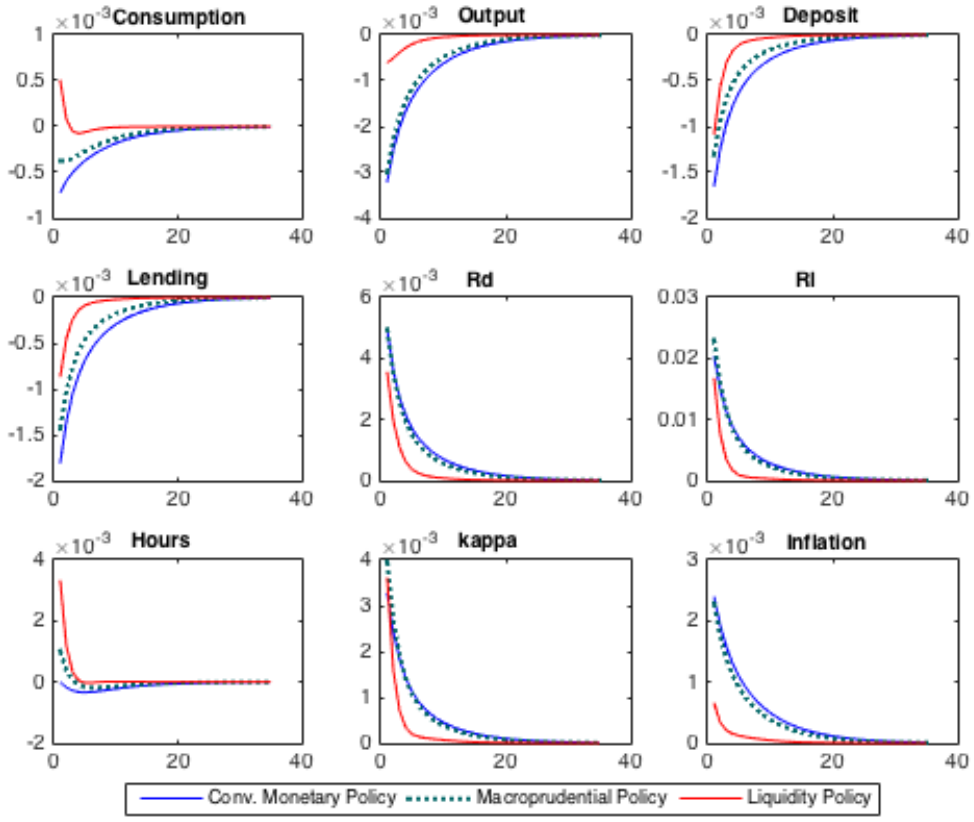


Figure 3.10: Marginal cost shock

3.5.2 Volatility and policies

Below we plot unconditional variances of the inflation, output and lending for different levels of reaction for the liquidity policy (figure, 3.11) and the LTV rule (figure, 3.12). Increasing level of the liquidity intervention reaction substantially reduces the volatility of output, and lending, whereas, the variance of inflation initially drops for low levels of reaction but eventually increases for higher intervention levels. The LTV rule, however, decreases the volatility of output and lending but raises the volatility of inflation. The volatility reduction in output is also smaller with the LTV rule than the liquidity policy.

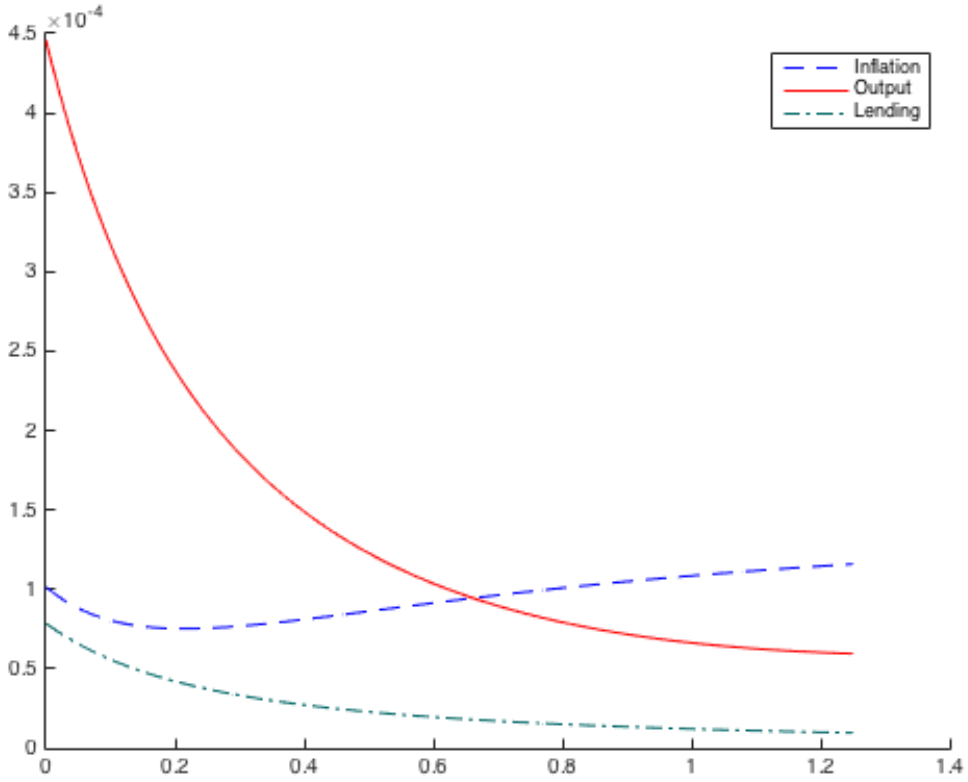


Figure 3.11: Volatility and the Liquidity Policy

We also consider volatility effects of the shocks on key macroeconomic variables in economies with high and low bank leverages. Following Mumtaz & Theodoridis (2015) and Rudebusch & Swanson (2012) we define conditional heteroscedastic response of the variable x_t as,

$$\sigma_{x,t} = \frac{\left[E_t\big(x_t - E_t(x_{t+1})\big)^2\right]^{\frac{1}{2}}}{\sigma_x} \tag{3.138}$$

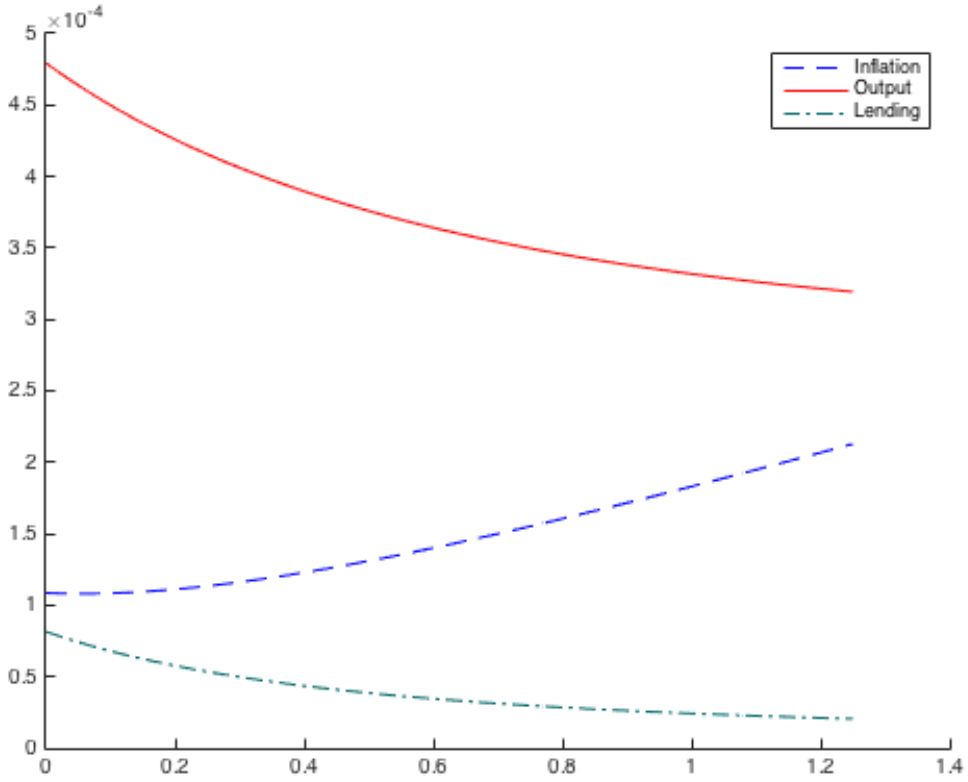


Figure 3.12: LTV rule and volatility

where σ_x is the unconditional (steady state) standard deviation of the variable x_t. Rudebusch & Swanson (2012) shows that higher order moments of the economy's endogenous state varies over time due to non-linearities in the model. Therefore, we consider volatility effects of contractionary monetary policy and credit shocks. For both shocks - increase in the policy rate and credit tightening raises the volatility of all variables except the policy rare and inflation. Increased volatility following adverse monetary policy shock is also consistent with Mumtaz & Theodoridis (2015). Nonetheless, volatility responses of the economy with higher lending are considerably lower than the economy with less lending to deposit ratio. Although, using the stochastic volatility shock in the model implies an exogenous change in second-order moments, availability of higher lending during the "moderation"(Figure 3.13, 3.14) possibly contributed to endogenous compression in the volatility of business cycle fluctuations.

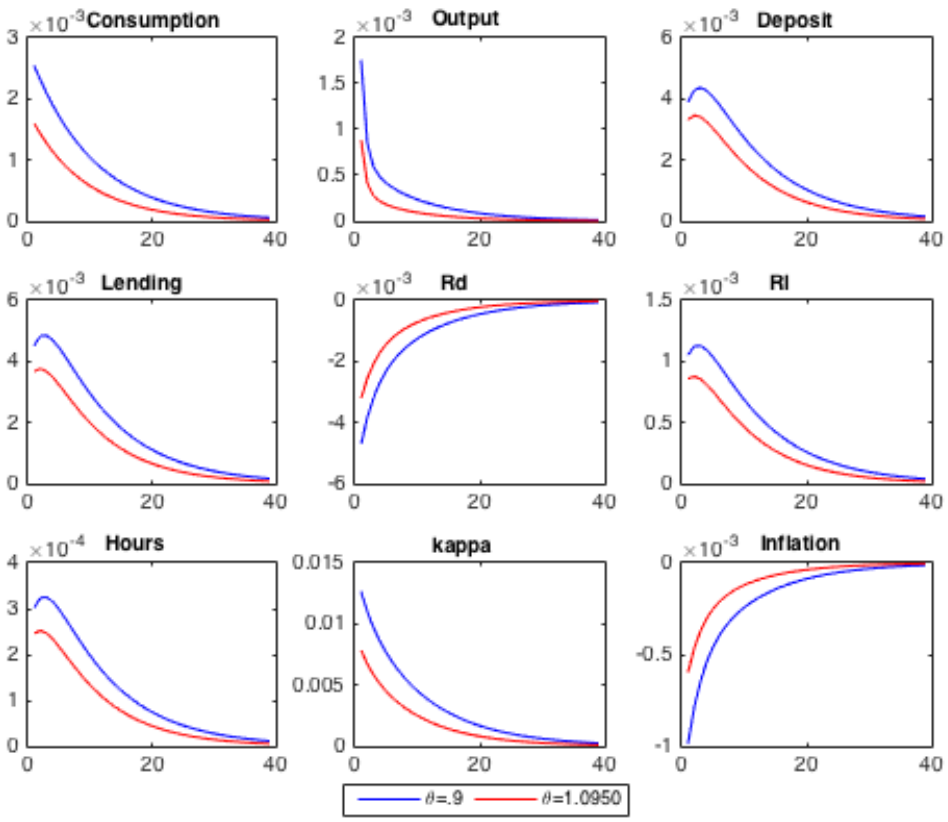


Figure 3.13: Volatility responses to an increase in the policy rate

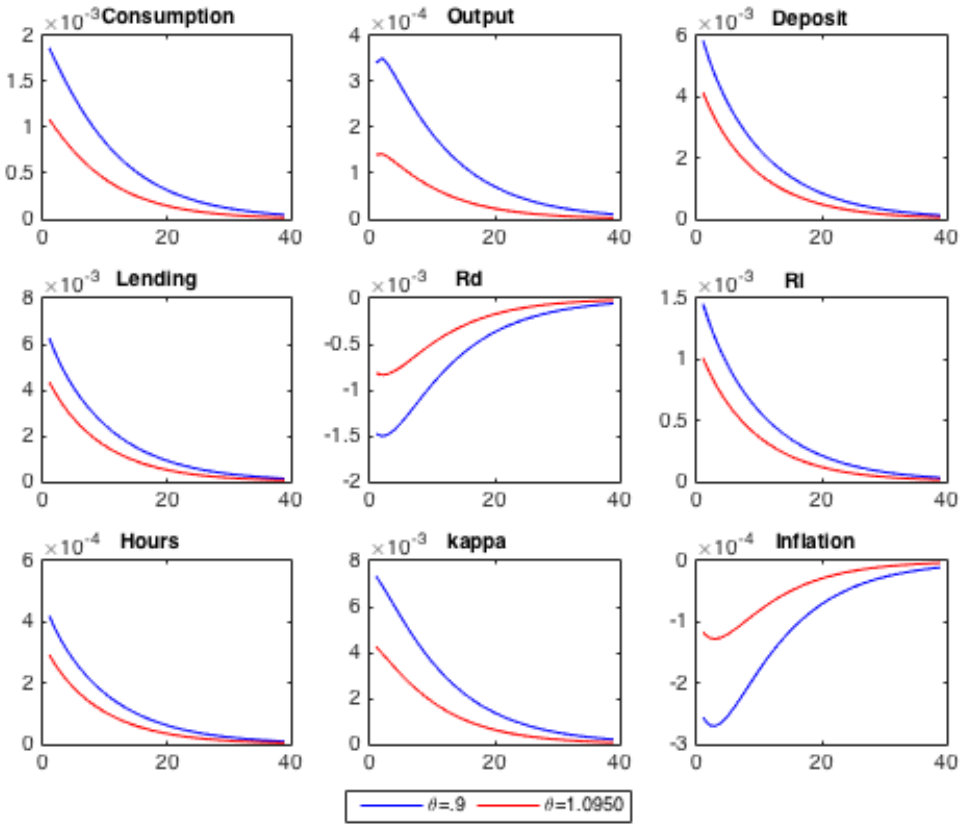


Figure 3.14: Volatility responses to a negative credit shock

3.5.3 Sensitivity Analyses

In this section, we analyse implications of various assumptions on parameters for transmission of uncertainty and credit shocks. We also look at how these parameters affect the term premium in the model. Figure (3.15) plots impulse responses for different values of banks' leverage ratio (θ). We find that the lower deposit to lending ratio exacerbates the transmission of the credit shock. Particularly, the lower ratio expands responses of consumption, both interest rates, employment, default rate, and inflation. Whereas, it decreases the lending without considerably changing responses of output and deposits. However, for the uncertainty shock (figure 3.16), a lower deposits to lending ratio reduces responses of all variables, except the lending rate on impact.

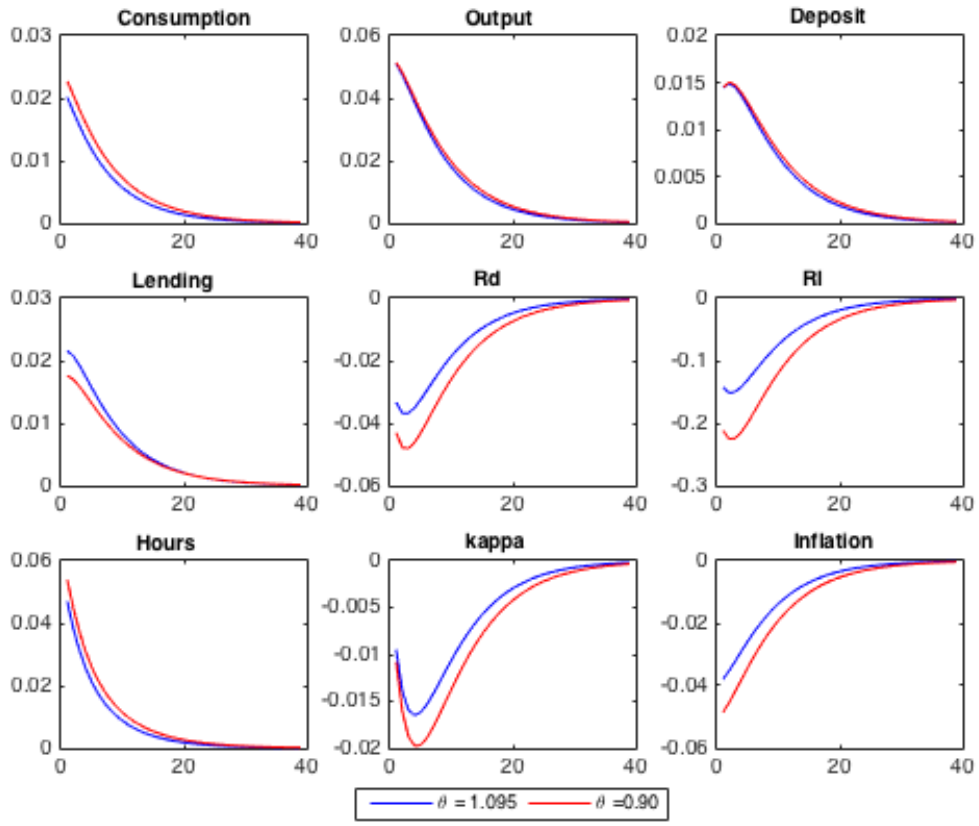


Figure 3.15: Banks leverage and the credit shock

Figures (3.17, 3.18) show impulse responses for different levels of firms' leverage ratio. Differently from banks' leverage, higher firms' leverage increases responses of both, credit and uncertainty shocks.

In figure (3.19) we plot average term premium for different values of the stochastic volatility and firms' leverage ratio. For estimated values of the uncertainty and the leverage of firms (dashed blue line), the term premium is around 36 basis points. The

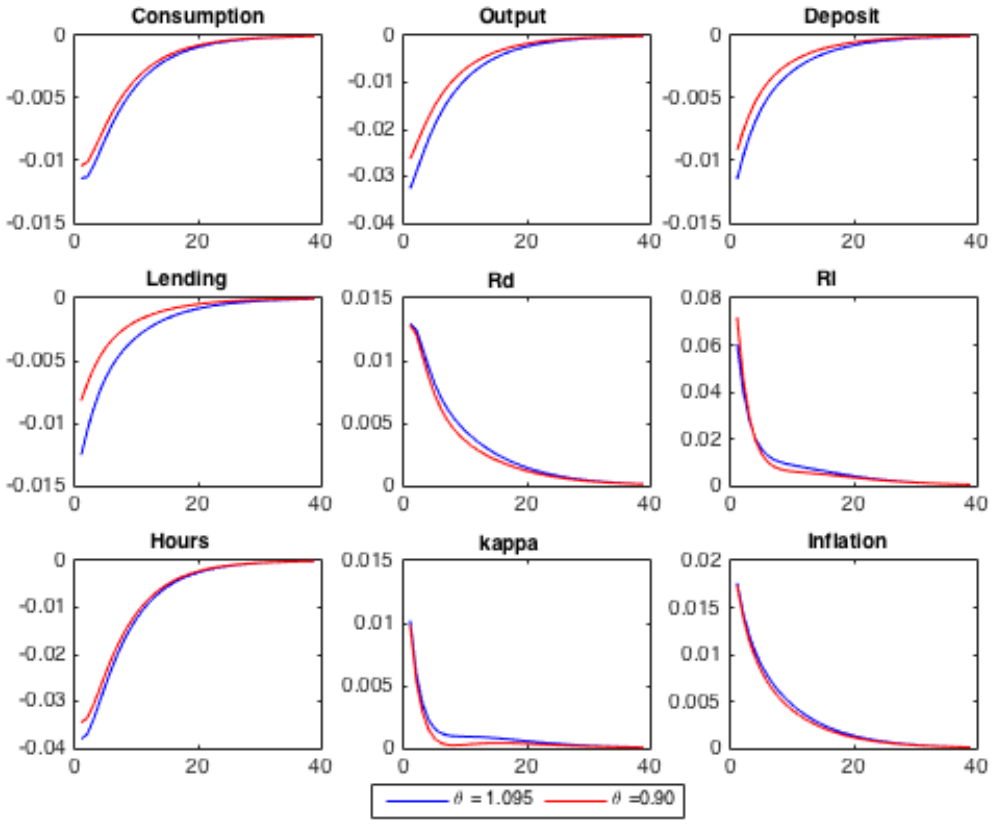


Figure 3.16: Banks leverage and the uncertainty shock

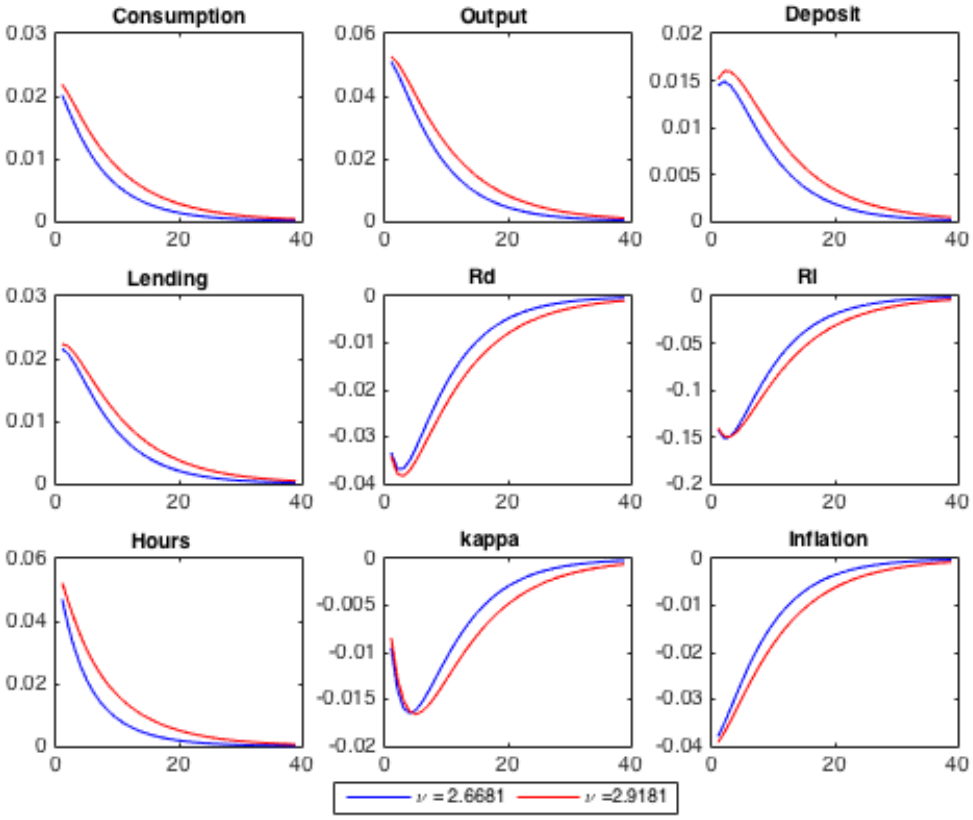


Figure 3.17: Firms' leverage and the credit shock

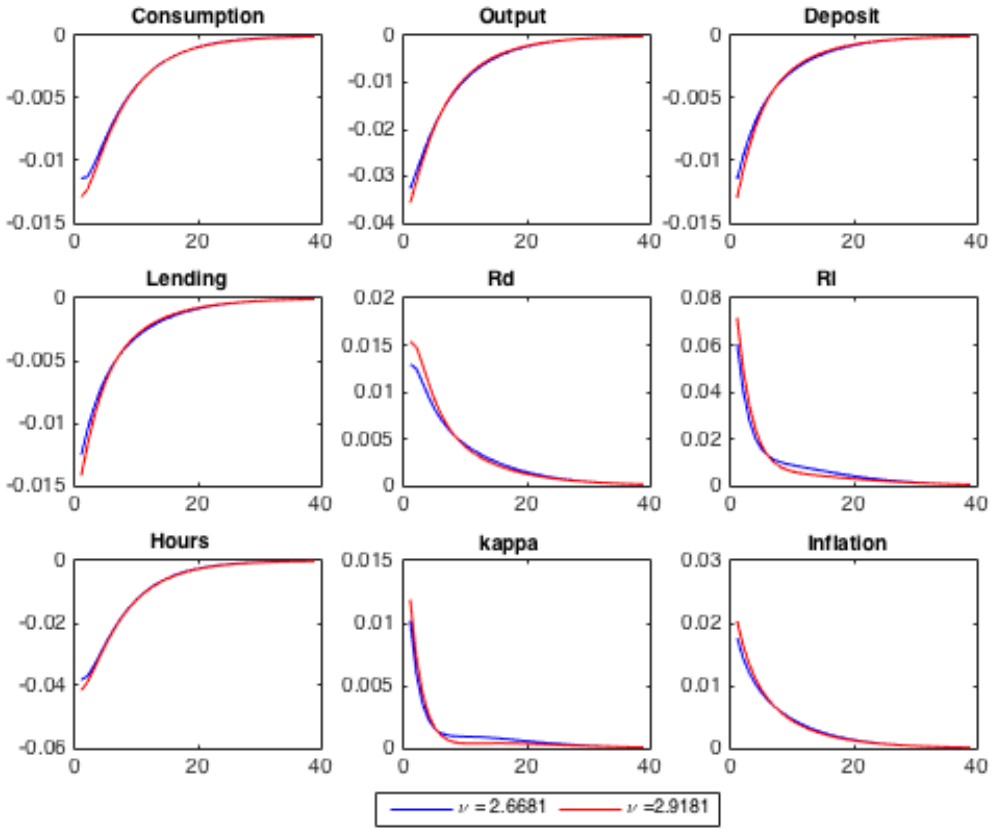


Figure 3.18: Firms' leverage and the uncertainty shock

term premium increases with firms leverage. When the leverage is 69% $(1 - 1/3.2)$, the term premium increases to 60 basis points. Higher uncertainty increases the mean term premium to 75 basis points for the lower value of firm's leverage ratio (60%). The gap in the mean term premium also increases between higher and lower leveraged economies with increasing uncertainty. Consequently, when firm leverage increases to 69%, the term premium rises to 1.4% with the higher level of volatility. Higher firms leverage implies higher default probability together with higher inflation. Therefore, the term premium increases with the leverage of firms.

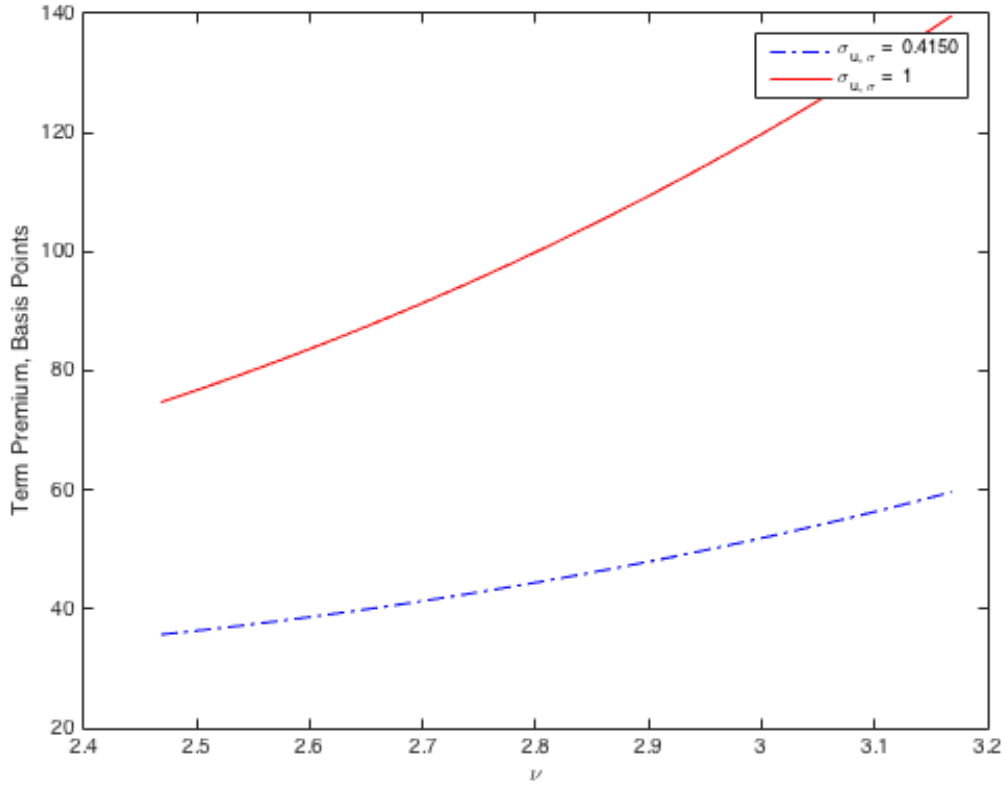


Figure 3.19: Uncertainty, Term Premium and Firm's leverage

Figure (3.20) plots the relationship between the term premium and banks' leverage. As opposed to the firms' leverage, banks' leverage considerably reduces the term premium in the economy. For the fixed firms' leverage (at its estimated value), 90% loans to deposit ratio implies 46.5 basis term premium. For the higher level of uncertainty (one standard deviation stochastic volatility shock), the term premium increases to 97 basis points. However, when banks' leverage is negative, the term premium substantially reduces under both levels of volatility.

Figure (3.21) plots the relation between mean term premium and monetary policy

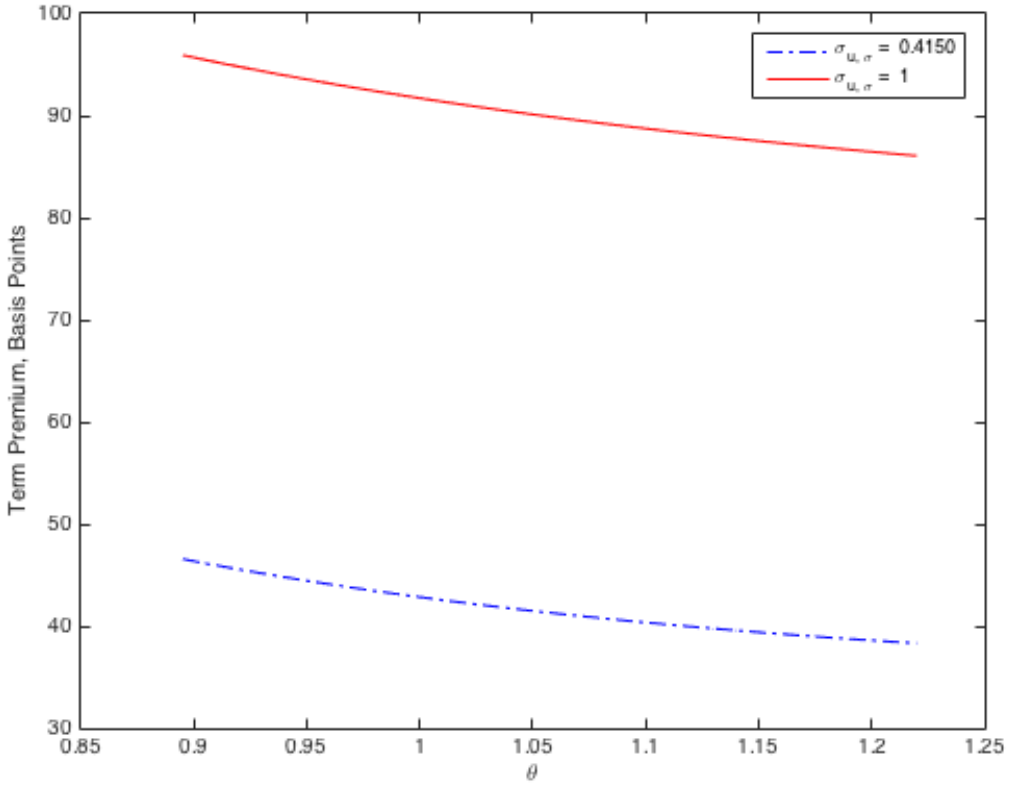


Figure 3.20: Uncertainty, Term Premium and Banks' leverage

for the fixed Epstein-Zin parameter at the calibrated value ($\gamma_{EZ} = 10$). The term premium is very sensitive to the stance of monetary policy and is decreasing with the inflation parameter in the Taylor rule. Whereas, higher output gap targeting increases the term premium when combined with low inflation targeting. This in line with findings of Ajello & Tanaka (2016). When the central bank reacts more to fluctuations in the inflation, this reduces uncertainty around expected inflation and also affects the correlation between consumption growth and inflation. This, in turn, reduces the term premium. However, increased focus on output fluctuations reduces the mean credit premium and therefore generates higher term premium.

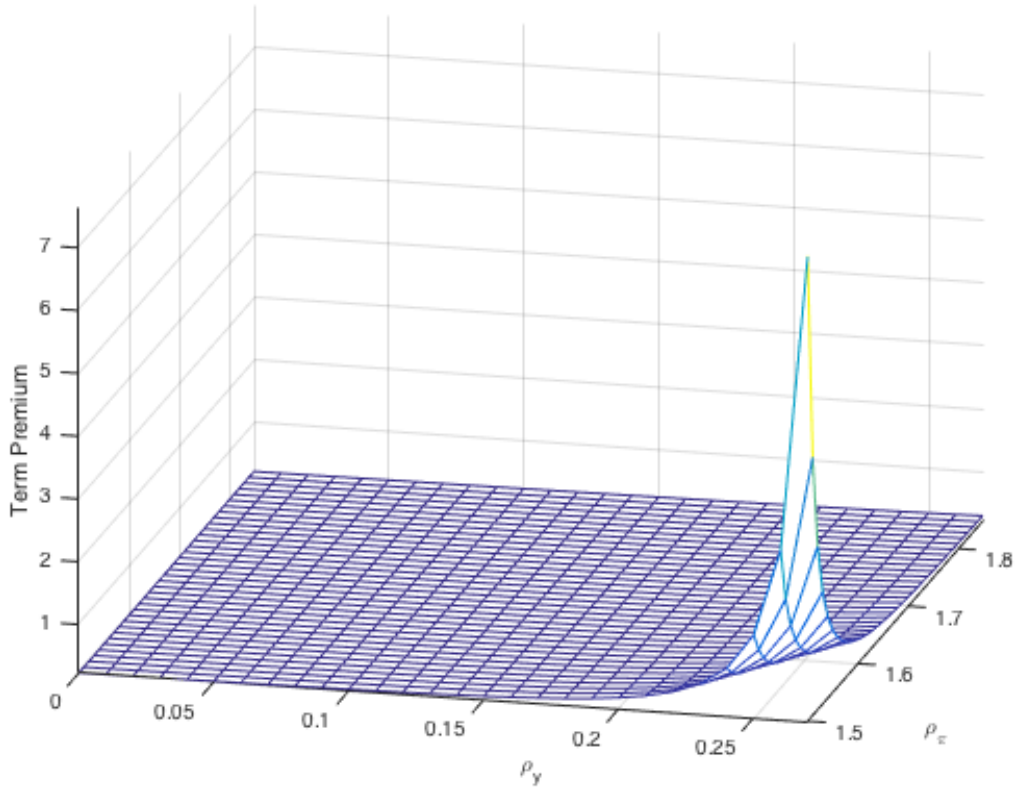


Figure 3.21: Monetary Policy and Term Premium

We also plot impulse responses with expected ($\gamma_{EZ} = 0$) and recursive utility preferences (Epstein-Zin parameter, $\gamma_{EZ} = 10$) for uncertainty shock (fig. 3.22). The Epstein-Zin parameter does not considerably alter dynamics of the model when the inter-temporal elasticity is unity (log consumption temporal utility) for low values of the risk aversion. Therefore to exacerbate implications of the recursive preferences, we set the inter-temporal elasticity of substitution to 1.5. For the uncertainty shock Epstein-Zin parameter implies higher precautionary behaviour and therefore largely

dampens responses of all variables, particularly of the inflation and, thus, the policy rate. For level shocks, the recursive preference largely gives similar dynamics as the expected utility.

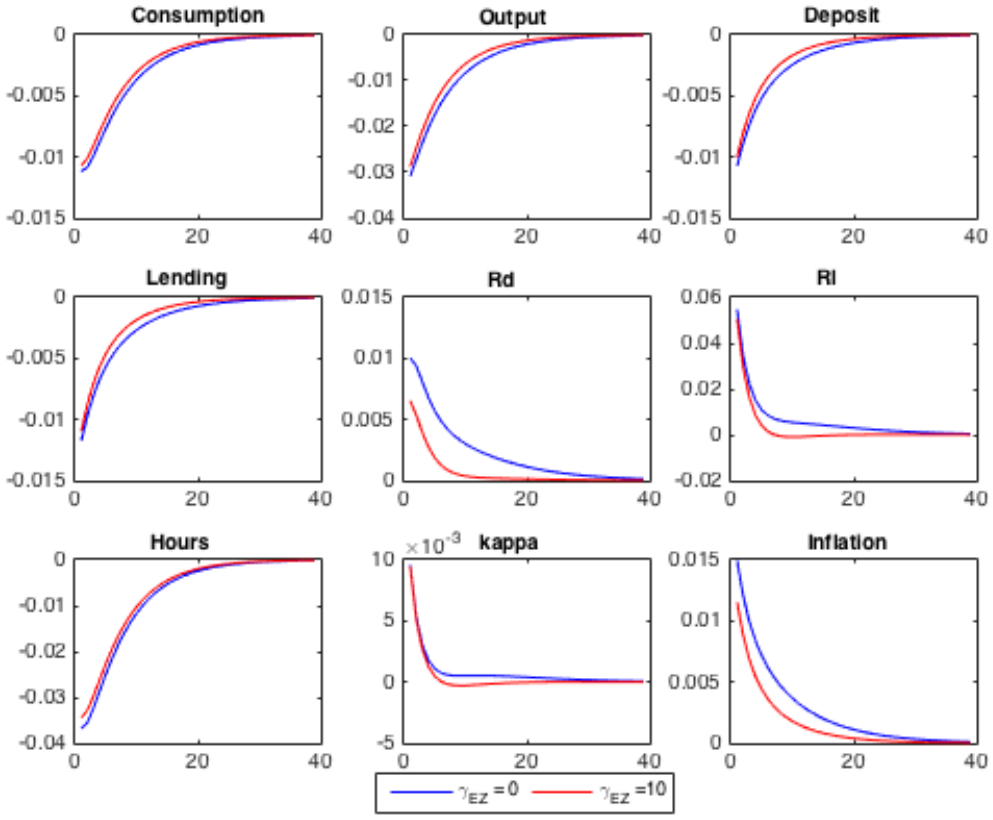


Figure 3.22: Uncertainty shock and EZ parameter

3.5.4 Optimal Policy and Welfare analysis

In this section, we first look at optimal policies and policy mixes that maximize the welfare measure of the representative agent and then analyse optimal policies in a non-cooperative institutional design based on an ad-hoc, loss linked mandates. We also investigate implications of the recursive utility for optimal policies in our model. Therefore, we look at optimal policies under two welfare assumptions. First welfare case corresponds to the expected utility. Second, we assume Epstein-Zin parameter that is fixed at it's calibrated value. For welfare analyses, we assume that the central bank and the macroprudential regulator jointly set policy parameters to maximize the lifetime welfare of households. Welfare gains are calculated in consumption equivalents relative to the baseline model.[9] Results are presented in table (3.4). In general, both policies (liquidity policy and LTV rule) deliver higher welfare outcomes than the optimal monetary policy alone, regardless the underlying welfare assumption. The liquidity policy, however, delivers considerably higher welfare pay-offs. The LTV rule also gives welfare improvements, but to the lesser extent, particularly for the households with recursive preferences. The LTV rule increases the reaction to output for both utility case[10]. The liquidity policy also increases the inflation parameter for the recursive utility case. Overall, the liquidity rule brings the highest welfare improvement across all welfare assumptions and policies.

Table 3.4: Welfare optimal policies

Welfare assumption	Optimized parameters [Consumption Equivalents]		
	$[\rho^*_{r,\pi},\ \rho^*_{r,y}]$	$[\rho^*_{r,\pi},\ \rho^*_{r,Y},\ \rho^*_{\nu,LTV}\ ]$	$[\rho^*_{r,\pi},\ \rho^*_{r,Y},\ \rho^*_{\theta,Liquidity}]$
$\gamma_{EZ}=0$	[9.8967 0.0004] [0.0380]	[9.8696 0.2067 8.1343] [0.0386]	[1.5005 0.002 9.9873] [0.044]
$\gamma_{EZ}=10$	[9.8382 0.0019] [0.0380]	[9.9986 0.2492 8.4964] [0.0384]	[8.6602 0.0133 4.9098] [0.0612]

Next, we look at optimal policy under non-cooperative game between the macro-prudential institution and the central bank. For this purpose, we choose loss based

[9] As a baseline model, we consider an economy with an unoptimized -standard Taylor rule and no macro-prudential policy

[10] To hold the model within the determinacy region we restrict the inflation parameter at 1.3 from below and the output parameter to 0.25 at upper bound

ad-hoc mandates for both institutions. Under this design, the central bank minimizes volatilities of, both, inflation and output. Whereas, the macroprudential authority targets the volatility of total lending in the economy. Specifically, we assume following mandates for the central bank and macroprudential authority:

$$L_{CB} = \sigma_{\pi}^2 + 0.1\sigma_Y^2 \tag{3.139}$$

$$L_{MP} = \sigma_L^2 \tag{3.140}$$

In the central bank's mandate, we assign a higher weight for the inflation than output which is consistent with inflation targeting, practically, being a primary concern for many central banks. Following analyses in the first chapter, we assume that for every possible policy option of the macroprudential institution, the central bank forms a vector of best responses that minimize its' loss function. Similarly, the macro-prudential authority also constructs its' best responses for each and single policy option available for the central bank. For simplicity of analyses, we assume that the central bank optimizes over a single parameter, which is the inflation parameter in the Taylor rule where available policy options for the central bank is a subset of the vector $P_{CB} = [1.3, ..., 10]$. Macroprudential authority reacts to the lending to carry out its mandate where policy options are also restricted to the vector of $P_{MP} = [0, ..., .10]$. Thus, without going too many technical details, in this set-up, the definition of the Nash equilibrium is standard and corresponds to the intersection of best responses of both policymakers.

Figure (3.23) and table (3.5) summarizes policy and loss outcomes for the Nash equilibrium and coordination with and without the liquidity policy. It is always optimal for the macroprudential authority to choose the highest policy option available, regardless of the central bank's policy stance either on inflation or on the liquidity. Whereas central bank's best response is decreasing for the lower LTV parameters, but then increasing for higher values of the LTV rule, without the liquidity policy. We repeat this game with an active, but fixed liquidity policy that complements the standard monetary policy ($\rho_\theta = 10$). Irrespective of the additional policy tool, it is still optimal for the macroprudential institution to opt for the highest policy choice. With a fixed liquidity policy, the central banks' best responses comprise of the highest reaction parameter irrespective of the LTV policy. Therefore, the highest policy options for both institutions is a Nash equilibrium with an active liquidity policy. The collaborative institutional design also implies same policy outcomes regardless the liquidity policy. Thereby, the liquidity policy renders the institutional design irrelevant in this case[11].

[11]This outcome also holds for a moderate level of liquidity intervention ($\rho_{\theta=5}$)

Table 3.5: Coordination vs Nash - Taylor rule and LTV

Institutional arrangement	Policy strategy $[\rho_\nu \quad \rho_\pi]$	CB loss	MP loss	Total loss
Nash: ρ_θ=0	[10 6.4748]	1.5714	0.5260	2.0059
Nash: ρ_θ=10	[10 10]	0.2516	0.0151	0.3670
Coordination: ρ_θ=0	[10 10]	1.3541	0.2516	1.5467
Coordination: ρ_θ=10	[10 10]	0.2516	0.0151	0.3670

Furthermore, an active reserve requirement considerably decreases loss proxies for both institutions[12]. We also calculate aggregate losses when two institutions cooperate and jointly minimize a common and additive loss function. Without the liquidity policy, the non-cooperative game always results in sub-optimal outcomes (Table, 3.5) for both institutions and at the aggregate level. An active reserve requirement policy substantially decreases both the total loss and losses at institutional levels when policy-makers do not cooperate. Whereas, without the reserve requirement non-coordinative set-up results in a much higher total loss (2.0059 vs. 1.5467). Therefore, complementing the standard Taylor rule with a reserve requirement policy, may also substantially reduce loss based ad-hoc penalties in non-cooperative institutional design, both at an institutional and aggregate level. Most importantly, the reduction in the total loss does not come at the expense of the increased penalty for one of the institutions.

Next, we analyse interactions between the liquidity policy and the LTV rule. Interactions between unconventional monetary policies (such as the liquidity policy) with the macro-prudential policy are fundamental, because of ever increased usage of both policies globally. For this reason, we analyse outcomes of the Nash game with an alternative policy set for the central bank. Henceforth, we assume that the central bank commits to a certain level of reaction to inflation, and only uses the reserve requirement policy to achieve its optimal mandate. First, we fix the inflation parameter to 1.75 in the Taylor rule and allow the central bank to choose from policy set of $P_{CB}{}^{\rho_\theta} = [0, ..., 10]$ for the liquidity intervention. The outcome of this game is summarized in figure (3.24) and table (3.6) together with optimal coordinated institutional arrangement.

For both levels of the inflation parameter in the Taylor rule, the highest response is the dominant strategy for the macro-prudential institution. Whereas, central bank's

[12]Note that the total loss will be different from the sum of losses at institutional levels. In general, when variables correlated the sum of variances is not same as the variance of the sum, because the covariance term will also enter into the variance of the sum

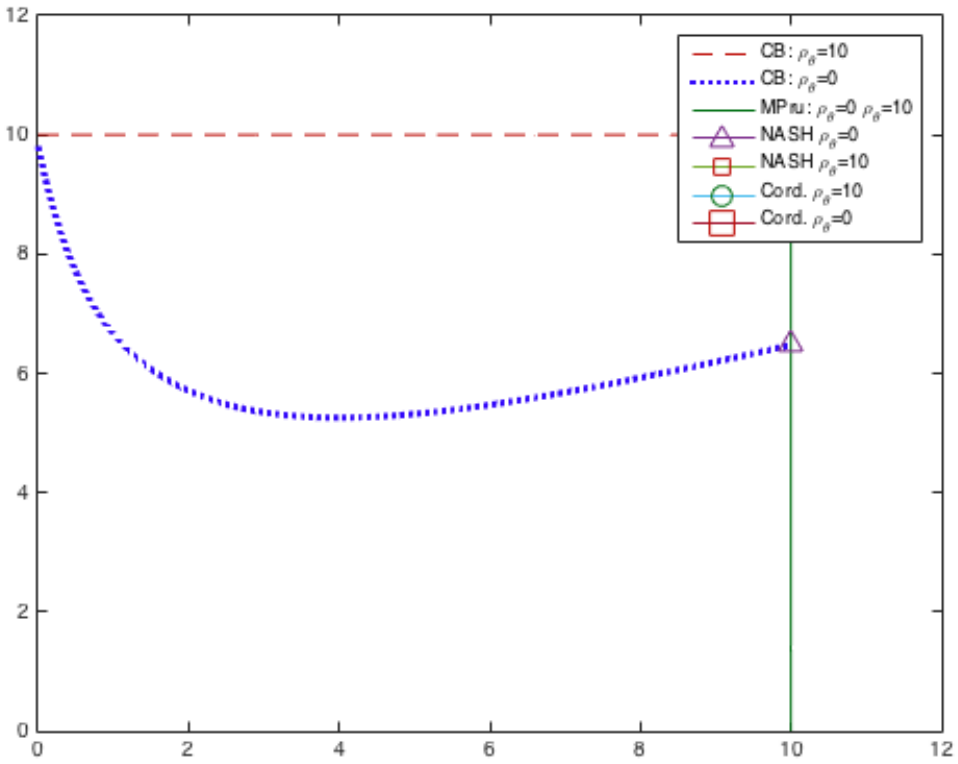


Figure 3.23: Nash game and cooperative institutional design

Table 3.6: Coordination vs Nash - Reserve Requirement and LTV

Institutional arrangement	Policy strategy $[\rho_\theta \quad \rho_\nu]$	CB loss	MP loss	Total loss
Nash: ρ_π=1.75	[10 10]	4.1236	0.0598	4.1828
Nash: ρ_π=10	[10 8.9106]	0.3514	0.0177	0.3692
Coordination: ρ_π=1.75	[2.7867 0.0001]	2.3534	0.5648	2.9185
Coordination: ρ_π=10	[6.4123 0.0005]	0.2665	0.0599	0.3263

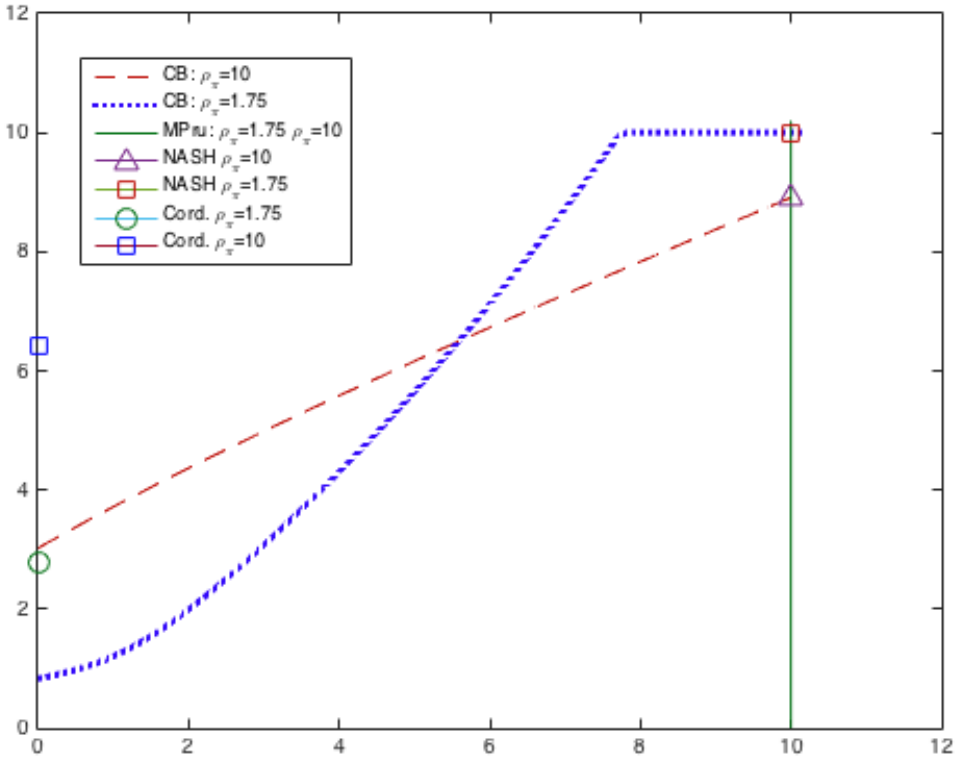


Figure 3.24: Nash game and cooperative institutional design

best responses increase with the LTV rule for both levels of the inflation targeting, but with varying degrees. When the inflation parameter of the Taylor rule is low (1.75), the Nash equilibrium comprises strongest policy strategies by two policy institutions. However, this leads to the biggest loss for the central bank and also to the highest loss measure on the aggregate level. When we fix the inflation parameter at its upper bound ($\rho_\pi = 10$), the Nash equilibrium opts relatively lower level of liquidity policy ($\rho_\theta = 8.9106$). This decreases the loss not only for the central bank but also for the macro-prudential institution. The coordinative institutional arrangement implies minimal feedback for the LTV rule, but moderate levels of intervention for the liquidity policy depending on the level of inflation targeting. For a lower level of inflation targeting the coordinative institutional design significantly reduces the loss measure of the central bank but also results in a higher loss for the prudential body. Whereas, the coordination with a higher level of inflation feedback reduces losses for both institutions. This also delivers the lowest level of loss at the aggregate level, but slightly higher loss for the prudential body than the Nash equilibrium under strong inflation targeting.

Overall, while the first arrangement (fixed liquidity policy and active inflation policy to respond to changes in LTV) allows for a Pareto optimal equilibrium, the second arrangement implies higher costs of non-cooperative design. The coordination in the second game also delivers higher loss measures at institutional levels. Whereas coordination and Nash equilibrium in the first design gives a slightly higher loss on the aggregate level, but still lower losses at institutional levels. This due to a higher correlation between individual loss measures in the first arrangement.

3.6 Conclusion

In this chapter we build a model that features firm default and price adjustment frictions. The model allows to analyse interactions among financial frictions, uncertainty and nominal rigidities. The present, in particular, augments the model of Pesaran & Xu (2016) with stochastic volatility and price rigidities. Following Pesaran & Xu (2016) we abstract from capital accumulation and attached rigidities to study the independent role of financial and nominal frictions in the transmission of the uncertainty shock. We also solve and estimate the non-linear model around the ergodic mean following Meyer-Gohde (2014). The model generates a substantial recession following the uncertainty shock which is further amplified by the presence of nominal rigidities. In particular, an increased uncertainty of the productivity process widens the default probability and thereby increases the cost of bank lending. Stochastic volatility, combined with nominal frictions dynamically affect the default threshold, whereas, it is fixed in Pesaran & Xu (2016). Therefore there is a feedback from nominal side to financial conditions and policies together with wider economic developments affect the default probability along the dynamic path. We find that the contractionary effect of the uncertainty shock rises with firms' leverage while it decreases with banks' leverage ratio. We also analyse implications of recursive preferences for the propagation of innovations and, thereby, allowing the precautionary behaviour to have a wider role in economic outcomes, beyond the default channel. Specifically, the model still generates a significant reduction in economic activity despite a higher precautionary behaviour.

We also augment the model with two additional policy tools, the liquidity policy and the LTV policy. The liquidity policy delivers higher stabilization benefits and welfare outcomes across both welfare assumptions - expected utility and recursive utility. Whereas, the welfare improvements and the convergence pressure of the LTV policy are only moderate over the optimal monetary policy alone. Welfare gains of the liquidity policy increase with the Epstein-Zin parameter, whereas welfare gains of the LTV rule over optimal monetary policy decrease with higher risk-aversion. Moreover, we use a game theoretical approach to analyse interactions among three policy choices. We find that the presence of liquidity policy allows having a Pareto-optimal Nash equilibrium between the inflation targeting central bank and lending targeting macro-prudential body where the institutional design is irrelevant given that the central bank also commits to a higher liquidity intervention. On the contrary, committing to a certain level of inflation targeting together with active liquidity policy as a response to the prudential body causes sizeable losses in non-cooperative arrangement and with slightly higher losses at institutional levels in the cooperative set-up.

We further investigate volatility implications of the monetary policy and credit shocks for varied levels of banks' leverage ratio. Our results suggest that higher a lending to deposit ratio decreases volatility outcomes of both innovations. This combined with largely leveraged banking sector during the "great moderation" possibly contributed to lower volatility in economic fluctuations observed in the data since the early 1980's.

The model also generates a moderate term premium with estimated and calibrated set of parameters that increases with underlying volatility shock. We also analyse effects of financial conditions on the term premium. The term premium, especially, rises with firms' leverage but decreases with banks' leverage ratio.

Our model, however, lacks habit formation and other rigidities attached to the capital accumulation process. Thus, the model can be extended to include these features as well. International spillovers of uncertainty and credit shocks would equally be useful extensions. Another critical issue of having more elaborate banking sector with interbank frictions, and rigidities between households and banks.

3.7 Appendix

Derivation of the steady state

The discount factor in the steady state is given as

$$\Xi = \beta(1+\mu)^{-\gamma} \tag{3.141}$$

where μ is the deterministic growth rate of the economy. Therefore, the deposit rate and the risk free rate in the steady state is given as, respectively

$$R_d = \frac{\bar{\Pi}}{\Xi}, \qquad R_f = \frac{1}{\Xi} \tag{3.142}$$

where $\bar{\Pi}$ is the steady state inflation. The steady state marginal cost is recovered from equations 3.94 - 3.96,

$$\overline{MC} = \left(1 - \frac{1}{\zeta}\right) \frac{1 - \xi\bar{\Xi}\bar{\Pi}^{\zeta}}{1 - \xi\bar{\Xi}\bar{\Pi}^{\zeta-1}} \left(\frac{1 - \xi\bar{\Pi}^{\zeta-1}}{1-\xi}\right)^{\frac{1}{1-\zeta}} \tag{3.143}$$

Equations 3.96-3.97 gives the steady state price dispersion,

$$\bar{\Delta} = \frac{(1-\xi)\left(\frac{1-\xi\bar{\Pi}^{\zeta-1}}{1-\xi}\right)^{\frac{-\zeta}{1-\zeta}}}{1 - \xi\bar{\Pi}^{\zeta}} \tag{3.144}$$

Steady state values of the marginal cost and the prices dispersion along with model parameters give steady states of variables related to the firm default,

$$x_{\kappa} = \frac{\ln\left(1 - \frac{\alpha}{\upsilon}\right) + \ln\left(\overline{MC}\right) + \frac{1}{\xi}\ln(\bar{\Delta})}{\varphi\sigma_{\vartheta}} + \frac{1}{2}\varphi\sigma_{\vartheta} \tag{3.145}$$

$$\kappa = \Phi\big(x_{\kappa}\big) \tag{3.146}$$

$$x_{\tau} = \frac{\ln\left(1-\alpha\right) + \ln\left(\overline{MC}\right)}{\varphi\sigma_{\vartheta}} + \frac{1}{2}\varphi\sigma_{\vartheta} \tag{3.147}$$

$$\tau = \Phi\big(x_{\tau}\big) \tag{3.148}$$

To derive the steady state lending rate we use the equation for the Bank's profit. Replacing the R.H.S. of the equation (3.62) with the definition of the Bank's profit ($\Pi_b = R_d D$) we have

$$R_d D = R_k L(1-\kappa) - WN(\kappa - \tau) + \big[\iota_1(u) - \iota_2(u)\big]Y^W \tag{3.149}$$

Then using the relation among deposits, lending and capital together with households' F.O.C. for wages gives the following result

$$\frac{R_d}{\theta}\left(1-\frac{1}{\nu}\right)K = R_k\left(1-\frac{1}{\nu}\right)K(1-\kappa) - \chi_0(\kappa-\tau)N^{\chi} + \big[\iota_1(u)-\iota_2(u)\big]Y^W \quad (3.150)$$

By using firms demand for the capital (equation, 3.23), and noting that $Y^W = \frac{R_k K}{\alpha MC}$, we have

$$\frac{R_d}{\theta}\left(1-\frac{1}{\nu}\right)K = R_k\left(1-\frac{1}{\nu}\right)K(1-\kappa) - KR_k(\kappa-\tau)\frac{1-\alpha}{\alpha} + \big[\iota_1(u)-\iota_2(u)\big]\frac{R_k K}{\alpha MC} \quad (3.151)$$

Dividing both side of the equation (3.151) by K allows to write R_k in terms of R_d and parameters

$$R_k = \frac{\frac{R_d}{\theta}\left(1-\frac{1}{\nu}\right)}{\left(1-\frac{1}{\nu}\right)(1-\kappa) - (\kappa-\tau)\frac{1-\alpha}{\alpha} + \frac{\iota_1(u)-\iota_2(u)}{\alpha MC}} \quad (3.152)$$

The rest of model variables can recursively be recovered using the rental rate of the capital - equation 3.75 gives the steady state of labour, then equation 3.74 gives the steady state capital, etc.

Derivation of the stochastic steady state and ergodic mean

For the second order σ approximation of the stochastic steady state, differentiate $\bar{y} = g(\bar{y}(\sigma), 0, \sigma)$ once at the deterministic steady state ($\sigma = 0$)

$$\bar{y}^{'}(0) = g_y \bar{y}^{'}(0) + g_\sigma = \big[I - g_y\big]^{-1} g_\sigma = 0 \tag{3.153}$$

and twice for

$$\bar{y}^{''}(0) = g_{yy}\bar{y}^{'}(0) \otimes \bar{y}^{'}(0) + 2g_{y\sigma}\bar{y}_\sigma + g_y \bar{y}^{''}(0) + g_{\sigma\sigma} = \big[I - g_y\big]^{-1} g_{\sigma\sigma} \tag{3.154}$$

Therefore, the second order σ approximation to the stochastic steady state is calculated as

$$\bar{y}^{stoch} \approx \bar{y} + \frac{1}{2}\big[I - g_y\big]^{-1} g_{\sigma\sigma} \tag{3.155}$$

The second order σ approximation for the ergodic mean is also related to the third order pruned state-space model using implicit function theorem. Differentiate $\bar{y}(\sigma) = E\big[g\big(y_{t-1}, \sigma\epsilon_t, \sigma\big)\big]$ once at $y_{t-1} = \bar{y}(0)$, and $\sigma = 0$

$$\bar{y}^{'}(0) = E\Bigg[g_y \frac{\partial y_{t-1}}{\partial \sigma} + g_\epsilon \epsilon_t + g_\sigma\Bigg] = \Bigg[I - g_y\Bigg]^{-1} g_\sigma = 0 \tag{3.156}$$

and twice for

$$\begin{aligned} \bar{y}^{''}(0) = E\Bigg(g_y \frac{\partial^2 y_{t-1}}{\partial \sigma^2} + g_{yy}\frac{\partial}{\partial \sigma}\big[y_{t-1} \otimes y_{t-1}\big] + 2g_{y\epsilon}\epsilon_t \otimes \frac{\partial y_{t-1}}{\partial \sigma} + \\ + 2g_{y\sigma}\frac{\partial y_{t-1}}{\partial \sigma} + 2g_{\epsilon\sigma}\epsilon_t + g_{\epsilon\epsilon}(\epsilon_t \otimes \epsilon) + g_{\sigma\sigma}\Bigg) = \\ = \big[I - g_y\big]^{-1}\Bigg[g_{yy}E\Bigg(\frac{\partial}{\partial \sigma}\big(y_{t-1} \otimes y_{t-1}\big)\Bigg) + g_{\epsilon\epsilon}E\big(\epsilon_t \otimes \epsilon_t\big) + g_{\sigma\sigma}\Bigg] \end{aligned} \tag{3.157}$$

and using the following result,

$$\begin{aligned} E\Bigg[\frac{\partial}{\partial \sigma}\big(y_t \otimes y_t\big)\Bigg] = E\Bigg[\big(g_y \frac{\partial y_{t-1}}{\partial \sigma} + g_\epsilon \epsilon_t + g_\sigma\big) \otimes \big(g_y \frac{\partial y_{t-1}}{\partial \sigma} + g_\epsilon \epsilon_t + g_\sigma\big)\Bigg] = \\ = g_y \otimes g_y E\Bigg[\frac{\partial}{\partial \sigma}\big(y_{t-1} \otimes y_{t-1}\big)\Bigg] + g_\epsilon \otimes g_\epsilon E\big[\epsilon_t \otimes \epsilon_t\big] \end{aligned} \tag{3.158}$$

we have,

$$\bar{y}^{''}(0) = \big[I_{n_y} - g_y\big]^{-1}\Bigg[g_{\sigma\sigma} + \Bigg(g_{\epsilon\epsilon} + \big(I_{n_y \times n_y} - g_y \otimes g_y\big)^{-1} g_\epsilon \otimes g_\epsilon\Bigg) E\big(\epsilon_t \otimes \epsilon_t\big)\Bigg] \tag{3.159}$$

which gives the second order σ approximation to the ergodic mean,

$$\bar{y}^{mean} \approx \bar{y} + \frac{1}{2}\big[I_{n_y} - g_y\big]^{-1}\Bigg[g_{\sigma\sigma} + \Bigg(g_{\epsilon\epsilon} + \big(I_{n_y \times n_y} - g_y \otimes g_y\big)^{-1} g_\epsilon \otimes g_\epsilon\Bigg) E\big(\epsilon_t \otimes \epsilon_t\big)\Bigg] \tag{3.160}$$

Pricing decision of the firm.

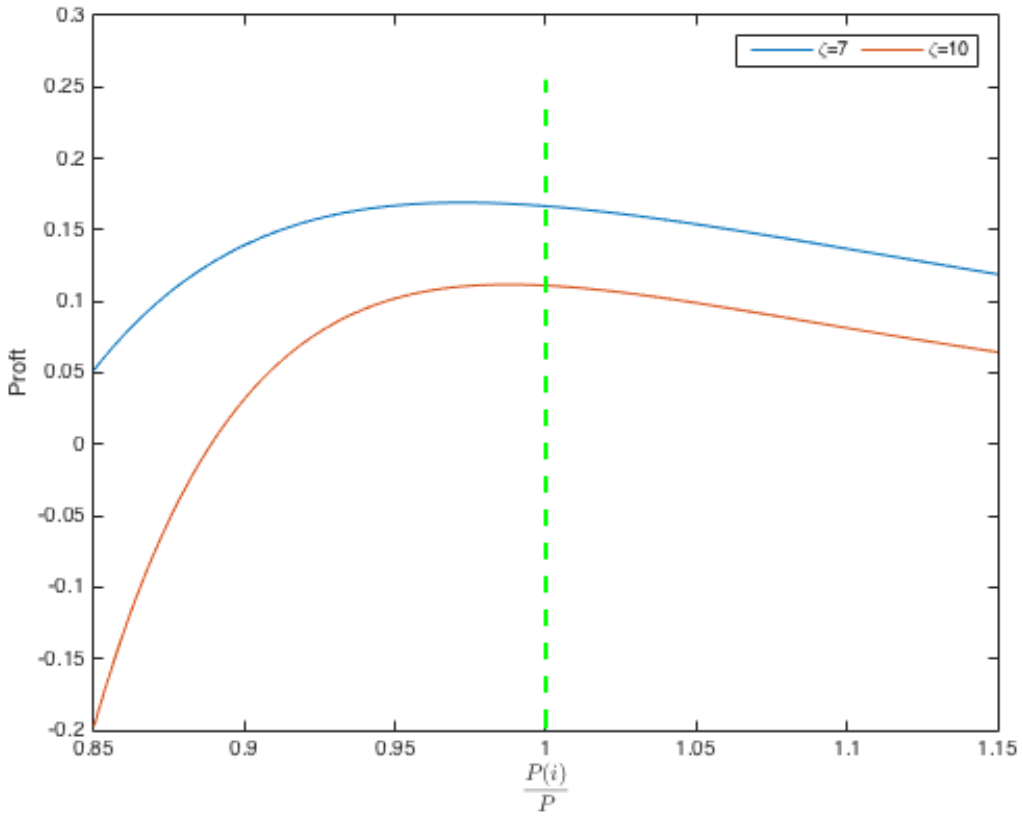


Figure 3.25: Firm profit and the relative price

Conslusion

In the present study, we attempt to shed light on a number of important questions related to the interactions between various policy tools in the DSGE modelling framework in the presence of heterogeneity and volatility shocks. Furthermore, we also provide a clear analysis of the implications on both the conduct of optimal policies and institutional arrangement of the models put forward. Specifically, the first chapter of our study assumes heterogeneity in households level, while the second chapter incorporates ex-post heterogeneous firms with idiosyncratic productivity innovations. The first model offers a framework where the welfare of both agents is tractable and allows policies to bear uneven consequences on individual agents where the distribution of wealth has a significant role in economic outcomes. Using the notion of partially distorted steady-state we derive a theoretical, welfare-based loss function for this economy and show that this welfare measure depends on the consumption gap between agents along with other variables (collateral of borrowers, deviation of output from its efficient risk-sharing level) that are present because of financial distortions. Therefore we are able to propose welfare based mandates to the central bank and the macroprudential institution that targets related inefficiencies. We analyse the interactions between macroprudential and monetary policy in a game theoretical framework and show that when welfare based mandates are assigned to each institution, the equilibrium of this game will be Pareto optimal which will coincide with the outcome of the cooperative institutional design.

In the second chapter, however, idiosyncratic-heterogeneous technology shocks at firms' level create a wedge between lending and deposit rates owing to the costly default. We also augment the model with stochastic volatility in the common technology process (systemic or system-wide risk) which increases the default probability along the dynamic path. We analyse the role of uncertainty, credit shocks and recursive preferences in the model, on one hand, and the implications of monetary, liquidity and macroprudential policies in the model subject to the risk, on the other. In fact, the present offers a unified framework for studying interactions among uncertainty, financial frictions, and

nominal rigidities. In our proposed model, nominal variables enter into the default condition together with the stochastic volatility and therefore dynamically alter financial conditions. We find that an increase in the volatility of common technology process creates a slowly building and persistent drop in economic activity where feedbacks from the demand and supply side work together to extend the recession. More to the point, increased uncertainty raises both interest rates, however unevenly, where the lending rate raises considerably more because banks seek compensation for the elevated level of uncertainty over productivity outcomes. This causes uneven responses where the cost of the reduction in demand is less than the increase in the cost of capital, and therefore profit-maximizing firms choose fewer quantities with higher prices which significantly increases the inflation. Moreover, the initial decrease in demand and higher inflation further tightens financial conditions because of the feedback loop from the nominal side. An expansionary credit shock, however, reverses this loop and thereby does not put inflationary pressure on the economy.

We also investigate the impacts of liquidity policy as a proposed unconventional monetary instrument together with the LTV rule on macroeconomic stability and welfare. We find that the liquidity policy delivers higher stabilization benefits across all shocks, except for the monetary policy shock, and overall higher consumption equivalents across both utility assumptions. The LTV rule is also welfare improving to some extent and has positive stabilization pressure which, however, is moderate compared the liquidity policy. Moreover, we analyse interactions among three policy options using loss-based mandates. We find that adopting the liquidity policy as an additional policy instrument allows having a Pareto optimal Nash equilibrium between the central bank who targets the volatility of inflation and the macroprudential body who targets the volatility of the lending, given that the central bank commits to the highest level of liquidity intervention. The liquidity policy also in moderate intervention levels can help to reduce the unconditional volatility of the inflation, still, raising it only slightly with higher reaction parameters.

Finally, we analyse volatility implications of the bank leverage in the economy. Our results suggest that increased leverage implies sizeable reductions in volatility responses of the economy to shocks. Therefore, highly leveraged banking sector (possibly as a response to suppressed volatility) during the "great moderation" potentially augmented further the reduction in the volatility of macroeconomic variables.

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