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Modelling, Analysis and Control of Tendon- driven Continuum Robots

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Abstract

Continuum robots (CRs) outperform the conventional rigid-link industrial manipulators owing to their hyper-redundant and compliant properties, enabling them the capability of accessing to confined environments (e.g., aero-engines and pipeline systems) and applications requiring flexible interactions such as minimal invasive surgeries. As such, an Innovate UK project was setup to validate the concept of utilizing slender continuum robots for the in-situ maintenance of aero-engines. This research firstly follows the development of the control algorithms for the continuum robot constructed in the project, which is a 715 mm long continuum robot with ten 55 mm long one-DoF (Degree of freedom) body-sections and three 55 mm long two-DoF sections at the tip, together with a linear stage for feed-in motion. The robot has been demonstrated within a range of aerospace scenarios. Besides, precise control of single-section continuum robots is also within the scope of this research, aiming to develop universal high-performance controllers for various designs.

Through this work, model-less controllers and model-based hybrid controllers are designed for slender multi-section and single-section continuum robots, respectively. Inspired by the tug of war, a novel local model-less controller utilizing fuzzy logic algorithm is proposed for the tip sections control on the slender long continuum robot. This implements the control policies directly from the task space to the actuation space, avoiding the model mismatch of the PCC (Piecewise-Constant-Curvature) assumption owing to the explicit call of arc parameters. Experiments on a single section of the tendon-driven continuum robot, in comparison with PCC-based method, validate the stability of the developed controller, which can reach ± 1 mm overall positioning accuracy and ± 0.5 mm positional accuracy for 75% of the test points in both X and Y

directions. Further, a set of trails on two distal sections of a long robot demonstrate that the controller can also effectively minimise the section coupling issue.

In terms of single-section continuum robots, hybrid control scheme is proposed after the analysis of the regular control modes of continuum robots and the advantage of such type of robots, which enables tension supervision to avoid slack state owing to the cumulative control errors during operation. Also, a novel differential kinematics is derived directly from the task space to actuation space by using polar coordinate system to replace the Cartesian coordinate system in conventional kinematics and applying accurate piecewise linear approximation. The proposed controller is experimentally compared with the PCC-based method on the continuum robot with twin-pivot joints. The experimental results demonstrate that the proposed controller had more comprehensive stability for positioning, while both controllers show their advantages on different performance indices such as RMSE(Root-Mean-Square-Error) and maximum error in terms of path tracking. Further performance characterisation of the proposed controller is conducted on the dual-revolute and soft-robot designs.

Publications

The research in this thesis has contributed in part or full for the following publications:

- **Ba, W.**, Dong, X., Mohammad, A., Wang, M., Axinte, D., & Norton, A. (2021). Design and validation of a novel fuzzy-logic-based static feedback controller for tendon-driven continuum robots. IEEE/ASME Transactions on Mechatronics. (Corresponding to the work in Chapter 5)
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Nomenclature

The following notation and abbreviations are used throughout this work.

Short	Full name
CR	Continuum robot
DoF	Degree of freedom
PCC	Piecewise-constant-curvature
RMSE	Root-mean-square-error
NiTi	Nitinol
T·S	Task space
C·S	Configuration space
A·S	Actuation space
IK	Inverse kinematics
SMA	Shape memory alloy

Symbol	Description	Unit
$l = [L_1, L_2, L_3]^T$	Cable lengths	mm
$q = [\theta, \varphi, S]^T$	Arc parameters: bending angle, direction angle, backbone length	[°, °, mm]
$P = [x, y, z]^T$	Tip position in Cartesian coordinate system	mm
$P = [p, \phi, z]^T$	Tip position in Cylindrical coordinate system	[mm, °, mm]
$(\vec{e}_x, \vec{e}_y)$	Vectors of error components in Cartesian coordinate system	mm
$(\vec{e}_r, \vec{e}_\theta)$	Vectors of error components in Polar coordinate system	mm
α_i	Cable phase angle (i is cable index)	°
R	Radius of single section	mm
r	Radius of the circle where the cables are distributed	mm
n	Number of segments within single section	1
T	Transformation matrix	$I_{4 \times 4}$
$R_{3 \times 3}$	Rotation matrix	$I_{3 \times 3}$
β	Joint angle	°
K	Gain coefficient in the PCC-based static controller	1
u	Step-size in the empirical controller	mm
μ	Gain coefficient in the fuzzy-logic-based controller	1
$d\delta L_i$	Cable displacement change (i is cable index)	mm
η	Gain coefficient in the hybrid controller	1

1 Introduction

1.1 Background

The term ‘Robot’ was first applied for artificial automata in a 1920 science-fiction play *R.U.R.* [1] by the Czech writer Karel Čapek, which was originated from the Slavic language as *robota*. However, it was until 1954 that the first programmable and digitally operated robot was invented, which was ultimately called *Unimate* [2] and laid the foundations of modern robotics industry. In fact, modern robots have entered rapid and prosperous development with the emergence of integrated circuits and the advancement of computer technology and control theory.

Autonomous robots have play essential role for the fast development of industrialization, which are capability of performing repetitive and heavy-duty tasks, for instance, manufacture and assembly in industry, with high speed, precision, and reliability. Further, the application field of robots started to expand outside of factories and served in areas such as agriculture, service, aerospace, and military. Overall, those developed robots can be divided into two main categories: industrial and service robots [3, 4]. According to ISO 8373 by the International Federation of Robotics (IFR) [5], the category of industrial robot can be defined as “an autonomous controlled, reprogrammable, multipurpose manipulator programmable in three or more axes, which may be wither fixed in place or mobile for use in industrial automation applications”, which can be classified into several sub-categories by mechanical structure, including articulated robot, SCARA robot, Delta robot, and Cartesian/Cylindrical/Spherical coordinate robot. While service robot is defined by the International Organization for Standardization as a robot that “performs useful tasks for

humans or equipment excluding industrial automation applications”, including frontline service robot, domestic robot, and scientific robot.

Over the last two decades, a new type of robot – continuum robot, also called snake arm, has been created owing to its flexible and hyper-redundant characters. The main advantage of continuum robots over conventional robots is the ability of getting access into narrow environments; the other advantage is that they are capable of performing compliant human/object-robot interactions. Those advantages enable them promising applications in high value-added fields such as in-situ aero-engine maintenance and minimal invasive surgery. However, compared with conventional robots, this new brand ones have their own disadvantages such as low stiffness and repeatability, which makes it challenging for modelling and control. As a result, more research on control is required to develop the performance of continuum robots for practical applications.

1.2 Objectives of this study

The research topic of this thesis focuses on the modelling, analysis, and control of tendon-driven continuum robots. Novel general numerical framework is firstly proposed for the kinematic modelling of various designs of continuum robots. Furthermore, two distinct prototypes of continuum robotic systems are constructed as the research objects in this work based on whether the cable tension can be precisely obtained or not, with which different control approaches are developed according to their own characteristics. The specific objectives of this study are illustrated as below:

- Propose a general framework for the inverse kinematics (from task space to configuration space) of continuum robots that can adapt to different tasks and designs. Considering the fact that the forward kinematics (from configuration space to task space) of continuum robots can usually be directly derived, the inverse kinematics problem can be converted to a numerical optimization

problem. Therefore, the traditional complicated analysis for the analytical solution can be circumvented, which usually does not exist especially for multiple sections. The numerical framework will be implemented on different prototypes of continuum robots performing the simulation of position and orientation control.

- Design novel model-less controllers for slender long tendon-driven continuum robots. The snake robot developed in the project ‘FLARE’ (more details about the project are introduced in Chapter 3) will be used as the research object, where the control challenges of such long continuum robots, including the model mismatch, the cable elongation, and the section coupling, will be analysed, and discussed. Preliminary test will be conducted to demonstrate the defect of the conventional model-based method, which gives rise to the necessity of novel model-less approaches.
- Develop universal control approach for various single-section continuum robots, where hybrid control scheme is proposed taking advantage of the accurate measurement of cable tensions of the robots of this type. In addition, innovative differential kinematics will be proposed by modifying the PCC model. The hybrid controller will be implemented on different prototypes of continuum robots for experimental validation and comparison, and comprehensive performance characterization will be conducted for tip positioning and path tracking.

1.3 Structure of the thesis

Chapter 2 conducted a comprehensive literature review on continuum robots. This chapter cites a wide range of publications and categorizes the development of continuum robots from four different aspects, which are mechanical design, kinematic

and mechanics modelling, feedback control, and shape sensing. Finally, research opportunities of continuum robots are identified from the perspective of control.

Chapter 3 details the design of continuum robotic systems utilized in this work. The construction of two prototypes is firstly introduced including the design of continuum robots and actuation pack. Then the hardware control system and the control architecture are illustrated. Lastly, the measurement devices for control and performance evaluation are presented.

Chapter 4 presents the kinematic modelling and analysis of continuum robots. The kinematic models are firstly formulated based on the PCC assumption and the geometry of twin-pivot and dual-revolute designs. Moreover, the workspace comparisons are made between the PCC-based and the geometry-based kinematics. Further, a general numerical framework is proposed for the inverse kinematics of continuum robots, which is validated by the tip position/orientation control of single section (twin-pivot and dual-revolute designs) and the step control of the three-section manipulator (PCC model).

Chapter 5 introduces the development of novel model-less controllers for slender long continuum robots. The control challenges are firstly discussed to analyse the defects of conventional model-based controllers that are then validated in the preliminary test and to illustrate the necessity of model-less control. The following sections come to the development of novel model-less controllers. An empirical static controller is initially designed inspired by the tug of war. Further, in order to solve the discontinuity problem of the empirical controller, a novel fuzzy-logic-based controller is developed.

Chapter 6 presents the development of novel universal model-based hybrid controllers for single-section continuum robots. Firstly, the hybrid control scheme is

proposed after analysing the common control modes of general tendon-driven continuum robots and the unique advantage of single-section continuum robots. Then, a novel differential kinematic model is derived by replacing the Cartesian coordinate system with the polar coordinate system in the task space and applying accurate approximation to avoid the explicit call of arc parameters in the configuration space. Further, two forms of hybrid controllers are developed based on the conventional PCC-based and the proposed kinematics, which are compared on the twin-pivot design in terms of positioning and path tracking. In addition, the proposed controller is also implemented on the dual-revolute and soft-robot designs for comprehensive performance characterization and to demonstrate its versatility.

Chapter 7 concludes the achievements of this research work including the academic and engineering contributions in this thesis and discuss the potential research aspects for further development of continuum robots.

1.4 Highlights and Contributions

The main highlights and contributions of this work can be categorised into three aspects, the general numerical solution for inverse kinematics of CRs, novel model-less controllers for slender long tendon-driven CRs, and novel high-performance hybrid model-based controllers for single-section CRs, which are further illustrated as below:

1.4.1 General numerical solution for inverse kinematics problems

A general numerical framework is proposed in this work for the inverse kinematics of continuum robots. Instead of complicated analytical analysis, the framework converts the inverse kinematic problems into optimization problems that can be solved numerically by the optimization toolbox in Matlab/LabVIEW with high precision. Moreover, the framework is versatile to different tasks, which can be modified to adapt to position/orientation control for either single or multiple sections.

1.4.2 Novel model-less control with application to slender long continuum robots

Since the conventional model-based control suffers from the non-convergence issue on the slender long continuum robots owing to the control challenges of model mismatch, cable elongation and section coupling, the necessity of model-less control arises. Two novel model-less controllers are developed in this work: an empirical controller is initially designed inspired by the tug of war, which is then optimized with the fuzzy logic algorithm to solve the discontinuity issue.

1.4.3 Novel model-based hybrid control with application to single-section continuum robots

Hybrid control scheme is proposed in this work to design high-performance controllers for single-section continuum robots. Moreover, a novel kinematic model is derived based on the PCC assumption, where the analytical differential kinematics directly from task space to actuation space can be obtained. Two forms of hybrid controllers are developed based on the PCC model and the proposed differential kinematics.

2 Literature survey

2.1 Introduction

Nowadays, robots play an essential role in the production and life activities of human beings, which can be roughly divided into two broad groups, that is, industrial and service robots, according to their functionalities and application scenarios. In factories, the conventional industrial robots have proved their excellent performance in terms of high precision, repeatability, speed, and payload, and achieved great success in increasing productivity. Meanwhile, new demands of robots, especially service robots, increasingly arise in many other sectors except for the conventional factories. For example, industrial products from aircraft, ships to railway, heavily rely on routine maintenance and repair for securing the safety and efficiency of these high-value systems through their service lives. According to IBISWorld Industry report (2015-2020) on Global Engineering Services [6], the global market will reach £710 billion by 2025. However, many of the industrial environments are difficult to access or hazardous to work there for human, requiring robots or smart tools to assist the process. In aviation sector, for instance, novel service robots are in need for the in-situ maintenance of aero-engines, which requires a high diameter-to-length ratio design in order to access into the cluttered inner environment through small entrances. In some sectors of industry, e.g., energy and nuclear, several off-the-shelf technologies have been applied for repair and maintenance using industrial robots [3] and specialist tools [7]. However, those existing solutions defect owing to the limited DoFs and large dimension of the devices, restricting their further applications in more constraint environments, e.g., aero-engines. In the last two decades, a novel type of flexible hyper-redundant robot, continuum robot, has been developed, presenting a great possibility to be used for the repair and

maintenance in confined space [8-10]. In this chapter, a wide range of literature survey is conducted to introduce the current research status of continuum robots in aspects of mechanical design, modeling, control methodology, and approaches for measurement and sensing.

2.2 Design of continuum robots

Continuum robots (CRs) outperform the conventional industrial manipulators in aspects of hyper-redundant and compliant features. They provide universal and efficient solutions to access to constrained environments, e.g., aero-engines and industrial vessels. Unlike industrial robots, composed of revolute and translation joints with high positioning accuracy and good dynamic performance, continuum robots employ compliant materials (soft material [11-13], spring [14, 15], and NiTi (Nitinol) alloy [16, 17], etc.) or compliant mechanism [18, 19] to construct the arm and actuators, which prevent them from rigid collisions with the surrounding environments.

As shown in Figure 2-1, the family of continuum robots can be divided into four main categories according to actuation mechanism and mechanical features, among which the pneumatic and tendon-driven continuum manipulators are two of the most essential types, while the other two categories include robots constructed by functional materials and rigid joints. In the following sections, those categories will be discussed in detail.

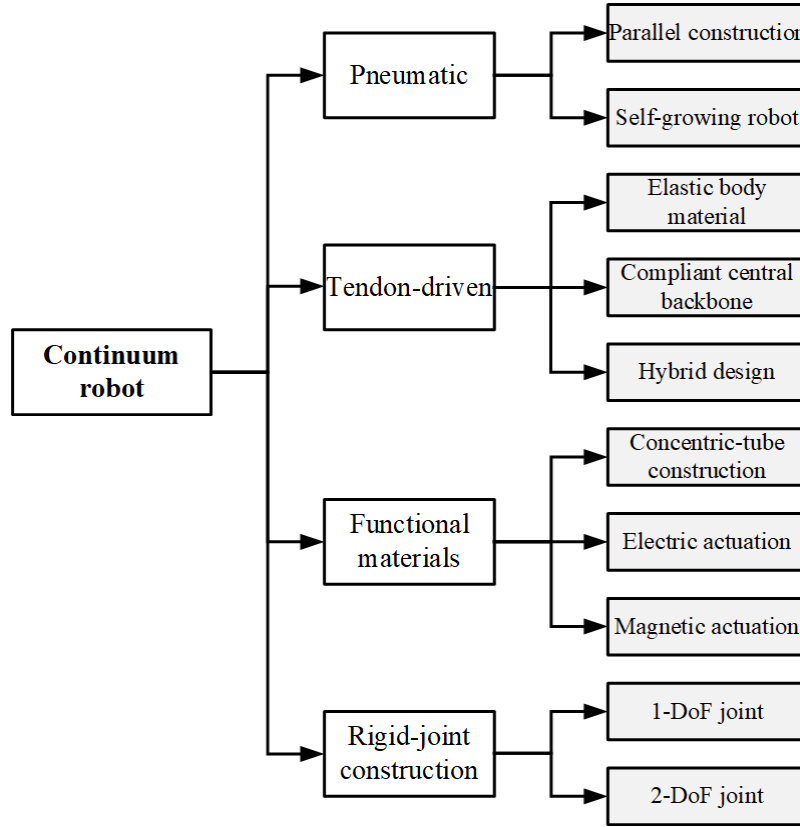


Figure 2-1 Classification of continuum robots.

2.2.1 Pneumatic CRs

Pneumatic continuum robots are usually intrinsically actuated, which are constructed by combining multiple pneumatic/hydraulic actuators in parallel as single three-DoF joint [20]. By adjusting the pressure, single pneumatic actuator can realize elongation and compression motion along the length direction, which can be regarded to be one-DoF.

- Parallel construction

It is a typical way to construct a single-section continuum robot with three parallel pneumatic actuators. Spatial manipulation can be achieved by controlling the pressure of each actuator independently.

The first prototype adopting this concept, Flexible Microactuator (FMA), was developed in 1991 [20]. Single FMA (Figure 2-2(b)) is designed with three degrees of freedom: pitch, yaw and stretch. Serially connected FMAs act as a miniature robot

manipulator (Figure 2-2(c)), while FMAs combined in parallel act as a multi-fingered robot hand (Figure 2-2(d)).

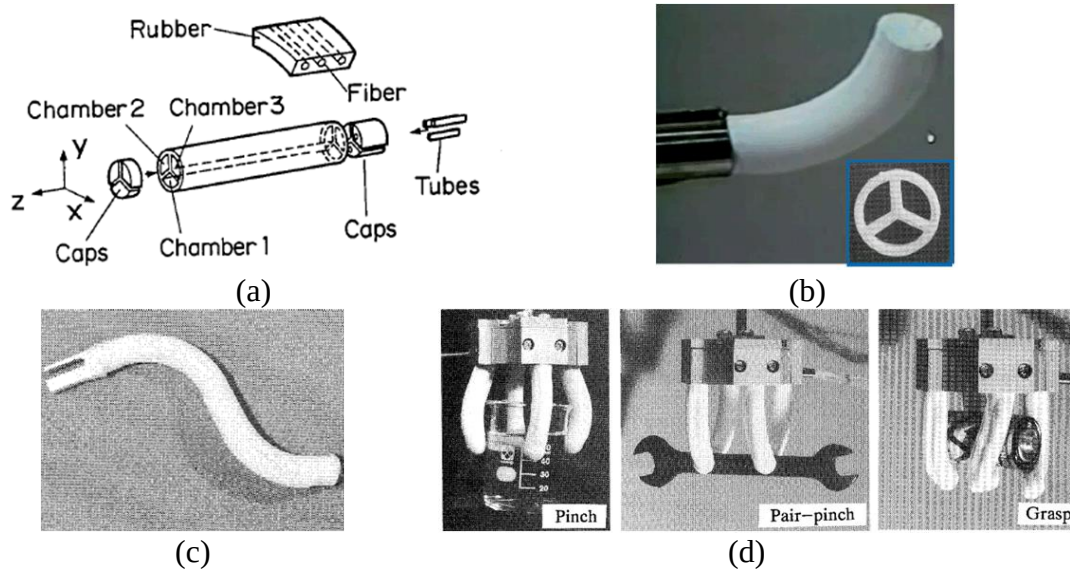


Figure 2-2 Flexible Micro-actuator (FMA) [20]: (a) structure of an FMA; (b) single section with three-DoF (12 mm in diameter); (c) 2-section manipulator (120 mm in length); (d) multi-fingered robot hand using FMAs.

Based on the concept of parallel construction, a group of large-dimension pneumatic continuum manipulators have been created, which are shown in Figure 2-3. Since 2006, a series of pneumatic manipulators have been developed for the family of OctArm [21, 22] by Ian Walker. OctArm IV (Figure 2-3(a)) contains four sections while OctArm (Figure 2-3(b)) has only three. The number of air muscle actuators varies for different sections. For the proximal two sections, three pairs of actuators are deployed to reach higher stiffness and load capacity, while three single actuators are used to construct the distal section for larger curvature. Each section is three-DoF, 1 for extension in the length direction and 2 for bending in space. Owing to the large vertical (minimum: 90 N) and transverse (minimum: 20 N) loading capability, the pneumatic arm can complete tasks such as cone stacking and grasp of large spherical or elongated objects [21].

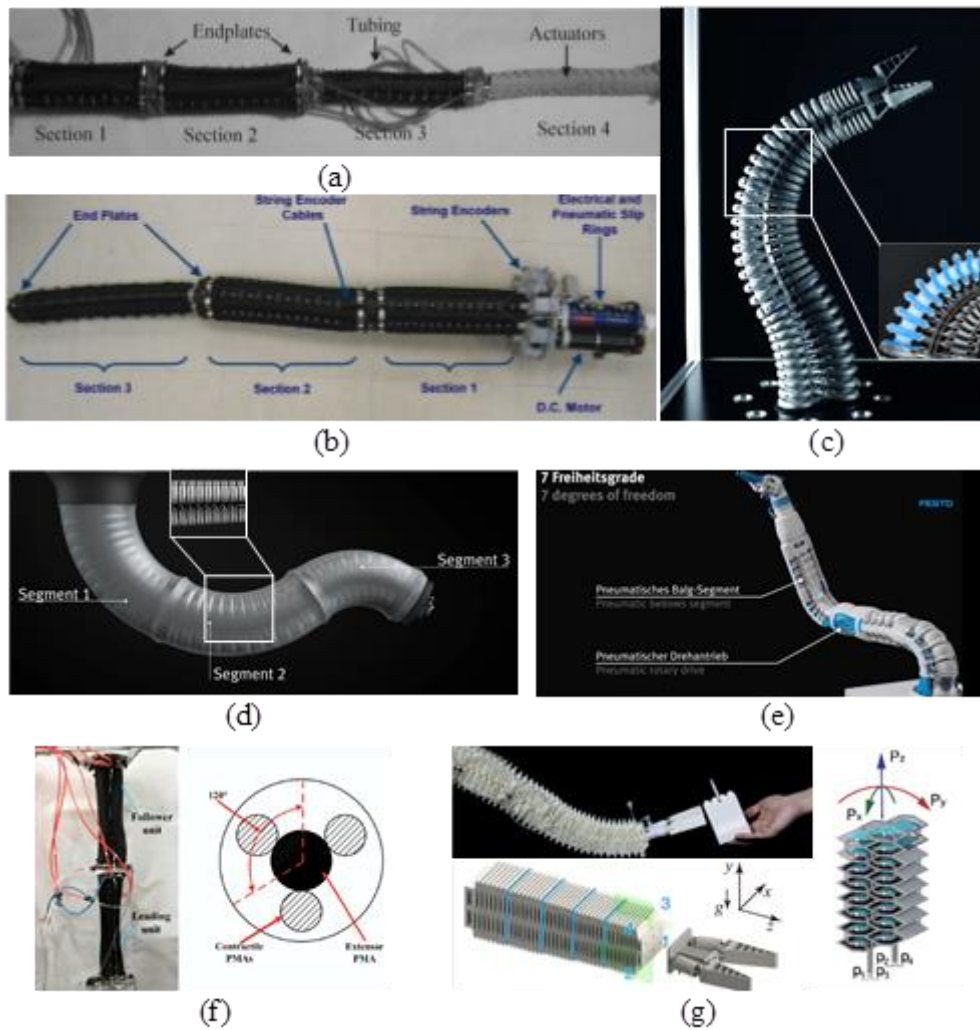


Figure 2-3 A group of large-dimension pneumatic continuum manipulators: (a) OctArm IV [21]; (b) OctArm VI [22]; (c) the Bionic-Handling-Assistant (BHA) by FESTO [23-26]; (d) the BionicMotionRobot by FESTO [27, 28]; (e) the BionicSoftArm by FESTO [29, 30]; (f) two-section continuum manipulator with pneumatic muscle actuator (PMA) [31, 32]; (g) the Honeycomb-Pneumatic-Network-Arm (HPN Arm) [33, 34].

With the same concept but different manufacturing approach (e.g., 3D printing), FESTO has successfully developed and commercialised three generations of pneumatic manipulators. The first generation is the Bionic-Handling-Assistant (BHA) in Figure 2-3(c), which consists of three serial sections built with selective laser sintering. Each three-DoF section is made up of three pneumatically actuated bellows to mimic the shape of elephant's trunk. Further, apart from the pressure sensors in each bellow, wire-cable potentiometers are also equipped so that the length of each bellow can be measured and feedback to the controller of the robot. The manipulator can extent up to

1.1 m from the origin length (0.75 m) under 0.5 Kg payload. In the second generation, the BionicMotionRobot (Figure 2-3(d)), sturdy elastomer is used for the bellow design to further increase the maximum payload to 3 Kg, which can carry approximately the manipulator's own weight. In the third generation, the BionicSoftArm (Figure 2-3(e)), the length of each section is shortened while adjacent sections are connected with rigid links, which essentially strengthens the manipulator's stiffness and improves the repetition accuracy to 5 mm.

An innovative structure-based strategy is proposed in [31, 32] to address the trade-off between workspace and stiffness during the design of pneumatic manipulators. By changing the angle between a braided thread and the axis of a long cylinder outside an inner rubber, specific contractile and extensor pneumatic muscle actuators (PMAs) can be manufactured. A variable stiffness pneumatic manipulator (Figure 2-3(f)) is designed with two serial sections, with each section made up of three contractile PMAs and one extensor PMA in the centre. Experiments demonstrate that the stiffness of the manipulator can reach about 2800 N/m with 2.5 bar inflated pressure of the extensor PMA when a 2 Kg weight is place on the tip. Another novel honeycomb pneumatic networks manipulator (HPN) is proposed in [33, 34], where inflatable airbags are used to construct the manipulator. As shown in Figure 2-3(g), the manipulator consists of five sections; each section is made up of four groups of airbags with square cross-section. Single section's movement can be decomposed into three elementary motions: bending in two side planes and axial elongation-retraction. The manipulator is capable of completing tasks such as opening drawers, turning hand wheels and cleaning glass.

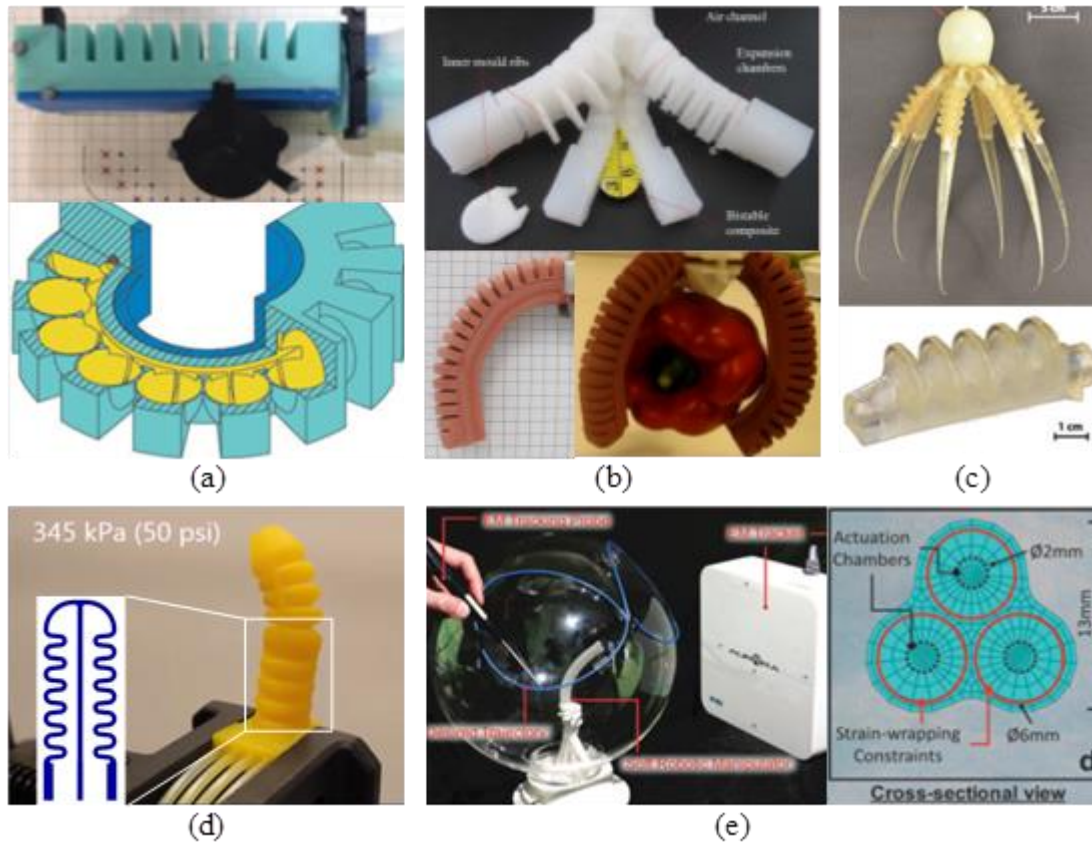


Figure 2-4 A group of small-dimension pneumatic continuum robots: (a) a soft planar robot [35, 36]; (b) a soft gripper with three strengthened pneumatic actuators [37]; (c) an octopus-like robot [38]; (d) a soft pneumatic actuator printed with elastomeric precursor (EP) [39, 40]; (e) a soft robotic endoscope prototype with parallel bellows [41].

In addition to large-dimension pneumatic manipulators for situations with large payload, a great amount of research has also been conducted for developing miniaturized design to explore new applications of pneumatic robots. Characteristics such as large-scale deformation, adjustable impedance, and high compliance make soft robots well-suited for tasks requiring high dexterity and secure interactions with structured or unstructured environments. For instance, a soft planar fluid-powered gripper was developed in [35, 36] to complete tasks with low payloads, e.g., grasping objects of unknown geometry. To avoid the bend under soft robot's own weight at zero input pressure, a bi-stable material, such as a strip of tape measure, is embedded into a soft actuator so that the system can behave stiff under no actuation [37]. As shown in Figure 2-4(b), a three-fingered gripper was developed by using the enhanced actuator to hold capsicum and

grapefruit, which shows potential in the applications for the harvest of agricultural products. In addition, an octopus-like robot was also designed in [38] with six tentacle-like one-DoF bending pneumatic actuators controlled by electrorheological valve.

Figure 2-4(d) and Figure 2-4(e) illustrate two types of two-DoF pneumatic robots. One is constructed with two serial one-DoF actuators printed with elastomeric precursor [39, 40], while the other one still applies the aforementioned parallel-construction concept to increase extra DoF for single section [41].

- Self-growing robot

Another noteworthy example of pneumatic robot is illustrated in Figure 2-5, which is a vine-like self-grow robot [42-44]. The robot was developed by researchers from Stanford University, which is inspired by natural organisms such as nerve cells, fungi and vines that reach distance area by growing. With soft material folded inside itself as the robot body, the growing mechanism can be implemented when the material at the tip everts. Figure 2-5(a) shows a basic version of the robot grows straightforward in free space, which is manufactured by thin and cheap plastic. In addition, as illustrated in Figure 2-5(b), series Pneumatic-Artificial-Muscles (sPAMs) are placed around the robot along the whole length, which enables active steering for navigation in constraint environments. With the assistance of a camera at the tip, a demonstration of the robot is performed under different scenarios, e.g., high temperature and narrow space (Figure 2-5(c)), indicating its wide potential applications in the realms of search, rescue, and medical equipment.

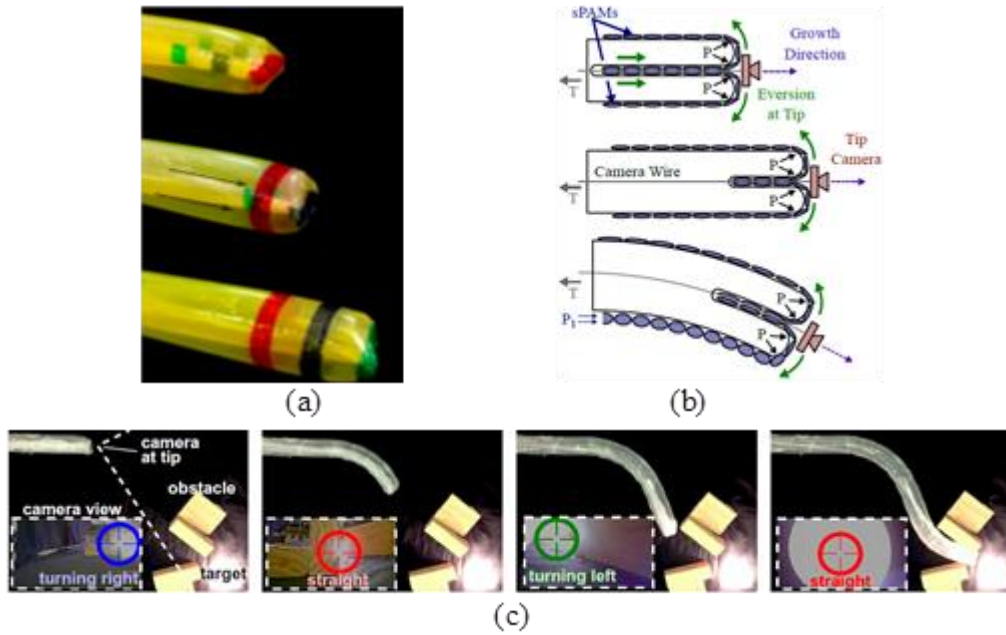


Figure 2-5 Vine-like self-growing robot [42-44]: (a) images of the lengthening body (2.5 cm in diameter); (b) principle of tip steering; (c) demo of active steering.

2.2.2 Tendon-driven CRs

In addition to pneumatic actuation, tendon-driven approach is another essential category of continuum robot. Considering the mechanical property of materials constructing the arm of the robot, the tendon driven CRs can be divided into three types, which will be introduced explicitly in the following parts.

- Elastic body

Figure 2-6 shows several examples for the first type of tendon driven CRs, within which the robot body is made up of elastic material (e.g., resin, rubber, plastic, and spring). A conical shape soft arm (Figure 2-6(a)) was developed [13, 45], which is manufactured by a single casting mould using silicone. Apart from moulding, additive manufacturing is also a widely used approach. For example, 3D-printed planar spring (Figure 2-6(b)) and helical spring (Figure 2-6(c)) were also utilized to construct continuum manipulator with three-DoF for single section of the robot [15, 46]. Moreover, a novel ‘hose in hose’ design concept was adopted to develop a multi-section continuous-backbone manipulator, Air-Octor (Figure 2-6(d)), which combines the features of pneumatic and

tendon actuation [47]. Pneumatic-pressurized chambers are utilized to provide the structural support in the centre of the manipulator, which are manufactured by a loose helix metallic spring to provide the chamber with lateral structural support and the ability of longitudinal collapse.

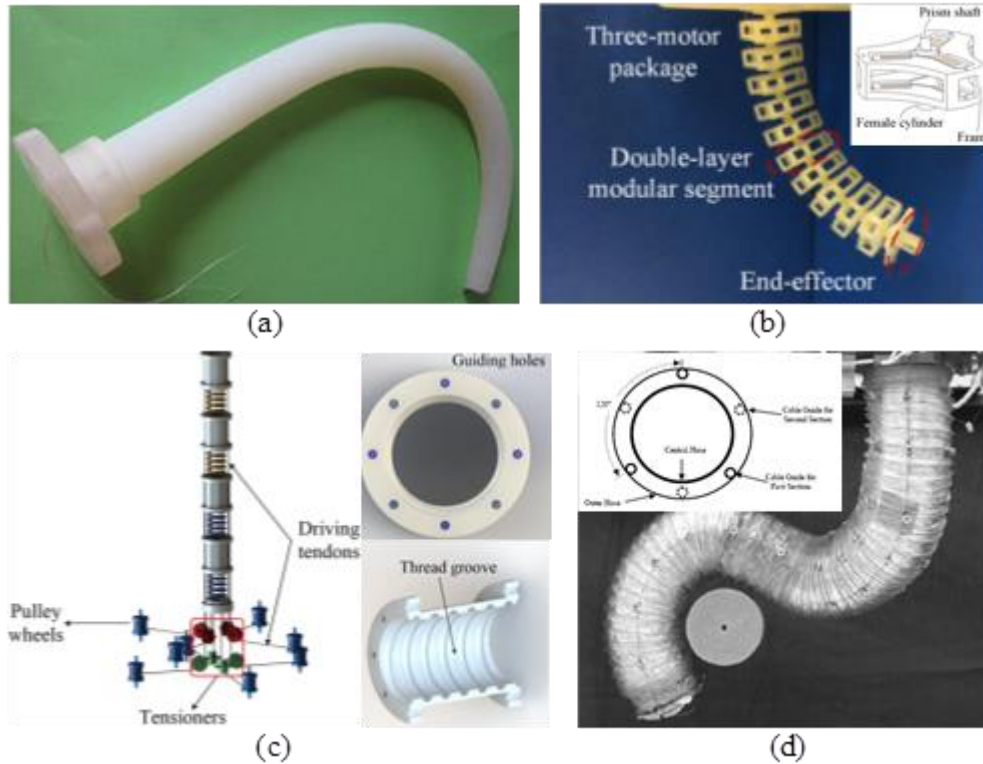


Figure 2-6 Tendon-driven CRs with elastic body material: (a) a soft arm with conical shape [13, 45]; (b) a 3D-printed continuum manipulator with planar springs [46, 48]; (c) a two-module continuum robot with helical spring [15, 49]; (d) the two-section Air-Octor manipulator [47].

- Compliant central backbone

The second category of tendon driven continuum robots are those with compliant central backbones. Three examples are presented in Figure 2-7 with such kind of design. In Figure 2-7(a), a two-section robotic tail constructed from a thin and elastic core was developed with rigid disks evenly mounted along its length [50]. Hybrid actuation scheme is employed with the base section actuated by elastic rods and the tip section actuated by cables. The advantage of utilizing rods to actuate the proximal section is that it can help to resist the twist deformation caused by the gravity of the distal section.

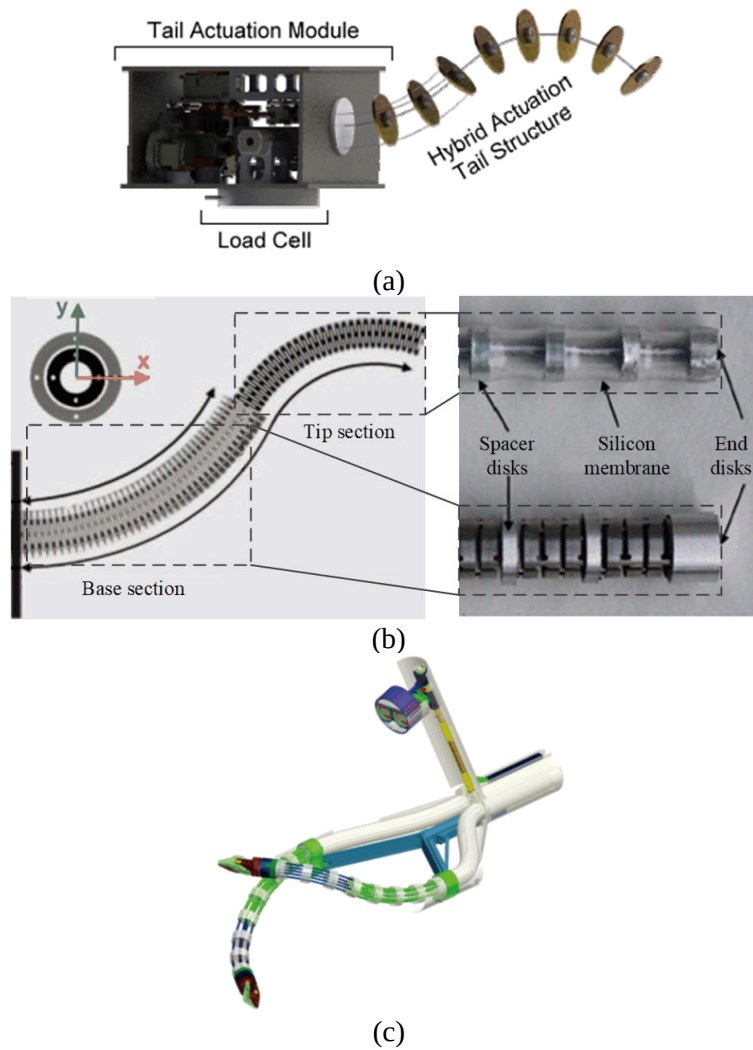


Figure 2-7 Tendon-driven CRs with flexible central backbones: (a) a continuum robotic tail [50]; (b) a concentric continuum robot with a NiTi tube as the backbone [51]; (c) the IREP robot [52, 53].

The concentric design is employed to develop a hybrid 2-section robot as a combination of tendon-driven tubular continuum robots with a three-DoF design for each section [51]. Figure 2-7(b) illustrates the conceptual structure of the robot. The base section utilizes a patterned NiTi tube (outer diameter: 5 mm; inner diameter: 4.79 mm) as backbone, which is equipped with evenly distributed aluminium disks with routing channels, such that the tendons can run outside the tube. As for the tip section, disks with smaller outer diameter are applied so that the entire section can enter the base section freely. A silicone membrane is inserted between two section to fulfil the gap

and reduce the friction. In this system, each section is designed with two DoFs for bending and one DoF for translation motion.

A surgical dual arm, the Insertable Robotic Effector Platform (IREP) [52, 53], is shown in Figure 2-7(c). The key component of the IREP robot is a two-section continuum robot in each arm. Each section of single continuum robot consists of one primary NiTi backbone located in the centre and four secondary NiTi backbones for actuation. Compared to the common cable-driven method, the usage of NiTi rods can essentially increase the stiffness of the whole robot.

- Hybrid body with rigid component and compliant joint

Apart from the aforementioned two sub-categories, the concept of a hybrid construction with rigid component and compliant joint is widely utilized for the development of slender tendon-driven continuum robots. On the one hand, the compliant joints enable the elastic bending ability of the robot; on the other hand, the rigid component ensure the overall structural stiffness.

The Electrical-Discharge-Machining (EDM) is innovatively imported into the manufacture of continuum robots. As shown in Figure 2-8(a), a one-DoF continuum manipulator is fabricated by the EDM process on a Nitinol tube with triangular notches. The V-shape cuts are equally spaced and interleaved to enable bending in one plane and guarantee the out-plane stiffness. Figure 2-8(b) illustrates a prototype with U-shape wire-EDM cutting optimized using finite element calculations to reduce the strain and fatigue of the joint. The cuts are orthogonally deployed to enable spatial bending. Moreover, the torsional and axial stiffness is essentially enhanced by using a spiral component as the outer tube. The last example is an elliptic notched two-DoF manipulator shown in Figure 2-8(c), where a series of rigid disks are arranged outside the notched backbone to guide driven cables.

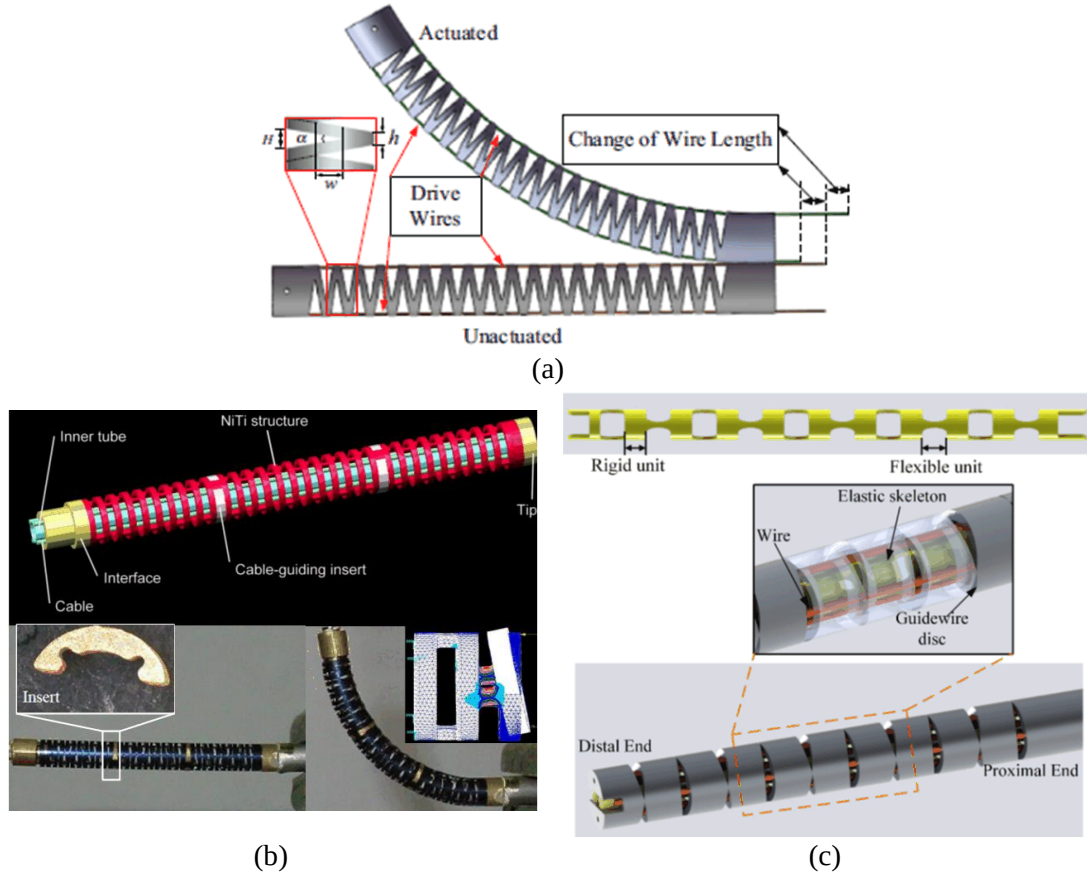


Figure 2-8 Continuum robots fabricated by EDM process: (a) a V-shape notched manipulator [19]; (b) a U-shape notched manipulator [54]; (c) an elliptical notched manipulator [55].

The advantage of the EDM is quite straightforward. It not only provides a convenient and effective method of manufacturing the flexible backbone for continuum manipulators, but also the manufacturing quality can be well guaranteed, and the assembly process can be eliminated. However, it is not suitable for manufacturing robots with complicated design and multiple sections.

The other method, the discretized construction, is also proposed to combine rigid components and compliant joints for continuum robots. Figure 2-9(a) shows a three-section continuum manipulator (diameter: 15 mm; length: 280 mm) using twin-pivot joint, which is formed by a pair of antagonistic Nitinol backbones between two adjacent disks. Since single twin-pivot joint only allows for bending in the neutral plane of two flexible backbones, the spatial free bending of single section is achieved by arranging

the adjacent joints orthogonally. Experiments demonstrate that the twin-pivot construction essentially increases the twist stiffness of the structure: the twisting angle of the proximal section is 1.33 degrees for 90 degrees bend with 450 g payload.

Further, a slender 1270 mm-long 12-section continuum robot (Figure 2-9(b)) is developed using the same joint design for the in-situ inspection and maintenance of aero-engines. Since each section is two-DoF, the robot has a total of 24 DoFs. The diameter of different sections adopts gradient design, ranging from 40 mm (the tip section) down to 13 mm (the base section), to increase the overall stiffness so that the robot can be straightened under its own gravity and keep rigid and stable in operation. The robot is designed to enter the gas turbine engine, reach the first stage of the lower pressure compressor (LPC), and perform the inspection tasks. Figure 2-9(b) presents a trial on a Rolls-Royce aero-engine mock-up. The slender continuum robot can navigate into the mock-up through the gaps between fan blades owing to its hyper-redundant design of up to 24 DoFs, with the feed-in of a linear stage at the base. However, the robot suffers from difficulties such as low stiffness and high friction. The first issue disables the robot to perform movements with large bending angles and leads to the robot shaking in operation, not to mention performing tasks with contact such as grinding; while the second problem can result in serious cable elongation issue and cause time delay of the cable displacements from the actuation terminate to the distal section, hence significantly increase the control challenge.

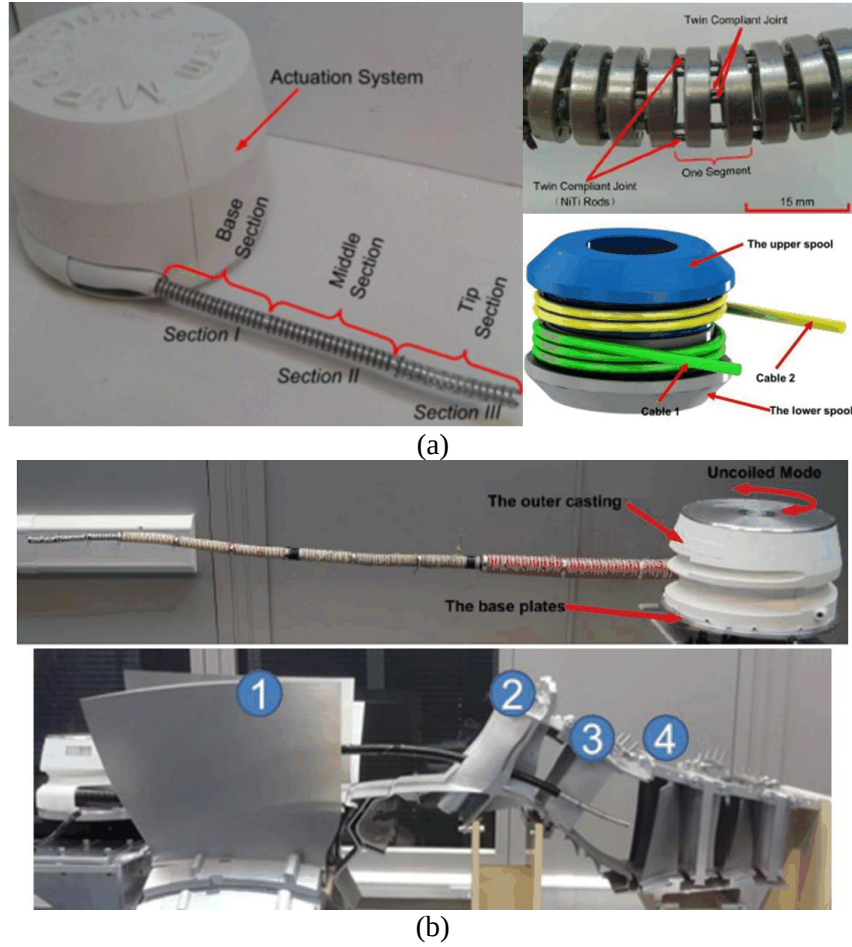


Figure 2-9 Continuum robots with discretized structure: (a) three-section continuum manipulator with twin-pivot joint [56, 57]; (b) slender 12-section continuum robot [10, 58].

2.2.3 Functional materials

Except for the regular actuation approaches (pneumatic and tendon-driven), various special continuum robots are also developed according to the unique features of some functional materials.

- Concentric-tube construction

Considering the super-elastic feature of SMA (shape memory alloy), the novel concentric structure is proposed to construct continuum robots utilizing pre-curved SMA tubes (Figure 2-10(a)). The movement principle of the concentric-tube robot is in accordance with the structural features. As shown in Figure 2-10(b), each tube can retract/extend relative to one another, as well as rotate axially. To implement those elementary motions, a compact modular actuation design is developed utilizing gear

sets. Figure 2-10(c) illustrates a single actuation module for one concentric tube, which mainly consists of three customized gears, a roller gear, a translation spur gear, and a rotation spur gear. The roller gear is placed in a tube to maintain axial alignment and also keep the ability to rotate and translate freely with the concentric tube mounted on the top. Hence, it can be easily extended for multiple concentric tubes by deploying the actuation module in serial.

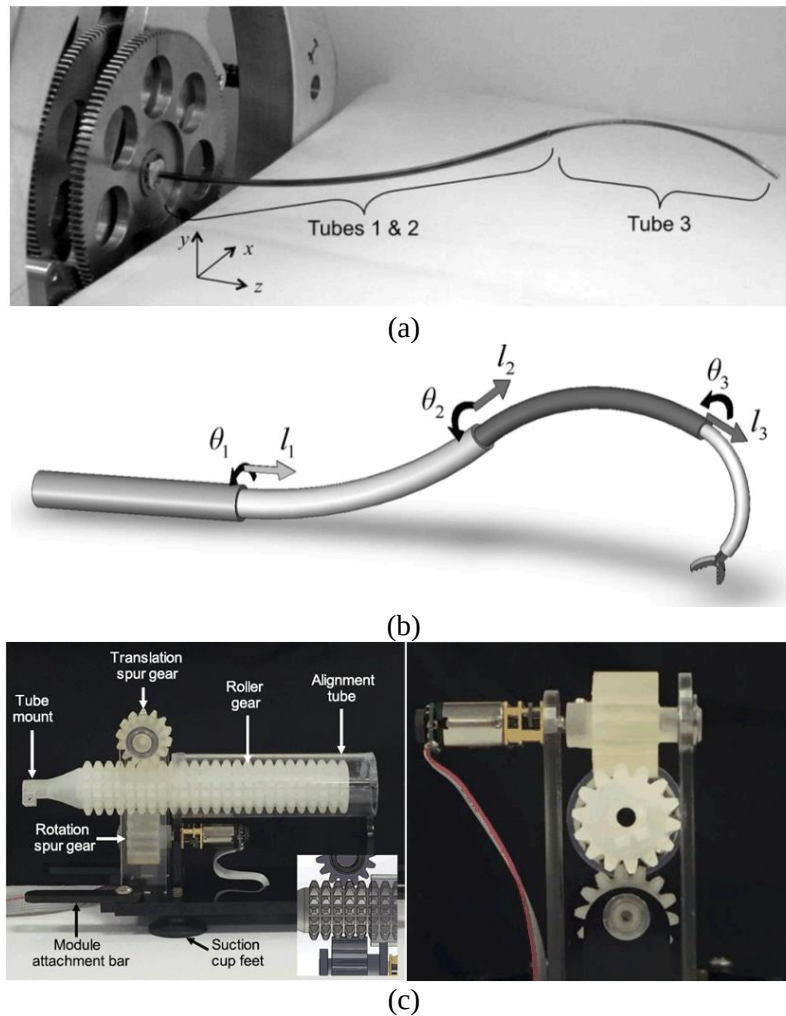


Figure 2-10 Concentric-tube robot with SMA: (a) a three-section concentric robot [59-61]; (b) movement principle of concentric-tube robots [59]; (c) modular actuation design [62].

Despite that the construction of the concentric-tube robot with SMA and the actuation system is simple and straightforward, the precise shape control of such robot is quite challenging. Since all the tubes are inserted inside each other, the shape of the common section conforms to a resultant curvature. Two cases for interactions are illustrated in

Figure 2-11 when the ratio of the bending stiffness for the two tubes varies. Figure 2-11 (a) gives the case when the outer tube dominates the stiffness so that the shape of the common portion conforms to the shape of it, while Figure 2-11 (b) shows the case with balanced-stiffness tube pair. To sum up, the complicated stiffness coupling of multiple concentric tubes makes it difficult to set up accurate models for shape prediction.

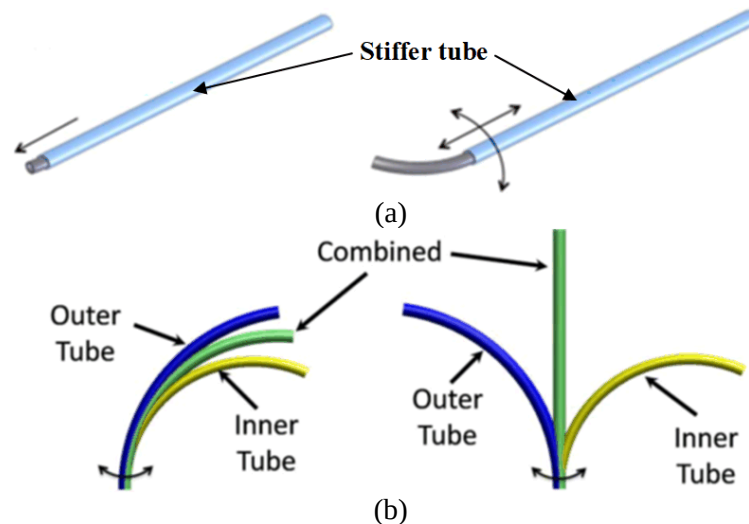


Figure 2-11 Interactions between tube pair [59, 61]: (a) Case 1: dominating stiffness; (b) Case 2: balanced stiffness.

- Electric actuation

With the development of material science, different kinds of functional materials have been found and composited with special physical and chemical characteristics, which brings more possibilities for the design of continuum robots.

Figure 2-12(a) illustrates a continuum manipulator actuated by the Electro-Active Polymers (EAP) [63]. As shown in the figure, the manipulator consists of four concentric layers. The most inner layer is a catheter with thin thickness, which acts as the elementary skeleton of the manipulator and the hollow channel reserves space for installing end effectors on the tip. The electrolyte fulfils the outer space of the catheter and forms the second layer, which is axially evenly surrounded by four pieces of thin polymer electrodes. The outer layer is a thin tube for encapsulation. The working principle of the EAP is given in Figure 2-12(b). By alternating the voltage applied

across one pair of polymer electrodes, the state of the polymer electrode will switch between oxidation and reduction, which corresponds to the expansion of one electrode and contraction of the other electrode and hence cause the whole structure to bend in one direction. As such, the manipulator shown in Figure 2-12(a) can bend in space by vertically deploying two pairs of antagonistic polymer electrodes.

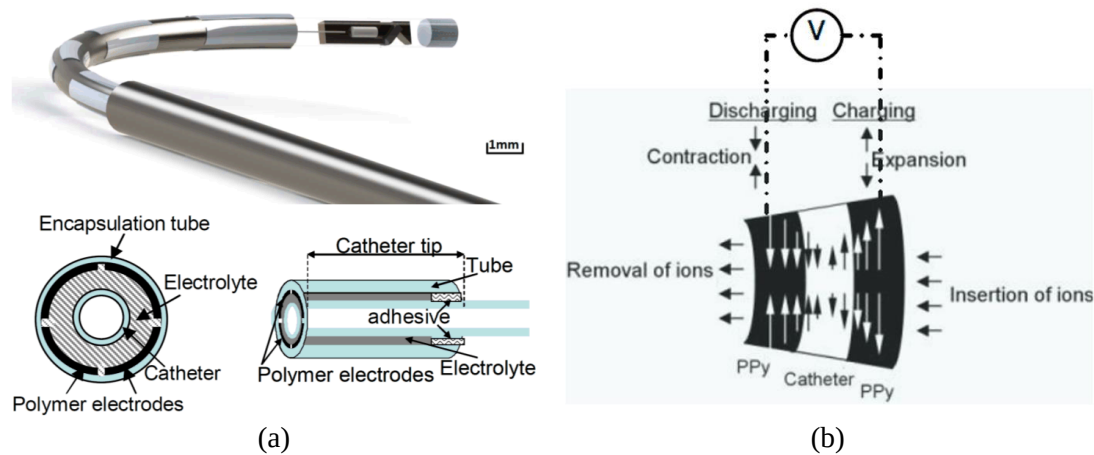


Figure 2-12 (a) Continuum manipulator constructed with EAP [63]; (b) working principle of EAP [64].

Taking advantage of the phenomenon that phase transition can occur when the temperature changes, the SMA wire is utilized in continuum robots for actuation. Figure 2-13 illustrates the schematic of a steerable cannula, which is composed of three straight copper links (inner diameter: 1.651 mm; outer diameter: 3.175 mm) and is actuated by two one-DoF bending joints made of two antagonistic SMA wires. The links from the base to tip are in length of 38 mm, 38 mm, and 25 mm respectively, with a 5 mm gap in between for the joints. The SMA wire can be heated up with the electrical connections at the end and the temperature is monitored by the RTD (resistance temperature detector) sensor attached in the middle. By adjusting the heating temperature, the bending angle of single joint can range between $\pm 21^\circ$.

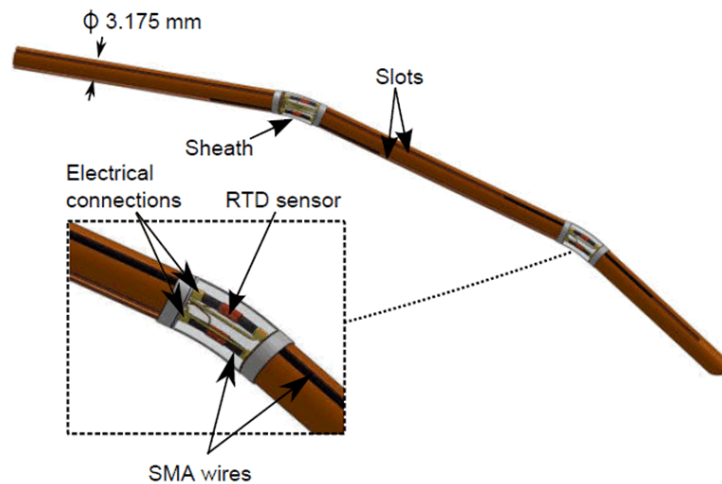


Figure 2-13 Steerable cannula discretely actuated by SMA-wire [65].

- Magnetic actuation

Figure 2-14 presents a novel design of extensible continuum robots utilizing permanent circular Neodymium magnets for space disks, which is composed of three sections from which the distal two sections are extensible. The backbone tube of the base section is fabricated from polycaprolactone to provide structural stiffness for the extensible sections. As shown in Figure 2-14(b), the interaction between the spacer disks comes from the repulsive force of same magnetic poles. Figure 2-14(c) shows the control strategy to deploy the robot along a planar tortuous path. The robot is manoeuvred section by section: the middle section is extended and bend to make the turn by controlling tendons, after which the tip section reaches the target position by extending to the full length and bending simultaneously.

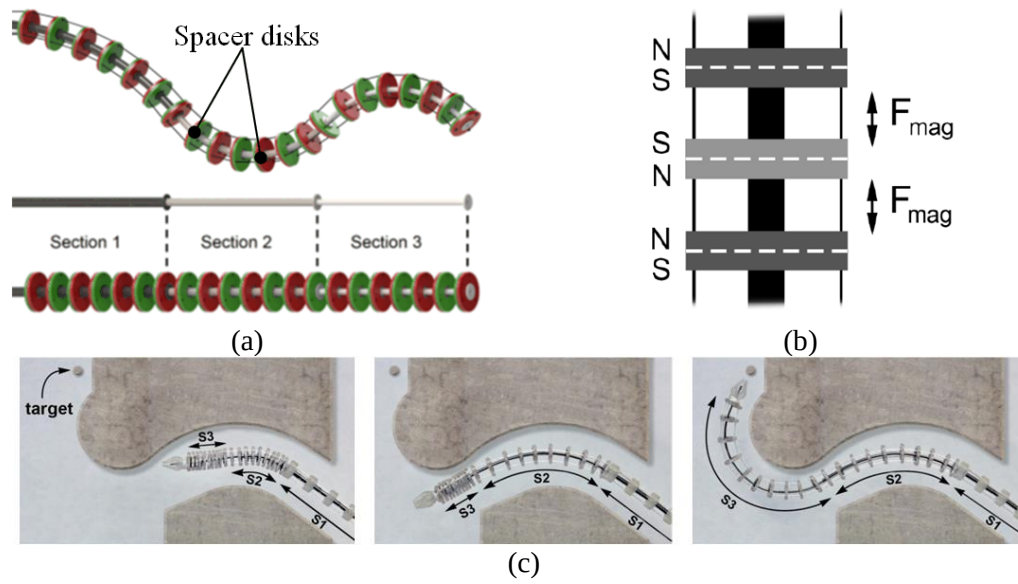


Figure 2-14 (a) Tendon-driven extensible three-section continuum robot with magnetic spacer disks [66]; (b) interaction mechanism between disks; (c) control strategy with follow-the-leader motion.

A small-scale ferromagnetic soft robot is shown in Figure 2-15, which is constructed using magnetic composite polymers characterized by doping magnetic micro or nanoparticles into the polymer matrix. The robot is fabricated by coating hydrogel skin on the magnetized ink, which can be miniaturized to a submillimetre scale. The schematic of the soft robot is illustrated in Figure 2-15(a), where active steering of the tip of the robot can be achieved by applying magnetic field in a specific direction. Figure 2-15(b) shows a demonstration performed a cerebrovascular phantom. By adjusting the direction of the applied magnetic field, the tip of the robot can be controlled to pass through sharp corners. Advancement of the magnetically steered tip is then made by pushing the proximal end manually. The demonstration shows the promising application of this technology in minimally invasive surgery for cerebrovascular diseases.

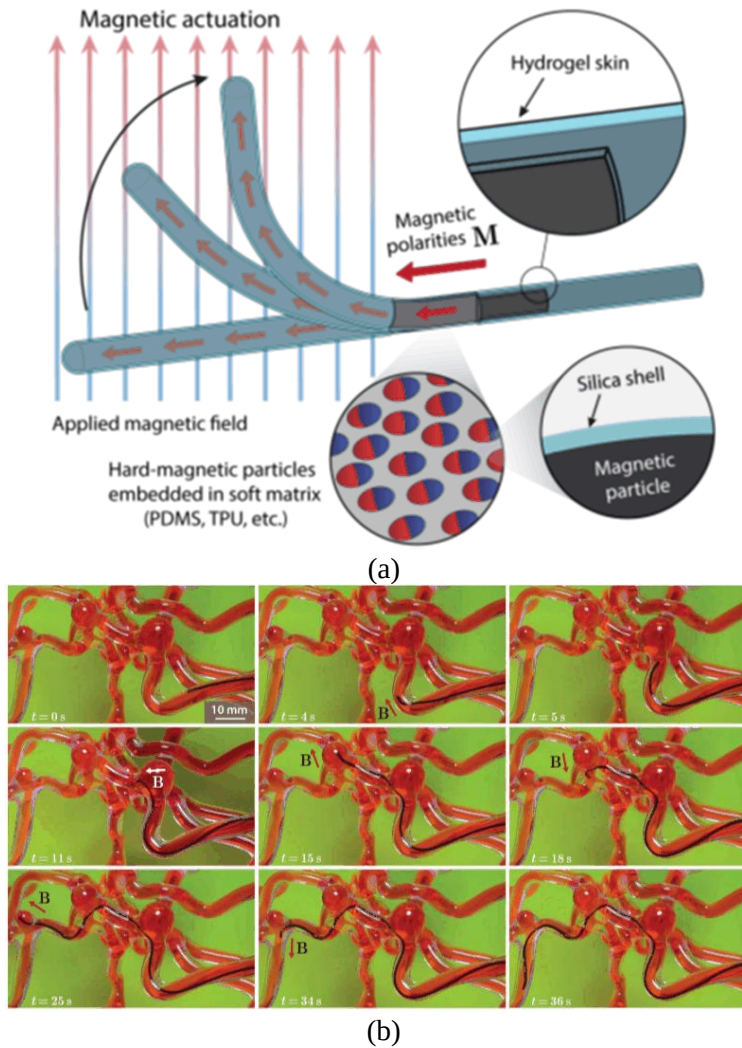


Figure 2-15 Ferromagnetic continuum robot [67]: (a) schematic illustration of the active steering of continuum robot in applied magnetic field; (b) demonstration of navigation in a cerebrovascular phantom.

2.2.4 Rigid-joint construction

The last category of continuum robots are those hyper-redundant ones with rigid links, which are also called snake arms. Comparing to continuum robots constructed with compliant joint or flexible material, the rigid-link hyper-redundant robot is superior in aspects of mechanical behaviours, which is capable of performing tasks with relatively high pay load. According to the DoFs of applied joints, it can be divided into two sub-categories, those with one-DoF joint (revolute joint) and the others with two-DoF joint (universal/spherical joint), which are to be discussed as below.

- One-DoF joint

Considering snake arms with one-DoF rigid joint, the regular revolute joint is usually utilized for construction. Figure 2-16(a) illustrates a prototypes of snake manipulator with revolute joint developed in early stage, the CT ARM-1. The manipulator consists of seven serial links connected with revolute joints, which are actuated by tendons through pulley sets. Since the tendons for distal links inevitably pass through all the proximal links, multiple parallel pulleys have to be deployed to guide all the tendons on the proximal revolute joint, resulting in an increase in width.

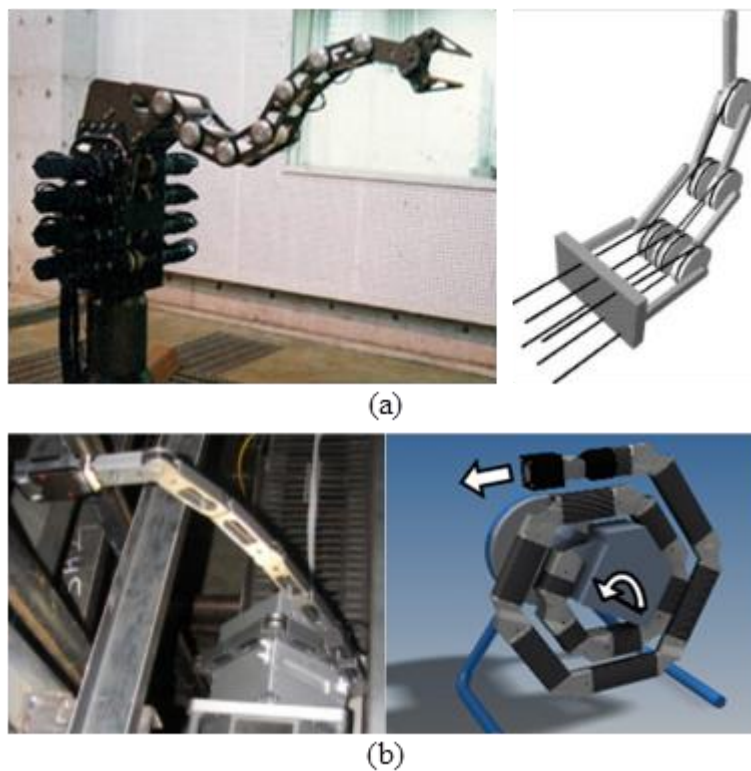


Figure 2-16 Rigid-link snake robots with revolute joint (a) CT ARM-1 [70, 71]; (b) planar snake-arm robot [68, 69].

With the development of technology and material, a more mature design of planar snake-arm robot is proposed [68, 69]. The snake arm is 2.2 m long and 12.5 mm wide, which is developed for applications in nuclear energy industry. As shown in the right figure of Figure 2-16(b), the design of the snake arm is relatively compact since it can coil around the actuation pack. Therefore, the snake arm is ideal for applications in constraint environments as the installation of a linear stage for feed-in is not needed.

Figure 2-17 presents a novel design of single-section continuum manipulator using gear teeth. Both ends of the main links are manufactured to be in the profile of gear, while the auxiliary links connect the rolling centre of two adjacent main links. The gear shape enables a stable rolling motion between links, which is robust against slippage compared to other non-skid mechanisms such as ribbons [72] and elastic fixtures [73]. The final prototype is in a diameter of 3.7 mm and the total length is about 25 mm with three segments, which can keep the deflection under 1.5 mm in the rotational direction with a 500 g payload on the tip. However, the defects of the gear-teeth scheme are quite obvious. On the one hand, it required high manufacturing accuracy to minimize the gear meshing clearance, which can be costly. On the other hand, the lifetime of the mechanism will be a big concern since the gear can be worn heavily owing to the large torque applied on the gear teeth.

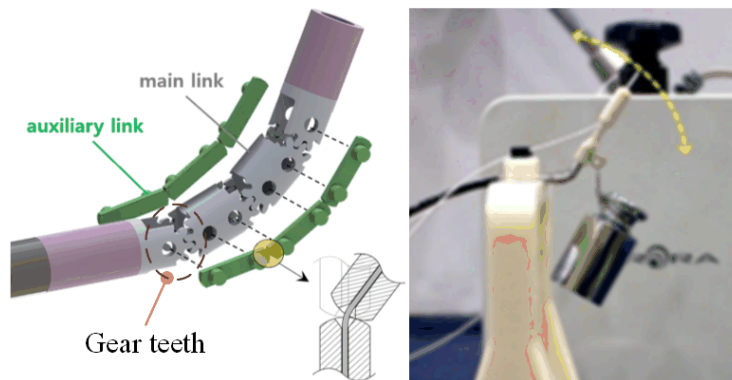


Figure 2-17 Constraint continuum manipulator with gear teeth [74, 75].

A modular snake robot is illustrated in Figure 2-18(a), which is capable of climbing inside a pipe [76] and performing on-site rescue tasks [77]. As shown in Figure 2-18(b), the snake robot applies servo motor for actuation. Each servo motor acts as a revolute joint mounted on one rigid link, while the output arm on the output shaft is fixed on the other link. With the modular design, the snake robot can be easily extended to 16 sections. Since the snake robot is intrinsically actuated, there is no actuation pack at the base, enabling it the ability to have access to the depth of constraint environments.

However, owing to its own weight and the limited output torque of servo motors, the robot cannot support itself and needs to contact with the environment when in operation.

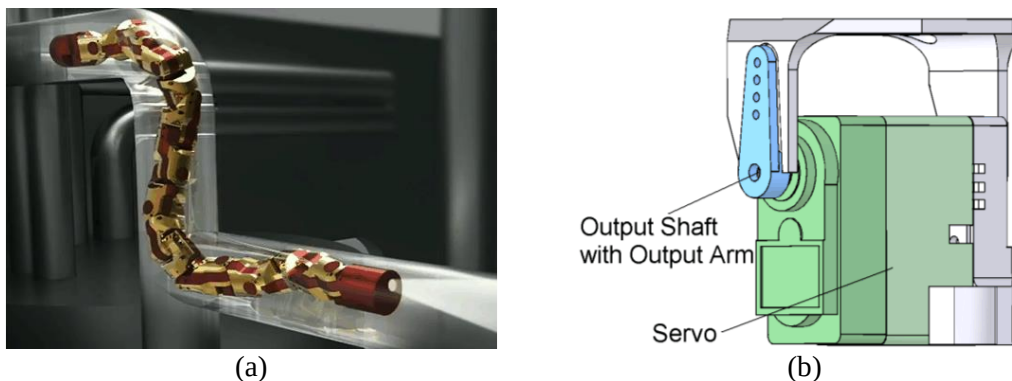


Figure 2-18 Modular snake robot actuated by servo motor [76, 77].

- Two-DoF joint

Figure 2-19 presents a novel wire-driven continuum manipulator with spherical joint. As shown in the figure, the manipulator is actuated by two pairs of antagonistic wires, which are deployed vertically for spatial maneuverer. Moreover, balls are used to prevent the rotation about the axial direction while springs are applied between two adjacent spherical joints for resilience to obtain a more constant curvature. Experimental result illustrates that the positioning errors without payload are smaller than 0.5 mm (standard deviation: 0.11 mm), in comparison with the constant curvature assumption.

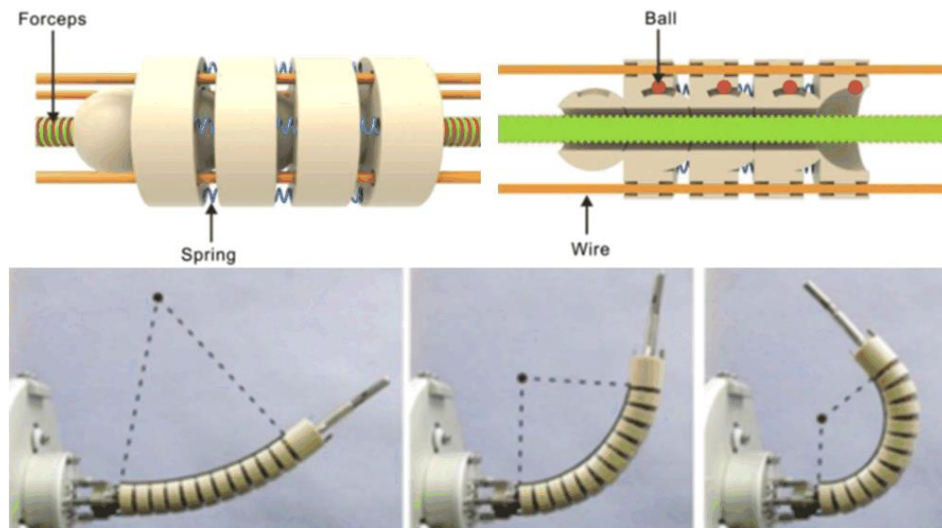


Figure 2-19 Wire-driven manipulator with spherical joint [78].

There have been a few prior attempts to construct hyper-redundant manipulators with universal joints. The concept was firstly proposed in the late 1960s where a tensor arm manipulator was developed. As shown in Figure 2-20(a), the manipulator consists of a series of plates interconnected by universal joints. All the joints are actuated by nylon tendons. The idea was further optimized in 1999 [79] by adding springs to increase stiffness.

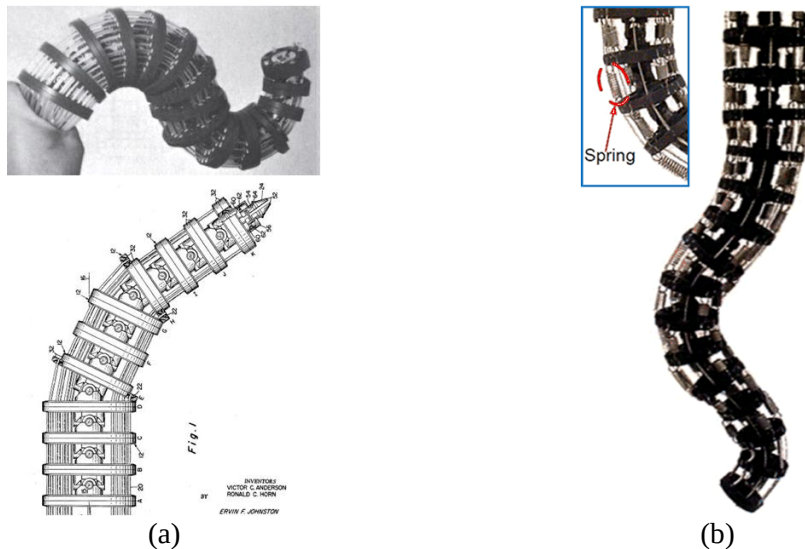


Figure 2-20 Rigid-link snake robots with universal joint in early stage: (a) tensor arm manipulator [80, 81]; (b) elephant's trunk manipulator [79].

More mature design of snake arms with universal joint (Figure 2-21(a)) has been proposed by OC Robotics [82], which has successfully commercialized snake arms as

industrial solutions. The company has developed off-the-shelf Explorer Series ranging from 150 mm down to 40 mm in diameter, with full lengths from 1 m to 3.25 m and payload up to 32 Kg. The explicit structure of the universal joint [83-87] is illustrated in Figure 2-21(b), which is actuated by steel wires. The hollow core design of the snake arms allows for installation of various end effectors at the tip.

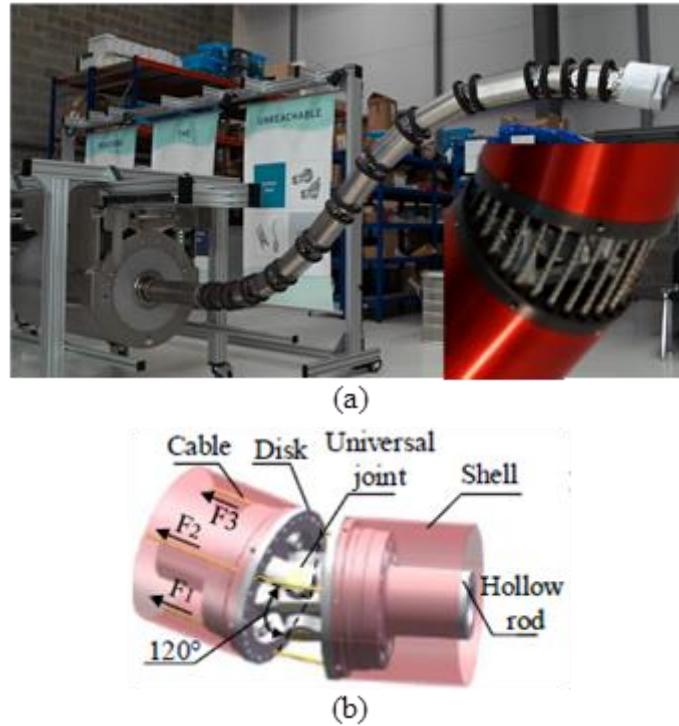


Figure 2-21 Mature rigid-link snake robots with universal joint: (a) Spatial snake arm by OC Robotics [68, 88, 89]; (b) single cable-driven universal joint [83-87].

2.3 Modelling of continuum robots

Beyond structural design, modelling is also an essential research field for continuum robots, which deals with the relationship of parameters among different operating spaces shown in Figure 2-22 and consists of two main categories. The kinematic framework is the most basic category used to represent the geometry of continuum robots, while the other category is the mechanical framework involving the constitutive relationships of mechanics between force (actuation and external loads) and geometry

(shape and motion). In the following sections, various approaches will be introduced and discussed in aspects of both categories.

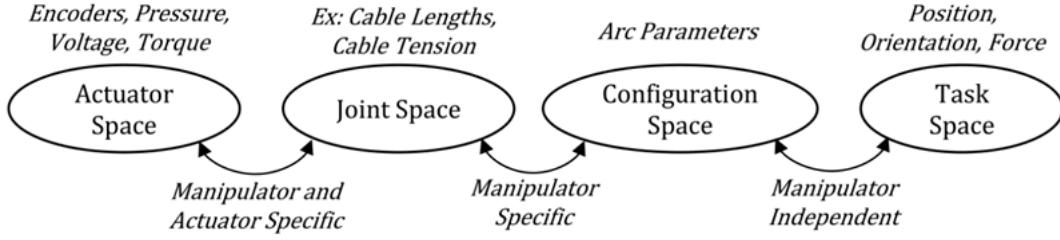


Figure 2-22 Hierarchy of the operating spaces of a continuum robot [90].

2.3.1 Kinematic framework

- Variable-curvature model

Without loss of generality, the shape of continuum robot can be accurately modelled as a continuous curve with variable curvature along the robot backbone [91]. As depicted in Figure 2-23, the rotation matrix $R(s)$ and position vector $p(s)$ evolve along the backbone length s according to the differential kinematic formula $\frac{dR}{ds} = R(s) \times u(s)$ and $\frac{dp}{ds} = R(s)v(s)$, where $u(s)$ denotes the curvature vector and $v(s)$ is analogous to the linear velocity vector expressed in body-frame coordinates.

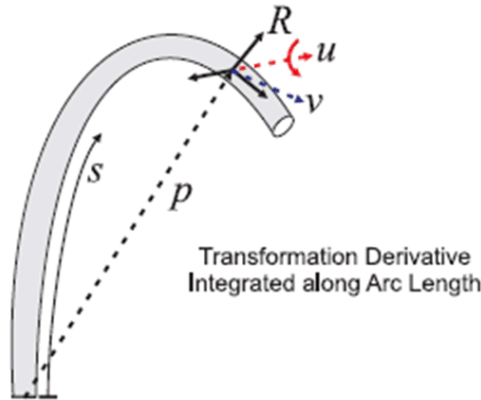


Figure 2-23 Illustration of the variable-curvature approach [91].

Figure 2-24 gives an example utilizing the variable-curvature approach for the geometric calibration of continuum robots [92]. In this work, the in-plane shape deviation from circular bending is reduced by interpolation from two curvature profiles

(Figure 2-24(a)). The twist along the central backbone (Figure 2-24(b)) is also considered to calibrate the orientation of end-effector. The kinematic error propagation is then derived considering both factors and thus the model calibration approach is developed, which significantly reduces the shape deviation from the circular assumption (Figure 2-24(c)(d)).

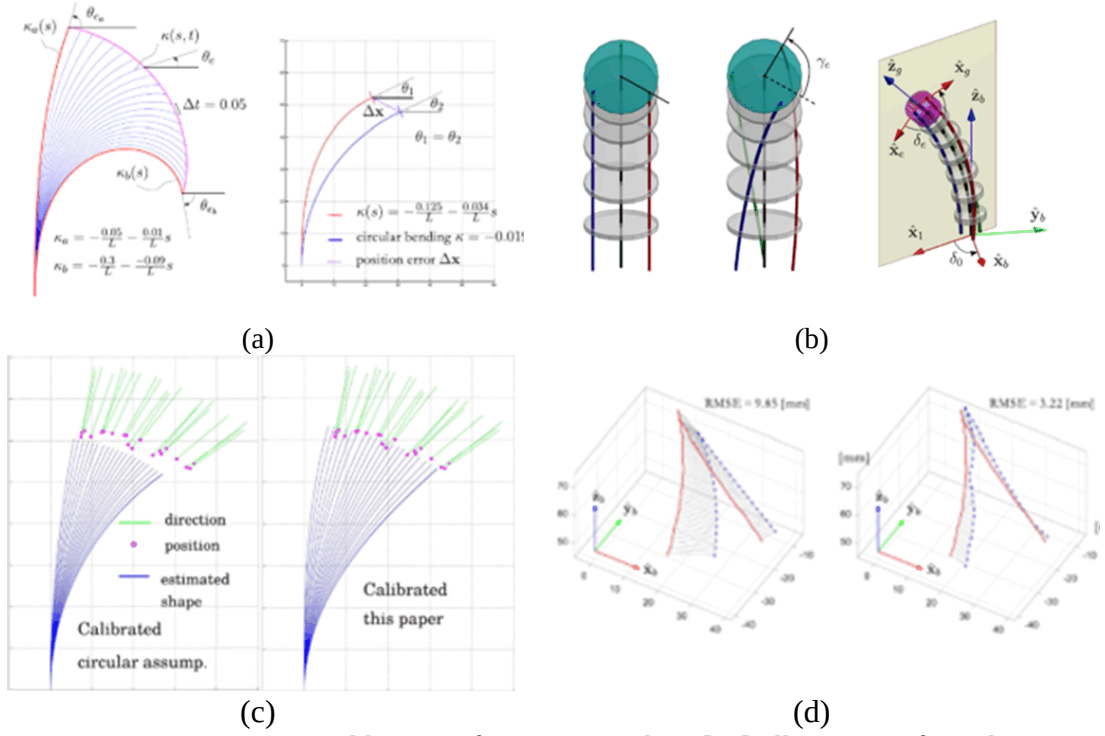


Figure 2-24 Geometric calibration of continuum robots [92]: illustration of (a) Shape deviation from circular bending and (b) Twist assumption. The calibration approach is experimentally validated by (c) in-plane bending and (d) ‘V’ shape trajectory following, in comparison with the circular assumption.

Despite of its precision, the application of the variable-curvature approach is extremely restricted since the function of the rotation matrix $R(s)$ and the linear position vector $p(s)$ are usually unavailable.

- Constant-curvature assumption

The constant-curvature assumption is widely applied for the kinematic modelling of continuum robots owing to its simplicity. This assumption can deal with most types of continuum robots with decent accuracy. In this model, bending curvature, backbone length and direction angle of each section form the arc parameters (Figure 2-25) in the

configuration space. Therefore, the shape profile of multiple serial sections can be described as piecewise-constant-curvature (PCC).

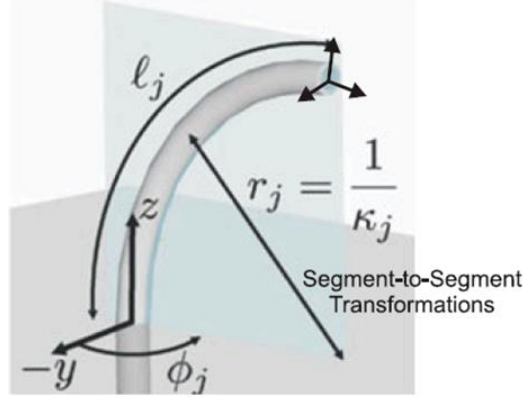


Figure 2-25 Illustration of the constant-curvature assumption [93].

a) Analytical method

To analytically derive the kinematics based on the constant-curvature assumption, different methods have been proposed for the modelling process. The first method is illustrated in Figure 2-25(a). In order to apply the well-established Denavit-Hartenberg (D-H) [94] approach on conventional rigid-link robots, a virtual rigid-link is created between the section beginning to the section end point as the translation joint, together with two virtual revolute joints at the beginning and end point, respectively. The second method is mathematically direct, which treats the backbone as a curve in space (Figure 2-25(b)) and is quite similar to the modelling of the variable-curvature approach but with the curvature being constant. Both modelling processes can lead to the same homogeneous transformation matrix $[H_j]$ of section j (Figure 2-25) in the plane:

$$[H_j] = \begin{bmatrix} \cos(s_j k_j) & -\sin(s_j k_j) & 0 & \cos(s_j k_j) - 1/k_j \\ \sin(s_j k_j) & \cos(s_j k_j) & 0 & \sin(s_j k_j)/k_j \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.1)$$

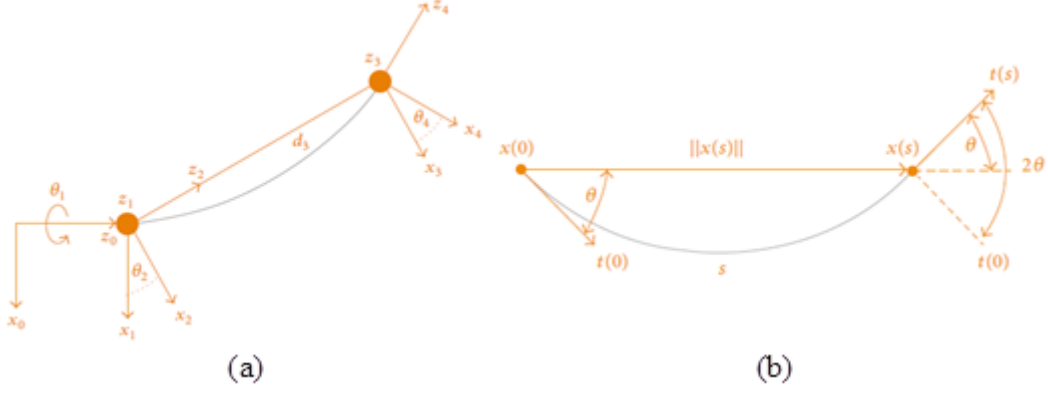


Figure 2-26 (a) Virtual rigid-link modelling method [95]; (b) the direct kinematic approach [93].

b) *Modal approach*

The previous analytical approach establishes kinematics directly modelling the nominal shape of continuum robots, which are fairly complicated for formulation and manipulation. An alternative approach is to build the kinematics using as backbone curvature with a finite number of modal functions [93, 96]:

$$k(s) = \sum_{i=1}^n \mu_i \cdot \phi_i(s) \quad (2.2)$$

where μ_i are coefficients and $\phi_i(s)$ denote the modal functions.

As shown in Figure 2-27(a), two basis sets are used to synthesize the kinematics for a four-section planar backbone. With the modal kinematics, the Jacobian can then be derived to solve the inverse kinematics. One example is given in Figure 2-27(b) with a straight tip trajectory.

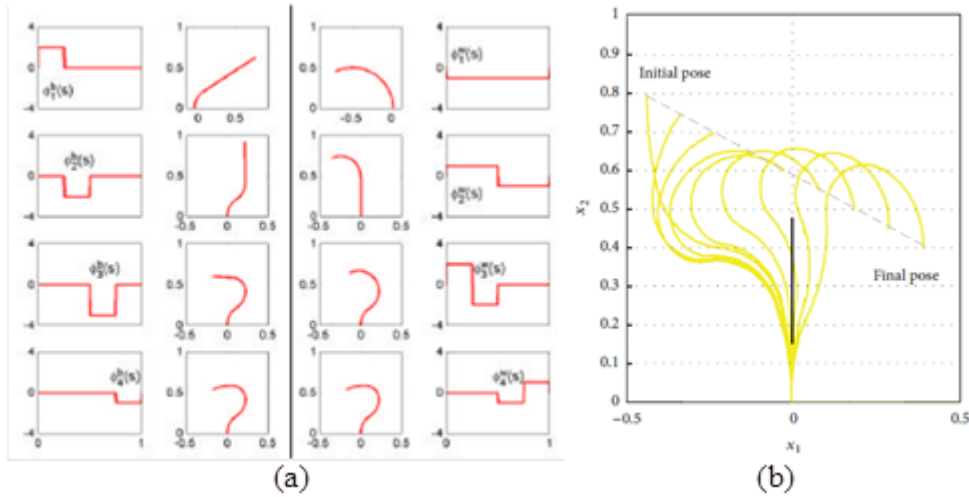


Figure 2-27 Illustration of the modal approach [93, 96]: (a) backbone shape generation based on the natural basis set (left) and the Haar basis set (right); (b) inverse kinematics solution for planar straight line tip trajectory.

Instead of using basis sets of modal functions, novel modal kinematics has been developed in [97] by using multivariable Taylor series expansion, which successfully extent the modal approach from plane to space. Simulations have been conducted considering both trajectory following and object manipulation (Figure 2-28) for algorithm validation.

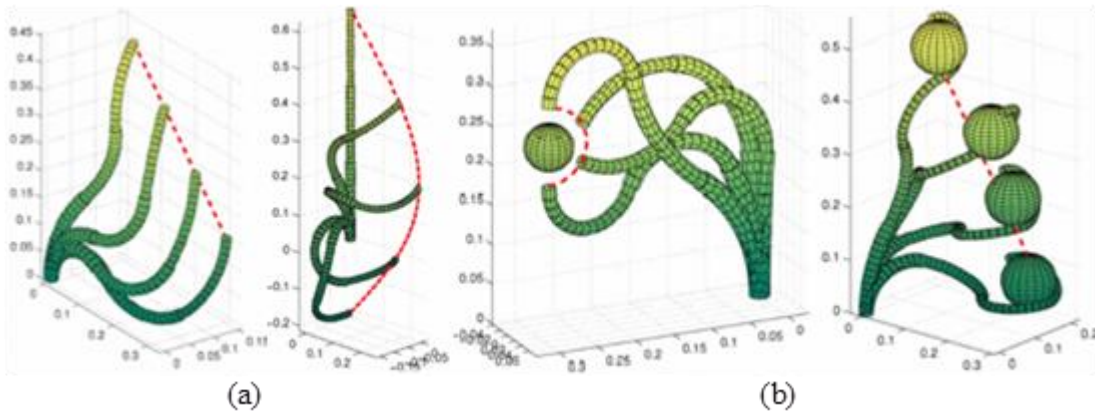


Figure 2-28 Improved modal approach [97]: (a) inverse kinematics solution for spatial trajectories; (b) simulation for object inspection and grasping.

- Learning-based kinematics

In addition to the variable-curvature model and the constant-curvature assumption, learning-based approach is also developed for the kinematic modelling of continuum robots. As shown in Figure 2-29(a), feedforward-neural-network (FNN) is applied to

learn the inverse statics of a conical shape soft robot driven by two cables, which builds the mapping from the desired tip position to the approximated cable tensions. Experiments reveal that FNN is more accurate than the model-based method owing to modelling uncertainties. A more complicated case is given in Figure 2-29(b), where distal learning (DSL) is used to approximate the inverse kinematics model of a pneumatic manipulator. Both forward and inverse kinematics model are firstly learnt between the tip position and wire potentiometer voltage, which are then improved online according to the performance error and prediction error.

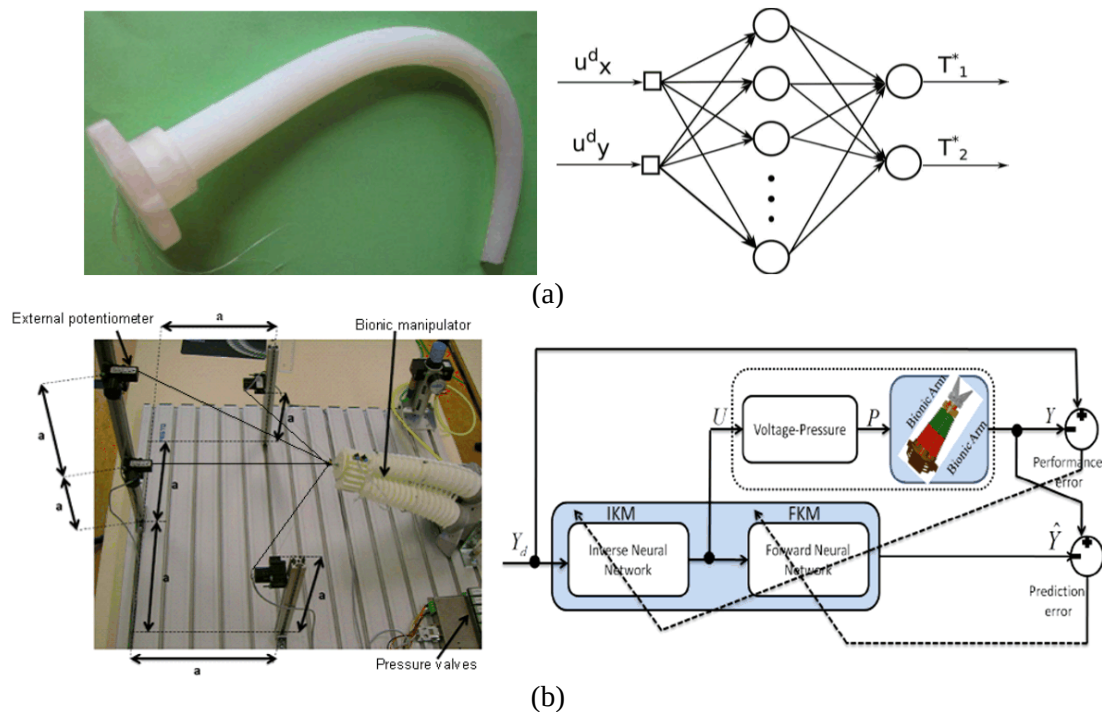


Figure 2-29 NN-based kinematics learning: (a) supervised learning between tip position and cable tension [13, 45, 98]; (b) DSL approach for the inverse kinematics [99].

In this section, three types of methods are introduced in this section for the kinematic modelling of continuum robots. Even though the variable-curvature model can precisely describe the shape profile of continuum robots, its further application is restricted owing to the requirement of the rotation matrix and position matrix along the backbone. Instead, the constant-curvature assumption is widely applied in most of the research because of its simplicity and efficacy, which assumes the shape of single

section as a circular arc. Recently, learning-based approach is also imported to learn the authentic kinematics by neural network. However, it is quite time consuming and of high computational burden owing to data sampling and pre-training. Since this work focuses on the feedback control of continuum robots, the constant-curvature assumption is chosen for the kinematic modelling to reach the balance between modelling accuracy and computational complexity.

2.3.2 Dynamic framework

While kinematics framework focuses on geometrical description, dynamics framework deals with more authentic models to consider internal and external interactions for both the statics and dynamics of continuum robots. It is necessary to mention that here the dynamics of continuum robots is presented and discussed as an introduction of another essential modelling approach for continuum robots, but it is not involved in the following chapters in this work. In the following, three different types of methods will be reviewed in this aspect.

- Lumped-parameter approach

Although lumped-parameter approach is developed to analyse the mechanics of discrete-link structure, it can also be imposed on top of kinematic model (usually the constant-curvature assumption) for the mechanics model of continuum robots, which considers elements such as point masses, dampers, and springs to approximate their mechanical behaviour.

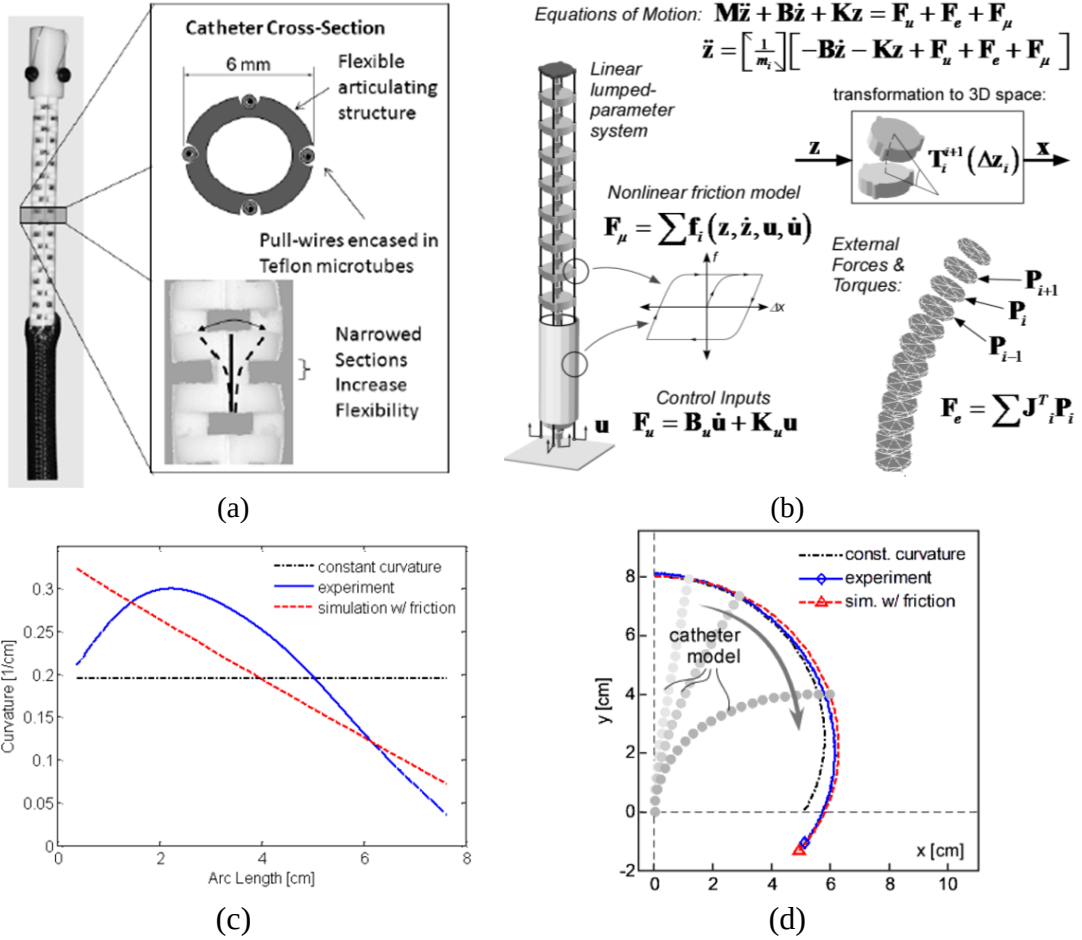


Figure 2-30 Lumped-parameter model using Newton-Euler equation [100, 101]: (a) the flexible catheter; (b) overview of the catheter mechanical modelling; comparison of (c) the curvature along the arc length and (d) the tip positions between the constant-curvature model, the experimental results and the simulation.

Two different approaches are utilized to obtain the governing equations for the lumped-parameter model, which are explicitly illustrated in Figure 2-30 and Figure 2-31, respectively. On the one hand, the classical Newton-Euler equations are used to model the mechanics of the flexible catheter. The motion equations are set up incorporating the control inputs (F_u), nonlinear friction effects (F_μ), and external disturbances (F_e) in separate forcing terms, which are experimentally validated by simulating the curvature along the catheter length (Figure 2-30(c)) and tip positions (Figure 2-30(d)). On the other hand, the lumped-parameter approach can also be performed from the perspective of energy. For instance, the principle of virtual power is utilized to implement the dynamic model of a cable-driven manipulator shown in Figure 2-31(a). Different from

the Newton-Euler method, virtual power builds the system dynamic model in terms of energy equations instead of complicated force analysis. The static virtual power continuum robot model is experimentally validated with various tensions applied on single cable as shown in Figure 2-31(c).

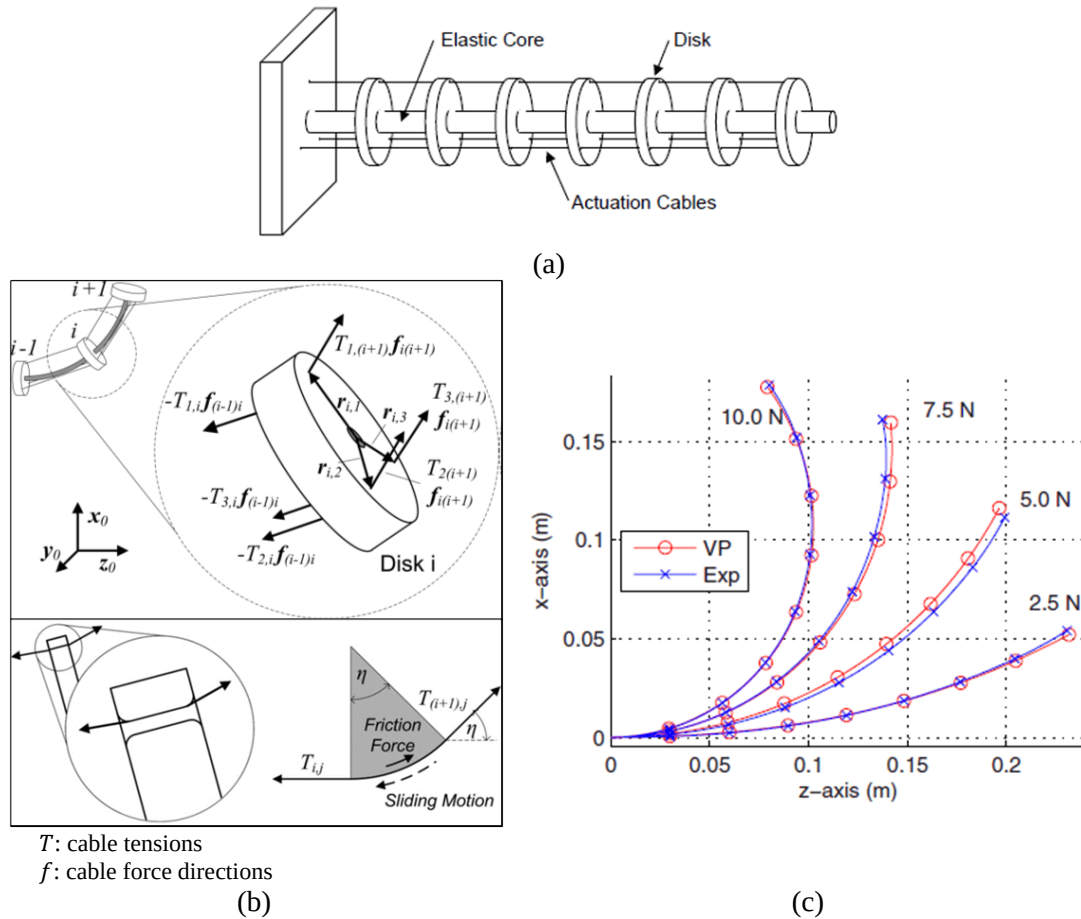


Figure 2-31 Lumped-parameter model with principle of virtual power [102-104]: (a) the cable-driven manipulator; (b) schematic diagram of the cabling force and friction; (c) experimental validation in static equilibria.

- Elasticity theory

While the lumped-parameter approach can achieve precise modelling of continuum robots by considering mechanical elements comprehensively, its performance is significantly restricted owing to the difficulty of parameters calibration. As such, elasticity theory is introduced to simplify the modelling of continuum robots.

Concentric-tube robot is one type of continuum robots constructed by pre-curved SMA tubes, which is well suited for modelling with Cosserat rod theory [105, 106]. The

schematic of the theory is illustrated in Figure 2-32(a), which is applied on multiple tubes with point and/or distributed loads. The model is validated on a two-tube cannula, where nuts are equally spaced along the backbone as the distributed load.

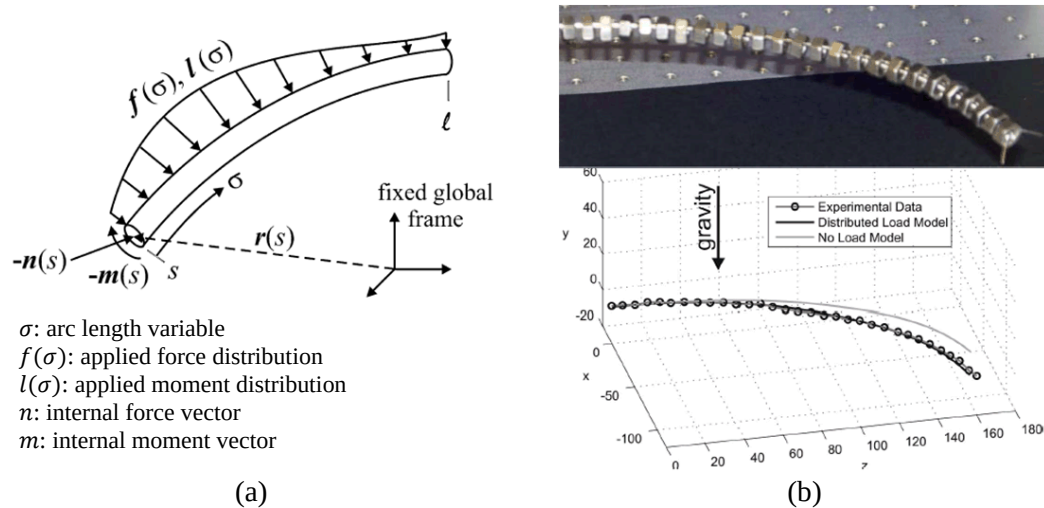


Figure 2-32 Application of Cosserat rod theory on concentric-tube robot [105, 106]: (a) schematic diagram; (b) illustration of the prototype and experiments with distributed load.

The elasticity theory is also applied on a three-section tendon-driven continuum robots with variable backbone length [107]. Both Beam mechanics (Figure 2-33(a)) and Cosserat rod theory (Figure 2-33(b)) are implemented for static modelling. Experiments were performed to compare the shape errors and end-effector errors with both approaches (Figure 2-33(c) and (d)) for the purpose of comparison, demonstrating that Cosserat rod slightly outperforms Beam mechanics in aspect of accuracy, while the latter significantly reduces the computation time.

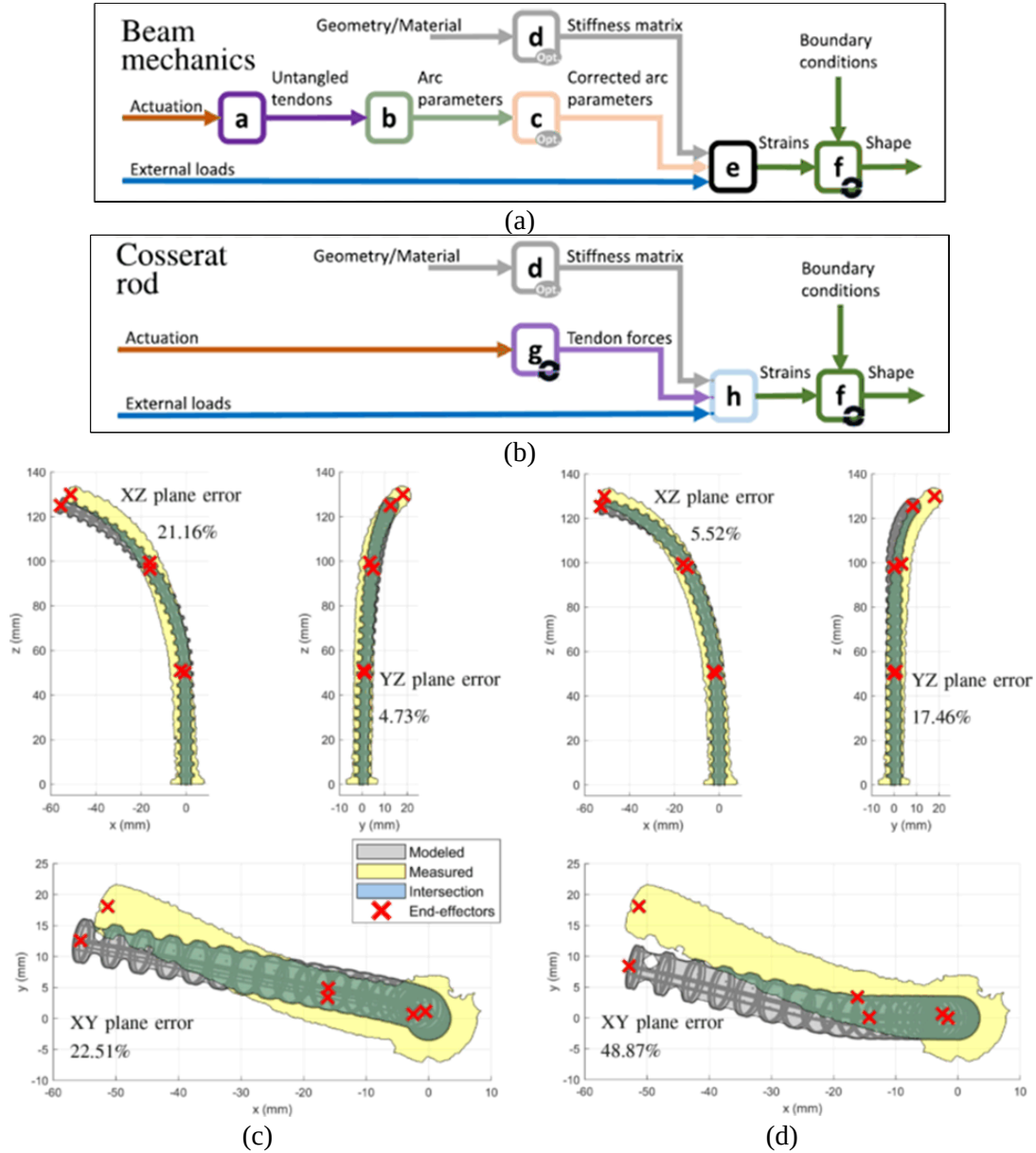


Figure 2-33 Approach comparison of elasticity theory [107]: (a) flowchart of Beam mechanics; (b) flowchart of Cosserat rod model; (c) experimental results of the shape errors and end-effector errors with Beam mechanics; (d) experimental results of the shape errors and end-effector errors with Cosserat rod model.

- Finite-Element-Analysis

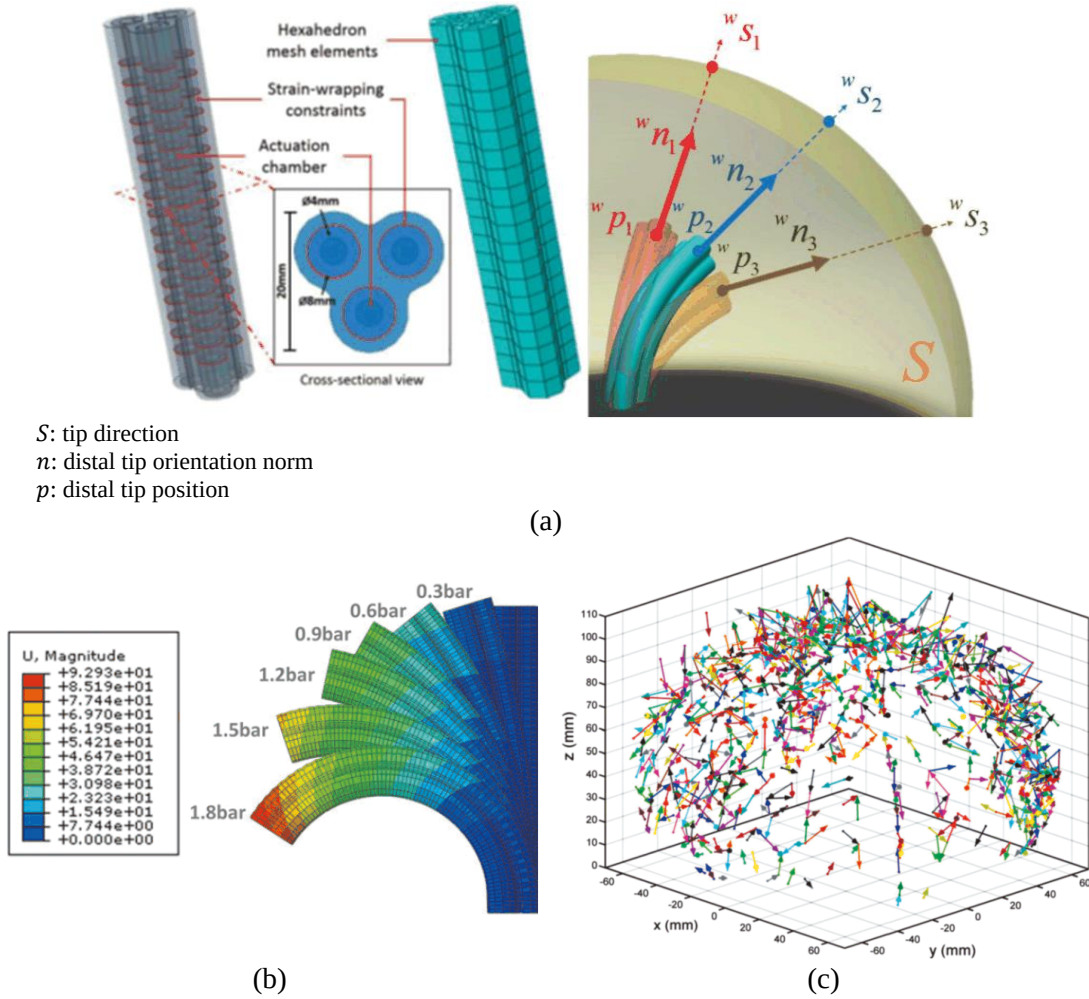


Figure 2-34 FEA simulation [41, 108]: (a) soft pneumatic robot with three parallel bellows made of silicon rubber; (b) deformation simulation of the robot with a single chamber in various inflation pressure; (c) simulation of the robot to obtain data for pre-training covering the entire workspace.

The last approach in the mechanical framework is finite-element-analysis (FEA), which is especially suitable for pneumatic continuum manipulators. Given necessary parameters of the constructing materials and the control input (actuating pressure), the deformation of the manipulator can be accurately simulated. As shown in Figure 2-34(a), the pneumatic manipulator consists of three parallel chambers, whose finite element model is tessellated with 12000 linear hexahedron elements. The FEA model is simulated by applying seven different levels of inflation pressure on a single chamber, which exhibits similar deformation characteristics in experiments under the same

corresponding pressure (Figure 2-34(b)). By varying the pressure applied on the chambers, the kinematics data of the soft robot covering the entire workspace can be obtained (Figure 2-34(c)).

In conclusion, different approaches are reviewed in this section for the mechanical modelling of continuum robots. Lumped-parameter approaches are developed for continuum robots with discrete-link structure since they are capable of dealing with elements such as masses, dampers, and frictions, while the elasticity theory and FEA are applied for continuum robots with continuous elastic backbones and pneumatic manipulators, respectively.

2.4 Control of continuum robots

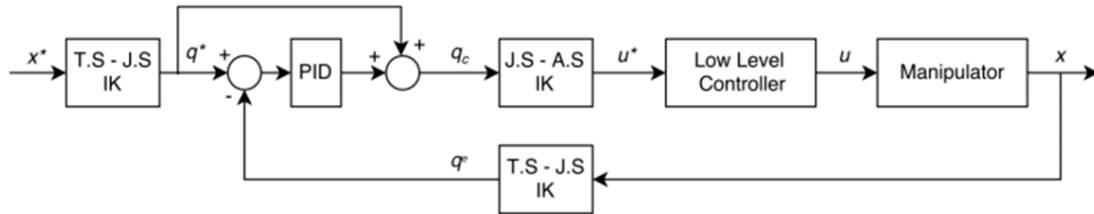
Owing to the variations of structure design and actuation method, different control approaches have been developed for continuum robots. It is necessary to declare that in this section, the open-loop control approaches are excluded because of the low repeatability of the majority of continuum robots resulted from modelling uncertainties such as nonlinear friction [109, 110], tendon coupling [111] and the stochastic nature of soft robots [12, 112, 113] and so forth. The state-of-the-art close-loop control will be introduced in the following, which can be divided into two groups, that is, model-based and model-less. Model-based methods are usually developed with kinematic models, where the expression of the controllers can be explicitly derived. As for model-less approaches, advanced algorithms are applied to learn/train the control models, either online or offline.

2.4.1 Model-based control

- PCC-based approach

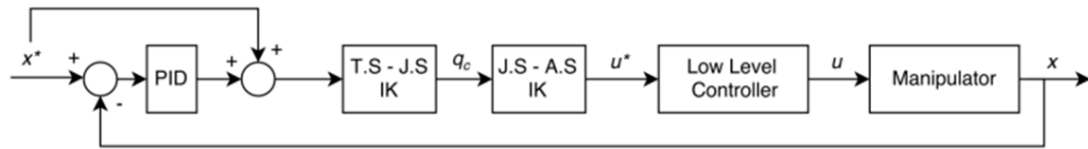
In terms of model-based control, a series of close-loop controllers have been proposed with the PCC assumption described in Section 2.3.1. Although more complex models

from the mechanical framework in Section 2.3.2 can be utilized to develop controllers for feedback control, the performance improvement (e.g., control accuracy) was not significant compared to their cost for computation and parameter calibration and therefore their usage have been limited.

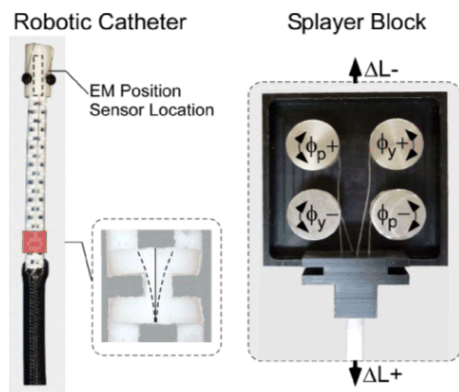


x^*, x : desire position, actual position
 q^*, q, q_e : desire arc parameter, actual arc parameter, arc parameter error
 u^*, u : desire actuation variable, actual actuation variable

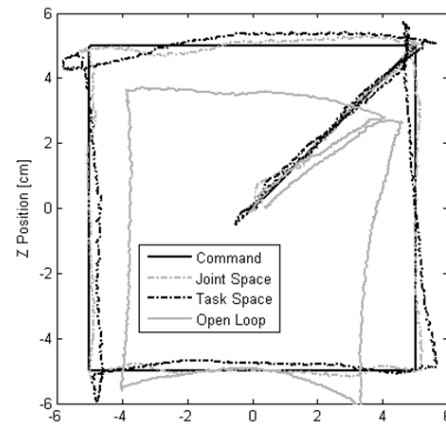
(a)



(b)



(c)



(d)

Figure 2-35 PCC-based position control in task space [90, 100, 114]: (a) implementation in task space; (b) implementation in configuration space; (c) mechanics of the robotic catheter; (d) experimental comparison tracking a square path.

There are mainly two different ways to develop the close-loop control using the PCC assumption, considering in which space the controller is implemented [100, 114]. On the one hand, the control loop can be directly closed in the task space for tip positioning. As shown in Figure 2-35(a), a simple integral controller is applied based on the error calculated in the task space, then the inverse Jacobian is utilized to transform the action

into joint space and actuation space sequentially. On the other hand, the controller can also be implemented in the configuration space (Figure 2-35(b)), where the inverse kinematics, rather than the inverse Jacobian, is utilized to convert the variables from task space to configuration space. Figure 2-35(c) gives a two-DoF single-section catheter with one extra DoF for translation, where both controllers are implemented and compared by tracking a square path. The experimental results in Figure 2-35(d) illustrates that the configuration-space controller performs slightly better than the task-space controller, while both controllers effectively improve the performance relative to the open loop control.

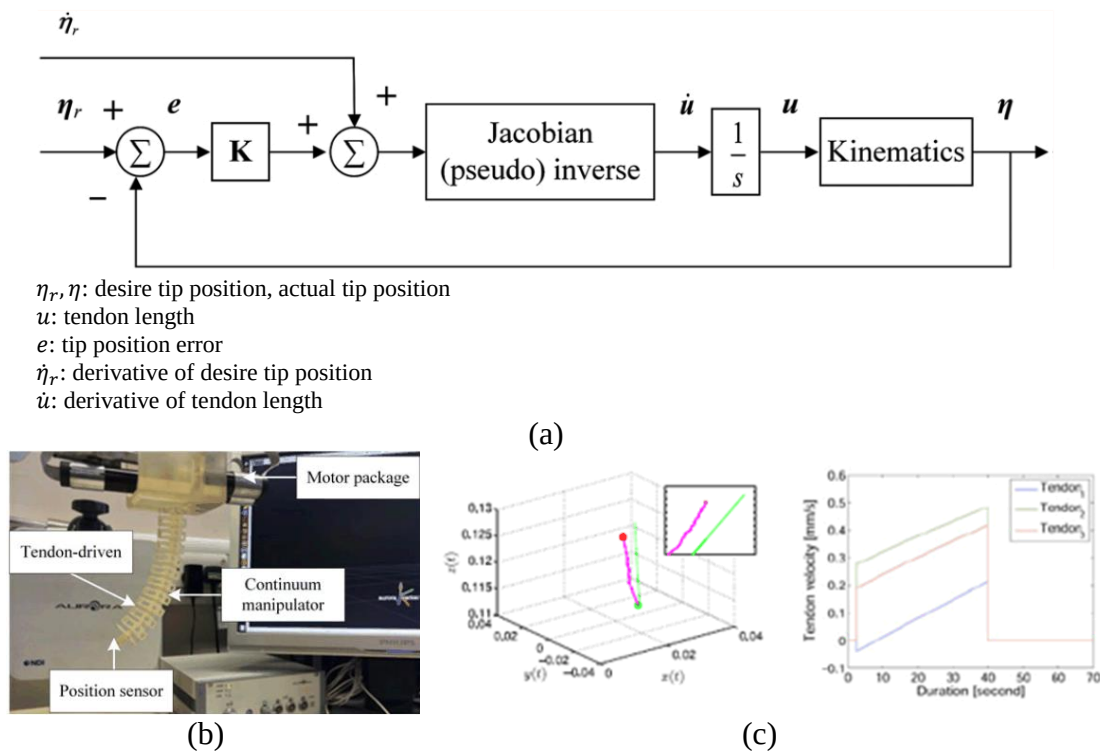
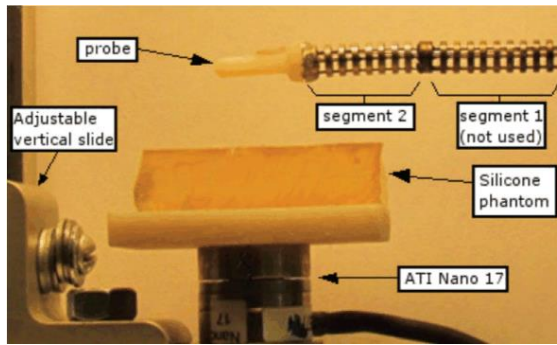
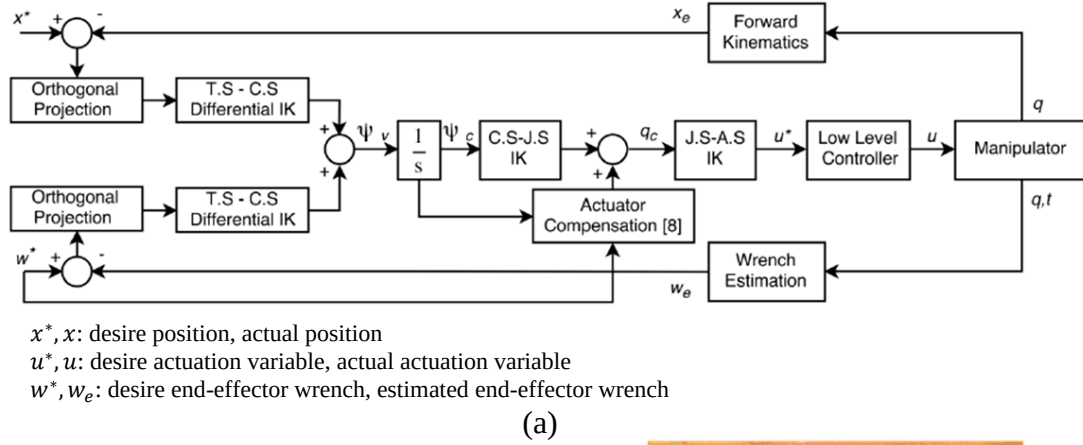


Figure 2-36 PCC-based position and velocity control [48]: (a) control architecture based on the (pseudo) inverse Jacobian method; (b) the extensible 3D-printed continuum manipulator; (c) experimental results tracking a linear trajectory.

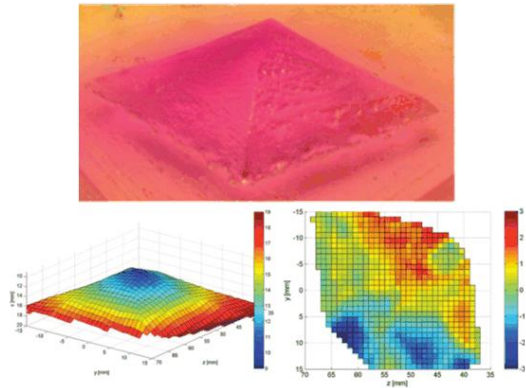
Even though that a square path is tracked in the case mentioned above, it is achieved by updating the discretized desired tip position in sequence and hence the controllers are still intrinsically for positioning. In [48], a controller based on the PCC assumption is proposed to operate continuum robots in the velocity level so that continuous path

tracking can be achieved. The schematic diagram of the controller is given in Figure 2-36(a), which can be formulated as $\dot{u} = J(u)^{-1}(\dot{\eta}_r + K(\eta_r - \eta))$, where u and η denote the tendon length and the tip position, respectively. The test is conducted on an extensible single-section continuum manipulator (Figure 2-36(b)) for linear trajectory tracking. The experimental results are shown in Figure 2-36(c), where the targeting precision is 3.1 mm and the integral absolute error (IAE $\int_0^t (|x - x_r| + |y - y_r| + |z - z_r|) dt$) is 296.8 mm·s.

Furthermore, hybrid motion/force control is developed with the PCC assumption [115]. The controller is implemented in the configuration space, where the desired twist and wrench vectors are projected orthogonally for decoupling the control effort into feasible motions and then merged using the differential inverse kinematics (motion) and the configuration-space compliance matrix (force) mapping the variable change in the configuration space to the tip wrenches, respectively. The hybrid framework is evaluated on a single-section continuum robot shown in Figure 2-37(b), which is 17.5 mm in length and 5 mm in diameter and equipped with an ATI Nano force/torque sensor at the tip. The robot is able to estimate and regulate the force at the tip, which can be used to perform force-regulated shape estimation of unknown flexible objects. As illustrated in Figure 2-37(c), the manipulator is operated to scan the surface of the silicon phantom while maintaining a 10 gram-force at the tip, with the shape error within ± 3 mm.



(b)



(c)

Figure 2-37 PCC-based hybrid motion/force control [90, 115]: (a) control architecture; (b) the two-DoF one-section multi-backbone continuum manipulator with the 3rd DoF for axial insertion; (c) experiments of shape estimation under 10-gram force-regulated scan.

- Jacobian update and estimation

While the PCC-based control is suitable for continuum robots whose shape profile are approximately a circular arc, control methods are developed by Jacobian update and estimation to deal with interactions with environment.

The first approach is about learning the local Jacobian online while traversing the environment [116-118]. Figure 2-38(a) illustrates the hybrid controller which comprises two components: a close-loop optimal controller solving for actuator displacements, and an online estimator learning the local Jacobian. The manipulator (length: 280 mm; diameter: 12 mm) is driven by two tendons for planar motion, with another DoF for axial insertion (Figure 2-38(c)). Since the manipulator is designed to operate within the lubricated anatomy of the body where sharp corners or deep

indentations are not encountered in the environment, the Coulomb friction is considered to not significantly change the surface normal measured from the sensor placed at the tip of the manipulator. The optimal control of tip position and contact force is achieved by minimizing the sum of three components: two different portions of cable displacements for the position control and the contact force control, and the cable tensions. It is necessary to mention that here the cable tension is regarded to be proportional to the cable displacement, which is not a general assumption in the research field of continuum robots. The experiment is given in Figure 2-38(d), which demonstrates that the manipulator is able to complete the linear ablation task on the tissue phantom with the contact force approximately regulated to 100 mN.

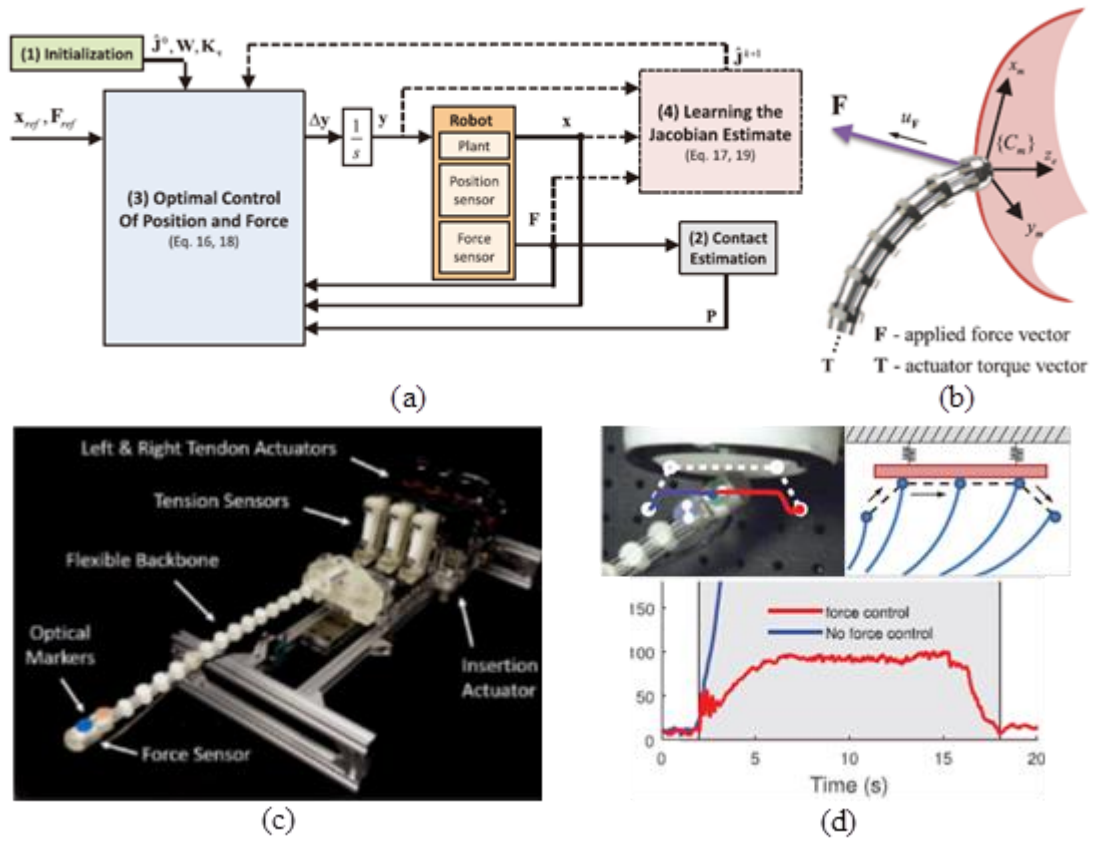


Figure 2-38 Hybrid position/force control with online Jacobian estimation [116-118]: (a) control architecture; (b) the illustration of the interaction between the manipulator and environment; (c) the planar continuum manipulator with one DoF for linear insertion, and a force sensor is placed at the tip; (d) experiments of tracing a path along the constraint with and without the force control.

The other way to estimate the Jacobian is based on the specific structure of the continuum manipulator [33, 34]. Figure 2-39(a) illustrates a novel design of pneumatic manipulator inspired by the geometry of honeycomb. The manipulator consists of 5 similar sections with each section composed of four groups of airbags well aligned along the X and Y axes. Based on the structural feature and several assumptions for simplification, the Jacobian of the manipulator can be approximately defined. As the control flow chart (Figure 2-39(b)) illustrates, the Jacobian will be further updated during the feedback process using the actual change of tip coordinates and the airbag pressure. The path tracking task is performed to follow a square path in the XY plane (Figure 2-39(c)), reaching 5 mm positional error (in both XY plane and Z axis) and 4 degrees orientation error, respectively.

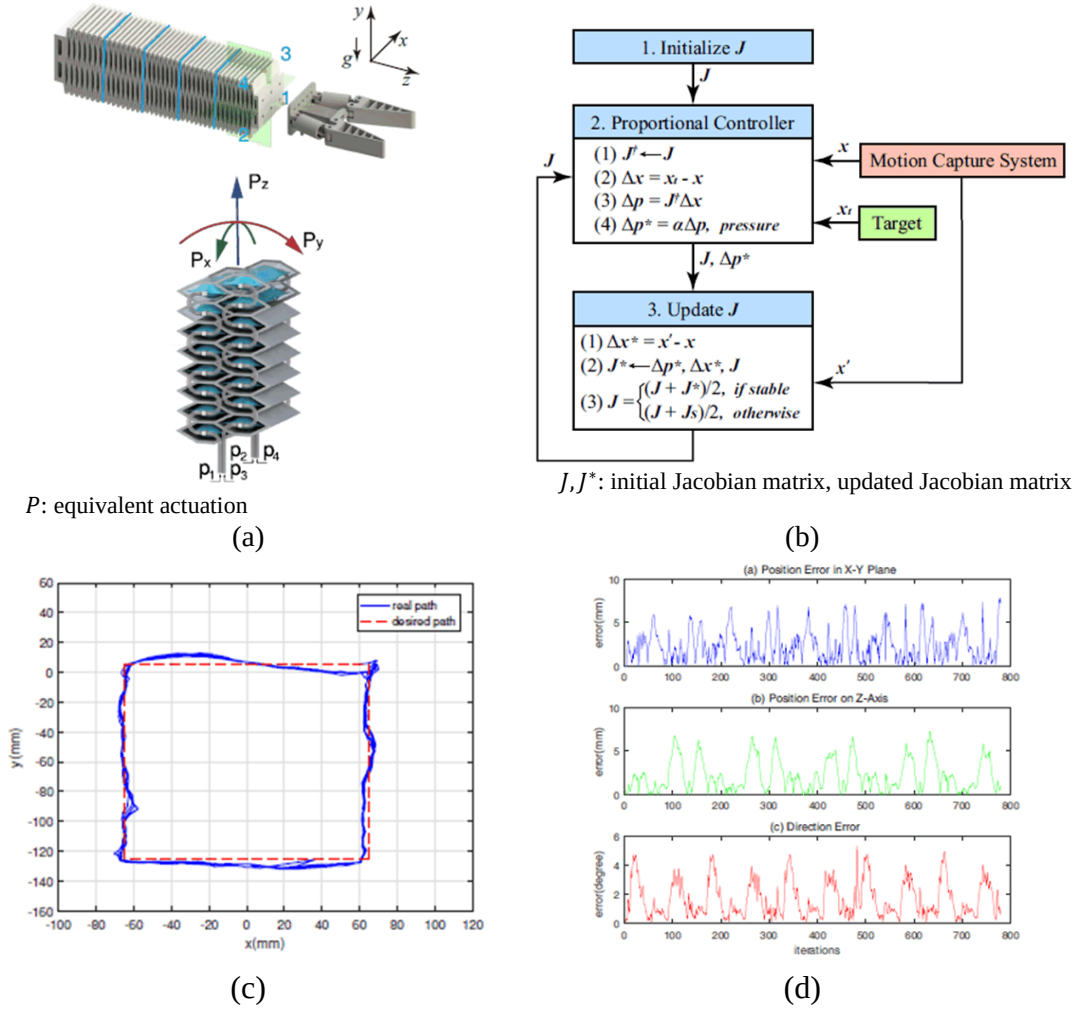


Figure 2-39 Robot-specific Jacobian estimation [33, 34]: (a) the novel honeycomb networks manipulator; (b) the framework of a proportional feedback controller based on estimated Jacobian; (c) desired and actual trajectories for path tracking task in the XY plane; (d) position and direction errors in path tracking task.

2.4.2 Model-less control

Even though model-based approaches can formulate the relationships mathematically between the causes and effects utilizing the fundamental equations in aspects of kinematics or dynamics, the existence of model uncertainties and difficulties of parameter calibrations lead to the emergence of model-less control, which can give more accurate models.

- Neural Network(NN)-based approach

Owing to the universal approximation property of neural network, it has been widely applied for the identification and control of multiple-input-multiple-output (MIMO)

nonlinear systems. Different from the introduction of using neural network to learn the static kinematic model of continuum robots in 2.3.1, neural network can also be embedded into the closed control loop to update the learnt model online.

The first implementation of the NN-based approach is shown in Figure 2-40(a), where neural network is utilized for iterative control. A unique formulation is proposed to learn the inverse kinematic (IK) mapping $(x_{i+1}, q_i, x_i) \rightarrow (q_{i+1})$ with the end effector feedback integrated. The proposed controller is experimentally validated on an extensible tendon-driven manipulator, which consists of two three-DoF sections. Figure 2-40(c) shows the result of trajectory following with the target path being a straight line (200 mm in length) divided into 40 waypoints and each step with 1 second to settle. In addition, the controller is also tested in the presence of obstacle during trajectory following (Figure 2-40(d)), exhibiting its adaptive behaviour in unstructured environment.

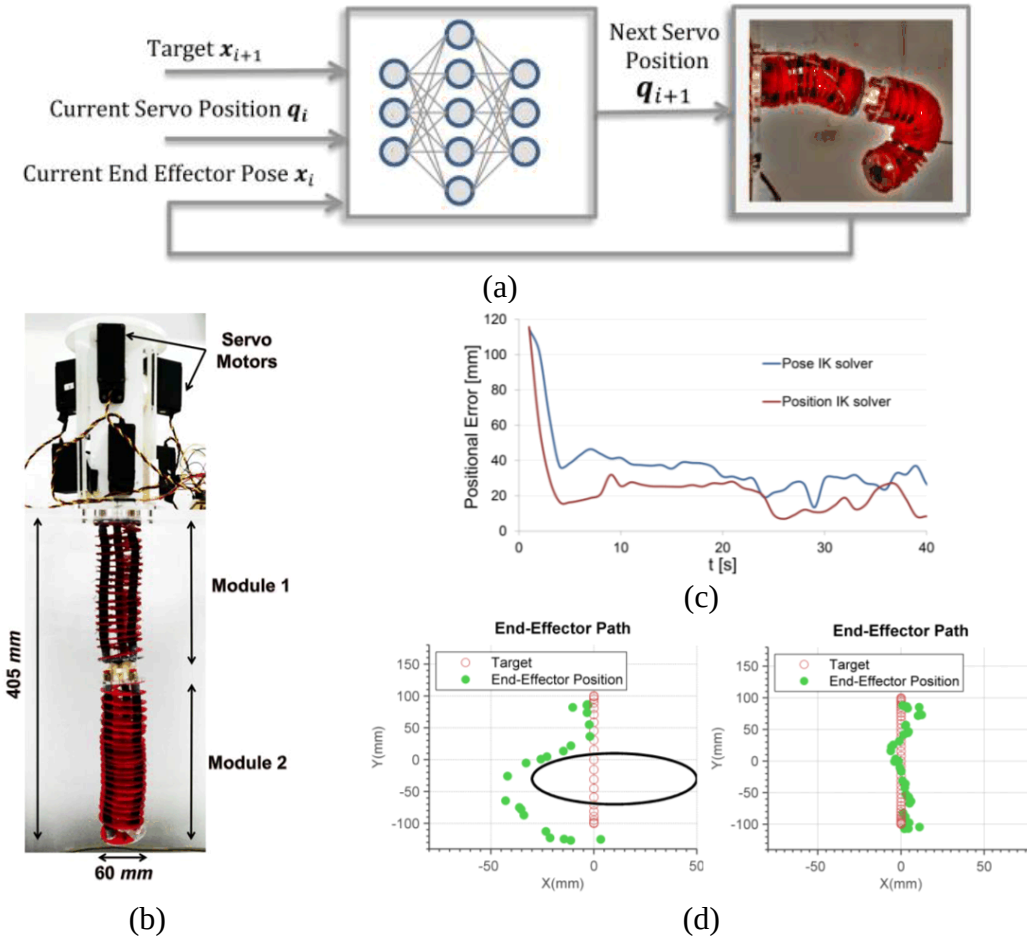


Figure 2-40 NN-based iterative control [113]: (a) schematic diagram of the learning architecture; (b) the six-DoF cable-driven continuum manipulator; (c) experimental evaluation of the learnt inverse kinematics solver; (d) linear trajectory following task with (left) and without (right) obstacle.

Further, another NN-based control model has been proposed to deal with the nonlinearities and hyper-redundancy of the Compact Bionic Handling Arm (CBHA) in Figure 2-41(a), together with the hysteresis behaviour of pneumatic actuators and the memory phenomenon of the polyamide materials for construction. To investigate the nonstationary behaviours of the robot, the kinematic control (Figure 2-41(b)) and the adaptive NN-based control (Figure 2-41(c)) are developed for the purpose of comparison. The NN-based control scheme includes two sub-controllers: a distal-supervised-learning controller comprising the forward and inverse kinematic models and an adaptive controller consisting of a controller and an emulator. A path-tracking task is conducted to compare the kinematic and adaptive controllers in Figure 2-41(d).

The tracking error ($\Delta X, \Delta Y, \Delta Z$) shown in Figure 2-41(c) demonstrates that the proposed controller significantly develops the accuracy of path tracking.

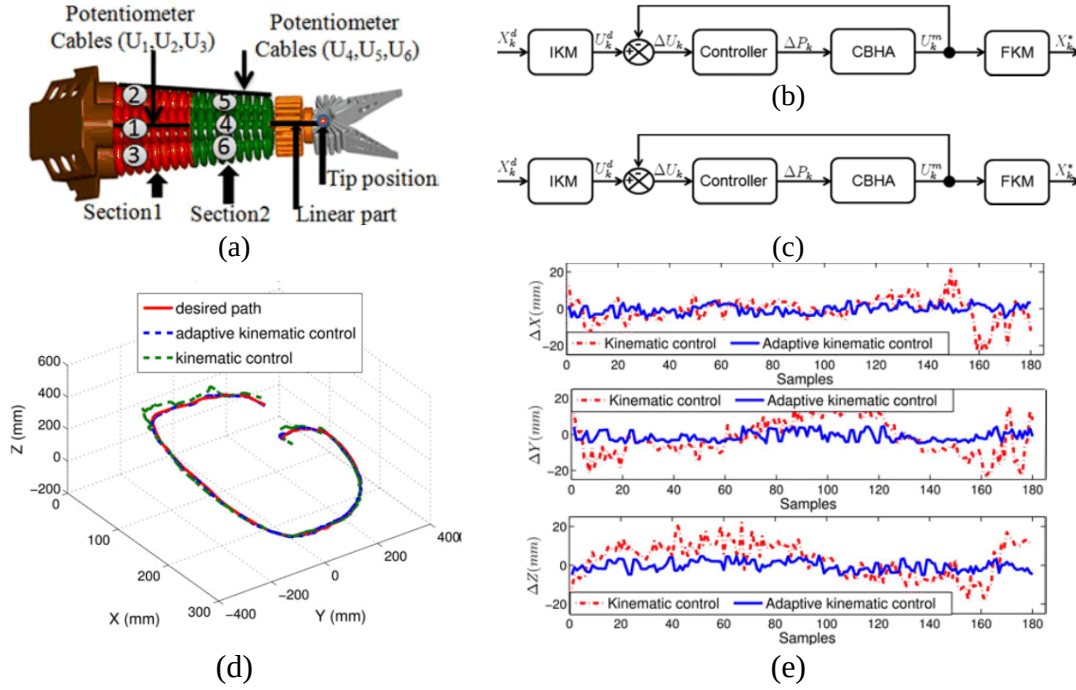


Figure 2-41 NN-based adaptive control [12]: (a) the Compact Bionic Handling Arm (CBHA); (b) kinematic control scheme; (c) adaptive kinematic control scheme; (d) experiments of path tracking; (e) Euclidean errors of the tip position.

- Locally-weighted-projection-regression (LWPR)

While it is introduced in previous part that the global kinematic model of continuum robots is usually trained directly with neural network, it can be also obtained by linearly combining multiple locally learnt models [41]. Figure 2-42(a) shows the schematic diagram of utilizing LWPR to approximate local inverse models based on the updated forward models. This nonparametric local learning technique is applied on a single-section fluid-driven manipulator (Figure 2-42(b)) to achieve orientation-based path tracking. The tracking performance of the online learning controller is experimentally validated in free space (Figure 2-42(c)) and under constraints (Figure 2-42(d)) by three different data sources: pre-trained by FEA data (top), pure online learning by random exploration (middle), and their combinations (bottom). It turns out that pure online learning performs best for the trajectory tracking in free space with an accuracy of mean

$\pm 1.13^\circ$ and maximum $\pm 4.24^\circ$ in the first cycle and an average of $\pm 0.87^\circ$ and maximum of $\pm 1.92^\circ$ after; while in the constrained experiment, the combined mode reaches the best overall performance (mean $\pm 2.49^\circ$ and maximum $\pm 11.03^\circ$).

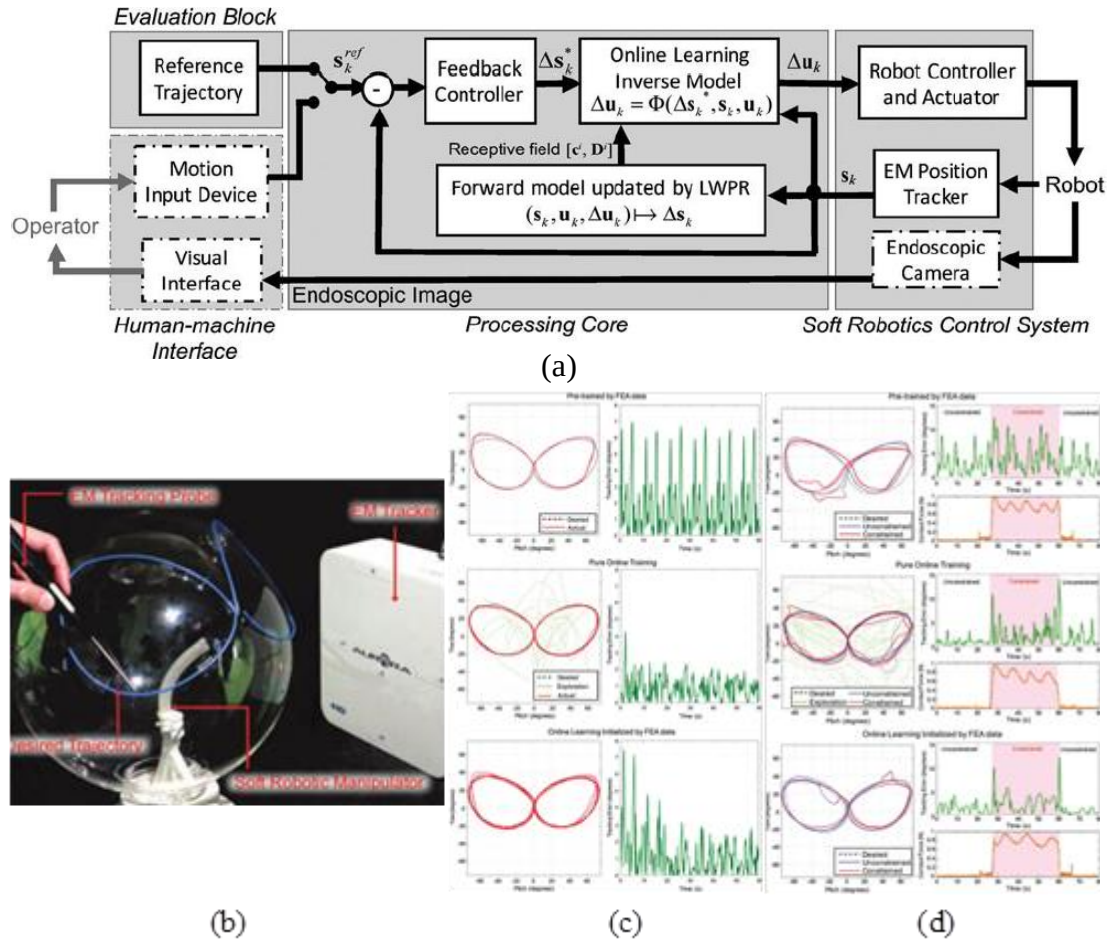


Figure 2-42 LWPR-based online learning controller for direction navigation [41]: (a) schematic diagram of the controller; (b) the pneumatic soft manipulator; (c) experimental comparison in free space; (d) experimental comparison under constraints.

- Fuzzy-logic algorithm

There are two different examples applying fuzzy logic algorithm for the control of continuum robots. The first example is shown in Figure 2-43, where empirical control policies are generated using fuzzy logic to achieve feedback control of a two-DoF SMA-actuated continuum robot (Figure 2-43(b)) in configuration space [119, 120]. In the control architecture (Figure 2-43(a)), both the desired variables and their errors are fed to the fuzzy controller, which outputs the control voltages of the SMA wires to

operate the robot. The proposed fuzzy controller is experimentally validated under a series of configurations for positioning. Figure 2-43(c) gives the result at the desired configuration $(\theta, \varphi) = (70^\circ, 10^\circ)$.

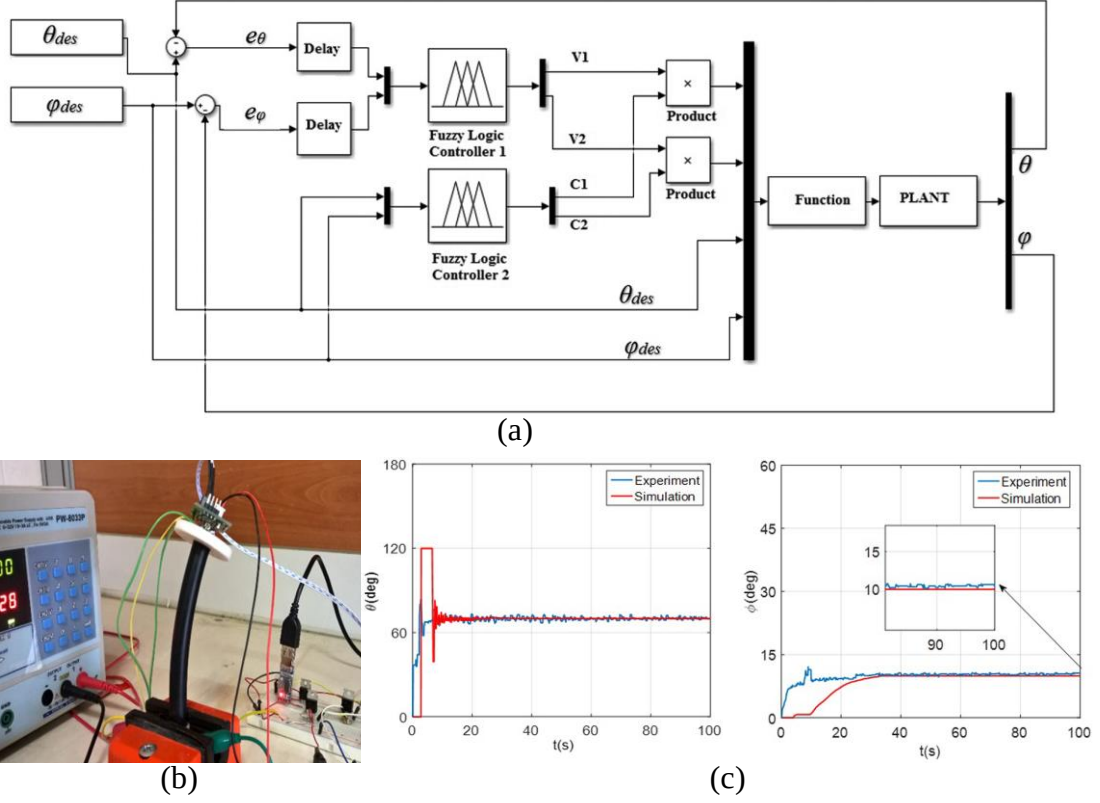
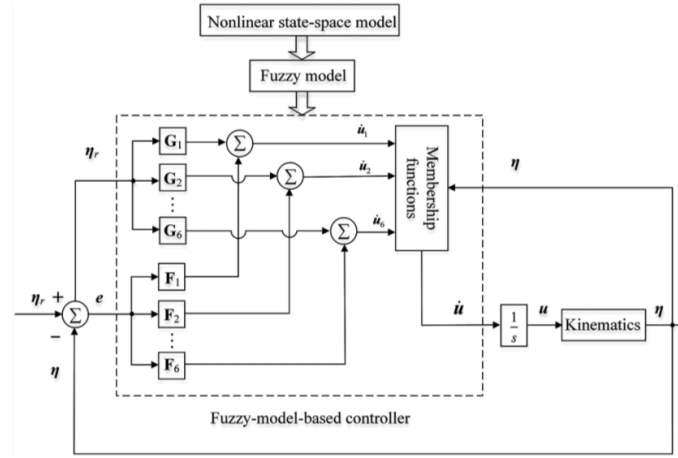
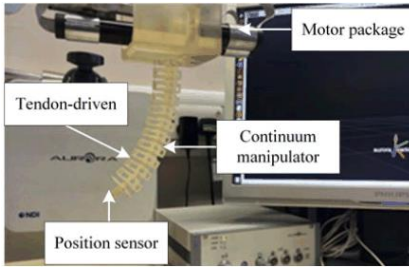


Figure 2-43 Fuzzy-rules-based empirical positioning controller in configuration space [119, 120]: (a) schematic diagram of the controller; (b) the two-DoF module actuated by the SMA wire; (c) the positioning result at the configuration $(\theta, \varphi) = (70^\circ, 10^\circ)$.

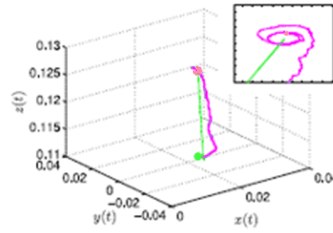
Figure 2-44(a) illustrates another way to implement fuzzy-logic-based controller for continuum robots, where the fuzzy algorithm was utilized to learn the nonlinear model based on the Jacobian of multiple local operating points for path tracking on a tendon-driven three-DoF single-section continuum manipulator (Figure 2-44(b)) in [48]. The proposed controller is experimentally compared with the Jacobian-based controller in 2.4.1, where the integral absolute error (IAE) was reduced from $296.8 \text{ mm} \cdot \text{s}$ (Jacobian method) to $244.6 \text{ mm} \cdot \text{s}$ (fuzzy controller) for tracking a 1 cm long linear path within 40 s while the targeting precision was decreased from 3.1 mm (Jacobian method) to 0.72 mm (fuzzy controller).



(a)



(b)



(c)

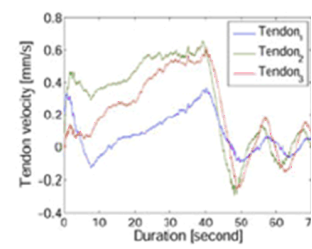


Figure 2-44 Fuzzy-model-based path-tracking controller in task space [48]: (a) schematic diagram of the controller; (b) the extensible 3D-printed continuum manipulator; (c) experimental results tracking a linear trajectory.

2.5 Position and shape sensing

The sensing of position and shape is another main research aspect in the field of continuum robots. On the one hand, tendon-driven continuum robots suffer from friction and section coupling issues; on the other hand, the stochastic feature exists in pneumatic manipulators. Those factors result in modelling uncertainties, low repeatability of continuum robots, not to mention precise manipulation under external disturbances. As such, various techniques are applied to measure the position/shape of continuum robots for the benefit of modelling and control. Figure 2-45 illustrates the classification of those techniques including the direct methods and indirect methods, which will be introduced in the following section.

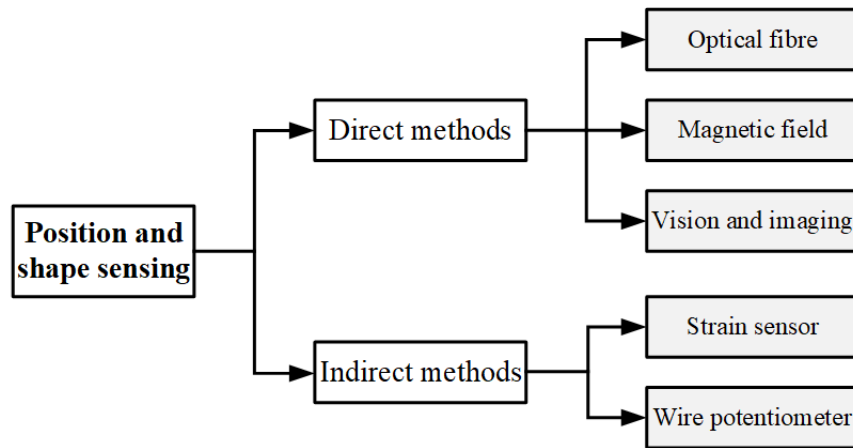


Figure 2-45 Classification of the position and shape sensing techniques for continuum robots.

2.5.1 Direct approaches

- Optical fibre

Optical fibre shows great potential for the shape measurement of continuum robots in aspects of practicality, precision, and sampling frequency and so on. The working principle is based on the Fibre Bragg Grating (FBG), which is a kind of distributed Bragg reflector constructed in a short length of optical fibre, reflecting particular wavelengths of lights while transmitting all others [121]. The shape deformation of the optical fibre will induce strain on the FBG node, which can cause a shift in the Bragg wavelength. By measuring the wavelength shift of all the FBG nodes, the 3D profile of the optical fibre can be reconstructed. The measurement performance can be further developed by Optical Frequency Domain Reflectometry (OFDR), which is an interferometric technique that works in the frequency domain and uses continuous wave light sources (e.g. 1535 nm – 1565 nm) to reach higher signal to noise ratios [122]. In addition, since the Bragg wavelength is also sensitive to temperature change, the scheme of a triplet of optical fibres [123, 124] is usually deployed in a triangular configuration to eliminate the temperature drift.

A comparative study is conducted with the regular FBG-based sensing (Figure 2-46 (a)) and the OFDR distributed sensing (Figure 2-46 (b)) for the shape reconstruction of a 35 mm long continuum robot [123, 124]. Figure 2-46(c) and (d) illustrate the experiments when the continuum manipulator bends in free environment and interacts with obstacles, respectively. It is found that the OFDR technique can reach smaller tip position error than the regular FBG method for both free-bending (OFDR: 0.32 mm (mean error) / 0.96 mm (max error); FBG: 0.83 mm (mean error) / 2.34 mm (max error)) and obstacle interacting (OFDR: 0.45 mm (mean error) / 0.72 mm (max error); FBG: 2.27 mm (mean error) / 3.40 mm (max error)).

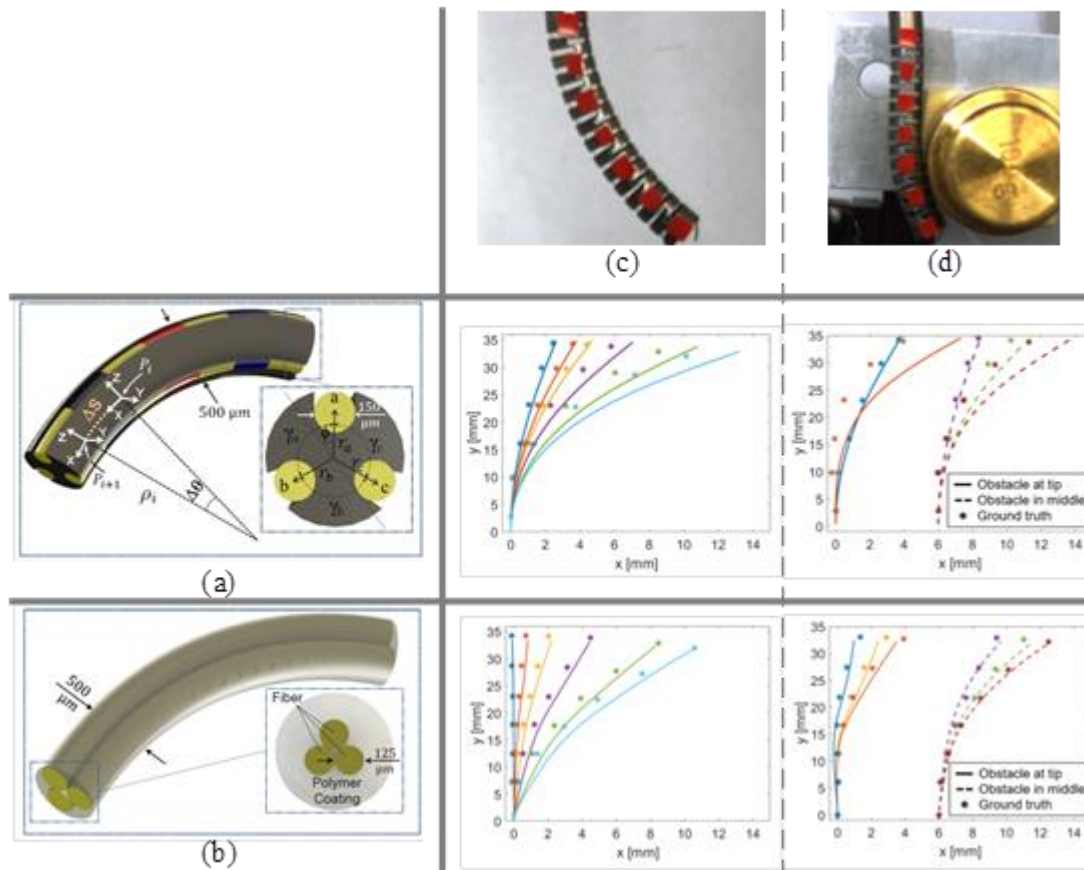


Figure 2-46 Comparison between the (a) FBG- and (b) OFDR-based sensors for the shape reconstruction of continuum robots (c) in free environment and (d) when interacting with obstacles [123, 124].

Despite their undeniable advantages in aspects of sensor integration and the capability of sensing the full shape of continuum robots, the requirements of the interrogator (data

acquisition unit) and customised sensing fibre with FBG nodes prevent them from being widely applied in the research field of continuum robots. In addition, the narrow laser wavelength range (up to 80 nm for most commercial products) significantly reduces the maximum sensing length (below 1.5 m), not mentioning that the measuring error increases drastically as the sensing length grows.

- Electromagnetic (EM) sensor

The EM tracking system have also been employed for the shape sensing of continuum robots, especially in the surgical field. The whole system mainly consists of multiple EM receivers and the EM field generator. The field generator can emit a low-intensity, varying EM filed and establishes the position of the measurement volume, while the receivers are miniaturized to the shape of pins, which can be attached on the continuum robot for both position and orientation measurement (Figure 2-47 (a)).

Figure 2-47(b) presents examples that EM sensors are mounted on a wire-driven continuum robot (top), as well as a concentric-tube robot (bottom). Each sensor is placed at the distal end of each section and base to capture the positional and directional information. As shown in Figure 2-47 (c), the three-order Bezier curve is proposed for curve fitting to reconstruct the shape of single section with unknown payload. The proposed algorithm is verified by experiments, which can achieve a mean position error of 1.7 mm for the wire-driven robot and approximately 1 mm for the concentric-tube robot.

Obviously, the EM sensor is inappropriate for industrial applications due to the interference of the metallic environment. Instead, it can be applied on surgical continuum robots for minimally invasive surgery.

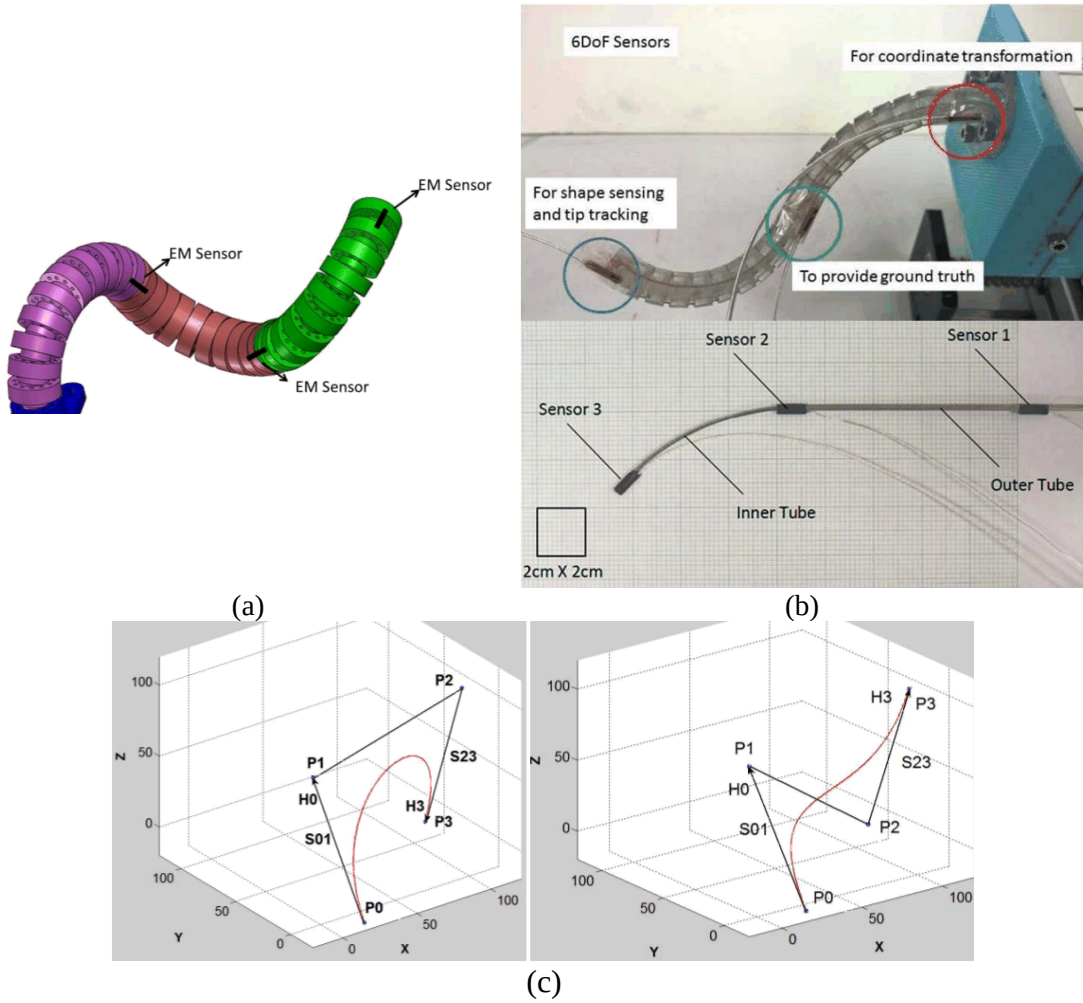


Figure 2-47 EM tracking-based shape sensing [125-129]: (a) schematic diagram of EM sensors integrated in continuum robots; (b) two-section wire-driven (top) and concentric-tube (bottom) robots with EM sensors mounted; (c) Bezier curve with C shape (left) and S shape (right) for the curve fitting to reconstruct the shape of the robot.

- Vision and imaging

The last category of the direct measurement is those vision-based approaches, including the binocular vision (Figure 2-48 (a)), the stereo vision using reflective markers (Figure 2-48 (b)), and the X-ray fluoroscopy (Figure 2-48 (c)).

As shown at the top of Figure 2-48 (a), two cameras are deployed from two arbitrary perspective views for the real-time 3D reconstruction of continuum robots [130, 131]. The abstract mathematical model of the binocular vision is present at the bottom of the figure. The shape sensing algorithm is developed based on the close-form analytical solution for the reconstruction of spatial quadratic curves extracted from two cameras,

which successfully achieves the feature recognition and therefore avoids the need of accurate point correspondence between two images. The experiments demonstrate that the algorithm can run at high speed (200 Hz) with good accuracy (maximum errors: 0.6 mm (position) / 0.5° (direction)).

The stereo vision, a more mature commercial solution with reflective markers, is imported for the shape sensing of continuum robots [25] in Figure 2-48 (b). The principle of the system is still based on the binocular vision. The difference is that here the markers are mounted on the surface of the pneumatic manipulator to reflect the infrared light emitted from cameras, which can avoid the need of accurate feature recognition and eventually develop the system robustness. Further, by deploying multiple cameras, the working volume of the system can be largely expanded, and the redundant algorithm can be applied to reach accuracy in submillimetre.

However, despite of their good performance, the application of binocular and stereo vision is restricted owing to the requirement of externally deployed cameras, making them only suitable for algorithm verification in laboratory environments and tasks in open space. To this, the X-ray fluoroscopy is developed for the intraoperative shape sensing of surgical continuum robots [132]. Figure 2-48 (c) illustrates one implementation of real-time shape sensing of a concentric-tube robot for trans-nasal surgery by combining monoplanar fluoroscopy with the kinematic model [132]. The demonstration on a skull phantom can reach an average error of 0.88 mm for the shape construction, revealing potential clinical value of this work.

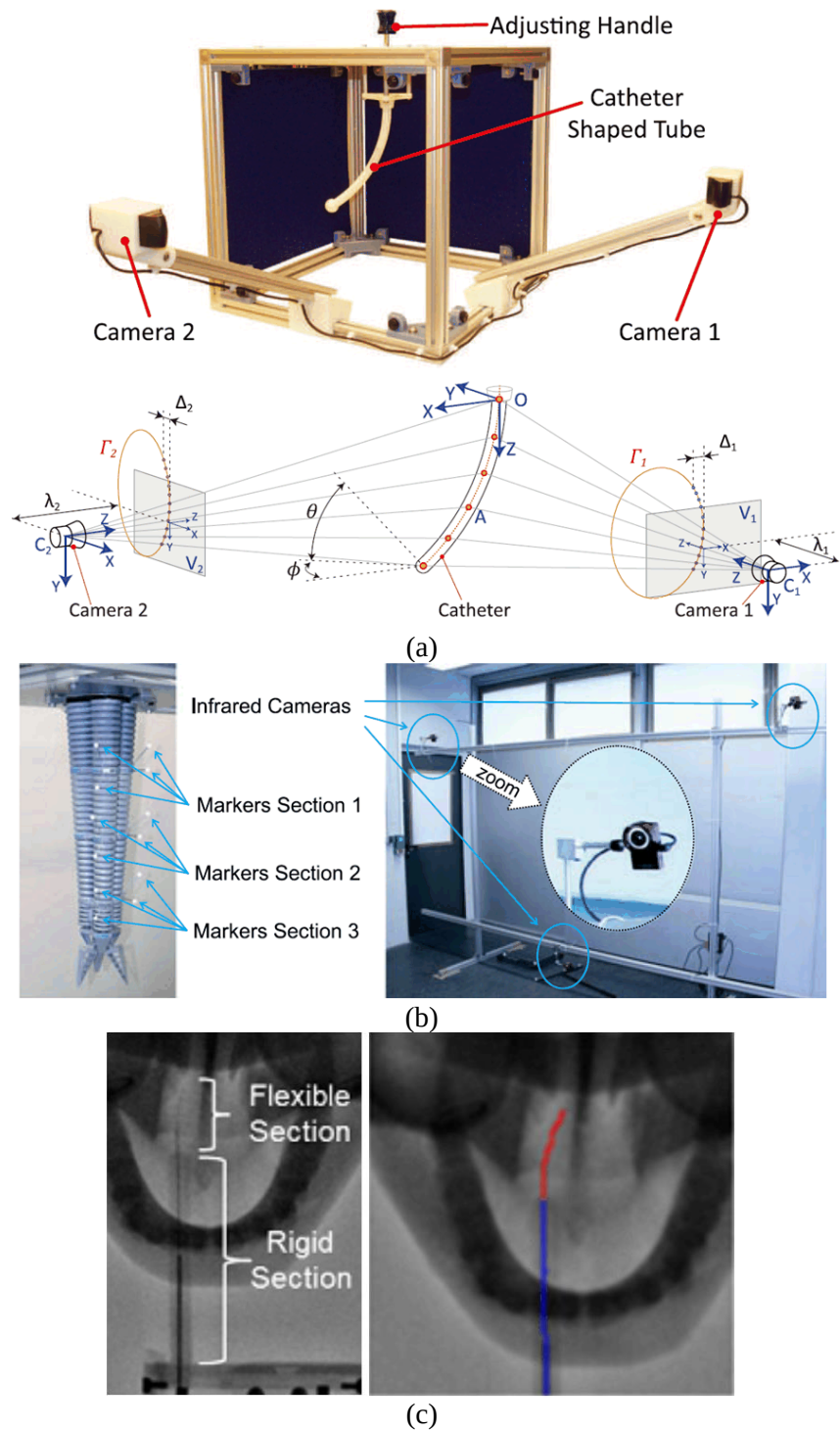


Figure 2-48 Vision-based approaches: (a) binocular vision [130, 131]; (b) stereo vision using infrared reflective marker [25]; (c) X-ray fluoroscopy [132, 133].

To sum up, the regular vision-based approaches are not applicable in confined space since the deployment of cameras is impractical. While the X-ray fluoroscopy solves the problem, it is only suitable for medical applications.

2.5.2 Indirect approaches

While direct approaches for measurement are usually implemented by deploying external devices, the indirect approaches are developed by embedding sensors into the structure of continuum robots, especially for the pneumatic and soft manipulators.

As shown in Figure 2-49(a), wire potentiometer is arranged surrounding the external surface of each bellows of the pneumatic manipulator to measure its instantaneous length [134, 135], which can provide accurate joint parameters for the kinematic calibration on the one hand, and be used as the feedback signal to achieve close-loop control of the actuators in the actuation level on the other hand.

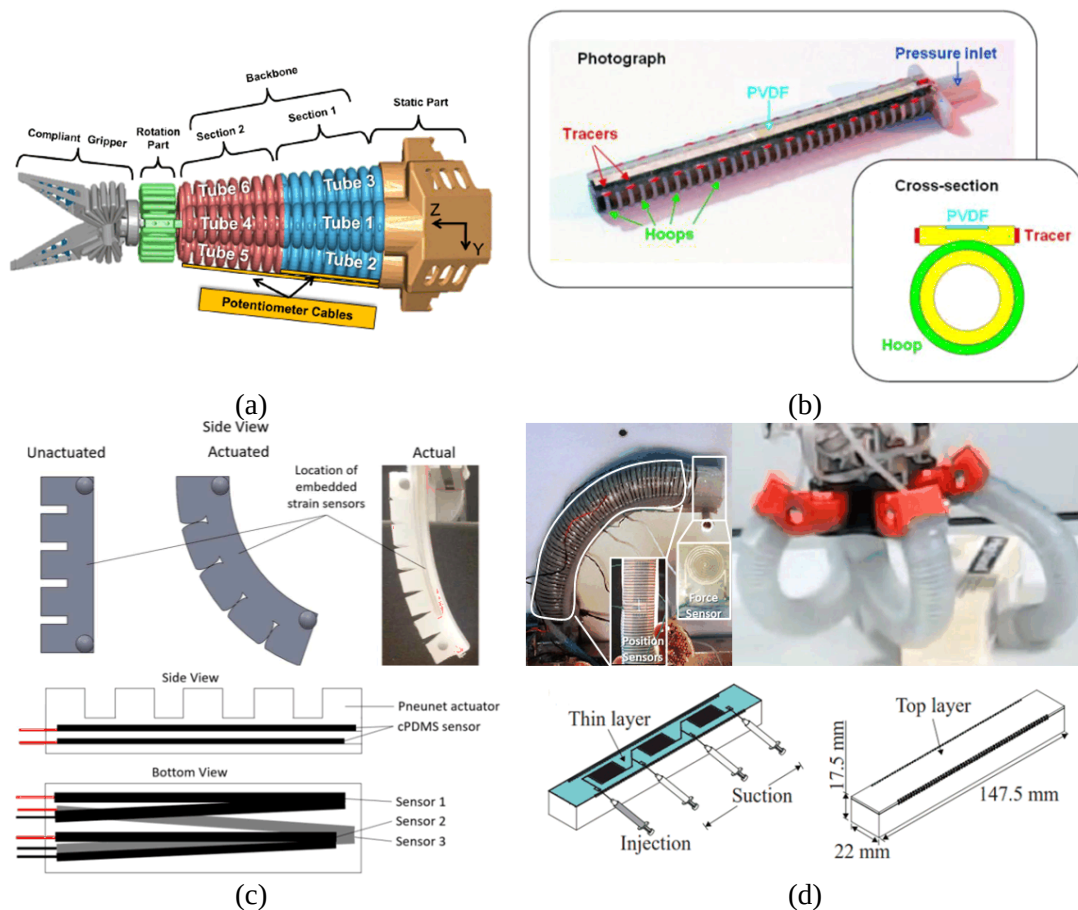


Figure 2-49 Approaches of indirect measurement: (a) the pneumatic manipulator with internal wire potentiometers [134, 135]; (b) the bi-bellows structure with PVDF films inside [136, 137]; (c) the planar soft robot with embedded cPDMS sensors [138] ; (d) the soft gripper with eGaIn sensors [139, 140].

Further, various strain sensors are integrated into the fabrication of soft robots for intrinsic position/force measurement. For instance, Polyvinylidene fluoride (PVDF)

film is embedded onto a one-DoF pneumatic compliant actuator for force sensing and deflection tracking [136, 137] (Figure 2-49(a)). Although the experiments reveal good correspondence of the PVDF strain in the 0-90° bending range, its further application is restricted by the high signal-to-noise ratio and drift owing to low working frequency (< 5 Hz). Similarly, as shown in Figure 2-49 (b), the soft resistive sensor consisting of layers of polydimethylsiloxane (PDMS) impregnated with conductive carbon nanotubes (cPDMS) is fabricated into a planar soft robot [138], where the long short-term memory network, a type of recurrent neural network, is used for learning the time series kinematic mapping because that it is able to train long time-lagged data. For the purpose of comparison, both the cPDMS sensor and the commercial flex sensor are tested for the kinematic modelling, where the former illustrates more stable performance than the latter in contact and non-contact situations. Lastly, liquid metal has been integrated into soft actuators for position and force feedback control [139, 140]. As illustrated in Figure 2-49(d), the spiral pattern is applied to design the micro-channels filled with eGaIn for force sensing at the tip, while the other three strain sensors are directly embedded serially into dorsal surface of the robot locating off the neural axis for the bending measurement. The prototype can be utilized as fingers in a soft robotic hand, which can grasp objects securely without damaging them by force monitoring.

In conclusion, various measuring techniques are reviewed in this section for the position and shape sensing of continuum robots. Generally, the direct approaches enable real-time tip localization and shape estimation with reasonable accuracy, despite of different strengths and drawbacks. Regarding the indirect methods, they are usually built-in for auxiliary measurement, which can be applied on pneumatic and soft robots for performance improvement.

2.6 Opportunities for future research

Through the comprehensive literature review in this chapter, the state-of-the-art development of continuum robots has been investigated in aspects of design, control, and shape sensing. There is no doubt that the structural design and shape sensing approaches play a significant role on the overall performance of the continuum robotic system. However, the former factor represents the intrinsic mechanical feature for a particular design of continuum robots, while the latter one is largely related to the working principle of the deployed sensors themselves. Actually, the control approaches applied have great effect on the performance for a specific continuum robot. As such, some potential research opportunities will be discussed as below to further fulfil the research field of continuum robots from the perspective of control.

- Need for general solution to the inverse kinematics of various CRs

As shown in Section 2.2, a series of continuum robots with different geometric designs have been developed for specific application requirements and scenarios, resulting in various expressions of kinematics for accurate kinematic modelling and analysis. Furthermore, usually only the expressions of the forward kinematics can be explicitly formulated, while the inverse kinematics for most designs (excluding the PCC assumption) can't be solved analytically owing to the complicated mathematical expressions, especially for multiple sections. As such, a general approach is necessary to unify the solution for the inverse kinematics of various continuum robots. To be specific, a mathematical framework can be developed by converting the analytical problem to a numerical optimization problem.

- Need to develop the tip-sections control performance of slender long CRs

To have access to confined environments, for instance, the combustor of aero-engines and pipelines, a slender shape profile is required for the structural design of continuum

robots, which can usually be divided into two stages: the proximal body-sections and the distal tip-sections. Even though the existence of body-sections enables the whole robot to navigate into the narrow space so that the tip-sections can perform tasks either for inspection or maintenance, it also brings in several challenges for the control of tip-sections. To be specific, the total length of the driving cables for tip-sections will inevitably increase, resulting in the inaccuracy of cable displacements owing to the negligible cable elongation. In addition, the friction between the cable and the guiding holes will further exacerbate the nonlinearity and even cause motion delay to cable displacement. As a result, the necessity of novel model-less controller for tip-sections arises to deal with those challenges on such kind of slender continuum robots.

- Needs for high-performance control of single-section CRs

The other research field goes to the design of high-performance controller for single-section continuum robots. Compared to slender continuum robots, the single-section ones are not subject to the severe motion delay caused by friction and extremely long driving cables. Instead, the actual cable tension can be accurately measured in real time by deploying force cells on the actuators. Hence, hybrid controllers integrated with tension supervision can be further developed to improve the performance of continuum robots.

3 Construction of the continuum robotic systems

In Chapter 3, the continuum robots used for evaluating and validating the control approaches developed in this thesis, are introduced, including the design of the arms and actuation systems, the architecture of the low-level controls and the equipment of measuring the configurations of the arms for the close-loop control.

3.1 Hardware of the continuum robots

In this chapter, two continuum robot prototypes, which are equipped with one 715 mm long and a class of single-section short continuum robots, respectively, are employed as the test benches for evaluating the performance of the control methods developed in the thesis. The first demonstrator was developed in an Innovate UK project - FLARE (Flame Spray Adder for in-situ Patch Repair of Aero-Engine Combustors) [141], which is designed to verify the concept of utilizing slender long continuum robots for in-situ combustor maintenance of aero-engines. Further, the second one was built as a test bench where different control approaches were developed and evaluated on several designs of single-section continuum arms, which can benefit the fundamental research on the control of continuum robot [142], especially for high-performance control. The publications relative to the design of the continuum robots utilized on those two prototypes are included in [17, 56]. The composition of those two prototypes is explicitly introduced in the following sections.

3.1.1 Prototype One: a multi-section continuum robot with dual-revolute joints

In this part, the demonstrator with a 715 mm long continuum arm is introduced. The design of the prototype of the tendon-driven continuum robotic system is shown in Figure 3-1. The continuum robot consists of two parts, the robot body and tip. The robot body is made up with ten one-DoF sections, which can achieve continuous motion in a

plane, while the robot tip has three two-DoF sections for operation in space. The sections of the body and tip are actuated by high-strength steel wires (0.4~0.6 mm), and each steel wire is controlled separately by a single motor module. A 3D-printed cable adapter with multiple independent internal channels (2 mm in diameter) is utilized to guide the steel wires from the arm to the actuation pack. Compared to other methods of guiding cables, e.g., pulley systems [15], the 3D-printed adapter significantly decreases the structural complexity and simplify the tasks for robot maintenance such as cable replacements. Lastly, the whole robotic system is mounted on a linear stage, which can precisely control the feed-in displacements when accessing to confined environments, etc. the combustor of aero-engines, for performing inspection or maintenance tasks.

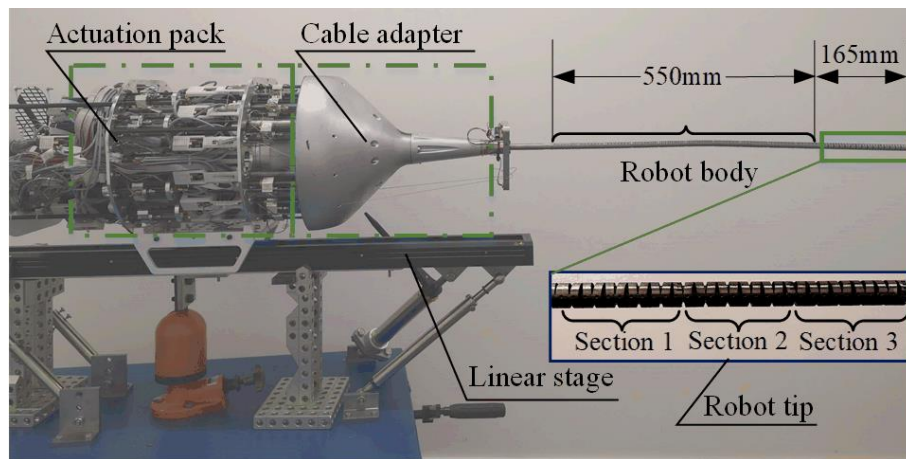


Figure 3-1 The overall system of Prototype One, including a continuum arm comprised of a robot body with ten one-DoF sections and a robot tip with three two-DoF sections, a cable adapter, and an actuation pack.

In terms of the design of the continuum arm, a single body section with the designs of disks used in the robot body are presented in Figure 3-2(a) and (b), respectively. The figure shows that a revolute joint is formed at the junction of two adjacent disks. Also, bevelled disk design with specific angled surfaces is applied so that the revolute joint can move freely in a certain range. Each body section is made up of several disks, which are connected serially by NiTi (nitinol) rods. In addition, the NiTi rods are locked from the side of the disks at both ends within single section by screws to keep the whole

structure compact. Since all the revolute joints are arranged in the same direction, single section is constrained to bend in a plane, which is regarded to be one-DoF.

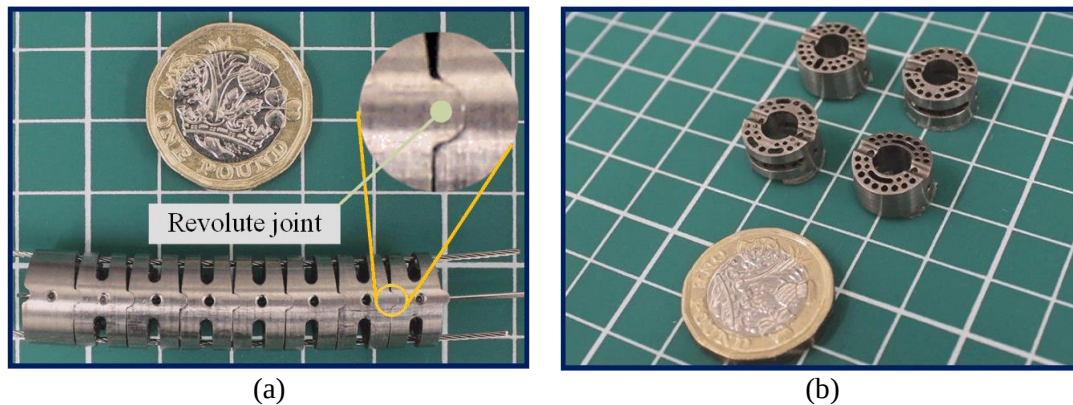


Figure 3-2 The design of a single body section (a) a single body section comprising of eight disks connected by revolute joints and flexible rods (super-elastic Nitinol) and (b) different disks used within the robot body.

Further, the disk design of the tip sections is different with that in the body sections so that the robot tip is enabled for two-DoF spatial mobility. To be specific, the corresponding two parts constructing the revolute joint are perpendicularly located on the top and bottom surfaces of the disk. Two disks make up a single segment shown in Figure 3-3(b), which is the elementary unit of the tip sections. The structure of a single tip section is illustrated in Figure 3-3(a), which consists of five segments in serial. Obviously, single section cannot either be compressed or extended, which is designed with two DoFs.

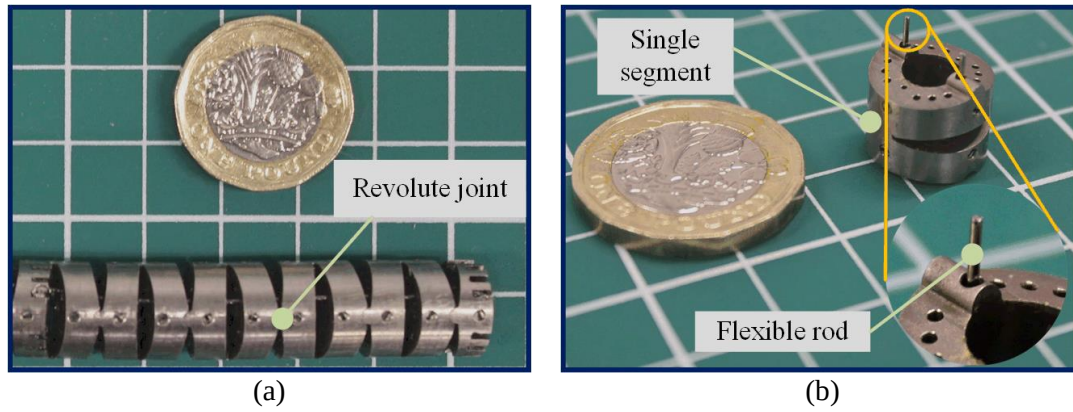


Figure 3-3 The design of a single tip section (a) a single tip section comprising of five segments and (b) the design of a single segment connected by revolute and compliant joints (super-elastic Nitinol).

Figure 3-4 shows the design of the 3D-printed cable adapter. Each cable has independent channel within the adapter to be guided smoothly from the arm to the corresponding motor in the actuation pack. To decrease the tension loss caused by friction, except for the use of lubricating grease, soft metal spring is also installed inside each channel. The friction force can be significantly reduced owing to the low friction coefficient between the steel cable and the metal spring.

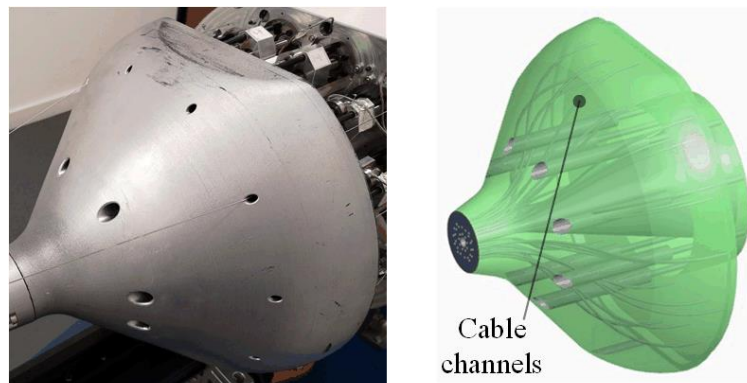


Figure 3-4 3D-printed cable adapter of Prototype One.

Finally, the actuation system of the 1st prototype is illustrated in Figure 3-5(a), which contains multiple motor modules shown in Figure 3-5(b). Each motor module is made up of a DC brushed motor with a threaded spindle (110 mm), a load cell (0-500 N), and a magnetic linear encoder (100 mm). Two supporting shafts are used to prevent the rotation of the customized slider on the motor spindle so that the rotary motion of the

motor is transferred to the linear motion of the slider. The load cell is mounted on the top of the customized slider at one of its ends, and the other end of the cell is connected to the steel wire with a nut. Considering the backlash of the motor gearbox, a magnetic linear encoder (resolution: $2.5\ \mu\text{m}$) is utilized to increase the control accuracy of cable displacements.

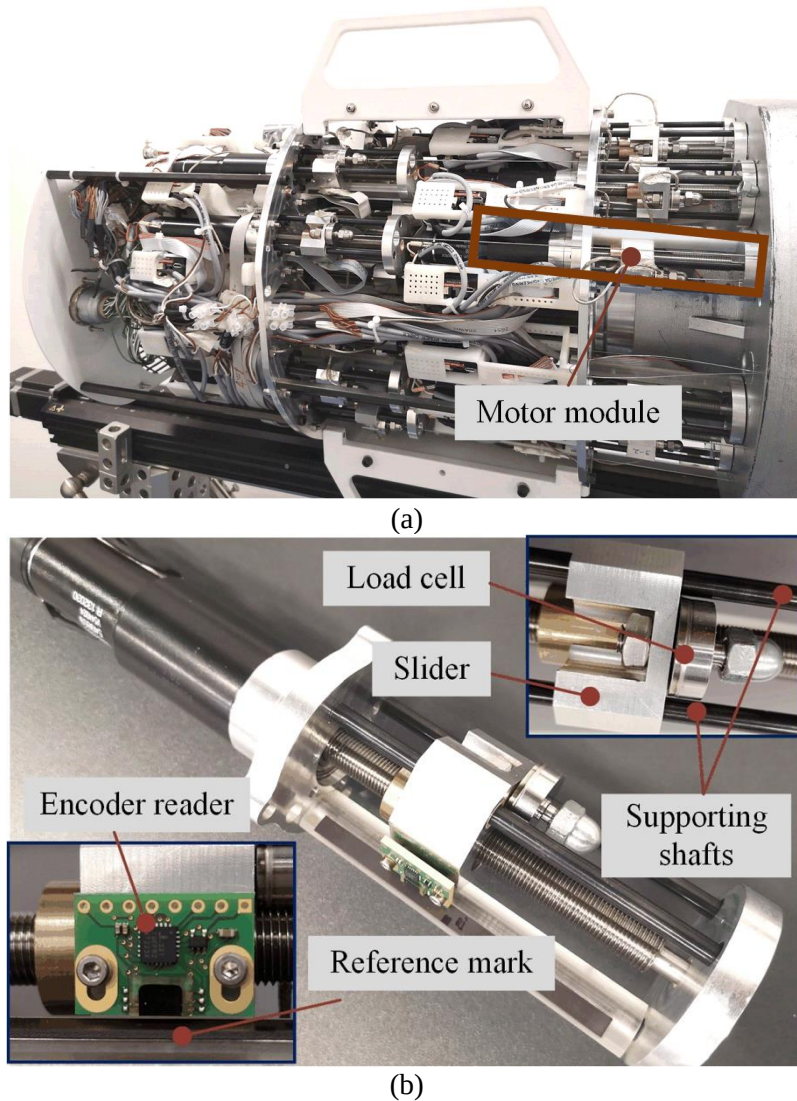


Figure 3-5 The actuation system of Prototype One (a) the overall design of the actuation system ; (b) the detail design of a single motor module consisting of a motor with a spindle, a load cell and a linear endcoder (including a reader and a reference mark).

The main contributions of the candidate for the development of Prototype One include the system assembly, the implementation and modification of the hardware control system, and the development and optimisation of the control program. Special thanks

to Dr Mingfeng Wang and Dr Xin Dong for their contributions of hardware design and maintenance, and Dr David Palmer's work on the control system in the initial stage of the project.

3.1.2 Prototype Two: a single-section continuum manipulator

In this part, the 2nd continuum robot prototype is presented, which can be installed with different single section continuum arms. The prototype (Figure 3-6) is used as a general simplified test bench for evaluating the performance of the control methods developed in the thesis on a group of continuum arms. The whole system of Prototype 2 is setup by the candidate independently.

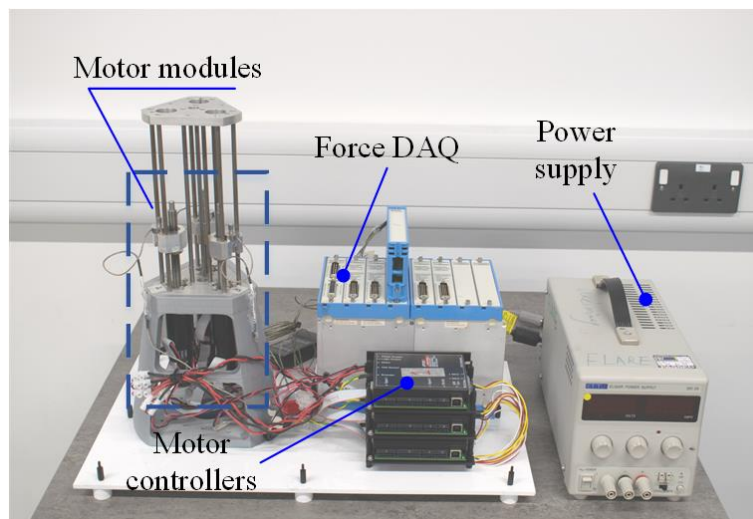


Figure 3-6 The overall system of Prototype Two for studying the high performance controllers for various designs of continuum robots.

On this prototype, single-section manipulators with different structural design were utilized for the performance characterization of the controllers developed in this thesis, including two types of continuum robots (Figure 3-7(a-b)) and a 3D-printed soft robot (Figure 3-7(c)). The continuum robot shown in the Figure 3-7(a) is known as twin-pivot design, where the bending capability is achieved by two antagonistic flexible Nitinol rods. The structure of the second continuum robot in Figure 3-7(b) has been explicitly illustrated in Section 3.1.1. Regarding the soft robot, it is made from flexible resin by

3D printing, which can not only be bent freely in space, but also be extent or compressed axially. The kinematics of those robots will be analysed and modelled in Chapter 4.

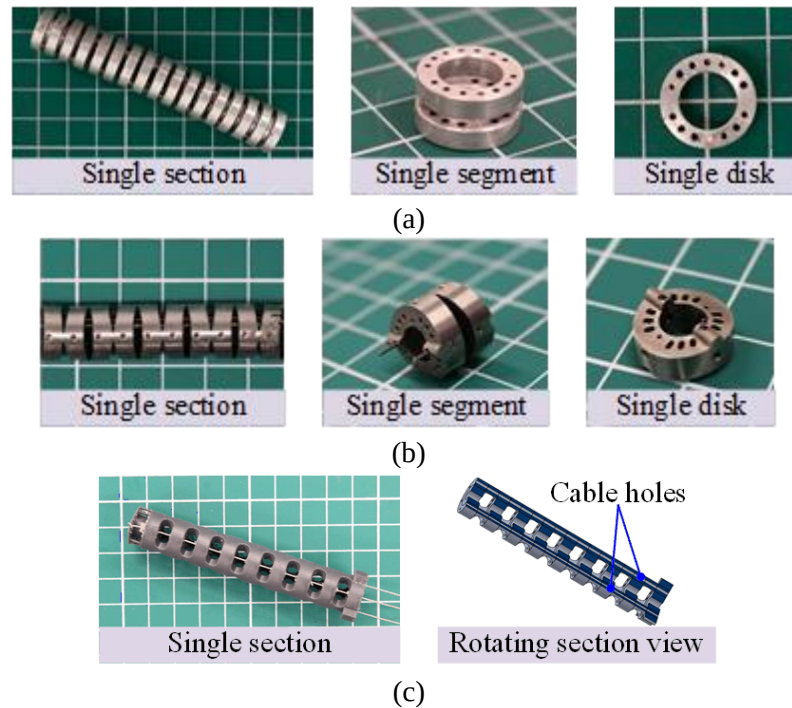


Figure 3-7 Three designs of single-section continuum manipulators: (a) a single-section continuum arm with a twin-pivot design, (b) a single section continuum arm with a dual-revolute design, (c) a single section continuum arm made from flexible resin.

Similar to the design of the cable adapter and motor module of Prototype One, the corresponding parts of Prototype Two are illustrated in Figure 3-8 and Figure 3-9 respectively. The cable adapter is fabricated by additive manufacturing approach with plastic, containing two components, within which the structure of the smaller component can be slightly adjusted to adapt to different manipulators. In terms of the actuation system, the motor module of Prototype Two is adapted from the design of Prototype One, where the magnetic linear encoder is removed.

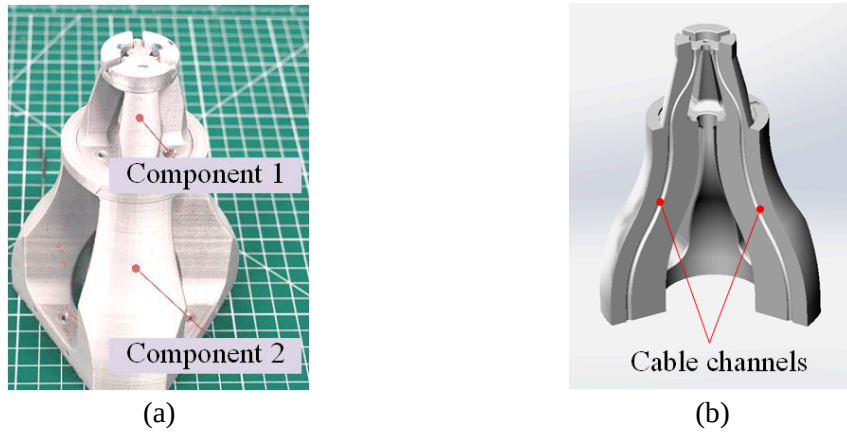


Figure 3-8 3D-printed cable adapter of Prototype Two: (a) the overall design of the cable adapter; (b) the cable channels of the adapter.

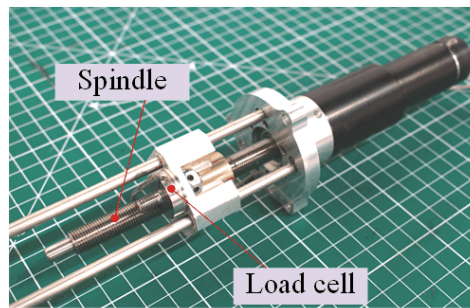


Figure 3-9 A motor module of Prototype Two, including a motor with a spindle and a load cell.

3.2 Control architecture of the continuum robots

The hierarchy of the control architecture for both prototypes can be classified into two levels: the first level is the control of the cable displacement of the motor modules, which is related to the hardware system and hence is also called as low-level control; while the second level focuses on applying various algorithms to improve the performance of continuum robots.

3.2.1 Cable displacement control

For the operation of tendon-driven continuum robots, accurate, repeatable, and deterministic cable displacement control is fundamental and significant. Figure 3-10(a) and (b) illustrate two different motor control schemes, which are deployed in Prototype

One and Two, respectively. In Prototype One, a speed controller (ESCON 70/10) is utilized to realize the internal speed loop of motor control with the velocity feedback from the motor encoder, while the outer position loop is implemented on the NI sbRIO device (sbRIO-9627) with the magnetic linear encoder providing the actual cable displacement. The speed controller receives the velocity command from the sbRIO by PWM signals. In terms of Prototype Two, a position controller (EPOS2 50/5) is applied for the motor control, which directly receives the positional commands from the host computer. The host computer sends commands to the controller via USB cable while the communication among several controllers is on CAN bus protocol.

For the low-level control of the prototypes, two proposed schemes are suitable for different application scenarios. The first scheme was applied to multi-section continuum robots where a group of motors need to be controlled with low time latency and high synchronization, such as Prototype One. As for the second scheme, the use of position controllers significantly reduces the system complexity by removing the sbRIO, which is applicable for promptly developing demonstrators of continuum robots, such as Prototype Two.

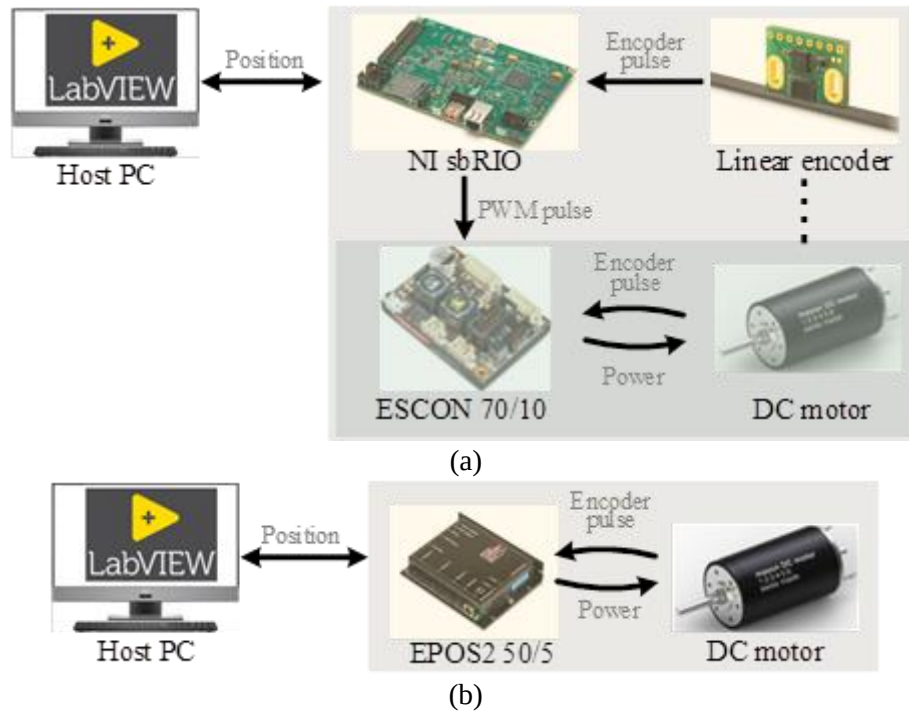


Figure 3-10 Motor control schemes with (a) speed controller and (b) position controller.

3.2.2 Manipulation in task space

The second level of the control architecture is the manipulation of continuum robots in task space (tip position/orientation), which can also be noted as the high-level control of the system. Different from the cable displacement control, which is usually independent from the main body of continuum robots and integrated into the actuation system, the manipulation control is related to the behaviour study of continuum robots. In the control block diagram of continuum robots shown in Figure 3-11, the research of the manipulation in task space corresponds to the ‘Controller’ block in the figure, where x_d and x_a denote the desired and actual tip position/orientation in task space, u is the expected cable displacements in actuation space.

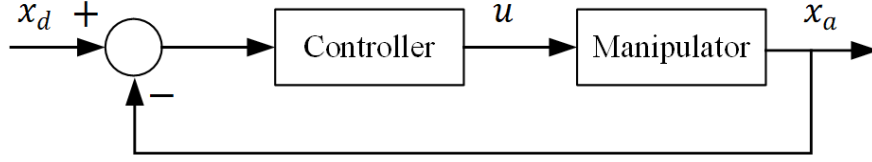


Figure 3-11 Block diagram of a close-loop control of continuum manipulators in task space.

This research of this thesis will mainly focus on the development of the controllers for the manipulation of continuum robots in task space. For the two prototypes of continuum robots, different controllers are designed according to the specific characteristics of the arms, which are presented in Chapter 5 and 6, respectively.

3.3 Measuring equipment for control and evaluation

In the continuum robots, Prototype One and Two, there are a set of sensors integrated in these systems for measuring the parameters, such as the configurations of the continuum arms and the force applied on the cables from the motors, used in the close-loop control systems developed in this thesis. Figure 3-12 illustrates the measuring equipment utilized in this research for position feedback and cable tension measurement. A vision system, consisting of four VICON cameras, is applied to provide the feedback of the tip position of each section for the robot manipulation in task space. Figure 3-12(a) shows the reflective marker and the camera used for the system deployment. The whole system works with active light source to reduce the disturbances from the ambient light. To be specific, the cameras omit infrared light to the environment, then markers reflect the light back and hence can be detected by the cameras. Each marker is made up of 5 reflective hemispheres adhered on a flat plastic board, which needs to be placed on the measuring point. In the software of the vision system, a fixed Cartesian coordinate system can be defined on each object, whose position and orientation can be streamed to the high-level control in a frequency of 50 or 100 Hz. The details of the setup of the

vision system on two prototypes are presented in Chapter 5 and 6, respectively. Further, load cells are also utilized to measure the tension of the driving cables. Figure 3-12(b) shows two components for the tension measurement of the cables. The first component is a load cell, whose base is amounted in the motor module and the other measuring head is connected to the cable (Figure 3-5 and Figure 3-9). While the second component is a force DAQ (Data acquisition), which samples the output signal of each load cell and send them to the high-level control. The details of how the load cells are integrated in the robot system are illustrated in Section 3.1.



Figure 3-12 The measuring equipment used for the close-loop control of Prototype One and Two: (a) a vision system comprised of multiple VICON cameras for visual feedback; (b) a Load cell and the DAQ (Data acquisition) for sensing the cable tension of the robots.

It is necessary to mention that this work focuses on the validation of the novel feedback controllers developed in this work. The open configuration of the experimental scenarios shown in Chapter 5 and Chapter 6 are made to comply with the application condition of the VICON system. Actually, the controllers can directly adapt to other

types of position sensing systems (etc. the EM sensors and optical fibre) for practical applications in confined space.

3.4 Chapter summary

In this chapter, the continuum robots used for evaluating the close-loop controllers developed in this thesis are illustrated in three aspects, the hardware components, the control architecture, and the equipment for measurement. Two systems, Prototype One and Two, are introduced, one is a slender multi-section continuum robot (715 mm long), which is utilised for the trials of the model-less controller developed in chapter 5, and the other one is a single section continuum manipulator (55 mm long), which is used as a test bench of the model-based controllers presented in chapter 6. Further, the architecture of the control systems is presented and divided into two levels, the low level for the control of the cable displacement and the high level for the control of the manipulation of the robot in task space (tip position/orientation). In terms of the measuring equipment, a vision system consisting of multiple VICON cameras, is deployed as the visual feedback device while load cells are used for the measurement of the cable tensions in the actuation system.

4 Kinematic modelling and analysis

Kinematics describes the geometry relationship of mechanism motion, without reference to the causes of motion (i.e., forces), which therefore can also be regarded as the geometry constraints of mechanism parameters. For tendon-driven continuum or soft robots, the conventional kinematics is about the geometry relationship among three levels, which are actuation space, configuration space and task space, respectively. The actuation space denotes cable displacements on the hardware level, while the task space corresponds to the tip position and orientation of the robot, which is usually the control target in operation. As for the configuration space, also called joint space, it depicts the geometry relationship in joint level, which is the bridge linking actuation space and task space. In this chapter, three kinematic models are derived for different design of continuum and soft robots developed in this work.

4.1 PCC-based kinematic model

The kinematic model with constant-curvature assumption is widely used for the research of continuum robots owing to its simplicity and effectiveness. When applied on multi-section continuum robots, the shape of the robot can be described as piecewise-constant-curvature (PCC) arcs. Figure 4-1 shows four types of continuum robots that are suitable for this kinematic method, including tendon-driven robot with central flexible backbone, pneumatic manipulator with parallel bellow, electrode-actuated concentric tube, and tendon-driven manipulator with air backbone.

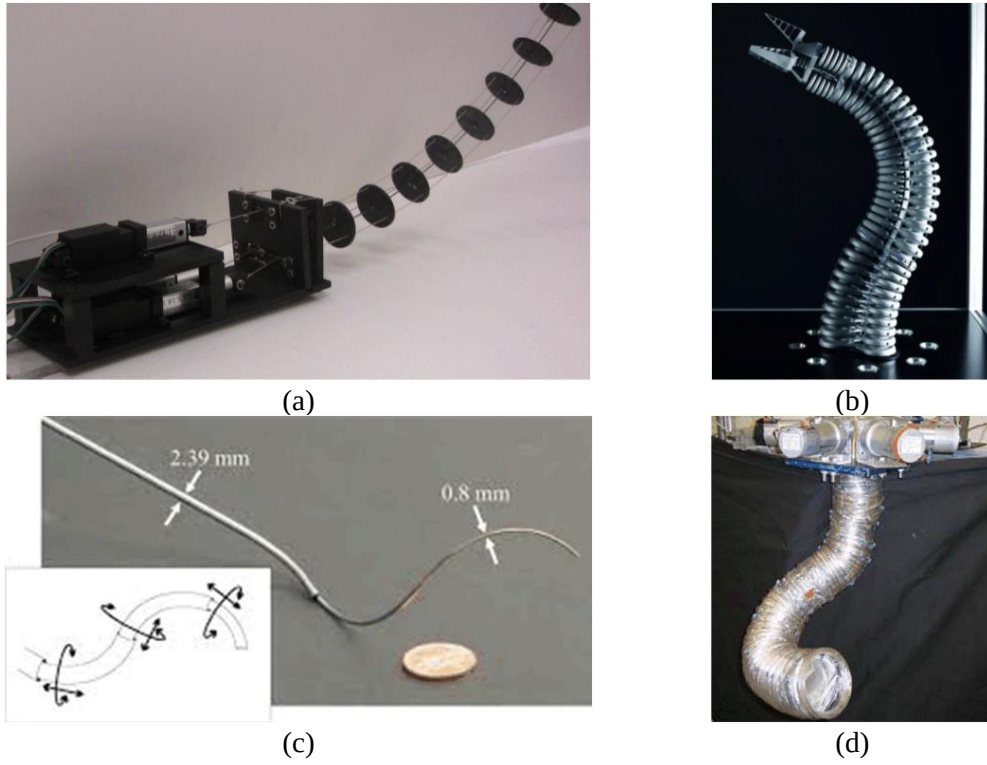


Figure 4-1 Continuum robots suitable for constant curvature assumption: (a) central flexible backbone; (b) pneumatic parallel bellow; (c) concentric-tube robot; (d) pneumatic backbone.

As shown in Figure 4-2, the kinematic mapping consists of two levels. The first level is between actuation space and configuration space while the second one is between configuration space and task space. In the configuration space, the bending angle θ , direction angle φ and the backbone length S are used to describe the shape of single section. For incompressible and inextensible continuum robots, the backbone length S is regarded to be constant.

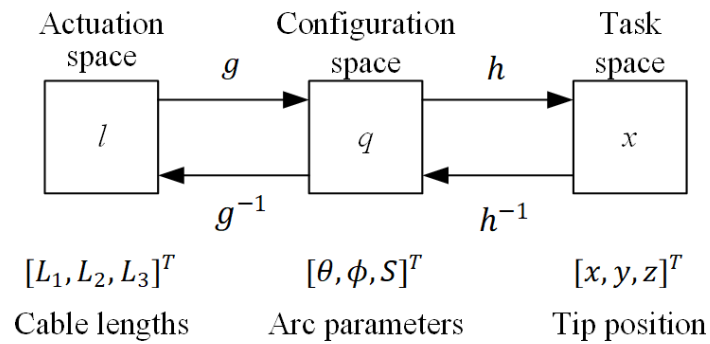


Figure 4-2 Illustration of the PCC-based kinematic mapping.

4.1.1 Cable kinematics

With the constant-curvature assumption, the cable kinematics from configuration space to actuation space can be derived. There are two standard methods for operating single section of continuum robots: 3-cables and 4-cables. Actually, three independently actuated cables are sufficient for manoeuvring single section to reach any position within the workspace. For the 4-cables scheme, it is usually applied to reduce the number of motors by controlling the cables in pairs [56]. Both cables in one pair are symmetrically distributed relative to the central backbone so that the overall cable lengths can keep approximately constant in operation. Therefore, one pair of cables can be actuated with one motor by using spool system [56]. However, the performance of such actuation scheme can be significantly affected by various factors, for instance, the assembly accuracy and cable elongations. As such, the 3-cables method is adopted for the design of the actuation system in this work.

Two situations need to be considered for the cable kinematics according to the structure of continuum robots. The first case is shown in Figure 4-3(a), where the profile of the driving cables is consistent with the shape of the robot. This situation applies to continuum robots within which continuous channels are reserved for the guidance of cables. Also, it can be observed that a coordinate system is attached at the base of the section, whose Z-axis is along the direction of the backbone. Figure 4-3(b) gives the cross-section view of the section to illustrate the cable distribution. All the cables are located on a circle in radius of r , which is slightly smaller than the dimension of the robot. The cable location is noted by phase angle $\alpha_i (i = 1, 2, 3)$, which is the angle between the X-axis and the vector from the central point to the cable hole.

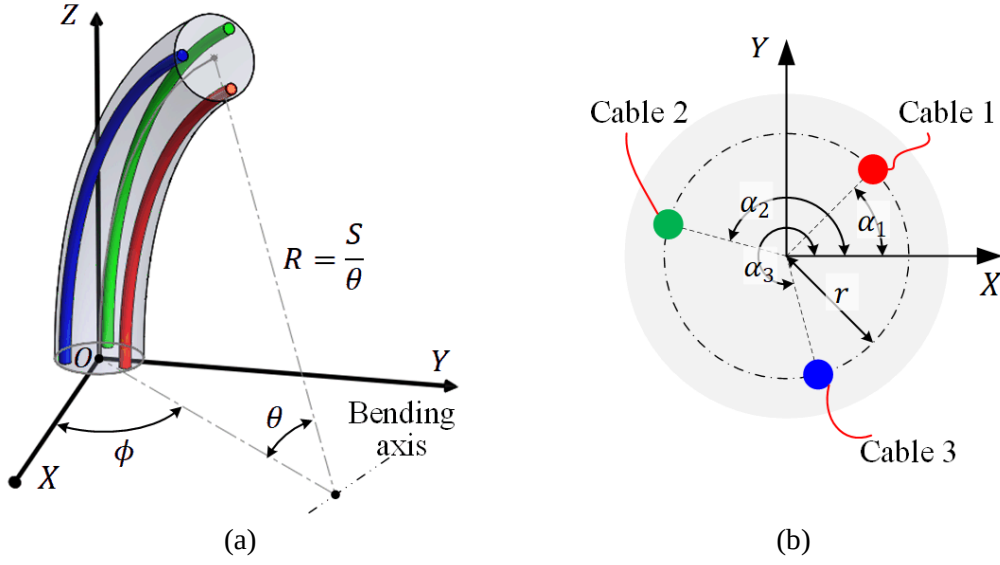


Figure 4-3 Case I of the cable kinematics: (a) illustrative 3D profile; (b) cross-section view of the robot.

As shown in Figure 4-3(a), the robot and the cables are all assumed to bend around the bending axis, the expression of the cable length $L_1 \sim L_3$ can then be computed as below:

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} S \\ S \\ S \end{bmatrix} - r\theta \cdot \begin{bmatrix} \cos(\alpha_1 - \varphi) \\ \cos(\alpha_2 - \varphi) \\ \cos(\alpha_3 - \varphi) \end{bmatrix} \quad (4.1)$$

Figure 4-4 illustrates the second case, where the shape of the cable is segmented lines within single section. As Figure 4-4(a) demonstrates, disks are evenly deployed on the flexible backbone to guide the cables. In the figure, the whole section consists of n ($= 5$) segments, whose bending angle θ_{seg} equals to θ/n .

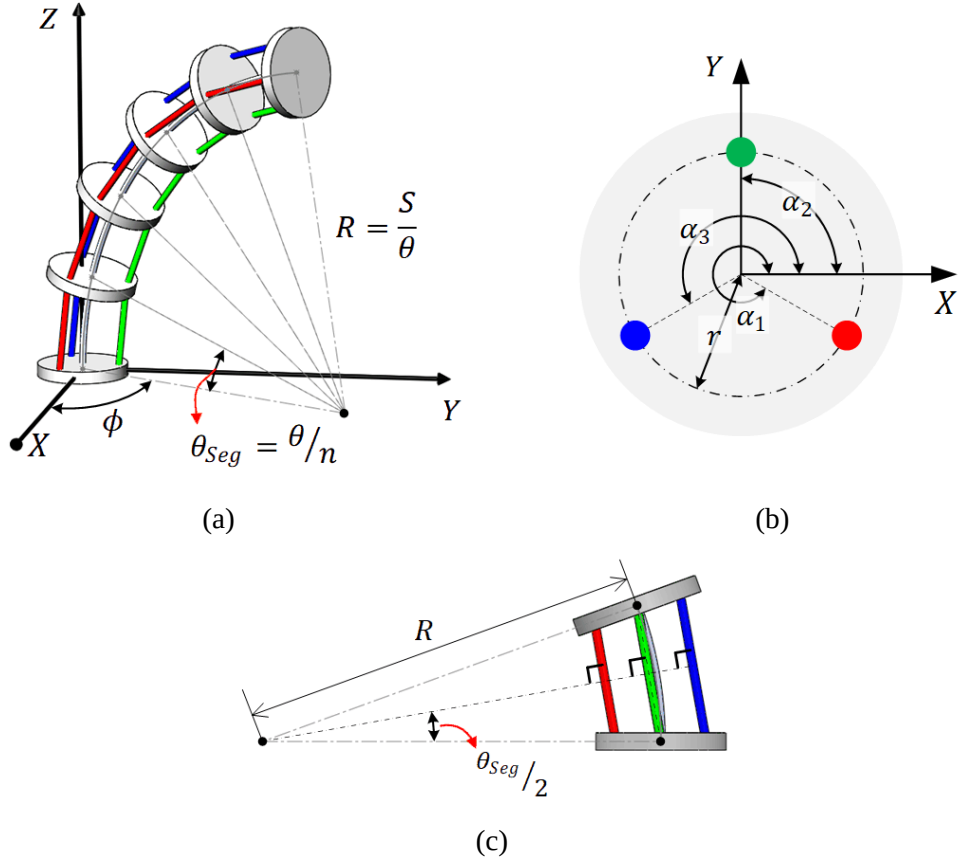


Figure 4-4 Case II of the cable kinematics: (a) illustrative 3D profile of single section; (b) cross-section view of the disk; (c) side view of single segment.

Figure 4-4(c) gives the side view of single segment, within which the cable shape is straight in tension state. Therefore, the cable length for the section depicted can be derived as:

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = 2n \cdot \sin \frac{\theta}{2n} \cdot \left(\frac{S}{\theta} - r \cdot \begin{bmatrix} \cos(\alpha_1 - \varphi) \\ \cos(\alpha_2 - \varphi) \\ \cos(\alpha_3 - \varphi) \end{bmatrix} \right) \quad (4.2)$$

It is necessary to mention that Eq. (4.2) ignores the cable length within the disks for the benefit of simplicity. Hence, the thickness of the disk should be reduced for the robot construction of this type to guarantee the modelling accuracy.

4.1.2 Geometric kinematics

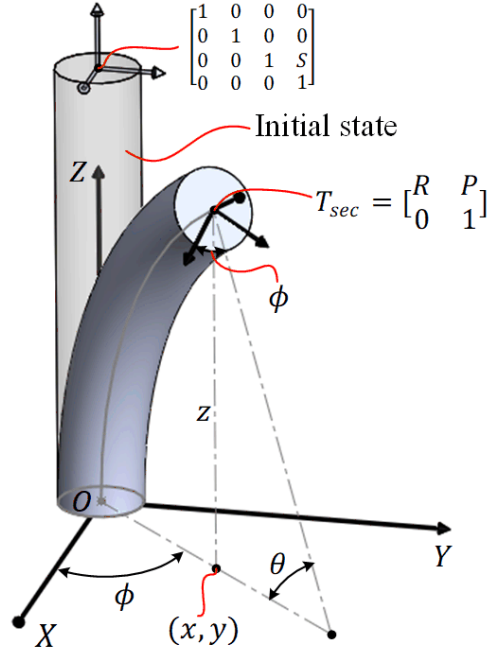


Figure 4-5 Schematic of the geometric kinematics with the constant-curvature assumption.

Cable kinematics is about the modelling between configuration space and actuation space, whereas the geometric kinematics deals with the mapping between configuration space and task space. Figure 4-5 illustrates the schematic of the constant-curvature-based geometric kinematics for single section. Two coordinate systems are defined at the section base and tip, respectively. The shape of single section can be fully represented by the backbone length S , the bending angle θ , and the direction angle ϕ .

From these parameters, the tip position can be derived as:

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{S}{\theta} \cdot (1 - \cos \theta) \cdot \cos \phi \\ \frac{S}{\theta} \cdot (1 - \cos \theta) \cdot \sin \phi \\ \frac{S}{\theta} \cdot \sin \theta \end{bmatrix} \quad (4.3)$$

Except for the constant-curvature assumption, it is also assumed that there is no axial distortion along the robot. As such the rotation matrix from the base coordinate frame to the tip is

$$R_{3 \times 3} = Rot_z(\phi)Rot_y(\theta)Rot_z(-\phi)$$

$$= \begin{bmatrix} c^2\phi \cdot c\theta + s^2\phi & c\phi \cdot s\phi \cdot (c\theta - 1) & c\phi \cdot s\theta \\ c\phi \cdot s\phi \cdot (c\theta - 1) & s^2\phi \cdot c\theta + c^2\phi & s\phi \cdot s\theta \\ -c\phi \cdot s\theta & -s\phi \cdot s\theta & c\theta \end{bmatrix} \quad (4.4)$$

where $c\theta = \cos \theta$, $s\theta = \sin \theta$, $c\phi = \cos \phi$, $s\phi = \sin \phi$.

Therefore, the transformation matrix for single section can be expressed as

$$T_{sec} = \begin{bmatrix} R_{3 \times 3} & P \\ 0 & 1 \end{bmatrix} \quad (4.5)$$

4.2 Geometry-based kinematic model

Even though the PCC assumption provides a general approach for the kinematic modeling of continuum robots, more accurate kinematics can be established for some continuum robots with specific geometry. In the following section, the geometry-based kinematic modelling will be performed on the two prototypes of continuum robots introduced in Chapter 3.

4.2.1 Dual-revolute design

The dual-revolute joint is a novel type of design for the development of slender continuum robots, which is applied on the tip sections of Prototype 1 in Chapter 3. Specifically, the semi-cylindrical groove and bulge on two adjacent disks make up one revolute joint. One pair of short NiTi rods passes through the through holes in the revolute joints and are locked by screws so that the whole structure can keep compact.

Further, the spatial bending is achieved by arranging the revolute joints orthogonally in sequence.

- **Cable kinematics**

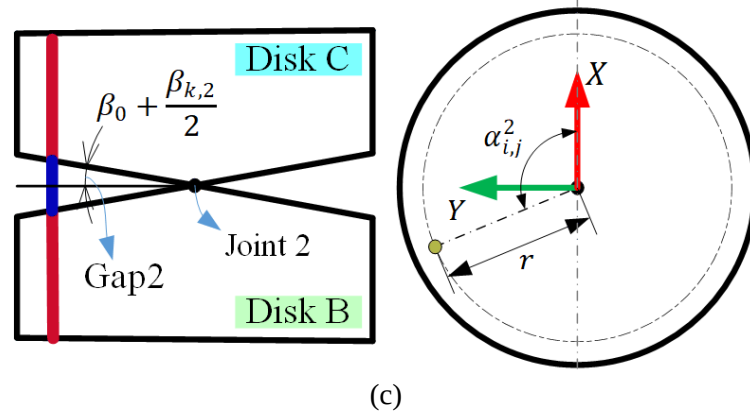
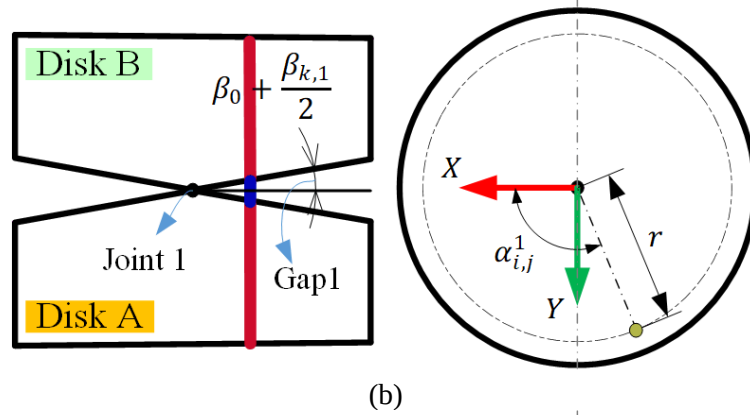
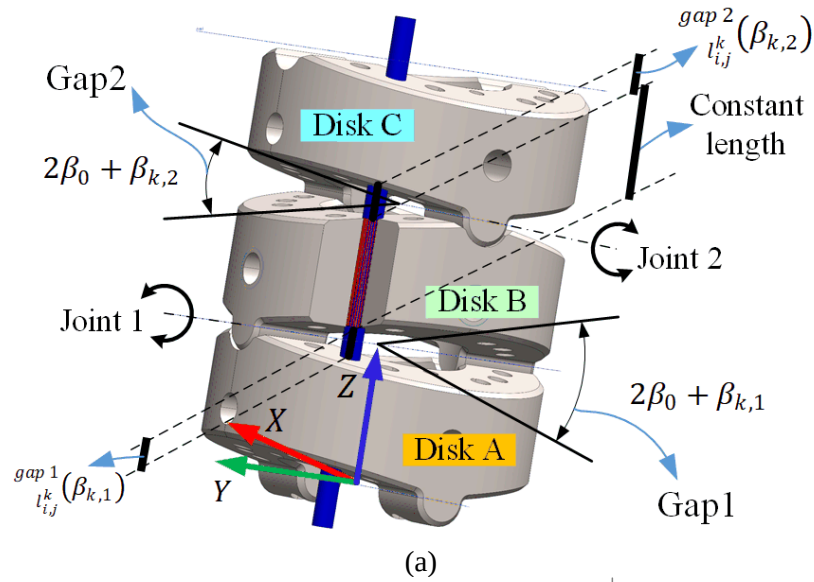


Figure 4-6 Schematic of the cable kinematics with dual-revolute design: (a) parameter annotation within single segment; (b) side view and top view of joint 1 between disk A and B; (c) side view and top view of joint 2 between disk B and C.

a) *Kinematics formulation*

Firstly, we derive the cable displacement change for single segment, whose explicit structure is given in Figure 4-6(a). Since the length of cable within disks keep constant, here we only consider the length of those between gaps, which can be written as:

$$l_{i,j}^k(\beta_{k,1}, \beta_{k,2}) = {}^{gap1}_{l_{i,j}^k}(\beta_{k,1}) + {}^{gap2}_{l_{i,j}^k}(\beta_{k,2}) \quad (4.6)$$

where i is the section index, k, j denote the segment index and cable index within single section. $\beta_{k,1}, \beta_{k,2}$ are the angle variations of Gap 1 and Gap 2 from the initial angle of both gaps β_0 .

The explicit expressions of the cable lengths in Gap 1 and Gap 2, ${}^{gap1}_{l_{i,j}^k}(\beta_{k,1})$ and ${}^{gap2}_{l_{i,j}^k}(\beta_{k,2})$, are as below:

$$\begin{aligned} {}^{gap1}_{l_{i,j}^k}(\beta_{k,1}) &= \frac{2r \sin \alpha_{i,j}^1}{\cos \beta_0} \sin \left(\beta_0 + \frac{\beta_{k,1}}{2} \right) \\ {}^{gap2}_{l_{i,j}^k}(\beta_{k,2}) &= \frac{2r \sin \alpha_{i,j}^2}{\cos \beta_0} \sin \left(\beta_0 + \frac{\beta_{k,2}}{2} \right) \end{aligned} \quad (4.7)$$

where r is the radius of the circle where the cable holes locate, $(\alpha_{i,j}^1, \alpha_{i,j}^2)$ are single cable's phase angles in Disk A and Disk B.

The expression of cable length variations is given in Eq.(4.8) when the robot moves from the straight shape to the desired state:

$$\Delta l_{i,j}^k = l_{i,j}^k(\beta_{k,1}, \beta_{k,2}) - l_{i,j}^k(0,0) \quad (4.8)$$

Since $(\frac{\beta_{k,1}}{2}, \frac{\beta_{k,2}}{2}) \in [-5^\circ, 5^\circ]$, the following approximations are applied to Eq.(4.7):

$$\sin(\frac{\beta_{k,1}}{2}) \approx \frac{\beta_{k,1}}{2}, \sin(\frac{\beta_{k,2}}{2}) \approx \frac{\beta_{k,2}}{2}, \cos(\frac{\beta_{k,1}}{2}) \approx 1, \cos(\frac{\beta_{k,2}}{2}) \approx 1$$

Thus, Eq.(4.8) can be simplified as:

$$\Delta l_{i,j}^k = c_{i,j}^1 \beta_{k,1} + c_{i,j}^2 \beta_{k,2}, (c_{i,j}^1 = r \sin \alpha_{i,j}^1, c_{i,j}^2 = r \sin \alpha_{i,j}^2) \quad (4.9)$$

Here the constant-curvature assumption is still applied, which means the segments within single section are assumed to bend equally. Hence, the total cable displacements in a single section with five segments are:

$$\Delta L_{i,j}^k = 5 \Delta l_{i,j}^k \quad (4.10)$$

Substituting Eq.(4.9) into Eq.(4.10), the matrix format of the equation can be written as below:

$$\Delta \mathbf{L}_i^k = \mathbf{C}_i^k \boldsymbol{\beta}_k \quad (4.11)$$

where $\Delta \mathbf{L}_i^k = [\Delta L_{i,1}^k \quad \Delta L_{i,2}^k \quad \Delta L_{i,3}^k]^T$, $\mathbf{C}_i^k = 5 \begin{bmatrix} c_{i,1}^1 & c_{i,1}^2 \\ c_{i,2}^1 & c_{i,2}^2 \\ c_{i,3}^1 & c_{i,3}^2 \end{bmatrix}$ and $\boldsymbol{\beta}_k = [\beta_{k,1} \quad \beta_{k,2}]^T$.

Finally, it comes to the calculation of the overall length variations of the driven cables for section i , which can be written as:

$$\Delta \mathbf{L}_i = \sum_{k=1}^i \Delta \mathbf{L}_i^k = \sum_{k=1}^i \mathbf{C}_i^k \boldsymbol{\beta}_k \quad (4.12)$$

b) Range of cable displacements for single section

Since the numerical relationship is obtained in Eq. (4.11) from the joint parameters to cable displacements, the theoretical variation range of displacements for all the cables can be calculated within single section, which is essential for the design of the actuation module. Figure 4-7 shows the calculation results of selected cables when the joint parameters vary within $[-10^\circ, 10^\circ]$, which illustrates that the maximum cable displacement is below 5 mm.

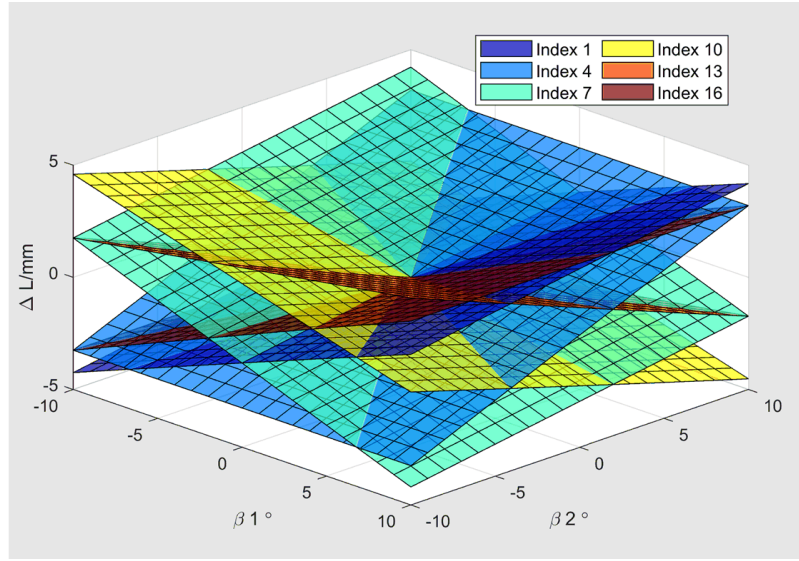


Figure 4-7 Theoretical calculation of cable displacements with the joint parameters $[\beta_1, \beta_2]$ varying in range of $[-10^\circ, 10^\circ]$. The index denotes the cable distributions on the disk.

- **Geometric kinematics**

Figure 4-8(a) shows the definition of all local coordinate systems attached on a single segment. In the figure, two coordinate systems (base and tip frames) are defined at the bottom and up surfaces of each disk.

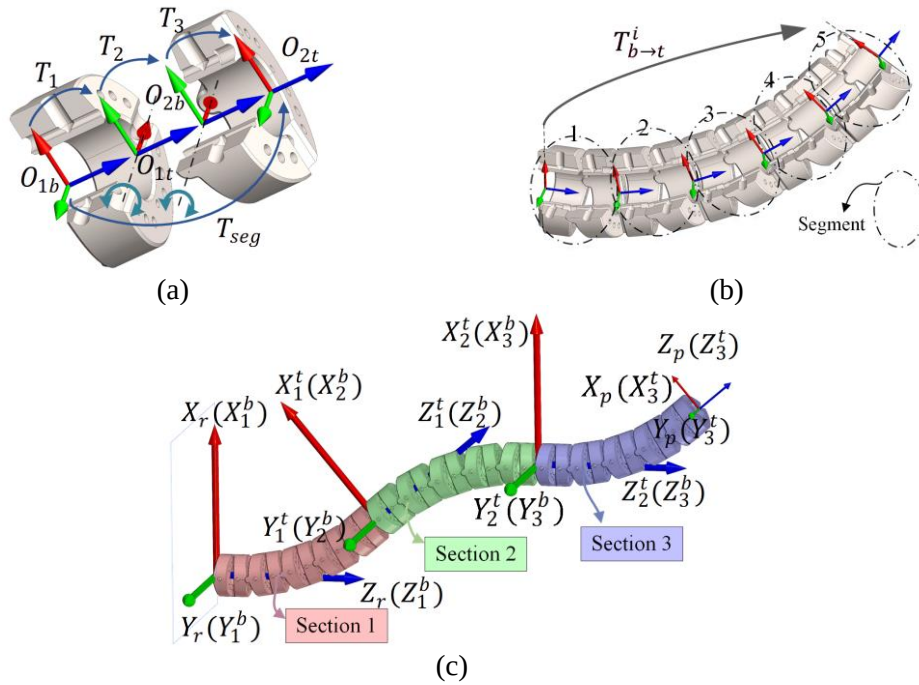


Figure 4-8 Definition of coordinate systems for (a) single segment; (b) single section; (c) the three-section continuum manipulator. The axes in red, green and blue are X, Y and Z axes, respectively.

The whole transformation process for a single segment is given by:

$$XYZ_{1b} \xrightarrow{T_1} XYZ_{1t} \xrightarrow{T_2} XYZ_{2b} \xrightarrow{T_3} XYZ_{2t}$$

Then, the HTM (homogeneous transformation matrix) of a single segment can be written as below:

$$T_{seg} = T_1 T_2 T_3 \quad (4.13)$$

Since every single section is made up of five serially linked segments which are assumed to bend equally, the local HTM for section i can be derived as:

$$T_{sec}^i = T_{b \rightarrow t}^i = T_{seg}^5 (i = 1, 2, 3) \quad (4.14)$$

Similarly, the global HTM could be obtained for each section in Figure 4-8(c) relative to the world frame:

$$T^j = \prod_{i=1}^j T_{sec}^i (j \in [1, 2, 3]) \quad (4.15)$$

4.2.2 Twin-pivot design

Twin-pivot design is another type of continuum robot to be considered in this research, which will be utilized for the development of the novel hybrid controller on the test bench of Prototype 2. Similar to the dual-revolute design, the two-DoF spatial bending capability of the twin-pivot design is implemented by deploying two bending planes orthogonally. The only difference is that, in this design, the bending joint is a pair of antagonistic NiTi rods. In this model, the bending profile of the NiTi rod is assumed to be circular arc with constant arc length, which is the basis of the geometry-based kinematics.

- **Cable kinematics**

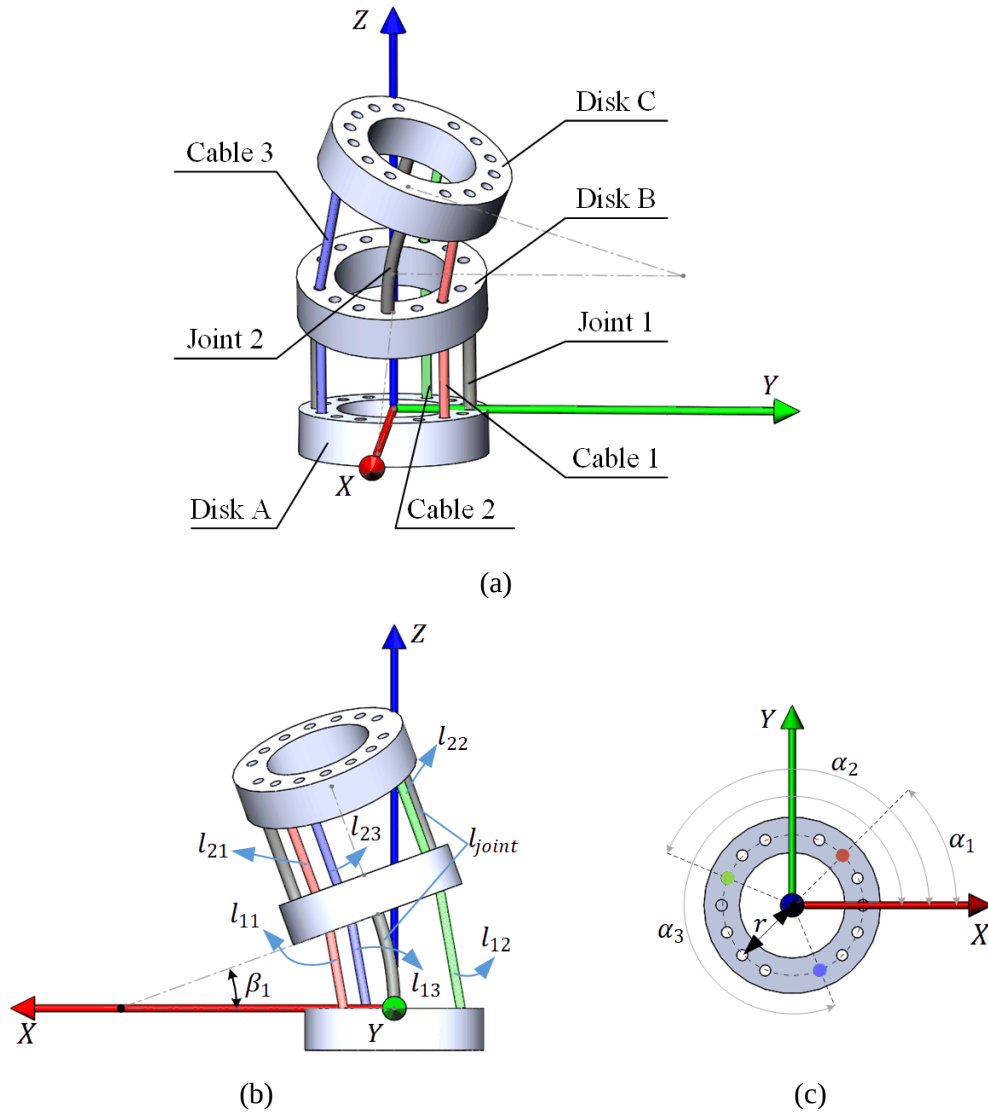


Figure 4-9 Schematic of the cable kinematics with twin-pivot design: (a) illustration of single segment; (b) parameter notation within single segment; (c) cable distribution on the base disk.

a) Kinematics formulation

As each section consists of multiple identical segments in serial, the kinematic formulation of the twin-pivot construction can also be simplified based on the analysis of single segment illustrated in Figure 4-9(a). A local coordinate frame is defined on the upper surface of the base disk within single segment, where the origin locates at the centre of the disk and the Z axis is along the length direction.

Since the length of the cable in the disk is regarded as a constant, only the cable length in joint 1 and 2 is considered when calculating the cable length within single segment:

$$l_j(\beta_1, \beta_2) = l_j^{joint1}(\beta_1) + l_j^{joint2}(\beta_2) \quad (4.16)$$

The expressions of cable length within two joints are given separately as below:

$$\begin{aligned} l_j^{joint1}(\beta_1) &= l_{1j} = 2 \sin \frac{\beta_1}{2} \cdot \left(\frac{l_{joint}}{\beta_1} - r \cos \alpha_j \right) \\ l_j^{joint2}(\beta_2) &= l_{2j} = 2 \sin \frac{\beta_2}{2} \cdot \left(\frac{l_{joint}}{\beta_2} - r \cos \alpha_j \right) \end{aligned} \quad (4.17)$$

where l_{joint} denotes the length of the NiTi rod, r is the radius of the circle where the cable holes distribute and α_j is the phase angle showing the exact location of cable j .

In the actual system, the pose of the segment is usually initialized to be straight(β_1, β_2) = (0,0), where the cable lengths $l_j(j = 1,2,3)$ are equal to the joint length l_{joint} . Hence, the cable displacement $\Delta l_j(\beta_1, \beta_2)$ from the initial position $\Delta l_j(0,0)$ can be computed as:

$$\Delta l_j(\beta_1, \beta_2) = l_j(\beta_1, \beta_2) - 2l_{joint} \quad (4.18)$$

Therefore, the cable displacement within a single section consisting of eight segments (assumed to bend equally) can be obtained:

$$\Delta L_j(\beta_1, \beta_2) = 8\Delta l_j(\beta_1, \beta_2) \quad (4.19)$$

b) Range of cable displacements for single section

To estimate the maximum cable displacement for single section, the numerical computation is performed for all the cables when the joint parameters (β_1, β_2) vary in range of $[-10^\circ, 10^\circ]$. The results are plotted in Figure 4-10, showing that the maximum cable displacement is below 12 mm.

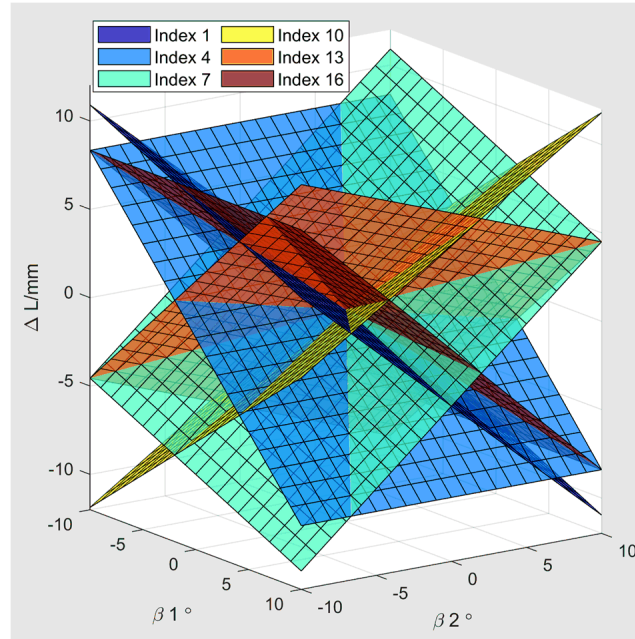


Figure 4-10 Theoretical calculation of cable displacements with the joint parameters $[\beta_1, \beta_2]$ varying in range of $[-10^\circ, 10^\circ]$. The index number denotes the cable distributions on the disk.

- Geometric kinematics**

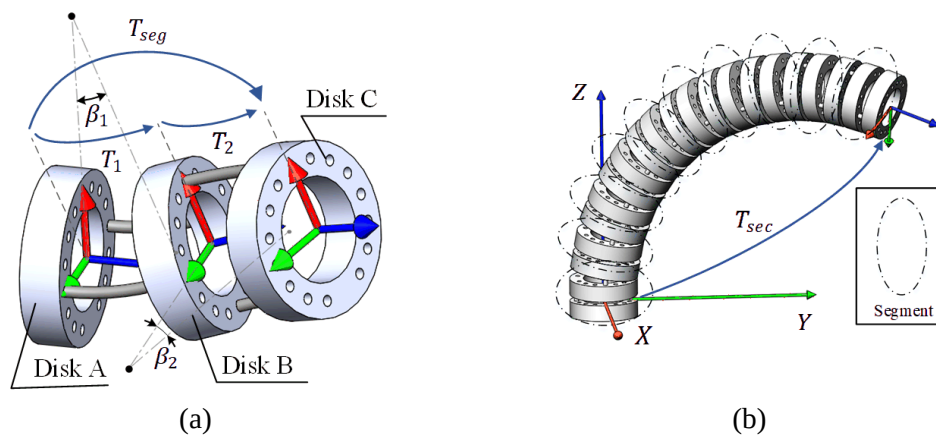


Figure 4-11 Definition of coordinate systems for (a) single segment; (b) single section. The axes in red, green and blue denote X, Y and Z axes, respectively.

As shown in Figure 4-11(a), the transformation matrix of single segment T_{seg} can be decomposed into two sequential transformations between two adjacent disks:

$$T_{seg} = T_1 T_2 \quad (4.20)$$

where the expressions of T_1 and T_2 are given as below:

$$T_1 = \begin{bmatrix} \cos \beta_1 & 0 & \sin \beta_1 & \frac{l_{joint}}{\beta_1} (1 - \cos \beta_1) \\ 0 & 1 & 0 & 0 \\ -\sin \beta_1 & 0 & \cos \beta_1 & \frac{l_{joint}}{\beta_1} \cdot \sin \beta_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \beta_2 & \sin \beta_2 & \frac{l_{joint}}{\beta_2} (1 - \cos \beta_2) \\ 0 & -\sin \beta_2 & \cos \beta_2 & \frac{l_{joint}}{\beta_2} \cdot \sin \beta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that the translation vectors of the above equations encounter singularity when the joint parameters β_1 and β_2 are equal to 0, which can be solved by computing their limits. Moreover, when implementing the kinematics numerically, the singularity issue can be circumvented by performing Taylor series expansion, and the precision can be guaranteed by choosing proper truncation order.

Therefore, the HTM of single section with 8 equally bending segments can be written as:

$$T_{sec} = T_{seg}^8 \quad (4.21)$$

4.3 Kinematic analysis and simulation

In this section, the kinematic analysis will be performed based on the kinematic models developed in 4.1 and 4.2, which mainly includes two parts. The first part is about the workspace comparison of single section between the PCC-based and the geometry-

based kinematics to illustrate the theoretical deviations of different kinematic models. The other part comes to the general solution to the inverse kinematics for both the PCC-based and geometry-based models.

4.3.1 Workspace analysis

Since the kinematics of continuum robots have been modelled with different approaches, it is necessary to analyse the difference between them. To be specific, comparison can be made between the workspace of single section obtained by the PCC assumption and the geometry-based approaches, including the translational and orientation offset of the tip position.

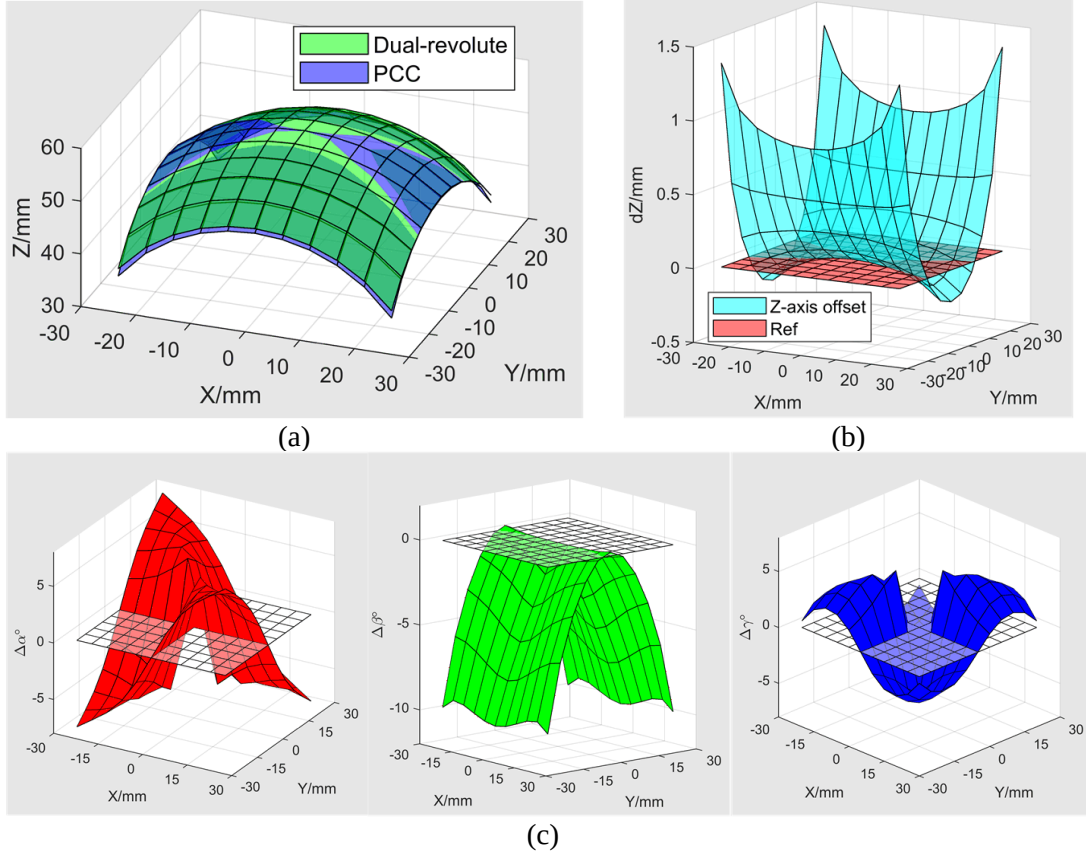


Figure 4-12 Workspace analysis of the dual-revolute design: (a) comparison with the PCC-based model; (b) offset in the Z axis; (c) offset of the normal vector represented in Euler angles (α, β, γ).

The comparison results are illustrated in Figure 4-12 between the kinematics of the dual-revolute design and the PCC assumption. Figure 4-12(a) plots the tip position of

single section (length: 55 mm) in range of $\{(x, y) | x, y \in [-25 \text{ mm}, 25 \text{ mm}]\}$ computed by two kinematic models, while the offset along the Z-axis dZ is given in Figure 4-12 (b). It can be seen that the overall distribution of the offset dZ is highly nonlinear in the XY plane, which increases drastically when the tip position deviates from the X axis and reaches the peak value 1.38 mm at four corner points $(\pm 25 \text{ mm}, \pm 25 \text{ mm})$. Further, to quantize the orientation deviation between two workspaces, the rotation matrix of the transformation matrix, R , is expressed by the Euler angles (α, β, γ) [143] with the equation $R = Rot_x(\alpha)Rot_y(\beta)Rot_z(\gamma)$. As shown in Figure 4-12 (c), the maximum absolute angular deviation $\Delta\alpha, \Delta\beta, \Delta\gamma$ are 7.65° , 9.83° and 5.09° , respectively.

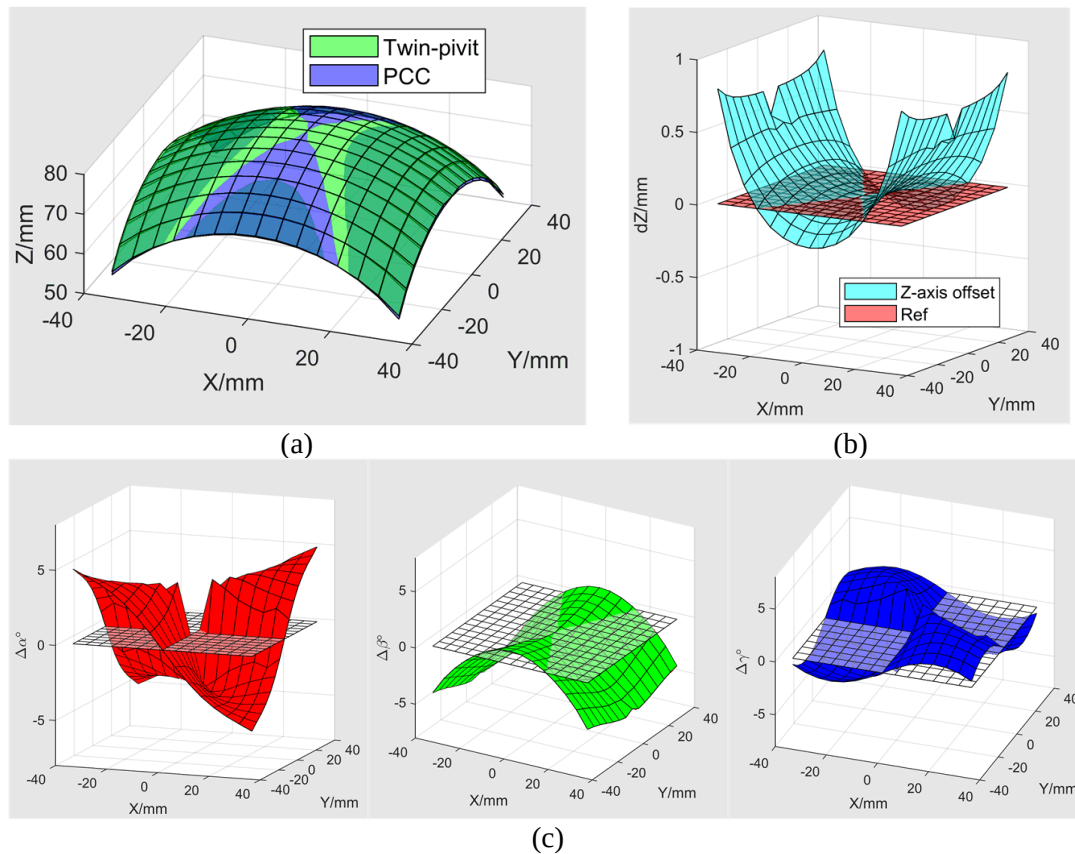


Figure 4-13 Workspace analysis of the twin-pivot design: (a) comparison with the PCC-based model; (b) offset in the Z axis; (c) offset of the normal vector represented in Euler angles (α, β, γ) .

Likewise, the twin-pivot design is also compared to the PCC assumption for the workspace comparison of single section in length of 80 mm in range of $\{(x, y) | x, y \in [-35 \text{ mm}, 35 \text{ mm}]\}$. The results are illustrated in Figure 4-13, from which it can be seen that the maximum deviations $dZ, \Delta\alpha, \Delta\beta, \Delta\gamma$ are 0.79 mm, 4.98°, 5.53° and 3.39°, respectively.

Therefore, the model deviation between the PCC assumption and the geometry-based kinematic models is validated theoretically. In addition, it can also be observed that all the deviation indices ($dZ, \Delta\alpha, \Delta\beta, \Delta\gamma$) of the twin-pivot design (0.79 mm, 4.98°, 5.53°, 3.39°) are smaller than those of the dual-revolute design (1.38 mm, 7.65°, 9.83°, 5.09°), though the section length of the former structure (80 mm) is longer than the latter one (55 mm). In other words, the kinematic deviation of the twin-pivot design and the PCC assumption is smaller than that of dual-revolute design and the PCC assumption.

4.3.2 General solution for inverse kinematics

Similar to the kinematic modelling of industrial rigid-link robots, the formulation of the forward kinematics of continuum robots is straightforward, either based on the PCC assumption or the geometric structures, while the analytical solution for the inverse kinematics usually does not exist owing to its high complicity and nonlinearity. As such, a general solution framework is developed in the following section, which can adapt to various designs of continuum robots for different inverse kinematic problems.

- ***Optimization approach***

Considering the fact that the expression of the forward kinematics of continuum robots is easy to obtain, the inverse kinematic problem can then be turned into a general numerical optimization problem that can be solved by the Optimization Toolbox in Matlab [144].

Owing to the high nonlinearity of the forward kinematics, the nonlinear solver, *fmincon* [145], is chosen for the optimization problem. In order to apply the solver correctly, the problem should be specified as below:

$$\min f(x) \text{ such that } \begin{cases} c(x) \leq 0 \\ ceq(x) = 0 \\ A \cdot x \leq b \\ Aeq \cdot x = beq \\ lb \leq x \leq ub, \end{cases} \quad (4.22)$$

where $f(x)$ is the subject function that returns a scalar, $c(x)$ and $ceq(x)$ are functions that return vectors, A and Aeq are matrices, b and beq are vectors, and lb, ub are the lower bound and upper bound of the vector variable x . $f(x)$, $c(x)$ and $ceq(x)$ can be nonlinear functions.

Since there exists no equality or inequality constraints for the inverse kinematics, the actual syntax is as shown as below:

$$x = fmincon(f, x_0, [], [], [], [], lb, ub, [])$$

where A, b, Aeq, beq and $nonlcon = [c(x), ceq(x)]$ are set to be null, x_0 is the pre-specified initial point for the vector variable x .

- **Simulation on single section**

With regard to the control of single-section continuum robots, the control target can be either position or orientation of the section tip. Therefore, the inverse kinematics of single section also includes two aspects, namely, the tip position control and the tip orientation control, whose schematic diagrams are illustrated in Figure 4-14(a) and (b), respectively. Since each section has two DoFs, only x, y coordinates of the section tip P are independent from each other while z can be regarded as a function of x, y . Therefore, the coordinate pair (x, y) is necessary and sufficient for the inverse kinematics of position control. In terms of the inverse kinematics of orientation control,

the target point P' is required as the input, and the goal is to make the norm vector $\vec{R_z}$ aligned with the vector $\vec{PP'}$.

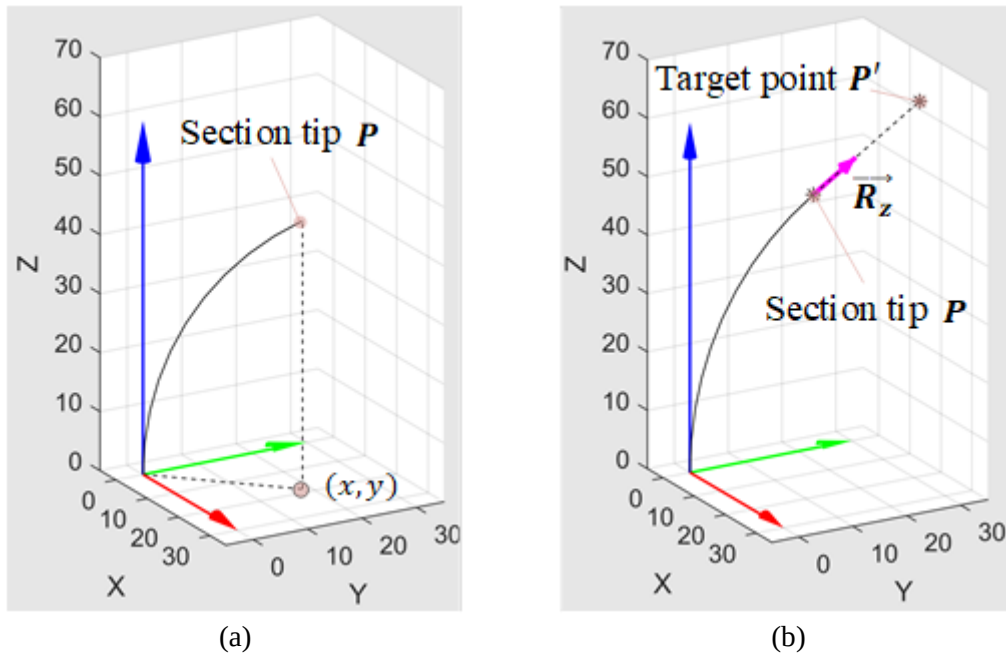


Figure 4-14 Schematic diagrams of the inverse kinematics on single section in aspects of (a) tip position control and (b) tip orientation control.

Based on the above analysis, the objective optimization function for the tip position control $f_p(\beta)$ shall be the 2D Euclidean distance between the actual tip position $P(x_p, y_p, z_p)$ and the target position (x, y) in XY plane:

$$f_p(\beta) = \sqrt{(x_p - x)^2 + (y_p - y)^2} \quad (4.23)$$

where $\beta = [\beta_1, \beta_2]^T$ is the vector variable of the joint parameters. The actual tip position $P(x_p, y_p, z_p)$ is computed with the geometric kinematics developed in 4.2.1(the dual-revolute design) and 4.2.2(the twin-pivot design), which is a function of the joint parameters β .

As shown in Figure 4-15, a triangular path is chosen in the XY plane to validate the proposed optimal approach for tip position control of dual-revolute (a) and twin-pivot

(b) design. It can be seen that the maximum value of the objective function $f_p(\beta)$ is about $0.8 \times 10^{-3} \text{ mm}$, which is less than the set tolerance (0.01 mm).

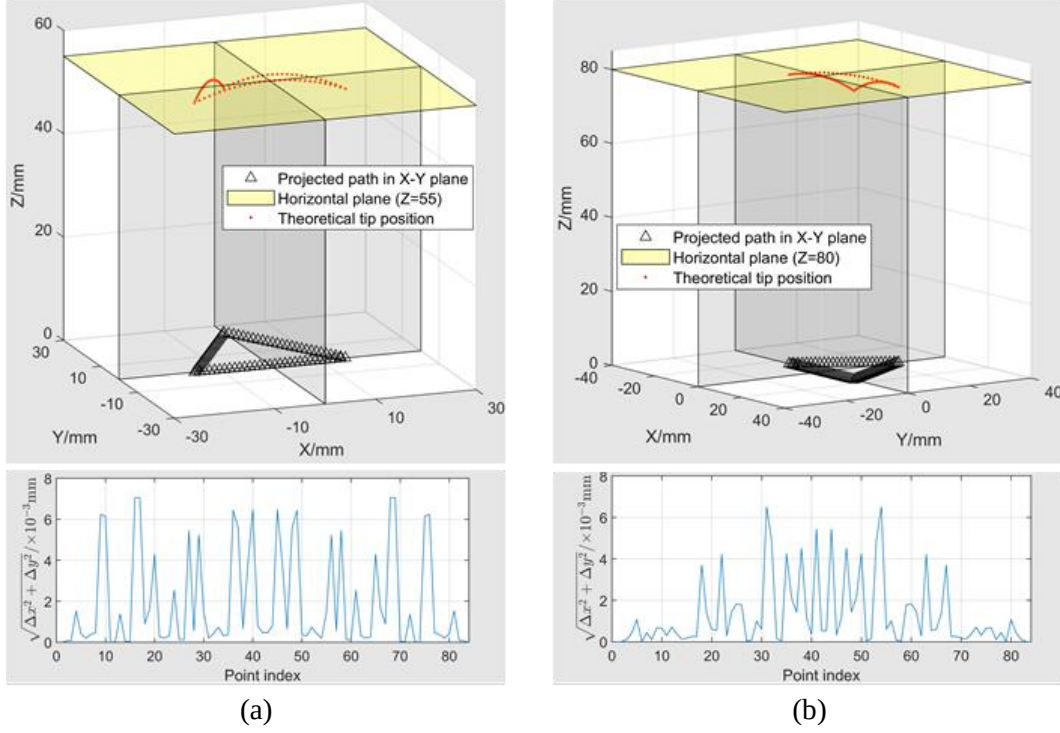


Figure 4-15 Inverse kinematics for tip position control with (a) the dual-revolute design and (b) the twin-pivot design. The projection of the test trajectory (red dotted line) on the XY plane is a triangle (black line with triangular symbol).

Likewise, the objective function for the tip orientation control can be derived as below:

$$f_o(\beta) = \arccos(\vec{R}_z \cdot \frac{\vec{PP'}}{|\vec{PP'}|}) \quad (4.24)$$

where $f_o(\beta)$ is the angle between vector $\vec{R}_z$ and vector $\vec{PP'}$, which reaches the minimum value 0° when the section orientation $\vec{R}_z$ points to the target point P' . The results of the numerical validation are given in Figure 4-16 utilizing a triangle as the target path for the tip orientation control, which can keep the maximum angular deviation below 0.025° for both the dual-revolute (a) and twin-pivot (b) design.

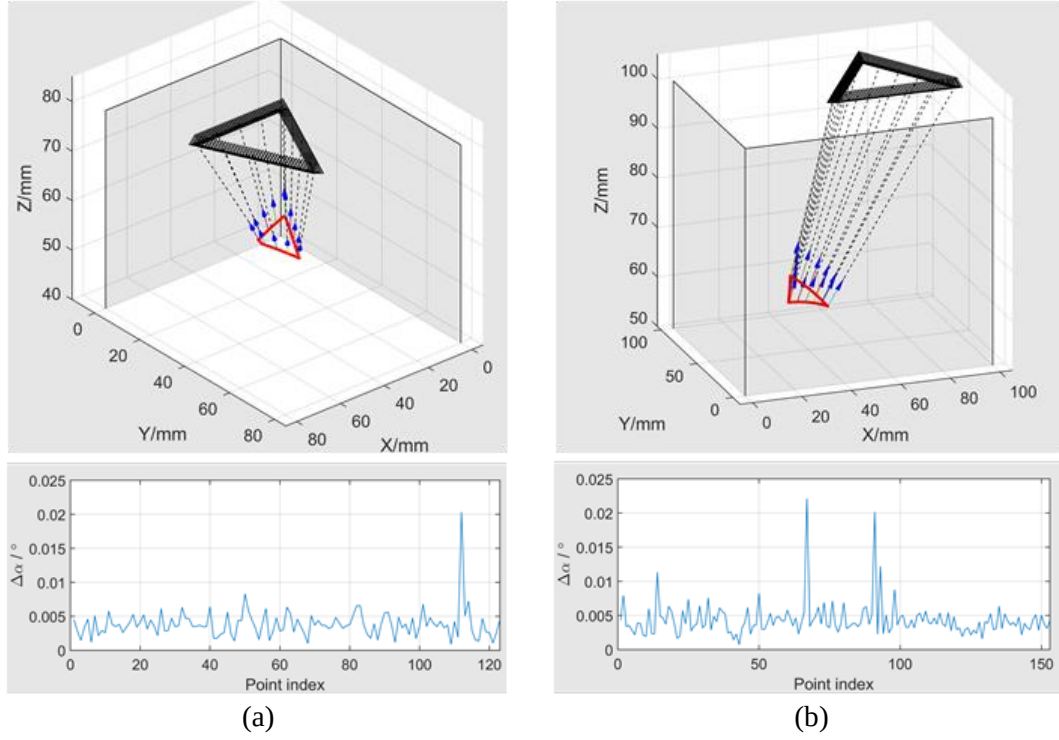


Figure 4-16 Inverse kinematics for tip orientation control with (a) the dual-revolute design and (b) the twin-pivot design.

Therefore, the efficacy of the optimal approach has been demonstrated for the inverse kinematics of single section, which can reach high accuracy for both the tip position (< 0.01 mm) and orientation ($< 0.025^\circ$) control.

- **Simulation on three sections**

In terms of the inverse kinematics for three sections, it becomes more complicated since the total number of joint variables increase to $3 \times 2 = 6$. Theoretically, there exists multiple possible solutions given a desired tip position for a three-section continuum robot. To simplify the process of analysis, here only the PCC-based model is considered to solve the inverse kinematics of multiple sections. Both the position and orientation constraints can be integrated into the objective function:

$$f(q) = e_P + k e_{R_z} \quad (4.25)$$

where e_P denotes the positional error, e_{R_Z} represents the angular error of the norm vector R_Z . The coefficient k is a weighting factor.

e_P and e_{R_Z} can be computed as below:

$$\begin{aligned} e_P &= |PP'| \\ e_{R_Z} &= \angle \langle \overrightarrow{R_Z}, \overrightarrow{R'_Z} \rangle \end{aligned} \tag{4.26}$$

where P and $\overrightarrow{R_Z}$ are the desired tip position and orientation, P' and $\overrightarrow{R'_Z}$ are the estimated tip position and orientation.

Usually, an initial configuration $q^0 = (\theta_1^0, \varphi_1^0, \theta_2^0, \varphi_2^0, \theta_3^0, \varphi_3^0)$ is given as the start point to ensure a feasible pair of $(P, \overrightarrow{R_Z})$, while the desired tip position P' is set by a position shift ΔP relative to P , namely, $P' = P + \Delta P$. In addition, P' and $\overrightarrow{R'_Z}$ are both functions of the estimated joint variables $q = (\theta_1, \varphi_1, \theta_2, \varphi_2, \theta_3, \varphi_3)$. Hence, the syntax of calling function *fmincon* can be written as:

$$q = \text{fmincon}(f, q^0, [], [], [], [], lb, ub, [])$$

where $lb, ub \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

As shown in Figure 4-17, the simulation is conducted to validate the developed solution for inverse kinematics. The initial configuration q^0 is $(60^\circ, 25^\circ, -45^\circ, 25^\circ, 45^\circ, 60^\circ)$, and the position shift ΔP varies to manoeuvre the desired tip position along X, Y and Z axes. Further, the performance of the proposed solution is compared by altering the weighting coefficient k .

Figure 4-17(a) and (b) illustrate the simulation results when k is set to 0 and 1, respectively. It can be seen that smaller maximum/average positional/orientation errors can be obtained when the positional (e_P) and orientation (e_{R_Z}) constraints are both applied for the optimization problem.

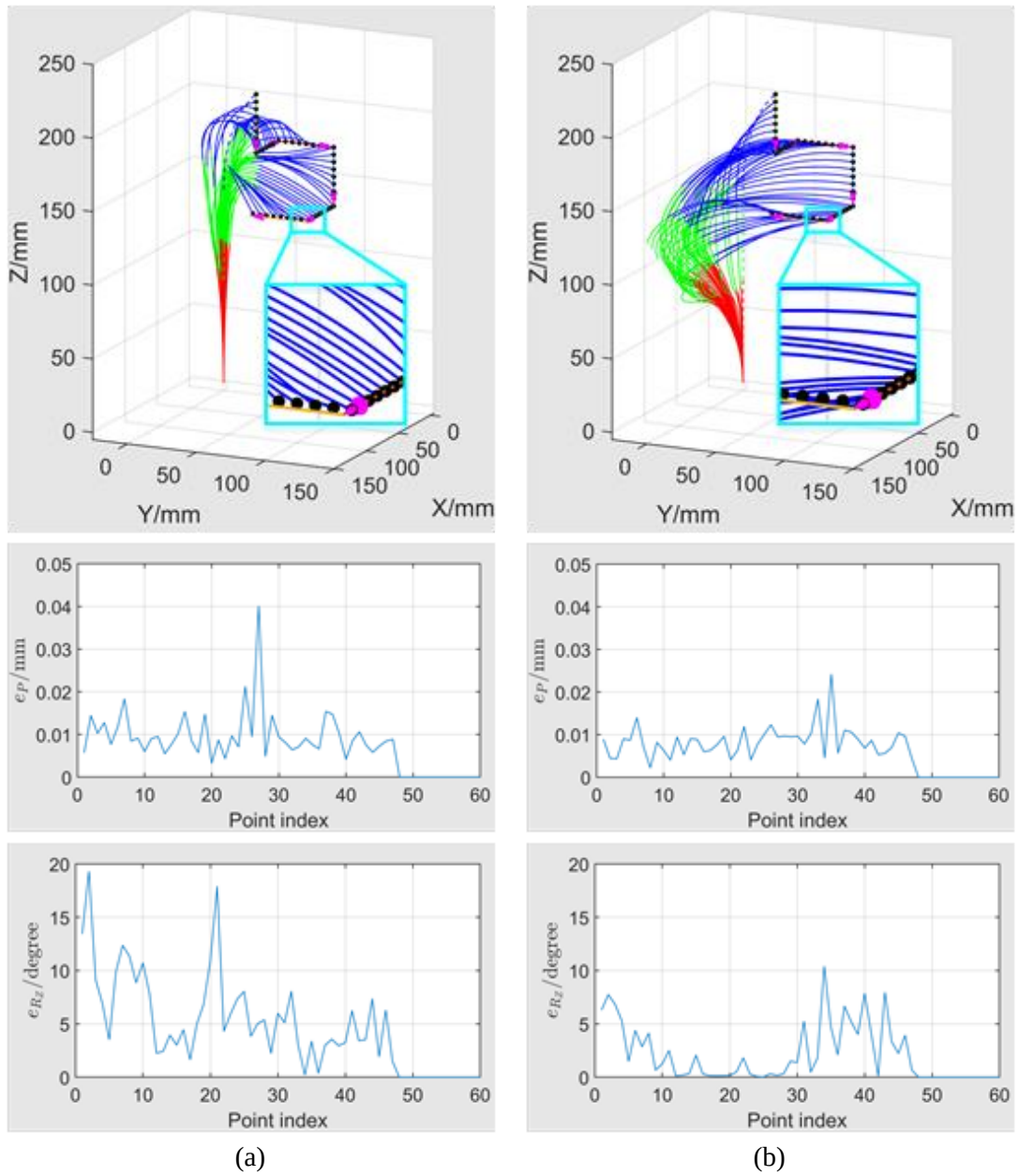


Figure 4-17 Simulation of the inverse kinematics for three sections based on the PCC-based model with (a) $k = 0$ and (b) $k = 1$.

Overall, it is demonstrated that the optimization approach is able to solve the inverse kinematics of three sections with high accuracy (< 0.05 mm), which yields one solution out of many with given constraints. In general, smoother pose transition can be achieved when both the positional and orientation constraints are applied.

4.4 Chapter summary

In this chapter, a comprehensive analysis has been performed for the kinematics of continuum robots. On the one hand, the kinematics is formulated based on the widely used PCC assumption, which is suitable for continuum robots whose shape profile of each section is similar to a circular arc. On the other hand, more accurate geometry-based kinematic models are derived for specific design of continuum robots, including the dual-revolute and twin-pivot design introduced in the previous chapter.

Further, a universal framework is developed with numerical optimization, which provides a general solution for the inverse kinematics of continuum robots. The proposed approach is firstly validated on single section for tip positioning and orientating, achieving a maximum error of 0.01 mm and 0.025° , respectively. Then, the performance of the approach is tested by manoeuvring the tip position of three sections for step control, which can keep the maximum position error below 0.03 mm with position and orientation constraints.

5 Novel model-less control of slender long CRs

With the rapid development of industrialization, huge demand emerges for various types of robots in different application scenarios. Although the conventional rigid-link robots, including serial industrial manipulators and parallel robotic platforms, have been widely applied in traditional industrial tasks (e.g., manufacture, assembly, and transport) requiring high speed, high precision, and large load capability, they are not suitable for some lightweight tasks in confined environments such as the aero-engines and pipeline systems. As a potential solution, the slender long continuum robot has been proposed for performing tasks in narrow space. For instance, as introduced in Chapter 3, Prototype One has been developed to verify the concept of utilizing continuum robots for in-situ maintenance of aero-engine combustors.

In this chapter, the work focuses on designing novel feedback controllers for slender multi-section continuum robots, where the tip sections of Prototype One are utilised as the research target. The control challenges of such kind of slender tendon-driven continuum robots are firstly analysed. Then the preliminary test is performed to demonstrate the defect of the conventional model-based method, leading to the need of model-less controllers. Finally, two stages are presented to illustrate the development and experimental validation of the proposed novel model-less controllers.

5.1 Control challenges

Regarding the control challenges of slender tendon-driven continuum robots, there are three main factors, including the model mismatch, cable elongation and section coupling, which are to be discussed in this section.

- **Model (PCC assumption) mismatch**

In most of the existed research, each section of continuum robots is treated as a circular arc to describe the shape and to derive the kinematic model. The PCC assumption is widely utilized for controlling short continuum robots when the deviation between the actual shape of single section and the assumed circular arc is negligible or the accuracy requirement is not high. However, for the continuum robot proposed in this work, there exists non-negligible deviation between the actual shape and the PCC-based model, which is called the PCC assumption mismatch and can be explained from two aspects. Firstly, given the same bending angle θ , the error between the PCC-based model and the geometry of the robot is obvious, which is shown in Figure 5-1(a). Secondly, Figure 5-1(b) reveals that segments within a single section are unevenly bent, which further contributes to the model mismatch issue.

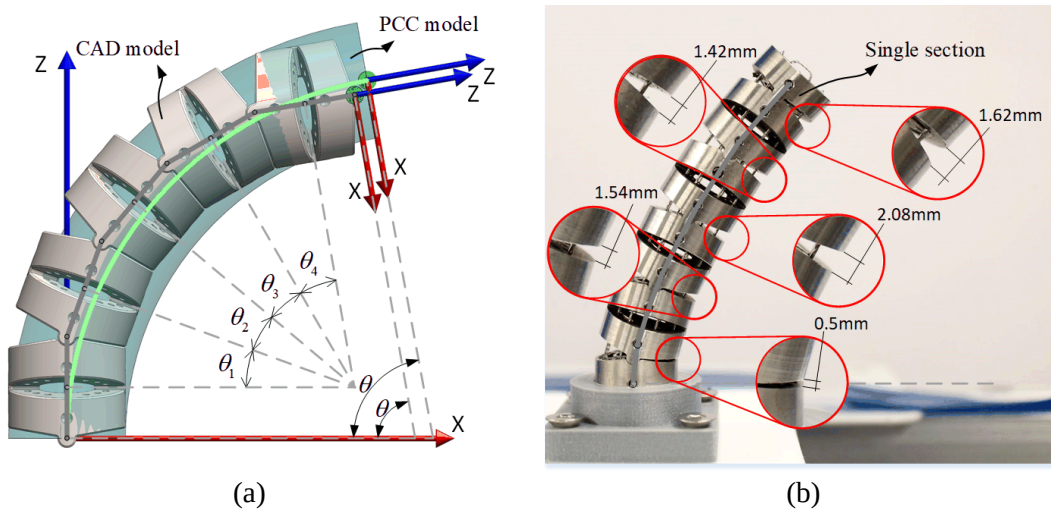


Figure 5-1 Illustration of the PCC assumption mismatch issue: (a) Comparison of the PCC model and CAD model with the same bending angle θ ; (b) One configuration of the robot depicting uneven bending of different segments within single section.

- **Cable elongation affecting control precision of cable displacements**

For all cable-driven continuum robots, the control accuracy of cable displacements is the basis of precise control of the robot. Usually, the nominal cable displacements are calculated according to the inverse kinematics from configuration space to actuation

space. However, this method is purely based on the geometry of the robot regardless of the disturbances caused by cable elongation.

The elastic extension $el(T)$ of the steel cable can be calculated as follows (Hooke's law):

$$el(T) = \frac{TL}{EA} \quad (5.1)$$

where T is the applied force on cable (N), L is the original length of the driving cable (mm), E is the strand modulus (kN/mm²), and A is the cross-section area of cable (mm²).

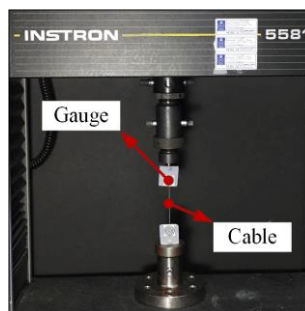
For a certain type of steel cable, E and A keep constant. Then (4.16) can be rewritten as $el(T) = \eta TL$, where $\eta = \frac{1}{EA}$ is the cable change rate.

As the cable tension is initialised to be T_0 , the initial cable length is $L(T_0) = L + el(T_0)$.

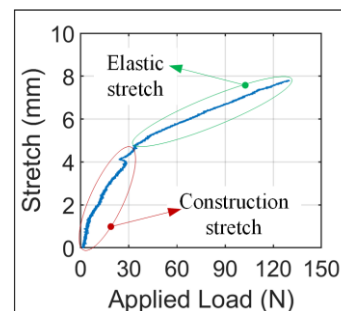
When the tension goes to $T = T_0 + \Delta T$ as the robot moves, the length turns to be $L(T) = L + el(T)$. Then the cable length change can be expressed as $\Delta L = L(T) - L(T_0) = \eta \cdot L \cdot \Delta T$, which is proportional to the original cable length and the tension change ΔT .

The tensile test is conducted to measure the generic cable elongations under different tensions shown in Figure 5-2. The steel cable used in the test is 120 mm in length and 0.8 mm in diameter. From the diagram we can calculate the exact cable change rate $\eta =$

$$\frac{1}{9000} N^{-1}.$$



(a)



(b)

Figure 5-2 Cable stretch test: (a) the tensile testing machine; (b) curve of cable elongation relative to applied load.

The cables utilised on the manipulator are the same with the one in the test while the average length $|\Delta T|$ is about 1000 mm. All the cables are pre-tensioned to be 30 N. According to the record from the force cells during the experiment, the maximum tension change N can reach as much as 25 N. Thus, the maximum cable elongation is $\frac{1}{9000} N^{-1} \cdot 1000 \text{ mm} \cdot 25 \text{ N} \approx 2.8 \text{ mm}$. In addition, according to Section 4.2.1, the variation range of the cable displacements is between -4.57 mm to 4.57 mm .

Above analysis reveals that the elastic elongation of the driven cable has the same order of magnitudes with respect to the desired displacement from the kinematics, which has great effect on the control of cable displacements. Hence, the pure kinematics-based cable displacement control has become invalid.

- **Section coupling issue affecting the stability and convergence of controllers**

Since the driving cables for the distal sections of continuum robots inevitably pass through the proximal ones, the coupling issue is brought in for the control of multi-section continuum robots, which leads to two major problems. (i) The first problem is the physical coupling, namely, the driving cables for the distal sections will apply extra torque and force on proximal sections. The physical coupling is the intrinsic property of tendon driven CRs, which has a great effect on the stability of multi-section system. (ii) The second problem is kinematic coupling. The total cable lengths for controlling the distal section (cable one: L_1+L_2 ; cable two: L_3+L_4) are determined by both proximal and distal sections. If there is a difference between the theoretical and actual pose of the proximal section, the lengths of the cables shaping the pose of the distal section (L'_1 and L'_3) will not equal to the theoretical values (L_1 and L_3). Hence, the configuration of the distal section will be affected, regardless of its modelling accuracy. Obviously, the

PCC assumption mismatch will be magnified on multiple sections because of the kinematic coupling issue.

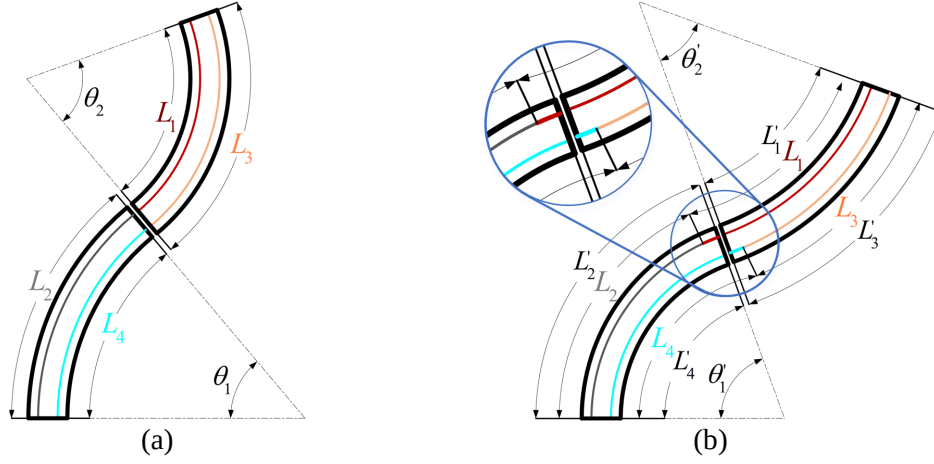


Figure 5-3 Illustration of kinematic section-coupling issue (a) theoretical pose of two sections with bending angles (θ_1, θ_2) ; (b) actual pose with a control error $(\theta'_1 > \theta_1)$ on the base section.

Among the three control challenges mentioned above, the first one is related to the specific robot design, while the second and third ones are the inherent shortcomings of the tendon-driven approach and multi-section continuum robots. In the following sections, the performance of the conventional model-based controller and the developed novel model-less ones against those challenges are to be explored.

5.2 Preliminary test with the conventional model-based controller

In this section, the preliminary test is conducted to experimentally validate the defects of the conventional model-based controller on Prototype One, which is a typical slender long tendon-driven continuum robot.

5.2.1 PCC-based static controller

In the research of continuum robots, the conventional model-based controller usually refers to controllers designed based on the PCC assumption. As illustrated in Figure 5-4, a static implementation of the PCC-based controller is utilized for the test in this work. In the figure, the desired cable length L_d and actual cable length L_a are computed

utilizing the inverse kinematics from task space to configuration space (“T·S-C·S IK”, corresponding to equations in Section 4.1.2) and from configuration space to actuation space (“C·S-A·S IK”, corresponding to equations in Section 4.1.1), where the difference ΔL will then be multiplied by the gain coefficient $K(\in (0,1])$.

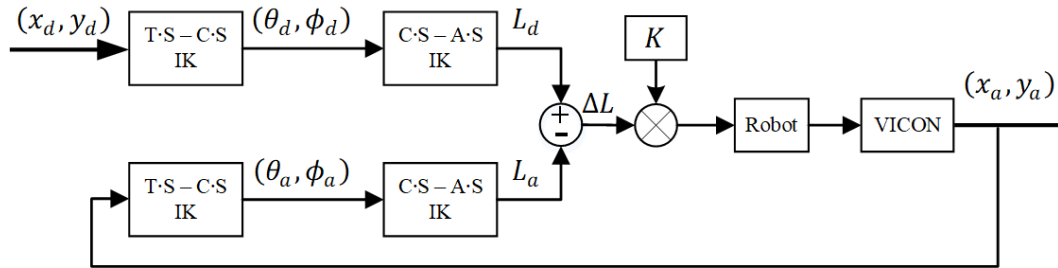


Figure 5-4 Control architecture of the PCC-based static controller.

5.2.2 Experiments

In this section, the test-rig setup is illustrated for the preliminary test, as well as the other experiments in this chapter. Then the results for tip positioning is presented using the aforementioned PCC-based controller.

- **Test-rig setup**

A 64-bit Windows 7 based computer with an Intel Xeon E5-1620 processor is used as the host controller to run the main control algorithm at an execution rate of 20 Hz using LabVIEW in 32-bit version. The experimental data (cable displacements, tip position) is also recorded at the same frequency. The desired cable displacements calculated by the host computer are sent to the sb-RIO board (National Instrument), which is utilised as the low-level controller for the close-loop control of motors to realise precise displacements of all the driven cables. In addition, VICON system, is deployed to measure the tip position and orientation of each section, which is grasped and pre-processed by the program in 64-bit version LabVIEW running on the host computer. The measuring data is transferred from the 64-bit LabVIEW project to the 32-bit version control algorithm for the position feedback with a sampling rate of 50 Hz.

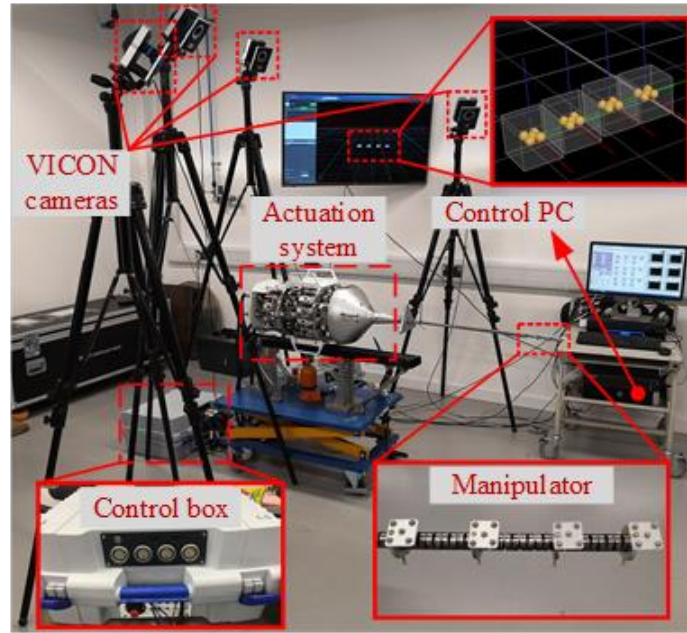


Figure 5-5 Test-rig setup. The proposed controller is validated on the continuum manipulator with VICON as the feedback device.

- **Results for tip positioning**

In the preliminary test, the PCC-based static controller is implemented with the gain factor K set to 0.1. Figure 5-6 shows the 3D and 2D views of all the 17 target points used for the test, whose projections in the XY plane are evenly distributed within the semicircle in a radius of 30 mm and cantered at the origin. It is necessary to declare that the test zone is restricted to half of the workspace of the section (the 3rd and 4th quadrants) due to the measuring range of the VICON system.

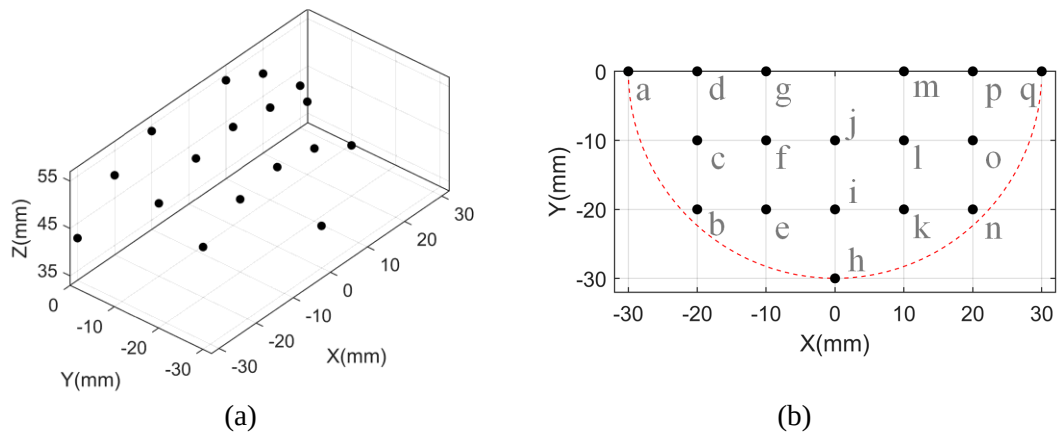
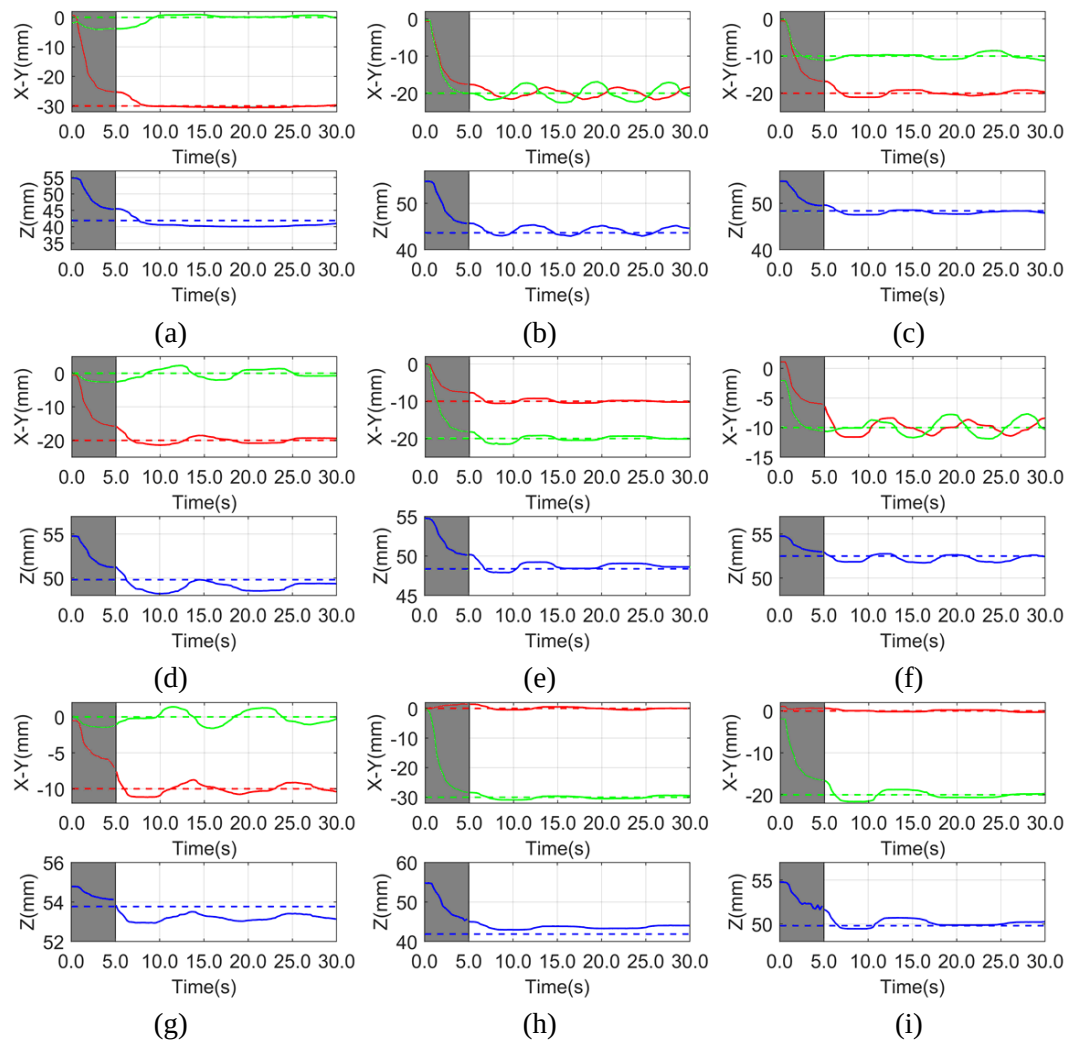


Figure 5-6 Target points for the positioning test: (a) 3D view; (b) plane view.

The experimental results are illustrated in Figure 5-7, where the subfigures (a)-(q) correspond to the target points (a-q) marked in Figure 5-6(b). Each subfigure depicts the actual Cartesian coordinates of the section tip in X (in red), Y (in green) and Z (in blue) axes in 30 seconds, which consists of two stages: the open-loop stage (the grey area) and the PCC-based close-loop stage (the white area). It can be found that the controller can keep converged at all the target points. Oscillation occurs at point b, d, f, g, k, l, m, n, and o.



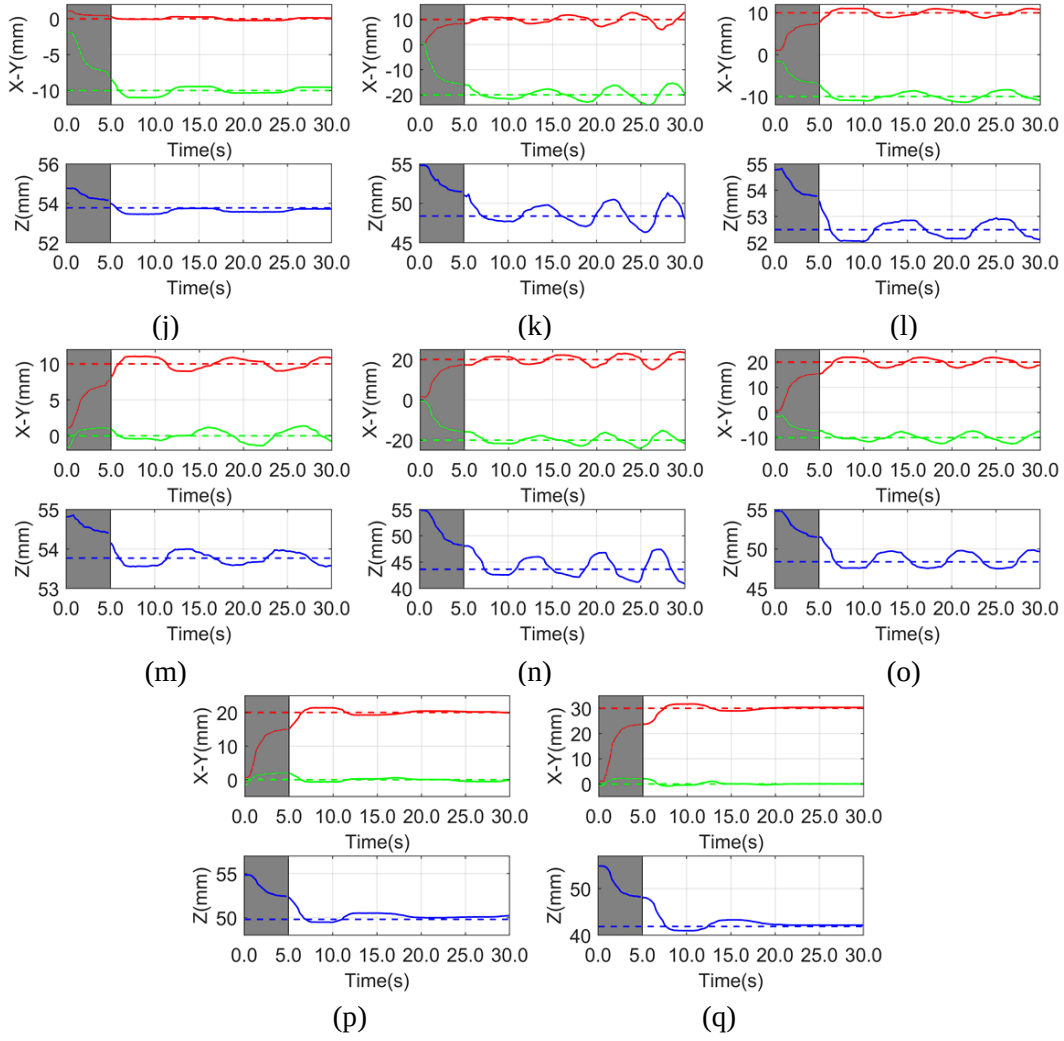


Figure 5-7 Experiments of tip positioning with the PCC-based static controller. The subfigures (a)-(q) correspond to the target points (a-q) in Figure 5-6 (b).

A qualitative explanation for this phenomenon is that modelling error is imported when arc parameters (θ, ϕ) in configuration space are utilized as the intermediate to derive the control model from task space to actuation space based on the PCC assumption. As introduced in Section 5.1, the PCC model cannot precisely describe the actual shape of the proposed robot. The experimental result reveals that the PCC-based controller takes effect at some target points. However, it cannot keep convergent for the target points in the whole workspace.

Therefore, the defect of the conventional model-based controller is experimentally demonstrated for the control of the tip section on Prototype One. As such, the need arises for novel model-less controllers for slender long tendon-driven continuum robots.

5.3 Empirical model-less static controller

In this section, a novel model-less static controller is designed based on a series of empirical control policies directly from task space (tip positional errors) to actuation space (cable displacements). The analysis, development, and validation of the controller are to be presented below.

5.3.1 Control analysis of tendon-driven continuum robots

Even though the PCC assumption is widely adopted for the kinematics modelling of CRs, arc parameters (θ, φ) are dispensable for the derivation of feedback controllers. Inspired by the tug of war shown in Figure 5-8(a), the movement of tendon driven CRs can be also seen as the tug among three driving cables (Figure 5-8 (b)). The control law in the tug of war is quite straightforward to keep the target point in the middle: when the target is bias towards one side, pull the rope on the other side. Likewise, similar control policies can be summarized to guide the displacements of three cables in a single section of a continuum robot, according to the error vector from actual position P_a to desired one P_d , which can minimise the kinematics error caused by cable elongation.

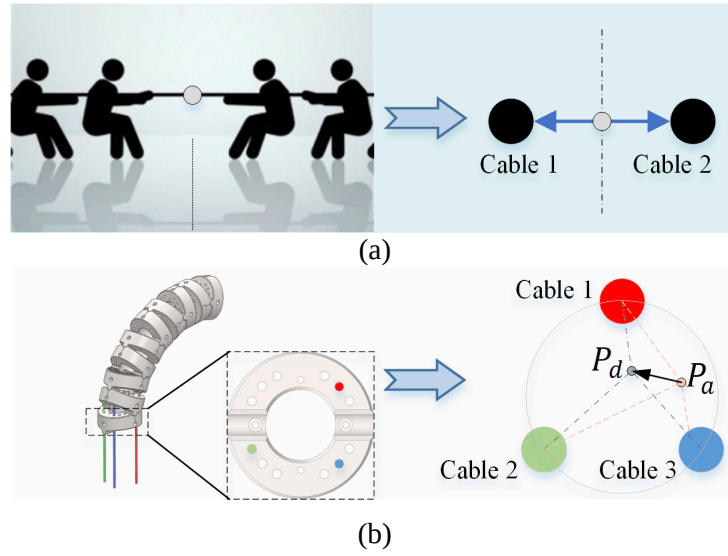


Figure 5-8 Illustration of the movement mechanism of (a) the tug of war and (b) tendon driven CRs. For a two-DoF CR, the tip position control problem in 3D space can be regarded as the tug among three driving cables in a 2D plane, which avoids the model error caused by cable elongation when utilizing arc parameters (θ, φ) in the PCC model.

5.3.2 Controller design

The main purpose of the robot control is to drive the specific cable-driven continuum manipulator to achieve accurate snake-tip positioning by tuning the actuator lengths with visual feedback. Continuum robots are usually redundantly actuated, and theoretically there are multiple configurations to reach the same tip position. However, some configurations can drive the robot into unnatural states, such as high cable tension and over deformation of the robot body, and so forth. Further, manufacturing error and compliant structure enlarges the difficulty to develop the static and dynamic performance of the robot even with feedback signals. In this section, a novel neighbourhood algorithm for the close-loop control of single section, which is based on the geometric construction of the cable-driven manipulator, is proposed to increase the quasi-static control precision of the robot.

- **Workspace division**

Firstly, the workspace of single section is divided into several sections according to the layout of the driven cables. In Figure 5-9, there are three cables roughly evenly distributed in single section, which are in red, green, and blue, respectively. The whole space is divided into 6 parts, which are noted as I, II, III, IV, V, and VI. Sections I, III and V represent the neighbourhoods of each cable while sections II, IV and VI are the medium areas in between two cables.

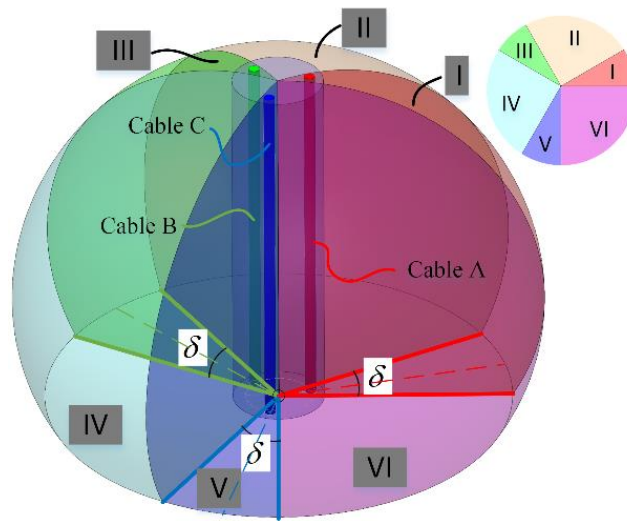


Figure 5-9 Workspace division of single section based on the layout of driven cables. I,III,V and II,IV,VI are the local and in-between areas of Cable A, Cable B and Cable C, respectively. The dimension of local areas is defined by radial angle δ , which needs to be tuned during experiments. Additionally, the workspace division can be asymmetric owing to the actual distributions of the driving cables.

By dividing the workspace into 6 sections according to the layout of driven cables, the proposed local empirical control policies can be generalized into the whole workspace for feedback control.

- **Basic Motion Patterns and Control Policy**

In the configuration space, the motion of a single section can be decomposed into two basic types of motion, which are one rotation around the Z-axis in the base coordinate system (direction angle θ) and one bend in the bending plane which is perpendicular to the X-Y plane (bending angle φ), shown in Figure 5-10. All the motions of the single

section can be completed by either conducting the rotation and then bend or in the opposite order.

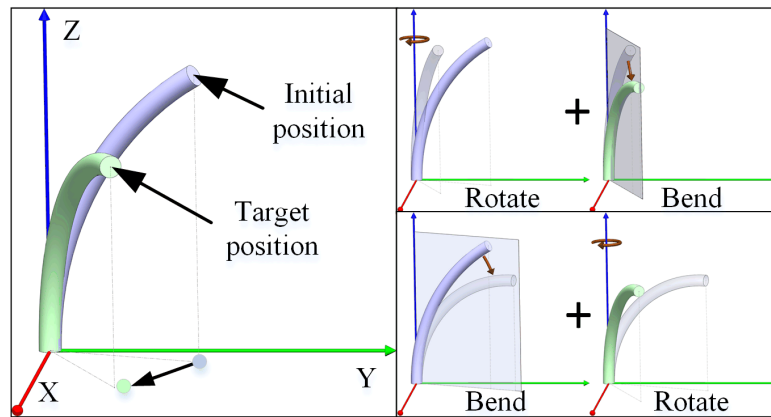


Figure 5-10 Schematic diagram of motion decomposition for single section. The expected adjustment from actual position to desired position can be divided into the combination of rotary and bending motion.

The projection of the tip position can either locate at the nearby region of single section or interval between two cables when the section moves. Hence, the basic motion of a single section can be divided into two cases (Figure 5-11). For example, in Case I, the section's tip is projected in the nearby area of a cable (e.g., Cable 3 - Blue) while the projection of the section tip locates in the interval between two cables (e.g., Cable 1 – Red and Cable 3) in Case II. For both cases, there are four basic motion patterns in the configuration space for adjusting the tip position to a desired location in the task space, representing the increment or decrement of the bending angle (motion ① and ③) and direction angle (motion ② and ④), respectively (**Table 5-1**).

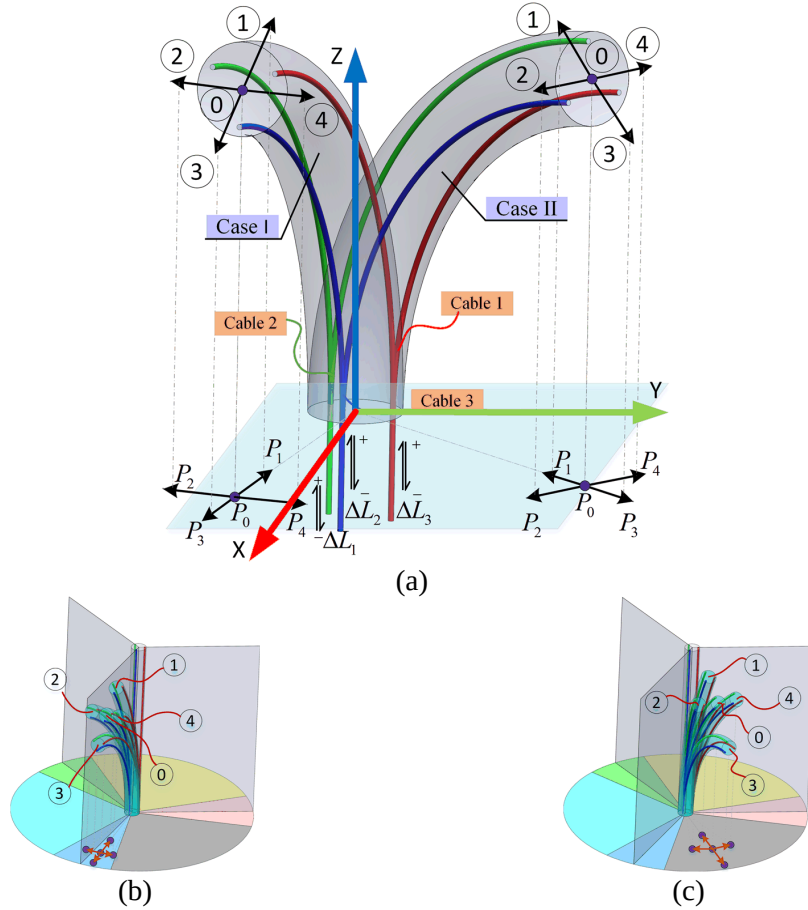


Figure 5-11 (a) Motion patterns of a single section considering the basic motions and cable layout of continuum robot. Cable A – Red, Cable B – Green and Cable C- Blue; (b) and (c) are motion patterns generalized in the nearby regions (I, III, V) and the intervals (II, IV, VI) in Figure 5-9, respectively.

Table 5-1 Classification of the motion patterns and the table of control actions.
All the control policies are derived empirically.

Task space	Configuration Space			Actuation Space		
	Motion pattern	Bend angle (°)	DIR angle (°)	ΔL_a	ΔL_b	ΔL_c
Current position to a target in the Cartesian frame	Case I	①→②	↓	-u	-u	+2u
		②→③	—	+u	-u	0
		③→④	↑	+u	+u	-2u
		④→①	—	-u	+u	0
	Case II	①→②	↓	+u	-2u	+u
		②→③	—	+u	0	-u
		③→④	↑	-u	+2u	-u
		④→①	—	-u	0	+u

Then the cable displacements for each motion patterns are derived, which is given in **Table 5-1**, to build the mapping relationship between the task space and actuation space.

To explain how the mapping relationship is derived, take for example the motion patterns ①→③ in Case I and ①→④ in Case II. In the first pattern, the section is controlled to bend without any rotation, thus the blue cable should be further tensioned ('-') with $2u$ (unit of displacements) while the red and green cables should be released ('+') equally with $1u$. Here $1u$ represents the step-size utilized in the controller. As for the second pattern, the section is expected to rotate in clockwise direction, then the red cable should be tensioned while the blue cable should be released both with $1u$, with the green cable remained static. The rest of the motion patterns can be analysed in similar way. In addition, from **Table 5-1** it can be observed that the sum of all the three cables' displacement changes is zero in each motion pattern, which is designed to avoid the over-tension of the driven cables in the local area.

Hence, the basic control policy of a single section is analyzed to quantize the mapping relationship from motion patterns to changes of cable displacements with one specific cable arrangements. This method can avoid the computation of the bending and direction angles in the configurations, leading to a significant simplification of the controller and high positioning accuracy, which was validated in the experiments.

- **Algorithm integration and formulation**

In this part, the whole algorithm will be integrated and formulated based on the analysis above. As is shown in Figure 5-12, the desired tip position and actual tip position which is measured by VICON system are projected to the X-Y plane in the section's base coordinate system. The error vector $\vec{e}$ is decomposed into two components $\vec{e}_r$, and $\vec{e}_\theta$ in the polar coordinate system, which are utilised in the model-less controller for selecting the motion pattern from the control policy.

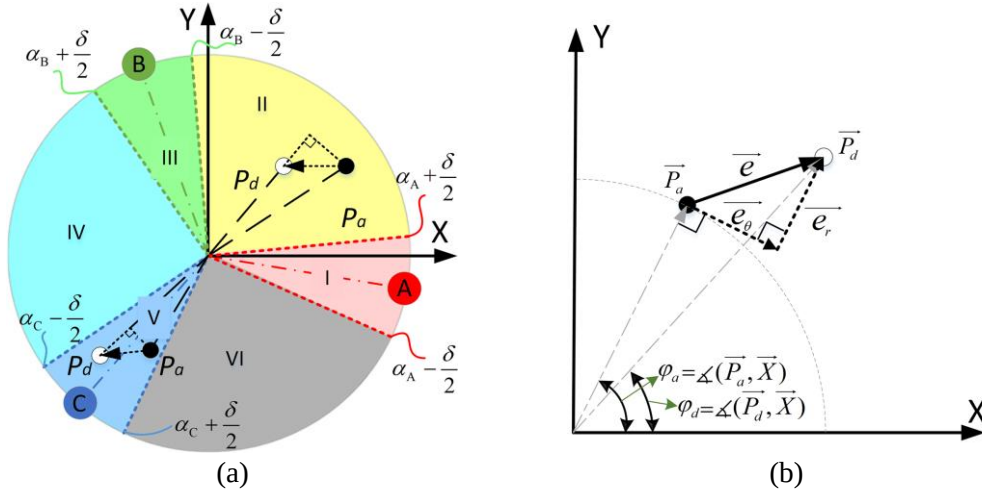


Figure 5-12 (a) Projection view of the desired and actual tip position in the X-Y plane with workspace division; (b) the error between desired and actual position $\vec{e}$. $\vec{e}_r$ and $\vec{e}_\theta$ are the errors in the radial and angular coordinates, respectively; φ_a and φ_d is the actual and desired direction angles, respectively.

Figure 5-13 shows the pseudo-code for the proposed feedback controller, which demonstrates how it is implemented in the control of a single section of the continuum robot. After the open-loop control stage completes, the feedback controller starts to take effect. Vicon system was utilised to measure the actual tip position $\vec{P}_a$ (Line 2) in the Cartesian coordinate, with which the error vector $\vec{e}$ (Line 4) from the actual tip position to the desired one can be obtained. Then, $\vec{e}$ will be decomposed into two forms (in Cartesian (Line 6) and Polar coordinate (Line 14)) on the X-Y plane.

The error components in Cartesian form ($\vec{e}_x, \vec{e}_y$) are used to judge if the control error is less than the desired error constraint e (Line 8), while the polar error components ($\vec{e}_r, \vec{e}_\theta$) are applied for the closed-loop control which indicate the expected radial and angular motion to eliminate the positioning error (Line 22 to 32).

It is necessary to mention that when the current tip error $|\vec{e}|$ is beyond five times of the error constraint e , three times of the unit-step-size is applied to the step-size, u , which speeds up the convergence of the control process (Line 16 to 20). Here the unit-step-size utilized in the controller is an empirical value and was determined by the tuning trials (0.05 mm is used in the experiment).

By comparing the radial and angular errors (from Line 22 to 32), the motion patterns are determined (Line 34 and 36). Finally, the motors of the robot conduct the displacement based on the control policy (Line 38 to 40).

```

1: % read actual tip position from Vicon system
2:  $\vec{P}_a \leftarrow \text{Vicon}$ 
3: % calculate error vector
4:  $\vec{e} = \vec{P}_d - \vec{P}_a$ 
5: % decompose error vector in Cartesian coordinate system on X-Y plane
6:  $[\vec{e}_x, \vec{e}_y] \leftarrow \vec{e}$ 
7: % while-loop for feedback control
8: while  $|\vec{e}_x| > e \parallel |\vec{e}_y| > e$  do
9:   % compute actual direction angle
10:   $\theta_a \leftarrow \vec{P}_a$ 
11:  % select section(I,II,III,IV,V,VI) based on actual direction angle
12:  SectionIndex = SectionSelector( $\theta_a$ )
13:  % decompose error vector in Polar coordinate system on X-Y plane
14:   $[\vec{e}_r, \vec{e}_\theta] \leftarrow \vec{e}$ 
15:  % adjust Stepsize according to the error's magnitude
16:  if  $|\vec{e}| > 5e$  then
17:     $u = 3 * \text{Unit-Step-size}$ 
18:  else
19:     $u = \text{Unit-Step-size}$ 
20:  end if
21:  % judge the direction of radial motion
22:  if  $\angle(\vec{P}_a, \vec{e}_r) > 0$  then
23:    Patternr = 3
24:  else
25:    Patternr = 1
26:  end if
27:  % judge the direction of angular motion
28:  if  $\angle(\vec{P}_a \times \vec{e}_\theta, \vec{Z}) > 0$  then
29:    Patternθ = 4
30:  else
31:    Patternθ = 2
32:  end if
33:  % select the control policy for radial motion
34:   $U_r = \text{ControlPolicy}(\text{SectionIndex}, \text{Pattern}_r, u)$ 
35:  % select the control policy for angular motion
36:   $U_\theta = \text{ControlPolicy}(\text{SectionIndex}, \text{Pattern}_\theta, u)$ 
37:  % compose cable displacements for both component motions
38:   $[\Delta L_1, \Delta L_2, \Delta L_3] = \frac{\vec{e}_r}{e} U_r + \frac{\vec{e}_\theta}{e} U_\theta$ 
39:  % send the command to the robot's actuation system
40:  Robot  $\leftarrow [\Delta L_1, \Delta L_2, \Delta L_3]$ 
41:   $\vec{P}_a \leftarrow \text{Vicon}$ 
42:   $\vec{e} = \vec{P}_d - \vec{P}_a$ 
43:   $[\vec{e}_x, \vec{e}_y] \leftarrow \vec{e}$ 
44: end while

```

Figure 5-13 Pseudo-code for the empirical model-less feedback controller.

5.3.3 Experimental validation

In this section, the proposed controller for manipulating an extra-long continuum robot is applied to the Prototype One in Chapter 3, which was designed for navigation and operation into narrow space (e.g., aero engine combustor). A series of experiments were performed to validate the effect and applicability of the feedback control law on single section and the three-section manipulator, respectively.

- **Experiments on single section**

In this section, we conduct comprehensive experiments on single section to test the performance of the designed controller. The variation ranges of θ and ϕ are $[5^\circ, 75^\circ]$ and $[-180^\circ, 0^\circ]$ with the increments of 5° and 10° respectively, covering half of the workspace with 285 discrete points.

In Figure 5-14(a), both measured actual and PCC-based theoretical workspaces of a single section are displayed to demonstrate the model-failure issue presented in 5.1. It is necessary to declare that we only analyse half of the workspace owing to the measurement range restriction of the VICON system. In the figure, the blue surface represents the theoretical workspace generated by the PCC-based model while the red one illustrates the measured workspace of the single section utilising the proposed controller. The error constraints for the control precision in the X direction and Y direction are set to be 1mm, respectively. It can be observed that there are clear asymmetric offsets in the Z direction between the actual workspace and the theoretical workspace. Also, the difference gets larger as the bending angle increases, which can reach 6 mm at most when the bending angle is 75° .

There are several aspects contributing to the workspace offset, including the PCC model failure, infeasible tension configuration of the actuating cables and unevenly distributed layout of the actuation cables and so forth, which cause the uneven bending of the robot.

Figure 5-14(b) illustrates the vector map of the tip position error generated by projecting the section tip to the X-Y plane. For the benefit of the analysis, all vectors are translated so that their initial points coincide with the origin of the coordinate system. From the figure, we can see that the overall precision of the single section in both X and Y directions can reach ± 1 mm within the measured workspace. For the majority of the sampling points, the errors in both directions can be kept below 0.5 mm.

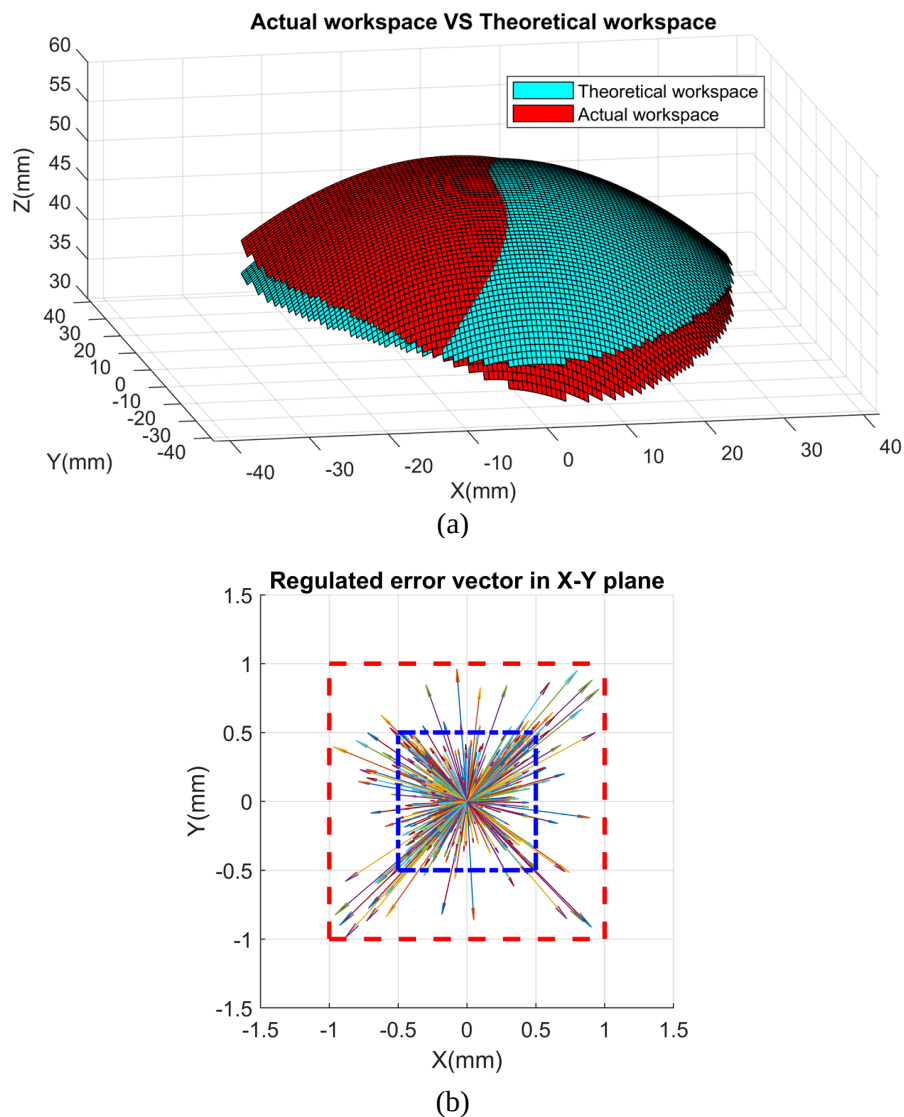


Figure 5-14 (a) Workspace of the single section: the theoretical geometry-based workspace (in light blue) and the actual workspace (in deep red) measured utilising the developed model-less controller; (b) Regulated positioning error vector in the X-Y plane for all measured 285 points.

In Figure 5-15, three examples with different configurations are given to illustrate the efficacy of the controller, compared to the result of open-loop control and the desired positions. Detailed data processing and performance analysis are also completed based on the experimental results. From the experiments, it can be concluded that the proposed controller can address the error caused by the PCC-model failure and long elongation identified from the extra-long continuum robot built for the in-situ repair of aero-engine combustors.

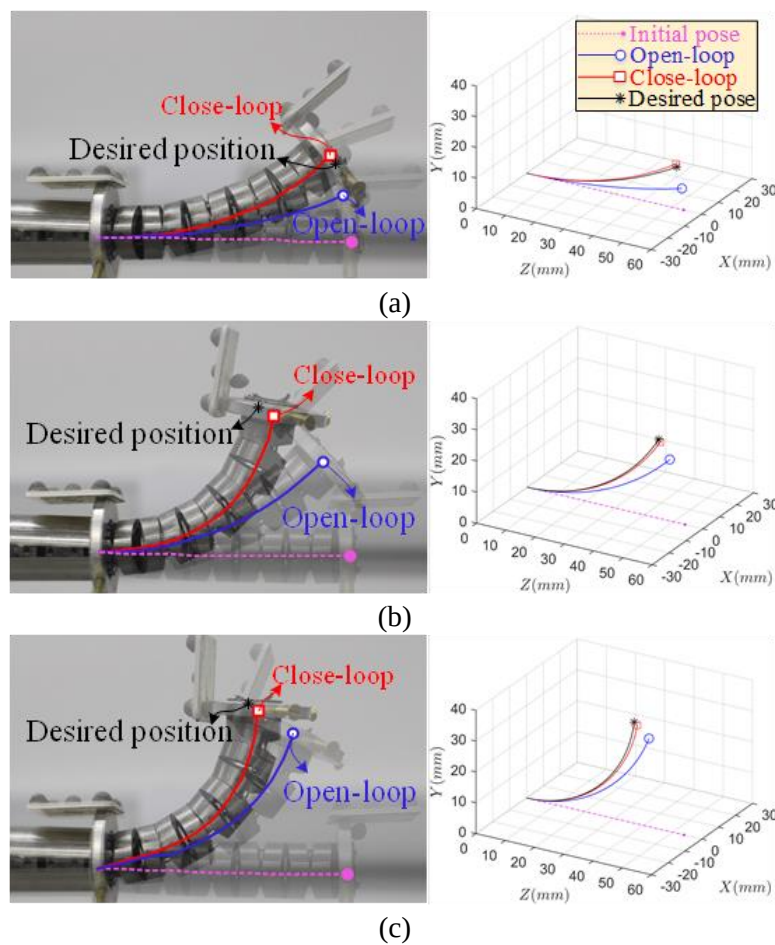


Figure 5-15 Comparison of the open-loop and closed-loop model-less control on a single section. Three cases are shown: (a) $(\theta, \varphi) = (30^\circ, 90^\circ)$, (b) $(\theta, \varphi) = (60^\circ, 90^\circ)$ and (c) $\text{abs}(\theta, \varphi) = (85^\circ, 90^\circ)$.

- **Trials on three sections**

In this section, trials are proceeded on the three-section manipulator to study the applicability of the designed controller on multiple sections. At the beginning of the

experiments, various configurations of bending angles are tested on the manipulator. However, when the bending angles arise, the controller starts to lose effectiveness and the whole manipulator cannot reach a steady position. This is mainly caused by the interference between different sections, especially when the bending angles of the distal sections are larger than 50°. So finally, the configurations for the experiments are selected as below: the bending angles of the 1st and 2nd proximal sections remain 25° while the 3rd section alters between 25° and 50°; as for the direction angles of all the three sections, they vary among -30°, -60° and -90°, as shown in **Table 5-2**.

Table 5-2 Configurations for all eighteen trials on the three-section manipulator

Trials	θ_1	φ_1	θ_2	φ_2	θ_3	φ_3
1						-30°
2					25°	-60°
3						-90°
4				-30°		-30°
5					50°	-60°
6						-90°
7						-30°
8					25°	-60°
9						-90°
10	25°	-30°	25°	-60°		-30°
11					50°	-60°
12						-90°
13						-30°
14					25°	-60°
15						-90°
16				-90°		-30°
17					50°	-60°
18						-90°

a) Performance analysis in the local coordinate system

Three cases are presented in Figure 5-16 to show the manipulator's pose before and after the controller is applied. It can be found that the control accuracy of the multiple sections is significantly improved. Further, the test results of the eighteen trials in their local coordinate systems are illustrated in Figure 5-17. From the figure, it can be seen that the mean error of the distal section in X, Y and Z directions of the local coordinate

system are 0.14 mm, -0.03 mm, and -0.17 mm, respectively (**Table 5-3**). The mean errors for sections 2 and 3 are slightly larger but still below 1 mm. Also, it can be observed that the absolute values of the max error and min error for all three sections increase from section 1 (around 1.1 mm) to section 3 (about 2.5 mm), which clearly reveals the disturbances on the distal section by the middle and proximal sections.

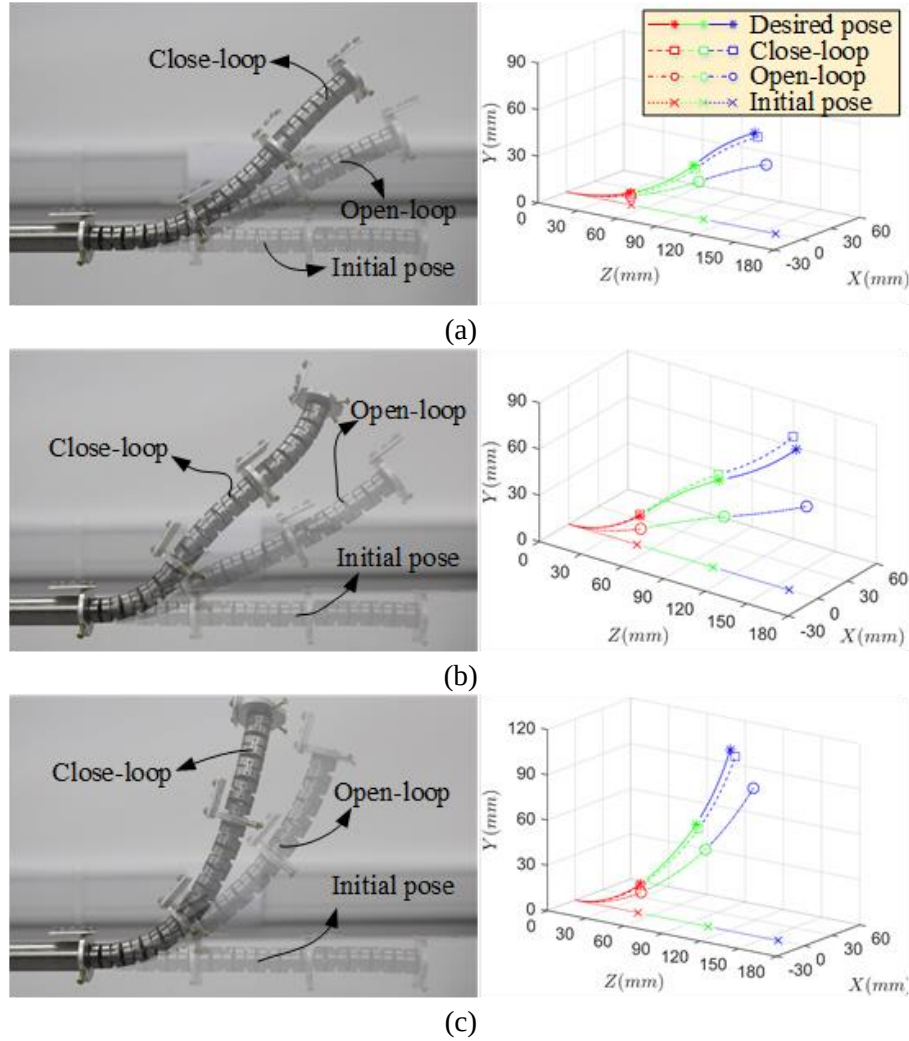


Figure 5-16 Comparison of the open-loop control and close-loop control utilising proposed controller on three sections. Three different configurations are shown:
 $[\theta_1, \varphi_1, \theta_2, \varphi_2, \theta_3, \varphi_3] =$ (a) $[20^\circ, 90^\circ, 20^\circ, 90^\circ, 20^\circ, -90^\circ]$, (b) $[45^\circ, 60^\circ, 20^\circ, -90^\circ, 30^\circ, 120^\circ]$ and (c) $[45^\circ, 60^\circ, 20^\circ, 120^\circ, 20^\circ, 60^\circ]$.

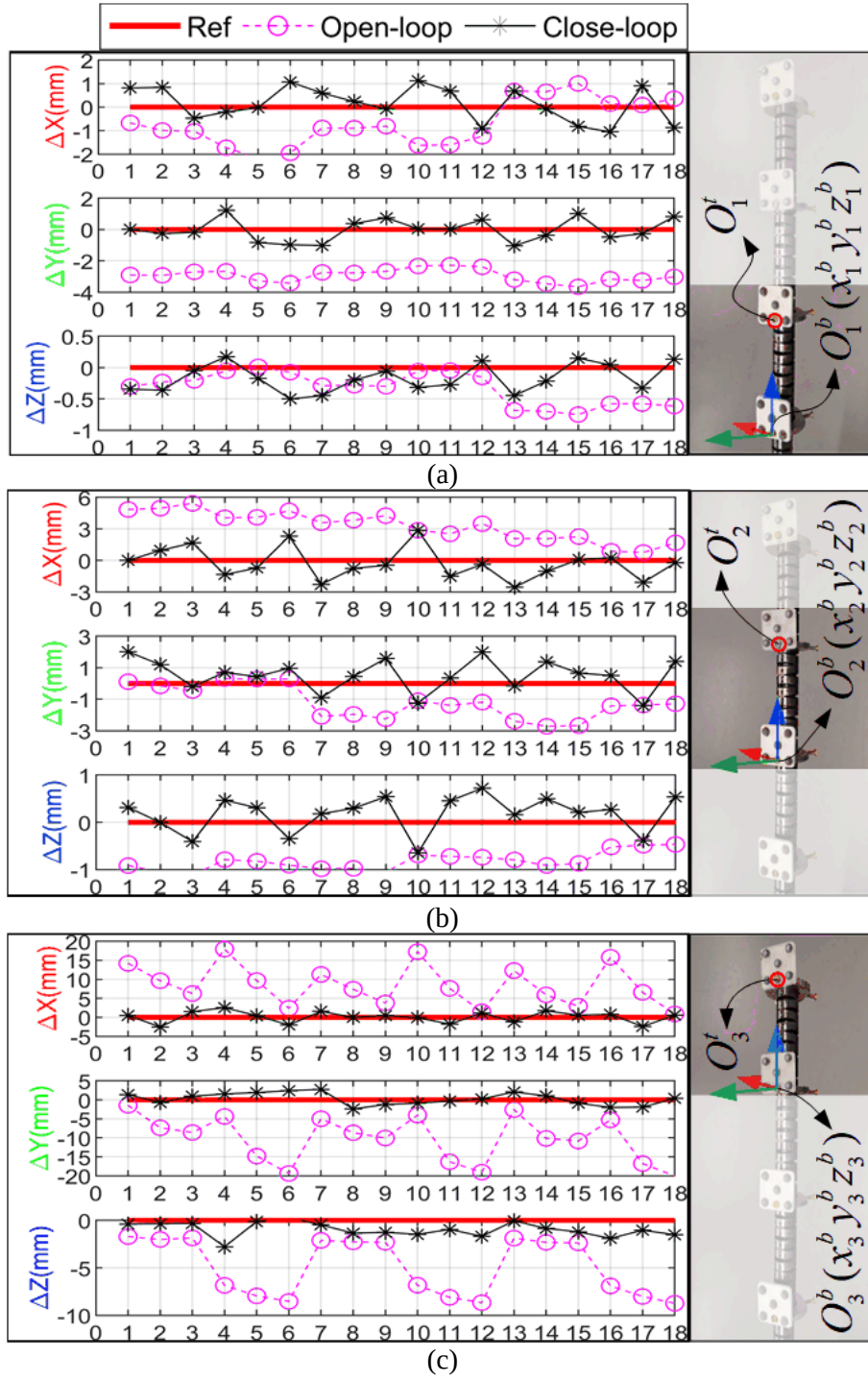


Figure 5-17 Experimental results (Actual tip position and positioning error in the Cartesian coordinate system) of (a) Section 1, (b) Section 2 and (c) Section 3 in the local coordinate systems, respectively.

Table 5-3 Mean, max and min error in local coordinate systems

		<i>Mean Error (mm)</i>	<i>Max Error (mm)</i>	<i>Min Error (mm)</i>
Section 1 (Distal)	X	0.14	1.12	-1.05
	Y	-0.03	1.22	-1.04
	Z	-0.17	0.17	-0.5
Section 2 (Middle)	X	-0.29	2.86	-2.56
	Y	0.54	2.01	-1.39
	Z	0.18	0.72	-0.65
Section 3 (Proximal)	X	0.09	2.55	-2.52
	Y	0.24	2.75	-2.4
	Z	-0.97	0.3	-2.78

In conclusion, the proximal sections have a great effect on the distal sections owing to the coupling issue; but the proposed controller still performs well with the disturbances.

b) Performance analysis in the global coordinate system

In this part, the performance of the controller for minimizing the multiple section coupling issue in the global coordinate system is discussed.

Figure 5-18 illustrates the measurement results of all the three sections in the global coordinate system, from which it clearly shows the performance improvement after the new controller was deployed. It was found that the mean tip positioning error is decreased up to 69% compared to the result of the open-loop control.

Further, it can also be seen that the error in the global coordinate system increases from section 1 to 3, which mainly results from two aspects. Firstly, since there exists non-negligible deviation between the theoretical workspace and actual workspace of every single section, the model-failure issue brings in systematic error in the local coordinate system. Secondly, since all sections are serially linked in space, the position error increases through the amplification of the proximal sections in the global coordinate system, compared to the error in the local coordinate systems.

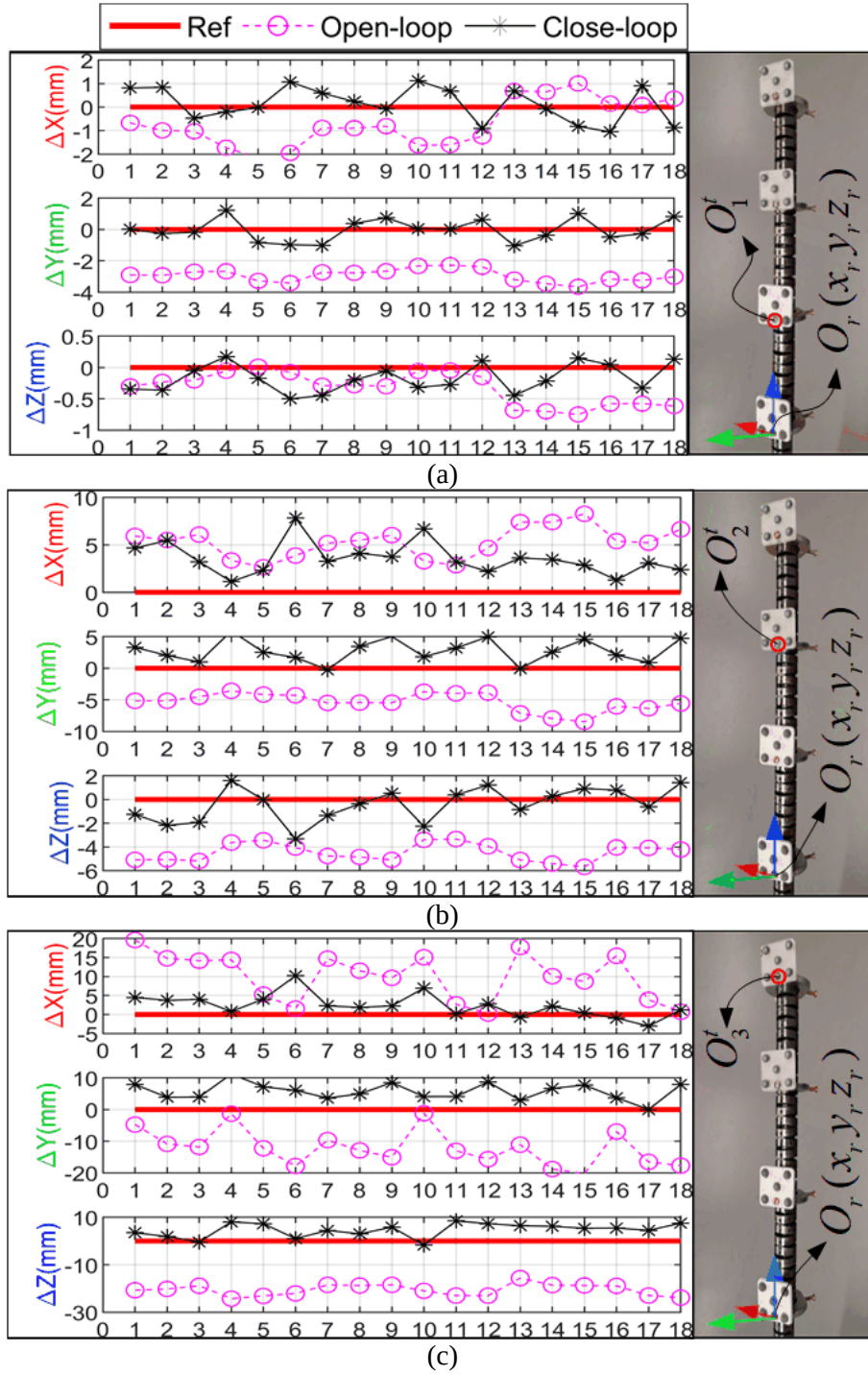


Figure 5-18 Experimental results (Actual tip position and positioning error in the Cartesian coordinate system) of (a) Section 1, (b) Section 2 and (c) Section 3 in the global coordinate system.

Based on the experiments on a single section and the tip three sections, it can be found that the position error in X and Y directions of a single section can be maintained within ± 1 mm (1.82%, the position error over the section length), which demonstrates that the empirical controller can address the error caused by PCC-model failure and long cable

elongation. while the mean positioning error (Euclidean distance) of each section of the tip three sections varies from 0.22 mm to 1.0 mm in the local coordinate systems for all three section and the average tip positioning error of the three-section manipulator can be reduced up to 69% in the global coordinate system, compared to model-based open-loop control, which proves the new controller can significantly minimize the multiple-section coupling issue.

However, since the control policies are defined for the discrete sections by workspace division, the empirical controller theoretically suffers from the discontinuity for the target points locating at the section boundaries, though the instable phenomenon was not observed in the experiment.

5.4 Fuzzy-logic-based static controller

To solve the discontinuity problem of the empirical controller, fuzzy logic algorithm is imported to further optimize the model-less controller design, which is to be explicitly introduced in this section.

The overall control architecture is illustrated in Figure 5-19, which consists of two stages, a kinematics-based open-loop stage followed by a close-loop stage implemented by the proposed model-less controller. In the first stage, the desired position (x_d, y_d) is given as the system input, which is used to compute the desired cable displacements with the kinematic model developed in Section 4.2.1. Once all the motors complete the displacements computed based on the kinematic model, the robot reaches a position close to the desired one; then the operation enters into the 2nd stage where the conventional PCC-based controller is substituted with a fuzzy-logic-based approach, leading the robot to the desired position with minimised positional error and preventing the non-convergent problem caused by PCC assumption mismatch (see details in 5.1).

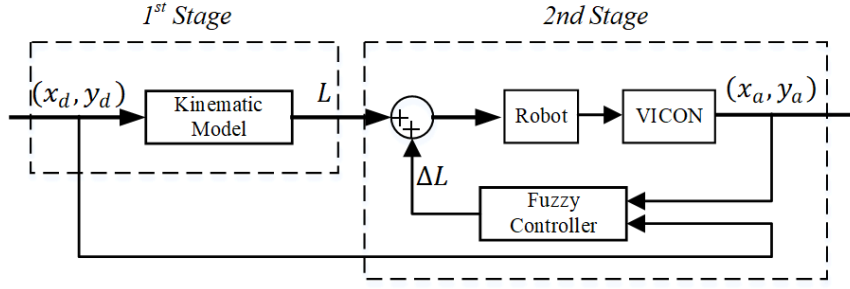


Figure 5-19 Architecture of the control framework including two stages. The purpose of the first stage is to drive the robot tip close to the desired position so that the adjustments of cable displacements in the second stage can be reduced, which can lower the risk of over-tensioning or over-releasing the driving cables in the control process.

5.4.1 Controller design

Based on the analysis of the movement mechanism for tendon-driven CRs, a fuzzy logic algorithm is adopted to design the model-less feedback controller, which excludes the bending angle φ from the controller. In the fuzzy controller design procedure, the first step is to realize the fuzzification of input variables and defuzzification of output variables.

- **Fuzzification and defuzzification**

By describing the tip position in a cylindrical coordinate system (Figure 5-20(a)), the PCC-based kinematics from configuration space to task space can be rewritten as below:

$$\begin{bmatrix} p \\ \varphi \\ z \end{bmatrix} = \begin{bmatrix} S \cdot \frac{1 - \cos \theta}{\theta} \\ \phi \\ S \cdot \frac{\sin \theta}{\theta} \end{bmatrix} \quad (5.2)$$

Since the single section is designed with two-DoF for inextensible CRs, there are only two independent variables among (p, φ, z) in the task space. Actually, (p, φ) is the projection of the point (p, φ, z) on the bottom plane, which can realize full mapping from the spatial workspace to the XY plane. Therefore, the variable z will not be utilized in the controller design.

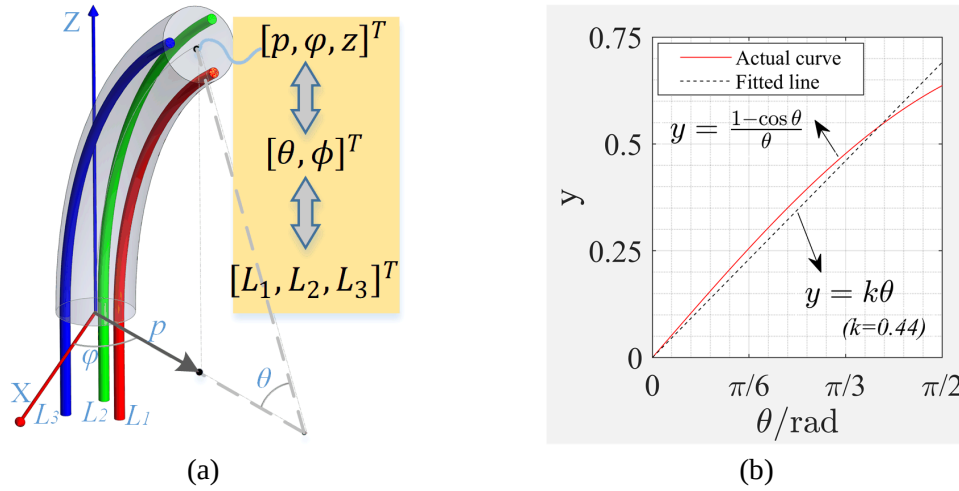


Figure 5-20 (a) Schematics of the kinematic model; (b) the function of the quotient of the radial coordinate p and the backbone length S : $p/S = \frac{1-\cos\theta}{\theta}$. Here the cylindrical coordinate system $[p, \phi, z]^T$ is applied to replace the Cartesian coordinate system $[x, y, z]^T$ in the task space.

Figure 5-20(b) plots the curve of $p/S = \frac{1-\cos\theta}{\theta}$ (red solid line) and the fitted line $y = k\theta$ ($k = 0.44$) (black dash line) when $\theta \in [0, \frac{\pi}{2}]$. It can be observed that $\frac{1-\cos\theta}{\theta}$ is approximately linear to θ . Hence, it can be concluded that the radial distance p is proportional with the bending angle θ with high linearity while the angular position ϕ coincides with the direction angle ϕ , the following relationships can be obtained:

$$\begin{aligned} p &\propto \theta; \phi \equiv \phi \\ dp &\propto d\theta; d\phi \equiv d\phi \end{aligned} \quad (5.3)$$

The vector form of the PCC-based kinematics from configuration space to actuation space is given as:

$$L = S - r\theta \cos(\alpha - \phi) \quad (5.4)$$

where L is the cable length within single section, r denotes the distance from the robot centre to the driving cable, and α is the phase angle depicting the location of the driving cable (Figure 4-6 (b-c)).

The differential component of (5.4) to θ can be derived as below:

$$dL_\theta = -r \cos(\alpha - \phi) \cdot d\theta \quad (5.5)$$

Combining Eq.(5.3) and Eq.(5.5), it can be concluded that

$$dL_\theta \propto -\cos(\alpha - \phi) \cdot dp \quad (5.6)$$

Equation (5.6) implies that given the increment $d\theta$, which is equivalent to dp according to (5.4), the cable displacement is the function of the angular position φ and the radial error increment dp . Hence, the expression of the first controller can be noted as $\Delta l_p = \text{Fuzzy}_p(\varphi_a, e_p)$, where φ_a denotes the actual angular position while e_p is used to replace dp . Similarly, the expression of the second controller for minimising the angular position error component $p_d e_\varphi$ can be obtained: $\Delta l_\varphi = \text{Fuzzy}_\varphi(\varphi_a, p_d e_\varphi)$. Therefore, the input variables for design of the fuzzy controllers are φ_a , e_p and $p_d e_\varphi$, excluding the arc parameters (θ and ϕ) from the controller.

The variation range of the angular position φ is defined to be the interval $[-180^\circ, 180^\circ]$, which covers the whole workspace and is in consistent with the function domain of $\text{atan2}(x, y)$. As shown in Figure 5-21(b), the locations of the cable holes are selected in the whole range of φ to be the centre of 17 districts (N8-N1, S0, P1-P8). The triangular membership function (MF) in Figure 5-21(c) is employed to describe the membership grades of certain points belonging to different districts. Further, the variation of the positional error components $e_p(\text{mm})$ and $p_d e_\varphi(\text{mm})$ in Figure 5-21(b) is categorized into three ranges (Figure 5-21(d)), negative $(-\infty, -0.5]$, zero $[-0.5, +0.5]$ and positive $[+0.5, +\infty)$, which are noted using N, Z and P. Here the threshold value is set to be 0.5 mm, which can be adjusted according to the control accuracy requirement.

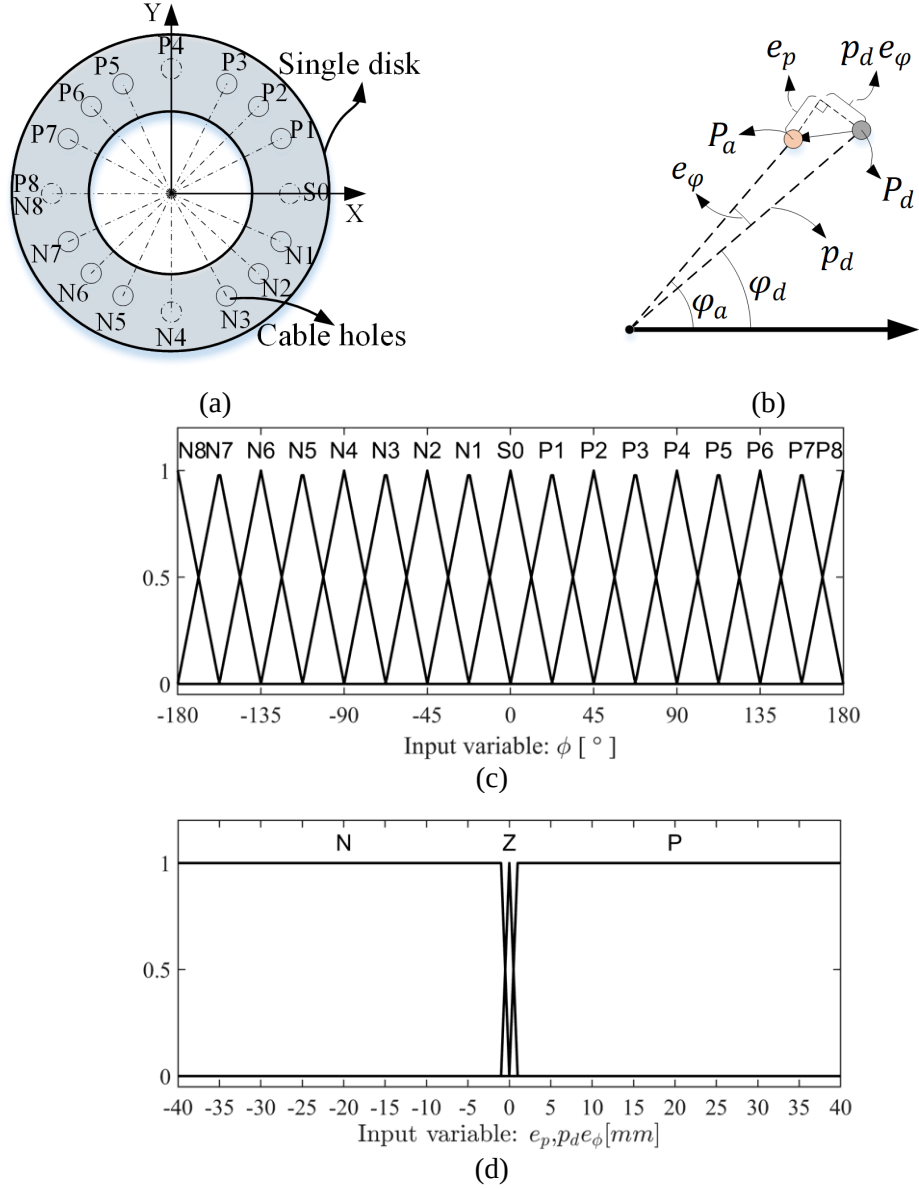


Figure 5-21 (a) Error ($P_a - P_d$) decomposition in the polar coordinate system, the error component in the radial direction is e_p while in the angular direction is $p_d e_\varphi$; (b) District divisions of a single section's workspace based on the distribution of cable holes in the X-Y plane; (c) MFs associated with the angular position φ ; (d) MFs associated with the error components e_p and $p_d e_\varphi$.

Owing to the axisymmetric and rotationally symmetric features for the distribution of cable holes, there are five different cable displacements in any elementary motions (Figure 5-10), regardless of the cable moving directions. Those different cable displacements can be expressed as $\{0, \pm k_1, \pm k_2, \pm k_3, \pm 1\}$ by regularization in the interval of $[-1, 1]$. Since the phase angle of the cable holes $\alpha \in \{-180^\circ, -167.5^\circ, \dots, 167.5^\circ, 180^\circ\}$ and the angular position φ also belongs to the

same set when the projection of the tip position locates at the centre of each district shown in Figure 5-21(a), the variation range of $(\alpha - \phi)$ can be determined, which is consistent with the range of α after regularization in the interval of $[-180^\circ, 180^\circ]$. According to Eq.(5.6), the trigonometric function is utilized to determine the value of k_1 , k_2 and k_3 , which is shown in Figure 5-22(a). The MFs of the outputs for both fuzzy controllers are depicted in Figure 5-22(b), where the terms NF (negative full), NH (negative high), NM (negative medium), NL (negative low), Z (zero), PL (positive low), PM (positive medium), PH (positive high) and PF (positive full) denote $-1, -k_3, -k_2, -k_1, 0, k_1, k_2, k_3, 1$.

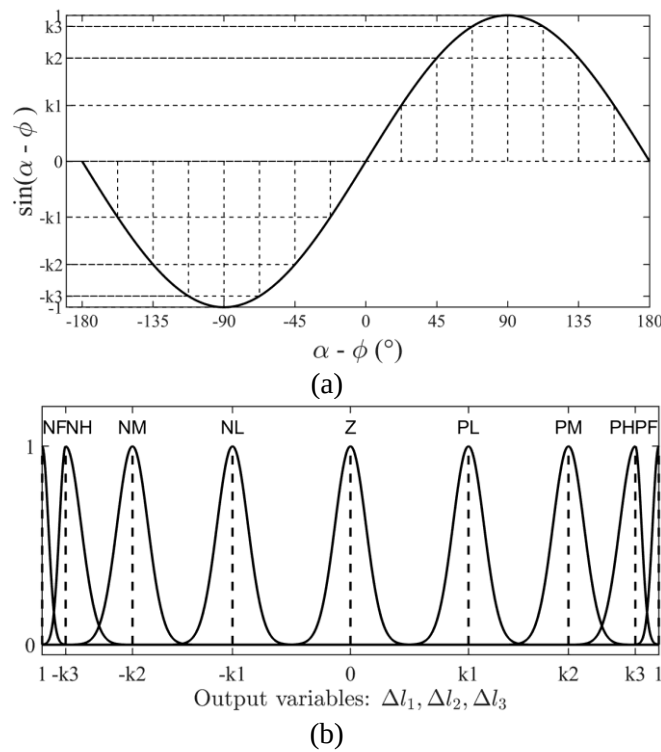


Figure 5-22 (a) Trigonometric function utilized to determine the value of k_1 , k_2 and k_3 ; (b) MFs utilized in the output variables Δl_1 , Δl_2 and Δl_3 for both fuzzy controllers $Fuzzy_p$ and $Fuzzy_\phi$.

- **Construction of fuzzy rules**

In this part, the control rules are set up for a series of discretized points locating at the centre of different districts (N8-N1, S0, P1-P8), which is utilized to develop the nonlinear controllers $Fuzzy_p$ and $Fuzzy_\varphi$ by fuzzy-logic algorithm.

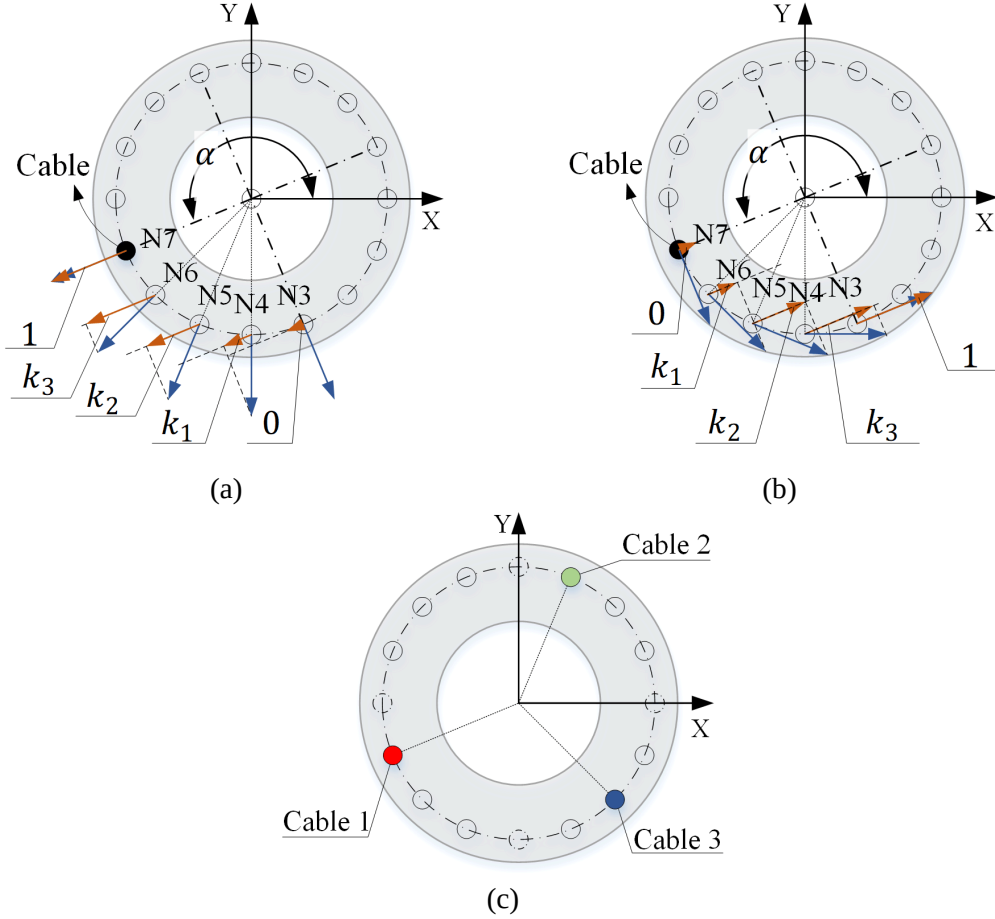


Figure 5-23 Schematic diagram constructing the control policies of (a) $Fuzzy_p$ and (b) $Fuzzy_\varphi$ for the cable located at N7; (c) Cable distribution for section 3.

Without loss of generality, the cable at N7 (in Figure 5-23), which does not locate on the X-axis or Y-axis, is selected to derive the control policies for both the position error components (e_p and $p_d e_\varphi$) in the polar coordinate system. As shown in Figure 5-23(a), the components (brown array) along the vector from the origin to the cable location varies when an identical radial error e_p (blue array) is given to the centre of different districts. It is feasible to empirically assume that the larger the component along the

vector, the greater the cable displacement will be. If the maximum displacement, which is at N7, is regularized to be 1, then for N3-N6 the regularized cable displacements are $0, k_1, k_2, k_3$. Moreover, by the axisymmetric and rotational symmetric features of the robot structure, the regularized cable displacements can be derived based on N3-N7. The cases for cables at other locations can be analysed similarly. Therefore, the control policies of the first controller $Fuzzy_p$ for any cable can be obtained. Likewise, the same analysis method was applied for the derivation of the second controller ($Fuzzy_\varphi$).

Finally, based on the cable distribution of Section 3 in Figure 5-23(c), an example is given for the control policy construction of a single section, which is shown in **Table 5-4** and **Table 5-5**.

Table 5-4 The fuzzy rules for the first controller $\Delta l_p = Fuzzy_p(\varphi_a, e_p)$

φ_a	$e_p = N$			$e_p = Z$			$e_p = P$		
	Δl_p^1	Δl_p^2	Δl_p^3	Δl_p^1	Δl_p^2	Δl_p^3	Δl_p^1	Δl_p^2	Δl_p^3
N8	NH	PL	PM	Z	Z	Z	PH	NL	NM
N7	NF	PM	PL	Z	Z	Z	PF	NM	NL
N6	NH	PH	Z	Z	Z	Z	PH	NH	Z
N5	NM	PF	NL	Z	Z	Z	PM	NF	PL
N4	NL	PH	NM	Z	Z	Z	PL	NH	PM
N3	Z	PM	NH	Z	Z	Z	Z	NM	PH
N2	PL	PL	NF	Z	Z	Z	NL	NL	PF
N1	PM	Z	NH	Z	Z	Z	NM	Z	PH
S0	PH	NL	NM	Z	Z	Z	NH	PL	PM
P1	PF	NM	NL	Z	Z	Z	NF	PM	PL
P2	PH	NH	Z	Z	Z	Z	NH	PH	Z
P3	PM	NF	PL	Z	Z	Z	NM	PF	NL
P4	PL	NH	PM	Z	Z	Z	NL	PH	NM
P5	Z	NM	PH	Z	Z	Z	Z	PM	NH
P6	NL	NL	PF	Z	Z	Z	PL	PL	NF
P7	NM	Z	PH	Z	Z	Z	PM	Z	NH
P8	NH	PL	PM	Z	Z	Z	PH	NL	NM

Table 5-5 The fuzzy rules for the second controller
 $\Delta l_\varphi = \text{Fuzzy}_\varphi(\varphi_a, pe_\varphi)$

φ_a	$pe_\varphi = N$			$pe_\varphi = Z$			$pe_\varphi = P$		
	Δl_φ^1	Δl_φ^2	Δl_φ^3	Δl_φ^1	Δl_φ^2	Δl_φ^3	Δl_φ^1	Δl_φ^2	Δl_φ^3
N8	NL	PH	NM	Z	Z	Z	PL	NH	PM
N7	Z	PM	NH	Z	Z	Z	Z	NM	PH
N6	PL	PL	NF	Z	Z	Z	NL	NL	PF
N5	PM	Z	NH	Z	Z	Z	NM	Z	PH
N4	PH	NL	NM	Z	Z	Z	NH	PL	PM
N3	PF	NM	NL	Z	Z	Z	NF	PM	PL
N2	PH	NH	Z	Z	Z	Z	NH	PH	Z
N1	PM	NF	PL	Z	Z	Z	NM	PF	NL
S0	PL	NH	PM	Z	Z	Z	NL	PH	NM
P1	Z	NM	PH	Z	Z	Z	Z	PM	NH
P2	NL	NL	PF	Z	Z	Z	PL	PL	NF
P3	NM	Z	PH	Z	Z	Z	PM	Z	NH
P4	NH	PL	PM	Z	Z	Z	PH	NL	NM
P5	NF	PM	PL	Z	Z	Z	PF	NM	NL
P6	NH	PH	Z	Z	Z	Z	PH	NH	Z
P7	NM	PF	NL	Z	Z	Z	PM	NF	PL
P8	NL	PH	NM	Z	Z	Z	PL	NH	PM

- **Integration of the fuzzy-logic controller:**

Figure 5-24 illustrates the block diagram of the fuzzy-logic controller which integrates two sub-controllers together (Fuzzy_p and Fuzzy_φ). The desired tip position (x_d, y_d) and the actual tip position (x_a, y_a) are converted to polar coordinates (p_d, φ_d) and (p_a, φ_a) to compute the radial position error $e_p = p_a - p_d$ and the angular position error $p_d e_\varphi = p_d(\varphi_a - \varphi_d)$. Then (φ_a, e_p) and $(\varphi_a, p_d e_\varphi)$ are utilised as the inputs of the 1st controller $\Delta l_p = \text{Fuzzy}_p(\varphi_a, e_p)$ and the 2nd controller $\Delta l_\varphi = \text{Fuzzy}_\varphi(\varphi_a, p_d e_\varphi)$. Since the outputs of both sub-controllers, Δl_p and Δl_φ , are regularized within $[-1, 1]$, the coefficients μ_p and μ_φ are utilised to convert the outputs to the cable displacements. For simplicity, a fixed-step control is achieved by setting μ_p and μ_φ to be static ($\mu_p = \mu_\varphi = 0.01 \text{ mm}$) in this work, which can also be adjusted dynamically to be proportional to the value of e_p and $p_d e_\varphi$ to increase the speed of convergence.

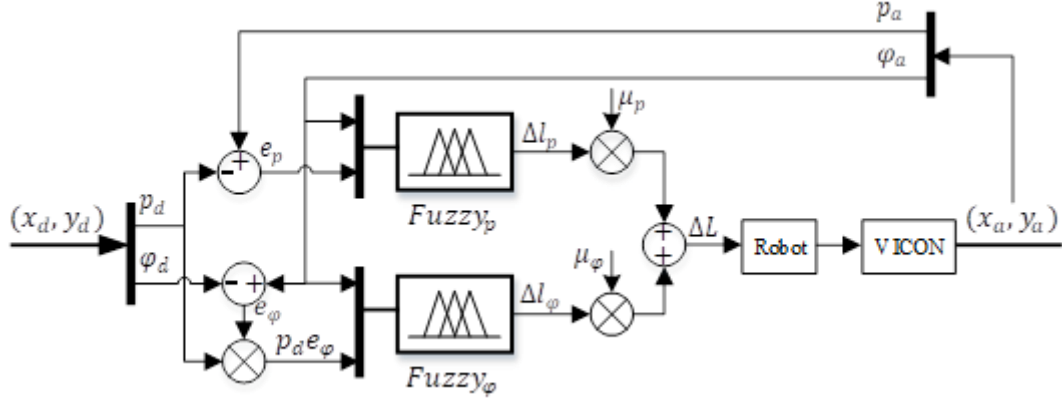


Figure 5-24 Block diagram of the fuzzy-logic controller. The controller is made up of two sub-controllers, corresponding to the error correction in the radial and angular directions in the polar coordinate system.

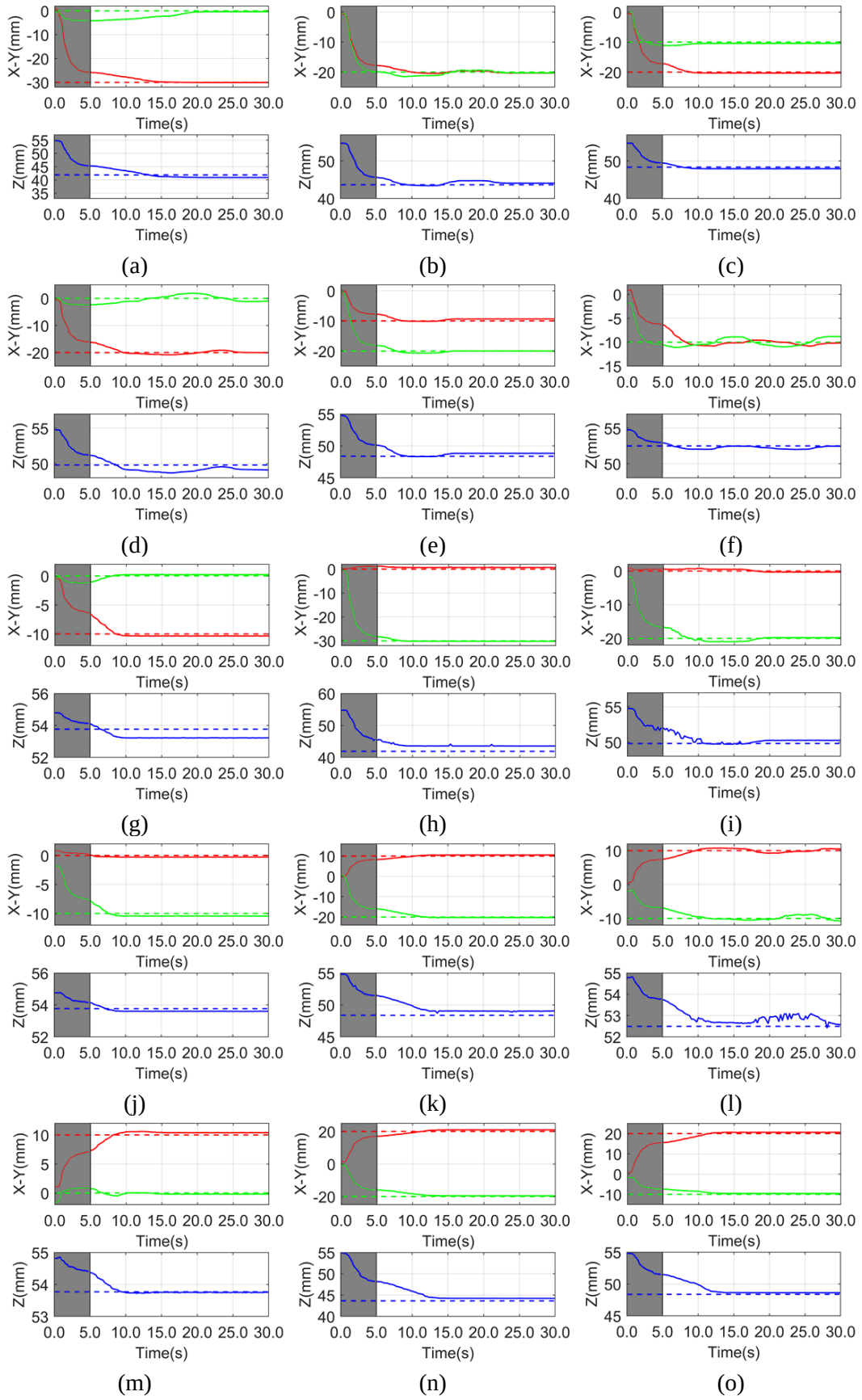
5.4.2 Experimental validation

In this section, both the proposed fuzzy-logic controller and the PCC-based approach were implemented on a single tip section of the slender continuum robot to evaluate the impact of the mismatch. Then, a set of points evenly distributed in the workspace were tested as the target position to characterize the overall positioning performance of the fuzzy-logic controller, thus the actual workspace of the single section was obtained. Besides, several trials applying the proposed controller on two sections were conducted to explore its capability for diminishing the section coupling issue.

- **Tests on single section**

- a) **Positioning performance**

The proposed fuzzy-logic controller (Figure 5-24) was firstly implemented for tip positioning. The experimental results are shown in Figure 5-25, where the subfigures (a)-(q) correspond to the target points (a-q) marked in Figure 5-6(b). Each subfigure depicts the actual Cartesian coordinates of the section tip in X (in red), Y (in green) and Z (in blue) axes in 30 seconds, which consists of two stages: the open-loop stage (the grey area) and the fuzzy-logic-based close-loop stage (the white area). The experiment illustrates that the controller can keep convergent for all the target points.



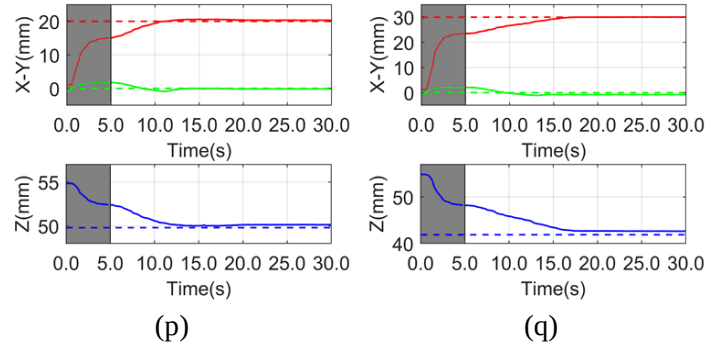


Figure 5-25 Experiments of tip positioning with the proposed fuzzy-logic-based static controller. The subfigures (a)-(q) correspond to the target points (a-q) in Figure 5-6 (b).

b) Overall positioning performance

Further, the proposed controller is applied on a set of target points marked with blue circles in Figure 5-26(a) to comprehensively characterize its positioning performance. Figure 5-26(b) depicts the error map in both X and Y axes after the controller running for 20 seconds at every target point, which indicates that the overall positioning accuracy of the controller can reach $\pm 1 \text{ mm}$ in X and Y directions while for 75% points the positioning error can be kept within 0.5 mm along both axes. After the test, the actual workspace was obtained, which is marked with red stars in Figure 5-26(a). In addition, there exists remarkable offset in the Z axis between the PCC-based workspace and the actual workspace of the robot, demonstrating the PCC assumption mismatch.

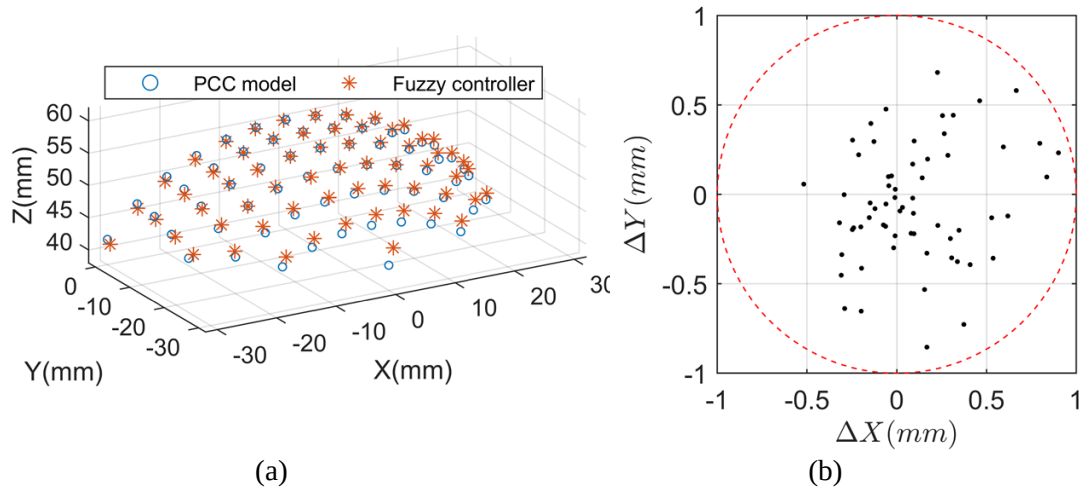


Figure 5-26 (a) Comparison between theoretical PCC-based workspace and actual workspace measured by the fuzzy controller; (b) The scatter diagram showing the positioning accuracy of all the 63 testing points in the X-Y plane.

- **Trials on two sections**

Although the control performance and stability of the proposed controller has been validated on a single section, there still exists uncertainty whether it can take effect when applied on multiple sections owing to the section coupling issue presented in 5.1. Hence, four trials on two sections have been conducted to research the effect of the controller on this issue. The target point's configurations of both sections for the trails are given in Table 5-6.

Table 5-6 Target points configuration for the trails on two sections

	Section 2			Section 3		
	X(mm)	Y(mm)	Z(mm)	X(mm)	Y(mm)	Z(mm)
Trial 1	10	-10	52.50	15	-15	49.10
Trial 2	10	-10	52.50	-15	-15	49.10
Trial 3	10	-20	48.38	15	-15	49.10
Trial 4	10	-20	48.38	-15	15	49.10

Fig. 19(a)-(d) present the experimental results of Trials 1-4. The adjustment processes in yellow area in (a) and (c) indicate that both sections affect each other during their motion to reduce the error under the control of the fuzzy controller and thus verify the existence of the kinematic section coupling issue, which also demonstrates the proposed

controller can effectively reduce the positioning error (median positioning error: 0.7 mm; minimal positioning error: 0.13 mm) along X and Y axes in local coordinate system for both sections.

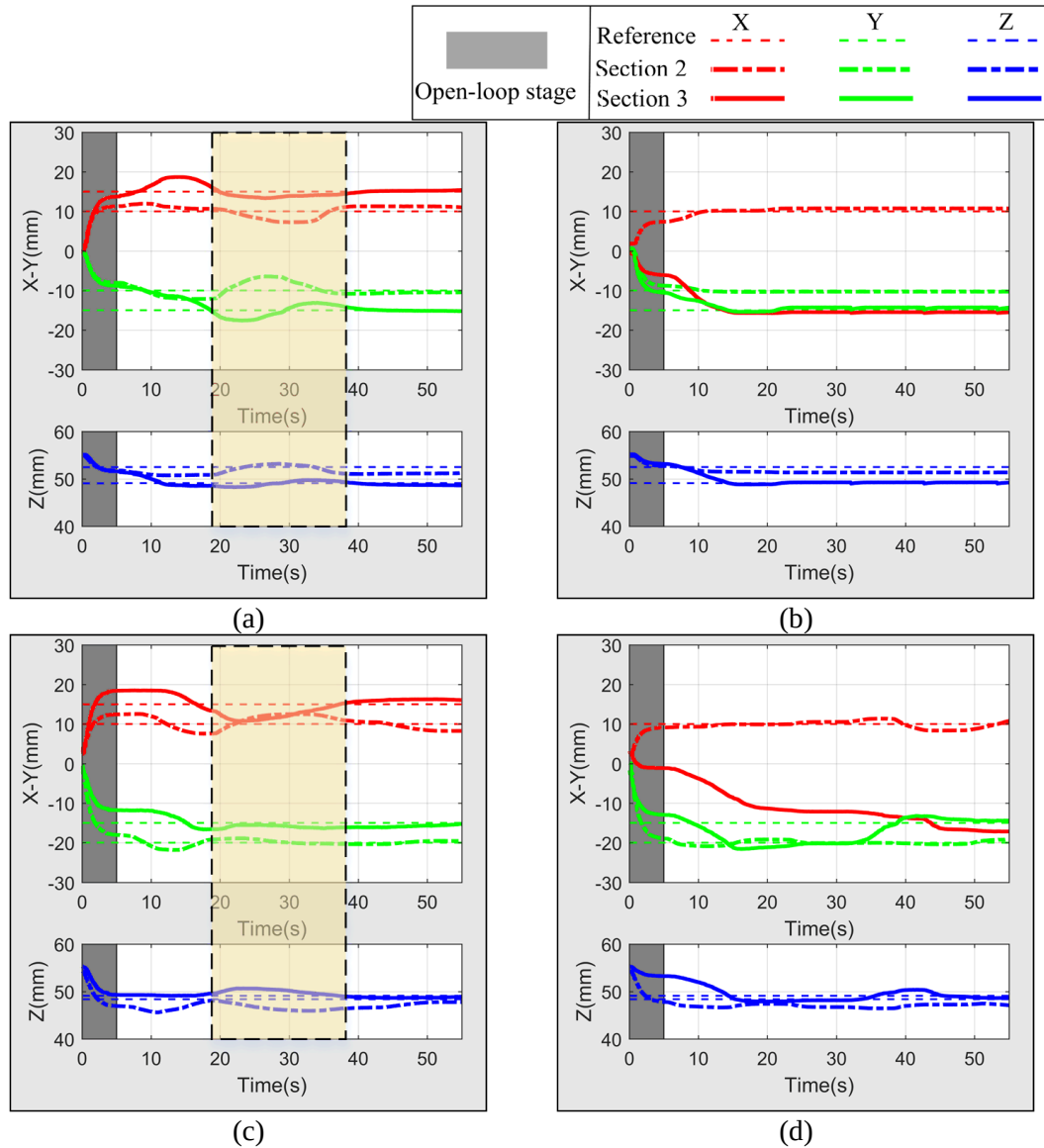


Figure 5-27 Experimental results on two tip sections: (a) Trial 1, (b) Trial 2, (c) Trial 3 and (d) Trial 4. The yellow area marked in (a) and (c) reveals the physical coupling issue between two sections, which will in turn affect the stability and converging speed of each section.

These trials demonstrate the efficacy of the proposed controller with respect to the kinematic section coupling issue. However, it should also be recognized that the performance of the proposed controller is greatly reduced (maximum positioning error: -2.17 mm) by the physical coupling issue. A possible solution is to adopt the gradient

stiffness of the robot along the length to physically reduce the interference between sections, which could be applied in the future design.

5.5 Chapter summary

This chapter has focused on the design and validation of novel model-less controllers for the tip sections control of Prototype One. Three main control challenges have been analysed qualitatively for slender long tendon-driven continuum robots, including the model mismatch, the cable elongation as well as the section coupling issue. Since the first and second challenges imply that the conventional model-based approach may lose effectiveness on slender continuum robots, the preliminary test is conducted with the PCC-based static controller. The result shows the controller fails to converge at some of the testing points (12 out of 17) caused by the PCC assumption mismatch, verifying the PCC assumption mismatch issue on the prototype and illustrating the necessity of developing novel model-less controllers.

The development process of the model-less controllers consists of two stages. In the first stage, an empirical model-less controller is designed based on a series of control policies directly from the task space (tip positional error) to actuation space (cable displacements), which are inspired by the tug of war. Experiments on single- and multi-sections have been conducted to validate the effectiveness of the empirical controller. Further, in order to solve the discontinuity problem of the empirical controller, fuzzy logic algorithm is imported to optimize the controller in the second stage. The proposed fuzzy-logic controller was tested on a single section of the slender continuum robot, which can reach $\pm 1\text{ mm}$ overall positioning accuracy and $\pm 0.5\text{ mm}$ for 75% test points. Further, the controller was performed on two sections, demonstrating the capability of the controller on minimising the section coupling issue.

Since it has been revealed that the proposed model-less controllers cannot completely eliminate the effect of the section coupling issue, future work can come to the mechanical improvement of the slender continuum robots to solve the problem. One possible solution is to apply the gradient stiffness design for the tip sections of slender continuum robots, where the physical coupling can be weakened by reducing the cable tension for distal sections. In addition, hybrid actuation approaches can also be considered for multiple sections by applying pneumatic actuation for distal sections, which can fully avoid the mechanical interactions between sections.

6 Novel model-based hybrid control of single-section CRs

Continuum robots can not only serve as specialist tools to perform tasks in narrow space with multiple serial sections, but also be mounted on rigid-link manipulators as end effector in the form of single section or multiple parallel sections. Compared with the former form, the latter has better controllability owing to its shorter overall length, which shows great potentials in applications such as minimal invasive surgeries and object grabbing.

This chapter focuses on developing high-performance universal controllers for precise manipulation of single-section tendon-driven continuum robots. On the one hand, by replacing the Cartesian coordinate system with the Polar coordinate system in the task space and applying accurate piecewise linear approximation, an analytical differential kinematic model is developed directly from task space to actuation space, based on which a novel static feedback controller is proposed. On the other hand, tension supervision is integrated into the controller for hybrid control (or called as cascade control since the position control (task space) and tension supervision (actuation space) are running at different level), preventing the driving cables from being slack owing to the cumulative errors of cable displacements. For the purpose of comparison, both the PCC-based and the proposed controllers are implemented on the twin-pivot design. Further, the performance of the proposed controller is comprehensively characterised on the dual-revolute design and the soft robot to prove its versatility for various designs of continuum robots.

6.1 Preliminary analysis of single-section continuum robots

In this section, the common control modes of tendon-driven continuum robots are firstly presented. Then, combining the mechanical characteristics of single-section continuum

robots, a novel hybrid control scheme is developed, which will be integrated into the novel model-based controllers design in the following sections.

6.1.1 Common control modes

In general, the control algorithms of tendon-driven continuum robots are mostly based on cable displacements (Figure 6-1(a)), which are the only variables controlled directly at the hardware level. This control mode is used in most of the research for close-loop control of continuum robots as it is easy to implement and control. Besides, considering that the shape deformation of continuum robots is essentially the result of the interactions between the robots and driving cables, an alternative control mode of continuum robots is by cable tensions (Figure 6-1(b)). However, this mode is hardly applied in feedback control owing to the great challenge of designing controllers between the robot shape (tip position/orientation) and cable tensions, especially for multi-section continuum robots where complex mechanics interactions exist between sections and cables.

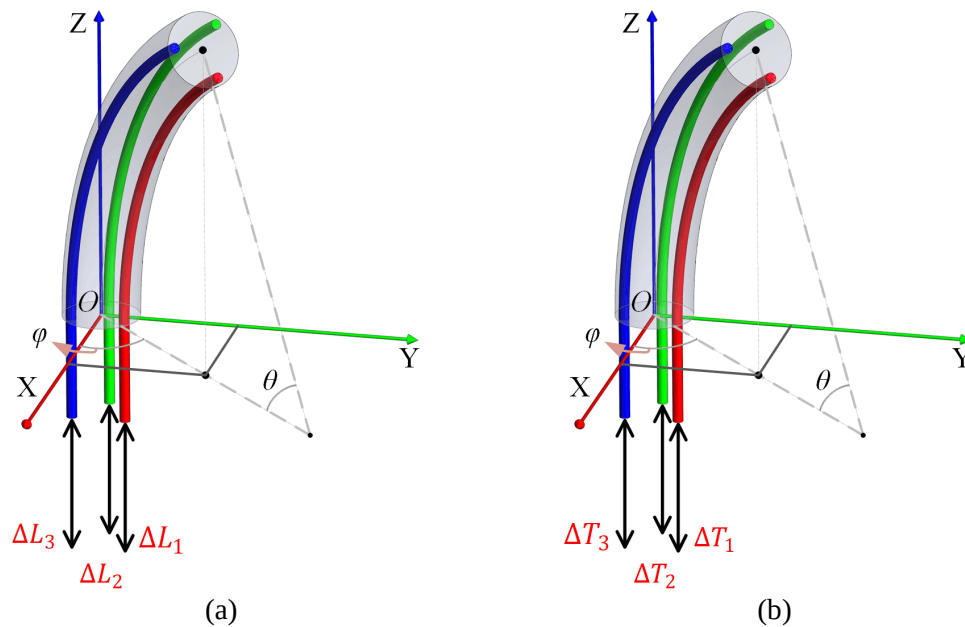


Figure 6-1 Illustration of two control modes for tendon-driven continuum robots: (a) by cable displacements ($\Delta L_1, \Delta L_2, \Delta L_3$); (b) by cable tensions ($\Delta T_1, \Delta T_2, \Delta T_3$).

6.1.2 Novel hybrid control scheme of single-section continuum robots

Regarding single-section continuum robots, the tension loss of driving cables is negligible. As such, the actual cable tensions can be measured relatively accurately with load cells at the actuation end. Therefore, cable tensions can also be utilized to design controllers for single-section continuum robots.

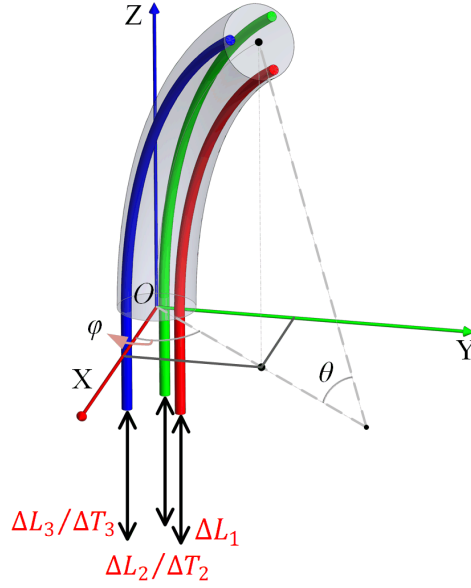


Figure 6-2 Schematic diagram of hybrid control scheme for tendon-driven continuum robots.

One example is given in Figure 6-2 to illustrate the schematic diagram of hybrid control. For the robot configuration indicated in the figure, the red cable dominates the section's bending motion, while the other two cables (green and blue) are responsible for the circumferential motion. However, it is obvious that there is no need to control the green and blue cables simultaneously for the circumferential motion. Actually, the section motion can be fully operated with two active cables at any moment, with the third passive cable in a proper tension. With such control scheme, the displacements of three driving cables can be decoupled in the kinematic level. More importantly, by importing tension control, to be more precise, tension supervision, ill cables states, such as over-tension or slack, can be avoided.

6.2 Model-based hybrid controllers

In this section, two different model-based hybrid controllers are introduced with tension supervision integrated. The first controller is designed with the PCC assumption, where the arc parameters (bending angle and direction angle) are explicitly called in the configuration space. While the second controller is developed based on the novel modified differential kinematics, which achieves close-loop control directly from the task space to actuation space.

6.2.1 PCC-based controller with tension supervision

Figure 6-3 shows the control diagram integrating the tension supervision into the conventional PCC-based static controller introduced in Section 5.2.1. In the figure, the desired cable length L_d and actual cable length L_a are computed utilizing the inverse kinematics from task space to configuration space (“T·S-C·S IK”) and from configuration space to actuation space (“C·S-A·S IK”), whose difference ΔL will be multiplied by the gain coefficient $K (\in (0,1])$ and then be regulated by the tension supervision block.

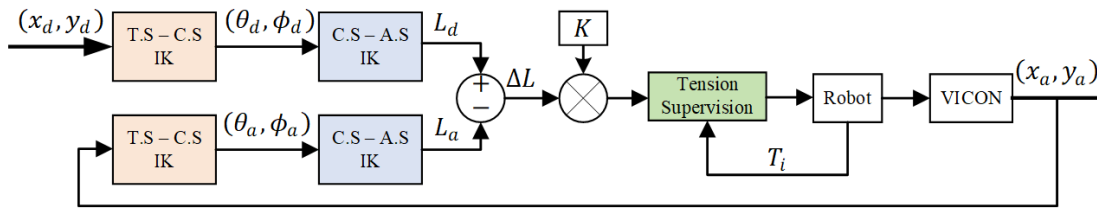


Figure 6-3 Control architectures of the PCC-based hybrid static controller. In the controller, the arc parameters in configuration space are used as a transition to perform the differential inverse kinematics from task space to actuation space.

With regard to the tension supervision block for hybrid control, it is worth mentioning that maintaining a certain cable with a constant tension is unreachable for the control of continuum robots since the state switching between active/passive cables in Section 6.1.2 occurs to all three cables when the robot moves in the whole workspace and the movement direction of the section tip varies. Instead, a lower limit τ is set to the

tensions T_i of all the cables to prevent them from being slack, which are implement as below in the form of pseudo-code:

```

for  $i = 1, 2, 3$                                 %  $i$ : cable index
    if ( $T_i \leq \tau$  &  $d\delta L_i > 0$ )             %  $d\delta L_i$ : change of cable displacements
         $d\delta L_i = 0$ 
    endif
endfor

```

6.2.2 Novel modified kinematic controller with tension supervision

Although the conventional PCC-based model is widely used on continuum robots, there still exists some problems that need to be resolved. On one hand, the conventional kinematic model explicitly calls the arc parameters (θ, φ) in configuration space, which contributes to the model mismatch in the task space. On the other hand, when deriving the Jacobian matrix from the kinematic model, its analytical expression only exists on the condition that the driving cables are evenly distributed around the central longitudinal axis. Therefore, the novel modified kinematic model is proposed to address the above-mentioned challenges.

- **Formulation of the modified kinematic model**

The schematic diagram of the modified kinematic model is illustrated in Figure 6-4, where the tip position is described with cylindrical coordinates (p, φ, z) instead of Cartesian coordinates (x, y, z) (Figure 6-3). Further, since single section of inextensible continuum robots has only two DoFs, the variables (p, φ, z) in task space are not totally independent from each other. Actually, once (p, φ) is chosen, the value of z is determined, even though the expression of the function $z = f(p, \varphi)$ is unknown. Hence, the parameter pair of (p, φ) is necessary and sufficient for the derivation of the modified kinematic model.

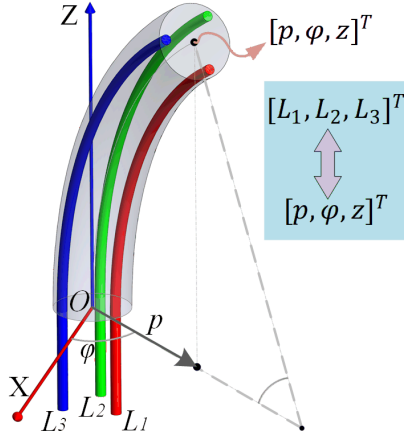


Figure 6-4 Schematic of the modified kinematic model. The target tip position is converted from Cartesian coordinates to Polar coordinates, with which the direct inverse kinematics is implemented without the explicit call of arc parameters.

Firstly, based on the constant-curvature assumption, the kinematics from configuration space to task space can be written as:

$$\begin{bmatrix} p \\ \varphi \end{bmatrix} = \begin{bmatrix} S \cdot \frac{1 - \cos \theta}{\theta} \\ \theta \end{bmatrix} \quad (6.1)$$

where S denotes the section length.

Figure 6-5(a) plots the curve (in red) of function of $f(\theta) = \frac{p}{S} = \frac{1 - \cos \theta}{\theta} \left(\theta \in \left[0, \frac{\pi}{2}\right] \right)$, which is nonlinear but highly positively related to θ . Thus, a linear fitting approach is considered to linearize $f(\theta)$ for the benefit of following derivation. To reach low deviation ratio $e(\theta) = \left| \frac{f(\theta) - l(\theta)}{f(\theta)} \right|, \forall \theta \in \left[0, \frac{\pi}{2}\right]$, a piece-wise line with three segments is adopted for the curve fitting. The whole variation range of θ is divided into three subsets: $U_1 = [0, \frac{\pi}{6})$, $U_2 = [\frac{\pi}{6}, \frac{\pi}{3})$, $U_3 = [\frac{\pi}{3}, \frac{\pi}{2}]$. In each subset, $f(\theta)$ is approximated by single segment whose endpoints overlap with those of $f(\theta)$. Figure 6-5(b) gives the deviation ratio in the whole range, of which the maximum deviation ratio (2.26%) is below 2.5%.

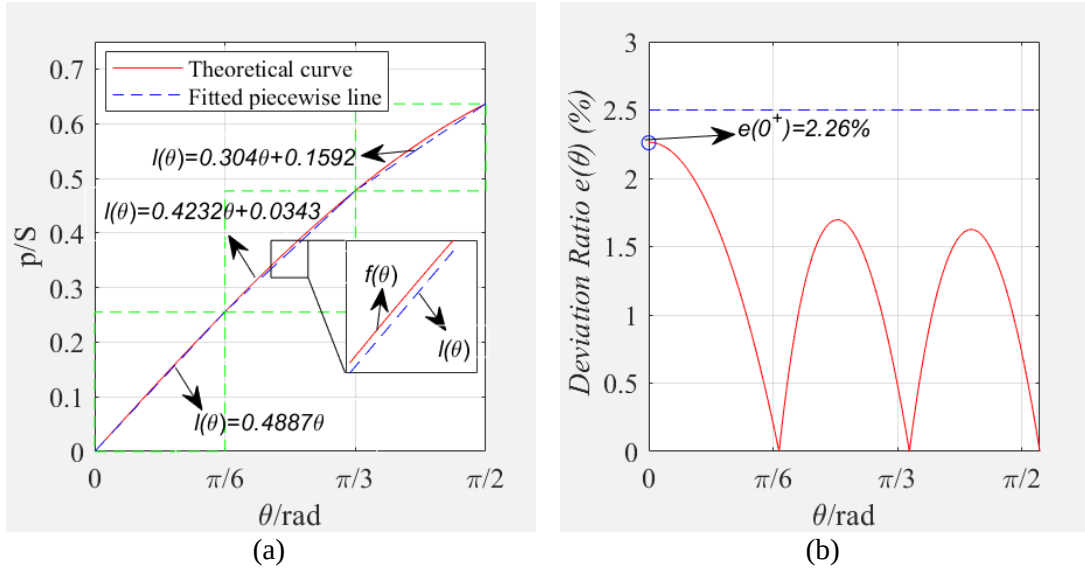


Figure 6-5 Curve fitting of the function $f(\theta)$: (a) $f(\theta)$ is approximated by three segmented lines $l(\theta)$; (b) the maximum fitting error $e(\theta)$ is below 2.5%.

Thus, $f(\theta)$ can be linearized with high accuracy as below:

$$f(\theta) \approx l(\theta) = k_j \theta + b_j, \theta \in U_j (j = 1, 2, 3) \quad (6.2)$$

$$\text{where } \begin{bmatrix} k_1 & b_1 \\ k_2 & b_2 \\ k_3 & b_3 \end{bmatrix} = \begin{bmatrix} 0.4887 & 0 \\ 0.4232 & 0.0343 \\ 0.304 & 0.1592 \end{bmatrix}.$$

Combining Eq.(6.1) and Eq.(6.2), the expression of the radical coordinate p can be rewritten as:

$$p = S \cdot (k_j \theta + b_j), \theta \in U_j (j = 1, 2, 3) \quad (6.3)$$

Hence, the derivative of p with respect to the bending angle θ can be obtained:

$$\frac{dp}{d\theta} = S k_j, \theta \in U_j (j = 1, 2, 3) \quad (6.4)$$

Equation (6.5) gives the kinematics from configuration space (θ, φ) to actuation space (L_1, L_2, L_3) :

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} S \\ S \\ S \end{bmatrix} - r\theta \cdot \begin{bmatrix} \cos(\alpha_1 - \varphi) \\ \cos(\alpha_2 - \varphi) \\ \cos(\alpha_3 - \varphi) \end{bmatrix} \quad (6.5)$$

where r denotes the radius of circle on which all the driving cables are distributed while $\alpha_1, \alpha_2, \alpha_3$ are the phase angles with respect to X axis in XY plane indicating the location of each cable.

Since continuum robots are usually initialized to be straight, the initial values of L_1, L_2 and L_3 are all equal to S . Therefore, the increment of all the cables within the continuum robot can be written as below:

$$\begin{bmatrix} \delta L_1 \\ \delta L_2 \\ \delta L_3 \end{bmatrix} = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} - \begin{bmatrix} S \\ S \\ S \end{bmatrix} \quad (6.6)$$

Put Eq.(6.5) into Eq.(6.6), we can get:

$$\begin{bmatrix} \delta L_1 \\ \delta L_2 \\ \delta L_3 \end{bmatrix} = -r\theta \cdot \begin{bmatrix} \cos(\alpha_1 - \varphi) \\ \cos(\alpha_2 - \varphi) \\ \cos(\alpha_3 - \varphi) \end{bmatrix} \quad (6.7)$$

Then, we can start to derive the differential kinematics from task space (p, φ) to actuation space (L_1, L_2, L_3) , based on which the feedback controller is designed. The differential of the cable displacement changes $d\delta L_i$ with respect to the polar coordinates p and φ can be written as below:

$$d\delta L_i = \frac{\partial \delta L_i}{\partial p} \cdot dp + \frac{\partial \delta L_i}{\partial \varphi} \cdot d\varphi, i = 1, 2, 3 \quad (6.8)$$

According to the chain rule for differential, together with Eq.(6.4), the first coefficient on the right side of Eq.(6.8) can be computed as following:

$$\begin{aligned}\frac{\partial \delta L_i}{\partial p} &= \frac{\partial \delta L_i}{\partial \theta} \bigg/ \frac{dp}{d\theta} = \frac{r}{S k_j} \cdot \cos(\alpha_i - \varphi) \\ \frac{\partial \delta L_i}{\partial \varphi} &= \frac{r}{k_j} \cdot \left(\frac{p}{S} - b_j\right) \cdot \sin(\alpha_i - \varphi)\end{aligned}\tag{6.9}$$

Noticing that θ is not explicitly called in Eq.(6.9), which is required for the selection of k_j . According to Eq.(6.3), p is positively correlated to θ with high linearity, which can also be utilized to determine the value of k_j . The interval mapping relationship of θ and p is given below:

$$\begin{aligned}U_1 &= [0, \frac{\pi}{6}) \leftrightarrow \Omega_1 = S \cdot [0, 0.2559) \\ U_2 &= [\frac{\pi}{6}, \frac{\pi}{3}) \leftrightarrow \Omega_2 = S \cdot [0.2559, 0.4775) \\ U_3 &= [\frac{\pi}{3}, \frac{\pi}{2}] \leftrightarrow \Omega_3 = S \cdot [0.4775, 0.6366]\end{aligned}$$

Thus, we obtain the explicit expression of the differential kinematics directly from task space to actuation space:

$$d\delta L_i = \frac{r}{k_j} \cdot \left[\frac{\cos(\alpha_i - \varphi)}{S} \cdot dp + \left(\frac{p}{S} - b_j\right) \cdot \sin(\alpha_i - \varphi) \cdot d\varphi \right] \tag{6.10}$$

Here, it can be found that a direct mapping between actuation and task space is established, while the bending and direction angles are not involved.

Further, since Cartesian coordinate system is more intuitive and commonly used for both input and feedback, the following transformations are necessary for algorithm implementation:

$$\begin{aligned}p &= \sqrt{x_d^2 + y_d^2} \\ \varphi &= \tan^{-1}\left(\frac{y_d}{x_d}\right)\end{aligned}$$

$$dp = \sqrt{x_a^2 + y_a^2} - \sqrt{x_d^2 + y_d^2}$$

$$d\varphi = \text{atan2}(y_a, x_a) - \text{atan2}(y_d, x_d)$$

where (x_d, y_d) denotes the desired tip position while (x_a, y_a) is the actual tip position. In addition, since the angular coordinate $\varphi \in (-\pi, \pi]$, the domain of $d\varphi$ becomes discontinuous for quadrant switch between quadrant 2 and quadrant 4, which can be solved by the rules shown below:

$$d\varphi < -\pi, d\varphi + 2\pi \rightarrow d\varphi$$

$$d\varphi > \pi, d\varphi - 2\pi \rightarrow d\varphi$$

- **Controller design**

Based on the modified kinematic model, a novel static hybrid controller can be developed, which is shown in Figure 6-6. In the controller, the target position (x_d, y_d) and actual position (x_a, y_a) is firstly converted to (p_d, φ_d) and (p_a, φ_a) from the Cartesian coordinates to the Polar coordinates. Then the desired position (p_d, φ_d) and position error $(\Delta p, \Delta \varphi)$ are used as the input of the modified differential inverse kinematics directly from task space to actuation space (“T·S-A·S Differential IK”), whose output, the cable displacements ΔL , will multiply by the gain coefficient $\eta (\in (0,1])$ and then be regulated by the tension supervision block introduced in 6.2.1.

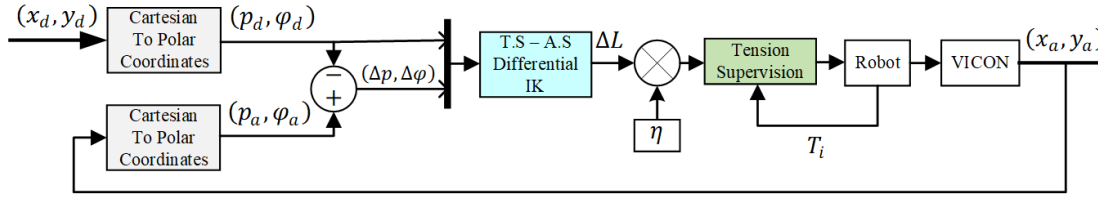


Figure 6-6 Control architectures of the novel hybrid modified kinematic controller. In the controller, the target tip position is converted from Cartesian coordinates to Polar coordinates, with which the direct inverse kinematics is implemented without the explicit call of arc parameters.

6.3 Experimental validation

In this section, a series of experiments are conducted to validate the hybrid controllers developed in previous section. The test-rig is firstly introduced, which can be used as a universal operating platform for continuum robots. Then, for the purpose of comparison, both the PCC-based and proposed hybrid controllers are implemented on the twin-pivot design for tip positioning and path tracking. Further experiments are also conducted on the other two prototypes (dual-revolute design and soft robot) to prove the versatility of the proposed controller.

6.3.1 Test-rig setup

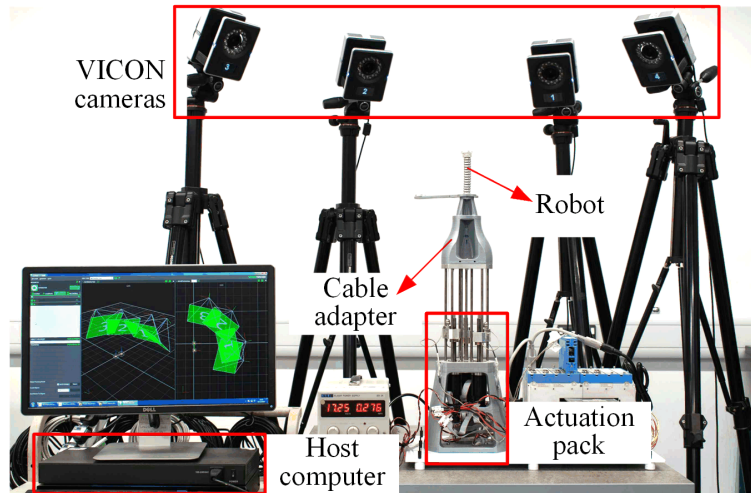


Figure 6-7 Test-rig setup.

The whole test-rig is shown in Figure 6-7, which consists of a single-section continuum robot, an actuation system, and a host controller as well as a visual feedback device.

The actuation system is made up of a cable adapter and three sets of motor modules. The adapter is 3D printed by plastic with three channels inside to guide the driving cables from the robot to the motor modules smoothly. While each motor module is made up of a Maxon DC motor, an EPOS2 50/5 position controller and a load cell (LCM201-200N). The load cell is taken as the intermediate between the slider and the driven cable, with one side mounted on the slider and the other side connected to the cable for tension measurement.

Three prototypes of continuum robots for testing are illustrated in Figure 6-8. The size of 3D-printed adapter is fine-tuned to fit different prototypes. Moreover, two sets of VICON reflective markers are used for measurement. One set locates on a cantilever fixedly connected to the adapter for reference, while the other set is place on the tip for position and orientation measurement.

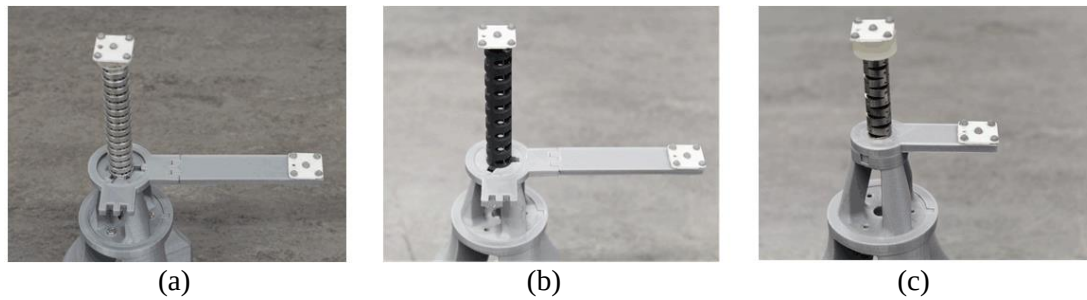


Figure 6-8 Three prototypes of continuum robots for testing: (a) the twin-pivot design; (b) the dual-revolute design; (c) the 3D-printed soft robot.

6.3.2 Experiments for tip positioning

- **Parameter tuning**

In order to achieve the optimal performance, the parameter tuning process is needed for the gain coefficients K in the PCC-based controller and η in the proposed controller before the comprehensive experiments.

Without loss of generality, two sets of motions, one with small bending angle $((-5,5) \rightarrow (-10,10))$ and the other one with large bending angle $((-30,30) \rightarrow$

$(-35,35)$), are chosen for two controllers. Figure 6-9 illustrates the results of both motions for the PCC-based controller, with K varying between 0.05, 0.1 and 0.3. It can be found that the optimal value of K is 0.1 from the perspective of responding speed and stability. Similarly, as shown in Figure 6-10, the optimal value is 0.02 for $\eta(\in \{0.01,0.02,0.04\})$ in the proposed controller.

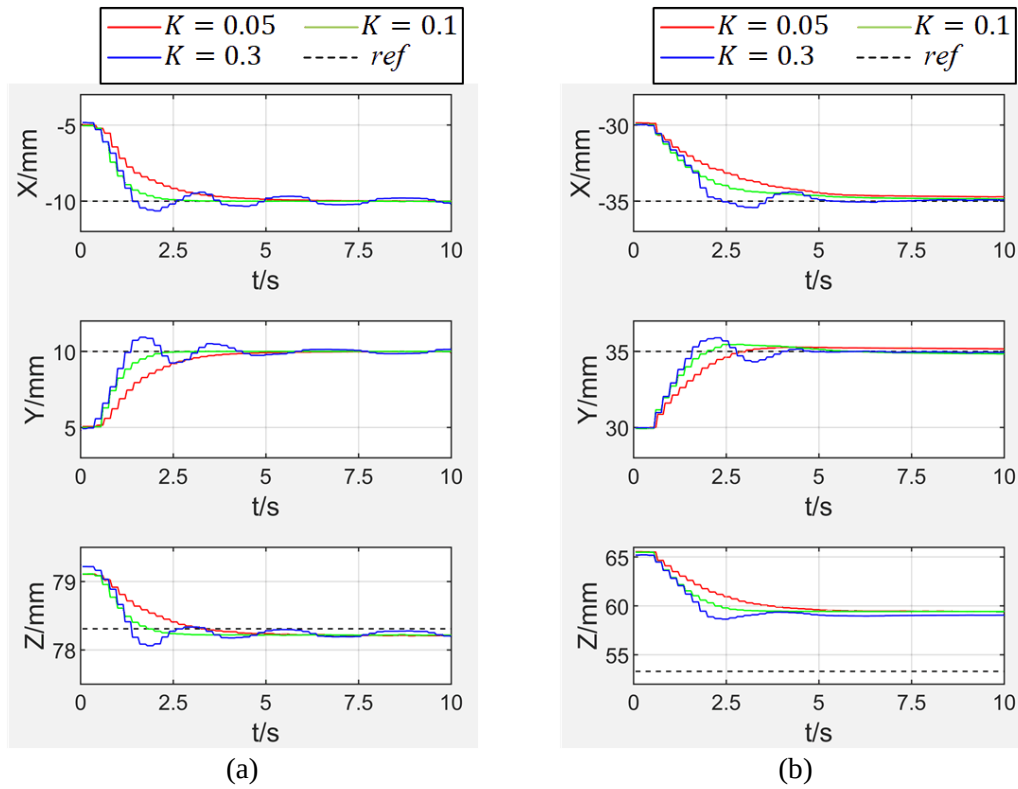


Figure 6-9 Parameter tuning of K in the PCC-based hybrid controller.

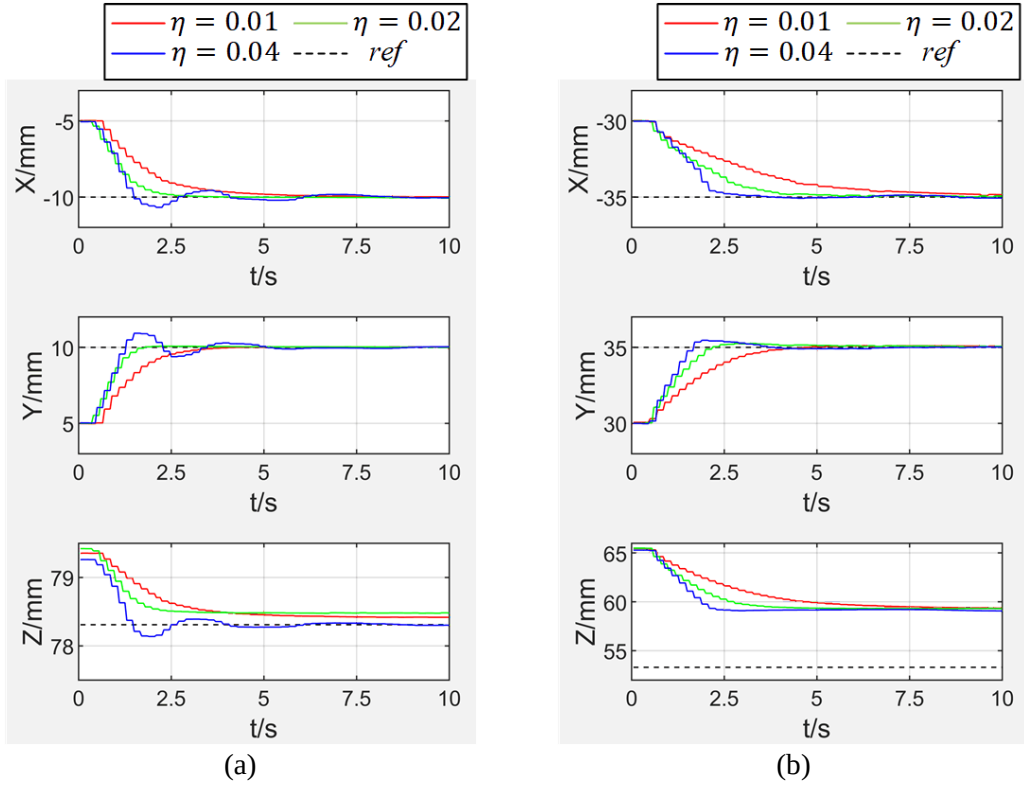


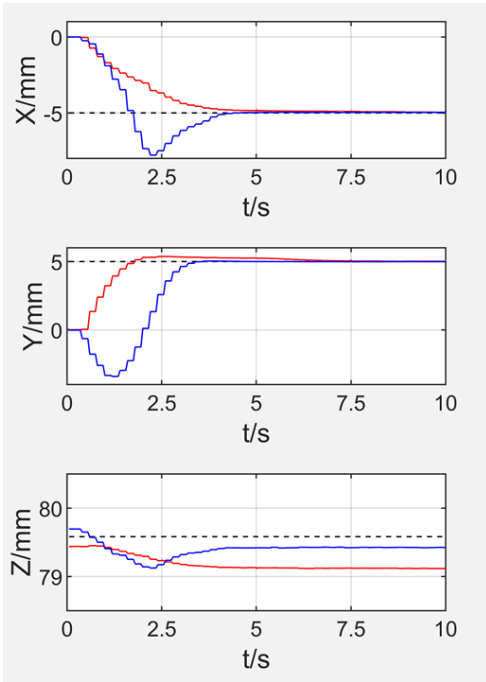
Figure 6-10 Parameter tuning of η in the proposed hybrid controller.

- **Comparison of both controllers**

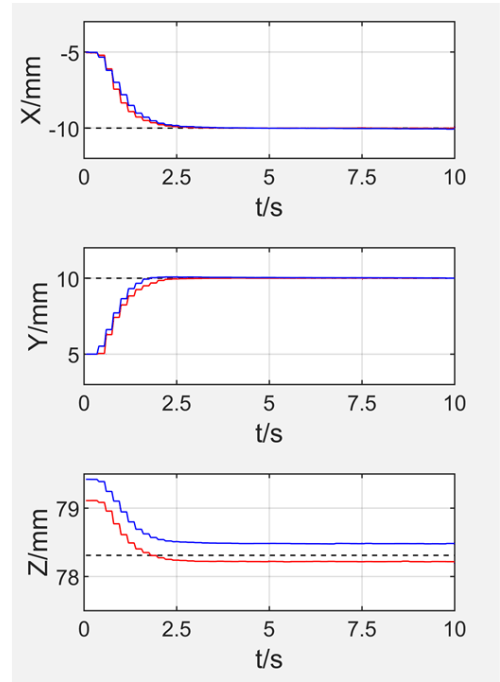
Further comparisons of tip positioning were made between both controllers after the parameter tuning process. Eight sets of different motions are chosen within the workspace for testing, which are shown in Figure 6-11(a)-(h), respectively. In the initial motion $(0,0) \rightarrow (-5,5)$ (Figure 6-11(a)), the PCC-based controller performs better than the proposed one, which suffers from overshoot. As the target point gets far away from the origin, the performances of both controllers are similar. However, when the motion comes to the boundary of the workspace $(-35,35) \rightarrow (-40,40)$, oscillation occurs to the PCC-based controller while the proposed controller keeps steady with fast converging speed.

Overall, both controllers behave similarly to each other for most of the target points, yet the proposed one are superior in terms of the global stability.

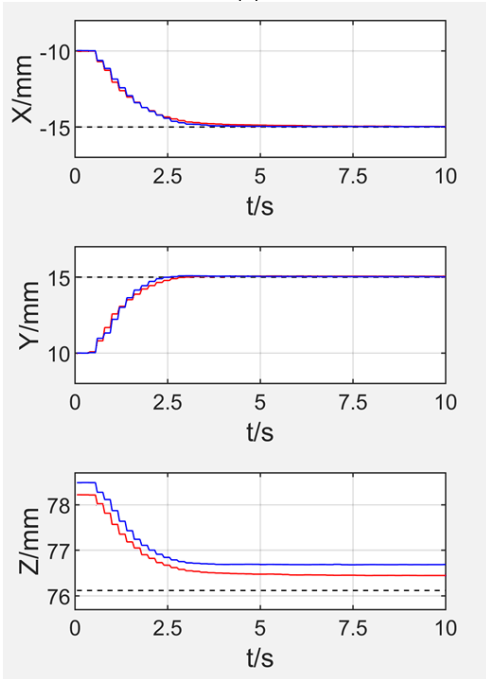
-----Reference — PCC-based controller — Proposed controller



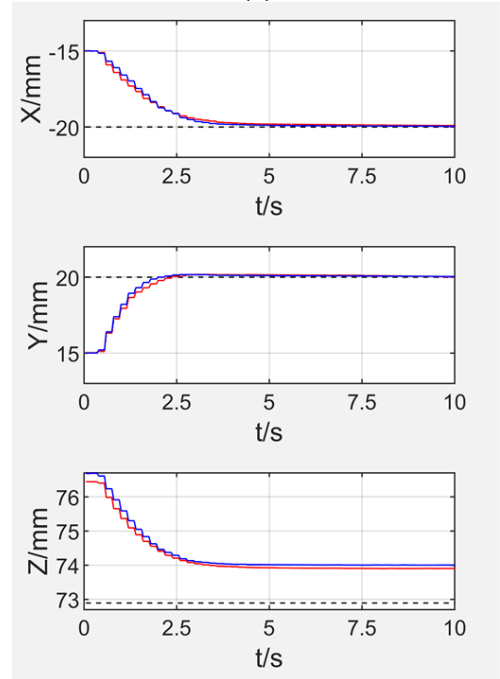
(a)



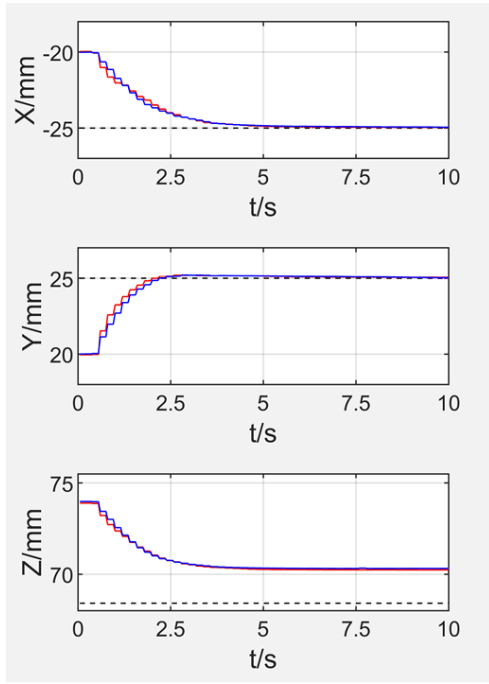
(b)



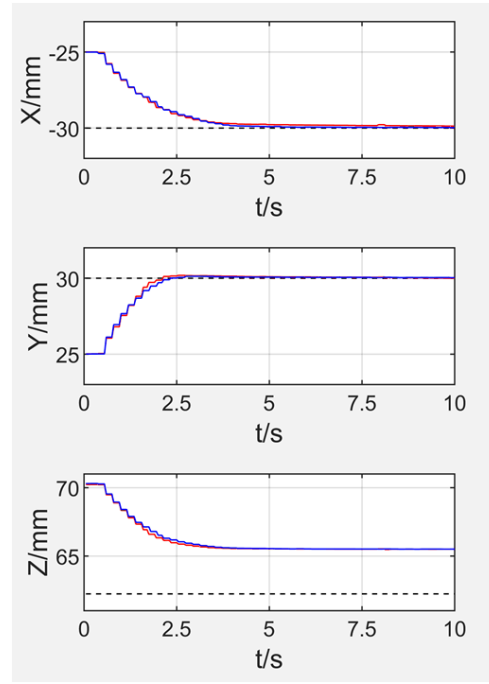
(c)



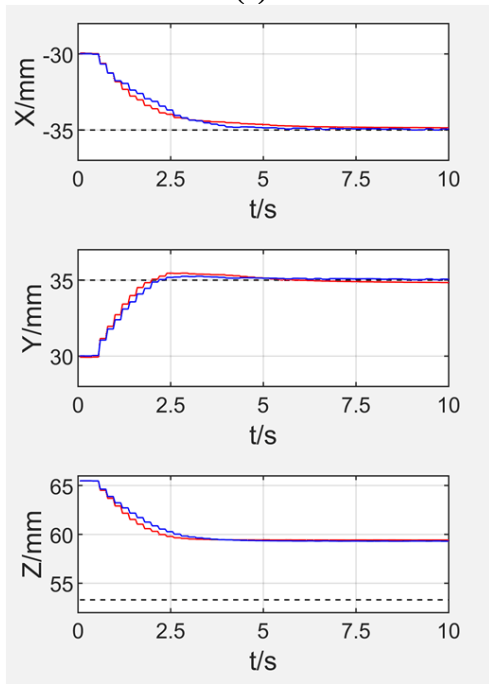
(d)



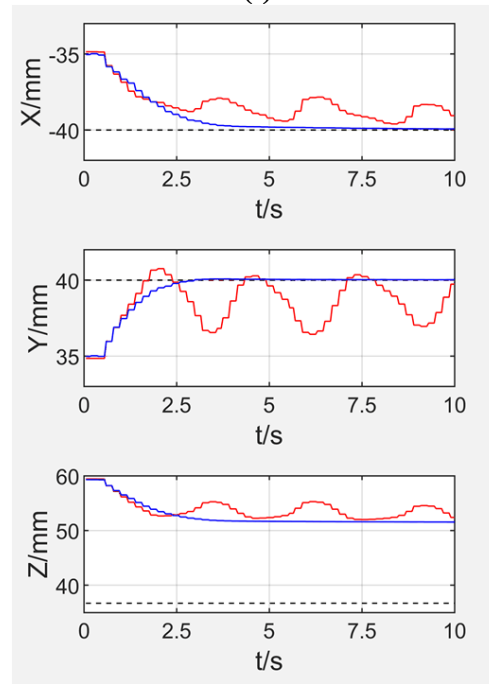
(e)



(f)



(g)



(h)

Figure 6-11 Experimental comparison between the PCC-based and proposed hybrid controllers for tip positioning. Figures (a)-(h) correspond to the motions $(0,0) \rightarrow (-5,5)$, $(-5,5) \rightarrow (-10,10)$, $(-10,10) \rightarrow (-15,15)$, $(-15,15) \rightarrow (-20,20)$, $(-20,20) \rightarrow (-25,25)$, $(-25,25) \rightarrow (-30,30)$, $(-30,30) \rightarrow (-35,35)$, $(-35,35) \rightarrow (-40,40)$, respectively.

- **Overall positioning performance of the proposed controller**

After the comparison with the PCC-based approach, a comprehensive set of tests were performed to characterize the overall positioning performance of the proposed controller.

As shown in Figure 6-12(a), the scanning path covering the whole workspace of the continuum robot was divided into four quadrants mainly owing to the measuring range and stability of the VICON system, which are marked using different colors (red, green, blue, and black). Figure 6-12(b) gives the top view of the scanning paths, which are composed of discrete points whose projection in the XY plane are evenly distributed in the region $\Omega = \{(x, y) | x, y \in [-40, 40]\}$ in the interval of 2.5 mm. And the time interval updating the target point is 3.5 s in the experiment.

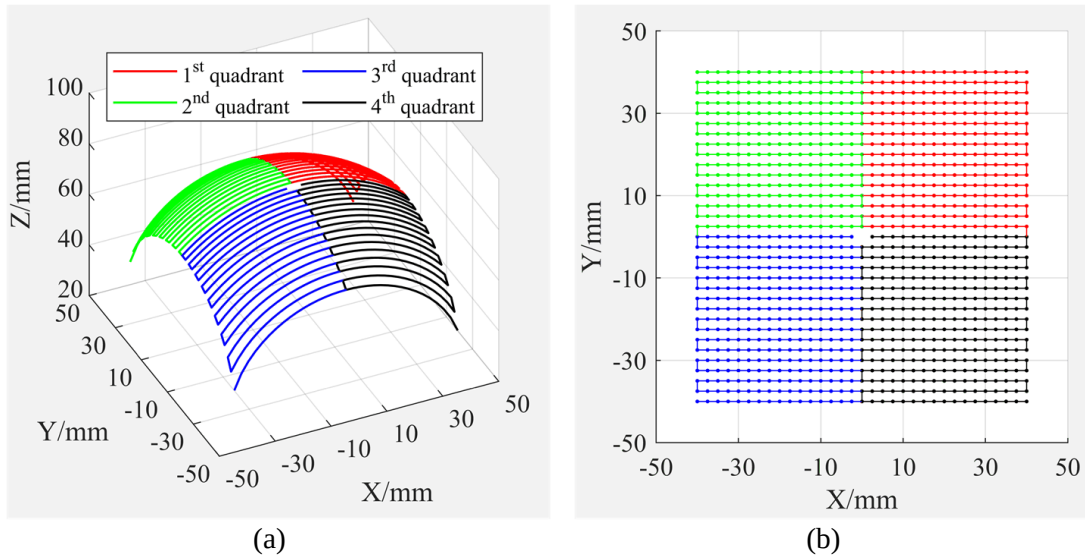
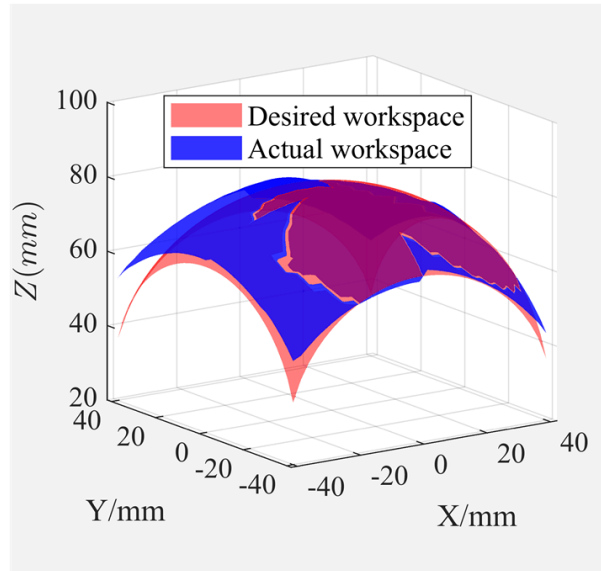


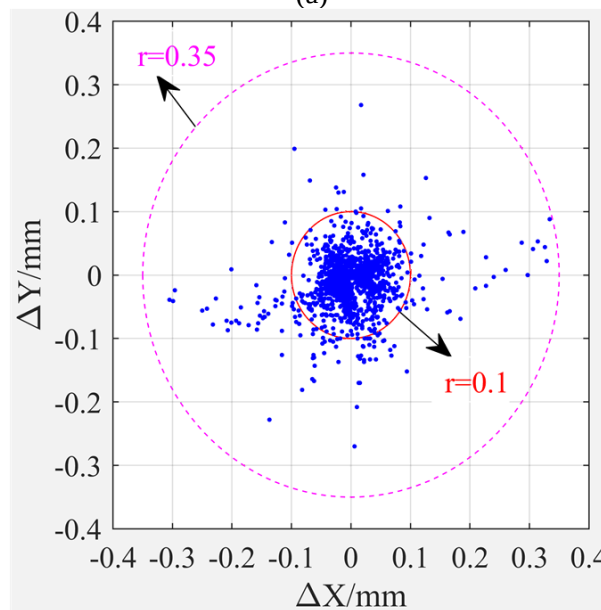
Figure 6-12 3D view (a) and top view (b) of the scanning path for tip positioning indicated by coloured dots.

Figure 6-13(a) compares the actual workspace with the theoretical workspace. It can be observed that there is a positive offset in the Z axis between the actual and theoretical workspaces, and the offset value becomes larger as the bending angle increases. The factor contributing to this phenomenon is that all the disks are uncompressible rigid bodies, which cannot deform elastically as the NiTi rods bend to form arc-like shape.

While Figure 6-13(b) shows the scatter diagram of the positioning error ($\Delta X, \Delta Y$) for all target points, which illustrates that the overall positioning accuracy is $\pm 0.35 \text{ mm}$ in the XY plane, and $\pm 0.1 \text{ mm}$ control accuracy can be reached for over 60% of the tested points.



(a)



(b)

Figure 6-13 Overall positioning performance of the proposed controller: (a) Comparison between the theoretical workspace and actual workspace; (b) scatter diagram of the positioning error in X and Y axes for all 1088 test points.

6.3.3 Experiments for path tracking

This section presents the experimental results of path tracking. First of all, comparisons are conducted between the PCC-based and the proposed controllers with the twin-pivot design. Then, the proposed controller is also implemented on the dual-revolute and soft-robot designs for comprehensive performance characterization.

- **Comparison of both controllers (on the twin-pivot design)**

Two different spatial trajectories are used for the experiments of path tracking: the first path (*Path I*) is a segmented curve whose projection is a 20×20 mm square centred at (−15,15) in the XY plane while the second path (*Path II*) is a smooth curve whose projection is a 20 mm diameter circle centred at (−15,15). Since both approaches are essentially static controllers, each path is discretized into a series of points, and the path tracking is achieved by updating the target position in a constant time interval (1 s). In addition, both controllers are running in the frequency of 20 Hz, which is mainly restricted by the communication speed and responding time of the actuation system.

Figure 6-14 gives the experimental results of *Path I*. The desired path for reference and the actual trajectories obtained by the PCC-based and the proposed controllers are shown in Figure 6-14(a), whose projections in the XY plane are illustrated in Figure 6-14(c). Figure 6-14(b) is the time response of the tip position (X, Y, Z), from which it can be observed that both controllers achieve accurate tracking in the X and Y axes while there exists slight offset in the Z axis between the desired and actual paths, demonstrating the deviation of the robot's workspace from the PCC model. Also, for the purpose of comparison, the tracking errors ($\Delta X, \Delta Y$) of both controllers in the X and Y axes are plotted in Figure 6-14(d). Furthermore, the real-time changes of cable tensions are monitored for both controllers, which are given in Figure 6-14(a) and Figure 6-14(b), respectively. The fluctuations of cable tensions for the proposed

controller are much smoother than the PCC-based controller. Likewise, the experimental results of *Path II* are illustrated in Figure 6-15.

The numerical comparisons of both controllers are given in **Table 6-1**, which includes RMSE (root mean square error) and maximum error as the performance indices. The RMSE of the PCC-based approach is slightly smaller than the proposed method, whose performances exceeds the former one in aspect of maximum error.

Table 6-1 Target points configuration for the trails on two sections

	<i>Path I</i>		<i>Path II</i>	
	RMSE(mm)	Max error(mm)	RMSE(mm)	Max error(mm)
PCC-based controller	0.17	0.67	0.21	0.97
Proposed controller	0.21	0.47	0.31	0.72

To sum up, the path-tracking performance of the PCC-based and proposed controllers are equivalent to each other in aspects of RMSE and maximum error, while the latter can achieve smaller tension fluctuations than the former.

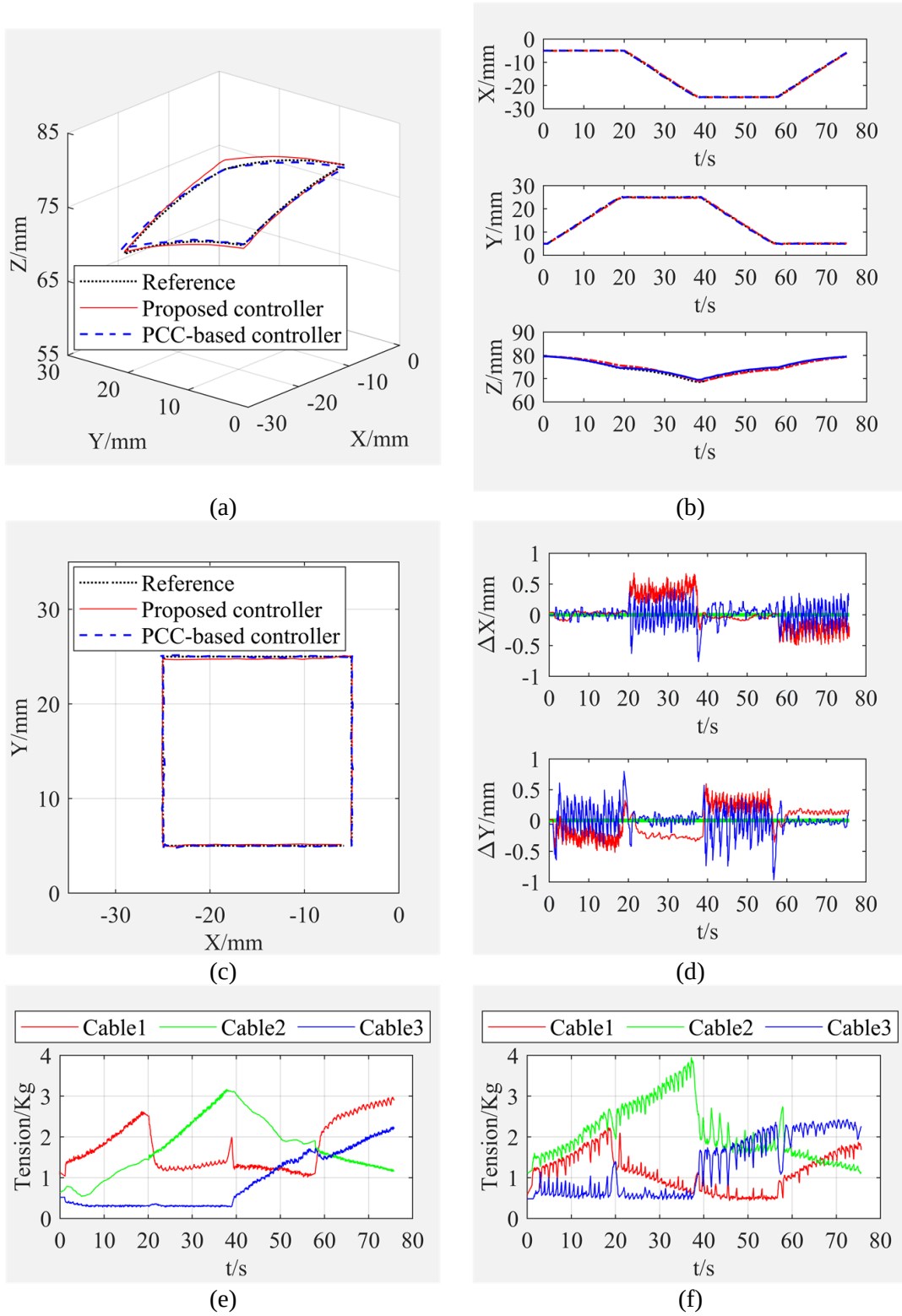


Figure 6-14 Experimental results of *Path I*: (a) desired trajectory (dotted line in black), actual trajectories based on the PCC-based controller (dashed line in blue) and the proposed controller (dash-dot line in red); (b) time responses of the tip position $[X, Y, Z]$; (c) trajectory projection in the XY plane; (d) positioning error in X and Y axes; (e) cable tensions with proposed controller; (f) cable tensions with PCC-based controller.

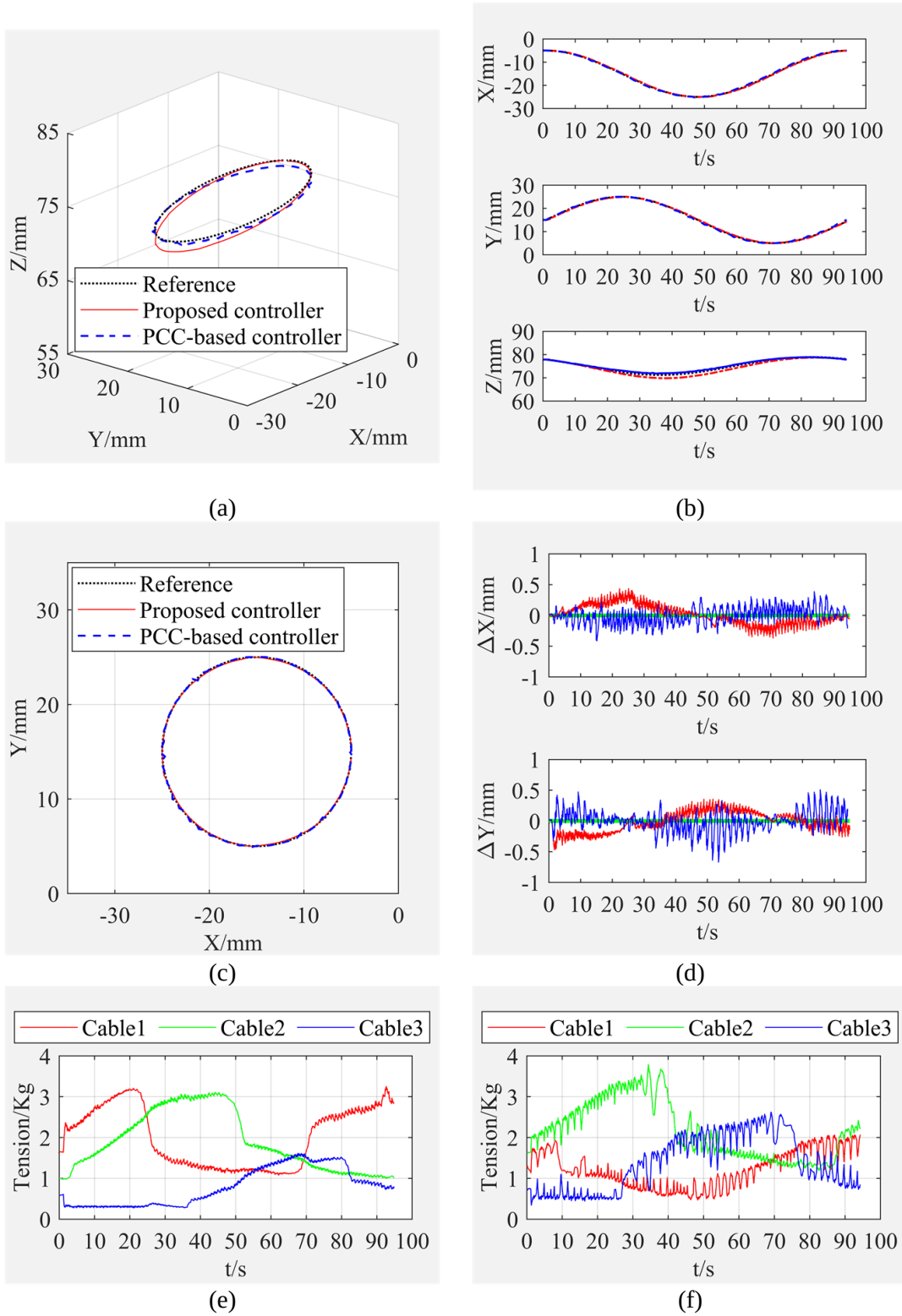


Figure 6-15 Experimental results of *Path II*: (a) desired trajectory (dotted line in black), actual trajectories based on the PCC-based controller (dashed line in blue) and the proposed controller (dash-dot line in red); (b) time responses of the tip position $[X, Y, Z]$; (c) trajectory projection in the X-Y plane; (d) positioning error in X and Y axes; (e) cable tensions with proposed controller; (f) cable tensions with PCC-based controller.

- **Performance characterisation of the proposed controller (on the dual-revolute and soft-robot designs)**

After the comparison with the PCC-based controller on twin-pivot design, the proposed controller is further applied on the dual-revolute and soft-robot designs to demonstrate its versatility on different designs of continuum robots.

The experimental results of *Path I* and *Path II* on the dual-revolute design are shown in Figure 6-16 and Figure 6-17, respectively. It is found that the tracking error increases significantly at the upper left and bottom right corners of *Path I*. While the tracking performance of *Path II* is quite smooth and stable except for the initial disturbance. Moreover, Figure 6-18 (*Path I*) and Figure 6-19 (*Path II*) present the experiments results on the soft-robot design. Owing to the compressibility of the soft robot, significant offset on the Z axis can be observed from the figures. Finally, the numerical results on both designs are listed in **Table 6-2**.

Table 6-2 Trajectory tracking performance of the proposed controller

	Dual-revolute design		Soft-robot design	
	RMSE(mm)	Max error(mm)	RMSE(mm)	Max error(mm)
<i>Path I</i>	0.7	2.14	1.04	2.02
<i>Path II</i>	0.4	1.62	0.53	1.13

In conclusion, the effectiveness of the novel proposed controller is experimentally validated on the twin-pivot, the dual-revolute and the soft-robot designs. Among the three prototypes, the best control performance has been achieved on the twin-pivot design.

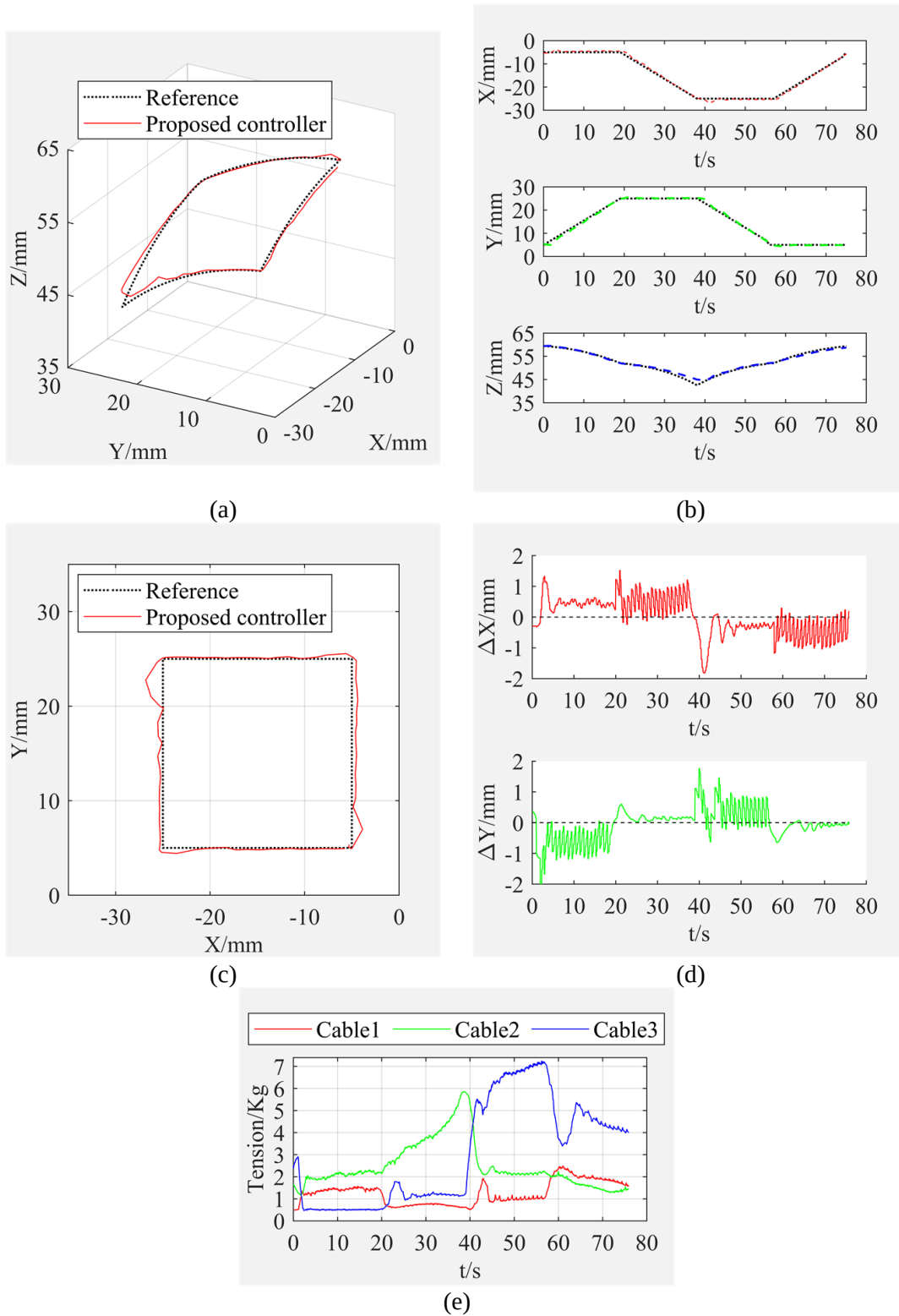


Figure 6-16 Experimental results of *Path I* on the dual-revolute design: (a) desired trajectory (dotted line in black), actual trajectories based on the PCC-based controller (dashed line in blue) and the proposed controller (dash-dot line in red); (b) time responses of the tip position $[X, Y, Z]$; (c) trajectory projection in the X-Y plane; (d) positioning error in X and Y axes; (e) cable tensions with proposed controller.

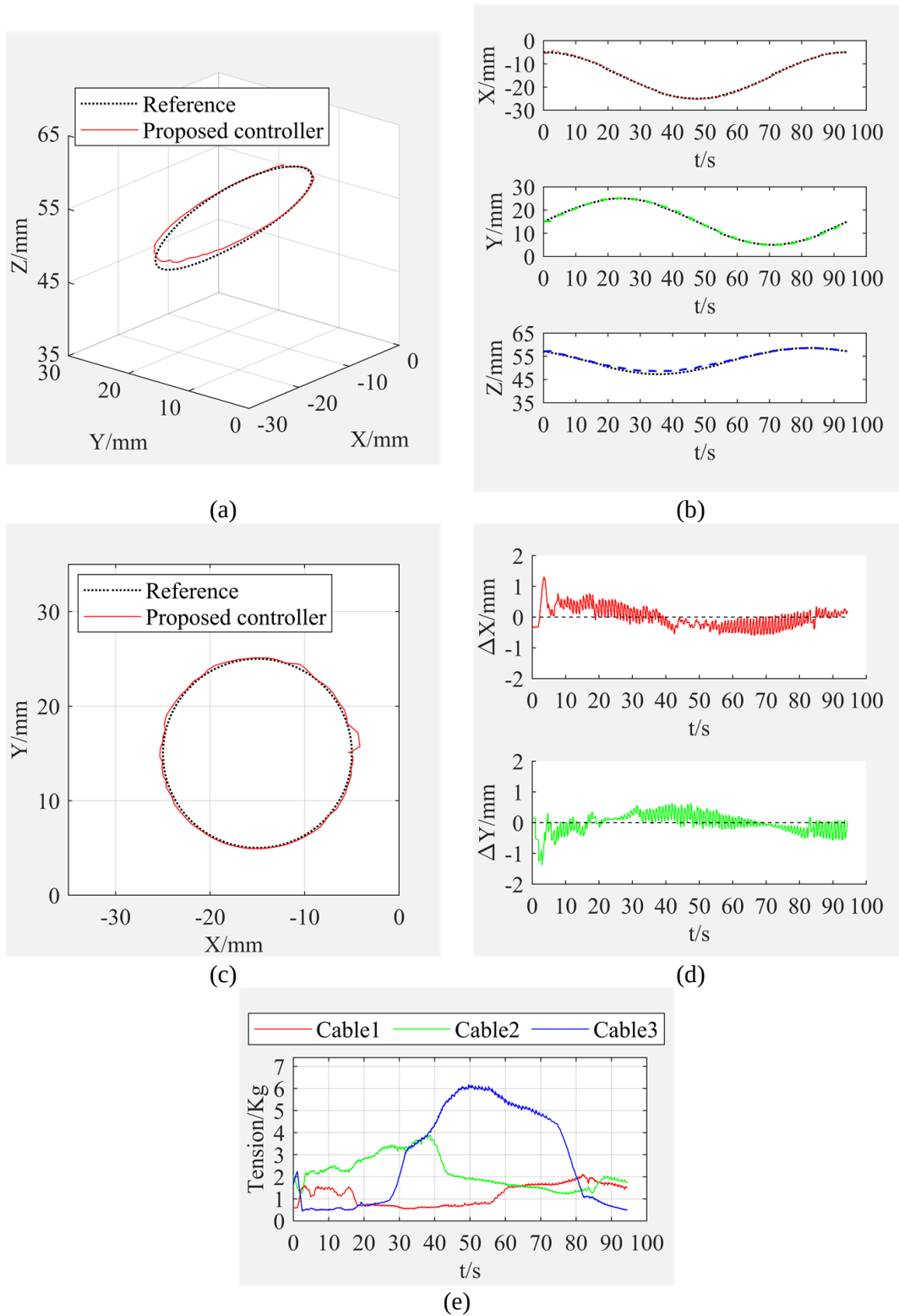


Figure 6-17 Experimental results of *Path II* on the dual-revolute design: (a) desired trajectory (dotted line in black), actual trajectories based on the PCC-based controller (dashed line in blue) and the proposed controller (dash-dot line in red); (b) time responses of the tip position $[X, Y, Z]$; (c) trajectory projection in the X-Y plane; (d) positioning error in X and Y axes; (e) cable tensions with proposed controller.

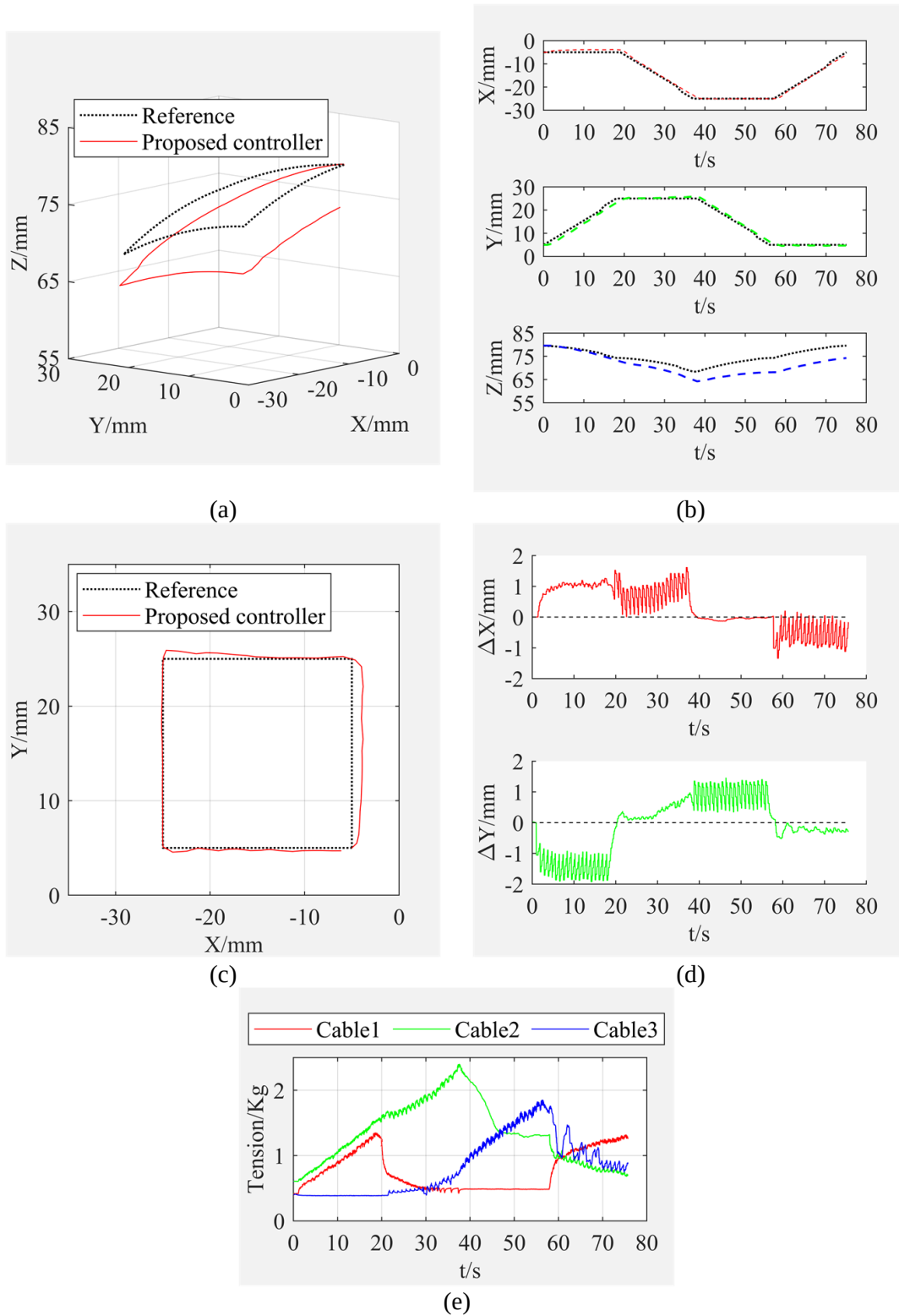


Figure 6-18 Experimental results of *Path I* on the soft-robot design: (a) desired trajectory (dotted line in black), actual trajectories based on the PCC-based controller (dashed line in blue) and the proposed controller (dash-dot line in red); (b) time responses of the tip position $[X, Y, Z]$; (c) trajectory projection in the X-Y plane; (d) positioning error in X and Y axes; (e) cable tensions with proposed controller.

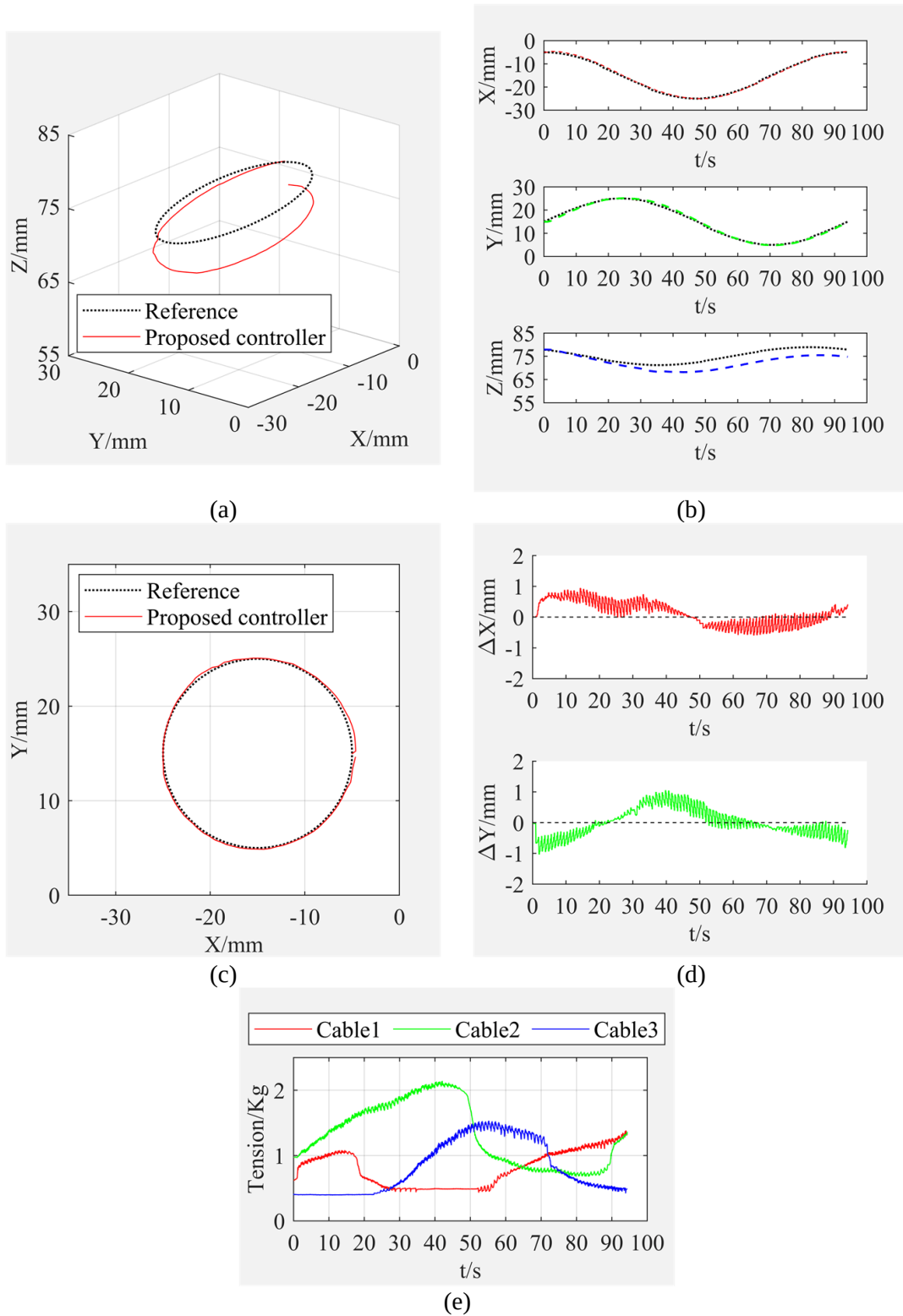


Figure 6-19 Experimental results of *Path II* on the soft-robot design: (a) desired trajectory (dotted line in black), actual trajectories based on the PCC-based controller (dashed line in blue) and the proposed controller (dash-dot line in red); (b) time responses of the tip position $[X, Y, Z]$; (c) trajectory projection in the X-Y plane; (d) positioning error in X and Y axes; (e) cable tensions with proposed controller.

6.4 Chapter summary

In this chapter, novel model-based hybrid controllers have been developed for precise manipulation of single-section continuum robots. Firstly, based on the preliminary analysis of common control modes for tendon-driven continuum robots and combining the advantage of single-section continuum robots, a hybrid control scheme is proposed to decouple the control of cable displacements so that tension supervision can be achieved to avoid slack or over-tension states of driving cables. Then, a novel differential kinematic model directly from task space to actuation space is developed, which describes the tip position in polar coordinate system instead of the Cartesian coordinate system and applies piecewise linear approximation to avoid the explicit call of arc parameters. By applying the hybrid control scheme on both the PCC-based and the proposed kinematics, two different forms of model-based hybrid controllers are obtained.

The experimental validation is carried out from two aspects. On one hand, the proposed hybrid controller is compared with the PCC-based one on the twin-pivot. It turns out that both controllers are almost equivalent to each other, but the proposed controller outperforms the PCC-based one in terms of global stability for tip positioning and the tension fluctuations for path tracking. On the other hand, comprehensive experiments are conducted on three prototypes of continuum robots for the performance characterization of the proposed controller. The versatility of the proposed controller is demonstrated. Also, it can be found that the controller performs best on the twin-pivot design in aspects of RMSE and maximum deviation error for path tracking.

7 Conclusions and future work

This chapter summarises the overall outcomes and achievements of the work presented in this thesis, emphasising on how these advances address aforementioned control challenges and promote developments in the field of continuum robots. Further, potential research and future outlook of continuum robots are discussed from different aspects.

7.1 Academic and engineering contributions

This work mainly focuses on the control of tendon-driven continuum robots. The major academic contributions are the development of novel control methodologies for different continuum robots, involving model-less controllers for slender long continuum robots and generalized model-based hybrid controllers for single-section continuum robots. Additionally, secondary contributions have also been made to the solution of the inverse kinematics and the advances of robotic system and control approach.

In the following subsections, the achievements in this thesis are explicitly illustrated.

7.1.1 General numerical solution to the inverse kinematics of continuum robots

Chapter 4 presents a general numerical framework to solve a batch of inverse kinematic problems for continuum robots.

The main advantage of this approach is that it circumvents complicated analysis for the solution of the inverse kinematic problem by converting it to an optimization problem, which can then be solved numerically. The approach can not only be applied to various designs of continuum robots as long as their forward kinematics can be formulated, but also can be modified to adapt to different kinematic problems.

The versatility of the novel numerical framework has been proved on the twin-pivot and dual-revolute designs for tip positioning (tolerance constraints: 0.01 mm) and orientating (tolerance constraints: 0.025°). Furthermore, the step control simulation of the three-section manipulator has been conducted to demonstrate the effectiveness of the framework to deal with the inverse kinematics for multiple sections.

7.1.2 Novel model-less control exemplified on a slender long continuum robot

Chapter 5 of this thesis deals with the control of slender long continuum robot. The control challenges for such continuum robots are firstly analysed, including the model (PCC assumption) mismatch, cable elongation and section coupling. Those challenges reduce the reliability of the conventional model-based controller, which is experimentally validated in the preliminary test where oscillation occurs at some target points when the controller is implemented for tip positioning.

To solve the problem, novel model-less controllers are proposed to improve the overall control performance of multi-section continuum robots with long full length, which consists of two stages. In the first stage, an empirical model-less controller, which achieves the close loop control directly from task space (tip position) to actuation space (cable displacements), is developed with a series of control policies inspired by the tug of war. Even though the effectiveness of the empirical controller is experimentally demonstrated by the positioning test (maximum error: $\pm 1\text{ mm}$ in both X and Y axes) on single section and several trials on three sections, it suffers from theoretical discontinuity issue at the boundaries between workspace sections. As such, fuzzy logic algorithm is proposed for the controller design in the second stage to solve the problem. To be specific, the workspace is divided into multiple sections according to the distribution of cable holes. Taking the PCC-based kinematics as reference, discretized control policies are derived for the centre lines of each section and act as the fuzzy rules

in the fuzzy-logic controller, which are globally converged with $\pm 1\text{ mm}$ overall positioning accuracy in the XY plane and $\pm 0.5\text{ mm}$ for 75% test points.

7.1.3 Novel model-based hybrid control exemplified on a single-section continuum robot

Chapter 6 presents the development of novel high-performance hybrid controllers for single-section continuum robots.

The first novelty is the hybrid control scheme. Different from slender long continuum robots, the cable tensions of single-section continuum robots can be accurately measured. Therefore, hybrid control can be applied which combines the advantages of kinematic control and tension supervision. Moreover, a novel differential kinematic model is proposed based on the modification of the conventional kinematics. Specifically, on the one hand the polar coordinate system is utilized to replace the Cartesian coordinate system in task space, on the other hand an accurate approximation is applied for piecewise linear fitting so that the explicit call of the arc parameters in configuration space can be circumvented. As a result, an analytical differential kinematics is derived directly from task space to actuation space. Consequently, two forms of model-based hybrid controllers are developed, which are based on the PCC assumption and the proposed novel differential kinematics, respectively.

The experimental validation is carried out from two aspects. Comparisons are firstly made between two hybrid controllers on the twin-pivot design. It is demonstrated that the performances of both controllers are equivalent to each other for tip positioning, while the proposed controller is superior to the other because of smaller tension fluctuations in path tracking. Moreover, the proposed controller is further implemented on the dual-revolute and soft-robot designs for comprehensive performance

characterization. The experiments demonstrate the robustness and versatility of the proposed hybrid controller.

In conclusion, this work focuses on developing novel feedback controllers for different types of tendon-driven continuum robots, which can be categorised into slender long continuum robots with multiple sections and those single-section continuum robots. Generally, the proposed model-less controllers are appropriate to multi-section continuum robots with long full length, which usually suffers from three main control challenges explicitly discussed in Section 5.1; while the hybrid model-based controllers are specific for various single-section continuum robots whose cable tensions can be accurately measured and hence can be utilized for the high-performance control.

7.2 Future work

The proposed general kinematic solution and novel feedback controllers for different designs of continuum roots have been validated by numerical simulations and physical experiments in this thesis. Furthermore, some challenges and opportunities have been identified in the process of research. In this section, the potential research topics are discussed in briefly for further progress and study.

7.2.1 Mechanical aspect

- **Hybrid design of CRs** – As discussed in Section 5.1, the coupling issue exists on multi-section tendon-driven continuum robots. Although the trials on two/three sections reveal that the proposed model-less controllers can still take effect under the disturbances of the coupling issue, the convergence time is significantly extended. To solve the problem, a hybrid design of tendon-driven (proximal sections) and pneumatic (distal sections) continuum robots can be considered to physically eliminate the section coupling issue.

- **Variable damping actuation for tendon-driven CRs** – Usually, low-stretch cables are utilized in tendon-driven continuum robots to guarantee fast response speed. However, this will cause remarkable tension fluctuations during operation. Therefore, variable damping control could be implemented in the actuation system, which will also facilitate the hybrid control of continuum robots.
- **Minimising actuation pack by clutch for semi-passive CRs** – In addition to regular fully-actuated tendon-driven continuum robots with bulky actuation pack, novel semi-passive continuum robots have also been developed for inspection/maintenance tasks in narrow space. To further improve the practicality of these robots, the clutch-like mechanism could be integrated into the robot to manipulate multiple sections with less driving cables so that the size of the actuation pack can be reduced, thus the mobility of robot will be significantly improved.

7.2.2 Sensor & control & application

- **Real-time shape sensing and manipulation with the FBG fibre sensor** – According to the literature survey in Section 2.5, the FBG fibre sensor is the optimal option for the shape sensing of continuum robots in terms of measuring performance (sampling frequency, precision) and practicality. As such, the integration of such sensor would be significantly beneficial for continuum robots, which is also the prerequisite for designing advanced algorithms to further improve the control accuracy and automation.
- **Collision avoidance with tactile sensors and side cameras** – Despite the undeniable advantages of the FBG fibre sensor, the expensive price prevents it from being regularly deployed on continuum robots. Actually, the overall shape

sensing is not indispensable for some auxiliary tasks in confined space such as inspection and rescue in hazard and hard-to-reach environments (nuclear plants, aero-engines, ruins, etc.). Instead, tactile sensors and side cameras are more effective for collision avoidance in those scenarios, which would be another research hotspot promoting the applications of continuum robots.

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