



CONTROL STRATEGIES FOR IMPROVING POWER QUALITY AND PLL STABILITY EVALUATION IN MICROGRIDS

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Abstract

In recent years the interest in environmental protection and energy sustainability has steadily increased; this fact has promoted research activities, and projects focused on non-conventional renewable energy (NCRE) sources as a replacement for fossil fuels. In this context, NCRE-based technologies offer a solution for integrating distributed energy resources where the Microgrid (MG) concept has been introduced to facilitate the integration of a large number of micro-generators, energy storage units and loads. This integration is enabled by power electronic converters, and therefore there is an increase in the number of these devices in the modern electrical power systems. The proliferation of power electronic-based systems will inevitably affect both the power quality and the stability of electrical systems which have power electronic interfaces. In this context, two key issues that need to be addressed are the following: (i) the imbalances and harmonics inherently present in MG due to the random nature of loads and (ii) since MGs are weak systems, the stability between the power electronic-interface and the Point of Common Coupling (PCC). These two critical issues in MGs are studied in this thesis. The former is analysed considering 4-leg systems while the latter is studied considering a 3-leg system. The 3-wire system is used for developing a methodology to study the effects of the weak nature of MGs on a simple system. As future work, this methodology can be modified to analyse specific control schemes proposed in this thesis to manage imbalances and harmonic in MGs.

To manage imbalance and harmonic issues in 4-leg MGs, two approaches are proposed. First, a novel methodology based on the application of a 4-leg shunt Active Power Filter (APF) is proposed. The control for the compensator is based on the Conservative Power Theory (CPT) augmented by resonant controllers. Experimental

validation of the APF will be performed with a focus on the transient behaviour and on its performance when frequency variations occur. In the second approach, a cooperative control scheme based on the CPT is proposed for sharing imbalances and harmonics in three-phase four-wire droop-controlled systems. With this approach, it is demonstrated that the CPT can be an effective tool to develop imbalance sharing algorithms in isolated MGs. The behaviour of proposed control schemes is demonstrated using experimental prototypes deployed in a laboratory scale 4-leg MG, and using unbalanced and distorted conditions of a real isolated MG located in Canada.

To study how the weak nature of MGs affect the stability of the interface between a converter and the PCC, a stability analysis focused on the phase-locked loop (PLL) has been undertaken based on a simple dq model proposed in this thesis. The aim of this proposal is to understand the effect of the weak nature of the MG (measured with the short circuit ratio) and the PLL bandwidth on the stability of the system. The system studied corresponds to a 3-leg converter connected to a balanced three-phase weak grid. The performance of this method is validated through simulation using Plexim PLECS software, and experimental validation using a laboratory-scale system is performed.

The main contributions of this thesis can be summarised as follows:

- In microgrids, relatively large variations in the electrical frequency may occur. The sensitivity to grid frequency variations is a known limitation of the CPT, and this work proposes and validates an implementation approach that shows the limited impact of frequency variation on the APF performance.
- The CPT is applied to 4-wire systems considering full control of the 4 legs in the APF, to regulate the positive, negative and zero sequence voltages synthesised by the power converter. This includes the operation of the CPT compensator with single-phase non-linear loads.

- The proposed sharing algorithm uses a novel approach based on the CPT methodology that can be used to implement a simple identification of the balanced, unbalanced and non-linear components of the currents and powers. Separation of the sequence components is not required. This produces a more robust imbalance sharing algorithm, particularly because most of the sequence separation algorithms are strongly affected by noise, harmonic distortion, small variations in the sampling time, etc.
- A very simple method to share harmonic distortion between the generation sources is realised considering the application of the CPT transform. In this thesis, this methodology is discussed and experimentally validated.
- A control algorithm is proposed based on the CPT which is very robust to issues such as distortion, noise, changes in the sampling time etc. As is demonstrated in this work, it is very simple to extend the proposed methodology to include harmonic distortion.
- A systematic PLL design process is proposed to be used in weak grids to ensure system stability. This design process is based on a proposed low-complexity small signal model of the system in the dq reference frame. Based on this proposal, it could be possible (in future work), analyse the effects of the weak nature of MGs on the performance of the CPT-based controllers proposed in this thesis.

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List of Acronyms

- APF: Active Power Filter
- ARF: Actual Reference Frame
- BEV: Battery Electric Vehicle
- BPF: Band-Pass Filter
- BW: Bandwidth
- CPT: Conservative Power Theory
- CRF: Converter Reference Frame
- DG: Distributed Generation
- DSP: Digital Signal Processor
- ESS: Energy Storage System
- EMS: Energy Management System
- FPGA: Field-Programmable Gate Array
- HEV: Hybrid Electric Vehicle
- KVL: Kirchhoff's Current Law
- KVL: Kirchhoff's Voltage Law
- LPF: Low-Pass Filter
- LV: Low Voltage
- LVUR: Line Voltage Unbalance Rate
- MG: Microgrid
- NCRE: Non-Conventional Renewable Energy
- PCC: Point of Common Coupling
- PI: Proportional–Integral

- PLL: Phase Locked Loop
- PR: Proportional - Resonant
- PV: Photovoltaic
- PVUR: Phase Voltage Unbalance Rate
- PWM: Pulse Width Modulation
- RMS: Root Mean Square
- SCR: Short Circuit Ratio
- THD: Total Harmonic Distortion
- VA: Volt-Ampere
- VSI: Voltage-Source-Inverter
- VUF: Voltage Unbalanced Factor

List of Terms Used in Equations

- C_1 : Filter capacitor
- D : Void power
- E_n : Nominal voltage of the MG
- E_{1dq}^c : dq components of the capacitor voltage in the converter reference frame
- E_{1dq}^a : dq components of the capacitor voltage in the actual reference frame
- G_{cj} : transfer function of the PR controller
- I_{1dq}^c : Inverter output current in the converter reference frame (dq reference frame)
- I_{1dq}^a : Inverter output current in the actual reference frame (dq reference frame)
- I_{1dq}^{*c} : Reference values of currents in the converter reference frame (dq reference frame)
- I_{gdq}^c : Grid current in the converter reference frame (dq reference frame)
- I_{gdq}^a : Grid current in the actual reference frame (dq reference frame)
- I_{c1dq}^c : Current in the capacitor (converter reference frame)
- I_{c1dq}^a : Current in the capacitor (actual reference frame)
- $i_{a\mu}^b$: Balanced active currents ($\mu = a,b,c$)
- $i_{r\mu}^b$: Balanced reactive currents ($\mu = a,b,c$)
- i_{μ}^u : Unbalanced currents ($\mu = a,b,c$)
- $i_{v\mu}$: Void currents ($\mu = a,b,c$)
- $\underline{i}^b$: Balanced current vector
- $\underline{i}^u$: Unbalanced current vector
- L_1 : Inverter side filter inductor

- L_g : Inductive component of grid impedance
- N : Unbalanced power
- N_x : Unbalanced power in converter “x”
- P : Active power
- p : Instantaneous power
- Q : Reactive power
- R_g : Resistive component of grid impedance
- V_{gdq}^c : dq components of the grid voltage in the converter reference frame
- V_{gdq}^a : dq components of the grid voltage in the actual reference frame
- V_{1dq}^c : dq components of the voltage at the converter output (converter reference frame)
- V_{1dq}^a : dq components of the voltage at the converter output (actual reference frame)
- W : Reactive energy
- w : Instantaneous reactive energy
- $\hat{v}$: Unbiased voltage
- ω_n : Nominal frequency of the MG
- ω_{nat} : Natural frequency
- ζ : Damping ratio
- ω_{PLL} : Frequency obtained by the PLL
- $\Delta\theta$: Perturbation in the angle between the converter and the actual reference frame
- θ_{PLL} : Phase obtained by the PLL

Chapter 1. Introduction

Generation of electrical energy based on non-conventional renewable energy (NCRE) systems is an area that has motivated a large research effort in recent years. Factors motivating this effort have been: (i) policies in most countries to promote green-energy [1], for example the implementation of targets for greenhouse emissions [2], tradable green certificates, tax credits, subsidies [1] etc., and (ii) NCRE-based technologies are efficient solutions when distributed generation resources are available [3] [4]. In fact, NCRE-based technologies offer a solution for integrating distributed energy resources [4], as for instance, plug-in hybrid electric vehicles (HEVs), battery electric vehicles (BEVs), and microgrids (MGs) [5]. In particular, MGs are considered to be effective solutions for electrification in rural zones, industrial parks, commercial and institutional campuses, and other places where distributed renewable resources exist [4] [6] [7].

A microgrid can be defined as a cluster of loads, distributed generation (DG) units and energy storage systems (ESSs), which are connected to the main power system at the point of common coupling (PCC) [1], as shown Fig. 1.1. All these units are usually connected to the PCC through voltage-source-inverter (VSI)-based interfaces, and they are managed by an energy management system (EMS) [8] [9]. The EMS is essentially a control algorithm designed to regulate the power supply of each MG generation unit. This is usually achieved by minimising a cost function in which operational costs, losses, renewable energy forecasting, the status of the ESSs and DGs, etc. are considered [4] [10]. Fig. 1.2, shows an example of an EMS implemented in a MG. Note that a MG can operate in both grid-connected and stand-alone modes [11]. Moreover, the MG has to be

able to manage the transition between these two modes [1]. Note that MGs may be considered as controllable entities with respect to the grid. (See Fig. 1.1 and Fig. 1.2)

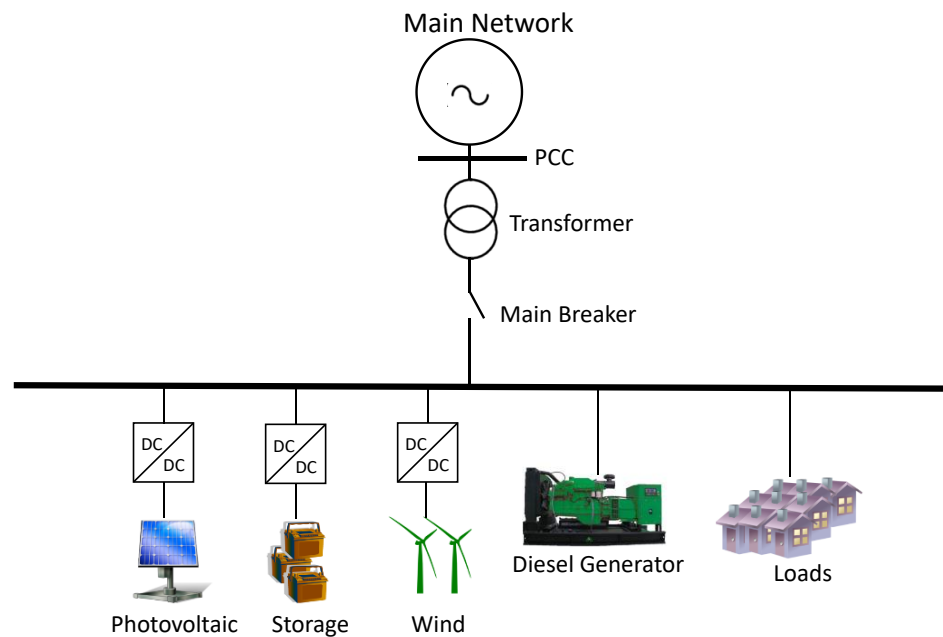


Fig. 1.1. Example of an AC Microgrid

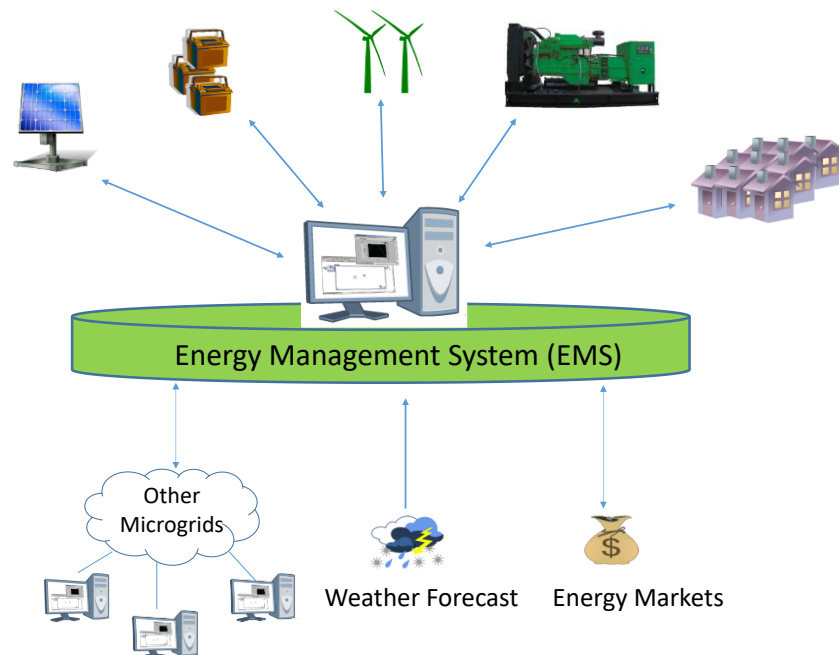


Fig. 1.2. Example of the architecture of an EMS in a Microgrid

MGs can also be implemented as low voltage (LV) power distribution systems [9] [12] [13]. Therefore, they need to have the capability of feeding single-phase linear and non-linear loads using a three-phase 4-wire topology [14]. The neutral connection can be provided by (i) zigzag or delta-star (Δ -Yn) transformers, which prevent the propagation of the zero sequence component to the transformer primary winding; (ii) a three-leg split-capacitor converter topology; (iii) a three-phase four-leg topology in which the neutral connection is provided by a fourth leg in the (VSI)-based interfaces [15] [16]. For microgrids, the latter option is usually considered the best as no large capacitors are required, and full utilisation of the dc link could be achieved (using suitable modulation algorithms) [16] [15]. Although this option is more suitable for MGs than the other solutions, very few researchers propose methodologies for controlling these types of power converter; this in the sense of managing power quality issues such as harmonics, and zero and negative sequences, which could be produced by loads.

When a microgrid is implemented as a low voltage network, it is inherently an unbalanced system. Indeed, in an LV network, there are two main issues: (i) it may have asymmetrical impedances per phase and (ii) there is a constant switching on and off of single-phase loads. Both issues could produce unbalanced loads [17]. In this context, since the MG is an unbalanced electrical system, it produces oscillations in the converter's DC-link. Also, in an unbalanced MG, neutral currents will circulate through the neutral wire [18], which will produce (i) an increase of the losses in the distribution grid, (ii) a reduction of the efficiency of grid assets, (iii) overloading of the neutral conductor, and (iv) malfunctioning of sensitive electronic equipment since the ground voltage will also be fluctuating due to the large neutral line current [18] [19]. Moreover, if a synchronous and/or a doubly fed induction generator-based micro-generator is connected to the MG, unbalanced loading will result in a double system frequency ($2\omega_e$)

oscillating active power component which translates into an oscillating torque component [17]. The latter will produce mechanical stress, torque pulsations and noise, affecting the efficiency and lifetime span of these machines [20] [21]. Additionally, the authors in [21] suggest that unbalanced loads affect the stability of the MG.

Typical loads connected to low-voltage MGs are usually computers, lighting ballasts, appliances, battery chargers, etc. [19] [22] [23]. Most of these loads have nonlinear behaviour, generating distorted load currents. In this situation, one of the most serious problems produced by single-phase loads is zero-sequence third harmonic current components (or triple order harmonics) that flow through the neutral conductor [18] [19]. This neutral current generates the same problems as those produced by unbalanced loads and others, e.g. (i) harmonic interaction and harmonic resonance between the DG inverters and the network [24] [25] (ii) harmonics that contribute to decreasing the maximum load which can be drawn from DG units (resulting high loading levels and possibly instability), and (iii) since an islanded MG is a weak network, harmonic loads could produce harmonic voltages, which could spread through the MG.

Another characteristic of MGs is their weak nature, which is produced because in some cases renewable sources are installed in remote locations, far from the main power grid. In other cases, renewables are connected to distribution feeders designed for a relatively low power rating, with transformers characterised by non-negligible impedance. In the worst case, renewables contribute to the power generation in autonomous remote MGs, typically powered by diesel gensets. In all these cases, the AC grid seen by the inverters is weak. In this context, it has recently been reported that the Phase-Locked Loop (PLL) algorithm used by inverters to synchronise to the grid affects the stability of the system, leading to the disconnection of the inverter from the PCC [26] [27].

In short, imbalances, harmonic and weakness issues affect power quality and stability of MGs, and consequently, must be taken into account to ensure robust microgrid operation. These phenomena are quantified using indexes. Specifically, a common index for measuring the degree of imbalance in an electrical system is defined by the European Standard EN50160. This is named the Percentage Voltage Unbalanced Factor (%VUF) and is defined in (1) [17]. Harmonic distortion is measured using the Total Harmonic Distortion (THD) index, which is presented in (2). In this equation, X could represent either voltage or current, h corresponds to the harmonics order and, finally, X_1 is the fundamental RMS component of the signal. In the MG context, there is the IEEE Std. 1547-2203 which establishes a MG operating mode to ensure good power quality from DGs. It designates that for distributed resource technologies with an added capacity of 10MVA or less at the PCC, it should not exceed the injection of current harmonics shown in Table 1.1 [17]. Finally, the level of weakness of a grid can be characterised by the Short Circuit Ratio (SCR) which is shown in (3). In this index, V_g is the line to line RMS voltage at the PCC, Z_g is the grid impedance, and P_{rated} is the nominal power of the converter. Based on the SCR, a weak grid is conventionally identified by a short circuit ratio in the range $2 \leq SCR \leq 3$ [28] [29] and a very weak grid is when $SCR < 2$.

$$\%VUF = \frac{\text{negative sequence voltage component}}{\text{positive sequence voltage component}} \times 100 \quad (1)$$

$$\%THD = \sqrt{\sum_{h=2}^{\infty} \left(\frac{X_h}{X_1}\right)^2} \quad (2)$$

Table 1.1. Maximum allowed Harmonic Current Injection according to IEEE Std. 1547-2203

Individual harmonic order h	h<11	11≤h<17	17≤h<23	23≤h<35	h≥35	THD
Percentage (%)	4.0	2.0	1.5	0.6	0.3	5.0

$$SCR = \frac{V_g^2}{Z_g \cdot P_{rated}} \quad (3)$$

In order for the microgrid concept to become widely accepted, it is necessary to ensure that the quality of the power is within the recommended limits given in the IEEE Std. 1547-2203, (and in the European Standard EN50160) and it is also necessary to ensure the stability in the interface converter-PCC, specifically understanding how the PLL dynamics affects this interface when the grid is weak. These issues have been studied in the literature, and a summary of the methods to address them is next provided.

To manage imbalance and harmonics in a MG:

- The use of active power filters (APFs) to inject currents and/or voltages to compensate imbalance and/or harmonics, which are present in the microgrid. In particular, the use of APFs in shunt connection (see Fig. 1.3) have been widely used in MGs [30] [31] [32] [33] [34] [35] [36] [37] [38] To use this topology, it is necessary to have a power converter in the microgrid which can inject harmonics and/or negative/zero sequences. Obtaining the unwanted current components which need to be eliminated or reduced in the MG is required for using an APF. In this sense, some methods reported in the literature for determining the unwanted currents are: the p-q theory proposed by Akagi et al. in [31], the modified p-q theory [32], the FBD theory [33], Lagrange multiplier-based

decomposition [34], the estimation based on neural networks [35], the use synchronous frame [36] [37], and the Conservative Power Theory (CPT), recently proposed by Tenti in [30]. Among these methods, the most used for APF applications are [38]: the p-q theory, the FBD theory, and recently the CPT. In [38], a comparison among these three theories is performed, concluding that their results are very similar under ideal conditions (strong grids). If these theories are used in weak grids, it is suggested that both the p-q and the FBD theory produce the phenomenon called “hidden currents” [31] [38] [39]. It means that currents which are not present originally in the load are generated by these theories. Certainly, this is an important drawback, especially in the MGs context, since an MG is inherently a weak system [40]. The hidden currents are avoided when the CPT is used [38].

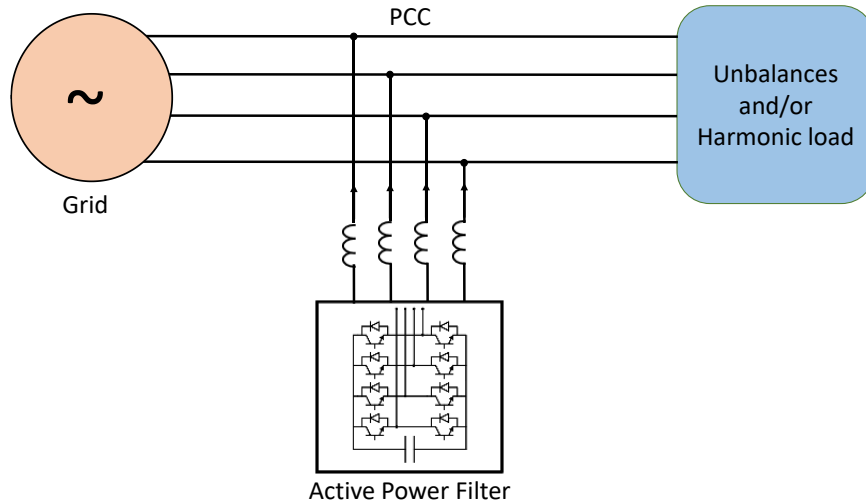


Fig. 1.3. Schematic of the active power filter studied in this thesis.

- To embed imbalance and harmonic compensation capabilities in the control algorithms of the converters that are already available in the MG, thus maximising the utilisation of the hardware. In this scheme, additional hardware is not required. It is worth considering that using the capability of the existing DG to

compensate harmonic and/or unbalanced distortions, involves two points of view: the first is to achieve cooperative imbalance/harmonic compensation among the inverters when the load is unbalanced and/or non-linear. The second is to improve the unbalanced power load sharing among the inverters located in a MG according to their power rating. This latter requirement is to prevent a disconnection of an inverter due to overload. These control algorithms are generally developed for parallel converters feeding a single common load. (See Fig. 1.4)

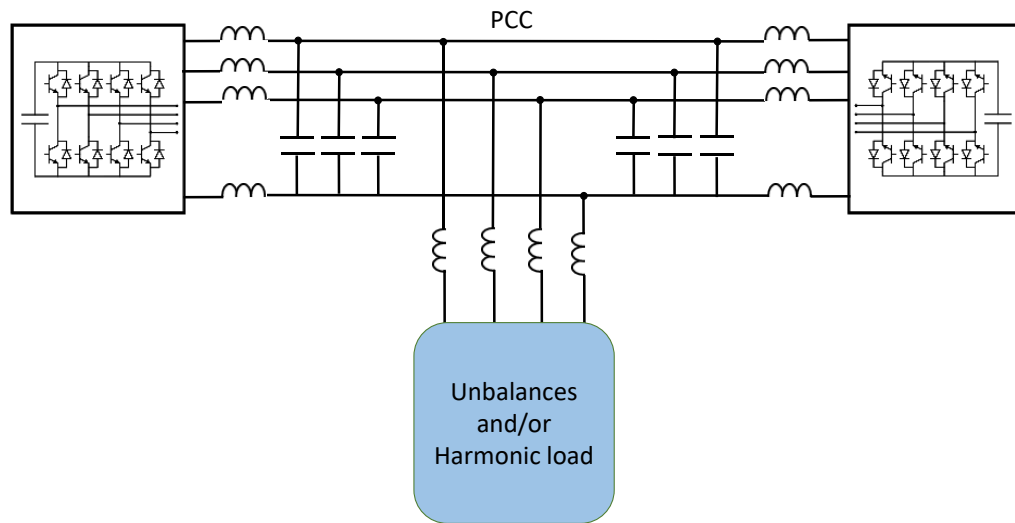


Fig. 1.4. Schematic of the aislanded MG studied in this thesis.

Stability of the interface converter-PCC with the focus in the PLL dynamics

- Regarding the effects of the PLL on the stability of the system, specifically on the interface between the power converter and the PCC (see Fig. 1.5), this is a well-known phenomenon that has been addressed by several recent publications with the common aim to provide models that explain the interaction between the PLL and a weak grid. Based on these models, the stability of the system is studied considering a certain level of weakness of the grid and a given PLL. Some tools

used for studying the stability of the system are the Nyquist criterion, eigenvalue analysis, among others. Studying the effects of the weak nature of MGs and the PLL bandwidth on the stability of power converters connected to weak PCCs is critical to prevent the power converters from tripping.

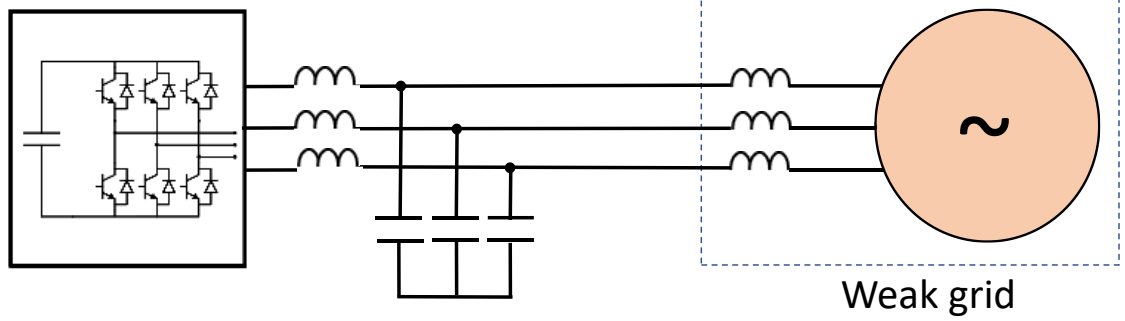


Fig. 1.5. Schematic of the converter connected to a weak MG studied in this thesis.

In this thesis, the use of shunt active power filters and the use of the residual apparent power (VA) capability of existing DGs in coordinated schemes will be investigated. This is to address the imbalances and harmonics issues present in MGs. This study will be based on a three-phase four-wire isolated Microgrid, which has not been completely studied in the literature. Special focus will be placed on (i) studying the CPT to generate references for a shunt connected compensator, (ii) designing a methodology for controlling multiple units in order to share imbalances and harmonics among the converters which are part of the MG. To address the effects of the PLL bandwidth in the stability between the converter and the PCC, a novel dq small signal model of the system is proposed. Based on this model, the stability can be studied by analysing the eigenvalues of the proposed model. This study is performed considering a 3-leg converter connected to a balanced three-phase three-wire weak grid.

1.1 Project Hypotheses

The following are the working hypotheses that guide this research project:

- The Conservative Power Theory (CPT) can be used to generate reference currents for a shunt active power filter implemented in three-phase four-wire microgrids to mitigate both unbalanced and distorted currents generated by unbalanced and non-linear loads present in MGs. This theory presents advantages over the p-q theory, which is commonly used in active power filters. These benefits are: (i) the CPT makes it relatively simple to separate the unwanted current into several decoupled components, e.g. calculating which components are produced by unbalanced or non-linear loads, and which are caused by power supply distortion, and (ii) the “hidden” currents associated with p-q power theory discussed in [31] [41] are completely avoided when the CPT is used. Notice that in islanded MGs, the hidden current could be a serious problem since it is a weak network [31]. Therefore, the use of CPT could improve the active power filter performance, at avoiding the “hidden currents” issue.
- CPT can be used to develop cooperative control schemes, in which both imbalances and harmonics can be shared among the 4-leg converters which are present in a MG, according to their residual VA capacity. Based on the CPT, it is possible to deduce a mathematical derivation to prove that the proposed controller can effectively achieve the sharing of both imbalances and harmonics among parallel converters feeding a single common load.

- A relationship between the CPT components and the positive, negative and zero sequence components (at the fundamental frequency) could be derived. That relationship could be extended to harmonic components.
- A detailed stability analysis could be added with the design of the PLL algorithm. Especially, to quantify the effects of the PLL bandwidth on the stability of a converter connected to a weak grid.

1.2 Project Objectives

The PhD research project aims to investigate imbalances, harmonics and weakness issues which affect power quality and stability of MGs, and propose new methodologies for facing and solving them. The research described in the thesis pursues the following objectives:

- To evaluate the CPT-based APF performance in steady state, transient states, and the decoupling of unbalanced and harmonics effects.
- To derive a small signal model for studying the stability of a converter connected to a three-phase three-wire weak grid concerning the PLL dynamics.
- To design and implement a shunt active power filter (APF) based on the conservative power theory (CPT) for an isolated three-phase four-wire laboratory prototype microgrid.
- To design a cooperative imbalance/harmonic sharing control scheme based on the CPT to be used in an isolated three-phase four-wire MG.
- To evaluate the performance of the cooperative control scheme in steady state, transient states, decoupling of unbalanced and harmonics effects using a microgrid prototype.
- To validate the proposed small signal model using experimental testing.

1.3 Contributions

This thesis aims to investigate control algorithms and topologies to address the power quality issues related to imbalances and harmonics in MGs and investigate the effects of the weak nature of MG on the control of the power converters. Regarding imbalance and harmonic issues, two approaches are investigated: (i) the use of active power filters to compensate unbalanced and distorted currents produced by loads, and (ii) the use of cooperative controllers to improve the sharing of imbalances and harmonic currents among the converters in a MG, according to their residual VA capacity. To study the effects of the weak nature of MGs on the previous two approaches, a methodology for a simple controller is studied, and its extension, as future work, to look at the specific CPT-based controllers discussed in this thesis is proposed.

The following are considered “contributions to knowledge” resulting from this thesis.

Active Power Filters:

- In this thesis, the CPT is applied to 4-wire systems considering full control of the 4 legs in the APF. Unlike previous publications where a four-leg converter is controlled using a fixed duty cycle of 50% in the fourth leg (see [42]), in this thesis, full utilisation of the dc link is achieved using proper modulation algorithms and control systems.
- An important remark related to the practical implementation of the CPT algorithm is the impact of grid frequency variations. In previous publications [42] [43] [30], moving average filters must be made frequency adaptive using the frequency measured by a

phase-locked loop (PLL). This leads to a considerable computational burden that might reduce the attractiveness of CPT-based filters. In this thesis, a different approach has been considered: rather than implementing highly selective and adaptive filters, fixed filters with lower selectivity have been adopted. Considering that the range of frequency variations is expected to be narrow, the impact on filtering will also be limited. This choice simplifies the structure of the CPT algorithm, with limited impact on the effectiveness of the algorithm, as demonstrated in the experimental results under grid frequency transients.

- The current control is realised using seven paralleled-connected self-tuning resonant controllers in each phase. The tracking of the resonant frequency is realised using a tuning algorithm, based on the real-time discretisation of controllers implemented using the bilinear transform with frequency prewarping.

Cooperative imbalance sharing method:

- The proposed sharing algorithm uses a novel approach based on the CPT methodology that can be used to implement a simple identification of the balanced, unbalanced and non-linear components of the currents and powers. Separation of the sequence components is not required.
- A simple method to share harmonic distortion between the generation sources is realised considering the application of the CPT transform. This methodology is discussed and experimentally validated.

- To the best of the author's knowledge, there is only one paper where a control scheme for cooperative imbalance sharing in 4-wire MGs is discussed (see [44]). The other control schemes are proposed for 3-wire MGs. The control system discussed in [44] considers sequence separation, which could be a drawback because most of the sequence separation algorithms are strongly affected by, noise, harmonic distortion, small variations in the sampling time, etc. [45] [46]. Moreover, the application of the scheme proposed in [44] to a system with relatively high harmonic distortion could be difficult to realise considering (i) the complexity of the extraction of the main harmonics and (ii) the harmonics extraction is sequence-separation-dependant. The control algorithm proposed in this work is based on the CPT which is very robust to issues such as distortion, noise, differences in the sampling time etc. As is demonstrated in this thesis, it is very simple to extend the proposed methodology to include harmonic distortion.

Effects of the PLL in weak connections

- A systematic PLL design process is proposed for use in weak grids to ensure system stability. It is based on a proposed low-complexity small signal model of the system in the dq reference frame.
- A comprehensive study of the effects of PLL bandwidth on the system stability is performed. These effects are also studied for different levels of grid weakness.

1.4 Thesis Structure

The rest of this work is divided into six chapters as follows:

- **Chapter 2:** In this chapter, an extensive literature about power quality and stability issues in microgrids is reported and discussed.
- **Chapter 3:** The proposed control algorithm based on the CPT for a shunt active power filter is introduced, explained and experimentally validated. Additionally, a literature review for APFs is presented.
- **Chapter 4:** The proposed control algorithm based on the Conservative Power Theory for cooperative sharing of imbalances and harmonics in 4-wire MGs is presented. The mathematical framework of this method is reported, and both simulations and experimental validations are performed which show the effectiveness of the proposed control algorithm. Moreover, a literature review about cooperative methods to manage imbalances and harmonics is presented.
- **Chapter 5:** The proposed scheme to design a PLL to be used in weak grids is reported. This design procedure is based on a low-complexity model developed in the dq reference frame. In this chapter, the mathematical derivation of the model is performed, and the effectiveness of the proposed design scheme is validated through both simulation and experimental tests. A literature review with a focus on works that study the effects of the PLL on the system stability is provided.
- **Chapter 6:** The conclusions of this PhD Project and an outlook to the future research on these topics are presented in this section.

Chapter 2. Literature Review

Power systems are undergoing a paradigm change, moving from centralised distribution to distributed generation including microgrids (MGs) and smart grids. In distributed generation systems it is expected that a relatively high penetration of electrical energy provided by Non-Conventional Renewable Energy (NCRE) sources of intermittent nature will be present, usually connected to the MG using power electronic converters which can affect the stiffness of the network. Droop control is one of the methods typically used to share active and reactive power between these generating units [11] [47] and this may lead to some variations in the microgrid fundamental frequency: this must be considered when designing control systems for MGs.

A MG could be implemented as a low voltage power distribution system operating in grid-connected or islanded mode [13] [48] [49] feeding single-phase linear and non-linear loads, typically using three-phase 4-wire topologies [14] [50]. These inherently unbalanced loads produce negative and zero sequence components in the currents and voltages of the network. According to [20], the ratio of unbalanced to balanced loads will increase in the future with higher penetrations of hybrid and electric vehicles, heat pumps, etc. In this context, some well-known problems related to load imbalance could appear [18]. For instance, negative sequence voltage components produce oscillations in the torque of induction machines and synchronous generators. This, in turn, may reduce the efficiency and useful life of these machines [20] [21]. Other effects of imbalance are localised heating in machines and power converters, and reduced loadability of conventional synchronous generators (see [21] [51]) which affects the stability of the MGs [51]. The zero sequence current flowing through the neutral wire can produce additional problems such as overloading of the neutral conductor [18] [19] and

malfunction of sensitive equipment due to ground voltage fluctuations if the current flowing through the neutral wire is large and/or the impedance of the neutral connection has a relatively large magnitude [18] [19]. In addition, the load served by a 4-leg MG could be of non-linear nature [19] [23] [52], generating distorted load currents which could also diminish the loadability of the power generating sources in the microgrid.

Several solutions have been proposed in the literature to compensate imbalance and harmonic distortion in MGs [25] [53] [54]. In particular, in this thesis two approaches will be studied, these are: (i) the use of Active power Filters (APFs), and (ii) the use of cooperative algorithms for sharing imbalances and harmonics among the converters which are part of a MG. Both approaches will be studied using a three-phase four-wire MG, and they are based on the Conservative Power Theory (CPT). The experimental validation of these control systems and hardware topologies proposed in this thesis has been partly based on the operating conditions of a real MG implemented in the north of Canada. The values of Total Harmonic Distortion (THD), current imbalance, variations of the load, etc. are based on the experimental data measured at this location. From this data (see Table 3.1 in section 3.4) it is concluded that the zero sequence current components can have a relatively large magnitude when a real case of an isolated microgrid operating at a remote location is considered.

Active Power Filters (APFs) are considered appropriate topologies with several papers discussing issues such as the design of APF's passive components [14] [19], compensation objectives [12] [20], serial and/or parallel connection of APFs [14], etc. The performance of the APF is heavily dependent on the control system applied which is usually based on a power theory, with many applications using the $p-q$ or modified $p-q$ power theories [32]. However, one of the reported disadvantages of these methodologies is their sensitivity to distortion in the signals and the generation of unwanted current

components in certain operating conditions [55] [56]. Most of the previous publications related to the use of active power filters in distributed generation are focused on 3-leg applications without discussing the problems associated with low voltage distribution systems which are mainly 4-wire systems. Therefore in this work, the control of a 4-leg APF based on the Conservative Power Theory (CPT) is presented. Unlike previous publication where a 4-leg converter is controlled using a fixed-duty cycle of 50% in the 4th leg (see [42]) in this work full utilisation of the dc-link is achieved using proper modulation algorithms and control systems. In addition, resonant controllers instead of controllers with low dynamic response, e.g. those reported in [42] [57], are designed and implemented in this application. Using a CPT based algorithm, it is relatively simple to separate the unwanted current into several decoupled components, for instance calculating which components are produced by unbalanced or non-linear loads and which are due to power supply distortion (for accountability purposes [30] [43]). This is important considering that a typical microgrid has several power converters interfacing the intermittent NCRE sources to the MG. Therefore it would be advantageous to use the residual VA capability of these converters to share the compensation of unwanted currents. This, in turn, could reduce the rating of the APF.

On the other hand, cooperative sharing of active (P) and reactive (Q) power in microgrids has been widely studied and can be achieved using the droop control method [11]. However, there is no straightforward solution to collaborative sharing of imbalances among inverters. Moreover, very few papers discuss collaborative control systems for 4-wire microgrids which contain single-phase loads, where the imbalances are produced not only by the presence of negative sequence components but also by the zero sequence. In fact, the presence of unbalanced loads in a MG could produce voltages imbalances at other points of the 4-wire system due to the circulation of unbalanced

currents through the line and converter impedances [44] [58] in the microgrid. To solve the problems produced by imbalances, cooperative methods for sharing imbalances and/or harmonics have been proposed [54] [59] [60] [61]. It means to embed imbalance and harmonic compensation capabilities in the control algorithms of the inverters that are already available in the MG, thus maximising the utilisation of the hardware [54] [59] [60] [61]. For example, in [60] a cooperative method for compensating unbalanced voltages at the inverters' output is proposed. The goal is to use the remaining VA capability of the converters as part of a negative sequence droop control scheme (called $Q^- - G$). The compensation reference is injected at the output of the voltage control loop and thus can be considered as a disturbance to be rejected by this control loop. Moreover, the cooperative performance is affected by the line impedance and the droop coefficients. In [54] the unbalanced output voltage of the inverters is compensated by adding a negative sequence voltage component to their references. The paper proposes the direct change of the voltage reference of the inverters to compensate the voltage imbalances in a MG. In this case, the compensation reference is considered a command to be followed by the voltage controllers. However, these references depend on the load characteristics, and it is difficult to calculate them in MGs with a high number of converters and with variable loads. In [61], a control method for reducing the voltage imbalances at the inverters' outputs is proposed. The voltage compensation introduces negative sequence currents through the current-controlled converters. This method is not suitable for voltage-controlled converters. In [59] a method for compensating harmonics and imbalances in a MG is proposed that modifies both the virtual impedance loop and the inner voltage control loop. The method is effective; however, it is more suitable for a single DG operation. Note that, the works of [54] [59] [60] [61] are performed for three-phase three-wire MGs, and they require an algorithm for sequence separation. This issue could be a

drawback considering that sequence separation algorithms are affected by noise, harmonic distortion, etc. [45] [46]. Moreover, in [54] [59] [60] [61] zero sequence components are not considered because three-phase three-wire systems are studied.

The aim of the work discussed in [54] [59] [60] [61] is predominantly to achieve cooperative imbalance voltage compensation among the inverters when the load is unbalanced. Another interesting goal is to improve the unbalanced power load sharing among the inverters located in a MG according to their power rating, to prevent a disconnection of some of them due to overload and therefore to prevent stability issues in the whole MG, especially under heavy load conditions. This aim can only be achieved at the expense of increasing voltage imbalance at the inverter outputs as discussed in [44] [58] [62]. Obviously, the maximum unbalanced voltage and voltage distortion allowed in the microgrid has to be regulated to avoid power quality issues. For instance, using the standards ANSI C84.1-2006 [63] (stating maximum voltage imbalances) and IEEE 519-1992 [64] (stating maximum THD values). However, in the planning of a 4-wire microgrid, the fact that not all the loads have the same sensitivity to the voltage imbalances should be considered [65]. Therefore, buses, where sensitive loads may be located, have to be subjected to more stringent voltage requirements than those buses where no sensitive loads are connected. Therefore, imbalance sharing methods should be designed to fulfil the voltage requirements of buses where sensitive loads may be located. Power quality in other non-critical buses of the MG could be improved using a hierarchical control system [58] [62] [65].

In the context of voltage imbalance control of microgrids considering sensitive and non-sensitive loads, an algorithm to improve unbalanced current sharing for a three-phase four-wire MG is proposed in [44]. Since the voltage quality could be degraded when the proposed method is enabled, the authors proposed a decentralized control

scheme to improve selectively the voltage quality in buses where sensitive loads are connected. It is argued in [44] that critical loads should be placed close to inverters and therefore the increase of voltage imbalance at the PCC when the proposed control scheme is working, is not an important concern.

On the other hand, in [58] it is assumed that a sensitive load is placed at the PCC. A method is proposed for improving the current sharing for both imbalances and harmonics in a three-phase three-wire MG, and because the voltage imbalance is increased at the PCC when the proposed method is enabled, the authors proposed a secondary control scheme for improving the voltage quality at this point. In [44] [58] the use of a sequence extraction algorithm is required. Cooperative imbalance sharing is achieved, defining both negative and zero sequence impedances in [44], while in [58], only a negative sequence impedance is used since a three-phase three-wire system is discussed.

Based on the literature review for APFs and cooperative algorithms, the objectives of this thesis are:

In this work, the control of a 4-leg APF based on the Conservative Power Theory (CPT) is presented. Unlike previous publication where a 4-leg converter is controlled using a fixed-duty cycle of 50% in the 4th leg (see [42]) in this work full utilisation of the dc-link is achieved using proper modulation algorithms and control systems. In addition, resonant controllers instead of controllers with low dynamic response, e.g. those reported in [42] [57], are designed and implemented in this application.

In this thesis, a decentralized collaborative control scheme for sharing load imbalance in a 4-wire MG is proposed. The term collaborative is used to indicate the capability of each inverter to supply unbalanced power (as defined by the CPT in section 2.1) to the load according to its residual power capacity, which changes with time. The control scheme proposed in this paper is performed in the fixed *abc* coordinate reference frame,

and sequence separation is not required. The Conservative Power Theory (CPT) [42] [43] [66] [67] [68] is used as the theoretical framework to achieve imbalance sharing among the inverters.

Due to in this thesis the proposal the shunt active power filter and the proposal related to cooperative algorithms are based on the CPT, in the next subsection, this power theory is discussed.

2.1 Conservative Power Theory

The CPT has been proposed as a new power theory to analyse distribution systems and microgrids, where single-phase loads, non-linear loads, and strong harmonic distortion could be present [43] [66]. Using the CPT, an orthogonal separation of the unwanted current components can be achieved to allow selective compensation of them. Moreover, the implementation of a CPT-based control algorithm can be realised in the stationary *a-b-c* frame: it does not require a synchronous rotating reference frame [43]. The CPT has been discussed before (e.g. [43] [69]). However, for completeness, a brief introduction is presented here.

To implement the CPT two instantaneous quantities are defined [43] [69]. The first is the instantaneous power which corresponds to the scalar product:

$$p(t) = v \cdot i = [v_a \ v_b \ v_c] \cdot [i_a \ i_b \ i_c]^T \quad (4)$$

The second is the instantaneous reactive energy defined as:

$$w(t) = \hat{v} \cdot i = [\hat{v}_a \ \hat{v}_b \ \hat{v}_c] \cdot [i_a \ i_b \ i_c]^T \quad (5)$$

where $\hat{v}$ is the vector containing the unbiased integral of the phase voltages. This vector $\hat{v}$ is calculated as:

$$\hat{v}_\mu = \int_0^t v_\mu(\tau) d\tau - \bar{v}_{f\mu}, \quad \bar{v}_{f\mu} = \frac{1}{T} \int_0^T v_\mu(\tau) d\tau, \quad \mu = a, b, c \quad (6)$$

Notice that the index “ μ ” ($\mu = a, b, c$..) represents the variable per phase and the term $\bar{v}_{f\mu}$, correspond to the average component of each phase voltage integral. For the application depicted in Fig. 3.1, phase voltages are measured with respect to the neutral conductor at the PCC [43]. Considering the mean values of (4)-(5), the active power (P) and reactive energy (W) are defined as:

$$P = \frac{1}{T} \int_0^T v \cdot i \, dt \quad W = \frac{1}{T} \int_0^T \hat{v} \cdot i \, dt \quad (7)$$

It should be highlighted that definitions (4)-(7) hold irrespective of voltage and current waveforms, provided that they are periodic [43] [69]. Moreover, it is important to note that in sinusoidal conditions, the active power P given by (7) coincides, at any moment, with the conventional active power definition ($P_{conv} = VI \cos \varphi$) [43] [69]. Conversely, the reactive energy W shown in (7) is related to conventional reactive power (Q_{conv}) through the fundamental frequency ω by the following relation: $W = \frac{Q_{conv}}{\omega} = \frac{VI \sin \varphi}{\omega}$. [43] [69]

Based on (4)-(7), the load current i_μ ($\mu = a, b, c$) can be decomposed into five orthogonal components. These are: (i) balanced active currents ($i_{a\mu}^b$), (ii) balanced reactive currents

($i_{r\mu}^b$), (iii) unbalanced active currents ($i_{a\mu}^u$), (iv) unbalanced reactive currents ($i_{r\mu}^u$) and (v) void currents ($i_{v\mu}$) [43] [69].

The *balanced active currents* are defined as:

$$i_{a\mu}^b = \frac{P}{V^2} v_\mu = G^b v_\mu \quad (8)$$

such that $V = \sqrt{V_a^2 + V_b^2 + V_c^2}$ is the collective RMS value (Euclidean norm) of the voltage, the parameter G^b is the equivalent balanced conductance and P is the active power.

The *balanced reactive currents* are defined as:

$$i_{r\mu}^b = \frac{W}{\hat{V}^2} \hat{v}_\mu = \mathcal{B}^b \hat{v}_\mu \quad (9)$$

such that $\hat{v}_\mu$ is the unbiased voltage integral, $\hat{V}$ is the collective RMS value of the unbiased voltage integral, $\mathcal{B}^b$ is the equivalent balanced reactivity and W is the reactive energy (see [43] [69]).

The *unbalanced currents* ($i_{a\mu}^u$ and $i_{r\mu}^u$) are defined as:

$$i_\mu^u = i_{a\mu}^u + i_{r\mu}^u = (G_\mu - G^b) v_\mu + (\mathcal{B}_\mu - \mathcal{B}^b) \hat{v}_\mu \quad (10)$$

where the per-phase conductance and per-phase reactivity are defined as:

$$G_\mu = \frac{P_\mu}{V_\mu^2} v_\mu \quad \text{and} \quad \mathcal{B}_\mu = \frac{W_\mu}{\hat{V}_\mu^2} \hat{v}_\mu \quad (11)$$

Note that the unbalanced currents (10) only exist if $G_\mu \neq G^b$ and/or if $\mathcal{B}_\mu \neq \mathcal{B}^b$, i.e. when the load is unbalanced. In single-phase circuit, the currents of (10) disappear. Note that P_μ and W_μ (see (11)) are respectively, the active power and the reactive energy per phase.

The *void currents* are the remaining currents that do not transfer active or reactive energy and are given by:

$$i_{v\mu} = i_\mu - i_{a\mu}^b - i_{r\mu}^b - i_\mu^u \quad (12)$$

All the current components (i.e. $i_{a\mu}^b$, $i_{r\mu}^b$, $i_{a\mu}^u$, $i_{r\mu}^u$ and $i_{v\mu}$) are orthogonal [43] [69]. Moreover, based on the RMS values of currents and voltage, the CPT defines power terms [43], where $P = V \cdot I_a^b$ is the active power, $Q = V \cdot I_r^b$ is the reactive power, $N = V \cdot I^u$ is the unbalanced power and $D = V \cdot I^v$ is the void power. Notice that the capital letters I and V denote RMS values. The term I^u is the RMS value of the sum of $i_{a\mu}^u$ and $i_{r\mu}^u$ [see (10)]. Finally, the apparent power S , is related with the powers defined by the CPT through the following expression: $S^2 = P^2 + Q^2 + N^2 + D^2$.

Finally, this paragraph discusses the definition of reactive power (W) given by the CPT [see (7)] with the one given by Instantaneous Power Theory. First of all, it is important to clarify that the definition of reactive power is not identical in all the power theories published in the literature [31]. For instance, in the Instantaneous Power Theory, the reactive power (Q_{conv}) is defined as the imaginary part of $S = k \cdot \underline{v} \cdot \underline{i}^c$, where k is a scalar, and the superscript “c” indicates the complex conjugate operator ($\underline{v}$ and $\underline{i}$ are the

voltage and current vector respectively). However, one of the main criticism to this definition of the instantaneous power theory is that in some unbalanced operating conditions a resistor bank can produce reactive power [31], and even when this is correct in the context of the Instantaneous Power Theory, it is seen as “unnatural” by electrical engineers. The conservative power theory uses a different definition. The Reactive Energy (W), defined by (7), which represents the average of the energy stored by different devices in the multiphase grid in generic conditions, including waveform distortions and imbalances [43] [69]. As was discussed earlier in this chapter, under sinusoidal conditions, the reactive energy (W) is related to the conventional reactive power (Q_{conv} , given by the Instantaneous Power Theory) by $W = \frac{Q_{conv}}{\omega} = \frac{VI\sin\phi}{\omega}$ [43] [69]. If the reactive energy W is compared with Q_{conv} , the following conclusions can be established [70] [43]: the reactive energy (W) does not generate any work (similar to Q_{conv}), and (ii) using the reactive power definition of the CPT, a resistor bank does not produce reactive power. (Unlike to Q_{conv})

2.2 Effects of the PLL bandwidth in weak grids

As was stated at the beginning of this thesis, MGs are weak electrical systems; this feature may produce stability issues on the converters connected to them. To study the effects of the weak nature of MGs on the control of converters connected to weak PCCs, in this thesis, a methodology for a simple controller will be derived. This methodology will be a start point for in the future study the effects of the weak nature of MGs in the performance (for low SCRs) of the controllers, based on the conservative power theory, proposed in this thesis (see chapter 3 and chapter 4). In particular, the study the effects

of the stability of a converter connected to a weak grid as a function of the PLL bandwidth and the short circuit ratio (level of weakness) will be performed in this thesis.

In the MG context, the Phase-locked Loop is fundamental to allow the integration of renewable energy into the MG. Indeed, power converters are connected to the MG at the Point of Common Coupling (PCC), and they are usually controlled as current or power sources. To achieve this, both the frequency and angle of the current injected by the inverter into the grid must be synchronised with the grid voltage measured at the PCC. This synchronisation stage is usually based on a Phase-Locked Loop (PLL) [40]. Based on that, and taking into account the weak nature of MGs, stability issues could appear. In particular, it has been reported that the PLL dynamic affects the stability of the interface between the converter and the PCC when the grid is weak [71] [72]. Based on this issue, the study of the PLL dynamic, considering weak MGs is necessary to ensure the stability of the control algorithms based on the CPT propose in this thesis.

The conventional and simplest PLL design process assumes: (i) a simple PLL small signal model and (ii) an ideal grid with negligible impedance, i.e. $Z_g \approx 0$. Based on these assumptions, the parameters of the PLL controllers are obtained by Linear Time Invariant (LTI) design methods such as those based on the natural frequency and the damping ratio. However, in weak grids with high Z_g , stability issues caused by the PLL dynamics could appear, leading to unstable operating modes that will ultimately cause the disconnection of the converter due to the operation of the overcurrent and/or overvoltage protections. Therefore, the PLL design process should be improved to account for non-negligible grid impedances. In fact, when the grid impedance is high, the PCC voltage depends on: (i) grid voltage, (ii) grid impedance and (iii) current injected by the inverter into the grid. As a result, there is a coupling between the PLL and the grid [71] [72]. For this reason, above certain grid impedance values, a PLL design neglecting the grid

impedance produces unstable behaviour leading to loss of synchronisation. This is a well-known phenomenon that has been addressed by several publications which aim to provide models that explain the interaction between the PLL and a weak grid. These are discussed below.

In [73], a model of a three-phase system in the dq reference frame considering the effects of the PLL dynamics is proposed. The authors analyse the inverter output impedance and conclude that a high PLL bandwidth increases the negative real part of the impedance and therefore stability issues can appear [74]. The model presented in [73] does not consider the effects of the duty cycle and the coupling terms in the current feedback control. In [75], the model reported in [73] is improved by considering the following effects: (i) coupling terms, (ii) duty cycle, (iii) voltage feedback control. It is shown that the PLL bandwidth has an influence on the inverter output impedance Z_{qq} (see [75]) which behaves as a negative incremental resistor. It should be noted that the model reported in [75] is evaluated injecting only active power. Based on that, the authors in [76] extended the model in [75] by considering the injection of reactive power and the effects of a power control loop. Subsequently, in [77] the model reported in [76] is used together with the generalised Nyquist criterion to study the stability of converters connected to a weak grid with a focus on the PLL parameters. Experimental results validated the proposed stability analysis. In [78], the authors propose an improvement in the design of current controller parameters for an LCL-type grid-connected converter for reducing the negative effects of the PLL bandwidth on the system stability when the grid is weak. The proposed guideline for the current controller tuning is experimentally validated. The authors in [26] [79] [27] use the dynamic phasor technique to derive an average model of a single-phase inverter feeding into a grid. Based on this model, an output impedance matrix is derived. It is concluded that the imaginary impedance Z_{II} (see

[26]) is negative for low frequencies because of the presence of the PLL. The Nyquist criterion is used then to predict the stability of the system under different values of the integral gain of the PLL. In this model, only the first order phasors were taken into account, and the zero order phasors were not considered, which leads to a reduction of the model performance.

All the previously discussed works are based on the derivation of the output impedance of the inverter in the Laplace domain and on the analysis of its dependence on the PLL dynamics. The precision of these type of models is good. However, they are not always intuitive, and their complexity is high. Additionally, the usual tool to analyse stability issues is the Nyquist criterion and therefore it could be required to measure or estimate the output impedances. An alternative approach is to model the whole system in which the converter is connected and derive its state space representation, allowing the utilisation of some of the well-known linear control methods associated with state equations, e.g. eigenvalues analysis, participation matrix, state feedback, etc. In this context, in [80] [81] a single-phase inverter connected to a weak grid is modelled in the AC frame, and the PLL dynamics are considered. Since the model is derived in the AC frame, it is linearised using harmonic linearisation techniques leading to a linear time-periodic model. Then, the stability of the system is studied analysing the eigenvalues of the transition matrix, deriving the stability boundaries. This model requires full knowledge of the system. In [82], an impedance-conditioning term in the voltage used by the PLL to improve the stability of the system is proposed. To validate the approach reported in [82], a small signal model of the system in the dq reference frame is developed. However, experimental validation of the method is not provided. The same is realised in [83], where a model of the inverter in the dq reference frame is developed to analyse the performance of the proposed current sharing controller. In [84], the authors modelled an islanded

microgrid composed of two inverters with droop control and considering the PLL dynamic. The accuracy of the proposed model is compared with both simulations and experimental results, finding a good match. Although works [82] [83] [84] have developed small signal models of the system considering the PLL dynamics, the main goals of these works were not to study the effects of the PLL on the stability of the system. As a result, their use is not straightforward, and their complexity is typically high. In this context, [29] proposed a small signal model in the dq reference frame to study the impact of the short-circuit ratio (SCR) and PLL parameters in a VSC-HVDC converter. The authors concluded that the PLL parameters, particularly at low SCRs, greatly affect the stability of the converter. Some comments about this work are: (i) validation is only based on simulations, and no experimental validation is shown and (ii) to evaluate the effects of the PLL parameters, the controller gains are changed but a rigorous procedure to obtain appropriate values for these gains is not discussed.

Based on the literature review discussed in this section, it is concluded that the PLL dynamics affect the stability of power converters connected to weak grids. For this reason, in the context of MGs, which are inherently weak systems, it is essential to study the effects of this weak nature on the stability of the power converters in the MG. The latter to avoid stability issues produced by the weak nature of MGs on the control loops of the power converters. Based on this observation, the objective of this thesis, in this area is:

To study the effects of the PLL dynamics versus the level of weakness of the grid (characterized by the SCR) for a simple application to develop a methodology to analyze the stability of the proposed control algorithms based on the CPT discussed in this thesis. Indeed, if those controllers are implemented in a weak MG, it is necessary to establish stability boundaries in which they can work without affecting the stability of the whole MG.

Chapter 3. Proposed Unbalance and Harmonic Compensator

3.1 Introduction

Four-wire microgrids and distribution systems are inherently unbalanced with the presence of negative and zero sequence components in voltages and currents. In small autonomous systems the imbalance, in addition to the harmonic distortion produced by non-linear loads, can significantly affect power quality, loadability and stability of the system. Furthermore, in isolated networks with significant generation from intermittent renewable energy sources, the stiffness of the system is reduced, and this could amplify the effects of imbalance on the stability and power quality. To mitigate some of these problems, a novel methodology based on the application of a 4-leg Active Power Filter (APF) is proposed in this chapter. The control of the compensator is based on the Conservative Power Theory (CPT) augmented by Resonant Controllers. The behaviour of the proposed system is demonstrated using an experimental prototype deployed in a laboratory scale microgrid.

The rest of the chapter is organized as follows. In section 3.2 the system under study is introduced. In section 3.3 the proposed control system for the 4-leg APF is discussed. The operating conditions for the small microgrid in northern Canada are briefly described in section 3.4. In section 3.5 experimental results obtained from a small prototype are presented. Finally, an appraisal of the proposed methodology discussed in this chapter is summarised in the Conclusions. (Section 3.6)

3.2 System Under Study

The proposed compensating topology is shown in Fig. 3.1. On the left-hand side, the Thévenin equivalent combination of the 4-leg power supplies, obtained at the Point of Common Coupling (PCC) is shown. The linear/non-linear load and the compensator are connected at the PCC. If an energy storage system is required, this could be connected to the dc-link of the APF to provide active power to the MG. The system studied in this chapter is shown in Fig. 3.1. It can be found in both grid-connected and islanded-mode microgrids. In this context, Fig. 3.2(a) shows an example of a grid-connected MG. In this case, the main grid imposes the frequency of the system and the voltage at the PCC. On the other hand, Fig. 3.2(b) shows an example of an islanded microgrid. In this case, (contrary to the grid-connected mode), the 4-leg converters impose both the frequency of the system and voltage at the PCC; this is usually achieved through droop controllers [11] [54]. In both cases, the proposed compensator shown in Fig. 3.1 can be placed at the PCC (in shunt connection), to improve the quality of the currents that the power supply [Fig. 3.2(a)] or the 4-leg converter [Fig. 3.2(b)] injects to the load.

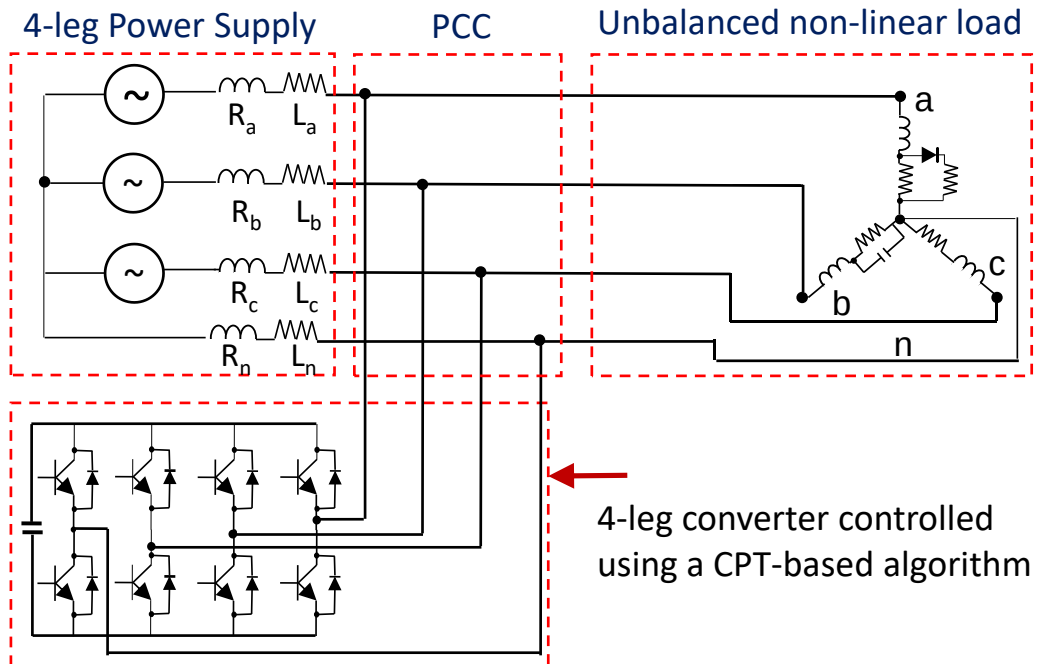


Fig. 3.1. Proposed compensation strategy for a 4-leg compensator connected to a distribution network.

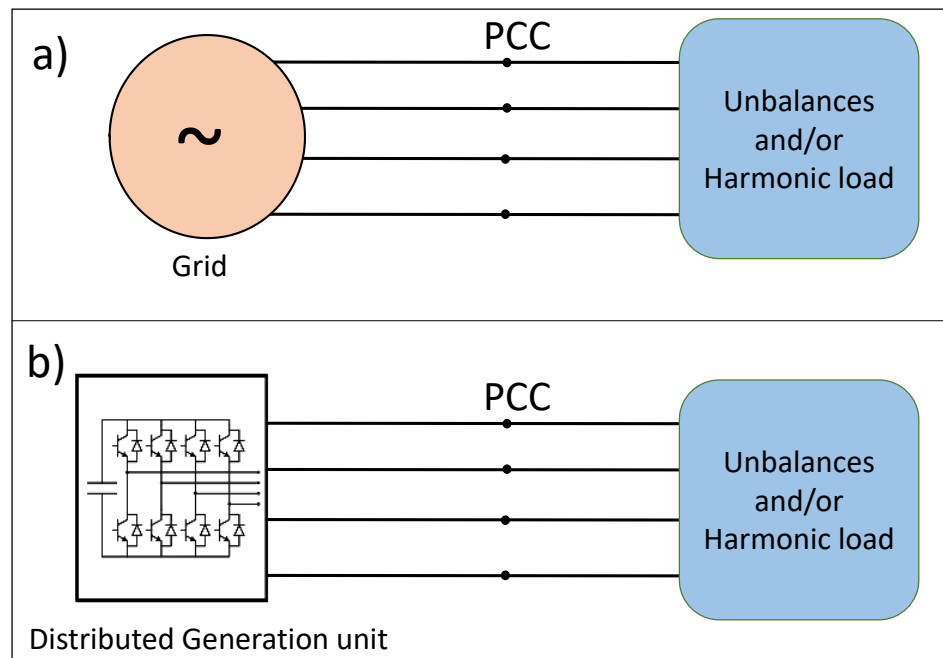


Fig. 3.2. (a) grid-connected microgrid, (b) islanded-mode microgrid

The implementation of the CPT used in this thesis is shown in Fig. 3.3. Voltage and current transducers are used to measure the load currents and voltages at the PCC. In this work, a Band-Pass Filter (BPF) is used to obtain the unbiased integral of the voltages (i.e. $[\hat{v}_a \ \hat{v}_b \ \hat{v}_c]$). Notice that this technique is widely used for the estimation of the magnetic flux in electrical machines [85]. The BPF eliminates the dc component of the signals and behaves as an ideal integrator for frequencies above $\approx 3\text{Hz}$ (see [85]).

Low Pass Filters (LPFs) are used to obtain the average value of the power P and the reactive energy W in each phase. The effective voltages and the Euclidian norms required by the CPT algorithm are calculated in the block labelled “RMS Calculator” shown at the top right of Fig. 3.3. This RMS calculator utilises a filter with a transfer function equal to that used in the others LPFs shown in Fig. 3.3. This filter has been designed with a relatively high cut-off frequency to avoid jeopardising the overall dynamic response. It should be highlighted that low pass filters are going to have an impact on the dynamic performance of the control system. However, filters are usually required to implement control systems for APFs, because it is necessary to separate the wanted from the unwanted current components. After separating them the unwanted current signals could be compensated. Therefore, this is not an issue limited only to the implementation of the CPT algorithm. Filters are also required when APFs based on the instantaneous p-q power theory are implemented [31] [55].

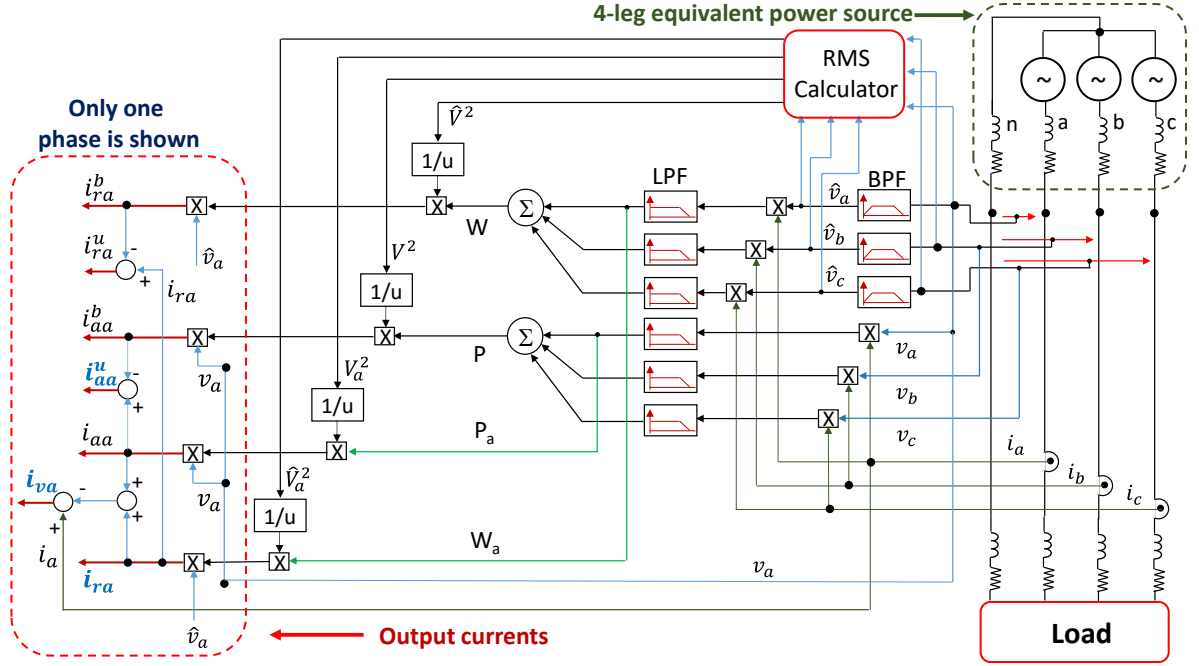


Fig. 3.3. Calculation of the compensating currents using the conservative power theory. Only currents of phase-a are shown ($i_{aa} = i_{aa}^b + i_{aa}^u$ and $i_{ra} = i_{ra}^b + i_{ra}^u$).

As shown in the left-hand side of Fig. 3.3, the outputs of the CPT algorithm are the currents $[i_{a\mu}^b, i_{a\mu}^u, i_{r\mu}^b, i_{r\mu}^u, i_{v\mu}]$. These currents are used to calculate the references required by the current control.

An important remark related to the practical implementation of the CPT algorithm is the impact of grid frequency variations. As discussed above, the calculation of the unbiased integral (6) of the active power and reactive energy (7), and of the Euclidean norms requires bandpass filters or low pass filters. In previous works [42], these filters have been implemented using moving average digital filters, resulting in selective frequency responses but highly sensitive to grid frequency variations. From a practical perspective, moving average filters must be made frequency adaptive using the frequency measured by a PLL. This leads to considerable computational burden that might reduce the attractiveness of CPT based filters. In this work, a different approach has been considered: rather than implementing highly selective and adaptive filters, fixed

filters with lower selectivity have been adopted. Considering that the range of frequency variations is expected to be narrow, also the impact on filtering will be limited. This choice simplifies the structure of the CPT algorithm, with limited impact on the effectiveness of the algorithm, as demonstrated in the experimental results under grid frequency transients.

A final comment about the implementation of the CPT is related with the practical need for complete current decomposition. If the APF is designed to fully compensate the load non-idealities, only the balanced active components (8) of the load current must be evaluated, and all the remaining unwanted components can be derived subtracting the active balanced current from the load current. In this case, the computational burden of the CPT algorithm will be much reduced. On the other hand, if the APF has small power rating or the active filtering is performed using the residual power capability of distributed generators, it might not be possible to compensate for the non-idealities in full, but an external command could be sent to another APF or power converter (with residual VA capability available) to demand for the compensation of a specific set of unwanted components. For the sake of generality, this work addresses the case where a full current decomposition is implemented, giving maximum flexibility in the selection of the compensation objectives.

The capability of separating the unwanted signals into several orthogonal components (which can be compensated using several power converters) is a very powerful feature of the CPT algorithm described in (4)-(12).

3.3 Proposed Control System

The proposed control system, for one of the phases, is shown in Fig. 3.4. In each phase, a PI controller and seven resonant controllers have been implemented in a parallel

topology. The PI controller is required to compensate the load dc-components produced when non-linear loads such as diode rectifiers are connected to the MG. The control system has been implemented in $a-b-c$ coordinates.

The seven paralleled resonant controllers (per phase) shown in Fig. 3.4 correspond to self-tuning resonant controllers. The tracking of the resonant frequency is realised using a tuning algorithm, based on the real-time discretisation of controllers implemented using the bilinear transform with frequency prewarping. As discussed in [86], this discretisation method has important advantages, in terms of preserving the resonant frequency accuracy, particularly when high-order harmonics have to be compensated. Self-tuning adaptive algorithms for the bilinear transform with frequency prewarping have not been discussed in the literature where most of the reported applications are based on the conventional bilinear transform [87]. In appendix A, a comparison between the effects of the use of the bilinear transform with frequency prewarping and other transforms in the implementation of discrete PR controllers is presented. Finally, It should be pointed out that self-tuning adaptive PR controllers have been proposed in this thesis, since in islanded microgrids, variations in the electrical frequency may occur (see Fig. 3.8) [11] [62] [54].

Resonant controllers have been selected for this application because they can regulate sinusoidal signals with zero steady-state error in $a-b-c$ coordinates. Moreover, they have a better dynamic response than that obtained with other control topologies (e.g. repetitive controllers [57]). In each phase, a PI controller and seven resonant controllers, tuned to regulate the fundamental and the 2nd, 3rd, 4th, 5th, 6th and 7th order harmonics, have been implemented in this work. These frequencies have been selected considering the harmonic distortion presented in the case-study microgrid discussed in

authors have implemented high order resonant control systems using the low-cost Texas Instrument processor, TMS320C6713 DSK augmented with an Altera FPGA board (see [89]). A detailed analysis of implementation issues is considered outside the scope of this thesis.

In a parallel topology, each resonant controller can be separately designed, assuming that the response of each controller is selective and only alters the frequency response around its resonant peak [90]. This simplifies the design procedure significantly.

3.3.1 Control System Design

The design and digital implementation of proportional resonant (PRs) controllers have been broadly discussed in previous publications (see [86] [89] [90] [91]) and is briefly summarised here.

In the s plane a PR controller, designed to regulate a current of frequency ω_j can be written as:

$$G_{cj}(s) = k_{pj} + \frac{k_{ij}s}{s^2 + 2\zeta\omega_j s + \omega_j^2} \quad (13)$$

where ζ is the controller damping and k_{pj}, k_{ij} are the controller gains. Notice that (13) is equivalent to the transfer function of a PI controller when $\omega_j = 0$.

The poles and zero of the resonant part of the PR controller [see the right side term of (13)] are shown in Fig. 3.5(a). Notice that in this graphic ω_j is defined as $\omega_j = \omega_n$. Using Fig. 3.5(a) and the pole-zero placement method is relatively simple to design the term $\omega_c = \zeta\omega_j$ in order to achieve a given bandwidth [17].

Assuming that the term ω_c in (13) is very small compared to ω_n , the peak amplitude (P) of the resonant part of the controller is obtained when $s = j\omega_n$. From Fig. 3.5(a) it is also simple to conclude that for $s = j(\omega_n + \omega_c)$ and $s = j(\omega_n - \omega_c)$ the gain is reduced in $\approx 3\text{db}$, corresponding to an amplitude value of $P/\sqrt{2}$. Therefore, the term ω_c could be used to set the band-pass of the resonant frequency and helps to modify the selectivity of the PR controller of (13) (see [17]). In this work, $\omega_c = 5\text{rads}^{-1}$ has been selected and this value allows the current control loop to achieve a good dynamic performance [17], as demonstrated by the experimental results presented in this thesis.

The control diagram for design purposes is shown in Fig. 3.5(b). Assuming that each controller could be separately designed and that the voltage at the PCC is an external perturbation (unaffected by the operation of the APF), the following transfer function can be derived from Fig. 3.5(b).

$$\frac{i_{fa}}{i_a^*} = \frac{G_{cj}(s)Plant(s)e^{-sT_s}}{1 + G_{cj}(s)Plant(s)e^{-sT_s}} \quad (14)$$

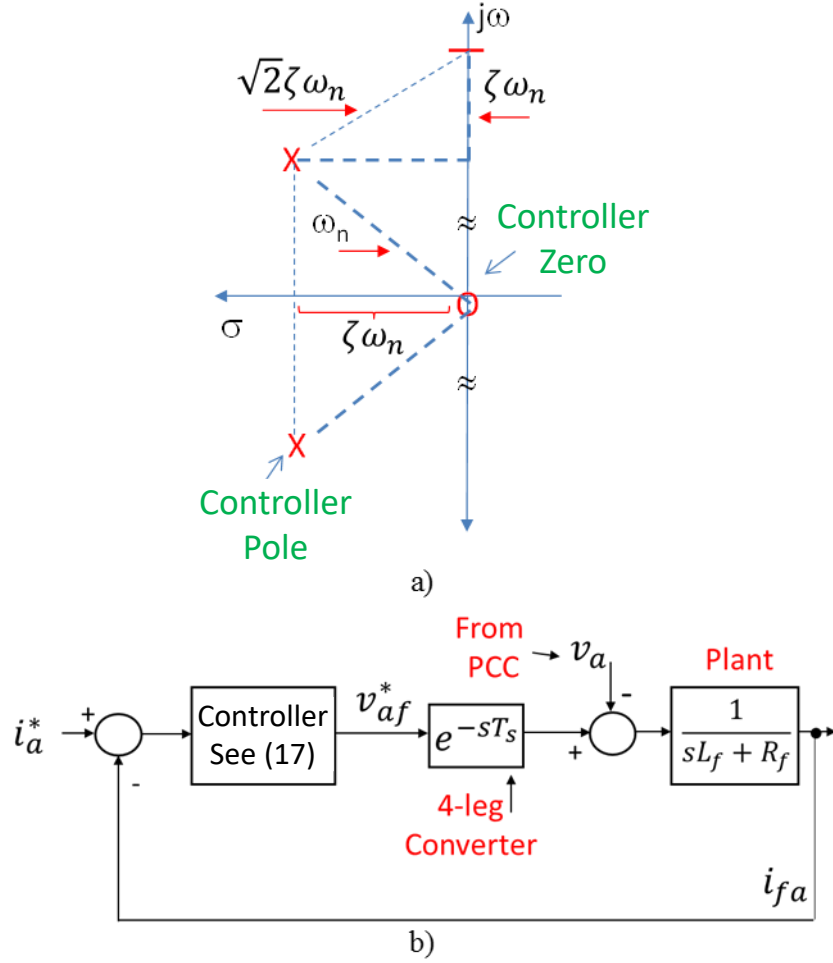


Fig. 3.5. . a) Pole and zero placement of the resonant part of the PR controller [see the right side term of (13)]. Control system considering the plant and the converter.

In (14), the term plant is defined as $plant(s) = 1/(sL_f + R_f)$ and i_{fa} is the current injected by the active filter in the phase “a” (see Fig. 3.4) . Notice that L_f , R_f are the inductance and the resistance per phase (respectively) of the impedance connecting the APF to the PCC. A delay of one sample in (14) is used to represent the transfer function of the 4-leg APF. This delay is a simplified representation of the effects associated to the power converter modulation algorithm.

Adding the transfer functions of the controllers depicted in Fig. 3.4, the transfer function of the whole current control system (phase A) is obtained as:

$$\frac{v_{af}^*(s)}{[i_a^*(s) - i_{fa}(s)]} = k_{pT} + \sum_{j=0}^7 \left[\frac{k_{ij}s}{s^2 + 2\zeta\omega_j s + \omega_j^2} \right] \quad (15)$$

where $k_{pT} = \sum k_{pj}$ is the overall controller gain. Notice that (15) includes the PI controller.

There are many control design methods reported in the literature for determining the controller gains k_{pj} and k_{ij} [see (13)] [89] [90]. In this thesis, linear control tools such as Bode Diagrams and Root Locus reshaping have been applied to design each of the eight parallel controllers shown in Fig. 3.4. Assuming that the resonant controllers alter only the frequency response around its resonant frequency, each controller (as shown in Fig. 3.4) has been designed for a phase margin of at least 50 degrees and a bandwidth of $\approx 30\text{rad/s}^{-1}$. The Bode diagram of the proposed control system is shown in Fig. 3.6.

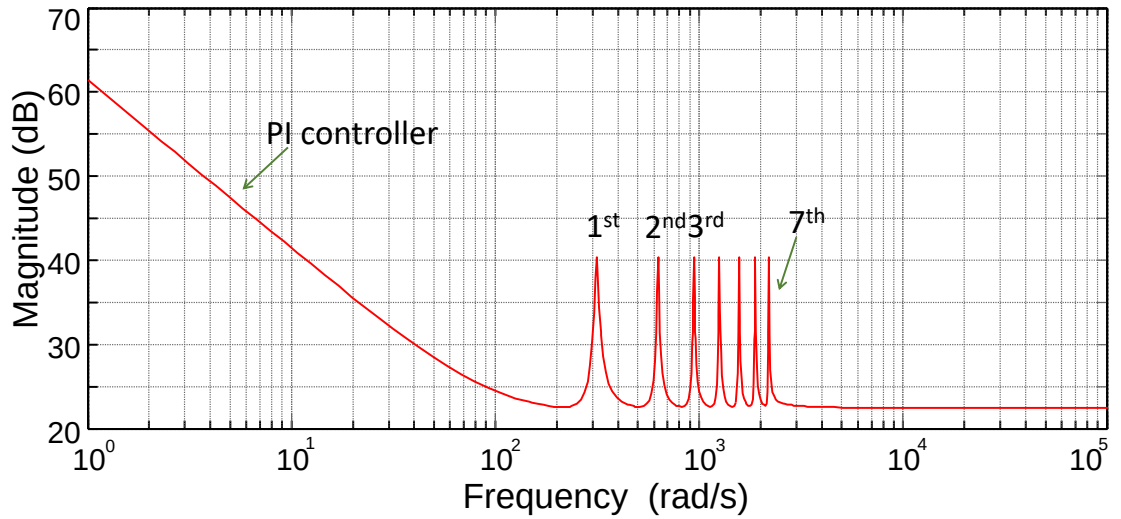


Fig. 3.6. Bode diagram of the current system.

In this thesis, the z-plane implementation of (13) is realised by applying the bilinear transform with frequency pre-warping to (15). This discretisation method has been selected because it preserves, with relatively good accuracy, the resonant frequency and

the phase margin of the original design in the “s” plane [86] [90]. Therefore, the digital version of (15) is obtained by replacing the Laplace operator “s” using:

$$s = \frac{\omega_j}{\tan\left(\omega_j \frac{T_s}{2}\right)} \left(\frac{z-1}{z+1}\right) = \frac{h\omega_e}{\tan\left(h\omega_e \frac{T_s}{2}\right)} \left(\frac{z-1}{z+1}\right) = k_{T\omega_j} \left(\frac{z-1}{z+1}\right) \quad (16)$$

where ω_j ($\omega_j = h\omega_e$) is the j -th controller resonant frequency, T_s is the sampling period and $k_{T\omega_j}$ is a non-linear function of the resonant and sampling frequencies. Replacing (16) in (15), the z-plane transfer function of the current control system is obtained as:

$$\frac{v_{af}^*(z)}{[i_a^*(z) - i_{fa}(z)]} = k_{pT} + \sum_{h=0}^7 \left[\frac{k_{T\omega_j} k_{ij} (z^2 - 1)}{(2\zeta\omega_j k_{T\omega_j} + k_{T\omega_j}^2 + \omega_j^2) z^2 + 2(\omega_j^2 - k_{T\omega_j}^2) z + (k_{T\omega_j}^2 + \omega_j^2 - 2\zeta\omega_j k_{T\omega_j})} \right] \quad (17)$$

Defining:

$$a_{0h} = k_{T\omega_j} k_{ij}, \quad h = 0, 1, \dots, 7 \quad (18)$$

$$b_{0h} = k_{T\omega_j}^2 + (h\omega_e)^2 - 2\zeta h\omega_e k_{T\omega_j}, \quad h = 0, 1, \dots, 7 \quad (19)$$

$$b_{1h} = 2((h\omega_e)^2 - k_{T\omega_j}^2), \quad h = 0, 1, \dots, 7 \quad (20)$$

$$b_{2h} = 2\zeta h\omega_e k_{T\omega_j} + k_{T\omega_j}^2 + (h\omega_e)^2, \quad h = 0, 1, \dots, 7 \quad (21)$$

Equation (17) can be rewritten as shown in (22).

$$\frac{v_{af}^*(z)}{[i_a^*(z) - i_{fa}(z)]} = k_{pT} + \sum_{h=0}^7 \left[\frac{a_{0h}(z^2 - 1)}{b_{2h}z^2 + b_{1h}z + b_{0h}} \right] \quad (22)$$

Note that (22) is the transfer function of a self-tuning resonant control system, which can be used to maintain the control system tuned in spite of changes in the fundamental frequency ω_e . In this thesis, the value of ω_e is estimated using a PLL (as shown in Fig. 3.4) and used to recalculate the values of the discrete coefficients a_{0h} , b_{0h} , b_{1h} and b_{2h} [using equations (16),(18)-(21)], updating the controller coefficients. To the best of the author's knowledge, the implementation of self-tuning resonant controllers, using the bilinear transform with frequency-pre-warping, has not been reported before.

3.3.2 Current Components Supplied by the APF

As shown in Fig. 3.3 and from (8)-(12), the CPT algorithm separates the load current into five orthogonal components $[i_{a\mu}^b, i_{a\mu}^u, i_{r\mu}^b, i_{r\mu}^u, i_{v\mu}]$. It is assumed that the load active balanced current $i_{a\mu}^b$ is supplied by the power source, and all the other components are supplied by the APF. Therefore the unwanted current components being compensated by the APF are obtained as:

$$i_{\mu}^* = i_{a\mu}^u + i_{r\mu} + i_{v\mu} \quad (23)$$

where $i_{r\mu} = i_{r\mu}^b + i_{r\mu}^u$ [see (9)-(10)] and $i_{a\mu}^u$ are the unbalanced active currents [see (10)-(11)]. If the dc-link capacitor of Fig. 3.1 is floating, the 4-leg APF has to supply the converter losses from the grid. Therefore a balanced active current component has to be added to (23). This is obtained as:

$$i_{a\mu_dc}^b = \frac{P_{dc}}{V^2} v_{\mu} \quad (24)$$

where P_{dc} is obtained from a PI controller (as shown in Fig. 3.4). Using (23)-(24), the current references are obtained as:

$$i_{\mu}^* = i_{a\mu}^u + i_{r\mu} + i_{v\mu} + i_{a\mu-dc}^b \quad (25)$$

As shown in Fig. 3.4, the current references (i_a^*, i_b^*, i_c^*) are processed by 24 controllers (8 per phase, see Fig. 3.4) generating three instantaneous voltages v_{af}^*, v_{bf}^* and v_{cf}^* . These voltages are modulated by the 4-leg converter using a sine-weighted PWM algorithm implemented in the DSPACE experimental platform discussed in section 3.5. In order to achieve full utilisation of the dc-link, a common voltage component is added to the modulation signals of all the four legs, including the neutral leg. This additional component is derived according to the well-known Min-Max algorithm using the methodology proposed in [92] [93]. This algorithm is briefly summarised below:

The phase to neutral voltages v_{af}^*, v_{bf}^* and v_{cf}^* are calculated as:

$$v_{af}^* = v_{az}^* - v_{fz}^* \quad (26)$$

$$v_{bf}^* = v_{bz}^* - v_{fz}^* \quad (27)$$

$$v_{cf}^* = v_{cz}^* - v_{fz}^* \quad (28)$$

where the subscripts a,b,c, f represent the four legs of the APF, and the subscript “z” is used to identify a fictitious neutral point which is located at a mid-point in the dc link capacitors. The additional component v_{fz}^* of (26)-(28) is derived using the Min-Max algorithm [92], where the common component added to each modulation signal is:

$$v_{fz}^* = -\frac{\min(v_{af}^*, v_{bf}^*, v_{cf}^*) + \max(v_{af}^*, v_{bf}^*, v_{cf}^*)}{2} \quad (29)$$

This guarantees maximum line to neutral voltage generation capability, and therefore maximum utilisation of the dc-link voltage, which is favourably compared to simpler solutions where the 4th leg is modulated at 50% duty cycle, and the peak line to neutral voltage is constrained to half the dc-link voltage [42]. It can also be shown that the Min-Max method discussed in [92] [93] is equivalent to Space Vector Modulation and, as a result, the line to neutral voltage could increase up to 15.4% when compared to the case where a 50% duty cycle is used for the 4th leg.

3.4 Operating Condition of the Case-Study Microgrid

To validate the performance of the proposed compensator, the operating conditions of a microgrid located in the North of Canada have been analysed. The population of this community is about 1000 inhabitants.

The microgrid power sources are three diesel generators (0.4 MW, 0.6 MW, 1 MW), 3x10 kW Bergey wind turbines, a 30 kW Wenvor wind turbine and a 10kW solar PV array. Fig. 3.7 shows the single-line diagram of the microgrid power system. The operating conditions, measured during the summer season are presented in Table 3.1. For the experimental work, the unbalanced currents shown on this table were scaled, using per unit analysis, to currents which could be handled by the experimental prototype built at the Power Electronic Lab, University of Chile.

It is important to highlight that the values shown in Table 3.1 were used to set the minimum operating conditions for the proposed APF. However, in order to test the performance of the proposed system considering the worst case situation, in some of the experimental results discussed in the next section, higher values of THDs, than those shown in Table 3.1, were used.

The frequency variation of the microgrid used as the case study in this work is shown in Fig. 3.8. However, for the experimental work presented in this thesis, the base frequency is set to 50Hz corresponding to the nominal frequency used in Chile.

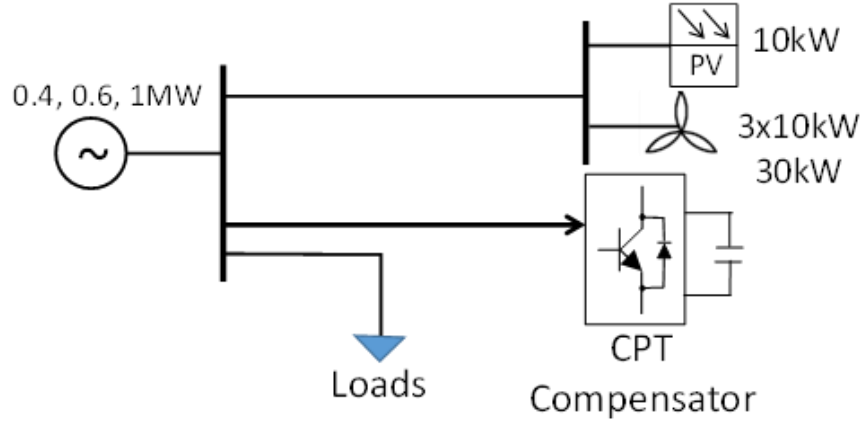


Fig. 3.7. Unifilar diagram of the case-study Microgrid.

Table 3.1. Microgrid Operating Conditions.

Description	Values	Units
Active power abc	458.41	kW
Reactive power abc	78.69	kVAr
Voltage RMS	605.09	V
Current a RMS	172.50	A
Current b RMS	333.07	A
Current c RMS	247.14	A
Current imbalance neg seq	18.70	A
Current imbalance zero seq	20.74	A
Current a THD	7.49	%
Current b THD	5.63	%
Current c THD	5.25	%

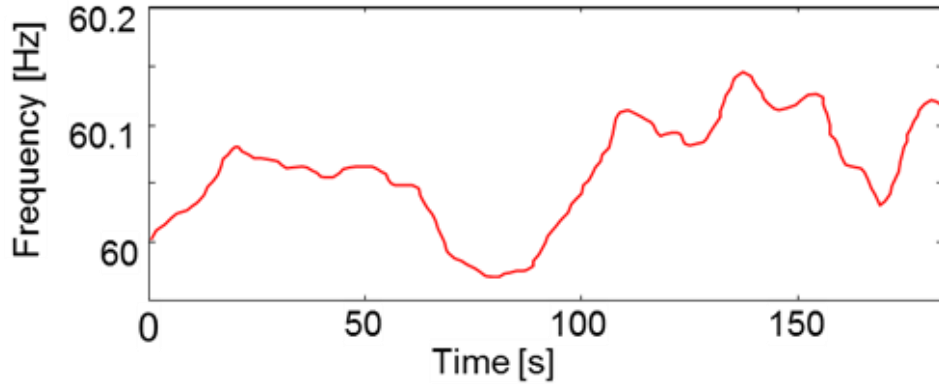


Fig. 3.8. Frequency variation in a real MG located in Canada.

3.5 Experimental Results

The control systems depicted in Fig. 3.4 have been implemented in the experimental system shown in Fig. 3.9. In this work, an Ametek 4-wire programmable power supply emulates the 4-leg power supply shown in Fig. 3.1. An Ametek 3-phase programmable load of 9kW (left side of Fig. 3.9) is used to emulate linear/non-linear load profiles replicating the operation conditions discussed in section 3.4. (See Table 3.1)

A 4-leg voltage source power converter, based on the Semikron SKM50GB123D IGBT module, was used for this work. The control system shown in Fig. 3.4 is embedded in a DSPACE 1103 platform. The computational time of the proposed control system composed of resonant controllers, low-pass/band-pass filters, etc. is less than $50\mu\text{s}$, which means that the proposed controller can be executed in one cycle of the system shown in Fig. 3.9 ($T_s = 200\mu\text{s}$). Hall effect transducers are used to measure the voltages and currents (at the load) required for the implementation of the CPT algorithm shown in

Fig. 3.3. A switching frequency of 5kHz^1 is utilised in the 4-leg power compensator. To analyse the power quality, a Hioki 3196 Power Analyser has been used in this work.

Optical fibre links are used to transmit the switching signals S_0 - S_7 (see Fig. 3.4) required to modulate the voltages v_{az}^* , v_{bz}^* , v_{cz}^* and v_{fz}^* in the 4-leg power converter. It should be pointed out that the experimental system depicted in Fig. 2.8 was built by Master student Carlos Hernandez [94]. In particular, I programmed the CPT algorithm in that experimental rig, and I obtained the experimental results.

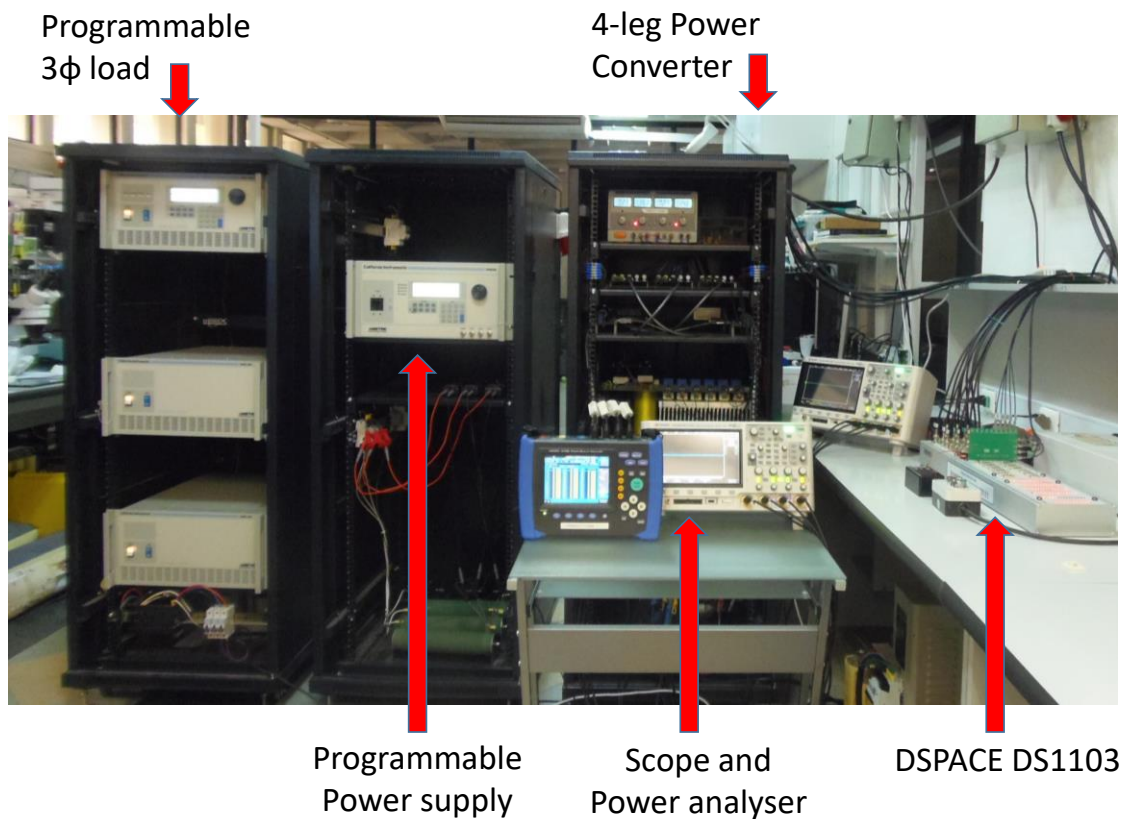


Fig. 3.9. Experimental system for validating the performance of the active power filter.

¹ This switching frequency is determined by the DSPACE platform

3.5.1 Control System Performance for Unbalanced Operation

The performance of the CPT-based 4-leg APF compensating a linear unbalanced system is shown in Fig. 3.10. An unbalanced 4-wire resistive load of 10Ω , 19.6Ω and 13.5Ω has been programmed in the 3ϕ load. With these resistors scaled values of the load currents shown in Table 3.1 (i.e. 333.07A, 247.14A and 172.5A) are produced. The power source is controlled to supply a balanced voltage of 120V peak, 50Hz. The dc-link voltage of the 4-leg converter is regulated at 400V.

Fig. 3.10(a) shows the power supply currents. Notice that between 0s-0.25s the compensator control system is disabled and the power supply current is heavily unbalanced with a peak current (phase-A) of about 12A. In $t=0.25$ s the control system is enabled, and the APF supplies to the PCC the currents shown in Fig. 3.10(b), eliminating unwanted components from the power supply currents.

Fig. 3.10(c) shows an amplified view of Fig. 3.10(a). Notice that the current waveforms have little distortion and the dynamic performance is good. The APF compensates the power supply currents with a settling time of about 30-35ms.

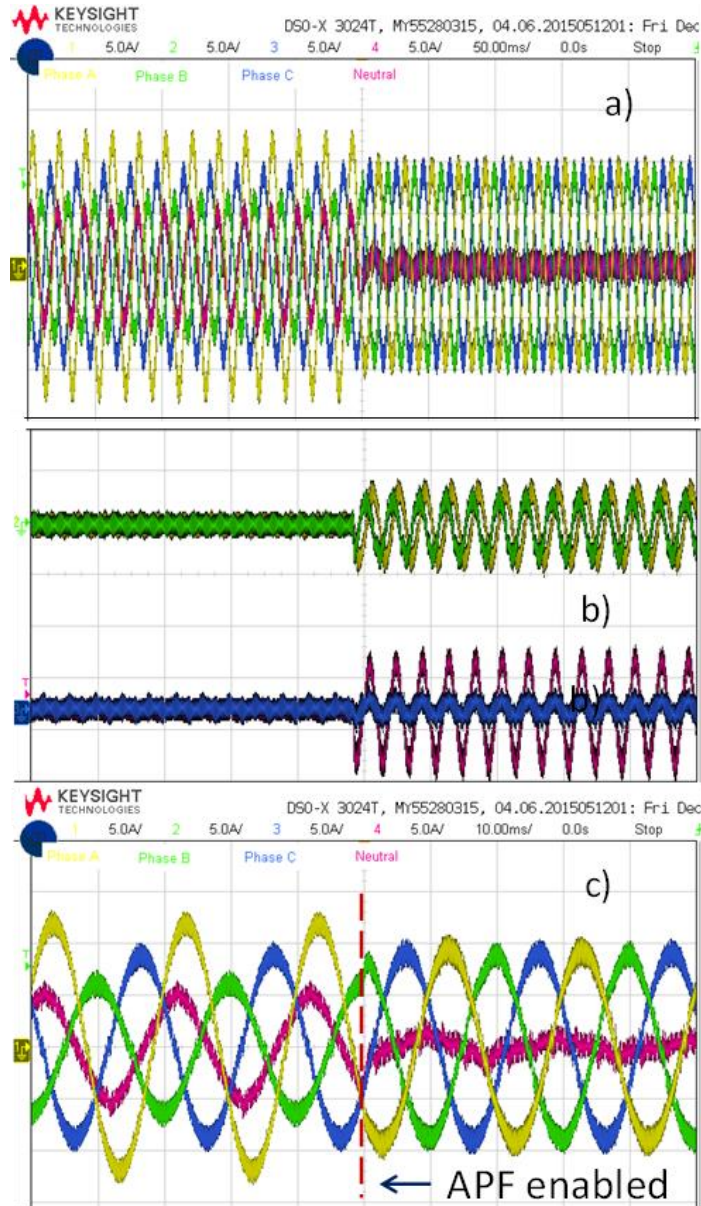


Fig. 3.10. Performance of the proposed APF imbalances produces by a linear resistive load. a) Currents in the power supply (5A/50ms/div). b) current supplied by the power compensator (5A/50ms/div). c) Amplified view of a) (5A/10ms/div).

The dynamic performance of the CPT-based control algorithm has also been analysed considering a load step in one of the phases. For this experimental test, the results were captured using a data acquisition interface. The experimental results are shown in Fig. 3.11. In this case, the APF control system is enabled for the whole test duration. Fig. 3.11(a) shows the step change in phase-B, where the phase to neutral resistance is

changed from 19.6Ω to 8Ω ($t=40ms$) changing the phase-B current from 6A to 14A (peak). The magnitude of this load-step variation has been selected based on the values of the negative/zero sequence currents presented in Table 3.1. Notice that the peak currents in the other phases (A, C and N) are not modified during this test.

Fig. 3.11(b) shows the power supply currents. After the load step, the APF compensates the power supply currents in less than one cycle. For this test 5th order low-pass filters with a cut-off frequency of 20Hz have been used in the implementation of the CPT algorithm shown in Fig. 3.3 (see the block labelled LPF). The BPF filter is also implemented using a 5th order transfer function. As discussed in section 3.2, this BPF eliminates the dc component and behaves as an ideal integrator for frequencies above $\approx 3Hz$.

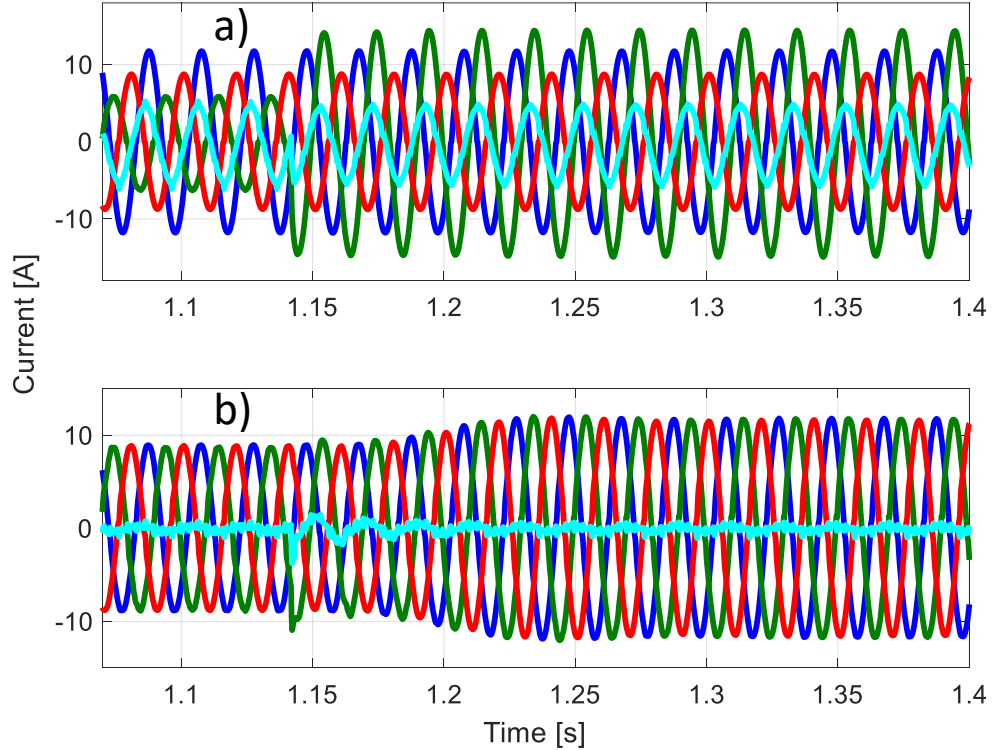


Fig. 3.11. Performance of the proposed control system for an unbalanced load-step. a) Load currents. b) Power supply currents- Matlab data logging of the experimental waveforms.

3.5.2 Control System Performance Considering a Non-Linear Load

To evaluate the performance of the proposed control system to reduce the harmonic distortion in the power supply currents, a non-linear load was implemented using a star-connected, 4-wire load of 10Ω per phase. A rectifier diode was serially connected to the resistor of phase A. In Fig. 3.12(a) the power supply currents are shown. Initially, the control system is disabled, and only the positive semi-cycle of the phase-A current is circulating. In $t=50ms$, the APF is enabled, and after a relatively fast transient, the current in the power supply becomes well balanced and undistorted. Fig. 3.12(b) depicts the neutral current, showing that before compensation, the power supply current has a strong zero sequence current component which is heavily distorted.

Fig. 3.12(c) shows the steady-state currents supplied by the APF. The currents in phase-A and in the neutral wire have large peak values which are injected into the system in order to eliminate unwanted components (imbalances, reactive and void currents) from the power supply. The Total Harmonic Distortion (THD) is compensated using the proposed methodology. Without the compensator [for $t < 50ms$ in Fig. 3.12(a)] the THDs for the power supply currents are 44.47%, 0.4% and 0.4% (phases A, B, C respectively). After the control system is enabled the THD in phase-A is reduced to 2.27%. In the other two phases, the THD is little affected by the operation of the APF, maintaining similar values during the whole test. The spectrum of the power supply currents, corresponding to the experimental test depicted in Fig. 3.12, is shown in Fig. 3.13. Fig. 3.13(a) shows the magnitude of the harmonics before the APF control system is enabled. Notice that in phase-A, where a non-linear load is connected, there is a strong dc-component and even harmonics (mainly 2nd and 4th) produced by the rectifier diode. When the CPT-based

control system is enabled, the harmonics are almost eliminated, and there is a significant reduction in the distortion of the power supply currents. [See Fig. 3.13(b)]

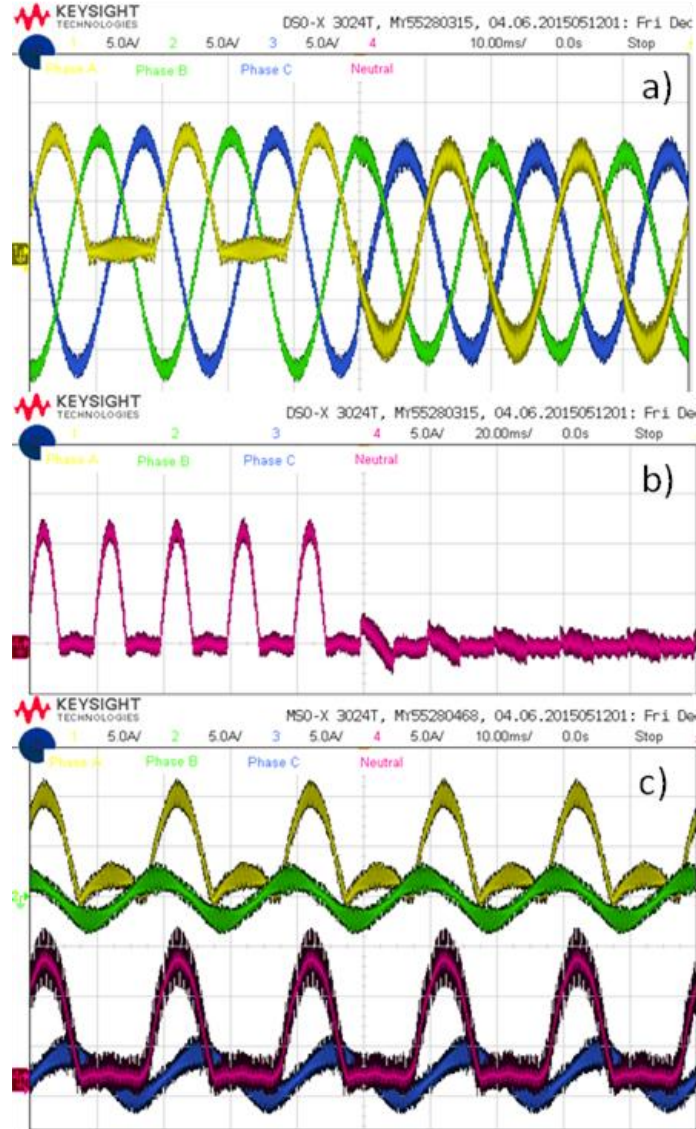


Fig. 3.12. Performance of the proposed control algorithm compensating non-linear loads. a) power supply currents with and without compensations (10ms/5A/div). b) Neutral current corresponding to the test of Fig. 3.12(a) (20ms/5A/div). c) Currents supplied by the APF to the PCC (10ms/div-5A/div).

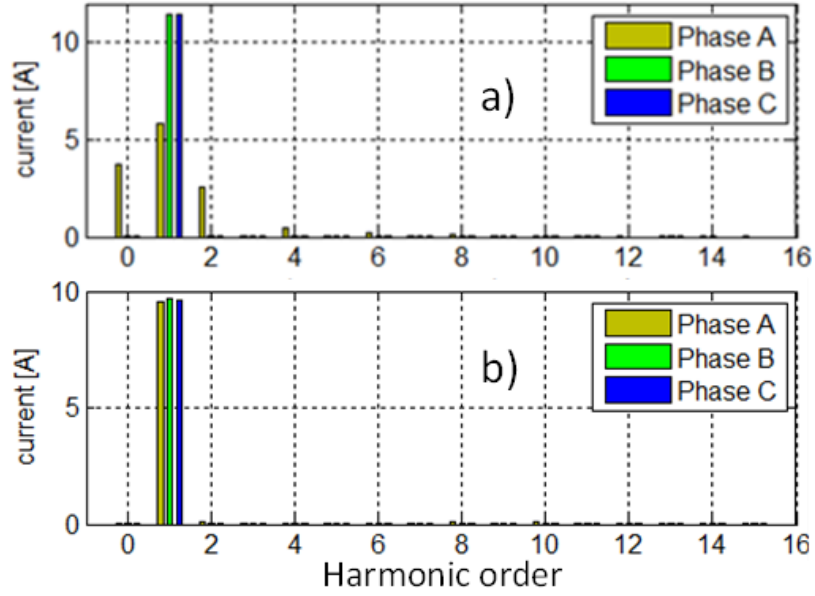


Fig. 3.13. Spectral estimation of the power supply currents. a) Without compensation. b) With the proposed 4-leg APF.

One of the reported advantages of the CPT [66] [69] is that the load current can be separated into current components which can be compensated by different power converters located in the microgrid, e.g. using the residual VA capability of the power converters interfacing PV and wind energy in Fig. 3.7.

Fig. 3.14(a) shows a distorted and unbalanced load-current which has been obtained by modifying the crest factor, in each phase, of the Ametek programmable load. For this load, the current has been separated into the currents defined by (8)-(10). This separation has been realised by the DSPACE platform in real-time operation. Fig. 3.14(b) shows the balanced active currents $i_{a\mu}^b$ [see (8)]. As mentioned before, this current is supplied by the 4-leg power source (as shown in Fig. 3.1). Fig. 3.14(c) shows the unbalanced active current component $i_{a\mu}^u$ [as shown in (10)] which is supplied by the APF. The unbalanced reactive current components $i_{r\mu}^u$ are shown in Fig. 3.14(d), finally the void current $i_{v\mu}$ is shown in Fig. 3.14(e). For this experimental test, the balanced reactive currents have a negligible magnitude, and they are not shown in Fig. 3.14. Finally in Fig. 3.15 the

compensated power supply currents, corresponding to the load currents of Fig. 3.14(a), are shown.

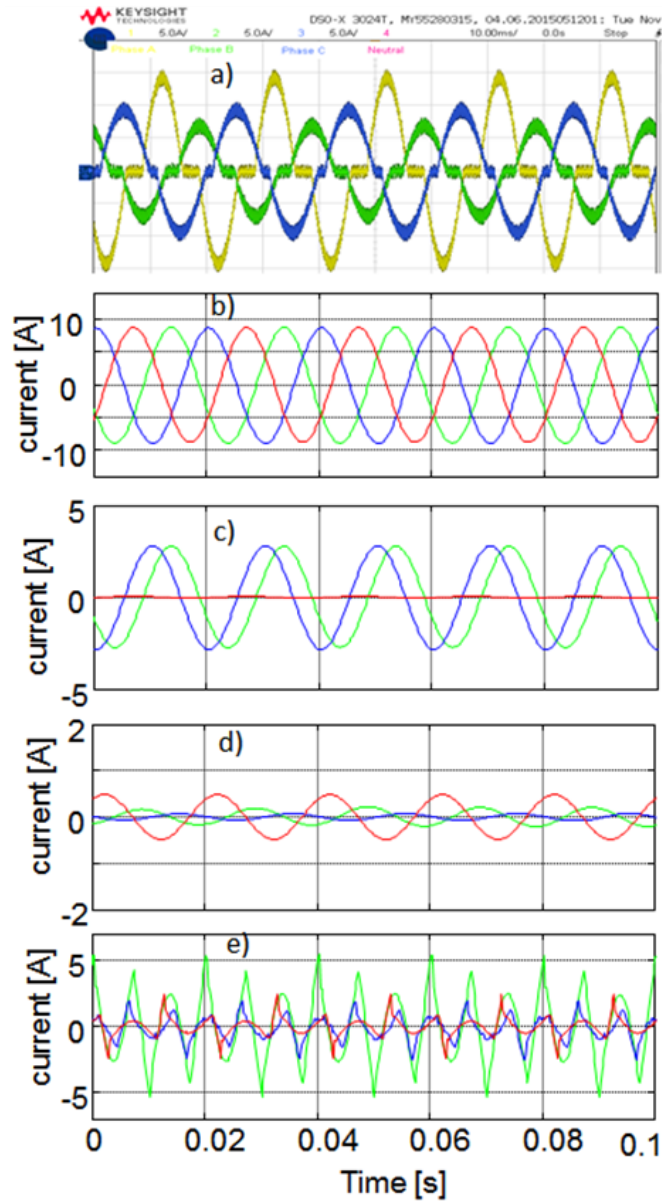


Fig. 3.14. Separation of the load current components using the CPT, a) load currents. b) Balanced active current. c) unbalanced active currents. d) Unbalanced reactive currents. e) void currents.

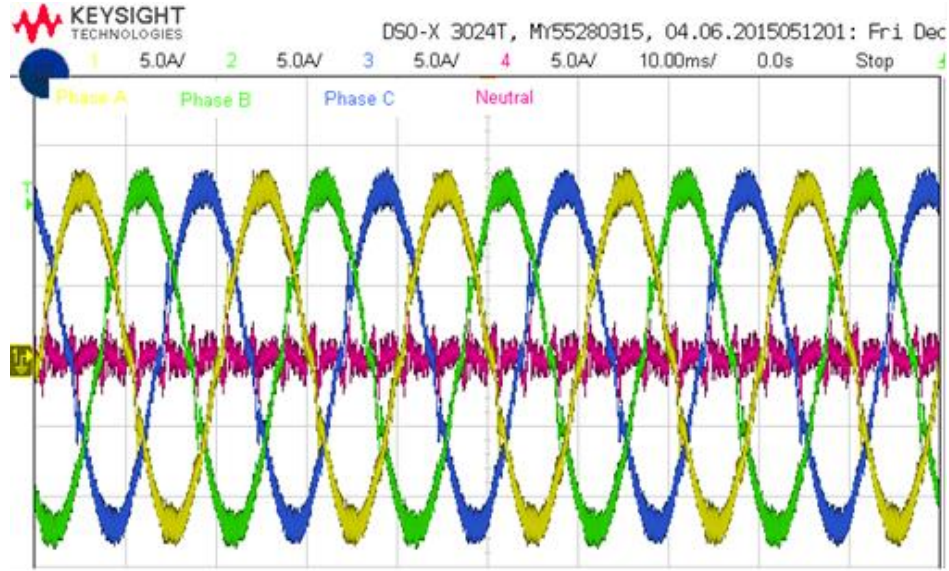


Fig. 3.15. Power supply currents corresponding to the experimental test of Fig. 3.14(a) (10ms/div-5A/div).

3.5.3 Operation of the Proposed 4-leg APF Considering Frequency Variations

The electrical frequency of a microgrid can be subjected to relatively large variations, e.g. produced by the application of P - f droop control or due to the effect of intermittent generation from renewable sources (see Fig. 3.8). For the correct operation of the proposed control system, self-tuning resonant controllers have been implemented in this thesis.

In the experimental implementation discussed in this work, the tuning of the resonant controller is achieved by the online calculation of the $k_{T\omega_j}$ values [as shown in (16)-(17)] required by the bilinear transform with frequency pre-warping. In addition, a PLL is necessary to estimate the fundamental frequency of the network (as shown in Fig. 3.4).

To test the performance of the self-tuning resonant control system, the programmable power supply used in this work is controlled using the frequency profile depicted in Fig.

3.16. Notice that this profile is a piece-wise linearization of that shown in Fig. 3.8 with some modifications. For instance, the duration of the profile has been reduced to 10s, and the frequency variation is larger (48Hz to 52Hz). Moreover (as mentioned before) the nominal electrical frequency used in this work is 50Hz instead of the 60Hz utilised for the frequency profile shown in Fig. 3.8.

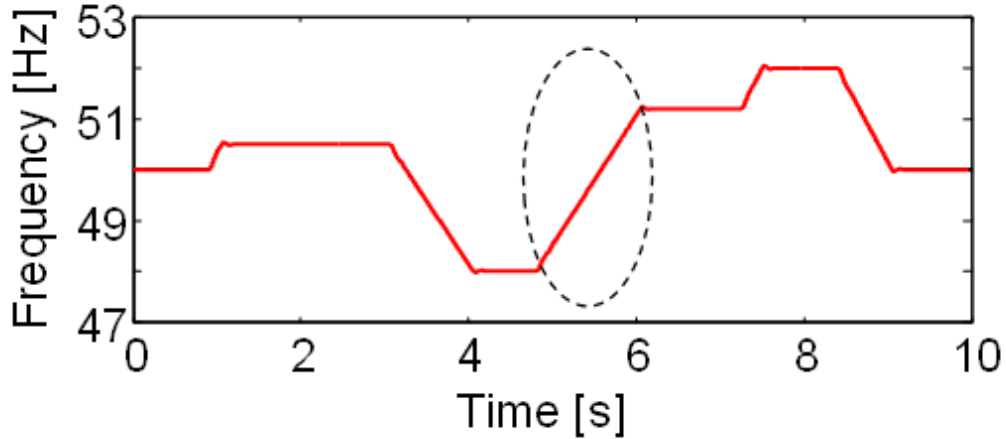


Fig. 3.16. Frequency profile used in this work.

The performance of the self-tuning algorithm has been tested considering the operation of the system, feeding a load programmed with the current profile depicted in Fig. 3.14(a) and subjected, during 10s, to the frequency variations shown in Fig. 3.16. Fig. 3.17 shows the currents in the programmable power supply (captured using the DSPACE data acquisition system). Notice that effective currents are shown because of the problems associated with displaying the instantaneous currents for 10s.

When the power compensator is enabled the neutral current shown in Fig. 3.17(a) is reduced to a small (RMS) value produced by the zero-sequence harmonics which are not compensated by the APF. As shown in Fig. 3.17(b) in the phases of the programmable power supply the effective currents are well balanced. However, because effective currents by themselves cannot be used to demonstrate that the self-tuning algorithm is

well tuned, an additional test is proposed and realised in this thesis. Frequency-adaptive filters have been programmed to process the data captured by the DSPACE platform. The filters are designed to isolate the 5th and 7th harmonics of phase-B in the load and power source currents.

In Fig. 3.18(a)-(b) the 5th harmonics of the load and power supply currents are depicted between $t=5$ to $t=6$ (see Fig. 3.16). Notice that the frequency is varying between $f_e=48\text{Hz}$ to $f_e=51\text{Hz}$ during that period of time. As shown in Fig. 3.18(a) the amplitude of the 5th harmonic in the load current is about 7.1% of the peak current. Fig. 3.18(b) shows the 5th harmonic component in the power supply current. Even with the relatively large frequency variation produced in this time period, the harmonic has been compensated, and its amplitude is only 0.075(A), a reduction of 91% with respect to the uncompensated value. An amplified view of the 5th harmonic present in the power supply current is shown in Fig. 3.18(c)

Using the proposed methodology, the performance of the APF to reject the 7th harmonic has also been evaluated. Again the performance of the APF based self-tuning resonant controllers is excellent, eliminating most of this harmonic from the power supply current as shown in Fig. 3.18(d)-(f). The amplitude of the 7th harmonic in the load current is about 3.33% of the peak current. However, the amplitude of the 7th harmonic in the power supply current is only 0.05(A), a reduction of 87.5% respect to the uncompensated value.

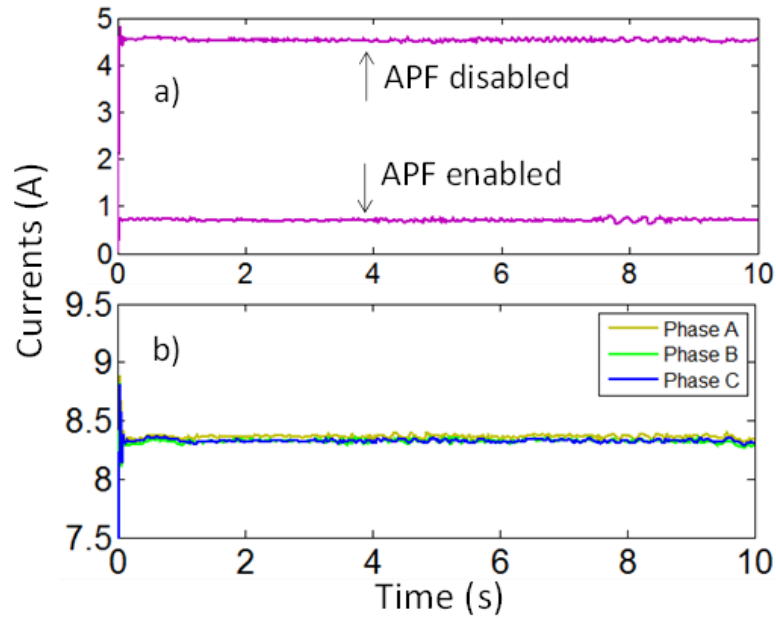


Fig. 3.17. Effective (RMS) currents circulating in the programmable power supply. a) Power supply neutral current with and without enabling the active power filter. b) Power supply phase currents considering the active filter enabled.

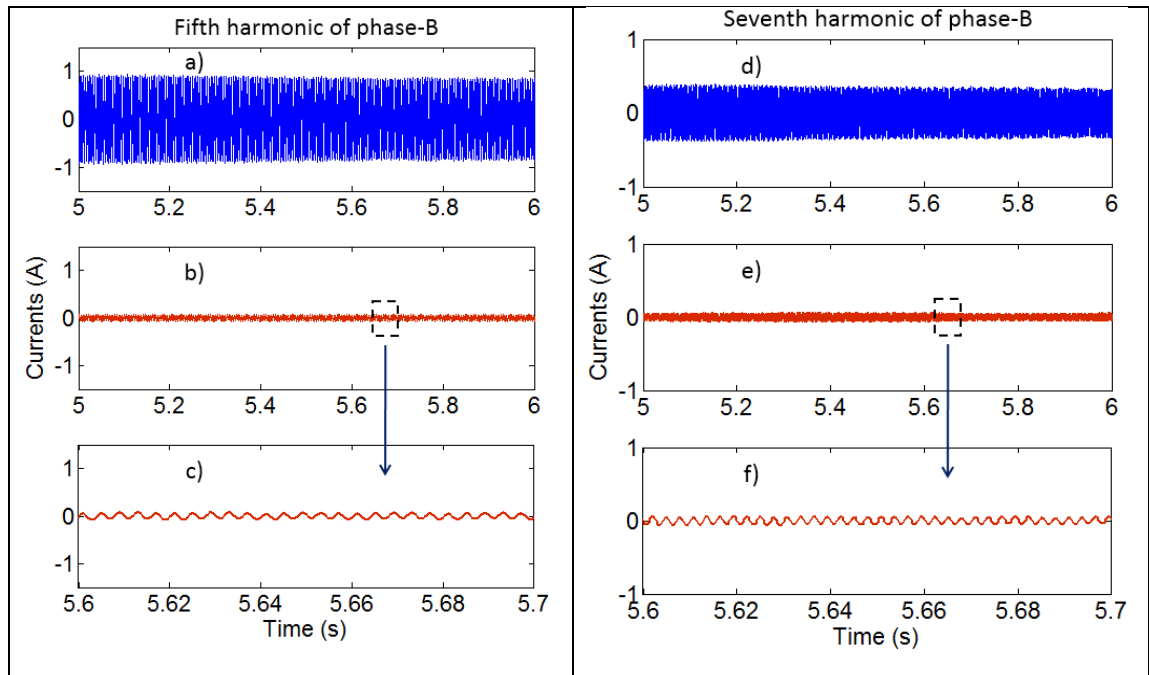


Fig. 3.18. a) 5th harmonic produced by the load current of Fig. 3.14(a). b) 5th harmonic component compensated in the power supply during a relatively fast frequency transient. c) Amplified view of Fig. 3.18b. d) 7th harmonic produced by the load current of Fig. 3.14(a). e) 7th harmonic component compensated in the power supply during a relatively fast frequency transient. f) Amplified view of Fig. 3.18e.

3.5.4 Influence of the self-tuning resonant controllers on the behaviour of the APF

In this section, the performance of the active power filter (APF) will be studied in the following two scenarios: (i) the APF is controlled using self-tuning resonant controllers, and (ii) the APF is controlled using non-self-tuning resonant controllers. Both scenarios will be studied through simulation work (based on PLECS software) and considering frequency variations in the power supply.

The simulation test assumes that a non-linear load with the same characteristics as the experimental test shown in section 3.5.2 (a rectifier diode is connected to phase A), is being fed by the power supply. In this condition, the performance of the APF is studied considering frequency variations, and for the two scenarios discussed in the previous paragraph (self-tuning and non-self-tuning PR controllers). Four steps were studied, these are: step 1 ($0s \leq t < 1s$) where the APF is not connected to the system (composed of the power supply and the non-linear load), and the electrical frequency ω_e is $2\pi \cdot 50 \text{ rad/s}$; step 2 ($1s \leq t < 2s$) where the active power filter is connected to the system and $\omega_e = 2\pi \cdot 50 \text{ rad/s}$; step 3 ($2s \leq t < 3s$) the APF is working and ω_e is changed to $2\pi \cdot 48 \text{ rad/s}$, finally in step 4 ($3s \leq t < 4s$), the APF is working, and ω_e is changed from $2\pi \cdot 48 \text{ rad/s}$ to $2\pi \cdot 52 \text{ rad/s}$ at $t = 2s$. Based on these steps, the quality of the current injected by the power supply to the load is studied considering the THD as a quality index. These steps are evaluated for the two scenarios discussed at the beginning of this section. (Implementation of the APF using self-tuning and non-self-tuning PR controllers)

Fig. 3.19 shows the frequency profile of the system in the four steps considered in the simulation work. Fig. 3.20 shows the currents injected by the power supply before and after the activation of the APF. From this figure, it can be appreciated that before the activation of the APF, the positive semi-cycle of phase “A” current is flowing. Then when

the APF is working (see step 2 in Fig. 3.20), the current injected by the power supply to the load becomes well balanced and undistorted. It should be pointed out that in steps 1 and 2 it does not matter if the APF is working with self-tuning or non-self-tuning PR controllers since in those steps the electrical frequency is the nominal one ($\omega_e = 2\pi \cdot 50 \text{ rad/s}$).

Fig. 3.21 shows the currents injected by the power supply to the non-linear load, in step 3 ($\omega_e = 2\pi \cdot 48 \text{ rad/s}$), for the case where the APF is implemented using self-tuning PR controllers [see Fig. 3.21(a)], and for the case where the APF uses non-self-tuning PR controllers [see Fig. 3.21(b)]. Similarly, Fig. 3.22 shows the same waveforms but now for the step 4 considered in this test ($\omega_e = 2\pi \cdot 52 \text{ rad/s}$). From Fig. 3.21 and Fig. 3.22, it can be concluded that the use of self-tuning PR controllers in the implementation of the APF, improves the quality of currents (measure through the THD) injected by the power supply to the load, in comparison with the case where non-self-tuning PR controllers are used.

Regarding implementation issues, it is well-known that Digital Signal Processors (DSPs), have experienced an exponential development in processing power [88]. Therefore even though the proposed control algorithm, based on 21 self-tuning resonant controllers, seems relatively complex it is simple to implement using a commercial DSP with a good processing capability and suitable FPGA platforms.

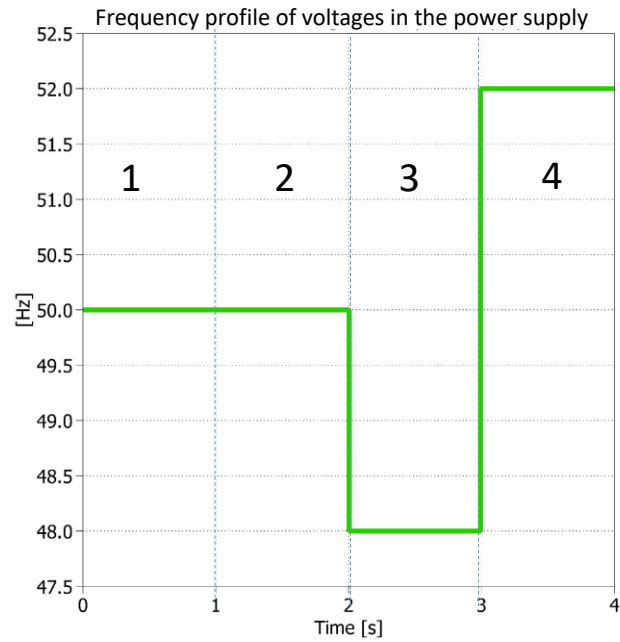


Fig. 3.19. Frequency profile at the power supply

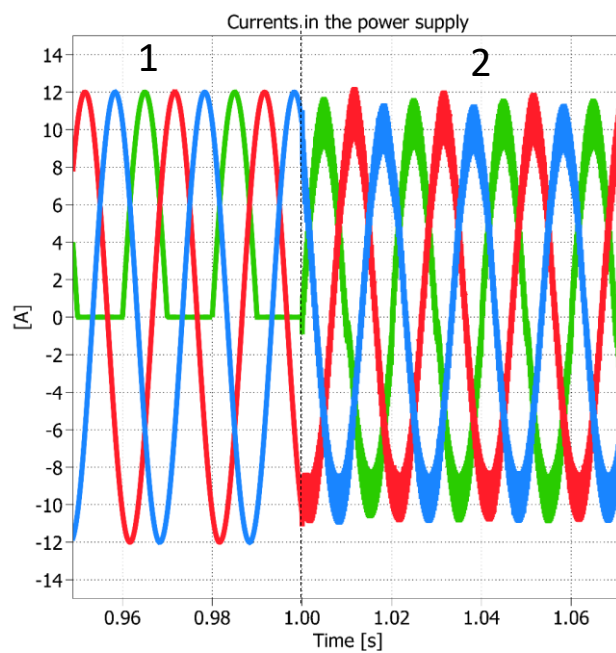


Fig. 3.20. Currents in the power supply before and after the activation of the AFP

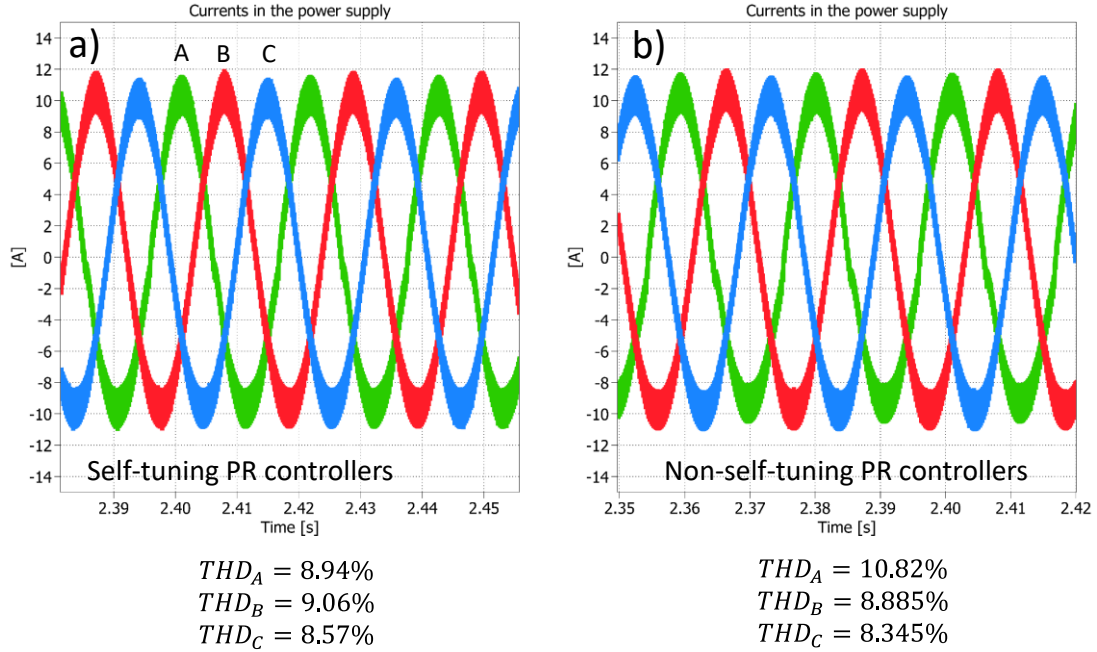


Fig. 3.21. (a) Current injected by the power supply to the load, in step 3 ($2\pi \cdot 48 \text{ rad/s}$) when the APF is implemented using self-tuning PR controllers, (b) Current injected by the power supply to the load, in step 3 ($2\pi \cdot 48 \text{ rad/s}$) when the APF is implemented using non-self-tuning PR controllers

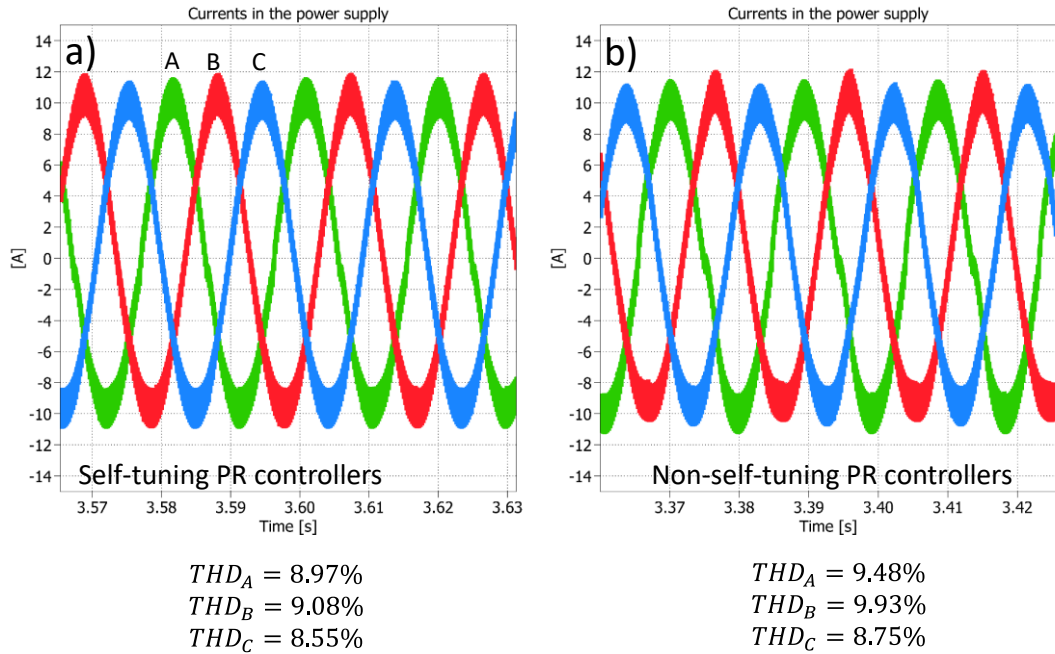


Fig. 3.22. (a) Current injected by the power supply to the load, in step 3 ($2\pi \cdot 52 \text{ rad/s}$) when the APF is implemented using self-tuning PR controllers, (b) Current injected by the power supply to the load, in step 3 ($2\pi \cdot 52 \text{ rad/s}$) when the APF is implemented using non-self-tuning PR controllers

3.6 Chapter Conclusions

A new methodology for the control of a 4-leg active power filter has been presented in this chapter. The proposed control system is based on the conservative power theory augmented with a PI and seven self-tuning resonant controllers in each phase. The compensation is achieved by regulating the positive, negative and zero sequence voltages modulated by a 4-leg voltage source converter.

The performance of the proposed control system has been tested considering the operating conditions of a microgrid located in northern Canada. Using per-unit analysis, the current and voltages have been referred to values which could be handled by the prototype implemented at the University of Chile. To evaluate the performance of the control system, balanced and unbalanced loads of linear/non-linear nature, considering fixed/variable frequency operation have been applied at the PCC. In all the cases the performance achieved by the proposed 4-leg APF has been excellent.

The advantages of the CPT, as a new power theory for applications such as microgrid where strong distortion and imbalances could be present, have also been demonstrated in this thesis.

Chapter 4. Proposed Cooperative Imbalance Sharing Control Algorithm

4.1 Introduction

In this chapter, a cooperative control scheme based on the Conservative Power Theory (CPT) is proposed for sharing imbalances in three-phase four-wire droop-controlled systems. By utilising the CPT, the balanced, unbalanced and distorted components of the currents and powers in a micro-grid could be identified. Using control loops, based on virtual impedances and implemented in the stationary a-b-c frame, the sharing of imbalances and also harmonics, between the different 4-leg inverters in the microgrid, can be realised. A secondary control loop is implemented to regulate the maximum voltage imbalance/distortion at the Point of Common Coupling (PCC) or any other point in the microgrid. The theoretical background of the method is presented, and experimental validation is demonstrated using a laboratory-scale microgrid.

The rest of this chapter is organised as follows: Section 4.2 introduces the proposed unbalanced power-sharing concept; section 4.3 discusses in detail the proposed imbalance sharing control scheme. Section 4.4 reports both simulation and experimental validation of the proposed imbalance control and sharing scheme. To realise the experimental validation, load conditions and the voltage/currents measured in a real microgrid located in the north of Canada are considered (see [40] and section 3.4). In section 4.5 the extension of the proposed control scheme for harmonic sharing is provided. In section 4.6 a comparison between the proposed control scheme and previous methods reported for imbalance sharing is performed. Finally, in section 4.7, the main conclusions of the proposed algorithm discussed in this chapter are shown.

4.2 Unbalanced Power Sharing Concept

4.2.1 Analysis of an Inverter Feeding an Unbalanced Load

Consider the 4-leg inverter in Fig. 4.1, where the load current $\underline{i}_o$ includes positive, negative, and zero sequence components as in (30), and considering the fact that the maximum allowed voltage imbalance in power systems is 3% as defined by ANSI C84.1-2006 standard (see [63]), the positive sequence voltage is dominant. Therefore, the voltage at the converter's outputs can be considered as shown in (31) [62] [95]. This can be corroborated using the voltages and currents measured in a real microgrid (see [40] and section 3.4).

Some comments about the topology used in this thesis (see Fig. 4.1) are:

- The system studied in this chapter corresponds to a three-phase four-wire droop-controlled microgrid in islanded mode. Based on that, it is assumed that each 4-leg converter in the MG must regulate their respective bus voltage and share active and reactive powers using the well-known droop control [11] [62] [54]. Therefore if the grid is connected at the PCC (see Fig. 4.1), the proposed controller, could not be used, since the grid will impose the frequency and the voltage at the PCC.
- Based on the first comment, it is necessary to place an LC filter at the output of the 4-leg converter as shown in Fig. 4.1 to ensure that voltage E meets requirements related to the voltage quality [96] (reducing the harmonics produced by the pulse-width modulation of the power converter). Moreover, the use of an LCL output filter instead of an LC does not affect the performance of the proposed controller, since the inductance of that filter on the PCC side can be considered as part of the line impedance. (See Fig. 4.1)

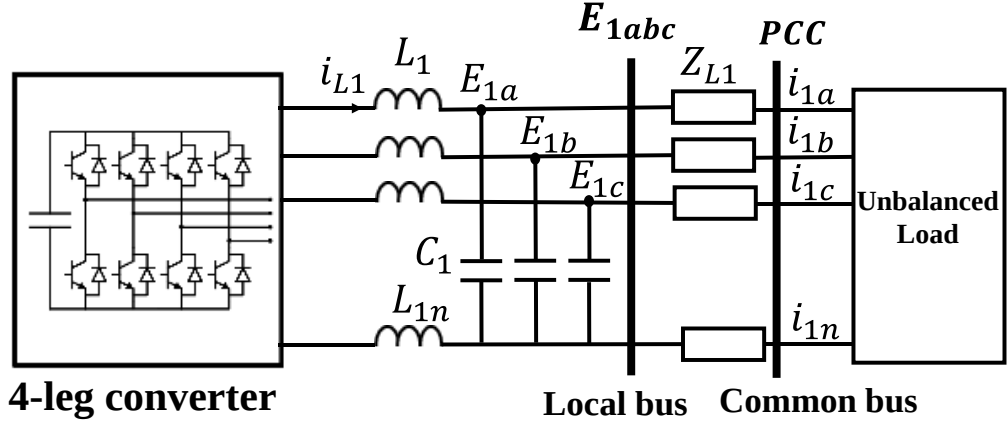


Fig. 4.1. Example of 4-leg converter feeding unbalanced load in an islanded microgrid.

$$\underline{i}_o = \begin{bmatrix} \sqrt{2}I^+ \sin(\omega t + \varphi^+) + \sqrt{2}I^- \sin(\omega t + \varphi^-) + \sqrt{2}I^0 \sin(\omega t + \varphi^0) \\ \sqrt{2}I^+ \sin\left(\omega t - \frac{2\pi}{3} + \varphi^+\right) + \sqrt{2}I^- \sin\left(\omega t + \frac{2\pi}{3} + \varphi^-\right) + \sqrt{2}I^0 \sin(\omega t + \varphi^0) \\ \sqrt{2}I^+ \sin\left(\omega t + \frac{2\pi}{3} + \varphi^+\right) + \sqrt{2}I^- \sin\left(\omega t - \frac{2\pi}{3} + \varphi^-\right) + \sqrt{2}I^0 \sin(\omega t + \varphi^0) \end{bmatrix} \quad (30)$$

$$\underline{E} = \begin{bmatrix} \sqrt{2}E \cdot \sin(\omega t + \delta) \\ \sqrt{2}E \cdot \sin\left(\omega t - \frac{2\pi}{3} + \delta\right) \\ \sqrt{2}E \cdot \sin\left(\omega t + \frac{2\pi}{3} + \delta\right) \end{bmatrix} \quad (31)$$

With these assumptions, the system shown in Fig. 4.1 can be analysed using the CPT definitions given in chapter 2, section 2.1. Using (6) and (31) the unbiased voltage $\hat{\underline{E}}$ is calculated as shown in (32).

$$\hat{\underline{E}} = \begin{bmatrix} -\frac{\sqrt{2}E}{\omega} \cdot \cos(\omega t + \delta) \\ -\frac{\sqrt{2}E}{\omega} \cdot \cos\left(\omega t - \frac{2\pi}{3} + \delta\right) \\ -\frac{\sqrt{2}E}{\omega} \cdot \cos\left(\omega t + \frac{2\pi}{3} + \delta\right) \end{bmatrix} \quad (32)$$

Using (30) and (31) in (8), the balanced active current vector ($\underline{i}_a^b$) is calculated as shown in (33). Note that $\underline{i}_a^b$ is a positive sequence vector current.

Similarly, using (30) and (32) to evaluate (9), the balanced reactive current vector ($\underline{i}_r^b$) is calculated by (34). Note that the vector $\underline{i}_r^b$ is also a positive sequence vector.

$$\underline{i}_a^b = \begin{bmatrix} \frac{\sqrt{2}I^+}{2} \sin(\omega t + \varphi^+) \\ \frac{\sqrt{2}I^+}{2} \sin\left(\omega t - \frac{2\pi}{3} + \varphi^+\right) \\ \frac{\sqrt{2}I^+}{2} \sin\left(\omega t + \frac{2\pi}{3} + \varphi^+\right) \end{bmatrix} + \begin{bmatrix} \frac{\sqrt{2}I^+}{2} \sin(\omega t + 2\delta - \varphi^+) \\ \frac{\sqrt{2}I^+}{2} \sin\left(\omega t - \frac{2\pi}{3} + 2\delta - \varphi^+\right) \\ \frac{\sqrt{2}I^+}{2} \sin\left(\omega t + \frac{2\pi}{3} + 2\delta - \varphi^+\right) \end{bmatrix} \quad (33)$$

$$\underline{i}_r^b = \begin{bmatrix} \frac{\sqrt{2}I^+}{2} \sin(\omega t + \varphi^+) \\ \frac{\sqrt{2}I^+}{2} \sin\left(\omega t - \frac{2\pi}{3} + \varphi^+\right) \\ \frac{\sqrt{2}I^+}{2} \sin\left(\omega t + \frac{2\pi}{3} + \varphi^+\right) \end{bmatrix} - \begin{bmatrix} \frac{\sqrt{2}I^+}{2} \sin(\omega t + 2\delta - \varphi^+) \\ \frac{\sqrt{2}I^+}{2} \sin\left(\omega t - \frac{2\pi}{3} + 2\delta - \varphi^+\right) \\ \frac{\sqrt{2}I^+}{2} \sin\left(\omega t + \frac{2\pi}{3} + 2\delta - \varphi^+\right) \end{bmatrix} \quad (34)$$

Based on (33) and (34), a new current vector can be defined, named the “balanced current vector $\underline{i}^b$ ”, corresponding to the sum of $\underline{i}_a^b$ and $\underline{i}_r^b$. This new vector is shown in (35) and it only contains the positive sequence of $\underline{i}_o$ [see (30)].

$$\underline{i}^b = \underline{i}_a^b + \underline{i}_r^b = \begin{bmatrix} \sqrt{2}I^+ \sin(\omega t + \varphi^+) \\ \sqrt{2}I^+ \sin\left(\omega t - \frac{2\pi}{3} + \varphi^+\right) \\ \sqrt{2}I^+ \sin\left(\omega t + \frac{2\pi}{3} + \varphi^+\right) \end{bmatrix} \quad (35)$$

If the same procedure used to calculate the vectors $\underline{i}_a^b$ and $\underline{i}_r^b$ is also used to determine the unbalanced current vectors $\underline{i}_a^u$ and $\underline{i}_r^u$ [defined by (10)], this results in (36) and (37). Based on these equations, a new unbalanced vector $\underline{i}^u$ is defined as the sum of $\underline{i}_a^u$ and $\underline{i}_r^u$. This new unbalanced vector is shown in (38). Note that $\underline{i}^u$ contains both the negative and the zero sequence current components of $\underline{i}_o$.

$$\underline{i}_a^u = \begin{bmatrix} \frac{\sqrt{2}I^-}{2} [\sin(\omega t + 2\delta - \varphi^-) + \sin(\omega t + \varphi^-)] + \frac{\sqrt{2}I^0}{2} [\sin(\omega t + 2\delta - \varphi^0) + \sin(\omega t + \varphi^0)] \\ \frac{\sqrt{2}I^-}{2} \left[\sin(\omega t + 2\delta - \varphi^-) + \sin\left(\omega t + \frac{2\pi}{3} + \varphi^-\right) \right] + \frac{\sqrt{2}I^0}{2} \left[\sin\left(\omega t + \frac{2\pi}{3} + 2\delta - \varphi^0\right) + \sin(\omega t + \varphi^0) \right] \\ \frac{\sqrt{2}I^-}{2} \left[\sin(\omega t + 2\delta - \varphi^-) + \sin\left(\omega t - \frac{2\pi}{3} + \varphi^-\right) \right] + \frac{\sqrt{2}I^0}{2} \left[\sin\left(\omega t - \frac{2\pi}{3} + 2\delta - \varphi^0\right) + \sin(\omega t + \varphi^0) \right] \end{bmatrix} \quad (36)$$

$$\underline{i}_r^u = \begin{bmatrix} -\frac{\sqrt{2}I^-}{2} [\sin(\omega t + 2\delta - \varphi^-) - \sin(\omega t + \varphi^-)] - \frac{\sqrt{2}I^0}{2} [\sin(\omega t + 2\delta - \varphi^0) - \sin(\omega t + \varphi^0)] \\ -\frac{\sqrt{2}I^-}{2} \left[\sin(\omega t + 2\delta - \varphi^-) - \sin\left(\omega t + \frac{2\pi}{3} + \varphi^-\right) \right] - \frac{\sqrt{2}I^0}{2} \left[\sin\left(\omega t + \frac{2\pi}{3} + 2\delta - \varphi^0\right) - \sin(\omega t + \varphi^0) \right] \\ -\frac{\sqrt{2}I^-}{2} \left[\sin(\omega t + 2\delta - \varphi^-) - \sin\left(\omega t - \frac{2\pi}{3} + \varphi^-\right) \right] - \frac{\sqrt{2}I^0}{2} \left[\sin\left(\omega t - \frac{2\pi}{3} + 2\delta - \varphi^0\right) - \sin(\omega t + \varphi^0) \right] \end{bmatrix} \quad (37)$$

$$\underline{i}^u = \underline{i}_a^u + \underline{i}_r^u = \begin{bmatrix} \sqrt{2}I^- \sin(\omega t + \varphi^-) + \sqrt{2}I^0 \sin(\omega t + \varphi^0) \\ \sqrt{2}I^- \sin\left(\omega t + \frac{2\pi}{3} + \varphi^-\right) + \sqrt{2}I^0 \sin(\omega t + \varphi^0) \\ \sqrt{2}I^- \sin\left(\omega t - \frac{2\pi}{3} + \varphi^-\right) + \sqrt{2}I^0 \sin(\omega t + \varphi^0) \end{bmatrix} \quad (38)$$

Finally, as the newly proposed vectors are orthogonal, this result verifies that the CPT also provides a robust methodology for decoupling the positive sequence current from both negative and zero sequence currents in the stationary abc frame.

4.2.2 Two Inverters Feeding a Common Unbalanced Load

The scenario discussed in Fig. 4.1 for a single 4-leg converter feeding an unbalanced load is now extended to the case of two inverters as is shown in Fig. 4.2. In this figure, the load is modelled as a current source, and the distributed generation units are modelled as voltage sources (generated in the capacitors of the inverter output filter, see Fig. 4.1). Based on the discussion presented in section 4.2.1 it is possible to calculate² the balanced and unbalanced current vectors [given by (35) and (38)] for each inverter. It is worth remembering that, under the assumption of negligible voltage imbalance, the two current vectors are orthogonal, and decoupled from a control perspective. Note that these current vectors depend on $\underline{E}_1$ and $\underline{i}_1$ for inverter 1 and on $\underline{E}_2$ and $\underline{i}_2$ for inverter 2 (see Fig. 4.2). It should be pointed out that $\underline{i}_1$ and $\underline{i}_2$ are respectively, the output current of inverter 1 and the output current of inverter 2 (after LC filter). Furthermore, E_1 shown in Fig. 4.2 refers to the voltage across the capacitor C_1 in Fig. 4.1 (for E_2 it is the same, but referred to the voltage across C_2). In Fig. 4.2, by applying the superposition principle, $\underline{i}_1$ and $\underline{i}_2$ can be written as (39) and (40).

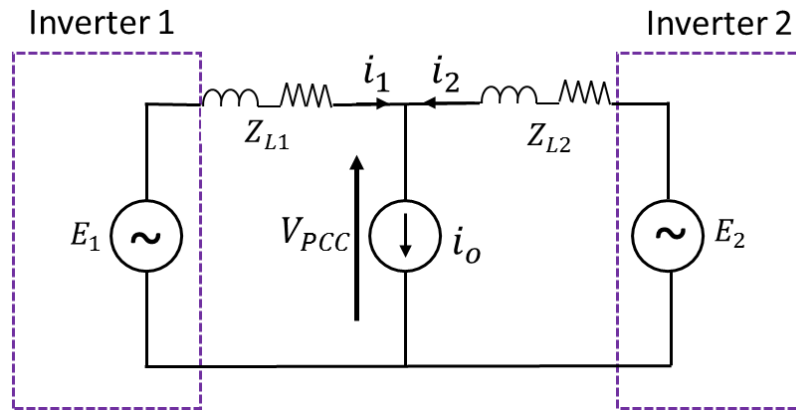


Fig. 4.2. Two inverters feeding an unbalanced load – unifilar equivalent circuit.

² Using the CPT definitions described in section 2.1

$$\underline{i}_1 = \underline{i}_{11} + \underline{i}_c \quad (39)$$

$$\underline{i}_2 = \underline{i}_{22} - \underline{i}_c \quad (40)$$

where $\underline{i}_{11}$ and $\underline{i}_{22}$ are the current components due to $\underline{i}_o$ while $\underline{i}_c$ is a circulating current component depending on the voltage difference between $\underline{E}_1$ and $\underline{E}_2$. The two currents $\underline{i}_1$ and $\underline{i}_2$ can be rewritten explicitly as:

$$\underline{i}_1 = \frac{Z_{L2}}{Z_{L1} + Z_{L2}} \cdot \underline{i}_o + \frac{\underline{E}_1 - \underline{E}_2}{Z_{L1} + Z_{L2}} \quad (41)$$

$$\underline{i}_2 = \frac{Z_{L1}}{Z_{L1} + Z_{L2}} \cdot \underline{i}_o - \frac{\underline{E}_1 - \underline{E}_2}{Z_{L1} + Z_{L2}} \quad (42)$$

From these equations, it can be observed that when the impedances $\underline{Z}_{L1}$ and $\underline{Z}_{L2}$ are predominantly inductive, only the circulating current $\underline{i}_c$ produced by the different voltage references of the two inverters, causes a phase shift between $\underline{i}_1$ and $\underline{i}_2$. Considering that in a typical microgrid, $\underline{E}_1 \approx \underline{E}_2$ because the magnitudes are similar [54] and the phase difference is small [11], it is assumed that $\underline{i}_c$ is smaller than $\underline{i}_o$ [44]. Based on that, calculating both balanced and unbalanced current vectors given by (35) and (38) respectively in the inverters and the load, the following relationships can be obtained.

$$\underline{i}_o^b \approx \underline{i}_1^b + \underline{i}_2^b \quad (43)$$

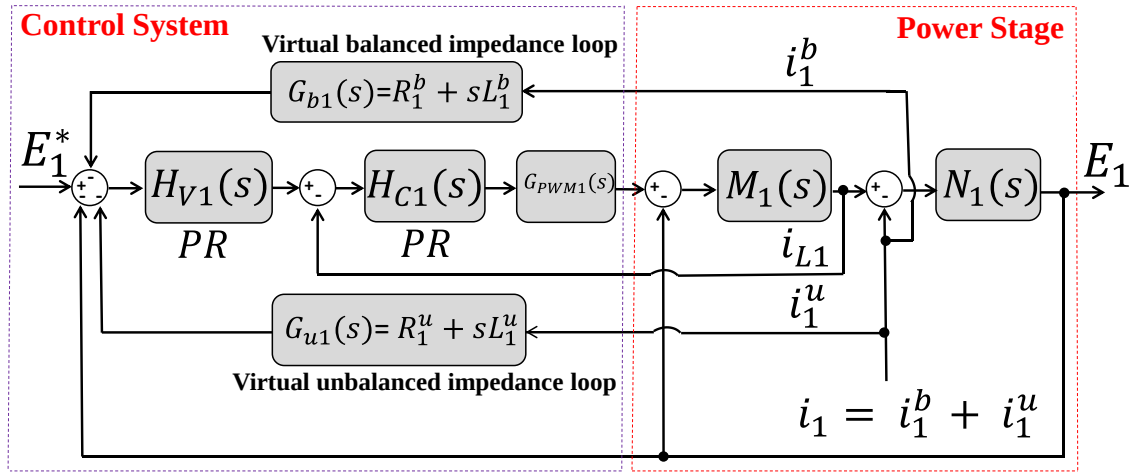
$$\underline{i}_o^u \approx \underline{i}_1^u + \underline{i}_2^u \quad (44)$$

From (43)-(44) it can be concluded that the current imbalance in the load ($\underline{i}_o^u$) depends mainly on the unbalanced current vectors calculated in both inverters ($\underline{i}_1^u, \underline{i}_2^u$). Conversely,

the positive sequence current in the load (i_o^b) is mainly a function of the balanced current vectors in both inverters (i_1^b, i_2^b).

4.3 Proposed Imbalance Sharing Control Scheme

The proposed control scheme is based on the property described in section 4.2 where the positive current sequence is decoupled from both negative and zero current sequences through the current vectors i^b and i^u [see (35) and (38)]. The proposed imbalance sharing control scheme implemented in inverter 1 of Fig. 4.2 is shown in Fig. 4.3 (in the Laplace domain), where the inverter output current³ $i_1(s)$ is divided into $i_1^b(s)$ and $i_1^u(s)$. With the balanced current $i_1^b(s)$ the well-known virtual impedance loop [54] [65] is implemented to achieve decoupling between active and reactive powers and also for stability purposes. [54] [65]



PR: proportional resonant controller

Fig. 4.3. Proposed control scheme of each four-leg inverter.

³ Current injected by the converter to the load (after LC filter, see Fig. 4.1)

A new virtual unbalanced impedance loop is proposed for the unbalanced current $i_1^u(s)$ to control the unbalanced voltage at the inverter output. The proposed control scheme shown in Fig. 4.3, is implemented in the abc reference frame and sequence separation is not required (this could be an advantage because most of the sequence separation algorithms are strongly affected by, noise, harmonic distortion, small variations in the sampling time, etc. [45] [46]). Notice that the virtual impedance loops are equivalent to output impedances where voltage drops are produced by the circulation of balanced and unbalanced currents respectively. These voltage drops are given in (45).

The descriptions of the parameters depicted in Fig. 4.3, are as follows:

- i_{L1} is the current in the inductance of the inverter LC output filter (see Fig. 4.1).
- $H_{V1}(s)$ is the transfer function associated with the voltage control loop.
- $H_{C1}(s)$ is the transfer function related to the current control loop.
- $G_{PWM1}(s)$ is the PWM transfer function.
- $M_1(s)$ and $N_1(s)$ are the transfer functions of the LC output filter (see Fig. 4.1).
- $G_{b1}(s)$ is the transfer function for the implementation of the virtual balanced impedance loop.
- $G_{u1}(s)$ is the transfer function of the virtual unbalanced impedance loop (see Fig. 4.3).

Note that in this work, PR controllers will be used because the proposed control scheme is defined in the stationary abc frame and PR controllers can provide zero steady state error to sinusoidal signals. Therefore, $H_{V1}(s)$ and $H_{C1}(s)$ are PR controllers (see Fig. 4.3). The closed-loop transfer function of the proposed control scheme (see Fig. 4.3) is shown in (45), where $E_1^*(s)$ is the voltage reference obtained from the droop control system; $K_1(s)$ is the closed-loop transfer function of the voltage controller [see (46)];

$Z_{out1}^b(s)$ is the balanced output impedance; and $Z_{out1}^u(s)$ is the unbalanced output impedance.

Based on (45), the proposed scheme achieves a decoupled control of the voltage drop produced by the balanced current and the voltage drop caused by the unbalanced current. The first voltage can be controlled through R_1^b and L_1^b , as shown in Fig. 4.3 and (47). These parameters are set to be constant and, they are used to implement the virtual impedance loop [54] [65]. The unbalanced voltage drop at the output of the inverter can be controlled with $Z_{out1}^u(s)$ through the transfer function $G_{u1}(s)$. From Fig. 4.3 and (48) it can be concluded that to control $Z_{out1}^u(s)$ there are two degrees of freedom represented by R_1^u and L_1^u respectively (see Fig. 4.3). In this work, it will be assumed that $G_{u1}(s)$ is a resistance, i.e $G_{u1}(s) = R_1^u$ (see [44]).

$$E_1(s) = K_1(s)E_1^*(s) - Z_{out1}^b(s)i_1^b(s) - Z_{out1}^u(s)i_1^u(s) \quad (45)$$

$$K_1(s) = \frac{N_1(s)M_1(s)G_{PWM1}(s)H_{C1}(s)H_{V1}(s)}{1 + N_1(s)M_1(s)G_{PWM1}(s)H_{C1}(s)H_{V1}(s) + M_1(s)G_{PWM1}(s)H_{C1}(s) + N_1(s)M_1(s)} \quad (46)$$

$$Z_{out1}^b(s) = \frac{N_1(s) + N_1(s)M_1(s)G_{PWM1}(s)H_{C1}(s) + N_1(s)M_1(s)G_{PWM1}(s)H_{C1}(s)H_{V1}(s)G_{b1}(s)}{1 + N_1(s)M_1(s)G_{PWM1}(s)H_{C1}(s)H_{V1}(s) + M_1(s)G_{PWM1}(s)H_{C1}(s) + N_1(s)M_1(s)} \quad (47)$$

$$Z_{out1}^u(s) = \frac{N_1(s) + N_1(s)M_1(s)G_{PWM1}(s)H_{C1}(s) + N_1(s)M_1(s)G_{PWM1}(s)H_{C1}(s)H_{V1}(s)G_{u1}(s)}{1 + N_1(s)M_1(s)G_{PWM1}(s)H_{C1}(s)H_{V1}(s) + M_1(s)G_{PWM1}(s)H_{C1}(s) + N_1(s)M_1(s)} \quad (48)$$

Equation (45) can be rewritten in terms of balanced and unbalanced terms as shown in (49). In this equation $E_1^b(s)$ corresponds to the balanced terms in (45) and $E_1^u(s)$ is the unbalanced voltage drop produced by the unbalanced output impedance. These voltages are given by (50), where it can be appreciated that the matrix associated to both balanced and unbalance currents is a diagonal matrix and therefore can be concluded that based on the assumptions discussed in section 4.2, the voltages $E_1^b(s)$ and $E_1^u(s)$ are decoupled.

In this section, inverter 1 of Fig. 4.2 was used as an example to show the mathematical analysis of the proposed control scheme. This scheme has also been implemented for inverter 2 of Fig. 4.2 and in a general way, it can be implemented in “n” 4-leg converters feeding a common load.

$$E_1(s) = E_1^b(s) + E_1^u(s) \quad (49)$$

$$\begin{bmatrix} E_1^b(s) \\ E_1^u(s) \end{bmatrix} = \begin{bmatrix} K_1(s)E_1^*(s) \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{out1}^b(s) & 0 \\ 0 & Z_{out1}^u(s) \end{bmatrix} \begin{bmatrix} i_1^b(s) \\ i_1^u(s) \end{bmatrix} \quad (50)$$

4.3.1 Cooperative Sharing of Imbalance between Inverters

Assuming that the inverters shown in Fig. 4.2 are controlled using the control methodology depicted in Fig. 4.3, the load voltage V_{PCC} can be written as the sum of a balanced and an unbalanced voltage. V_{PCC} as a function of the voltages and currents of inverter 1 is shown in (51). Analogously, V_{PCC} as a function of the voltages and currents of inverter 2 is given by (52). Equating these two expressions yields (53).

$$\begin{bmatrix} V_{PCCb} \\ V_{PCCu} \end{bmatrix} = \begin{bmatrix} K_1(s)E_1^*(s) \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{out1}^b(s) + Z_{L1}(s) & 0 \\ 0 & Z_{out1}^u(s) + Z_{L1}(s) \end{bmatrix} \begin{bmatrix} i_1^b(s) \\ i_1^u(s) \end{bmatrix} \quad (51)$$

$$\begin{bmatrix} V_{PCCb} \\ V_{PCCu} \end{bmatrix} = \begin{bmatrix} K_2(s)E_2^*(s) \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{out2}^b(s) + Z_{L2}(s) & 0 \\ 0 & Z_{out2}^u(s) + Z_{L2}(s) \end{bmatrix} \begin{bmatrix} i_2^b(s) \\ i_2^u(s) \end{bmatrix} \quad (52)$$

$$[Z_{out1}^u(s) + Z_{L1}(s)] \cdot i_1^u(s) = [Z_{out2}^u(s) + Z_{L2}(s)] \cdot i_2^u(s) \quad (53)$$

Based on (44) and (53) the sharing principle of the proposed control scheme can be derived. This principle is shown in (54) and (55) where the line impedances are neglected for simplicity. From these equations, it is concluded that the unbalanced current in the load $i_o^u(s)$ can be shared between the inverters according to a factor which depends on the unbalanced output impedances defined in this section. These impedances can be actively controlled, through $G_{u1}(s) = R_1^u$ in inverter 1 [see (48)] and through $G_{u2}(s) = R_2^u$ in inverter 2, thus providing active control of the imbalance sharing.

$$i_1^u(s) \approx \frac{Z_{out2}^u(s)}{Z_{out1}^u(s) + Z_{out2}^u(s)} \cdot i_o^u(s) \quad (54)$$

$$i_2^u(s) \approx \frac{Z_{out1}^u(s)}{Z_{out1}^u(s) + Z_{out2}^u(s)} \cdot i_o^u(s) \quad (55)$$

Using (54)-(55) to calculate the unbalanced power in each inverter (this power is defined by the CPT as described in chapter 2, section 2.1), equations (56) and (57) are obtained, where N_1 and N_2 are the unbalanced powers in inverter 1 and inverter 2 respectively. The unbalanced power is shared between the inverters according to the same principle as the unbalanced currents.

$$N_1 \approx 3 \cdot \left| \frac{Z_{out2}^u}{Z_{out1}^u + Z_{out2}^u} \right| \cdot E_1 \cdot I_o \quad (56)$$

$$N_2 \approx 3 \cdot \left| \frac{Z_{out1}^u}{Z_{out1}^u + Z_{out2}^u} \right| \cdot E_2 \cdot I_o \quad (57)$$

4.3.2 Practical Implementation of the Proposed Scheme

In section 4.3.1 the unbalanced sharing principle has been discussed, based on the insertion of virtual “unbalanced” output impedances in the control of the inverters. However, the insertion of large unbalanced impedances might induce large voltage imbalance at the output of the inverters, possibly exceeding regulatory limits. For this reason, the Phase Voltage Unbalance Rate index (PVUR, see (58)) [97] [98] is introduced to control imbalance sharing, by adjusting the magnitude of the unbalanced output impedance in each inverter, according to the unbalanced voltage requirement at the output of each inverter. Note that in (58) the voltages are measured with respect to the neutral point.

$$PVUR = \frac{\max(|E_{an}| - \bar{E}, |E_{bn}| - \bar{E}, |E_{cn}| - \bar{E})}{\bar{E}} \quad (58)$$

$$\bar{E} = (|E_{an}| + |E_{bn}| + |E_{cn}|)/3$$

The practical implementation of the proposed control scheme in the inverter 1 of Fig. 4.2 is shown in Fig. 4.4. Both the output currents and output voltages of the 4-leg inverter are measured. With these signals and using the CPT definitions described in section 2.1 (chapter 2), the following quantities are calculated: (i) Active (P_1) and Reactive (Q_1)

powers, (ii) balanced currents (i_{1abc}^b), (iii) unbalanced currents (i_{1abc}^u) and (iv) the $PVUR_1$ index at the output of the inverter. To share and regulate P_1 and Q_1 , standard droop controllers are implemented. The implementation of these controllers is shown in (59), where m_1 and n_1 are the frequency and voltage droop coefficients and ω_n and E_n are respectively, the nominal frequency and voltage of the MG. Additionally, using i_{1abc}^b the voltage drop due to the virtual impedance loop is calculated [54] [65] (see Fig. 4.3), thus improving decoupling between active and reactive power control. With i_{1abc}^u and the $PVUR_1$ index, the virtual resistance computation block shown in Fig. 4.4 and Fig. 4.5 is implemented, to adjust dynamically the value of R_1^u and therefore, the magnitude of Z_{out1}^u [see (48)]. As shown in Fig. 4.5, the reference for this block is the desired $PVUR_1^*$ which can be tolerated by the inverter. The control is based on a PI controller plus a low pass filter (for removing noise), whose output is R_1^u . This is multiplied by the unbalanced current to generate the unbalanced voltage reference. Doing that, the magnitude of the output impedance Z_{out1}^u in inverter 1 is controlled through $G_{u1}(s) = R_1^u$ according to (48). Similarly, in the control algorithm of inverter 2 (see Fig. 4.2) the $PVUR_2^*$ generates the output impedance Z_{out2}^u and, based on Z_{out1}^u and Z_{out2}^u the load imbalance is shared between the inverters according to (54)-(57). Note that the proposed control can adjust dynamically and in real-time the values of Z_{out1}^u and Z_{out2}^u in each inverter to operate with different degrees of load imbalance, maintaining the PVUR references ($PVUR_1^*$ and $PVUR_2^*$) in each inverter. Moreover, as shown in Fig. 4.7, a secondary control system could be used to dynamically regulate the values of $PVUR_1^*$ and $PVUR_2^*$ controlling the $PVUR$ at another point of the microgrid, for instance in the PCC.

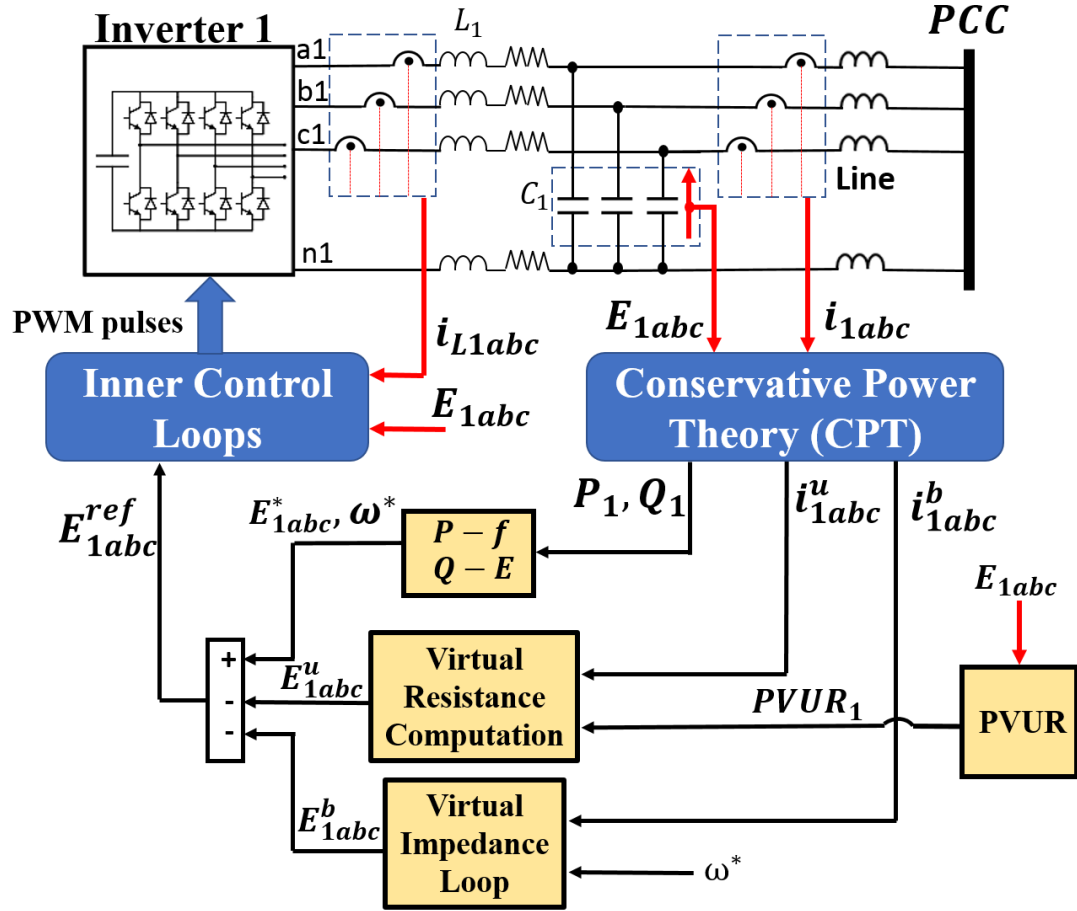


Fig. 4.4. Practical implementation of the proposed control scheme. (The inverter 1 was used as an example)

$$\begin{cases} \omega^* = \omega_n - m_1 P_1 \\ E_1^* = E_n - n_1 Q_1 \end{cases} \quad (59)$$

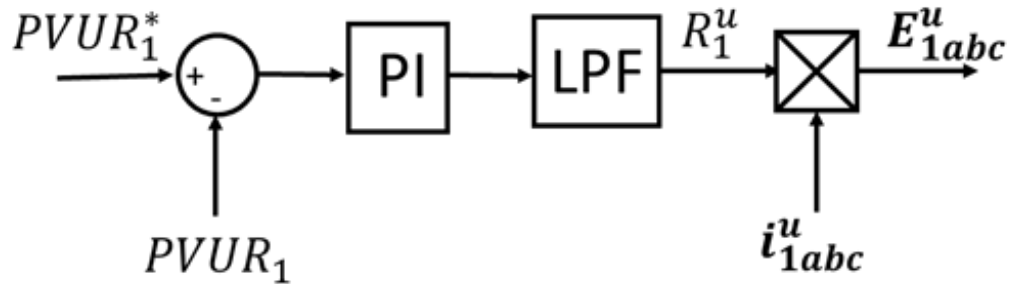


Fig. 4.5. Details of the virtual resistance computation block shown in Fig. 4.4. (Only inverter 1 is shown)

The implementation of the virtual impedance loop in the inverter 1 of Fig. 4.2 is shown in Fig. 4.6 (the same methodology is used for inverter 2). In this work, R_1^b and L_1^b (see Fig. 4.6) are chosen to maintain the inverter output voltage within 5% [99] of the nominal value. More information about the characteristics of this loop is presented in [54] [65]. It is worth considering that the virtual impedance loop and the virtual resistance computation block shown in Fig. 4.4 and Fig. 4.5 are decoupled since the former is implemented with the balanced current vector, and the latter utilises the unbalanced current vector, and these vectors are decoupled (see section 4.2.2). The inner voltage and current control loops shown in Fig. 4.4 are implemented in the abc reference frame using self-tuning proportional plus resonant (PR) controllers in each phase [40] (more information about the design of this type of controller can be found in chapter 3, subsection 3.3.1). Finally, it is worth remembering that an important characteristic of the proposed control scheme (see Fig. 4.3) is its capacity to be embedded and locally implemented at each converter. Therefore, a communication channel among the converters is not required for the primary control system.

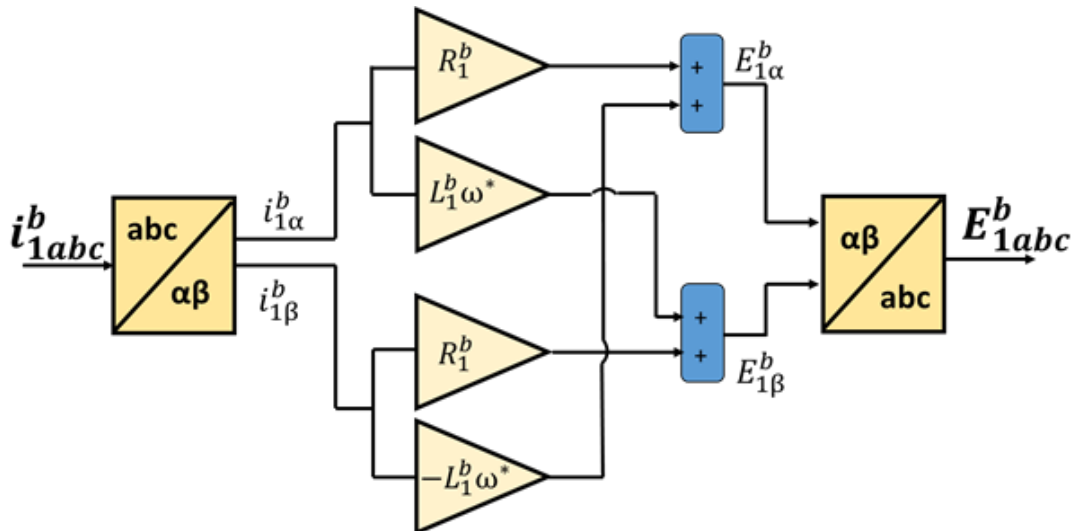


Fig. 4.6. Implementation of the virtual impedance loop shown in Fig. 4.4.

4.3.3 Methodology to choose the PVUR set points for each inverter

To validate the effectiveness of the proposed cooperative imbalance sharing method, in this section, a general methodology for calculating the PVUR set points for “n” converters feeding a common load is proposed. It is worth remembering that the choice of these references will determine the effort of imbalance sharing in each inverter. This sharing effort is performed according to the residual power capacity of each inverter.

In this section is assumed that the unbalanced load located at the PCC (see Fig. 4.2) is fed using “n” 4-leg inverters. In this case, the unbalanced powers defined in (56) and (57) can be generalised as shown in (60). From it, the unbalanced power N_x (see section 2.1 in chapter 2) in the converter “x” is given by the multiplication between the RMS voltage E_x at the output of this inverter, the RMS value of the unbalanced current at the load I_o^u and an equivalent impedance given by the current divider when “n” converters are feeding a common load. (See [100])

$$N_x = 3 \cdot \left| \frac{\prod_{m=1}^n Z_{out_m}^u}{Z_{out_x}^u \cdot \sum_{i=1}^n \frac{\prod_{j=1}^n Z_{out_j}^u}{Z_{out_i}^u}} \right| \cdot E_x \cdot I_o^u \quad (60)$$

In this thesis, the initial $PVUR_x^*$ is obtained by minimizing the cost function of (61), subjected to the restrictions shown in (61). The solution to this problem are the PVUR set points to the inverters which minimise the difference between the unbalanced powers supplied by the 4-leg converters. It should be pointed out that (61) considers the residual power capacity of each inverter. This is realised through N_x^{res} [see (61) and (62)], which correspond to the residual power capability in the inverter “x”. Note that N_x^{res} is given by

the difference between the maximum unbalanced power that this inverter can inject to the load N_x^{max} [given by (62)] [43] [66] [68] and the actual unbalanced power (N_x^{actual}) that this inverter is injecting to the load. Based on that, the imbalances on the three-phase four-leg MG studied in this work, are shared according to the residual power capacity of each inverter. Moreover, the optimisation problem shown in (61) has inequality restrictions, which characterise the voltage requirements at the output of each inverter. These requirements depend on the type of load which is connected at the output of them. In this work, if a sensitive load is connected at the output of the inverter “x”, the maximum PVUR allowed is 1% ($a_x = 1\%$) [44]. In contrast, if a non-sensitive load is connected at the output of the inverter “x”, the maximum PVUR allowed is 3% ($a_x = 3\%$) [63]. Finally, based on the PVUR restrictions in each inverter, the optimisation problem is solved⁴ and the optimal imbalance sharing among the converters is achieved. Note that in (61)-(62), “n” represents the number of converters which are feeding the common load. It should be pointed out that in (62), S_x is the apparent power of inverter x, P_x , and Q_x are respectively the active and reactive powers in that inverter, which are calculated using the CPT definitions. (See section 2.1)

$$\begin{cases} Min = \sum_{x=1}^n \sum_{y>x}^n \left| \frac{N_x}{N_x^{res}} - \frac{N_y}{N_y^{res}} \right| \\ s.t. \\ PVUR_x \leq a_x, \quad x = 1, 2, \dots, n \end{cases} \quad (61)$$

$$N_x^{res} = N_x^{max} - N_x^{actual} \quad , \quad N_x^{max} = \sqrt{S_x^2 - P_x^2 - Q_x^2} \quad , \quad x = 1, 2, \dots, n \quad (62)$$

⁴ Using an optimization solver such as the function fmincon or genetic algorithm [106], both contained in MATLAB

In (61), the unbalanced powers in the inverters are calculated using (60). Therefore, to solve this optimisation problem, it is only necessary to find the explicit relation for the PVURs in each inverter. For the sake of simplicity, the PVUR index of (58) could be approximated by (63). In this case, E_n is the nominal voltage of the MG. The difference between (58) and (63) is that the average voltage $\bar{E}$ defined in (58) is replaced by the nominal system voltage E_n in (63). This assumption is inspired by: (i) the NEMA definition for voltage imbalance, also known as the line voltage unbalance rate (LVUR), where it is assumed that the average voltage is always equal to the rated value [101] [102] and (ii) since the maximum imbalance at the inverters' output is regulated according to the ANSI C84.1-2006 standard, the average voltage $\bar{E}$ is close to the nominal voltage E_n of the MG.

$$PVUR = \frac{\text{Max}(|E_a| - E_n, |E_b| - E_n, |E_c| - E_n)}{E_n} \quad (63)$$

Using (63), an explicit equation for the PVUR will be derived. This is realised for inverter “x” (for the rest of the converters it is the same procedure). Using Fig. 4.4 it is concluded that the voltage at the output of the inverter “x” ($\underline{E}_x^{ref}$) is given by (in vector format): $\underline{E}_x^{ref} = \underline{E}_x^* - \underline{E}_x^b - \underline{E}_x^u$ (see Fig. 4.4), where $\underline{E}_x^*$ is the voltage reference given by the conventional $P - f$ and $Q - E$ droop controllers; $\underline{E}_x^b$ is the voltage drop produced by the virtual impedance loop (see Fig. 4.6); and $\underline{E}_x^u$ is the unbalanced voltage reference produced by the virtual resistance computation block shown in Fig. 4.4. Using Fig. 4.5 it is concluded that $\underline{E}_x^u$ is obtained by the product of the unbalanced current vector at the output of the inverter (in this case $\underline{i}_x^u$) and the virtual resistance R_x^u programmed in the

control system of that inverter. Using (63), the $PVUR_x$ of the inverter “x” is given by (64) [considering voltages and currents in phasor format].

$$PVUR_x = \frac{\text{Max}(|\dot{E}_{a-x}^* - \dot{E}_{a-x}^b - R_x^u \cdot i_{a-x}^u| - E_n, |\dot{E}_{b-x}^* - \dot{E}_{b-x}^b - R_x^u \cdot i_{b-x}^u| - E_n, |\dot{E}_{c-x}^* - \dot{E}_{c-x}^b - R_x^u \cdot i_{c-x}^u| - E_n)}{E_n} \quad (64)$$

Using, the properties of the Max function to (64) yields.

$$PVUR_x = \frac{\text{Max}(|\dot{E}_{a-x}^* - \dot{E}_{a-x}^b - R_x^u \cdot i_{a-x}^u|, |\dot{E}_{b-x}^* - \dot{E}_{b-x}^b - R_x^u \cdot i_{b-x}^u|, |\dot{E}_{c-x}^* - \dot{E}_{c-x}^b - R_x^u \cdot i_{c-x}^u|)}{E_n} - 1 \quad (65)$$

Equation (65) can be written as a function of the infinite norm and the Euclidean norm, such as shown in (66).

$$PVUR_x = \frac{1}{E_n} \left\| \begin{pmatrix} \dot{E}_{a-x}^* - \dot{E}_{a-x}^b - R_x^u \cdot i_{a-x}^u \\ \dot{E}_{b-x}^* - \dot{E}_{b-x}^b - R_x^u \cdot i_{b-x}^u \\ \dot{E}_{c-x}^* - \dot{E}_{c-x}^b - R_x^u \cdot i_{c-x}^u \end{pmatrix} \right\|_{\infty} - \frac{1}{\sqrt{3}} \left\| \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\|_2 \quad (66)$$

Because these norms meet the triangular inequality, $PVUR_x$ has an upper bound as shown in (67).

$$PVUR_x \leq \frac{1}{E_n} \left\| \begin{pmatrix} \dot{E}_{a-x}^* - \dot{E}_{a-x}^b \\ \dot{E}_{b-x}^* - \dot{E}_{b-x}^b \\ \dot{E}_{c-x}^* - \dot{E}_{c-x}^b \end{pmatrix} \right\|_{\infty} + R_x^u \cdot \frac{1}{E_n} \left\| \begin{pmatrix} i_{a-x}^u \\ i_{b-x}^u \\ i_{c-x}^u \end{pmatrix} \right\|_{\infty} - \frac{1}{\sqrt{3}} \left\| \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\|_2 \quad (67)$$

Finally, (67) can be rewritten as is shown in (68).

$$PVUR_x \leq \frac{1}{E_n} \text{Max}(|\dot{E}_{a-x}^* - \dot{E}_{a-x}^b|, |\dot{E}_{b-x}^* - \dot{E}_{b-x}^b|, |\dot{E}_{c-x}^* - \dot{E}_{c-x}^b|) + R_x^u \cdot \frac{1}{E_n} \text{Max}(|i_{a-x}^u|, |i_{b-x}^u|, |i_{c-x}^u|) - 1 \quad (68)$$

In (68), the magnitude of the balanced voltage phasors $\dot{E}_{a_x}^*$, $\dot{E}_{b_x}^*$ and $\dot{E}_{c_x}^*$ are equal since these values are manipulated by the reactive power droop controller. Moreover, because the voltage variation is small, these magnitudes can be approximated to the nominal voltage E_n of the MG [54] [65]. Moreover, assuming that the voltage drop caused by the virtual impedance loop (see Fig. 4.6) is negligible since it has been designed to reduce the output voltage by less than 5% of the nominal value [99] [103], it can be assumed that $|\dot{E}_{a_x}^* - \dot{E}_{a_x}^b| = |\dot{E}_{b_x}^* - \dot{E}_{b_x}^b| = |\dot{E}_{c_x}^* - \dot{E}_{c_x}^b| \approx E_n$. Based on that, (68) can be rewritten as shown in (69) (restriction of this problem). From (69) it is concluded that the $PVUR_x$ at the output of the inverter “x” has an upper bound which is a function of: (i) nominal voltage of the MG (E_n), (ii) the module of the unbalanced current at the output of that inverter ($|i_{a_x}^u|, |i_{b_x}^u|, |i_{c_x}^u|$), and (iii) the virtual output resistance of the inverter (R_x^u) (see Fig. 4.5 and section 4.3). Considering that in this work the magnitude of the unbalanced output impedance ($Z_{out_x}^u$) of the inverter “x” is controlled through the virtual resistance (R_x^u) (as discussed in section 4.3), it can be assumed that $Z_{out_x}^u \approx R_x^u$ [44] [104] and therefore (60) can be written as a function of virtual resistances instead of virtual impedances. Based on that, the optimisation problem introduced in (61) can be explicitly formulated as shown in (69), where the optimisation variables are the virtual resistances ($R_x^u, x = 1, \dots, n$) which are used in the controllers of each inverter. To solve this optimization problem, it is necessary to measure the current at the converters’ output, the RMS value of the voltage at the output of each of them and the current on the load. This can be achieved by a simulation stage or it can be implemented in a tertiary control scheme [62].

Once (69) is solved, the PVUR set points for each converter are obtained evaluating these optimisation variables in the expression of the PVUR of each converter [given by

the upper bounds of the PVURs shown in (69)]. It means that the optimisation problem is solved considering the worst case in the inequality restrictions.

$$\left\{ \begin{array}{l} Min J = 3I_o^u \sum_{x=1}^n \sum_{y>x}^n \left| \frac{\prod_{m=1}^n R_m^u}{R_x^u \cdot \sum_{i=1}^n \frac{\prod_{j=1}^n R_j^u}{R_i^u}} \cdot \frac{E_x}{N_x^{res}} - \frac{\prod_{m=1}^n R_m^u}{R_y^u \cdot \sum_{i=1}^n \frac{\prod_{j=1}^n R_j^u}{R_i^u}} \cdot \frac{E_y}{N_y^{res}} \right| \\ s. t. \\ PVUR_x \leq \frac{R_x^u}{E_n} \cdot Max(|i_{a_x}^u|, |i_{b_x}^u|, |i_{c_x}^u|) \leq a_x, \quad x = 1, 2, \dots, n \end{array} \right. \quad (69)$$

Finally, as stated in the introduction of this chapter, cooperative imbalance sharing in a MG is provided at the expense of increasing voltage imbalance at the voltage buses. In this work, it is assumed that sensitive loads are located close to the inverter buses, and the proposed control scheme achieves cooperative imbalance sharing among the 4-leg inverters and at the same time, maintains the voltage imbalances within acceptable levels at the inverter outputs. Obviously, the unbalanced voltage requirements at the inverter outputs will determine the cooperative sharing effort of them. In this context, the unbalanced voltage requirements at the PCC is a less important concern. This is because it is assumed that non-sensitive loads are connected at this point. In addition, a secondary control scheme is proposed to regulate the unbalanced voltage at the PCC: it is shown in Fig. 4.7 considering three 4-leg inverters. In this figure, the voltage at the PCC is measured, and the voltage unbalance index ($PVUR_{PCC}$) is calculated. Using the $PVUR_{PCC}$ and a desired imbalance index at the PCC ($PVUR_{PCC}^*$), a secondary control loop is implemented. The output of this secondary control is a $\Delta PVUR$ which is sent to the inverters. This $\Delta PVUR$ is subtracted from the PVUR references ($PVUR_1^*$, $PVUR_2^*$ and $PVUR_3^*$) given initially to the inverters. Fig. 4.7 shows that this secondary control system

is simple to implement considering the afore discussed cooperative control scheme. Moreover, a relatively low communication bandwidth can be used by this centralised secondary controller (see [11]). In summary, in this thesis, a hierarchical control scheme is proposed and discussed. This hierarchical scheme consists of two control levels. In the primary level, the converters are controlled locally with the scheme shown in Fig. 4.3 (based on currents and voltages measured at the output of each inverter) while the secondary level corresponds to the centralised controller shown in Fig. 4.7 which sends $\Delta PVUR$ references signals to the inverters in order to keep the PVUR at the PCC below a given value. This secondary control topology can be easily extended to “n” converters.

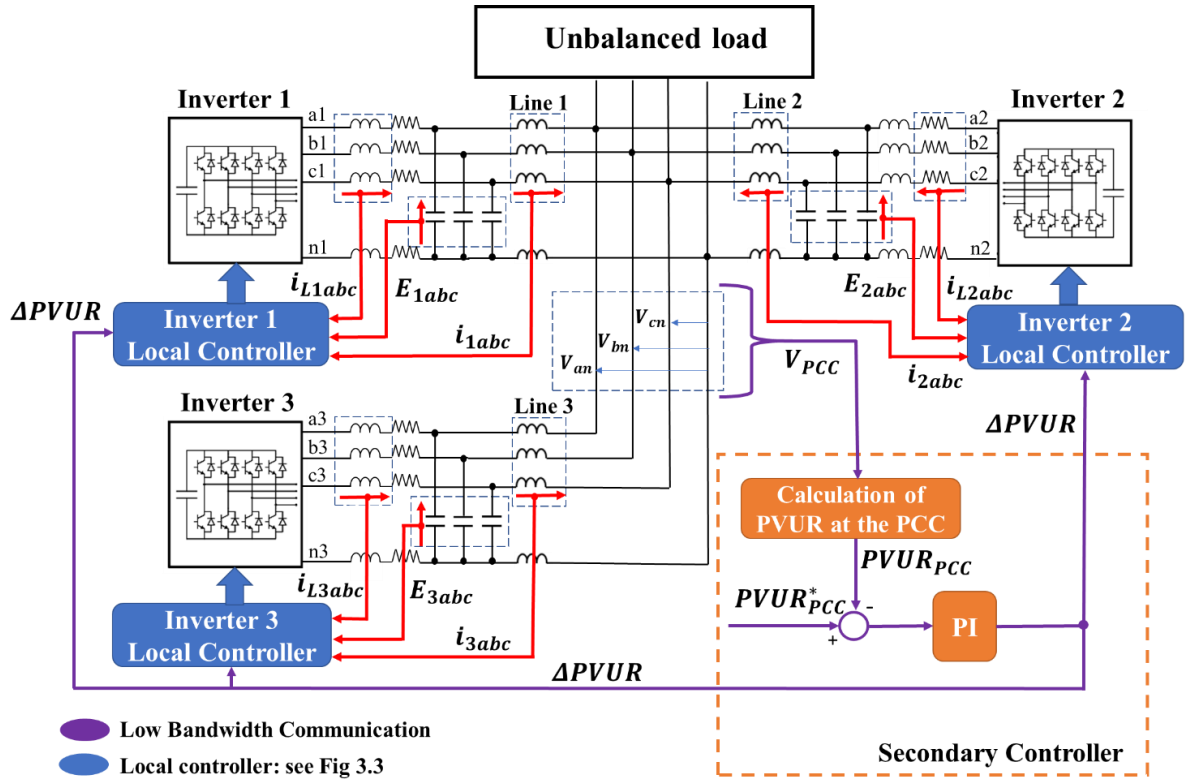


Fig. 4.7. Proposed secondary control scheme to regulate the voltage unbalance rate (PVUR) at the PCC in isolated MGs.

4.4 Simulation and Experimental Results

4.4.1 Simulation Results

In this section, both the primary and secondary controllers depicted in Fig. 4.3 and Fig. 4.7 respectively are validated. The system shown in Fig. 4.7 is simulated using Plexim PLECS software, and it is assumed that the converters are connected to the load through inductive line impedances of 2 mH. The three 4-legs converters are controlled considering the control scheme described in Fig. 4.3, which is implemented locally in each device. For the implementation of the controllers depicted in Fig. 4.3, no measurement of the voltage and currents at the PCC are required. However, when the secondary control system is considered the PVUR at the PCC has to be measured.

In the simulation work, it is assumed that the converters have different power rating and they are droop-controlled systems with the following droop coefficients [see (59)]: $m_1 = m$ and $n_1 = n$ for inverter 1, $m_2 = 2m$ and $n_2 = 2n$ for inverter 2, and $m_3 = 3m$ and $n_3 = 3n$ for inverter 3. The value of m and n are depicted in Table 4.1. The PVUR set points for each inverter are calculated using the methodology proposed in section 4.3.3 as 1% for inverter 1, 2% for inverter 2 and 3% for inverter 3. These results were obtained considering a genetic algorithm [105] to solve the optimisation problem [see (69)]. It should be pointed out that the proposed control scheme needs PVUR set points so that imbalances can be shared among the converters according to their residual VA power capacity. These PVUR set points are calculated according to the methodology discussed in section 4.3.3 which is based on the optimisation problem given by (69). Since this optimisation problem is non-linear, it can be solved (among others) using the function `fmincon` or genetic algorithm [105], both contained in MATLAB. In particular, in this thesis, genetic algorithms are used for solving (69). (In Appendix B, the MATLAB script used for

solving (69) is shown). Finally, At the PCC an unbalanced RL load is connected and the following power rating for each inverter are assumed: 5000VA (inverter 1), 2500VA (inverter 2) and 1666VA (inverter 3).

Four simulation steps are considered in this work, i.e. step 1 ($0 \leq t < 3s$) in which the converters operate with the proposed control scheme shown in Fig. 4.3 but with the virtual unbalanced impedance loop disabled; step 2 ($3s \leq t < 6s$) in which the virtual unbalanced impedance loop in each converter is enabled; step 3 ($6s \leq t < 9s$) where the secondary control described in Fig. 4.7 is activated to regulate the PVUR in the PCC at 3%; and finally step 4 ($9s \leq t \leq 12s$), where converter 1 trips at $t=9s$. This latter case is to evaluate the performance of the control system considering this critical situation.

Fig. 4.8 shows both the active power (P) and the unbalanced power (N) measured at the output of each inverter for the four steps considered. From Fig. 4.8(b), it is shown that in step 1, the unbalanced power supplied by the three converters to the load are equal because the line impedances are identical (2mH). Then, in step 2, when the proposed imbalance sharing algorithm is enabled (see Fig. 4.3) in all the converters, the unbalanced power is supplied by the inverters to the load according to their residual power capacity. An interesting characteristic of the proposed control method is that it does not affect the performance of the $P - f$ droop controller [see Fig. 4.8(a)].

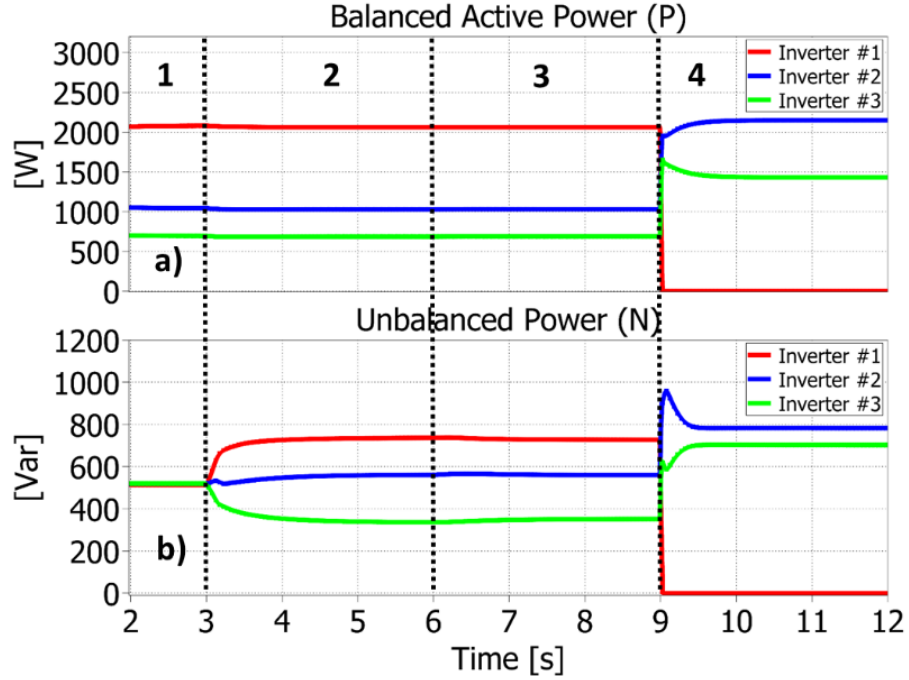


Fig. 4.8 (a) Active Power in the converters for the three steps studied, (b) Unbalanced Power in the converters for the three steps studied.

Fig. 4.9 shows both the three-phase unbalanced currents and the neutral current at the output of each inverter for the steps 1 and 2. It is worth remembering that the unbalance current defined in this work contain both negative and zero sequences (see section 4.2.1 and 4.2.2). From Fig. 4.9 it is concluded that in step 1, the unbalanced currents injected by the converters (including neutral currents) to the load are identical because the line impedances are identical. In step 2, after the proposed control scheme is enabled, both three-phase and neutral currents are supplied by the converters to the load according to their residual power capacity.

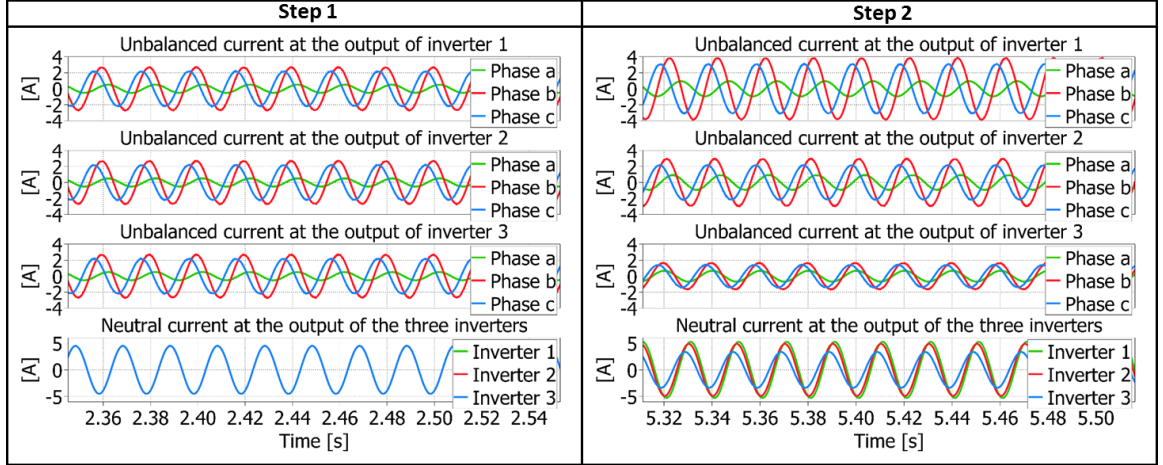


Fig. 4.9. Inverters' unbalanced output current waveforms at different simulation steps.

Fig. 4.10 shows the PVURs at the output of the converters and at the PCC in the four steps studied. From this figure can be concluded that in step 1, the PVUR at the converters' output is close to zero which means that the virtual unbalanced impedance loop in each converter is not activated (see Fig. 4.3). Moreover, the PVUR at the PCC is near 1.7% which is produced because the load is unbalanced. From Fig. 4.10, can be deduced that in step 2, the three PVUR set points in each converter are achieved. In this step, the PVUR at the PCC increases to around 3.3% respect to step 1 (see Fig. 4.10). To manage this issue, in step 3, the secondary control scheme shown in Fig. 4.7 is activated to regulate the PVUR at the PCC in 3%. From Fig. 4.10, can be concluded that the proposed secondary controller effectively regulates the PVUR at the PCC to 3%. This secondary control does not interfere in the unbalanced power sharing (see step 3 in Fig. 4.8). Finally, in Fig. 4.8 and Fig. 4.10 step 4 is examined, where inverter 1 trips at $t=9s$. In this case, the trip occurs when both the primary and secondary control schemes are working (see Fig. 4.3 and Fig. 4.7 respectively). From Fig. 4.10, it is possible to conclude that after the trip, the PVUR reference set points for inverter 2 and inverter 3 are automatically changed by the secondary control to ensure a 3% of imbalance at the PCC.

These new PVUR set points are approximately 0.2% for inverter 2 and 1.2% for inverter 3. From this figure can be seen that in $t = 9\text{s}$, there is a peak in the PVURs. These peaks are not actual peaks, and they are due to the PVUR formula [see (58)], which contain a division. Indeed, the division function is sensitive to variations in the magnitudes. For corroborating this, Fig. 4.11 and Fig. 4.12 shown respectively, the RMS voltages and the instantaneous voltages of the inverters and the load in $t = 9\text{s}$. From these figures can be concluded that there are not considerable peaks in the voltages.

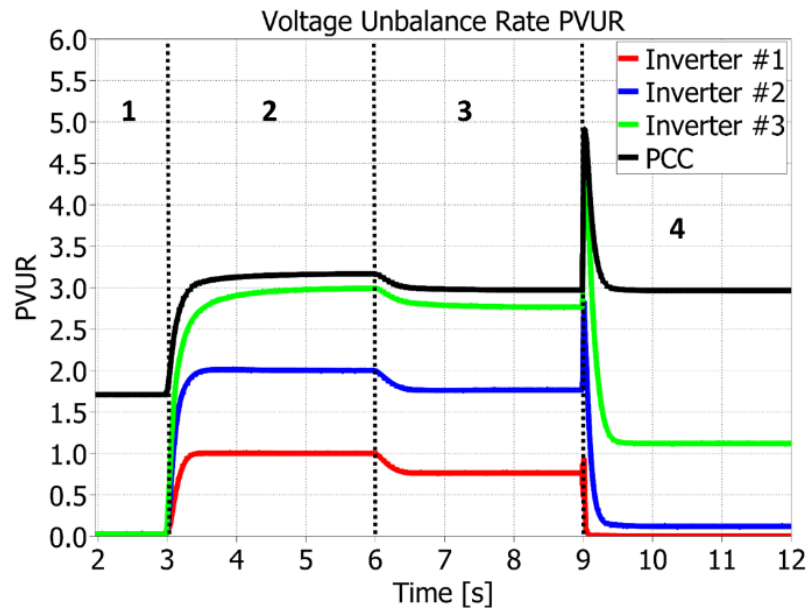


Fig. 4.10. PVURs in the converters and at the PCC for the three steps studied.

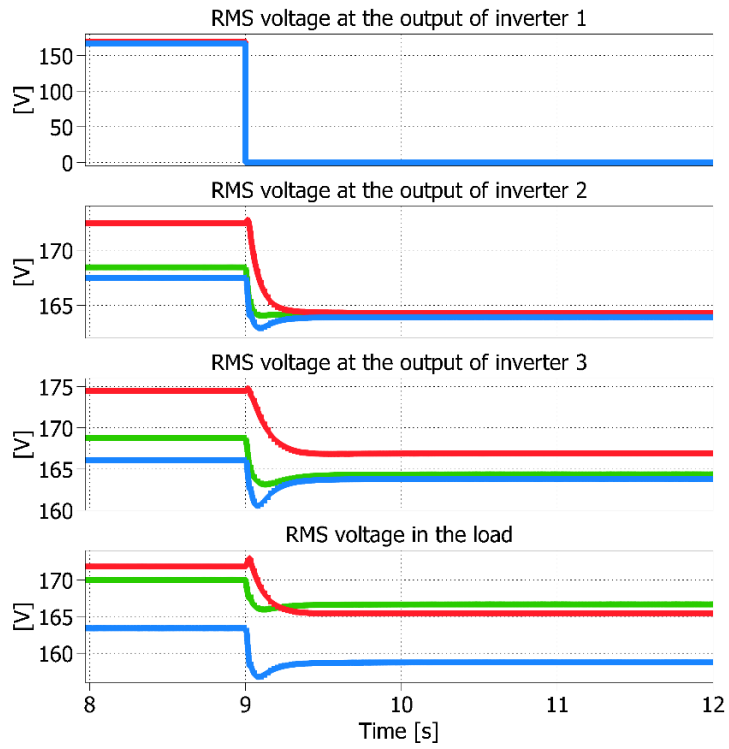


Fig. 4.11. RMS voltages of inverters and load around $t = 9\text{s}$

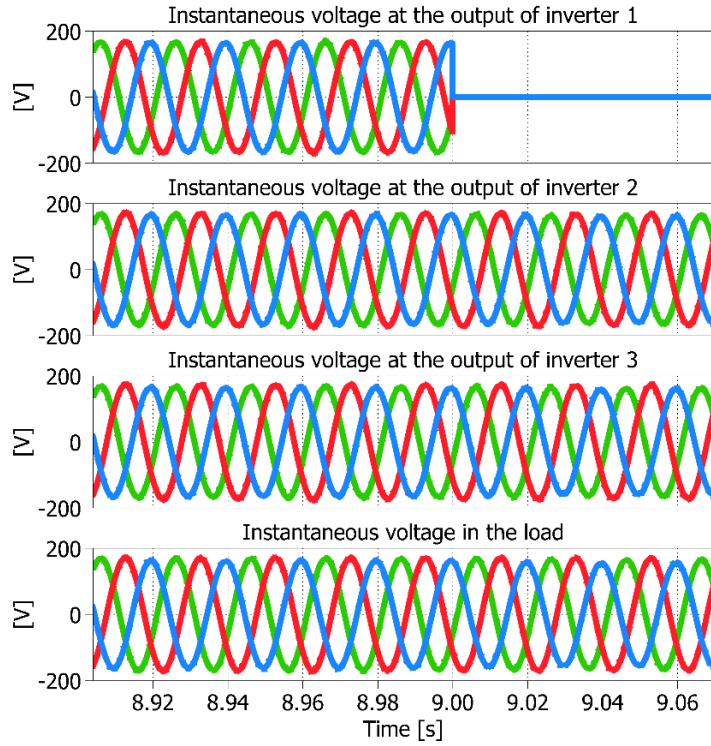


Fig. 4.12. Instantaneous voltages of inverters and load around $t = 9\text{s}$

Finally, it should be highlighted that if the unbalanced load shown in Fig. 4.7, is now a balanced load, the performance of both primary (see Fig. 4.4 and Fig. 4.5) and secondary (see Fig. 4.7) proposed controllers do not affect the stability of the system. In this case, the unbalanced output voltage (E_{1abc}^u) induced by the proposed primary control scheme (see Fig. 3.4) is equal to zero because this voltage is given by the product between R_1^u and i_{1abc}^u (see Fig. 4.5), and in this case i_{1abc}^u is zero (the same occurs for the rest of the 4-leg converters shown in Fig. 4.7).

In this case (balanced load), the proposed secondary controller shown in Fig. 4.7 is not working since this controller is activated if and only if, the $PVUR_{PCC}$ at the PCC (see Fig. 4.7) exceeds the PVUR set point ($PVUR_{PCC}^*$) previously established for the voltage quality in this bar, and in this case, $PVUR_{PCC}$ is close to zero (balanced load).

For corroborating the latter discussion, the system of Fig. 4.7 is simulated, and two steps are considered, they are (i) step 1 ($0s \leq t < 6s$) where the primary and the secondary proposed control schemes are working (the secondary control scheme is working to regulate the PVUR in the PCC at 3%). This step is similar to step 3 of the simulations shown at the beginning of this subsection. Finally, in step 2 ($6s \leq t \leq 12s$), the unbalanced load present in step 1 is changed for a balanced load to study the performance of the proposed controllers in this situation.

From Fig. 4.13 and Fig. 4.14, it can be appreciated the performance of the proposed controllers (primary and secondary) when the converters are feeding an unbalanced load (step 1), and the performance of them feeding a balanced load (step 2). As was discussed in the latter paragraphs, when the load is balanced, the proposed controllers do not affect the stability of the system, and their output is automatically changed to zero. It can be seen in (i) step 2 of Fig. 4.13, where the PVURs at the output of the converters is close

to zero, and (ii) in step 2 of Fig. 4.14(b), where it can be concluded that the unbalanced power injected by the converters to the load is zero.

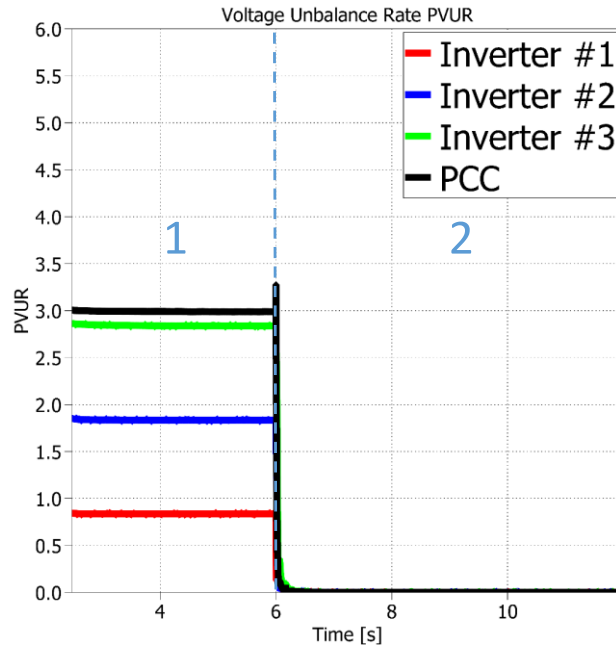


Fig. 4.13. PVURs in the converters and at the PCC for the two steps studied. In step 2 the converters are feeding an unbalanced load, while in step 2 the converters are feeding a balanced load

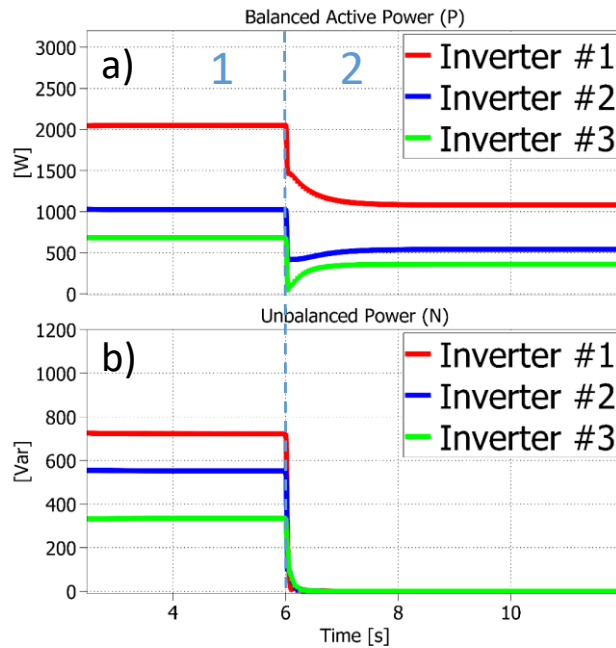


Fig. 4.14. (a) Active Power in the converters for the two steps studied, (b) Unbalanced Power in the converters for the two steps studied.

4.4.2 Experimental Results

In this section, the performance of the control algorithm shown in Fig. 4.3 is experimentally validated. The system of Fig. 4.2 has been implemented on the experimental system shown in Fig. 4.15. Two Triphase units are used as 4-leg inverters. Inverter 1 is a Triphase PM15F120 unit (22kW) while inverter 2 is a Triphase PM5F42R (5kW) unit. The load at the PCC is emulated by an Ametek (9kW) programmable load. The proposed control systems are implemented in the real-time target computers controlling each of the 4-leg inverters of Fig. 4.15. The inner control loops are based on self-tuning voltage and current PR controllers as shown in Fig. 4.16. In this investigation, non-ideal PR controllers are used as shown (70) [40] [106] (for more details see section 3.3.1). The parameters of the experimental system and control loops are given in Table 4.1.

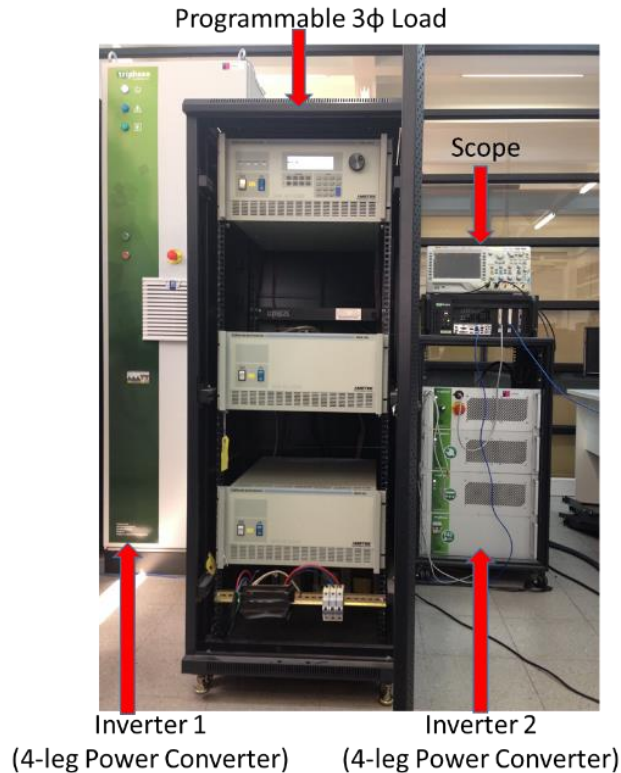


Fig. 4.15. Experimental system. (Both converters are standard two-level converters)



Table 4.1. System Parameters in unit PM15F120* and unit PM5F42R**

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	Table 4.1	Continued
Droop coefficients	m	$1 \cdot 10^{-4} \text{rad}/(\text{W} \cdot \text{s})$
	n	$1 \cdot 10^{-3} \text{V}/(\text{Var})$
Active	R_{D1}	3.75Ω
Damping	R_{D2}	2.50Ω
Balanced	R^b	$1^* \Omega / 1^{**} \Omega$
virtual impedance	L^b	$4^* \text{mH} / 4^{**} \text{mH}$
Unbalanced	PI	$(0.055s + 5)/s$
virtual impedance	LPF	$1/(1 + 0.031831927s)$
control loop		

To validate the performance of the proposed controller, the unbalanced conditions of a MG located in Canada have been emulated at the PCC (see [40] and section 3.4). For the experimental work, the unbalanced currents reported in [40] were scaled, using per unit analysis, to currents that could be handled by the experimental prototype shown in Fig. 4.15. The pattern of the unbalanced load connected at the PCC is shown in Fig. 4.17. In this, thesis, two cases are evaluated, these are discussed below.

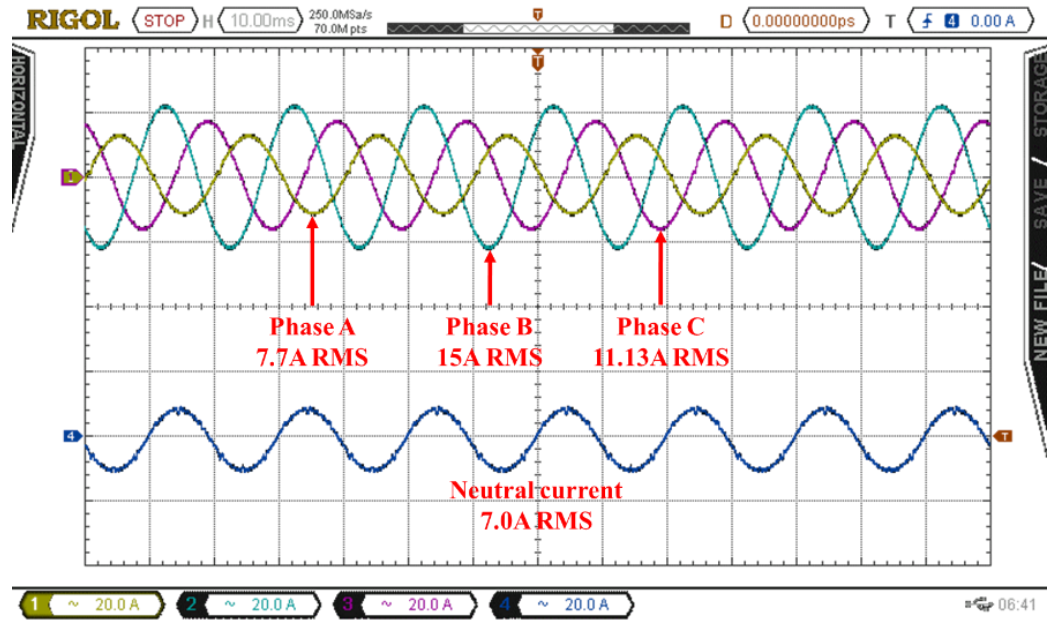


Fig. 4.17. Unbalanced current in the common load.

4.4.2.1 CASE 1: INVERTERS CONNECTED TO THE COMMON LOAD THROUGH DIFFERENT INDUCTIVE LINE IMPEDANCES

For the experimental test, inverter 1 is connected using an inductive line with $L=1.25\text{mH}$ and inverter 2 with a line inductance of 2.5mH . It is assumed that both inverters have the same power rating, and therefore it would be desirable that both inverters inject the same unbalanced power. Using the methodology discussed in section 4.3.3 the references values $PVUR_1^*$ and $PVUR_2^*$ are obtained: 2.8% for inverter 1 and 0.6% for inverter 2 (both values are permitted by ANSI C84.1-2006). The first PVUR set point means that a load less sensitive to unbalances could be placed at the output of inverter 1 while the second means that a sensitive load could be connected at the output of inverter 2.

The performance of the proposed control is shown in Fig. 4.18. Before enabling the control system ($t < 72\text{s}$) the unbalanced powers are different because of the line impedances. After activating the control system, the unbalanced powers in both inverters are equal [see Fig. 4.18(a)], meaning that the power imbalance sharing effort among the inverters is equal since they have the same power rating. This shows the effectiveness of the proposed control scheme. Moreover, Fig. 4.18(b) shows that the PVUR references are correctly achieved by each inverter. Notice that, in the experimental test shown in Fig. 4.18 an additional unbalanced load is connected to the PCC, at $t=87$ (increasing the level of current imbalance in the system). Therefore, after $t=87\text{s}$ [see Fig. 4.18(a)], the unbalanced powers in both inverters are increased. However, the PVUR set points in each inverter are successfully regulated by the proposed control scheme, even though the unbalanced currents are increased in both inverters after the load connection. This is achieved because each inverter reduces dynamically its respective unbalanced impedance to keep its respective voltage imbalance requirements, as shown in Fig.

4.18(c). Obviously, there is a trade-off between unbalanced power-sharing and fulfilling the voltage imbalance requirements, which is shown in Fig. 4.18(a) after $t=87s$, where the unbalanced power is not shared in the same proportion.

Finally, Fig. 4.19 shows how active and reactive powers are not affected by the activation of the proposed controller, confirming the expected decoupling feature, i.e. conventional $P - f$ and $Q - E$ droop controllers are not influenced by the imbalance sharing control. This means also that there is a decoupling between the virtual balanced impedance loop and the virtual unbalanced impedance loop shown in Fig. 4.3.

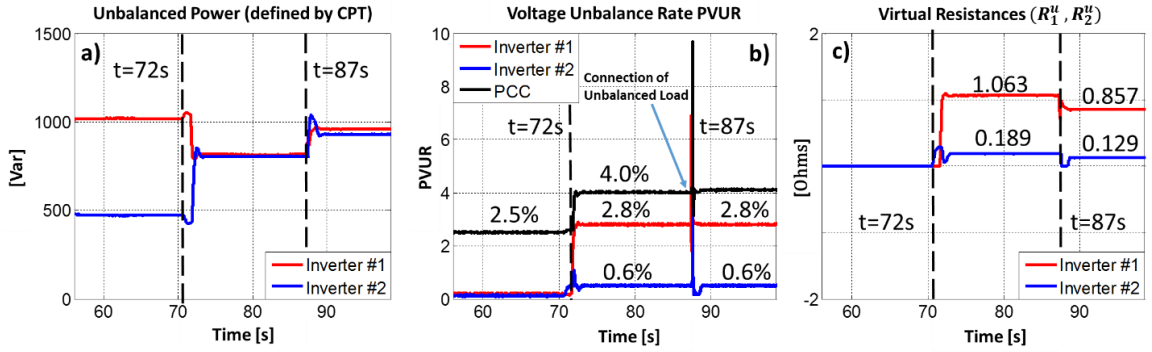


Fig. 4.18. (a) Unbalanced power, (b) PVUR index in both inverters and load, (c) unbalanced impedances evolution - Matlab data logging of the experimental waveforms.

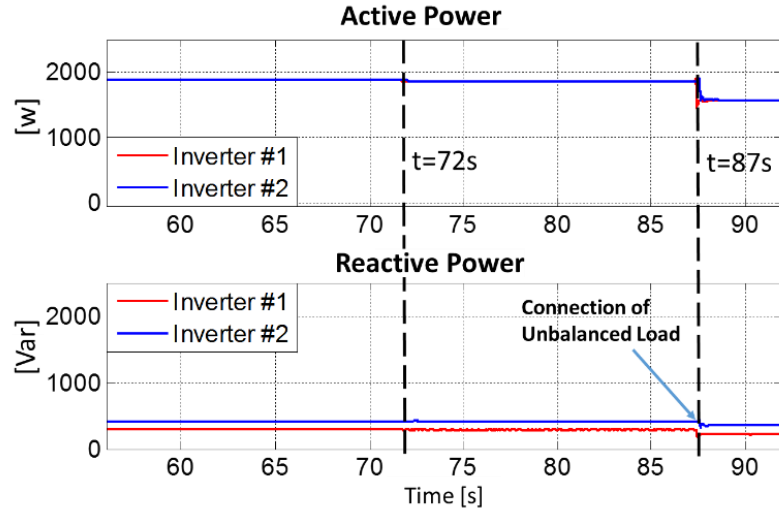


Fig. 4.19. Active and Reactive Powers in both inverters before and after the activation of the proposed control (captured by the data acquisition system of Triphase units) – Matlab data logging of the experimental waveforms.

Fig. 4.20 shows the effective (RMS) voltages at the output of the inverters during this test, where it is possible to see the dynamic performance of the proposed cooperative imbalance sharing method. The control is enabled at $t=72s$, and after that, the unbalanced voltage in the inverters is increased, to achieve a cooperative current imbalance sharing among the inverters. After 72s, it is possible to conclude that the transient response of the proposed scheme is less than 3s. The same is concluded after $t=87s$ when an additional unbalanced load is connected to phase “A” at the PCC.

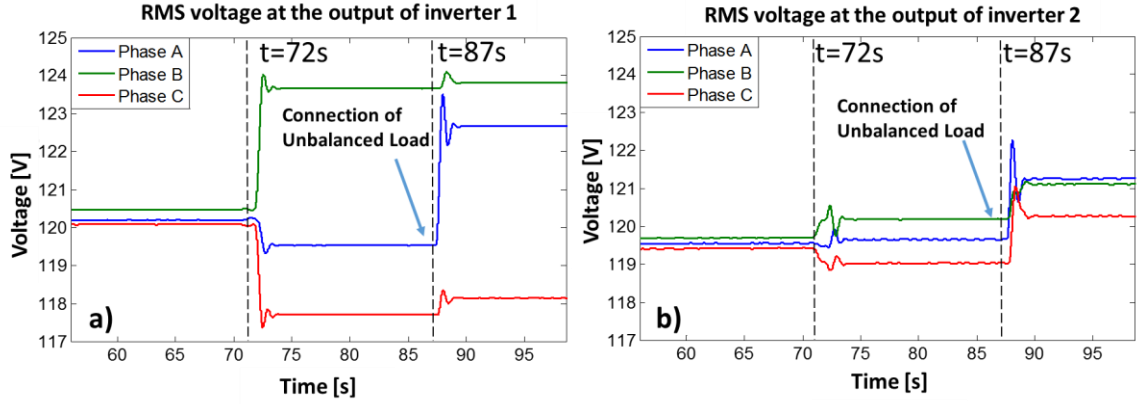


Fig. 4.20. (a) RMS voltages at the output of inverter 1, (b) RMS voltages at the output of inverter 2, (the load impact is applied to phase A) - Matlab data logging of the experimental waveforms.

In this experimental test, the proposed control scheme causes an increase in the PVUR at the PCC, from 2.5% to 4%, as shown in Fig. 4.18(b). This issue can be explained based on equations (51) and (52). The unbalanced voltage at the PCC, V_{PCC_u} , depends on the unbalanced impedances Z_{out1}^u and Z_{out2}^u . Since the unbalanced power sharing between the inverters is controlled through the addition of these impedances, it will modify the unbalanced voltage component at the PCC and its PVUR index. This issue can be solved if the proposed secondary control shown in Fig. 4.7 is implemented in the 3 ϕ four-

wire MG. In order to study the performance of the proposed secondary control in this condition, the MG used in this experimental test is simulated (with the secondary control shown in Fig. 4.7, using the PI parameters $k_p = 0.3$ and $k_i = 6.6$) using Plexim PLECS software and with the operating conditions given by the experimental test. The simulation is performed after $t=72s$ (see Fig. 4.18), where it can be appreciated that the proposed control scheme achieves a cooperative imbalance sharing among the 4-leg inverters but the PVUR at the PCC is increased from 2.5% to 4% in the experimental test [see Fig. 4.18(b)]. This condition is simulated and it is shown in Fig. 4.21(a) before $t=2s$. At $t=2s$, the proposed secondary control shown in Fig. 4.7 is enabled to regulate the PVUR at the PCC to 3%. In Fig. 4.21(a), after $t=2s$, the proposed secondary control effectively achieves this reference, by decreasing the PVUR references in the inverters. In Fig. 4.21(b) the RMS voltages at the PCC are shown before and after the activation of the proposed secondary control. Unbalanced voltage is decreased with the activation of the secondary control. Therefore, it can be concluded that the proposed control scheme for achieving a cooperative imbalance sharing can be easily extended to work together with a secondary control scheme as that shown in Fig. 4.7.

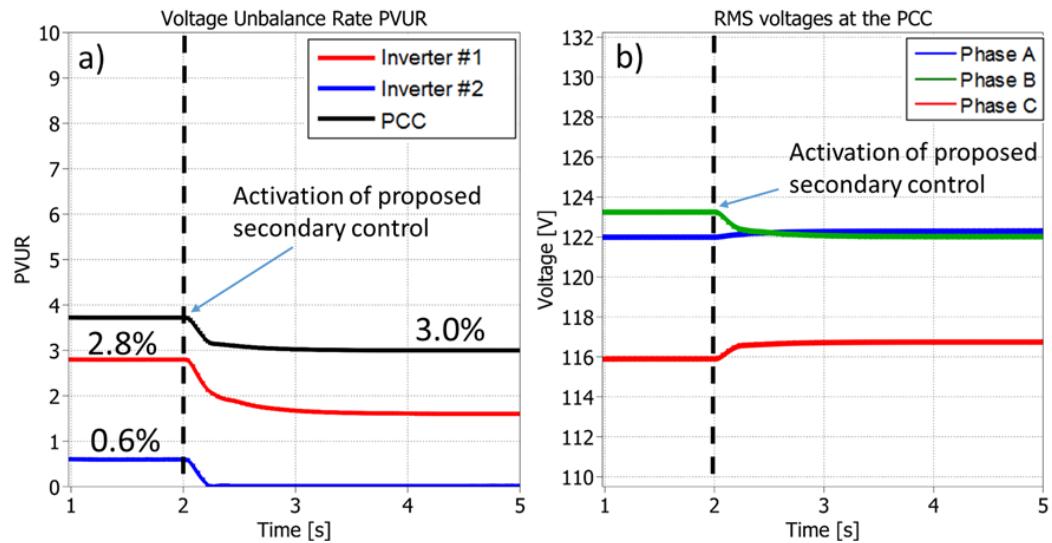


Fig. 4.21. Performance of the proposed secondary control scheme in the experimental test- Simulation results.

4.4.2.2 CASE 2: INVERTERS CONNECTED TO THE COMMON LOAD THROUGH EQUAL INDUCTIVE LINE IMPEDANCES

In this case, both inverters shown in Fig. 4.15 are connected to the common load with inductive line impedances equal to 2.5mH. Based on that, the unbalanced power is distributed between the inverters in the same proportion as shown in Fig. 4.22(a) before $t=67s$. Taking into account that for this case, it is assumed that inverter 1 has a power rating of 22kW and inverter 2 has 5kW of power rating, it would be desirable that inverter 1 injects almost all of the unbalanced power because it has a higher power rating than inverter 2. To achieve this, the following references for the voltage imbalance index ($PVUR_1^*$ and $PVUR_2^*$) are given to the inverters: 0.6% for inverter 1 and 3% for inverter 2. The first PVUR set point means that a sensitive load could be placed at the output of inverter 1 while the PVUR set point to inverter 2 corresponds to the maximum voltage imbalance permitted by the ANSI C84.1-2006 (3% of imbalance). It means that sensitive loads should not be placed at the output of inverter 2.

The proposed control scheme is enabled at $t=67s$ (referred to the timescale of the Matlab data logging of the experimental waveforms shown in Fig. 4.22 , Fig. 4.23 and Fig. 4.25) with a PVUR reference of 0.6% for inverter 1 and a PVUR reference of 3.0% for inverter 2. Fig. 4.23 shows how active and reactive powers are not affected by the activation of the proposed controller, confirming the expected decoupling feature, i.e. conventional $P - f$ and $Q - E$ droop controllers are not influenced by the imbalance sharing control. It means also that there is a decoupling between positive current sequence and both negative and zero current sequences.

Fig. 4.22(b) shows that the PVUR references are correctly achieved. It can also be noted that before the proposed control is initiated, the PVUR at the PCC is 2.2%, and

after that, this index increases to 3.5%. This increase is due to the specific values selected for $PVUR_1^*$ and $PVUR_2^*$.

Fig. 4.22(a) shows that the unbalanced powers in both inverters before $t=67s$ are equal because the load current is distributed between the inverters in equal proportion, as shown in Fig. 4.24(a) and Fig. 4.24(c). With the proposed control, the unbalanced power in the converters is distributed in a different proportion according to their different unbalanced output impedances: in Fig. 4.22(a) after $t=67s$, inverter 1 supplies a larger share of the unbalanced power. This can be also seen in Fig. 4.24(b) and Fig. 4.24(d), where the degree of imbalance in the current of inverter 1 is increased respect to the case shown in Fig. 4.24(a) [case where the proposed control scheme is disabled] and the imbalance in the output current in inverter 2 is decreased respect to the case shown in Fig. 4.24(c) [where the proposed control is not working]. It means that the proposed control algorithm achieves an imbalance load power/current sharing according to the inverter power rating.

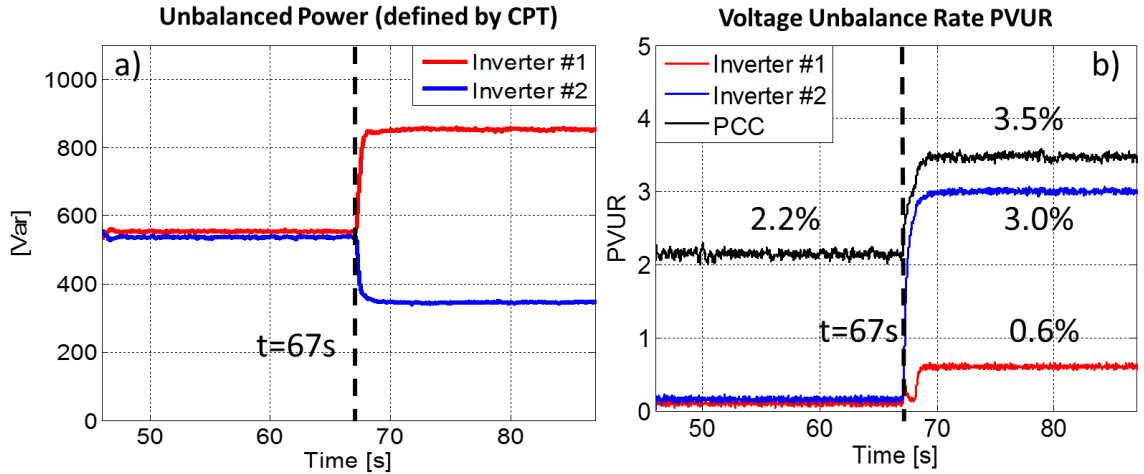


Fig. 4.22. (a) Unbalanced power for case 2, (b) PVUR in inverters and PCC– Matlab data logging of the experimental waveforms.

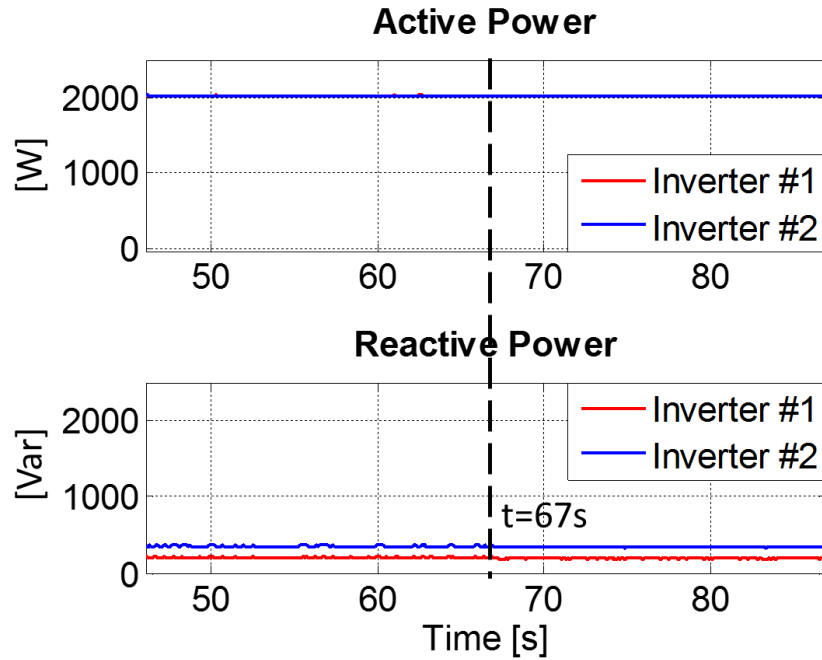


Fig. 4.23. Active and Reactive Powers in both inverters before and after the activation of the proposed control – Matlab data logging of the experimental waveforms.

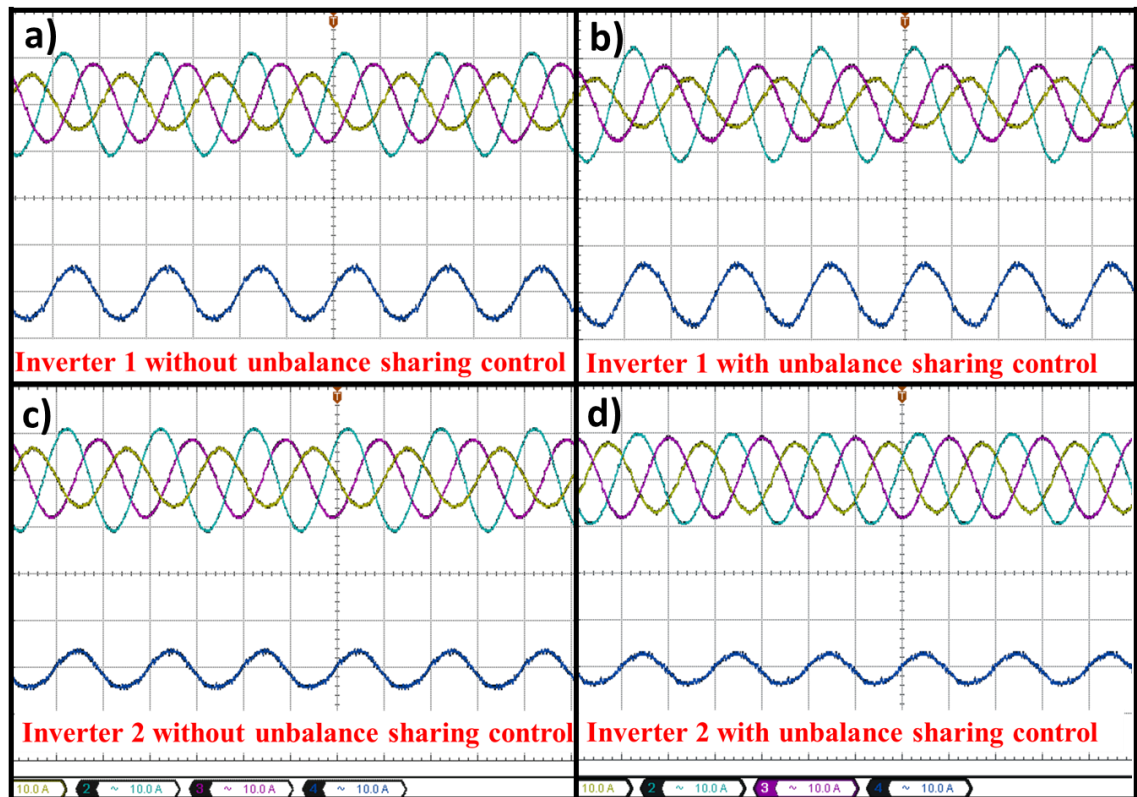


Fig. 4.24. (a), (c) - Output current in inverters 1 and 2 without the proposed control, (b), (d) - output current in inverters 1 and 2 with the proposed control activated. (10A/div)

Fig. 4.25 shows the effective RMS voltages at the output of the inverters (captured using the Triphase data acquisition system) before and after the activation of the proposed control scheme. In this figure, before $t=67s$, the imbalances at the output of the inverters are small, and thus the imbalances among the inverters are sharing at a proportion which depends on the inverters' line impedance [see Fig. 4.22(a)]. After $t=67s$ (see Fig. 4.25), when the proposed method is enabled, the imbalances at the output of the inverters are increased to achieve the PVUR references shown in Fig. 4.22(b) [0.6% for inverter 1 and 3% for inverter 2]. Therefore, the proposed control scheme shares the imbalances among the inverters cooperatively in a proportion which does not depend on the inverters' line impedance.

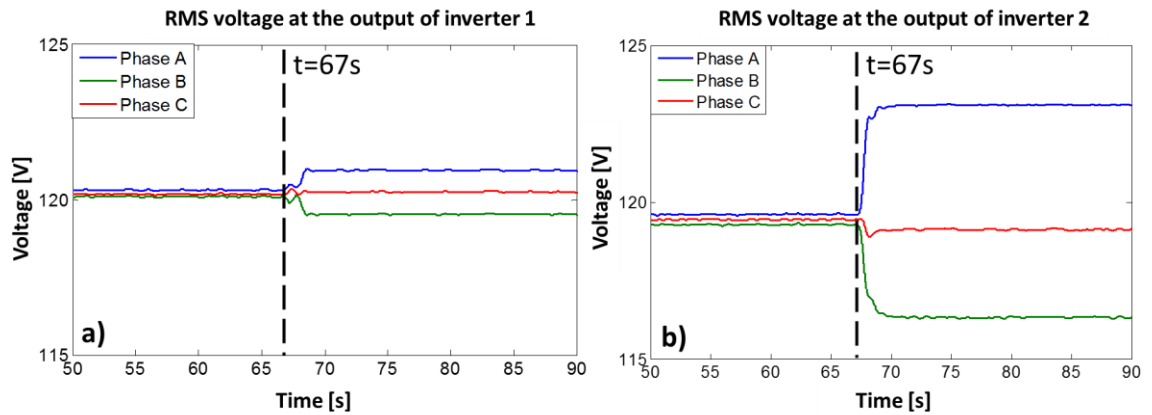


Fig. 4.25. (a) RMS voltages at the output of inverter 1, (b) RMS voltages at the output of inverter 2- Matlab data logging of the experimental waveforms-.

4.5 Extension to harmonic sharing

The proposed imbalance sharing control scheme can be used to share harmonics if the common load is non-linear. Therefore, the proposed control scheme shown in Fig. 4.3 can be implemented using the void current [see (12) in section 2.1], instead of the unbalanced current (see Fig. 4.3). Moreover, the unbalance index PVUR can be replaced by the Total Harmonic Distortion of the voltage at the inverter output. This THD is calculated in real-time by the microprocessor used to control the inverters shown in Fig. 4.15. The practical implementation of the proposed control algorithm for harmonic sharing is depicted in Fig. 4.26 and Fig. 4.27. As was stated above, to use the proposed control algorithm for sharing harmonics among the inverters it is necessary to use the void current instead of using the unbalanced current. Moreover, the PVUR index used for imbalance sharing must be replaced by the total harmonic distortion index (THD). It is worth noticed that in Fig. 4.26 and Fig. 4.27, inverter 1 is used as an example to show the implementation of the proposed control scheme for achieving harmonic sharing.

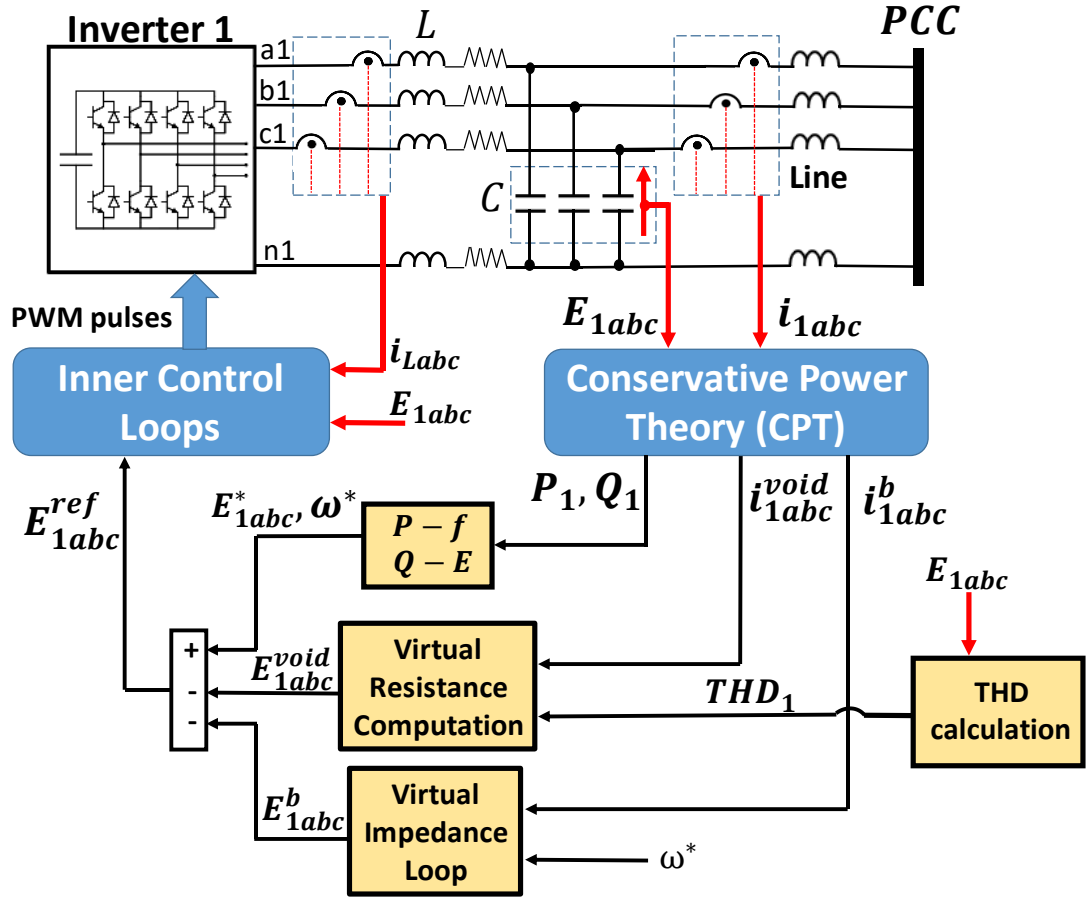


Fig. 4.26. Practical implementation of the proposed control scheme for harmonic sharing. (The inverter 1 was used as an example)

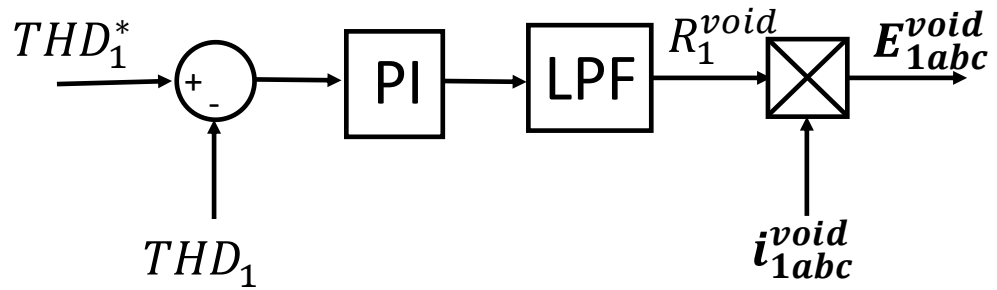


Fig. 4.27. Details of the virtual resistance computation block shown in Fig. 4.26. (Only inverter 1 is shown)

4.5.1.1 SIMULATIONS FOR HARMONIC SHARING

In 4-wire systems, third harmonic currents are an important concern since they will flow for the neutral wire, which can produce overloading of the neutral conductor, among other problems [18] [19]. For this reason, the performance of the proposed control algorithm applied for third harmonic current sharing is validated through simulation work. To do that, the experimental system described in the section 4.4.2 is simulated using Plexim PLECS software, and it is assumed that the 4-leg inverters are feeding a load which produces a third harmonic current component. Moreover, it is assumed that the inverters are connected to the load through equal line impedances of 0.8mH. In these conditions, the load current has third harmonic, and because the system is a 4-leg system, a third harmonic current also flows through the neutral wire. This can be appreciated in the load current used in the simulation work which is shown in Fig. 4.28 (in this work $\omega_e = 2\pi 50$). Fig. 4.29 shows the results of a Fourier analysis of the currents shown in Fig. 4.28. From these figures can be appreciated the load current has both fundamental and third harmonic currents. Moreover, only third harmonic current flows for the neutral conductor. [See Fig. 4.29(b)]

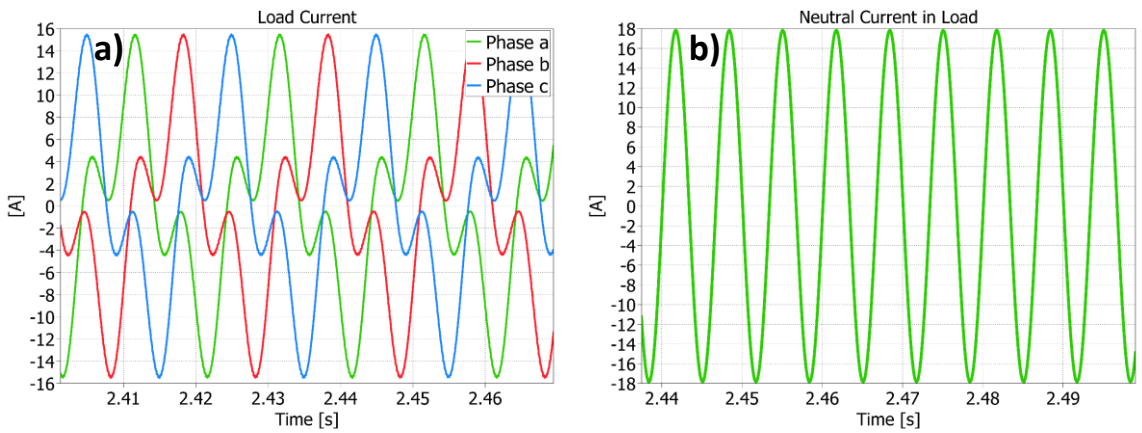


Fig. 4.28. (a) Phase currents in the load, (b) neutral current in the load–

Simulation results

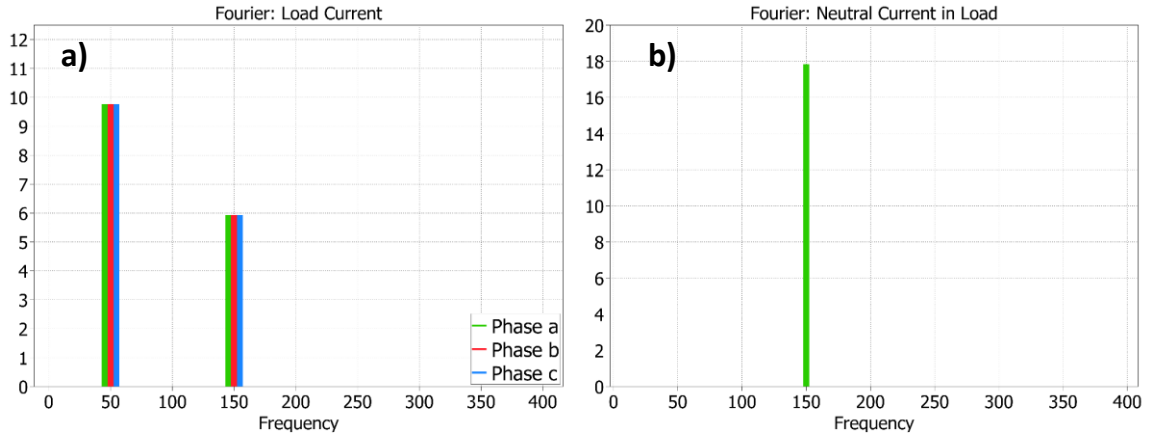


Fig. 4.29. (a) Fourier analysis of the phase currents in the load, (b) Fourier analysis of the neutral current on the load– Simulation results

The simulation work is divided into two steps: (i) step 1 ($0s \leq t < 3s$) where the proposed control system is not enabled, and (ii) step 2 ($3s \leq t < 6s$) where the proposed control scheme is activated to enable the third harmonic current sharing between the inverters. This control scheme is activated with the following voltage THD for the 4-leg inverters⁵: 5% for inverter 1 and 1.5% for inverter 2; both values agree with the IEEE Std. 519-1992 for voltage distortion [64]. Fig. 4.30 shows the void power (defined in section 2.1 contained in chapter 2) in the two steps studied. From this figure, it can be concluded that in step 1, both inverters have the same void power since they are connected to the load with the same line impedances (0.8mH). In step 2, when the proposed control scheme is enabled, the void power is sharing between the inverters in a different proportion compared to step 1. In fact, in step 2, inverter 2 inject more void power than inverter 1. Fig. 4.31 shows the Fourier analysis of the phase currents at the output of each inverter for the two steps studied. From this figure, it can be appreciated that in step 1, the

⁵ These THD set points were chosen based on simulation results

magnitude of the fundamental and the third harmonic currents in both inverters are equal. Conversely, in step 2, when the proposed controller is working, the third harmonic current is shared among inverters in a different proportion. In this case, inverter 2 inject more third harmonic current into the load than inverter 1. The same trend can also be appreciated in Fig. 4.32 where the third harmonic neutral currents at the output of the inverters are shown. From this figure, it can be concluded that in step 1, both neutral currents are equal. In contrast, in step 2, inverter 2 injects more neutral current than inverter 1. It should be pointed out that the fundamental current magnitude in both inverters is not altered when the proposed control scheme is working. Finally, from this simulation work, it can be seen the effectiveness of the proposed control scheme to manage third harmonic currents.

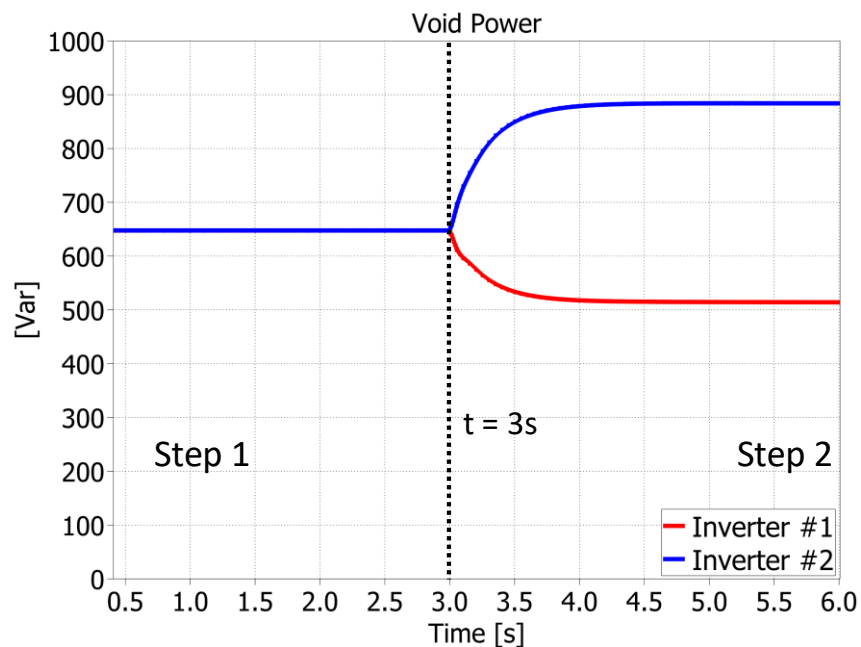


Fig. 4.30. Void Powers in both inverters– Simulation results

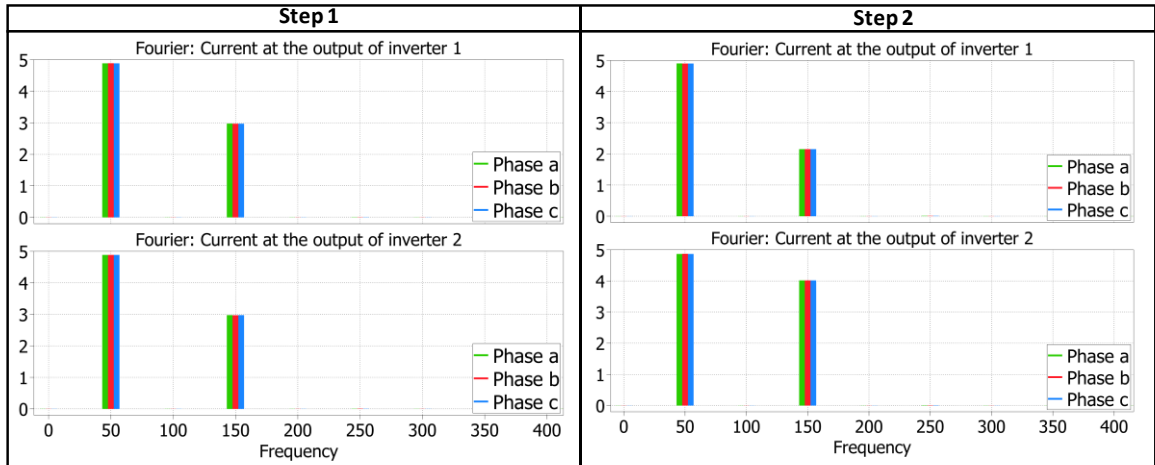


Fig. 4.31. Fourier analysis of the phase currents in both inverters (step 1), (b) Fourier analysis of the phase currents in both inverters (step 2) – Simulation results

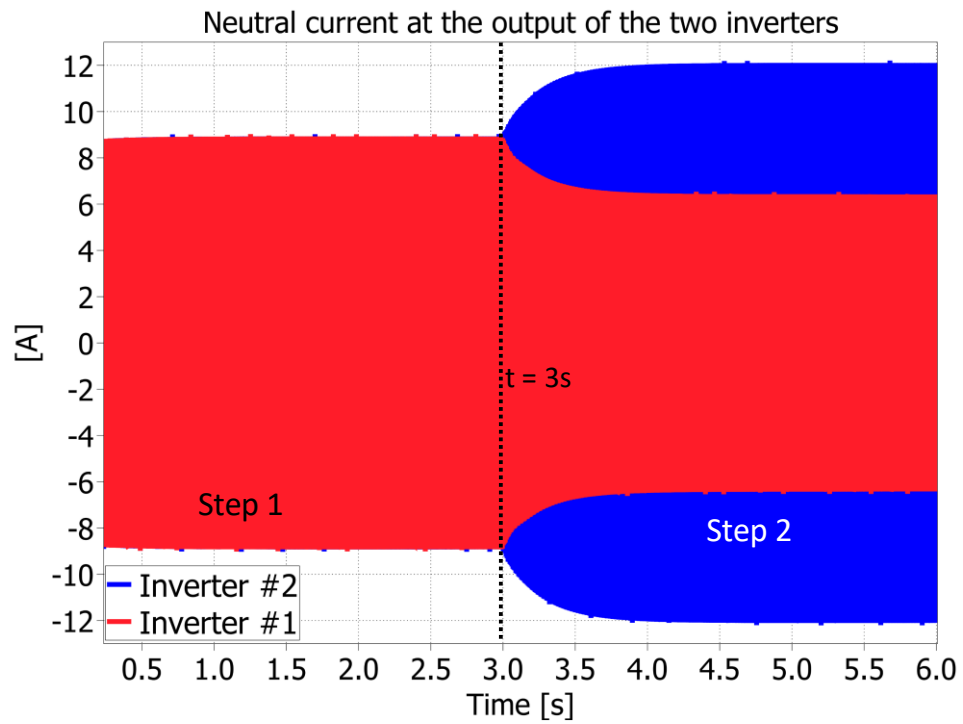


Fig. 4.32. Neutral current at the output of each inverter in the two steps analysed– Simulation results

4.5.1.2 EXPERIMENTAL VALIDATION FOR HARMONIC SHARING

To validate the control system proposed for harmonic sharing, the line impedances from the experimental test discussed in the section 4.4.2.1 are maintained⁶, and a programmable load is connected at the PCC of the experimental system. This load is controlled to operate as a 3 ϕ nonlinear load without neutral, absorbing the distorted currents shown in Fig. 4.33. Results are depicted only for the 5th harmonic component of the current because this is the main contributor to the THD for the current shown in Fig. 4.33. In Fig. 4.34, void powers (section 2.1) – including the effect of all the harmonics – and 5th harmonic currents are shown before and after the activation of the proposed control at t=83s. Before enabling the control system, noting the inverters have different line impedances with line impedance of inverter 1 being smaller than that of inverter 2, the void power and the fifth harmonic current supplied by inverter 1 to the load is higher than inverter 2. After activating the sharing control with an output voltage THD set point of 3.0% for inverter 1 and a THD of 2.8% for the output voltage in inverter 2 (both set points were calculated based on simulation results and with the aim of changing the pattern in which the void power is shared between the converters), the sharing profile is changed and now, inverter 2 injects most of the harmonic current content of the load. Fig. 4.34(c) shows that these THD set points are achieved by the local controllers of the inverters. To analyse the THD of the 5th harmonic of the current in both inverters before and after the activation of the proposed control, a Hioki 3196 Power Analyser has been used. Before t=83s, the 5th harmonic current THD is 14.67% in inverter 1 and 7.45% in

⁶ Inverter 1 is connected to the unbalanced load using an inductive line with L=1.25 mH and inverter 2 with a line inductance of 2.5mH

inverter 2. After that time, this index is 7.74% in inverter 1 and 15.43% in inverter 2. This result shows that the 5th harmonic power and current can be shared effectively with the proposed control scheme. These results match with the results shown in Fig. 4.34(b). It is worth noting that the objective here is not to reduce the THD of the voltage at the PCC but to modify how the harmonic components of the load current are shared between the two inverters. This is achieved by controlling the THD in the voltage at the output of the inverters.

If the nonlinear load has neutral, third harmonic current will flow through the neutral wire. In this case, the implementation of the proposed control scheme discussed in this section is the same, but now the 5th harmonic is replaced by the 3rd harmonic. The proposed control scheme achieves a sharing of the 3rd harmonic between the phases of the inverters and also between the neutrals of them. It is worth remembering that the void current defined by the CPT [see (12) in section 2.1] contains the harmonics present in the current [43] [66] [68] and therefore a specific harmonic can be selected from this current by using a suitable filter and based on it, the proposed control scheme depicted in Fig. 4.26 and Fig. 4.27 can be implemented.

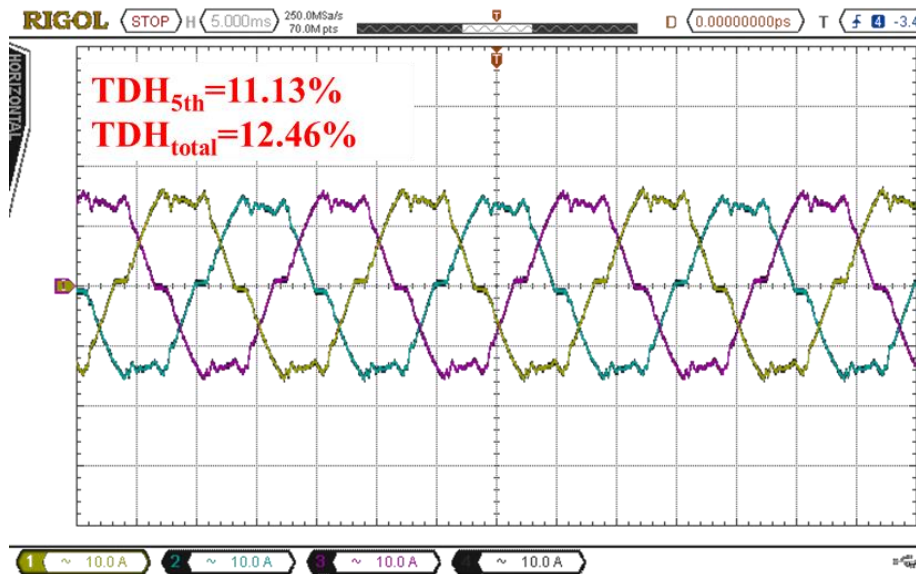


Fig. 4.33. Characteristics of the experimental non-linear load.

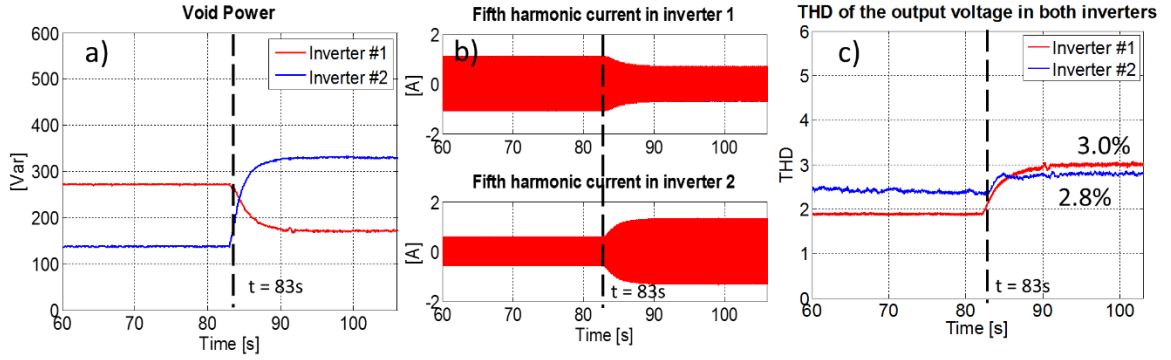


Fig. 4.34. (a) Void powers in both inverters, b) 5th harmonic current at the output of the inverters before and after the activation of the proposed control, and c) THD of the voltage at the output of the inverters – Matlab data logging of the experimental waveforms.

4.6 Comparison between the proposed control scheme and previous methods reported

As was discussed in the introduction of this chapter, the proposed control scheme for sharing imbalances in 4-wire microgrids is based on the Conservative Power Theory (CPT). Using this theory, it is possible to perform a simple identification, in the abc reference frame, of the balanced, unbalanced and non-linear components of the currents and powers. The main differences between the proposed control algorithm and those reported in the literature can be summarised as follows:

- Contrary to the previous control algorithms for managing imbalances in MGs, the proposed controller does not require the use of an algorithm for separation of sequence components. It produces a more robust imbalance sharing algorithm, particularly because most of the sequence separation algorithms are strongly affected by, noise, harmonic distortion, small variations in the sampling time, etc.
- [45] [46]

- The CPT can be used to identify distorted components of current and power, and therefore it can be used for extending the proposed imbalance sharing controller to achieve the sharing of harmonics among the converters. In this thesis, the extension for harmonic sharing was discussed and experimentally validated.
- To the best of the author's knowledge, there is only one paper where a control scheme for cooperative imbalance sharing in 4-wire MGs is discussed in [44]. The other control schemes are proposed for 3-wire MGs. Therefore, the proposed controller is a contribution to the study of issues related to zero sequence components in MGs. Contrary to [44], which is based on sequence separation, the proposed controller proposes a novel methodology (based on the CPT) to address these issues.

To explain in the best way the differences between the proposed controller and the previous ones reported in the literature, in this section, a comparison between the proposed algorithm and that reported in [44] is performed. Fig. 4.35 shows the implementation of the proposed control scheme, while Fig. 4.36 shows the implementation of the controller proposed in [44]. From these figures, and as was discussed at the beginning of this subsection, it can be appreciated that the proposed controller uses the CPT to obtain directly the magnitudes necessities to perform the control of imbalances (see Fig. 4.35). On the other hand, the controller proposed in [44] requires a previous step, to perform the identification of the sequence components of the system (see Fig. 4.36).

Another difference between the proposed controller and that reported in [44] is the fact that the inner control loops in the proposed controller, can be directly performed in the abc reference frame (see Fig. 4.35). However, the inner control loops for the controller

reported in [44], are implemented in the dq reference frame, and therefore the Park transform must be used.

Finally, from Fig. 4.35 and Fig. 4.36, it can be appreciated the virtual resistance (R_1^u) for each power converter is dynamically adjusted by the control loop depicted in Fig. 4.35. This control loop ensures that the imbalances in the voltage at the output of each converter does not exceed regulatory limits. In contrast, from Fig. 4.36, it can be concluded that the virtual resistances R_1^- and R_1^0 are not dynamically adjusted, and they are set to be constant. Based on that, the unbalanced voltage at the output of the power converters are not regulated, and it can exceed regulatory limits in some operational conditions (level of imbalances at the load). The latter does not happen with the proposed controller since the imbalances at the output of the converters are always regulated.

It is worth remembering that in the planning of a 4-wire microgrid, the fact that not all the loads have the same sensitivity to the voltage imbalance should be considered. Therefore, buses, where sensitive loads may be located, have to be subjected to more stringent voltage requirements than those buses where no sensitive loads are connected. Based on that, at solving the proposed optimisation problem (69), the PVUR set points for each power converter are calculated, and therefore an optimal imbalance sharing is achieving. (See section 4.3.3)

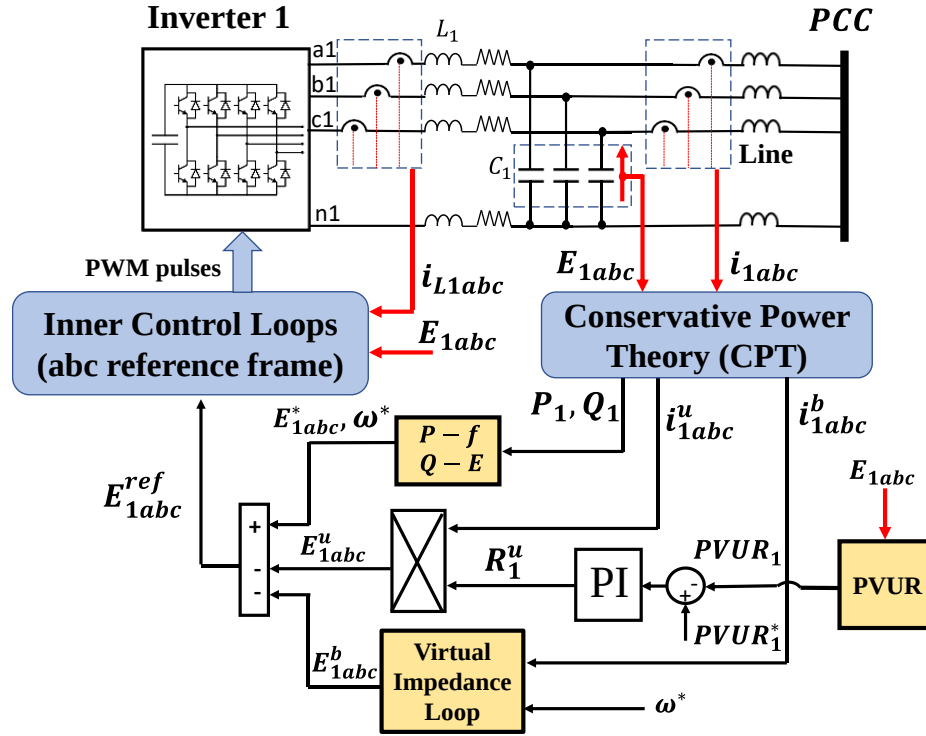


Fig. 4.35. Proposed control algorithm for sharing imbalances in 4-wire MGs

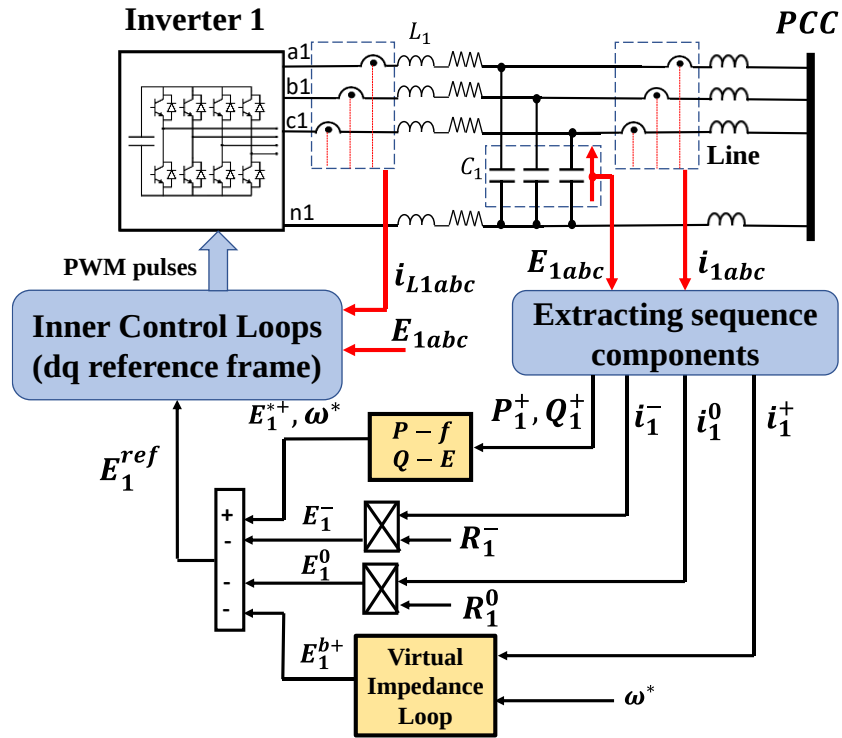


Fig. 4.36. Control scheme for sharing imbalances in 4-wire MGs proposed in [44]

The proposed control scheme shown in Fig. 4.35 and that reported in [44] (see Fig. 4.36) were simulated using PLECS software to perform a comparison between them. Both control schemes were simulated using the parameters of the MG⁷ of case 1 discussed in the section experimental results (see section 4.4.2.1).

As was discussed earlier in this section, the proposed control scheme (see Fig. 4.35) requires PVUR set points in order to calculate proper virtual resistances (R_1^u, R_2^u) to achieve imbalance sharing. These set points were calculated using the methodology discussed in section 4.3.3. Based on this methodology, the set points for each converter were established as follow $PVUR_1^* = 2.4\%$ and $PVUR_2^* = 0.6\%$. On the other hand, to implement the method reported in [44] (see Fig. 4.36), the delay signal cancellation method [45] [46] is used for extracting the sequence components. Moreover, since in [44] a procedure to calculate virtual resistances (see R_1^-, R_1^0) is not given, in this comparison, similar values to those used in [44] are used, they are $R_1^- = R_1^0 = R_2^- = R_2^0 = 1.5\Omega$.

In the simulation work, three steps were studied: step 1 ($0s \leq t < 5s$), where the imbalance sharing algorithms are disabled, step 2 ($5s \leq t < 10s$), where the imbalance sharing algorithms are enabled at $t=5s$, and step 3 ($10s \leq t < 15s$) where an additional unbalanced load is connected to the PCC at $t=10s$ (increasing the level of current imbalance in the system). The performance of both imbalance sharing algorithms depicted in Fig. 4.35 and Fig. 4.36 is studied in these three steps, and a comparison between them is performed.

⁷ Two power converters are feeding a single common load. Converter 1 is connected to the unbalanced load using an inductive line with $L=1.25$ mH and Converter 2 with a line inductance of 2.5 mH

Fig. 4.37(a) shows the unbalanced power of the MG when the converters are controlled with the proposed control system [see Fig. 4.35]. Fig. 4.37(b) shows the unbalanced power of the MG when the converters are controlled using the control scheme reported in [44] (see Fig. 4.36). From step 2 shown Fig. 4.37 (when both imbalance sharing schemes are enabled), it can be concluded that the proposed control scheme achieves better imbalance sharing than that reported in [44]. It is because when the proposed control scheme is used, the virtual resistances are optimally calculated (according to the methodology discussed in section 4.3.3) to achieve an optimum imbalance sharing. This does not happen with the method reported in [44], where a methodology for determining these virtual resistances is not provided.

Fig. 4.38(a) shows the PVUR index of the voltage at the output of the converters (in the three steps studied) when the converters are implemented with the proposed control scheme (see Fig. 4.35). Fig. 4.38(b) shows the PVUR index of the voltage at the output of the converters (in the three steps studied) when the converters are implemented with the control scheme depicted in Fig. 4.36. From step 2 shown in Fig. 4.38, it can be concluded that the proposed controller achieves imbalance sharing, inducing lower PVUR values at the output of the converters than those produced by the method reported in [44]. Analysing step 3 in the same figure, it can be appreciated that the PVURs in the converters, when they are controlled using the method reported in [44], increase noticeably as shown in Fig. 4.38(b) [step 3]. The latter may produce a voltage-quality issue. This phenomenon does not happen when the converters are controlled with the proposed control scheme as shown in step 3 of Fig. 4.38(a).

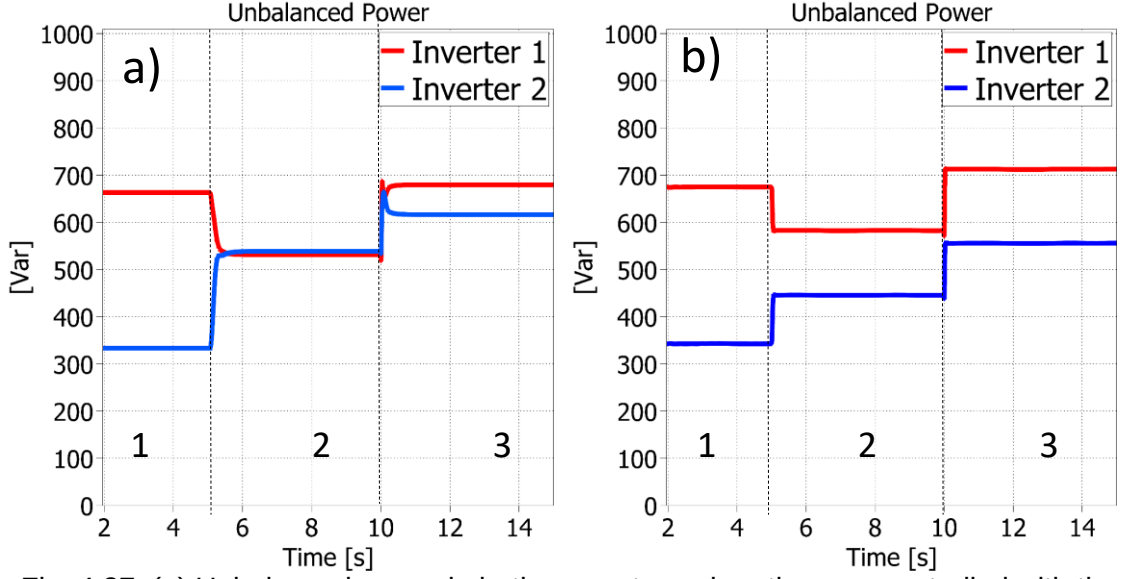


Fig. 4.37. (a) Unbalanced power in both converters when they are controlled with the proposed control scheme, (b) Unbalanced power in both converters when they are controlled with the controller reported in [44]

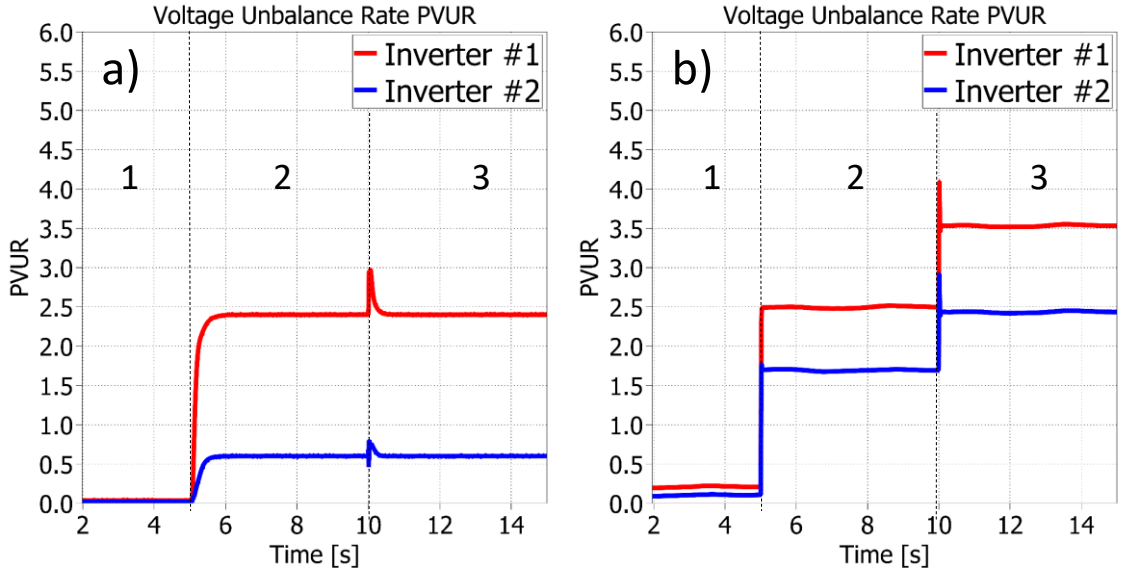


Fig. 4.38. (a) PVUR of the voltage at the output of the converters when they are controller using with the proposed control scheme, (b) PVUR of the voltage at the output of the converters when they are controlled with the controller reported in [44]

4.7 Chapter Conclusions

In this chapter, a new control algorithm for sharing load imbalance between inverters in a droop controlled MG has been presented. The proposed control scheme is based on the conservative power theory (CPT) to achieve the sharing objective in the abc reference frame, decoupled from the other control loops. The operating principle is based on decomposing the inverter current into balanced and unbalanced components according to the CPT, and on the concept of unbalanced virtual output impedance, implemented in the inverters' control loops. These unbalanced impedances are only seen by the unbalanced components of the inverter currents, and the impedance can be actively controlled to modify the sharing of the unbalanced components of the load current between the two inverters. Experimental results obtained from a laboratory scale microgrid confirmed the effectiveness of the sharing strategy, as well as the absence of any interference with the droop control. The extension of the proposed control scheme for sharing harmonics is discussed, and both numerical and experimental validation of this extension is provided.

The main differences of the proposed imbalance sharing algorithm with the previous ones reported in the literature can be summarized as: (i) The proposed scheme does not require sequence separation algorithms, (ii) a new method to define the preliminary unbalanced voltage set points is proposed, and (iii) contrary with the only paper where a control scheme for cooperative imbalance sharing in 4-wire MGs [44], the proposed control scheme can be easily extended to harmonics due to it does not require sequence separation algorithms.

Chapter 5. Proposed Small Signal Model to Study the Effects of the PLL on the System Stability

5.1 Introduction

In this chapter is proposed an improved design process for the commonly used dq-PLL that accounts for the potential stability issues arising in weak grids. The proposed design method is based on a new small signal model of a 3-leg converter connected to a balanced three-phase three-wire weak grid and synchronised with a dq-PLL. Based on the model, an exhaustive analysis of the system stability as a function of the PLL bandwidth and the Short Circuit Ratio (SCR) of the grid is performed. With the proposed approach it is possible to optimise the PLL design to find the fastest PLL that can operate stably with the SCR of the grid. In addition, the proposed model is particularly simple, resulting in a low-complexity design tool that could also be used for online stability monitoring. The theoretical background of the proposed dq-PLL model and design process is first presented. The method is then validated through a combination of Plexim PLECS simulations and experimental results from a 5kW laboratory-scale system. Experimental evidence confirms the validity of the model and the effectiveness of the design method.

As was stated in the previous paragraph, the system studied in this chapter corresponds to a 3-leg converter connected to a balanced three-phase three-wire weak grid. Based on that, the proposed PLL design process can only be used in this type of system (because the positive sequence component only is considered in the modelling

process). However, the proposed PLL design process could be extended to consider unbalanced and/or the presence of harmonic in weak grids. Based on that, the work developed in this chapter aims to consider the effects of the PLL bandwidth versus the short circuit ratio for a simple application to establish a methodology to analyse the stability of the proposed control algorithms discussed in chapters 3 and 4.

The rest of this chapter is organised as follows: in section 5.2 the typical dq-PLL design process is discussed alongside the objectives of the improved design. Section 5.3 introduces the proposed low-complexity small signal model. Section 5.4 validates the proposed model stability prognosis against simulation results. Section 5.5 reports the experimental validation. Finally, in section 5.6 the main conclusions about the proposed PLL design scheme shown in this chapter are presented.

5.2 Enhanced PLL Design Process

5.2.1 Proposed PLL Design for Weak Grids

The widely adopted method for the design of a dq-PLL is the simple linearised model shown in Fig. 5.1. The closed-loop transfer function can be calculated as in (71), where the parameters k_{pPLL} and k_{iPLL} are proportional and integral gains of the PI controller and E_m is the magnitude of the PCC voltage. Equation (71) is a second order transfer function characterized by a natural frequency ω_{nat} and a damping ratio ζ . Comparing (71) with a generic second order system, equations (72) and (73) are obtained. To tune k_{pPLL} and k_{iPLL} , the transient response of the phase-locked loop is set through the design parameters ω_{nat} and ζ . Since this design method uses a simple PLL model, the gains k_{pPLL} and k_{iPLL} obtained with it will always result in the system shown in (71) being stable.

It is worth remembering that the choice of ω_{nat} and ζ will determine the dynamics of the PLL. From a general perspective, PLLs with low bandwidth are required when the grid voltage is unbalanced/distorted (for cancelling ripple). In contrast, when the three-phase system is clean and balanced, three-phase PLLs can be implemented with a high bandwidth, which results in a very fast transient response⁸ [107] [108]. In this thesis, the latter case is studied to develop a methodology for understanding the effects of the PLL bandwidth in a simple system. This methodology, as future work, can be modified to look at the CPT-based controllers proposed in chapter 3 and 4, and considering unbalanced and distorted conditions.

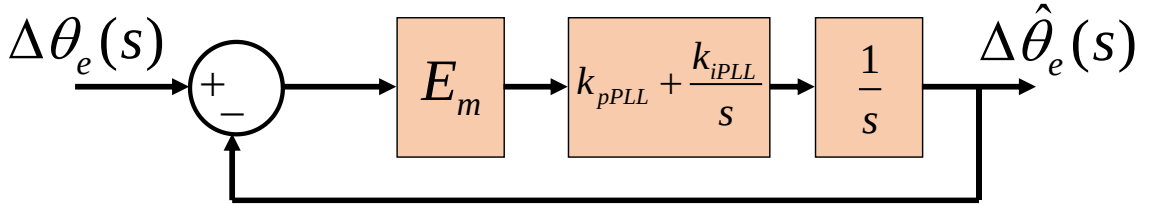


Fig. 5.1. Basic small signal model of a dq-PLL.

$$\frac{\Delta\hat{\theta}_e(s)}{\Delta\theta_e(s)} = \frac{E_m k_{pPLL} s + E_m k_{iPLL}}{s^2 + E_m k_{pPLL} s + E_m k_{iPLL}} \quad (71)$$

$$\omega_{nat} = \sqrt{E_m k_{iPLL}} \quad (72)$$

$$\zeta = \frac{k_{pPLL}}{2} \sqrt{\frac{E_m}{k_{iPLL}}} \quad (73)$$

The PLL design process described above assumes an ideal grid with negligible impedance. However, this is no longer the case when the PLL is used in a weak grid with non-negligible impedance. For this reason, in this investigation, an enhanced two-stage PLL design process is proposed, whose basic flow-chart is shown in Fig. 5.2. The first

⁸ Providing fast and clean phase-angle estimation

stage is the ideal PLL design discussed above, whose outputs are the proportional and the integral gains calculated using (72) and (73). In addition, at the output of the design block, the PLL bandwidth (BW) associated with k_{pPLL} and k_{iPLL} is calculated to characterize the PLL performance in terms of its transient response.

The second stage is the stability analysis block that determines whether the PLL designed in the previous block will be stable or unstable when it is used in a weak grid with known parameters. The stability analysis block is based on the simple state space model of the system in the dq reference frame proposed in this paper and discussed in section 5.3. Both the system studied in this work, and the proposed small signal model will be discussed in the following sections. It is worth noting that the PLL design process shown in Fig. 5.2 is flexible and can be used with any other PLL design procedure.

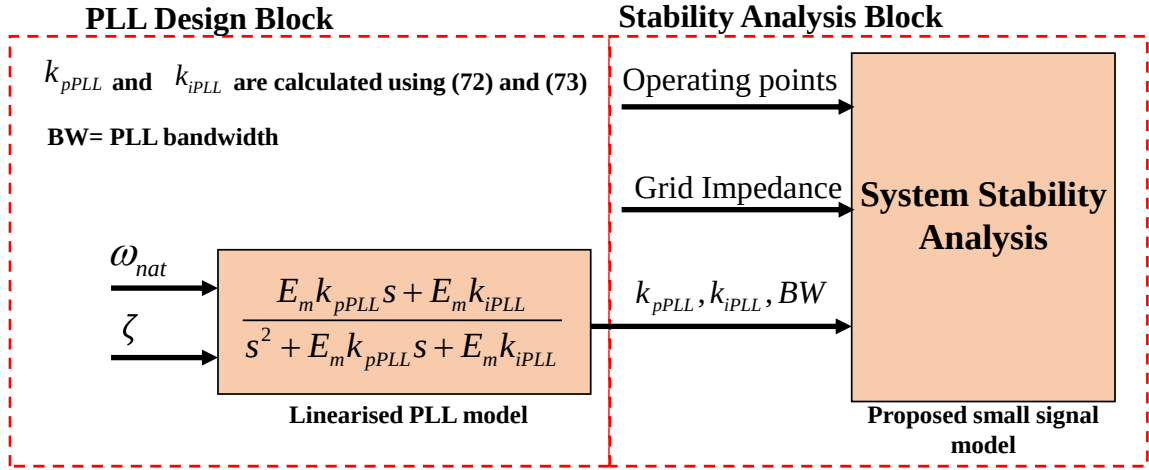
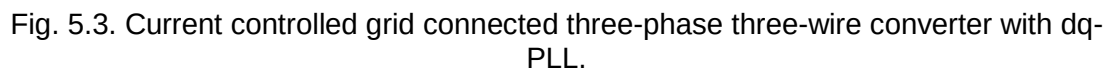


Fig. 5.2. Flow-chart of the proposed PLL design scheme to be used in weak grids.

5.2.2 System Under Study

Fig. 5.3 shows the diagram of the three-phase three-wire balanced system considered for this work. This topology is commonly used in power systems based on distributed generation (DG) units, e.g. photovoltaic. In some cases, renewable sources are installed

In all these cases, the AC grid seen by the converter is weak. It is therefore important to study the effects of the PLL on the system stability to prevent the inverter from tripping. In this work, it is assumed that the inverter shown in Fig. 5.3 is connected to a balanced three-phase three-wire weak grid. In this type of system, it is desirable to have a PLL with high bandwidth to ensure a fast-transient response. However, the weaknesses of the grid impose a trade-off between the maximum PLL bandwidth and the stability of the system.



It is worth recalling that a weak grid can be defined by the Short Circuit Ratio (SCR) [28] [29] shown in (74), where V_g is the line to line RMS voltage at the PCC, Z_g is the grid impedance, and P_{rated} is the nominal power of the inverter. Based on the SCR, a weak grid is conventionally identified by a short circuit ratio in the range $2 \leq SCR \leq 3$ [28] [29] and a very weak grid is when $SCR < 2$. Some examples of SCR values in wind power plants connected to weak grids in Australia are discussed in [109]: (i) Musselroe ($SCR=1.74$), (ii) Collgar ($SCR=2.65$), and Silverton ($SCR=1.24$).

$$SCR = \frac{V_g^2}{Z_g \cdot P_{rated}} \quad (74)$$

5.3 Linearised State-Space Model

In this section, the proposed low-complexity small signal model of the system described in Fig. 5.3 is developed. To achieve this, two dq reference frames are defined. The first one is named the Converter Reference Frame (CRF, $d^c - q^c$) and uses the angle estimated by the PLL from the measured capacitor voltages E_{1abc} in Fig. 5.3. The second one, is the Actual Reference Frame (ARF, $d^a - q^a$), aligned with the actual angle of the capacitor voltages E_{1abc} . At steady state, the CRF is aligned with the ARF. When small signal perturbations are added to the capacitor voltages (E_{1abc}), CRF and ARF are no longer aligned because of the PLL dynamics. In this case, the phase relationship between the frames is given by (75). In this equation, X^c corresponds to the dq complex variable in the CRF, X^a is the dq complex variable in the ARF, and $\Delta\theta$ is the difference between the angle of the ARF and the one of the CRF (Fig. 5.3). As shown later in the

analysis, this angle allows the easy inclusion of the PLL dynamics in the system modelling.

$$X^c = e^{-j \cdot \Delta \theta} \cdot X^a \quad (75)$$

5.3.1 Converter Model in the CRF

5.3.1.1 CURRENT CONTROL LOOP

As shown in Fig. 5.3, the current control is a conventional dq control with PI regulators and feed-forward terms. The corresponding state equations are given in (76)-(78).

$$\dot{\gamma}_{1d}^c = I_{1d}^{*c} - I_{1d}^c, \quad \dot{\gamma}_{1q}^c = I_{1q}^{*c} - I_{1q}^c \quad (76)$$

$$V_{1d}^{*c} = -\omega_{PLL} L_1 I_{1q}^c + k_{p1} \dot{\gamma}_{1d}^c + k_{i1} \int \dot{\gamma}_{1d}^c \quad (77)$$

$$V_{1q}^{*c} = \omega_{PLL} L_1 I_{1d}^c + k_{p1} \dot{\gamma}_{1q}^c + k_{i1} \int \dot{\gamma}_{1q}^c \quad (78)$$

Using (76) in (77) and (78), the latter two equations can be rewritten as shown in (79) and (80) respectively.

$$V_{1d}^{*c} = -\omega_{PLL} L_1 I_{1q}^c + k_{p1} (I_{1d}^{*c} - I_{1d}^c) + k_{i1} \gamma_{1d}^c \quad (79)$$

$$V_{1q}^{*c} = \omega_{PLL} L_1 I_{1d}^c + k_{p1} (I_{1q}^{*c} - I_{1q}^c) + k_{i1} \gamma_{1q}^c \quad (80)$$

Finally, using (76), (79) and (80), the matrix representation of the current control loop of the converter depicted in Fig. 5.3, in the time domain, is calculated. This representation is given by (81) and (82). [In the CRF]

$$\begin{pmatrix} \dot{\gamma}_{1d}^c \\ \dot{\gamma}_{1q}^c \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \gamma_{1d}^c \\ \gamma_{1q}^c \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} I_{1d}^{*c} \\ I_{1q}^{*c} \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} I_{1d}^c \\ I_{1q}^c \end{pmatrix} \quad (81)$$

$$\begin{pmatrix} V_{1d}^{*c} \\ V_{1q}^{*c} \end{pmatrix} = \begin{pmatrix} k_{i1} & 0 \\ 0 & k_{i1} \end{pmatrix} \begin{pmatrix} \gamma_{1d}^c \\ \gamma_{1q}^c \end{pmatrix} + \begin{pmatrix} k_{p1} & 0 \\ 0 & k_{p1} \end{pmatrix} \begin{pmatrix} I_{1d}^{*c} \\ I_{1q}^{*c} \end{pmatrix} + \begin{pmatrix} -k_{p1} & -\omega_{PLL} L_1 \\ \omega_{PLL} L_1 & -k_{p1} \end{pmatrix} \begin{pmatrix} I_{1d}^c \\ I_{1q}^c \end{pmatrix} \quad (82)$$

Equations (83)-(84) represent the linearised state-space form of equations (81)-(87). I_{1qo}^c and I_{1do}^c are the operating points associated to the inverter output current (I_{1abc}) in the “c” reference frame (see Fig. 5.3); ω_n is the nominal grid frequency and $\Delta\omega_{PLL}$ is a small signal frequency perturbation caused by the PLL algorithm. (More information about the linearisation process is presented in Appendix C)

$$\begin{pmatrix} \Delta \dot{\gamma}_{1d}^c \\ \Delta \dot{\gamma}_{1q}^c \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta \gamma_{1d}^c \\ \Delta \gamma_{1q}^c \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^{*c} \\ \Delta I_{1q}^{*c} \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^c \\ \Delta I_{1q}^c \end{pmatrix} \quad (83)$$

$$\begin{pmatrix} \Delta V_{1d}^{*c} \\ \Delta V_{1q}^{*c} \end{pmatrix} = \begin{pmatrix} k_{i1} & 0 \\ 0 & k_{i1} \end{pmatrix} \begin{pmatrix} \Delta \gamma_{1d}^c \\ \Delta \gamma_{1q}^c \end{pmatrix} + \begin{pmatrix} k_{p1} & 0 \\ 0 & k_{p1} \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^{*c} \\ \Delta I_{1q}^{*c} \end{pmatrix} + \begin{pmatrix} -k_{p1} & -\omega_n L_1 \\ \omega_n L_1 & -k_{p1} \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^c \\ \Delta I_{1q}^c \end{pmatrix} + \begin{pmatrix} -I_{1qo}^c L_1 \\ I_{1do}^c L_1 \end{pmatrix} \Delta\omega_{PLL} \quad (84)$$

5.3.1.2 LC FILTER MODEL

From Fig. 5.3, applying Kirchhoff's Voltage Law (KVL) to the converter side, yields (85) [in the $\alpha\beta$ reference frame]. In this equation $\vec{V}_1 = V_{1\alpha} + jV_{1\beta}$ (voltage at the inverter output), $\vec{I}_1 = I_{1\alpha} + jI_{1\beta}$ (current at the inverter output) and $\vec{E}_1 = E_{1\alpha} + jE_{1\beta}$ (voltage across the capacitor), and j represents the complex operator. Equation (87) shows the linearised small signal state-space form of (86).

$$\vec{V}_1 = R_1 \vec{I}_1 + L_1 \frac{d\vec{I}_1}{dt} + \vec{E}_1 \quad (85)$$

Transforming (85) to the converter dq reference frame (see Appendix D), and rearranging it, yields (91). Finally, Equation (87) shows the linearised small signal state-space form of (86) (see Appendix C):

$$\begin{pmatrix} \dot{I}_{1d}^c \\ \dot{I}_{1q}^c \end{pmatrix} = \begin{pmatrix} -\frac{R_1}{L_1} & \omega_{PLL} \\ -\omega_{PLL} & -\frac{R_1}{L_1} \end{pmatrix} \begin{pmatrix} I_{1d}^c \\ I_{1q}^c \end{pmatrix} + \begin{pmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_1} \end{pmatrix} \begin{pmatrix} V_{1d}^c \\ V_{1q}^c \end{pmatrix} + \begin{pmatrix} -\frac{1}{L_1} & 0 \\ 0 & -\frac{1}{L_1} \end{pmatrix} \begin{pmatrix} E_{1d}^c \\ E_{1q}^c \end{pmatrix} \quad (86)$$

$$\begin{pmatrix} \Delta \dot{I}_{1d}^c \\ \Delta \dot{I}_{1q}^c \end{pmatrix} = \begin{pmatrix} -\frac{R_1}{L_1} & \omega_n \\ -\omega_n & -\frac{R_1}{L_1} \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^c \\ \Delta I_{1q}^c \end{pmatrix} + \begin{pmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_1} \end{pmatrix} \begin{pmatrix} \Delta V_{1d}^c \\ \Delta V_{1q}^c \end{pmatrix} + \begin{pmatrix} -\frac{1}{L_1} & 0 \\ 0 & -\frac{1}{L_1} \end{pmatrix} \begin{pmatrix} \Delta E_{1d}^c \\ \Delta E_{1q}^c \end{pmatrix} + \begin{pmatrix} I_{1qo}^c \\ -I_{1do}^c \end{pmatrix} \Delta \omega_{PLL} \quad (87)$$

Assuming that $\Delta V_{1d}^{*c} \approx \Delta V_{1d}^c$ and $\Delta V_{1q}^{*c} \approx \Delta V_{1q}^c$, i.e. the voltage references given by (77) and (78) are effectively produced by the converter, and evaluating (84) in (87), yields (88):

$$\begin{pmatrix} \Delta \dot{I}_{1d}^c \\ \Delta \dot{I}_{1q}^c \end{pmatrix} = \begin{pmatrix} -\left(\frac{R_1}{L_1} + \frac{k_{p1}}{L_1}\right) & 0 \\ 0 & -\left(\frac{R_1}{L_1} + \frac{k_{p1}}{L_1}\right) \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^c \\ \Delta I_{1q}^c \end{pmatrix} + \begin{pmatrix} \frac{k_{p1}}{L_1} & 0 \\ 0 & \frac{k_{p1}}{L_1} \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^{*c} \\ \Delta I_{1q}^{*c} \end{pmatrix} + \begin{pmatrix} \frac{k_{i1}}{L_1} & 0 \\ 0 & \frac{k_{i1}}{L_1} \end{pmatrix} \begin{pmatrix} \Delta \gamma_{1d}^c \\ \Delta \gamma_{1q}^c \end{pmatrix} + \begin{pmatrix} -\frac{1}{L_1} & 0 \\ 0 & -\frac{1}{L_1} \end{pmatrix} \begin{pmatrix} \Delta E_{1d}^c \\ \Delta E_{1q}^c \end{pmatrix} \quad (88)$$

5.3.1.3 PLL AND CONVERTER MODELS

Considering now the PLL and assuming that an instantaneous frequency deviation $\Delta \omega_{PLL}$ exists at the output of the PI controller of the PLL [73] [83], (89) can be written (in the time-domain). From this equation, a small perturbation in ΔE_{1q}^c will produce a small perturbation $\Delta \omega_{PLL}$ at the PLL output. This perturbation will generate a small rotation $\Delta \theta$ between the CRF and the ARF as discussed at the beginning of this section (see Fig. 5.3). Defining $\Delta \dot{g} = \Delta E_{1q}^c$, (89) can be written in the matrix form shown in (90).

$$\frac{d\Delta \theta}{dt} = \Delta \omega_{PLL} = k_{p1} \Delta E_{1q}^c + k_{i1} \int \Delta E_{1q}^c \quad (89)$$

$$\begin{pmatrix} \Delta \dot{\theta} \\ \Delta \dot{g} \end{pmatrix} = \begin{pmatrix} 0 & k_{iPLL} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta \theta \\ \Delta g \end{pmatrix} + \begin{pmatrix} 0 & k_{pPLL} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta E_{1d}^c \\ \Delta E_{1q}^c \end{pmatrix} \quad (90)$$

Finally, combining equations (83), (88) and (90), the overall state-space model of the grid-connected converter with current control and PLL, derived in the CRF, is given by (91).

$$\begin{pmatrix} \Delta \dot{I}_{1d}^c \\ \Delta \dot{I}_{1q}^c \\ \Delta \dot{\gamma}_{1d}^c \\ \Delta \dot{\gamma}_{1q}^c \\ \Delta \dot{\theta} \\ \Delta \dot{g} \end{pmatrix} = \begin{pmatrix} -\left(\frac{k_{p1}}{L_1} + \frac{R_1}{L_1}\right) & 0 & \frac{k_{i1}}{L_1} & 0 & 0 & 0 \\ 0 & -\left(\frac{k_{p1}}{L_1} + \frac{R_1}{L_1}\right) & \frac{k_{i1}}{L_1} & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{ipLL} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^c \\ \Delta I_{1q}^c \\ \Delta \gamma_{1d}^c \\ \Delta \gamma_{1q}^c \\ \Delta \theta \\ \Delta g \end{pmatrix} + \begin{pmatrix} \frac{k_{p1}}{L_1} & 0 & \frac{-1}{L_1} & 0 \\ 0 & \frac{k_{p1}}{L_1} & 0 & \frac{-1}{L_1} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & k_{pPLL} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^{*c} \\ \Delta I_{1q}^{*c} \\ \Delta \gamma_{1d}^{*c} \\ \Delta \gamma_{1q}^{*c} \\ \Delta E_{1d}^c \\ \Delta E_{1q}^c \end{pmatrix}; \quad \begin{pmatrix} \Delta I_{1d}^c \\ \Delta I_{1q}^c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^c \\ \Delta I_{1q}^c \\ \Delta \gamma_{1d}^c \\ \Delta \gamma_{1q}^c \\ \Delta \theta \\ \Delta g \end{pmatrix} \quad (91)$$

5.3.2 Converter Model in the ARF

To refer the converter model (91) to the ARF, equation (75) is used. This equation, as small signal quantities, is given by (92).

$$\begin{pmatrix} \Delta x_d^c \\ \Delta x_q^c \end{pmatrix} = \begin{pmatrix} \cos(\Delta \theta_o) & \sin(\Delta \theta_o) \\ -\sin(\Delta \theta_o) & \cos(\Delta \theta_o) \end{pmatrix} \begin{pmatrix} \Delta x_d^a \\ \Delta x_q^a \end{pmatrix} + \begin{pmatrix} -x_{do}^a \cdot \sin(\Delta \theta_o) + x_{qo}^a \cdot \cos(\Delta \theta_o) \\ -x_{do}^a \cdot \cos(\Delta \theta_o) - x_{qo}^a \cdot \sin(\Delta \theta_o) \end{pmatrix} \cdot \Delta \theta \quad (92)$$

In (92), the subscripts “o” correspond to steady state quantities. Note that the angle $\Delta \theta_o$ is zero in steady-state and therefore, (92) can be rewritten as (93):

$$\begin{pmatrix} \Delta x_d^c \\ \Delta x_q^c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta x_d^a \\ \Delta x_q^a \end{pmatrix} + \begin{pmatrix} x_{qo}^a \\ -x_{do}^a \end{pmatrix} \cdot \Delta \theta \quad (93)$$

$\Delta x_d^c, \Delta x_q^c$ are small signal quantities in the CRF. $\Delta x_d^a, \Delta x_q^a$ are small signal quantities in the ARF. x_{do}^a, x_{qo}^a are dependent on the operating point in the ARF. Finally, $\Delta \theta$ is the small perturbation in the angle between the converter and the actual reference frames, that enables the inclusion of the PLL dynamics in the system model. (See Fig. 5.3)

Using (93), voltages ($\Delta E_{1d}^c, \Delta E_{1q}^c$) and currents ($\Delta I_{1d}^c, \Delta I_{1q}^c$) of the converter model shown in (91) can be referred to the ARF as shown in (94).

$$\begin{pmatrix} \Delta E_{1d}^c \\ \Delta E_{1q}^c \end{pmatrix} = \begin{pmatrix} \Delta E_{1d}^a + E_{1qo}^a \cdot \Delta \theta \\ \Delta E_{1q}^a - E_{1do}^a \cdot \Delta \theta \end{pmatrix}, \quad \begin{pmatrix} \Delta I_{1d}^a \\ \Delta I_{1q}^a \end{pmatrix} = \begin{pmatrix} \Delta I_{1d}^c - I_{1qo}^a \cdot \Delta \theta \\ \Delta I_{1q}^c + I_{1do}^a \cdot \Delta \theta \end{pmatrix} \quad (94)$$

In these relationships, E_{1do}^a , E_{1qo}^a and I_{1do}^a , I_{1qo}^a are the currents and voltages of the quiescent operating point in the ARF (currents are measured at the inverter output, see Fig. 5.3), and they are calculated following the methodology discussed in section 5.3.3

Finally, using (94) and assuming that $\Delta I_{1d}^{*c} \approx \Delta I_{1q}^{*c} \approx 0$, the small signal model in (91) is referred to the ARF to derive the model shown in (95). The assumption $\Delta I_{1d}^{*c} \approx \Delta I_{1q}^{*c} \approx 0$ is utilised because the current references are not affected by small variations in the PLL states [73] [74] [75].

$$\begin{pmatrix} \Delta \dot{I}_{1d}^c \\ \Delta \dot{I}_{1q}^c \\ \Delta \dot{I}_{1d}^{*c} \\ \Delta \dot{I}_{1q}^{*c} \\ \Delta \dot{\theta} \\ \Delta \dot{g} \end{pmatrix} = \begin{pmatrix} -\left(\frac{k_{p1}}{L_1} + \frac{R_1}{L_1}\right) & 0 & \frac{k_{i1}}{L_1} & 0 & -\frac{E_{1qo}^a}{L_1} & 0 \\ 0 & -\left(\frac{k_{p1}}{L_1} + \frac{R_1}{L_1}\right) & 0 & \frac{k_{i1}}{L_1} & \frac{E_{1do}^a}{L_1} & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_{pPLL} \cdot E_{1do}^a & k_{iPLL} \\ 0 & 0 & 0 & 0 & -E_{1do}^a & 0 \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^c \\ \Delta I_{1q}^c \\ \Delta I_{1d}^{*c} \\ \Delta I_{1q}^{*c} \\ \Delta \theta \\ \Delta g \end{pmatrix} + \begin{pmatrix} -\frac{1}{L_1} & 0 \\ 0 & -\frac{1}{L_1} \\ 0 & 0 \\ 0 & 0 \\ 0 & k_{pPLL} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta E_{1d}^a \\ \Delta E_{1q}^a \end{pmatrix}; \quad \begin{pmatrix} \Delta I_{1d}^a \\ \Delta I_{1q}^a \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & -I_{1qo}^a & 0 \\ 0 & 1 & 0 & 0 & I_{1do}^a & 0 \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^c \\ \Delta I_{1q}^c \\ \Delta I_{1d}^{*c} \\ \Delta I_{1q}^{*c} \\ \Delta \theta \\ \Delta g \end{pmatrix} \quad (95)$$

5.3.3 Steady State Operating Points

From (95) it is concluded that the proposed model requires a steady-state operating point to be evaluated (due to the system is non-linear). In particular, an operating point of (95) is composed of four variables; they are E_{1do}^a , E_{1qo}^a , I_{1do}^a , and I_{1qo}^a (see Fig. 5.3). All of them are defined in the ARF – that however corresponds to the CRF in steady state. The variable E_{1qo}^a is equal to zero because the converter control is oriented in the d axis. Moreover, as discussed above, the variables of the operating point associated to the currents (I_{1do}^a , I_{1qo}^a) in the ARF can be approximated with the ones in the CRF: $I_{1do}^a = I_{1d}^{*c}$ and $I_{1qo}^a = I_{1q}^{*c}$. The remaining variable, E_{1do}^a , is derived in this subsection.

As the grid impedance (Z_g) used in this work is mainly inductive and considering that the grid voltage has an angle, δ , with respect to the capacitor voltage (see Fig. 5.3), the active power injected to the grid by the converter, can be expressed as shown (96) [38-39]. In this equation, $|E_{1abc}|$ and $|V_{gabc}|$, are respectively, the peak values of E_{1abc} and V_{gabc} . (See Fig. 5.3)

$$P = \frac{3}{2} \cdot \frac{|E_{1abc}| \cdot |V_{gabc}| \cdot \sin(\delta)}{\omega_n L_g} \quad (96)$$

On the other hand, since the control of the converter shown in Fig. 5.3 is oriented in the d axis ($E_{1q}^a = 0$), it can be demonstrated that $E_{1d}^a = |E_{1abc}|$ [110], and therefore the active power given by (96), in the dq reference frame, can be written as shown (97).

$$P = \frac{3}{2} \cdot E_{1d}^a I_{gd}^a = \frac{3}{2} \cdot |E_{1abc}| \cdot I_{gd}^a \quad (97)$$

Finally, using (96) and (97), equation (98) is obtained. In this equation, I_{gd}^a is the current of the grid in the d axis, ω_n is the nominal frequency of the system, and L_g is the grid inductance.

$$\sin(\delta) = \frac{\omega_n L_g I_{gd}^a}{|V_{gabc}|} \quad (98)$$

Applying KVL to the grid side of Fig. 5.3, the system equation in steady state is that shown in (99). In this equation, I_{gdo}^a and I_{gqo}^a are operating points for the current on the grid side (see Fig. 5.3). These operating points must be written as a function of the inverter output current (I_{1do}^a , I_{1qo}^a , see Fig. 5.3) as shown in (100) and (101) because the converter model (95) utilises these currents.

$$E_{1do}^a = R_g \cdot I_{gdo}^a + V_{gdo}^a - \omega_n L_g I_{gqo}^a \quad (99)$$

$$I_{C1do}^a = \text{Re}(j \cdot E_{1do}^a \omega_n C_1) = 0; \quad I_{C1qo}^a = \text{Im}(j \cdot E_{1do}^a \omega_n C_1) = E_{1do}^a \omega_n C_1 \quad (100)$$

$$I_{1do}^a = I_{gdo}^a; \quad I_{1qo}^a = E_{1do}^a \omega_n C_1 + I_{gqo}^a \quad (101)$$

Using (101) in (99), yields (102). In this equation, V_{gdo}^a is the d component of the grid voltage in the ARF, as shown in (103), where V_g is the magnitude of the grid voltage and the angle δ is given by (98) [see Fig. 5.4]. Finally, using (98), (102)-(103) a theoretical expression for the variable E_{1do}^a can be obtained (104).

$$E_{1do}^a = R_g \cdot I_{1do}^a - \omega_n L_g (I_{1qo}^a - E_{1do}^a \omega_n C_1) + V_{gdo}^a \quad (102)$$

$$V_{gdo}^a = |V_{gabc}| \cdot \cos(\delta) \quad (103)$$

$$E_{1do}^a = \frac{R_g \cdot I_{1do}^a - \omega_n L_g I_{1qo}^a + \sqrt{|V_{gabc}|^2 - \omega_n^2 L_g^2 (I_{1do}^a)^2}}{1 - \omega_n^2 C_1 L_g} \quad (104)$$

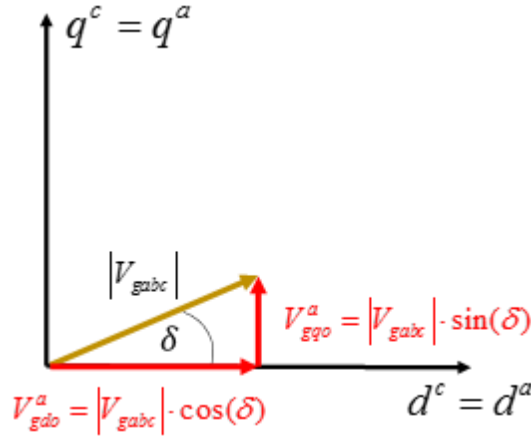


Fig. 5.4. Projection of grid voltage in the inverter reference frame. In this figure the $d^c - q^c$ reference frame corresponds converter reference frame, and $d^a - q^a$ reference frame corresponds actual reference frame. (See section 5.3)

5.3.4 System Equations

In this subsection, the remaining equations associated with the system in Fig. 5.3 will be derived. Note that all the equations are in the ARF illustrated in Fig. 5.3.

5.3.4.1 CAPACITOR EQUATIONS

From Fig. 5.3, by applying Kirchhoff's Current Law (KCL) to the capacitor C_1 , yields (105) [in the $\alpha\beta$ reference frame]. In this equation, $\vec{E}_1 = E_{1\alpha} + jE_{1\beta}$ (voltage across the capacitor), $\vec{I}_1 = I_{1\alpha} + jI_{1\beta}$ (current at the inverter output) and $\vec{I}_g = I_{g\alpha} + jI_{g\beta}$ (current grid).

$$C_1 \frac{d\vec{E}_1}{dt} = \vec{I}_1 - \vec{I}_g \quad (105)$$

Referring (105) to the actual dq reference frame (ARF), and rearranging it, yields (106). (See Appendix D)

$$\begin{pmatrix} \dot{E}_{1d}^a \\ \dot{E}_{1q}^a \end{pmatrix} = \begin{pmatrix} 0 & \omega_n \\ -\omega_n & 0 \end{pmatrix} \begin{pmatrix} E_{1d}^a \\ E_{1q}^a \end{pmatrix} + \begin{pmatrix} 1/C_1 & 0 \\ 0 & 1/C_1 \end{pmatrix} \begin{pmatrix} I_{1d}^a \\ I_{1q}^a \end{pmatrix} + \begin{pmatrix} -1/C_1 & 0 \\ 0 & -1/C_1 \end{pmatrix} \begin{pmatrix} I_{gd}^a \\ I_{gq}^a \end{pmatrix} \quad (106)$$

Based on (106), the small signal model of the voltage across the capacitor shown in Fig. 5.3 in the ARF is given by:

$$\begin{pmatrix} \Delta \dot{E}_{1d}^a \\ \Delta \dot{E}_{1q}^a \end{pmatrix} = \begin{pmatrix} 0 & \omega_n \\ -\omega_n & 0 \end{pmatrix} \begin{pmatrix} \Delta E_{1d}^a \\ \Delta E_{1q}^a \end{pmatrix} + \begin{pmatrix} 1/C_1 & 0 \\ 0 & 1/C_1 \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^a \\ \Delta I_{1q}^a \end{pmatrix} + \begin{pmatrix} -1/C_1 & 0 \\ 0 & -1/C_1 \end{pmatrix} \begin{pmatrix} \Delta I_{gd}^a \\ \Delta I_{gq}^a \end{pmatrix} \quad (107)$$

The output of the inverter model shown in (95) is used in (107) to replace ΔI_{1d}^a and ΔI_{1q}^a , leading to (108).

$$\begin{pmatrix} \Delta \dot{E}_{1d}^a \\ \Delta \dot{E}_{1q}^a \end{pmatrix} = \begin{pmatrix} 0 & \omega_n \\ -\omega_n & 0 \end{pmatrix} \begin{pmatrix} \Delta E_{1d}^a \\ \Delta E_{1q}^a \end{pmatrix} + \begin{pmatrix} 1/C_1 & 0 \\ 0 & 1/C_1 \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^c \\ \Delta I_{1q}^c \end{pmatrix} + \begin{pmatrix} -I_{1q0}^a / C_1 \\ I_{1d0}^a / C_1 \end{pmatrix} \Delta \theta + \begin{pmatrix} -1/C_1 & 0 \\ 0 & -1/C_1 \end{pmatrix} \begin{pmatrix} \Delta I_{gd}^a \\ \Delta I_{gq}^a \end{pmatrix} \quad (108)$$

5.3.4.2 GRID EQUATIONS

From Fig. 5.3, by applying Kirchhoff's Voltage Law (KVL) to the grid side, yields (109) [in the $\alpha\beta$ reference frame]. In this equation $\vec{E}_1 = E_{1\alpha} + jE_{1\beta}$, $\vec{I}_g = I_{g\alpha} + jI_{g\beta}$ and $\vec{V}_g = V_{g\alpha} + jV_{g\beta}$ (grid voltage).

$$\vec{E}_1 = R_g \vec{I}_g + L_g \frac{d\vec{I}_g}{dt} + \vec{V}_g \quad (109)$$

Referring (109) to the actual dq reference frame, and rearranging it, yields (110). (See Appendix D)

$$\begin{pmatrix} \dot{I}_{gd}^a \\ \dot{I}_{gq}^a \end{pmatrix} = \begin{pmatrix} -R_g / L_g & \omega_n \\ -\omega_n & -R_g / L_g \end{pmatrix} \begin{pmatrix} I_{gd}^a \\ I_{gq}^a \end{pmatrix} + \begin{pmatrix} 1 / L_g & 0 \\ 0 & 1 / L_g \end{pmatrix} \begin{pmatrix} E_{1d}^a \\ E_{1q}^a \end{pmatrix} + \begin{pmatrix} -1 / L_g & 0 \\ 0 & -1 / L_g \end{pmatrix} \begin{pmatrix} V_{gd}^a \\ V_{gq}^a \end{pmatrix} \quad (110)$$

Linearising (110), yields (111). (See Appendix C)

$$\begin{pmatrix} \Delta \dot{I}_{gd}^a \\ \Delta \dot{I}_{gq}^a \end{pmatrix} = \begin{pmatrix} -R_g / L_g & \omega_n \\ -\omega_n & -R_g / L_g \end{pmatrix} \begin{pmatrix} \Delta I_{gd}^a \\ \Delta I_{gq}^a \end{pmatrix} + \begin{pmatrix} 1 / L_g & 0 \\ 0 & 1 / L_g \end{pmatrix} \begin{pmatrix} \Delta E_{1d}^a \\ \Delta E_{1q}^a \end{pmatrix} + \begin{pmatrix} -1 / L_g & 0 \\ 0 & -1 / L_g \end{pmatrix} \begin{pmatrix} \Delta V_{gd}^a \\ \Delta V_{gq}^a \end{pmatrix} \quad (111)$$

5.3.5 Whole System Model

Using (95), (108) and (111), the whole small signal model of the system shown in Fig. 5.3 is obtained. This model is shown in (112) and the required operating points (E_{1do}^a , E_{1qo}^a , I_{1do}^a , I_{1qo}^a) were discussed in 4.3.3. The model can be represented as shown in (113) in the Laplace domain. The stability of the system can be studied by analysing the eigenvalues of the transition matrix "A". In this work, the stability analysis will be performed as a function of the PLL bandwidth and the weakness level of the grid, quantified by the SCR.

$$\begin{pmatrix} \Delta \dot{I}_{1d}^c \\ \Delta \dot{I}_{1q}^c \\ \Delta \dot{\gamma}_{1d}^c \\ \Delta \dot{\gamma}_{1q}^c \\ \Delta \dot{\theta} \\ \Delta \dot{g} \\ \Delta \dot{E}_{1d}^a \\ \Delta \dot{E}_{1q}^a \\ \Delta \dot{I}_{gd}^a \\ \Delta \dot{I}_{gq}^a \end{pmatrix} = \begin{pmatrix} -(k_{p1}/L_1 + R_1/L_1) & 0 & k_{i1}/L_1 & 0 & -E_{1q0}^a/L_1 & 0 & -1/L_1 & 0 & 0 & 0 \\ 0 & -(k_{p1}/L_1 + R_1/L_1) & 0 & k_{i1}/L_1 & E_{1d0}^a/L_1 & 0 & 0 & -1/L_1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_{pPLL} \cdot E_{1d0}^a & k_{iPLL} & 0 & k_{pPLL} & 0 & 0 \\ 0 & 0 & 0 & 0 & -E_{1d0}^a & 0 & 0 & 1 & 0 & 0 \\ 1/C_1 & 0 & 0 & 0 & -I_{1q0}^a/C_1 & 0 & 0 & \omega_n & -1/C_1 & 0 \\ 0 & 1/C_1 & 0 & 0 & I_{1d0}^a/C_1 & 0 & -\omega_n & 0 & 0 & -1/C_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/L_g & 0 & -R_g/L_g & \omega_n \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/L_g & -\omega_n & -R_g/L_g \end{pmatrix} \begin{pmatrix} \Delta I_{1d}^c \\ \Delta I_{1q}^c \\ \Delta \gamma_{1d}^c \\ \Delta \gamma_{1q}^c \\ \Delta \theta \\ \Delta g \\ \Delta E_{1d}^a \\ \Delta E_{1q}^a \\ \Delta I_{gd}^a \\ \Delta I_{gq}^a \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1/L_g & 0 \\ 0 & -1/L_g \end{pmatrix} \begin{pmatrix} \Delta V_{gd}^a \\ \Delta V_{gq}^a \end{pmatrix} \quad (112)$$

$$s \cdot X(s) = A \cdot X(s) + B \cdot U(s) \quad (113)$$

5.4 Stability Analysis and Simulation Results

Using the small signal model of (112), the proposed PLL design methodology for weak grids shown in Fig. 5.2 can be implemented. In (112) the parameters associated with the inverter output filter and the inverter controllers are kept constant. To study the system stability under different PLL bandwidths, the gains k_{pPLL} and k_{iPLL} are modified based on the PLL design requirements ω_{nat} and ζ (see the method described in subsection 5.2.1). Also, the weakness of the grid, characterised by the SCR, is modified utilising different values for the grid impedance (L_g , see Fig. 5.3). Results obtained using (112) are first compared with simulation results to validate the model in an environment where all the parameters are known. The model is later validated experimentally in a practical application scenario discussed in section 5.5.

The parameters of the system studied are shown in Table 5.1. First, ten PLLs with different bandwidths in the range 10.2Hz to 102.648Hz are designed (using the method described in section 5.2), obtaining the gains listed in Table 5.2. Notice from this table

that, the phase margin (P.M °) for each design is close to $P.M^\circ \approx 65^\circ$ to allow a fair comparison. Finally, using the stability analysis block shown in Fig. 5.2, which is based on the study of the eigenvalues of the model (112), the stability of these PLL designs is assessed.

In addition, the small signal model depends on the operating conditions of the inverter, hence stability will change accordingly. For each of the ten PLL designs, it is possible to calculate the maximum active power component of the current that the inverter can inject into the weak grid, before reaching instability, as a function of the PLL bandwidth and the SCR of the grid. The values of L_g used in this work and the SCRs associated with them are shown in Table 5.3.

Unless otherwise stated, it is assumed in this work that the reactive power component of the current supplied by the converter is zero, i.e. $I_{1q}^{*c} = 0$. (See Fig. 5.3), i.e. the current supplied by the converter corresponds to the active power component.

Based on this study, it is possible to find the following information about the system stability: (i) the maximum active power component of the current that the inverter can supply to the grid as a function of the PLL bandwidth and the SCR of the grid before going into the unstable zone, (ii) the maximum PLL BW which can be implemented in the system of Fig. 5.3 as a function of the grid SCR. Finally, using the proposed PLL design of Fig. 5.2 it is possible to optimise the PLL design to ensure a fast and stable system.

It needs to be pointed out that the current controller of the inverter shown in Fig. 5.3 was designed for $SCR=2.5942$, and controllers were kept constant for different SCRs. In this case, the closed-loop bandwidth of the current control loop is 200 Hz. The proportional and the integral gains are: $k_{p1} = 23.5422$ and $k_{i1} = 10701$ (see Fig. 5.3). These PI controller gains are used to evaluate the model shown in (112). The same controller is used in the simulation work as well as in the experimental work.

Table 5.1. System Parameters

L_1	R_1	C_1	R_g	V_g (line to line)
2.3 [mH]	0.2 [Ω]	10 [μ F]	0.8 [Ω]	$230\sqrt{2}$ [V]

Table 5.2. PLL Designs

k_{pPLL}	k_{iPLL}	P.M.° (Phase Margin)	PLL Bandwidth [Hz]
0.1388025	3.0845	65.5	10.277
0.2710840	12.322	64.7	20.334
0.4176300	27.842	65.6	30.898
0.5432020	49.382	64.7	40.723
0.6963750	77.375	65.6	51.514
0.8334000	111.12	65.5	61.697
0.9735680	152.12	65.5	72.136
1.1116560	198.51	65.5	82.388
1.24620000	249.24	65.5	92.336
1.38564000	307.92	65.5	102.648

Table 5.3. SCRs used in this investigation

L_g [mH]	SCR
25.2	2.5942
30.4	2.1652
35.4	1.8577
40.4	1.6265
45.6	1.4463

5.4.1 Comparison Between Model and Simulation Results

To validate the proposed dq-PLL design process shown in Fig. 5.2, in this subsection, a comparison between the results achieved with the proposed model and simulation results, in terms of stability as a function of the PLL bandwidth and the grid SCR is provided. The simulation is implemented in Plexim PLECS software, using the non-linear system of Fig. 5.3 and the parameters listed in Table 5.1. Considering that the proposed small signal model is based on an average model of the inverter, the simulation model,

therefore, neglects the PWM effects, using controllable voltage sources to represent the converter.

The proposed small signal model is verified studying the eigenvalues of the transition matrix (113). The results are generated as follows: all the PLL designs shown in Table 5.2 are evaluated for each of the SCRs shown in Table 5.3.

In each condition, the maximum active power current that the inverter can inject into the weak grid before the system becomes unstable is calculated with the proposed analytical model and with the PLECS simulation. Finally, theoretical and simulation results are compared. The nominal active current of the inverter shown in Fig. 5.3 is 18A.

Fig. 5.5 shows the maximum active current - as given by simulations and by the proposed analytical model - that the inverter of Fig. 5.3 can inject into the grid as a function of the PLL bandwidths described in Table 5.2 and for the SCRs shown in Table 5.3. (In appendix E, the points shown in Fig. 5.5 are provided)

The figure confirms that the proposed small signal model (112) can effectively represent the effects of the PLL dynamics when the converter operates connected to a weak grid.

Moreover, it is possible to conclude that with a high PLL bandwidth and a small SCR, the converter is not able to inject the nominal active current into the grid. In fact, in some conditions, the inverter can supply less than 50% of the rated active current, leading to under-utilisation of the installed power capacity. The proposed design process shown in Fig. 5.2, can effectively prevent incorrect PLL designs. For example, if the renewable system is connected to a weak grid with a SCR equal to 1.6265, the proposed model shows that the fastest PLL that can be implemented without affecting the stability and power transfer capability has a bandwidth of about 30 Hz (see Fig. 5.5).

To use the proposed dq-PLL design process (see Fig. 5.2), it is necessary to have the following information: (i) the parameters of the PI current controller, (ii) the parameters of the converter output filter and (iii) the weakness of the grid (SCR). The first two are known because they are characteristics of the converter, while the SCR can be estimated as is discussed in [111] [112].

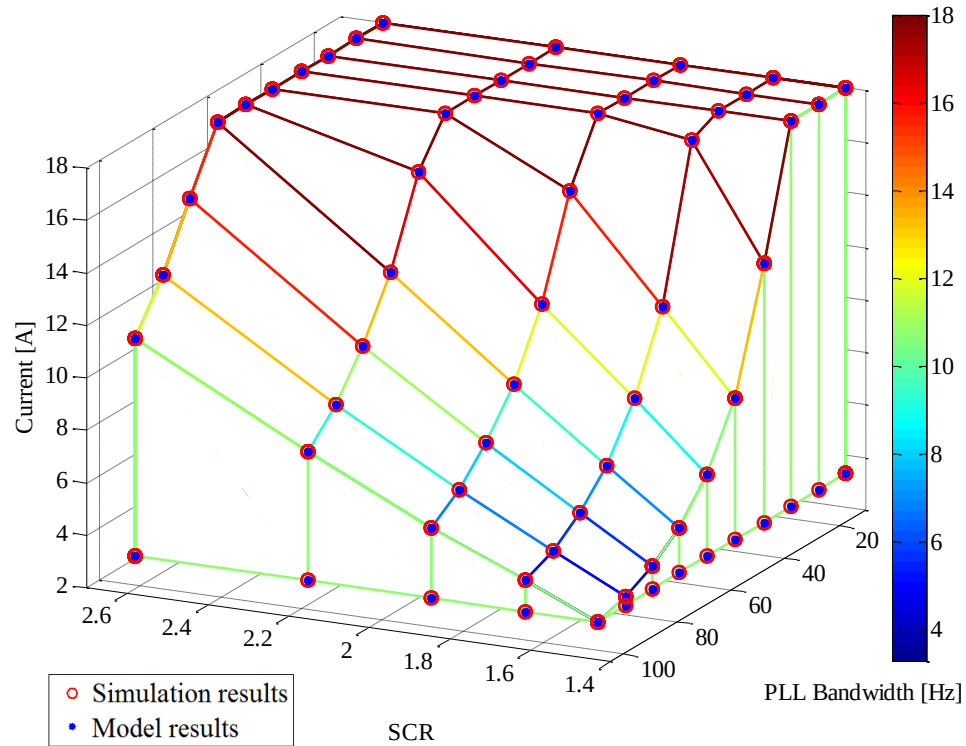


Fig. 5.5. Maximum active current that can be injected into the grid as a function of the PLL bandwidth and the SCR of the grid – Comparison between PLECS simulation and the proposed small signal model (Nominal active current = 18A).

5.4.2 Eigenvalue Analysis

In this section, the eigenvalues of the proposed model (112) are shown as a function of PLL bandwidth and of grid SCR. The analysis is performed at nominal conditions, with the converter injecting the nominal active power current (18A) into the weak grid. The results are depicted in Fig. 5.6, where it is shown that the modes 7 and 8 are heavily affected by the PLL bandwidth and by the SCR of the grid, and are responsible for the instability. For this reason, the damping ratio of the proposed model modes as a function of the PLL bandwidth and the SCR of the grid has been calculated. This is performed for the nominal conditions of the non-linear system in Fig. 5.3. Fig. 5.7 shows the damping ratio of the proposed model modes. As it is shown, the damping ratio of modes 7 and 8 - the damping is the same for both modes since they are a complex conjugate pair - with the ten PLL bandwidths under study and the different grid SCRs. Fig. 5.7 shows that for all the SCRs, the damping of mode 7 and 8 is an approximately linear function of the PLL bandwidth, with the damping decreasing as the bandwidth increases. Moreover, the lower the SCR, the sooner the damping crosses the imaginary axis to the right half-plane, bringing the system into instability. For the strongest grid, with $SCR=2.5942$, the damping crosses the imaginary axis with a PLL bandwidth of 72.136 Hz. For the weakest grid, with $SCR=1.6265$, the crossing occurs at 30.898 Hz.

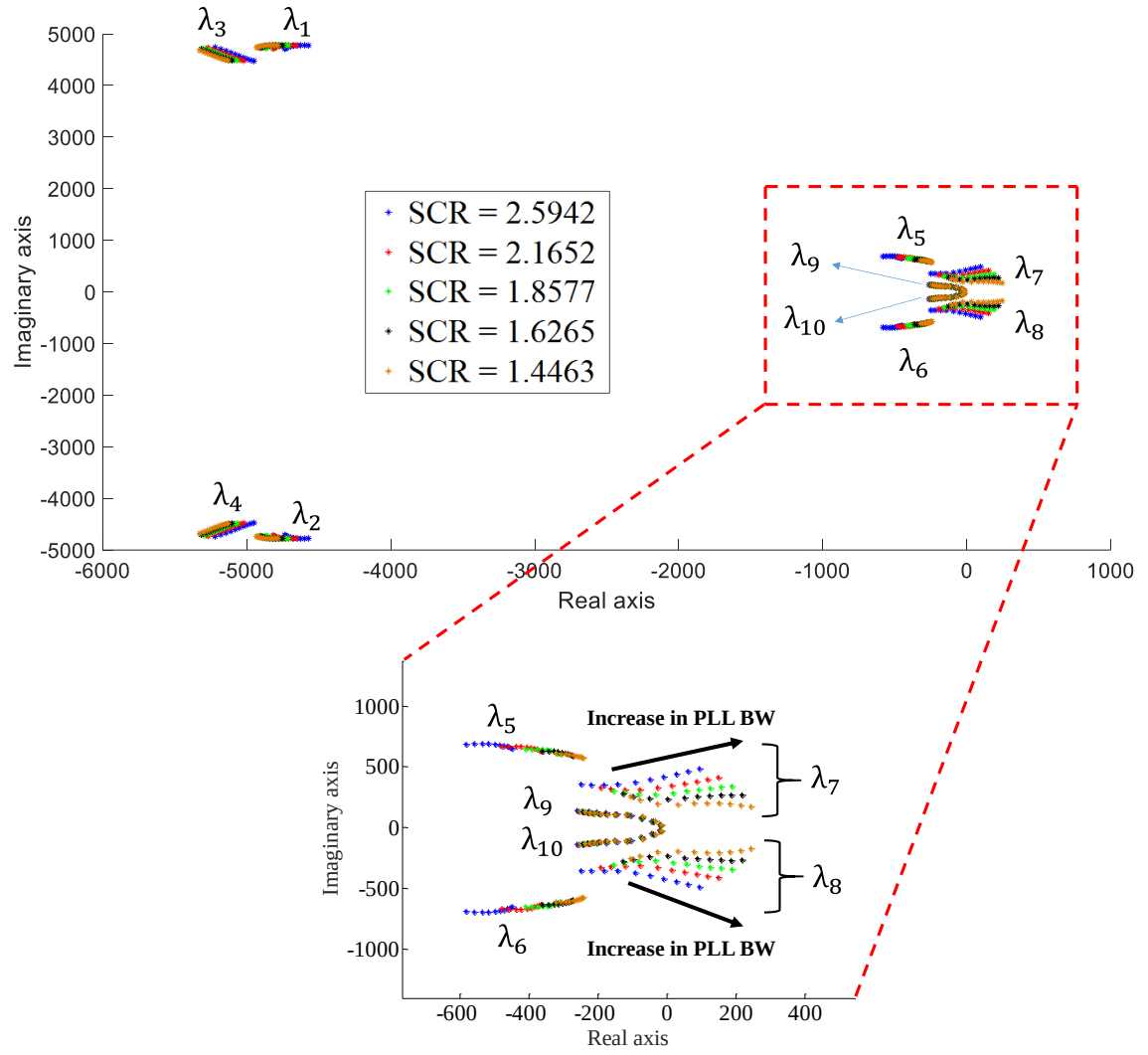


Fig. 5.6. Main eigenvalues of the proposed small signal model as a function of PLL bandwidths and SCRs.

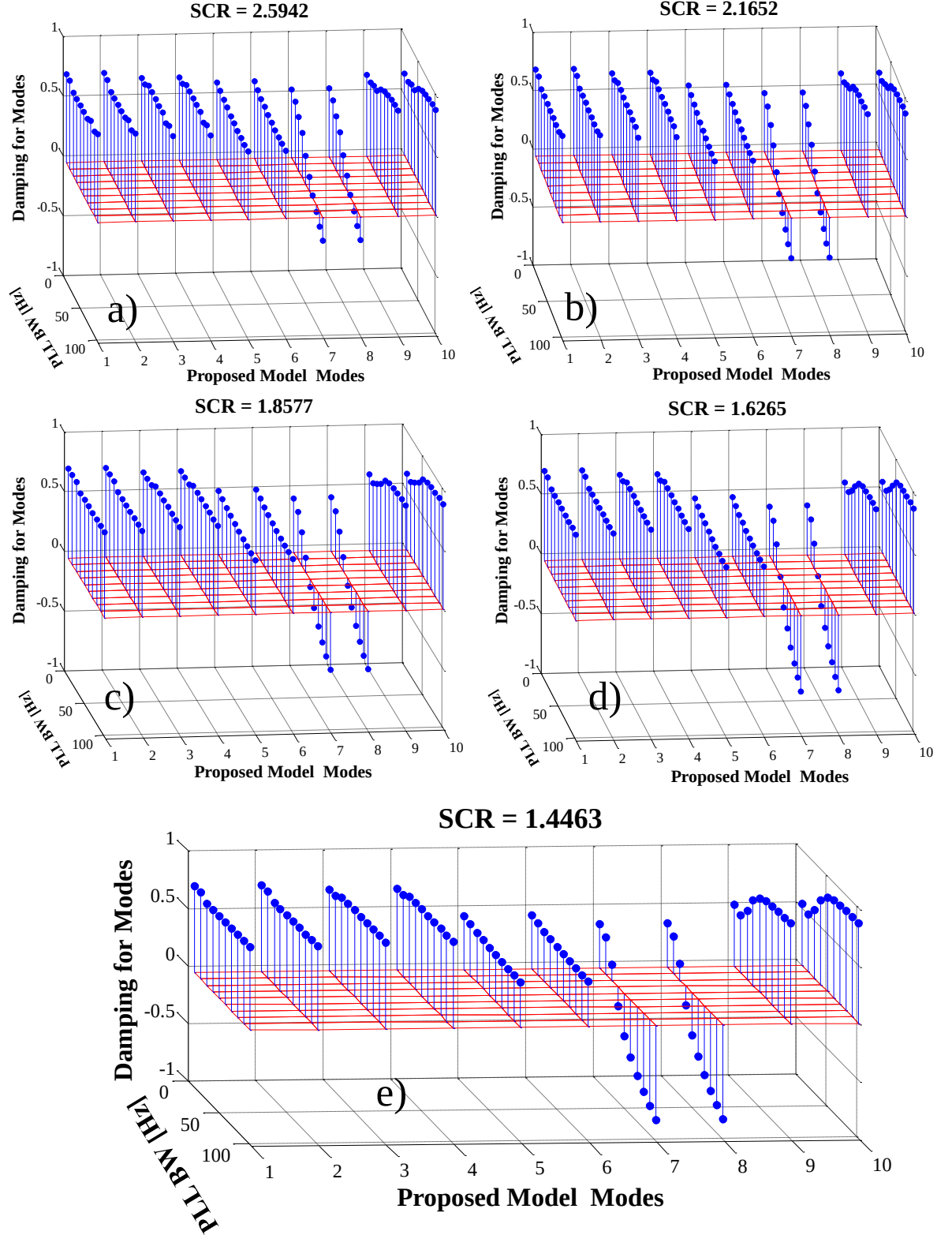


Fig. 5.7. Damping of modes of the proposed small signal model as a function of the PLL bandwidth and the grid SCR.

To explore stability under different operating points other than the nominal active current, Fig. 5.8 shows the damping ratio of modes 7 and 8 as a function of the PLL bandwidth and the active current supplied by the converter into the grid⁹. The figure confirms that stability is adversely affected also by the amount of injected active current, with the damping of the modes decreasing as the current increases. Moreover, the decrease is steeper as the SCR decreases and as the PLL design bandwidth increases.

Finally, the participation factor analysis discussed in [113] is applied to modes 7 and 8 to understand their relationship with the state variables of the proposed model. Based on this analysis (see Appendix F), omitted here for brevity, it is concluded that in the case of a high PLL bandwidth and low SCR, the states $\Delta\theta$ and Δg which are associated with the PLL model in (112) dominate modes 7 and 8, further confirming that the PLL dynamic is the main cause of instability.

⁹ These points are shown in Appendix G

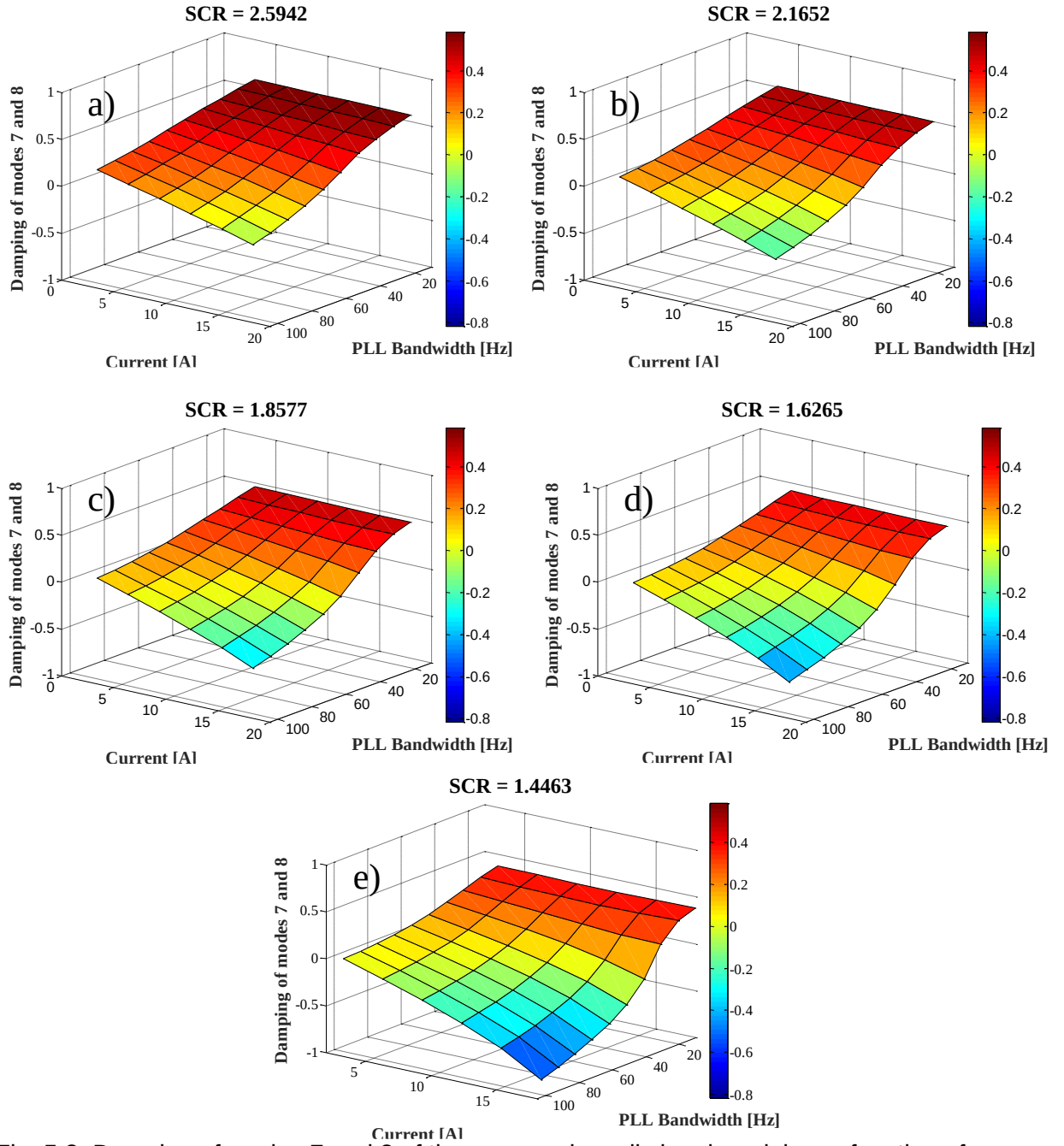


Fig. 5.8. Damping of modes 7 and 8 of the proposed small signal model as a function of the PLL bandwidth and the active current injected into the grid.

5.5 Experimental Results

The control system and converter topology of Fig. 5.3 is embedded in a Triphase unit PM15F42C (5 kW) [17] [114] (configured as a 3-leg converter) as is shown in Fig. 5.9. This unit has the same output filter parameters as those shown in Table 5.1. Inductances are used to modify the grid impedance, emulating the SCRs described in Table 5.3.

The converter is controlled according to the scheme shown in Fig. 5.3. The PI gains used in both the simulation and theoretical work described in the previous section are used to control the experimental system. The overcurrent protection of the system shown in Fig. 5.9 is set at 20A (instantaneous current).

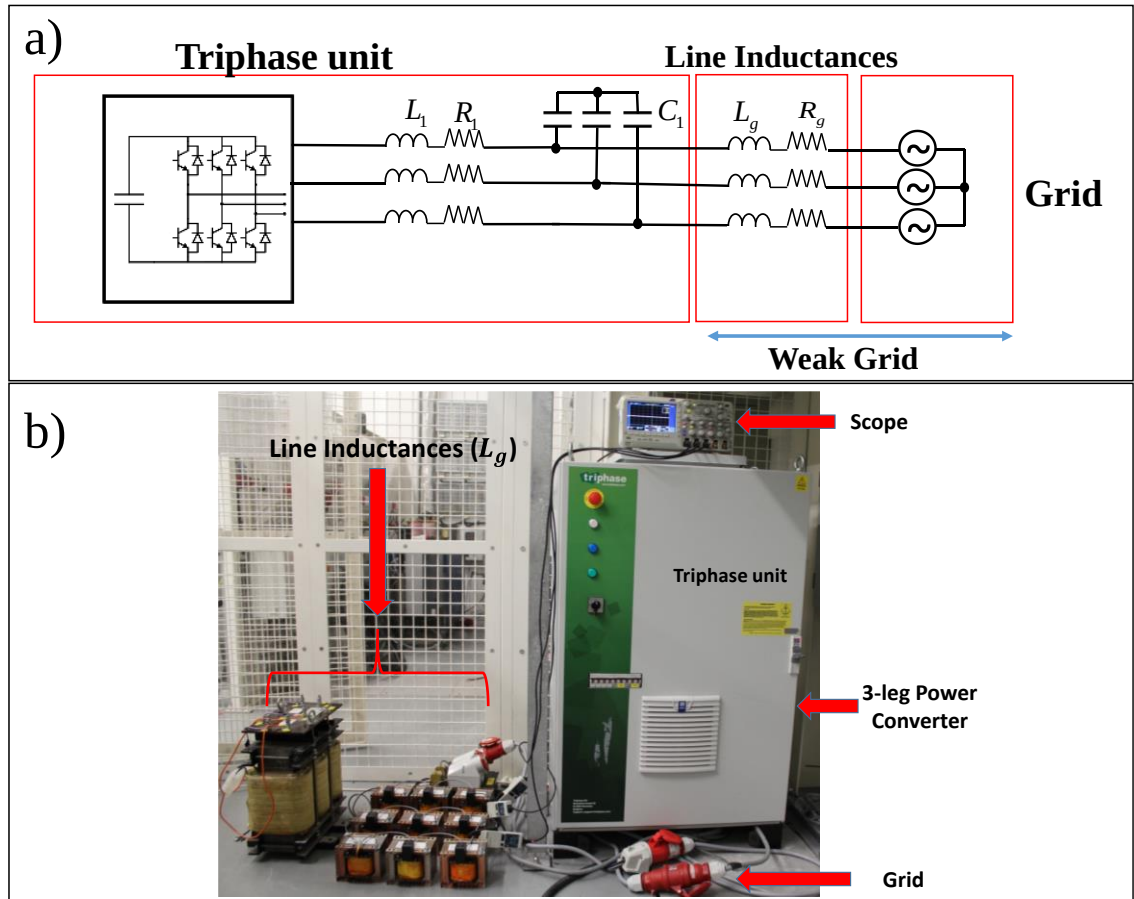


Fig. 5.9. (a) Schematic of the system studied in this chapter, (b) experimental system implemented in the laboratory to validate the proposal discussed in this chapter.

To validate the proposed small signal model of (112), PLL bandwidths in the range of approximately [10-50 Hz] (see Table 5.2) are considered. These values are used because it is the typical bandwidth of PLL implemented in real systems [107] [108] [115]. Moreover, all the SCRs shown in Table 5.3 are emulated in the experimental rig. First, the maximum active current that the converter can inject into the grid as a function of the PLL bandwidth and the SCR is experimentally measured and compared with that predicted by the proposed model. In the experimental tests shown in this section, current steps of 1A around the quiescent operating points are used.

According to the model results described in section 5.4, the experimental converter can supply the nominal active current (18A) into the grid without becoming unstable when SCRs of 2.5942 and 2.1652 are considered. This occurs for the whole range of PLL bandwidths situation, [$\approx$ 10-50 Hz]. Fig. 5.10(a) shows the nominal active current injected by the inverter for the worst case, i.e. for a PLL bandwidth of $\approx$ 51.5 Hz. The same trend is shown in Fig. 5.10(b) where the SCR of the grid is equal to 2.1652. In this case, the parameters of the PLL implemented in the converter of Fig. 5.9 are the same parameters used to generate the experimental results shown in Fig. 5.10(a).

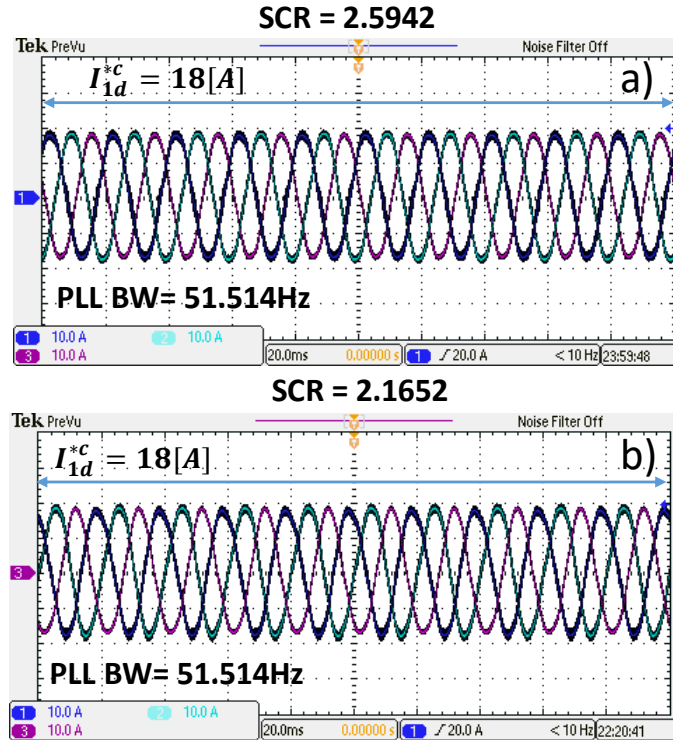


Fig. 5.10. Experimental results: (a) Maximum active current injected into the grid for SCR=2.5942 and PLL bandwidth of 51.514 Hz (10A/div), (b) Same as a) but with SCR=2.1652 (10A/div).

In Table 5.4, the predictions from the proposed model are compared with the experimental results for a grid with a SCR=1.8577. When a PLL with bandwidth 51.514 Hz is used, the proposed theoretical model predicts a maximum active current at the converter output of 15.7 A. On the other hand, the maximum active current that the experimental system can inject to the grid without becoming unstable is 14.5 A. Fig. 5.11(b) shows that the system is stable when injecting 14.5 A. When the reference current is changed to 15.5 A (current step of 1A), the system becomes unstable, showing a good match with the theoretical prediction. If the PLL bandwidth is reduced below 40.723 Hz, the instability issue is avoided, and the converter can fully supply the nominal current to the grid, as shown in Fig. 5.11(a).

Table 5.4. Comparison between the proposed model and experimental system results for a Short Circuit Ratio (SCR) equal to 1.8577

PLL Bandwidth [Hz]	Maximum active current - proposed model [A]	Maximum active current - experimental rig [A]
10.277-20.334-30.898-40.723	18	18
51.514	15.7	14.5

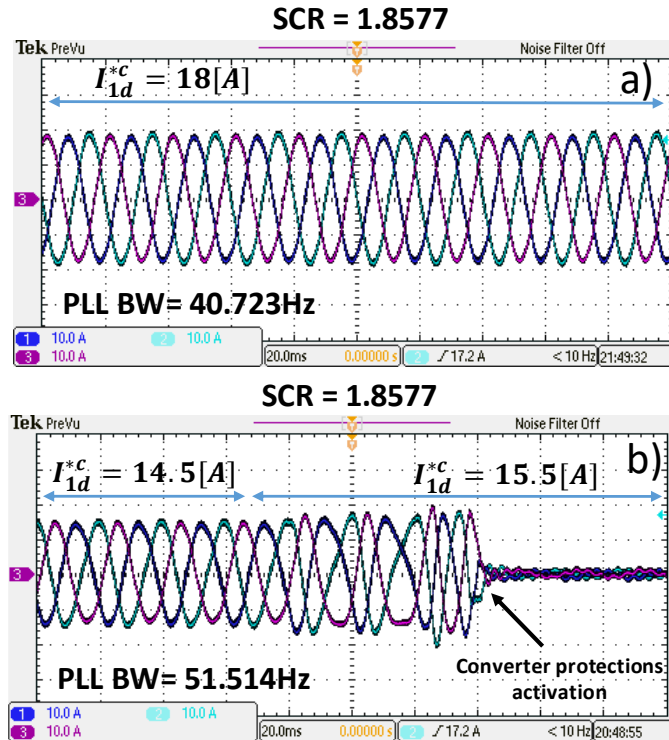


Fig. 5.11. Experimental results: (a) Maximum active current injected into the grid with a PLL bandwidth of 40.723 Hz (10A/div), (b) Maximum active current injected with a PLL bandwidth of 51.514 Hz (10A/div). (Converter protections activation means that a contactor is opened for disconnecting the converter from the grid)

In Table 5.5, the experimental results are compared with those produced by the theoretical model for the case of $SCR=1.6265$, confirming the good match between the model and the experimental evidence. Fig. 5.12 shows the experimental current waveforms corresponding to the three limit conditions shown in Table 5.5. For the rest of the cases, the converter can inject its nominal active current.

Table 5.5. Comparison between the proposed model and experimental system results for a Short Circuit Ratio (SCR) equal to 1.6265

PLL Bandwidth [Hz]	Maximum active current - proposed model [A]	Maximum active current - experimental rig [A]
10.277-20.334-30.898	18	18
40.723	17.5	16.5
51.514	11.8	10

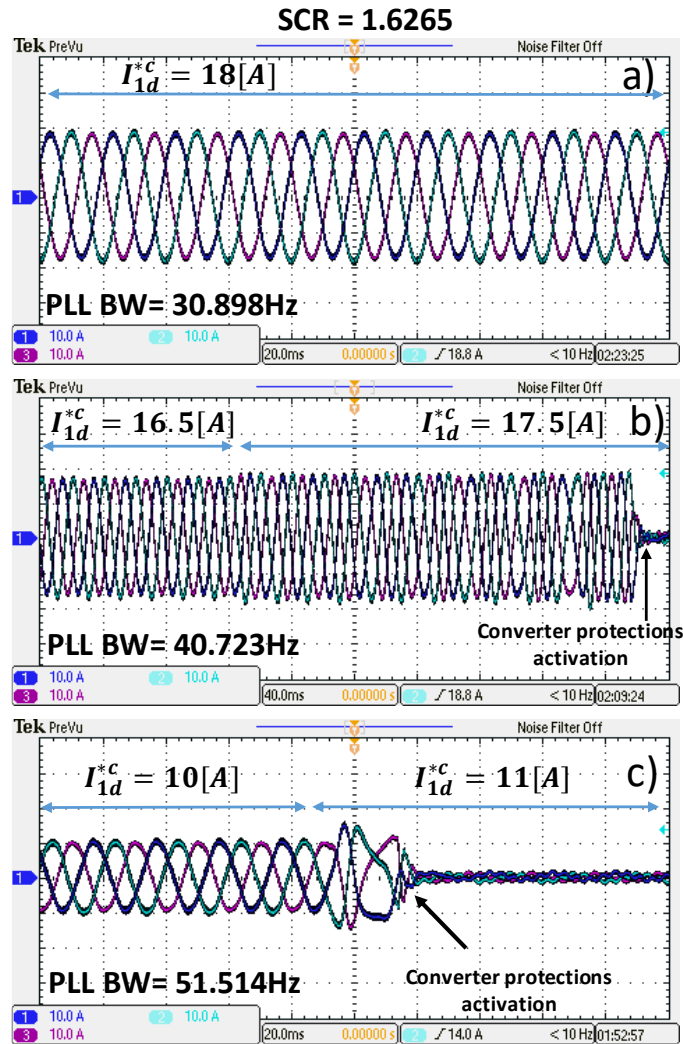


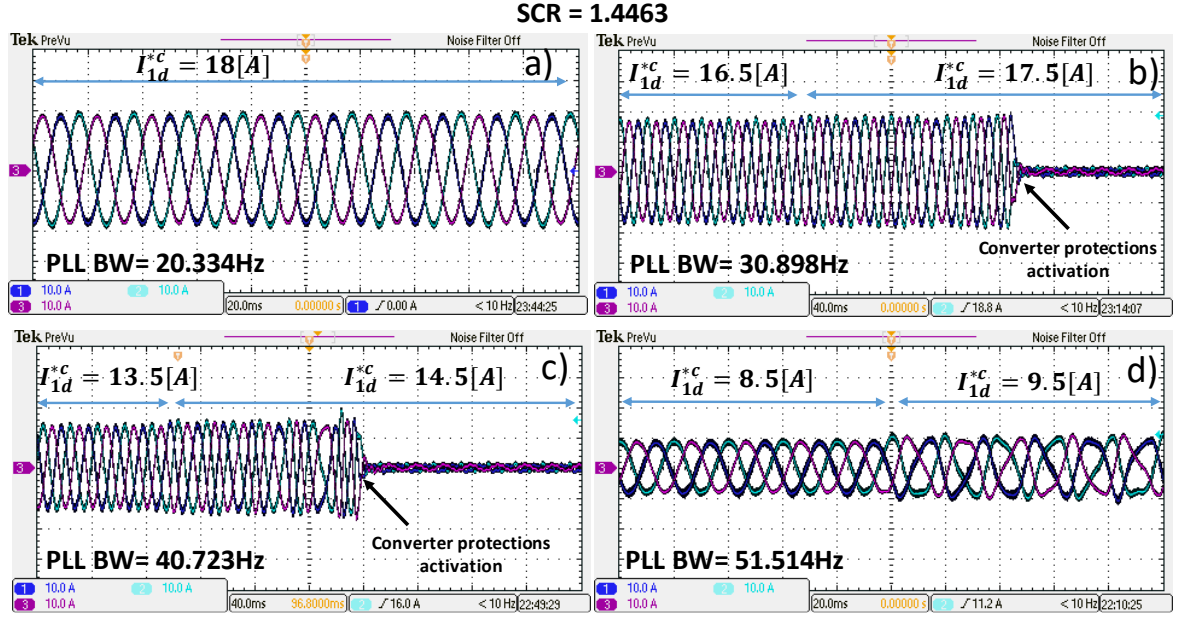
Fig. 5.12. Experimental results: (a) Maximum active current injected into the grid with PLL bandwidth of 30.898 Hz (10A/div), (b) Maximum active current with PLL bandwidth of 40.723 Hz (10A/div), (c) Maximum active current with PLL bandwidth of 51.514 Hz (10A/div).

The last case studied is with a SCR equal to 1.4463, representing a very weak grid. Both model results and experimental results are shown in Table 5.6, again confirming that experimental results agree with the theoretical expectations. For instance, if a PLL with a bandwidth of 51.514 Hz is used in the experimental system, the current that the inverter can inject into the weak grid before becoming unstable is less than the 50% of its nominal current. This case is depicted in Fig. 5.13(d) where the currents of the experimental rig are shown. To ensure the system stability in nominal conditions, a PLL with a maximum bandwidth of 20.334 Hz must be used as shown in Fig. 5.13(a). Finally, the current waveforms supplied by the converter when the PLL has a bandwidth of 30.898 Hz and 40.723 Hz are shown in Fig. 5.13(b) and Fig. 5.13(c) respectively.

It is, therefore, possible to conclude that the proposed model [see (112)] can effectively predict the system stability when the PLL bandwidth and the SCR of the weak grid are taken into account.

Table 5.6. Comparison between the proposed model and experimental system results for a Short Circuit Ratio (SCR) equal to 1.4463

PLL Bandwidth [Hz]	Maximum active current - proposed model [A]	Maximum active current - experimental rig [A]
10.277-20.334	18	18
30.898	18	16.5
40.723	13.2	13.5
51.514	8.7	8.5



5.6 Chapter Conclusions

In this chapter, a low-complexity small signal model of the system which enables the inclusion of the PLL bandwidth and the weakness of the grid [characterised using the SCR defined in (74)] is proposed. The proposed model has been exhaustively validated with simulations implemented in PLECS and experimentally for a wide range of PLL bandwidths and different SCRs.

Moreover, based on the proposed model a dq-PLL design procedure is proposed to ensure the stability of converters connected to weak grids. The improvement of the proposed design scheme over the schemes reported in previous studies is the fact that this is the first work where a systematic procedure to design a PLL to be used in weak grids is proposed.

Experimental results obtained from a 5 kW laboratory scale system confirm that the proposed model and design process can be effective tools to optimise the system operation in terms of finding the fastest PLL that can be used in a weak grid with known SCR without affecting the system stability.

The fact that the proposed model is simple enables it to be used for online stability monitoring. Indeed, it was demonstrated that only modes 7 and 8 of the model are affected by the PLL bandwidth and the SCR of the grid. Then, if an algorithm for monitoring the grid impedance is added, the stability of the system can be monitored online through the calculation of modes 7 and 8.

It is worth remembering that the proposed PLL design process is suitable for three-phase three-wire balanced weak grids. Its extension for considering unbalanced and/or harmonic weak grids is proposed as future work. In unbalanced weak grids, it is necessary to consider both positive and negative sequence components of currents and voltages of the system in the modelling process. While in harmonic weak grids, it is necessary to consider the voltages and currents of the main harmonics present in the system in the modelling process. In references [116] [117] [118] the output impedance of a converter connected to an unbalanced grid is developed (taking into account the PLL dynamics). Based on that, both positive and negative sequence impedances are derived, suggesting that there is a cross-coupling of them, which depends on the PLL bandwidth, and ultimately will affect the stability of the system. To the best of the author's knowledge, these type of studies, considering weak grids have not been performed so far.

Finally, it should highlight that the proposed methodology in this chapter to study the stability of the system taking into account the PLL bandwidth and the short circuit ratio of the weak grid is based in a simple model. In the modelling process, a balanced three-phase three-wire weak grid was considered, and also it was assumed that the power

converter is controlled using PI controllers. The proposed methodology, as future work can be modified to study at specific resonant structure presented for the control schemes discusses in chapter 3 and 4, and linked to maximum harmonic compensation capability as a function of the short circuit ratio.

Chapter 6. Conclusions and Future work

6.1 Conclusions

In recent years, a significant increase in electrical systems based on non-conventional renewable energy has occurred. To help facilitate the integration of NCRE-based technologies in power systems, the concept of the microgrid has appeared. Microgrids are considered efficient solutions when there are distributed resources. For instance, let us consider the Chilean case, where the Atacama Desert, the Andes Mountains, and the long coastline provide to the country abundant distributed renewable energy resources, which are spread along the Chile geographic; in this situation, microgrids offer a solution for gathering these distributed energy resources either feeding small towns far from the main power grid or feeding the main power grid.

MGs could be implemented as a low-voltage power distribution system operating in grid-connected or islanded mode. In these electrical systems, renewable energy resources are connected to loads and/or the distribution network using power electronic interfaces. Therefore, the widespread use of power electronic converters, the unbalanced and/or nonlinear nature of loads and the fact that MGs are weak systems, will produce power quality issues on MGs, which must be properly addressed. Power quality issues that require careful consideration are imbalances and harmonics. Indeed, typical loads connected to MGs are computers, lighting ballasts, appliances, battery chargers, and so on. Most of these loads have a nonlinear behaviour. On the other hand, MGs will likely operate under highly unbalanced conditions due to single-phase loads in each phase are

not balanced. Moreover, both imbalances and harmonics issues should be studied, considering the fact that MGs are weak electrical systems.

Therefore, the aim of this thesis was to investigate control algorithms and topologies to address the power quality issues related to imbalances and harmonics in MGs and investigate the effects of weak nature of MG on the control of the power converters. The main research objectives set forth are:

- ✓ Demonstrate the effectiveness of a 4-leg active power filter, controlled using the CPT, in mitigating the negative effects of imbalances and harmonics in three-phase four-wire microgrids.
- ✓ Demonstrate that the CPT can be used in the design of cooperative imbalance/harmonic sharing controllers, giving some advantages in comparison with the previous ones reported in the literature.
- ✓ Derivate a methodology to study the effects of the weak nature of MGs in the performance of the control of a power converter.

To meet the first objective, an active power filter (APF) composed of a three-phase four-leg connected in shunt at the PCC, and controlled using the conservative power theory (CPT) has been proposed. The performance of the APF, and in particular, the use of the CPT to generate the unwanted current components to achieve an effective compensation of imbalances and harmonics in a MG have been validated experimentally. Special focus was centred on the study of the control system during transient operation and under grid frequency variations. Experimental results showed that the performance of the proposed 4-leg active power filter has been excellent, concluding that the CPT as a new power theory is suitable for applications such as microgrids where strong distortion,

imbalances, and frequency variations. The control system was based on self-tuning proportional-resonant (PR) controllers based. Simulation work shows that the quality of currents, in term of their THD, injected by the power supply to the unbalanced/non-linear, when the APF is working, is improved with the use of self-tuning PR controllers. (In comparison with the use of non-tuning PR controllers)

To validate the proposed compensator, extensive experimental work was performed using the experimental system depicted in Fig. 3.9. The characteristics of the unbalanced and non-linear load were based in real data from an isolated MG located in Canada. Moreover, the performance of the proposed controller to compensate harmonics produced by a diode rectifier was evaluated. The latter scenario was used to evaluate the proposed controller in the worst case situation. All the experimental tests showed the excellent performance of the proposed active power filter in compensating current imbalance and neutral current.

To meet the second objective, a new control algorithm to achieve a cooperative imbalance sharing among the converters present in a three-phase four-wire microgrid has been proposed. This novel control scheme is based on the CPT, and therefore it is implemented in the abc reference frame. In the development of the proposed sharing algorithm, it was demonstrated that the orthogonal current components defined by the CPT [see (8)-(11)] are related with positive, negative and zero current sequence components through equations (35) and (38). This result constitutes a novel tool for decoupling the positive sequence component from both negative and zero current components in the abc reference frame and without the use of sequence separation algorithms. Based on that, the main characteristic of the proposed control scheme is the fact that it does not require the use of any sequence separation algorithm to achieve a

current separation. Moreover, the control scheme is implemented locally in each 4-leg converter based on local measurements of currents and voltage, and thus a communication network is not required. Finally, the extension of the proposed imbalance sharing controller for considering harmonics sharing was discussed and experimentally validated.

To validate the proposed controller to achieve the sharing of imbalances and harmonics among the converters present in an isolated MG, extensive experimental work was performed using the experimental system depicted in Fig. 4.15. The characteristics of the unbalanced and non-linear load were based in real data from an isolated MG located in Canada. All the experimental test showed the excellent performance of the proposed controller for achieving imbalances and harmonics sharing among the 4-leg converters. Based on these experimental tests, the validation of the controller was performed for the following conditions: (i) sharing of negative and zero sequence components (at the fundamental frequency), (ii) sharing of third harmonic components, and (ii) the evaluation of the controller when load impacts occur (those which increased the level of imbalanced in the MG).

Finally, to meet the last objective, a methodology to consider the effects of the weak nature of MG, and the influence of the PLL bandwidth for a simple controller has been derived. This thesis proposed a low-complexity small signal model for a 3-leg converter connected to a balanced three-phase, three-wire weak grid and synchronised to this grid using a PLL implemented in a synchronous rotating d-q axis. A thorough analysis of the system stability as a function of the PLL bandwidth and the short circuit ratio (SCR) of the grid was performed based on a linearised model. The proposed model has been extensively validated with simulations implemented in PLECS and experimentally, using

the experimental system depicted in Fig. 5.9, for a wide range of PLL bandwidths and different SCRs. Particularly, it was validated that the proposed model and the design process can be effective tools to optimise the system operation in terms of finding the fastest PLL that can be used in a weak grid with known SCR without affecting the system stability

In summary, the main conclusions drawn from this research are as follows:

- ✓ The use of the CPT as a power theory for controlling a 4-leg active power filter to compensate imbalances and harmonics in a 4-wire microgrid has been experimentally validated.
- ✓ A novel imbalance and harmonic sharing controller to achieve that both imbalances and harmonics produced at the load, are shared by the 4-leg converters present in an MG according to their remaining VA power capacity have been proposed and experimentally validated. The main differences of the proposed imbalance sharing algorithm with those previously reported in the literature can be summarized as: (i) the proposed scheme does not require sequence separation algorithms, (ii) a new method to define the preliminary unbalanced voltage set points is proposed, and (iii) contrary to the only paper where a control scheme for cooperative imbalance sharing in 4-wire MGs [44], the proposed control scheme can be easily extended to harmonics as it does not require sequence separation algorithms.
- ✓ A novel methodology based on a simple small signal model has been proposed to analyse the effects of the weak nature of MGs and the PLL bandwidth on the stability of the controller of a converter connected to a weak grid.

6.2 Future Work

In this thesis, the mitigation of technical issues associated with both unbalanced and harmonic operation of a three-phase four-wire isolated microgrid has been successfully achieved through: (i) the development of an advanced control system for a shunt connected three-phase four-leg VSC-based compensator, and (ii) an novel control algorithm for sharing both imbalances and harmonics among the 4-leg converters which are placed in an 4-wire microgrid. Moreover, a methodology to evaluate the effects of the weak nature of MGs have been derived for a simple controller; as future work, this methodology could be extended to evaluate these effects on the controllers proposed in this thesis. The experimental results show the validity of the proposed control schemes. Several areas for further research on improving the performance of them have been identified and are listed below:

Active power filter

- ✓ Study if the CPT can be used to control an APF with a series connection to the grid. In this case, the references to the APF must be voltage references instead of current references as was the case study in this thesis.
- ✓ Extend the APF control scheme to mitigate the double system frequency (100Hz) oscillation of the DC-link voltage.
- ✓ Study the integration of the APF control scheme to a community battery storage system in order to improve the functionality of a conventional three-phase battery charge/discharge power converter controller.

- ✓ Study the effects of the weak nature of microgrid in the performance of the APF. Especially, in terms of the maximum harmonic compensation capability for low short circuit ratios.
- ✓ In microgrids with many voltage bars, the use of APF is a suitable solution for mitigating imbalances and harmonics. Based on that, as future work, it is proposed to study an optimal location where APFs for optimising the power quality in whole MG.

Cooperative imbalance sharing

- ✓ To develop a methodology to choose the THD set points for each inverter when the proposed control scheme is used for sharing harmonics among the inverters present in a MG. In fact, in section 4.3.3 a methodology is proposed for calculating the PVUR reference points for each 4-leg converter to achieve imbalance sharing. However, a methodology for calculating the THD reference points is missing.
- ✓ Extend the proposed controller for improving the power quality at the PCC. As was discussed through the thesis, imbalances/harmonics sharing can only be achieved at the expense of increasing voltage imbalance/distortion at the converters outputs and the PCC. Based on that, it is proposed as future work, to extend the proposed controller to introduce some compensation capabilities in order to improve the power quality at the PCC. (In terms of imbalances and harmonic distortion in the voltage at that point)

- ✓ In this thesis, both primary and secondary control schemes proposed in chapter 4 are validated considering a centralised MG. As future work is proposed the extension of the proposed control schemes to be used in a distributed MG. Indeed, distributed schemes permit high flexibility to add new distributed generation units, and it can provide a plug-and-play operation of the units.
- ✓ In this thesis, AC microgrids have been studied. However, recently the study of DC microgrids and hybrid microgrids have increased considerably. The latter because power losses in these type of MGs are significantly reduced as the number of DC-AC conversions is reduced. In this context, it is proposed as future work, the study of cooperative controllers to achieve proper load current sharing among the distributed generations units in DC and hybrid MGs.
- ✓ Study the effects of the weak nature of MGs in the performance of the proposed controller for imbalances and harmonic sharing. Especially, analysing the effects on the proportional resonant controllers.

Effects of the PLL bandwidth in the system stability

- ✓ In this thesis, a methodology for a simple controller has been derived to analyse the effects of the level of weakness of the grid (measured using the SCR) on the stability of the system. This methodology, as future work, can be modified to look at specific resonant control schemes based on the CPT proposed in this thesis to ensure a stable operation of the proposals for low short circuit ratios.

- ✓ In this thesis, the system stability was study considering the weakness of the grid (SCR), PLL bandwidth and the maximum active power component of the current injected by the converter to the weak grid. Future work is proposed to study the stability of the system taking into account the maximum reactive current injected by the converter to the grid.
- ✓ Extend the small signal model to consider additional outer control loops as, active power loop, reactive power loop, and voltage loop.
- ✓ Study the generalisation of the proposed small signal model for considering the case where multiple parallel power converters are connected to a weak grid.
- ✓ Extend the small signal model to consider the case when the converter is connected to an unbalanced three-phase four-wire weak grid.

6.3 Publications Resulting from the Research

1. **C. Burgos-Mellado**, R. Cárdenas, D. Sáez, A. Costabeber, M. Sumner, "A Control Algorithm Based on the Conservative Power Theory for Cooperative Sharing of Imbalances in 4-Wire Systems", Accepted for publication to the *IEEE Transactions on Power Electronics*, 2018.
2. **C. Burgos-Mellado**, C. Hernandez, R. Cárdenas, D. Saez, M. Sumner, A. Costabeber, H. Morales, "Experimental Evaluation of a CPT-Based 4-leg Active Power Compensator for Distributed Generation", *IEEE Journal of Emerging and Selected Topics in Power Electronics*, (Special Issue on Distributed Generation), Vol. 2, no. 2, pp. 747-759, 2017
3. **C. Burgos-Mellado**, A. Costabeber, M. Sumner, R. Cárdenas, D. Sáez " Small-signal Modelling and Stability Assessment of Phase-locked Loop in Weak Grids", Accepted for publication to *Energies*.

Appendix A: Effects of discretisation methods on the performance of resonant controllers

In [119], a comprehensive study of the effects of discretisation methods on the performance of proportional resonant controllers is performed. As discussed in [119], the bilinear transform with frequency prewarping has important advantages over other discretisation methods, in terms of preserving the resonant frequency accuracy, particularly when high-order harmonics have to be compensated for. The following table shows a comparison between several discretisation methods [119]. From this table, it can be concluded that a proportional-resonant (PR) controller implemented using the Tustin transform has a large resonance deviation. In contrary, if the PR controller is implemented using the Tustin with pre-warping transform, there is not resonance deviation.

Table A.1. Performance of the discrete-time implementations

Discrete Transform	Infinite gain	Resonance deviation	Effect on zeros	Delay compensation
Forward Euler, Backward Euler	No	---	---	---
First-order hold, Tustin	Yes	Very Large	---	---
Zero-order hold, zero-pole matching	Yes	No	Phase lag at ω_o	Not effective
Impulse invariant	Yes	No	Accurate at ω_o and phase lead at $\uparrow \omega$	Accurate
Tustin with pre-warping	Yes	No	Accurate	Accurate

Appendix B: Code used to solve the optimisation problem introduced in section 4.3.3

Using the toolbox called genetic algorithm contained in Matlab, the optimisation problem given by (69) is solved using the objective function and the restrictions shown below.

Objective Function

```
function = fun_optimizar_3(x)
f=50;
Z1_eq=x(1);
Z2_eq=x(2);
Z3_eq=x(3);

Ztot1=(Z2_eq*Z3_eq)/(Z2_eq+Z3_eq);
aux1=(1*Ztot1)/(Z1_eq+Ztot1);

N1=3*norm(aux1)*10;

Ztot2=(Z1_eq*Z3_eq)/(Z1_eq+Z3_eq);
aux2=(1*Ztot2)/(Ztot2+Z2_eq);

N2=3*norm(aux2)*10;

Ztot3=(Z1_eq*Z2_eq)/(Z1_eq+Z2_eq);
aux3=(1*Ztot3)/(Z3_eq+Ztot3);

N3=3*norm(aux3)*10;

f=abs(N1-N2)+abs(N1-N3)+abs(N2-N3);
end
```

Restrictions of the optimization problem

```
function [c,ceq] = rectriciones_3(x)

%By using the Conservative power theory (CPT), the current measured
at the PCC is decomposed in the unbalanced components, giving:

Iunb_a=4.68;
% Iunb_b=3.936;
% Iunb_c=1.26;
% In=6;

%% Line impedance parameters
f=50;
we=2*pi*f;
R=1;
```

```

L=4e-3;

E1=180;
E2=180;
E3=180;
%%Unbalanced currents (calculated using the CPT)
Ia_u=0.9777;
Ib_u=8.7710;
Ic_u=7.7143;
Ib=16.3;

Z1_eq=x(1);
Z2_eq=x(2);
Z3_eq=x(3);

%Voltage at the output of converter 1

Ztot1=(Z2_eq*Z3_eq)/(Z2_eq+Z3_eq);
aux1=Ztot1/(Z1_eq+Ztot1);

PVUR1=100*x(1)*abs(aux1)/(abs(E1-
abs(aux1)*Ib*sqrt(R^2+L^2*we^2)))*max([Ia_u Ib_u Ic_u]);

%voltage at the output of converter 2

Ztot2=(Z1_eq*Z3_eq)/(Z1_eq+Z3_eq);
aux2=Ztot2/(Ztot2+Z2_eq);

PVUR2=100*x(2)*abs(aux2)/(abs(E2-
abs(aux2)*Ib*sqrt(R^2+L^2*we^2)))*max([Ia_u Ib_u Ic_u]);

%Voltage at the output of converter 3

Ztot3=(Z1_eq*Z2_eq)/(Z1_eq+Z2_eq);
aux3=Ztot3/(Z3_eq+Ztot3);

PVUR3=100*x(3)*abs(aux3)/(abs(E3-
abs(aux3)*Ib*sqrt(R^2+L^2*we^2)))*max([Ia_u Ib_u Ic_u]);

c(1)=PVUR1-3/1;
c(2)=PVUR2-3/1;
c(3)=PVUR3-3/1;
ceq=0;

end

```

Appendix C: Linearisation of a system represented in state space form

Let us consider that a time-invariant system is modelled in the state space representation shown in C.1

$$\begin{aligned}\dot{x} &= f(x, u) \\ y &= g(x, u)\end{aligned}\tag{C.1}$$

To linearise C.1, it is necessary to linearise both f and g around an equilibrium point.

An equilibrium point (x_0, u_0) is that where all the derivatives of C.1 are simultaneously zero. It means that $\dot{f}(x_0, u_0) = 0$. Due to C.1 is a nonlinear system, there may be more than one equilibrium point [113]. Function f linearised around (x_0, u_0) , yields:

$$f(x, u) = f(x_0, u_0) + J_{f,x} \Big|_{x=x_0} (x - x_0) + J_{f,u} \Big|_{u=u_0} (u - u_0) + f_1(x, u)\tag{C.2}$$

Where $J_{f,x}$ and $J_{f,u}$ are the Jacobians of f with respect to the state and input variables respectively, and f_1 is a remainder that absorbs all the differences that the first order approximation cannot represent [120]. A similar process can be applied to g , obtaining a linear approximation of it.

Let us define $\Delta x = x - x_0$ and $\Delta u = u - u_0$, and let us consider that the remainder functions f_1 and g_1 are neglected. Based on that, C.1 can be rewritten in terms of Δx and Δu as shown C.3.

$$\begin{aligned}\Delta \dot{x} &\approx A \Delta x + B \Delta u \\ \Delta y &\approx C \Delta x + D \Delta u\end{aligned}\tag{C.3}$$

Where $J_{f,x} \triangleq A$, $J_{f,u} \triangleq B$, $J_{g,x} \triangleq C$ and $J_{g,u} \triangleq D$. All of them are constant matrices since the system is time-invariant. Finally, it can be concluded that the linearised form of C.1 corresponds to the system depicted in C.3.

Appendix D: Transformation from $\alpha\beta$ reference frame to dq reference frame

Let us consider the following system in the $\alpha\beta$ reference frame:

$$\vec{V}_{\alpha\beta} = R_1 \vec{I}_{\alpha\beta} + L_1 \frac{d\vec{I}_{\alpha\beta}}{dt} + \vec{E}_{\alpha\beta} \quad \text{D.1}$$

Where $\vec{V}_{\alpha\beta} = V_\alpha + jV_\beta$, $\vec{I}_{\alpha\beta} = I_\alpha + jI_\beta$ and $\vec{E}_{\alpha\beta} = E_\alpha + jE_\beta$. Transforming D.1 to a dq synchronous reference frame, rotating at θ_e (with $\theta_e = \int \omega_e dt$), yields D.2

$$\left[\vec{V}_{\alpha\beta} \right] e^{-j\theta_e} = \left[R_1 \vec{I}_{\alpha\beta} + L_1 \frac{d\vec{I}_{\alpha\beta}}{dt} + \vec{E}_{\alpha\beta} \right] e^{-j\theta_e} \quad \text{D.2}$$

Equation D.2, can be rewritten as:

$$\vec{V}_{dq} = R_1 \vec{I}_{dq} + L_1 \frac{d\vec{I}_{\alpha\beta}}{dt} \cdot e^{-j\theta_e} + \vec{E}_{dq} \quad \text{D.3}$$

Due to θ_e is time-dependent, the term $e^{-j\theta_e}$ cannot be included in the derivative. Using the rule for the derivative of a product, D.4 is obtained.

$$\frac{d\vec{I}_{dq}}{dt} = \frac{d(\vec{I}_{\alpha\beta} \cdot e^{-j\theta_e})}{dt} = e^{-j\theta_e} \frac{d\vec{I}_{\alpha\beta}}{dt} + \vec{I}_{\alpha\beta} \frac{d(e^{-j\theta_e})}{dt} = e^{-j\theta_e} \frac{d\vec{I}_{\alpha\beta}}{dt} - j\omega_e \vec{I}_{dq} \quad \text{D.4}$$

Finally, using D.4, the system D.3 can be written fully in the dq reference frame as shown D.5.

$$\vec{V}_{dq} = R_1 \vec{I}_{dq} + L_1 \frac{d\vec{I}_{dq}}{dt} + j\omega_e L_1 \vec{I}_{dq} + \vec{E}_{dq} \quad \text{D.5}$$

Where $\vec{V}_{dq} = V_d + jV_q$, $\vec{I}_{dq} = I_d + jI_q$ and $\vec{E}_{dq} = E_d + jE_q$. Finally, it can be concluded that the system shown in D.1, in the synchronous dq reference frame (rotating at θ_e), corresponds to that shown in D.5

Appendix E: Comparison Between Proposed Model and Simulation Results

In this appendix, the points used for creating figure Fig. 5.5 are shown. These points were obtained from the proposed model (112) for short circuit ratios shown in Table 5.3 and the PLL designs depicted in Table 5.2.

Short Circuit Ratio: 2.5942

PLL Bandwidth [Hz]	Maximum active current that the inverter can inject to the grid before become unstable (Simulation Results) [A]	Maximum active current that the inverter can inject to the grid before become unstable (Proposed Model Results) [A]
10.277	18	18
20.334	18	18
30.898	18	18
40.723	18	18
51.514	18	18
61.697	18	18
72.136	18	18
82.388	15.7	15.7
92.336	13.4	13.4
102.648	11.6	11.6

Short Circuit Ratio: 2.1652

PLL Bandwidth [Hz]	Maximum active current that the inverter can inject to the grid before become unstable (Simulation Results) [A]	Maximum active current that the inverter can inject to the grid before become unstable (Proposed Model Results) [A]
10.277	18	18
20.334	18	18
30.898	18	18
40.723	18	18
51.514	18	18
61.697	16.4	16.4
72.136	13.2	13.2
82.388	11	11
92.336	9.4	9.4
102.648	8.2	8.2

Short Circuit Ratio: 1.8577

PLL Bandwidth [Hz]	Maximum active current that the inverter can inject to the grid before become unstable (Simulation Results) [A]	Maximum active current that the inverter can inject to the grid before become unstable (Proposed Model Results) [A]
10.277	18	18
20.334	18	18
30.898	18	18
40.723	18	18
51.514	15.7	15.7
61.697	12	12
72.136	9.6	9.6
82.388	8	8
92.336	6.8	6.8
102.648	6	6

Short Circuit Ratio: 1.6265

PLL Bandwidth [Hz]	Maximum active current that the inverter can inject to the grid before become unstable (Simulation Results) [A]	Maximum active current that the inverter can inject to the grid before become unstable (Proposed Model Results) [A]
10.277	18	18
20.334	18	18
30.898	18	18
40.723	17.5	17.5
51.514	11.8	11.8
61.697	8.9	8.9
72.136	7	7
82.388	5.8	5.8
92.336	5	5
102.648	4.5	4.5

Short Circuit Ratio: 1.4463

PLL Bandwidth [Hz]	Maximum active current that the inverter can inject to the grid before become unstable (Simulation Results) [A]	Maximum active current that the inverter can inject to the grid before become unstable (Proposed Model Results) [A]
10.277	18	18
20.334	18	18
30.898	18	18
40.723	13.2	13.2
51.514	8.7	8.7
61.697	6.4	6.4
72.136	5	5
82.388	4.2	4.2
92.336	3.6	3.6
102.648	3.3	3.3

Appendix F: Participation Factor Analysis

In this appendix, the information obtained from the participation factor analysis discussed in the thesis is shown. This analysis is performed for the PLLs designs shown Table 5.2 in and for the short circuit ratios depicted in Table 5.3. In particular, the effects of modes 7 and 8 on the states $\Delta\theta$ and Δg (those related to the PLL modelling) are highlighted.

6.3.1 Analysis for a SCR of 2.5942

PLL Bandwidth: 10.277Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.384729	0.384729	0.406235	0.406235	0.047546	0.047546	0.015909	0.015909	1.95E-06	1.95E-06
ΔI_{1q}^c	0.430992	0.430992	0.360748	0.360748	0.039364	0.039364	0.016821	0.016821	4.23E-07	4.23E-07
$\Delta \gamma_{1d}^c$	0.040194	0.040194	0.043245	0.043245	0.272974	0.272974	0.393638	0.393638	0.009006	0.009006
$\Delta \gamma_{1q}^c$	0.045027	0.045027	0.038403	0.038403	0.226001	0.226001	0.416202	0.416202	0.001949	0.001949
$\Delta\theta$	0.00459	0.00459	0.004364	0.004364	0.029269	0.029269	0.070245	0.070245	0.746894	0.746894
Δg	1.53E-05	1.53E-05	1.47E-05	1.47E-05	0.000734	0.000734	0.003705	0.003705	0.715381	0.715381
ΔE_{1d}^a	0.35057	0.35057	0.428863	0.428863	0.056413	0.056413	0.031638	0.031638	0.000876	0.000876
ΔE_{1q}^a	0.390823	0.390823	0.379107	0.379107	0.045263	0.045263	0.031567	0.031567	8.39E-06	8.39E-06
ΔI_{gd}^a	0.029687	0.029687	0.042067	0.042067	0.46596	0.46596	0.215262	0.215262	2.93E-05	2.93E-05
ΔI_{gq}^a	0.032769	0.032769	0.037033	0.037033	0.443841	0.443841	0.295086	0.295086	0.035048	0.035048

PLL Bandwidth: 20.334Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.366014	0.366014	0.434231	0.434231	0.049037	0.049037	0.014114	0.014114	1.98E-05	1.98E-05
ΔI_{1q}^c	0.457773	0.457773	0.343693	0.343693	0.034032	0.034032	0.015664	0.015664	1.05E-05	1.05E-05
$\Delta \gamma_{1d}^c$	0.0379	0.0379	0.045931	0.045931	0.288422	0.288422	0.378655	0.378655	0.02083	0.02083
$\Delta \gamma_{1q}^c$	0.047402	0.047402	0.036354	0.036354	0.20017	0.20017	0.420259	0.420259	0.011026	0.011026
$\Delta\theta$	0.00944	0.00944	0.008052	0.008052	0.05254	0.05254	0.161386	0.161386	0.797241	0.797241
Δg	6.43E-05	6.43E-05	5.55E-05	5.55E-05	0.002772	0.002772	0.0186	0.0186	0.734337	0.734337
ΔE_{1d}^a	0.333074	0.333074	0.45627	0.45627	0.060175	0.060175	0.032934	0.032934	0.001837	0.001837
ΔE_{1q}^a	0.41266	0.41266	0.357931	0.357931	0.039311	0.039311	0.032735	0.032735	9.12E-05	9.12E-05
ΔI_{gd}^a	0.028157	0.028157	0.044471	0.044471	0.48426	0.48426	0.194635	0.194635	3.09E-04	3.09E-04
ΔI_{gq}^a	0.034217	0.034217	0.034598	0.034598	0.438926	0.438926	0.362239	0.362239	0.072839	0.072839

PLL Bandwidth: 30.898Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.47308	0.47308	0.347522	0.347522	0.050966	0.050966	0.012373	0.012373	9.25E-05	9.25E-05
ΔI_{1q}^c	0.327361	0.327361	0.494471	0.494471	0.029138	0.029138	0.014356	0.014356	9.05E-05	9.05E-05
$\Delta \gamma_{1d}^c$	0.049741	0.049741	0.03561	0.03561	0.308707	0.308707	0.361857	0.361857	0.039118	0.039118
$\Delta \gamma_{1q}^c$	0.03442	0.03442	0.050667	0.050667	0.176489	0.176489	0.419869	0.419869	0.038232	0.038232
$\Delta \theta$	0.01172	0.01172	0.015565	0.015565	0.074324	0.074324	0.303839	0.303839	0.898107	0.898107
Δg	1.18E-04	1.18E-04	1.55E-04	1.55E-04	0.005917	0.005917	0.054267	0.054267	0.800059	0.800059
ΔE_{1d}^a	0.494254	0.494254	0.316059	0.316059	0.064594	0.064594	0.035282	0.035282	0.003037	0.003037
ΔE_{1q}^a	0.337346	0.337346	0.443241	0.443241	0.033674	0.033674	0.034993	0.034993	4.32E-04	4.32E-04
ΔI_{gd}^a	0.047831	0.047831	0.026693	0.026693	0.508393	0.508393	0.174907	0.174907	1.44E-03	1.44E-03
ΔI_{gq}^a	0.03222	0.03222	0.036342	0.036342	0.434436	0.434436	0.461812	0.461812	0.119016	0.119016

PLL Bandwidth: 40.723Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.513946	0.513946	0.332302	0.332302	0.052853	0.052853	0.010865	0.010865	2.85E-04	2.85E-04
ΔI_{1q}^c	0.314274	0.314274	0.532601	0.532601	0.025521	0.025521	0.013122	0.013122	4.31E-04	4.31E-04
$\Delta \gamma_{1d}^c$	0.053812	0.053812	0.033726	0.033726	0.32902	0.32902	0.341721	0.341721	0.062355	0.062355
$\Delta \gamma_{1q}^c$	0.032906	0.032906	0.054055	0.054055	0.158875	0.158875	0.412701	0.412701	0.094142	0.094142
$\Delta \theta$	0.014535	0.014535	0.021617	0.021617	0.089518	0.089518	0.473417	0.473417	0.938754	0.938754
Δg	2.00E-04	2.00E-04	2.93E-04	2.93E-04	0.010006	0.010006	0.1193	0.1193	0.82784	0.82784
ΔE_{1d}^a	0.534129	0.534129	0.302278	0.302278	0.068513	0.068513	0.037562	0.037562	0.004278	0.004278
ΔE_{1q}^a	0.320827	0.320827	0.475489	0.475489	0.029329	0.029329	0.037705	0.037705	1.29E-03	1.29E-03
ΔI_{gd}^a	0.051375	0.051375	0.025525	0.025525	0.532299	0.532299	0.157697	0.157697	4.38E-03	4.38E-03
ΔI_{gq}^a	0.030323	0.030323	0.038647	0.038647	0.429186	0.429186	0.574095	0.574095	0.163784	0.163784

PLL Bandwidth: 51.514Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.574339	0.574339	0.311533	0.311533	0.055267	0.055267	0.008866	0.008866	5.76E-04	5.76E-04
ΔI_{1q}^c	0.296551	0.296551	0.588066	0.588066	0.021711	0.021711	0.011039	0.011039	1.28E-03	1.28E-03
$\Delta \gamma_{1d}^c$	0.059907	0.059907	0.031223	0.031223	0.355989	0.355989	0.279129	0.279129	0.078801	0.078801
$\Delta \gamma_{1q}^c$	0.030932	0.030932	0.058939	0.058939	0.139847	0.139847	0.347553	0.347553	0.175821	0.175821
$\Delta \theta$	0.017474	0.017474	0.030289	0.030289	0.104958	0.104958	0.616734	0.616734	0.877894	0.877894
Δg	2.94E-04	2.94E-04	5.00E-04	5.00E-04	0.014766	0.014766	0.183431	0.183431	0.814401	0.814401
ΔE_{1d}^a	0.592835	0.592835	0.283644	0.283644	0.073372	0.073372	0.037296	0.037296	0.004446	0.004446
ΔE_{1q}^a	0.299151	0.299151	0.522799	0.522799	0.024691	0.024691	0.039978	0.039978	2.31E-03	2.31E-03
ΔI_{gd}^a	0.056602	0.056602	0.023962	0.023962	0.563336	0.563336	0.13301	0.13301	8.61E-03	8.61E-03
ΔI_{gq}^a	0.027912	0.027912	0.04208	0.04208	0.42198	0.42198	0.659405	0.659405	0.172172	0.172172

PLL Bandwidth: 61.697Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.635369	0.635369	0.288522	0.288522	0.05746	0.05746	0.007348	0.007348	7.98E-04	7.98E-04
ΔI_{1q}^c	0.276618	0.276618	0.642769	0.642769	0.018672	0.018672	0.009586	0.009586	2.35E-03	2.35E-03
$\Delta \gamma_{1d}^c$	0.066134	0.066134	0.028568	0.028568	0.381841	0.381841	0.216328	0.216328	0.079993	0.079993
$\Delta \gamma_{1q}^c$	0.028793	0.028793	0.063643	0.063643	0.124083	0.124083	0.282186	0.282186	0.236009	0.236009
$\Delta \theta$	0.019417	0.019417	0.039215	0.039215	0.114871	0.114871	0.652195	0.652195	0.719861	0.719861
Δg	3.93E-04	3.93E-04	7.74E-04	7.74E-04	0.019961	0.019961	0.215497	0.215497	0.733377	0.733377
ΔE_{1d}^a	0.651778	0.651778	0.26302	0.26302	0.077518	0.077518	0.034488	0.034488	0.003805	0.003805
ΔE_{1q}^a	0.276053	0.276053	0.569573	0.569573	0.020877	0.020877	0.041658	0.041658	2.93E-03	2.93E-03
ΔI_{gd}^a	0.061836	0.061836	0.022236	0.022236	0.592028	0.592028	0.113344	0.113344	1.17E-02	1.17E-02
ΔI_{gq}^a	0.025463	0.025463	0.045476	0.045476	0.411689	0.411689	0.681556	0.681556	0.151039	0.151039

PLL Bandwidth: 72.136Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.697389	0.697389	0.258465	0.258465	0.059564	0.059564	0.006381	0.006381	9.44E-04	9.44E-04
ΔI_{1q}^c	0.249871	0.249871	0.696229	0.696229	0.015885	0.015885	0.008905	0.008905	3.43E-03	3.43E-03
$\Delta \gamma_{1d}^c$	0.072528	0.072528	0.025251	0.025251	0.408379	0.408379	0.168724	0.168724	0.075666	0.075666
$\Delta \gamma_{1q}^c$	0.025987	0.025987	0.06802	0.06802	0.108912	0.108912	0.235488	0.235488	0.274995	0.274995
$\Delta \theta$	0.020426	0.020426	0.049047	0.049047	0.12142	0.12142	0.647687	0.647687	0.579412	0.579412
Δg	4.85E-04	4.85E-04	1.13E-03	1.13E-03	0.025455	0.025455	0.227595	0.227595	0.660151	0.660151
ΔE_{1d}^a	0.711051	0.711051	0.235929	0.235929	0.081328	0.081328	0.031651	0.031651	0.003089	0.003089
ΔE_{1q}^a	0.246677	0.246677	0.614961	0.614961	0.017332	0.017332	0.044749	0.044749	3.22E-03	3.22E-03
ΔI_{gd}^a	0.067062	0.067062	0.019961	0.019961	0.620156	0.620156	0.100848	0.100848	1.36E-02	1.36E-02
ΔI_{gq}^a	0.022501	0.022501	0.048703	0.048703	0.397425	0.397425	0.691106	0.691106	0.127151	0.127151

PLL Bandwidth: 82.388Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.223404	0.223404	0.748036	0.748036	0.061368	0.061368	0.005768	0.005768	1.06E-03	1.06E-03
ΔI_{1q}^c	0.73685	0.73685	0.217845	0.217845	0.013442	0.013442	0.008728	0.008728	4.44E-03	4.44E-03
$\Delta \gamma_{1d}^c$	0.021523	0.021523	0.077818	0.077818	0.433216	0.433216	0.135949	0.135949	0.071735	0.071735
$\Delta \gamma_{1q}^c$	0.07099	0.07099	0.022662	0.022662	0.094893	0.094893	0.205722	0.205722	0.301323	0.301323
$\Delta \theta$	0.058528	0.058528	0.020307	0.020307	0.124545	0.124545	0.636999	0.636999	0.479689	0.479689
Δg	1.53E-03	1.53E-03	5.53E-04	5.53E-04	0.030671	0.030671	0.232565	0.232565	0.608502	0.608502
ΔE_{1d}^a	0.204111	0.204111	0.758597	0.758597	0.084464	0.084464	0.02936	0.02936	0.002574	0.002574
ΔE_{1q}^a	0.648583	0.648583	0.212906	0.212906	0.014219	0.014219	0.049114	0.049114	3.41E-03	3.41E-03
ΔI_{gd}^a	0.017276	0.017276	0.07119	0.07119	0.645005	0.645005	0.093167	0.093167	1.51E-02	1.51E-02
ΔI_{gq}^a	0.050942	0.050942	0.019227	0.019227	0.379995	0.379995	0.703536	0.703536	0.110059	0.110059

PLL Bandwidth: 92.336Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.781544	0.781544	0.188173	0.188173	0.0628	0.0628	0.005319	0.005319	1.16E-03	1.16E-03
ΔI_{1q}^c	0.185002	0.185002	0.760068	0.760068	0.011349	0.011349	0.008776	0.008776	5.39E-03	5.39E-03
$\Delta \gamma_{1d}^c$	0.081385	0.081385	0.017876	0.017876	0.45515	0.45515	0.112388	0.112388	0.069222	0.069222
$\Delta \gamma_{1q}^c$	0.019265	0.019265	0.072206	0.072206	0.082254	0.082254	0.185429	0.185429	0.320894	0.320894
$\Delta \theta$	0.019335	0.019335	0.066789	0.066789	0.124865	0.124865	0.626696	0.626696	0.409316	0.409316
Δg	5.91E-04	5.91E-04	1.95E-03	1.95E-03	0.035321	0.035321	0.234891	0.234891	0.571864	0.571864
ΔE_{1d}^a	0.789095	0.789095	0.17197	0.17197	0.086838	0.086838	0.027419	0.027419	0.002227	0.002227
ΔE_{1q}^a	0.179189	0.179189	0.666386	0.666386	0.011574	0.011574	0.05416	0.05416	3.59E-03	3.59E-03
ΔI_{gd}^a	0.073762	0.073762	0.014553	0.014553	0.665496	0.665496	0.08769	0.08769	1.64E-02	1.64E-02
ΔI_{gq}^a	0.016045	0.016045	0.051888	0.051888	0.360636	0.360636	0.717561	0.717561	0.09867	0.09867

PLL Bandwidth: 102.648Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.155072	0.155072	0.801058	0.801058	0.063934	0.063934	0.004922	0.004922	1.28E-03	1.28E-03
ΔI_{1q}^c	0.769331	0.769331	0.153662	0.153662	0.009471	0.009471	0.008916	0.008916	6.38E-03	6.38E-03
$\Delta \gamma_{1d}^c$	0.014518	0.014518	0.083536	0.083536	0.474961	0.474961	0.093515	0.093515	0.067907	0.067907
$\Delta \gamma_{1q}^c$	0.072026	0.072026	0.016024	0.016024	0.070359	0.070359	0.16938	0.16938	0.337685	0.337685
$\Delta \theta$	0.074097	0.074097	0.017879	0.017879	0.122977	0.122977	0.61633	0.61633	0.354512	0.354512
Δg	2.39E-03	2.39E-03	6.09E-04	6.09E-04	0.039612	0.039612	0.236114	0.236114	0.542565	0.542565
ΔE_{1d}^a	0.141662	0.141662	0.805844	0.805844	0.088591	0.088591	0.02556	0.02556	0.001977	0.001977
ΔE_{1q}^a	0.67133	0.67133	0.147585	0.147585	0.009235	0.009235	0.059849	0.059849	3.80E-03	3.80E-03
ΔI_{gd}^a	0.011981	0.011981	0.075091	0.075091	0.68256	0.68256	0.082893	0.082893	1.79E-02	1.79E-02
ΔI_{gq}^a	0.051771	0.051771	0.013117	0.013117	0.339177	0.339177	0.731626	0.731626	0.09059	0.09059

6.3.2 Analysis for a SCR of 2.1652

PLL Bandwidth: 10.277Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.382551	0.382551	0.40109	0.40109	0.035645	0.035645	0.011566	0.011566	2.87E-06	2.87E-06
ΔI_{1q}^c	0.425015	0.425015	0.359022	0.359022	0.029339	0.029339	0.012097	0.012097	5.15E-07	5.15E-07
$\Delta \gamma_{1d}^c$	0.039304	0.039304	0.041961	0.041961	0.245382	0.245382	0.372356	0.372356	0.0136	0.0136
$\Delta \gamma_{1q}^c$	0.043666	0.043666	0.03756	0.03756	0.201975	0.201975	0.389426	0.389426	0.002438	0.002438
$\Delta \theta$	0.00428	0.00428	0.004098	0.004098	0.027717	0.027717	0.082296	0.082296	0.747237	0.747237
Δg	1.42E-05	1.42E-05	1.37E-05	1.37E-05	0.000761	0.000761	0.00495	0.00495	0.707033	0.707033
ΔE_{1d}^a	0.3435	0.3435	0.41596	0.41596	0.047888	0.047888	0.03206	0.03206	0.001328	0.001328
ΔE_{1q}^a	0.379874	0.379874	0.370715	0.370715	0.038272	0.038272	0.031715	0.031715	1.40E-05	1.40E-05
ΔI_{gd}^a	0.023709	0.023709	0.033168	0.033168	0.437724	0.437724	0.197634	0.197634	5.35E-05	5.35E-05
ΔI_{gq}^a	0.025956	0.025956	0.029422	0.029422	0.417192	0.417192	0.28353	0.28353	0.044042	0.044042

PLL Bandwidth: 20.334Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.365601	0.365601	0.42707	0.42707	0.036856	0.036856	0.010185	0.010185	3.10E-05	3.10E-05
ΔI_{1q}^c	0.449828	0.449828	0.343395	0.343395	0.025302	0.025302	0.010999	0.010999	1.44E-05	1.44E-05
$\Delta \gamma_{1d}^c$	0.037252	0.037252	0.044406	0.044406	0.259582	0.259582	0.366548	0.366548	0.032582	0.032582
$\Delta \gamma_{1q}^c$	0.045834	0.045834	0.035705	0.035705	0.178208	0.178208	0.395834	0.395834	0.01514	0.01514
$\Delta \theta$	0.008775	0.008775	0.007593	0.007593	0.049592	0.049592	0.1959	0.1959	0.822207	0.822207
Δg	5.93E-05	5.93E-05	5.18E-05	5.18E-05	0.002861	0.002861	0.026073	0.026073	0.738352	0.738352
ΔE_{1d}^a	0.327912	0.327912	0.441	0.441	0.051289	0.051289	0.033978	0.033978	0.002892	0.002892
ΔE_{1q}^a	0.399855	0.399855	0.351597	0.351597	0.033268	0.033268	0.033	0.033	1.74E-04	1.74E-04
ΔI_{gd}^a	0.022606	0.022606	0.034961	0.034961	0.456072	0.456072	0.177866	0.177866	5.97E-04	5.97E-04
ΔI_{gq}^a	0.027029	0.027029	0.027615	0.027615	0.413978	0.413978	0.363694	0.363694	0.094819	0.094819

PLL Bandwidth: 30.898Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.463259	0.463259	0.349142	0.349142	0.038334	0.038334	0.008824	0.008824	1.57E-04	1.57E-04
ΔI_{1q}^c	0.328711	0.328711	0.48404	0.48404	0.02158	0.02158	0.009662	0.009662	1.39E-04	1.39E-04
$\Delta \gamma_{1d}^c$	0.047887	0.047887	0.03523	0.03523	0.277523	0.277523	0.359885	0.359885	0.063821	0.063821
$\Delta \gamma_{1q}^c$	0.033979	0.033979	0.048843	0.048843	0.156229	0.156229	0.394058	0.394058	0.056464	0.056464
$\Delta \theta$	0.011109	0.011109	0.014423	0.014423	0.069797	0.069797	0.385343	0.385343	0.957416	0.957416
Δg	1.11E-04	1.11E-04	1.43E-04	1.43E-04	0.006065	0.006065	0.080317	0.080317	0.822157	0.822157
ΔE_{1d}^a	0.475844	0.475844	0.313025	0.313025	0.05519	0.05519	0.037471	0.037471	0.004962	0.004962
ΔE_{1q}^a	0.33325	0.33325	0.428042	0.428042	0.028497	0.028497	0.035485	0.035485	8.72E-04	8.72E-04
ΔI_{gd}^a	0.037477	0.037477	0.02157	0.02157	0.479024	0.479024	0.15874	0.15874	3.00E-03	3.00E-03
ΔI_{gq}^a	0.025864	0.025864	0.028621	0.028621	0.41062	0.41062	0.490066	0.490066	0.160158	0.160158

PLL Bandwidth: 40.723Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.501553	0.501553	0.335905	0.335905	0.039721	0.039721	0.007458	0.007458	4.84E-04	4.84E-04
ΔI_{1q}^c	0.317268	0.317268	0.519881	0.519881	0.01882	0.01882	0.008169	0.008169	6.89E-04	6.89E-04
$\Delta \gamma_{1d}^c$	0.05163	0.05163	0.033597	0.033597	0.295039	0.295039	0.334404	0.334404	0.099588	0.099588
$\Delta \gamma_{1q}^c$	0.032659	0.032659	0.051998	0.051998	0.139792	0.139792	0.366266	0.366266	0.141594	0.141594
$\Delta \theta$	0.013851	0.013851	0.019988	0.019988	0.083678	0.083678	0.594654	0.594654	0.982649	0.982649
Δg	1.89E-04	1.89E-04	2.69E-04	2.69E-04	0.010186	0.010186	0.172911	0.172911	0.843062	0.843062
ΔE_{1d}^a	0.512647	0.512647	0.301261	0.301261	0.058587	0.058587	0.039919	0.039919	0.006707	0.006707
ΔE_{1q}^a	0.31881	0.31881	0.458035	0.458035	0.024791	0.024791	0.037681	0.037681	2.52E-03	2.52E-03
ΔI_{gd}^a	0.040145	0.040145	0.020765	0.020765	0.500975	0.500975	0.138647	0.138647	9.07E-03	9.07E-03
ΔI_{gq}^a	0.024484	0.024484	0.030369	0.030369	0.406058	0.406058	0.619948	0.619948	0.211445	0.211445

PLL Bandwidth: 51.514Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.55892	0.55892	0.318149	0.318149	0.041425	0.041425	0.005812	0.005812	7.46E-04	7.46E-04
ΔI_{1q}^c	0.302142	0.302142	0.572887	0.572887	0.015934	0.015934	0.006367	0.006367	1.62E-03	1.62E-03
$\Delta \gamma_{1d}^c$	0.05731	0.05731	0.031454	0.031454	0.317549	0.317549	0.24951	0.24951	0.100606	0.100606
$\Delta \gamma_{1q}^c$	0.030981	0.030981	0.056639	0.056639	0.122149	0.122149	0.273318	0.273318	0.21899	0.21899
$\Delta \theta$	0.016803	0.016803	0.027977	0.027977	0.097511	0.097511	0.649613	0.649613	0.807965	0.807965
Δg	2.80E-04	2.80E-04	4.59E-04	4.59E-04	0.014898	0.014898	0.212994	0.212994	0.761731	0.761731
ΔE_{1d}^a	0.567605	0.567605	0.28565	0.28565	0.062707	0.062707	0.03652	0.03652	0.005501	0.005501
ΔE_{1q}^a	0.300202	0.300202	0.502832	0.502832	0.020863	0.020863	0.037978	0.037978	3.47E-03	3.47E-03
ΔI_{gd}^a	0.044137	0.044137	0.019708	0.019708	0.528434	0.528434	0.112045	0.112045	1.36E-02	1.36E-02
ΔI_{gq}^a	0.022755	0.022755	0.03303	0.03303	0.399603	0.399603	0.650004	0.650004	0.180178	0.180178

PLL Bandwidth: 61.697Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.61849	0.61849	0.298405	0.298405	0.042938	0.042938	0.004973	0.004973	8.76E-04	8.76E-04
ΔI_{1q}^c	0.285157	0.285157	0.626854	0.626854	0.013651	0.013651	0.005667	0.005667	2.56E-03	2.56E-03
$\Delta \gamma_{1d}^c$	0.063272	0.063272	0.029173	0.029173	0.338785	0.338785	0.192335	0.192335	0.089859	0.089859
$\Delta \gamma_{1q}^c$	0.029172	0.029172	0.061284	0.061284	0.107706	0.107706	0.219187	0.219187	0.262398	0.262398
$\Delta \theta$	0.018886	0.018886	0.036297	0.036297	0.10629	0.10629	0.634236	0.634236	0.636473	0.636473
Δg	3.79E-04	3.79E-04	7.12E-04	7.12E-04	0.019998	0.019998	0.224399	0.224399	0.676132	0.676132
ΔE_{1d}^a	0.624359	0.624359	0.268318	0.268318	0.066193	0.066193	0.033109	0.033109	0.004184	0.004184
ΔE_{1q}^a	0.280416	0.280416	0.548651	0.548651	0.017639	0.017639	0.039958	0.039958	3.80E-03	3.80E-03
ΔI_{gd}^a	0.048252	0.048252	0.018537	0.018537	0.553327	0.553327	0.098604	0.098604	1.57E-02	1.57E-02
ΔI_{gq}^a	0.021006	0.021006	0.035767	0.035767	0.390461	0.390461	0.654288	0.654288	0.143311	0.143311

PLL Bandwidth: 72.136Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.68171	0.68171	0.271643	0.271643	0.044374	0.044374	0.004522	0.004522	9.55E-04	9.55E-04
ΔI_{1q}^c	0.261518	0.261518	0.682358	0.682358	0.011582	0.011582	0.005504	0.005504	3.48E-03	3.48E-03
$\Delta \gamma_{1d}^c$	0.069661	0.069661	0.02623	0.02623	0.360406	0.360406	0.154285	0.154285	0.079945	0.079945
$\Delta \gamma_{1q}^c$	0.026723	0.026723	0.065889	0.065889	0.094065	0.094065	0.187793	0.187793	0.291108	0.291108
$\Delta \theta$	0.02016	0.02016	0.045673	0.045673	0.112122	0.112122	0.616479	0.616479	0.514891	0.514891
Δg	4.75E-04	4.75E-04	1.05E-03	1.05E-03	0.025363	0.025363	0.228871	0.228871	0.617126	0.617126
ΔE_{1d}^a	0.684057	0.684057	0.244658	0.244658	0.069415	0.069415	0.030624	0.030624	0.003245	0.003245
ΔE_{1q}^a	0.254486	0.254486	0.595637	0.595637	0.014665	0.014665	0.043984	0.043984	3.93E-03	3.93E-03
ΔI_{gd}^a	0.052554	0.052554	0.016926	0.016926	0.577525	0.577525	0.091975	0.091975	1.69E-02	1.69E-02
ΔI_{gq}^a	0.018841	0.018841	0.038542	0.038542	0.378117	0.378117	0.668284	0.668284	0.116658	0.116658

PLL Bandwidth: 82.388Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.736254	0.736254	0.238495	0.238495	0.045615	0.045615	0.00424	0.00424	1.02E-03	1.02E-03
ΔI_{1q}^c	0.231426	0.231426	0.727588	0.727588	0.00979	0.00979	0.005586	0.005586	4.37E-03	4.37E-03
$\Delta \gamma_{1d}^c$	0.075237	0.075237	0.022731	0.022731	0.380714	0.380714	0.128045	0.128045	0.07307	0.07307
$\Delta \gamma_{1q}^c$	0.023649	0.023649	0.069348	0.069348	0.081708	0.081708	0.168682	0.168682	0.311797	0.311797
$\Delta \theta$	0.020331	0.020331	0.054973	0.054973	0.115073	0.115073	0.604648	0.604648	0.432457	0.432457
Δg	5.48E-04	5.48E-04	1.43E-03	1.43E-03	0.030466	0.030466	0.231848	0.231848	0.577279	0.577279
ΔE_{1d}^a	0.734784	0.734784	0.215094	0.215094	0.072137	0.072137	0.028727	0.028727	0.002644	0.002644
ΔE_{1q}^a	0.222988	0.222988	0.633325	0.633325	0.01207	0.01207	0.049199	0.049199	4.05E-03	4.05E-03
ΔI_{gd}^a	0.056162	0.056162	0.014898	0.014898	0.59904	0.59904	0.088322	0.088322	1.79E-02	1.79E-02
ΔI_{gq}^a	0.016331	0.016331	0.040675	0.040675	0.363379	0.363379	0.689268	0.689268	0.09949	0.09949

PLL Bandwidth: 92.336Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.774148	0.774148	0.203115	0.203115	0.046626	0.046626	0.004025	0.004025	1.09E-03	1.09E-03
ΔI_{1q}^c	0.198612	0.198612	0.755642	0.755642	0.008268	0.008268	0.005767	0.005767	5.22E-03	5.22E-03
$\Delta \gamma_{1d}^c$	0.079176	0.079176	0.019105	0.019105	0.398941	0.398941	0.108661	0.108661	0.068607	0.068607
$\Delta \gamma_{1q}^c$	0.020313	0.020313	0.071076	0.071076	0.070742	0.070742	0.155716	0.155716	0.327868	0.327868
$\Delta \theta$	0.019553	0.019553	0.063226	0.063226	0.11571	0.11571	0.596766	0.596766	0.374579	0.374579
Δg	5.92E-04	5.92E-04	1.84E-03	1.84E-03	0.035057	0.035057	0.234234	0.234234	0.548753	0.548753
ΔE_{1d}^a	0.769107	0.769107	0.183325	0.183325	0.074306	0.074306	0.027115	0.027115	0.002251	0.002251
ΔE_{1q}^a	0.189676	0.189676	0.655602	0.655602	0.009869	0.009869	0.05502	0.05502	4.18E-03	4.18E-03
ΔI_{gd}^a	0.058543	0.058543	0.012706	0.012706	0.617194	0.617194	0.085765	0.085765	1.89E-02	1.89E-02
ΔI_{gq}^a	0.013761	0.013761	0.041778	0.041778	0.347222	0.347222	0.712288	0.712288	0.088331	0.088331

PLL Bandwidth: 102.648Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.168351	0.168351	0.796844	0.796844	0.047467	0.047467	0.003821	0.003821	1.17E-03	1.17E-03
ΔI_{1q}^c	0.768366	0.768366	0.165846	0.165846	0.006907	0.006907	0.005995	0.005995	6.11E-03	6.11E-03
$\Delta \gamma_{1d}^c$	0.015617	0.015617	0.081606	0.081606	0.415885	0.415885	0.092678	0.092678	0.065685	0.065685
$\Delta \gamma_{1q}^c$	0.071276	0.071276	0.016985	0.016985	0.060517	0.060517	0.145415	0.145415	0.342053	0.342053
$\Delta \theta$	0.070531	0.070531	0.018168	0.018168	0.114532	0.114532	0.590503	0.590503	0.329394	0.329394
Δg	2.27E-03	2.27E-03	6.13E-04	6.13E-04	0.039357	0.039357	0.236471	0.236471	0.525493	0.525493
ΔE_{1d}^a	0.151961	0.151961	0.788647	0.788647	0.076057	0.076057	0.025555	0.025555	0.001969	0.001969
ΔE_{1q}^a	0.663958	0.663958	0.157069	0.157069	0.007918	0.007918	0.061552	0.061552	4.36E-03	4.36E-03
ΔI_{gd}^a	0.010533	0.010533	0.059829	0.059829	0.632939	0.632939	0.083414	0.083414	2.02E-02	2.02E-02
ΔI_{gq}^a	0.041942	0.041942	0.011299	0.011299	0.329342	0.329342	0.736208	0.736208	0.080434	0.080434

6.3.3 Analysis for a SCR of 1.8577

PLL Bandwidth: 10.277Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.381065	0.381065	0.397402	0.397402	0.028827	0.028827	0.009008	0.009008	3.89E-06	3.89E-06
ΔI_{1q}^c	0.420902	0.420902	0.357752	0.357752	0.023611	0.023611	0.009283	0.009283	5.95E-07	5.95E-07
$\Delta \gamma_{1d}^c$	0.03872	0.03872	0.041095	0.041095	0.227625	0.227625	0.363961	0.363961	0.019222	0.019222
$\Delta \gamma_{1q}^c$	0.042768	0.042768	0.036995	0.036995	0.186438	0.186438	0.375066	0.375066	0.00294	0.00294
$\Delta \theta$	0.004014	0.004014	0.003865	0.003865	0.026548	0.026548	0.095592	0.095592	0.745045	0.745045
Δg	1.32E-05	1.32E-05	1.28E-05	1.28E-05	0.00078	0.00078	0.00644	0.00644	0.695643	0.695643
ΔE_{1d}^a	0.338878	0.338878	0.407397	0.407397	0.042816	0.042816	0.032661	0.032661	0.001888	0.001888
ΔE_{1q}^a	0.372695	0.372695	0.365258	0.365258	0.034128	0.034128	0.031922	0.031922	2.24E-05	2.24E-05
ΔI_{gd}^a	0.019859	0.019859	0.027532	0.027532	0.422293	0.422293	0.185653	0.185653	8.66E-05	8.66E-05
ΔI_{gq}^a	0.021626	0.021626	0.024565	0.024565	0.402665	0.402665	0.278588	0.278588	0.053742	0.053742

PLL Bandwidth: 20.334Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.365178	0.365178	0.421707	0.421707	0.029848	0.029848	0.007885	0.007885	4.54E-05	4.54E-05
ΔI_{1q}^c	0.444098	0.444098	0.342977	0.342977	0.020302	0.020302	0.008206	0.008206	1.85E-05	1.85E-05
$\Delta \gamma_{1d}^c$	0.036819	0.036819	0.043353	0.043353	0.240995	0.240995	0.369126	0.369126	0.048073	0.048073
$\Delta \gamma_{1q}^c$	0.044776	0.044776	0.035259	0.035259	0.163917	0.163917	0.384195	0.384195	0.019588	0.019588
$\Delta \theta$	0.008208	0.008208	0.007179	0.007179	0.047306	0.047306	0.237377	0.237377	0.850542	0.850542
Δg	5.52E-05	5.52E-05	4.87E-05	4.87E-05	0.002919	0.002919	0.035931	0.035931	0.741592	0.741592
ΔE_{1d}^a	0.324408	0.324408	0.430585	0.430585	0.045934	0.045934	0.035483	0.035483	0.004313	0.004313
ΔE_{1q}^a	0.391221	0.391221	0.347421	0.347421	0.029645	0.029645	0.033341	0.033341	3.01E-04	3.01E-04
ΔI_{gd}^a	0.018993	0.018993	0.028944	0.028944	0.440654	0.440654	0.166515	0.166515	1.04E-03	1.04E-03
ΔI_{gq}^a	0.022467	0.022467	0.023131	0.023131	0.400396	0.400396	0.37528	0.37528	0.120949	0.120949

PLL Bandwidth: 30.898Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.349869	0.349869	0.455398	0.455398	0.031045	0.031045	0.006743	0.006743	2.57E-04	2.57E-04
ΔI_{1q}^c	0.475964	0.475964	0.329184	0.329184	0.017244	0.017244	0.006811	0.006811	2.08E-04	2.08E-04
$\Delta \gamma_{1d}^c$	0.034956	0.034956	0.046552	0.046552	0.257393	0.257393	0.376621	0.376621	0.099776	0.099776
$\Delta \gamma_{1q}^c$	0.047555	0.047555	0.03365	0.03365	0.142967	0.142967	0.380417	0.380417	0.08074	0.08074
$\Delta \theta$	0.013443	0.013443	0.010532	0.010532	0.066205	0.066205	0.495386	0.495386	1.034894	1.034894
Δg	1.32E-04	1.32E-04	1.05E-04	1.05E-04	0.006146	0.006146	0.119077	0.119077	0.849707	0.849707
ΔE_{1d}^a	0.310688	0.310688	0.462693	0.462693	0.049444	0.049444	0.04085	0.04085	0.007797	0.007797
ΔE_{1q}^a	0.417259	0.417259	0.330382	0.330382	0.025353	0.025353	0.036112	0.036112	1.65E-03	1.65E-03
ΔI_{gd}^a	0.018185	0.018185	0.030913	0.030913	0.462786	0.462786	0.147495	0.147495	5.79E-03	5.79E-03
ΔI_{gq}^a	0.023713	0.023713	0.021745	0.021745	0.397573	0.397573	0.541875	0.541875	0.213982	0.213982

PLL Bandwidth: 40.723Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.490931	0.490931	0.337766	0.337766	0.032133	0.032133	0.005285	0.005285	7.22E-04	7.22E-04
ΔI_{1q}^c	0.318631	0.318631	0.509284	0.509284	0.014976	0.014976	0.005068	0.005068	9.90E-04	9.90E-04
$\Delta \gamma_{1d}^c$	0.04998	0.04998	0.033471	0.033471	0.273048	0.273048	0.327958	0.327958	0.140443	0.140443
$\Delta \gamma_{1q}^c$	0.032438	0.032438	0.050467	0.050467	0.127257	0.127257	0.314509	0.314509	0.192617	0.192617
$\Delta \theta$	0.01317	0.01317	0.018566	0.018566	0.078985	0.078985	0.704375	0.704375	0.983572	0.983572
Δg	1.79E-04	1.79E-04	2.49E-04	2.49E-04	0.010256	0.010256	0.230357	0.230357	0.825174	0.825174
ΔE_{1d}^a	0.496503	0.496503	0.300034	0.300034	0.052453	0.052453	0.041762	0.041762	0.009274	0.009274
ΔE_{1q}^a	0.317149	0.317149	0.444918	0.444918	0.022008	0.022008	0.036417	0.036417	4.21E-03	4.21E-03
ΔI_{gd}^a	0.032995	0.032995	0.017569	0.017569	0.483365	0.483365	0.120287	0.120287	1.59E-02	1.59E-02
ΔI_{gq}^a	0.020662	0.020662	0.02508	0.02508	0.39331	0.39331	0.663471	0.663471	0.250092	0.250092

PLL Bandwidth: 51.514Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.544282	0.544282	0.32195	0.32195	0.033426	0.033426	0.0041	0.0041	8.62E-04	8.62E-04
ΔI_{1q}^c	0.305113	0.305113	0.55877	0.55877	0.01262	0.01262	0.003896	0.003896	1.84E-03	1.84E-03
$\Delta \gamma_{1d}^c$	0.055193	0.055193	0.031562	0.031562	0.292619	0.292619	0.230614	0.230614	0.11637	0.11637
$\Delta \gamma_{1q}^c$	0.03094	0.03094	0.054779	0.054779	0.110482	0.110482	0.21912	0.21912	0.247842	0.247842
$\Delta \theta$	0.016064	0.016064	0.025889	0.025889	0.091464	0.091464	0.660673	0.660673	0.73518	0.73518
Δg	2.67E-04	2.67E-04	4.23E-04	4.23E-04	0.014875	0.014875	0.233186	0.233186	0.706253	0.706253
ΔE_{1d}^a	0.547125	0.547125	0.286282	0.286282	0.056032	0.056032	0.03589	0.03589	0.006247	0.006247
ΔE_{1q}^a	0.300493	0.300493	0.486426	0.486426	0.018483	0.018483	0.03606	0.03606	4.59E-03	4.59E-03
ΔI_{gd}^a	0.036119	0.036119	0.016783	0.016783	0.508339	0.508339	0.096927	0.096927	1.85E-02	1.85E-02
ΔI_{gq}^a	0.019328	0.019328	0.027176	0.027176	0.387154	0.387154	0.646173	0.646173	0.17982	0.17982

PLL Bandwidth: 61.697Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.600476	0.600476	0.304771	0.304771	0.034549	0.034549	0.003663	0.003663	9.24E-04	9.24E-04
ΔI_{1q}^c	0.290387	0.290387	0.610045	0.610045	0.010771	0.010771	0.003631	0.003631	2.65E-03	2.65E-03
$\Delta \gamma_{1d}^c$	0.060741	0.060741	0.029571	0.029571	0.310789	0.310789	0.181111	0.181111	0.097379	0.097379
$\Delta \gamma_{1q}^c$	0.029374	0.029374	0.05919	0.05919	0.096891	0.096891	0.17953	0.17953	0.279727	0.279727
$\Delta \theta$	0.018201	0.018201	0.033534	0.033534	0.099263	0.099263	0.625767	0.625767	0.577852	0.577852
Δg	3.63E-04	3.63E-04	6.55E-04	6.55E-04	0.019833	0.019833	0.233971	0.233971	0.634652	0.634652
ΔE_{1d}^a	0.600197	0.600197	0.271396	0.271396	0.059031	0.059031	0.032671	0.032671	0.004508	0.004508
ΔE_{1q}^a	0.283218	0.283218	0.529688	0.529688	0.015606	0.015606	0.039056	0.039056	4.67E-03	4.67E-03
ΔI_{gd}^a	0.039389	0.039389	0.015935	0.015935	0.53057	0.53057	0.089186	0.089186	1.96E-02	1.96E-02
ΔI_{gq}^a	0.018007	0.018007	0.029378	0.029378	0.378649	0.378649	0.652614	0.652614	0.137408	0.137408

PLL Bandwidth: 72.136Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.662204	0.662204	0.281426	0.281426	0.035604	0.035604	0.00345	0.00345	9.64E-04	9.64E-04
ΔI_{1q}^c	0.269908	0.269908	0.664957	0.664957	0.009113	0.009113	0.003662	0.003662	3.48E-03	3.48E-03
$\Delta \gamma_{1d}^c$	0.066892	0.066892	0.026996	0.026996	0.329108	0.329108	0.148994	0.148994	0.083694	0.083694
$\Delta \gamma_{1q}^c$	0.027265	0.027265	0.063785	0.063785	0.084241	0.084241	0.158139	0.158139	0.302591	0.302591
$\Delta \theta$	0.019682	0.019682	0.042288	0.042288	0.10442	0.10442	0.606019	0.606019	0.473316	0.473316
Δg	4.61E-04	4.61E-04	9.65E-04	9.65E-04	0.025015	0.025015	0.235753	0.235753	0.588542	0.588542
ΔE_{1d}^a	0.658062	0.658062	0.251033	0.251033	0.061802	0.061802	0.030548	0.030548	0.003418	0.003418
ΔE_{1q}^a	0.260626	0.260626	0.576001	0.576001	0.012973	0.012973	0.044062	0.044062	4.69E-03	4.69E-03
ΔI_{gd}^a	0.042938	0.042938	0.014764	0.014764	0.551969	0.551969	0.086391	0.086391	2.02E-02	2.02E-02
ΔI_{gq}^a	0.016377	0.016377	0.031725	0.031725	0.367495	0.367495	0.675935	0.675935	0.110187	0.110187

PLL Bandwidth: 82.388Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.718713	0.718713	0.251474	0.251474	0.036516	0.036516	0.003319	0.003319	1.00E-03	1.00E-03
ΔI_{1q}^c	0.242911	0.242911	0.713036	0.713036	0.007693	0.007693	0.003817	0.003817	4.29E-03	4.29E-03
$\Delta \gamma_{1d}^c$	0.072581	0.072581	0.023833	0.023833	0.346298	0.346298	0.126427	0.126427	0.074529	0.074529
$\Delta \gamma_{1q}^c$	0.024531	0.024531	0.067577	0.067577	0.072959	0.072959	0.145378	0.145378	0.319853	0.319853
$\Delta \theta$	0.020174	0.020174	0.051232	0.051232	0.107095	0.107095	0.596378	0.596378	0.402826	0.402826
Δg	5.41E-04	5.41E-04	1.33E-03	1.33E-03	0.029938	0.029938	0.238599	0.238599	0.557024	0.557024
ΔE_{1d}^a	0.71038	0.71038	0.224659	0.224659	0.064174	0.064174	0.028942	0.028942	0.002739	0.002739
ΔE_{1q}^a	0.23233	0.23233	0.616166	0.616166	0.010693	0.010693	0.050162	0.050162	4.72E-03	4.72E-03
ΔI_{gd}^a	0.046113	0.046113	0.013233	0.013233	0.571025	0.571025	0.085401	0.085401	2.07E-02	2.07E-02
ΔI_{gq}^a	0.014439	0.014439	0.033704	0.033704	0.354508	0.354508	0.706848	0.706848	0.092967	0.092967

PLL Bandwidth: 92.336Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.761019	0.761019	0.217702	0.217702	0.037271	0.037271	0.003216	0.003216	1.04E-03	1.04E-03
ΔI_{1q}^c	0.211791	0.211791	0.746129	0.746129	0.006497	0.006497	0.004023	0.004023	5.08E-03	5.08E-03
$\Delta \gamma_{1d}^c$	0.076898	0.076898	0.020379	0.020379	0.361855	0.361855	0.10943	0.10943	0.068408	0.068408
$\Delta \gamma_{1q}^c$	0.021401	0.021401	0.069845	0.069845	0.063075	0.063075	0.136889	0.136889	0.333644	0.333644
$\Delta \theta$	0.019698	0.019698	0.059423	0.059423	0.10782	0.10782	0.592232	0.592232	0.353223	0.353223
Δg	5.93E-04	5.93E-04	1.72E-03	1.72E-03	0.034387	0.034387	0.241947	0.241947	0.53399	0.53399
ΔE_{1d}^a	0.748729	0.748729	0.194691	0.194691	0.066122	0.066122	0.027578	0.027578	0.002297	0.002297
ΔE_{1q}^a	0.200812	0.200812	0.642998	0.642998	0.008768	0.008768	0.056889	0.056889	4.80E-03	4.80E-03
ΔI_{gd}^a	0.048394	0.048394	0.011481	0.011481	0.587325	0.587325	0.084972	0.084972	2.14E-02	2.14E-02
ΔI_{gq}^a	0.012358	0.012358	0.034925	0.034925	0.34051	0.34051	0.740172	0.740172	0.08173	0.08173

PLL Bandwidth: 102.648Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.78838	0.78838	0.182661	0.182661	0.03792	0.03792	0.003113	0.003113	1.10E-03	1.10E-03
ΔI_{1q}^c	0.178954	0.178954	0.763874	0.763874	0.005432	0.005432	0.004262	0.004262	5.92E-03	5.92E-03
$\Delta \gamma_{1d}^c$	0.079756	0.079756	0.016877	0.016877	0.376574	0.376574	0.095193	0.095193	0.064073	0.064073
$\Delta \gamma_{1q}^c$	0.018104	0.018104	0.070576	0.070576	0.053943	0.053943	0.130325	0.130325	0.34606	0.34606
$\Delta \theta$	0.018509	0.018509	0.066798	0.066798	0.10704	0.10704	0.591046	0.591046	0.314414	0.314414
Δg	6.21E-04	6.21E-04	2.14E-03	2.14E-03	0.03859	0.03859	0.24592	0.24592	0.51484	0.51484
ΔE_{1d}^a	0.772582	0.772582	0.163423	0.163423	0.067774	0.067774	0.026264	0.026264	0.001979	0.001979
ΔE_{1q}^a	0.168292	0.168292	0.656027	0.656027	0.007064	0.007064	0.064512	0.064512	4.94E-03	4.94E-03
ΔI_{gd}^a	0.049758	0.049758	0.009643	0.009643	0.601836	0.601836	0.084631	0.084631	2.23E-02	2.23E-02
ΔI_{gq}^a	0.010263	0.010263	0.035351	0.035351	0.325146	0.325146	0.775973	0.775973	0.073703	0.073703

6.3.4 Analysis for a SCR of 1.6265

PLL Bandwidth: 10.277Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.379961	0.379961	0.394506	0.394506	0.024271	0.024271	0.007282	0.007282	4.98E-06	4.98E-06
ΔI_{1q}^c	0.417753	0.417753	0.356764	0.356764	0.019798	0.019798	0.007367	0.007367	6.39E-07	6.39E-07
$\Delta \gamma_{1d}^c$	0.038299	0.038299	0.040449	0.040449	0.214546	0.214546	0.362439	0.362439	0.026332	0.026332
$\Delta \gamma_{1q}^c$	0.042108	0.042108	0.03658	0.03658	0.17501	0.17501	0.366629	0.366629	0.003379	0.003379
$\Delta \theta$	0.003748	0.003748	0.003627	0.003627	0.025397	0.025397	0.110511	0.110511	0.739888	0.739888
Δg	1.23E-05	1.23E-05	1.20E-05	1.20E-05	0.00079	0.00079	0.008262	0.008262	0.68049	0.68049
ΔE_{1d}^a	0.335512	0.335512	0.401053	0.401053	0.03927	0.03927	0.033413	0.033413	0.002607	0.002607
ΔE_{1q}^a	0.367436	0.367436	0.361338	0.361338	0.031258	0.031258	0.032162	0.032162	3.42E-05	3.42E-05
ΔI_{gd}^a	0.017083	0.017083	0.023518	0.023518	0.412754	0.412754	0.176469	0.176469	1.30E-04	1.30E-04
ΔI_{gq}^a	0.018533	0.018533	0.021091	0.021091	0.393659	0.393659	0.277369	0.277369	0.064997	0.064997

PLL Bandwidth: 20.334Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.364741	0.364741	0.417242	0.417242	0.025149	0.025149	0.00635	0.00635	6.44E-05	6.44E-05
ΔI_{1q}^c	0.439442	0.439442	0.342494	0.342494	0.016973	0.016973	0.006302	0.006302	2.25E-05	2.25E-05
$\Delta \gamma_{1d}^c$	0.0365	0.0365	0.042544	0.042544	0.227259	0.227259	0.383057	0.383057	0.069622	0.069622
$\Delta \gamma_{1q}^c$	0.043975	0.043975	0.034922	0.034922	0.153378	0.153378	0.380168	0.380168	0.024328	0.024328
$\Delta \theta$	0.007642	0.007642	0.006747	0.006747	0.045044	0.045044	0.289811	0.289811	0.885904	0.885904
Δg	5.12E-05	5.12E-05	4.56E-05	4.56E-05	0.002938	0.002938	0.049668	0.049668	0.744803	0.744803
ΔE_{1d}^a	0.321726	0.321726	0.42258	0.42258	0.042139	0.042139	0.037547	0.037547	0.00635	0.00635
ΔE_{1q}^a	0.384663	0.384663	0.344365	0.344365	0.027122	0.027122	0.033745	0.033745	4.99E-04	4.99E-04
ΔI_{gd}^a	0.016367	0.016367	0.024658	0.024658	0.43109	0.43109	0.158034	0.158034	1.71E-03	1.71E-03
ΔI_{gq}^a	0.019213	0.019213	0.01991	0.01991	0.391932	0.391932	0.39611	0.39611	0.155217	0.155217

PLL Bandwidth: 30.898Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.44837	0.44837	0.350053	0.350053	0.026146	0.026146	0.005351	0.005351	4.34E-04	4.34E-04
ΔI_{1q}^c	0.329115	0.329115	0.468892	0.468892	0.014359	0.014359	0.004816	0.004816	3.21E-04	3.21E-04
$\Delta \gamma_{1d}^c$	0.045472	0.045472	0.034735	0.034735	0.242441	0.242441	0.416828	0.416828	0.158843	0.158843
$\Delta \gamma_{1q}^c$	0.033378	0.033378	0.046528	0.046528	0.133151	0.133151	0.375142	0.375142	0.117709	0.117709
$\Delta \theta$	0.009912	0.009912	0.012463	0.012463	0.062657	0.062657	0.665722	0.665722	1.155154	1.155154
Δg	9.82E-05	9.82E-05	1.22E-04	1.22E-04	0.006142	0.006142	0.185351	0.185351	0.892323	0.892323
ΔE_{1d}^a	0.452022	0.452022	0.308619	0.308619	0.045313	0.045313	0.046402	0.046402	0.012544	0.012544
ΔE_{1q}^a	0.328104	0.328104	0.408569	0.408569	0.023146	0.023146	0.036941	0.036941	3.16E-03	3.16E-03
ΔI_{gd}^a	0.026228	0.026228	0.015696	0.015696	0.452508	0.452508	0.138853	0.138853	1.13E-02	1.13E-02
ΔI_{gq}^a	0.018765	0.018765	0.020206	0.020206	0.389302	0.389302	0.632925	0.632925	0.296686	0.296686

PLL Bandwidth: 40.723Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.480847	0.480847	0.338551	0.338551	0.027025	0.027025	0.0037	0.0037	9.57E-04	9.57E-04
ΔI_{1q}^c	0.318957	0.318957	0.499381	0.499381	0.012423	0.012423	0.002981	0.002981	1.28E-03	1.28E-03
$\Delta \gamma_{1d}^c$	0.048573	0.048573	0.033339	0.033339	0.256613	0.256613	0.316167	0.316167	0.178106	0.178106
$\Delta \gamma_{1q}^c$	0.03222	0.03222	0.049177	0.049177	0.117967	0.117967	0.254693	0.254693	0.237653	0.237653
$\Delta \theta$	0.01241	0.01241	0.017137	0.017137	0.074367	0.074367	0.781604	0.781604	0.931062	0.931062
Δg	1.68E-04	1.68E-04	2.29E-04	2.29E-04	0.010183	0.010183	0.282606	0.282606	0.767587	0.767587
ΔE_{1d}^a	0.482695	0.482695	0.298527	0.298527	0.04799	0.04799	0.04253	0.04253	0.011547	0.011547
ΔE_{1q}^a	0.315556	0.315556	0.4337	0.4337	0.020043	0.020043	0.033607	0.033607	6.23E-03	6.23E-03
ΔI_{gd}^a	0.027869	0.027869	0.015189	0.015189	0.471901	0.471901	0.100648	0.100648	2.43E-02	2.43E-02
ΔI_{gq}^a	0.017874	0.017874	0.021283	0.021283	0.385109	0.385109	0.693639	0.693639	0.272575	0.272575

PLL Bandwidth: 51.514Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.529287	0.529287	0.323951	0.323951	0.028037	0.028037	0.003008	0.003008	9.58E-04	9.58E-04
ΔI_{1q}^c	0.306353	0.306353	0.544425	0.544425	0.010424	0.010424	0.002425	0.002425	1.98E-03	1.98E-03
$\Delta \gamma_{1d}^c$	0.053258	0.053258	0.031587	0.031587	0.273882	0.273882	0.221121	0.221121	0.130308	0.130308
$\Delta \gamma_{1q}^c$	0.030826	0.030826	0.053085	0.053085	0.101822	0.101822	0.178236	0.178236	0.268612	0.268612
$\Delta \theta$	0.01518	0.01518	0.023752	0.023752	0.085559	0.085559	0.67357	0.67357	0.671735	0.671735
Δg	2.51E-04	2.51E-04	3.87E-04	3.87E-04	0.014649	0.014649	0.253574	0.253574	0.655208	0.655208
ΔE_{1d}^a	0.528333	0.528333	0.285882	0.285882	0.051114	0.051114	0.035943	0.035943	0.006943	0.006943
ΔE_{1q}^a	0.300143	0.300143	0.471223	0.471223	0.016789	0.016789	0.034393	0.034393	5.81E-03	5.81E-03
ΔI_{gd}^a	0.030316	0.030316	0.014562	0.014562	0.494792	0.494792	0.085113	0.085113	2.38E-02	2.38E-02
ΔI_{gq}^a	0.016798	0.016798	0.022927	0.022927	0.378994	0.378994	0.659881	0.659881	0.178813	0.178813

PLL Bandwidth: 61.697Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.580499	0.580499	0.308765	0.308765	0.028899	0.028899	0.002808	0.002808	9.73E-04	9.73E-04
ΔI_{1q}^c	0.293305	0.293305	0.591395	0.591395	0.008867	0.008867	0.002378	0.002378	2.71E-03	2.71E-03
$\Delta \gamma_{1d}^c$	0.058261	0.058261	0.029822	0.029822	0.289642	0.289642	0.178375	0.178375	0.10516	0.10516
$\Delta \gamma_{1q}^c$	0.029437	0.029437	0.05712	0.05712	0.08887	0.08887	0.151067	0.151067	0.292812	0.292812
$\Delta \theta$	0.017298	0.017298	0.030625	0.030625	0.092429	0.092429	0.634325	0.634325	0.533758	0.533758
Δg	3.44E-04	3.44E-04	5.97E-04	5.97E-04	0.019403	0.019403	0.24954	0.24954	0.600748	0.600748
ΔE_{1d}^a	0.576394	0.576394	0.272811	0.272811	0.053701	0.053701	0.0332	0.0332	0.004887	0.004887
ΔE_{1q}^a	0.284786	0.284786	0.510609	0.510609	0.014151	0.014151	0.038653	0.038653	5.68E-03	5.68E-03
ΔI_{gd}^a	0.032891	0.032891	0.013916	0.013916	0.514789	0.514789	0.08206	0.08206	2.38E-02	2.38E-02
ΔI_{qq}^a	0.015764	0.015764	0.024671	0.024671	0.370869	0.370869	0.678369	0.678369	0.134425	0.134425

PLL Bandwidth: 72.136Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.638184	0.638184	0.288733	0.288733	0.029698	0.029698	0.002731	0.002731	9.82E-04	9.82E-04
ΔI_{1q}^c	0.275812	0.275812	0.643236	0.643236	0.007485	0.007485	0.002487	0.002487	3.47E-03	3.47E-03
$\Delta \gamma_{1d}^c$	0.063945	0.063945	0.027598	0.027598	0.305351	0.305351	0.15049	0.15049	0.088205	0.088205
$\Delta \gamma_{1q}^c$	0.027636	0.027636	0.061483	0.061483	0.076964	0.076964	0.137089	0.137089	0.311613	0.311613
$\Delta \theta$	0.018915	0.018915	0.038561	0.038561	0.096926	0.096926	0.618335	0.618335	0.44309	0.44309
Δg	4.41E-04	4.41E-04	8.78E-04	8.78E-04	0.024337	0.024337	0.251328	0.251328	0.565505	0.565505
ΔE_{1d}^a	0.630203	0.630203	0.255498	0.255498	0.056082	0.056082	0.031499	0.031499	0.00365	0.00365
ΔE_{1q}^a	0.265322	0.265322	0.554144	0.554144	0.011757	0.011757	0.044842	0.044842	5.57E-03	5.57E-03
ΔI_{gd}^a	0.035763	0.035763	0.013055	0.013055	0.533829	0.533829	0.082379	0.082379	2.38E-02	2.38E-02
ΔI_{qq}^a	0.014523	0.014523	0.026597	0.026597	0.360556	0.360556	0.716601	0.716601	0.106795	0.106795

PLL Bandwidth: 82.388Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.694122	0.694122	0.262882	0.262882	0.030387	0.030387	0.002694	0.002694	9.95E-04	9.95E-04
ΔI_{1q}^c	0.252691	0.252691	0.691832	0.691832	0.006312	0.006312	0.002666	0.002666	4.23E-03	4.23E-03
$\Delta \gamma_{1d}^c$	0.069507	0.069507	0.024852	0.024852	0.320033	0.320033	0.130659	0.130659	0.076839	0.076839
$\Delta \gamma_{1q}^c$	0.025304	0.025304	0.065404	0.065404	0.066481	0.066481	0.129307	0.129307	0.326334	0.326334
$\Delta \theta$	0.019724	0.019724	0.046897	0.046897	0.099275	0.099275	0.615156	0.615156	0.381702	0.381702
Δg	5.27E-04	5.27E-04	1.22E-03	1.22E-03	0.029012	0.029012	0.256115	0.256115	0.540691	0.540691
ΔE_{1d}^a	0.681874	0.681874	0.232969	0.232969	0.058133	0.058133	0.030264	0.030264	0.002882	0.002882
ΔE_{1q}^a	0.240903	0.240903	0.594742	0.594742	0.0097	0.0097	0.052221	0.052221	5.51E-03	5.51E-03
ΔI_{gd}^a	0.0385	0.0385	0.011924	0.011924	0.550755	0.550755	0.083885	0.083885	2.39E-02	2.39E-02
ΔI_{qq}^a	0.013045	0.013045	0.028365	0.028365	0.348873	0.348873	0.763991	0.763991	0.089223	0.089223

PLL Bandwidth: 92.336Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.739931	0.739931	0.232468	0.232468	0.030963	0.030963	0.00267	0.00267	1.01E-03	1.01E-03
ΔI_{1q}^c	0.224888	0.224888	0.729333	0.729333	0.005331	0.005331	0.00288	0.00288	4.97E-03	4.97E-03
$\Delta \gamma_{1d}^c$	0.074112	0.074112	0.021729	0.021729	0.333366	0.333366	0.115661	0.115661	0.069056	0.069056
$\Delta \gamma_{1q}^c$	0.022525	0.022525	0.068173	0.068173	0.057396	0.057396	0.124745	0.124745	0.338377	0.338377
$\Delta \theta$	0.019642	0.019642	0.054848	0.054848	0.099979	0.099979	0.619447	0.619447	0.338364	0.338364
Δg	5.89E-04	5.89E-04	1.59E-03	1.59E-03	0.033244	0.033244	0.262674	0.262674	0.522097	0.522097
ΔE_{1d}^a	0.723524	0.723524	0.206251	0.206251	0.059849	0.059849	0.029272	0.029272	0.00238	0.00238
ΔE_{1q}^a	0.21262	0.21262	0.625525	0.625525	0.007971	0.007971	0.060484	0.060484	5.52E-03	5.52E-03
ΔI_{gd}^a	0.040674	0.040674	0.010571	0.010571	0.56535	0.56535	0.085807	0.085807	2.42E-02	2.42E-02
ΔI_{gq}^a	0.011401	0.011401	0.029643	0.029643	0.336526	0.336526	0.816132	0.816132	0.077615	0.077615

PLL Bandwidth: 102.648Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.773071	0.773071	0.198918	0.198918	0.031468	0.031468	0.002648	0.002648	1.04E-03	1.04E-03
ΔI_{1q}^c	0.193686	0.193686	0.753438	0.753438	0.004461	0.004461	0.003131	0.003131	5.76E-03	5.76E-03
$\Delta \gamma_{1d}^c$	0.077499	0.077499	0.018369	0.018369	0.346117	0.346117	0.103156	0.103156	0.063291	0.063291
$\Delta \gamma_{1q}^c$	0.019417	0.019417	0.069576	0.069576	0.049065	0.049065	0.121941	0.121941	0.349416	0.349416
$\Delta \theta$	0.018796	0.018796	0.06227	0.06227	0.099436	0.099436	0.629569	0.629569	0.304399	0.304399
Δg	6.28E-04	6.28E-04	2.00E-03	2.00E-03	0.037266	0.037266	0.271351	0.271351	0.506311	0.506311
ΔE_{1d}^a	0.752842	0.752842	0.176602	0.176602	0.06135	0.06135	0.028386	0.028386	0.002016	0.002016
ΔE_{1q}^a	0.18167	0.18167	0.644332	0.644332	0.006442	0.006442	0.070198	0.070198	5.59E-03	5.59E-03
ΔI_{gd}^a	0.042162	0.042162	0.00906	0.00906	0.578576	0.578576	0.088069	0.088069	2.47E-02	2.47E-02
ΔI_{gq}^a	0.009652	0.009652	0.030319	0.030319	0.32314	0.32314	0.875706	0.875706	0.069203	0.069203

6.3.5 Analysis for a SCR of 1.4463

PLL Bandwidth: 10.277Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.379096	0.379096	0.392033	0.392033	0.020898	0.020898	0.006009	0.006009	6.06E-06	6.06E-06
ΔI_{1q}^c	0.415121	0.415121	0.355957	0.355957	0.016995	0.016995	0.005939	0.005939	6.13E-07	6.13E-07
$\Delta \gamma_{1d}^c$	0.037975	0.037975	0.03993	0.03993	0.20396	0.20396	0.365955	0.365955	0.035956	0.035956
$\Delta \gamma_{1q}^c$	0.041584	0.041584	0.036256	0.036256	0.165867	0.165867	0.361688	0.361688	0.003637	0.003637
$\Delta \theta$	0.003442	0.003442	0.00335	0.00335	0.024027	0.024027	0.127645	0.127645	0.730272	0.730272
Δg	1.13E-05	1.13E-05	1.10E-05	1.10E-05	0.000785	0.000785	0.010557	0.010557	0.659778	0.659778
ΔE_{1d}^a	0.332867	0.332867	0.395934	0.395934	0.036502	0.036502	0.034316	0.034316	0.003603	0.003603
ΔE_{1q}^a	0.363257	0.363257	0.358337	0.358337	0.029066	0.029066	0.032417	0.032417	5.01E-05	5.01E-05
ΔI_{gd}^a	0.014913	0.014913	0.020411	0.020411	0.406425	0.406425	0.168826	0.168826	1.82E-04	1.82E-04
ΔI_{gq}^a	0.016136	0.016136	0.018396	0.018396	0.387602	0.387602	0.278724	0.278724	0.079532	0.079532

PLL Bandwidth: 20.334Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.364274	0.364274	0.413134	0.413134	0.021661	0.021661	0.005241	0.005241	8.98E-05	8.98E-05
ΔI_{1q}^c	0.435245	0.435245	0.34194	0.34194	0.014534	0.014534	0.004889	0.004889	2.53E-05	2.53E-05
$\Delta \gamma_{1d}^c$	0.03625	0.03625	0.041864	0.041864	0.216081	0.216081	0.411022	0.411022	0.102495	0.102495
$\Delta \gamma_{1q}^c$	0.043312	0.043312	0.03465	0.03465	0.144983	0.144983	0.38342	0.38342	0.028935	0.028935
$\Delta \theta$	0.006996	0.006996	0.006236	0.006236	0.042392	0.042392	0.360621	0.360621	0.934368	0.934368
Δg	4.67E-05	4.67E-05	4.20E-05	4.20E-05	0.002903	0.002903	0.070379	0.070379	0.748508	0.748508
ΔE_{1d}^a	0.319478	0.319478	0.41579	0.41579	0.039129	0.039129	0.040414	0.040414	0.009613	0.009613
ΔE_{1q}^a	0.379193	0.379193	0.341972	0.341972	0.025198	0.025198	0.034201	0.034201	8.16E-04	8.16E-04
ΔI_{gd}^a	0.0143	0.0143	0.021338	0.021338	0.424676	0.424676	0.151501	0.151501	2.75E-03	2.75E-03
ΔI_{gq}^a	0.016694	0.016694	0.017405	0.017405	0.386131	0.386131	0.428457	0.428457	0.206714	0.206714

PLL Bandwidth: 30.898Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.441409	0.441409	0.349814	0.349814	0.022499	0.022499	0.004354	0.004354	8.68E-04	8.68E-04
ΔI_{1q}^c	0.328598	0.328598	0.461999	0.461999	0.012253	0.012253	0.003237	0.003237	5.89E-04	5.89E-04
$\Delta \gamma_{1d}^c$	0.044506	0.044506	0.034544	0.034544	0.230174	0.230174	0.515194	0.515194	0.287032	0.287032
$\Delta \gamma_{1q}^c$	0.033132	0.033132	0.045622	0.045622	0.125349	0.125349	0.383061	0.383061	0.194961	0.194961
$\Delta \theta$	0.009162	0.009162	0.011351	0.011351	0.058579	0.058579	1.018532	1.018532	1.409177	1.409177
Δg	9.05E-05	9.05E-05	1.11E-04	1.11E-04	0.006024	0.006024	0.339195	0.339195	0.985216	0.985216
ΔE_{1d}^a	0.442368	0.442368	0.306579	0.306579	0.041982	0.041982	0.05808	0.05808	0.023132	0.023132
ΔE_{1q}^a	0.326125	0.326125	0.40079	0.40079	0.021461	0.021461	0.038386	0.038386	7.15E-03	7.15E-03
ΔI_{gd}^a	0.022585	0.022585	0.013718	0.013718	0.44535	0.44535	0.132498	0.132498	2.59E-02	2.59E-02
ΔI_{gq}^a	0.016436	0.016436	0.017483	0.017483	0.383429	0.383429	0.832876	0.832876	0.46895	0.46895

PLL Bandwidth: 40.723Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.470336	0.470336	0.338501	0.338501	0.023215	0.023215	0.002505	0.002505	1.18E-03	1.18E-03
ΔI_{1q}^c	0.31843	0.31843	0.489169	0.489169	0.010568	0.010568	0.001586	0.001586	1.52E-03	1.52E-03
$\Delta \gamma_{1d}^c$	0.047246	0.047246	0.033196	0.033196	0.243	0.243	0.304857	0.304857	0.211309	0.211309
$\Delta \gamma_{1q}^c$	0.031987	0.031987	0.047971	0.047971	0.110613	0.110613	0.193053	0.193053	0.272106	0.272106
$\Delta \theta$	0.011471	0.011471	0.015518	0.015518	0.069139	0.069139	0.834979	0.834979	0.841089	0.841089
Δg	1.55E-04	1.55E-04	2.07E-04	2.07E-04	0.009919	0.009919	0.331735	0.331735	0.680719	0.680719
ΔE_{1d}^a	0.469524	0.469524	0.296632	0.296632	0.04434	0.04434	0.042967	0.042967	0.013525	0.013525
ΔE_{1q}^a	0.313883	0.313883	0.423036	0.423036	0.018549	0.018549	0.029613	0.029613	8.56E-03	8.56E-03
ΔI_{gd}^a	0.023863	0.023863	0.013276	0.013276	0.463542	0.463542	0.080515	0.080515	3.41E-02	3.41E-02
ΔI_{gq}^a	0.015682	0.015682	0.01832	0.01832	0.379133	0.379133	0.724437	0.724437	0.282263	0.282263

PLL Bandwidth: 51.514Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.512782	0.512782	0.324538	0.324538	0.024016	0.024016	0.00223	0.00223	1.07E-03	1.07E-03
ΔI_{1q}^c	0.306156	0.306156	0.528699	0.528699	0.008835	0.008835	0.001448	0.001448	2.08E-03	2.08E-03
$\Delta \gamma_{1d}^c$	0.051315	0.051315	0.031543	0.031543	0.258236	0.258236	0.2229	0.2229	0.146196	0.146196
$\Delta \gamma_{1q}^c$	0.030638	0.030638	0.051386	0.051386	0.094997	0.094997	0.144715	0.144715	0.285117	0.285117
$\Delta \theta$	0.014045	0.014045	0.021328	0.021328	0.079005	0.079005	0.708551	0.708551	0.615441	0.615441
Δg	2.32E-04	2.32E-04	3.47E-04	3.47E-04	0.01415	0.01415	0.284746	0.284746	0.604313	0.604313
ΔE_{1d}^a	0.509285	0.509285	0.284506	0.284506	0.047038	0.047038	0.037413	0.037413	0.007808	0.007808
ΔE_{1q}^a	0.299173	0.299173	0.455733	0.455733	0.015503	0.015503	0.032883	0.032883	7.35E-03	7.35E-03
ΔI_{gd}^a	0.025738	0.025738	0.012743	0.012743	0.484437	0.484437	0.074767	0.074767	3.03E-02	3.03E-02
ΔI_{gq}^a	0.014785	0.014785	0.019577	0.019577	0.372881	0.372881	0.709662	0.709662	0.180678	0.180678

PLL Bandwidth: 61.697Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.55732	0.55732	0.310815	0.310815	0.024682	0.024682	0.002196	0.002196	1.04E-03	1.04E-03
ΔI_{1q}^c	0.294228	0.294228	0.5697	0.5697	0.007499	0.007499	0.001514	0.001514	2.74E-03	2.74E-03
$\Delta \gamma_{1d}^c$	0.055627	0.055627	0.029953	0.029953	0.271868	0.271868	0.186957	0.186957	0.115357	0.115357
$\Delta \gamma_{1q}^c$	0.029367	0.029367	0.054901	0.054901	0.082598	0.082598	0.128865	0.128865	0.303432	0.303432
$\Delta \theta$	0.016061	0.016061	0.027287	0.027287	0.084942	0.084942	0.677794	0.677794	0.497015	0.497015
Δg	3.18E-04	3.18E-04	5.31E-04	5.31E-04	0.018618	0.018618	0.281092	0.281092	0.567397	0.567397
ΔE_{1d}^a	0.550871	0.550871	0.272691	0.272691	0.04924	0.04924	0.035573	0.035573	0.005427	0.005427
ΔE_{1q}^a	0.285224	0.285224	0.489882	0.489882	0.013055	0.013055	0.038928	0.038928	6.98E-03	6.98E-03
ΔI_{gd}^a	0.027699	0.027699	0.012228	0.012228	0.502315	0.502315	0.076367	0.076367	2.92E-02	2.92E-02
ΔI_{gq}^a	0.013957	0.013957	0.020905	0.020905	0.364995	0.364995	0.755322	0.755322	0.135031	0.135031

PLL Bandwidth: 72.136Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.608179	0.608179	0.293796	0.293796	0.02529	0.02529	0.002233	0.002233	1.03E-03	1.03E-03
ΔI_{1q}^c	0.279348	0.279348	0.615776	0.615776	0.006323	0.006323	0.001667	0.001667	3.44E-03	3.44E-03
$\Delta \gamma_{1d}^c$	0.06059	0.06059	0.028051	0.028051	0.285277	0.285277	0.16403	0.16403	0.094923	0.094923
$\Delta \gamma_{1q}^c$	0.02783	0.02783	0.058794	0.058794	0.071329	0.071329	0.122439	0.122439	0.318896	0.318896
$\Delta \theta$	0.017723	0.017723	0.03416	0.03416	0.088776	0.088776	0.678249	0.678249	0.418478	0.418478
Δg	4.12E-04	4.12E-04	7.78E-04	7.78E-04	0.023224	0.023224	0.288524	0.288524	0.542452	0.542452
ΔE_{1d}^a	0.598137	0.598137	0.258044	0.258044	0.051252	0.051252	0.034807	0.034807	0.004008	0.004008
ΔE_{1q}^a	0.268557	0.268557	0.528364	0.528364	0.010852	0.010852	0.047284	0.047284	6.72E-03	6.72E-03
ΔI_{gd}^a	0.029922	0.029922	0.011588	0.011588	0.519125	0.519125	0.080626	0.080626	2.85E-02	2.85E-02
ΔI_{gq}^a	0.013007	0.013007	0.022407	0.022407	0.355354	0.355354	0.828702	0.828702	0.106583	0.106583

PLL Bandwidth: 82.388Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.659993	0.659993	0.272562	0.272562	0.025812	0.025812	0.002305	0.002305	1.02E-03	1.02E-03
ΔI_{1q}^c	0.260473	0.260473	0.661573	0.661573	0.005333	0.005333	0.001878	0.001878	4.15E-03	4.15E-03
$\Delta \gamma_{1d}^c$	0.065686	0.065686	0.025771	0.025771	0.297729	0.297729	0.148705	0.148705	0.081082	0.081082
$\Delta \gamma_{1q}^c$	0.025924	0.025924	0.062553	0.062553	0.061515	0.061515	0.121173	0.121173	0.33147	0.33147
$\Delta \theta$	0.018795	0.018795	0.041529	0.041529	0.090775	0.090775	0.699092	0.699092	0.364894	0.364894
Δg	5.00E-04	5.00E-04	1.08E-03	1.08E-03	0.027574	0.027574	0.303227	0.303227	0.524057	0.524057
ΔE_{1d}^a	0.645943	0.645943	0.239678	0.239678	0.052988	0.052988	0.034681	0.034681	0.003122	0.003122
ΔE_{1q}^a	0.248381	0.248381	0.566539	0.566539	0.00897	0.00897	0.057725	0.057725	6.54E-03	6.54E-03
ΔI_{gd}^a	0.032159	0.032159	0.01078	0.01078	0.534012	0.534012	0.08643	0.08643	2.80E-02	2.80E-02
ΔI_{gq}^a	0.011911	0.011911	0.023884	0.023884	0.344757	0.344757	0.922648	0.922648	0.088239	0.088239

PLL Bandwidth: 92.336Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.706603	0.706603	0.247207	0.247207	0.026252	0.026252	0.002407	0.002407	1.01E-03	1.01E-03
ΔI_{1q}^c	0.237512	0.237512	0.701178	0.701178	0.004509	0.004509	0.002146	0.002146	4.85E-03	4.85E-03
$\Delta \gamma_{1d}^c$	0.07031	0.07031	0.02314	0.02314	0.309038	0.309038	0.138642	0.138642	0.071395	0.071395
$\Delta \gamma_{1q}^c$	0.023633	0.023633	0.065634	0.065634	0.053084	0.053084	0.123603	0.123603	0.342022	0.342022
$\Delta \theta$	0.019158	0.019158	0.048875	0.048875	0.091407	0.091407	0.737841	0.737841	0.326909	0.326909
Δg	5.72E-04	5.72E-04	1.42E-03	1.42E-03	0.031516	0.031516	0.324925	0.324925	0.509782	0.509782
ΔE_{1d}^a	0.688478	0.688478	0.217607	0.217607	0.054458	0.054458	0.035123	0.035123	0.002539	0.002539
ΔE_{1q}^a	0.224754	0.224754	0.599246	0.599246	0.007395	0.007395	0.07061	0.07061	6.44E-03	6.44E-03
ΔI_{gd}^a	0.034129	0.034129	0.009801	0.009801	0.546907	0.546907	0.09377	0.09377	2.77E-02	2.77E-02
ΔI_{gq}^a	0.010678	0.010678	0.025118	0.025118	0.333803	0.333803	1.039112	1.039112	0.075917	0.075917

PLL Bandwidth: 102.648Hz

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8	Mode 9	Mode 10
ΔI_{1d}^c	0.745251	0.745251	0.217617	0.217617	0.026643	0.026643	0.002563	0.002563	1.02E-03	1.02E-03
ΔI_{1q}^c	0.210267	0.210267	0.731826	0.731826	0.003781	0.003781	0.002514	0.002514	5.60E-03	5.60E-03
$\Delta \gamma_{1d}^c$	0.074187	0.074187	0.02015	0.02015	0.319918	0.319918	0.132869	0.132869	0.063987	0.063987
$\Delta \gamma_{1q}^c$	0.020931	0.020931	0.067763	0.067763	0.045399	0.045399	0.130333	0.130333	0.351889	0.351889
$\Delta \theta$	0.018824	0.018824	0.056119	0.056119	0.091019	0.091019	0.803304	0.803304	0.297071	0.297071
Δg	6.26E-04	6.26E-04	1.80E-03	1.80E-03	0.035278	0.035278	0.35864	0.35864	0.497302	0.497302
ΔE_{1d}^a	0.723137	0.723137	0.191699	0.191699	0.055772	0.055772	0.036403	0.036403	0.002112	0.002112
ΔE_{1q}^a	0.197491	0.197491	0.623946	0.623946	0.006003	0.006003	0.088285	0.088285	6.41E-03	6.41E-03
ΔI_{gd}^a	0.035708	0.035708	0.008644	0.008644	0.55874	0.55874	0.104162	0.104162	2.77E-02	2.77E-02
ΔI_{gq}^a	0.009299	0.009299	0.025991	0.025991	0.322111	0.322111	1.200323	1.200323	0.066825	0.066825

Appendix G: Damping of Modes 7 and 8 as a Function of the PLL BW and the Active Current Injected into the Grid

In this appendix, the points used for creating figure Fig. 5.8 are shown. These points were obtained from the proposed model (112) for short circuit ratios shown in Table 5.3 and the PLL designs depicted in Table 5.2. These tables show damping of modes 7 and 8 of the proposed small signal model as a function of the PLL bandwidth and the active current injected into the grid.

Short Circuit Ratio: 2.5942

	PLL BW 10.27Hz	PLL BW 20.33Hz	PLL BW 30.89Hz	PLL BW 40.72Hz	PLL BW 51.51Hz	PLL BW 61.69Hz	PLL BW 72.13Hz	PLL BW 82.38Hz	PLL BW 92.33Hz	PLL BW 102.64Hz
$I_{1d}^* = 3A$	5.82E-01	5.50E-01	5.10E-01	4.74E-01	4.26E-01	3.82E-01	3.38E-01	3.00E-01	2.67E-01	2.38E-01
$I_{1d}^* = 6A$	5.80E-01	5.45E-01	4.99E-01	4.55E-01	3.96E-01	3.40E-01	2.85E-01	2.38E-01	1.97E-01	1.60E-01
$I_{1d}^* = 9A$	5.78E-01	5.39E-01	4.88E-01	4.36E-01	3.63E-01	2.94E-01	2.27E-01	1.70E-01	1.21E-01	7.72E-02
$I_{1d}^* = 12A$	5.77E-01	5.35E-01	4.76E-01	4.15E-01	3.26E-01	2.42E-01	1.63E-01	9.67E-02	4.13E-02	-9.00E-03
$I_{1d}^* = 15A$	5.75E-01	5.30E-01	4.65E-01	3.94E-01	2.86E-01	1.85E-01	9.43E-02	1.93E-02	-4.26E-02	-9.85E-02
$I_{1d}^* = 18A$	5.74E-01	5.26E-01	4.55E-01	3.73E-01	2.41E-01	1.23E-01	2.02E-02	-6.27E-02	-1.31E-01	-1.92E-01

Short Circuit Ratio: 2.1652

	PLL BW 10.27Hz	PLL BW 20.33Hz	PLL BW 30.89Hz	PLL BW 40.72Hz	PLL BW 51.51Hz	PLL BW 61.69Hz	PLL BW 72.13Hz	PLL BW 82.38Hz	PLL BW 92.33Hz	PLL BW 102.64Hz
$I_{1d}^* = 3A$	5.12E-01	4.74E-01	4.28E-01	3.84E-01	3.31E-01	2.85E-01	2.43E-01	2.09E-01	1.82E-01	1.59E-01
$I_{1d}^* = 6A$	5.10E-01	4.68E-01	4.14E-01	3.60E-01	2.92E-01	2.33E-01	1.80E-01	1.36E-01	1.01E-01	6.99E-02
$I_{1d}^* = 9A$	5.08E-01	4.62E-01	3.99E-01	3.34E-01	2.48E-01	1.74E-01	1.10E-01	5.71E-02	1.40E-02	-2.47E-02
$I_{1d}^* = 12A$	5.06E-01	4.57E-01	3.85E-01	3.06E-01	1.99E-01	1.10E-01	3.33E-02	-2.83E-02	-7.88E-02	-1.24E-01
$I_{1d}^* = 15A$	5.05E-01	4.53E-01	3.72E-01	2.76E-01	1.43E-01	3.76E-02	-5.01E-02	-1.20E-01	-1.78E-01	-2.30E-01
$I_{1d}^* = 18A$	5.04E-01	4.50E-01	3.60E-01	2.44E-01	8.05E-02	-4.26E-02	-1.42E-01	-2.20E-01	-2.85E-01	-3.43E-01

Short Circuit Ratio: 1.8577

	PLL BW 10.27Hz	PLL BW 20.33Hz	PLL BW 30.89Hz	PLL BW 40.72Hz	PLL BW 51.51Hz	PLL BW 61.69Hz	PLL BW 72.13Hz	PLL BW 82.38Hz	PLL BW 92.33Hz	PLL BW 102.64Hz
$I_{1d}^* = 3A$	4.59E-01	4.16E-01	3.63E-01	3.14E-01	2.57E-01	2.10E-01	1.71E-01	1.41E-01	1.19E-01	1.01E-01
$I_{1d}^* = 6A$	4.57E-01	4.09E-01	3.45E-01	2.83E-01	2.09E-01	1.49E-01	9.86E-02	5.92E-02	2.84E-02	1.63E-03
$I_{1d}^* = 9A$	4.55E-01	4.02E-01	3.27E-01	2.48E-01	1.54E-01	7.94E-02	1.77E-02	-3.10E-02	-7.02E-02	-1.05E-01
$I_{1d}^* = 12A$	4.53E-01	3.97E-01	3.09E-01	2.11E-01	9.17E-02	1.60E-03	-7.16E-02	-1.30E-01	-1.77E-01	-2.19E-01
$I_{1d}^* = 15A$	4.53E-01	3.93E-01	2.93E-01	1.68E-01	2.03E-02	-8.60E-02	-1.71E-01	-2.38E-01	-2.94E-01	-3.44E-01
$I_{1d}^* = 18A$	4.53E-01	3.93E-01	2.81E-01	1.20E-01	-6.32E-02	-1.87E-01	-2.85E-01	-3.62E-01	-4.25E-01	-4.83E-01

Short Circuit Ratio: 1.6265

	PLL BW 10.27Hz	PLL BW 20.33Hz	PLL BW 30.89Hz	PLL BW 40.72Hz	PLL BW 51.51Hz	PLL BW 61.69Hz	PLL BW 72.13Hz	PLL BW 82.38Hz	PLL BW 92.33Hz	PLL BW 102.64Hz
$I_{1d}^* = 3A$	4.16E-01	3.67E-01	3.08E-01	2.53E-01	1.94E-01	1.49E-01	1.13E-01	8.68E-02	6.82E-02	5.45E-02
$I_{1d}^* = 6A$	4.13E-01	3.58E-01	2.85E-01	2.14E-01	1.37E-01	7.80E-02	3.09E-02	-4.82E-03	-3.22E-02	-5.55E-02
$I_{1d}^* = 9A$	4.11E-01	3.51E-01	2.63E-01	1.70E-01	7.08E-02	-2.60E-03	-6.12E-02	-1.07E-01	-1.43E-01	-1.76E-01
$I_{1d}^* = 12A$	4.11E-01	3.46E-01	2.40E-01	1.20E-01	-5.12E-03	-9.41E-02	-1.65E-01	-2.21E-01	-2.66E-01	-3.07E-01
$I_{1d}^* = 15A$	4.11E-01	3.45E-01	2.20E-01	6.19E-02	-9.44E-02	-2.00E-01	-2.84E-01	-3.50E-01	-4.05E-01	-4.54E-01
$I_{1d}^* = 18A$	4.14E-01	3.50E-01	2.10E-01	-1.17E-02	-2.05E-01	-3.30E-01	-4.29E-01	-5.08E-01	-5.72E-01	-6.31E-01

Short Circuit Ratio: 1.4463

	PLL BW 10.27Hz	PLL BW 20.33Hz	PLL BW 30.89Hz	PLL BW 40.72Hz	PLL BW 51.51Hz	PLL BW 61.69Hz	PLL BW 72.13Hz	PLL BW 82.38Hz	PLL BW 92.33Hz	PLL BW 102.64Hz
$I_{1d}^* = 3A$	3.77E-01	3.23E-01	2.57E-01	1.97E-01	1.38E-01	9.52E-02	6.25E-02	3.98E-02	2.47E-02	1.45E-02
$I_{1d}^* = 6A$	3.75E-01	3.13E-01	2.29E-01	1.50E-01	7.10E-02	1.44E-02	-2.96E-02	-6.22E-02	-8.68E-02	-1.07E-01
$I_{1d}^* = 9A$	3.73E-01	3.05E-01	2.00E-01	9.50E-02	-6.74E-03	-7.83E-02	-1.34E-01	-1.78E-01	-2.12E-01	-2.43E-01
$I_{1d}^* = 12A$	3.73E-01	3.00E-01	1.71E-01	3.04E-02	-9.82E-02	-1.86E-01	-2.55E-01	-3.10E-01	-3.54E-01	-3.94E-01
$I_{1d}^* = 15A$	3.76E-01	3.04E-01	1.46E-01	-5.00E-02	-2.11E-01	-3.17E-01	-4.01E-01	-4.68E-01	-5.24E-01	-5.74E-01
$I_{1d}^* = 18A$	3.82E-01	3.23E-01	1.40E-01	-1.65E-01	-3.66E-01	-4.96E-01	-5.99E-01	-6.83E-01	-7.52E-01	-8.16E-01

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