



Monitoring movement in MRI by measuring changes in the EMF induced in head-mounted coils

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Abstract

Image quality is degraded by involuntary movement of the subject in an MRI scanner. It is fairly challenging in MRI of the brain to monitor the involuntary head movement accurately. Though there are a few techniques to monitor head movement of the subject for prospective motion correction, it is still an unsolved problem in MRI. In this study, head movement inside an MR scanner is monitored via measurement of changes in the voltage induced in head-mounted coils by switched magnetic field gradients. The motion of a rigid body such as the human head is decomposed into two components: namely translation and rotation. There are three degrees of freedom (DOFs) for translational motion i.e. translation along the $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ axes and three rotational degrees of freedom for rotational motion i.e. rotation about the $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ axes. Head movement is monitored in a gradient field by measuring the change in induced voltage in head mounted coils. To calculate the change in induced voltage I follow two approaches: circular loops simulation, analytical as well as numerical calculations. I show that by using a standard method one can form a linear model to identify the position and orientation of the coils.

An experimental arrangement is set up to check the validity of the analytical and numerical calculations. Experiments carried out with a rig of five coils verified that the changes in induced voltage in the coils is linear with respect to the changes in position of the coils. The linear model is also verified by comparing estimated positions obtained by using the coils to those found by image realignment of fast field echo (FFE) images using Statistical Parametric Mapping (SPM).

We experimentally evaluate the new approach for monitoring head movement inside an MR scanner, which exploits the linear variation of the voltages induced in a set of coils by time-varying magnetic field gradients with respect to small changes in position/orientation. This approach was tested by attaching five coils to a structured agar phantom and a healthy volunteer's head. The results suggest that it is possible to estimate the position and orientation with 0.22 mm and 0.24° root-mean-square error using this set-up. The new approach could be used for prospective or retrospective motion correction.

An experiment is also carried out by using free running EPI (Echo Planar Imaging) to track the head movement inside an MR scanner. There is a strong relation between head movement and EPI waveforms, the central point of the experiment is to track the head displacements via measuring induced voltage in the coils by using EPI waveforms during execution of free running EPI. The results obtained from the experiment reveal that the method is promising.

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Contents

1	Introduction	1
1.1	Motivation	1
1.2	Outline	3
1.3	An Overview of Subject Motion in MRI	5
1.3.1	Motion Prevention	5
1.3.2	Motion Correction	7
1.3.3	Position Monitoring	7
1.4	Rigid Body Kinematics	11
1.4.1	Degrees of Freedom of a Rigid Body	11
2	Magnetic Resonance Imaging (MRI)	15
2.1	Spatial Encoding	15
2.1.1	Gradients in MRI	16
2.2	Gradient Echo and Spin Echo	20
2.3	k-space	21
2.4	Echo Planar Imaging	24
2.5	MR Instruments	26

2.5.1	Main Magnet	26
2.5.2	Shim Coil	28
2.5.3	Gradient Coil	28
2.5.4	RF Coil	29
2.5.5	Control and Processing System	29
2.6	MR Compatible EEG System	30
2.6.1	The BrainAmp ExG	31
2.6.2	The SyncBox	31
2.7	Safety	31
3	Theory: Analytical Expression for the Induced Voltage in a Coil	34
3.1	Specific Coils in a Plane	34
3.1.1	Analytical Calculations	34
3.1.2	Translation	36
3.1.3	Rotation	37
3.1.4	Rotation about the x-axis	39
3.1.5	Rotation about the y-axis	41
3.1.6	Rotation about the z-axis	44
3.1.7	The General Equation for Rotation	47
3.1.8	The General Expression for Translation and Rotation	48
3.1.9	Special Cases	52
3.1.10	Absolute EMF in the Coils	54
3.2	Arbitrary Coils in Space	57

3.2.1	Geometrical Shape and the Dipole Concept of a Coil	61
3.3	Discussion	71
3.4	Conclusion	72
4	Simulations: Calculation of the Induced Voltage in a Coil	73
4.1	Introduction	74
4.2	Theory Underlying the Simulations	75
4.3	Method and Analysis	76
4.3.1	Method 1: Planar Coil	76
4.3.2	Method 2: Arbitrary Coils in Space	110
4.3.3	Method 3: Numerical Simulation Based on Theory	126
4.4	Results	127
4.4.1	Results: Planar Coils	127
4.4.2	Results: Arbitrary Coils in space	129
4.4.3	Results: Numerical Simulation	132
4.5	Discussion	136
4.6	Conclusion	138
5	EPI Pre-pulses: Rig and Planar Coil	140
5.1	Introduction	140
5.2	Method and Materials	141
5.2.1	MR Scanning Parameters	143
5.2.2	The Gradient Pre-pulses	144
5.2.3	The change in induced voltage in the coils	147

5.2.4	The change in induced voltage and their linear fit	153
5.3	Results	158
5.3.1	Comparison between Simulation and Experiment	158
5.3.2	Small Mixed Movements	161
5.4	Discussion	164
5.5	Conclusions	165
6	Fast Field Echo: Gimbal, Spherical Phantom and Non-planar Coils	167
6.1	Introduction	167
6.2	Method and Materials	168
6.2.1	Characterising the effect of z-displacement	170
6.2.2	Characterizing the effect of a rotation about the x-axis	171
6.2.3	Characterizing the effect of rotation about the z-axis	171
6.2.4	Mixed Movements	171
6.2.5	Scanning Parameter	173
6.2.6	MR Images and Realignment	173
6.3	Results and Discussions	174
6.3.1	Movement Parameters via SPM	174
6.3.2	Movement Parameters via Voltage Change in the Coils	176
6.4	Conclusion	185
7	Optimization of Gradient Pre-pulses and Multivariate Analysis (MVA) for Prediction of Parameters	186
7.1	Introduction	186

7.2	Method and Materials	188
7.2.1	Multivariate Analysis	191
7.3	Results	191
7.3.1	Pre-pulse Optimization	191
7.3.2	Model Formation	193
7.3.3	Recordings for long TR period	197
7.3.4	Recordings for Short TR period	203
7.3.5	Discussion	208
7.4	Conclusion	210
8	Free Running EPI	211
8.1	Introduction	211
8.2	Method and Materials	213
8.2.1	Free Running EPI Model	215
8.3	Results	218
8.3.1	Phantom	218
8.3.2	Subject	218
8.4	Discussions	223
9	Conclusion	224
9.1	Summary	224
9.2	Future Work	226
	Appendix A Mathematical Calculations in Details	241

A.1	Rotation about the x-axis	241
A.1.1	Rotation about the y-axis	246
A.1.2	Rotation about the z-axis	251

List of Figures

1.1	Degrees of freedom of a rigid body(a bar) in a plane	13
1.2	Degrees of freedom of a rigid body in space	13
1.3	Degrees of freedom. (a) Three translational degrees of freedom. (b) Three rotational degrees of freedom.	14
2.1	Diagram shows the basic concepts of slice select gradient.	17
2.2	RF and gradient waveform for slice selection and slice rephasing gradient. . .	18
2.3	Frequency encoding in MRI: (left) field variation due to an x-gradient; (right) frequency variation in presence of x-gradient.	19
2.4	Phase encoding in MRI: (left) simple two-dimensional object; (right) signal variation when phase encoding gradient (G_y) is followed by a frequency encoding gradient (G_x).	20
2.5	A frequency encoding gradient waveform. (a) Gradient echo pulse sequence. (b) Spin echo pulse sequence.	22
2.6	Digram showing the trajectory for filling k-space in the EPI sequence.	23
2.7	Schematic diagram of gradient echo EPI sequence.	25
2.8	MR instrument.	27
2.9	Schematic diagram of the control and processing system.	30

2.10	A schematic diagram of USB connection between the SyncBox main unit and the computer is utilizing hardware clock synchronization with any particular scanner clock frequency.	32
3.1	The location and the sensitivity vector of a coil.	35
3.2	The position and the orientation of the five coils.	37
3.3	Structure of the five coils rig.	55
3.4	This Figure represents an irregularly shaped coil (blue) and it's location vector (black) and sensitivity vectors (red).	58
3.5	Drawing representing an irregularly shaped coil (blue) and the corresponding vectors (blue) before rotation and the coil (red) and the corresponding vectors (red) after rotation about the x-axis. The circles ($\circ$) represent the location vectors and the arrows ($\rightarrow$) indicate the sensitivity vectors of the coils.	59
3.6	Drawing representing three types of coils and their location vectors denoted by a circle ($\circ$) and sensitivity vectors denoted by ($\rightarrow$). (a) Regular geometrical shaped (circular) coil. (b) Irregular geometrical shaped coil. (c) Highly irregular geometrical shaped coil.	62
3.7	Data shows the geometrical shape, the location vector denoted by a circle ($\circ$) and a cross ($\times$) and the sensitivity vector denoted by an arrow ($\rightarrow$) of a coil before (blue) and after (red) translation along the x-axis. The black cross ($\times$) and the black arrow ($\rightarrow$) stand for the location and the sensitivity vectors respectively, when they are shifted individually (not along with the coils). (a) Regular shaped (circular) coil. (b) Irregular shaped coil. (c) Highly irregular shaped coil.	64

3.8	Data shows the geometrical shape, the location vector denoted by a circle ($\circ$) and a cross ($\times$) and the sensitivity vector denoted by an arrow ($\rightarrow$) of a coil before (blue) and after (red) translation along the y-axis. The black cross ($\times$) and the black arrow ($\rightarrow$) stand for the location and the sensitivity vectors respectively, when they are shifted individually (not along with the coils). (a) Regular shaped (circular) coil. (b) Irregular shaped coil. (c) Highly irregular shaped coil.	65
3.9	Data shows the geometrical shape, the location vector denoted by a circle ($\circ$) and a cross ($\times$) and the sensitivity vector denoted by an arrow ($\rightarrow$) of a coil before (blue) and after (red) translation along the z-axis. The black cross ($\times$) and the black arrow ($\rightarrow$) stand for the location and the sensitivity vectors respectively, when they are shifted individually (not along with the coils). (a) Regular shaped (circular) coil. (b) Irregular shaped coil. (c) Highly irregular shaped coil.	66
3.10	Data shows the geometrical shape, the location vector denoted by a circle ($\circ$) and a cross ($\times$) and the sensitivity vector denoted by an arrow ($\rightarrow$) of a coil before (blue) and after (red) rotation about the x-axis. The black cross ($\times$) and the black arrow ($\rightarrow$) stand for the location and the sensitivity vectors respectively, when they are rotated individually (not along with the coils). (a) Regular shaped (circular) coil. (b) Irregular shaped coil. (c) Highly irregular shaped coil.	68

3.11	Data shows the geometrical shape, the location vector denoted by a circle ($\circ$) and a cross ($\times$) and the sensitivity vector denoted by an arrow ($\rightarrow$) of a coil before (blue) and after (red) rotation about the y-axis. The black cross ($\times$) and the black arrow ($\rightarrow$) stand for the location and the sensitivity vectors respectively, when they are rotated individually (not along with the coils). (a) Regular shaped (circular) coil. (b) Irregular shaped coil. (c) Highly irregular shaped coil.	69
3.12	Data shows the geometrical shape, the location vector denoted by a circle ($\circ$) and a cross ($\times$) and the sensitivity vector denoted by an arrow ($\rightarrow$) of a coil before (blue) and after (red) rotation about the z-axis. The black cross ($\times$) and the black arrow ($\rightarrow$) stand for the location and the sensitivity vectors respectively, when they are rotated individually (not along with the coils). (a) Regular shaped (circular) coil. (b) Irregular shaped coil. (c) Highly irregular shaped coil.	70
4.1	A circular loop in a plane	77
4.2	A closed circular loop of perimeter, l , and an infinitesimal element of length, dl	78
4.3	Translation along the x-axis. (a) A circular coil in the xy-plane having 10 turns is shifted 10 mm along the x-axis (b) Linear changes in induced voltage due to an x-translation during application of an x-gradient	79
4.4	Translation along the y-axis. (a) A circular coil in the yz-plane is shifted by 10 mm along the y-axis (b) Linear changes in induced voltage due to y-translation during execution of a z-gradient.	81
4.5	Translation along the z-axis. (a) A circular coil in the zx-plane is shifted by 10 mm along the z-axis. (b) Linear changes in induced voltage due to z-translation during application of a z-gradient.	82

4.6	Rotation about the x-axis. (a) A circular coil in the xy-plane and 5° rotation about the x-axis. (b) The change in induced voltage in a coil placed in the xy-plane due to rotation about the x-axis during execution of a y-gradient.	83
4.7	Rotation about the y-axis. (a) A circular coil in the yz-plane and its rotation (5°) about the y-axis. (b) The change in induced voltage in a coil placed in the yz-plane during execution of execution of an x-gradient.	84
4.8	Rotation about the z-axis. (a) A circular coil in the zx plane and its rotation (5°) about the z-axis. (b) The change in induced voltage in a coil due to rotation about the z-axis during execution of x-, y- and z-gradients.	85
4.9	A system of five loops in their corresponding planes	86
4.10	Translation of the system along the three mutually perpendicular axes. The centres of the coils are denoted by a circle ($\circ$) before and by a crosses ($\times$) after translations. (a) Translation along the x-axis. (b) Translation along the y-axis. (b) Translation along the z-axis.	88
4.11	Rotation of the system along the three mutually perpendicular axes. The centres of the coils are denoted by circles ($\circ$) before and by crosses ($\times$) after rotations. (a) Rotation about the x-axis. (b) Rotation about the y-axis. (b) Rotation about the z-axis.	89
4.12	The change in voltage induced (μV) in the coils by the execution of x-, y- and z-gradients. C_{ng_α} denotes the change in induced voltage in coil n due to the gradient applied along the α (a) due to x-translation, (b) due to y-translation, and (c) due to z-translation.	90

4.13	The change in voltage induced (μV) in the coils by the execution of x-, y- and z-gradients. $C_n g_\alpha$ denotes the change in induced voltage in coil n due to the gradient applied along the α direction (a) sequential translation along the x-axis for coil C_1 exposed to an x-gradient and coils C_2 & C_3 when exposed to a z-gradient, (b) sequential translation along the y-axis for coil 1 exposed to a y-gradient and coils C_4 & C_5 when exposed to a z-gradient.	94
4.14	The change in induced voltage due to translation along the z-axis in the coils C_1 during the application of a z-gradient, in C_2 and C_3 during execution of an x-gradient, in C_4 and C_5 when exposed to a y-gradient.	95
4.15	The change in voltage induced (μV) in the coils by the execution of x-, y- and z-gradients. $C_n g_\alpha$ denotes the change in induced voltage in coil n due to the gradient applied along the α (a) due to rotation about the x-axis, and (b) due to rotation about the y-axis.	97
4.16	The change in voltage induced (μV) in the coils by the execution of x-, y- and z-gradients. $C_n g_\alpha$ denotes the change in induced voltage in coil n due to the gradient applied along the α due to rotation about x-axis (a) in the coils C_1 , C_4 and C_5 during execution of y-gradient, and (b) in the coils C_1 , C_4 and C_5 during execution of z-gradient.	99
4.17	The change in voltage induced (μV) in the coils by the execution of x-, y- and z-gradients. $C_n g_\alpha$ direction denotes the change in induced voltage in coil n due to the gradient applied along the α due to rotation about the y-axis (a) in the coils C_1 , C_2 and C_3 during execution of x-gradient, and (b) in the coils C_1 , C_2 and C_3 during execution of a z-gradient.	100

4.18	The changes in induced voltage in the coils and their linear fits. (a) The induced voltage changes occur in coil C_1 during execution of an x-gradient, whereas in C_2 and C_3 when exposed to a z-gradient due to translation along the x-axis. (b) The induced voltage changes occur in coil C_1 during execution of a y-gradient, whereas in coils C_4 and C_5 during the application of a z-gradient due to translation along the y-axis. (c) The induced voltage changes occur in coil C_1 during execution of a z-gradient, whereas in coils C_2 and C_3 during the application of an x-gradient, on the other hand in coils C_4 and C_5 during the application of a z-gradient due to translation along the z-axis. . .	101
4.19	The change in voltage in the coils and their linear and quadratic fit. (a) The change in induced voltage in the coils C_1 , C_4 and C_5 during execution of a y-gradient due to rotation about the x-axis. (b) The change in induced voltage in the coils C_1 , C_4 and C_5 during execution of a z-gradient due to rotation about the x-axis.	104
4.20	The change in induced voltage in the coils and their fit. (a) The change in induced voltage in the coils C_1 , C_4 and C_5 during execution of a y-gradient due to rotation about the y-axis. (b) The change in induced voltage in the coils C_1 , C_4 and C_5 during execution of a z-gradient due to rotation about the y-axis.	104
4.21	Figures representing translation along the x-axis. (a) A coil shifted 10 mm in 40 steps along the x-axis. (b) The change in induced voltage in a coil due to sequential translations up to 10 mm along the x-axis during execution of the x-, y- and z-gradients.	111
4.22	Data represents translation along the y-axis. (a) A coil shifted 10 mm in 40 steps along the x-axis. (b) The change in induced voltage in a coil due to sequential translations up to 10 mm along the y-axis during execution of the x-, y- and z-gradients.	112

4.23	Data represents translation along the z-axis. (a) A coil shifted 10 mm in 40 steps along the z-axis. (b) The change in induced voltage in a coil due to sequential translations up to 10 mm along the z-axis during execution of the x-, y- and z-gradients.	113
4.24	Data represents rotation about the x-axis. (a) A coil rotates 10° in 40 steps about the x-axis. (b) The change in induced voltage in a coil due to successive rotations up to 10° during execution of the x-, y- and z-gradients.	115
4.25	Data represents rotation about the y-axis. (a) A coil rotates 10° in 40 steps about the y-axis. (b) The change in induced voltage in a coil due to successive rotations up to 10° during execution of x-, y- and z-gradients.	116
4.26	Data represents rotation about the z-axis. (a) A coil rotates 10° in 40 steps about the z-axis. (b) The change in induced voltage in a coil due to successive rotations up to 10° during the application of the x-, y- and z-gradients. . . .	117
4.27	Data represents sequential translation along the x-axis during execution of x-, y- and z-gradients. The coils shifted 10 mm along the x-axis are shown in (a). The changes in induced voltage in the five coils (1-5) and their 2nd order polynomial fits are shown in (b), (c), (d), (e) and (f) respectively.	118
4.28	Data represents sequential translation along the y-axis during execution of x-, y- and z-gradients. The coils shifted 10 mm along the y-axis are shown in (a). The changes in induced voltage in the five coils (1-5) and their 2nd order polynomial fits are shown in (b), (c), (d), (e) and (f) respectively.	119
4.29	Data represents sequential translation along the z-axis during execution of x-, y- and z-gradients. The coils shifted 10 mm along the z-axis are shown in (a). The changes in induced voltage in the five coils (1-5) and their 2nd order polynomial fits are shown in (b), (c), (d), (e) and (f) respectively.	120

4.30	Data represents sequential rotation of the coils about the x-axis during execution of x-, y- and z-gradients. The coils rotated 10° in 40 steps about the x-axis are shown in (a). The changes in induced voltage in the five coils (1-5) and their polynomial fits are shown in (b), (c), (d), (e) and (f) respectively.	121
4.31	Data represents sequential rotation of the coils about the y-axis during execution of x-, y- and z-gradients. The coils rotated 10° in 40 steps about the y-axis are shown in (a). The changes in induced voltage in the five coils (1-5) and their polynomial fits are shown in (b), (c), (d), (e) and (f) respectively.	122
4.32	Data represents sequential rotation of the coils about the z-axis during execution of x-, y- and z-gradients. The coils rotated 10° in 40 steps about the z-axis are shown in (a). The changes in induced voltage in the five coils (1-5) and their polynomial fits are shown in (b), (c), (d), (e) and (f) respectively.	123
4.33	The actual movement parameters and the parameters calculated from numerical simulation by calculating the change in induced voltage in the coils due to translation and rotation of the coils during the application of x-, y- and z-gradients. (a) Data shows the actual and estimated movement parameters where there is no zero terms in the actual movements. (b) Data represents actual and estimated movement parameters where there are a few zero terms in the actual movements.	127
4.34	Errors in the estimated movement parameters from calculated induced voltages in the coils due to translation and rotation without adding noise voltages.	128
4.35	Estimated errors in position and orientation from calculated induced voltages due to translation and rotation by high noise level (0-20 μV) in (a) and low noise level (0-10 μV) in (b).	129

4.36	A comparison between the sum of the change in induced voltage due to translations and rotations and the change in induced voltage due to combined (in a single go) movements (simultaneous translation and rotation). (a) Small rotations up to 5° and translation. (b) Large rotation up to 10° and translation.	130
4.37	Estimated position and orientation where the maximum rotation is 5° and translation is 10 mm. (a) Estimated parameters from a random movement when there is no zero movement along/about an axis. (b) Estimated parameters from a random movement when there is at least one zero movement along/about an axis.	131
4.38	Estimated the movement parameters from calculated induced voltage in the coils due to translation and rotation without adding noise to the induced voltages.	132
4.39	Errors in estimated position and orientation due to addition of random noise voltages. (a) Errors in estimated movement parameters from calculated induced voltages due to translation and rotation by increasing noise level in the induced voltages (0-20 μV). (b) Errors in estimated movement parameters from calculated induced voltage in the coils due to translation and rotation by reducing noise level in the induced voltages (0-10 μV).	133
4.40	Estimated movement parameters from calculated induced voltage in the coils due to translation and rotation without noise (red line) and with increasing added noise (black circle) 1-10 μV in the induced voltages.	134
4.41	Data represents the estimated movement parameters (line) and their standard deviation (error bars) by using calculated voltage induced in the coils when an increasing noise voltage of 10-100 μV is added to the change in induced voltage in the coils. (a) Translations along the x-, y- and z-axes. (b) Rotations about the x- and y-axes.	135

4.42	Errors in estimated movement parameters are increasing as the noise voltage increases in the induced voltage of the coils.	136
5.1	A rig of five coils	142
5.2	The position and orientations of an experimental system of five coils used to measure position changes via recording of voltages induced by time-varying gradients.	142
5.3	Data represents the change in induced voltage during execution of gradient pre-pulses along RL, AP and FH axes. The change in induced voltage recorded on coil C_1 due to translation along the x-axis (a), y-axis (b) and z-axis (c). .	145
5.4	Data represents the change in induced voltage during execution of gradient pre-pulses along RL, AP and FH axes. The change in induced voltage recorded on coil C_1 due to rotation about the x-axis (a), y-axis (b) and z-axis (c). . . .	146
5.5	Experimental data indicate the change in induced voltage in the coils due to translation along the x-axis (a) during execution of x-gradient, (b) during execution of y-gradient, and (c) during execution of z-gradient.	148
5.6	Experimental data indicate the change in induced voltage in the coils due to translation along the y-axis (a) during the application of an x-gradient, (b) during the execution of a y-gradient, and (c) because of the application of a y-gradient. . . .	149
5.7	Experimental data indicate the change in induced voltage in the coils due to translation along the z-axis (a) during the execution of the x-gradient, (b) during the execution of the y-gradient, and (c) during execution of the z-gradient.	150
5.8	Experimental data indicate the change in induced voltage in the coils because of the application of the x-, y- and z-gradients (a) due to translation along the x-axis, (b) to translation along the y-axis, and (c) due to translation along the z-axis.	151

5.9	Experimental data represents the change in induced voltage in the coils due to rotation about the x-axis (a) because of the application of the x-gradient, (b) because of the application of the y-gradient, and (c) because of the application of the z-gradient. . .	152
5.10	Experimental data indicate the change in induced voltage in the coils due to rotation about the y-axis. (a) Experimental data indicate the change in induced voltage during the application of the x-gradient, (b) during the application of the y-gradient, and (c) during the application of the z-gradient.	154
5.11	Experimental data represents the change in induced voltage in the coils due to rotation about the z-axis (a) during the application of the x-gradient, (b) during the application of the y-gradient, and (c) during the application of the z-gradient. . . .	155
5.12	Plots showing the change in induced voltage in the coils during execution of x-, y- and z-gradients. (a) due to translation along the x-axis, (b) due to translation along the y-axis, and (c) due to translation along the z-axis. . . .	156
5.13	Experimental results showing induced voltage changes in the coils due to sequential rotation about the x-axis and their linear fit (a) during the application of the x-gradient, (b) during the application of the y-gradient, and (c) during the application of the z-gradient.	157
5.14	Experimental data indicate the change in induced voltage due to rotation about the y-axis and their linear fit (a) during the application of the x-gradient, (b) during the application of the y-gradient, and (c) during the application of the z-gradient.	159
5.15	Experimental results represent the change in induced voltage in the coil due to rotation about the z-axis and their linear fit (a) during execution of a x-gradient, (b) during execution of a y-gradient, and (c) during execution of a z-gradient.	160

5.16	The change in the voltage induced in the 5 coils due to sequential translation along the z-direction for application of x, y, z gradients. (a) Experiment. (b) Simulation.	161
5.17	The coefficients of the matrix $\mathbf{A}$ for a system of linear equations $\mathbf{Ax} = \delta\mathbf{v}$. Colour-bar shows $\mu\text{V}/\text{mm}$ for translation and $\mu\text{V}/\text{degree}$ for rotation. (a) Experiment. (b) Simulation.	162
5.18	Comparison between actual (A) movements and movements estimated (E) from experiment. (a) Movements are limited to 6 mm (translation) and 5° (rotation) for the six parameters. (b) Movements are limited to 8 mm (translation) and 5° (rotation) for the six parameters.	163
5.19	Comparison between actual movements and movements estimated from experimental measurement. (a) Movements are limited to translation along x, y and z, and rotation about the x-axis only. (a) Movements are limited to translation along x, y and z, and rotation about the x- and z-axes only. . . .	164
6.1	An MR image of a spherical agar phantom having a 3D cross of water.	169
6.2	The Gimbal allowing calibration of x- and z-rotation is shown in (A) and the spherical head phantom covered with EEG like cap mounted by five coils is shown in (B).	170
6.3	The movement parameters from the co-registered FFE images. Where (a) & (b) z-translation, (c) & (d) rotation about the x-axis and (e) & (f) rotation about the z-axis.	175
6.4	Movement parameters via SPM from image realignment. (a) Translational parameters via SPM using image realignment. (b) Rotational parameters via SPM using image realignment.	176
6.5	The change in induced voltage in the coils due to sequential z-translations. (a) Execution of x-gradient, (b) Execution of y-gradient, (c) Execution of z-gradient	177

6.6	Induced voltage change in the coils due to sequential rotation about the x-axis. (a) Execution of x-gradient. (b) Execution of y-gradient. (c) Execution of z-gradient.	179
6.7	The change in induced voltage in the coils due to sequential rotation about the z-axis. (a) Execution of x-gradient. (b) Execution of y-gradient. (c) Execution of z-gradient.	180
6.8	Experimental data indicate induced voltage change in the coils and their linear fittings. (a) The change in induced voltage in the coils due to execution of x-gradient and their linear fittings. (b) The change in induced voltage in the coils due to execution of y-gradient and their linear fittings. (c) The change in induced voltage in the coils due to execution of z-gradient and their linear fittings.	181
6.9	Experimental data indicate the change in induced voltage in the coils and their linear fittings. (a) The change in induced voltage in the coils during execution of x-gradient and their linear fittings. (b) The change in induced voltage in the coils during execution of y-gradient and their linear fittings. (c) The change in induced voltage in the coils during execution of z-gradient and their linear fittings.	182
6.10	Experimental data indicate induced voltage change in the coils due to sequential rotation about the z-axis and their linear fittings. (a) The change in induced voltage in the coils during execution of x-gradient and their linear fittings. (b) The change in induced voltage in the coils during execution of y-gradient and their linear fittings. (c) The change in induced voltage in the coils during execution of z-gradient and their linear fittings.	183
6.11	A comparison between movement parameters obtained via SPM and obtained via voltage measurement in the coils.	184

7.1	Set of five coils attached to (A) a spherical agar phantom (all coils had ~ 4 cm diameter and 2 turns) and (B) a healthy human subject (Coils 1-3 coils had ~ 4 cm diameter and 2 turns, while Coils 4 and 5 had ~ 3 cm diameter and 3 turns). Coils were connected to the ExG bipolar amplifier (Brain Products GmbH) using star quad cable (Van Damme) minimizing the induced voltage contribution from the cables. Coils are connected to the ExG amplifier through $100\text{ k}\Omega$ resistors.	188
7.2	Voltages induced in a single coil mounted on a phantom by application of a sequence of three trapezoidal gradient pulses (5 ms ramp-up/down at 4 T/m-s) successively applied along the RL, AP and FH directions. Position of the phantom was shifted away from iso-centre in the FH direction in steps of 2 mm causing the amplitude of the induced voltages to change. The results are shown for 4 different positions of the phantom. Inset A shows no appreciable change in the baseline recording with respect to successive movements, whereas inset B shows a linear increase in the voltage induced by the FH trapezoidal pulse as the FH shift increases.	190
7.3	Flow diagram for multivariate analysis.	192
7.4	Optimization of TR and time delay between pre-pulses, delays are 100 ms (a) 200 ms (b). A 50 ms period spanning each pre-pulse has been cut out and the all three periods were concatenated. The delay between pulses is thus not completely shown. TR was set to 10 s for these experiment.	194
7.5	Optimization of TR and time delay between pre-pulses delays are 400 ms (a) 500 ms (b). A 50 ms period spanning each pre-pulse has been cut out and the all three periods were concatenated. The delay between pulses is thus not completely shown. TR was set to 10 s for these experiment.	195

7.6	Optimization of TR and time delay between pre-pulses, where time delays 400 ms. A 50 ms period spanning each pre-pulse has been cut out and the all three periods were concatenated. The delay between pulses is thus not completely shown. TR was set to 7.5 s for these experiment.	196
7.7	Data represents the movements parameters estimated by measuring the changes in induced voltage in the coils by time varying gradient magnetic field. Parameters estimated by using the induced voltages ($\circ$) and SPM8 (*). 12 data points are estimated for 4 consecutive movement and excluding 3 data points, (3rd, 7th and 11th) one from each type of movement, end up with 9 data points that are used to form the model, which is then applied to find position for the 12 data points are shown in (A) for the phantom. The difference ($\times$) between the parameters estimated with the coils ($\circ$) and using SPM8 (*) is shown in (B) for the phantom.	198
7.8	Change in position and orientation for phantom (A). Data collection was split into a model-forming period during which positions were successively adjusted along/about a particular axis and an experimental period during which positions were randomly varied. The changes in voltages with position were characterised using data from the model-forming period. This characterisation was used to estimate the position change from the voltage recordings collected during the whole experiment. The differences ($\times$) between the position estimated with the coils ($\circ$) and using SPM8 (*) is shown in (B) for the phantom.	199
7.9	Root mean square error in the estimated parameters for spherical head phantom.	200

7.10	Change in position and orientation for subject (A). Data collection was split into a model-forming period during which positions were successively adjusted along/about a particular axis and an experimental period during which positions were randomly varied. The changes in voltages with position were characterised using data from the model-forming period. This characterisation was used to estimate the position change from the voltage recordings collected during the whole experiment. The differences ($\times$) between the position estimated with the coils ($\circ$) and using SPM8 ($*$) is shown in (B) for the phantom.	201
7.11	Root mean square error in the estimated parameters for the healthy volunteer.	202
7.12	Measurements were made with short TR (TR = 2s) during scanning of the spherical agar phantom with a single pre-pulse applied per slice acquisition and the direction of the pre-pulse gradient cycled across volume acquisition. Changes in position and orientation for phantom (A). Data collection was split into a model-forming period during which positions were successively adjusted along/about a particular axis and an experimental period during which positions were randomly varied. The changes in voltages with position were characterised using data from the model-forming period. This characterisation was used to estimate the position change from the voltage recordings collected during the whole experiment. The differences ($\times$) between the position estimated with the coils ($\circ$) and using SPM8 ($*$) is shown in (B) for the phantom.	205

7.13	Measurements were made with short TR ($TR = 2s$) during scanning of the healthy subjects with a single pre-pulse applied per slice acquisition and the direction of the pre-pulse gradient cycled across volume acquisition. Change in position and orientation for subject (A). Data collection was split into a model-forming period during which positions were successively adjusted along/about a particular axis and an experimental period during which positions were randomly varied. The changes in voltages with position were characterised using data from the model-forming period. This characterisation was used to estimate the position change from the voltage recordings collected during the whole experiment. The differences ($\times$) between the position estimated with the coils ($\circ$) and using SPM8 ($*$) is shown in (B) for the subject.	206
7.14	Change in position and orientation for subject (A). Measurements were made with short TR ($TR = 2s$) during scanning of the healthy subjects with a single pre-pulse applied per slice acquisition and the direction of the pre-pulse gradient cycled across slice acquisition. Data collection was split into a model-forming period during which positions were successively adjusted along/about a particular axis and an experimental period during which positions were randomly varied. The changes in voltages with position were characterised using data from the model-forming period. This characterisation was used to estimate the position change from the voltage recordings collected during the whole experiment. The differences ($\times$) between the position estimated with the coils ($\circ$) and using SPM8 ($*$) is shown in (B) for the subject.	207
7.15	Without dynamic stabilization image shift along the phase encoding direction.	209
8.1	Experimental arrangement to recorded voltages on a spherical head phantom (a) and on a healthy volunteer (b) on five coils during experiment of a multi-slice EPI sequence.	213

8.2	Voltage recorded on five coils during execution of a multi-slice EPI sequence. The R128 marker indicates the start of each slice acquisition.	214
8.3	Voltage changes produced by z-translation, head-noding (pitch) and head-shaking (yaw) when free running EPI is applied. Voltage changes are shown for a two slice acquisitions corresponding to voltages shown in Fig. 8.2. . . .	216
8.4	A portion of the three EPI model waveforms (voltage waveforms) and the voltage changes induced in the individual coil are fitted. Model waveform is formed from the changes in induced voltage in the coils and fitted waveform are obtained by using model.	217
8.5	Data obtained from phantom scan. Estimated position and orientation change by using SPM8 and by fitting to the coil voltages during cued movement in the experimental period.	219
8.6	Data obtained from the phantom scan. Estimated position and orientation change by using SPM8 and by fitting the voltages induced in the coils during free movement in the experimental period.	220
8.7	Data obtained from the subject scan. Estimated position and orientation change by using SPM8 and by fitting the voltages induced in the coils during cued movement in the experimental period.	221
8.8	Data obtained from subject scan. Estimated position and orientation change by using SPM8 and the coils during free movement in the experimental period.	222

List of Tables

1.1	Summary of the preprocessing techniques used to reduce motion artefacts in MRI.	12
3.1	The changes in induced voltage in the coils due to translation.	38
3.2	Summary of the changes in induced voltage in the coils due to a general rotation.	49
3.3	Summary of the change in induced voltage in the coils due to translation and rotation.	51
4.1	The fit parameters a_1 obtained from 2nd order polynomial fitting due to translations and rotations of the coils during execution of the x-,y- and z-gradients.	124
4.2	The fit parameters a_2 obtained from 2nd order polynomial fitting due to translations and rotations of the coils during execution of the x-,y- and z-gradients.	125
4.3	Sensitivity of the planar coils with respect to the movements and the applied gradients.	137
7.1	Summary of the scanning parameters to optimize pre-pulses.	192
7.2	Table showing the range and RMS amplitude of the differences between the SPM8 and the induced-voltage estimates of position for the phantom (calculated from 14 measurements) and the subject (determined from 5 measurements) over the experimental periods	202

8.1	Model and experiment for free running EPI.	212
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Abbreviation

- 2D - Two-dimensional
- AP - Anterior-Posterior
- CT - Computerized Tomography
- DOF - Degrees of Freedom
- EEG - Electroencephalography
- EM - Electromagnetic
- EPI - Echo Planar Imaging
- ETL - Echo Train Length)
- FH - Foot-Head
- fMRI - Functional Magnetic Resonance Imaging
- FFE - Fast Field Echo
- FOV - Field of View
- FSE - Fast Spin Echo
- FT - Fourier transform
- GE - Gradient Echo
- MR - Magnetic Resonance
- MRI - Magnetic Resonance Imaging
- NMR - Nuclear Magnetic Resonance Imaging
- PCA - Principal Component Analysis
- PMC - Prospective motion correction
- PROMO - Prospective motion correction
- PROPELLER - Periodically rotated overlapping parallel lines with enhanced reconstruction
- rf - Radio Frequency
- RL - Right-Left
- SNR - Signal to Noise Ratio
- SPM - Statistical Parametric Mapping
- TE - Echo time
- TR - Repetition time

Chapter 1

Introduction

1.1 Motivation

In spite of continuous improvements in Magnetic Resonance Imaging (MRI) through advances in image quality and acquisition speed, MR examinations are easily compromised by patient movements during a scan. Nearly one MRI scan out of five has to be repeated due to the poor quality of images caused by patient movement [1]. Consequently, the costs to the healthcare industry increase to almost \$1 billion per year [2]. Problems with patient motion are most often encountered when working with non-compliant patient populations, such as children or infants [3, 4], trauma cases or stroke patients [5], people agitated by anxiety or pain or people affected by movement disorders such as Parkinson's disease [6], and people affected by other diseases such as Alzheimer's disease [7], Huntington's disease [8], and multiple sclerosis [9]. Thus development of efficient motion correction would have huge beneficial impact for clinical MRI. By robust motion correction, MRI scans may eventually become less expensive and therefore more affordable, which means more people can benefit from MRI.

Motion artefacts are a serious problem in many MRI applications. Small head movements during image acquisition produce artefacts in magnetic resonance images. In functional MRI

(fMRI) time series, movement between consecutive acquisitions of the same slice leads to image mis-registration which can produce spurious activation in fMRI data. In anatomical MRI, movement between acquisitions of different data segments in k-space (the Fourier domain) generates image artefacts [10]. With additional information about the movement, it is possible to improve the re-alignment of images in fMRI time series and potentially to eliminate the artefacts in anatomical images by re-aligning the k-space data (known as “retrospective” correction). If movement information is recorded and processed during the image acquisition it is possible to adjust the image acquisition geometry [10] in “real time” so that the artefacts are eliminated. This is known as “prospective” motion correction.

Various methods [11–15] have been proposed for prospective motion correction, and a lot of research has been carried out on this topic in the last two decades. Motion sensitivity is still however an unsolved problem in MRI, as well as in fMRI. Researchers are hopeful regarding prospective motion correction. This technique has benefits such as less medical treatment costs and shows far improved diagnostic results. However, there can be some problems with implementing this method which still need to be solved. For instance, it becomes problematic when any shifting of the object creates changes in the magnetic field leading to changes in image distortion [10].

The challenge is to track involuntary movements of the patient’s head in an MR scanner. In this study I aim to track involuntary head movement by measuring the voltages induced by gradient switching in a set of coils attached to the head. By considering the human head as a rigid body this will allow us to apply prospective motion correction in MRI.

A variety of approaches [16–20] to monitoring motion of the head inside the MR scanner have been developed. These include using line of sight optical detection and small NMR coils. Moreover, some other researchers have carried out prospective motion correction due to head motions. For instance, real time intra-volume motion correction in EPI using active markers [21], head position prediction in EPI during rapid subject motion [22], real-time slice-by-slice motion correction for fMRI in freely moving subjects [23], combining prospective

motion and distortion correction for EPI [24].

This study proposes using a set of inductive coils woven into a cap placed on the scalp to measure motion. This will be accomplished by monitoring the voltages induced in the coils by the time-varying magnetic field gradients applied in the MRI sequence and relating the changes in the induced voltages to changes in head position and orientation. In the experimental works, the voltages can be recorded by connecting an MR compatible bipolar EEG amplifier(Brain Products GmbH) to the coils.

One approach in fMRI would be to measure the voltages induced in the coils on the scalp with the head held still at a number of different positions or orientations. By characterizing the positional changes through analysis of the MRI data and comparing with the pattern of voltage changes across coils, it should be possible to associate the pattern of voltage changes with particular movements. In subsequent free-running data acquisition by fitting the measured voltages changes to these patterns it should be possible to provide motion correction of the fMRI data on a slice-by-slice basis. It may also be possible to use measurements from the coils to interpolate slice-to-slice changes in position across volume acquisitions, where the movement between volumes are characterized by standard motion correction-characterization, which is also used to calibrate the coils. For anatomical imaging, a similar approach to measuring motion should be possible, though it will be necessary to avoid or model the effect of any phase encoding gradients.

1.2 Outline

Overall the project involves: theoretical work in establishing the optimal configuration of the coils, and deriving equations to calculate the change in induced voltage in the coils. The project also involves computing work, in numerical simulations of the effect of the time-varying gradients on the coils and calculation of motion parameters from measured data. Experimental work involves validating the method in an MR system with a simple set-up and performing measurements using the MR scanner and EEG system. Experiments also

involve optimization and establishment of the precision of the estimated movements and their multivariate analysis. A free running EPI model is established to estimate the movement parameters by measuring induced voltage in the coils and by using image co-registration with the help of SPM8.

A short introduction of this study is given in Chapter 1. In this chapter, Section 1.3 is devoted to a review of motion artefacts, prospective motion correction, real-time motion correction and Echo Planar Imaging (EPI). Rigid body kinematics are also briefly discussed in Section 1.4. For simulation it is necessary to understand the rigid body motion. A model is developed for tracking rigid body motion via the EMF's induced in a set of coils. For this purpose software is developed for tracking head motion. Chapter 4 describes the simulations in detail.

The motion tracking sequence and algorithms are discussed in detail in Chapters 3 and 4. I try to analyse and understand the systematic errors in the estimated position and orientation via measurement of the changes in induced voltage during movement monitoring in the MR system and develop models for capturing the systematic errors. Chapter 5 focuses on experiments designed to measure the model parameters with high SNR. The validity of the method and software are checked by using a simple experimental set-up. Chapter 6 is also devoted to checking the validity of the linear model by using image co-registration as well as measuring the changes in the induced voltage in the coils.

Optimization of EPI pre-pulses and the EPI pulse sequence, multivariate analysis, as well as the precision of the estimated movement parameters are discussed in Chapter 7. In Chapter 8 a free-running EPI model is established to estimate movement parameters.

Chapter 9 briefly discusses the findings from simulation, theory and experiment and also mentions some challenges for real-life implementation. Finally, this Chapter draws some conclusions from results and gives a direction for future work.

1.3 An Overview of Subject Motion in MRI

Subject motion has long been recognized as a problem in MR imaging, in diagnosis as well as in research. In MRI the data acquisition time varies depending on the particular sequence used. The problem can be solved by gating the image acquisition for some types of motion which are periodic, such as those due to blood flow, and due to cardiac and respiratory movements. Rigid body motion such as head motion is, however, still a severe problem in MRI.

There are some methods to avoid motion artefacts in MRI: namely motion prevention, pre-processing and post-processing. In this section these methods are discussed in brief. Motion correction applied during post-processing generally refers to an affine or sometimes a non-linear registration algorithm that seeks to maintain a constant object position in a stack of slices throughout a time series acquisition. In this study, attention is given to pre-processing methods, and mainly to prospective motion correction.

1.3.1 Motion Prevention

Preventing motion of the subject is the most common approach to avoiding motion artefacts in MRI. Physical restraints, sedation and training are effective techniques for motion prevention. In clinical MRI a physical restraint, such as cushions, bean bags or foam packed around the head and within the head coil is often used to prevent head motion. Though these measures are not successful for all types of patient, they are easy to use and cause little distress to the patient.

Several other types of physical restraint exist. Bite bars are sometime used; these involve making a dental mould and then having the subject to hold it firmly in their mouth [25,26]. As the mould is custom-made, it fits the user well and can greatly reduce motion. The inconvenience of constructing a personalized restraint is however a major disadvantage of this approach. Furthermore, the presence of an object in the mouth of a patient can increase

swallowing, leading to an increase in head motion; Edward *et al.* [27] report success using a plaster cast head holder. This custom-made head holder exerted an even pressure on the skull and fits comfortably, but it is expensive as well as time consuming to prepare a cast. Although possibly useful for research, this approach is not suitable for clinical MRI. In general, physical restraints can cause problems by adding to patient discomfort and feelings of claustrophobia. Claustrophobia is a major problem in MRI and patients sometimes need to have ‘breaks’ during their scan if they suffer from this. Any restraint must also be easy to use: if a system can only be used on a well-behaved and fully conscious patient, then it is of limited use clinically. Physical restraints are also often inappropriate for use in fMRI as they may interfere with the study itself.

Sedation is also used in some cases to prevent motion of the patient, but it is not suitable for healthy subjects in research. Besides this, there are several disadvantages associated with sedation. Monitoring sedated patients adds an unwanted complication to the scan and often requires the presence of an anesthetist. Sedation of children for MRI and computerized tomography (CT) is associated with risks of hypoxemia (an abnormally low concentration of oxygen in the blood) [28, 29]. Moreover, sedation is not useful for functional MRI (fMRI) due to the potential for interfering with experimental results usually requiring consciousness.

Training is another option to prevent motion, since it is possible to train subjects to hold themselves still prior to a scan. Slifer *et al.* [30] trained subjects to lie still in a simulator constructed from an inactive scanner. The situation was made as realistic as possible: pre-recorded scanner noise was played at high volume and a standard head support and restraining straps were used. Subjects (all children) watched a video which was halted whenever their head motion exceeded a certain threshold. Over time, this threshold was gradually reduced. While this approach has potential for studies of healthy subjects in research, it could not be generalized to acutely ill patients in clinical MRI. Training is useful to reduce the motion artefacts during scanning of the cooperative patient only.

Speeding up of the image acquisition using radial acquisition or echo planar imaging

reduces sensitivity to motion effects. Echo planar imaging is a useful technique, but can suffer from poor image quality in areas of low field homogeneity, whereas radial imaging has robustness to motion, but is not completely effective and increases acquisition time. Echo planar imaging is discussed in brief in this study because EPI pre-pulses and free running EPI are used for our data acquisition.

1.3.2 Motion Correction

There are two main types of motion correction in MRI: prospective and retrospective motion correction. Retrospective motion correction is a post-processing technique, while prospective motion correction involves modifying scanning parameters in real-time to negate the effects of motion. Real-time prospective motion correction was first proposed more than 20 years ago, but is still a challenge to implement in clinical MRI, and also in fMRI. Prospective motion correction generally refers to the same goal: conserving the anatomical image content over time, but, there are concomitant changes in the imaging signal, and perhaps the noise, when a head moves inside the magnet.

1.3.3 Position Monitoring

Over the last decade a number of motion monitoring techniques have been proposed. These include: optical tracking [31–33], image-based tracking [34–40], navigator echoes [41–53], and use of dedicated RF coils [54–56] etc. In this section, all these methods are discussed in brief. For prospective motion correction in brain imaging, several systems have been developed for tracking the head movement in an MRI scanner.

Optical methods include laser systems [57, 58], optical fibers [59], and camera systems. These methods are completely independent from the MR pulse sequence. Optical tracking work began with the use of out-of-bore camera systems by Zaitsev *et al.* [60], but it has now moved to in-bore camera systems. Camera systems have recently become popular, due to technology improvements in both the cameras and computing. Methods successfully used

for motion correction include out-of-bore stereo camera systems [23, 60], out-of-bore single camera systems [61], in-bore single camera systems [62, 63], and in-bore systems with multiple cameras [64, 65]. Out of-bore camera systems were useful for proof-of-concept studies, due to the lower requirements for MR compatibility, compared to in-bore cameras. However, they require optical line-of-sight and extremely high mechanical stability when the tracking marker is located several meters away from the camera. Although none of the systems are flawless, from these practical considerations, one can say in-bore camera systems are promising as a long-term solution. Schulz *et al.* [66] used a novel home-built optical embedded tracking system for motion-corrected magnetic resonance imaging at 7T. On the other hand Formana *et al.* [67] used a novel self-encoded marker to overcome the limited tracking range of the head position inside the camera’s narrow field of view (FOV). Almost all optical systems require a marker; which must be rigidly attached to the head. Researchers are now trying to develop markerless optical tracking [68, 69] systems, since this will form an ideal technique from a patient handling perspective.

Field detection methods have a long history in MRI. This principle can be used to encode the position of a marker when different locations in the scanner bore experience different magnetic fields. In particular the scanner’s gradient fields can be measured to localize the object. The method requires the use of short sequences of pulses to obtain position information from a small sample of MR visible material fixed inside a miniature receiver coil. Although Dumoulin *et al.* [70] pioneered developments in this area, this method was first introduced by Ackerman *et al.* [71] in 1986. A study of slice-by-slice prospective motion correction using such a system was published by Derbyshire *et al.* in 1998 [72]. Recent implementations, such as that of Ooi *et al.* [73, 74] characterise the RF coil/material combination as “active markers.” Active markers have been used for prospective motion correction in structural brain scans [73] and in EPI [74]. A similar technique was recently applied to measure gradient waveforms for prospective motion correction by Barmet *et al.* [75, 76], who decouple tracking from MR imaging by using RF-shielded probes and separate transmit or receive chains. Micro radio frequency coil “active markers” have also been used

by Melvyn *et al.* [77] for real-time slice-by-slice prospective correction. Recently Melvyn *et al.* [78] used inductively coupled wireless radio frequency (RF) coils (wireless markers) for position tracking. Lei *et al.* [79] used an array of intra-cavity imaging coils for prospective motion correction. Field detection methods require several probes or active markers to be attached to the subject (a minimum of three markers are required, positioned non-collinearly and connected in a rigid arrangement). In Refs. [73, 74], marker fixation was achieved by attaching the coils to a headband worn by the subject. There is a slight disadvantage over optical methods here, as the active markers (and hence the subject) are connected to the scanner by wires, which makes patient handling more difficult and could perhaps increase the patient's anxiety levels. The presence of cables also enhances the difficulties with the rigid marker fixation.

Navigator methods are a more traditional way of obtaining object position information during MRI, where the scanner is used for location tracking, in one or more dimensions, and the resulting data are compared over time. Recent examples include navigators operating in k-space, such as cloverleaf navigators [80], orbital navigators [43, 81], and spherical navigators [50]. Cloverleaf navigators [82] can be used to measure and correct rigid body motion and associated first-order shim changes in real time. An alternative approach employs FID navigators [83] and high channel-count RF coil arrays. It applies ultra-short (<100 msec) free induction decay signals, making use of the knowledge that motion induces variations in these signals when compared to a reference free induction decay signal. On the other hand spherical navigator echoes [84] use a 3D spherical navigator (SNAV) echo technique that can measure rigid body motion in all six degrees of freedom simultaneously by sampling a spherical shell in k-space.

Besides these methods, there are image-based tracking methods, such as PROMO [85] or EPI navigators [86, 87], Sparsity and Kalman Filtering [88] and PROPELLER [89]. K-space navigators repeatedly sample parts of k-space and quantify rotations and translations of the object by measuring rotations and phase shifts in the k-space data [22]. Depending on the trajectory used, this can allow motion quantification in all six degrees of freedom.

Image-based navigators generally use low-resolution images or volumes. These require longer acquisitions than k-space navigators, but allow the user to define the region of interest for motion quantification, thus avoiding non-rigid regions (e.g. the neck). Alternatively, it is possible to detect, but not quantify, motion by comparing the relative intensity of a free induction decay signal between multiple receive coils [90].

Navigator methods with sufficient accuracy for prospective motion correction all require unused time in the sequence to obtain accurate motion information, which makes them incompatible with some imaging sequences. This spoils one of the main advantages of prospective correction, namely that the technique can be applied to most MR sequences. Nonetheless, if time in the sequence is available, as is often the case in spectroscopy, this method is very practical. Navigator methods have an advantage over optical tracking and field detection methods, in that they require no additional hardware and there is no need for a marker to be attached to the subject. This is particularly important in terms of patient handling in clinical MRI. None of the existing methods is perfect, and the best choice of method will depend on the relative importance of the three criteria: high accuracy and precision, no patient interaction, and sequence independence [22].

Functional MR imaging (fMRI) is another application where motion detection, and instantaneous adjustment of the acquisition according to this information, are crucial. In fMRI, complete multi-slice EPI datasets of the head are acquired in rapid succession during presentation of various stimuli. In order for the statistical analysis to be successful, the datasets need to be aligned perfectly. For this purpose, each 3D dataset is compared with the previous one and the translation as well as the rotation of the head is calculated (and displayed) in real-time. The software is able to compensate for rotations and translations in all 6 degrees of freedom. The technique can therefore account, in real time, for any so-called “rigid-body motion”. For acquisition of the next dataset, slice position and orientation are adjusted according to the altered position of the head.

Although different techniques have been developed for preventing motion artefacts, in

the case of clinical scanning none of the methods work adequately at present. A summary of the relevant methods is shown in Table 1.1.

All these methods summarized in Table 1.1 have advantages and disadvantages. Optical tracking is accurate and fast, but costly and requires line of sight access to the head, whereas the navigator methods are fast, but require imaging sequence modification. On the other hand, image-based tracking is accurate, but slow. Some of the methods require additional hardware.

1.4 Rigid Body Kinematics

This study is devoted to tracking involuntary head movements in MRI. For this purpose the human head is considered as a rigid body. At the very beginning one has to know the rigid body kinematics. This section provides a review of rigid body kinematics.

1.4.1 Degrees of Freedom of a Rigid Body

The number of degrees of freedom (DOF) of a rigid body are defined as the number of independent parameters needed to describe its movements. A rigid body in a plane is shown in Fig. 1.1. To determine the DOF's of this body one has to consider how many distinct ways the bar can be moved in the plane. A rigid body in a two dimensional plane possesses three DOFs. It can be translated along the x-axis, translated along the y-axis, and rotated about its center of mass (when a physical object has uniform density).

A rigid body in three dimensional space has six degrees of freedom as shown in Fig. 1.2. These are three translating motions along the three mutually perpendicular x-, y- and z-axes and three rotary motions about the x, y and z-axes respectively. Two or more rigid bodies in space are collectively called a rigid body system.

The human head is considered as a rigid body. Head movement is involuntary 3D movement having six degrees of freedom, three for translations and three for rotations. To under-

Method	Type	Benefit	Shortcomings
Motion prevention	Physical restraints	Simple and fairly easy to use	Patient feels discomfort. Not completely effective.
	Sedation	Very effective in some cases	Safety concerns.
	Training	Safe and effective	Only suitable in very specific cases.
Optical tracking	External optical motion tracking	No special post-processing required	An external tracking mechanism is needed
	Optical embedded tracking	MRI compatible and comparatively low latency	Extra hardware needed, costly, increase patient anxiety
	Self-encoded marker	Advantageous over checker-board marker	Extra hardware such as camera and marker needed, expensive
Field detection	Wireless RF coils	Good alternative to wired markers, avoid safety risks	Limited applicability
	Tracking coils	Suitable for physiological motion artefacts	Limited success for rigid body motion artefacts
	Active markers	Improved image stability	Extra hardware required, limited to EPI
Navigators	Cloverleaf navigators	Can improve image quality, increased acquisition time	Limited success for rigid-body head motion.
	FID navigators	Can predict all six rigid body motion parameters	An optical tracking system is needed, complex technique
	Spherical navigators	No extra hardware is required	The accuracy and precision is limited
Image-based tracking	PROMO	Advantageous over navigators and Kalman filtering	Complicated, combination of navigator and Kalman filter
	Sparsity and Kalman Filtering	No external tracking hardware, no navigator	Combined method, time consuming
	PROPELLER	Effective in some situations.	Increased acquisition time and relatively poor SNR for high spatial frequencies.

Table 1.1: Summary of the preprocessing techniques used to reduce motion artefacts in MRI.

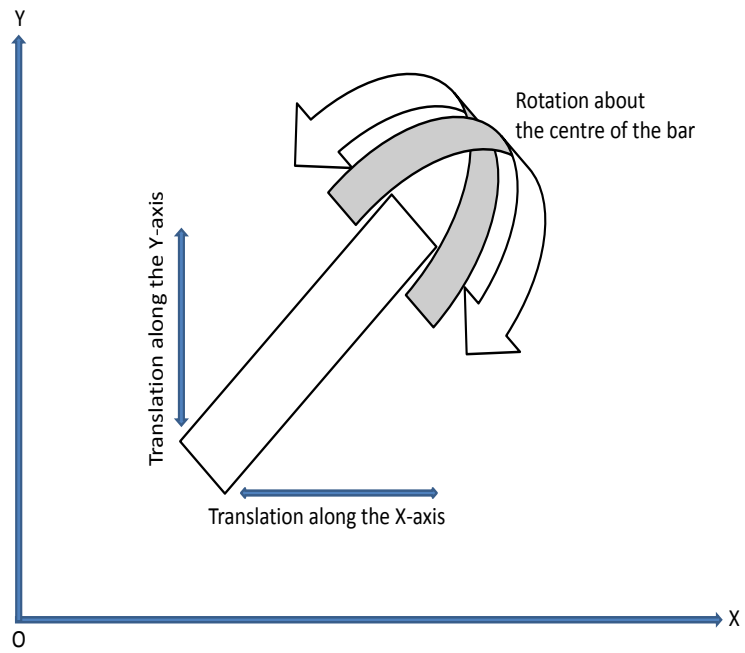


Figure 1.1: Degrees of freedom of a rigid body(a bar) in a plane

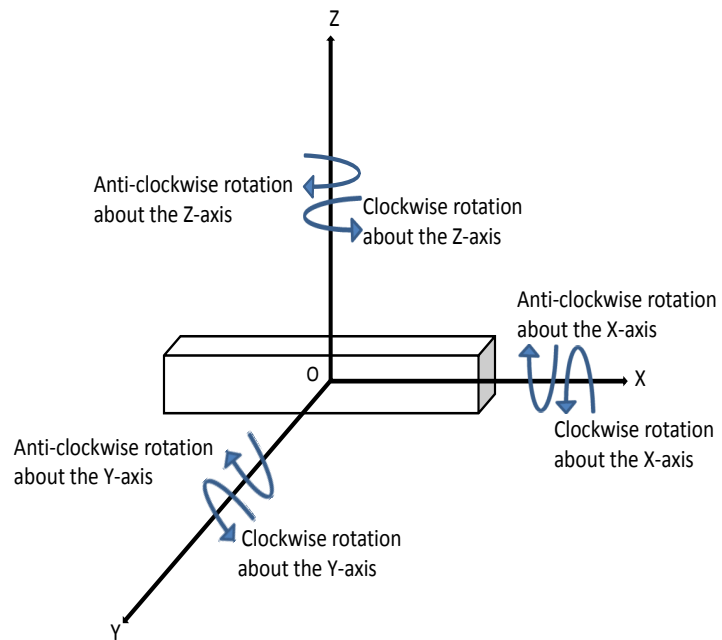


Figure 1.2: Degrees of freedom of a rigid body in space

stand the rigid body movement spatial translational transformation and spatial rotational transformation are discussed in the following sub-subsections.

A rigid body in space has six degrees of freedom which are shown in Fig. 1.3. Translational degrees of freedom's are surge, heave and sway whereas rotational degrees of freedom are roll, pitch and yaw, as shown in Fig. 1.3(a) and Fig. 1.3(b) respectively. Up-Down

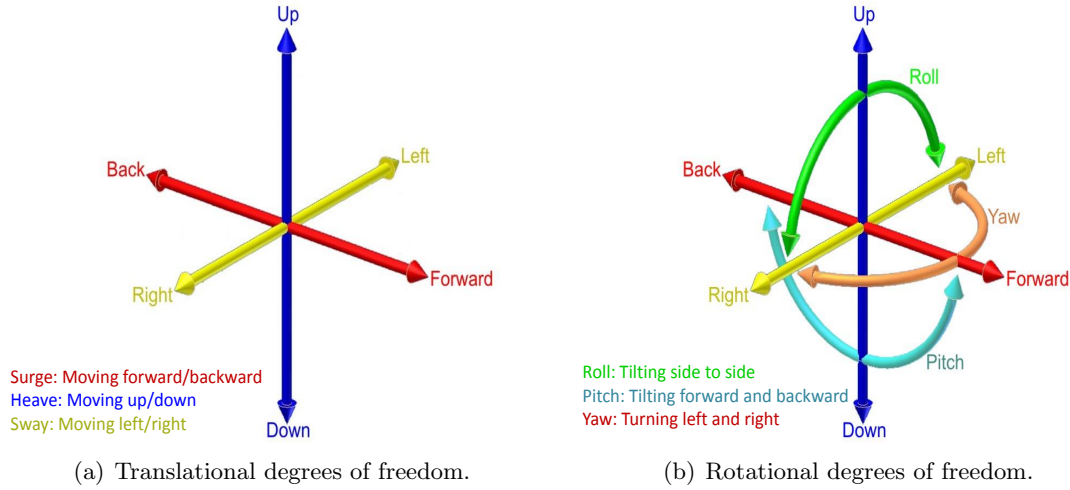


Figure 1.3: Degrees of freedom. (a) Three translational degrees of freedom. (b) Three rotational degrees of freedom.

(Heave), Forward-Back (Surge) and Right-Left (Sway) are known as Anterior-Posterior (AP) Foot-Head (FH) and Right-Left (RL) respectively in a MR scanner.

Chapter 2

Magnetic Resonance Imaging (MRI)

This chapter discusses the fundamentals of MRI which are relevant to my work, particularly the simple methods for producing images with magnetic resonance. In addition, this chapter focuses on measurement-related equipment and gradient-related terms that are mainly relevant to the work described in this thesis.

2.1 Spatial Encoding

Encoding means recognising the location of the MR signal and positioning it at the correct place in the image. The magnetic field dependence of the Larmor frequency is used to distinguish the signals coming from an object or subject. This task is performed by using magnetic field gradients, which are spatially varying magnetic fields. Three orthogonal physical gradients are generated by passing current through three different coils of wire located inside the bore of the magnet.

2.1.1 Gradients in MRI

Mansfield *et al.* [91] and Lauterbur [92] first proposed the use of magnetic field gradients to achieve spatial resolution in MRI. The gradient coils generate a magnetic field that varies linearly in strength with distance over a predefined FOV. Magnetic field gradients can be applied in any direction, but in MRI the gradients are applied along three mutually perpendicular directions. Three distinct gradient waveform (Gradient waveform refers to all gradient lobes on a single axis within a pulse sequence) are applied in MRI to localize signal from a three-dimensional volume: slice selection, frequency encoding and phase encoding gradients.

Slice select Gradient

Magnetic field gradients are used to select the slice plane in most imaging sequences. To excite a unique slice in 2D MR imaging two steps are required, namely, (A) a slice-select gradient is executed along an axis perpendicular to the plane of the chosen slice, producing a linear variation of resonance frequencies in that direction, and (B) an RF-pulse, whose frequency components match the narrow range of frequencies confined in the required slice is concurrently applied. The amalgamation of these two ensures that only protons within the chosen slice are excited.

For 2D imaging of a 3D sample, we need to select a “slice” as shown in Fig. 2.1. Generally, this is accomplished by selective excitation of NMR active molecules residing in the slice. This is based on the fact that the RF magnetic field, also known as B_1 -field, only interacts strongly with nuclei whose Larmor frequencies equal to those of the RF frequency pulse. An RF pulse made up of a frequency spectrum, centred at $\omega = \omega_a$ and spanning a frequency range, $\delta\omega$, is applied in the presence of a z-gradient, G_z , such that the z-component of the field, $B_z = G_z z$. This will excite signal from a slice of width, $\delta z = \frac{\delta\omega}{\gamma G_z}$, centred at position, $z_a = \frac{\omega_a}{\gamma G_z}$. γ is a constant for a specific nucleus, known as the gyromagnetic and depends on the charge-to-mass ratio.

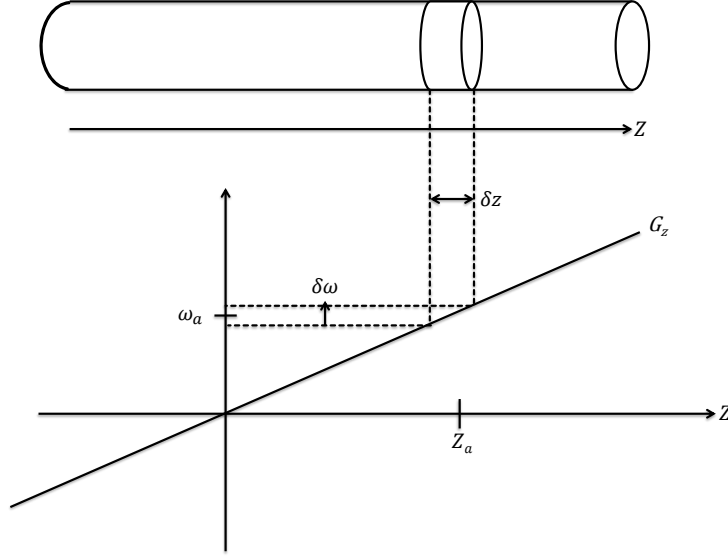


Figure 2.1: Diagram shows the basic concepts of slice select gradient.

In common practice $\delta \omega$ is held constant (about 1000 to 2000 radian per second) and slice thickness δz is varied by regulating G_z . Stronger gradients produce thinner slices.

Slice-Rephasing Gradient

To make pulse sequence diagrams understandable, the gradients are drawn as flawless rectangles. Such a gradient shape indicates that a gradient may change from zero to a positive value immediately, but this is impossible to achieve in practice. Therefore, all gradients should be drawn as trapezoids having finite ramp-up and ramp-down times.

There is a small lobe of reverse polarity in the slice-select gradient which is applied immediately after the RF-pulse, named the slice-rephasing lobe. This is shown in Fig. 2.2. It is needed to correct for the phase dispersion of transverse magnetization which arises concomitant with the application of the main slice-select gradient. Without this rephasing lobe, intra-voxel phase dispersion would occur across the slice with resulting signal loss.

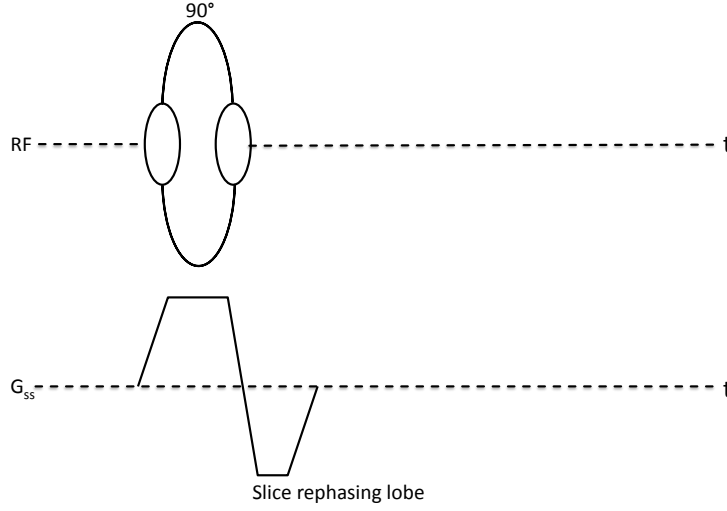


Figure 2.2: RF and gradient waveform for slice selection and slice rephasing gradient.

Frequency Encoding Gradient

A magnetic field gradient produces a linear variation of field with position as shown in Fig. 2.3. Magnetic field gradients $G_x = \frac{dB_z}{dx}$, $G_y = \frac{dB_z}{dy}$, and $G_z = \frac{dB_z}{dz}$ are formed in any direction to produce spatial variations in the main magnetic field B_0 to make the larmor frequency spatially dependent. The components of the magnetic field for the x-, y-, and z-gradients are $B_z = G_x x$, $B_z = G_y y$, and $B_z = G_z z$ respectively. With applied field, B_0 , and a gradient applied along the x-direction, we get the resulting field:

$$B = B_0 + G_x \times x, \quad (2.1)$$

so that $\Delta\omega = \gamma(B - B_0) = \gamma G_x \times x$ and in the presence of a field gradient, ω varies linearly with position. Fourier transformation of the signal acquired under a frequency encoding gradient thus indicates the position along the gradient direction from which the signal originated.

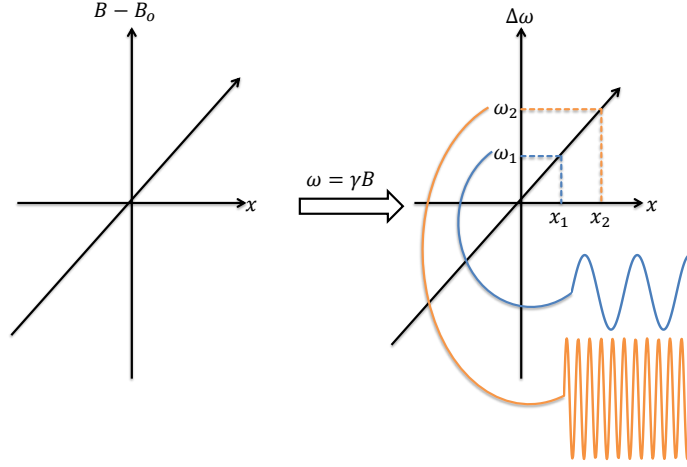


Figure 2.3: Frequency encoding in MRI: (left) field variation due to an x-gradient; (right) frequency variation in presence of x-gradient.

Phase Encoding Gradient

In imaging, all the signals of the same slice are recorded in a matrix then the detected signals undergo Fourier transformation to produce the signal frequency spectrum. In a single acquisition, frequency encoding indicates position in one dimension. Encoding in the 2nd and 3rd dimensions is usually accomplished *via* phase encoding (i.e. position is encoded in the phase of the NMR signal).

We consider a simple 2D object and look at the signal acquired under an x-gradient after application of a y-gradient pulse as shown in Fig. 2.4. The phase of the signal depends on the y-position ($\phi = \gamma G_y y t$, t stands for time), while the frequency depends on the x-position ($\omega = \gamma G_x x$), but one cannot unambiguously identify the y-position from a single measurement of phase. To achieve this, one has to make multiple measurements with different values of G_y or τ (length of the pulse).

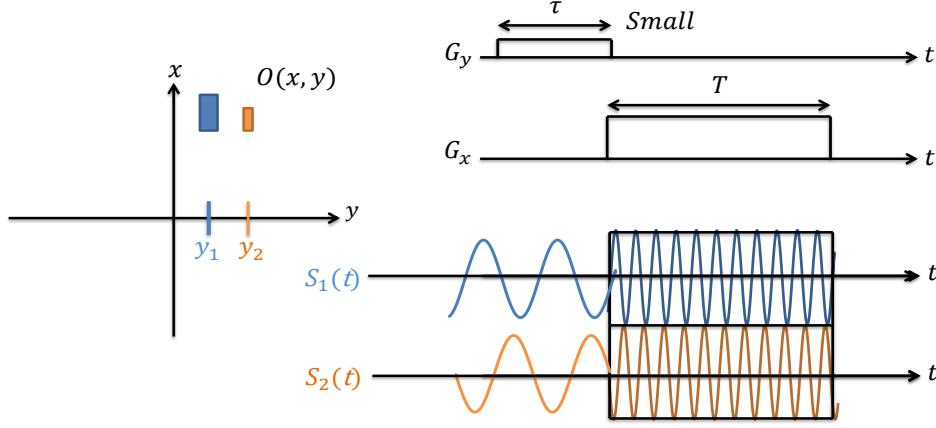


Figure 2.4: Phase encoding in MRI: (left) simple two-dimensional object; (right) signal variation when phase encoding gradient (G_y) is followed by a frequency encoding gradient (G_x).

Spoiler Gradient

In any acquisition, residual transverse magnetization can remain at the end of the pulse sequence. A spoiler gradient dephases this unwanted residual transverse magnetization that would otherwise produce artefacts in the image [93,94]. The area of the spoiler is large so it can adequately dephase the residual magnetization.

2.2 Gradient Echo and Spin Echo

A gradient echo (GE) is produced by applying a single RF pulse in combination with a gradient reversal. How a GE forms is demonstrated schematically in Fig. 2.5(a). Following the RF pulse, the first negative-going lobe of the gradient causes phase dispersion of the precessing spins. After the gradient reversal, the spins refocus and form a gradient echo.

One of the significant features of GE imaging is that the gradient reversal refocuses only

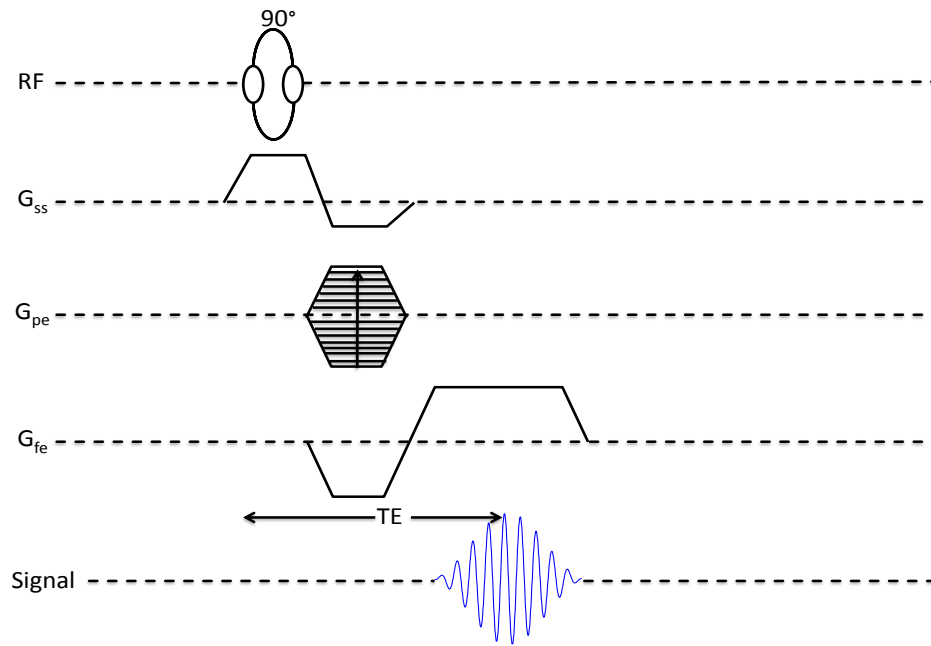
those spins that have been dephased by application of the gradient itself. Explicitly, phase shifts resulting from magnetic field inhomogeneities, static tissue susceptibility gradients, or chemical shifts are not refocused at the centre of the GE; in contrast to what happens in spin echo (SE) sequences. Image contrast is therefore dictated not by true T_2 -relaxation, but by the magnetic field inhomogeneities and tissue susceptibility which produce T_2^* signal decay.

GE and SE imaging are different in several aspects. Primarily, since only one RF-pulse is applied, the echo can be recorded much more rapidly in a GE sequence. Consequently, echo time (TE) is usually shorter for GE than for SE sequences. If GE sequences are used with low-flip-angle excitations, short values of the repetition time (TR) may also be used. The short TR and short TE combination allows for very rapid signal acquisition. As a result, GE sequences form the base of many rapid imaging and MR angiographic techniques.

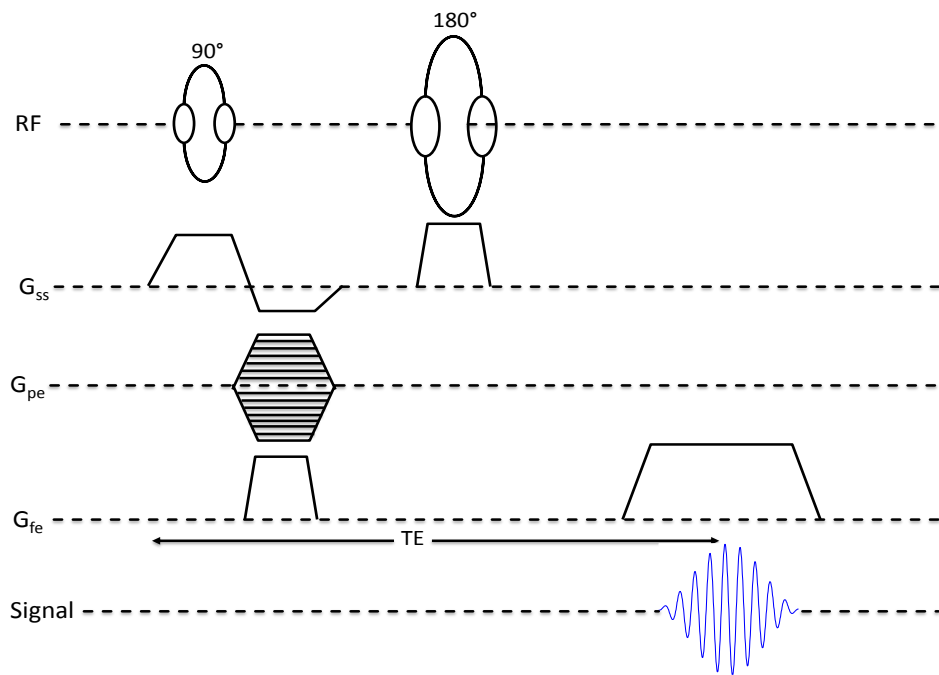
Spin echo imaging uses a 180 degree refocusing RF pulse to refocus spins producing a spin echo which is shown in Fig. 2.5. This refocuses the effects of magnetic field inhomogeneity so that SE images reflect true T_2 decay and are relatively insensitive to magnetic susceptibility effects.

2.3 k-space

The spatial frequencies in the MR images are represented by an array of numbers known as k-space. Fig. 2.6 shows k-space trajectory for frequency and phase encoding gradient. To produce an image, one needs to sample the spatial frequencies in the object. We represent spatial frequencies using the vector $\mathbf{k}$ and the signal variation in the k-space represents the spatial frequencies present in the object. High spatial frequencies represent fine detail, whereas low spatial frequencies represent rough structure. Image contrast depends on the weighting of low spatial frequencies. Applied gradients cause a translation in k-space. The rows and columns of k-space are generally displayed on a rectangular grid with principal axes k_x and k_y . The k_x and k_y axes of k-space correspond to the horizontal (x-) and vertical (y-)



(a)



(b)

Figure 2.5: A frequency encoding gradient waveform. (a) Gradient echo pulse sequence. (b) Spin echo pulse sequence.

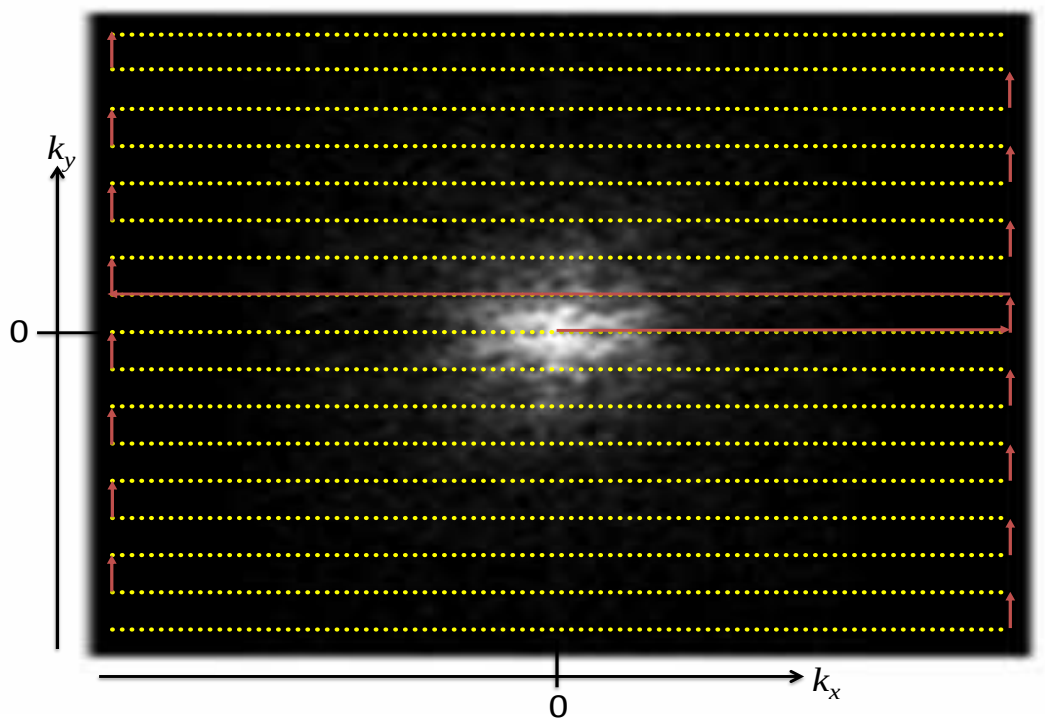


Figure 2.6: Digram showing the trajectory for filling k-space in the EPI sequence.

axes of the image. However, the k-axes represent spatial frequencies in the x- and y-directions rather than positions. Each (k_x, k_y) data point corresponds to the echo signal amplitude influenced by the combination of frequency and phase encoding gradient area [96,97]. Each k-space point comprises spatial frequency and phase information about every pixel in the final image. Conversely, each pixel in the image maps to every point in k-space.

2.4 Echo Planar Imaging

Sir Peter Mansfield proposed echo planar imaging (EPI) in 1977 [98]. EPI is a fast imaging technique, which involves imaging with multiple gradient echoes with different phase encoding in the short period after a single excitation of the spin system by an RF pulse. In this study, standard EPI pulse sequences were used for data acquisition.

In single-shot EPI, the (N_y) lines in k-space are filled by multiple gradient reversals, producing multiple gradient echoes in a single acquisition after a single radio frequency (RF) pulse, that is, in a single measurement or “shot”. In multi-shot EPI, the readout is divided into multiple “shots” or segments (N_s) , so that $N_y = N_s \times \text{ETL}$, where echo train length (ETL) is the number of lines in each segment. Since k-space is segmented into multiple acquisitions, this technique is also called segmented EPI. The multi-shot EPI technique places less stress on the gradients compared with single-shot EPI and phase errors have less time to build up compared with single-shot EPI, therefore, reducing susceptibility artefacts. However, multi-shot EPI takes longer to perform than does single-shot EPI. For this reason, multi-shot EPI is more susceptible to motion artefacts [99].

Phase errors may result from the multiple positive and negative passes through k-space (i.e., from the alternating polarity of the readout gradient). “Ghost” artefacts of the main image may appear along the phase axis, not caused by motion as in conventional spin echo, but by eddy currents, imperfect gradients, field non-uniformities, or a mismatch between the timing of the odd and even echoes. Magnetic susceptibility effects in EPI may result in variations in frequencies and phase errors. Chemical shift artefacts in EPI occur along

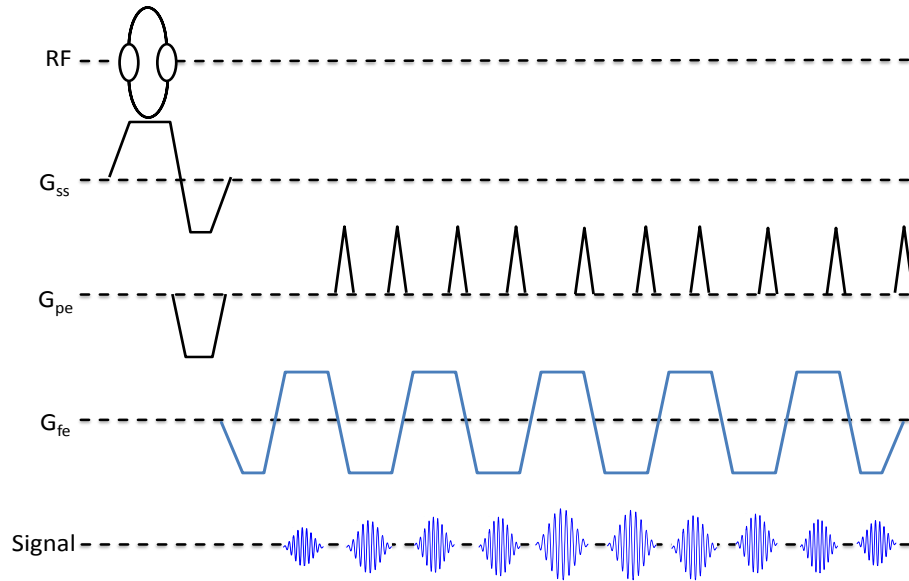


Figure 2.7: Schematic diagram of gradient echo EPI sequence.

the phase-encode axis rather than along the frequency-encode axis. These artefacts are much more pronounced in EPI than in other sequences, thus an effective fat suppression technique is necessary. Though EPI has artefacts like the Nyquist ghost, signal drop out, image distortion, it plays a crucial role in functional MRI, diffusion tensor weighted imaging, and perfusion imaging.

Even though EPI is the fastest MRI technique, it has some advantages as well as disadvantages. These are as follows:

Advantages of EPI:

- Scan time is approximately 100 msec or less.
- Cardiac and respiratory motion do not pose problems.
- Proton density, T_1 , and T_2 -weighted images can be produced free of motion artefacts.
- It allows the study of organ function rather than the mere depiction of organ anatomy.

Disadvantages of EPI:

- Since fat-water chemical shift artefacts (Dixon effect [100]) can be problematic, fat suppression with pre-saturation techniques is often required for EPI.
- Intrinsic non-uniformities in B_0 and susceptibility effects also result in variable resonance frequencies and increasing phase errors. However, this effect occurs less in multi-shot EPI.

2.5 MR Instruments

All the data for this thesis were acquired in the Sir Peter Mansfield Imaging Centre (SPMIC) at the University of Nottingham. The laboratory is equipped with a 3.0 T Philips Achieva scanner (Philips Medical Systems, Best, the Netherlands) with two-channel, multi-transmit and 32-channel receive capabilities. A detailed description of the system is avoided since it is commercially available. I mainly focus on the main magnet, gradient coils, shim coils and RF coils.

2.5.1 Main Magnet

The main magnet is the biggest and the most important component of an MR scanner which produces the main polarizing magnetic field B_0 with reasonable spatial homogeneity and high temporal stability. A superconducting magnet consists of many coils or windings of wires, usually composed of Niobium-titanium (Nb_3Ti) alloy, through which an electric current is passed to create a high magnetic field. Nb_3Ti becomes superconducting at about 10 K. To maintain the superconducting state, the wires are immersed in liquid helium at -269.1°C .

Depending on the design of the main magnet and the scanner room, a moderately large fringe field might spread for quite a few meters around, above, and below an MR scanner. From the magnet isocenter, the strength of a magnetic fringe field is inversely proportional to the third power of the distance ($\frac{1}{r^3}$). Hence, moving twice as far away from the magnet, the fringe field would fall by a factor of approximately eight.

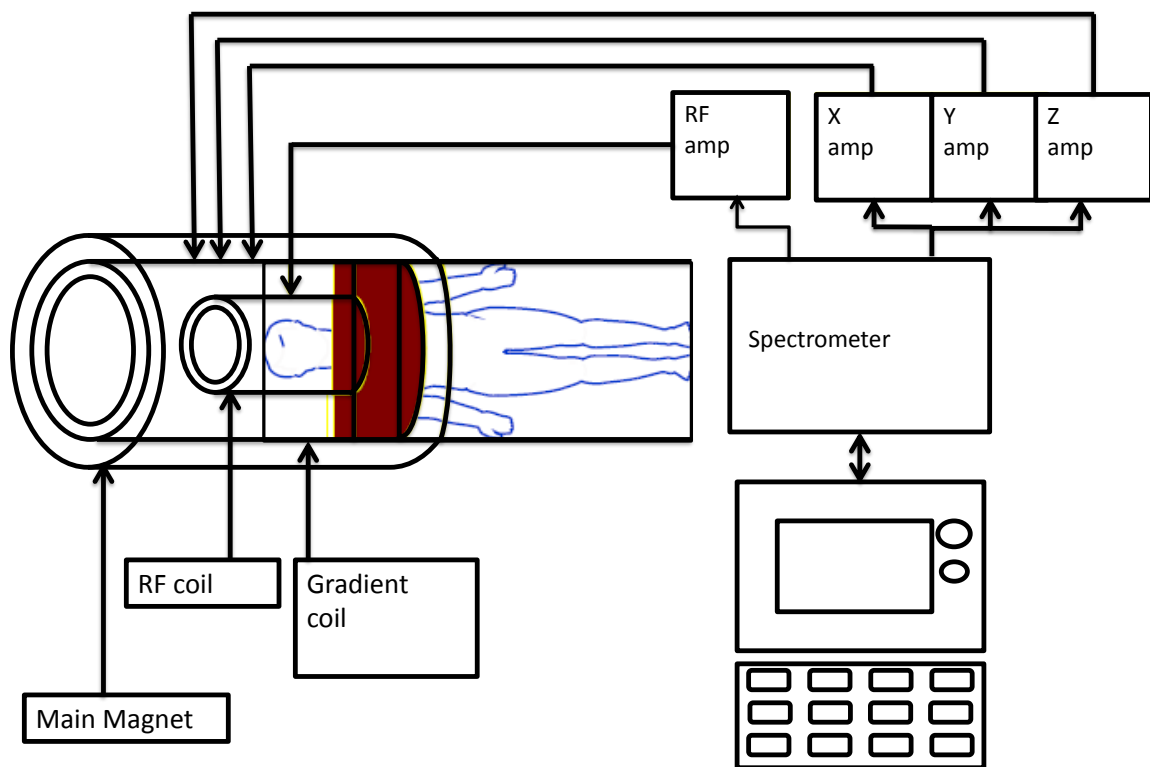


Figure 2.8: MR instrument.

2.5.2 Shim Coil

It is essential to have good homogeneity in the local magnetic field to localize the MR signals precisely and reduce image artefacts. Nonetheless, placement of an object or a patient within the main B_0 field generates local susceptibility effects, reducing the field homogeneity. Shimming refers to the process of removing small inhomogeneities which are present in the magnetic field. There are two types of shimming, namely, passive or active.

Passive shimming refers to the use of magnetic material to improve the field homogeneity. In a MR system, it is achieved during magnet installation by placing sheets or small pieces of ferromagnetic material at certain locations within the magnet bore. The ferromagnetic material concentrates magnetic field lines, and thus corrects the inhomogeneities in the static field [101].

Active shimming provides additional field correction around an object of interest through the use of shim coils. The magnetic field produced by each coil set is proportional to the current passing through the coils. During the shim process, a 3D gradient echo sequence acquires volumetric data of the region to be shimmed, controlled by the host computer; under the guidance of the scanner application software and the operator program analyzes the collected data to estimate the current needed in each shim coil to optimize the magnetic field homogeneity. Homogeneity and variation in B_0 is measured in parts per million (ppm), that is, a fraction, of the static magnetic field over a specified spherical volume. For a 1 T system with a homogeneity of 1 ppm over 40 cm, no two points within 20 cm of the magnet's isocenter differ by more than 1.0×10^{-6} T [102].

2.5.3 Gradient Coil

Gradient coils are one of the major components of the MR system. Gradient coils producing field variation along the three mutually perpendicular x-, y- and z-axes lie concentric to each other within the main magnet. Moreover, they are not supercooled and can operate relatively near to room temperature. A gradient coil has the ability to produce a magnetic

field in the same direction as B_0 , but with a strength that changes with position in the x-, y- or z-direction, depending on which gradient coil is used. The magnetic field produced by the gradient coils is added to B_0 so that the main magnetic field strength varies in the direction of the executed gradient field [104].

2.5.4 RF Coil

The RF coils generating the required RF pulses are arranged inside the gradient coils and normally lie concentric to them and to each other. They are considered to be the ‘antenna’ of the MRI system. They have two main purposes: to transmit RF energy to the desired tissue and to receive the induced RF signal back from the desired tissue [103].

Some RF coils perform the dual role of transmission and reception of RF energy. Some of them transmit or receive only. However, a separate RF receiver coil, custom-made to maximize the signal from the brain is usually applied around the patient’s head to detect the emitted MR signals in neuroimaging [104].

A time-varying RF field is generally referred to as the B_1 field. After it has been switched on, the B_1 field produces MR signals that are spatially localized and encoded by the gradient magnetic fields to generate an image [105]. The output signal picked up by the receive coil is digitized and then sent to a reconstruction computer processor to provide the image following complex mathematical manipulation [106].

2.5.5 Control and Processing System

Magnetic resonance imaging (MRI) systems include an integrated processing and control system, as shown in Fig. 2.9. Control and processing systems control the operation of an MRI scanner from the control room. The MRI scanner is located within an RF shielded room. Gradient coils are connected to the three gradient amplifiers and RF coils are connected to RF amplifiers *via* the RF electronics. The waveform generator which is connected to the amplifiers passes the information regarding the shape and size of the gradients and RF

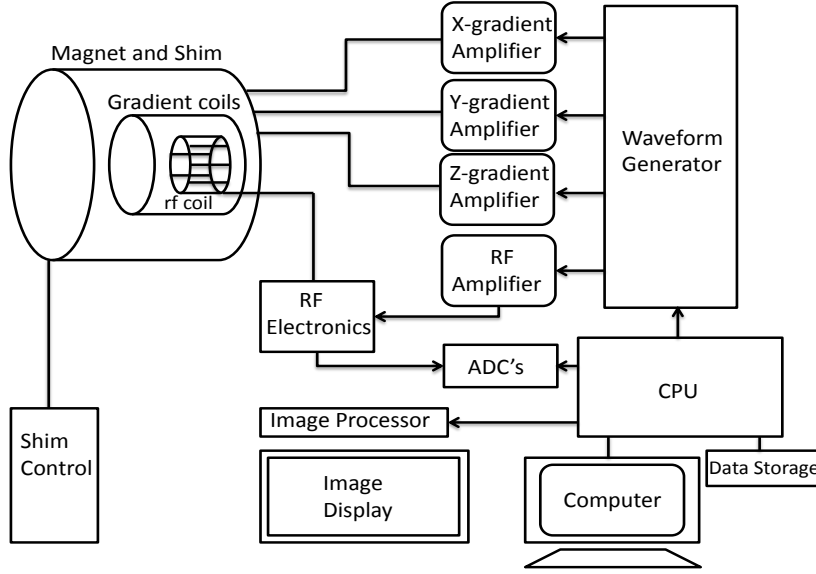


Figure 2.9: Schematic diagram of the control and processing system.

waveforms as well as timings to be used, and which signals to be amplified to send to the coils. The NMR signal is converted to a digital signal by an analogue to digital converter (ADC's) as shown in Fig. 2.9. The image processor receives this digital data for Fourier transformation and the image is displayed on a computer monitor. The data storage is connected to the CPU of the computer to store the raw data (i.e. the signal before Fourier transformation) to enable the application of corrections to the data in post-processing.

2.6 MR Compatible EEG System

An MR compatible EEG system was used as the recording apparatus in the work described in this thesis. This consists of the BrainAmp ExG (Bipolar Amplifier) that can measure voltage from few μV to 16 mV or GSR (Galvanic Skin Response), re-chargeable battery (PowerPack), SyncBox, USB 2 Adapter, pulse stretcher, ribbon cable and optical fibres. An Analyzer (USB dongle license) is used to analyze the recorded signal to get raw data from signals.

2.6.1 The BrainAmp ExG

The BrainAmp ExG is a bipolar amplifier which can be used to record EEG data, but here used to record induced voltages in the coils. We used a 16-channel ExG system. This can record bipolar signals with a sampling rate of up to 5000 Hz and a bandwidth ranging from DC to 1000 Hz or in some cases 250 Hz. The ExG amplifier is powered by a rechargeable battery and can be connected to a laptop allowing it to be used anywhere and for any type of recording.

2.6.2 The SyncBox

The SyncBox is used to improve the quality of data for synchronous EEG and fMRI recordings. It is a device which significantly decreases timing related errors and enhances the quality of MRI artefact correction by synchronizing the clock of the BrainAmp MR or BrainAmp MR plus amplifier with the clock driving the MRI scanner's gradient switching system.

Phase synchronization between the EEG recording equipment and the MRI scanner produces temporal stability of the EEG recordings in relation to the switching of the gradients during the MR acquisition which is shown in Fig.2.10. This leads to significant improvement of the recorded data quality. There is no time difference between the scanner's decision to start the next slice and the next data point acquired by the EEG amplifier (BrainAmp). Therefore, the BrainAmp will always record every data point throughout each slice and volume at the very exact same time as was used for that time point in the previous and subsequent slice and volume acquisitions.

2.7 Safety

The number of MRI scans of patients and healthy volunteers, as well as phantom scanning performed annually all over the world is consistently increasing. In accordance with the imaging technology advances, more subjects require monitoring during study because of the

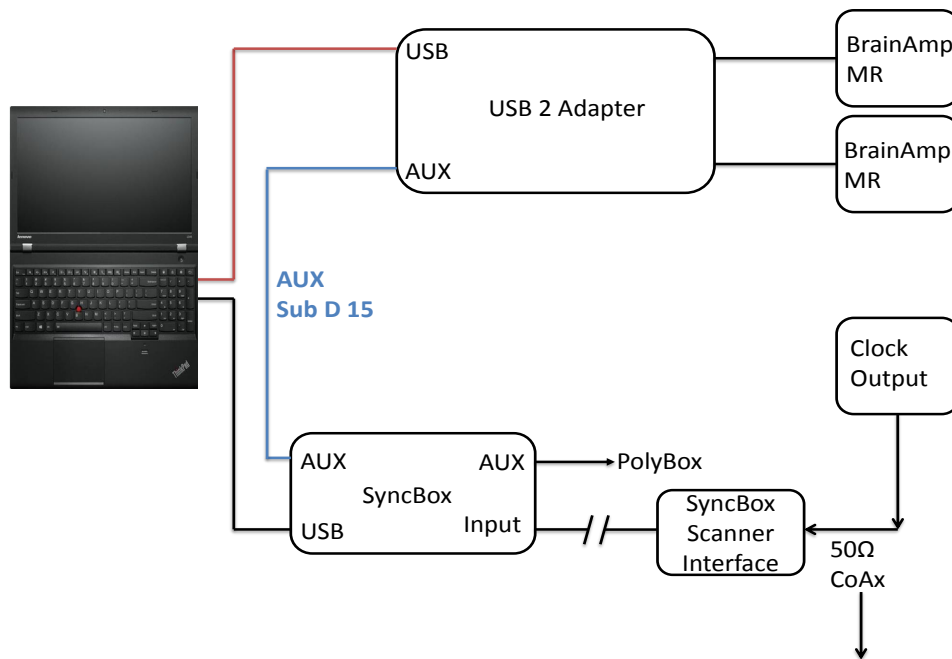


Figure 2.10: A schematic diagram of USB connection between the SyncBox main unit and the computer is utilizing hardware clock synchronization with any particular scanner clock frequency.

type of study or their general condition. Phantom scanning and subject monitoring requires the use of MRI-compatible equipment. To circumvent any sort of accident or subject-hostile events during an MRI, one must follow these steps:

- Consult with the MRI staff technologist in the centre before phantom scanning as well as the imaging study to confirm that cables and coils are available that have been verified for use in the MRI environment. It is always necessary to make sure that coils and cables used are in a good condition.
- Coils are made up of copper wires and the wires are covered with a thermal insulator to avoid any possibility of burns due to heat produced by the wires.
- Remove any coils and cables that are no longer being actively used for monitoring. This will prevent the possibility of burns from leftover coils and wires.
- Coils are connected to the EEG amplifier using star quad cable (Vandamme®) min-

imizing the induced voltage contribution from the cables. Coils are connected to the ExG reference (common ground) through 100 k Ω resistors. If the resistors are very low the induced voltage will drop across the resistors, whereas very high resistance could not serve the purpose of providing a common ground.

- If there is an air gap among the turns of the coils or the electrical pathway of the coils are broken (not completely disconnected) then heat can build up in the coils which can cause burns. Do not let the cables form a loop and keep them off the subject's skin by placing a cloth under them.

We discussed different methods of motion tracking in MRI briefly in Chapter 1. In recent years, some work has been done on motion tracking inside MR scanner. Mads Anderson *et al.* [108] used EEG-equipment to track head motion inside the MR scanner. This method was based on induced signals from gradient switching in an EEG-cap the subject wore during scanning. Rasmus *et al.* [109] used Tracoline system (currently scanning surfaces at up to 20 Hz) to track the head movement inside MR scanner during the application of the MR sequences. The tracking, geometric alignment, and a time synchronization between TracSuite and scanner allowed for synchronized motion correction of all the EPI 2D image slices.

The necessary theories has been discussed in the next chapter to calculate the change in induced voltage in a coil (planar or non-planar) placed in a time-varying magnetic field gradient.

Chapter 3

Theory: Analytical Expression for the Induced Voltage in a Coil

This chapter explores the theories needed to calculate the induced voltage in a general coil when placed at different positions and orientations in a time varying magnetic field. This theory can also be used to calculate the change in induced voltage as a result of changes in coil position and orientation due to translation and rotation. To study the kinematics of a rigid body, six parameters (three for translation and three for rotation) are required which are discussed in detail in Section 1.4 of Chapter 1. The relevant theories are developed that are needed to work out the six parameters characterizing movements from the measurement of the induced voltage in the coils.

3.1 Specific Coils in a Plane

3.1.1 Analytical Calculations

When considering electromagnetic (EM) induction due to gradient-fields (g-field) or uniform-fields, a planar coil can be represented as a magnetic dipole with a sensitivity vector $\mathbf{S}$ and a location vector $\mathbf{r}$. The location and sensitivity vectors of the coil are shown in Fig. 3.1.

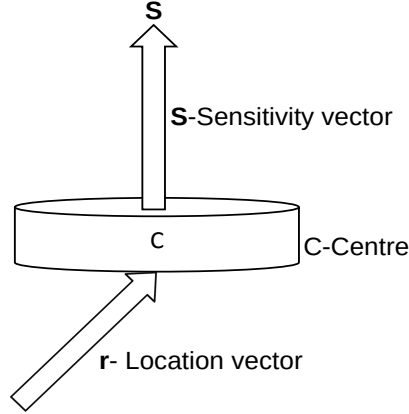


Figure 3.1: The location and the sensitivity vector of a coil.

Suppose the location vector $\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and the sensitivity vector $\mathbf{S} = S_x\hat{i} + S_y\hat{j} + S_z\hat{k}$. For a set of time-varying gradients, with amplitudes G_x , G_y and G_z , the rate of change of magnetic field is as follows:

$$\dot{\mathbf{B}} = \dot{\mathbf{B}}_x + \dot{\mathbf{B}}_y + \dot{\mathbf{B}}_z = g_x(z\hat{i} + x\hat{k}) + g_y(z\hat{j} + y\hat{k}) + g_z(-\frac{x}{2}\hat{i} - \frac{y}{2}\hat{j} + z\hat{k}) \quad (3.1)$$

where lower case letters are used to indicate time varying quantities thus: $g_x = \frac{dG_x}{dt}$, $g_y = \frac{dG_y}{dt}$ and $g_z = \frac{dG_z}{dt}$, and $\hat{i}, \hat{j}$ and $\hat{k}$ are Cartesian unit vectors. In the Eq. 3.1 the concomitant fields for the x-gradient are $B_x = G_x z$, for the y-gradient $B_y = G_y z$, and for the z-gradient $B_x = -G_z x/2$ and $B_y = -G_z y/2$.

Voltage induces in a coil either in a time-varying or within in a time-invariant (static) magnetic field gradient due to translation and rotation. The changes in induced voltage in a coil within in a time-varying magnetic field gradient due to change in orientation and position of the coil are calculated. The EMF induced in the coil is given by the equation,

$$-V = \dot{\mathbf{B}} \cdot \mathbf{S} = (g_x z - \frac{1}{2}g_z x)S_x + (g_y z - \frac{1}{2}g_z y)S_y + (g_x x + g_y y + g_z z)S_z. \quad (3.2)$$

The change in induced voltage in a coil due to translation/rotation in a time-varying

magnetic field gradient dominate over the time-invariant magnetic field gradient which is shown in the equations,

$$\phi_B = G \times r \times A \implies -V = \frac{d\phi_B}{dt} = \frac{dG}{dt} \times r \times A + G \times \frac{dr}{dt} \times A \quad (3.3)$$

or

$$\phi_B = G \times \theta \times A \implies -V = \frac{d\phi_B}{dt} = \frac{dG}{dt} \times \theta \times A + G \times \frac{d\theta}{dt} \times A. \quad (3.4)$$

Where ϕ_B stands for magnetic flux, G for gradient amplitude r for translation, θ for rotation and A for the area of the coil. For small translation/rotation of a coil, one can neglect the voltage induced by the movement of the head within a static field gradient.

3.1.2 Translation

For a general translation $\mathbf{T} = \delta x \hat{i} + \delta y \hat{j} + \delta z \hat{k}$, the change in induced voltage in a coil is calculated by simple mathematical manipulation which gives

$$\delta V = -\delta V_T = (g_x S_z - \frac{1}{2} g_z S_x) \delta x + (g_y S_z - \frac{1}{2} g_z S_y) \delta y + (g_x S_x + g_y S_y + g_z S_z) \delta z. \quad (3.5)$$

Where δV_T stands for the change in induced voltage in the coil due to translational movement. Let us consider a system of five coils in which the centres of the coils lie on the x-, y- or z-axes. The positions and orientations of the coils, which are denoted as S_1, S_2, S_3, S_4 and S_5 , are shown in Fig. 3.2. The coil S_1 is perpendicular to the z-axis and lies in an xy-plane, while the coils S_2 and S_3 are perpendicular to the x-axis and lie in a yz-plane. These two coils are mirror reflections of one another. The coils S_4 and S_5 are oriented perpendicular to the y-axis and lie in a zx-plane. Consequently $\hat{S}_1 = \hat{k}, \hat{S}_2 = \hat{S}_3 = \hat{i}$ and $\hat{S}_3 = \hat{S}_4 = \hat{j}$, where $\hat{S}_n$ is the unit vector along the director of S_n . The changes in induced voltage in the coils

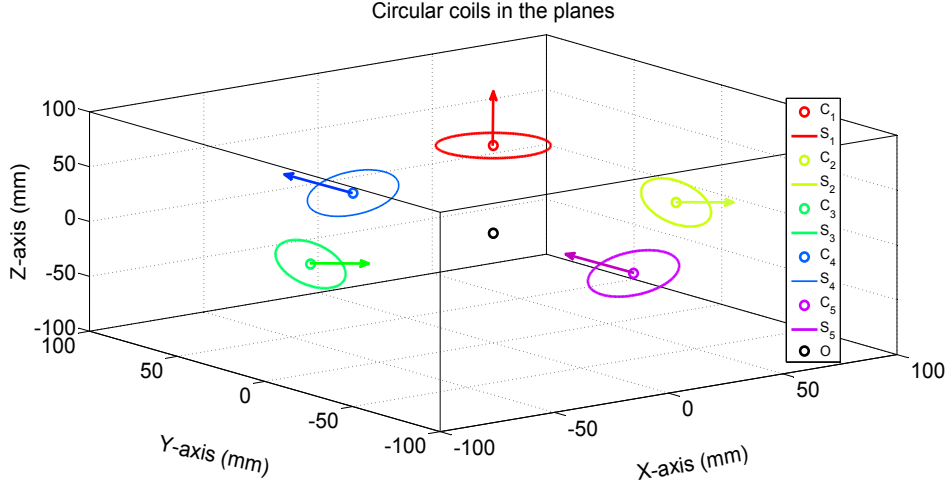


Figure 3.2: The position and the orientation of the five coils.

for a translation $\mathbf{T} = \delta x \hat{i} + \delta y \hat{j} + \delta z \hat{k}$ are calculated by using the five equations:

$$\begin{aligned} \delta V_1 &= (g_x \delta x + g_y \delta y + g_z \delta z) S_z, \\ \delta V_2 &= \left(-\frac{1}{2} g_z \delta x + g_x \delta z\right) S_x, & \delta V_3 &= \left(-\frac{1}{2} g_z \delta x + g_x \delta z\right) S_x, \\ \delta V_4 &= \left(-\frac{1}{2} g_z \delta y + g_y \delta z\right) S_y, & \delta V_5 &= \left(-\frac{1}{2} g_z \delta y + g_y \delta z\right) S_y. \end{aligned} \quad (3.6)$$

The magnitudes of the change in induced voltage in the two opposite coils are the same (the sign depends on the definition of the direction of S). Table 3.1 summarizes the changes in induced voltage in the five coils due to translations.

3.1.3 Rotation

I am interested to calculate the changes in the voltage induced in a coil by time-varying gradient fields when its position and orientation change due to rotation. A rotational matrix operator operates on the location vector as well as the sensitivity vector and gives us new location and sensitivity vectors. For this purpose, the new location and sensitivity vectors are found after rotation about a general axis. For a general rotation described by a rotation matrix, $\mathbf{R}$, the position and sensitivity vectors after rotation are given by $\mathbf{r}_{at} = \mathbf{R} \cdot \mathbf{r}$ and $\mathbf{S}_{at} = \mathbf{R} \cdot \mathbf{S}$. To find the location vector $\mathbf{r}$ and the sensitivity vector $\mathbf{S}$ after rotations about

Coil No.	Sensitivity	δx	δy	δz	Voltage
C_1	S_z	g_x	g_y	g_z	δV_1
C_2	S_x	$-\frac{1}{2}g_z$	0	g_x	δV_2
C_3	S_{-x}	$-\frac{1}{2}g_z$	0	g_x	δV_3
C_4	S_y	0	$-\frac{1}{2}g_z$	g_y	δV_4
C_5	S_{-y}	0	$-\frac{1}{2}g_z$	g_y	δV_5

Table 3.1: The changes in induced voltage in the coils due to translation.

the x, y or z-axis, three rotation matrices $\mathbf{R}_x$, $\mathbf{R}_y$ and $\mathbf{R}_z$, describing rotations through angle ϕ about the x-, y- and z-axes, respectively are defined as follows:

$$\mathbf{R}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{pmatrix}, \mathbf{R}_y = \begin{pmatrix} \cos \phi & 0 & -\sin \phi \\ 0 & 1 & 0 \\ \sin \phi & 0 & \cos \phi \end{pmatrix}, \mathbf{R}_z = \begin{pmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

To make mathematical manipulation easier by considering $\cos \phi = c$ and $\sin \phi = s$, one can write the matrices as follows:

$$\mathbf{R}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}, \mathbf{R}_y = \begin{pmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{pmatrix}, \mathbf{R}_z = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3.1.4 Rotation about the x-axis

After rotation about the x-axis by an angle, ϕ_x , the location vector $\mathbf{r}$ and the sensitivity vector $\mathbf{S}$ become $\mathbf{r}_{xat}$ and $\mathbf{S}_{xat}$ respectively. These can be written as follows:

$$\begin{aligned}\mathbf{r}_{xat} &= \mathbf{R}_x \cdot \mathbf{r} = x\hat{i} + [(y - y_0)c + (z - z_0)s]\hat{j} + [(z - z_0)c - (y - y_0)s]\hat{k} + y_0\hat{j} + z_0\hat{k} \\ \mathbf{S}_{xat} &= \mathbf{R}_x \cdot \mathbf{S} = S_x\hat{i} + (S_y c + S_z s)\hat{j} + (S_z c - S_y s)\hat{k},\end{aligned}\tag{3.7}$$

where it is assumed that the axis of rotation is displaced in the y- and z-direction by y_0 and z_0 with respect to the scanner's frame of reference in which the gradients are defined. By re-arranging and replacing $x = x'$, $y - y_0 = y'$ and $z - z_0 = z'$, one can obtain the location vector as follows:

$$\mathbf{r}_{xat} = x\hat{i} + (y'c + z's + y_0)\hat{j} + (z'c - y's + z_0)\hat{k}.\tag{3.8}$$

In terms of this location vector the rate of change of magnetic field can be calculated:

$$\begin{aligned}\dot{\mathbf{B}}_{xat} &= g_x[(z'c - y's + z_0)\hat{i} + x\hat{k}] + g_y[(z'c - y's + z_0)\hat{j} + (y'c + z's + y_0)\hat{k}] \\ &\quad + g_z[-\frac{x}{2}\hat{i} - \frac{1}{2}(y'c + z's + y_0)\hat{j} + (z'c - y's + x_0)\hat{k}],\end{aligned}\tag{3.9}$$

where $c = \cos \phi_x$, $s = \sin \phi_x$ and subscript 'xat' indicates rotation about the x-axis. Before and after rotation the induced voltages are denoted by V and V_{xat} which are given in the following equations respectively:

$$\begin{aligned}-V &= [g_x(z' + z_0) - \frac{1}{2}g_z x]S_x + [g_y(z' + z_0) - \frac{1}{2}g_z(y' + y_0)]S_y + [g_x x + g_y(y' + y_0) + g_z(z' + z_0)]S_z, \\ -V_{xat} &= \dot{\mathbf{B}}_{xat} \cdot \mathbf{S}_{xat} = [g_x(z'c - y's + z_0) - \frac{1}{2}g_z x]S_x + [g_y(z'c - y's + z_0) - \frac{1}{2}g_z(y'c + z's + y_0)](S_y c + S_z s) \\ &\quad + [g_x x + g_y(y'c + z's + y_0) + g_z(z'c - y's + z_0)](S_z c - S_y s).\end{aligned}\tag{3.10}$$

Thus after rotational transformation the change in induced voltage due to the application of the x-, y- and z-gradients can be obtained from the following equation:

$$\begin{aligned}
\delta V_x = & g_x [z' S_x c - y' S_x s + x S_z c - x S_y s - z' S_x - x S_z] \\
& + g_y [(z' c - y' s)(S_y c + S_z s) + (y' c + z' s)(S_z c - S_y s) - z' S_y - y' S_z] \\
& + g_y [(z_0 S_y + y_0 S_z)(c - 1) + (z_0 S_z - y_0 S_y)s] \\
& - \frac{1}{2} g_z [x S_x + (y' c + z' s)(S_y c + S_z s) - x S_x - y' S_y - 2(z' c - y' s)(S_z c - S_y s) + z' S_z] \\
& + g_z [(z_0 S_z + \frac{1}{2} y_0 S_y)(c - 1) - (\frac{1}{2} y_0 S_z + z_0 S_y)s].
\end{aligned} \tag{3.11}$$

The derivation of these expressions is described in detail in the Appendix A. To make mathematical manipulation easier, let us consider the contribution of the x-gradient as A_x which is given as follows:

$$A_x = g_x [(z' S_x + x S_z)(c - 1) - (y' S_x + x S_y)s]. \tag{3.12}$$

By rearranging and substituting $c = \cos \phi_x$ and $s = \sin \phi_x$, one obtains

$$A_x = -g_x [2(z' S_x + x S_z) \sin^2 \frac{\phi_x}{2} + (x S_y + y' S_x) \sin \phi_x]. \tag{3.13}$$

Similarly, the contribution of the y-gradient can be denoted by A_y and is given by:

$$\begin{aligned}
A_y = & g_y [z' S_y (c^2 - 1) + y' S_z (c^2 - 1) - y' S_z s^2 - z' S_y s^2 + 2(z' S_z - y' S_y)sc] \\
& + g_y [(z_0 S_y + y_0 S_z)(c - 1) + (z_0 S_z - y_0 S_y)s].
\end{aligned} \tag{3.14}$$

By replacing $c = \cos \phi_x$ and $s = \sin \phi_x$ and rearranging, one obtains

$$\begin{aligned}
A_y = & -g_y [2(z' S_y + y' S_z) \sin^2 \phi_x + (y' S_y - z' S_z) \sin(2\phi_x)] \\
& + g_y [-2(z_0 S_y + y_0 S_z) \sin^2 \frac{\phi_x}{2} + (z_0 S_z - y_0 S_y) \sin \phi_x].
\end{aligned} \tag{3.15}$$

Similarly, the contribution of the z-gradient, A_z , is

$$A_z = -\frac{1}{2}g_z[y'S_y(c^2 - 1) + z'S_zs^2 + (y'S_z + z'S_y)sc - 2z'S_z(c^2 - 1) - 2y'S_ys^2 + 2(y'S_z + z'S_y)sc] \\ + g_z[(z_0S_z + \frac{1}{2}y_0S_y)(c - 1) - (\frac{1}{2}y_0S_z + z_0S_y)s] \quad (3.16)$$

and by replacing $c = \cos \phi_x$ and $s = \sin \phi_x$, one obtains by simple mathematical manipulation:

$$A_z = \frac{3}{2}g_z[(y'S_y - z'S_z) \sin^2 \phi_x - \frac{1}{2}(y'S_z + z'S_y) \sin(2\phi_x)] \\ + g_z[-2(z_0S_z + \frac{1}{2}y_0S_y) \sin^2 \frac{\phi_x}{2} - (\frac{1}{2}y_0S_z + z_0S_y) \sin \phi_x]. \quad (3.17)$$

For application of the x-, y- and z-gradients, the change in induced voltage, following a rotation about the x-axis, is then given by

$$\delta V_x = A_x + A_y + A_z \\ \delta V_x = -g_x[2(z'S_x + xS_z) \sin^2 \frac{\phi_x}{2} + (xS_y + y'S_x) \sin \phi_x] \\ - g_y[2(z'S_y + y'S_z) \sin^2 \phi_x + (y'S_y - z'S_z) \sin(2\phi_x)] \\ + g_y[-2(z_0S_y + y_0S_z) \sin^2 \frac{\phi_x}{2} + (z_0S_z - y_0S_y) \sin \phi_x] \\ + \frac{3}{2}g_z[(y'S_y - z'S_z) \sin^2 \phi_x - \frac{1}{2}(z'S_y + y'S_z) \sin(2\phi_x)] \\ + g_z[-2(z_0S_z + \frac{1}{2}y_0S_y) \sin^2 \frac{\phi_x}{2} - (\frac{1}{2}y_0S_z + z_0S_y) \sin \phi_x]. \quad (3.18)$$

3.1.5 Rotation about the y-axis

After rotation about the y-axis by an angle, ϕ_y , the location vector, $\mathbf{r}$, and the sensitivity vector, $\mathbf{S}$, become $\mathbf{r}_{yat}$ and $\mathbf{S}_{yat}$ respectively, which can be written as follows:

$$\mathbf{r}_{yat} = \mathbf{R}_y \cdot \mathbf{r} = [(x - x_0)c - (z - z_0)s]\hat{i} + y\hat{j} + [(x - x_0)s + (z - z_0)c]\hat{k} + x_0\hat{i} + z_0\hat{k} \\ \mathbf{S}_{yat} = \mathbf{R}_y \cdot \mathbf{S} = (S_xc - S_zs)\hat{i} + S_y\hat{j} + (S_xs + S_zc)\hat{k}, \quad (3.19)$$

where I assume that the axis of rotation is displaced in the x- and z-directions by x_0 and z_0 with respect to the scanner frame of reference in which the gradients are defined. By

replacing $x - x_0 = x'$, $y = y'$ and $z - z_0 = z'$ one can obtain the coil position by rearranging as follows:

$$\mathbf{r}_{yat} = (x'c - z's + x_0)\hat{i} + y\hat{j} + (x's + z'c + z_0)\hat{k}. \quad (3.20)$$

In terms of this location vector the rate of change of magnetic field becomes:

$$\begin{aligned} \dot{\mathbf{B}}_{yat} = & g_x[(x's + z'c + z_0)\hat{i} + (x'c - z's + x_0)\hat{k}] + g_y[(x's + z'c + z_0)\hat{j} + y\hat{k}] \\ & + g_z[-\frac{1}{2}(x'c - z's + x_0)\hat{i} - \frac{1}{2}y\hat{j} + (x's + z'c + z_0)\hat{k}], \end{aligned} \quad (3.21)$$

where subscript ‘yat’ indicates rotation about the y-axis. Before and after the rotation about the y-axis the induced voltages are denoted by V and V_{yat} and can be obtained from the following equations respectively:

$$\begin{aligned} -V = & [g_x(z' + z_0) - \frac{1}{2}g_z(x' + x_0)]S_x + [g_y(z' + z_0) - \frac{1}{2}g_z y]S_y \\ & + [g_x(x' + x_0) + g_y y + g_z(z' + z_0)]S_z, \\ -V_{yat} = & [g_x(x's + z'c)(S_xc - S_zs) + g_x(x'c - z's)(S_xs + S_zc)] \\ & + g_xz_0(S_xc - S_zs) + g_xx_0(S_xs - S_zc) \\ & + [g_y(x's + z'c)S_y + g_yy(S_xs + S_zc) \\ & - \frac{1}{2}g_z[(x'c - z's)(S_xc - S_zs) + yS_y + g_z(x's + z'c)(S_xs + S_zc)] \\ & + g_zz_0(S_xs + S_zc) - \frac{1}{2}g_zx_0(S_xc - S_zs). \end{aligned} \quad (3.22)$$

The change in induced voltage due to rotation about the y-axis is then given as follows:

$$\begin{aligned}
\delta V_y &= -V_{yat} + V = \dot{\mathbf{B}}_{yat} \cdot \mathbf{S}_{yat} - \dot{\mathbf{B}} \cdot \mathbf{S} \\
&= g_x(x'S_x sc + z'S_x c^2 - x'S_z s^2 - z'S_z sc \\
&\quad + x'S_x cs - z'S_x s^2 + x'S_z c^2 - z'S_z cs - z'S_x - x'S_z) \\
&\quad + g_x(z_0 S_x c - z_0 S_z s + x_0 S_x s - x_0 S_z c - z_0 S_x - x_0 S_z) \\
&\quad + g_y(x'S_y s + z'S_y c + yS_x s + yS_z c - z'S_y - yS_z) \\
&\quad + \frac{1}{2}g_z(-x'S_x c^2 + z'S_x sc + x'S_z sc - z'S_z s^2 \\
&\quad - yS_y + 2x'S_x s^2 + 2z'S_x sc + 2x'S_z sc \\
&\quad + 2z'S_z c^2 + x'S_x + yS_y - z'S_z) \\
&\quad + \frac{1}{2}g_z(2z_0 S_x s + 2z_0 S_z c - x_0 S_x c + x_0 S_z s) - 2g_z z_0 S_z - x_0 S_x.
\end{aligned} \tag{3.23}$$

Again by simple mathematical manipulation, the contribution of the x-gradient becomes:

$$\begin{aligned}
B_x &= g_x[2(x'S_x - z'S_z S_z)sc + (z'S_x + z'S_z)(c^2 - 1) \\
&\quad - (x'S_z + z'S_x)s^2] \\
&\quad + g_x[(z_0 S_x + x_0 S_z)(c - 1) + (x_0 S_x - z_0 S_z)s].
\end{aligned} \tag{3.24}$$

By rearranging and replacing $c = \cos \phi_y$ and $s = \sin \phi_y$, one obtains

$$\begin{aligned}
B_x &= g_x[(xS_x - zS_z) \sin(2\phi_y) - 2(zS_x + xS_z) \sin^2 \phi_y] \\
&\quad - g_x[-2(z_0 S_x + x_0 S_z) \sin^2 \frac{\phi_y}{2} + (x_0 S_x - z_0 S_z) \sin \phi_y].
\end{aligned} \tag{3.25}$$

Similarly, the contribution of a y-gradient is

$$B_y = g_y[(x'S_y + yS_x) \sin \phi_y - 2(z'S_y + yS_z) \sin^2 \frac{\phi_y}{2}], \tag{3.26}$$

and the contribution of the z-gradient can be obtained as

$$\begin{aligned}
B_z = & +\frac{1}{2}g_z(-x'S_xc^2 + z'S_xsc + x'S_zsc - z'S_zs^2 \\
& -yS_y + 2x'S_xs^2 + 2z'S_xsc + 2x'S_zsc \\
& + 2z'S_zc^2 + x'S_x + yS_y - z'S_z) \\
& + g_zz_0(S_xs + S_zc) - \frac{1}{2}(S_xc - S_zs) - g_zz_0S_z + \frac{1}{2}g_zx_0S_x.
\end{aligned} \tag{3.27}$$

By replacing $c = \cos \phi_y$ and $s = \sin \phi_y$, one obtains

$$\begin{aligned}
B_z = & \frac{3}{2}g_z[(x'S_x - z'S_z)\sin^2 \phi_y + \frac{1}{2}(z'S_x + x'S_z)\sin(2\phi_y)] \\
& + g_z[(z_0S_x + \frac{1}{2}x_0S_z)\sin \phi_y - 2(z_0S_z - \frac{1}{2}x_0S_x)\sin^2 \frac{\phi_y}{2}].
\end{aligned} \tag{3.28}$$

Now the change in induced voltage due to rotation about the y-axis because of the application of the x-, y- and z-gradients becomes:

$$\begin{aligned}
\delta V_y = & B_x + B_y + B_z \\
\delta V_y = & g_x[(x'S_x - z'S_z)\sin(2\phi_y) - 2(z'S_x + x'S_z)\sin^2 \phi_y] \\
& - g_x[-2(z_0S_x + x_0S_z)\sin^2 \frac{\phi_y}{2} + (x_0S_x - z_0S_z)\sin \phi_y] \\
& + g_y[(x'S_y + yS_x)\sin \phi_y - 2(z'S_y + yS_z)\sin^2 \frac{\phi_y}{2}] \\
& + \frac{3}{2}g_z[\frac{1}{2}(z'S_x + xS_z)\sin(2\phi_y) + (xS_x - z'S_z)\sin^2 \phi_y] \\
& + g_z[(z_0S_x + \frac{1}{2}x_0S_z)\sin \phi_y - 2(z_0S_z - \frac{1}{2}x_0S_x)\sin^2 \frac{\phi_y}{2}].
\end{aligned} \tag{3.29}$$

3.1.6 Rotation about the z-axis

After rotation about the z-axis by an angle, ϕ_z , the location vector $\mathbf{r}$ and the sensitivity vector $\mathbf{S}$ become $\mathbf{r}_{zat}$ and $\mathbf{S}_{zat}$, respectively, given by the equations:

$$\begin{aligned}
\mathbf{r}_{zat} = \mathbf{R}_z \cdot \mathbf{r} = & [c(x - x_0) + s(y - y_0)]\hat{i} + [-s(x - x_0) + c(y - y_0)]\hat{j} + z\hat{k} + x_0\hat{i} + y_0\hat{j} \\
\mathbf{S}_{zat} = \mathbf{R}_z \cdot \mathbf{S} = & (S_xc + S_ys)\hat{i} + (S_yc - S_xs)\hat{j} + S_z\hat{k}.
\end{aligned} \tag{3.30}$$

By replacing $x - x_0 = x'$, $y - y_0 = y'$ and $z = z'$ one can obtain the location vector as follows:

$$r_{zat} = (x'c + y's + x_0)\hat{i} + (y'c - x's + y_0)\hat{j} + z\hat{k}. \quad (3.31)$$

In terms of this location vector, the rate of change of magnetic field becomes:

$$\begin{aligned} \dot{\mathbf{B}}_{zat} = & g_x[z\hat{i} + (x'c + y's + x_0)\hat{k}] + g_y[z\hat{j} + (y'c - x's + y_0)\hat{k}] \\ & + g_z[-\frac{1}{2}(x'c + y's + x_0)\hat{i} - \frac{1}{2}(y'c - x's + y_0)\hat{j} + z\hat{k}]. \end{aligned} \quad (3.32)$$

The voltage induced in the coil before and after the rotation about the z-axis can be denoted by V and V_{zat} which are given by the following equations:

$$\begin{aligned} -V = & [g_x z - \frac{1}{2}g_z(x' + x_0)]S_x + [g_y z - \frac{1}{2}g_z(y' + y_0)]S_y \\ & + [g_x(x'x_0) + g_y(y'y_0) + g_z z]S_z \\ -V_{zat} = & g_x[z(S_x c + S_y s) + S_z(x'c + y's)] + g_x x_0 S_z \\ & + g_y[z(S_y c - S_x s) + S_z(-x's + y'c)] + g_y y_0 S_z \\ & + \frac{1}{2}g_z[-(x'c + y's)(S_x c + S_y s) - (-x's + y'c)(S_y c - S_x s) + 2zS_z] \\ & - \frac{1}{2}g_z[x_0(S_x c + S_y s) + y_0(S_y c - S_x s)], \end{aligned} \quad (3.33)$$

where ϕ_z is the rotational angle. The change in induced voltage in the coil due to rotation about the z-axis can be represented by the equation:

$$\begin{aligned} \delta V_z = & -V_{zat} + V = \dot{\mathbf{B}}_{zat} \cdot \mathbf{S}_{zat} - \dot{\mathbf{B}} \cdot \mathbf{S} \\ = & g_x[z(S_x c + S_y s) + S_z(x'c + y's) - zS_x - x'S_z] \\ & + g_y[z(S_y c - S_x s) + S_z(-x's + y'c) - zS_y - y'S_z] \\ & + \frac{1}{2}g_z[-(x'c + y's)(S_x c + S_y s) - (-x's + y'c)(S_y c - S_x s) \\ & (S_y c - S_x s) + 2zS_z + x'S_x + y'S_y - 2zS_z] \\ & - \frac{1}{2}g_z[x_0(S_x c + S_y s) - x_0S_x + y_0(S_y c - S_x s) - y_0S_y]. \end{aligned} \quad (3.34)$$

To make the calculation simpler, let us consider the contribution of the x-gradient:

$$C_x = g_x[zS_x(c-1) + x'S_z(c-1) + (zS_y + y'S_z)s], \quad (3.35)$$

and by replacing $c = \cos \phi$ and $s = \sin \phi$, one obtains,

$$C_x = g_x[-2(zS_x + x'S_z) \sin^2 \frac{\phi_z}{2} + (zS_y + y'S_z) \sin \phi_z]. \quad (3.36)$$

Similarly, the contribution of the y-gradient is

$$C_y = g_y[zS_y(c-1) + y'S_z(c-1) - (zS_x + x'S_z)s], \quad (3.37)$$

and by replacing $c = \cos \phi_z$ and $s = \sin \phi_z$, one obtains,

$$C_y = g_y[-2(zS_y + y'S_z) \sin^2 \frac{\phi_z}{2} - (zS_x + x'S_z) \sin \phi_z]. \quad (3.38)$$

The contribution of the z-gradient can be obtained by using the following equation:

$$\begin{aligned} C_z = & \frac{1}{2}g_z[-x'S_x(c^2 + s^2) - y'S_y(c^2 + s^2) + x'S_x + y'S_y] \\ & - \frac{1}{2}g_z[(x_0S_x + y_0S_y)(c-1) + (x_0S_y - y_0S_x)s], \end{aligned} \quad (3.39)$$

and by replacing $c = \cos \phi_z$ and $s = \sin \phi_z$, one obtains

$$C_z = +g_z(x_0S_x + y_0S_y) \sin^2 \frac{\phi_z}{2} - \frac{1}{2}g_z(x_0S_y - y_0S_x) \sin \phi_z. \quad (3.40)$$

The change in induced voltage due to rotation about the z-axis is consequently as follows:

$$\begin{aligned}
\delta V_z &= C_x + C_y + C_z \\
\delta V_z &= -g_x[-(z'S_y + y'S_z) \sin \phi_z + 2(z'S_x + xS_z) \sin^2 \frac{\phi_z}{2}] \\
&\quad -g_y[(z'S_x + x'S_z) \sin \phi_z + 2(z'S_y + y'S_z) \sin^2 \frac{\phi_z}{2}] \\
&\quad +g_z(x_0S_x + y_0S_y) \sin^2 \phi_z - \frac{1}{2}g_z(x_0S_y - y_0S_x) \sin \phi_z.
\end{aligned} \tag{3.41}$$

3.1.7 The General Equation for Rotation

The general equation for the change in induced voltage when the x-, y-, and z-gradients are applied, due to a rotation by an angle ϕ_x about the x-axis, ϕ_y about the y-axis and ϕ_z about the z-axis is

$$\begin{aligned}
\delta V_R &= \delta V_x + \delta V_y + \delta V_z, \\
\delta V_R &= -g_x[2(z'S_x + xS_z) \sin^2 \frac{\phi_x}{2} + (xS_y + y'S_x) \sin \phi_x] - g_y[2(z'S_y + y'S_z) \sin^2 \phi_x + (y'S_y - z'S_z) \sin(2\phi_x)] \\
&\quad +g_y[-2(z_0S_y + y_0S_z) \sin^2 \frac{\phi_x}{2} + (z_0S_z - y_0S_y) \sin \phi_x] + \frac{3}{2}g_z[(y'S_y - z'S_z) \sin^2 \phi_x - \frac{1}{2}(z'S_y + y'S_z) \sin(2\phi_x)] \\
&\quad +g_z[-2(z_0S_z + \frac{1}{2}y_0S_y) \sin^2 \frac{\phi_x}{2} - (\frac{1}{2}y_0S_z + z_0S_y) \sin \phi_x] + g_x[(x'S_x - z'S_z) \sin(2\phi_y) - 2(z'S_x + x'S_z) \sin^2 \phi_y] \\
&\quad -g_x[-2(z_0S_x + x_0S_z) \sin^2 \frac{\phi_y}{2} + (x_0S_x - z_0S_z) \sin \phi_y] + g_y[(x'S_y + yS_x) \sin \phi_y - 2(z'S_y + yS_z) \sin^2 \frac{\phi_y}{2}] \\
&\quad +\frac{3}{2}g_z[\frac{1}{2}(z'S_x + xS_z) \sin(2\phi_y) + (xS_x - z'S_z) \sin^2 \phi_y] + g_z[(z_0S_x + \frac{1}{2}x_0S_z) \sin \phi_y - 2(z_0S_z - \frac{1}{2}x_0S_x) \sin^2 \frac{\phi_y}{2}] \\
&\quad -g_x[-(z'S_y + y'S_z) \sin \phi_z + 2(z'S_x + xS_z) \sin^2 \frac{\phi_z}{2}] - g_y[(z'S_x + x'S_z) \sin \phi_z + 2(z'S_y + y'S_z) \sin^2 \frac{\phi_z}{2}] \\
&\quad +g_z(x_0S_x + y_0S_y) \sin^2 \phi_z - \frac{1}{2}g_z(x_0S_y - y_0S_x) \sin \phi_z.
\end{aligned} \tag{3.42}$$

Now considering the arrangement of the five coils shown in Fig. 3.2, the change in induced voltage in the individual coils due to a rotation by an angle, ϕ_x, ϕ_y, ϕ_z about the x-, y- and

z-axes respectively, is found to be

$$\begin{aligned}
\delta V_1 &= +g_x S_z \delta x + g_y S_z \delta y + g_z S_z \delta z \\
&+ g_y (z' S_z) \sin(2\phi_x) - \frac{3}{2} g_z (z' S_z) \sin^2(\phi_x) - g_x (z' S_z) \sin(2\phi_y) - \frac{3}{2} g_z (z' S_z) \sin^2(\phi_y) \\
&+ g_x (z_0 S_z) \sin \phi_y - 2g_z (z_0 S_z) \sin^2 \frac{\phi_x}{2} - g_y (z_0 S_z) \sin \phi_x - 2g_z (z_0 S_z) \sin^2 \frac{\phi_y}{2} \\
\delta V_2 &= -\frac{1}{2} g_z S_x \delta x + g_x S_x \delta z + g_x (x' S_x) \sin(2\phi_y) + \frac{3}{2} g_z (x' S_x) \sin^2 \phi_y \\
&- g_x (x_0 S_x) \sin \phi_y + 2g_z (z_0 S_z) \sin^2 \frac{\phi_y}{2} + g_z (x_0 S_x) \sin^2 \phi_z \\
\delta V_3 &= -\frac{1}{2} g_z S_x \delta x + g_x S_x \delta z - g_x (x' S_x) \sin(2\phi_y) - \frac{3}{2} g_z (x' S_x) \sin^2 \phi_y; \\
&- g_x (x_0 S_x) \sin \phi_y + 2g_z (z_0 S_z) \sin^2 \frac{\phi_y}{2} + g_z (x_0 S_x) \sin^2 \phi_z \\
\delta V_4 &= -\frac{1}{2} g_z S_y \delta y + g_y S_y \delta z - g_y (y' S_y) \sin(2\phi_x) + \frac{3}{2} g_z (y' S_y) \sin^2 \phi_x; \\
&+ g_y (y_0 S_y) \sin \phi_x + g_z (\frac{1}{2} y_0 S_y) \sin^2 \frac{\phi_x}{2} + g_z (y_0 S_y) \sin^2 \phi_z \\
\delta V_5 &= -\frac{1}{2} g_z S_y \delta y + g_y S_y \delta z + g_y (y' S_y) \sin(2\phi_x) - \frac{3}{2} g_z (y' S_y) \sin^2 \phi_x \\
&+ g_y (y_0 S_y) \sin \phi_x + g_z (\frac{1}{2} y_0 S_y) \sin^2 \frac{\phi_x}{2} + g_z (y_0 S_y) \sin^2 \phi_z.
\end{aligned} \tag{3.43}$$

These equations are summarized in Table 3.2.

3.1.8 The General Expression for Translation and Rotation

Combining the voltage changes due to rotation and translation gives

$$\begin{aligned}
\delta V_{RT} &= \delta V_T + \delta V_R \\
&= (g_x S_z - \frac{1}{2} g_z S_x) \delta x + (g_y S_z - \frac{1}{2} g_z S_y) \delta y + (g_x S_x + g_y S_y + g_z S_z) \delta z \\
&- g_x [2(z' S_x + x S_z) \sin^2 \frac{\phi_x}{2} + (x S_y + y' S_x) \sin \phi_x] - g_y [2(z' S_y + y' S_z) \sin^2 \phi_x + (y' S_y - z' S_z) \sin(2\phi_x)] \\
&+ g_y [-2(z_0 S_y + y_0 S_z) \sin^2 \frac{\phi_x}{2} + (z_0 S_z - y_0 S_y) \sin \phi_x] + \frac{3}{2} g_z [(y' S_y - z' S_z) \sin^2 \phi_x - \frac{1}{2} (z' S_y + y' S_z) \sin(2\phi_x)] \\
&+ g_z [-2(z_0 S_z + \frac{1}{2} y_0 S_y) \sin^2 \frac{\phi_x}{2} - (\frac{1}{2} y_0 S_z + z_0 S_y) \sin \phi_x] + g_x [(x' S_x - z' S_z) \sin(2\phi_y) - 2(z' S_x + x' S_z) \sin^2 \phi_y] \\
&- g_x [-2(z_0 S_x + x_0 S_z) \sin^2 \frac{\phi_y}{2} + (x_0 S_x - z_0 S_z) \sin \phi_y] + g_y [(x' S_y + y S_x) \sin \phi_y - 2(z' S_y + y S_z) \sin^2 \frac{\phi_y}{2}] \\
&+ \frac{3}{2} g_z [\frac{1}{2} (z' S_x + x S_z) \sin(2\phi_y) + (x S_x - z' S_z) \sin^2 \phi_y] + g_z [(z_0 S_x + \frac{1}{2} x_0 S_z) \sin \phi_y - 2(z_0 S_z - \frac{1}{2} x_0 S_x) \sin^2 \frac{\phi_y}{2}] \\
&- g_x [-(z' S_y + y' S_z) \sin \phi_z + 2(z' S_x + x S_z) \sin^2 \frac{\phi_z}{2}] - g_y [(z' S_x + x' S_z) \sin \phi_z + 2(z' S_y + y' S_z) \sin^2 \frac{\phi_z}{2}] \\
&+ g_z (x_0 S_x + y_0 S_y) \sin^2 \phi_z - \frac{1}{2} g_z (x_0 S_y - y_0 S_x) \sin \phi_z.
\end{aligned} \tag{3.44}$$

Coil No.	Sensitivity	ϕ_x	ϕ_y	ϕ_z	Voltage
C_1	S_z	$+g_y(z'S_z)\sin(2\phi_x)$ $-\frac{3}{2}g_z(z'S_z)\sin^2(\phi_x)$ $-g_y(z_0S_z)\sin(\phi_x)$ $-2g_z(z_0S_z)\sin^2(\frac{\phi_x}{2})$	$-g_x(z'S_z)\sin(2\phi_y)$ $-\frac{3}{2}g_z(z'S_z)\sin^2(\phi_y)$ $+g_x(z_0S_z)\sin(\phi_y)$ $-2g_z(z_0S_z)\sin^2(\frac{\phi_y}{2})$	0	δV_1
C_2	S_x	0	$+g_x(x'S_x)\sin(2\phi_y)$ $+\frac{3}{2}g_z(x'S_x)\sin^2(\frac{\phi_y}{2})$ $+g_x(x_0S_x)\sin(\phi_y)$ $+2g_z(x_0S_x)\sin^2(\frac{\phi_y}{2})$	$g_z(x_0S_x)\sin^2(\phi_z)$	δV_2
C_3	S_{-x}	0	$+g_x(x'S_x)\sin(2\phi_y)$ $+\frac{3}{2}g_z(x'S_x)\sin^2(\frac{\phi_y}{2})$ $-g_x(x_0S_x)\sin(\phi_y)$ $+2g_z(x_0S_x)\sin^2(\frac{\phi_y}{2})$	$g_z(x_0S_x)\sin^2(\phi_z)$	δV_3
C_4	S_y	$-g_y(y'S_y)\sin(2\phi_x)$ $+\frac{3}{2}g_z(y'S_y)\sin^2(\phi_x)$ $+g_y(y_0S_y)\sin(\phi_x)$ $+2g_z(y_0S_y)\sin^2(\frac{\phi_x}{2})$	0	$+g_z(y_0S_y)\sin^2(\phi_z)$	δV_4
C_5	S_{-y}	$-g_y(y'S_y)\sin(2\phi_x)$ $+\frac{3}{2}g_z(y'S_y)\sin^2(\phi_x)$ $+g_y(y_0S_y)\sin(\phi_x)$ $+2g_z(y_0S_y)\sin^2(\frac{\phi_x}{2})$	0	$+g_z(y_0S_y)\sin^2(\phi_z)$	δV_5

Table 3.2: Summary of the changes in induced voltage in the coils due to a general rotation.

For small rotation angles such that ϕ_x , ϕ_y , and $\phi_z \leq 5^\circ$, then $\sin \phi_x = \phi_x$, $\sin \phi_y = \phi_y$, $\sin \phi_z = \phi_z$, when ϕ_x , ϕ_y , and ϕ_z are in radian; as a result the general equation for the change in induced voltage due to rotation about the x-, y- and z-axes by angle $\delta\phi_x$, $\delta\phi_y$ and $\delta\phi_z$ reduces to

$$\begin{aligned}
\delta V_{RT} = & (g_x S_z - \frac{1}{2} g_z S_x) \delta x + (g_y S_z - \frac{1}{2} g_z S_y) \delta y + (g_x S_x + g_y S_y + g_z S_z) \delta z \\
& - \frac{1}{2} g_x (z' S_x) \delta \phi_x^2 + \frac{1}{2} (x S_z) \delta \phi_x^2 + g_x (x S_y) \delta \phi_x + g_x (y' S_x) \delta \phi_x \\
& - 2g_y (z' S_y) \delta \phi_x^2 - 2g_y (y' S_z) \delta \phi_x - 2g_y (y' S_y) \delta \phi_x - 2g_y (z' S_z) \delta \phi_x \\
& - \frac{1}{2} g_y (z_0 S_y) \delta \phi_x^2 - \frac{1}{2} g_y (y_0 S_z) \delta \phi_x^2 + g_y (z_0 S_z) \delta \phi_x - g_y (y_0 S_y) \delta \phi_x \\
& + \frac{3}{2} g_z (y' S_y) \delta \phi_x^2 - \frac{3}{2} g_z (z' S_z) \delta \phi_x^2 - \frac{3}{2} g_z (z' S_y) \delta \phi_x + \frac{3}{2} g_z (y' S_z) \delta \phi_x \\
& - \frac{1}{2} g_z (z_0 S_z) \delta \phi_x^2 + \frac{1}{4} g_z (y_0 S_y) \delta \phi_x^2 - \frac{1}{2} g_z (y_0 S_z) \delta \phi_x - g_z (z_0 S_y) \delta \phi_x \\
& + 2g_x (x' S_x) - 2g_x (z' S_z) \delta \phi_y - 2g_x (z' S_x) \delta \phi_y^2 - 2g_x (x' S_z) \delta \phi_y^2 \\
& + \frac{1}{2} g_x (z_0 S_x) \delta \phi_y^2 - \frac{1}{2} g_x (x_0 S_z) \delta \phi_y^2 + g_x (x_0 S_x) \delta \phi_y - g_x (z_0 S_z) \delta \phi_y \\
& + g_y (x' S_y) \delta \phi_y + g_y (y S_x) \delta \phi_y - \frac{1}{2} g_y (z' S_y) \delta \phi_y^2 - \frac{1}{2} g_y (y S_z) \delta \phi_y^2 \\
& + \frac{3}{2} g_z (z' S_x) \delta \phi_y + \frac{3}{2} g_z (x S_z) \delta \phi_y + \frac{3}{2} g_z (x S_x) \delta \phi_y^2 - \frac{3}{2} g_z (z' S_z) \delta \phi_y^2 \\
& + g_z (z_0 S_x) \delta \phi_y + \frac{1}{2} g_z (x_0 S_z) \delta \phi_y - \frac{1}{2} g_z (z_0 S_z) \delta \phi_y^2 + \frac{1}{4} g_z (x_0 S_x) \delta \phi_y^2 \\
& + g_x (z' S_y) \delta \phi_z - g_x (y' S_z) \delta \phi_z + \frac{1}{2} g_x (z' S_x) \delta \phi_z^2 + \frac{1}{2} g_x (x S_z) \delta \phi_z^2 \\
& - g_y (z' S_x) \delta \phi_z - g_y (x' S_z) \delta \phi_z + \frac{1}{2} g_y (z' S_y) \delta \phi_z^2 + \frac{1}{2} g_y (y' S_z) \delta \phi_z^2 \\
& + g_z (x_0 S_x) \delta \phi_z^2 + g_z (y_0 S_y) \delta \phi_z^2 - \frac{1}{2} g_z (x_0 S_y) \delta \phi_z + \frac{1}{2} g_z (y_0 S_x) \delta \phi_z.
\end{aligned} \tag{3.45}$$

Let us again consider a coil in the different planes in which the centres lie on the x-, y- and z-axes and the axes of the coils is the x-, y- and z-axes (see Fig. 3.2). The change in induced voltage in each coil due to rotation about the x-, y- and z-axes can be calculated by taking only the terms xS_x , yS_y , zS_z , $x_0S_{x_0}$, $y_0S_{y_0}$ and $z_0S_{z_0}$. The changes in induced voltage in the five coils are then given by a general equation:

$$\begin{aligned}
\delta V_{RT} = & (g_x S_z - \frac{1}{2} g_z S_x) \delta x + (g_y S_z - \frac{1}{2} g_z S_y) \delta y + (g_x S_x + g_y S_y + g_z S_z) \delta z \\
& - 2g_y (y' S_y) \delta \phi_x - 2g_y (z' S_z) \delta \phi_x + g_y (z_0 S_z) \delta \phi_x - g_y (y_0 S_y) \delta \phi_x + \frac{3}{2} g_z (y' S_y) \delta \phi_x^2 - \frac{3}{2} g_z (z' S_z) \delta \phi_x^2 \\
& - \frac{1}{2} g_z (z_0 S_z) \delta \phi_x^2 + \frac{1}{4} g_z (y_0 S_y) \delta \phi_x^2 + 2g_x (x' S_x) - 2g_x (z' S_z) \delta \phi_y + g_x (x_0 S_x) \delta \phi_y - g_x (z_0 S_z) \delta \phi_y \\
& + \frac{3}{2} g_z (x S_x) \delta \phi_y^2 - \frac{3}{2} g_z (z' S_z) \delta \phi_y^2 - \frac{1}{2} g_z (z_0 S_z) \delta \phi_y^2 + \frac{1}{4} g_z (x_0 S_x) \delta \phi_y^2 + g_z (x_0 S_x) \delta \phi_z^2 + g_z (y_0 S_y) \delta \phi_z^2.
\end{aligned} \tag{3.46}$$

For small angles of rotation the linear terms dominate in the expression for the change

Coil No.	Sensitivity	δx	δy	δz	$\delta\phi_x$	$\delta\phi_y$	$\delta\phi_z$	Voltage
C_1	S_z	g_x	g_y	g_z	$+2g_y z' S_z - g_y z_0 S_z$	$-2g_x z' S_z + g_x z_0 S_z$	0	δV_1
C_2	S_x	$-\frac{1}{2}g_z$	0	g_x	0	$+2g_x x' S_x - g_x x_0 S_x$	0	δV_2
C_3	S_{-x}	$-\frac{1}{2}g_z$	0	g_x	0	$+2g_x x' S_x - g_x x_0 S_x$	0	δV_3
C_4	S_y	0	$-\frac{1}{2}g_z$	g_y	$-2g_y y' S_y + g_y y_0 S_y$	0	0	δV_4
C_5	S_{-y}	0	$-\frac{1}{2}g_z$	g_y	$-2g_y y' S_y + g_y y_0 S_y$	0	0	δV_5

Table 3.3: Summary of the change in induced voltage in the coils due to translation and rotation.

in induced voltages in a coil, giving:

$$\begin{aligned}
\delta V_{RT} = & (g_x S_z - \frac{1}{2}g_z S_x)\delta x + (g_y S_z - \frac{1}{2}g_z S_y)\delta y + (g_x S_x + g_y S_y + g_z S_z)\delta z \\
& -2g_y(y' S_y)\delta\phi_x - 2g_y(z' S_z)\delta\phi_x + g_y(z_0 S_z)\delta\phi_x - g_y(y_0 S_y)\delta\phi_x \\
& +2g_x(x' S_x) - 2g_x(z' S_z)\delta\phi_y + g_x(x_0 S_x)\delta\phi_y - g_x(z_0 S_z)\delta\phi_y.
\end{aligned} \tag{3.47}$$

The above general equation can be used to evaluate the change in induced voltage in each of the coils in the system shown in Fig. 3.2. For this coil system we have $\hat{S}_1 = \hat{k}$, $\hat{S}_2 = \hat{S}_3 = \hat{i}$ and $\hat{S}_4 = \hat{S}_5 = \hat{j}$, and the centres of the five coils are at $C_1(0 \ 0 \ z)$, $C_2(x \ 0 \ 0)$, $C_3(-x \ 0 \ 0)$, $C_4(0 \ y \ 0)$, and $C_5(0 \ -y \ 0)$. The equation for the change in induced voltage in the coils due to translation and rotations is then given by:

$$\begin{aligned}
\delta V_1 = & +g_x S_z \delta x + g_y S_z \delta y + g_z S_z \delta z \\
& -2g_y(z' S_z)\delta\phi_x - 2g_x(z' S_z)\delta\phi_y + g_x(z_0 S_z)\delta\phi_y - g_y(z_0 S_z)\delta\phi_x \\
\delta V_2 = & -\frac{1}{2}g_z S_x \delta x + g_x S_x \delta z + 2g_x x' S_x \delta\phi_y - g_x(x_0 S_x)\delta\phi_y \\
\delta V_3 = & -\frac{1}{2}g_z S_x \delta x + g_x S_x \delta z - \frac{1}{2}g_x(x' S_x)\delta\phi_y - g_x(x_0 S_x)\delta\phi_y \\
\delta V_4 = & -\frac{1}{2}g_z S_y \delta y + g_y S_y \delta z - 2g_y(y' S_y)\delta\phi_x + g_y(y_0 S_y)\delta\phi_x \\
\delta V_5 = & -\frac{1}{2}g_z S_y \delta y + g_y S_y \delta z + 2g_y(y' S_y)\delta\phi_x + g_y(y_0 S_y)\delta\phi_x.
\end{aligned} \tag{3.48}$$

All these are summarized in Table 3.3.

3.1.9 Special Cases

The change in induced voltage in the individual coil due to a rotation by an angle, ϕ_x , about the x-axis can be obtained by the given equations:

$$\begin{aligned}
\delta V_1 &= +g_y(z'S_z) \sin(2\phi_x) - \frac{3}{2}g_z(z'S_z) \sin^2 \phi_x + g_y(z_0S_z) \sin \phi_x - 2g_z(z_0S_z) \sin^2 \frac{\phi_x}{2} \\
\delta V_2 &= 0 \quad \delta V_3 = 0 \\
\delta V_4 &= -g_y(y'S_y) \sin(2\phi_x) - g_y(y_0S_y) \sin \phi_x + \frac{3}{2}g_z(y'S_y) \sin^2 \phi_x - 2g_z(y_0S_y) \sin^2 \frac{\phi_x}{2} \\
\delta V_5 &= -g_y(y'S_y) \sin(2\phi_x) - g_y(y_0S_y) \sin \phi_x + \frac{3}{2}g_z(y'S_y) \sin^2 \phi_x - 2g_z(y_0S_y) \sin^2 \frac{\phi_x}{2}.
\end{aligned} \tag{3.49}$$

For a small rotation angle ϕ_x , i.e. $\phi_x \leq 5^\circ$ one can write $\sin \phi_x = \phi_x$ (in radian) and as a result the general equation for the change of induced voltage in the coils due to rotation about the x-axis by an angle, $\delta\phi_x$ reduces to

$$\begin{aligned}
\delta V_1 &= +2g_y(z'S_z)\delta\phi_x - \frac{3}{2}g_z(z'S_z)\delta\phi_x^2 + g_y(z_0S_z)\delta\phi_x - \frac{1}{2}g_z(z_0S_z)\delta\phi_x^2 \\
\delta V_2 &= 0, \quad \delta V_3 = 0 \\
\delta V_4 &= -2g_y(y'S_y)\delta\phi_x - g_y(y_0S_y)\delta\phi_x + \frac{3}{2}g_z(y'S_y)\delta\phi_x^2 - \frac{1}{2}g_z(y_0S_y)\delta\phi_x^2 \\
\delta V_5 &= -2g_y(y'S_y)\delta\phi_x - g_y(y_0S_y)\delta\phi_x + \frac{3}{2}g_z(y'S_y)\delta\phi_x^2 - \frac{1}{2}g_z(y_0S_y)\delta\phi_x^2.
\end{aligned} \tag{3.50}$$

The linear terms dominate in the expressions for the change in induced voltages in the coils, so one can exclude the non-linear terms,

$$\begin{aligned}
\delta V_1 &= 2g_y(z'S_z)\delta\phi_x + g_y(z_0S_z)\delta\phi_x \\
\delta V_2 &= 0, \quad \delta V_3 = 0 \\
\delta V_4 &= -2g_y(y'S_y)\delta\phi_x - g_y(y_0S_y)\delta\phi_x \\
\delta V_5 &= -2g_y(y'S_y)\delta\phi_x - g_y(y_0S_y)\delta\phi_x.
\end{aligned} \tag{3.51}$$

The change in induced voltage in the individual coil due to a rotation by an angle, ϕ_y , about the y-axis can similarly be obtained by using the equations:

$$\begin{aligned}
\delta V_1 &= -g_x(z'S_z) \sin(2\phi_y) + g_x(z_0S_z) \sin \phi_y - \frac{3}{2}g_z(z'S_z) \sin^2 \phi_y - 2g_z(z_0S_z) \sin^2 \frac{\phi_y}{2} \\
\delta V_2 &= g_x(x'S_x) \sin(2\phi_y) + g_x(x_0S_x) \sin \phi_y + \frac{3}{2}g_z(x'S_x) \sin^2 \phi_y + g_z(x_0S_x) \sin^2 \frac{\phi_y}{2} \\
\delta V_3 &= g_x(x'S_x) \sin(2\phi_y) + g_x(x_0S_x) \sin \phi_y + \frac{3}{2}g_z(x'S_x) \sin^2 \phi_y + g_z(x_0S_x) \sin^2 \frac{\phi_y}{2} \\
\delta V_4 &= 0, \quad \delta V_5 = 0.
\end{aligned} \tag{3.52}$$

For a small rotation angle ϕ_y , i.e. $\phi_y \leq 5^\circ$, $\sin \phi_y = \phi_y$ (in radian); as a result the general equation for the change of induced voltage in the coils due to rotation about the y-axis by angle $\delta\phi_y$ reduces to

$$\begin{aligned}
\delta V_1 &= -2g_x(z'S_z)\delta\phi_y + g_x(z_0S_z)\delta\phi_y - \frac{3}{2}g_z(z'S_z)\delta\phi_y^2 - \frac{1}{2}g_z(z_0S_z)\delta\phi_y^2 \\
\delta V_2 &= 2g_x(x'S_x)\delta\phi_y + g_x(x_0S_x)\delta\phi_y + \frac{3}{2}(xS_x)\delta\phi_y^2 + \frac{1}{4}g_z(x_0S_x)\delta\phi_y^2 \\
\delta V_3 &= 2g_x(x'S_x)\delta\phi_y + g_x(x_0S_x)\delta\phi_y + \frac{3}{2}(xS_x)\delta\phi_y^2 + \frac{1}{4}g_z(x_0S_x)\delta\phi_y^2 \\
\delta V_4 &= 0, \quad \delta V_5 = 0.
\end{aligned} \tag{3.53}$$

The linear terms dominate in the expressions for the change in induced voltages in the coils, so one can exclude the non-linear terms,

$$\begin{aligned}
\delta V_1 &= -2g_x(z'S_z)\delta\phi_y + g_x(z_0S_z)\delta\phi_y \\
\delta V_2 &= 2g_x(x'S_x)\delta\phi_y + g_x(x_0S_x)\delta\phi_y \\
\delta V_3 &= 2g_x(x'S_x)\delta\phi_y + g_x(x_0S_x)\delta\phi_y \\
\delta V_4 &= 0, \quad \delta V_5 = 0.
\end{aligned} \tag{3.54}$$

The change in induced voltage in the individual coils due to rotation about the z-axis can then be obtained by the given equations:

$$\begin{aligned}
\delta V_1 &= 0 \\
\delta V_2 &= +g_z(x_0S_x)\sin^2\phi_z \quad \delta V_3 = +g_z(x_0S_x)\sin^2\phi_z \\
\delta V_4 &= +g_z(y_0S_y)\sin^2\phi_z \quad \delta V_5 = +g_z(y_0S_y)\sin^2\phi_z.
\end{aligned}$$

For a small rotation angle ϕ_z , i.e. $\phi_z \leq 5^\circ$, $\sin \phi_z = \phi_z$ (in radian); as a result the general equation for the change of induced voltage due to the rotation about z-axis by angle $\delta\phi_z$ reduces to

$$\begin{aligned}
\delta V_1 &= 0 \\
\delta V_2 &= +g_z(x_0S_x)\delta\phi_z^2 \quad \delta V_3 = +g_z(x_0S_x)\delta\phi_z^2 \\
\delta V_4 &= +g_z(y_0S_y)\delta\phi_z^2 \quad \delta V_5 = +g_z(y_0S_y)\delta\phi_z^2.
\end{aligned} \tag{3.55}$$

The linear terms dominate in the expressions for the change in induced voltages in the coils,

and excluding non-linear terms gives:

$$\delta V_1 = 0, \quad \delta V_2 = 0, \quad \delta V_3 = 0, \quad \delta V_4 = 0, \quad \delta V_5 = 0.$$

3.1.10 Absolute EMF in the Coils

Assuming that the positions of the coils are known relative to the origin of the system, the translations x , y and z along the x-, y- and z-axes as well as the rotations ϕ_x , ϕ_y and ϕ_z about the origin can be defined from the measured voltage changes in the five coils shown in Fig. 3.3. Let us consider the translational vector $\mathbf{R}' = (\delta \mathbf{x}, \delta \mathbf{y}, \delta \mathbf{z})$ and rotational matrices $\mathbf{R}_x$, $\mathbf{R}_y$ and $\mathbf{R}_z$ which characterise the movement of the rig.

$$\mathbf{R}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi_x & \sin \phi_x \\ 0 & -\sin \phi_x & \cos \phi_x \end{pmatrix}, \mathbf{R}_y = \begin{pmatrix} \cos \phi_y & 0 & -\sin \phi_y \\ 0 & 1 & 0 \\ \sin \phi_y & 0 & \cos \phi_y \end{pmatrix}, \mathbf{R}_z = \begin{pmatrix} \cos \phi_z & \sin \phi_z & 0 \\ -\sin \phi_z & \cos \phi_z & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Consider $\cos \phi_x = C_x$, $\sin \phi_x = S_x$, $\cos \phi_y = C_y$, $\sin \phi_y = S_y$, $\cos \phi_z = C_z$ and $\sin \phi_z = S_z$. The rotational matrices are reduced to

$$\mathbf{R}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_x & S_x \\ 0 & -S_x & C_x \end{pmatrix}, \mathbf{R}_y = \begin{pmatrix} C_y & 0 & -S_y \\ 0 & 1 & 0 \\ S_y & 0 & C_y \end{pmatrix}, \mathbf{R}_z = \begin{pmatrix} C_z & S_z & 0 \\ -S_z & C_z & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The structure of the five coils rig and the relative position of the coils with respect to the origin are shown in Fig. 3.3. After movements (rotation/translation or both) of the rig, the

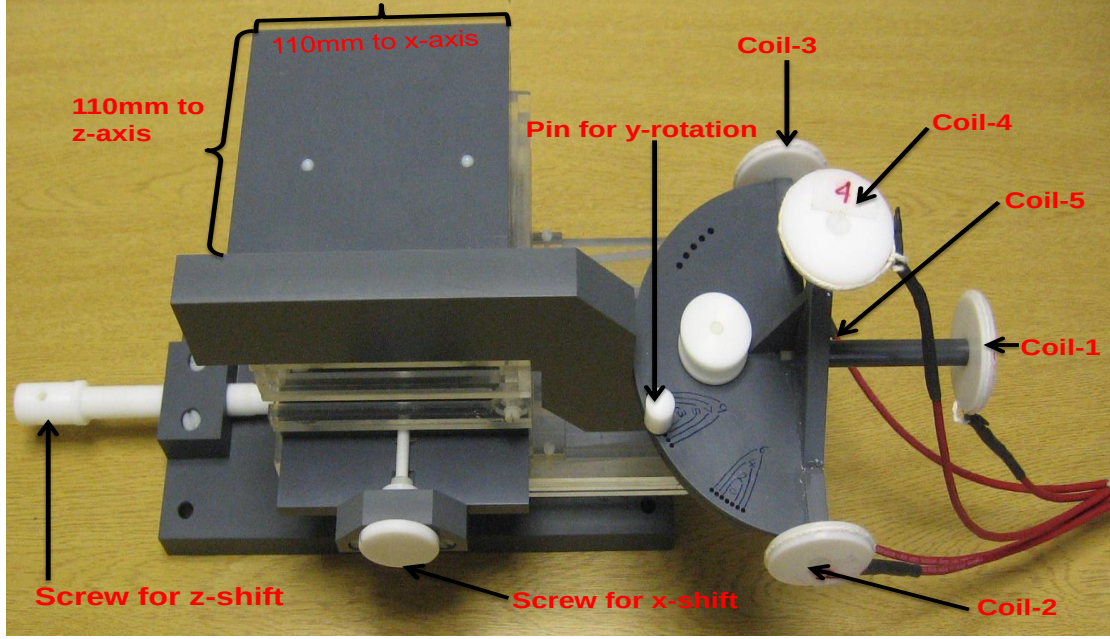


Figure 3.3: Structure of the five coils rig.

location and sensitivity vectors of coil C_1 become

$$\mathbf{r}_1 = \mathbf{R}_z \mathbf{R}_y \mathbf{R}_x \begin{pmatrix} 0 \\ 0 \\ L_1 \end{pmatrix} + \mathbf{R}', \quad \mathbf{S}_1 = \mathbf{R}_z \mathbf{R}_y \mathbf{R}_x \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad (3.56)$$

and the sensitivity vector can be rewritten as

$$\mathbf{S}_1 = \begin{pmatrix} S_x S_z - C_x C_z S_y \\ C_x S_y S_z + S_x C_z \\ C_x C_y \end{pmatrix} = \begin{pmatrix} S_{1x} \\ S_{1y} \\ S_{1z} \end{pmatrix}, \quad (3.57)$$

where L_1 represents the distance of coil 1 from the axis of rotation. The new position of the coil C_1 is given by a vector $\mathbf{r}_1$ whose elements are x_1 , y_1 and z_1 . On the other hand position

before rotation/translation (or both) of the coil is denoted by a vector $\mathbf{R}$ whose elements are x , y and z .

$$\mathbf{r}_1 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \mathbf{L}_1 \begin{pmatrix} S_x S_z - C_x C_z S_y \\ C_x S_y S_z + S_x C_z \\ C_x C_y \end{pmatrix} + \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \quad (3.58)$$

The voltage induced in the coil C_1 after movement is then denoted by the equation

$$-V_1 = S_{1x}(\dot{g}_x z_1 - \dot{g}_z \frac{x_1}{2}) + S_{1y}(\dot{g}_y z_1 - \dot{g}_z \frac{y_1}{2}) + S_{1z}(\dot{g}_x x_1 + \dot{g}_y y_1 + \dot{g}_z z_1). \quad (3.59)$$

Similarly for the coils C_2 and C_3 , the location and sensitivity vectors become

$$\mathbf{r}_{2,3} = \mathbf{R}_z \mathbf{R}_y \mathbf{R}_x \begin{pmatrix} L_{2,3} \\ 0 \\ 0 \end{pmatrix} + \mathbf{R}', \quad \mathbf{S}_{2,3} = \mathbf{R}_z \mathbf{R}_y \mathbf{R}_x \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad (3.60)$$

which can be rewritten as

$$\mathbf{r}_{2,3} = L_{2,3} \begin{pmatrix} C_z C_y \\ -S_z C_y \\ S_y \end{pmatrix} + \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{S}_{2,3} = \mathbf{R}_z \mathbf{R}_y \mathbf{R}_x \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} C_z C_y \\ -S_z C_y \\ S_y \end{pmatrix}, \quad (3.61)$$

where $L_2 = L_3$ describe the distances of the coils C_2 and C_3 from the axis of rotation.

The induced voltage in the coils C_2 and C_3 is then given by the following equations:

$$\begin{aligned} -V_2 &= S_{2x}(\dot{g}_x z_2 - \dot{g}_z \frac{x_2}{2}) + S_{2y}(\dot{g}_y z_2 - \dot{g}_z \frac{y_2}{2}) + S_{2z}(\dot{g}_x z_2 + \dot{g}_y y_2 + \dot{g}_z z_2) \\ -V_3 &= S_{3x}(\dot{g}_x z_3 - \dot{g}_z \frac{x_3}{2}) + S_{3y}(\dot{g}_y z_3 - \dot{g}_z \frac{y_3}{2}) + S_{3z}(\dot{g}_x z_3 + \dot{g}_y y_3 + \dot{g}_z z_3), \end{aligned} \quad (3.62)$$

where x_2 , y_2 and z_2 indicate the position of coil C_2 after a movement, whereas x_3 , y_3 and z_3 indicate the position of coil C_3 after a movement.

The location and sensitivity vectors of the coils C_4 and C_5 can be written as

$$\mathbf{r}_{4,5} = L_{4,5} \begin{pmatrix} C_x S_z + S_x S_y C_z \\ -S_x S_y S_z + C_x C_z \\ -S_x C_y \end{pmatrix} + \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{S}_{4,5} = \mathbf{R}_z \mathbf{R}_y \mathbf{R}_x \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} C_x S_z - S_x S_y C_z \\ S_x S_y S_z + C_x C_z \\ -S_x C_y \end{pmatrix}, \quad (3.63)$$

where $L_4 = L_5$ is the distance of the coils C_4 and C_5 from the axis of rotation.

The induced voltage in the coils C_4 and C_5 is given by the following equations-

$$\begin{aligned} -V_4 &= S_{4x}(\dot{g}_x z_4 - \dot{g}_z \frac{x_4}{2}) + S_{4y}(\dot{g}_y z_4 - \dot{g}_z \frac{y_4}{2}) + S_{4z}(\dot{g}_x z_4 + \dot{g}_y y_4 + \dot{g}_z z_4) \\ -V_5 &= S_{5x}(\dot{g}_x z_5 - \dot{g}_z \frac{x_5}{2}) + S_{5y}(\dot{g}_y z_5 - \dot{g}_z \frac{y_5}{2}) + S_{5z}(\dot{g}_x z_5 + \dot{g}_y y_5 + \dot{g}_z z_5), \end{aligned} \quad (3.64)$$

where x_4 , y_4 and z_4 indicate the position of coil C_4 after a movement, whereas x_5 , y_5 and z_5 indicate the position of coil C_5 after a movement.

Whatever the position and orientation of the coils is, one can calculate the voltage induced in the coils by using these non-linear equations. These equations can be solved using the ‘fsolve’ standard MatlabTM function.

3.2 Arbitrary Coils in Space

This section is devoted to explaining the dipole concept of a coil and its significance. In this study, a coil is considered as a magnetic dipole having a location vector $\mathbf{r}$ and a sensitivity vector $\mathbf{S}$ as shown in Fig. 3.4. A vector drawn from a reference point (origin) to a specific point in space where the centre of the coil is located, is known as a location vector. The magnitude and direction of the vector indicates the position of a coil placed in a space. On

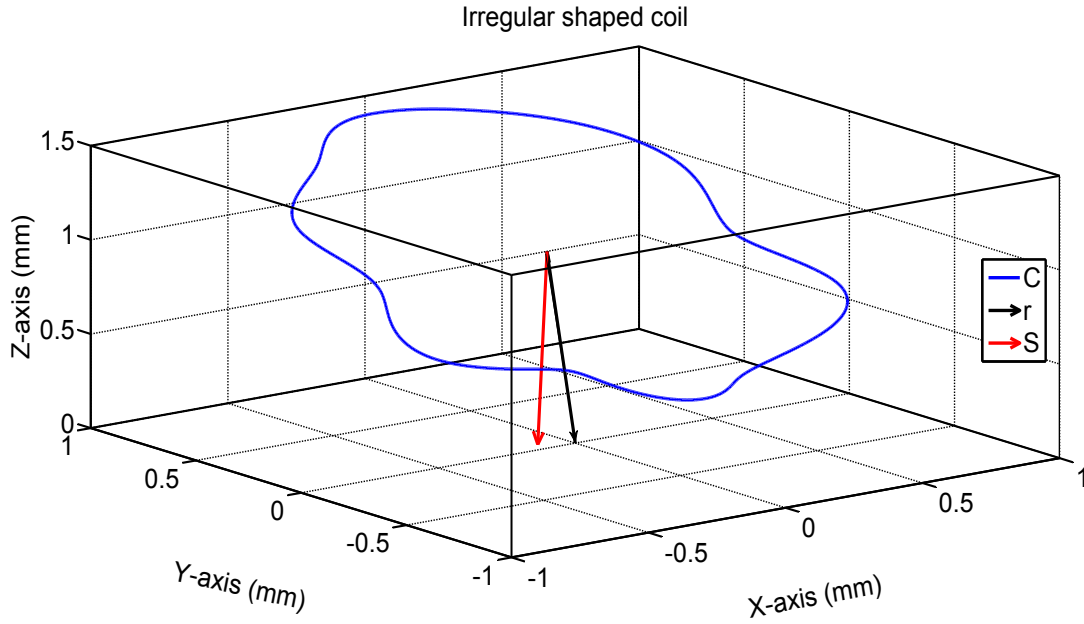


Figure 3.4: This Figure represents an irregularly shaped coil (blue) and its location vector (black) and sensitivity vectors (red).

the other hand, the direction of a sensitivity vector is a unit vector normal to the surface of the coil and the magnitude of the vector represents the effective area of the coil.

The size and the geometrical shape of a coil play an important role in the dipole concept. As the size of a coil decreases, it tends to a perfect dipole, whereas as the size increases it may no longer resemble a perfect dipole. On the contrary, a coil having a regular (circular) shape behaves as a perfect dipole. The dipole concept of a coil is useful to calculate the induced emf in a coil in space having irregular geometrical shape.

The pattern of the changes of location and sensitivity vectors depends on the geometrical shape as well as the types of movements of a coil. According to the dipole concept, the location vector changes its direction/magnitude or both, but the sensitivity vector may changes only its direction.

Let us consider a general coil to investigate the practicability of the dipole concept. At the very beginning, an equation has been derived to calculate the induced voltage in the coil

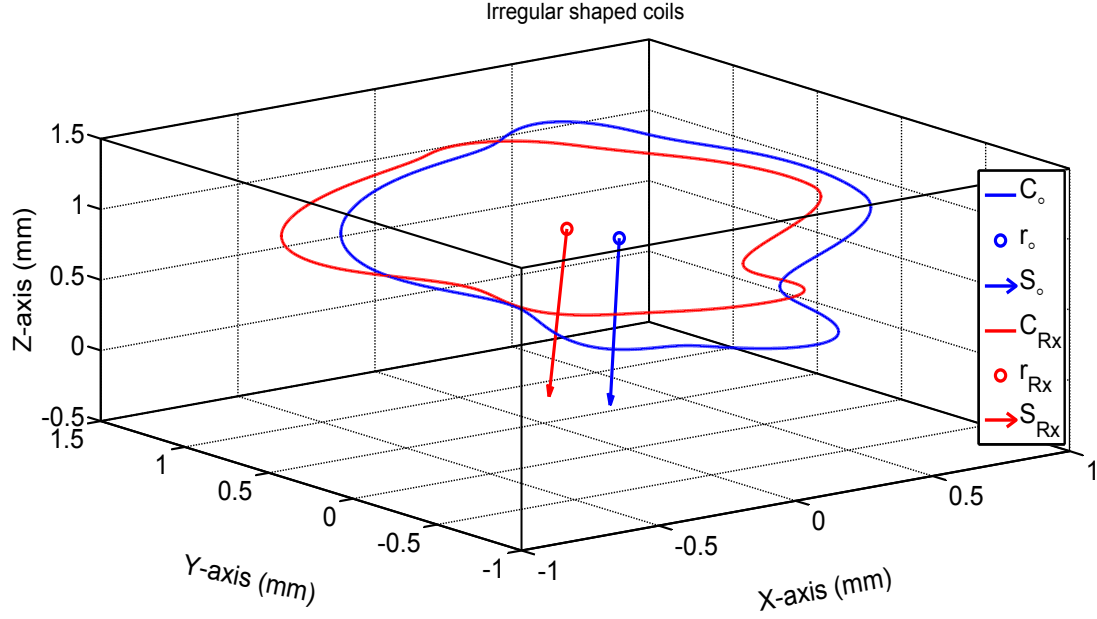


Figure 3.5: Drawing representing an irregularly shaped coil (blue) and the corresponding vectors (blue) before rotation and the coil (red) and the corresponding vectors (red) after rotation about the x-axis. The circles ($\circ$) represent the location vectors and the arrows ($\rightarrow$) indicate the sensitivity vectors of the coils.

due to translation and rotation. We consider the irregular shaped coil shown in Fig. 3.5, which is generated and then rotated about the x-axis. The location vectors are denoted by circles ($\circ$), whereas the sensitivity vectors are denoted by arrows ($\rightarrow$).

MatLab programmes are written to calculate the change in induced voltage in the coils by time varying gradient magnetic fields due to translations and rotations and also to evaluate the changes of the location and sensitivity vectors during the movements of the coils. In addition, the programs were also used to evaluate the changes of the vectors due to the same movement. It has observed that the location vector $\mathbf{r}_0$ always transforms correctly during translation and rotation irrespective of the size and shape of a coil. In contrast, the sensitivity vector $\mathbf{S}$ always transforms correctly during translation, but does not necessarily transform correctly during rotation when the shape of the coil is highly irregular.

However, first consider the effect of a time-varying uniform field $\dot{\mathbf{B}} = \dot{B}_x \hat{i} + \dot{B}_y \hat{j} + \dot{B}_z \hat{k}$,

so one can write the change in voltage as

$$-V = \int \frac{d\mathbf{B}}{dt} \cdot d\mathbf{S} = \dot{B}_x \int \hat{i} \cdot d\mathbf{S} + \dot{B}_y \int \hat{j} \cdot d\mathbf{S} + \dot{B}_z \int \hat{k} \cdot d\mathbf{S} = \dot{B}_x S_x + \dot{B}_y S_y + \dot{B}_z S_z = \dot{\mathbf{B}} \cdot \mathbf{S}, \quad (3.65)$$

where $S_x = \int \hat{i} \cdot d\mathbf{S}$, $S_y = \int \hat{j} \cdot d\mathbf{S}$, $S_z = \int \hat{k} \cdot d\mathbf{S}$. Since $\mathbf{B} = \nabla \times \mathbf{A} \neq \mathbf{0}$, where $\mathbf{A}$ is a suitable vector potential which gives us uniform field along the three mutually perpendicular directions, one can use $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = \int \mathbf{A} \cdot d\mathbf{l}$, which yields;

$$-V = \dot{\mathbf{B}} \cdot \mathbf{S} = \int \left((z\hat{i} + x\hat{k}) + (z\hat{j} + y\hat{k}) + \left(-\frac{x}{2}\hat{i} - \frac{y}{2}\hat{j} + z\hat{k}\right) \right) \cdot d\mathbf{l} \quad (3.66)$$

Now considering the effect of time-varying magnetic field gradients we can rewrite

$$\frac{d\mathbf{B}}{dt} = g_x(z\hat{i} + x\hat{k}) + g_y(z\hat{j} + y\hat{k}) + g_z\left(-\frac{x}{2}\hat{i} - \frac{y}{2}\hat{j} + z\hat{k}\right). \quad (3.67)$$

Let us consider the flux linked by a generally shaped coil and relate this to the flux experienced by an equivalent dipole at position $\mathbf{r}_0 = x_0\hat{i} + y_0\hat{j} + z_0\hat{k}$; this gives

$$\begin{aligned} F_x &= \int (z\hat{i} + x\hat{k}) \cdot d\mathbf{S} = z_0 S_x + x_0 S_z \\ F_y &= \int (z\hat{j} + y\hat{k}) \cdot d\mathbf{S} = z_0 S_y + y_0 S_z \\ F_z &= \int \left(-\frac{x}{2}\hat{i} - \frac{y}{2}\hat{j} + z\hat{k}\right) \cdot d\mathbf{S} = -\frac{x_0}{2} S_x - \frac{y_0}{2} S_y + z_0 S_z. \end{aligned} \quad (3.68)$$

Applying Stokes theorem and using the known form of the vector potentials [115] gives

$$\begin{aligned} F_x &= z_0 S_x + x_0 S_z = \int \left(-\frac{xy}{2}\hat{i} + \frac{x^2 - y^2}{4}\hat{j} + yz\hat{k}\right) \cdot d\mathbf{l} \\ F_y &= z_0 S_y + y_0 S_z = \int \left(\frac{x^2 + y^2}{4}\hat{i} - \frac{xy}{2}\hat{j} - xz\hat{k}\right) \cdot d\mathbf{l} \\ F_z &= -\frac{x_0}{2} S_x - \frac{y_0}{2} S_y + z_0 S_z = \int \left(-\frac{x}{2}yz\hat{i} + \frac{y}{2}xz\hat{j}\right) \cdot d\mathbf{l}. \end{aligned} \quad (3.69)$$

By using all these equations one can form a system of linear equations in matrix form

$$M \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} S_z & 0 & S_x \\ 0 & S_z & S_y \\ -\frac{S_x}{2} & -\frac{S_y}{2} & S_z \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} \quad (3.70)$$

By comparing the above matrix as $\mathbf{M}\mathbf{r}_0 = \mathbf{F}$ and taking the inverse of the matrix $\mathbf{M}$ one can find $\mathbf{r}_0$

$$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = M^{-1} \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix}. \quad (3.71)$$

3.2.1 Geometrical Shape and the Dipole Concept of a Coil

To explain the dipole concept of a coil, three types of coils and their location and sensitivity vectors are presented in Fig. 3.6. These are: regular shaped i.e. planar (circular), irregular and highly irregular shaped coils. Again, the location and the sensitivity vectors are denoted by a circle ($\circ$) and an arrow ($\rightarrow$) respectively.

At first, a movement is applied to the coil and the position and orientation of the coil is investigated. The magnitude and the direction of the location and the sensitivity vectors of the coils are also explored. It is observed that the vectors change position and orientation simultaneously with the movement of the coils. After that, the same amount of movement is applied individually to the original location and the sensitivity vectors. Then the magnitude and direction of these two vectors are investigated. As a result, three sets of location and sensitivity vectors are obtained, one set from the coil before movement, one set from the

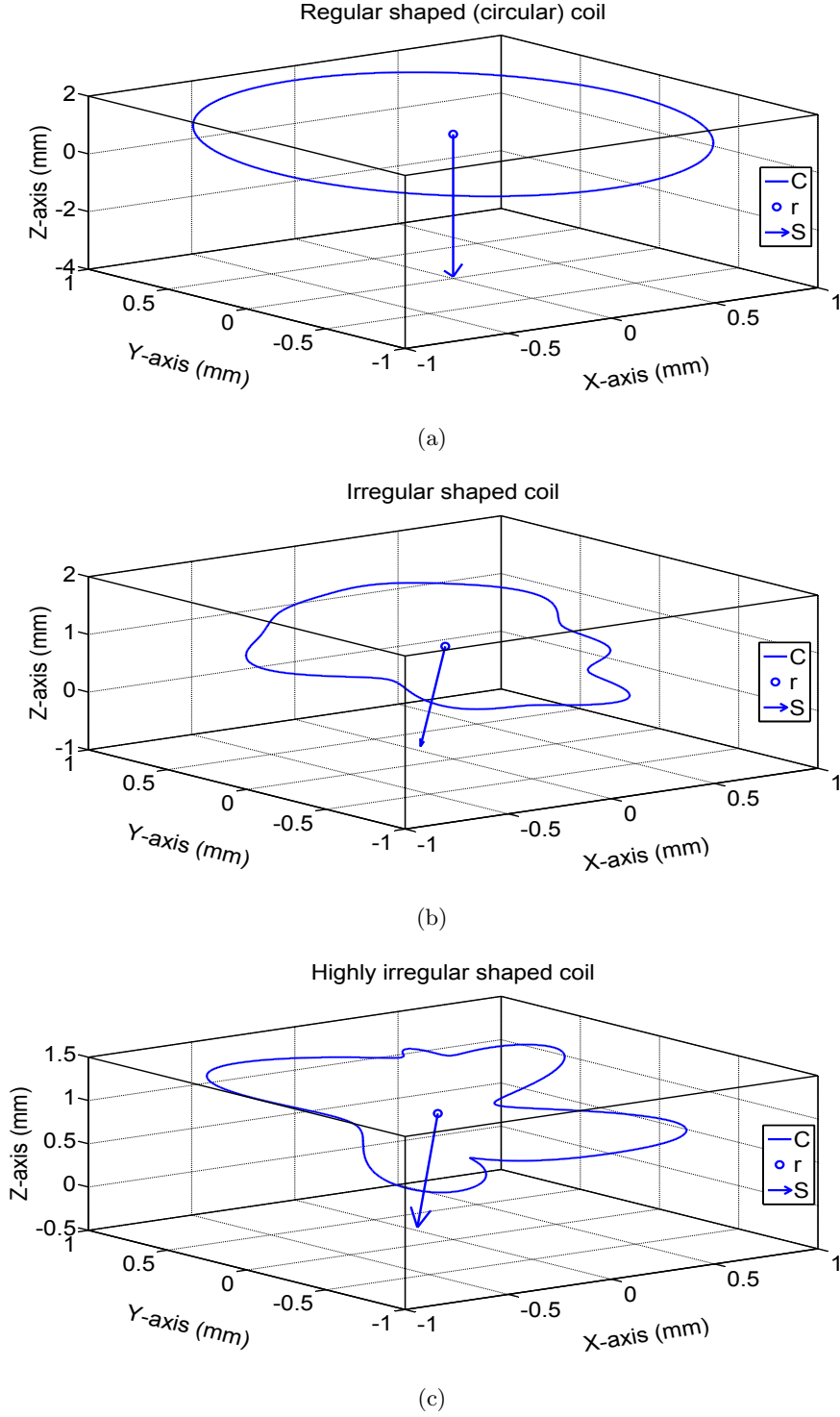


Figure 3.6: Drawing representing three types of coils and their location vectors denoted by a circle ($\circ$) and sensitivity vectors denoted by ($\rightarrow$). (a) Regular geometrical shaped (circular) coil. (b) Irregular geometrical shaped coil. (c) Highly irregular geometrical shaped coil.

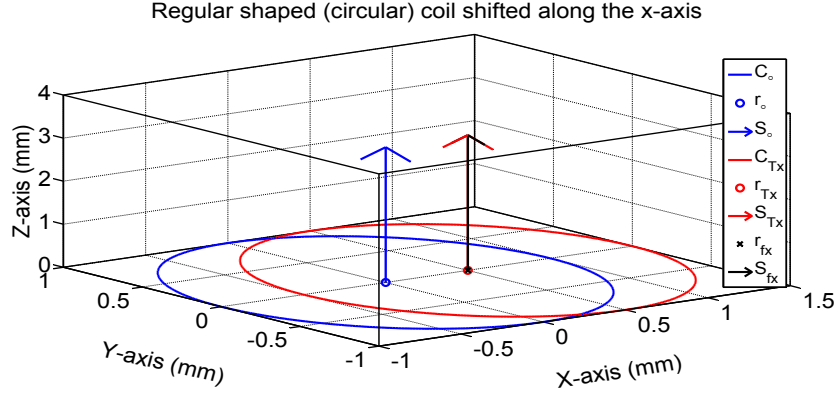
coils after movement and another from the individual movement of the vectors. Only the coincidence of the latter two sets of the vectors is the direct proof of the validity of the dipole concept.

In this subsection, in all figures a coil is denoted by C_0 , the location and the sensitivity vectors are denoted by $\mathbf{r}_0$ and $\mathbf{S}_0$ respectively. After a translation along the x-, y- and z-axes the coils are denoted by C_{Tx} , C_{Ty} and C_{Tz} , whereas r_{Tx} , r_{Ty} and r_{Tz} represent the location vectors of the corresponding coils, on the other hand S_{Tx} , S_{Ty} and S_{Tz} stand for the sensitivity vectors of the coils respectively. In addition, S_{fx} , S_{fy} and S_{fz} represent the sensitivity vectors after an individual translation along the x-, y- and z-axes. On the other hand, r_{fx} , r_{fy} and r_{fz} indicate the location vectors after an individual translation along the x-, y- and z-axes. In this case the location vector is denoted by a black cross ($\times$) and the sensitivity vector denoted by a black arrow ($\rightarrow$).

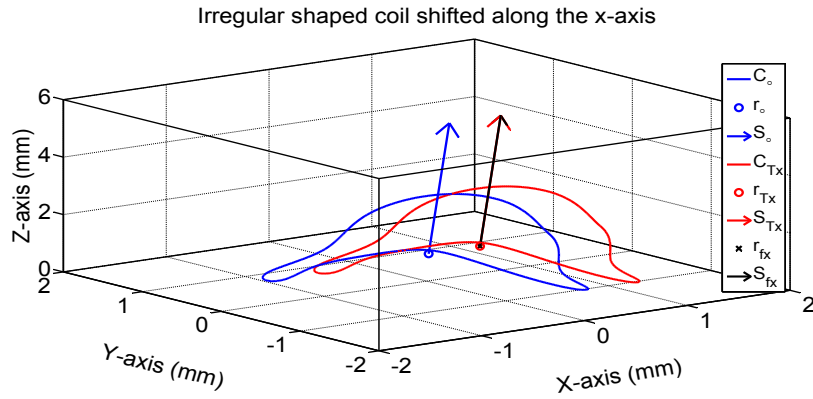
Figs. 3.7(a), 3.7(b) and 3.7(c) show regular (circular), irregular shaped and highly irregular shaped coils respectively. In this case highly irregular means coils that are not confined to a single plane. The coils and their location and sensitivity vectors are shifted 10 mm (inside a head coil a patient hardly can move head more than 10 mm) along the x-axis. It is worth noting that after a 10 mm translation the two sets of the location vectors (red circle and black cross) and the sensitivity vectors (red arrow and black arrow) coincide which is a direct proof of the validity of the dipole concept.

Again, Figs. 3.8(a), 3.8(b) and 3.8(c) show regular shaped (circular), irregular shaped and highly irregular shaped coils shifted 10 mm along the y-axis. The overlapping of the location vectors and the sensitivity vectors indicate again that the dipole concept is valid for the y-translation.

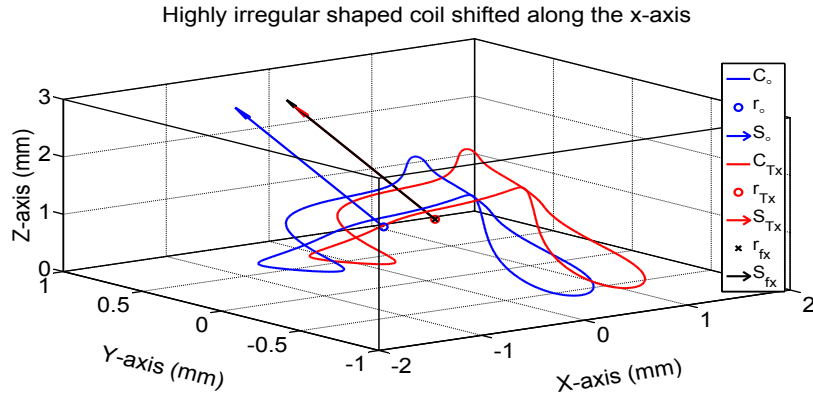
Similarly, Figs. 3.9(a), 3.9(b) and 3.9(c) show regular shaped (circular), irregular shaped and highly irregular shaped coils shifted along the z-axis. In these case the two sets of the location vectors and the sensitivity vectors also coincide which satisfies the dipole concept during z-translation.



(a)



(b)



(c)

Figure 3.7: Data shows the geometrical shape, the location vector denoted by a circle ($\circ$) and a cross ($\times$) and the sensitivity vector denoted by an arrow ($\rightarrow$) of a coil before (blue) and after (red) translation along the x-axis. The black cross ($\times$) and the black arrow ($\rightarrow$) stand for the location and the sensitivity vectors respectively, when they are shifted individually (not along with the coils). (a) Regular shaped (circular) coil. (b) Irregular shaped coil. (c) Highly irregular shaped coil.

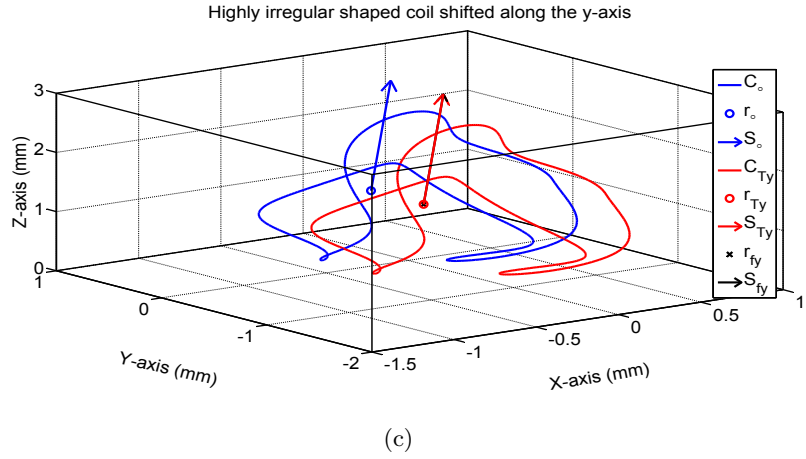
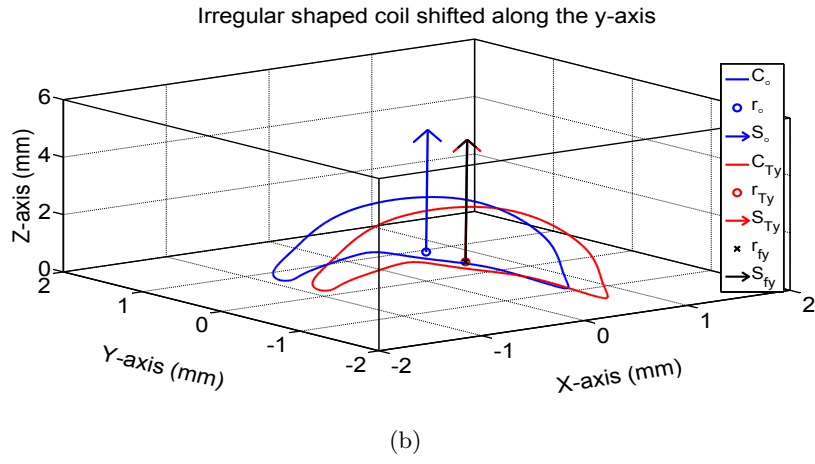
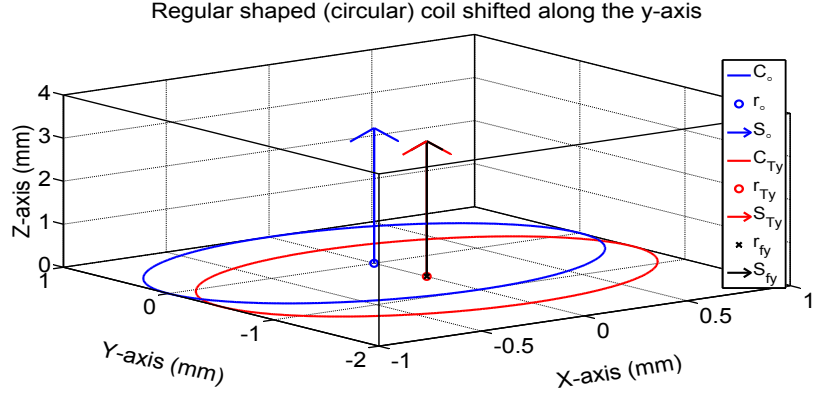
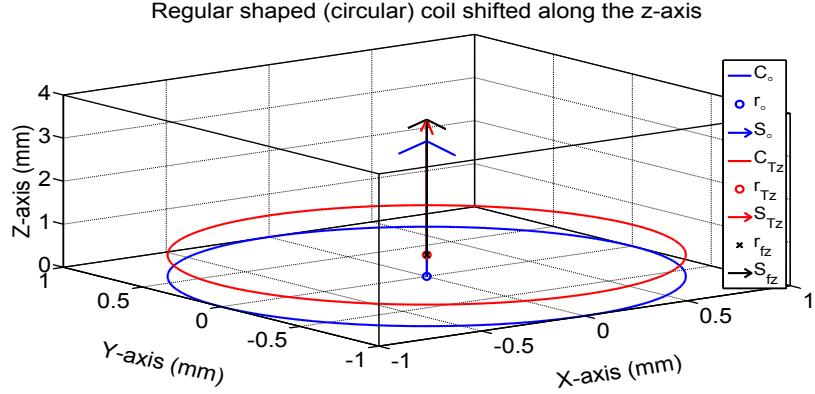
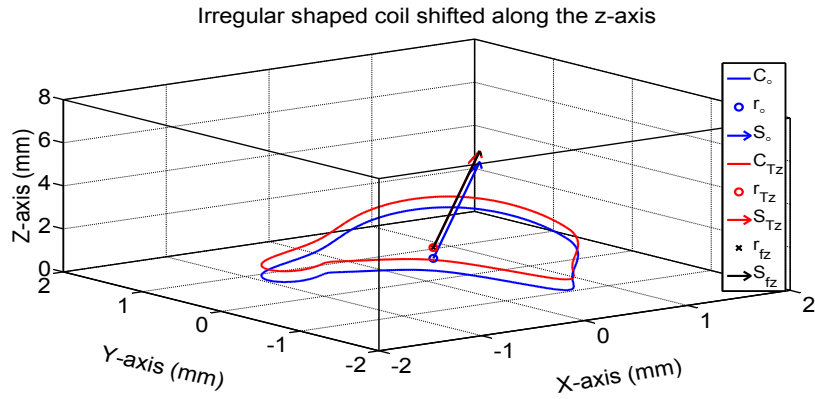


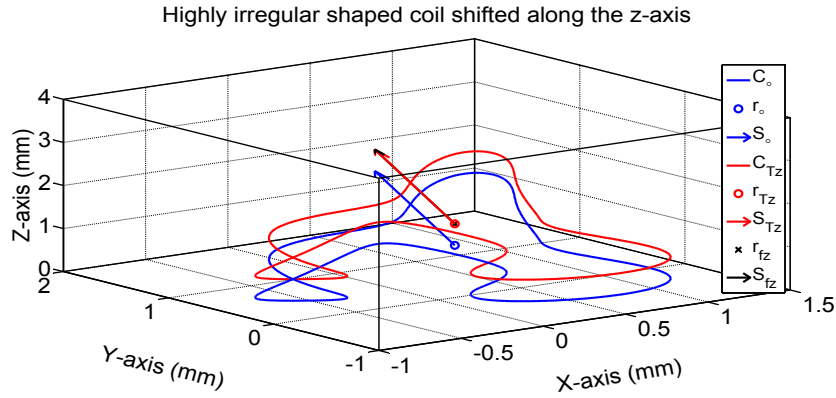
Figure 3.8: Data shows the geometrical shape, the location vector denoted by a circle ($\circ$) and a cross ($\times$) and the sensitivity vector denoted by an arrow ($\rightarrow$) of a coil before (blue) and after (red) translation along the y-axis. The black cross ($\times$) and the black arrow ($\rightarrow$) stand for the location and the sensitivity vectors respectively, when they are shifted individually (not along with the coils). (a) Regular shaped (circular) coil. (b) Irregular shaped coil. (c) Highly irregular shaped coil.



(a)



(b)



(c)

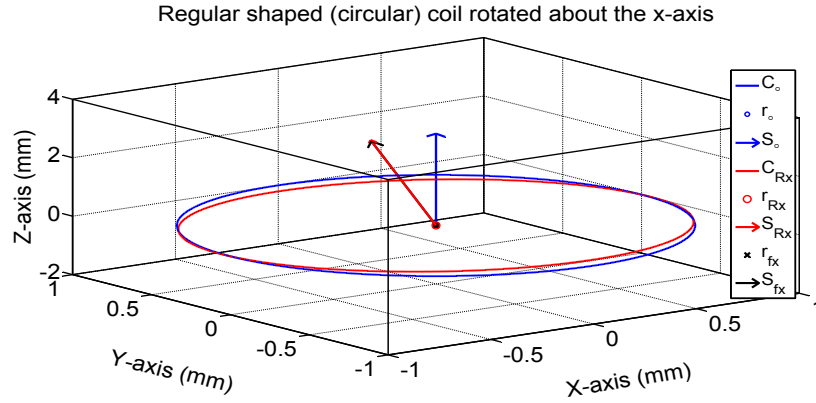
Figure 3.9: Data shows the geometrical shape, the location vector denoted by a circle ($\circ$) and a cross ($\times$) and the sensitivity vector denoted by an arrow ($\rightarrow$) of a coil before (blue) and after (red) translation along the z-axis. The black cross ($\times$) and the black arrow ($\rightarrow$) stand for the location and the sensitivity vectors respectively, when they are shifted individually (not along with the coils). (a) Regular shaped (circular) coil. (b) Irregular shaped coil. (c) Highly irregular shaped coil.

Fig. 3.10(a) shows a regular (circular) coil, whereas Fig. 3.10(b) and Fig. 3.10(c) show irregular and highly irregular shaped coils rotation about the x-axis respectively. After a 10° rotation the coils are denoted by C_{Rx} , sensitivity vector S_{Rx} and location vector r_{Rx} which are shown in Fig. 3.10. The location and the sensitivity vectors are denoted by a circle ($\circ$) and by an arrow ($\rightarrow$) before and after rotation about the x-axis. The location vectors are denoted by a black cross ($\times$) and the sensitivity vector denoted by a black ($\rightarrow$) when they move individually.

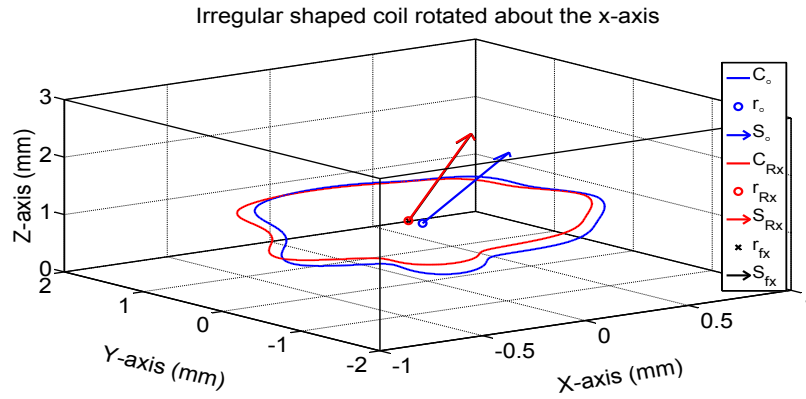
Fig. 3.11(a) shows a regular (circular) coil, whereas Fig. 3.11(b) shows irregular and Fig. 3.11(c) show and irregular highly irregular shaped coils rotating about the y-axis. After a 10° rotation about the y-axis the coil C_{Ry} , sensitivity vector S_{Ry} and location vector r_{Ry} are shown in Fig. 3.11. Again, the black cross ($\times$) and the black arrow ($\rightarrow$) indicates the vectors individual rotation about the y-axis.

Fig. 3.12(a) shows regular (circular) shaped coils and Fig. 3.12(b) shows irregular shaped coils, whereas Fig. 3.12(c) shows highly irregular shaped coils rotating about the z-axis. After a 10° rotation about the z-axis coil C_{Rz} , sensitivity vector S_{Rz} and location vector r_{Rz} are shown in Fig. 3.12. After an individual rotation about the z-axis the location and the sensitivity vectors are denoted by a black cross ($\times$) and by a black arrow ($\rightarrow$) respectively as shown in Fig. 3.12.

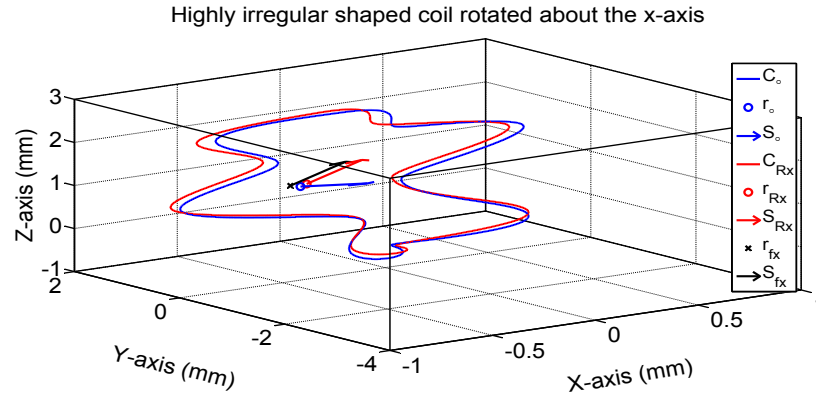
Whatever the shape of the coils the dipole concept is shown to be valid for all translations. The dipole concept is also true for a regular (planar) and an irregular shaped coils undergoing a rotation about the x-, y- and z-axes. But for a highly irregular shaped coil the dipole concept is no more valid when considering a rotation about the x-, y- and z-axes. Data presented in Figs. 3.10(c), 3.11(c) and 3.12(c) clearly shows that there is no coincidence between the two sets of location and sensitivity vectors. For a highly irregular shaped coil the rotation of the location and the sensitivity vectors and their individual rotation are producing different outcomes. In the case of a rotation about the x-, y- and z-axes the movements of the location and the sensitivity vectors break the dipole concept when the



(a)

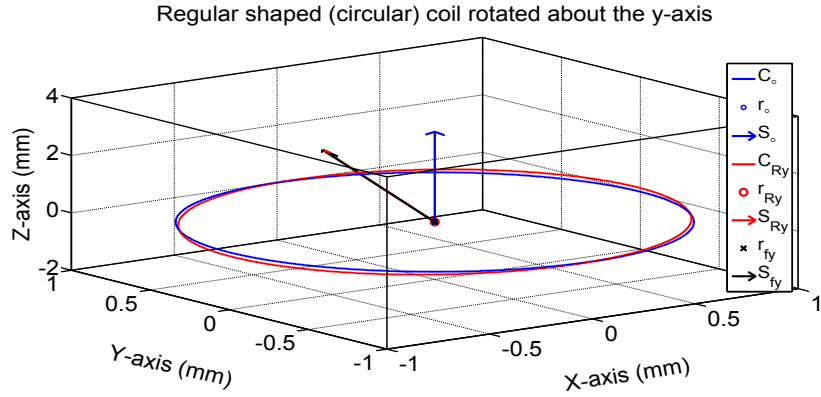


(b)

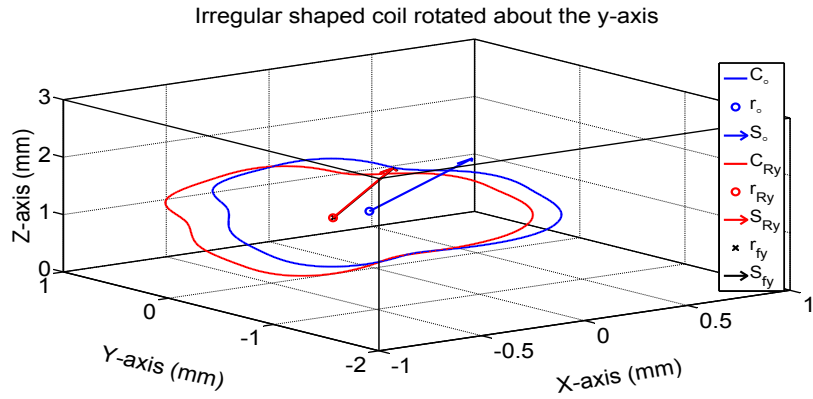


(c)

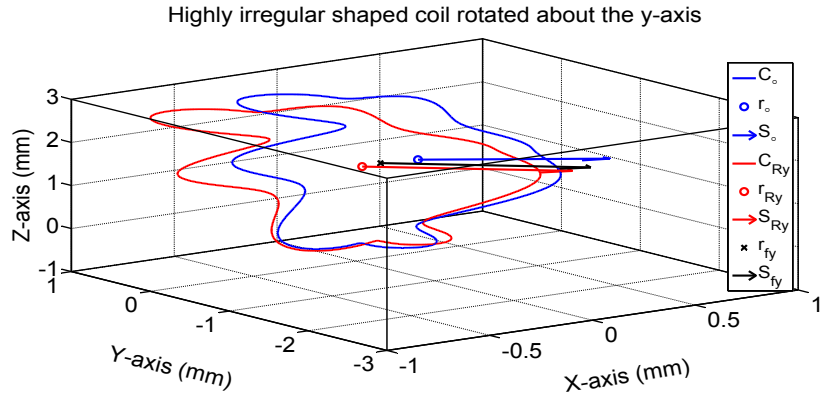
Figure 3.10: Data shows the geometrical shape, the location vector denoted by a circle ($\circ$) and a cross ($\times$) and the sensitivity vector denoted by an arrow ($\rightarrow$) of a coil before (blue) and after (red) rotation about the x-axis. The black cross ($\times$) and the black arrow ($\rightarrow$) stand for the location and the sensitivity vectors respectively, when they are rotated individually (not along with the coils). (a) Regular shaped (circular) coil. (b) Irregular shaped coil. (c) Highly irregular shaped coil.



(a)



(b)



(c)

Figure 3.11: Data shows the geometrical shape, the location vector denoted by a circle ($\circ$) and a cross ($\times$) and the sensitivity vector denoted by an arrow ($\rightarrow$) of a coil before (blue) and after (red) rotation about the y-axis. The black cross ($\times$) and the black arrow ($\rightarrow$) stand for the location and the sensitivity vectors respectively, when they are rotated individually (not along with the coils). (a) Regular shaped (circular) coil. (b) Irregular shaped coil. (c) Highly irregular shaped coil.

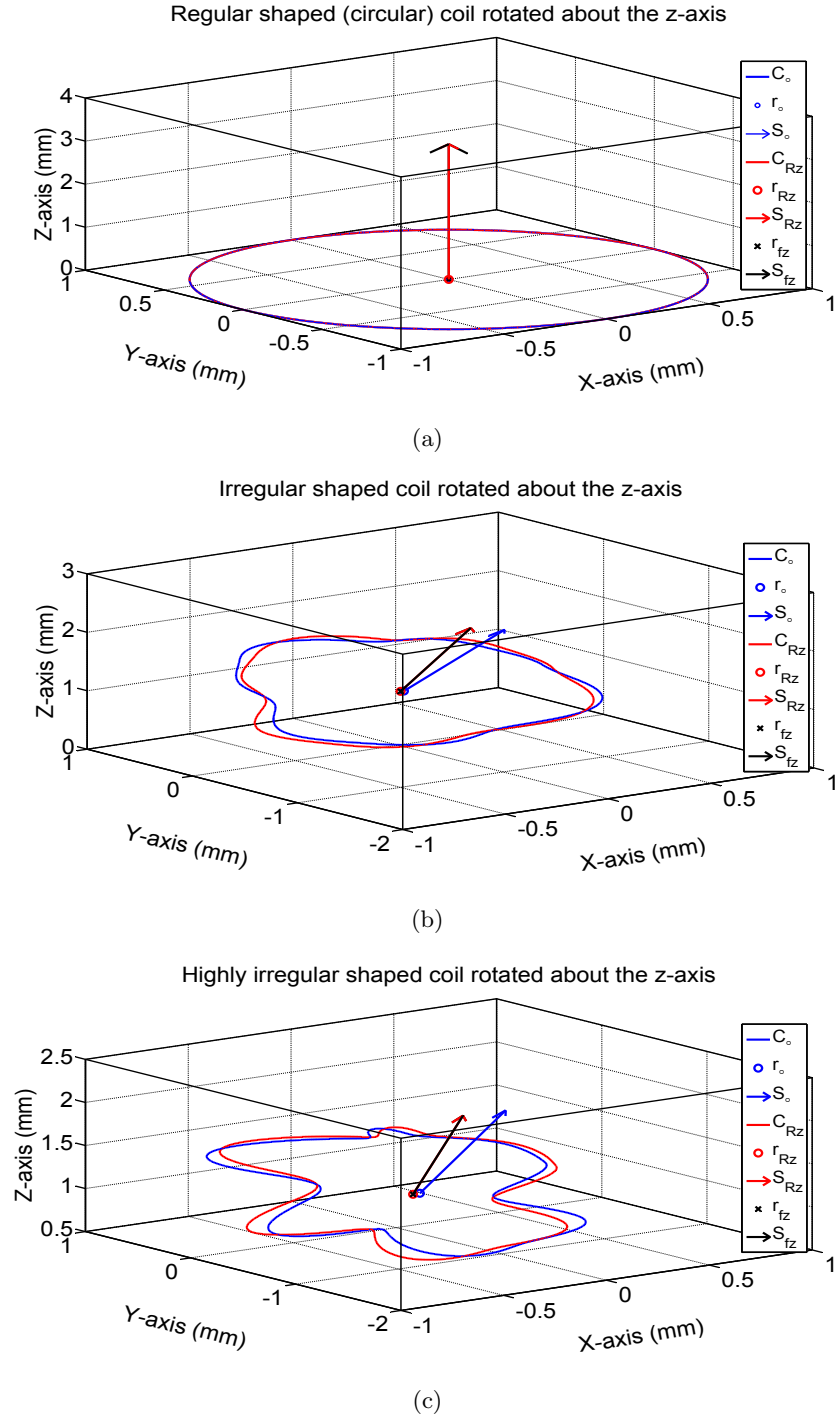


Figure 3.12: Data shows the geometrical shape, the location vector denoted by a circle ($\circ$) and a cross ($\times$) and the sensitivity vector denoted by an arrow ($\rightarrow$) of a coil before (blue) and after (red) rotation about the z-axis. The black cross ($\times$) and the black arrow ($\rightarrow$) stand for the location and the sensitivity vectors respectively, when they are rotated individually (not along with the coils). (a) Regular shaped (circular) coil. (b) Irregular shaped coil. (c) Highly irregular shaped coil.

coils have a highly irregular geometrical shape. This is a drawback of the dipole concept. This case is highly unlikely because nobody will be interested to make a highly irregular coil to perform experiment.

3.3 Discussion

This chapter describes a general expression for the change in voltage induced in a coil due to translational and rotational movement when the coil exposed to a time-varying magnetic-field gradient. An expression is also derived for the change in induced voltage in a coil for a five coil rig, where the coils are confined to the faces of a cube. Expressions for the voltage changes in the coils when the axes of the rig are displaced from the axes of the magnetic field gradient are derived. The simplified forms of equations for the special case of small movements i.e. for small changes in position (≤ 10 mm) and orientation ($\leq 10^\circ$) are derived. An equation is also derived to calculate the absolute induced voltage in a coil placed in a plane.

An expression is derived to calculate the induced voltage when an arbitrarily shaped coil is placed in space rather than in a plane. The dipole concept of a coil has been verified for a wide range of geometrical shapes of a coil. Numerical calculations have been done on the basis of analytical expressions.

One can conclude that the dipole concept is valid for a wide range of coil shapes and movements with a few limitations. This concept is in particular valid for a small circular coil placed in a plane. This is also valid for a non-planar irregular shaped coil. The more regular the geometrical shape of the coil is, the more accurate the dipole concept of a coil. This concept is not valid for rotation of a highly irregular coils (when the coils are not confined to a single plane).

3.4 Conclusion

This pilot project is devoted to a feasibility study of the head mounted coils to monitor head movement inside an MR scanner. It confirms the potential of the approach to estimate changes in position and orientation of the head inside a MR scanner by measuring the changing induced voltage in the coils array. This method and array of the coils may provide a real life solution to the long standing problem of head motion during MRI. The next step of this pilot project will use the coil array to estimate motion parameters. This technique will be combined with other MRI motion correction schemes as a method of estimating position and orientation during head motion. The prospective and retrospective motion correction could be both possible by using this technique.

Chapter 4

Simulations: Calculation of the Induced Voltage in a Coil

This chapter uses simulations to investigate the feasibility of tracking the position and orientation of the human head (considered as a rigid body) inside an MRI scanner by measuring the changes in the voltages induced in head-mounted coils by time varying magnetic field gradients as a result of head movements. Firstly, the voltage induced in a circular loop of wire that is exposed to time-varying x, y and z-magnetic field gradients, is calculated for varying positions and orientations of the coils. The sensitivity to rotations and translations of loops of varying sizes is also established. Similar steps are taken for a system of five coils confined to the faces of a cube and it is established whether the voltage changes due to gradient pulses applied along three directions can be used to calculate the change in position and orientation of the set of coils. Secondly, an arbitrary shaped coil (non-planar) is generated in space and the induced voltage is calculated for a range of coil positions. This induced voltage is then used to estimate the movement parameters.

4.1 Introduction

Our starting point is Faraday’s law of electromagnetic induction. The electromotive force (emf) induced in a coil by a change in its magnetic flux environment can be calculated using Faraday’s law of induction [112]

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int \mathbf{B} \cdot \hat{\mathbf{n}} da = -\frac{d}{dt} \left(\int_s \mathbf{B} \cdot d\mathbf{s} \right), \quad (4.1)$$

where $\mathbf{B}$ represents the magnetic field and $\mathbf{E}$ is the electric field. Defining the magnetic flux through a surface to be

$$\phi_B = \int \mathbf{B} \cdot d\mathbf{S}, \quad (4.2)$$

and considering the case where the surface is bounded by a wire loop, the emf or voltage induced across the ends of the wire is given by

$$V = \oint \mathbf{E} \cdot d\mathbf{l}. \quad (4.3)$$

When Eq. 4.2 and Eq. 4.3 are substituted into Eq. 4.1, a more familiar form of Faraday’s law is found:

$$V = -\frac{d\phi_B}{dt}. \quad (4.4)$$

Faraday’s law states that a time-varying magnetic flux through a loop of wire, or any circuit, generates a voltage whose amplitude is proportional to the rate-of change of the flux. The flux can be thought of as proportional to the number of field lines penetrating the effective area presented by the loops making up the coils.

The voltage induced in the circular coils and the change in the induced voltage in the coils is calculated due to positional changes. By characterising the positional changes through analysis of the data and comparison with the pattern of voltage changes across coils, it is possible to associate the pattern of voltage changes with particular movements.

4.2 Theory Underlying the Simulations

The simulations are based on using Faraday's law of electromagnetic induction, but it is troublesome to use Faraday's law directly to calculate the change in induced voltages. In the presence of a conductor it is not always straightforward to define the relevant area over which the rate of change of flux occurs. By using the following equation the induced voltage can be calculated directly from the rate-of-change of flux linked by the wire loop [113],

$$\begin{aligned} -V &= \frac{\partial}{\partial t} \left(\int_S \mathbf{B} \cdot d\mathbf{S} \right) = \oint \frac{\partial \mathbf{A}}{\partial t} \cdot d\mathbf{l} \\ &= \oint \left(\frac{\partial \mathbf{A}_x}{\partial t} + \frac{\partial \mathbf{A}_y}{\partial t} + \frac{\partial \mathbf{A}_z}{\partial t} \right) \cdot d\mathbf{l}, \end{aligned} \quad (4.5)$$

where, $\mathbf{A}$ is the vector potential, which is related to the magnetic field, $\mathbf{B}$, by $\mathbf{B} = \nabla \times \mathbf{A}$, and Stokes' theorem has been applied to convert the surface integral to a closed-line integral.

The following vector potentials [114–117] for the different gradients are used in these simulations.

Longitudinal (z) gradient:

$$\mathbf{A}_z = -\frac{1}{2}G_z yz\hat{i} + \frac{1}{2}G_z xz\hat{j}. \quad (4.6)$$

Transverse (x) gradient:

$$\mathbf{A}_x = -\frac{1}{2}G_x xy\hat{i} + \frac{1}{4}G_x(x^2 - y^2)\hat{j} + G_x yz\hat{k}. \quad (4.7)$$

Transverse (y) gradient:

$$\mathbf{A}_y = \frac{1}{4}G_y(x^2 - y^2)\hat{i} + \frac{1}{2}G_y xy\hat{j} - G_y xz\hat{k}, \quad (4.8)$$

where G is the amplitude of a given gradient and the rest of the symbols are conventional.

4.3 Method and Analysis

In this section two simulating methods are discussed, based on theory developed in Chapter 3. In the theory, equations are derived for the circular coils confined to planes and for arbitrary shaped coils in space. Method 1 and Method 2 focus on the planar coils and arbitrary coils (non-planar) in space respectively.

4.3.1 Method 1: Planar Coil

A circular coil in the specific plane is generated to calculate the change in induced voltage in the coils due to translations along the x-, y- and z-axes due to time varying x-, y- and z-gradients. A system of five coils confined to five of the surfaces of a cube is generated and the same calculations are done for the five coils. A linear model is used to estimate the change in position and orientation of the coils from the calculated induced voltage changes.

Simulation Parameters

A few parameters are used for simulations of the voltage induced in a loop. These are: the number of turns in each loop n ; the radius of the loop R , the amplitudes of the x-, y- and z-gradients G_x , G_y and G_z , the frequency (ν) at which the gradient changes in time, the number of points around the loop N . The translations along the x-, y- and z-axes are δx , δy and δz respectively, the rotations about the x-, y- and z-axes are $\delta\phi_x$, $\delta\phi_y$ and $\delta\phi_z$ respectively. In this study the number of turns are taken $n=10$, radius $R = 20$ mm; the gradient amplitudes $G_x = G_y = G_z = G = 0.636 \times 10^{-3}$ T/m (to get the measurable optimized induced voltage in a coil), angular frequency $\omega = (2\pi \times 1000)$ Hz, gradient strength $\frac{dG}{dt} = 4$ T/m-s, number points around the loop $N = 200$, maximum translation along the x-, y- and z-axes $\delta x = \delta y = \delta z = 10$ mm and maximum rotation about the x-, y- and z-axes $\delta\phi_x = \delta\phi_y = \delta\phi_z = 5^\circ$.

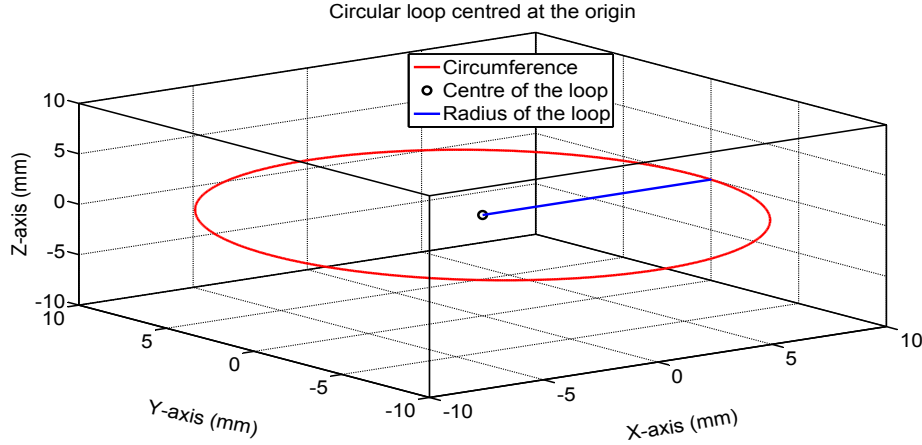


Figure 4.1: A circular loop in a plane

EMF in a Circular coil

A circular loop is formed from a small set of line elements at a given radius, as shown in Fig. 4.1, where O is the centre of the loop. One can obtain the change in induced voltage in the loop due to translation, rotation and change in size of the loop in a time-varying magnetic-field gradient. To describe the rigid body movement completely one has to handle six movement parameters as discussed in Chapter 1. Three of these parameters are for translation and three are for rotational motion, as shown in Fig. 1.3.

The voltage is calculated by numerical integration of the vector potential, $\mathbf{A}$, over a closed circular loop shown in Eq. 4.5. Let us consider a closed circular loop of circumference l and an infinitesimal length of the circumference to be $d\mathbf{l}$, where $d\mathbf{l} = \hat{i}dl_x + \hat{j}dl_y + \hat{k}dl_z$, as shown in Fig. 4.2. In Fig. 4.2, C is the centre of the loop, $\mathbf{r}$ is the location vector, $\mathbf{S}$ is the sensitivity vector, i.e. a vector normal to the surface of the loop and M is the mid-point of the infinitesimal length of the loop. For simulation purpose, N points are taken round the circumference that gives us N infinitesimal elements. The vector potential is calculated at the mid-point of each element. By summing up the contributions from all the elements the total induced voltage in the loop is calculated due to the application of the time-varying magnetic field gradients along the x-, y- and z-directions. The general equation for the

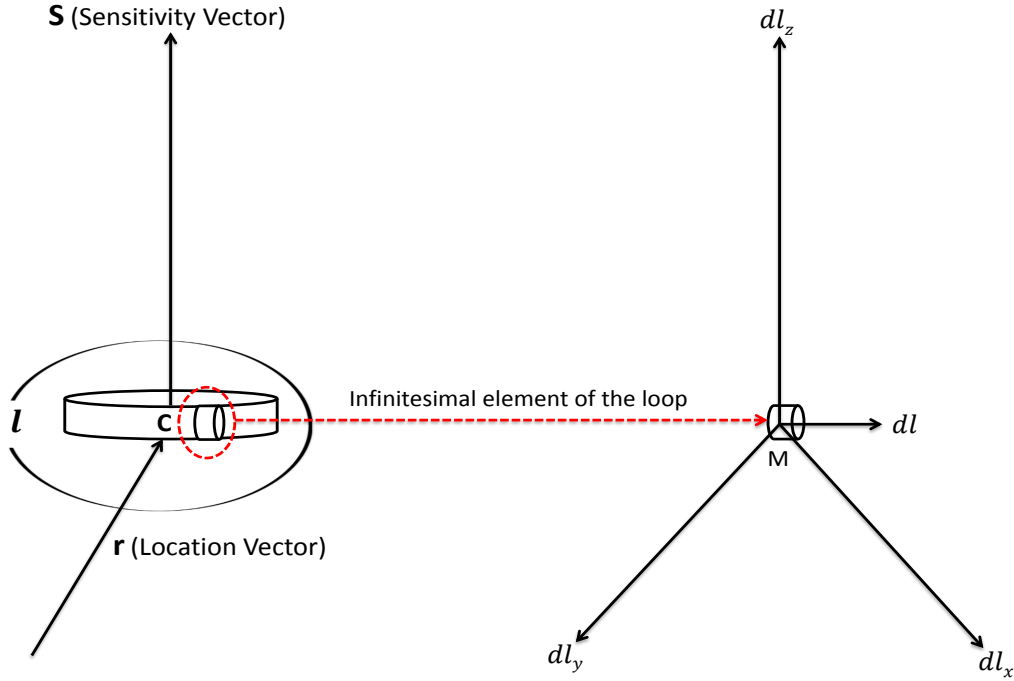


Figure 4.2: A closed circular loop of perimeter, l , and an infinitesimal element of length, dl .

induced voltage in a coil due to the applications of the magnetic field gradients is given as follows:

$$V = \sum V_{g_x} + V_{g_y} + V_{g_z}, \quad (4.9)$$

where V_{g_x} , V_{g_y} , and V_{g_z} are the contribution in the induced voltage in the coil due to x-, y- and z-gradients and subscripts g_x , g_y and g_z indicate the x, y and z-gradients respectively.

The Change in Induced EMF in a Circular Coil

A circular coil is generated in the xy -plane as shown in Fig. 4.3(a). The change in emfs induced in the coil due to a time-varying x-gradient is calculated for sequential translations along the x-axis. Fig. 4.3(b) shows the linear change in induced voltage in the coils. In Fig. 4.3, C_o stands for centre, L_o stands for the coil wire path before translation and R denotes the radius of the coil. On the other hand, C_{Tx} stands for centre, L_{Tx} stands for the coil wire path after 10 mm translation along the x-axis.

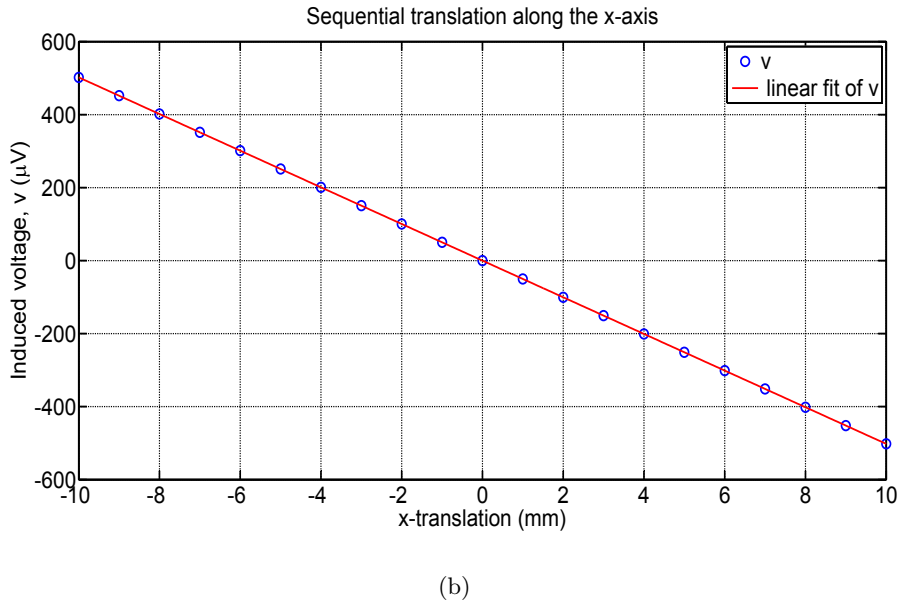
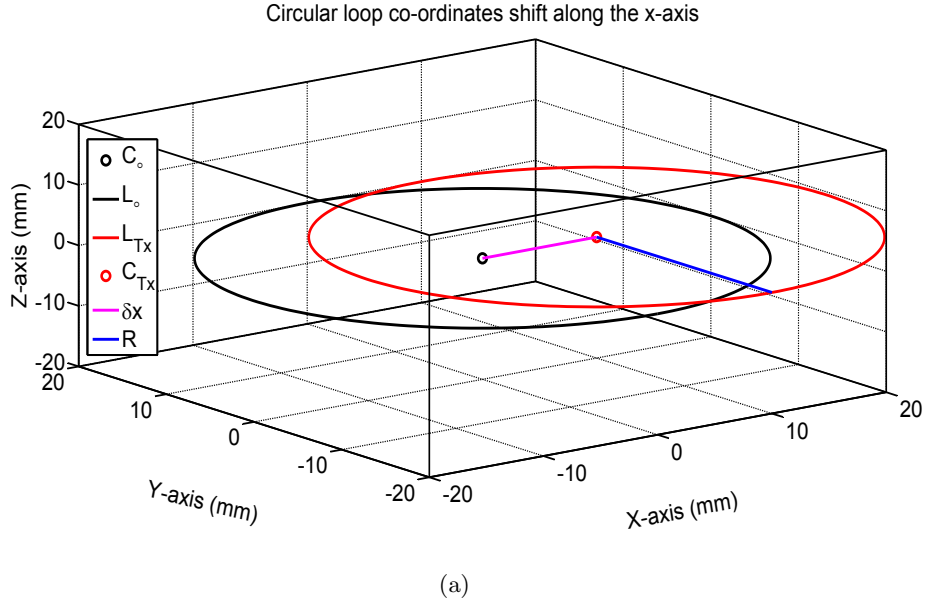


Figure 4.3: Translation along the x-axis. (a) A circular coil in the xy-plane having 10 turns is shifted 10 mm along the x-axis (b) Linear changes in induced voltage due to an x-translation during application of an x-gradient

Fig. 4.4(a) shows a circular coil in the yz -plane before and after it is shifted by 10 mm along the y -axis. The gradient-induced emf in the coil changes linearly with sequential shifts along the y -axis due to the application of a z -gradient as shown in Fig. 4.4(b). In Fig. 4.4, C_o stands for centre, L_o stands for the coil wire path before translation and r denote the radius of the coil. On the other hand, C_{Ty} stands for centre, L_{Ty} stands for the coil wire path after 10 mm translation along the x -axis

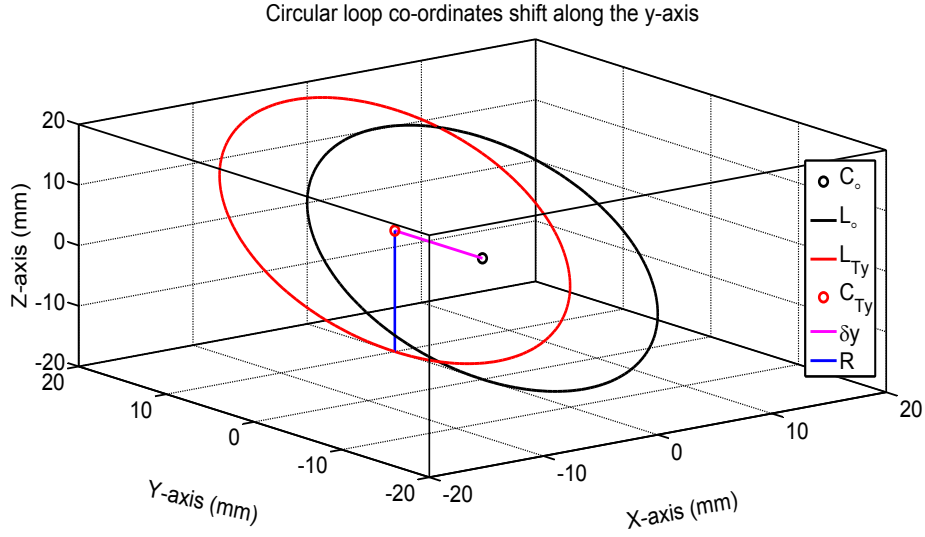
Similarly, Fig. 4.5(a) shows a circular coil in the zx -plane and Fig. 4.5(b) shows the gradient induced emf changes linearly with sequential shifts of a coil along the z -axis during the application of z -gradients. In Fig. 4.4 C_{Tz} stands for centre, L_{Ty} stands for the coil after wire path 10 mm translation along the z -axis .

These are the representative data. By analyzing all calculated data it is concluded that whatever the translation is, the voltage induced in a circular coil varies linearly with position during execution of x -, y - and z -gradients.

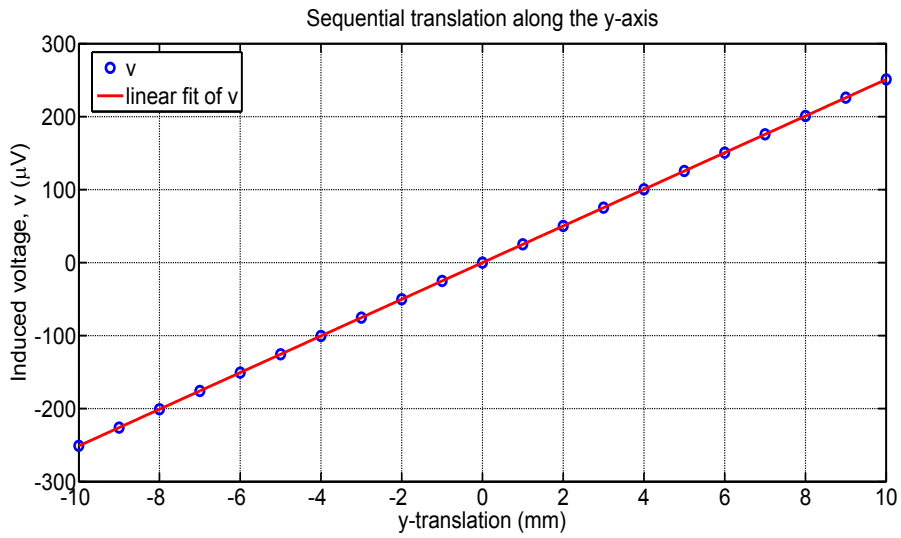
A circular coil in the xy -plane is generated and its rotation about the x -axis is shown in Fig. 4.6(a). The corresponding change in induced voltage in a coil is linear with respect to rotation about the x -axis during execution of a y -gradient for these small angles of rotation as shown in Fig. 4.6(b). In Fig. 4.6(a) C_{Rx} stands for centre, L_{Rx} stands for the coil wire path after 5° rotation about the x -axis.

Now, a circular coil is generated in the yz -plane and is rotated about the y -axis that is shown in Fig. 4.7(a). The corresponding change in induced voltage is shown in Fig. 4.7(b) during the execution of an x -gradient. The voltage changes again linearly with respect to rotation angles for these small angles of rotation. In Fig. 4.7(a) C_{Ry} stands for centre, L_{Ry} stands for the coil wire path after 5° rotation about the x -axis.

A circular coil in the zx -plane is considered and rotated by 5° about the z -axis as shown in Fig. 4.8(a). In this case no voltage change is observed during execution of x -, y - and z -gradients as shown in Fig. 4.8(b). In Fig. 4.8(a) C_{Rz} stands for centre, L_{Rz} stands for the coil wire path after 5° rotation about the z -axis.



(a)



(b)

Figure 4.4: Translation along the y-axis. (a) A circular coil in the yz-plane is shifted by 10 mm along the y-axis (b) Linear changes in induced voltage due to y-translation during execution of a z-gradient.

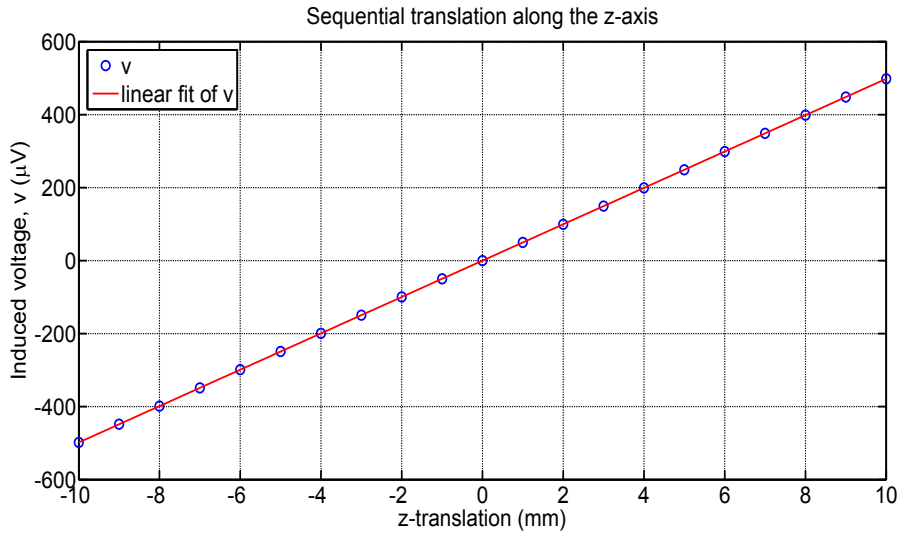
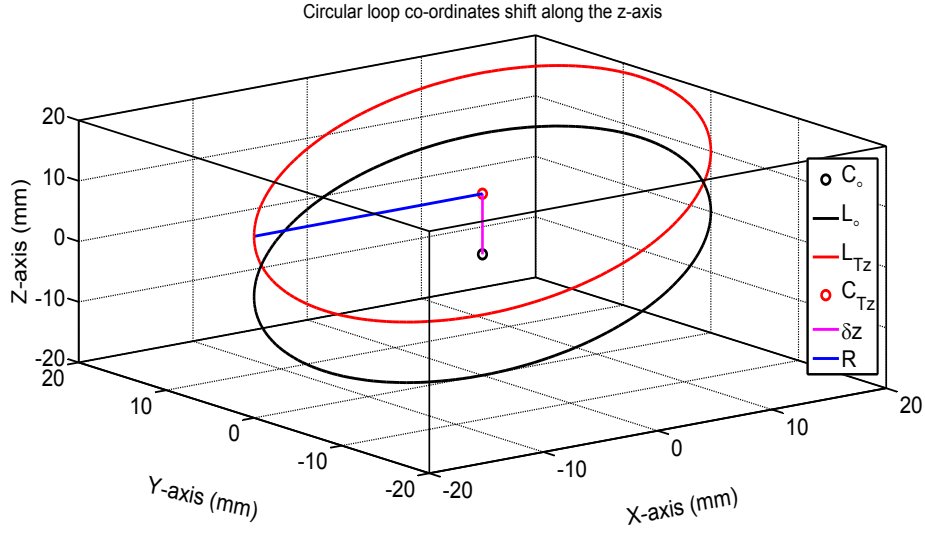


Figure 4.5: Translation along the z-axis. (a) A circular coil in the zx -plane is shifted by 10 mm along the z-axis. (b) Linear changes in induced voltage due to z -translation during application of a z -gradient.

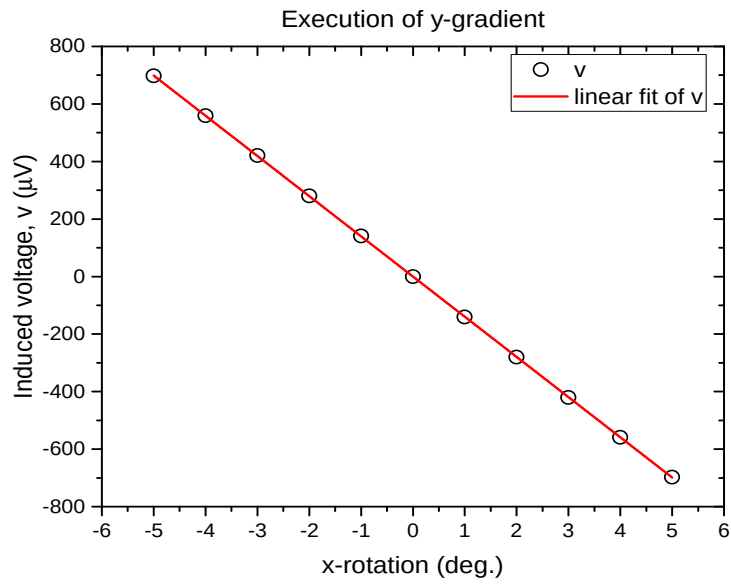
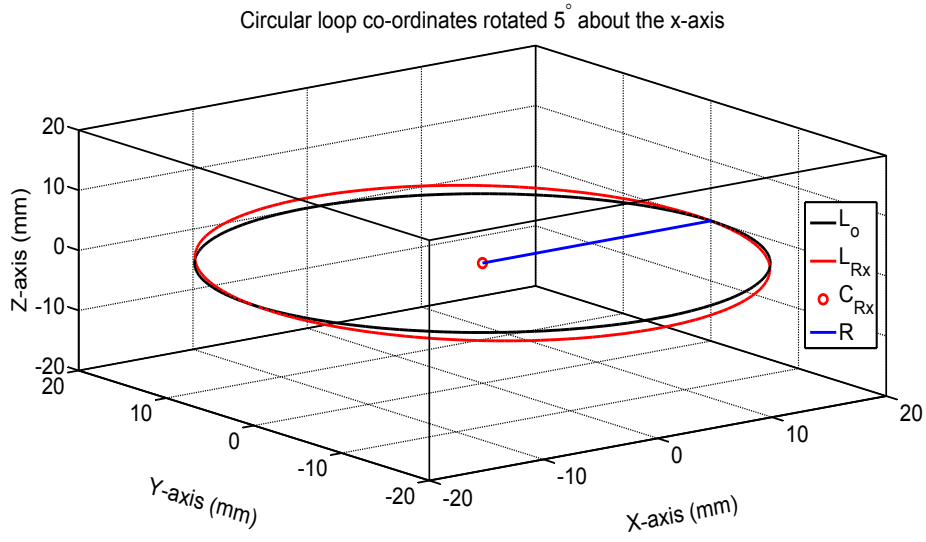
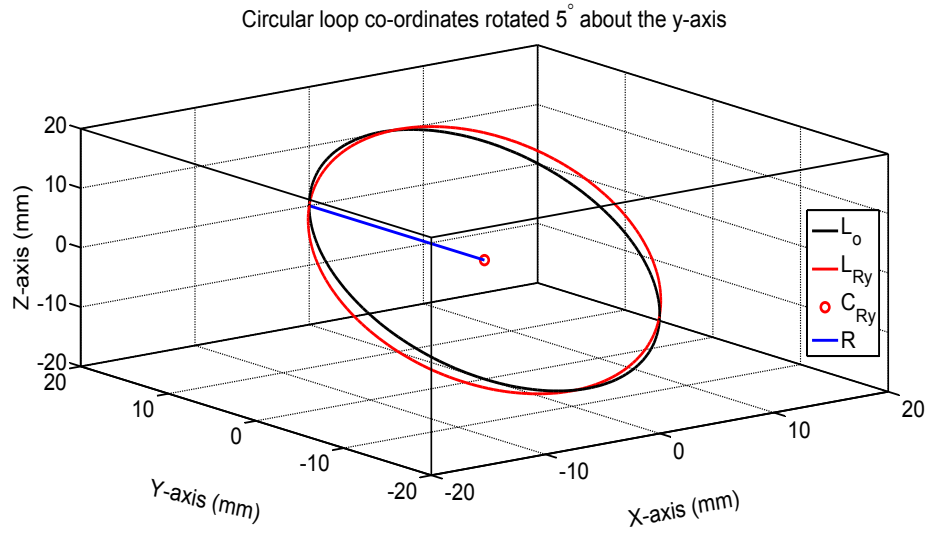
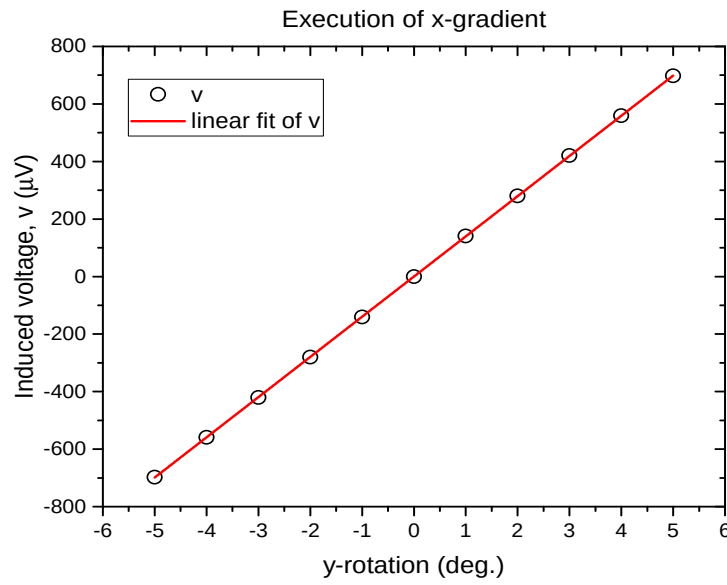


Figure 4.6: Rotation about the x-axis. (a) A circular coil in the xy-plane and 5° rotation about the x-axis. (b) The change in induced voltage in a coil placed in the xy-plane due to rotation about the x-axis during execution of a y-gradient.

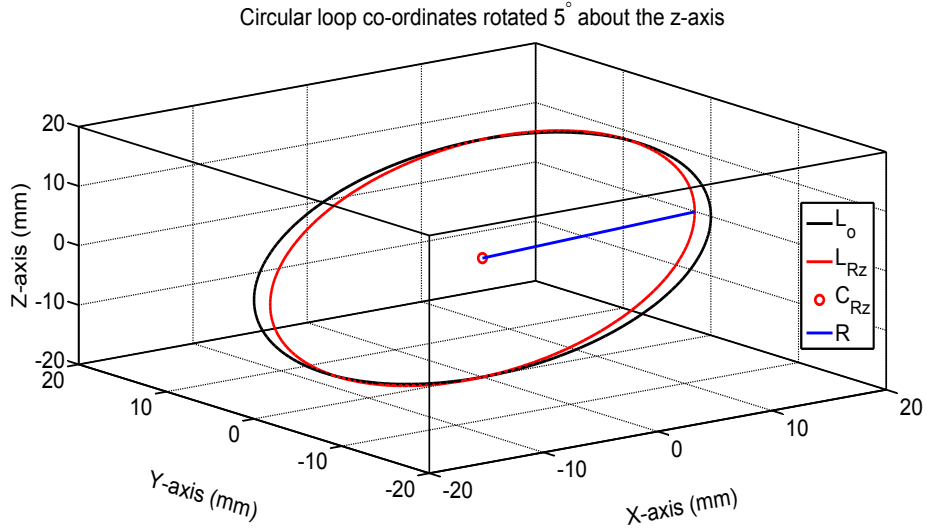


(a)

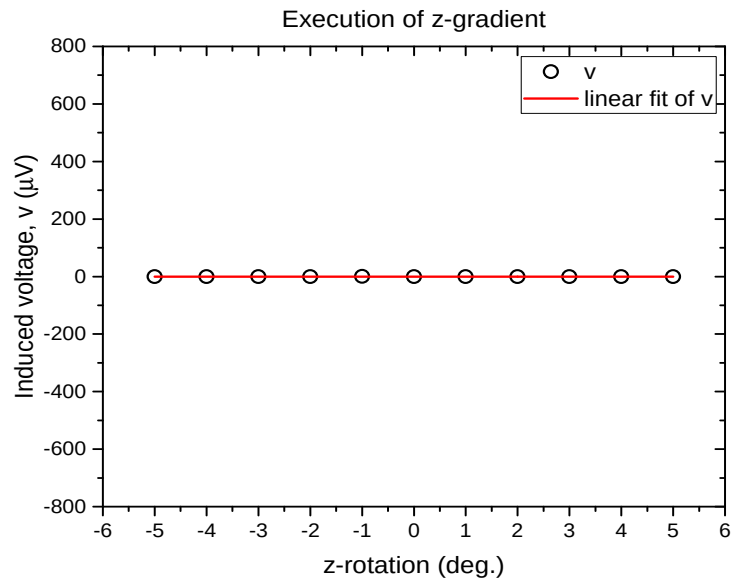


(b)

Figure 4.7: Rotation about the y-axis. (a) A circular coil in the yz-plane and its rotation (5°) about the y-axis. (b) The change in induced voltage in a coil placed in the yz-plane during execution of execution of an x-gradient.



(a)



(b)

Figure 4.8: Rotation about the z-axis. (a) A circular coil in the zx plane and its rotation (5°) about the z-axis. (b) The change in induced voltage in a coil due to rotation about the z-axis during execution of x-, y- and z-gradients.

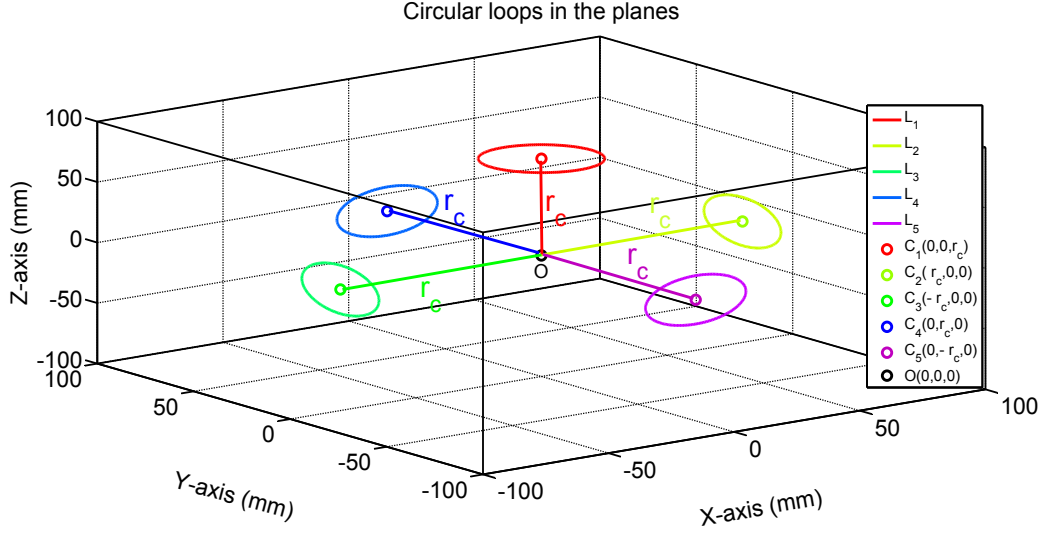


Figure 4.9: A system of five loops in their corresponding planes

All these are representative data. The change in induced voltage due to the three types of rotation is thoroughly calculated during the execution of x-, y- and z-gradients when the coils are in different planes. Due to x- and y-rotation during execution of a z-gradient the voltage change is quadratic in a coil for some special orientation even for small angles of rotation. This issue is addressed in detail in sub-subsection 4.3.1. It is worth noting that linear changes when present dominate over quadratic changes. According to theory and simulation there is no change in the voltage induced in the coils due to rotation about the z-axis.

A System of Five Coils

Exactly the same simulation parameters and 10 turns are used here for a single coil, but the corresponding MatLab code was generalized and restructured for a system of five coils. The coils are located on the faces of a cubic box of side $2r_c$ which is shown in Fig. 4.9, where $C_1(0,0,r_c)$, $C_2(r_c,0,0)$, $C_3(0,-r_c,0)$, $C_4(0,r_c,0)$, and $C_5(0,0,-r_c)$ are the centres of the respective coils and O is the origin. The change in induced voltage in each coil is estimated due to movement (rotation and translation) of the whole system. A random noise voltage of 1 to

10 μV has been added to the calculated voltage in each case. For this, the six parameters are also taken that are shown in Fig. 1.3. The five equations are obtained for the change in induced voltage in the five specific coils by using basic curve fitting (linear and quadratic). In this study, a special case of small rotations (rotation angle is not more than 5°) is considered because it is assumed that a patient cannot rotate his or her head by more than 5° easily inside the head coil.

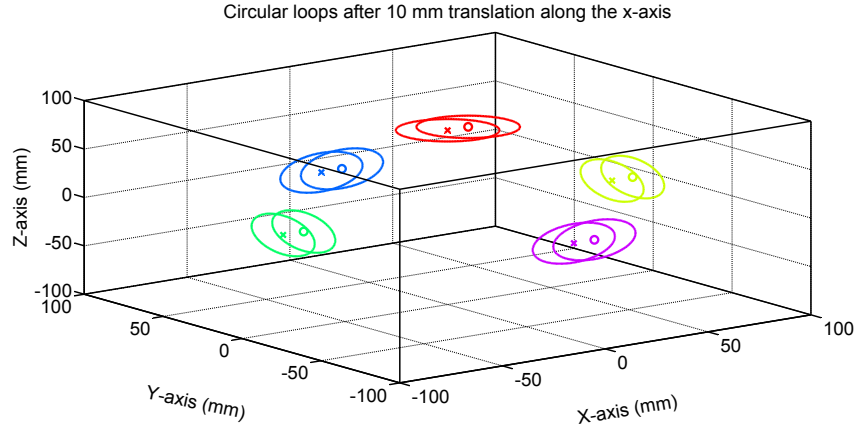
The whole system is translated 10 mm along the x- y- and z-axes, as shown in Fig.4.10. Rotation of the whole system about the x-, y- and z-axes is shown in Fig. 4.11. In this case the coils C_1 , C_4 and C_5 change their orientation due to x-rotation as shown in Fig. 4.11(a), but the coils C_2 and C_3 remain unchanged. Similarly, due to rotation about the y-axis the coils C_1 , C_4 and C_5 change their orientation but the coils C_2 and C_3 remain unchanged which is shown in Fig. 4.11(b). On the other hand, due to rotation about the z-axis the coils C_2 , C_3 , C_4 and C_5 all change their orientation but coil C_1 remains unchanged as shown in Fig. 4.11(c).

The Change in induced EMF due to Translation

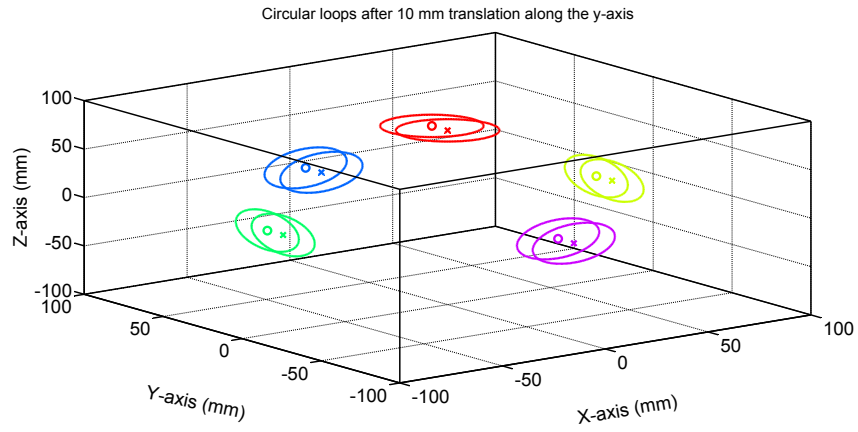
The change in induced emfs are calculated for the application of a gradient strength of $\frac{dG}{dt} = 4$ T/m-s. The change in induced voltage in the coils due to a 10 mm shift along the x-axis is shown in Fig. 4.12. For translations along the x-axis, the change in induced voltage due to the application of an x-gradient occurs in coil C_1 only; for a y-gradient there are no changes; but for a z-gradient, changes occur in coils C_2 and C_3 . The change in induced voltage in coil C_1 due to the execution of an x-gradient is $\delta V_x = 502.65\mu\text{V}$. According to the theory in Chapter 3, the change in induced voltage in the coil C_1 due to a shift along the x-axis is given by the equation:

$$\delta V_1 = -g_x \times S_z \times \delta x. \quad (4.10)$$

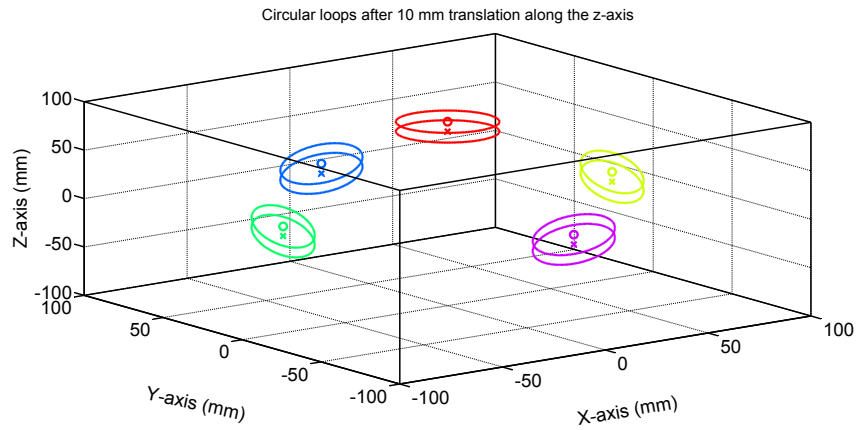
Where $g_x = \frac{dG_x}{dt}$ and area of the circular coil having 10 turns is S_z . In our simulation $g_x = 4$ T/m-s, area of the circular coil $S_z = 10 \times \pi R^2 = \pi(20)^2 \text{ mm}^2$ and $\delta x = 10 \text{ mm}$, that gives us



(a)

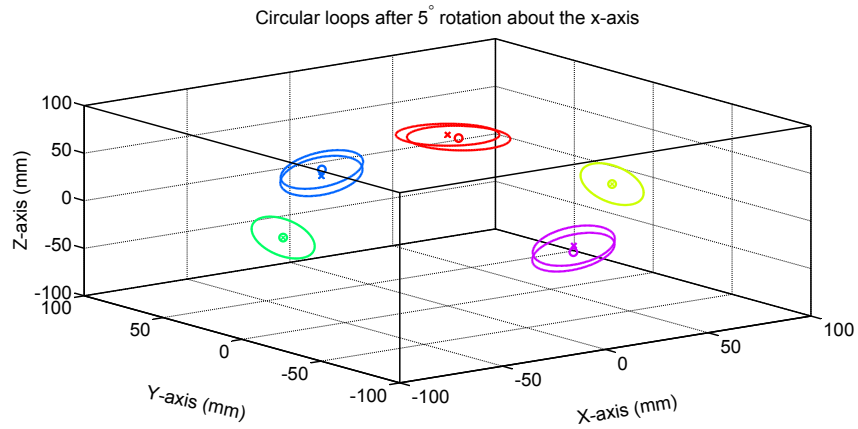


(b)

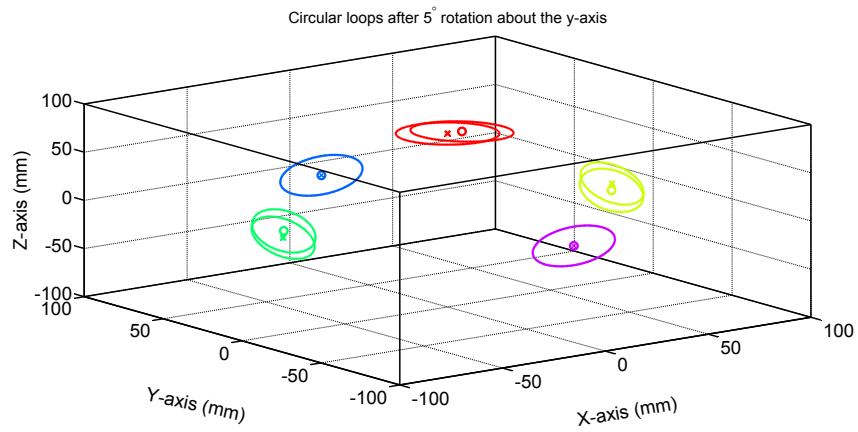


(c)

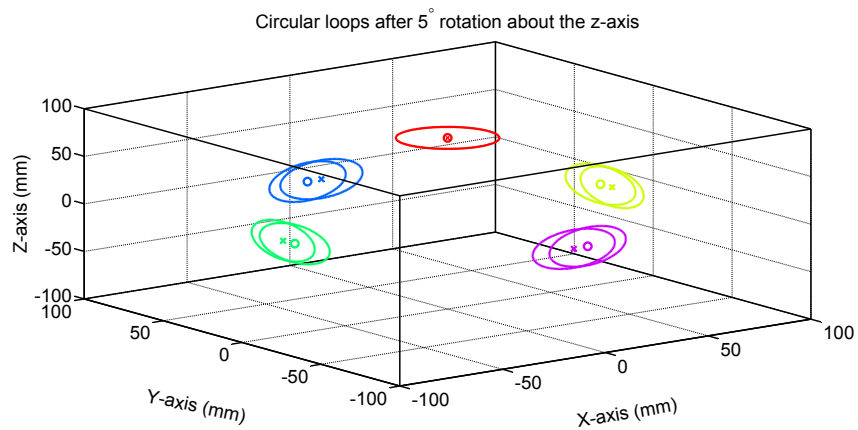
Figure 4.10: Translation of the system along the three mutually perpendicular axes. The centres of the coils are denoted by a circle (o) before and by a crosses (x) after translations. (a) Translation along the x-axis. (b) Translation along the y-axis. (c) Translation along the z-axis.



(a)

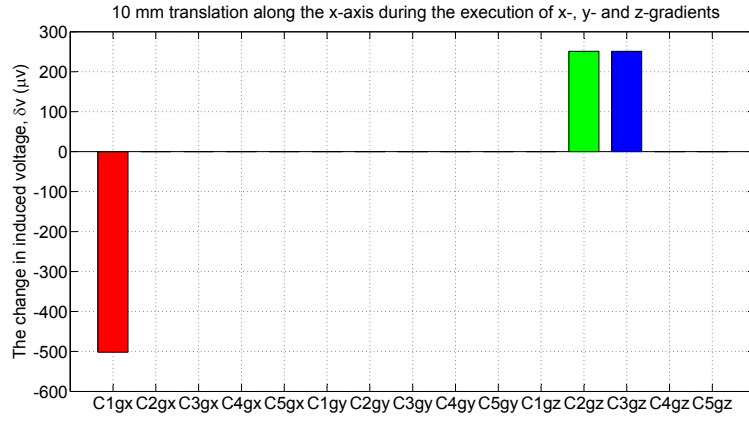


(b)

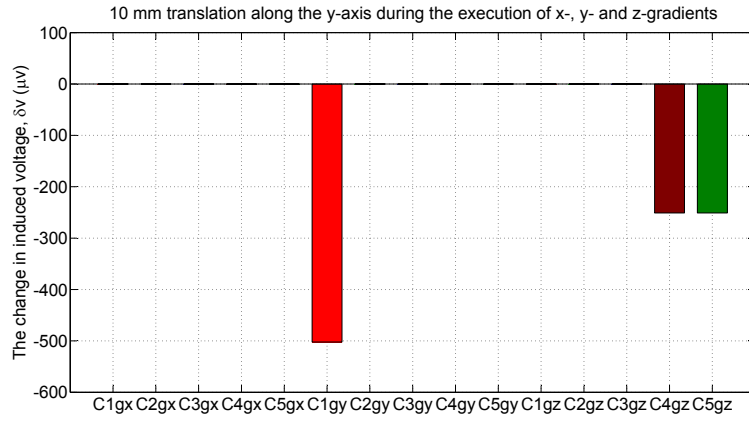


(c)

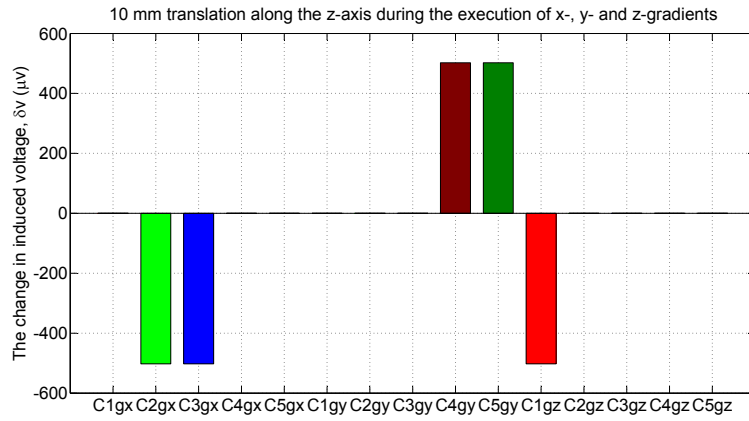
Figure 4.11: Rotation of the system along the three mutually perpendicular axes. The centres of the coils are denoted by circles (o) before and by crosses (x) after rotations. (a) Rotation about the x-axis. (b) Rotation about the y-axis. (c) Rotation about the z-axis.



(a)



(b)



(c)

Figure 4.12: The change in voltage induced (μV) in the coils by the execution of x-, y- and z-gradients. $C_{ng\alpha}$ denotes the change in induced voltage in coil n due to the gradient applied along the α (a) due to x-translation, (b) due to y-translation, and (c) due to z-translation.

$\delta V_1 = 502.65\mu\text{V}$ which is exactly what is obtained from the simulation shown in Fig. 4.12(a).

For the z-gradient, the change in induced voltage in the coils C_2 and C_3 are as follows:

$$\delta V_2 = \frac{1}{2}g_z \times S_x \times \delta x, \quad \delta V_3 = \frac{1}{2}g_z \times S_{-x} \times \delta x, \quad (4.11)$$

which gives us $\delta V_2 = \delta V_3 = 251.33\mu\text{V}$. Calculation gives us the same result as shown in Fig. 4.12(a).

The change in induced voltage due a to 10 mm shift along the y-axis is shown in Fig. 4.12(b). In translation along the y-axis, the induced voltage due to the x-gradient changes in coil C_1 only, for the y-gradient there is no change, but for the z-gradient changes occur in coil C_4 and C_5 . A change in induced voltage is observed in coil C_1 for execution of the x-gradient and in the coils C_4 and C_5 for the z-gradient. There are no voltage changes observed in the rest of the coils. In our simulation $g_y = 4\text{T/m-s}$, $S_z = 10 \times \pi R^2 = 10 \times \pi(20)^2\text{mm}^2$ and $\delta y = 10\text{mm}$, that gives us $\delta V_1 = -502.65\mu\text{V}$ which is exactly what is obtained from the simulation shown in Fig. 4.12(b).

$$\delta V_4 = -\frac{1}{2}g_z \times S_y \times \delta y, \quad \delta V_5 = -\frac{1}{2}g_z \times S_{-y} \times \delta y, \quad (4.12)$$

which gives us $\delta V_4 = \delta V_5 = 251.33\mu\text{V}$. Calculation therefore gives us the same result as are shown in Fig. 4.12(b).

The change in induced voltage due to a 10 mm shift along the z-axis is shown in Fig. 4.12(c). In translation along the z-axis, the induced voltage due to the z-gradient changes in coil C_1 only, for the x-gradient changes occur in the coils C_2 and C_3 , but for the y-gradient changes occurs in coil C_4 and coil C_5 . Data shown in Fig. 4.12(c) also agree well with numerical calculations based on theory described in Chapter 3.

The change in induced voltage due to sequential shifts along the x-axis during the application of the x- and z-gradients is shown in Fig. 4.13(a). The changes occur in coil C_1 during execution of an x-gradient and in the coils C_2 and C_3 during execution of a z-

gradient, whereas no changes occur in the rest of the coils. In this case, there is no change in the induced voltage in any of the coils during execution of a y-gradient.

Induced voltage changes in the coils due to sequential shifts along the y-axis for the application of the y- and z-gradients are shown in Fig. 4.13(b). The voltage changes occur in coil C₁ for a y-gradient and in the coils C₄ and C₅ for a z-gradient. No change is observed in other coils for both of the gradients. The change in induced voltage due to a sequential shift in the z-axis during the application of the x- and y-gradients is shown in Fig. 4.14. The voltage changes occur in the coils C₂ and C₃ for an x-gradient and in the coils C₄ and C₅ for a y-gradient. In this case there is no change in induced voltage in the rest of the coils for the both of the gradients. The change in induced voltage due to a sequential shift in the z-axis because of the application of z-gradient occurs in coil C₁.

By shifting the whole system in the y-axis sequentially linear changes in induced voltages in the coils are observed. According to the theory in Chapter 3, the change in induced voltage in the coil C₁ due to shift in the y-axis is given by the equation:

$$\delta V_1 = -g_y \times S_z \times \delta y. \quad (4.13)$$

Linear changes in induced voltage are observed due to sequential shifts along the z-axis. One can conclude that a change in induced voltage due to translation is always linear with change in position along x-, y- and z-axes. According to the theory in Chapter 3, changes in induced voltages in the coils due to a shift in the z-axis are given by the equations:

$$\begin{aligned} \delta V_1 &= -g_z \times S_z \times \delta z, & \delta V_2 &= -g_x \times S_x \times \delta z \\ \delta V_3 &= -g_x \times S_{-x} \times \delta z, & \delta V_4 &= +g_y \times S_y \times \delta z \\ \delta V_5 &= +g_y \times S_{-y} \times \delta z. \end{aligned} \quad (4.14)$$

In our simulation $g_x = g_y = g_z = 4\text{T/m-s}$, $S_z = S_x = S_{-x} = S_y = S_{-y} = \text{Number of turns} \times \pi R^2 = 10 \times \pi(20)^2\text{mm}^2$ and $\delta z = 10\text{mm}$, that gives us $\delta V_1 = \delta V_2 = \delta V_3 = -502.65\mu\text{v}$ and

$\delta V_4 = \delta V_5 = -502.65\mu\text{V}$ which is exactly what is obtained from the simulation shown in Fig. 4.12(c).

The change in induced voltage observed in the coils C_1 , C_2 and C_3 due to the application of the x-, y- and z-gradients is shown in Fig. 4.13(a) for the case where the whole system is sequentially shifted along the x-axis. When the whole system is sequentially shifted along the y-axis the change in induced voltage occurs in the coils C_1 , C_4 and C_5 due to the application of the x-,y- and z-gradients as shown in Fig. 4.13(b). The changes in induced voltage are linear and quantitatively agree with theoretical predictions.

The whole system is now sequentially shifted along the z-axis and the change in induced voltage observed in the coils C_1 , C_2 , C_3 , C_4 and C_5 due to the application of the x-, y- and z-gradients, are shown in Fig. 4.14.

The Change in Induced Voltage due to Rotation

The change in induced voltage in the coils due to rotations (of less than 5°) about the x- and y-axes is shown in Fig. 4.15. According to the theory in Chapter 3 in Eq. 3.51, changes in the induced voltages occurring in the coils C_1 , C_4 and C_5 due to the application of the y-gradient because of the rotation about the x-axis are as follows:

$$\begin{aligned}\delta V_1 &= +2g_y \times z \times S_z \times \delta\phi_x, & \delta V_4 &= -2g_y \times y \times S_y \times \delta\phi_x, \\ \delta V_5 &= -2g_y \times y \times S_y \times \delta\phi_x.\end{aligned}\tag{4.15}$$

Where $g_y = 4 \text{ T/m-s}$, $z = y = r_c = 80 \text{ mm} = \text{coil offset}$, $S_y = S_{-y} = S_z = \text{Number of turns (N)} \times \pi R^2 = 10 \times \pi(20)^2 \text{ mm}^2$ and $\delta\phi_x = 0.0873 \text{ radians}$, gives $\delta V_1 = -702.11\mu\text{V}$, $\delta V_4 = 702.11\mu\text{V}$ and $\delta V_5 = -702.11\mu\text{V}$ which is exactly what is obtained from simulation as shown in Fig. 4.15(a).

In this case for the z-gradient changes occur in the same coils and are given by the

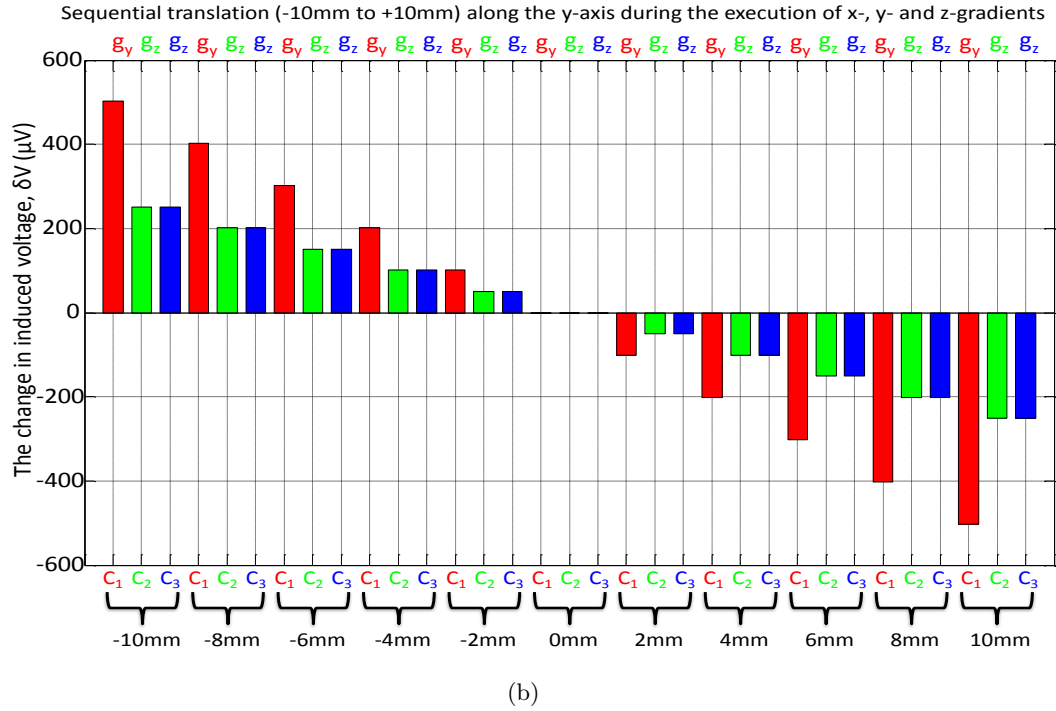
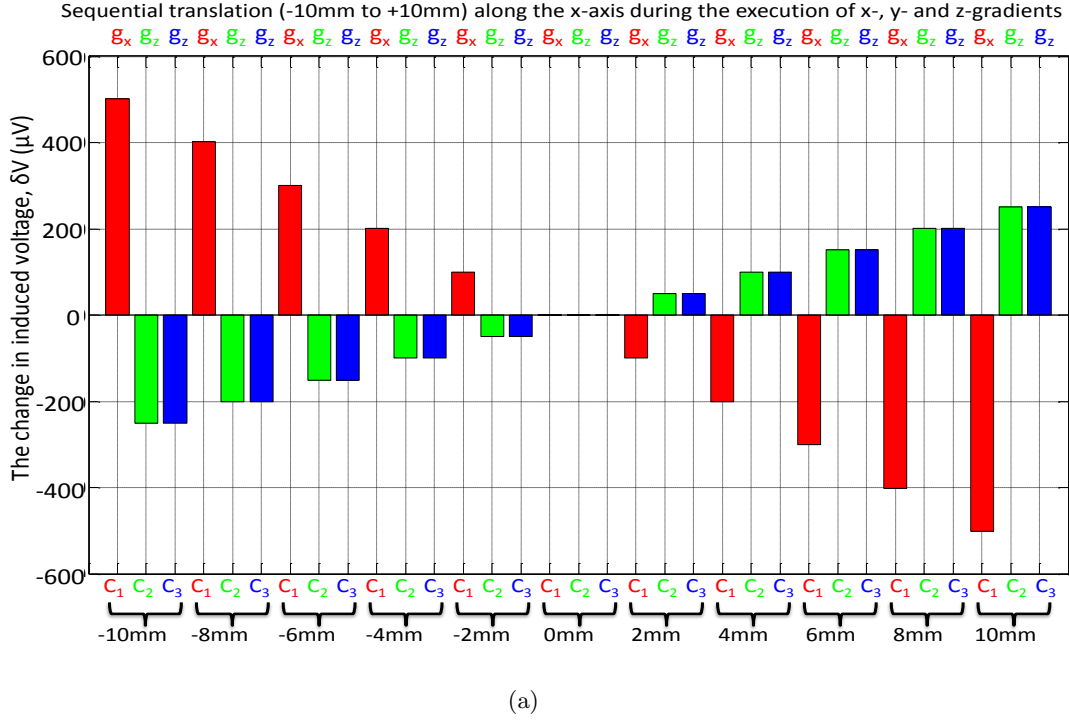


Figure 4.13: The change in voltage induced (μV) in the coils by the execution of x-, y- and z-gradients. $C_n g_\alpha$ denotes the change in induced voltage in coil n due to the gradient applied along the α direction (a) sequential translation along the x-axis for coil C_1 exposed to an x-gradient and coils C_2 & C_3 when exposed to a z-gradient, (b) sequential translation along the y-axis for coil 1 exposed to a y-gradient and coils C_4 & C_5 when exposed to a z-gradient.

following equations:

$$\begin{aligned}\delta V_1 &= -\frac{3}{2}g_z \times z \times S_z \times \delta\phi_x^2, & \delta V_4 &= \frac{3}{2}g_z \times y \times S_y \times \delta\phi_x^2, \\ \delta V_5 &= \frac{3}{2}g_z \times y \times S_y \times \delta\phi_x^2.\end{aligned}\tag{4.16}$$

Since $g_z = 4 \text{ T/m-s}$, $z = y = r_c = 80\text{mm} = \text{coil offset}$, $S_y = S_z = \text{Number of turns (N)} \times \pi R^2 = 10 \times \pi(20)^2\text{mm}^2$ and $\delta\phi_x^2 = 7.61 \times 10^3 \text{ radians}^2$, this gives us $\delta V_1 = -45.90\mu\text{V}$, $\delta V_4 = 45.90\mu\text{V}$ and $\delta V_5 = -45.90\mu\text{V}$ which is exactly what I get from simulation as shown in Fig. 4.15(a). On the other hand, the change in induced voltages occurring in the coils C₁, C₂ and C₃ due to rotation about the y-axis because of the application of the x-gradient is as follows:

$$\begin{aligned}\delta V_1 &= -2g_x \times z \times S_z \times \delta\phi_y, & \delta V_2 &= 2g_x \times x \times S_x \times \delta\phi_y, \\ \delta V_3 &= 2g_x \times x \times S_x \times \delta\phi_y,\end{aligned}\tag{4.17}$$

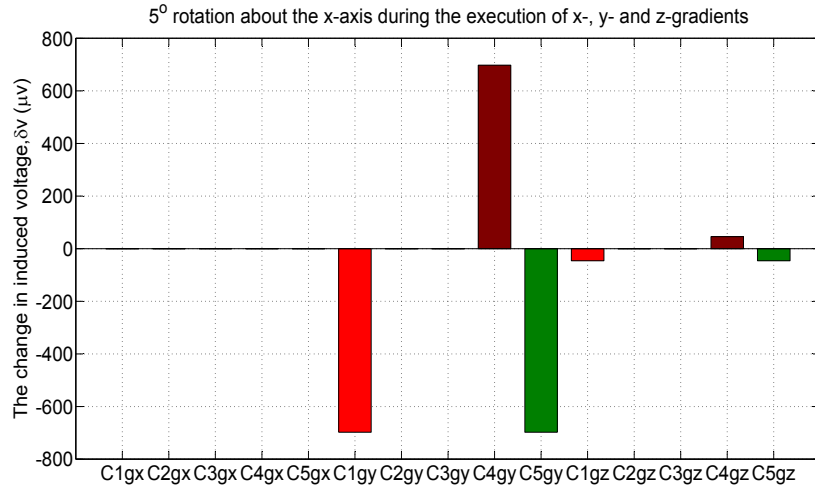
where $g_x = 4 \text{ T/m-s}$, $z = x = r_c = 80 \text{ mm} = \text{coil offset}$, $S_x = S_z = \text{Number of turns (N)} \times \pi R^2 = \pi(20)^2 \text{ mm}^2$ and $\delta\phi_y = 0.0873 \text{ radians}$ all these gives us $\delta V_1 = -702.11 \mu\text{V}$, $\delta V_2 = -702.11 \mu\text{V}$ and $\delta V_3 = 702.11 \mu\text{V}$ volts which is exactly what is obtained from the simulations, as shown in Fig. 4.15(b).

In this case for the z-gradient changes occur in the same coils and are given by the following equations:

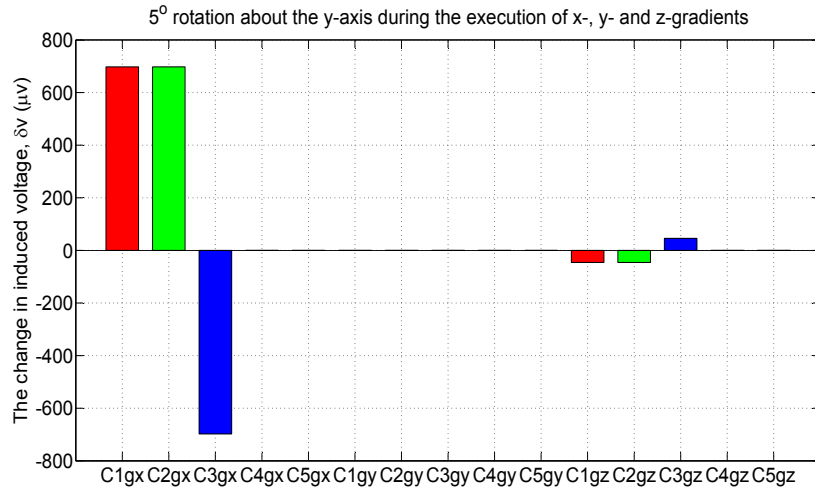
$$\begin{aligned}\delta V_1 &= -\frac{3}{2}g_z \times z \times S_z \times \delta\phi_y^2, & \delta V_2 &= \frac{3}{2}g_z \times x \times S_x \times \delta\phi_y^2, \\ \delta V_3 &= \frac{3}{2}g_z \times x \times S_x \times \delta\phi_y^2,\end{aligned}\tag{4.18}$$

where $g_z = 4\text{T/m-s}$, $z = x = r_c = 80 \text{ mm} = \text{coil offset}$, $S_x = S_z = \text{Number of turns (N)} \times \pi R^2 = 10 \times \pi(20)^2 \text{ mm}^2$ and $\delta\phi_y^2 = 7.61 \times 10^{-3} \text{ radians}^{-2}$. these gives us $\delta V_1 = -45.90 \mu\text{V}$, $\delta V_2 = -45.90 \mu\text{V}$ and $\delta V_3 = 45.90 \mu\text{V}$ which is exactly what is obtained from simulation that is shown in Fig. 4.15(b).

The theory and simulation agree well in that there is no change in the voltage induced



(a)



(b)

Figure 4.15: The change in voltage induced (μV) in the coils by the execution of x-, y- and z-gradients. $C_{n\alpha}$ denotes the change in induced voltage in coil n due to the gradient applied along the α (a) due to rotation about the x-axis, and (b) due to rotation about the y-axis.

in the coils due to rotation about the z-axis. The changes in induced voltage in the coils due to sequential rotation about the x- and z-axes are shown in Fig. 4.16 and Fig. 4.17 respectively.

For rotation, the change in induced voltage due to the x- and z-gradients is linear with rotation angle whereas for the y-gradient the change in induced voltage with rotation angle is quadratic in nature.

Although the change in induced voltage in the coils due to a rotation about the x and y-axes because of the application of z-gradient is quadratic with rotation angle, the changes are small for small rotation angles ($\phi \leq 5^\circ$). Analysing the induced voltage data obtained from simulation and by using basic linear fittings one can obtain the corresponding equations. A system of linear equations is developed by basic linear fittings. The change in positions and orientations of the coils can then be found by solving this system of linear equations.

The Change in emf and their Linear Fit

The changes in induced voltage in the five coils due to translations and rotations are analytically investigated in Chapter 3. The analytical equations are summarized in Tables 3.1 and 3.2. The changes in induced voltage in the coils are also calculated by numerical simulation in this Chapter. The aim of this study is to find the change in position and orientation of the coils by using the change in induced voltage in the coils. The change in induced voltage in the coils is calculated due to specific translations and rotations as well as sequential translation in the x-, y- and z-axes and sequential rotation about the x-, y- and z-axes. The graphs are plotted for the voltage change, δv versus δx , δy , δz , $\delta\phi_x$, $\delta\phi_y$ of the resulting data. Linear fitting of the change in induced voltage gives us the slopes of the graphs. These slopes are compared with the slopes obtained from theoretical calculations.

Linear fitting for the change in induced voltage in the coils due to sequential translation along the x-, y- and z-axes during the application of x-, y- and z-gradients are shown in Figs. 4.18(a), 4.18(b) and 4.18(c) respectively.

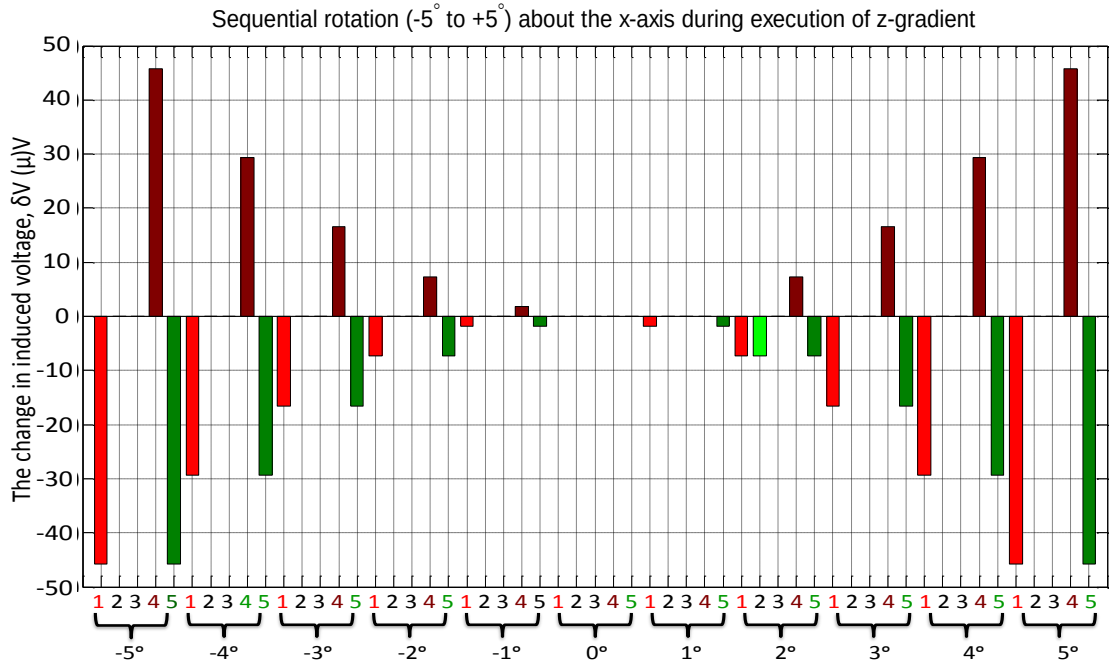
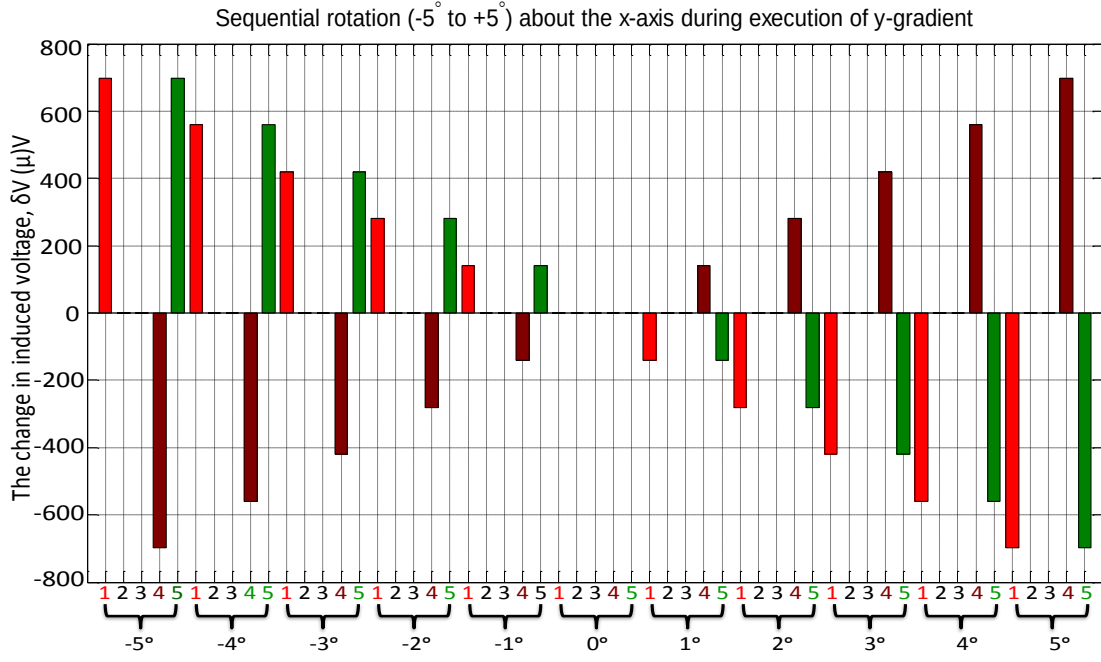
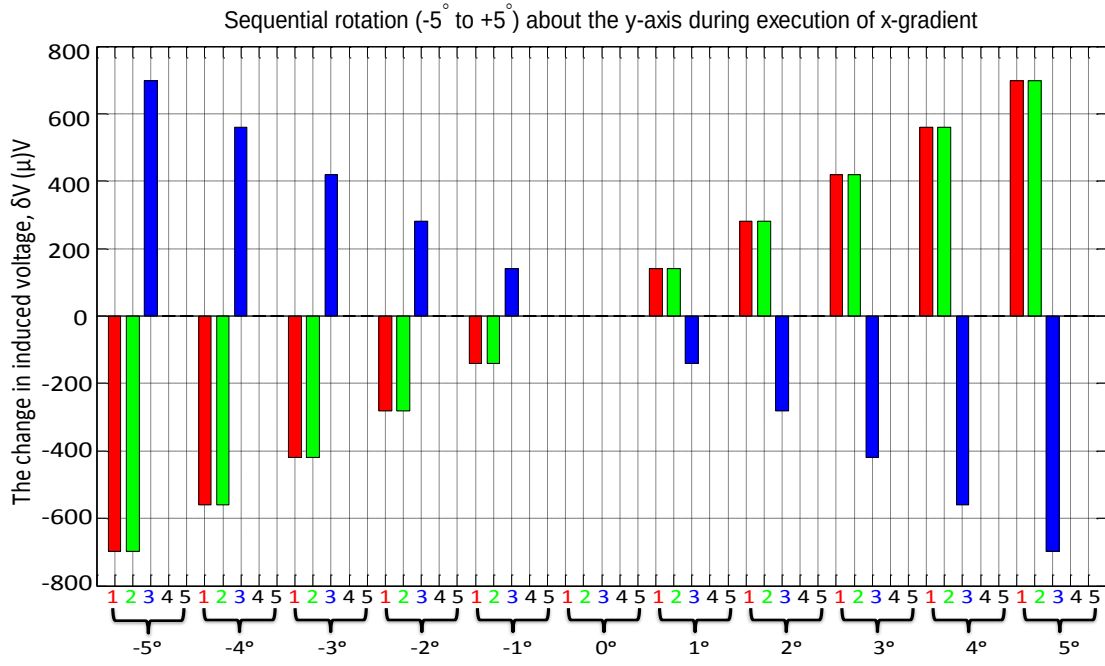
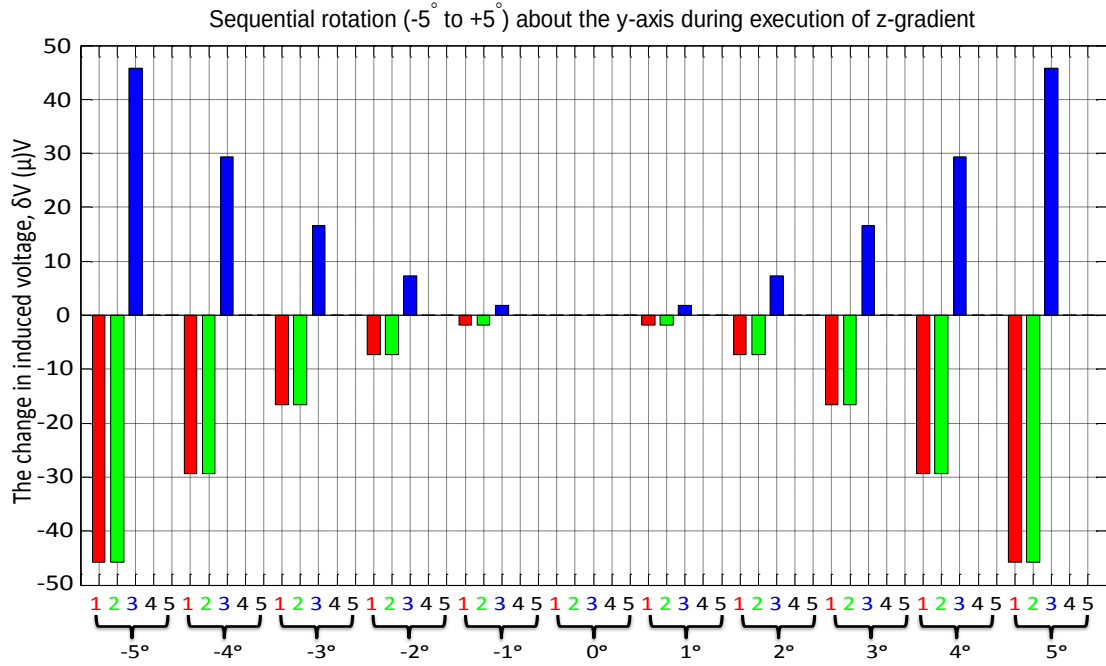


Figure 4.16: The change in voltage induced (μV) in the coils by the execution of x-, y- and z-gradients. $C_n g_\alpha$ denotes the change in induced voltage in coil n due to the gradient applied along the α due to rotation about x-axis (a) in the coils C_1 , C_4 and C_5 during execution of y-gradient, and (b) in the coils C_1 , C_4 and C_5 during execution of z-gradient.



(a)



(b)

Figure 4.17: The change in voltage induced (μV) in the coils by the execution of x-, y- and z-gradients. $C_n g_\alpha$ direction denotes the change in induced voltage in coil n due to the gradient applied along the α due to rotation about the y-axis (a) in the coils C_1 , C_2 and C_3 during execution of x-gradient, and (b) in the coils C_1 , C_2 and C_3 during execution of a z-gradient.

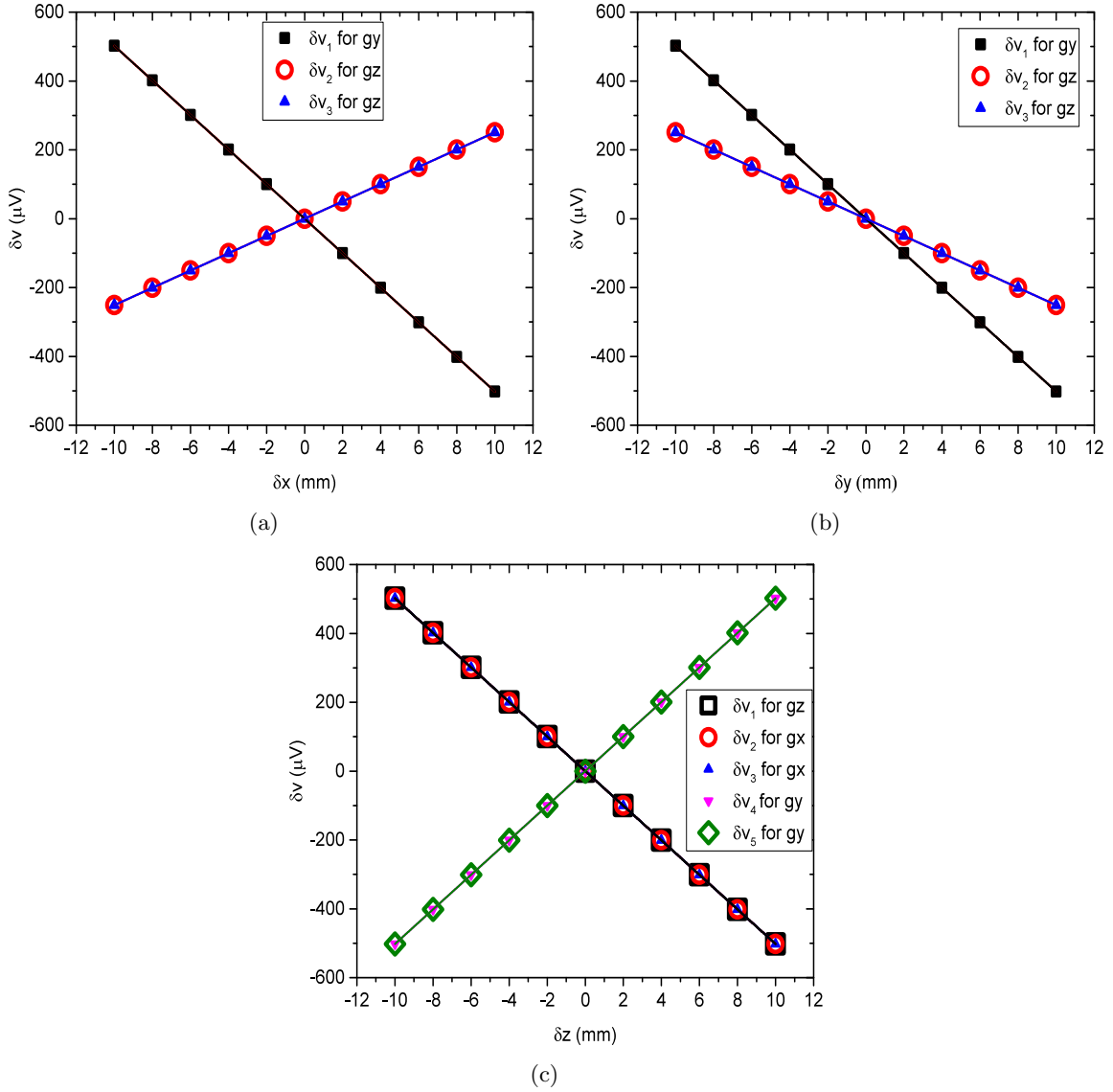


Figure 4.18: The changes in induced voltage in the coils and their linear fits. (a) The induced voltage changes occur in coil C_1 during execution of an x-gradient, whereas in C_2 and C_3 when exposed to a z-gradient due to translation along the x-axis. (b) The induced voltage changes occur in coil C_1 during execution of a y-gradient, whereas in coils C_4 and C_5 during the application of a z-gradient due to translation along the y-axis. (c) The induced voltage changes occur in coil C_1 during execution of a z-gradient, whereas in coils C_2 and C_3 during the application of an x-gradient, on the other hand in coils C_4 and C_5 during the application of a z-gradient due to translation along the z-axis.

If the whole system is shifted along the x-axis when an x-gradient is applied, there is a change in induced voltage in the coil C_1 , no voltage change during the application of y-gradient, and changes occur only in coils C_2 and C_3 during execution of a z-gradient. These results are shown in Fig. 4.18(a).

On the other hand, due to translation of the whole system along the y-axis the change in induced voltage occurs in coil C_1 during execution of a y-gradient, in coils C_4 and C_5 during execution of a z-gradient, but there is no voltage change during execution of an x-gradient.

In case of translation along the z-axis, the changes in induced voltage occur in coil C_1 , when exposed to a z-gradient, in coils C_2 and C_3 during execution of an x-gradient and in coils C_4 and C_5 during execution of a y-gradient. However, the change in induced voltages in the coils is calculated by numerical integration of the vector potential over the digitized coil.

The change in induced voltage versus the position change due to translation along the x-, y- or z-axis are plotted, and 11-curves and corresponding equations are obtained by using basic linear fittings. All these equations are given as follows, where the voltage are in μV and shifts are in mm and

$$\begin{aligned}
V_{1T}g_x &= -50.26\delta x, & V_{2T}g_x &= -50.26\delta z, & V_{3T}g_x &= -50.26\delta z \\
V_{1T}g_y &= -50.26\delta y, & V_{4T}g_y &= 50.26\delta z, & V_{5T}g_y &= 50.26\delta z \\
V_{1T}g_z &= -50.26\delta z, & V_{2T}g_z &= 25.13\delta x, & V_{3T}g_z &= 25.13\delta x \\
V_{4T}g_z &= -25.13\delta y, & V_{5T}g_z &= -25.13\delta y,
\end{aligned} \tag{4.19}$$

where the maximum translation is 10 mm along each axis. Subscript T -stands for translation and 1, 2, 3, 4, and 5 relates to the coil numbers. The rate of change of the three gradients is denote by g_x , g_y and g_z , whereas δx , δy and δz denoted the amount as well as the direction of the translation.

According to the theory in Chapter 3, the change in induced voltage in the coil C_1 due

to the shift along the x-axis is given as follows:

$$V_{1T}g_x = -\frac{dG_x}{dt} \times S \times \delta x, \quad (4.20)$$

where $\frac{dG_x}{dt} = 4 \text{ T/m-s}$, $S = \pi R^2 = \pi(20)^2 \text{ mm}^2$ and $\delta x = 10 \text{ mm}$ and the number of turns in each coil is 10, that gives us $V_{1T}g_x = -502 \text{ } \mu\text{V}$. On the other hand, by fitting data from simulation one obtains the equation as:

$$V_{1T}g_x = \frac{dV_{1x}}{dx} \delta x, \quad (4.21)$$

where the slope $\frac{dV_{1x}}{dx} = -50.1 \text{ } \mu\text{V/mm}$ and $\delta x = 10 \text{ mm}$ which gives us exactly the change in induced voltage that is obtained from the theory. Similarly, the change in induced voltages is calculated for the rest of the coils due to x-, y- and z-translations during execution of x-, y- and z-gradients and compares the results obtained from theory and simulation. It is worth noting that in our study there is a good agreement between theory and simulation.

The change in induced voltage in the coils is estimated for sequential rotations about the x-, y- and z-axes during the application of x-, y- and z-gradients. There is no change in induced voltage due to rotation about the z-axis in any of the coil during the application of the three gradients. However, changes in induced voltage occur due to rotations about the x- and y-axes.

In the case of a rotation about the x-axis, during the application of a y-gradient the change in induced voltage occurring in coils C_1 , C_4 and C_5 that varies linearly with rotation angle is shown in Fig. 4.19(b). On the other hand, in the case of a rotation about the x-axis during the application of a z-gradient, the change in induced voltage occurring in the same coils varies non-linearly (quadratic) with rotation angle as shown in Fig. 4.20(b).

If the whole system is rotated about the y-axis during the application of an x-gradient the change in induced voltage occurring in coils C_1 , C_2 and C_3 is linear with rotation angle as shown in Fig. 4.20(a). In the case of rotation about the y-axis during the application of

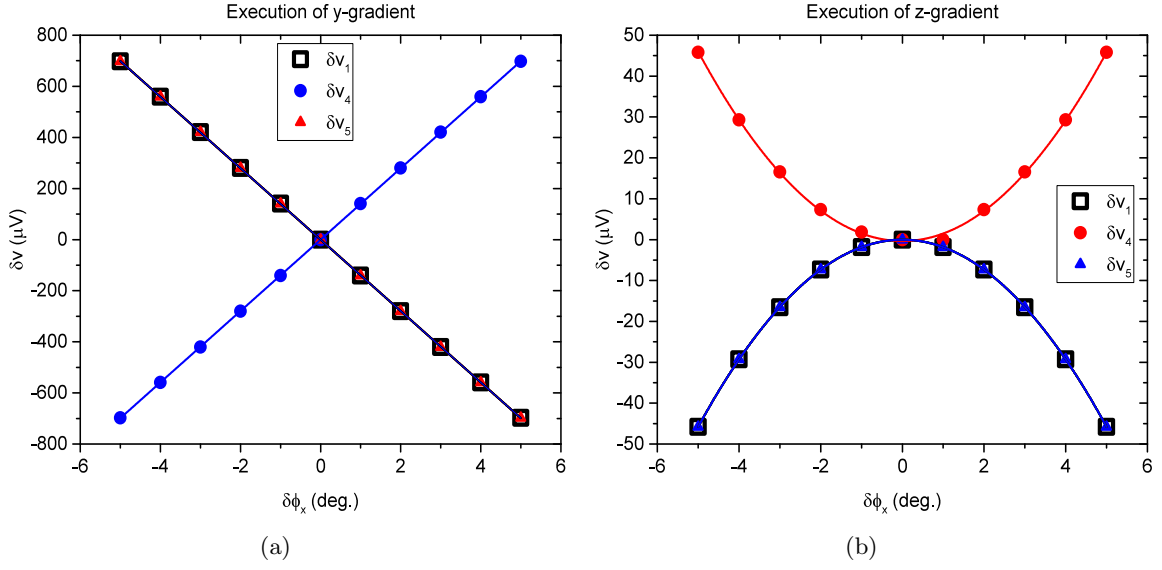


Figure 4.19: The change in voltage in the coils and their linear and quadratic fit. (a) The change in induced voltage in the coils C_1 , C_4 and C_5 during execution of a y-gradient due to rotation about the x-axis. (b) The change in induced voltage in the coils C_1 , C_4 and C_5 during execution of a z-gradient due to rotation about the x-axis.

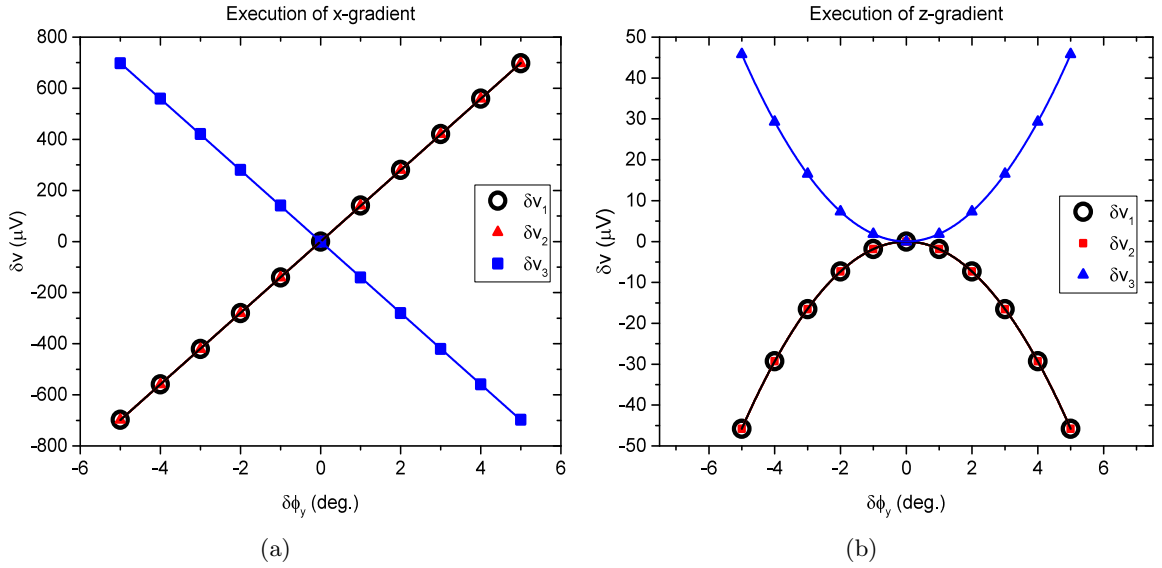


Figure 4.20: The change in induced voltage in the coils and their fit. (a) The change in induced voltage in the coils C_1 , C_4 and C_5 during execution of a y-gradient due to rotation about the y-axis. (b) The change in induced voltage in the coils C_1 , C_4 and C_5 during execution of a z-gradient due to rotation about the y-axis.

the z-gradient the change in induced voltage occurring in coils C₁, C₂ and C₃ is non-linear (quadratic) with rotation angle as shown in Fig. 4.20(b).

The graphs of the change in induced voltage versus the change of rotation angle due to rotation about the x- and y-axes are plotted. By fitting to the 12-curves the corresponding equations are obtained:

$$\begin{aligned}
V_{1R}g_x &= -140.78\delta\phi_y, & V_{2R}g_x &= 140.78\delta\phi_y, & V_{3R}g_x &= -140.78\delta\phi_y \\
V_{1R}g_y &= 140.78\delta\phi_x, & V_{4R}g_y &= 140.78\delta\phi_x, & V_{5R}g_y &= -140.78\delta\phi_x \\
V_{1R}g_z &= 1.82\delta\phi_y^2, & V_{2R}g_z &= -1.82\delta\phi_y^2, & V_{3R}g_z &= 1.82\delta\phi_y^2 \\
V_{1R}g_z &= 1.82\delta\phi_x^2, & V_{4R}g_z &= 1.82\delta\phi_x^2, & V_{5R}g_z &= -1.82\delta\phi_x^2.
\end{aligned} \tag{4.22}$$

where R stands for rotation and 1, 2, 3, 4, and 5 refer to the coils number. The rate of change of the three gradients is denoted by g_x , g_y and g_z whereas $\delta\phi_x$, $\delta\phi_y$ and $\delta\phi_z$ denote the rotation angles about the x-, y- and z-axes respectively, the angles are measured in degrees, and the voltages are quoted in μV .

According to the theory in Chapter 3, again the change in induced voltage in the coil C₁ due to rotation about the y-axis due to the application of the x-gradient is given as follows:

$$V_{1R}g_x = -2 \cdot \frac{dG_x}{dt} (S \times r_c) \times \delta\phi_y, \tag{4.23}$$

where $\frac{dG_x}{dt} = 4 \text{ T/m-s}$, $S = \text{Number of turns} \times \pi R^2 = 10 \times \pi(20)^2 \text{ mm}^2$ and $\delta\phi_y = 5\pi/180 = 0.0873$ radians, that gives us $V_{1R}g_x = -702.11\mu\text{V}$. On the other hand, by fitting data from simulation I obtain the equation as:

$$V_{1R}g_x = 2g_x \times z \times S_z \times \delta\phi_y, \tag{4.24}$$

where the slope $2g_x z S_z = -140 \mu\text{V/degree}$, due to 5° rotation about the y-axis, one can obtain $V_{1R}g_x = -700 \mu\text{V}$, this is exactly what is obtained from the theory. But in the theory, due to the application of z-gradient the change in induced voltage in the coils is represented

by the equation:

$$V_{1Rg_z} = -\frac{3}{2} \frac{dG_z}{dt} (S \times r_c) \times \delta\phi_y^2, \quad (4.25)$$

where $\delta\phi_y^2 = 7.6 \times 10^{-3}$ radian². Putting the values of the parameters, one obtain $V_{1Rg_z} = 45.90 \mu\text{V}$. On the other hand fitting the the change in induced voltage in coil C_1 due to rotation about the y-axis when exposed to a z-gradient, the equation is obtained as follows:

$$V_{1Rg_z} = 1.82\delta\phi_y^2. \quad (4.26)$$

Using the value of $\delta\phi_y$, one obtains $V_{1Rg_z} = 45.50 \mu\text{V}$, which is equal to the theoretical value. Similarly, it is possible to calculate for the rest of the coils and compare the results from the theory and experiment.

System of Linear Equations

For our generalized system (a system of five coils), 15 equations relating the measured voltage changes in 5 coils due to 3 gradients to the six parameters are obtained. Keeping only the linear terms in the expression, a system of linear equations is formed $\mathbf{v} = \mathbf{A}\mathbf{r}$, where $\mathbf{A}$ is a 15×6 matrix of the coefficients (slopes of the equations) and $\mathbf{r}$ is a vector of six degrees of freedom and $\mathbf{v}$ is a vector of the induced voltage in the five coils due to 3-gradients by using these equations. The system of linear equations is solved to find $\mathbf{r}$, i.e. $\mathbf{r} = \mathbf{A}^{-1}\mathbf{v}$ which

gives us six movement parameters (degrees of freedom) δx , δy , δx , $\delta\phi_x$, $\delta\phi_y$, and $\delta\phi_z$.

$$\begin{pmatrix} \delta V_{1x} \\ \delta V_{2x} \\ \delta V_{3x} \\ \delta V_{4x} \\ \delta V_{5x} \\ \delta V_{1y} \\ \delta V_{2y} \\ \delta V_{3y} \\ \delta V_{4y} \\ \delta V_{5y} \\ \delta V_{1z} \\ \delta V_{2z} \\ \delta V_{3z} \\ \delta V_{4z} \\ \delta V_{5z} \end{pmatrix} = \begin{pmatrix} \frac{dV_{1x}}{dx} & \frac{dV_{1x}}{dy} & \frac{dV_{1x}}{dz} & \frac{dV_{1x}}{d\phi_x} & \frac{dV_{1x}}{d\phi_y} & \frac{dV_{1x}}{d\phi_z} \\ \frac{dV_{2x}}{dx} & \frac{dV_{2x}}{dy} & \frac{dV_{2x}}{dz} & \frac{dV_{2x}}{d\phi_x} & \frac{dV_{2x}}{d\phi_y} & \frac{dV_{2x}}{d\phi_z} \\ \frac{dV_{3x}}{dx} & \frac{dV_{3x}}{dy} & \frac{dV_{3x}}{dz} & \frac{dV_{3x}}{d\phi_x} & \frac{dV_{3x}}{d\phi_y} & \frac{dV_{3x}}{d\phi_z} \\ \frac{dV_{4x}}{dx} & \frac{dV_{4x}}{dy} & \frac{dV_{4x}}{dz} & \frac{dV_{4x}}{d\phi_x} & \frac{dV_{4x}}{d\phi_y} & \frac{dV_{4x}}{d\phi_z} \\ \frac{dV_{5x}}{dx} & \frac{dV_{5x}}{dy} & \frac{dV_{5x}}{dz} & \frac{dV_{5x}}{d\phi_x} & \frac{dV_{5x}}{d\phi_y} & \frac{dV_{5x}}{d\phi_z} \\ \frac{dV_{1y}}{dx} & \frac{dV_{1y}}{dy} & \frac{dV_{1y}}{dz} & \frac{dV_{1y}}{d\phi_x} & \frac{dV_{1y}}{d\phi_y} & \frac{dV_{1y}}{d\phi_z} \\ \frac{dV_{2y}}{dx} & \frac{dV_{2y}}{dy} & \frac{dV_{2y}}{dz} & \frac{dV_{2y}}{d\phi_x} & \frac{dV_{2y}}{d\phi_y} & \frac{dV_{2y}}{d\phi_z} \\ \frac{dV_{3y}}{dx} & \frac{dV_{3y}}{dy} & \frac{dV_{3y}}{dz} & \frac{dV_{3y}}{d\phi_x} & \frac{dV_{3y}}{d\phi_y} & \frac{dV_{3y}}{d\phi_z} \\ \frac{dV_{4y}}{dx} & \frac{dV_{4y}}{dy} & \frac{dV_{4y}}{dz} & \frac{dV_{4y}}{d\phi_x} & \frac{dV_{4y}}{d\phi_y} & \frac{dV_{4y}}{d\phi_z} \\ \frac{dV_{5y}}{dx} & \frac{dV_{5y}}{dy} & \frac{dV_{5y}}{dz} & \frac{dV_{5y}}{d\phi_x} & \frac{dV_{5y}}{d\phi_y} & \frac{dV_{5y}}{d\phi_z} \\ \frac{dV_{1z}}{dx} & \frac{dV_{1z}}{dy} & \frac{dV_{1z}}{dz} & \frac{dV_{1z}}{d\phi_x} & \frac{dV_{1z}}{d\phi_y} & \frac{dV_{1z}}{d\phi_z} \\ \frac{dV_{2z}}{dx} & \frac{dV_{2z}}{dy} & \frac{dV_{2z}}{dz} & \frac{dV_{2z}}{d\phi_x} & \frac{dV_{2z}}{d\phi_y} & \frac{dV_{2z}}{d\phi_z} \\ \frac{dV_{3z}}{dx} & \frac{dV_{3z}}{dy} & \frac{dV_{3z}}{dz} & \frac{dV_{3z}}{d\phi_x} & \frac{dV_{3z}}{d\phi_y} & \frac{dV_{3z}}{d\phi_z} \\ \frac{dV_{4z}}{dx} & \frac{dV_{4z}}{dy} & \frac{dV_{4z}}{dz} & \frac{dV_{4z}}{d\phi_x} & \frac{dV_{4z}}{d\phi_y} & \frac{dV_{4z}}{d\phi_z} \\ \frac{dV_{5z}}{dx} & \frac{dV_{5z}}{dy} & \frac{dV_{5z}}{dz} & \frac{dV_{5z}}{d\phi_x} & \frac{dV_{5z}}{d\phi_y} & \frac{dV_{5z}}{d\phi_z} \end{pmatrix} \cdot \begin{pmatrix} \delta x \\ \delta y \\ \delta z \\ \delta\phi_x \\ \delta\phi_y \\ \delta\phi_z \end{pmatrix} \quad (4.27)$$

By inserting only the non-zero coefficients of the augmented matrix $\mathbf{A}$ obtain from the linear fitting of the change in induced voltages with respect to change in position and orientation

one can write

$$\begin{pmatrix} \delta V_{1x} \\ \delta V_{2x} \\ \delta V_{3x} \\ \delta V_{4x} \\ \delta V_{5x} \\ \delta V_{1y} \\ \delta V_{2y} \\ \delta V_{3y} \\ \delta V_{4y} \\ \delta V_{5y} \\ \delta V_{1z} \\ \delta V_{2z} \\ \delta V_{3z} \\ \delta V_{4z} \\ \delta V_{5z} \end{pmatrix} = \begin{pmatrix} \frac{dV_{1x}}{dx} & 0 & 0 & 0 & \frac{dV_{1x}}{d\phi_y} & 0 \\ 0 & 0 & \frac{dV_{2x}}{dz} & 0 & \frac{dV_{2x}}{d\phi_y} & 0 \\ 0 & 0 & \frac{dV_{3x}}{dz} & 0 & \frac{dV_{3x}}{d\phi_y} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{dV_{1y}}{dy} & 0 & \frac{dV_{1y}}{d\phi_x} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{dV_{4y}}{dz} & \frac{dV_{4y}}{d\phi_x} & 0 & 0 \\ 0 & 0 & \frac{dV_{5y}}{dz} & \frac{dV_{5y}}{d\phi_x} & 0 & 0 \\ 0 & 0 & \frac{dV_{1z}}{dz} & \frac{dV_{1z}}{d\phi_x} & \frac{dV_{1z}}{d\phi_y} & 0 \\ \frac{dV_{2z}}{dx} & 0 & 0 & 0 & \frac{dV_{2z}}{d\phi_y} & 0 \\ \frac{dV_{3z}}{dx} & 0 & 0 & 0 & \frac{dV_{3z}}{d\phi_y} & 0 \\ 0 & \frac{dV_{4z}}{dy} & 0 & \frac{dV_{4z}}{d\phi_x} & 0 & 0 \\ 0 & \frac{dV_{5z}}{dy} & 0 & \frac{dV_{5z}}{d\phi_x} & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \delta x \\ \delta y \\ \delta z \\ \delta\phi_x \\ \delta\phi_y \\ \delta\phi_z \end{pmatrix} \quad (4.28)$$

There is no change in induced voltage in the coils due to rotation about any axis during execution of z-gradient. On the other hand the change in induced voltage in the coils due

to a rotation about the x- and y-axes because of the application of the z-gradient is non-linear (quadratic) with change in orientation. These smaller coefficients are therefore further excluded for rotation about z-axis during application of z-gradient, yielding a new matrix as follows:

$$\begin{pmatrix} \delta V_{1x} \\ \delta V_{2x} \\ \delta V_{3x} \\ \delta V_{1y} \\ \delta V_{4y} \\ \delta V_{5y} \\ \delta V_{1z} \\ \delta V_{2z} \\ \delta V_{3z} \\ \delta V_{4z} \\ \delta V_{5z} \end{pmatrix} = \begin{pmatrix} \frac{dV_{1x}}{dx} & 0 & 0 & 0 & \frac{dV_{1x}}{d\phi_y} \\ 0 & 0 & \frac{dV_{2x}}{dz} & 0 & \frac{dV_{2x}}{d\phi_y} \\ 0 & 0 & \frac{dV_{3x}}{dz} & 0 & \frac{dV_{3x}}{d\phi_y} \\ 0 & \frac{dV_{1y}}{dy} & 0 & \frac{dV_{1y}}{d\phi_x} & 0 \\ 0 & 0 & \frac{dV_{4y}}{dz} & \frac{dV_{4y}}{d\phi_x} & 0 \\ 0 & 0 & \frac{dV_{5y}}{dz} & \frac{dV_{5y}}{d\phi_x} & 0 \\ 0 & 0 & \frac{dV_{1z}}{dz} & \frac{dV_{1z}}{d\phi_x} & \frac{dV_{1z}}{d\phi_y} \\ \frac{dV_{2z}}{dx} & 0 & 0 & 0 & \frac{dV_{2z}}{d\phi_y} \\ \frac{dV_{3z}}{dx} & 0 & 0 & 0 & \frac{dV_{3z}}{d\phi_y} \\ 0 & \frac{dV_{4z}}{dy} & 0 & \frac{dV_{4z}}{d\phi_x} & 0 \\ 0 & \frac{dV_{5z}}{dy} & 0 & \frac{dV_{5z}}{d\phi_x} & 0 \end{pmatrix} \cdot \begin{pmatrix} \delta x \\ \delta y \\ \delta z \\ \delta \phi_x \\ \delta \phi_y \end{pmatrix} \quad (4.29)$$

The slopes are obtained by processing inconsistent linear system for a meaningful least square solution [118]. By manipulating the above system of linear equations one can obtain $\mathbf{r}$ i.e.

the change in position of the coils as well as amount of rotation of the coils about the three mutually perpendicular (x, y and z) axes. The results are discussed later on in Section 4.4.

4.3.2 Method 2: Arbitrary Coils in Space

In this method an arbitrary-shaped coil is generated in space. The change in induced voltage has been calculated due to sequential translations and rotations during execution of x-, y- and z-gradients. By manipulating the calculated voltage, a linear model is established and movement parameters are estimated. The arbitrary coils considered here provide a better representation of the head-mounted coils that would be used in real experiments.

Translation of the coils

A coil is generated on the surface of a sphere of radius 40 mm and the effect of incremental translations is then calculated. Specially, I calculated the change in voltage for translation along the the x-, y- and z-axes by 10 mm in 40 steps due to the application of three time-varying gradients (g_x , g_y , and g_z). The results in Figs. 4.21(a), 4.22(a) and 4.23(a) show the outcome for translation of the spherical coils along the x-, y- and z-axis. respectively. Figs. 4.21(b), 4.22(b) and 4.23(b) show the changes in induced voltage in the coils due to translations during execution of x-, y- and z-gradients. The changes in voltage are linear with respect to the changes in position.

Rotation of the Coils

In Eq. 3.68, F is a function that calculates the voltages V plus sensitivity vector $\mathbf{S}$ and location vector $\mathbf{r}_0$ for a general coil. A coil is generated on the surface of a sphere of radius 40 mm to calculate incremental rotations- i.e. change in voltage for rotation about the x-, y- and z-axes by 5° in 40 steps due to the application of three time-varying gradients (g_x , g_y , and g_z). Fig. 4.24(a) shows the rotation of the coils about the x-axis and the change of the location and sensitivity vectors of the coils, whereas Fig. 4.24(b) shows the change in

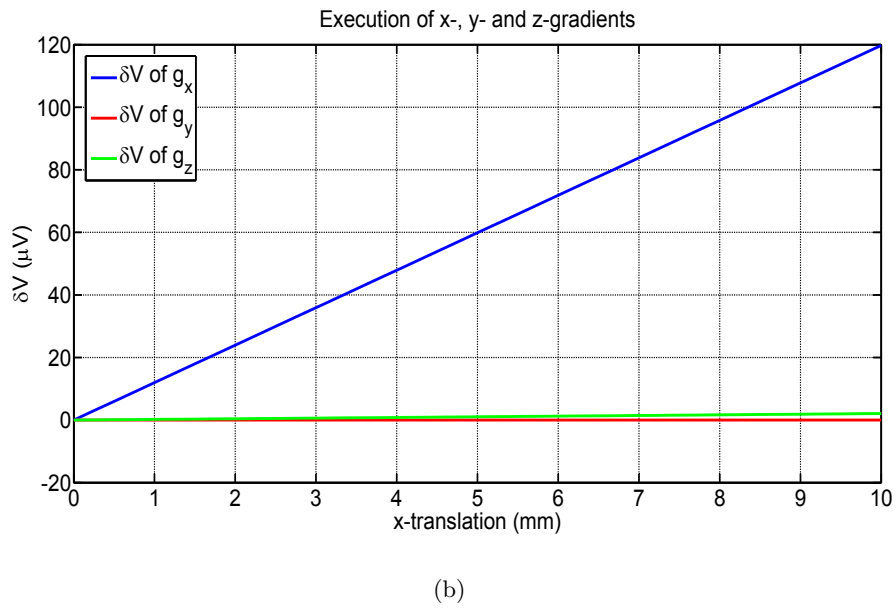
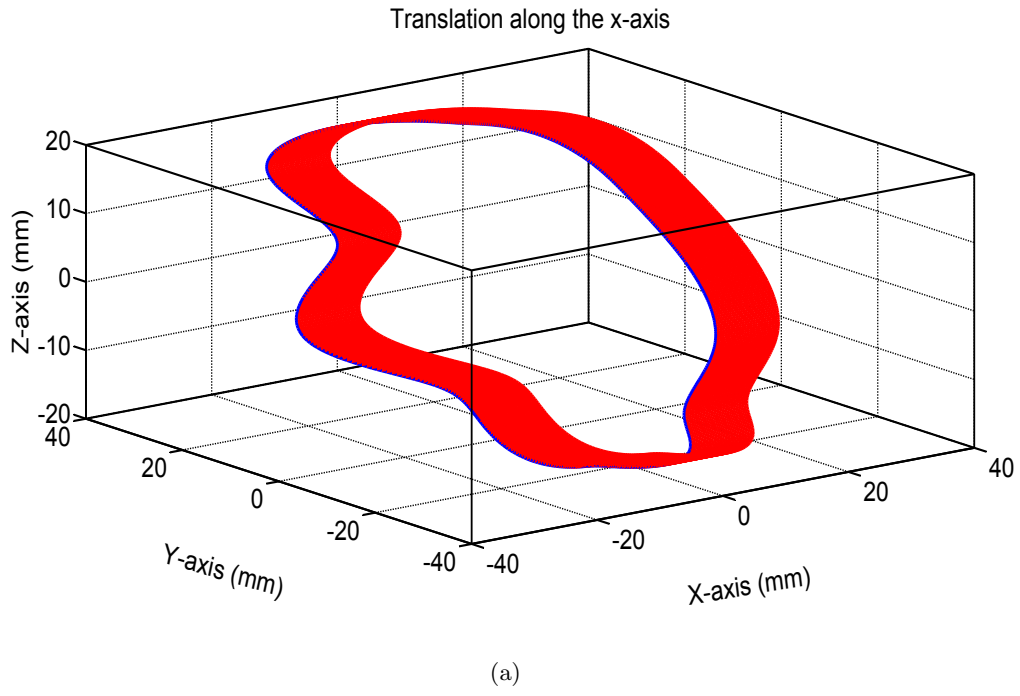
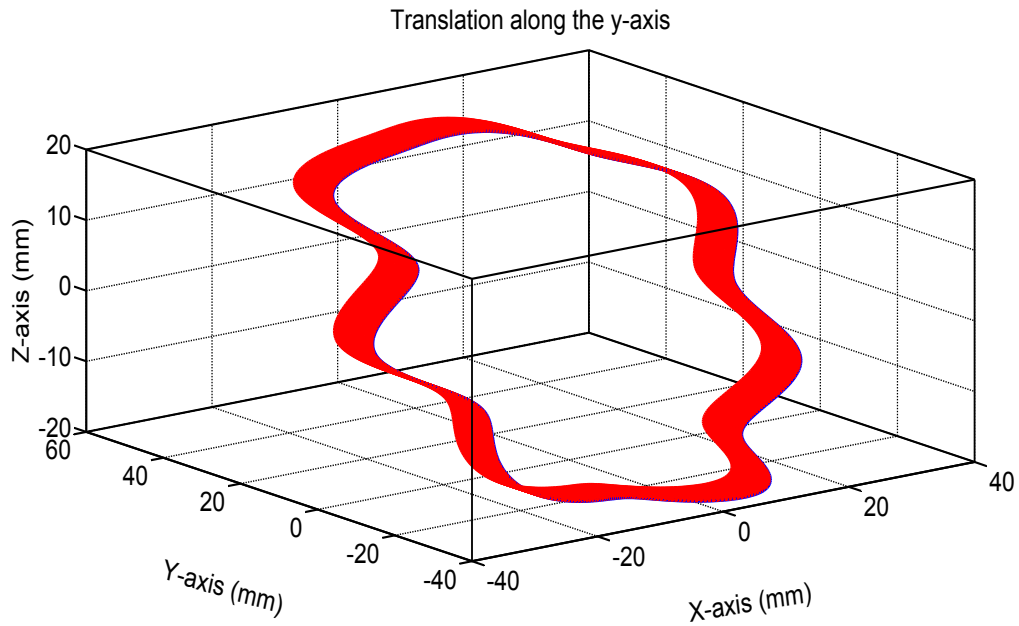
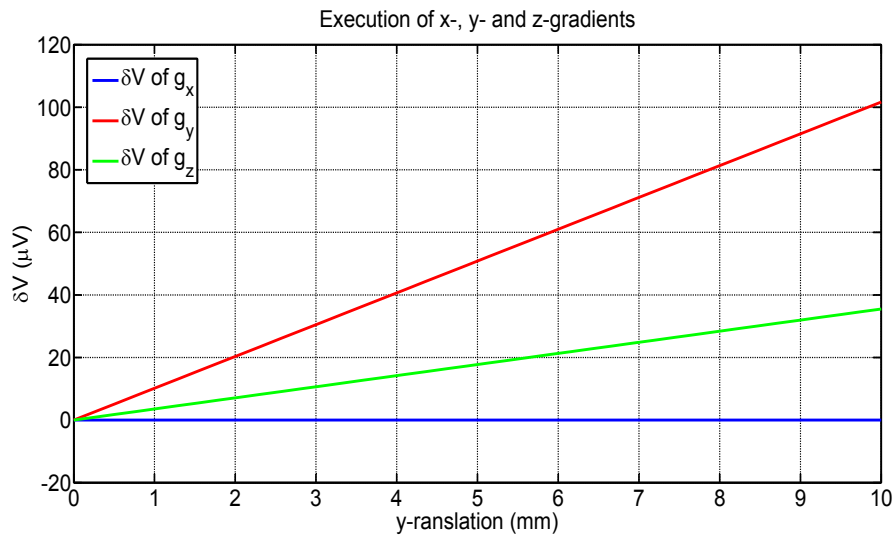


Figure 4.21: Figures representing translation along the x-axis. (a) A coil shifted 10 mm in 40 steps along the x-axis. (b) The change in induced voltage in a coil due to sequential translations up to 10 mm along the x-axis during execution of the x-, y- and z-gradients.

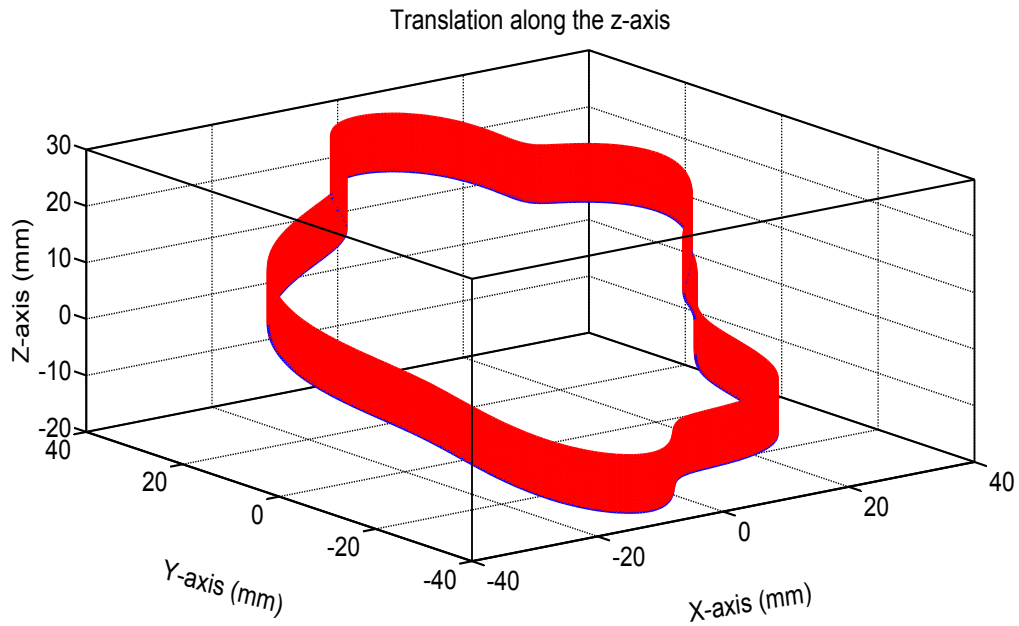


(a)

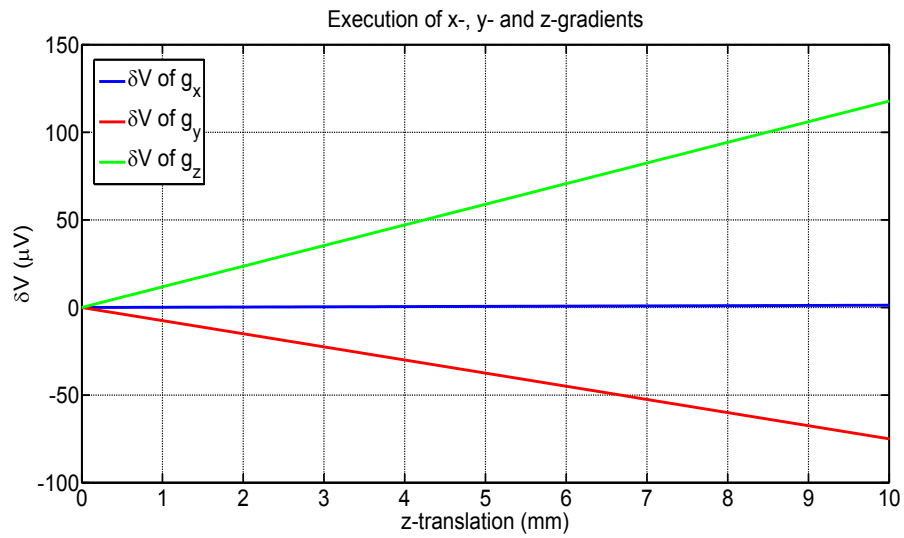


(b)

Figure 4.22: Data represents translation along the y-axis. (a) A coil shifted 10 mm in 40 steps along the x-axis. (b) The change in induced voltage in a coil due to sequential translations up to 10 mm along the y-axis during execution of the x-, y- and z-gradients.



(a)



(b)

Figure 4.23: Data represents translation along the z-axis. (a) A coil shifted 10 mm in 40 steps along the z-axis. (b) The change in induced voltage in a coil due to sequential translations up to 10 mm along the z-axis during execution of the x-, y- and z-gradients.

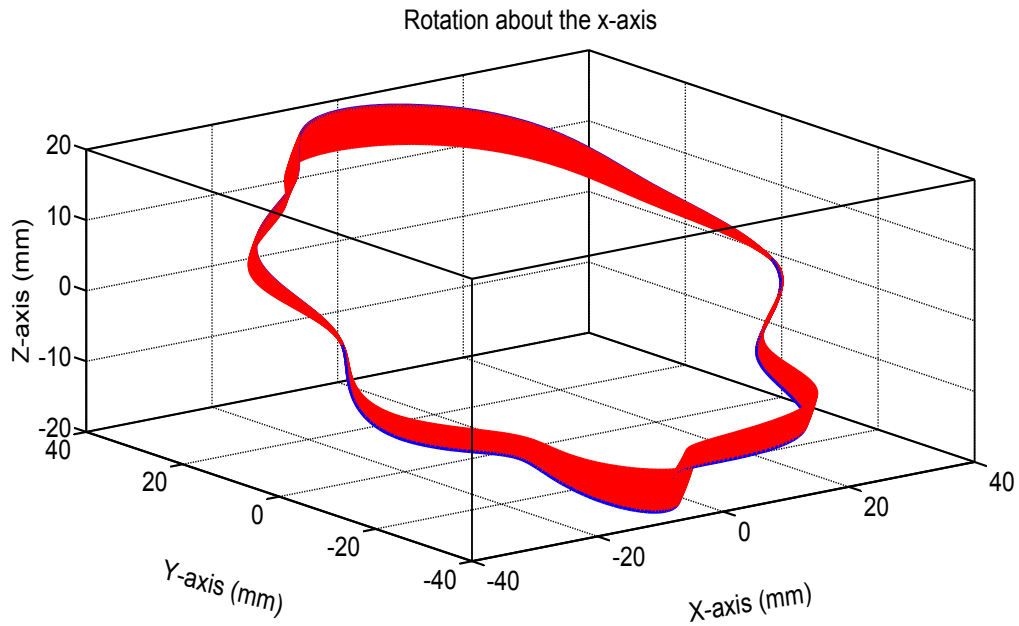
induced voltage with respect to the angle of x-rotation. Fig. 4.25(a) shows the rotation of the coil about the y-axis and the change of the location and sensitivity vectors of the coil, whereas Fig. 4.25(b) shows the change in induced voltage in the coil with respect to the angle of y-rotation. Similarly, Fig. 4.26(a) shows the rotation of the coil about the z-axis and also represents the change of the location and sensitivity vector. Fig. 4.26(b) indicates the voltage change with respect to the angle of z-rotations.

Five Coils on the Surface of a Sphere

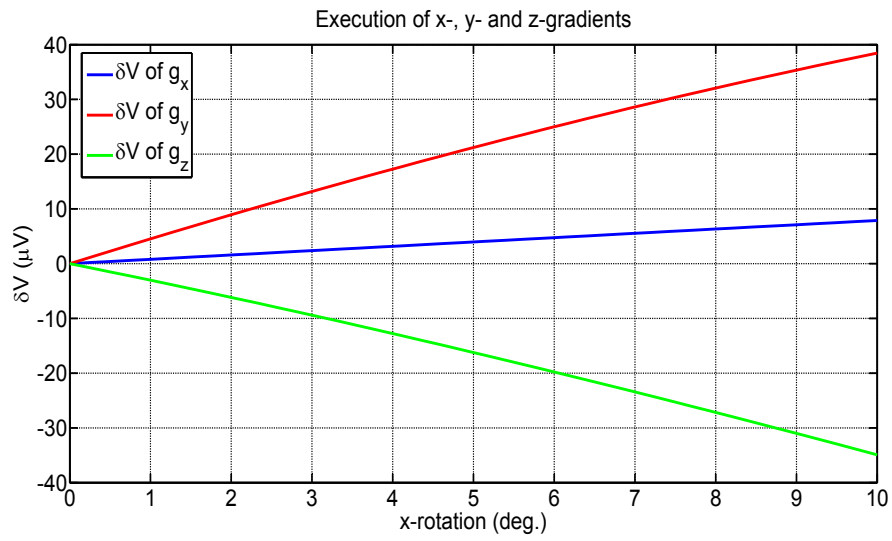
The five coils are defined on the surface of a standard sphere of radius 80 mm having one turn each, 2 coils are arranged with their sensitivity vectors along the RL direction; 2 along the AP and the one along the FH direction. Figs. 4.27(a), 4.28(a), 4.29(a), 4.30(a), 4.31(a), and 4.32(a), show the coils attached to a standard head size spherical head phantom having a radius of 80 mm. The voltages in all five coils were calculated for the execution of the gradients g_x , g_y and g_z as the coils are translated through 10 mm in 40 steps along the x-, y- and z-directions and rotated through 5° in 40 steps about the x-, y- and z-axes and. In each case the variation of the voltage with angle or distance is fitted to a 2^{nd} order polynomial of the form $V = a_1\delta r$ for translation and $V = a_1\delta\phi_x + a_2\delta\phi_x^2$, $V = a_1\delta\phi_y + a_2\delta\phi_y^2$, and $V = a_1\delta\phi_z + a_2\delta\phi_z^2$ for rotation, where ϕ_x , ϕ_y , and ϕ_z are rotation angles.

The following figures show the effect of translation and rotation on the coils, and the change in induced voltages in the coils for the different gradients due to translation along the x-axis (Fig. 4.27), y-axis (Fig. 4.28) & z-axis (Fig. 4.29) and rotation about the x-axis (Fig. 4.30), y-axis (Fig. 4.31) and z-axis (Figs. 4.32).

The 2^{nd} order polynomial fits of the changes in induced voltage of the coils are shown in the corresponding figures and the fit parameters a_1 and a_2 are summarized in the Tables 4.1 & 4.2 respectively.



(a)



(b)

Figure 4.24: Data represents rotation about the x-axis. (a) A coil rotates 10° in 40 steps about the x-axis. (b) The change in induced voltage in a coil due to successive rotations up to 10° during execution of the x-, y- and z-gradients.

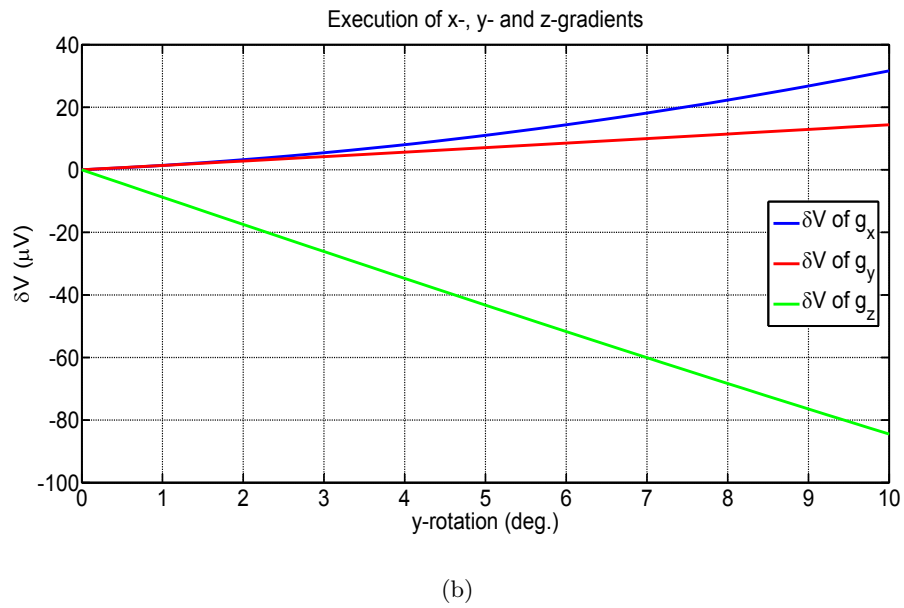
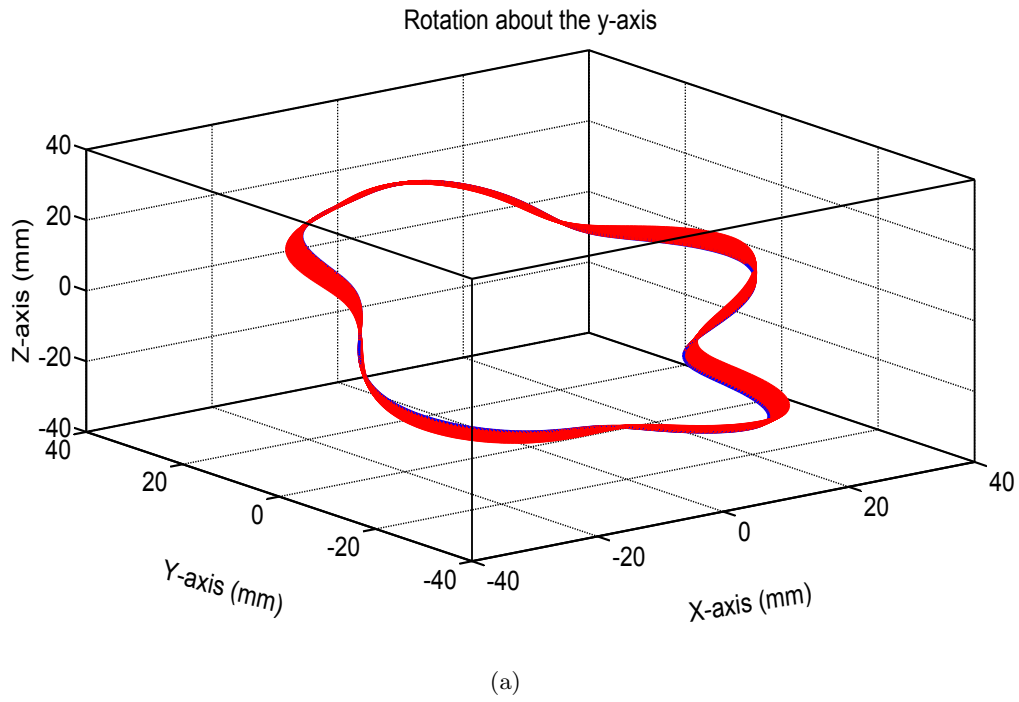


Figure 4.25: Data represents rotation about the y-axis. (a) A coil rotates 10° in 40 steps about the y-axis. (b) The change in induced voltage in a coil due to successive rotations up to 10° during execution of x-, y- and z-gradients.

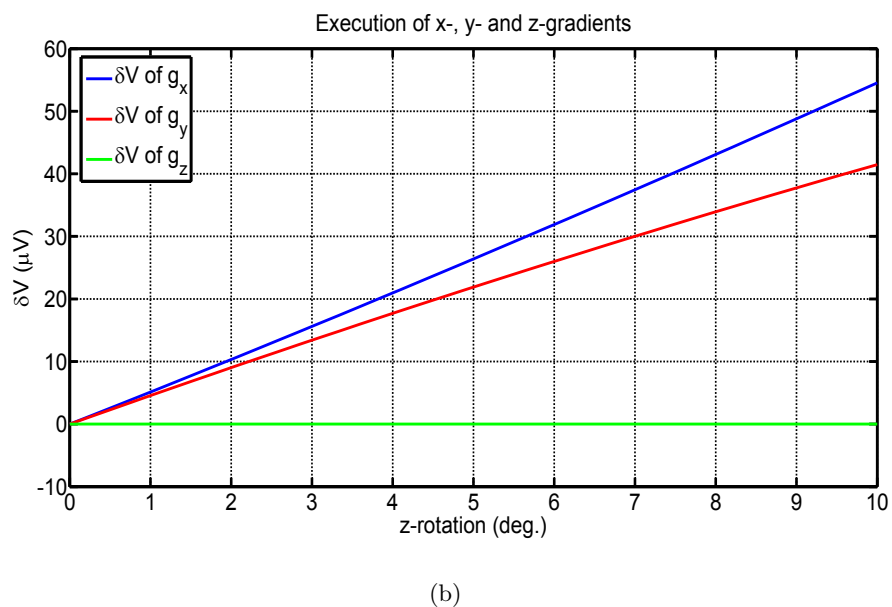
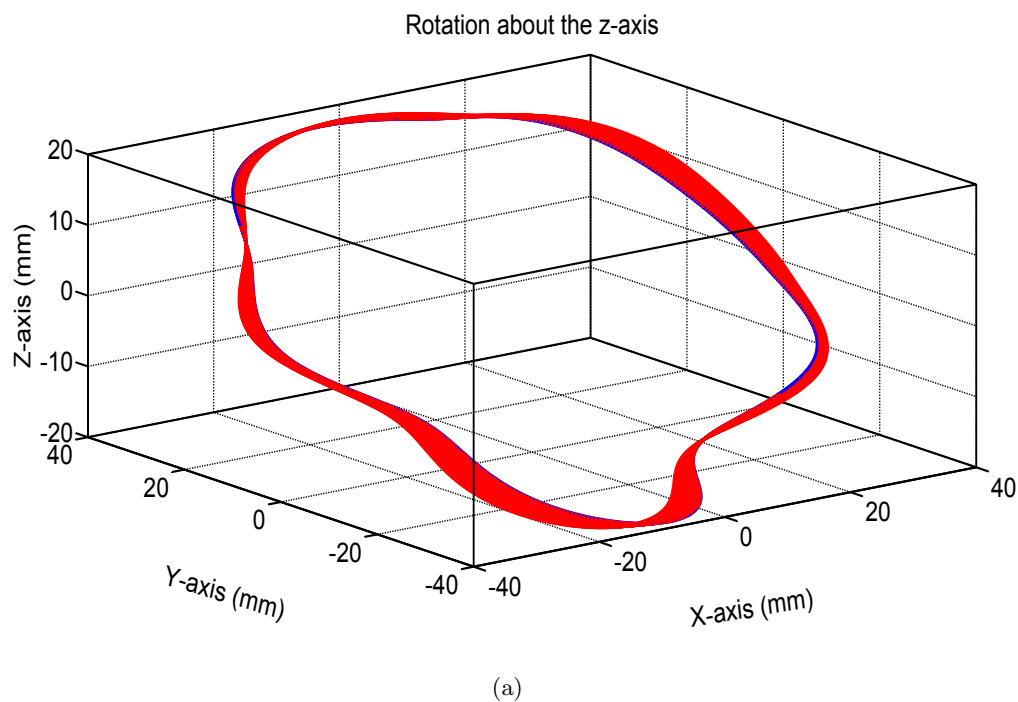
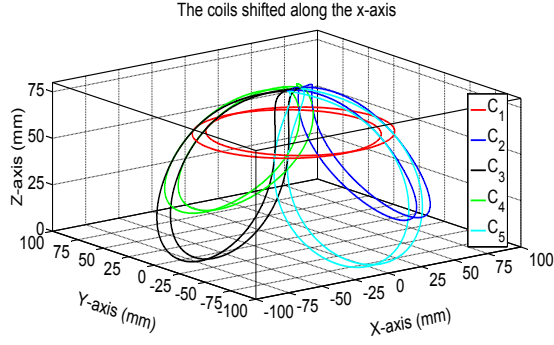
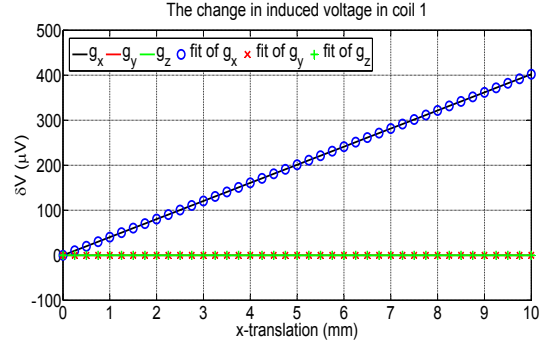


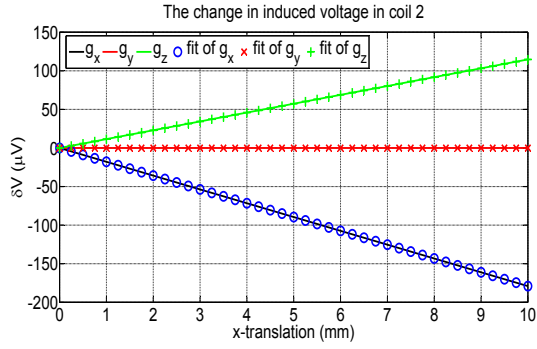
Figure 4.26: Data represents rotation about the z-axis. (a) A coil rotates 10° in 40 steps about the z-axis. (b) The change in induced voltage in a coil due to successive rotations up to 10° during the application of the x-, y- and z-gradients.



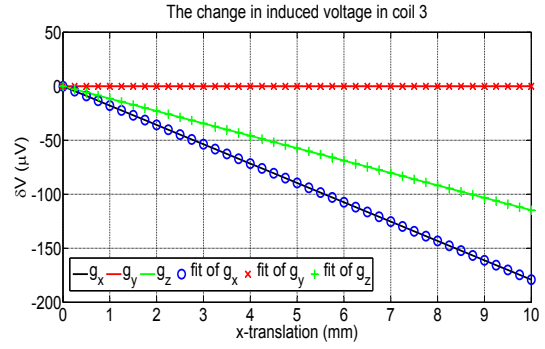
(a)



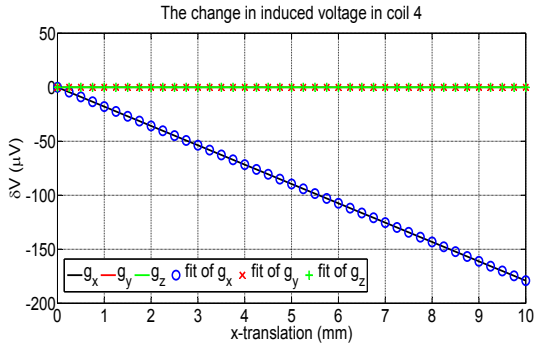
(b)



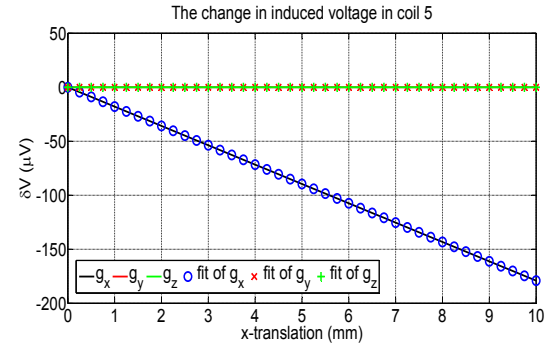
(c)



(d)



(e)



(f)

Figure 4.27: Data represents sequential translation along the x-axis during execution of x-, y- and z-gradients. The coils shifted 10 mm along the x-axis are shown in (a). The changes in induced voltage in the five coils (1-5) and their 2nd order polynomial fits are shown in (b), (c), (d), (e) and (f) respectively.

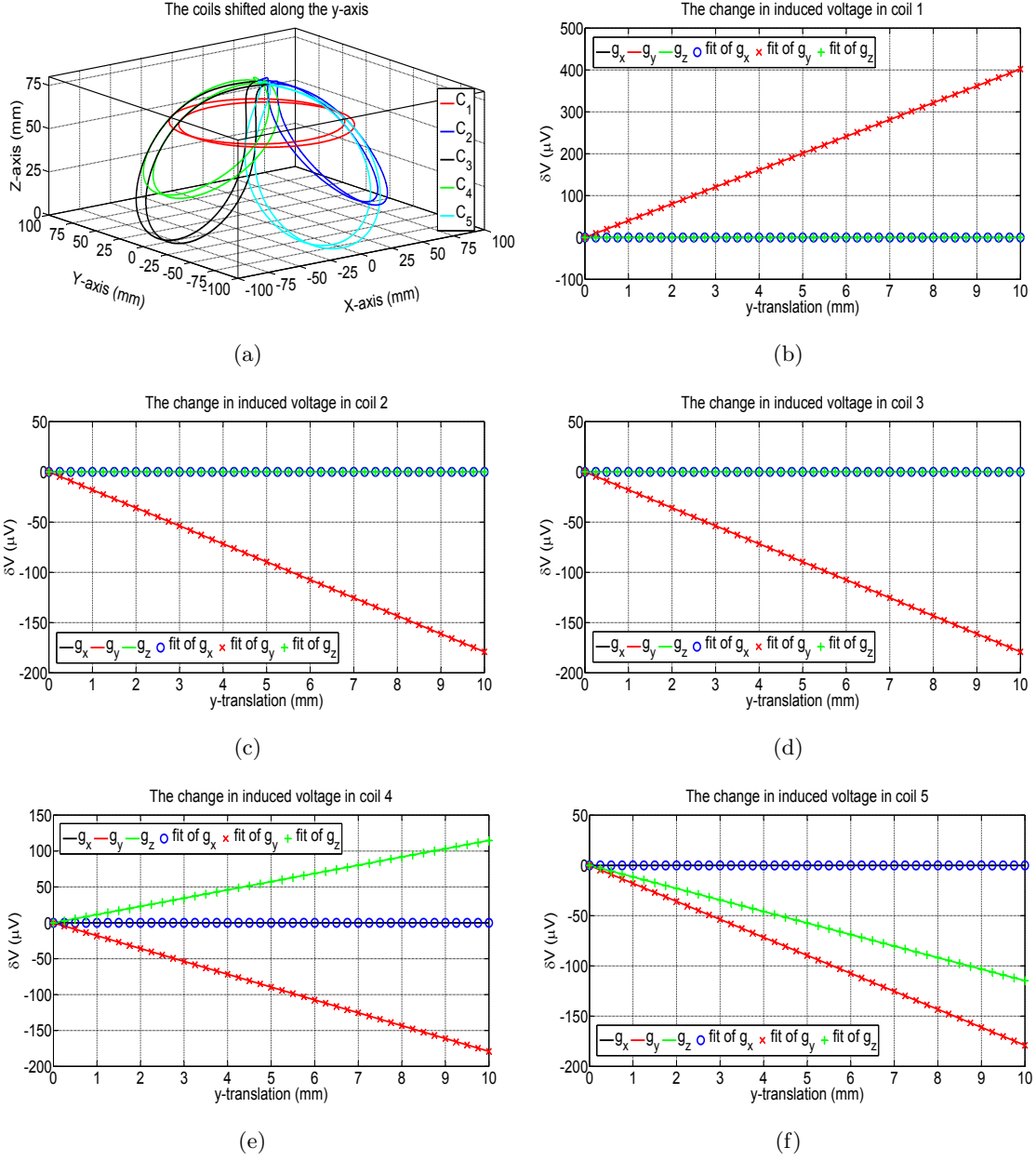


Figure 4.28: Data represents sequential translation along the y-axis during execution of x-, y- and z-gradients. The coils shifted 10 mm along the y-axis are shown in (a). The changes in induced voltage in the five coils (1-5) and their 2nd order polynomial fits are shown in (b), (c), (d), (e) and (f) respectively.

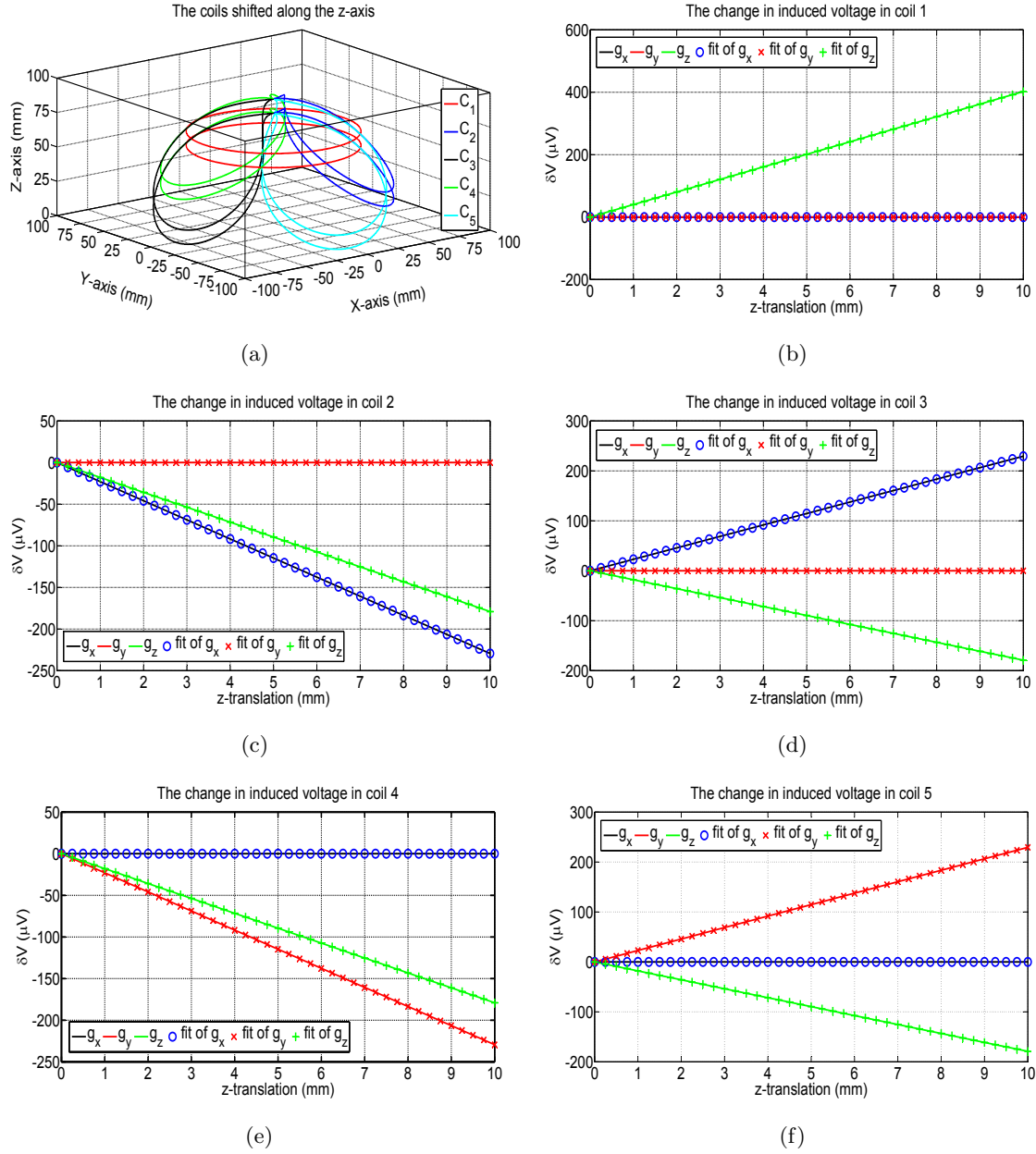


Figure 4.29: Data represents sequential translation along the z-axis during execution of x-, y- and z-gradients. The coils shifted 10 mm along the z-axis are shown in (a). The changes in induced voltage in the five coils (1-5) and their 2nd order polynomial fits are shown in (b), (c), (d), (e) and (f) respectively.

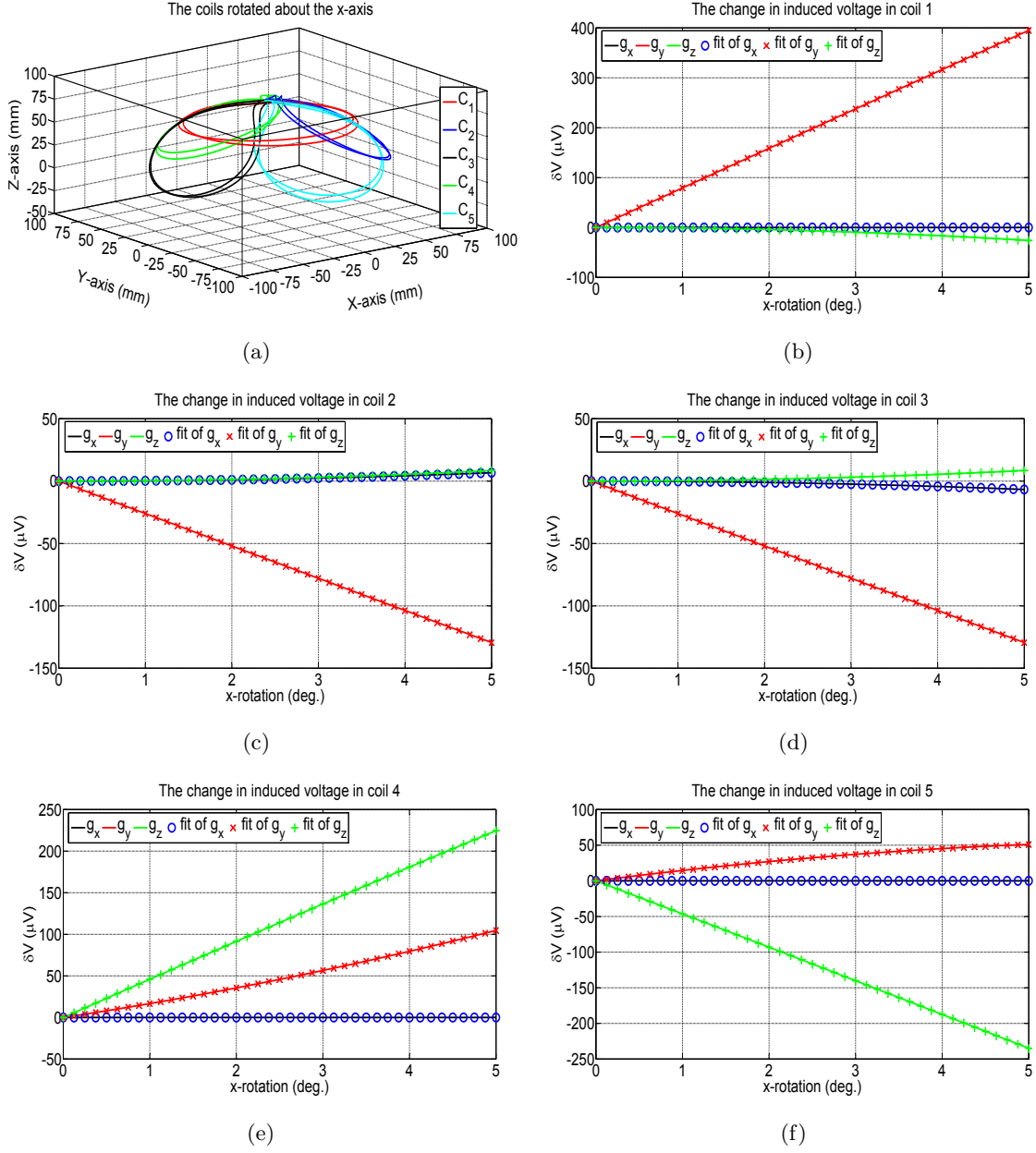
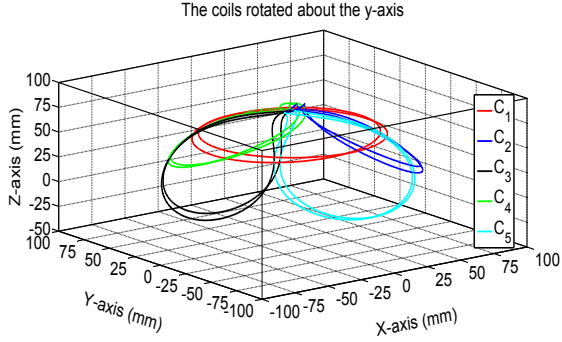
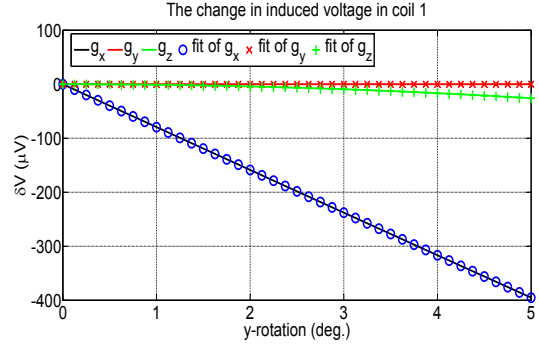


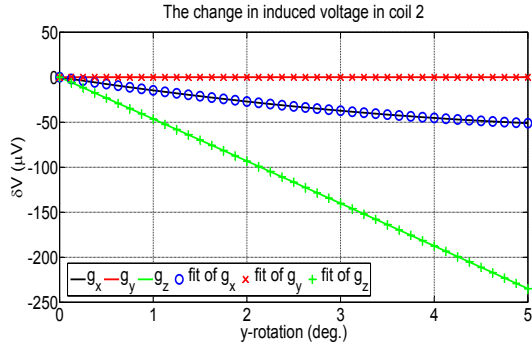
Figure 4.30: Data represents sequential rotation of the coils about the x-axis during execution of x-, y- and z-gradients. The coils rotated 10° in 40 steps about the x-axis are shown in (a). The changes in induced voltage in the five coils (1-5) and their polynomial fits are shown in (b), (c), (d), (e) and (f) respectively.



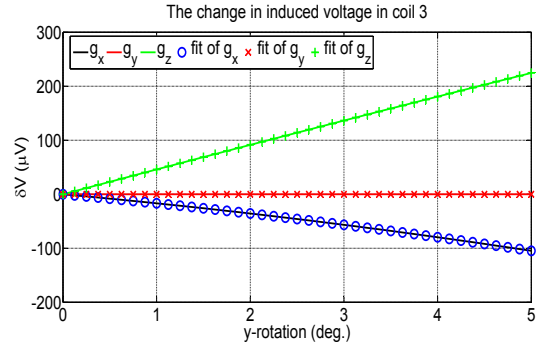
(a)



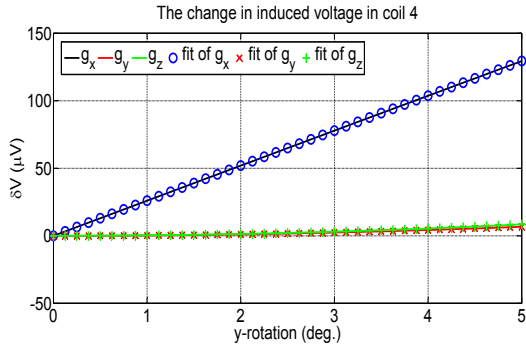
(b)



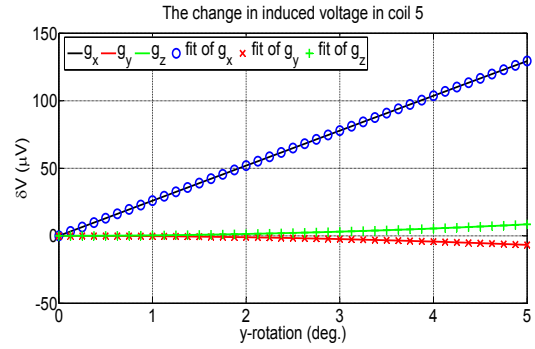
(c)



(d)



(e)



(f)

Figure 4.31: Data represents sequential rotation of the coils about the y-axis during execution of x-, y- and z-gradients. The coils rotated 10° in 40 steps about the y-axis are shown in (a). The changes in induced voltage in the five coils (1-5) and their polynomial fits are shown in (b), (c), (d), (e) and (f) respectively.

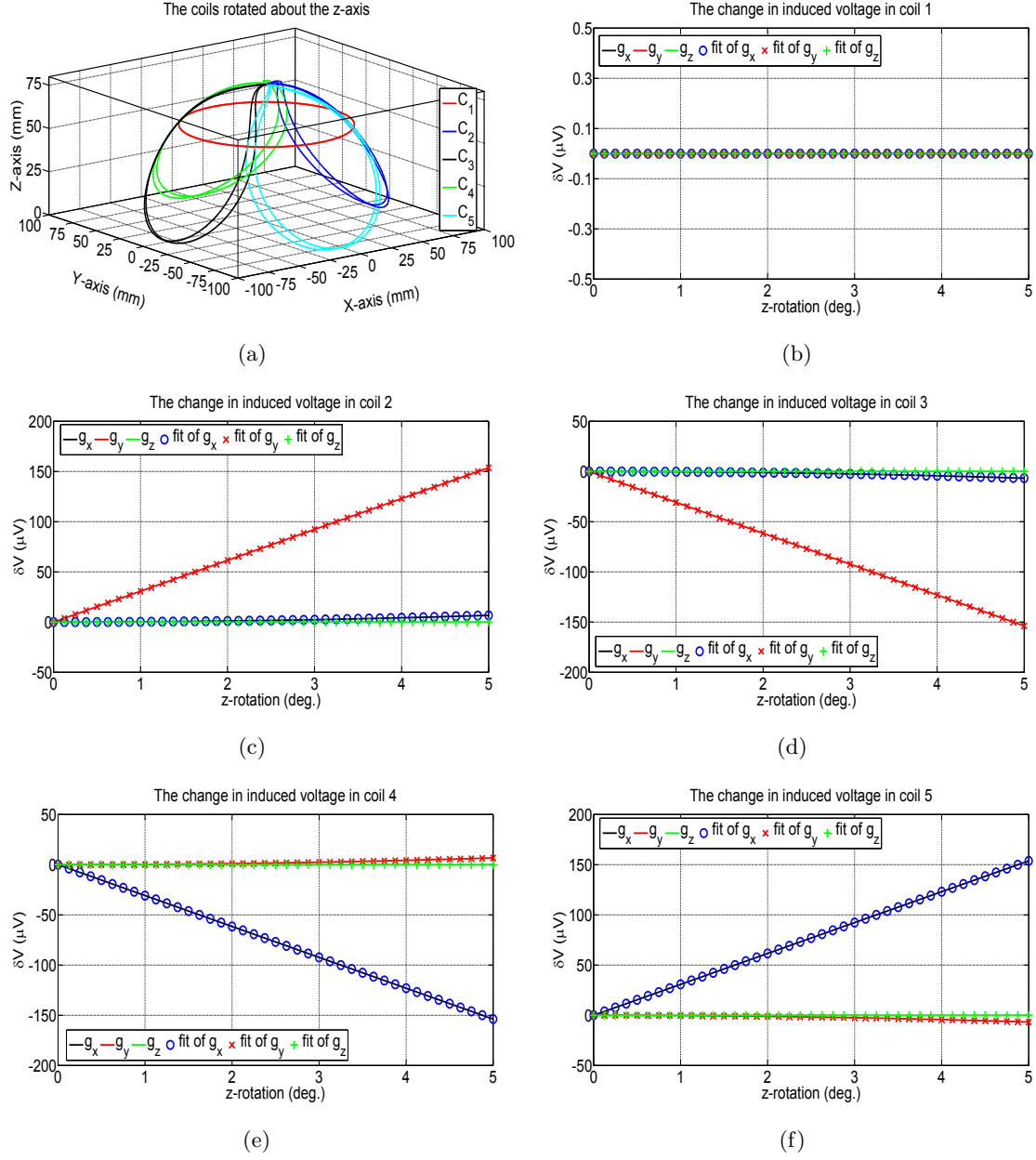


Figure 4.32: Data represents sequential rotation of the coils about the z-axis during execution of x-, y- and z-gradients. The coils rotated 10° in 40 steps about the z-axis are shown in (a). The changes in induced voltage in the five coils (1-5) and their polynomial fits are shown in (b), (c), (d), (e) and (f) respectively..

Coil and Gradient	$\frac{dV}{d\phi_x}$ ($\frac{\mu V}{deg.}$)	$\frac{dV}{d\phi_y}$ ($\frac{\mu V}{deg.}$)	$\frac{dV}{d\phi_z}$ ($\frac{\mu V}{deg.}$)	$\frac{dV}{dx}$ ($\frac{\mu V}{mm}$)	$\frac{dV}{dy}$ ($\frac{\mu V}{mm}$)	$\frac{dV}{dz}$ ($\frac{\mu V}{mm}$)
C_{1gx}	0	-79.640	0	40.211	0	0
C_{2gx}	0.001	-15.684	0.001	-17.901	0	-22.940
C_{3gx}	-0.001	-15.709	-0.001	-17.901	0	22.940
C_{4gx}	0	26.085	-30.807	-17.901	0	0
C_{5gx}	0	26.085	30.807	-17.901	0	0
C_{1gy}	79.640	0	0	0	40.211	0
C_{2gy}	-26.085	0	30.807	0	-17.901	0
C_{3gy}	-26.085	0	-30.807	0	-17.901	0
C_{4gy}	15.709	0.001	0.001	0	-17.901	-22.940
C_{5gy}	15.684	-0.001	-0.001	0	-17.901	22.940
C_{1gz}	-0.012	-0.012	0	0	0	40.211
C_{2gz}	0.004	-46.317	0	11.470	0	-17.901
C_{3gz}	-0.004	46.312	0	-11.470	0	-17.901
C_{4gz}	46.312	0.004	0	0	11.470	-17.901
C_{5gz}	-46.317	0.004	0	0	-11.470	-17.901

Table 4.1: The fit parameters a_1 obtained from 2nd order polynomial fitting due to translations and rotations of the coils during execution of the x-,y- and z-gradients.

Coil and Gradient	$\frac{dV}{d\phi_x}$ $(\frac{\mu V}{deg.})$	$\frac{dV}{d\phi_y}$ $(\frac{\mu V}{deg.})$	$\frac{dV}{d\phi_z}$ $(\frac{\mu V}{deg.})$	$\frac{dV}{dx}$ $(\frac{\mu V}{mm})$	$\frac{dV}{dy}$ $(\frac{\mu V}{mm})$	$\frac{dV}{dz}$ $(\frac{\mu V}{mm})$
C_{1gx}	0	0.121	0	0	0	0
C_{2gx}	0.268	1.094	0.268	0	0	0
C_{3gx}	-0.268	-1.046	-0.268	0	0	0
C_{4gx}	0	-0.040	0.012	0	0	0
C_{5gx}	0	-0.040	-0.012	0	0	0
C_{1gy}	-0.121	0	0	0	0	0
C_{2gy}	0.040	0	-0.012	0	0	0
C_{3gy}	0.040	0	0.012	0	0	0
C_{4gy}	1.046	0.268	0.268	0	0	0
C_{5gy}	-1.094	-0.268	-0.268	0	0	0
C_{1gz}	-1.035	-1.035	0.000	0	0	0
C_{2gz}	0.339	-0.134	0	0	0	0
C_{3gz}	0.339	-0.274	0	0	0	0
C_{4gz}	-0.274	0.339	0	0	0	0
C_{5gz}	-0.134	0.339	0	0	0	0

Table 4.2: The fit parameters a_2 obtained from 2nd order polynomial fitting due to translations and rotations of the coils during execution of the x-,y- and z-gradients.

4.3.3 Method 3: Numerical Simulation Based on Theory

Algebraic equations are described in Chapter 3. But it is not always possible to solve all algebraic equations analytically. For the five coils there are five equations (one for each) which form a system of linear equations having six unknown (x , y , z , ϕ_x , ϕ_y , and ϕ_z). It is not possible to solve these equations analytically in any exact way. Fortunately, such equations can often be solved numerically with computation in an easy way. To do so MatLab has a built in function called *fsolve*, which is a routine that solves equations of the form $f(x) = 0$ where one has to supply the function $f(x)$. Because it employs an iterative method, one has to give the routine a starting value, x_0 (a smart initial guess may reduce iteration time and also gives precise results, however, in our case initial guess was zero and a range of different values for x_0 also gives a good results), which is an initial guess at the value of the solution.

A program is written based on the derived equation 3.48 in Chapter 3 to calculate the change in induced voltage in a coil due to change in position and orientation of the coil when it is placed in a time-varying field gradient. By using this program the change in induced voltage in each coil of a system of five coils is calculated due to translation as well as rotation. It is quite interesting that there is no change in induced voltage in the coils (in a system of five coils) due to rotation about z -axis. As a result the degrees of freedom (unknown variables) reduce to five.

The change in position and orientation of the coils can be estimated from the calculated voltage. For this purpose, a system of linear equation $\delta V = A\delta r$ is formed and an standard MatLab command 'pinv' (pseudo-inverse) is used to find the values of $\mathbf{r}$ ($\delta x, \delta y, \delta z, \delta\phi_x, \delta\phi_y$).

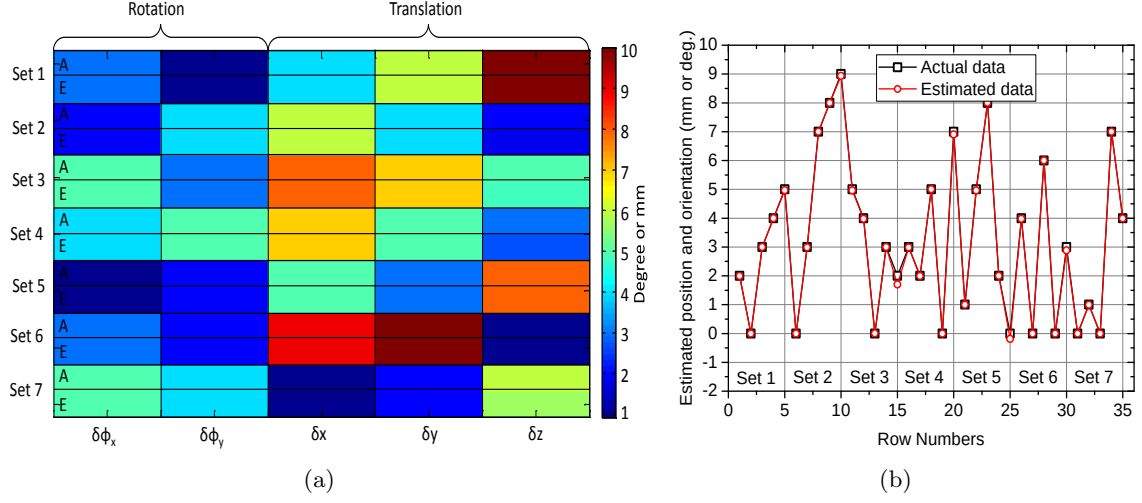


Figure 4.33: The actual movement parameters and the parameters calculated from numerical simulation by calculating the change in induced voltage in the coils due to translation and rotation of the coils during the application of x-, y- and z-gradients. (a) Data shows the actual and estimated movement parameters where there is no zero terms in the actual movements. (b) Data represents actual and estimated movement parameters where there are a few zero terms in the actual movements.

4.4 Results

4.4.1 Results: Planar Coils

The changes in induced voltages in the 5-coils during execution of the 3-gradients due to rotations and translations are calculated. By using the system of linear equations described in Eq. 4.29 in Sub-section 4.3.1, the five movement parameters are estimated. Comparison between the actual movements and the movements calculated from simulation by inverting the matrix equation are shown in Fig. 4.33. Fig. 4.33(a) shows the 7 sets of estimated parameters where there are no zero terms in the actual movements in any direction. On the other hand, in Fig. 4.33(b) a few zero terms in the actual movements in translation or rotation are included. In both cases, actual and estimated movement parameters have a reasonably good agreement.

Without adding any noise voltage to the changes in induced voltage, the estimated

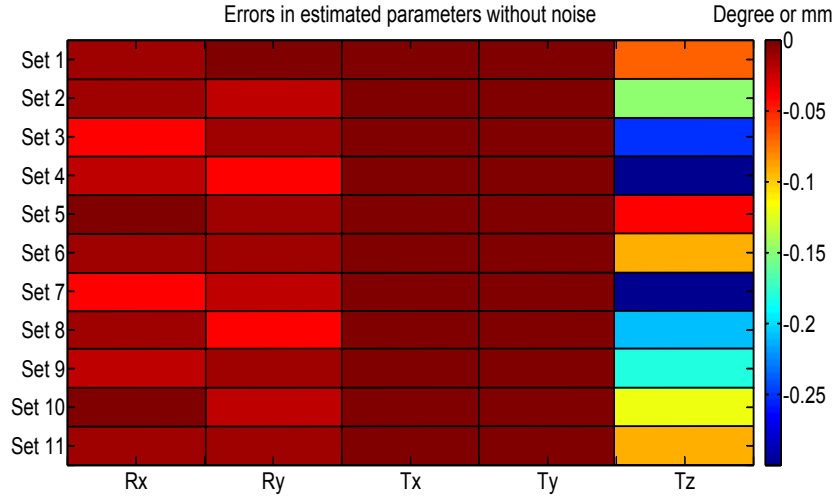


Figure 4.34: Errors in the estimated movement parameters from calculated induced voltages in the coils due to translation and rotation without adding noise voltages.

movements parameters have maximum errors in translations (~ 0.3 mm) and in rotations ($\sim 0.05^\circ$), as shown in Fig. 4.34. In this case the actual translation is 10 mm and rotation is 5° .

For our specification of the coils and the gradient strength, the change in induced voltage in the different coils and gradients was around 50-700 μV due to 10 mm translation or 5° rotation. To check the effect of noise voltage 1-20 μV random noise voltage is added to the change in induced voltage in the coils. A random noise voltage, 0-20 μV is added to the calculated voltage, and as a result errors are observed in the estimated parameters that are shown in Fig. 4.35(a). A reduced random voltage, 0-10 μV is added and the errors in estimated parameters are calculated and represented in Fig. 4.35(b). In spite of the addition of a random noise voltage (0-10 μV), the voltage data gives us a good estimation of the change in position and orientation of the coils. Even after addition of noise voltage (0-20 μV) estimated parameters remain remarkably meaningful.

For planar coils, there is no voltage induced in the coils 4 and 5 during execution of an x-gradient and in the coils 1 and 3 during execution of an y-gradient due to translation

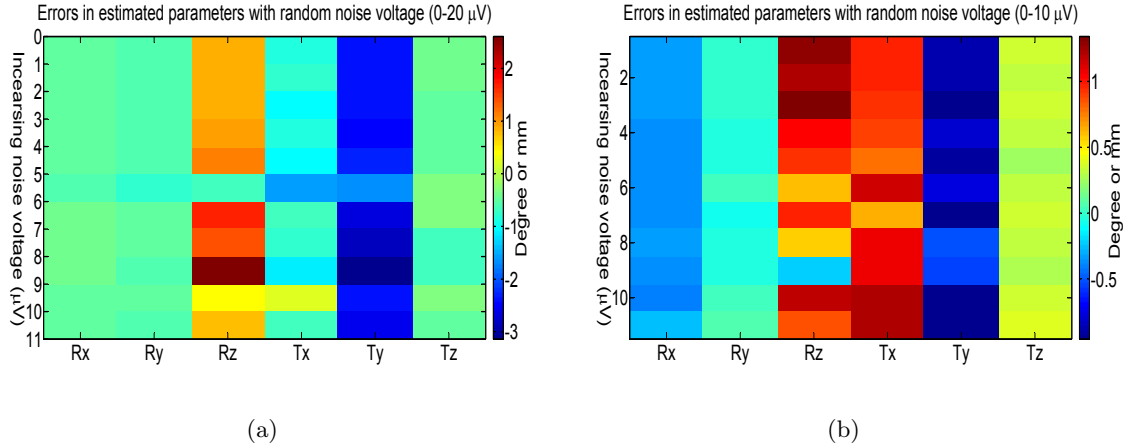


Figure 4.35: Estimated errors in position and orientation from calculated induced voltages due to translation and rotation by high noise level (0-20 μV) in (a) and low noise level (0-10 μV) in (b).

along the x-, y- and z-axes and rotation about the x-, y- and z-axes. Moreover, no voltage is induced in any coils due to z-rotation.

4.4.2 Results: Arbitrary Coils in space

Voltage variations are different for different types of movement. The question is how linear is this variation? To evaluate this question I use a sub-routine that generates a general movement. It passes the programme the translation vector and the rotation vectors (order Rx Ry Rz) and then compares the sum of voltages for individual position changes to the voltage change produced by a single composite position change.

The results of this comparison are shown in Fig. 4.36 indicating very good agreement for small rotations (Fig. 4.36(a)), and reasonable agreement for a rotations up to 10° (4.36(b)); (somewhat independent of translation) e.g. 5° and 10 mm (green = single added; blue = combined; red = difference) and the ratio which is shown in Fig. 4.36.

The data shows that the sum of the voltages induced in the coils due to individual rotation and translation is almost equal to the voltage induced in the coils due to a single movement

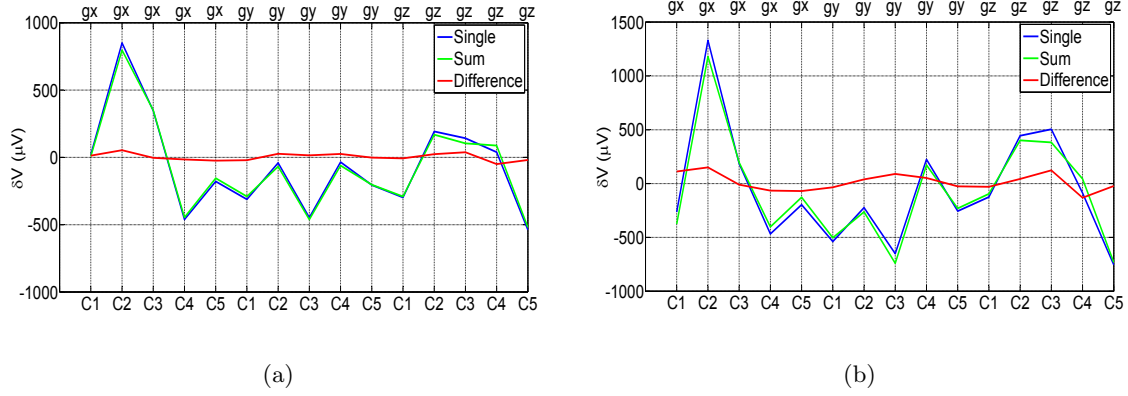


Figure 4.36: A comparison between the sum of the change in induced voltage due to translations and rotations and the change in induced voltage due to combined (in a single go) movements (simultaneous translation and rotation). (a) Small rotations up to 5° and translation. (b) Large rotation up to 10° and translation.

(simultaneous rotation and translation) during execution of x-, y- and z-gradients. This is a verification of the Eq. 3.48 described in Chapter 3.

The programme was changed to produce the coils on a standard head-sized sphere with radius $R = 80$ mm. This is done by using another MatLab programme which is a function of gradient amplitudes, noise and coordinates (x, y, z). The programme saves the matrix of values linking the coil voltages to the linear terms in the fits to variation with rotation angles and translations (fit over 5° ranges including quadratic term and 10 mm range of translation).

When no added noise, and radius of the sphere $R = 10$ mm shown in Fig. 4.1: 6 columns relate to movements R_x , R_y , R_z , T_x , T_y , T_z ; 15 rows show the effect of g_x for coils 1-5; g_y for coils 1-5; g_z for coils 1-5. The biggest linear terms are obtained for the five coils for x-translation in g_x , y-translation in g_y and z-translation in g_z . Zero linear terms are found for x-rotation and g_x , y-rotation and g_y , z-rotation and g_z .

By using the fit parameters a_1 summarized in Table 4.1 and the changes in induced voltage in the 5 coils due to 3-gradients, we construct a system of linear equations $\delta V = A\delta r$; where the A is a 15×6 matrix. Inversion of this matrix gives us the movement parameters

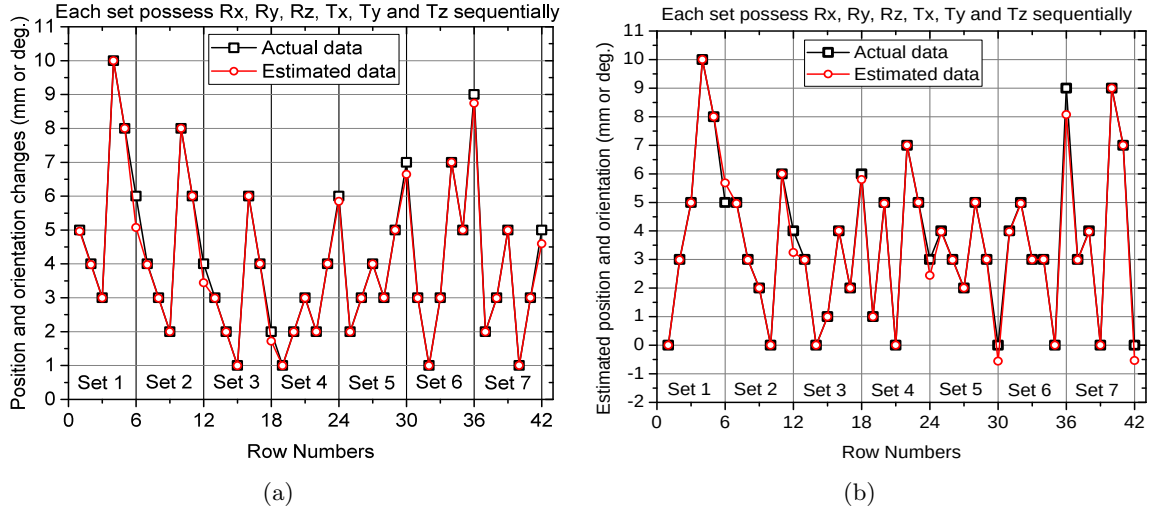


Figure 4.37: Estimated position and orientation where the maximum rotation is 5° and translation is 10 mm. (a) Estimated parameters from a random movement when there is no zero movement along/about an axis. (b) Estimated parameters from a random movement when there is at least one zero movement along/about an axis.

$r(\delta x, \delta y, \delta z, \delta \phi_x, \delta \phi_y, \delta \phi_z)$ which are shown in Fig. 4.37. In Fig. 4.37(a) and 4.37(b) there are seven sets of data having six movement parameters (3-rotations and 3-translations) in each. Data represents in Fig. 4.37(a) has non-zero parameters i.e. in the actual movements, none of the six parameters were zero in any of the sets, whereas Fig. 4.37(b) has a few zero parameters i.e. in the actual movement at least one of the parameters was zero in each set; the results show that both can estimate the movement parameters very accurately.

The precision level of the estimated parameters without adding noise voltage in the induced voltage is good though there is some uncertainty in rotation and translation which comes from the non-linear terms due to the rotation about the z-axis and also rotation about the x- and y-axis during the execution of z-gradients. The data shown in Fig. 4.38 indicates that the maximum uncertainty in y-translation is 0.8 mm, whereas the actual movement is 10 mm.

The modified programme calculates the voltage change due to a pre-set movement and then estimates the movement parameters using the inverse matrix by using the standard

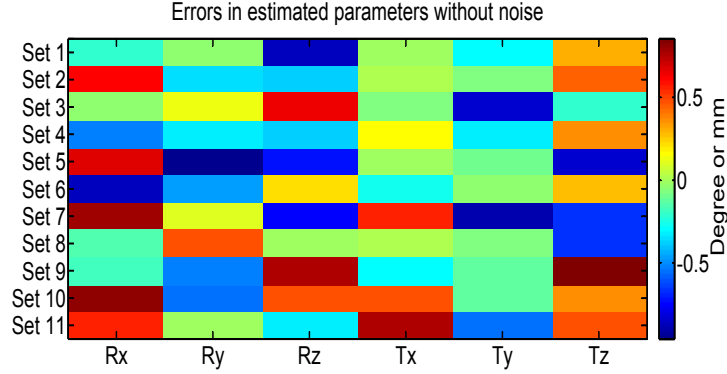


Figure 4.38: Estimated the movement parameters from calculated induced voltage in the coils due to translation and rotation without adding noise to the induced voltages.

MatLab command ‘pinv’(pseudo inverse). At the moment the programme is set to do this for a movement i.e. translations by 8 mm, 6 mm and 10 mm along the three mutually perpendicular axes respectively and rotation of 4° , 5° , and -3° degrees about the x-, y- and z-axes respectively; for noise levels varying from 0 to $20 \mu V$ in 11 steps of $2 \mu V$. The positional errors are plotted in Fig. 4.39(a) -column Rx, Ry, Rz and Tx, Ty, Tz -row 1 to 11 for increasing noise. Fig. 4.39(a) shows the behaviour of the six movement parameters for increasing noise voltage levels. Now reducing both noise a factor of 2 and the size of the movements by a factor of 2 it is observed that errors are reduced. Fig 4.39(b) shows rotational and translational parameters calculated for a reduced noise voltage. There is a non-zero error for zero noise as shown in Fig. 4.38 (particularly in z-rotation) presumably because of non-linear terms.

4.4.3 Results: Numerical Simulation

The changes in induced voltage in the five coils are calculated using Eqs. 3.48 described in Chapter 3. The five movements parameter are determined by using calculated induced

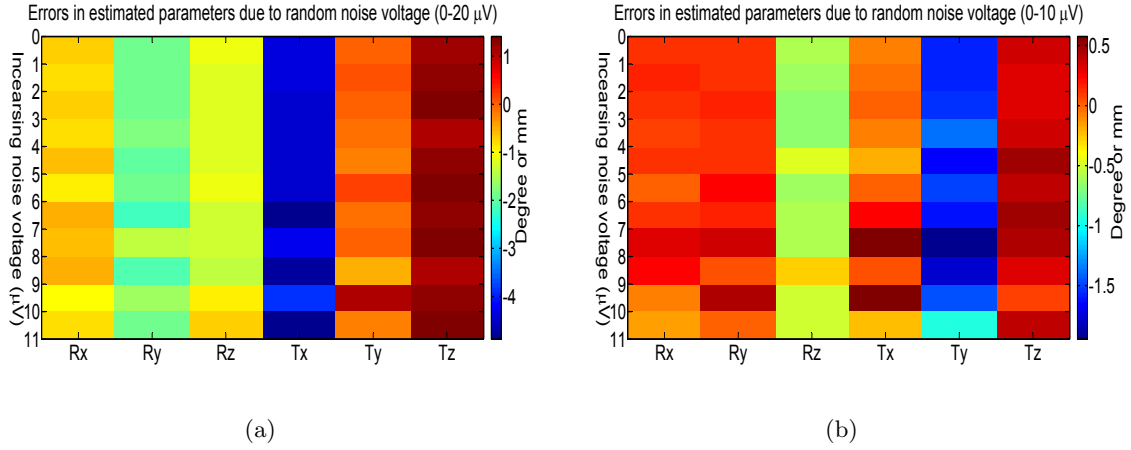


Figure 4.39: Errors in estimated position and orientation due to addition of random noise voltages. (a) Errors in estimated movement parameters from calculated induced voltages due to translation and rotation by increasing noise level in the induced voltages (0-20 μV). (b) Errors in estimated movement parameters from calculated induced voltage in the coils due to translation and rotation by reducing noise level in the induced voltages (0-10 μV).

voltage in the five coils by applying the standard MatLab function *fsolv* for non-linear and inconsistent linear equations.

Fig. 4.40 shows the affects of increasing the random noise voltage in translational movement parameters as well as on the rotational parameters. If the random noise voltage is less than 10 μV , it does not significantly effect movement parameters because the change in induced voltage in the coils dominate over the small noise voltage.

By adding a random noise voltage in each coil one can measure the standard errors in position and orientation of the coils by considering the standard deviation which is shown in Fig. 4.41. Noise voltages are added to a constant voltage to check how the specific movement parameter changes with increasing noise voltage.

A random noise voltage of 10 μV to 100 μV is introduced in each coil to test the sensitivity of the movement parameters with noise voltage. In that case the position of the whole system remains unchanged. As the noise voltage increases the error (standard deviation) in the movement parameters (Δx , Δy , Δz , $\Delta \phi_x$ and $\Delta \phi_y$) increases, as shown in Fig. 4.42.

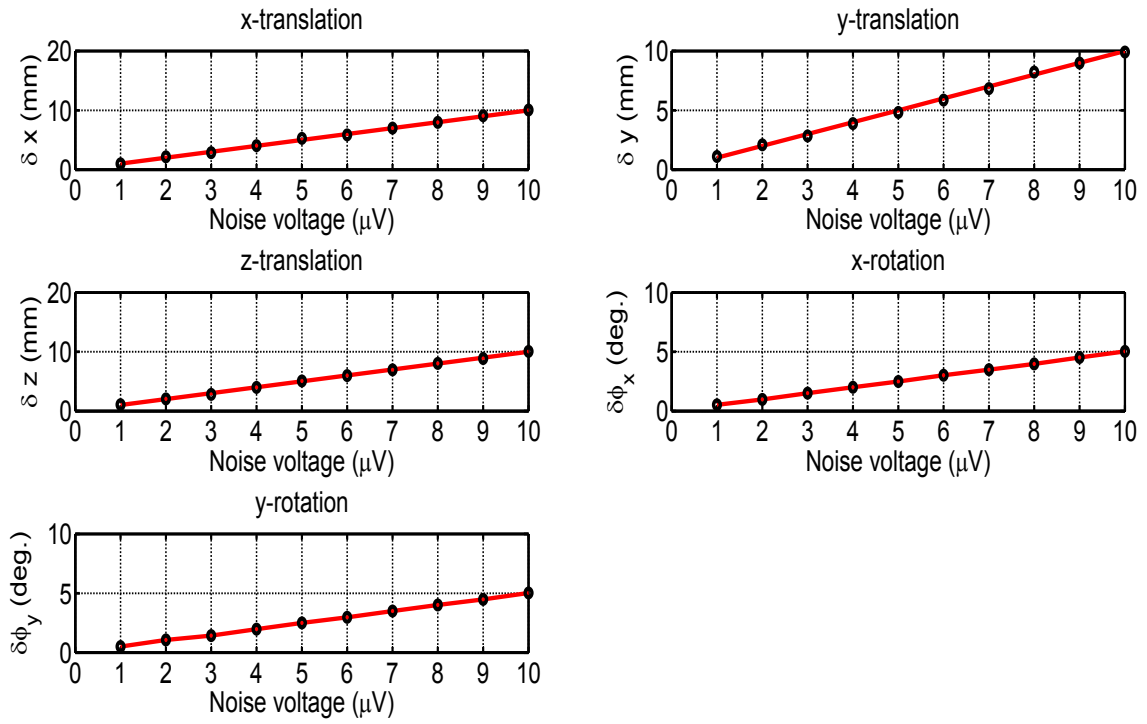
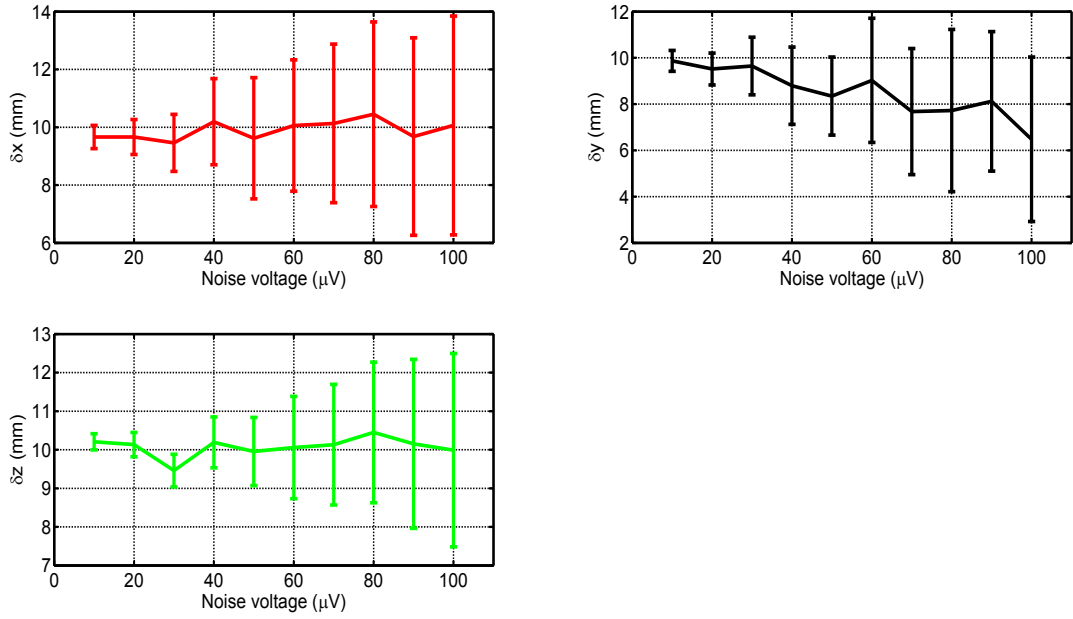
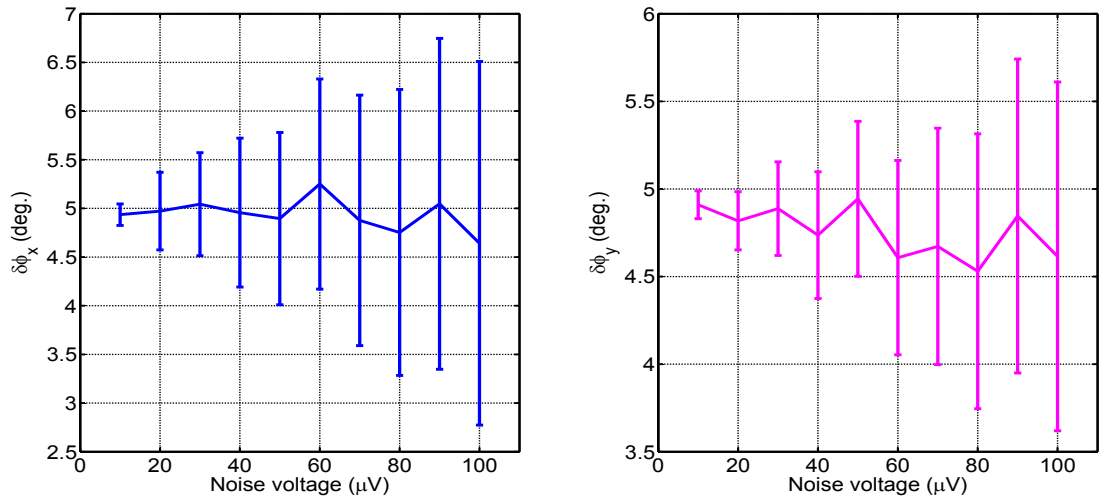


Figure 4.40: Estimated movement parameters from calculated induced voltage in the coils due to translation and rotation without noise (red line) and with increasing added noise (black circle) 1-10 μV in the induced voltages.



(a)



(b)

Figure 4.41: Data represents the estimated movement parameters (line) and their standard deviation (error bars) by using calculated voltage induced in the coils when an increasing noise voltage of 10-100 μV is added to the change in induced voltage in the coils. (a) Translations along the x-, y- and z-axes. (b) Rotations about the x- and y-axes.

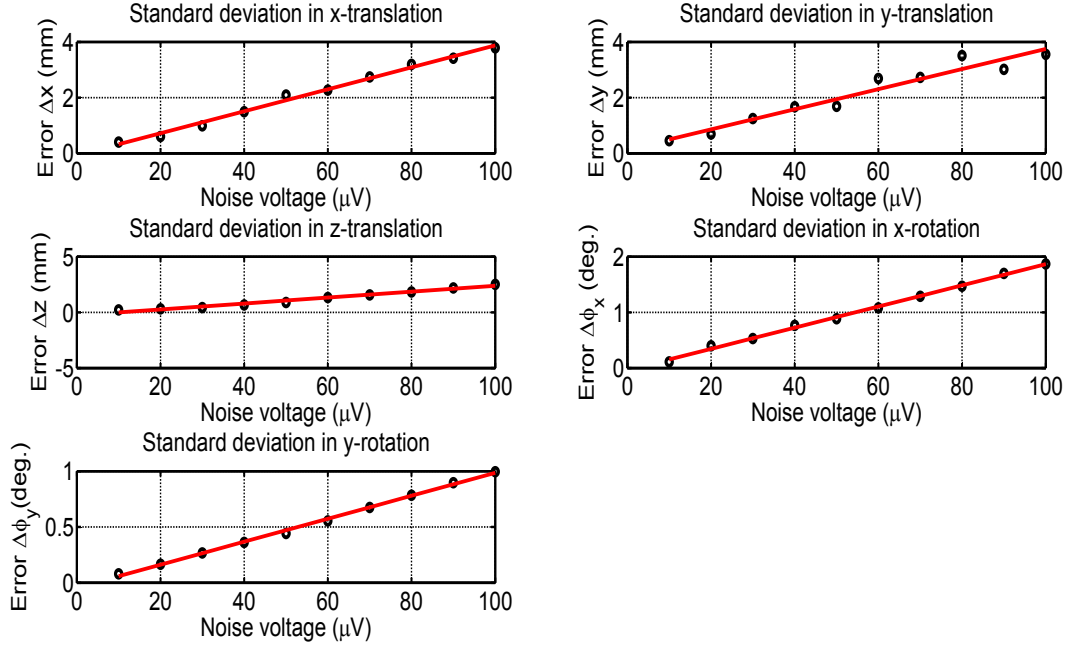


Figure 4.42: Errors in estimated movement parameters are increasing as the noise voltage increases in the induced voltage of the coils.

However, the change in induced voltage in the coils is calculated due to 10 mm translation along the x-, y- and z-axes and 5° rotation about the x- and y-axes. The maximum errors in translation is $\sim 4\text{mm}$ for 10 mm actual translation and in rotation is $\sim 2^\circ$ for 5° actual rotation due to the addition of random noise voltage ($100 \mu\text{V}$) are shown in Fig. 4.42. Even at the highest noise level ($100 \mu\text{V}$) the estimated parameters are meaningful.

4.5 Discussion

Firstly, a circular coil is generated in a specific plane shown in Fig. 4.1. Similarly, circular loops are generated in the xy, yz and zx planes. The change in induced voltage has been calculated in the coils due to translation and rotation due to the application of x-, y- and z-gradients.

	x-translation	y-translation	z-translation	x-rotation	y-rotation	z-rotation
x-gradient	C_1	0	C_2, C_3	C_1, C_4, C_5	0	0
y-gradient		C_1	C_4, C_5	0	C_1, C_2, C_3	0
z-gradient	C_2, C_3	C_4, C_5	C_1	C_1, C_4, C_5	C_1, C_2, C_3	0

Table 4.3: Sensitivity of the planar coils with respect to the movements and the applied gradients.

A system of five coils is generated in the planes of a cubic box as shown in Fig. 4.9. The coils C_1 , C_2 and C_3 are sensitive to x-translation; whereas coils C_1 , C_4 and C_5 are sensitive to y-translation and all of the five coils are sensitive to z-translation during execution of x-, y- and z-gradients. On the other hand, the coils C_1 , C_4 and C_5 are sensitive to x-rotations during execution of y- and z-gradients; whereas the coils C_1 , C_2 and C_3 are sensitive to y-rotation during execution of x- and z-gradients. None of the coils are sensitive to z-rotations during execution of any gradient. The sensitivity of the coils to translation along the three axes and rotation about the three axes during execution of x-, y- and z-gradients are summarized in Table 4.3.

By linear fitting of the change in induced voltage in the coils with respect to the change in position and orientation of the coils one can obtain the slopes of the graphs to form a system of linear equations described in subsection 4.3.1. By taking the pseudo inverse ('pinv') of the coefficient matrix formed from the slopes the changes in movement parameters are estimated. Except for z-rotations, the other five movement parameters can be estimated with fairly good accuracy. Although noise voltages effect the estimated parameters, a random noise voltage of 0-20 μV produce acceptable errors in the measured voltages. If the thermal noise voltages becomes more than 20 μV error in the estimated parameters $\sim 2\%$.

Secondly, an arbitrary shaped coil is generated on the surface of a sphere and the voltage changes are calculated due to successive translations and rotations of 10 mm and 10°

respectively.

The voltage changes in the coils are calculated and the change in the position and orientation are estimated. A random noise voltage is added to the calculated voltage and the parameters are estimated again to check the effect of the noise voltage in the movement parameters.

The change in induced voltage in the coils is quadratic due to rotation about the x- and y-axes for application of z-gradient. However, the induced voltage is small compared to the change in induced voltage in the coils due to translation, so that linear voltage change dominates over quadratic change.

Whatever the translation is, the change in induced voltage in the coils varies linearly. Moreover, in the case of rotation about the x- and y-axes, the changes in induced voltage in the coils during the application of x- and y-gradients is also linear with the angle of rotation. However, the change during execution of z-gradient is quadratic. Linear variation of the changes in induced voltage due to translation and rotation may help to calibrate the translation and rotations by measuring the change in induced voltage in the coils.

It is clear that whatever the translation is, the changes in induced voltage in the coils are linear in all of the methods. In the case of larger rotations ($>5^\circ$) though the linearity breaks down, one still can estimate movement parameters quite accurately due to rotation up to 10° by using any one of the methods. Within the limits of linearity, it is quite possible to estimate the changes in position and orientation of the coils precisely.

4.6 Conclusion

In this chapter an approach is developed to track motion of a rigid body (head) inside an MR scanner by measuring the change in induced voltage in head-mounted coils when the coils are in specific planes in a space or in a space. The approach is very fast (takes few seconds) and is compatible with an MR system. The method mainly depends on the geometry of the

coils and the time varying magnetic field gradients. The software is relatively simple and straightforward. It is developed with the potential application of the EPI pulse sequence and fMRI in mind, and in principle it can be adapted to any MRI application where rigid body motion is likely to take place. Monitoring head movement inside an MR scanner it is possible to estimate six motion parameters of a rigid body, and prospective motion correction could be implemented by passing six parameters (degrees of freedom) to the MR pulse programmer.

Chapter 5

EPI Pre-pulses: Rig and Planar Coil

The aim of this project is to develop a robust method through further refinement and innovation which forms the basis of a useful movement monitoring system, for tracking head motion in an MRI scanner for prospective motion correction. This Chapter presents an experiment based on theory and simulation when the coils are positioned in orthogonal planes in space. This chapter focuses also on the results obtained from experiment and a comparison between theory, simulation and experiment. To do so, an experimental arrangement is set up to check the validity of the analytical and numerical models.

5.1 Introduction

A theory is described to calculate the change in induced voltage in the coils due to translation and rotation during execution of the x-, y- and z-gradients in Chapter 3. Simulations were carried out in Chapter 4 based on the theory that is developed in Chapter 3. An experiment has designed to measure the changes in the voltages induced in the coils due to translation and rotation during execution of the x-, y- and z-gradients.

On the scanner the gradient directions are defined as RL, AP and FH rather than x, y and z; and this terminology is used throughout the experimental Chapters.

5.2 Method and Materials

According to the simulations (in Chapter 4) for a planar coil the change in induced voltage in a circular coil depends on the area i.e. radius and the number of turns in it. The more the radius and the turns in a coil the more the change in induced voltage in the coil. The maximum measuring limit of an ExG amplifier is 16 mV. On the other hand, small coils having less turns induced a measurable and suitable voltages (hundreds of μV) in the coils, but there is a limitation (difficult) to manufacture such a small coil. By considering all these limitations we choose a suitable coil of specific radius and having sufficient number of turns in it.

A rig containing five coils of 20 mm radius, each having 10 turns of copper wire, which is MR scanner compatible was built as shown in Fig. 5.1. Each of the 5 coils is connected to a bipolar MR-compatible EEG amplifier (Brain Products, Garching) by using star-quad cables.

According to the description of the software developed in Chapter 4, the positions and orientations of the five coils forming the system are shown in Fig: 5.2. The centres of the coils are located $C_1(0, 0, 210)$, $C_2(80, 0, 0)$, $C_3(-80, 0, 0)$, $C_4(0, 105, 0)$ and $C_5(0, -55, 0)$ where all co-ordinates are in mm.

The rig carrying the coils can be moved Head-Foot (HF) in the MR scanner by using a screw (screw for z-shift) as shown in Fig. 5.1. Right-left (sway) i.e. Right-Left (RL) movements of the rig in the MR scanner is controlled by turning another screw (screw for x-shift) shown in Fig. 5.1. In both cases the rig shifts 1 mm for a complete revolution (one turn) of the screw. Moreover, the rig can be moved in the Anterior-Posterior (AP) direction in the MR scanner (1 mm in each step) by pulling or pushing a plate attached to the rig.

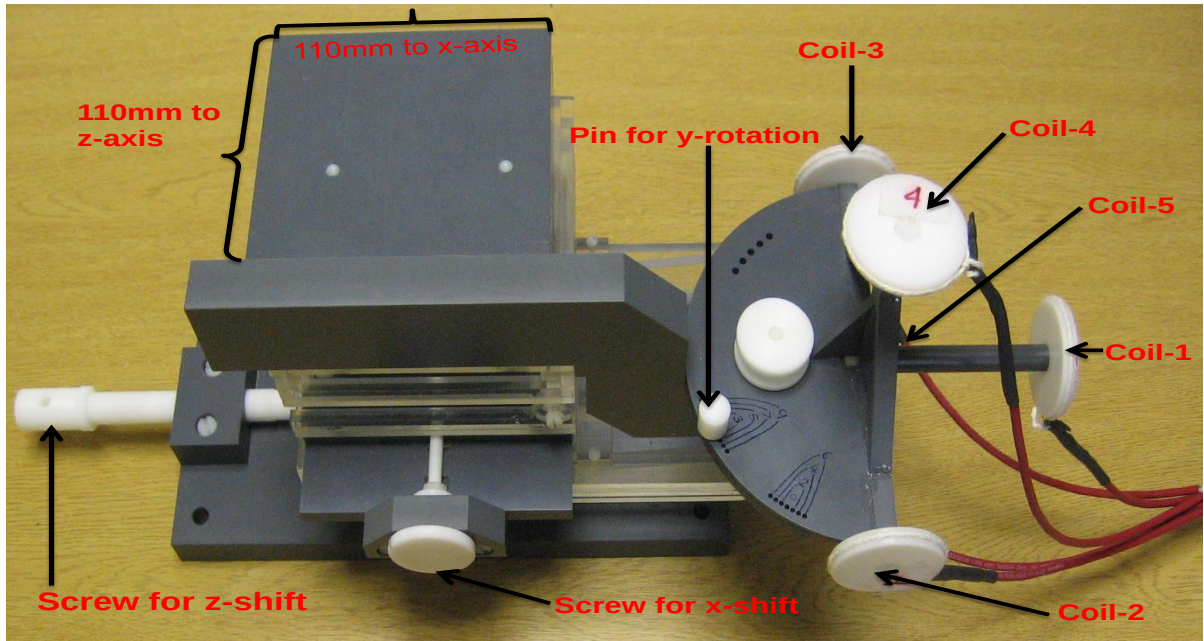


Figure 5.1: A rig of five coils

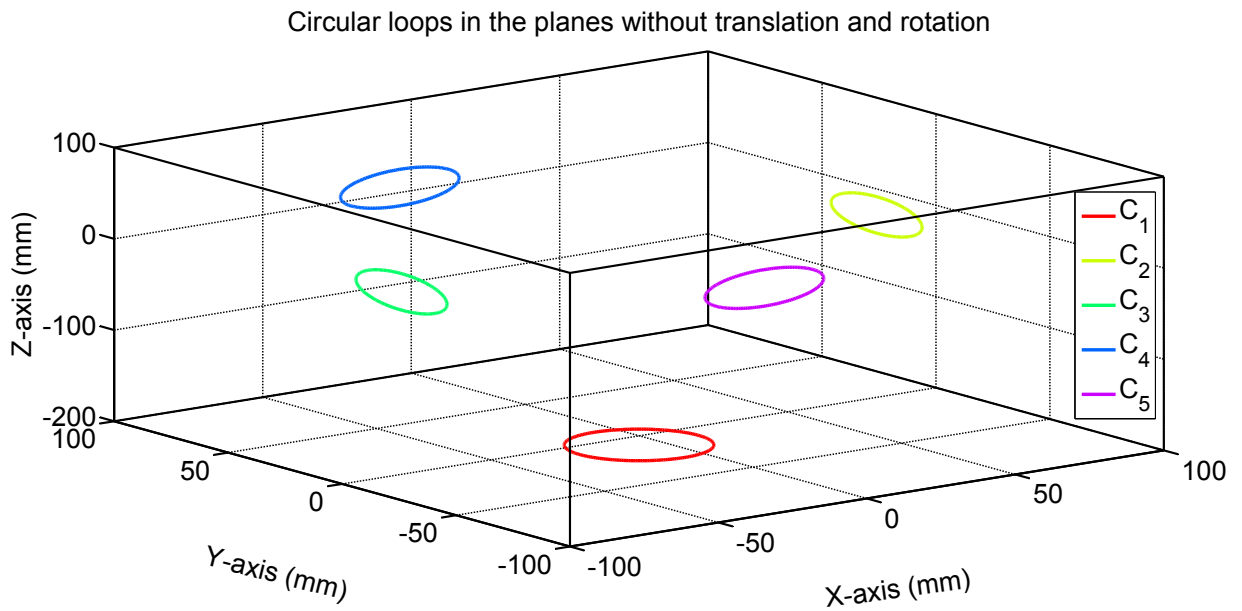


Figure 5.2: The position and orientations of an experimental system of five coils used to measure position changes via recording of voltages induced by time-varying gradients.

The rig can be rotated about the x-axis (Roll) by making an arc length 10 mm that makes an angle of 5.21° with the pivot, where the distance between pivot and arc length is 110 mm. Similarly, the rig can be rotated about the z-axis (pitch) by using a stair key. A y-rotation uses two sliding plates that are attached to the rig and calibrated in degrees by making holes in the plates. One can select a rotation in degrees by sliding the plates and then inserting a screw into the appropriate hole to fix the plates in position.

In the presence of magnetic field gradients (time varying or uniform) the coils pick up a noise voltage ($\sim 2 \mu\text{V}$) due to any kind of vibration, e.g. air flow (air cooling), or strong audio signals. This results from rotations of the coil in the strong B_0 field even in the absence of field gradients. In homogeneous B_0 , there is no induced voltage in the coils due to vibrations. A small noise voltage is also picked up by the coils due to the cold-head pump. The sensitivity of the coils strongly depends on their geometry. A large portion of the noise voltage comes from patient bed vibration. Therefore, the rig is put on to a beam which passes through the MR scanner bore from one end to the other. The beam is consequently decoupled from the MR scanner.

5.2.1 MR Scanning Parameters

Data was acquired in a Philips Achieva 3T MR scanner. An EPI pre-pulse [119] sequence was used although no images were produced in this experiment. Three pre-pulses (i.e RL, AP and FH) of gradient strength 4 T/m-s are included in this sequence. For the MR scanning that I use, the Body RF coil was employed for excitation, other parameters were set as follows: Field of View (FOV) = (RL $\times$ AP $\times$ FH) = (192 $\times$ 192 $\times$ 15) mm³, Voxel size = (RL $\times$ AP) = (3 $\times$ 3) mm², Slice thickness 3 mm, TE = 35 ms, TR = 2000 ms, Number of slices = 5, and the standard parameters were: Dynamics = 5, Flip angle = 80° , transverse slice orientation, Fold-over direction-AP and the slice scan order is descending.

5.2.2 The Gradient Pre-pulses

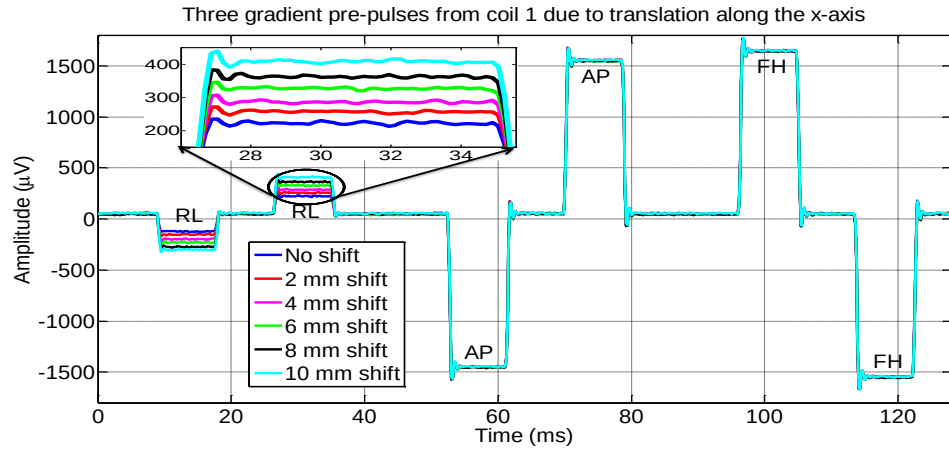
The rig carrying the five coils was first sequentially shifted 0-10 mm in the head to foot (HF) direction. The change in induced voltage is recorded by using a ExG (Brain Product GmbH) bipolar amplifiers. The recorded voltage signal were analyzed by using standard MatLab software to find the EPI gradient pre-pulses (RL (G_x), AP (G_y) and FH (G_z)) and to measure the induced voltage in each coil. The signals obtained during translation along the FH (Foot to Head) direction are shown in Fig. 5.3(a). Data clearly indicate that induced voltage changes occur in coil C_1 due to shift along the x-axis during execution of an x-gradient.

The experimental data presented in Fig. 5.3(b) indicates that the voltage change occurs in coil C_1 during execution of the three gradient pre-pulses. The signals obtained from the raw data due to translation along the z-axis are shown in Fig. 5.3(c). Data clearly indicate that the induced voltage change occurs in coil C_1 during the application of an x-gradient. There are small changes in the induced voltages found in the other coils, but it is negligible compared to the voltage induced in the other coils and it is also comparable to noise voltage ($\sim 10 \mu V$). One can consider or ignore these changes in induced voltages that does not affect linear model of the voltage change in the coils.

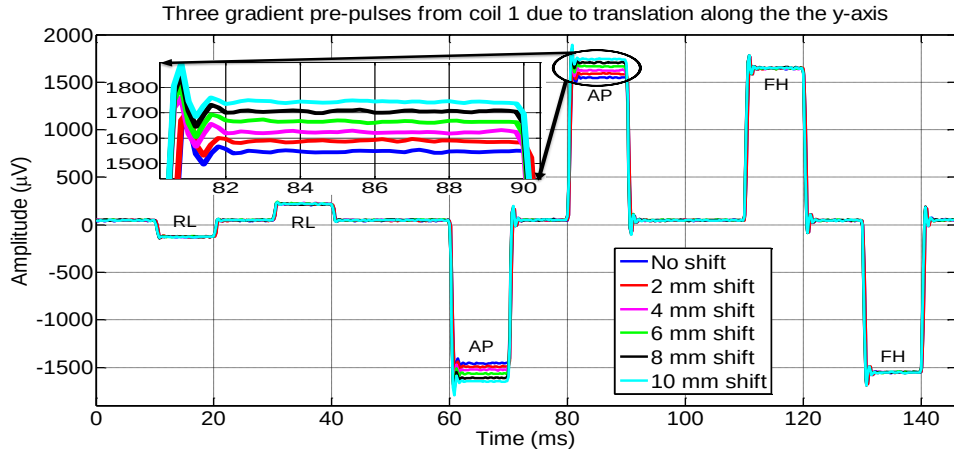
The induced voltage change is recorded by using an EEG apparatus during the executions of gradient pre-pulses along RL, AP and FH due to rotation about the x-axis. The data recorded because of the application of the x-, y- and z-gradient due to rotation about the y-axis is shown in Fig. 5.4(a). The recorded voltage signal clearly indicates that the change in induced voltage occur only along AP pre-pulse in coil C_1 . Experimental data were also acquired for the rest of the four coils.

Data recorded during the application of the x-, y- and z-gradients due to rotation about the y-axis is shown in Fig. 5.4(b). The recorded voltage clearly indicates that the change in induced voltage occur only along RL pre-pulse.

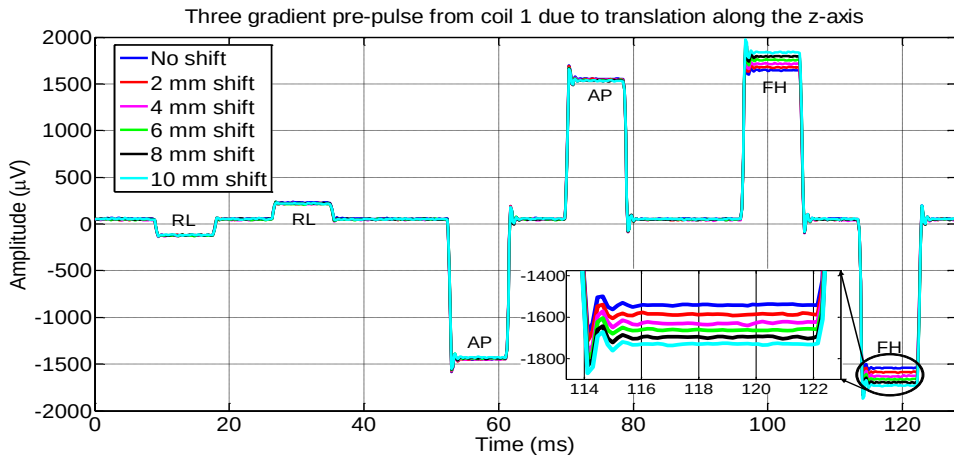
The induced voltage in the coils is recorded by using an ExG bipolar amplifier during



(a)

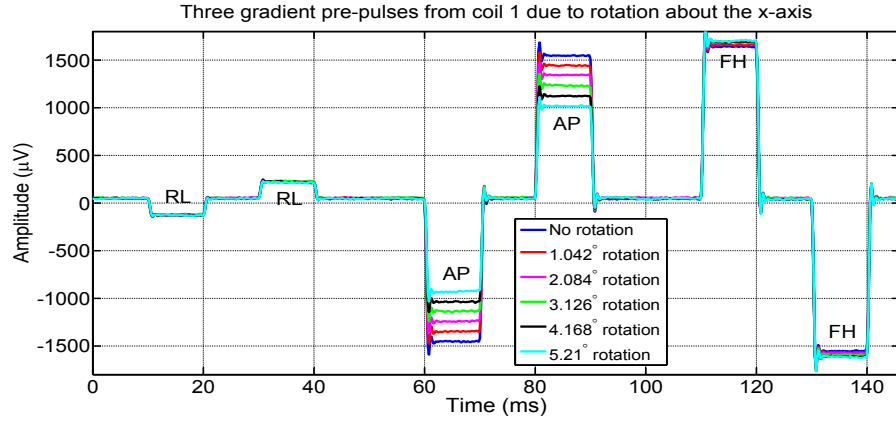


(b)

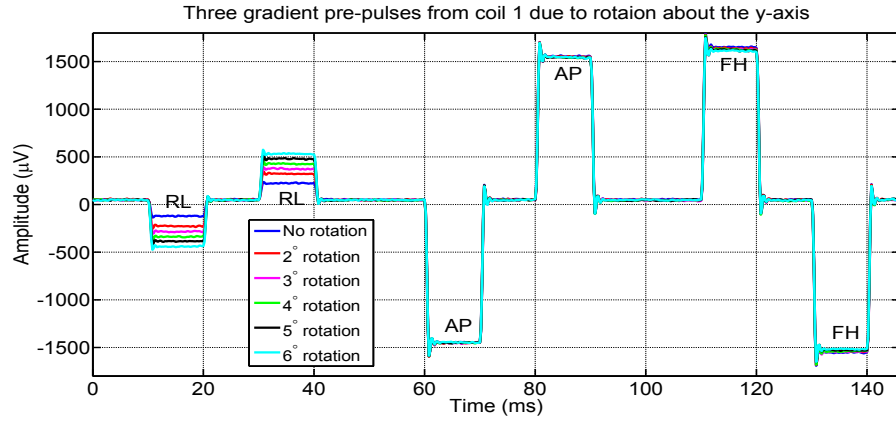


(c)

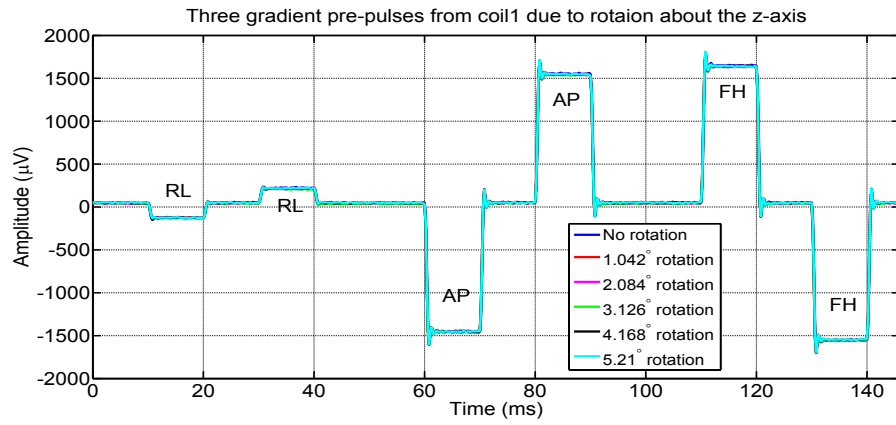
Figure 5.3: Data represents the change in induced voltage during execution of gradient pre-pulses along RL, AP and FH axes. The change in induced voltage recorded on coil C_1 due to translation along the x-axis (a), y-axis (b) and z-axis (c).



(a)



(b)



(c)

Figure 5.4: Data represents the change in induced voltage during execution of gradient pre-pulses along RL, AP and FH axes. The change in induced voltage recorded on coil C_1 due to rotation about the x-axis (a), y-axis (b) and z-axis (c).

execution of gradient pre-pulses along the RL, AP and FH due to rotation about the z-axis that is shown in Fig. 5.4(c). Recorded voltage signal clearly indicates that there is no change in the induced voltage in the coils.

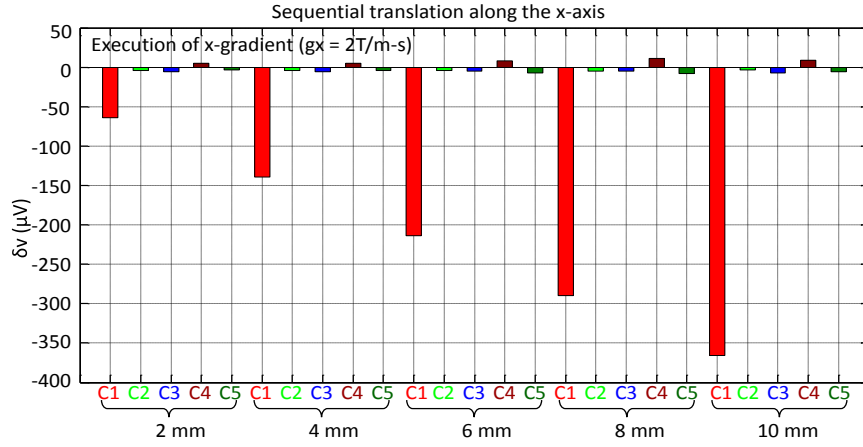
5.2.3 The change in induced voltage in the coils

The change in induced voltage occurring in coil C_1 due to sequential translation along the x-axis, because of the application of the x-gradient is shown in Fig. 5.5(a). During execution of y-gradient there is a little voltage change observed in the coils, shown in Fig. 5.5(b), but for the application of the z-gradient the induced voltage changes occur in the coils C_1 and C_2 , as shown in Fig. 5.5(c).

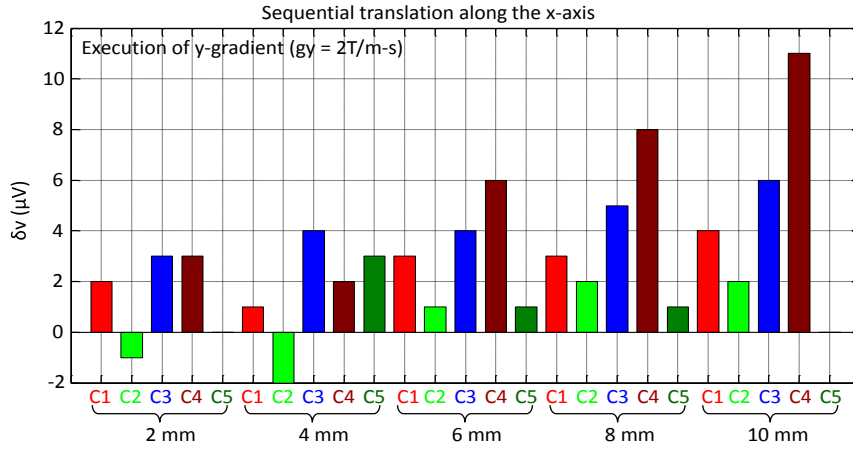
The change in induced voltage due to successive translation (0-10 mm) along the y-axis i.e. in the Posterior to Anterior (AP) direction are shown in Fig. 5.6. Due to sequential shift along the y-axis i.e. AP shift the changes in induced voltage occurring in coil C_1 because of the application of y-gradient are shown in Fig. 5.6(b) and in the coils C_4 and C_5 , and for the z-gradient they are shown in Fig. 5.6(c), whereas very little changes occurs because of the application of the x-gradient as shown in Fig. 5.6(a).

The change in induced voltage in the coils due to translation along the z-axis (FH) during the application of x-, y- and z-gradients is shown in Fig. 5.7. The changes in induced voltage recorded are the same in magnitude, but the sign is opposite.

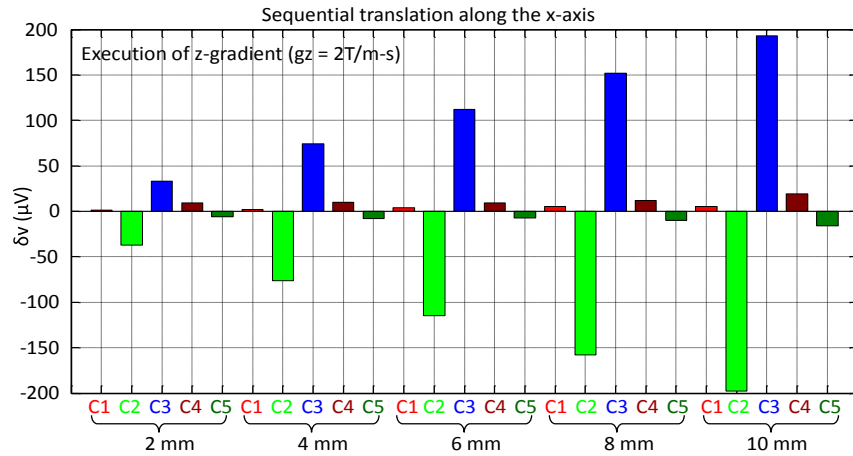
Data obtained due to successive translation of the system along the x-, y-, and z-axes for the application of x-, y- and z-gradients are summarized in Fig. 5.8. In the case of rotation about the x-axis, a signal is obtained from raw data which is shown in Fig. 5.13. In this case, significant changes in the induced voltage are measured in the coils C_1 , C_2 and C_3 because of the application of y- and z-gradients, as shown in Fig. 5.13. This agrees well with theory. However, a small voltage is observed in the coils C_4 and C_5 during execution of an x-gradient. This could happen due to inclination of the coils (coil offset) with respect to any of the three axes. According to the theory and simulation the change in induced voltage



(a)

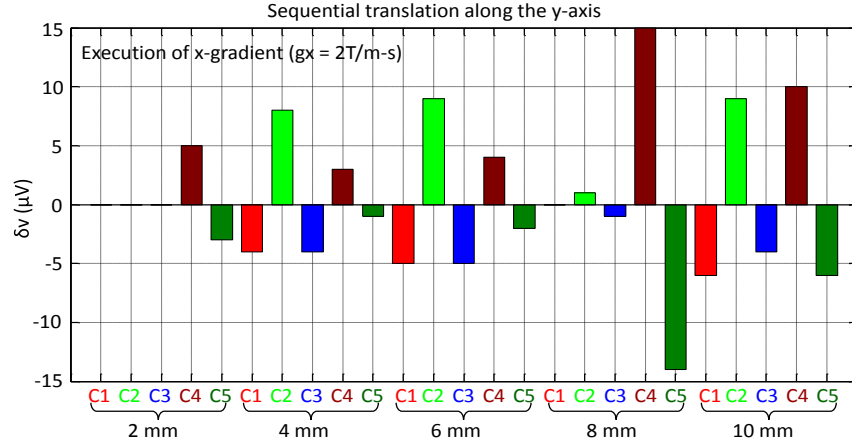


(b)

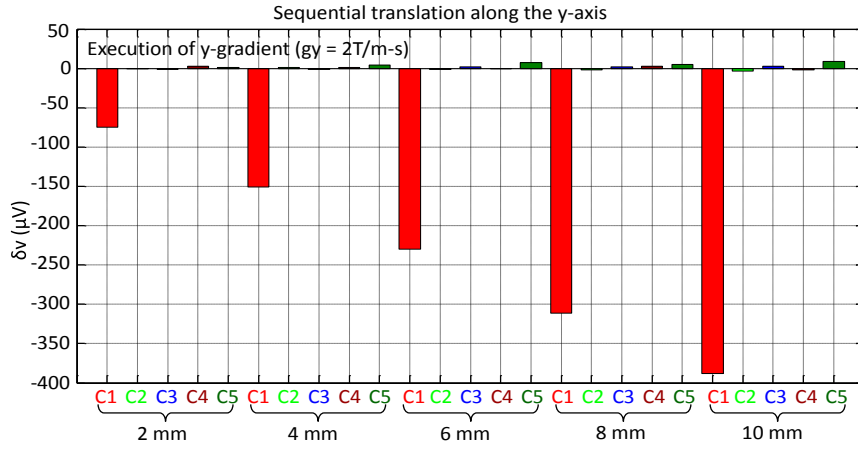


(c)

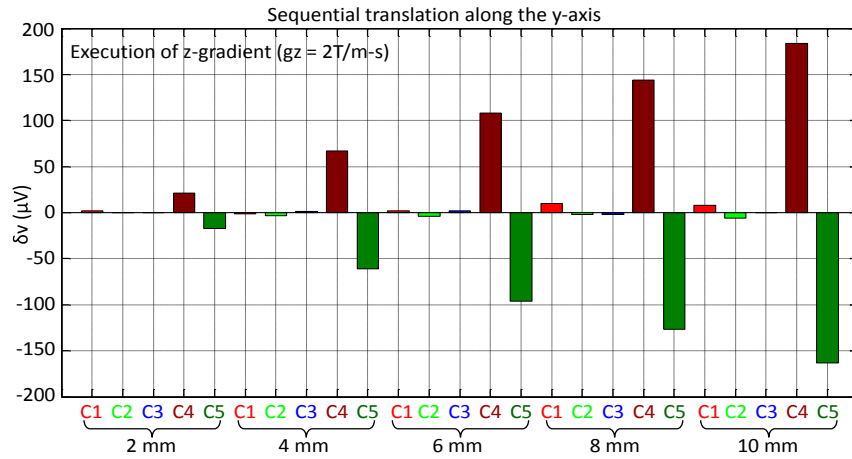
Figure 5.5: Experimental data indicate the change in induced voltage in the coils due to translation along the x-axis (a) during execution of x-gradient, (b) during execution of y-gradient, and (c) during execution of z-gradient.



(a)

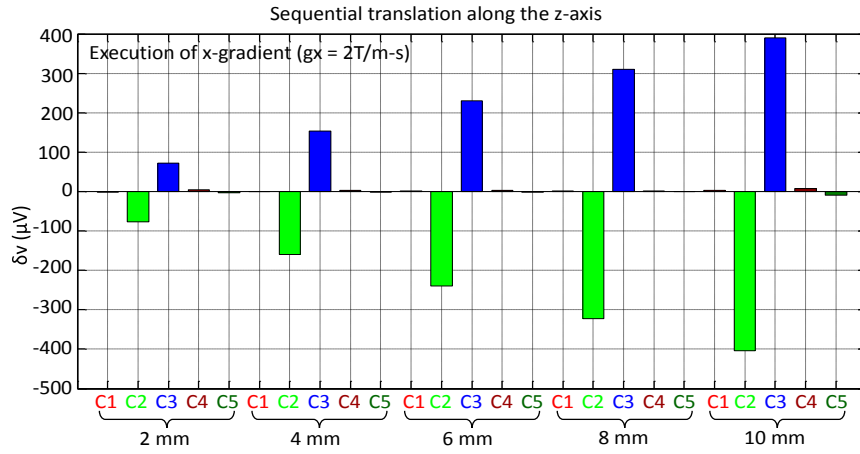


(b)

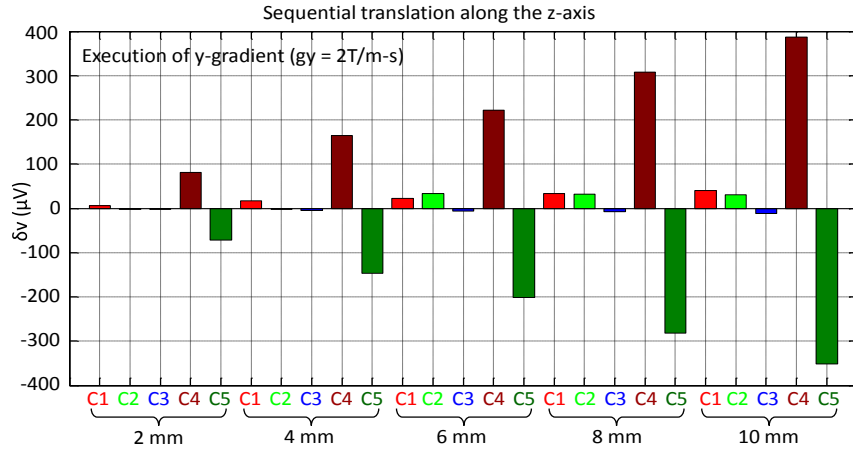


(c)

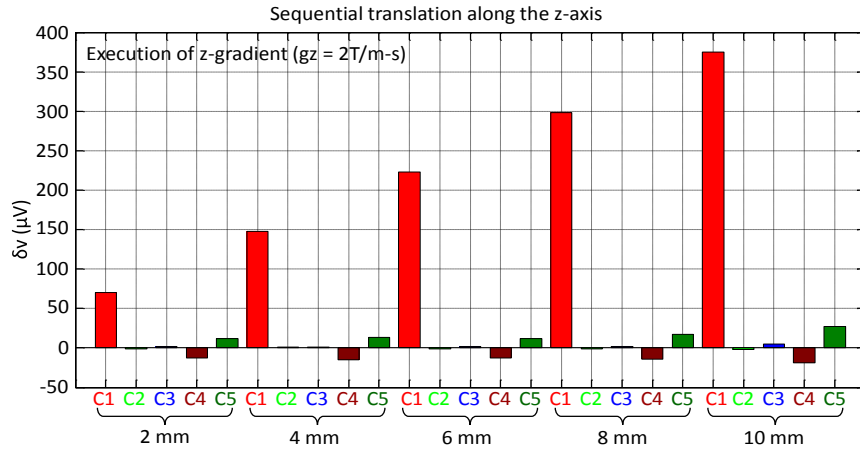
Figure 5.6: Experimental data indicate the change in induced voltage in the coils due to translation along the y-axis (a) during the application of an x-gradient, (b) during the execution of a y-gradient, and (c) because of the application of a y-gradient.



(a)

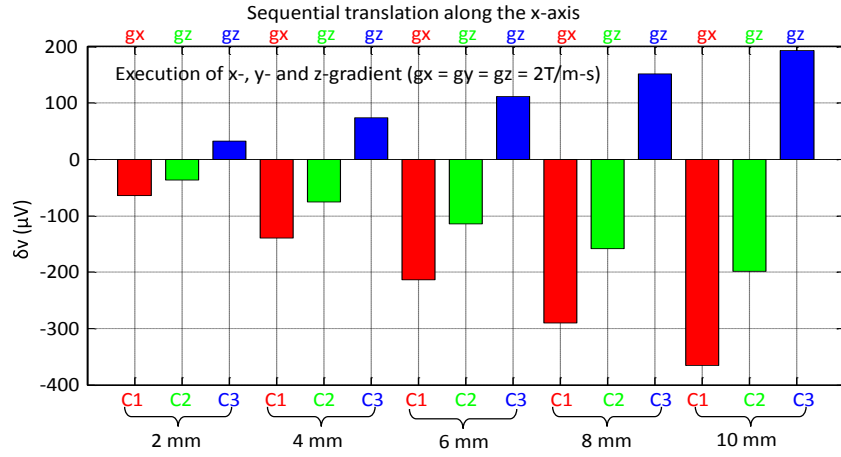


(b)

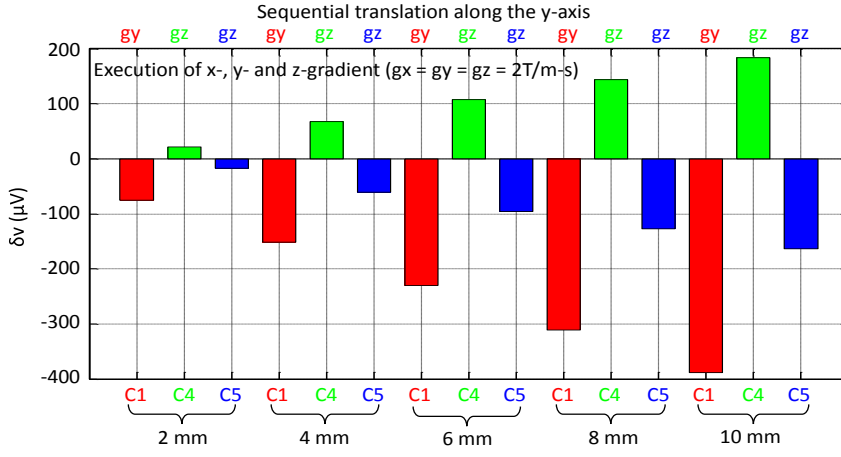


(c)

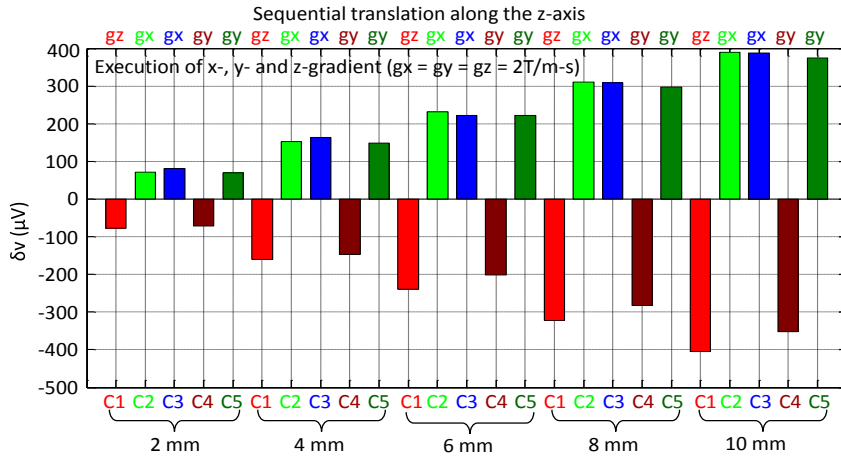
Figure 5.7: Experimental data indicate the change in induced voltage in the coils due to translation along the z-axis (a) during the execution of the x-gradient, (b) during the execution of the y-gradient, and (c) during execution of the z-gradient.



(a)

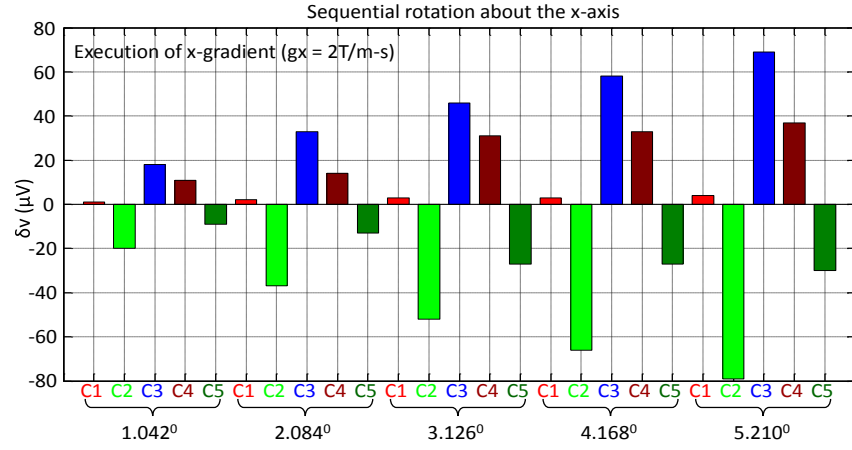


(b)

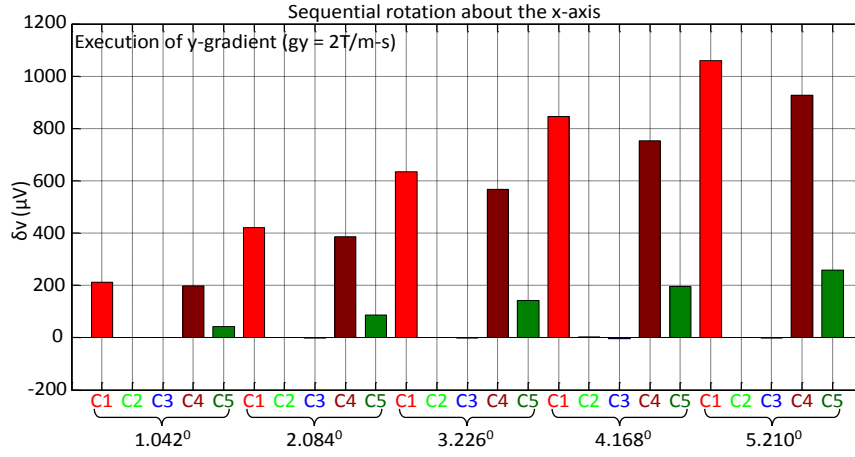


(c)

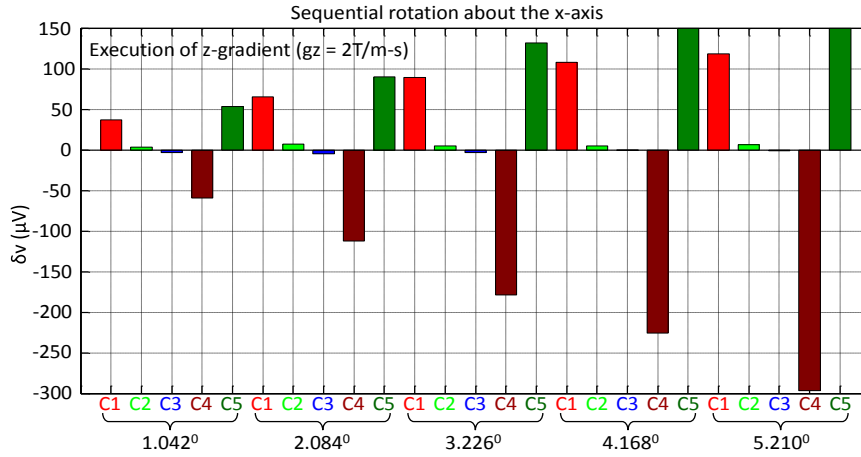
Figure 5.8: Experimental data indicate the change in induced voltage in the coils because of the application of the x-, y- and z-gradients (a) due to translation along the x-axis, (b) to translation along the y-axis, and (c) due to translation along the z-axis.



(a)



(b)



(c)

Figure 5.9: Experimental data represents the change in induced voltage in the coils due to rotation about the x-axis (a) because of the application of the x-gradient, (b) because of the application of the y-gradient, and (c) because of the application of the z-gradient.

in the coil C_1 due to rotation about the x-axis because of the application of y-gradient is $\delta V_1 = 678 \mu V$, whereas numerical simulation gives $\delta V_1 = 697 \mu V$.

The change in induced voltage in the coils due to rotation about the y-axis because of the application of the three pre-pulses is shown in Fig. 5.14. The changes occurring in the coils C_1 , C_2 and C_3 during execution of x- and z-gradients, and occurring in the coils C_2 and C_3 during execution of y-gradient, as shown in Fig. 5.10(b).

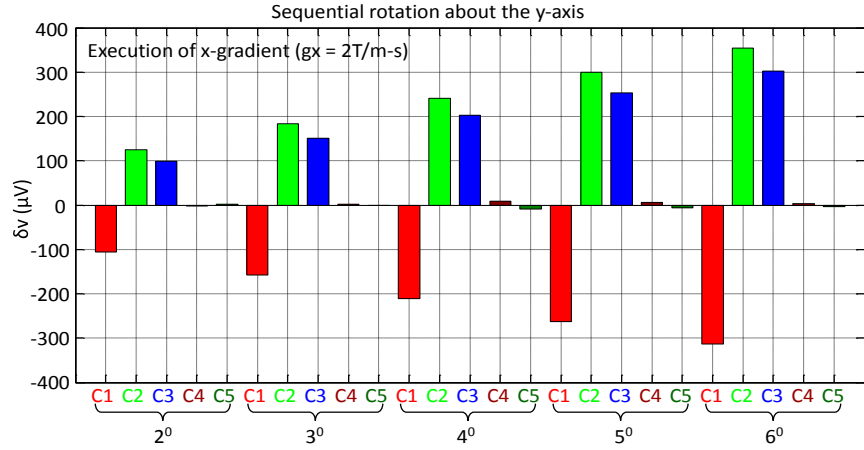
According to the theory and simulation there is no change in induced voltage due to rotation about the z-axis, but from experimental data small voltages are observed in the coils as shown in Fig. 5.15. It is perhaps due to the offset of the axes of the rig from the gradient axes. Either the coils are not properly positioned with respect to the gradient axes or inclined with respect to any of the three axes. However, the change in induced voltage is very small compared to the voltage induced in other coils and also comparable to noise voltage.

5.2.4 The change in induced voltage and their linear fit

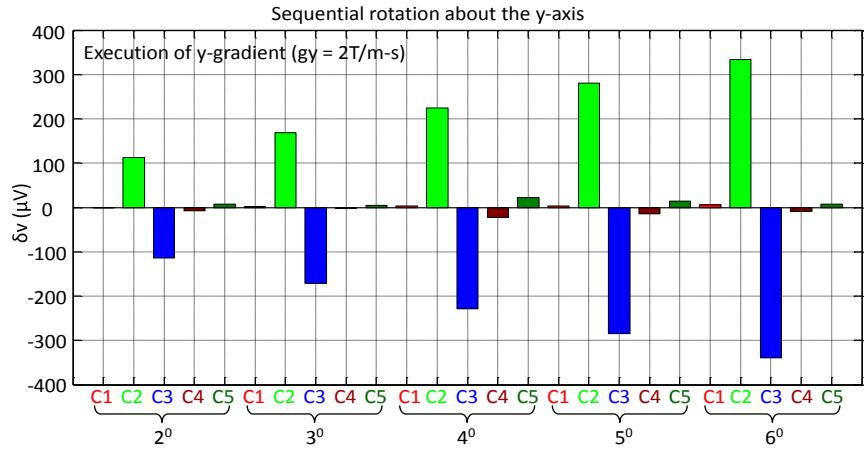
In the case of head to foot (FH) sequential shifts during the applications of the x-gradient, change in induced voltage occurs in the coils C_2 and C_3 , it occurs in the coils C_4 and C_5 during execution of a y-gradient, whereas it occurs in the coil C_1 due to the z-gradient. The change in induced voltage is purely linear with sequential change in position along the z-axis, as shown in Fig. 5.12. The change in induced voltage in the coils due to sequential translation along the three mutually perpendicular axes because of execution of x-, y- and z-gradients also varies linearly.

The experimental data in Fig. 5.13 indicate the change in induced voltage in the coils due to rotation about the x-axis during execution of y- and z-gradients and also indicates their linear fittings of the change in induced voltages. In Fig. 5.13 circles, triangles and squares indicate data obtained from experiment and solid lines indicate fitting of the data.

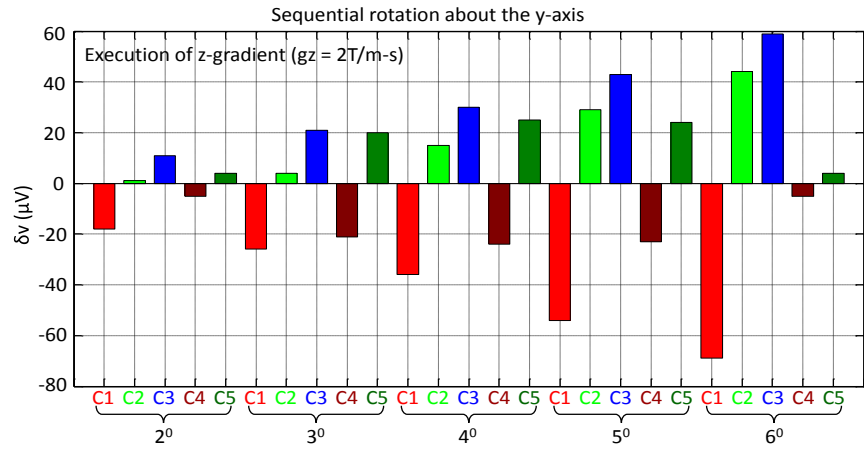
The changes in induced voltages measured from different coils due to rotation about the



(a)

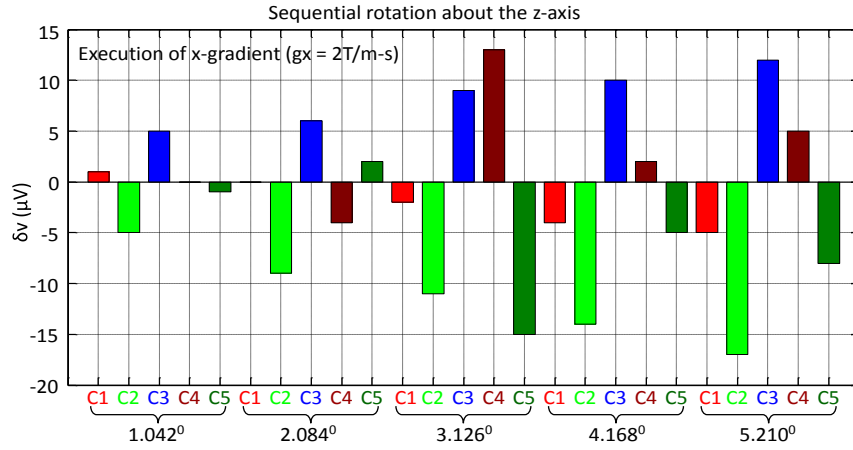


(b)

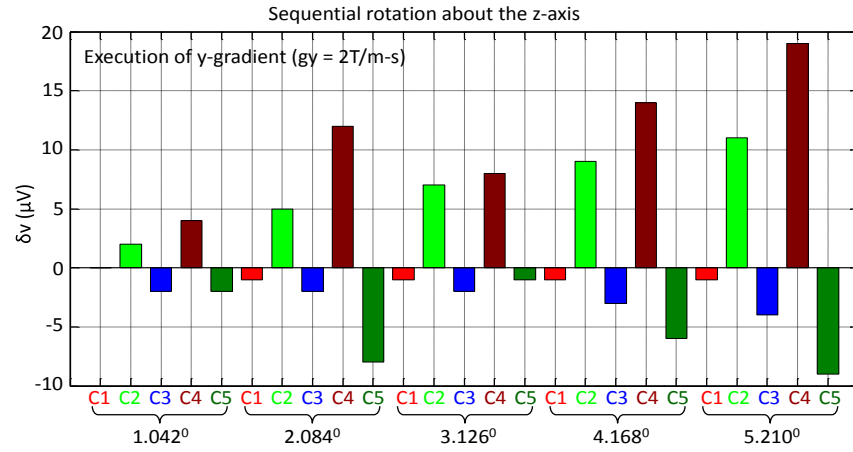


(c)

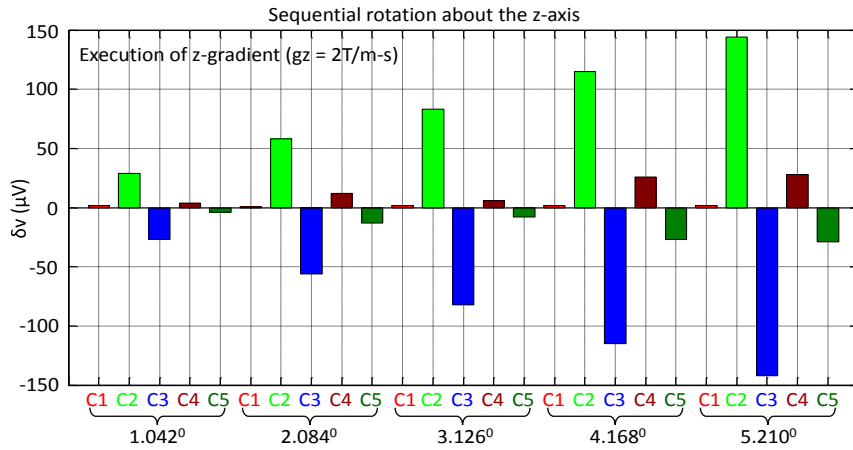
Figure 5.10: Experimental data indicate the change in induced voltage in the coils due to rotation about the y-axis. (a) Experimental data indicate the change in induced voltage during the application of the x-gradient, (b) during the application of the y-gradient, and (c) during the application of the z-gradient.



(a)



(b)



(c)

Figure 5.11: Experimental data represents the change in induced voltage in the coils due to rotation about the z-axis (a) during the application of the x-gradient, (b) during the application of the y-gradient, and (c) during the application of the z-gradient.

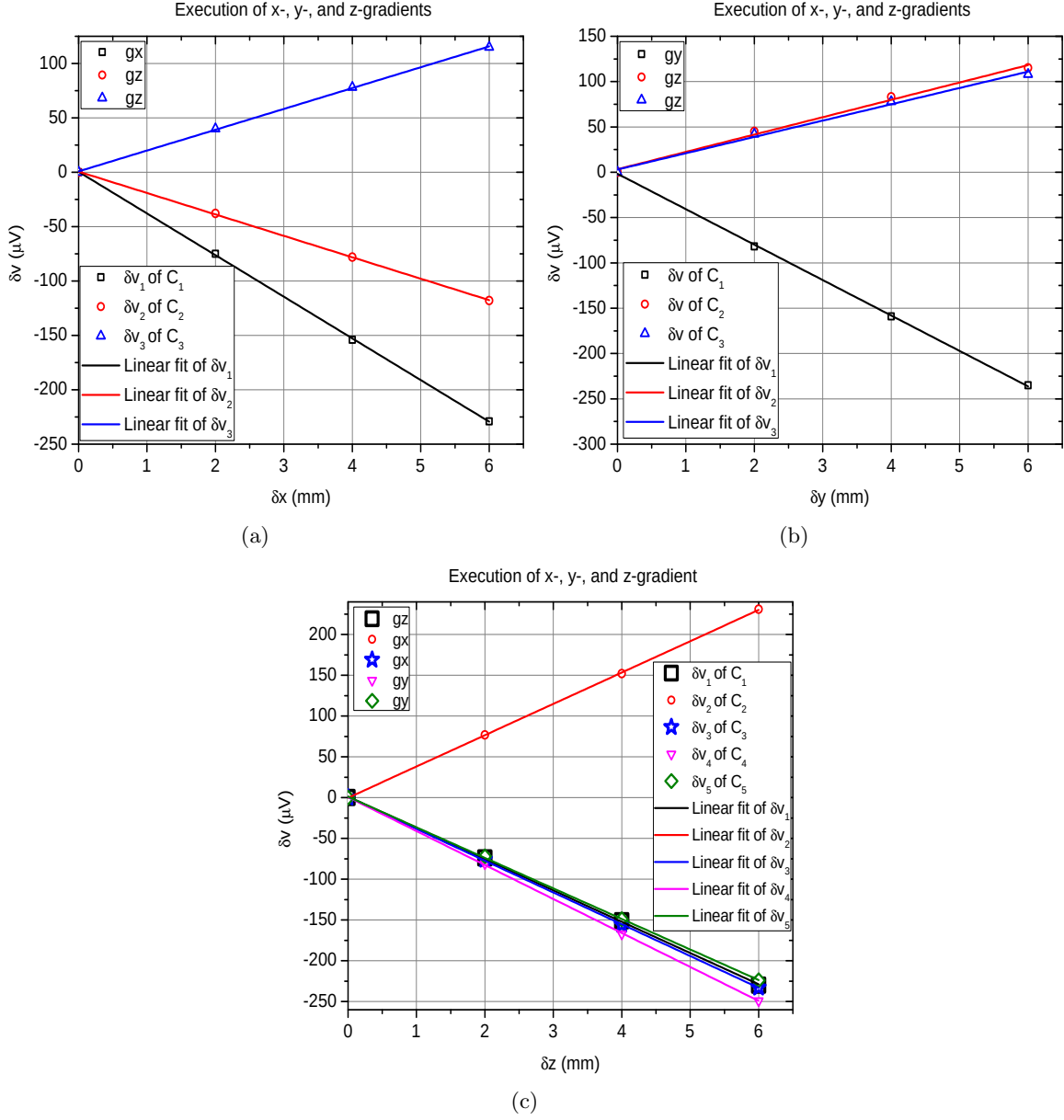


Figure 5.12: Plots showing the change in induced voltage in the coils during execution of x-, y- and z-gradients. (a) due to translation along the x-axis, (b) due to translation along the y-axis, and (c) due to translation along the z-axis.

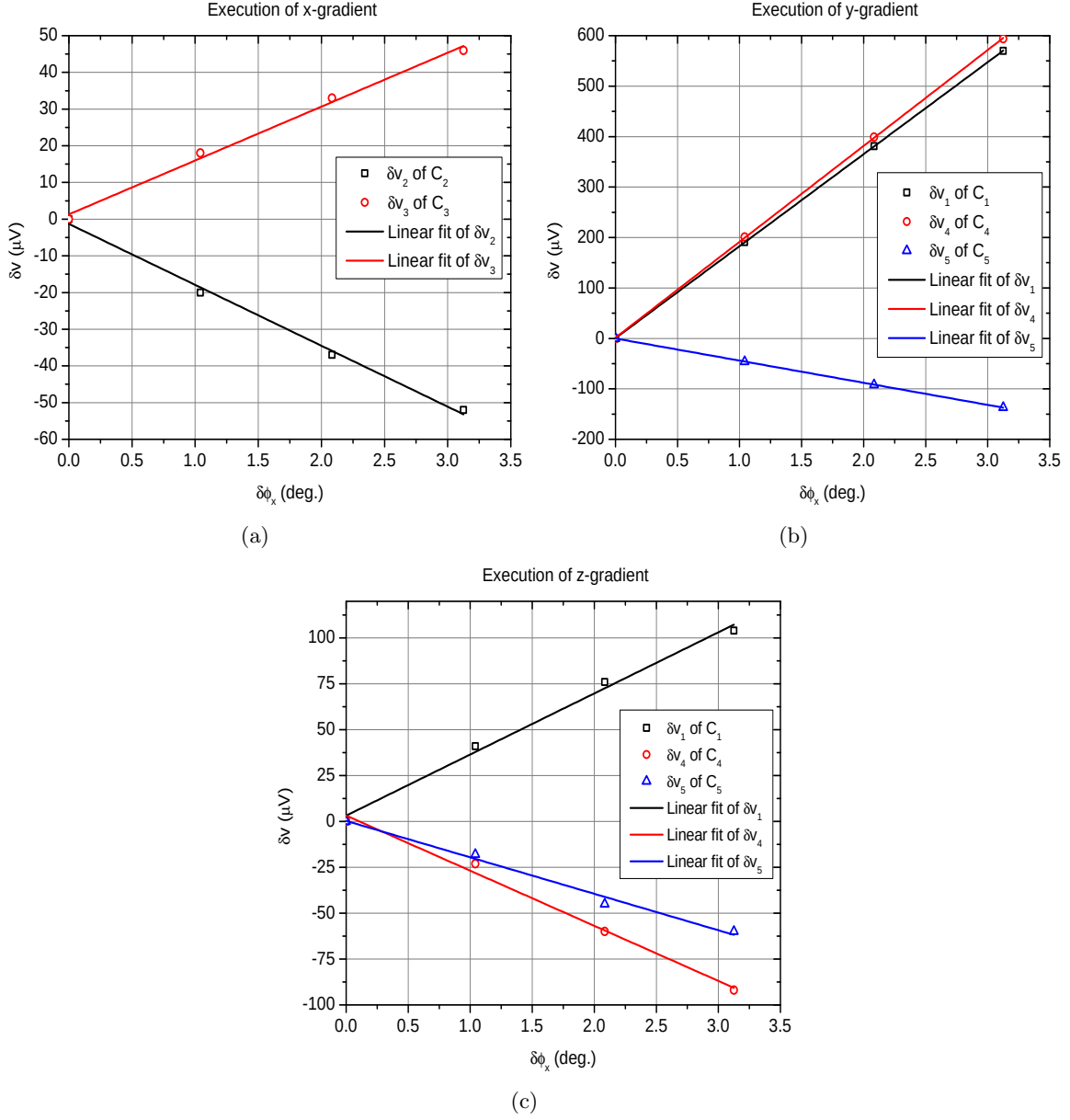


Figure 5.13: Experimental results showing induced voltage changes in the coils due to sequential rotation about the x-axis and their linear fit (a) during the application of the x-gradient, (b) during the application of the y-gradient, and (c) during the application of the z-gradient.

y-axis during execution of the x- and z-gradients are presented in Fig. 5.14. In Fig. 5.14 circles, triangles and squares represent measured data and the solid lines show the relevant linear fits.

In Fig. 5.15, circles, triangles and squares indicate the change in induced voltage in the coils obtain from experimental measurement due to rotation about the z-axis and solid lines indicate the relevant linear fits.

5.3 Results

5.3.1 Comparison between Simulation and Experiment

Fig. 5.16 shows voltages generated in the 5 different coils by the 3 different time-varying gradients as the rig is moved in 2 mm steps in the z-direction, for numerical simulations and experimental measurements. Simulations are in good agreement with measurements and it is clear that the largest changes in voltage with z-position occur in Coil 1 for the z-gradient, Coils 2 & 3 for the x-gradient and Coils 4 & 5 for the y-gradient.

This pattern of variation and those due to translation in the x-, y- and z-directions and for rotation about the x- and y-axes, are depicted in Fig. 5.17, which maps the coefficients of the matrix $\mathbf{A}$ (described in Chapter 4) that translates co-ordinate changes into the voltages induced by the 3 gradients in the 5 coils. Matrix $\mathbf{A}$ contains 90 coefficients obtained from 5 coils, 3 gradients and 6 parameters. These coefficients were found by linearly fitting the simulated and measured voltages with respect to the positional change. According to the theory and simulation only the 23 coefficients (11 for translation and 12 for rotation) are non-zero, as described in Eqs. 4.19 and 4.22.

There is again reasonable agreement between measurement and simulation, particularly for translations; the discrepancies for rotation may reflect the effect of non-linearity in the voltage variation over the range of angles studied. The significant differences in the patterns of voltages produced by the different types of movement are promising for the prospect of

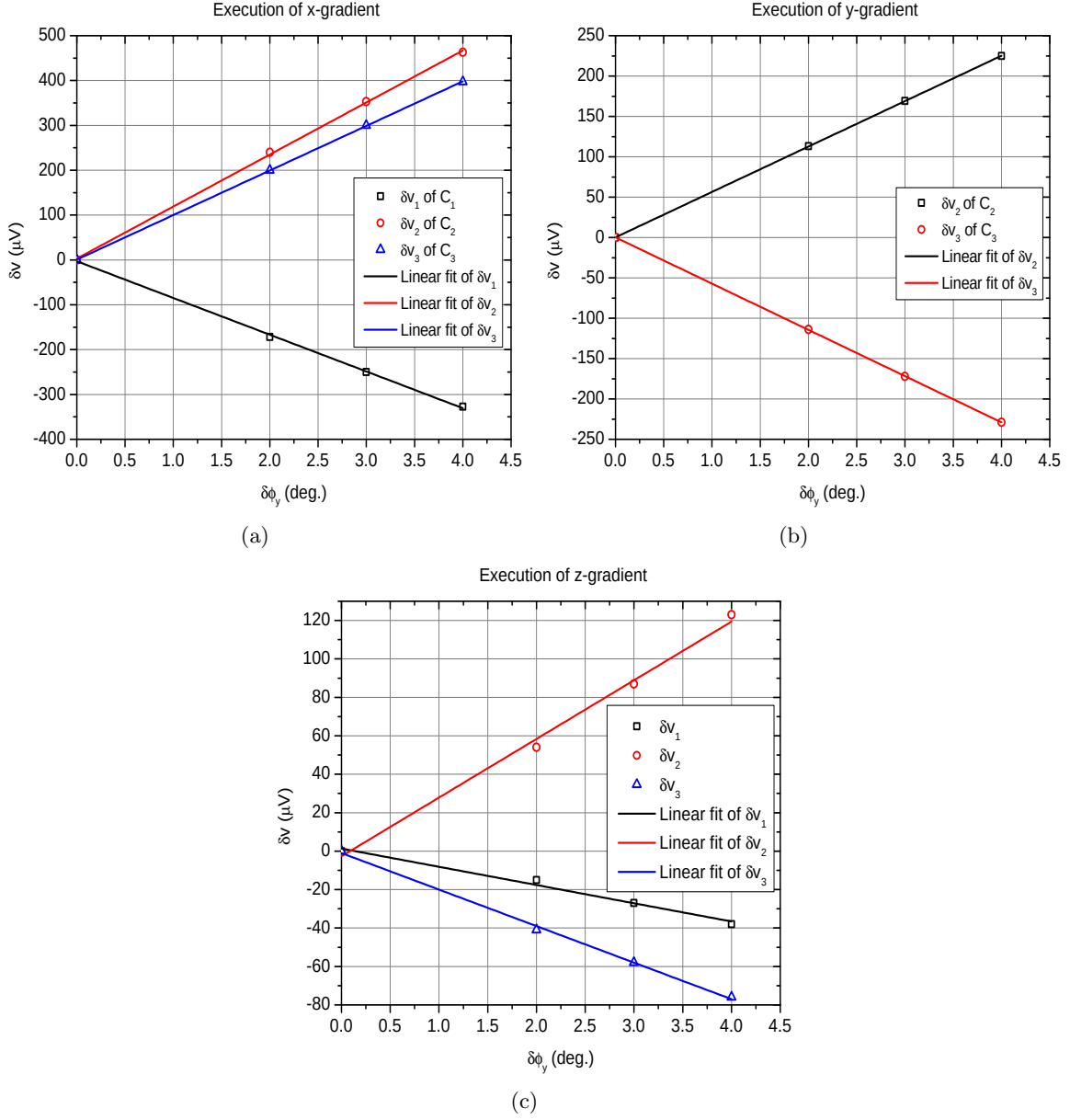


Figure 5.14: Experimental data indicate the change in induced voltage due to rotation about the y-axis and their linear fit (a) during the application of the x-gradient, (b) during the application of the y-gradient, and (c) during the application of the z-gradient.

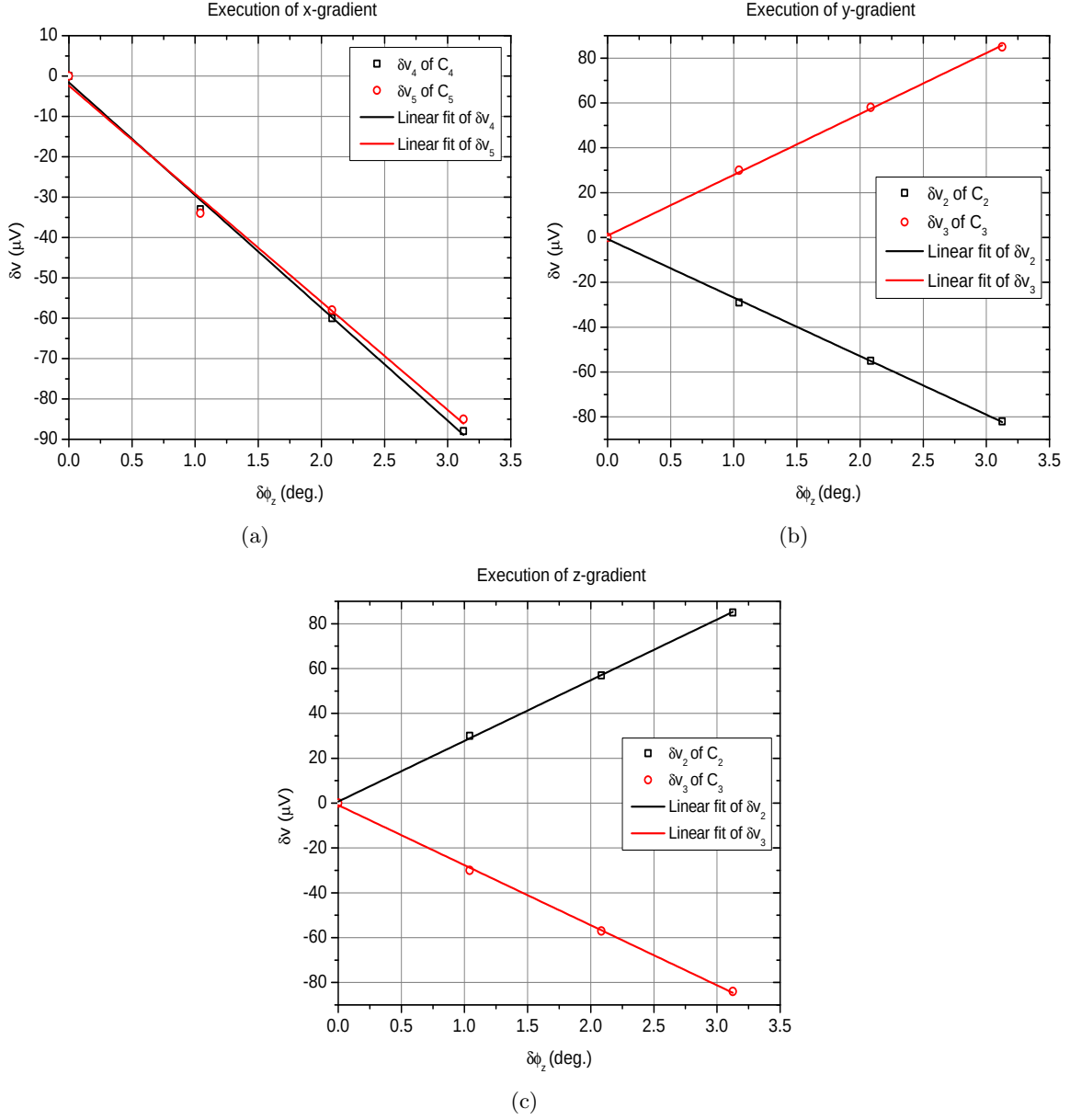


Figure 5.15: Experimental results represent the change in induced voltage in the coil due to rotation about the z-axis and their linear fit (a) during execution of a x-gradient, (b) during execution of a y-gradient, and (c) during execution of a z-gradient.

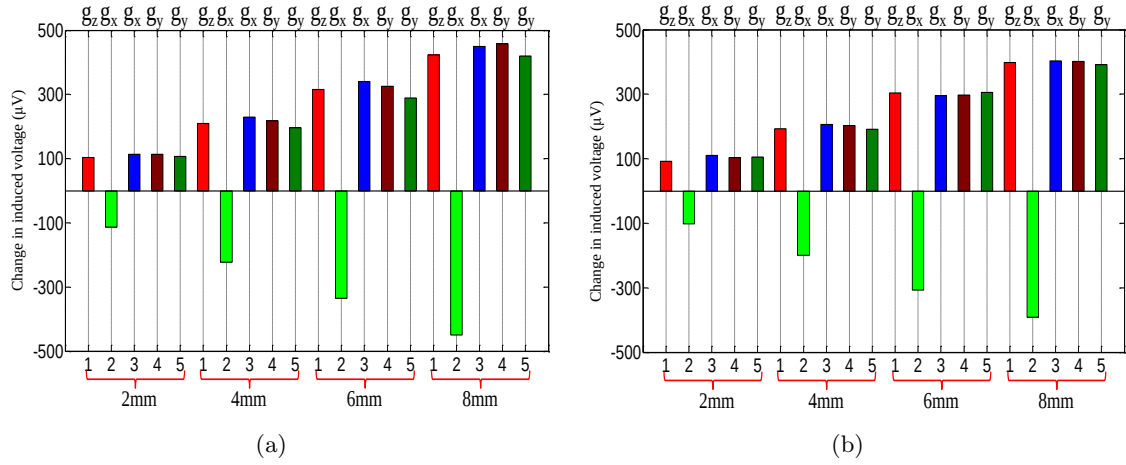


Figure 5.16: The change in the voltage induced in the 5 coils due to sequential translation along the z-direction for application of x, y, z gradients. (a) Experiment. (b) Simulation.

decomposing the voltages generated by a general small movement into a linear superposition of these patterns and thus identifying the motion parameters. Experimental results have good agreement with the results obtain from simulation both qualitatively and quantitatively.

5.3.2 Small Mixed Movements

This sub-section explores whether a matrix inversion can be used to find the positions and orientations of the coils from measurement of the induced voltage changes in the 5 coils. Here, small mixed movement means small random changes in position and orientation of more than at least 1 mm or 1° along/about two (a translation and a rotation) directions. For matrix inversion the standard MatLab command `pinv` (pseudo inverse) is used.

The change in induced voltage in the five coils is measured during execution of the three pre-pulses, involving RL, AP and FH gradients for successive individual movements of the rig along/about the x-, y- and z-axes. 15 voltages are measured at each position corresponding to the effect of 3 gradients on 5 different coils. By measuring the voltages at 3 values of each co-ordinate and employing basic linear fitting of the change in induced voltages (Fig. 5.12-Fig. 5.15), one can find the slopes and use these values to form a matrix A as described

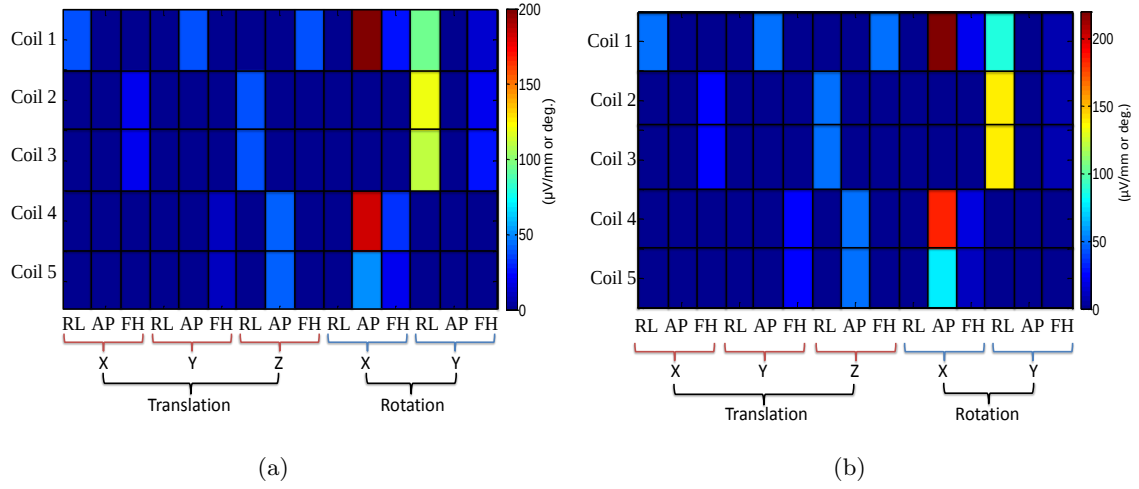


Figure 5.17: The coefficients of the matrix $\mathbf{A}$ for a system of linear equations $\mathbf{Ax} = \delta\mathbf{v}$. Colour-bar shows $\mu\text{V}/\text{mm}$ for translation and $\mu\text{V}/\text{degree}$ for rotation. (a) Experiment. (b) Simulation.

in Chapter 4. The equation relating the measured voltage δv to the position changes can be written as

$$\delta V = A\delta r^T, \quad (5.1)$$

where $\delta r^T = (\delta x, \delta y, \delta z, \delta\phi_x, \delta\phi_y)$. The matrix A could have 90 coefficients corresponding to the slopes of the linear equations obtained from basic linear fittings of the change in induced voltage in the 5 coils due to sequential changes in the 3 positions and 3 orientations. However, only 3 of these coefficients are non-zero.

A few general small movements, i.g. $\delta x, \delta z$, or $\delta y, \delta\phi_x$ of the coils are then generated by using the rig. The change in induced voltage in the coils is measured and one obtains δr^T by using the equation-

$$\delta r^T = A^{-1}\delta V \quad (5.2)$$

For small movements it is possible to work out δr^T precisely for a wide range of translations and small rotations. Figs. 5.18 and 5.18(b) show results for seven (1-7) sets of movement parameters. Each set contains two rows (A and E) of six $(\delta x, \delta y, \delta z, \delta\phi_x, \delta\phi_y, \delta\phi_z)$ movement parameters, where A stands for actual and E for estimated parameters. For the parameters,

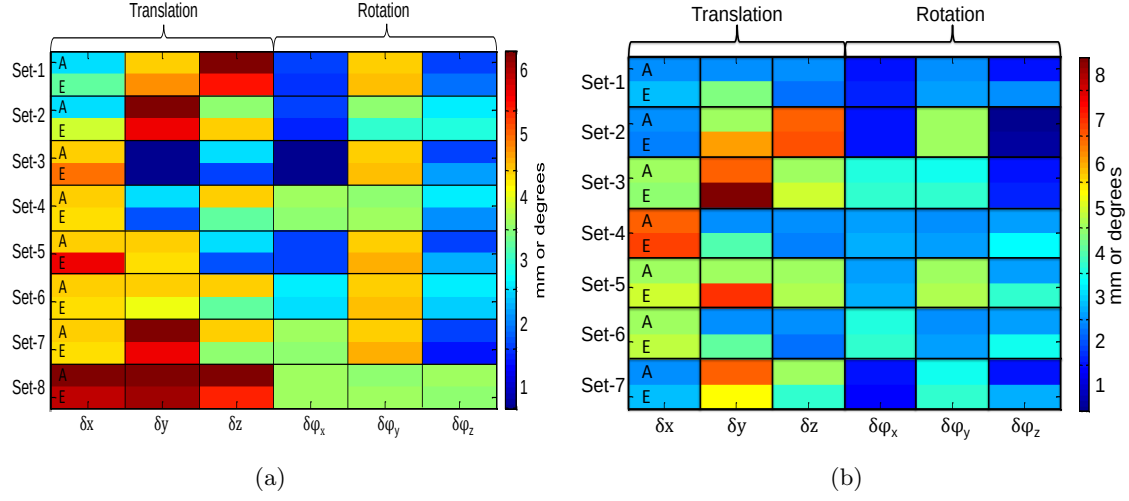


Figure 5.18: Comparison between actual (A) movements and movements estimated (E) from experiment. (a) Movements are limited to 6 mm (translation) and 5° (rotation) for the six parameters. (b) Movements are limited to 8 mm (translation) and 5° (rotation) for the six parameters.

when the actual and estimated parameters coincide then the small rectangle gets a single colour, there is a colour difference otherwise. One can compare the accuracy of the results obtained by using the inverse matrix. Deviation in δz is greater in the case of translation along the z-axis.

It is useful to check whether it is possible to estimate the movements parameters if I omit one or two of the six parameters (rotational parameters) measured. In Fig. 5.19(a) $\delta \phi_z$ and $\delta \phi_y$ are omitted because in planar coils there is no change in induced voltage due to z-rotation and in real life it is hard to rotate the head about the y-axis inside MR scanner head coil.

However, Fig. 5.19(a) shows seven sets of actual and estimated movement parameters. Each set contains four movement parameters $\delta x, \delta y, \delta z, \delta \phi_x$ sequentially. The data shows that a good estimation is possible, even when I omit two movement parameters. On the other hand, Fig. 5.19(b) shows four sets of movement parameters having $\delta x, \delta y, \delta z, \delta \phi_x, \delta \phi_z$ in each set and also shows that the actual and estimated data have a good agreement even

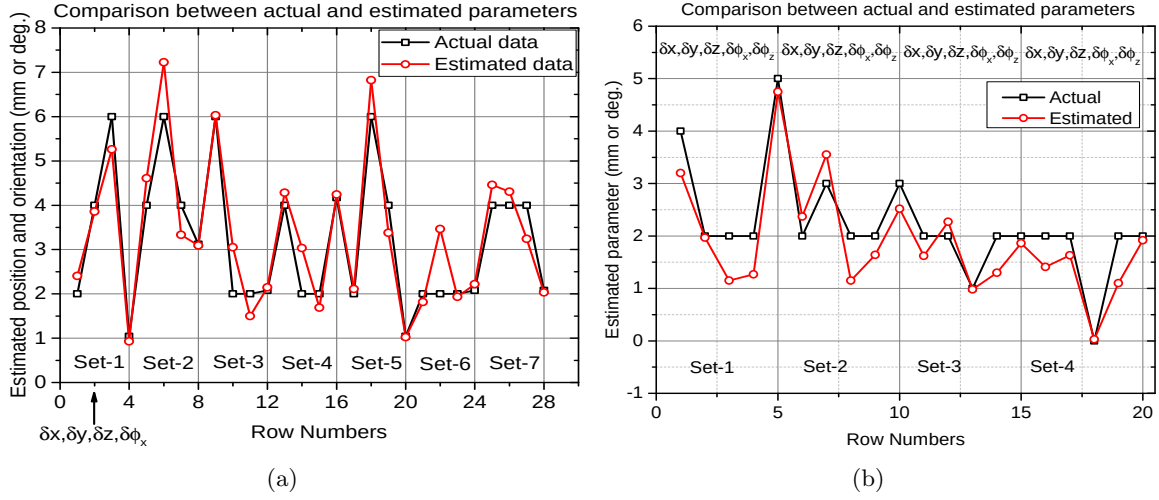


Figure 5.19: Comparison between actual movements and movements estimated from experimental measurement. (a) Movements are limited to translation along x, y and z, and rotation about the x-axis only. (a) Movements are limited to translation along x, y and z, and rotation about the x- and z-axes only.

when movements are limited to translation along x, y and z, and rotation about the x- and z-axes only.

5.4 Discussion

The change in induced voltage in the coils due to x-, y- and z-gradients following translation and rotation are measured. The changes in induced voltage mainly occur in the coils 1, 2 and 3 due to x-translation; in the coils 1, 4 and 5 due to y-translation and in the coils 1, 2, 3, 4 and 5 due to z-translation and also occurred in the coils 1, 4 and 5 due to x-rotation during execution of y- or z-gradients and in the coils 1, 2 and 3 due to y-rotation during execution of x- or z-gradient. Those results agree well with the theory described in Chapter 3 and the simulations described in Chapter 4. Changes in induced voltages are observed in the coils 2 and 3 due to x-rotation for the x-gradient, due to y-rotation for the y-gradient, due to z-rotation for y- or z-gradients and observed in the coils 4 and 5 due to z-rotation for x-gradient.

Voltages induced in a single coil mounted on a phantom by application of a sequence of three trapezoidal gradient pulses (10 ms ramp-up/down and 10 ms plateau at 2 T/m-s) successively applied along the RL, AP and FH directions. These pre-pulses produce bipolar square pulses in the EEG recordings. Within the short duration of ramp-up, head movement are very unlikely, but could occur. To reduce the effect of head movement during gradient ramp up, average are taken at the middle of the plateau and the induced voltage are measured by taking the difference between positive and negative pulses.

The measured induced voltage changes are plotted and found to be linear with the changes in position and orientation. Basic linear fitting of the change in induced voltage gives a system of linear equations. The change in position and orientation of the coils for a few general movements can then be estimated by taking the pseudo inverse of the system of linear equations. Estimated movement parameters are plotted. Actual movement parameters are compared with the estimated movement parameters and found to be in reasonable agreement.

5.5 Conclusions

The change in induced voltages in the coils mounted on the rig as shown in Fig. 5.1 are measured due to small movements (0-10 mm translation and 0-5° rotation) during execution of x-, y- and z-gradients. In order to check the validity of the linear model that is developed in Chapter 4, a system of linear equations is formed from the change in induced voltages in the coils. By using the matrix form of the system of linear equations and inverting the matrix, general changes in position and orientation of the coils in the MR scanner can also be estimated.

By analyzing the experimental data one can conclude as follows:

- The voltages induced in a set of coils by switched gradients provide a sensitive measure of translation and rotation ($10 \mu\text{V}$ per mm shift per cm^2 coil area per turn for $\frac{dG}{dt} = 100 \text{ T/m-s}$).

- There is a linear variation of the induced voltage for translation and small rotations.
- Measured voltages agree with simulations (and theory).
- We can measure translation and rotation from measured voltages.

Chapter 6

Fast Field Echo: Gimbal, Spherical Phantom and Non-planar Coils

The size and direction of movements made during MRI using a set of coils mounted onto a spherical head phantom are measured. Statistical Parametric Mapping (SPM) is used to estimate the movement parameters for head-nod, head-shake and foot-head movements of the phantom. The parameters are also estimated by measuring the EMF induced in the coils. A linear set of equations relating position change to induced voltage is again formed and inverted to allow position changes to be found from voltage measurement.

6.1 Introduction

In this chapter data-led approaches are explored which can use arbitrary coil shapes e.g. coils fixed in an EEG-type cap. The idea here is to precede an experiment by measurement of the voltages induced in the coils by three gradient pre-pulses (RL, AP and FH) on the three different gradient channels after each of a set of small, broadly defined movements (such as head-nod, head-shake and axial translation) for which head position can also be characterized from image data. Since the voltage changes will be linear with motion parameters for small

movements, this will allow a system of equations relating the pattern of voltage change to movement parameters to be established. In subsequent experiments the monitoring of voltage changes during pre-pulse execution will allow head movement to be monitored on a slice-by-slice basis in EPI or on a k-space line-by-line basis in Fourier imaging experiments.

In this study a gimbal [127] is used to hold a spherical head phantom onto which an EEG-like cap is mounted. The cap carries five arbitrary shaped coils. The gimbal allows the phantom to be translated along the z-axis and rotated about the x-axis (nod) and about the z-axis (shake). Simultaneously by using MR the image contrast and image realignment tools SPM8 (A major updated version of the SPM software, released *April 2009*), one can find out the change in position and orientation of the spherical phantom. In this study a linear model is formed from the change in induced voltage in the coils due to change in position and orientation of the coils. This linear model allowed us to find the position and orientation of the spherical head phantom from the measured voltage. The movement parameters obtained via measurement of the change in induced voltage in the coils and parameters obtained from SPM8 are then compared.

6.2 Method and Materials

A spherical agar phantom which has a diameter of 20 cm containing a 3D cross-shaped, water-filled structure was constructed, as shown in Fig. 6.1. Since the relaxation times of the water inside the cross and the agar are different, the MR images should show differences in contrast. The presence of the 3D cross in the agar phantom make it easier to co-register images of the phantom acquired at different positions. Initially the phantom was positioned so that coil 4 and 5 were at the scanner's isocentre [128]. This position will be considered as the reference position. All other changes in position and orientation are measured from this reference point. Positioning of the subject within the scanner can reduce the GA (Gradient Amplitude), decreasing the chance of amplifier saturation [128]. The gimbal [127] is used to hold a spherical agar phantom of diameter 20 cm which is shown in Fig. 6.2(A). The gimbal

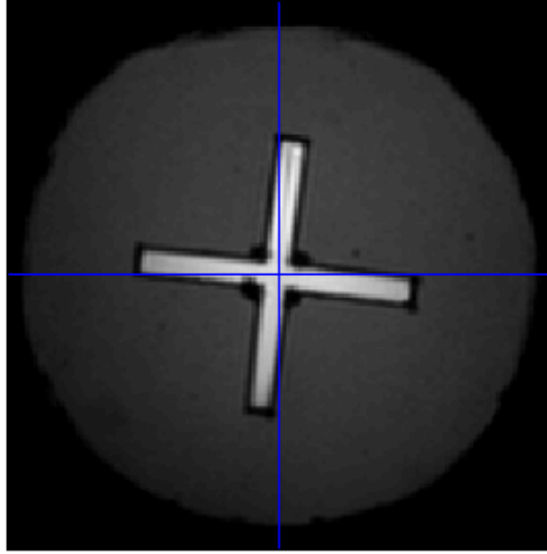


Figure 6.1: An MR image of a spherical agar phantom having a 3D cross of water.

allows the phantom to be moved along the z-axis as well as facilitating rotation about the x- and z-axes. A spherical head phantom was constructed and a cap carrying five coils of arbitrary shape, having two turns each, was placed on the phantom. The whole system is put into a 3T MR scanner by holding the phantom in the gimbal. The coils were connected to a bipolar amplifier (Brain Product) by star quad cables. Fig. 6.2(B) shows an EEG-like cap carrying five large coils each formed from two turns, connected to a bipolar amplifier by star quad cable.

Data-led approaches are explored which can use arbitrary coil shapes e.g. arbitrary coils fixed onto an EEG-type cap. The idea here is to precede an experiment by measurement of the voltages induced in the coils by pre-pulses on the three different gradient channels after each of a set of small, broadly defined movements (such as head-nod, head-shake and axial translation) for which head position can also be characterized from image data. Since the voltage changes are linear with motion parameters for small movements, this will allow a system of equations relating the pattern of voltage change to movement parameters to be established. In subsequent experiments the monitoring of the voltage changes during pre-pulse execution would allow head movement to be monitored on a slice-by-slice basis in EPI

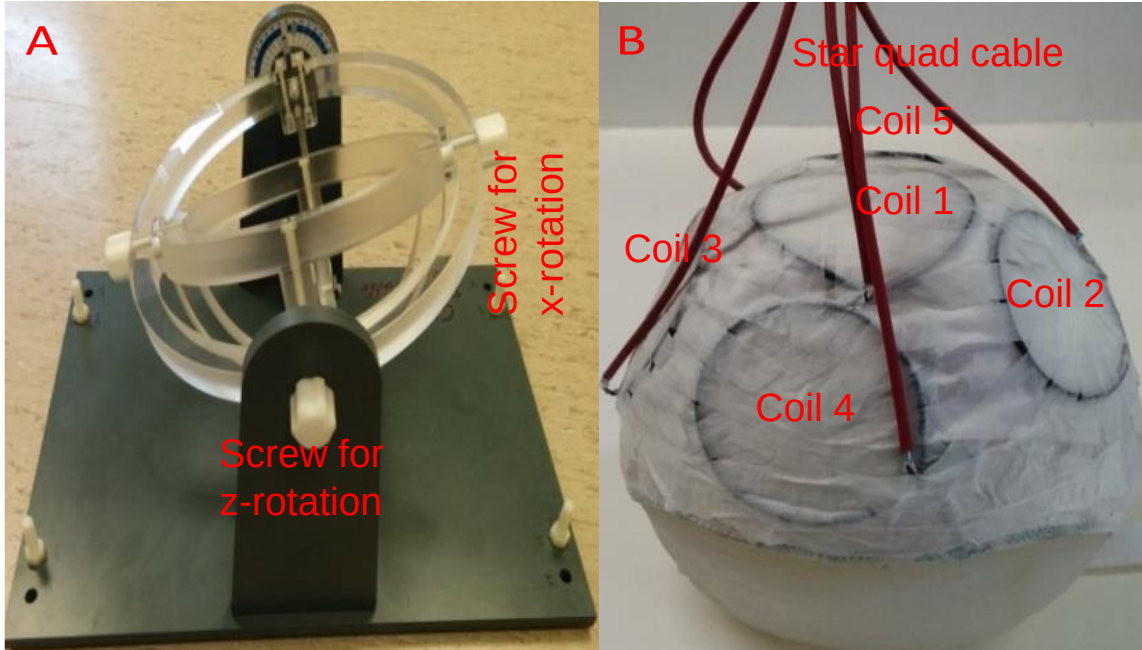


Figure 6.2: The Gimbal allowing calibration of x- and z-rotation is shown in (A) and the spherical head phantom covered with EEG like cap mounted by five coils is shown in (B).

or on a k-space line-by-line basis in Fourier imaging experiments.

A set of measurements are recorded that characterized the change in the voltages induced within a set of coils by the application of MRI gradients for a basis-set of typical head movements. Then this data is used to confirm whether the voltage change is linear with respect to small changes in position. This comprises three sets of recordings:

6.2.1 Characterising the effect of z-displacement

Varying the position of the phantom along the z-axis (a range of approximately -10 mm, 10 mm) the induced voltage in the coils are measured. A ruler is attached to patient bed by tape to calibrate the z-translation. The voltage changes are recorded using the bipolar square pulses induced by the custom pre-pulses and a set of T_2^* -weighted Gradient Echo (GE) images were acquired to estimate positions by using image co-registration. The position change can be varied by sliding the Gimbal up and down the scanner bed. The position changes are

marked on the bed by using a ruler before attaching the Gimbal.

6.2.2 Characterizing the effect of a rotation about the x-axis

The phantom sits in the gimbal which allows orientation of the head about the x-axis (simulating a nod type motion) to be adjusted. It is better to fix a range of rotation spanning approximately $[-10^\circ, 10^\circ]$. Both GE and custom EPI (pre-pulses) are acquired at each position using the body coil excitation. Variation in orientation can be achieved either by leaning into the bore or by moving the bed in and out taking care to return the bed to the same position using the scan position button (no laser realignment).

6.2.3 Characterizing the effect of rotation about the z-axis

This is similar to the characterization of the effect of a rotation about the x-axis, except now involving variation of the orientation about the z-axis (simulating a shake of the head). Using the GE images, the size of the position changes are estimated by using SPM8. The peak-peak voltage from the induced bipolar square pulses for each of the orthogonal gradients at each position. The voltage changes are plotted against position measured using SPM8 and the rate of change was determined by using a basic linear fit. A linear fit explains the relationship between the variation in position and the change in induced voltage.

6.2.4 Mixed Movements

Here the phantom position is varied using a mixture of rotations and translations. The aim is to see if the position changes can be measured using the change in the voltage recordings. Measurements will be acquired using the phantom mounted in the gimbal with the custom EPI pre-pulse and the GE sequence being executed at each position. Actual position changes relative to the initial position will be measured using the realignment parameters estimated by SPM8.

If the relationship between the voltage changes and the position are linear, then the following relationship will be true for the set of movements considered here.

$$\begin{pmatrix} V_{RL}^1 - V_{RL}^0 \\ V_{AP}^1 - V_{AP}^0 \\ V_{FH}^1 - V_{FH}^0 \end{pmatrix} = \begin{pmatrix} dV_{RL}/dz & dV_{RL}/d\theta & dV_{RL}/d\psi \\ dV_{AP}/dz & dV_{AP}/d\theta & dV_{AP}/d\psi \\ dV_{FH}/dz & dV_{FH}/d\theta & dV_{FH}/d\psi \end{pmatrix} = \begin{pmatrix} z^1 - z^0 \\ \theta^1 - \theta^0 \\ \psi^1 - \psi^0 \end{pmatrix} \quad (6.1)$$

This matrix can be written in the following form:

$$\delta V = A\delta r, \quad (6.2)$$

where $V_i^1 - V_i^0$ is the difference in the voltage measured for a gradient applied along the $i = RL, AP$ and FH directions at position 1 and the initial position (position 0) and $z^1 - z^0$, $\theta^1 - \theta^0$ and $\psi^1 - \psi^0$ are differences in position for a z-displacement, x-rotation and z-rotation respectively.

The differentials (e.g. dV_{RL}/dz) for the first analysis can be estimated from the linear fit parameters determined from experiment. Position changes can then be estimated by using a pseudo inverse ('pinv' in MatLab) of the design matrix and multiplying it by the voltage change vector: $\delta r \approx A^{-1}\delta V$

For the second analysis, the differentials can be estimated using two position change measurements along each direction from the data collected in each experiment: Explore Linearity, described above; i.e. $dV_{RL}/dz \approx (V_{RL}^1 - V_{RL}^0)/(z^1 - z^0)$. Position changes can again then be estimated using the pseudo inverse of the design matrix.

6.2.5 Scanning Parameter

EPI gradient pre-pulses are used to measure the change in induced voltage in the coils attached to an EEG cap and mounted onto a spherical head phantom. In the case of the gradient pre-pulses, the gradient strength is 20 mT/m, slope 10 ms and it is applied for a duration of 10 ms. For the scanning parameters I use $FOV = (192 \times 192 \times 15) \text{ mm}^3$, voxel size = $(3 \times 3) \text{ mm}^2$, slice thickness 3 mm, $TE = 35 \text{ ms}$ and $TR=2000 \text{ ms}$. For the geometry I took 5 slices, 5 dynamics, 80° flip angle, transverse slice orientation, fold over direction AP and descend slice scan order.

On the other hand for image acquisition I use Fast Field Echo (FFE), Q-body coil, $FOV = (AP \times RL \times FH) = (200 \times 200 \times 179) \text{ mm}^3$, voxel size = $(AP \times RL \times FH) = (1.52 \times 1.52 \times 1.52) \text{ mm}^3$, number of slices 118, transverse slice orientation, fold over direction, RL.

6.2.6 MR Images and Realignment

Images were acquired on a Philips Achieva 3T MR scanner. A 3D, T_2^* -weighted GE-sequence (Fast Field Echo [FFE]) was used to acquire structural data at each position. These images were used in the analysis to determine by how much the phantom had moved. Each pre-pulse consisted of a 10 ms ramp up at a rate of 2 T/m-s followed by a 10 ms plateau and 10 ms ramp down at the same rate. Simultaneously, induced voltage in each coils are recorded by using a 16-channel MR-compatible BrainProducts system with clock synchronization. The sampling rate was 5 kHz; high- and low-pass filter cut-off's were 0.016 Hz and 1 kHz respectively with a 30 dB/octave roll-off.

Accuracy of co-registration of scans was studied by Glyn [127] using the FFE sequence. That gave a measure of the error in co-registration of images. By using SPM8 the FFE images were co-registered which provide a precise ($\approx 0.01 \text{ mm}$ depending on pulse sequence) measure of the movement of the phantom based on x-, y- and z-translations and rotations respectively.

6.3 Results and Discussions

This section presents the experimental findings and discusses them in details. Movement parameters are estimated from image realignment by using Statistical Parametric Mapping (SPM). The change in induced voltage in the coils is measured by using EEG system. The change in induced voltage change *vs* movement parameters which varies linearly and the slopes are determined by basic linear fittings. A linear model is formed to estimate the movement parameters by analyzing the change in induced voltage in the coils. A matrix equation is formed by using the corresponding slopes and the movement parameters by taking matrix inversion and compare them with movement parameters obtained from image realignment by using SPM8.

6.3.1 Movement Parameters via SPM

In co-registered images the recording data set 1 is considered as the reference image. The rest of the FFE images are co-registered to this data set to sequential movement to quantify the actual change in position and orientation of the phantom.

Fig. 6.3 shows the results of the co-registration of the FFE images from datasets (a)-(f) of phantom data acquired during sequential movements. However, there are components of measured movement that are significantly larger than the net co-registration errors. Rotational motions could introduce translational errors in two possible ways. If the centre of the circular phantom is not exactly aligned with the centre of rotation of the gimbal, then rotations may also produce actual translations of the phantom. The other potential source of error is that the estimated centre of mass of the phantom from the realignment algorithm may not correspond to the physical centre of rotation of the phantom. Additionally, translational errors may be caused by heating of the passive shim elements in the magnet, which can alter the average Larmor frequency and thus shift the image. However, the effect is much smaller than that of the dominant movements. The phantom is adjusted to have a few positions/orientations that are made up of mixed small z-translation and x- and z-rotations.

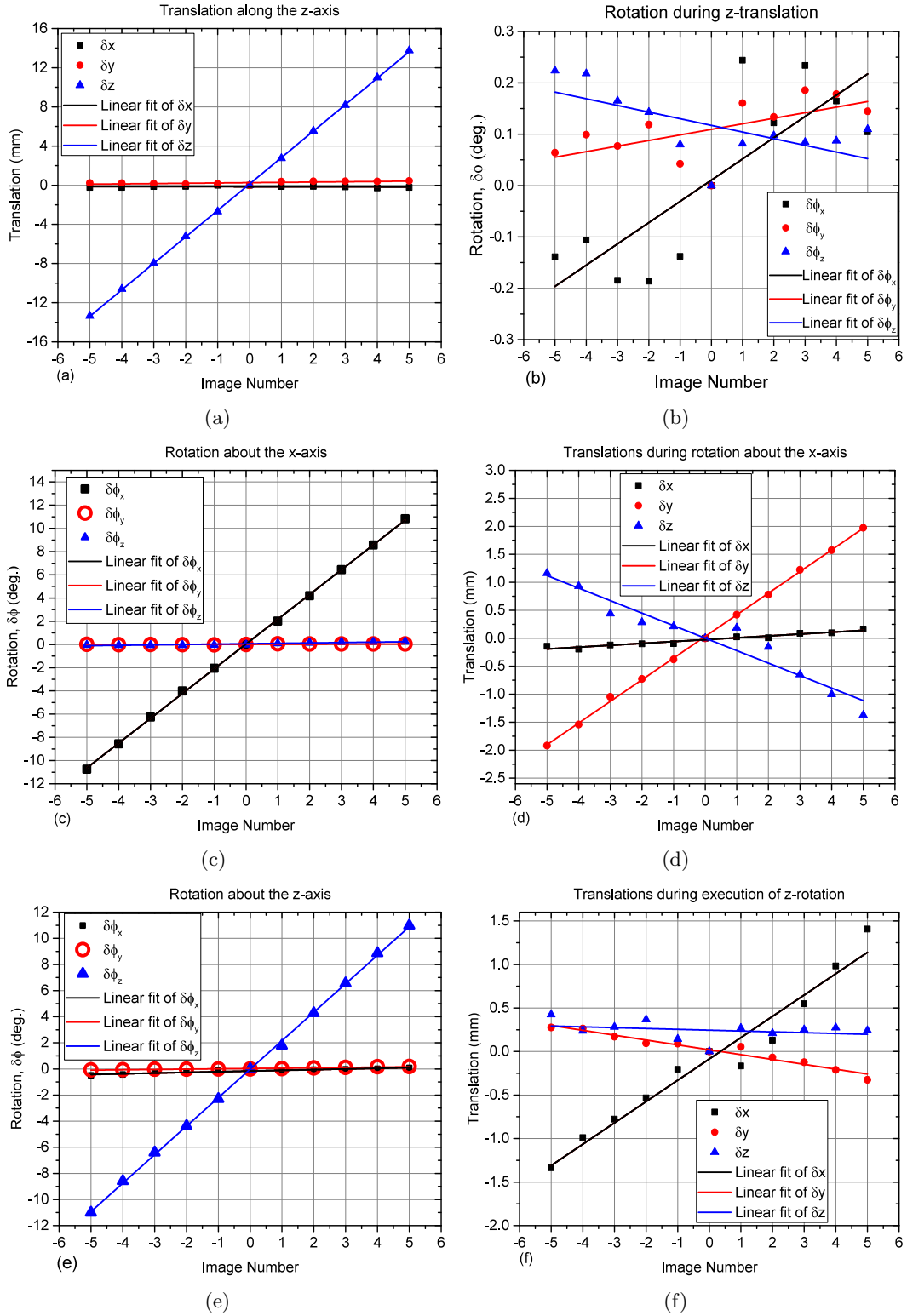


Figure 6.3: The movement parameters from the co-registered FFE images. Where (a) & (b) z-translation, (c) & (d) rotation about the x-axis and (e) & (f) rotation about the z-axis.

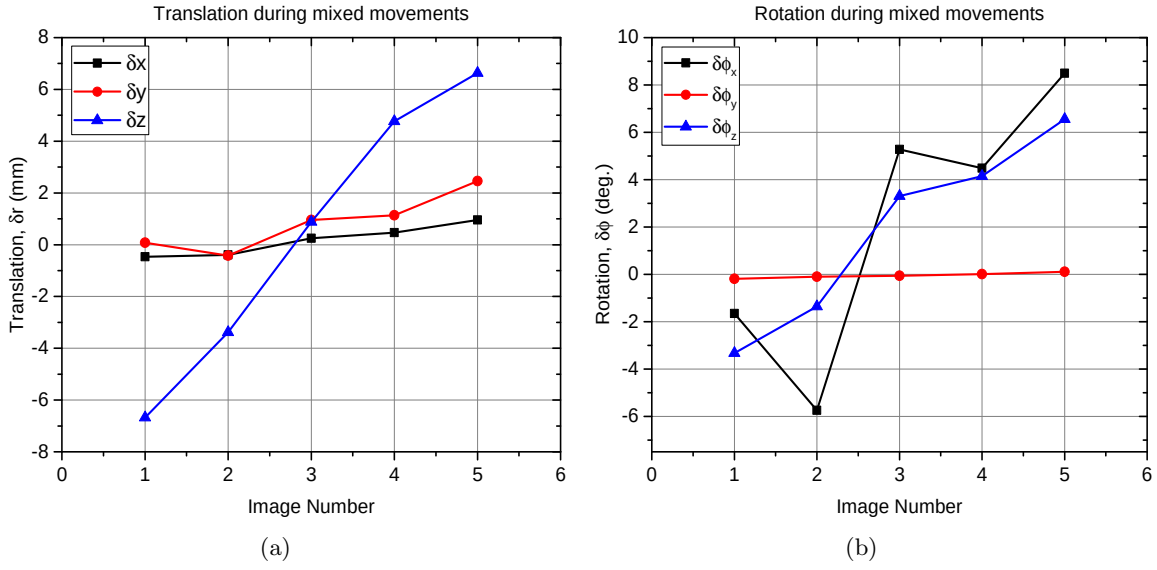
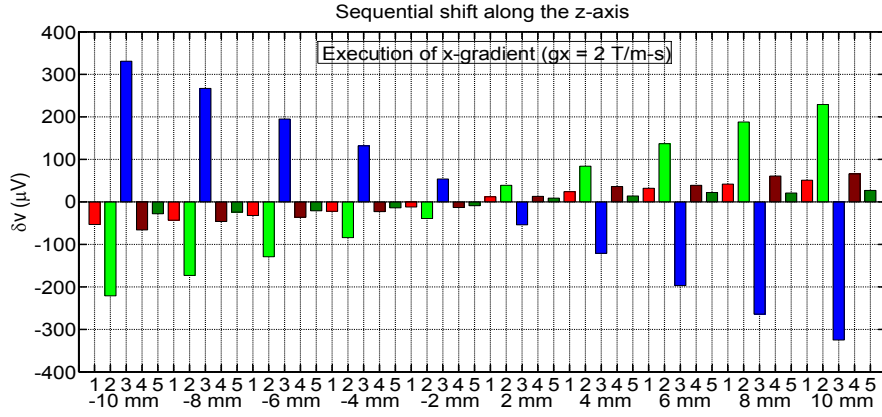


Figure 6.4: Movement parameters via SPM from image realignment. (a) Translational parameters via SPM using image realignment. (b) Rotational parameters via SPM using image realignment.

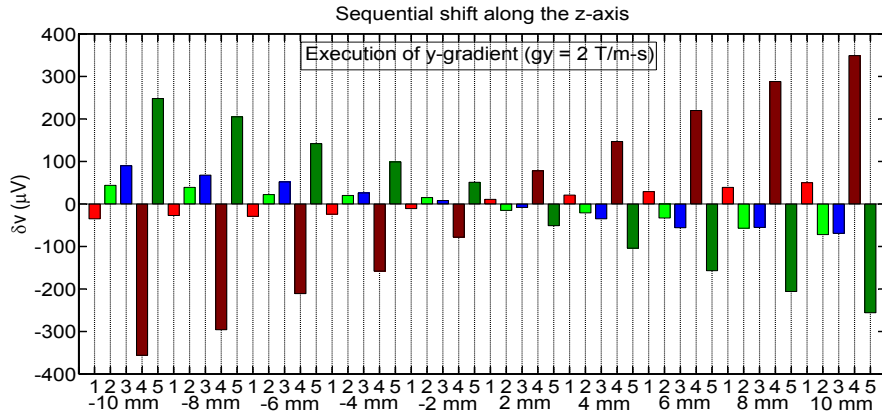
Images realignment by using SPM8, gives us three movement parameters. Fig. 6.4 shows the results obtained from image co-registration due to mixed movement.

6.3.2 Movement Parameters via Voltage Change in the Coils

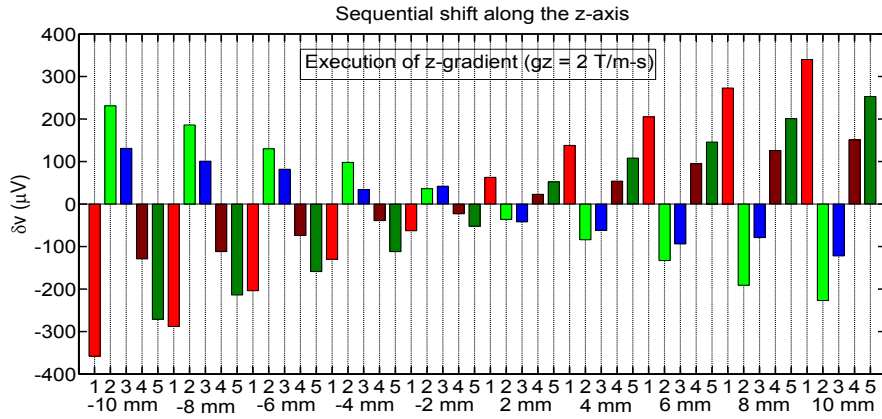
The voltage induced in the coils due to execution of three gradient pre-pulses (RL, AP and FH) are measured by using the MR-compatible EEG apparatus. Gradient pre-pulse of strength 20 mT/m and slope 10 ms is applied for 10 ms so that the induced voltage in the coils does not exceed the upper cut-off of the bipolar amplifier. Arbitrary-shaped coils having two turns (more turns induced too much voltage) are used to record voltages from pre-pulses for different position and orientation of the phantom. The change in induced voltage in the coils due to sequential shift along the z-axis is shown in Fig. 6.5 where x-, y- and z-gradients are applied respectively. It is interesting that the change in induced voltage occurred in all of the coils due to translation along the z-axis because of execution of a particular gradient, but for the x-gradient the major voltage is induced in coils 2 & 3, for the y-gradient in the



(a)



(b)



(c)

Figure 6.5: The change in induced voltage in the coils due to sequential z-translations. (a) Execution of x-gradient, (b) Execution of y-gradient, (c) Execution of z-gradient

coils 4 & 5 and for the z-gradient there is a significant change observed in all of the coils.

The change in induced voltage in the coils due to sequential rotation about the x-axis because of execution of three gradient pre-pulses is shown in Fig. 6.6. A significant voltage change occurred in the coils 2, 3, 4 and 5 for execution of x-gradient, for y-gradient it is in the coils 1, 2, 4 and 5, whereas for the z-gradient it occurred in all of the coils. The change in induced voltage in the coils due to sequential rotation about the z-axis (Yaw) is shown in Fig. 6.7. A significant voltage change is occurring in the coils 1, 2, 4 and 5 because of execution of x-gradient which is shown in Fig. 6.7(a), for y-gradient it is observed in the coils 1, 2, 3 and 4, but for the z-gradient it is in the coils 2 and 3 only. The shape and location of the coils are arbitrary so it is not possible to calibrate for x- and y-rotation. However, the change in induced voltage is linear with sequential translation and rotation. Linearity does not break for any of the three movements. As a result one can form a linear model.

The voltage changes in the coils and their linear fittings in case of translation along the z-axis for three gradient pre-pulses are shown in Fig. 6.8. Slopes are obtained from linear fits. Fig. 6.9 shows the voltage change in the coils and their linear fits. It is interesting that linearity persists for a wide range of rotations ranging from -10 to +10 degrees. Again, the change in induced voltage in the coils due to sequential rotation about the z-axis for a range of -10 to +10 degrees is linear with respect to rotational angle. Linearity does not break for a wide range of rotations about the x- and z-axis. On the other hand linearity persists for any range of translation along the z-axis. From image realignment using SPM8, the movement parameters are obtained for three movements (head-nod, head-shake and z-translation). The change in induced voltage in the coils versus movement parameters obtained from image realignment is plotted and the corresponding slopes are determined from linear fittings of the voltage. By using these slopes a 9×3 matrix is formed which is known as A. A system of linear equations is formed in the matrix form as $\delta V = A\delta r$, where $\delta r = [\delta z, \delta\theta, \delta\psi]$ and $\delta V = [\delta V_z, \delta V_\theta, \delta V_\psi]$.

It is possible to identify the position by inverting the matrix equation based on: (i)

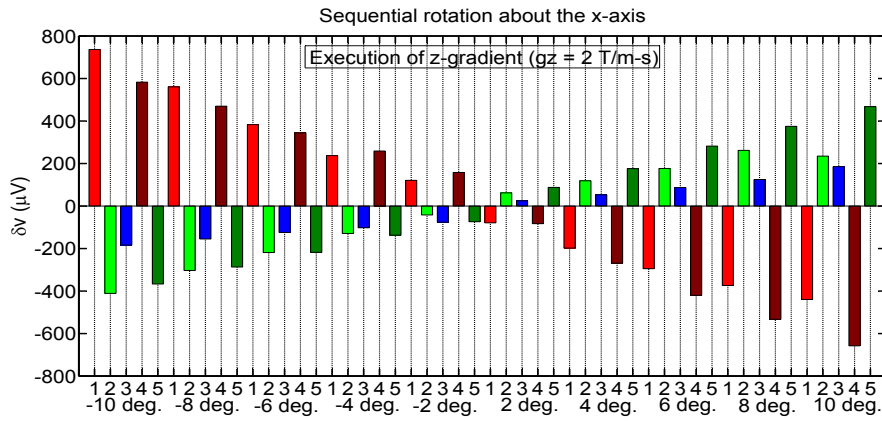
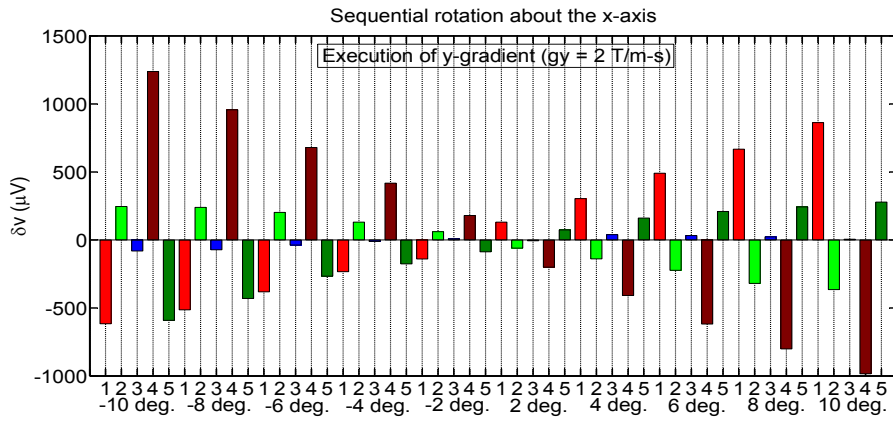
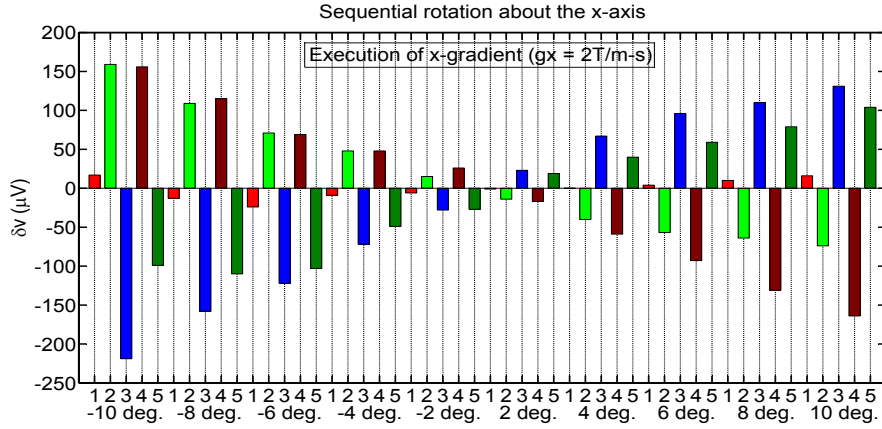


Figure 6.6: Induced voltage change in the coils due to sequential rotation about the x-axis. (a) Execution of x-gradient. (b) Execution of y-gradient. (c) Execution of z-gradient.

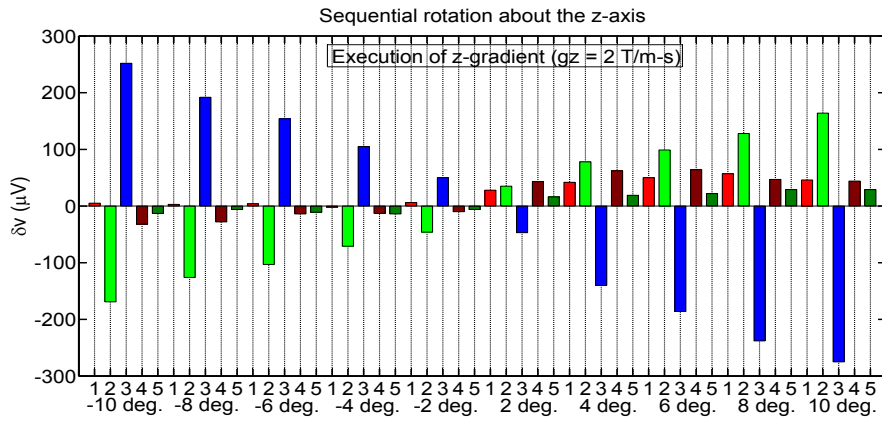
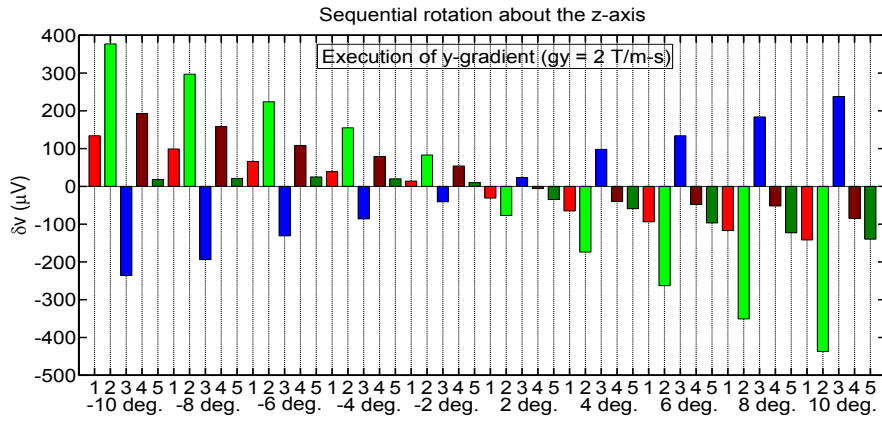
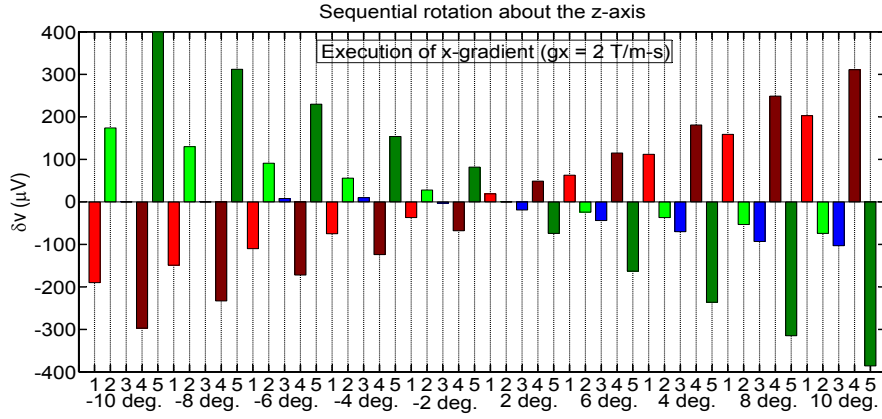


Figure 6.7: The change in induced voltage in the coils due to sequential rotation about the z-axis. (a) Execution of x-gradient. (b) Execution of y-gradient. (c) Execution of z-gradient.

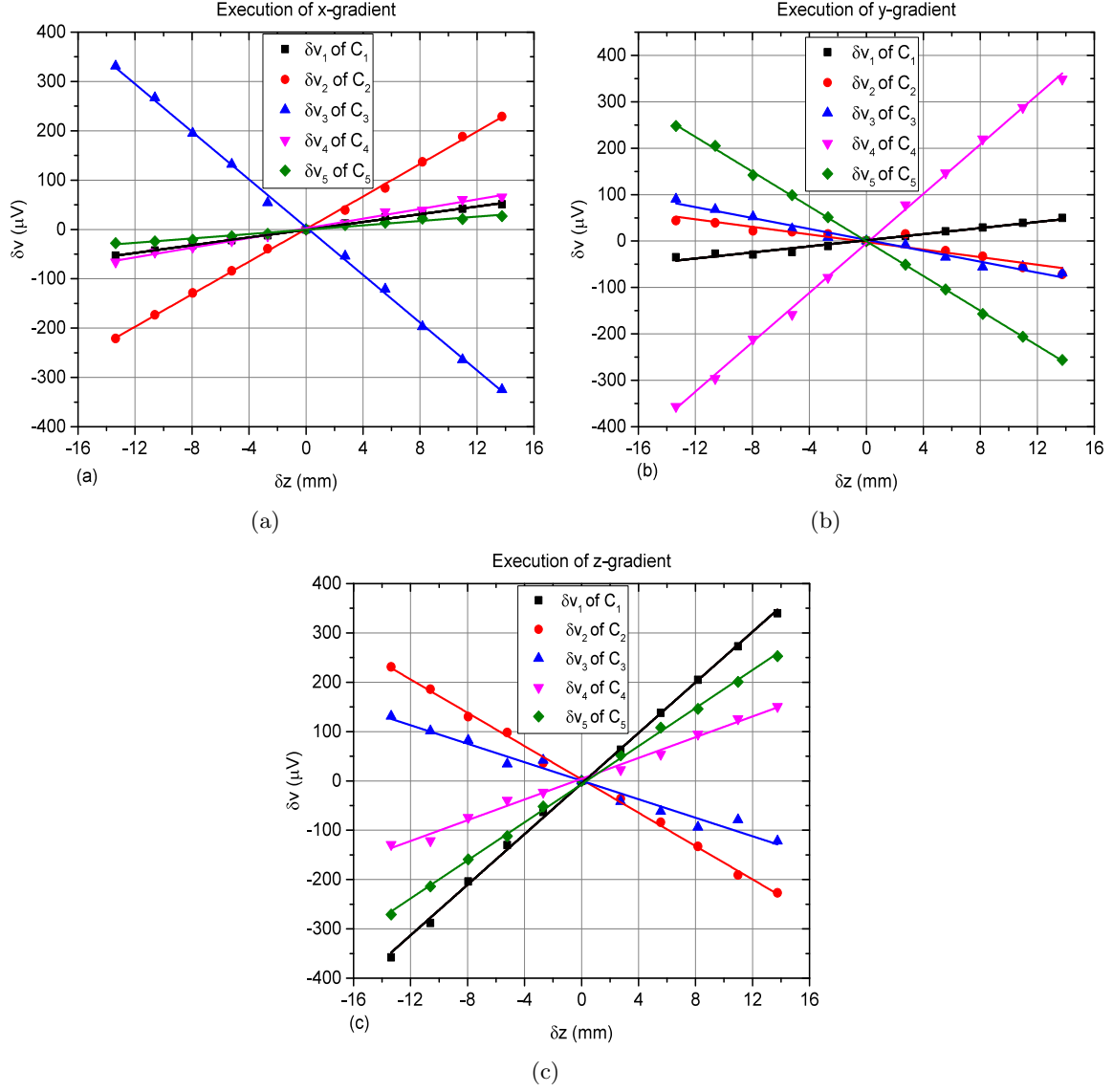


Figure 6.8: Experimental data indicate induced voltage change in the coils and their linear fittings. (a) The change in induced voltage in the coils due to execution of x-gradient and their linear fittings. (b) The change in induced voltage in the coils due to execution of y-gradient and their linear fittings. (c) The change in induced voltage in the coils due to execution of z-gradient and their linear fittings.

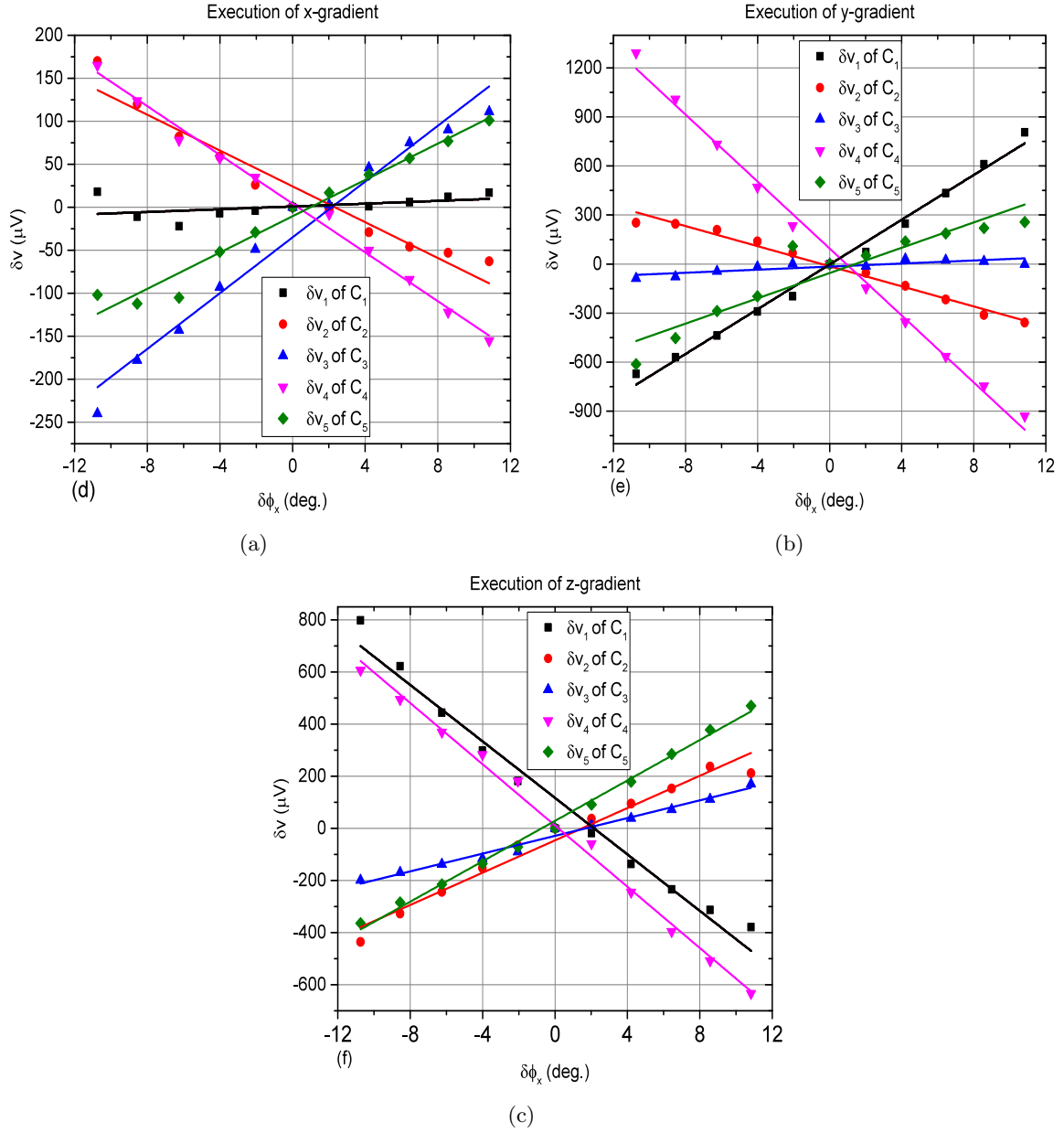


Figure 6.9: Experimental data indicate the change in induced voltage in the coils and their linear fittings. (a) The change in induced voltage in the coils during execution of x-gradient and their linear fittings. (b) The change in induced voltage in the coils during execution of y-gradient and their linear fittings. (c) The change in induced voltage in the coils during execution of z-gradient and their linear fittings.

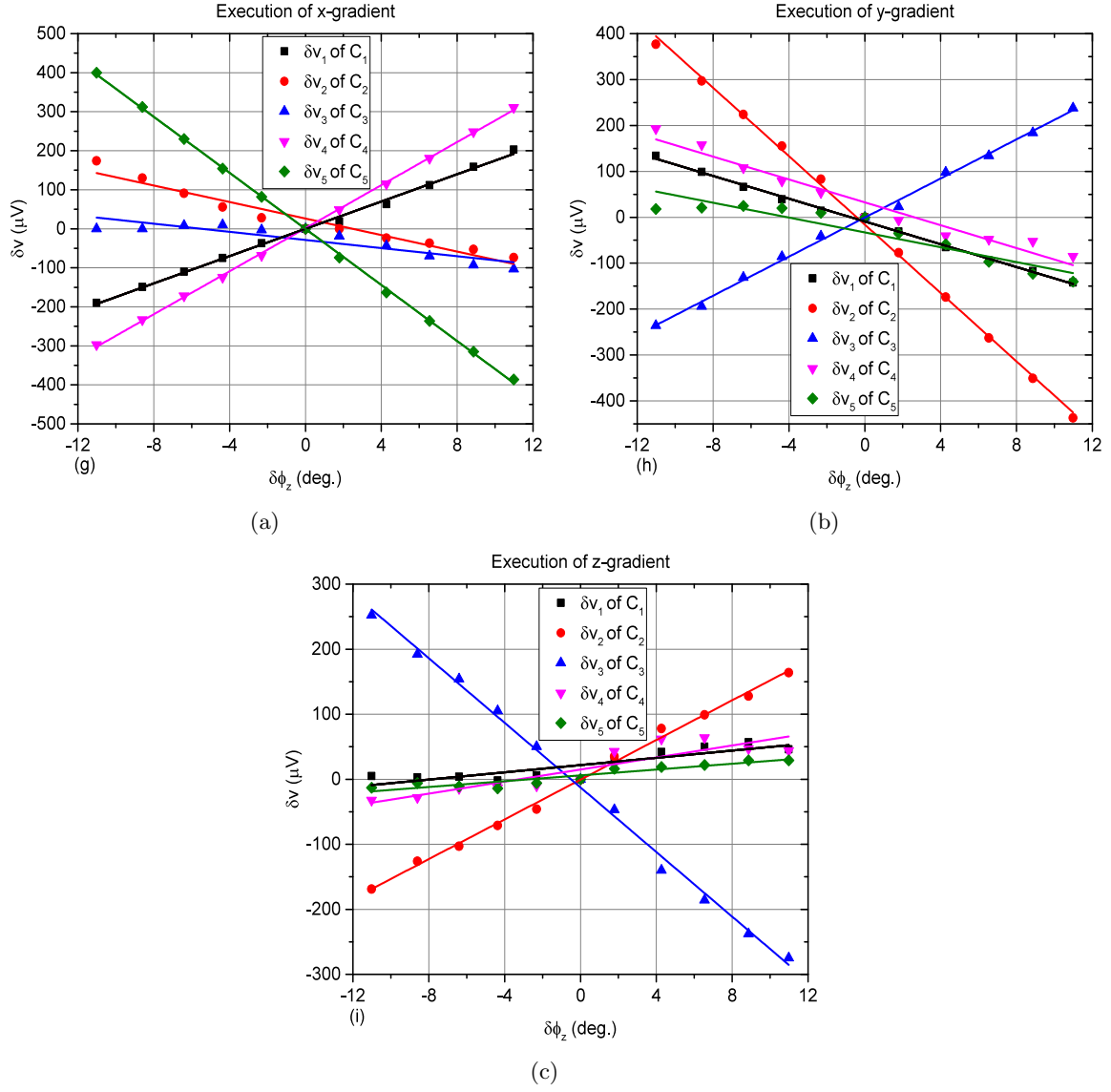


Figure 6.10: Experimental data indicate induced voltage change in the coils due to sequential rotation about the z-axis and their linear fittings. (a) The change in induced voltage in the coils during execution of x-gradient and their linear fittings. (b) The change in induced voltage in the coils during execution of y-gradient and their linear fittings. (c) The change in induced voltage in the coils during execution of z-gradient and their linear fittings.

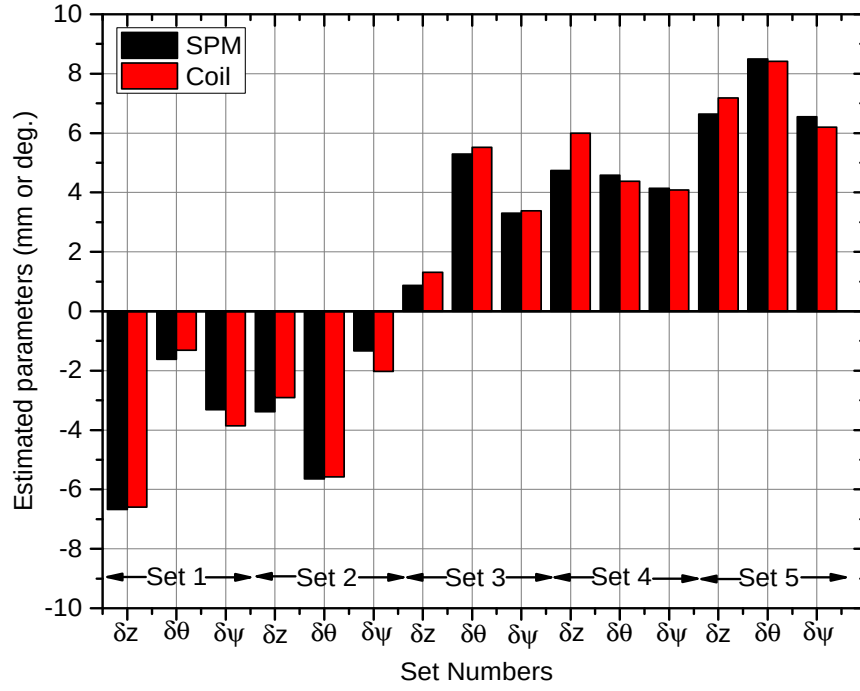


Figure 6.11: A comparison between movement parameters obtained via SPM and obtained via voltage measurement in the coils.

coefficients from linear variation; (ii) single measurements of voltage change due to single known movements (from realignment parameters).

There are two groups of movement parameters: one from image realignment by using SPM8 and another group obtained via measuring the change in induced voltage in the coils. By processing acquired data and following our procedure, in particular taking matrix inversion, one can estimate the movement parameters. The comparison between the two sets of movement parameters is shown in the bar plot in Fig. 6.11. This shows five sets of data each having z-translation, x-rotation and z-rotation. The black bars represents data obtained from SPM8, whereas red bars indicate estimated data by using sensor coils. In each set there are three movement parameters $\delta z, \delta \theta, \delta \psi$ sequentially which is shown in Fig. 6.11. The parameters obtained from SPM8 and the parameters obtained via measurement of induced voltage in the coils have a good agreement.

6.4 Conclusion

Change in induced voltage is observed in all of the coils due to movement of the spherical head-shape phantom during execution of the x-, y- and z-gradient pre-pulses. Whatever the shape of the coil and wherever it is built in onto an EEG like cap the change in induced voltage is always linear with the change in position and orientation into MR scanner.

When the coils built onto a spherical head phantom (small enough) mounted onto an EEG like cap, the linearity continue for a wide range of translations and rotations. It is quite interesting that linearity in the induced voltage change is observed in all of the coils during execution of the x-, y- and z-gradients. It could happen due to the influence of more than one gradient on a coil when it is placed in a space (real case) rather than in a plane of a cube (simple case). Perhaps linear change in the induced voltage in a coil blow up the quadratic changes. It agrees well with simulation done in the Chapter 4 for arbitrary coil.

It is very much possible to form a system of linear equations to estimate the positions and orientations of the spherical head phantom as well as the coils via measurement of the change in induced voltage in the coils.

On the other hand, if the coils are in a particular plane the change in induced voltage during execution of a z-gradient is non-linear. In that case the change in induced voltage is very small compared to the change in induced voltage in the coils exhibiting linearity. So, linear change in induced voltage in the coils dominates over non-linear changes. As a result the linear model does not fail in any case.

Chapter 7

Optimization of Gradient Pre-pulses and Multivariate Analysis (MVA) for Prediction of Parameters

7.1 Introduction

Previous work has suggested that head movement can be monitored by using measurements of the change with respect to position of the voltages induced in a set of coils by time-varying magnetic field gradients [108, 120, 121]. Particularly, in high field MRI the image resolution can be a less than a millimeter and even such a very small movement can cause substantial artefacts [122]. In recent years many motion correction techniques have been suggested, but none of the devices or methods proposed so far produces a workable solution for clinical use. The use of gradient-induced voltages in coils attached to the subject has potential advantages over other motion monitoring techniques [41, 109, 123, 124], as it does not require line-of-sight access to the subject or significant modification of the imaging sequence.

This chapter firstly focuses on optimization of gradient pre-pulses for position monitoring. A drift is observed in the voltages recorded from the EPI pre-pulses using the coils. It is found to depend mainly on the time delay between pulses. This study also checked the TR dependency of the voltages due to EPI pre-pulses. This was done by measuring the voltage induced in the coils by three gradient pre-pulses successively applied on the RL, AP and FH axes with varying delays between the pulses. In this experiment an EEG system that is described in Chapter 2 is used for recording the induced voltages.

Secondly, this chapter evaluates the use of principal component analysis (PCA) for simplifying the process of fitting the measured voltages to the head movement. PCA is probably the most popular multivariate statistical technique, which uses sophisticated underlying mathematical principles and is used in almost all scientific disciplines. It analyses a data table in which observations are described by several inter-correlated, dependent variables. Its goal is to extract the important information from the data, to represent it using a new set of orthogonal variables, called the principal components. In general terms, principal component analysis uses a vector space transform to reduce the dimensionality of large data sets. Using mathematical projection, the original data set, which may have involved many variables, can often be interpreted using just a few variables (the principal components) [125, 126].

The approach described here, which could be used for prospective or retrospective motion correction, involves characterising the relationship between head pose and the induced voltages in an initial set of imaging experiments, and subsequently using this relationship to calculate changes in head position and orientation from the measured voltages. This approach was tested by attaching five coils to a structured, spherical agar phantom, which was then placed in a gimbal system that allowed controlled rotation about the right-left ($RL = x$) and foot-head ($FH = z$) directions, as well as translation in the z -direction. The approach was then evaluated in experiments on a human subject, in which the coils were attached to the head.

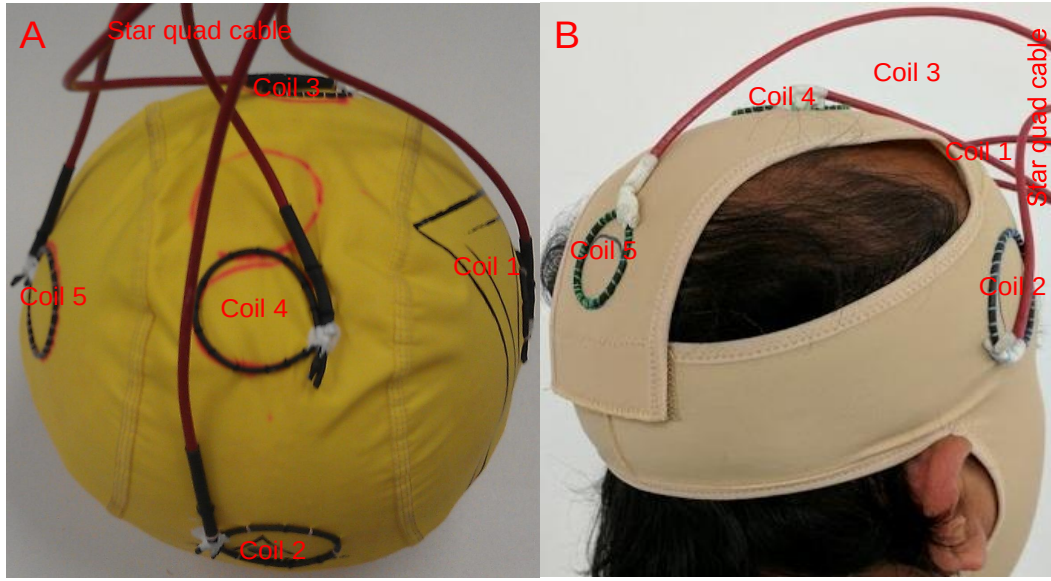


Figure 7.1: Set of five coils attached to (A) a spherical agar phantom (all coils had ~ 4 cm diameter and 2 turns) and (B) a healthy human subject (Coils 1-3 coils had ~ 4 cm diameter and 2 turns, while Coils 4 and 5 had ~ 3 cm diameter and 3 turns). Coils were connected to the ExG bipolar amplifier (Brain Products GmbH) using star quad cable (Van Damme) minimizing the induced voltage contribution from the cables. Coils are connected to the ExG amplifier through $100\text{ k}\Omega$ resistors.

7.2 Method and Materials

A spherical head phantom, a gimbal to hold the phantom and the coils mounted onto an EEG-like cap were used to perform the experiments. The gimbal is clamped on to the patient bed and the coils are mounted onto a cap fitted on the spherical head phantom, as shown in Fig. 7.1 (A). For the experiments on the human subject the coils were mounted onto a thin face mask, as shown in Fig. 7.1 (B). In both cases, the coils are made up of copper wire of 0.224 mm diameter coated with heat shrink to avoid deformation. The coils are connected to 5-channels of a 16 channel bipolar amplifier using star quad cable (Van Damme). The use of star quad cable reduces the magnitude of any voltages induced in the cables by gradient switching or movement in the B_0 -field. There are four cores of wires very tightly twisted with each other and rotate over a relatively short length. This arrangement ensures a magnetically coaxial structure and offers significantly enhanced rejection of electromagnetic interference related to standard two core mic cables. The body RF coil was used for experiments on the

phantom, while a 32-channel head Rx coil was used for experiments on the human subject. The coils were attached to the cap/mask by sewing using thread and needle.

For the initial experiments, we used an EPI sequence which included three additional gradient pulses successively applied along the right-left (RL), anterior-posterior (AP) and foot-head (FH) directions, applied in the quiet periods between slice acquisitions. Each pulse ramped-up/down over 5 ms at 4 T/m-s inducing voltages of several hundreds of μV in the coils, as shown in Fig. 7.2. This sequence was run with a long TR (48s) to allow large temporal separation of pre-pulses thus limiting any effect of vibration. In later experiments we changed the sequence so that a single gradient pre-pulse was applied with each EPI slice acquisition with the gradient direction cycling $\text{RL} \rightarrow \text{AP} \rightarrow \text{FH}$ across successive volume or slice acquisitions. This allowed us to reduce the TR to a more realistic value of 2s.

In the case of EPI pre-pulses applied in conjunction with a long TR period ($\text{TR} = 48\text{s}$), the position (x , y and z) and orientation (θ , ϕ and ψ) of the phantom was also measured by using SPM8 to co-register gradient echo (GE) images $[(1.5 \times 1.5 \times 1.5) \text{ mm}^3]$ resolution; $\text{FOV} = (240 \times 240 \times 96) \text{ mm}^3$; 144 s acquisition time for 2 dynamics]. For the human subject, the poses were also measured by using GE-EPI images of $(3.0 \times 3.0 \times 3.0) \text{ mm}^3$ resolution; $\text{FOV} = (240 \times 240 \times 96) \text{ mm}^3$; 624 s acquisition time for 12 dynamics.

For the repetition of pre-pulses in each dynamic and with a short TR period ($\text{TR} = 2 \text{ s}$), the positions (x , y and z) and orientations (θ , ϕ and ψ) of the phantom were also measured by co-registering images acquired during the experiments using the Statistical Parametric Mapping toolbox (SPM8). 3D gradient echo images with resolution $(3.0 \times 3.0 \times 3.0) \text{ mm}^3$ resolution; $\text{FOV} = (240 \times 240 \times 60) \text{ mm}^3$; scan duration is 366 s were used for the phantom, whereas the EPI images of $(3.0 \times 3.0 \times 3.0) \text{ mm}^3$; $\text{FOV} = (240 \times 240 \times 78) \text{ mm}^3$; 26 slices in each dynamic were used for the subject.

Before performing the experiment on phantom or on human subjects, the volunteers were given instructions regarding the nature of the movements during the model formation period (cued movement) and the experimental period (random movement).

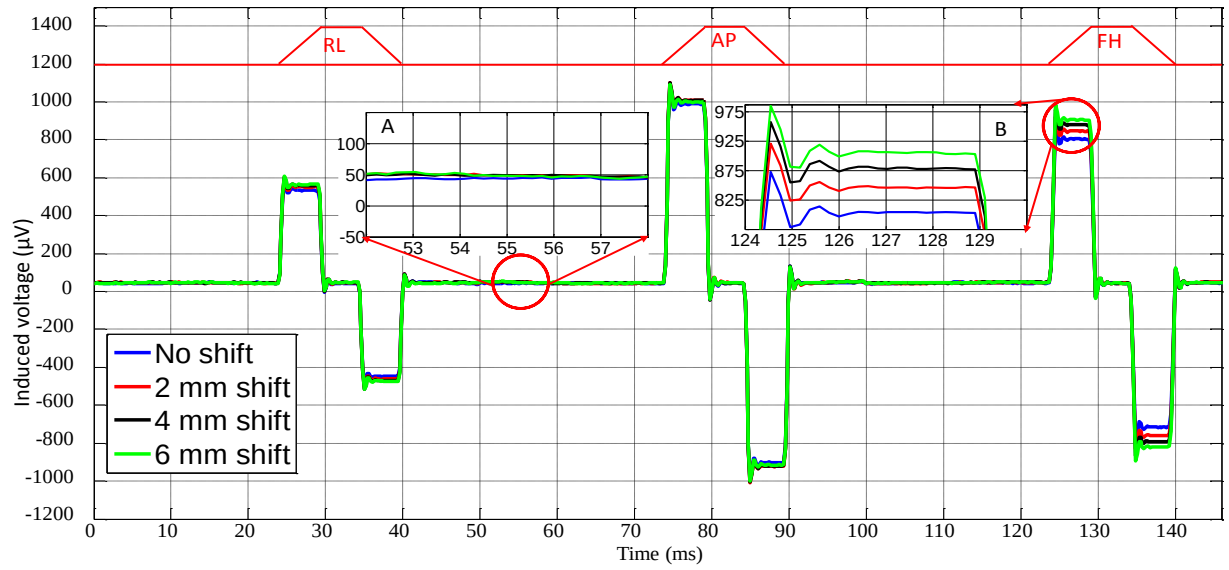


Figure 7.2: Voltages induced in a single coil mounted on a phantom by application of a sequence of three trapezoidal gradient pulses (5 ms ramp-up/down at 4 T/m-s) successively applied along the RL, AP and FH directions. Position of the phantom was shifted away from iso-centre in the FH direction in steps of 2 mm causing the amplitude of the induced voltages to change. The results are shown for 4 different positions of the phantom. Inset A shows no appreciable change in the baseline recording with respect to successive movements, whereas inset B shows a linear increase in the voltage induced by the FH trapezoidal pulse as the FH shift increases.

7.2.1 Multivariate Analysis

Multivariate analysis is used here to avoid the risk of over fitting and reduced the dimensionality of the movement parameters obtained from image co-registration by using SPM8. For this purpose, a training set of measurements was first made while the phantom/head was translated along the z-axis and then rotated about the x- and z-axes. Principal Component Analysis was applied to the position data estimated using SPM8 so as to identify the combination of parameter changes that best characterised the movements, and three (three movements are dominating: axial translation, head-shake and head-nod) principal components were then collected together into a design matrix along with a baseline term. A general linear model (GLM) is used to form the design matrix. The design matrix was fitted to the measured voltages (voltages were measured from 5 coils for three gradient pre-pulses, yielding 15 independent voltage measurements) using a pseudo-inverse to determine the coefficients relating the co-ordinate changes and induced voltages. Experimental data were then collected while the subject/phantom underwent random changes in position. The coefficients derived from the training set were used to estimate the position changes and the results compared to movement parameters found using image co-registration. The whole process is summarized in the flow diagram shown in Fig. 7.3.

7.3 Results

7.3.1 Pre-pulse Optimization

The parameters characterising the sequence that was used for optimising the pre-pulses are in Table 7.1.

In initial experiments we found that vibration produced by early pre-pulses affects the recording of voltages from later pre-pulses because any coil rotation in the strong B_0 -field produces additional induced voltages. By varying the time delay between pulses as well as the TR one can optimize the pre-pulses. Time delays between the pre-pulses of 100 ms, 200

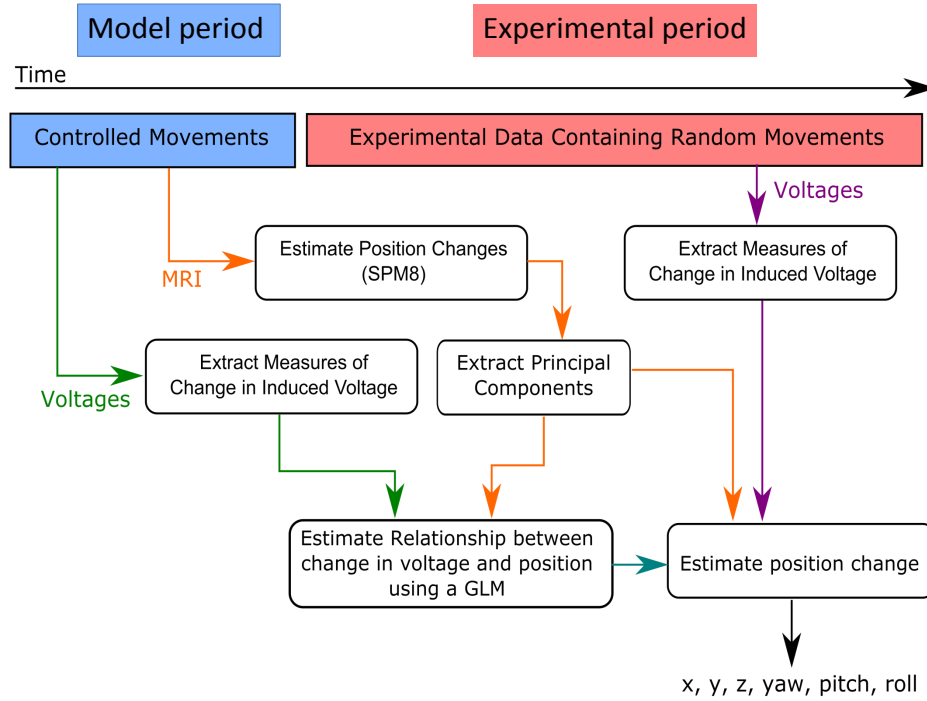


Figure 7.3: Flow diagram for multivariate analysis.

No. of Slice	Dynamics	TR (s)	Gradient Strength (mT/m)	Pre-pulse ramp time (ms)	Pre-pulse duration (ms)	Time between pre-pulses (ms)
10	20	7.5, 10	20	5	5	100, 200 400, 500

Table 7.1: Summary of the scanning parameters to optimize pre-pulses.

ms, 400 ms and 500 ms were tested. It is tested for TR values of 7.5 s and 10 s by taking 10 slices per TR period. In these two cases, the time between successive EPI slice acquisition was 0.75 s (TR = 7.5 s) or 1 s (TR = 10 s) respectively.

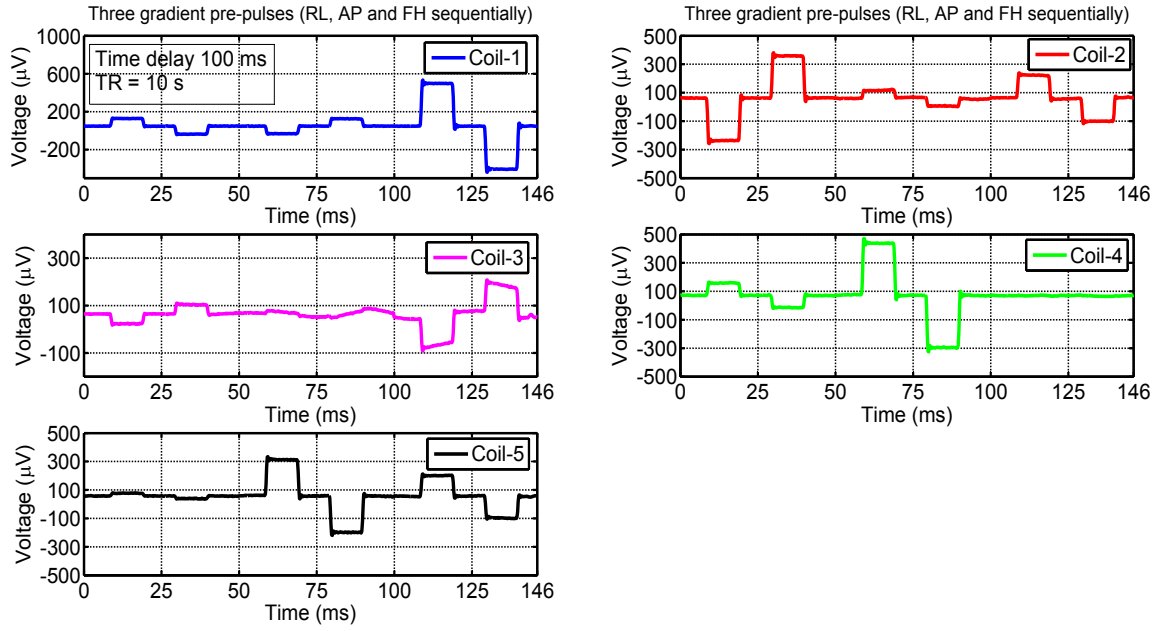
The voltages due to the three gradients pre-pulses obtained from the five coils placed on the phantom are shown in Figs. 7.4, 7.5 and 7.6. The sequence of gradients pre-pulses is RL, AP and FH in all figures. Fig. 7.4(a) shows the results for delays between pre-pulses of 100 ms, whereas Fig. 7.4(b) shows results for time delays of 200 ms and TR=10 s for both sets of data. Fig. 7.5(a) shows data for time delays of 400 ms, whereas Fig. 7.5(b) shows results for time delays of 500 ms and TR is 10 s for both these sets of data. Fig. 7.6 shows data when TR is 7.5 s and time delays of 400 ms are used.

From analysing the voltages induced by the pre-pulses one can conclude that the drift observed in the voltages due to the pre-pulses mainly depends on the time delays between pre-pulses rather than on the TR. One can reduce the drift in the expected recorded voltage pulse shapes by tuning the time delays between pulses. In this work we found that an inter-pulse delay of 400 ms gave acceptable results.

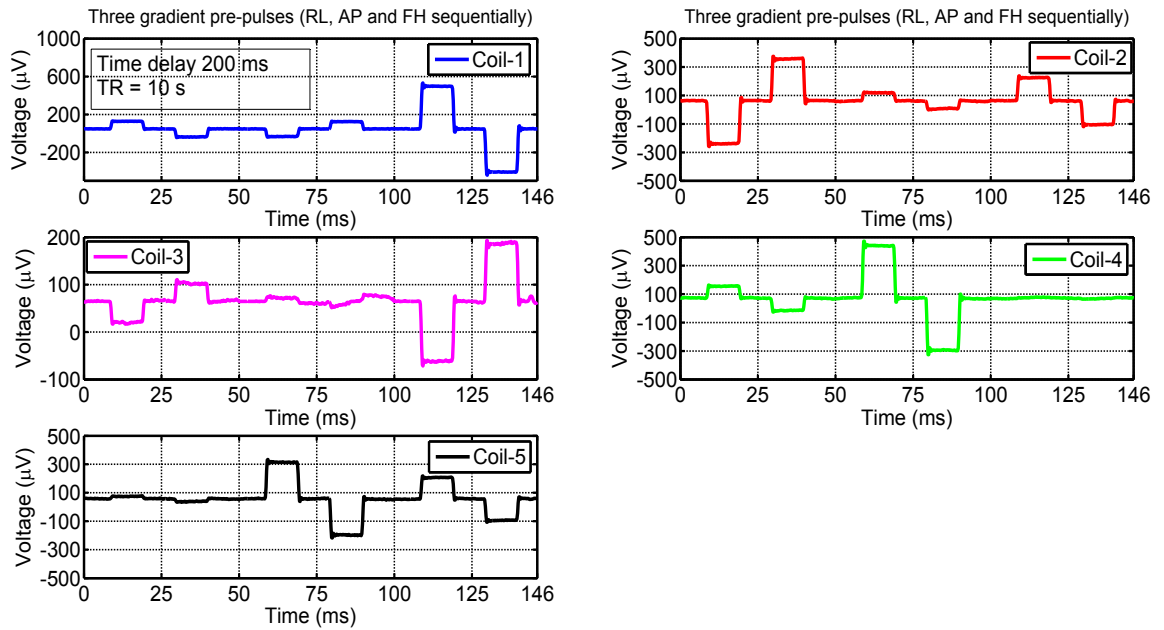
7.3.2 Model Formation

Here, twelve measurements were made with the phantom first rotated about the x-axis and then about the z-axis, before being translated to different z-positions. Principal Component Analysis was applied to these data to identify the combination of parameter changes that best characterised the movements, and three principal components were then collected together into a general linear model that was fitted to the measured voltage changes. Three measurements were subsequently left out of the model formation phase and the resulting model used to identify the phantom position from the voltage measurements. The results were then compared to parameters identified using the SPM8 registration.

Fig. 7.7(a) shows 12 data points, 4 from each of the 3 different types of movement parameters (head-nod, head-shake and axial-translation) for three PCA components, excluding

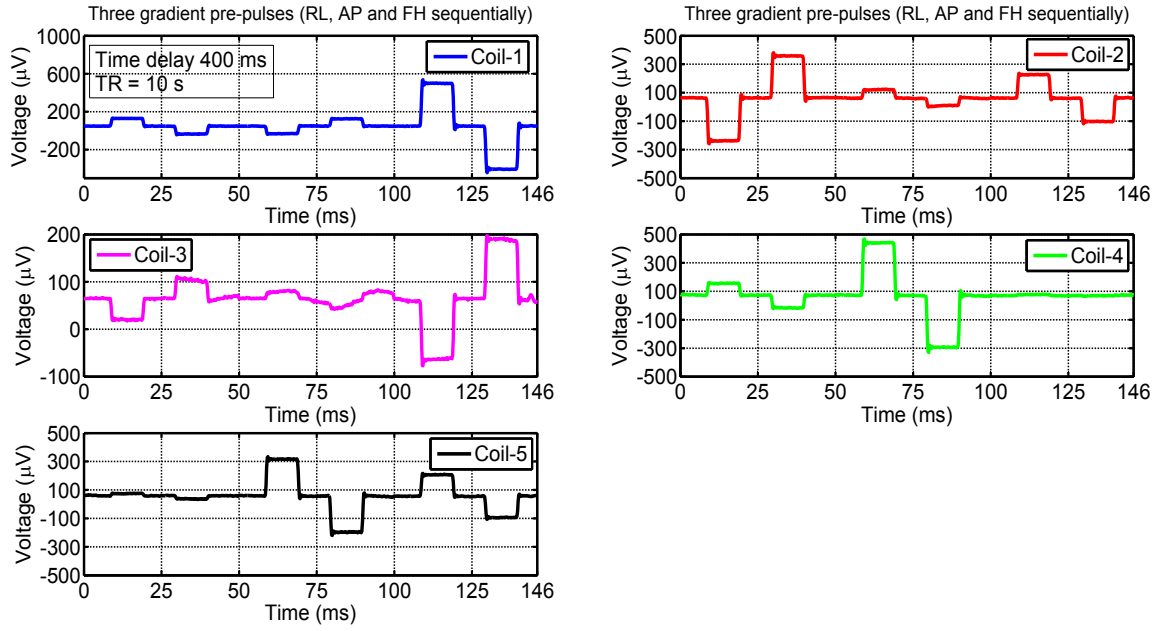


(a)

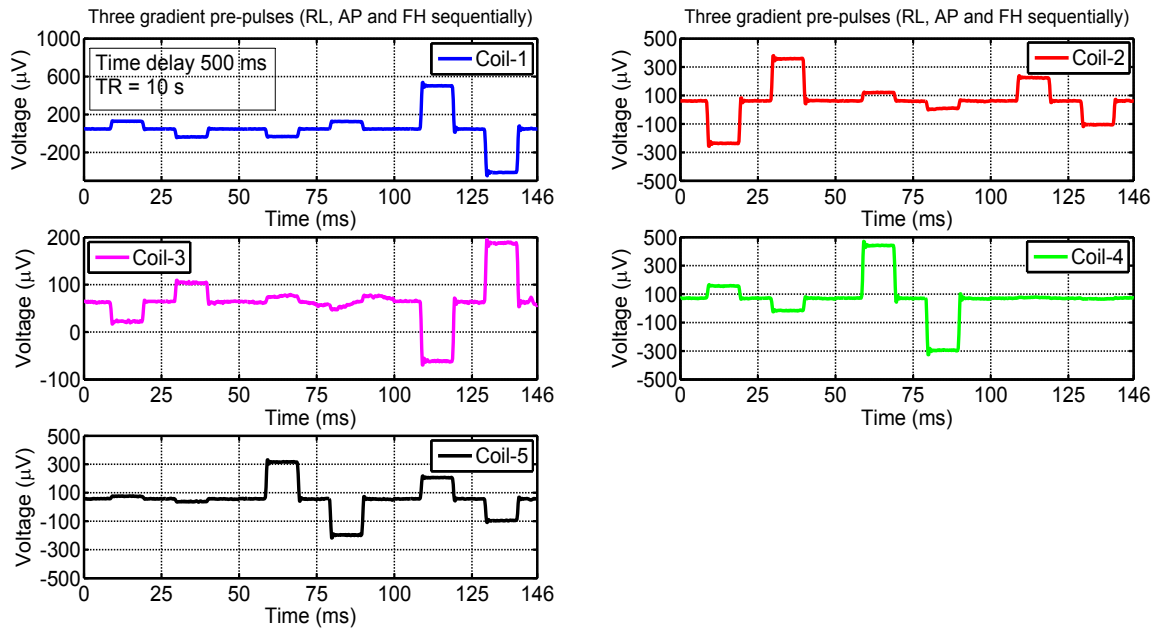


(b)

Figure 7.4: Optimization of TR and time delay between pre-pulses, delays are 100 ms (a) 200 ms (b). A 50 ms period spanning each pre-pulse has been cut out and the all three periods were concatenated. The delay between pulses is thus not completely shown. TR was set to 10 s for these experiment.



(a)



(b)

Figure 7.5: Optimization of TR and time delay between pre-pulses delays are 400 ms (a) 500 ms (b). A 50 ms period spanning each pre-pulse has been cut out and the all three periods were concatenated. The delay between pulses is thus not completely shown. TR was set to 10 s for these experiment.

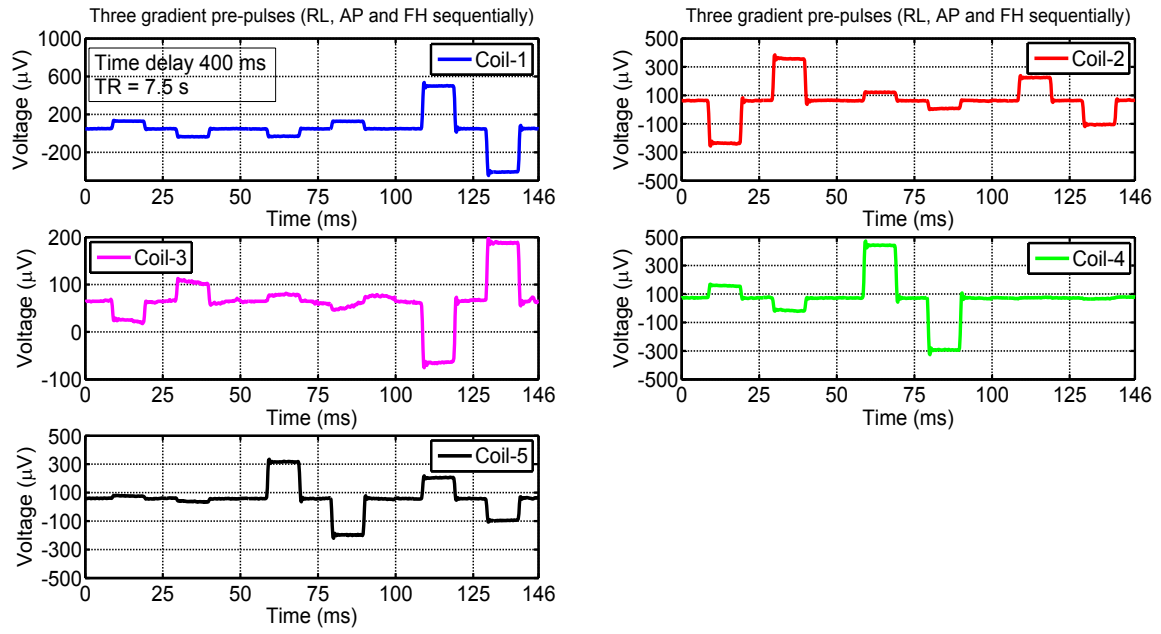


Figure 7.6: Optimization of TR and time delay between pre-pulses, where time delays 400 ms. A 50 ms period spanning each pre-pulse has been cut out and the all three periods were concatenated. The delay between pulses is thus not completely shown. TR was set to 7.5 s for these experiment.

three points (3rd, 7th and 11th). Fig. 7.7(b) shows the difference (errors) between parameters estimated by using coils and by using SPM8. In Fig. 7.7, the first 4 data points show the effects of x-rotation and the second 4 points show z-rotation effects; whereas the third set shows z-translation of the spherical head phantom.

7.3.3 Recordings for long TR period

Phantom

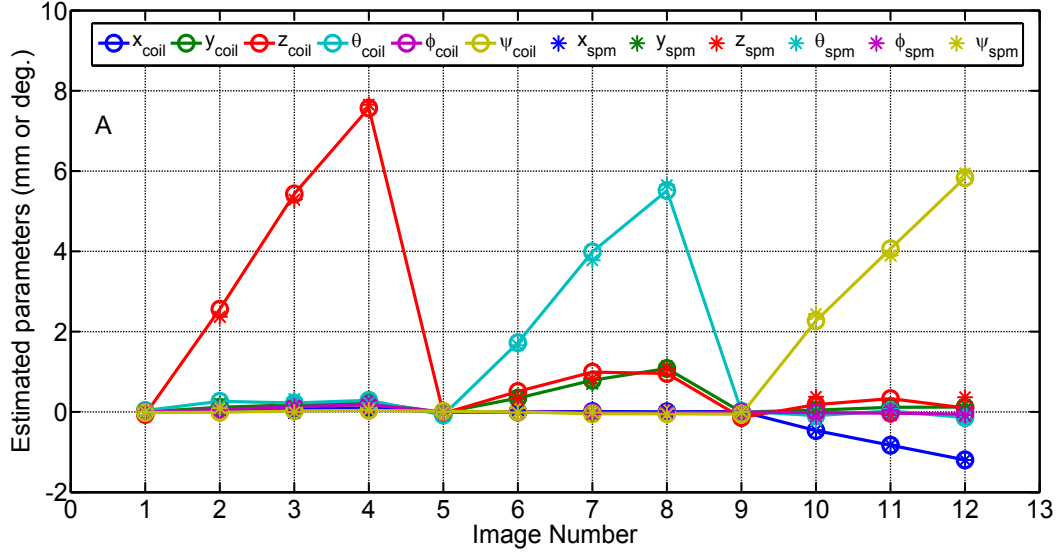
The model is formed by including data acquired at 12 time-points during the model formation period in which controlled movements are made. In the experimental period 14 points are taken for 14 random movements of the phantom. Analysis of data from the model formation period is used to predict the positions of 14 points in the experimental period. The results are shown in Fig. 7.8 (A). Fig. 7.8 (B) shows the errors in the estimated parameters.

In the case of the model formation period, the model can predict the excluded points precisely. Moreover, the difference between estimated parameters is not more than 0.4 mm in translation and not more than 0.3° in rotation. This model is successfully exploited in order to estimate the position and orientation of an object inside the MRI scanner to within an accuracy of 0.7 mm and 0.7 degrees

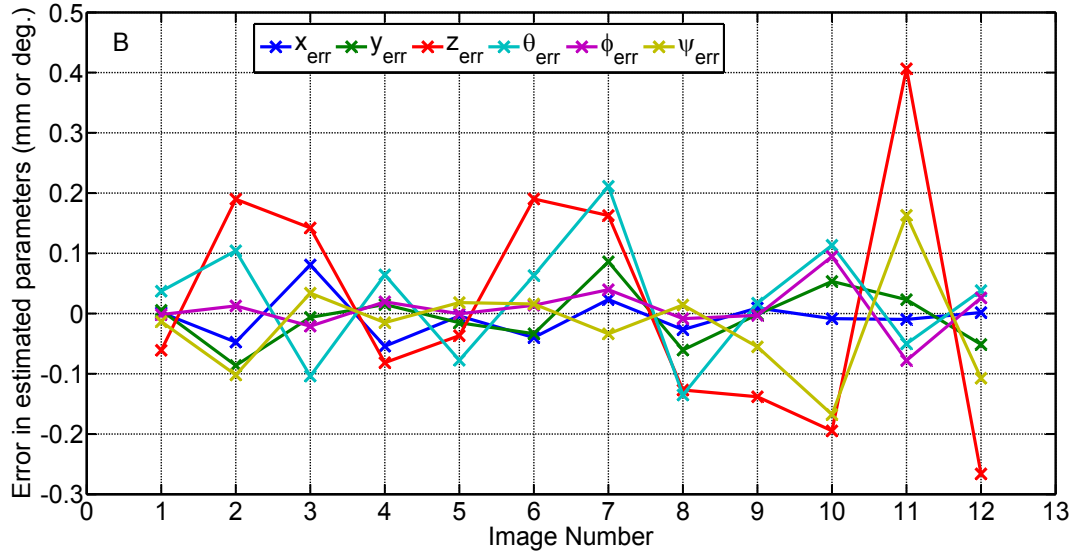
Root mean square errors in the estimation of the parameters are shown in Fig. 7.9. In summary, the estimated parameters from SPM8 and from the induced voltages were in strong agreement. These results imply that the amplitudes of the (induced voltage) vary linearly with respect to position and orientation.

Subject

In this case, the model is formed including 7 points, one for the initial position and 6 for the controlled movement (axial-translation, head-shake and head-nod) 2 for each, in the model formation period. In the experimental period 5 points are taken for 5 random movements



(a)



(b)

Figure 7.7: Data represents the movements parameters estimated by measuring the changes in induced voltage in the coils by time varying gradient magnetic field. Parameters estimated by using the induced voltages (o) and SPM8 (*). 12 data points are estimated for 4 consecutive movement and excluding 3 data points, (3rd, 7th and 11th) one from each type of movement, end up with 9 data points that are used to form the model, which is then applied to find position for the 12 data points are shown in (A) for the phantom. The difference ($\times$) between the parameters estimated with the coils (o) and using SPM8 (*) is shown in (B) for the phantom.

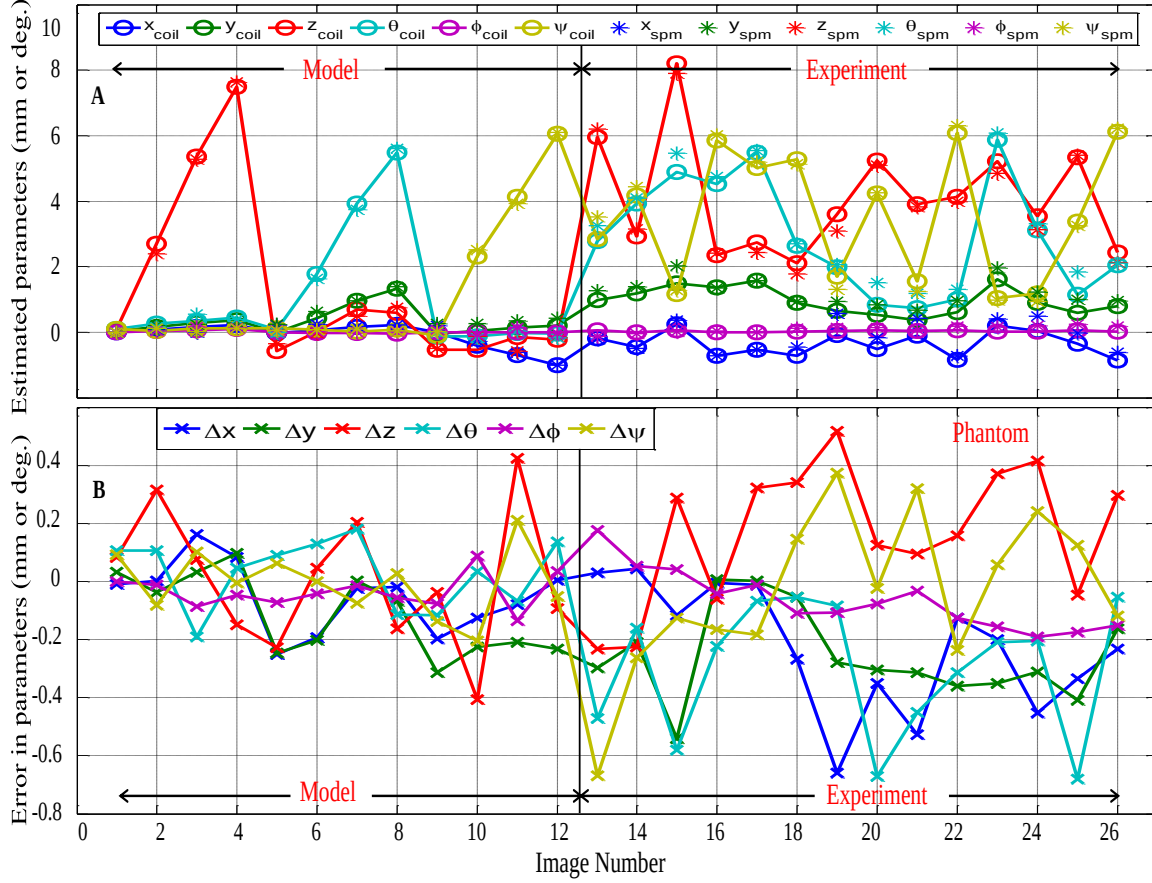


Figure 7.8: Change in position and orientation for phantom (A). Data collection was split into a model-forming period during which positions were successively adjusted along/about a particular axis and an experimental period during which positions were randomly varied. The changes in voltages with position were characterised using data from the model-forming period. This characterisation was used to estimate the position change from the voltage recordings collected during the whole experiment. The differences ($\times$) between the position estimated with the coils ($\circ$) and using SPM8 ($*$) is shown in (B) for the phantom.

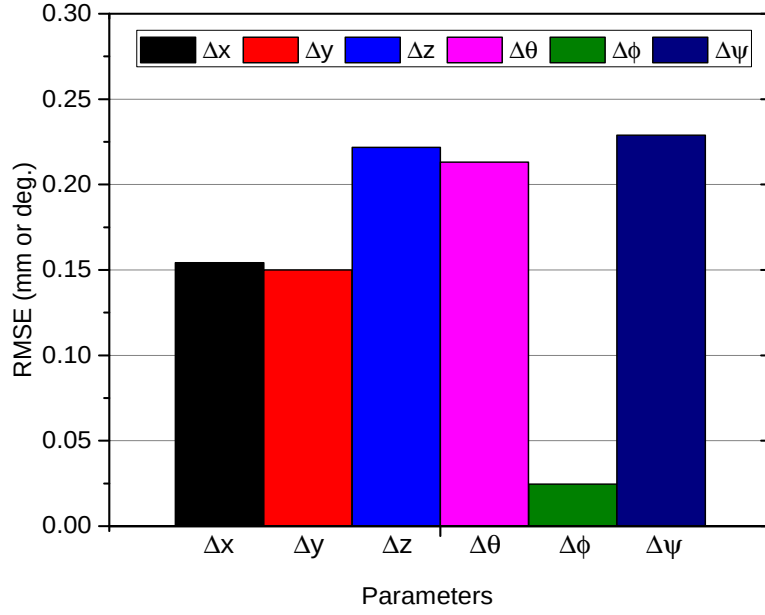


Figure 7.9: Root mean square error in the estimated parameters for spherical head phantom.

of the subject's head. A similar approach to that used in the model formation period is used to predict the 5 points in the experimental period only for the three PCA (Principal Component Analysis) as shown in Fig. 7.10 (A). Fig. 7.10 (B) shows error in the estimated parameters.

In the case of the model formation period, the model can predict the position at the excluded time-points precisely. Moreover, the difference between estimated parameters is not more than 0.4 mm in translation and not more than 0.3° in rotation. This model is successfully exploited in order to estimate the position and orientation of an object inside the MRI scanner during experimental period to within an accuracy of 0.7 mm and 0.7 degrees. Root mean square errors in estimation of the parameters are shown in Fig. 7.11.

The standard errors in the estimated positions of the phantom and the subject are summarised in the Table 7.2. Data shows that the range of the errors not more than 0.66 mm in translation and not more than 0.67° for the phantom, whereas it is not more than 0.99 in translation and 0.89° in rotation for the subject. The results from the phantom show that this approach can provide estimates of position and orientation with less than 0.21mm and

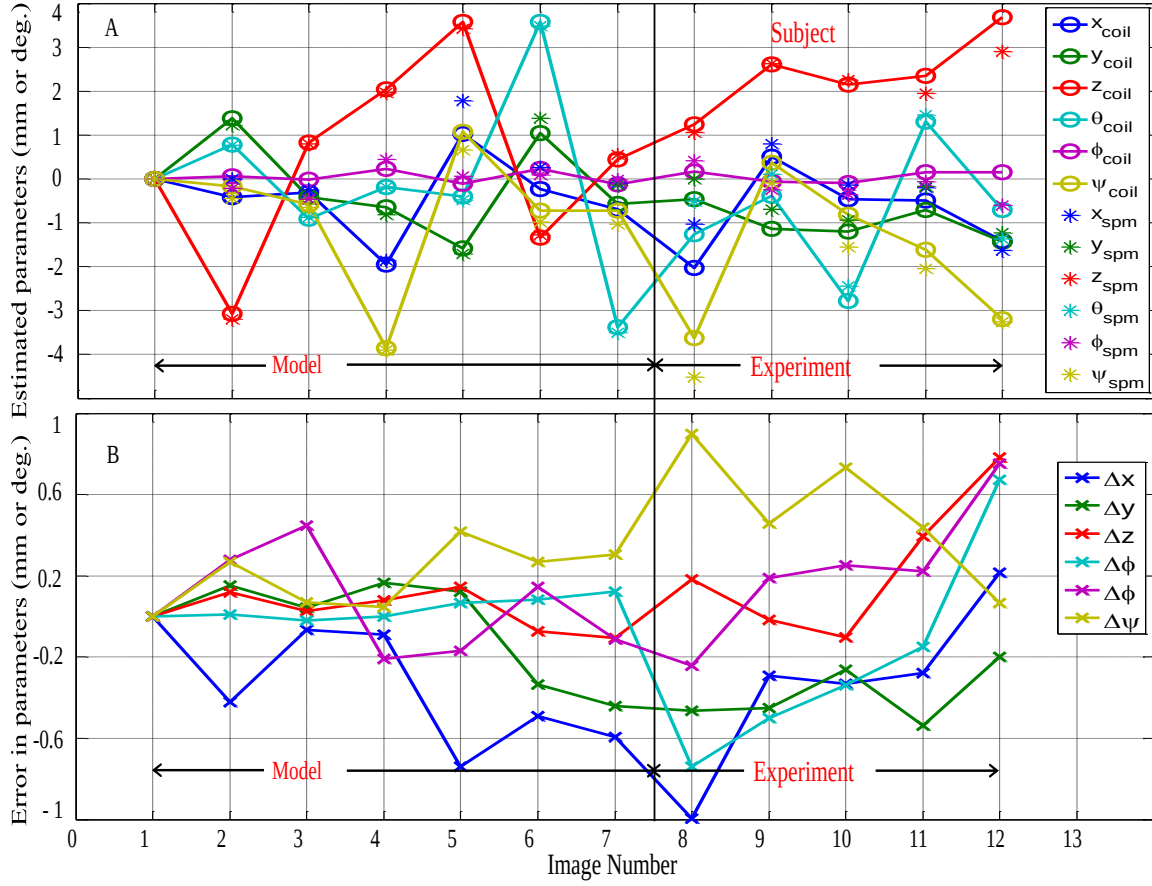


Figure 7.10: Change in position and orientation for subject (A). Data collection was split into a model-forming period during which positions were successively adjusted along/about a particular axis and an experimental period during which positions were randomly varied. The changes in voltages with position were characterised using data from the model-forming period. This characterisation was used to estimate the position change from the voltage recordings collected during the whole experiment. The differences ($\times$) between the position estimated with the coils ($\circ$) and using SPM8 ($*$) is shown in (B) for the phantom.

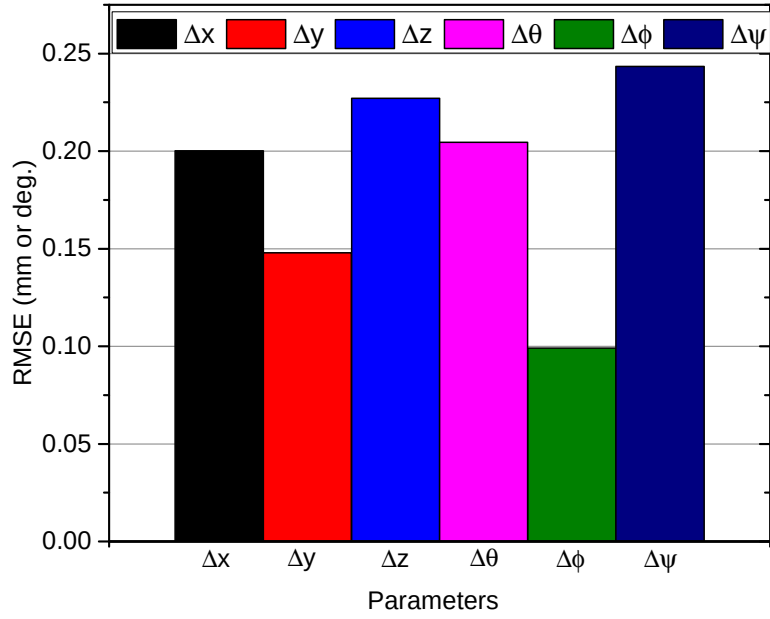


Figure 7.11: Root mean square error in the estimated parameters for the healthy volunteer.

Error	Phantom		Subject	
	Range	RMS	Range	RMS
x (mm)	0.66	0.15	0.99	0.20
y (mm)	0.54	0.15	0.53	0.14
z (mm)	0.52	0.22	0.78	0.22
Pitch (θ°)	0.66	0.21	0.73	0.20
Roll (ϕ°)	0.18	0.02	0.75	0.10
Yaw (ψ°)	0.67	0.23	0.89	0.24

Table 7.2: Table showing the range and RMS amplitude of the differences between the SPM8 and the induced-voltage estimates of position for the phantom (calculated from 14 measurements) and the subject (determined from 5 measurements) over the experimental periods

0.23° RMS error, whereas from the human subject it can provide estimates of position and orientation with less than 0.22mm and 0.24° RMS error for head movements in the range of ± 4 mm/degrees (Table 7.2).

7.3.4 Recordings for Short TR period

For these experiments, we used an EPI sequence which included one gradient pre-pulse which was cycled across the image volume or slice acquisition along the right-left (RL), anterior-posterior (AP) and foot-head (FH) directions. Each pulse ramped-up/down over 2 ms at $4 \text{ Tm}^{-1}\text{s}^{-1}$ inducing voltages of several 100 μV in the coils.

The positions (x, y and z) and orientations (θ , ϕ and ψ) of the head/phantom were estimated by measuring the induced voltages in the coils and also measured by co-registering images acquired during the experiments using the Statistical Parametric Mapping toolbox (SPM8). The EPI data ($3 \times 3 \times 3$) mm³ resolution, FOV = (240×240×60) mm³ for the phantom and FOV = (240×240×76) mm³ for the subject was used for image registration in this case.

Voltage measurements were made due to cued axial-translation, head-shake and head-nod and for a set of random movements. Data recorded at times when movements occurred were removed as they were affected by artefacts (depend how many times phantom or subject was moved), as a result voltages being induced in the coils by rotation in the strong B_0 field. 2 axial-translations; 2 head-shake rotations and 2 head-nod rotations were used to form a model and then 5 random movements were made, and the position changes due to the random movements were predicted from the model.

Phantom: Single Pre-pulses per Slice Acquisition Cycled over Volume Acquisition

For these recordings, I took TR=2000 ms, 180 volumes and 20 slices in each dynamic. The volumes at which movements occurred are taken out from the acquisition and averages are

taken for three consecutive RL, AP and FH gradients. Finally, the data were reduced to 49 volumes rather than 60 volumes which are shown in Fig. 7.12. Data points are taken for axial translation, head shake and head nod as well as for five random movements. In Fig. 7.12, 30 data points are used to form the model, which is then applied to find the positions for all data points. The maximum translation is around 9 mm whereas the maximum rotation is about 12° . The maximum error in translation is < 0.4 mm, whereas in rotation it is $< 1.3^\circ$.

Subject: Single Pre-pulses per Slice Acquisition Cycled over Volume Acquisition

The gradients are cycled on a volume-by-volume basis, where TR=2000 ms and 26 slices are taken in each of the 180 volumes. 11 volumes where movement occurred are taken out from the acquired data and averages are taken for three consecutive RL, AP and FH gradients. Finally, 49 data points are obtained, from these the first 30 data points are used to form a model. The 30 data points are used for model formation to estimate the positions at all time points. The results are shown in Fig. 7.13. In this case the error is more than the error observed in case of the phantom. The maximum errors (1.2 mm) observed along the y-translation. Apart from this, the errors were within ± 0.5 mm and $\pm 0.5^\circ$ for the y- and z-translations and the x-, y- and z-rotations, respectively.

Subject: Single Pre-pulses per Slice Acquisition Cycled over Slice Acquisition

To make the approach more practical, we modified the sequence so as to cycle the gradient pre-pulse axis across slice acquisitions. EPI data are again co-registered by using SPM8. We took 180 volumes and took out 15 volumes where movements occurred leaving 165 points. The first 100 volumes are used to form a model which is then applied to estimate the position during the remaining 165 volume acquisitions. The results are shown in Fig. 7.14.

The maximum errors in estimated positions were not more than 1.2 mm (1.2 mm for y-translation, but for x- and z-translation it was less than 0.6 mm or 0.6°) for the phantom, when the maximum movements were 9 mm or 11° during single pre-pulse per slice acquisition

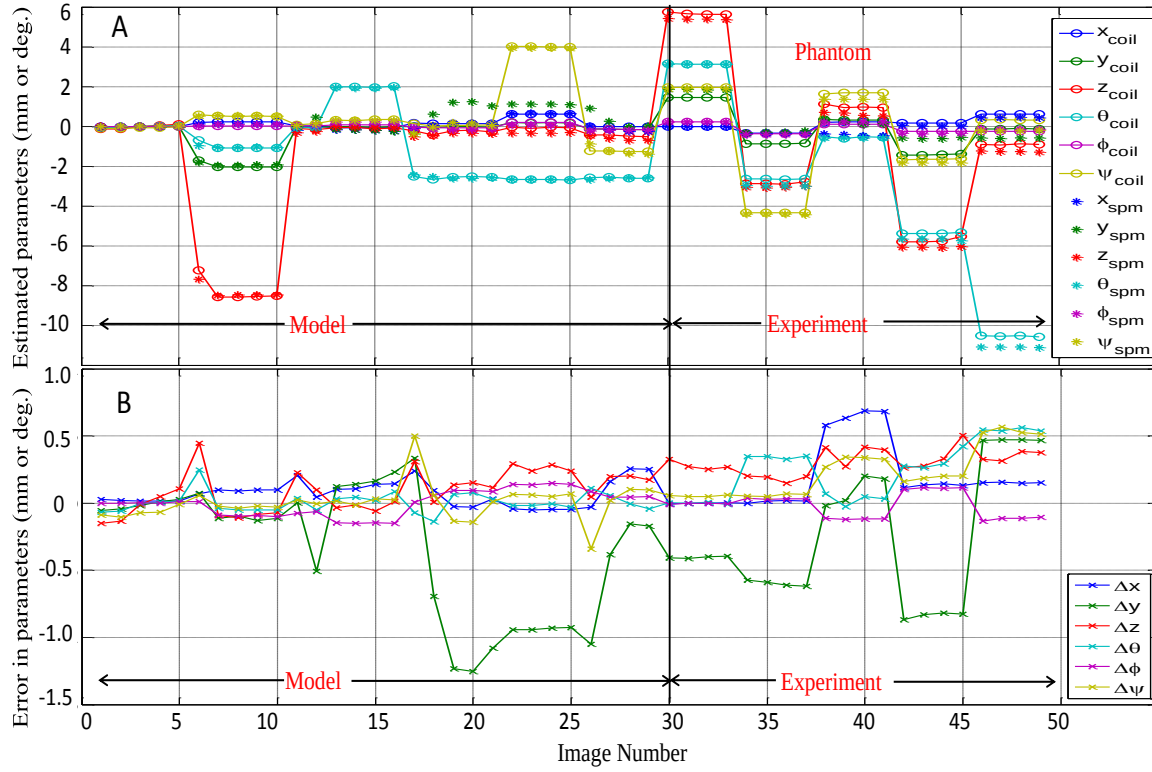


Figure 7.12: Measurements were made with short TR ($TR = 2s$) during scanning of the spherical agar phantom with a single pre-pulse applied per slice acquisition and the direction of the pre-pulse gradient cycled across volume acquisition. Changes in position and orientation for phantom (A). Data collection was split into a model-forming period during which positions were successively adjusted along/about a particular axis and an experimental period during which positions were randomly varied. The changes in voltages with position were characterised using data from the model-forming period. This characterisation was used to estimate the position change from the voltage recordings collected during the whole experiment. The differences ($\times$) between the position estimated with the coils ($\circ$) and using SPM8 ($*$) is shown in (B) for the phantom.

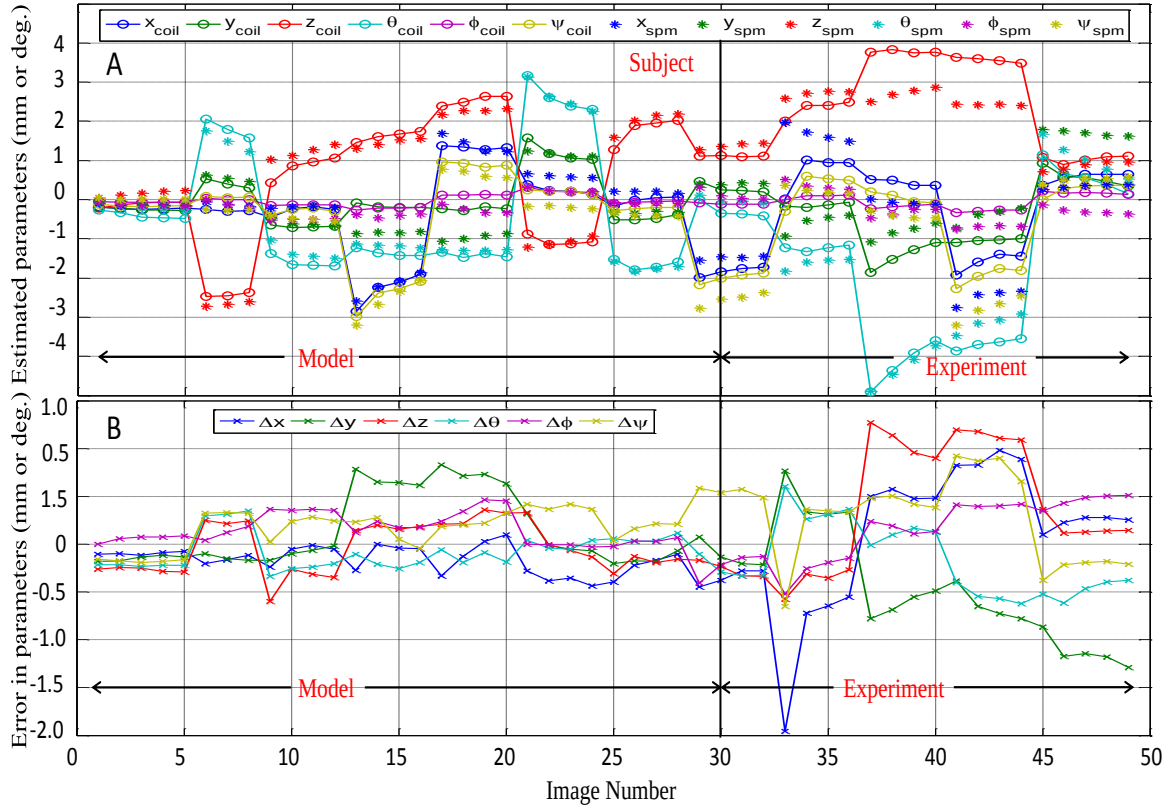


Figure 7.13: Measurements were made with short TR ($TR = 2s$) during scanning of the healthy subjects with a single pre-pulse applied per slice acquisition and the direction of the pre-pulse gradient cycled across volume acquisition. Change in position and orientation for subject (A). Data collection was split into a model-forming period during which positions were successively adjusted along/about a particular axis and an experimental period during which positions were randomly varied. The changes in voltages with position were characterised using data from the model-forming period. This characterisation was used to estimate the position change from the voltage recordings collected during the whole experiment. The differences ($\times$) between the position estimated with the coils ($\circ$) and using SPM8 ($*$) is shown in (B) for the subject.

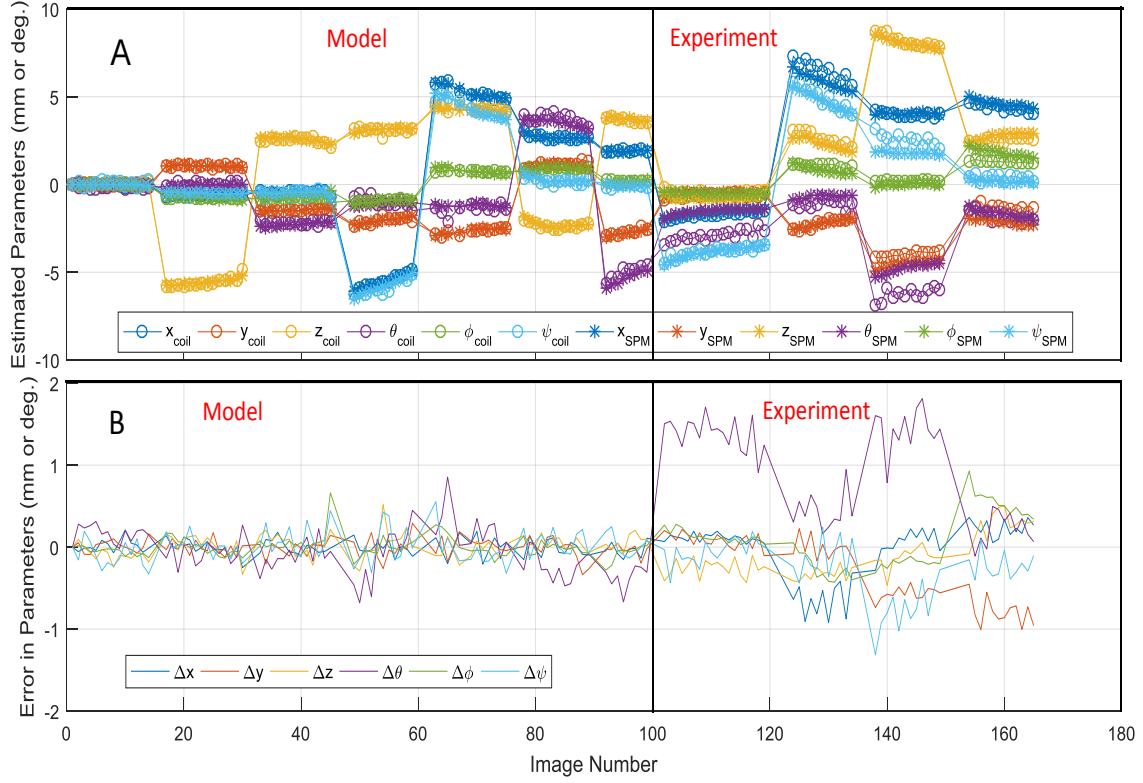


Figure 7.14: Change in position and orientation for subject (A). Measurements were made with short TR ($TR = 2s$) during scanning of the healthy subjects with a single pre-pulse applied per slice acquisition and the direction of the pre-pulse gradient cycled across slice acquisition. Data collection was split into a model-forming period during which positions were successively adjusted along/about a particular axis and an experimental period during which positions were randomly varied. The changes in voltages with position were characterised using data from the model-forming period. This characterisation was used to estimate the position change from the voltage recordings collected during the whole experiment. The differences ($\times$) between the position estimated with the coils ($\circ$) and using SPM8 ($*$) is shown in (B) for the subject.

cycled over volume acquisition; as shown in Fig. 7.12. In this case, for the subject, the errors were not more than 1.9 mm (1.9 mm for x-translation, but for y-translation it was ~ 1.2 and for z-translation it was less than 0.7 mm or 0.6°), when the maximum movements were 4 mm or 5° ; as shown in Fig. 7.13. On the other hand, the maximum errors in estimated positions were not more than 1 mm or 1.8° (1.8° for x-rotation, but for x- and z-rotation it was less than 1°) for the subject, when the maximum movements were 10 mm or 10° during single pre-pulse per slice acquisition cycled over slice acquisition; as shown in Fig. 7.14.

7.3.5 Discussion

The results of the experiments confirm that the amplitudes of the gradient-induced voltages vary linearly with changes in the coils' translational positions and with small changes in their orientations (Fig. 7.8) and that the pattern of variation across coils is different for different types of movement (Figs. 7.8 and 7.10). This allows movement information to be derived from coil measurements (Fig. 7.10) by using a linear model incorporating coefficients calculated from a training data set for which complementary information about movement is available (e.g. from image co-registration, as used here, or from optical or navigator-based position measurements). The results from the human subject show that this approach can provide estimates of position and orientation with less than 0.22 mm and 0.24° RMS error for head movements in the range of ± 4 mm/degrees (Figs. 7.8, 7.10 and in the Table 7.2). These results were obtained by applying three pre-pulses per slice acquisition with a 400 ms delay between pre-pulses, which necessitated the use of a long TR of 48 s. In initial experiments, we also tested the feasibility of using short TR periods, with a single pre-pulse per slice acquisition, producing promising results. The relatively larger errors for the subject data compared to the phantom may be due to the use of a smaller training data set and/or the greater variety of movements in this data set during experimental period. A practical implementation of this approach would involve measuring the position-sensitive voltages induced by the imaging gradients rather than additional gradient pulses. This would speed up the model formation phase and increase the temporal resolution of position monitoring.

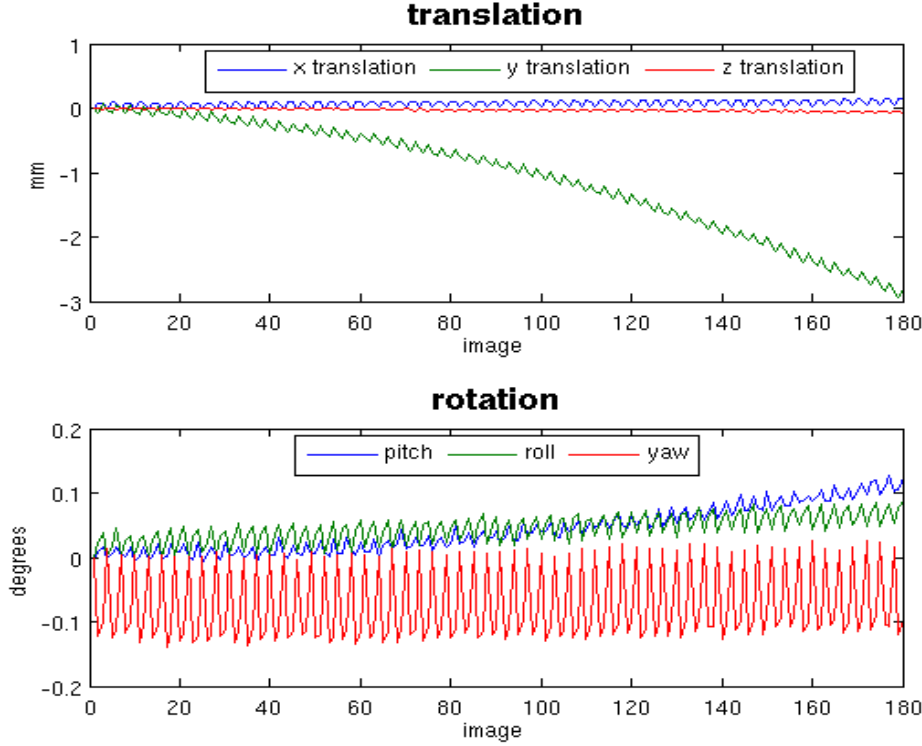


Figure 7.15: Without dynamic stabilization image shift along the phase encoding direction.

The approach used here has a few limitations. These are: image shift along the AP direction due to frequency changes, effects of vibration in the recorded voltages, effect of movement in the field and proper fixation of coils. We can overcome these by optimizing the pre-pulses, using dynamic stabilization, removing data where movement occurs and by using a tight fitting EEG-like cap. We initially reduced the effects of vibrations due to one gradient pre-pulses on the voltage measured due to later pre-pulses by using a large inter-pulse delay, which required a long TR. A more practical approach which we also tested involved using a single pre-pulse across successive acquisition.

Applying EPI with short TR produces more gradient heating which causes the passive shims to heat up, leading to a change in the B_0 field. The change in field leads to a change in resonance frequency and as a result the image shifts in the EPI phase encoding (AP) direction. The resulting image shifts along the AP direction in the absence of any kind of motion is shown in Fig. 7.15. Here the results are corrected by subtracting shift in image based on

prior measurements in the case of short TR both for phantom and subject. Dynamic stabilization measures the frequency in every volume acquisition and holds it constant. Dynamic stabilization will therefore help to avoid image shifts along the AP direction, but we did not use dynamic stabilization in this experiment.

7.4 Conclusion

In conclusion:

- Results show that the voltages induced in a set of coils fixed to a rigid body can be related to the position and orientation of the body
- This characteristic was successfully exploited in order to estimate the position and orientation of an object inside the MRI scanner to within an accuracy of 0.7 mm and 0.7 degrees
- While further work is required to fully optimise the approach, the principle of using a set of coils fixed to a subjects head could be used for prospective motion correction.

Further work is now needed to evaluate the efficacy of this approach for a broader range of movements of a phantom and human head. A realistic implementation of this approach would be to measure the position sensitive voltages induced by the imaging gradients of an echo planar sequence rather than a series of additional gradient pulses. This would speed up the model formation phase, increase the temporal resolution of the position monitoring (10's of ms) and could be implemented using standard MRI sequences. An initial experimental test of this approach is described below in Chapter 8.

Chapter 8

Free Running EPI

8.1 Introduction

The aim behind this experiment is to see if position changes can be quantified, by looking at how the voltages induced in a set of head-mounted coils by a standard EPI sequence change as a result of small movements. To do this, I followed the same procedure as used for the experiments described in Chapter 7 in which pre-pulses were applied, but used the voltage due to the EPI gradients for the analysis.

As before a set of position changes were made during the experiment and held for long enough to characterise the EPI-induced waveforms. These position changes were split into two periods; a model-forming period and an experimental period. During the initial model-forming period, position changes along the principal axes (z-translation, x- and z-rotations) were made sequentially. During the experimental period a set of random, mixed axial movements were made. Provided that the voltages induced by the standard EPI sequence scale

Model Forming Period				
Initial	Z-translation	Head-Shake	Head-Nod	Start
Experimental Period				
Move (1)	Move (2)	Move (3)	Move (4)	Move (5)

Table 8.1: Model and experiment for free running EPI.

linearly with respect to position changes, the following relation holds:

$$\begin{pmatrix} V^1[1] - V^0[1] \\ V^1[2] - V^0[2] \\ \dots\dots\dots \\ V^1[N-1] - V^0[N-1] \\ V^1[N] - V^0[N] \end{pmatrix} = \begin{pmatrix} dV[1]/dz & dV[1]/d\theta & dV[1]/d\psi \\ dV[2]/dz & dV[2]/d\theta & dV[2]/d\psi \\ \dots\dots & \dots\dots & \dots\dots \\ dV[N-1]/dz & dV[N-1]/d\theta & dV[N-1]/d\psi \\ dV[N]/dz & dV[N]/d\theta & dV[N]/d\psi \end{pmatrix} \cdot \begin{pmatrix} z^1 - z^0 \\ \theta^1 - \theta^0 \\ \psi^1 - \psi^0 \end{pmatrix}, \quad (8.1)$$

where $V^1[n]$ is the EPI induced waveform at head position 1 and sample time point n , where $n = 1, 2, 3, \dots, N$. The length of the vector (N) spans a single slice acquisition period of the EPI sequence. The differentials represent the change in the EPI-induced voltage with respect to the various position changes z (z-translation), θ (rotation about the x-axis) and ψ (rotation about the z-axis). By taking the difference in average waveform recorded between initial and z-translation periods and dividing by the average position change estimated by the SPM movement parameters, $dV[n]/dz$ can be estimated. The other differentials can also be estimated in a similar way using the rest of the data recorded during the model forming period. By taking the pseudo-inverse of the design matrix (formed by the differentials) and multiplying it by the difference in waveforms between the initial position and a new position,

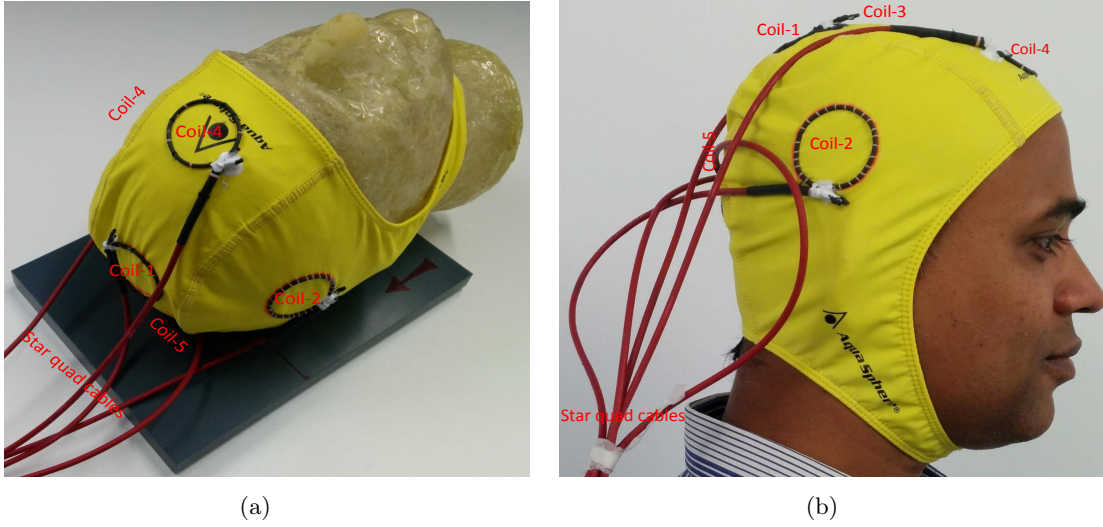


Figure 8.1: Experimental arrangement to recorded voltages on a spherical head phantom (a) and on a healthy volunteer (b) on five coils during experiment of a multi-slice EPI sequence.

the change in position can then be estimated.

8.2 Method and Materials

Recordings were made on a spherical head phantom (Fig. 8.1(a)) and on a healthy human volunteer (Fig. 8.1(b)). EEG data was acquired on a Philips Achieva 3T MR scanner for the phantom and for the healthy human subject. This was done by using a 16-channel bipolar amplifier, Brain Vision Recorder software (Brain Products GmbH), an EEG-like cap (swimming cap) carrying five arbitrary-shaped coils having two turns in each. Data were recorded with a 5 kHz sampling rate and a band-pass filter with a lower cut-off frequency 0.016 Hz and a higher cut off frequency of 250 Hz. Synchronisation of the MRI and EEG clocks was achieved by using the SyncBox (Brain Product GmbH). Multi-slice EPI data were recorded during the experiment. The whole Q-body coil was used for RF excitation with a flip angle of 80° . A standard EPI sequence was used to acquire 10 slices, 40 volumes of EPI data having $TE = 35$ ms and $TR = 4$ s. A portion of sequence recorded from the coils is shown in Fig. 8.2. In the experiments on the phantom, the matrix size was $(192 \times 192 \times$

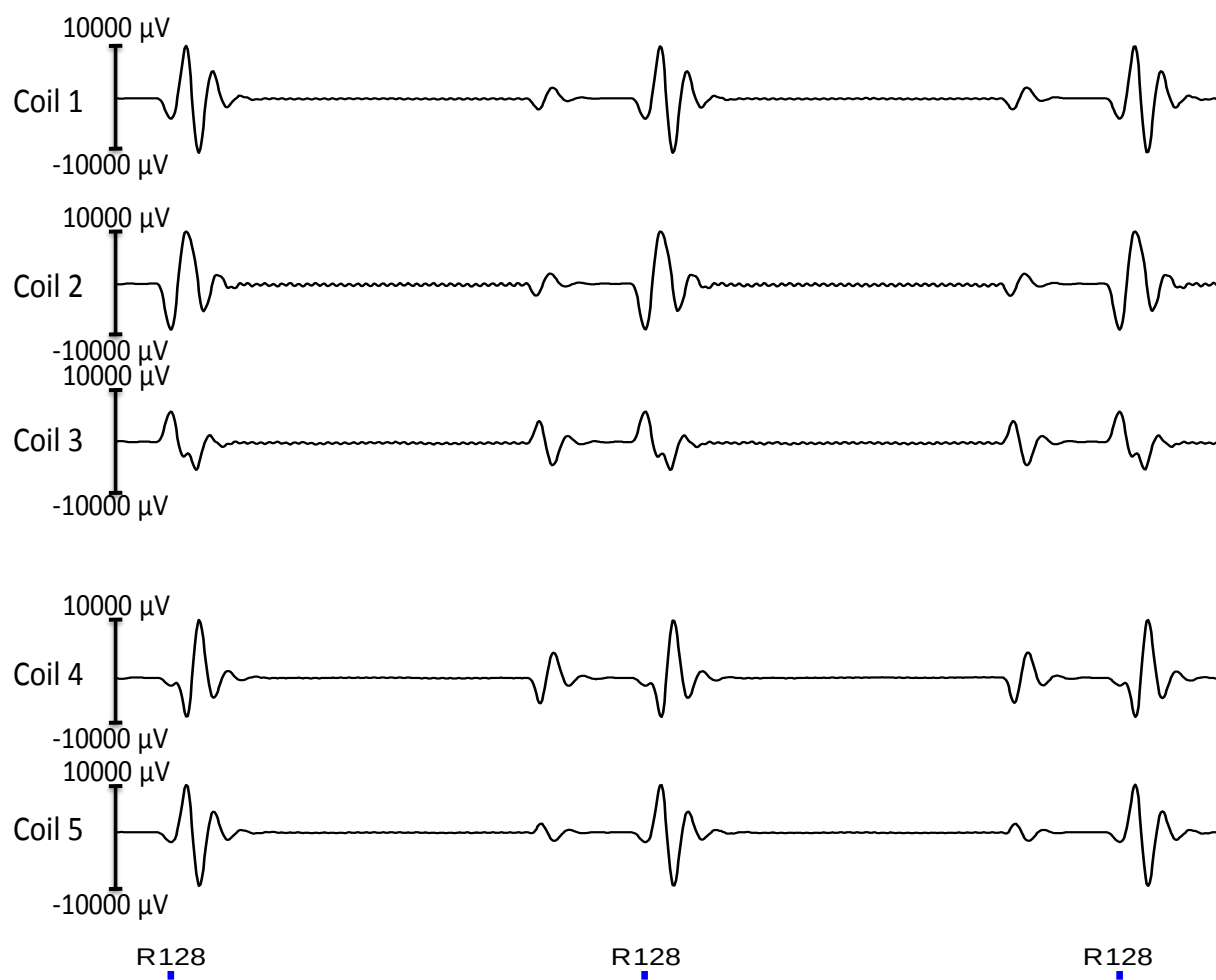


Figure 8.2: Voltage recorded on five coils during execution of a multi-slice EPI sequence. The R128 marker indicates the start of each slice acquisition.

30) mm^3 with a resolution of $(3 \times 3 \times 3) \text{ mm}^3$. Dynamic stabilization was switched off for both cases (phantom and subject).

In the experiment on the human subject 150 volumes of EPI data with $\text{TE} = 40 \text{ ms}$, number of slices $N_{SL}=25$ and $\text{TR} = 2 \text{ s}$ were recorded. In this case the matrix size was $(240 \times 240 \times 75)$ with a resolution of $(3 \times 3 \times 3) \text{ mm}^3$ resolution.

Each recording had two stages, comprising a model-forming period and an experimental period. During the model-forming period subjects were visually cued to make a small change in position of the head for three different types of movement (z -rotation, x -rotation and z -translation) before returning to the initial position. Each position was held for 10 TR-periods (40 s), giving 400 slices of voltage data for each position. During the experimental period, subjects were visually cued to make small, head movements to simulate the typical position changes that might occur during an EEG-fMRI experiment. Subjects were instructed to make small positional changes along a single axis for a recording, and subsequently in any direction for another recording. During each experimental data recording, the 3 positional changes were made 40 s apart so as not to be separated in time by a multiple of the TR-period. The MR images were co-registered using SPM8 to estimate the size and direction of the movements made by the subjects. To form templates of the change in induced voltage with respect to the different types of change in position, each set of 150 volumes were averaged together. These average waveforms were then subtracted from each other for each type of movement to form a set of difference models for each coil.

A similar experimental protocol was used for the experiments on the phantom. In both cases movements are estimated by inverting Eq. 8.1 having established the voltage changes that were produced by particular movements.

8.2.1 Free Running EPI Model

The change in the induced voltage with respect to movements was modelled from data acquired during the model-forming period. Models of the induced voltage changes with

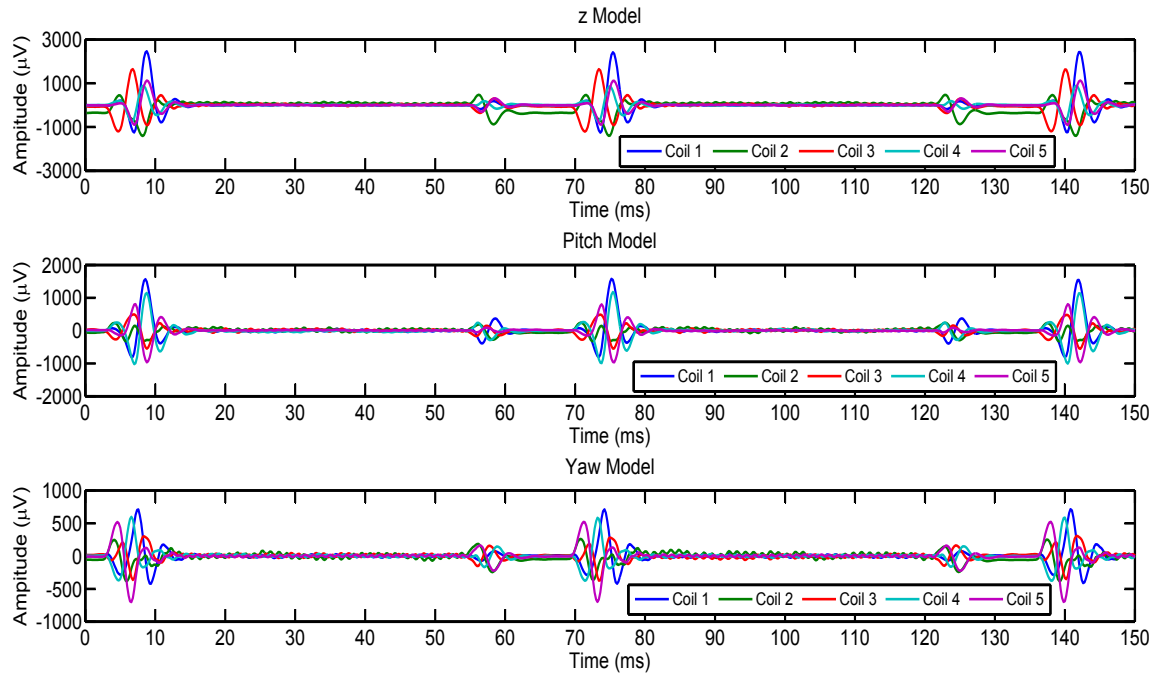


Figure 8.3: Voltage changes produced by z-translation, head-noding (pitch) and head-shaking (yaw) when free running EPI is applied. Voltage changes are shown for a two slice acquisitions corresponding to voltages shown in Fig. 8.2.

respect to three movement parameters were generated from data acquired in the model-formation period of the free running EPI. For example, models for the 5 coils and three specific movements are shown in Fig. 8.3. In the experimental period data are acquired for three mixed movements head-nod (pitch), head-shake (yaw) and axial translation. Three particular movement parameters are obtained from matrix inversion and by using these movement parameters a fitted waveform is formed. Fits to the voltages recorded in the five coils following head movements are shown in Fig. 8.4. Here, the voltage changes induced in the individual coil are fitted rather than the absolute artefact voltages.

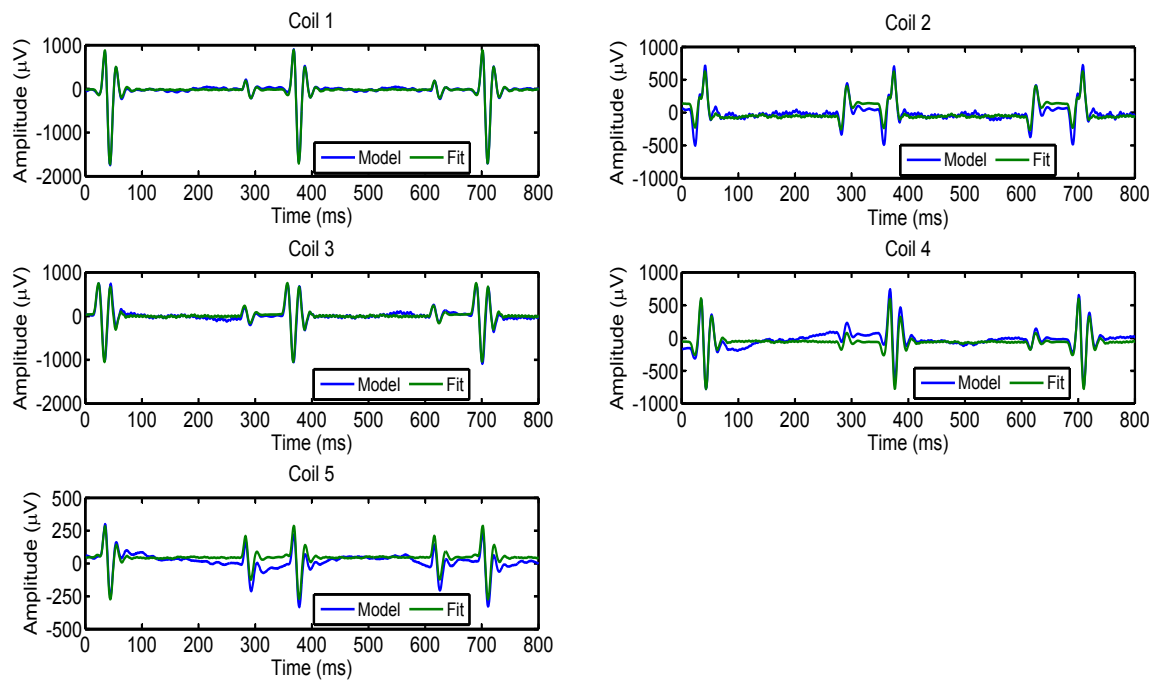


Figure 8.4: A portion of the three EPI model waveforms (voltage waveforms) and the voltage changes induced in the individual coil are fitted. Model waveform is formed from the changes in induced voltage in the coils and fitted waveform are obtained by using model.

8.3 Results

8.3.1 Phantom

Two sets movement parameters are obtained, one from image realignment carried out using SPM8 and another from wave function fittings by using the change in induced voltage in the coils. Each recording has two stages, a model-forming period and an experimental period. In all experiments subjects were visually cued to make small changes in the position of the head for each type of movement (x-rotation, z-rotation and z-translation) before returning to the initial positions. In one experiment phantom cued movements made during the experimental period (CMEP), whereas free movements were executed in the experimental period (FMEP) in a second experiment.

Data shown in Fig. 8.5 are estimated by using SPM8 and by measuring induced voltages in the coils due to the movement during execution of the x-, y- and z-gradients. In this case cued movements are executed in the experimental period. A comparison between two sets of movement parameters is also shown in Fig. 8.5. Fig. 8.5 also shows comparison between the two sets of data only for the x-, y- and z-translations and also for the x-, y- and z-rotations. But there is also x-, y-translations and y-rotation. In addition, there is a correlation among the movement parameters. There is quite good agreement with the movement parameters obtained from image realignment.

Data shown in Fig. 8.6 are estimated by using SPM8 and coils during free movement in the experimental period. A comparison between two the sets of movement parameters is also shown in Fig. 8.6

8.3.2 Subject

The coils are mounted on a swimming cap and the cap mounted onto the head of a healthy subject. In one experiment the subject was asked to perform cued movement in the model-forming period as well as in the experimental period, whereas in another experiment, the

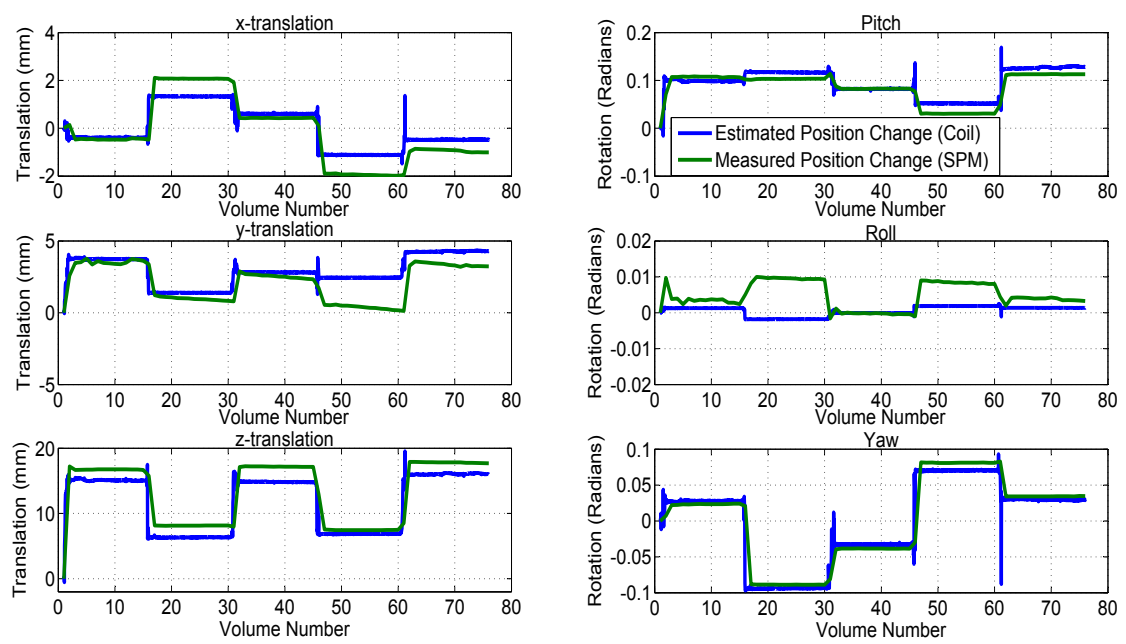


Figure 8.5: Data obtained from phantom scan. Estimated position and orientation change by using SPM8 and by fitting to the coil voltages during cued movement in the experimental period.

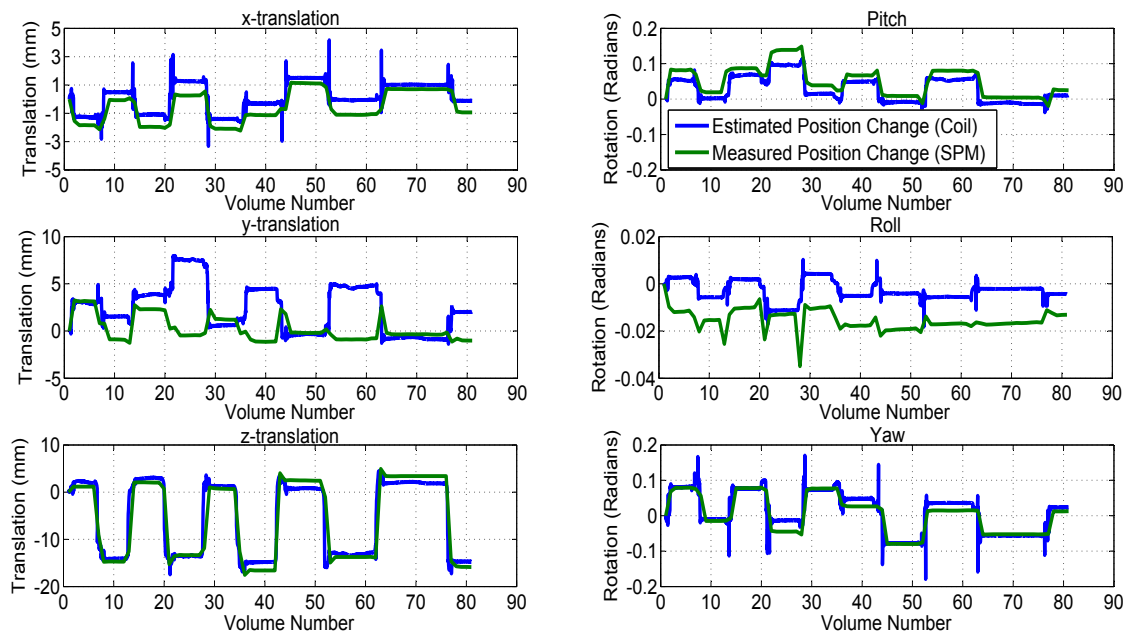


Figure 8.6: Data obtained from the phantom scan. Estimated position and orientation change by using SPM8 and by fitting the voltages induced in the coils during free movement in the experimental period.

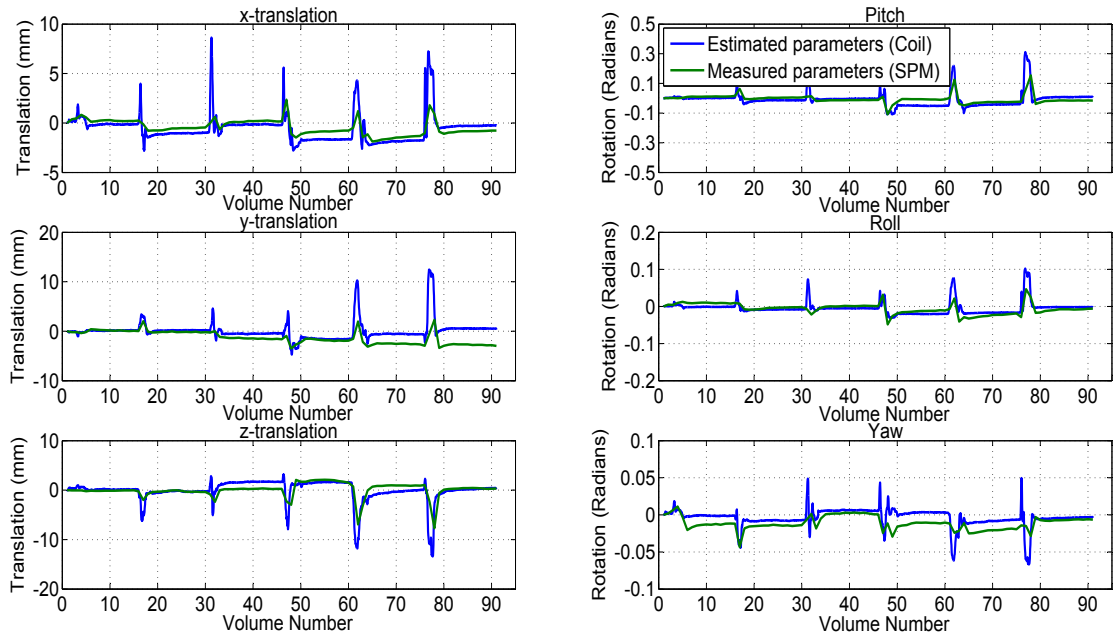


Figure 8.7: Data obtained from the subject scan. Estimated position and orientation change by using SPM8 and by fitting the voltages induced in the coils during cued movement in the experimental period.

subject performed cued movement in the model-forming period and free movement in the experimental period. The movement parameters were estimated for both of the models by using SPM8 and by analysing the induced voltages in the coils.

Fig. 8.7 shows estimated parameters due to cued movement in the experimental period, while Fig. 8.8 shows estimated parameters due to free movement in the experimental period. The subjects movements were small during the cued movement in the experimental period and also during the free movement in the experimental period. The net displacement from the initial position was no larger than -15 mm and -15° across all their recordings. A comparison of the data sets acquired during small, complex movements with those measured from SPM8 in Figs. 8.7 and 8.8 respectively. The performance for both methods is overall better for smaller subject movements.

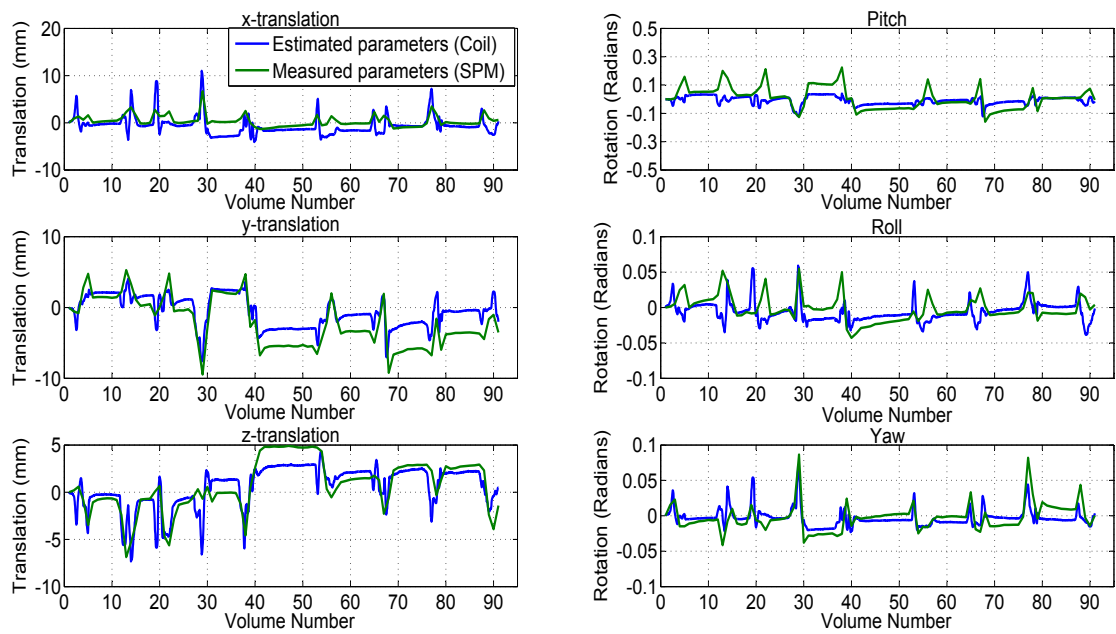


Figure 8.8: Data obtained from subject scan. Estimated position and orientation change by using SPM8 and the coils during free movement in the experimental period.

8.4 Discussions

In this study, the EPI waveform was used to estimate movement parameters for a spherical head phantom and a healthy subject moving inside an MR scanner, as shown in Figs. 8.5-8.8.

For simplification, it was assumed that the most likely head movements could be described by a basis set comprising rotations about the x- and z-axes as well as translation along the z-direction. This assumption appears to have worked well. Additional movements could easily be added to the model-forming period to capture the effects of other types of head movement.

The data was acquired using the Brain Products Sync Box to synchronize the MR scanner and EEG clocks and the slice TR was selected to be an integer multiple of the sampling period. This ensured that the artefact waveforms were sampled at the same times over all slices. It is important for the proposed approach that the sampling of the induced voltages is consistent across slice acquisitions [129]. This means that the data for forming the models needs to be acquired during the same run as the experimental data. This ensures that the sampling points of the models align perfectly with the sampling points of the induced voltages across all slice acquisitions.

Chapter 9

Conclusion

9.1 Summary

The objectives of my research work are briefly discussed in Chapter 1 and Chapter 2 is devoted to an overview of subject motion in MRI. Chapter 3 is dedicated to describing theories needed to calculate the induced voltage in a coil or a system of coils during execution of time-varying magnetic field gradients. A dipole concept is established to represent a coil having a location vector and a sensitivity vector. The validity of the dipole concept is checked for a wide range of geometrical shapes of a coil. It is worth noting that the dipole concept is not valid when a coil has highly irregular geometrical shape.

In Chapter 4 we developed software to calculate the change in induced voltage in the coils due to movement, whether the coils are in a plane or distributed in a space. Software is developed to track motion of a rigid body inside an MR scanner. One can estimate the position and orientation of the coils from the calculated voltage by using the developed software. It offers very high spatial resolution and is compatible with the MR system. The software is relatively simple and straightforward.

Chapter 5 is dedicated to developing a linear model by measuring the change in induced voltage in the coils with respect to position change. Translation and rotation are calibrated

by using the change in induced voltage in the coils due to sequential translations and rotations of a coil or a system of coils.

In Chapter 6 induced voltages and images are simultaneously recorded. A linear model to estimate the movement parameters from the induced voltages is established. Here analysing MRI data by using SPM8 we estimate the movement parameters and compare these to the movement estimated from the induced voltages.

Optimization of the EPI pre-pulse has been done in chapter 7. There are six movement parameters in a rigid body motion. In Chapter 7 Multivariate Analysis (MVA) is introduced to estimate the movement parameters by reducing the dimension of the parameters.

The movement parameters are estimated in Chapter 8 through investigation of the voltage due to the standard EPI sequence. In this chapter data are acquired both for the spherical head phantom and healthy subjects.

The challenges in movement monitoring via measurement of the changes in induced voltage in the head mounted coils are as follows:

- Induced voltages are sensitive to air flow, patient bed vibration, the geometry of the coils, and connecting wires.
- Head movement could happen without any movement of the coils or coils could move without head movement, so mounting coils properly onto the head is a vital issue.
- Image movement along the phase encoding direction during EPI time series acquisition as a result of the heating of the passive shims and consequent frequency shifts.
- Any unwanted distortion in the wave function may affect linearity.
- Signal clipping could happen in EPI pulse sequence during free running EPI.

There is no unique motion correction technique that can solve all sorts of motion problem in MRI. Obviously, there are some powerful techniques for particular situations. All these have some advantages and disadvantages. For example: optical tracking is accurate and

fast but costly, and requires line of sight access. On the other hand, navigators are fast, but requires image sequence modification; whereas image based tracking is accurate, but slow. There are some other techniques for specific problems. Moreover, it is expected that developing sophisticated techniques for specific problems will be what ultimately solves the motion artefacts in MRI, rather than a single technique that solves all problems.

Measurement of EMF induced in head mounted coils by time-varying gradients is fast and sequence modification is not necessarily required. This technique has now been tested for head movements by using EPI pre-pulses and EPI sequence.

9.2 Future Work

The next steps would involve carrying out prospective and retrospective motion correction using the proposed methods. Prospective motion correction will be implemented by passing six rigid-body motion parameters(degrees of freedom) to the scanner. The rest of the plans are itemized below:

- To compare and combine this method to other prospective motion correction specially with optical tracking (KinetiCor).
- It would be interesting to check our model in a higher magnetic field i.e. in 7T.
- Used neural network to localize the position and orientation of the coils by taking account of a wide range of movements (larger or shorter).

Bibliography

- [1] F. Godenschweger, U. Kägebein, D. Stucht, U. Yarach, A. Sciarra, R. Yakupov, F. Lüsebrink, P. Schulze. and O. Speck; Motion correction in MRI of the brain; *Phys. Med. Biol.* 61, R32-R56 (2016).
- [2] Pierre L. Young and Leigh Anne Olsen; The Healthcare Imperative: Lowering Costs and Improving Outcomes: Workshop Series Summary; Roundtable on Evidence-Based Medicine; ISBN: 0-309-14434-5, 668 pages, 6 × 9 (2010).
- [3] Poldrack R. A., Pare'-Blagoev E. J., Grant P. E.; Pediatric functional magnetic resonance imaging: progress and challenges; *Top Magn Reson Imaging* 13:61-70 (2002).
- [4] Brown T. T., Kuperman J. M., Erhart M., White N. S., Roddey J. C., Shankaranarayanan A, Han E. T., Rettmann D., Dale A. M.; Prospective motion correction of high-resolution magnetic resonance imaging data in children; *Neuroimage* 53:139-145 (2010).
- [5] Seto E., Sela G., McIlroy W. E., Black S. E., Staines W. R., Bronskill M. J., McIntosh A. R., Graham S. J.; Quantifying head motion associated with motor tasks used in fMRI; *Neuroimage* 14:284-297 (2001).
- [6] Cochrane C. J. C., Ebmeier K. P. K.; Diffusion tensor imaging in parkinsonian syndromes: a systematic review and meta-analysis; *Neurology* 80 (9), 857-864 (2013).
- [7] Stebbins G. T., Murphy C. M.; Diffusion tensor imaging in Alzheimer's disease and mild cognitive impairment; *Behav. Neurol.* 21 (1), 39-49 (2009).

- [8] Bohanna I., Georgiou-Karistianis N., Hannan A., Egan G.; Magnetic resonance imaging as an approach towards identifying neuropathological biomarkers for Huntington's disease; *Brain Res. Rev.* 58 (1), 209 (2008).
- [9] Inglese, M., Bester, M.; Diffusion imaging in multiple sclerosis: research and clinical implications; *NMR Biomed.* 23 (7), 865-872 (2010).
- [10] Julian Maclaren, Michael Herbst, Oliver Speck, and Maxim Zaitsev; Prospective Motion Correction in Brain Imaging: A Review; *Magnetic Resonance in Medicine* 69: 621-636 (2013).
- [11] James, G. Pipe; Motion Correction With PROPELLER MRI: Application to Head Motion and Free-Breathing Cardiac Imaging; *Magnetic Resonance in Medicine* 42:963-969 (1999).
- [12] M. Zaitsev, C. Dold, G. Sakas, J. Hennig and O. Speck; Magnetic resonance imaging of freely moving objects: Prospective real-time motion correction using an external optical motion tracking system; *NeuroImage* 31, 1038-1050 (2006).
- [13] Nathan White, Cooper Roddey, Ajit Shankaranarayanan, Eric Han, Dan Rettmann, Juan Santos, Josh Kuperman, and Anders Dale; PROMO: Real-Time Prospective Motion Correction in MRI Using Image-Based Tracking; *Magnetic Resonance in Medicine* 63:91-105 (2010).
- [14] Todd S. Sachs, Craig H. Meyer, Bob S. Hu, Jim Kohli, Dwight G. Nishimura, Albert Macovski; Real-Time Motion Detection in Spiral MRI Using Navigators; *Magnetic Resonance in Medicine* 32:639-645 (1994).
- [15] Lei Qin, Ehud J. Schmidt, Zion Tsz Ho Tse, Juan Santos, William S. Hoge, Clare Tempany-Afdhal, Kim Butts-Pauly, and Charles L Dumoulin; Prospective motion correction using tracking coils; *Magn Reson Med.* 69(3): 749-759 (2013).
- [16] Lei Qin, Peter van Gelderen, John Andrew Derbyshire, Fenghua Jin, Jongho Lee, Jacco A. de Zwart, Yang Tao, and Jeff H Duyn; Prospective Head Movement Correction for

- High Resolution MRI using an In-bore Optical Tracking System; *Magn Reson Med.* 62(4): 924-934 (2009).
- [17] MacFarlane *et al.*; Single Camera Motion Measurement and Monitoring for Magnetic Resonance Applications. US patent no. 0230755 A1 (2011).
 - [18] Maclaren *et al.*; Contact-Free Physiological Monitoring During Simultaneous Magnetic Resonance Imaging; US patent 0331239 A1 (2016).
 - [19] Tobias Kober, Jose P. Marques, Rolf Gruetter, and Gunnar Krueger; Head Motion Detection Using FID Navigators; *Magnetic Resonance in Medicine* 66:135-143 (2011).
 - [20] Dr. Rasmus Birn, Device for tracking Head Motion in an MRI Simulator; PhD Thesis; University of Wisconsin-Madison Department of Biomedical Engineering (2012).
 - [21] Ooi M. B., Krueger S., Thomas W. J., and Brown T. R.; Real-Time Intra-Volume Motion Correction in EPI Using Active Markers; *Proc. Intl. Soc. Mag. Reson. Med.* 18 (2010).
 - [22] Maclaren J., Boegle R., Herbst M., Hennig J., and Zaitsev M.; Head pose prediction for prospectively-corrected EPI during rapid subject motion; *Proc. Intl. Soc. Mag. Reson. Med.* 18 (2010).
 - [23] Speck O., Hennig J., Zaitsev M.; Prospective real-time slice-by-slice motion correction for fMRI in freely moving subjects; *Magn Reson Mater Phy* 19: 55-61 (2006).
 - [24] Rainer Boegle, Julian Maclaren, Maxim Zaitsev; Combining prospective motion correction and distortion correction for EPI: towards a comprehensive correction of motion and susceptibility-induced artifacts; *Magn Reson Mater Phy* 23:263-273 (2010).
 - [25] Bettinardi V., Scardaoni R., Gilardi M. C., Rizzo G., Perani D., Paulesu E., Striano G., Triulzi F., and Fazio F.; Head holder for PET, CT, and MR studies; *Journal of Computer Assisted Tomography*, vol. 15, no. 5, pp. 886-92 (1991).

- [26] Menon V., Lim K. O., Anderson J. H., Johnson J., and Pfefferbaum A.; “Design and efficacy of a head-coil bite bar for reducing movement-related artifacts during functional MRI scanning;” *Behavior Research Methods Instruments and Computers*, vol. 29, no. 4, pp. 589-594 (1997).
- [27] Edward V., Windischberger C., Cunningham R., Erdler M., Lanzenberger R., Mayer D., Endl W., and Beisteiner R.; Quantification of fMRI artifact reduction by a novel plaster cast head holder; *Human Brain Mapping*, vol. 11, no. 3, pp. 207-213 (2000).
- [28] Malviya S., Voepel-Lewis T., Eldevik O. P., Rockwell D. T., Wong J. H., and Tait A. R.; Sedation and general anaesthesia in children undergoing MRI and CT: adverse events and outcomes; *British Journal of Anaesthesia*, vol. 84, no. 6, pp. 743-748 (2000).
- [29] Hubbard A. M., Markowitz R. I., Kimmel B., Kroger M., and Bartko M. B., Sedation for pediatric patients undergoing CT and MRI; *Journal of Computer Assisted Tomography*, vol. 16, no. 1, pp. 3-6 (1992).
- [30] Slifer K. J., Cataldo M. F., Cataldo M. D., Llorente A. M., and Gerson A. C.; Behavior analysis of motion control for pediatric neuroimaging; *Journal of Applied Behavior Analysis*, vol. 26, no. 4, pp. 469-470 (1993).
- [31] Schulz J. *et al.*, An embedded optical tracking system for motion-corrected magnetic resonance imaging at 7 T *Magn. Reson. Mater. Phys. Biol. Med.* 25 44353 (2012).
- [32] Singh A. *et al.*, Optical tracking with two markers for robust prospective motion correction for brain imaging *Magn. Reson. Mater. Phys. Biol. Med.* 28 523-34 (2015).
- [33] Forman C. *et al.*, Self-encoded marker for optical prospective head motion correction in MRI *Med. Image Anal.* 15 708-19 (2011).
- [34] Kim K.*et al.*, Intersection based motion correction of multislice MRI for 3-D in utero fetal brain image formation *IEEE Trans. Med. Imag.* 29 146-58 (2010).

- [35] Thesen S. *et al.*, Prospective Acquisition Correction for head motion with image-based tracking for real-time fMRI Magn. Reson. Med. 44 457-65 (2000).
- [36] Kochunov P. *et al.*, Retrospective motion correction protocol for high-resolution anatomical MRI Human Brain Mapp. 27 957-62 (2006).
- [37] Jenkinson M. *et al.*, Improved optimization for the robust and accurate linear registration and motion correction of brain images *NeuroImage* 17 825-41 (2002).
- [38] Huizinga W. *et al.*, Groupwise registration for correcting subject motion and eddy current distortions in diffusion MRI using a PCA based dissimilarity metric MICCAI Workshop on Computational Diffusion MRI and Brain Connectivity ed T Schultz *et al.* (Berlin: Springer) pp 163-74 (2014).
- [39] Metz C. T. *et al.*, Nonrigid registration of dynamic medical imaging data using nD + t B-splines and a groupwise optimization approach Med. Image Anal. 15 238-49 (2011).
- [40] Wachinger C. and Navab, Simultaneous registration of multiple images: similarity metrics and efficient optimization IEEE Trans. Pattern Anal. Mach. Intell. 35 1221-33 (2013).
- [41] Welch E. B. *et al.*, Self-navigated motion correction using moments of spatial projections in radial MRI Magn. Reson. Med. 52 337-45 (2004).
- [42] Liu C. *et al.*, Self-navigated interleaved spiral (SNAILS): application to high-resolution diffusion tensor imaging Magn. Reson. Med. 52 1388-96 (2004).
- [43] Fu Z. W. *et al.*, Orbital navigator echoes for motion measurements in magnetic resonance imaging Magn. Reson. Med. 34 746-53 (1995).
- [44] McGee K. P. *et al.*, Rapid autocorrection using prescan navigator echoes Magn. Reson. Med. 43 583-8 (2000b).
- [45] Hu X. and Kim S. G., Reduction of signal fluctuation in functional MRI using navigator echoes Magn. Reson. Med. 31 495-503 (1994).

- [46] Ordidge R. J. *et al.*, Correction of motional artifacts in diffusion-weighted MR images using navigator echoes Magn. Reson. Imag. 12 455-60 (1994).
- [47] de Crespigny A. J. *et al.*, Navigated diffusion imaging of normal and ischemic human brain Magn. Reson. Med. 33 720-8 (1995).
- [48] Butts K. *et al.*, Diffusion-weighted interleaved echo-planar imaging with a pair of orthogonal navigator echoes Magn. Reson. Med. 35 763-70 (1996).
- [49] van der Kouwe A. J. W. and Dale A. M., Real-time motion correction using octant navigators Neuroimage 13 48 (2001).
- [50] Welch E. B. *et al.*, Spherical navigator echoes for full 3D rigid body motion measurement in MRI Magn. Reson. Med. 47 32-41 (2002).
- [51] van der Kouwe A. J. W., Benner T. and Dale A. M., Real-time rigid body motion correction and shimming using cloverleaf navigators Magn. Reson. Med. 56 1019-32 (2006).
- [52] Lin W. *et al.*, Motion correction using an enhanced floating navigator and GRAPPA operations Magn. Reson. Med. 63 339-48 (2010).
- [53] Aksoy M. *et al.*, A self-navigated spiral in and out pulse sequence design for retrospective motion correction Proc. 14th Scientific Meeting of ISMRM p 3197 (2006).
- [54] Krueger S. *et al.*, Prospective intra-image compensation for non-periodic rigid body motion using active markers Proc. 14th Scientific Meeting ISMRM p 3196 (2006).
- [55] Sengupta S. *et al.*, Prospective real-time head motion correction using inductively coupled wireless NMR probes Magn. Reson. Med. 72 971-85 (2013).
- [56] van der Kouwe A. *et al.*, Real-time prospective rigid-body motion correction with the EndoScout gradient-based tracking system Proc. 17th Scientific Meeting ISMRM p 4623 (2009).

- [57] Eviatar H., Saunders J. K., Hoult D.I.; Motion compensation by gradient adjustment; In Proceedings of the 5th Annual Meeting of ISMRM, Vancouver, Canada, p. 1898 (1997).
- [58] Eviatar H., Schattka B., Sharp J. C., Rendell J., Alexander M. E.; Real time head motion correction for functional MRI; In Proceedings of the 7th Annual Meeting of the ISMRM, Philadelphia, Pennsylvania, USA, p. 269 (1999).
- [59] Herbst M., Maclaren J., Korvink J. G., Zaitsev M.; A practical tracking system to avoid motion artifacts; In Proceedings of the 19th Annual Meeting of ISMRM, Montreal, Canada, p. 2683 (2011).
- [60] Zaitsev M., Dold C., Sakas G., Hennig J., and Specka O.; Magnetic resonance imaging of freely moving objects: Prospective real-time motion correction using an external optical motion tracking system; *NeuroImage* 31:1038-1050 (2006).
- [61] Andrews-Shigaki B. C., Armstrong B. S. R., Zaitsev M., Ernst T.; Prospective motion correction for magnetic resonance spectroscopy using single camera retro-grate reflector optical tracking; *J Magn Reson Imaging* 33:498-504 (2011).
- [62] Qin L., van Gelderen P., Derbyshire J. A., Jin F., Lee J., de Zwart J. A., Tao Y., Duyn J. H.; Prospective head-movement correction for high-resolution MRI using an in-bore optical tracking system; *Magn Reson Med* 62:924-934 (2009).
- [63] Aksoy M., Forman C., Straka M., Skare S., Holdsworth S., Hornegger J., Bammer R.; Real-time optical motion correction for diffusion tensor imaging; *Magn Reson Med* 66:366-378 (2011).
- [64] Yancey S. E., Rotenberg D. J., Tam F., Chiew M., Ranieri S., Biswas L., Anderson K. J. T., Baker S. N., Wright G. A., Graham S. J.; Spin-history artifact during functional MRI: potential for adaptive correction; *Med Phys* 38:4634-4646 (2011).
- [65] Schulz J., Siegert T., Reimer E., Zaitsev M., Maclaren J., Herbst M., Turner R.; First embedded in-bore system for fast optical prospective head motion-correction in MRI; In

- Proceedings of the 28th Annual Scientific Meeting of the ESMRMB, Leipzig, Germany, p. 369 (2011).
- [66] Jessica Schulz, Thomas Siegert, Enrico Reimer, Christian Labadie, Julian Maclaren, Michael Herbst, Maxim Zaitsev, Robert Turner; An embedded optical tracking system for motion-corrected magnetic resonance imaging at 7T; *Magn Reson Mater Phy* 25:443-453 (2012).
 - [67] Christoph Formana, Murat Aksoya, Joachim Horneggerb, Roland Bammera; Self-Encoded Marker for Optical Prospective Head Motion Correction in MRI; Springer-Verlag Berlin Heidelberg (2010).
 - [68] G. Simon, A. Fitzgibbon, and A. Zisserman, "Markerless tracking using planar structures in the scene," in *Proc. International Symposium on Augmented Reality*, pp. 120-128 (Oct. 2000).
 - [69] J  el Schaerer, Aurora Fassi, Marco Riboldi, Pietro Cerveri, Guido Baroni, and David Sarrut; Multi-dimensional respiratory motion tracking from markerless optical surface imaging based on deformable mesh registration; *Phys. Med. Biol.* 57, 357-373 (2012)
 - [70] Dumoulin C. L., Souza S. P., Darrow R. D.; Real-time position monitoring of invasive devices using magnetic resonance; *Magn Reson Med* 29:411-415 (1993).
 - [71] Ackerman J. L. , Offutt M. C. , Buxton R. B., Brady T. J.; Rapid 3D tracking of small RF coils; In *Proceedings of the 5th Annual Meeting of ISMRM*, Montreal, Canada, pp 1131-1132 (1986).
 - [72] Derbyshire J. A., Wright G. A., Henkelman R. M., Hinks R. S.; Dynamic scanplane tracking using MR position monitoring; *J Magn Reson Imaging* 8:924-932 (1998).
 - [73] Ooi M. B., Krueger S., Thomas W. J., Swaminathan S. V., Brown T. R.; Prospective real-time correction for arbitrary head motion using active markers; *Magn Reson Med* 62:943-954 (2009).

- [74] Ooi M. B., Krueger S., Muraskin J., Thomas W. J., Brown T. R.; Echo-planar imaging with prospective slice-by-slice motion correction using active markers; *Magn Reson Med* 66:73-81 (2011).
- [75] Barmet C., De Zanche N., Pruessmann K. P.; Spatiotemporal magnetic field monitoring for MR; *Magn Reson Med* 60:187-197 (2008).
- [76] Barmet C., De Zanche N., Wilm B. J., Pruessmann K. P.; A transmit/receive system for magnetic field monitoring of in vivo MRI; *Magn Reson Med* 62:269-276 (2009).
- [77] Melvyn B. Ooi, Sascha Krueger, Jordan Muraskin, William J. Thomas, and Truman R. Brown; Echo-Planar Imaging With Prospective Slice-by-Slice Motion Correction Using Active Markers; *Magnetic Resonance in Medicine* 66:73-81 (2011).
- [78] Melvyn B. Ooi, Murat Aksoy, Julian Maclaren, Ronald D. Watkins, and Roland Bammer; Prospective Motion Correction Using Inductively Coupled Wireless RF Coils; *Magnetic Resonance in Medicine* 69:621-636 (2013).
- [79] Lei Qin, Ehud J. Schmidt, Zion Tsz Ho Tse, Juan Santos, William S. Hoge, Clare Tempany-Afdhal, Kim Butts-Pauly, and Charles L. Dumoulin; Prospective motion correction using tracking coils; *Magnetic Resonance in Medicine* 69: 749-759 (2013).
- [80] Van der Kouf A. J. W., Benner T., Dale A. M.; Real-time rigid body motion correction and shimming using cloverleaf navigators; *Magn Reson Med* 56:1019-1032 (2006).
- [81] Ward H. A., Riederer S. J., Grimm R. C., Ehman R. L., Felmlee J. P., Jack C. R. Jr.; Prospective multiaxial motion correction for fMRI; *Magn Reson Med* 43:459-469 (2000).
- [82] Andre' J. W. van der Kouwe, Thomas Benner, and Anders M. Dale; Real-Time Rigid Body Motion Correction and Shimming Using Cloverleaf Navigators; *Magnetic Resonance in Medicine* 56:1019-1032 (2006).
- [83] Tobias Kober, Jose' P. Marques, Rolf Gruetter, and Gunnar Krueger; Head Motion Detection Using FID Navigators; *Magnetic Resonance in Medicine* 66:135-143 (2011).

- [84] Edward Brian Welch, Armando Manduca, Roger C. Grimm, Heidi A. Ward, and Clifford R. Jack Jr.; Spherical Navigator Echoes for Full 3D Rigid Body Motion Measurement in MRI; *Magnetic Resonance in Medicine* 47:32-41 (2002).
- [85] Nathan White, Cooper Roddey, Ajit Shankaranarayanan, Eric Han, Dan Rettmann, Juan Santos, Josh Kuperman, and Anders Dale; PROMO: Real-Time Prospective Motion Correction in MRI Using Image-Based Tracking; *Magnetic Resonance in Medicine* 63:91-105 (2010).
- [86] Tisdall M. D., Hess A. T., Reuter M., Meintjes E. M., Fischl B., van der Kouf A. J. W., Volumetric navigators for prospective motion correction and selective reacquisition in neuroanatomical MRI; *Magn Reson Med* 68:389-399 (2012).
- [87] Alhamud A., Tisdall M. D., Hess A. T., Hasan K. M., Meintjes E. M., van der Kouf A. J. W., Volumetric navigators for real-time motion correction in diffusion tensor imaging; *Magn Reson Med* 68:1097-1108 (2012).
- [88] Daniel S. Illera, Douglas C. Nollb, and Jeffrey A. Fessler; Prospective Motion Correction for Functional MRI Using Sparsity and Kalman Filtering; *Proc. of SPIE Vol. 8858* 885823-1 (2013).
- [89] Pipe J. G., Motion correction with PROPELLER MRI: application to head motion and free-breathing cardiac imaging; *Magnetic Resonance in Medicine*, vol. 42, no. 5, pp. 963-969 (1999).
- [90] Kober T., Marques J. P., Gruetter R., Krueger G.; Head motion detection using FID navigators; *Magn Reson Med* 66:1351-1363 (2011).
- [91] Mansfield, P. and Grannell, P. M., NMR 'diffractions' in solids; *Journal of Physics C: Solid State Physics* 6: L422-L425 (1973).
- [92] Lauterbur, P. C., Image formation by induced local interactions: examples employing nuclear magnetic resonance; *Nature* 242: 190-191 (1973).

- [93] Haacke, E. M., Brown, R. W. and Thompson, M. R.; Magnetic Resonance Imaging Physical Principles and Sequence Design, John Wiley & Sons, Canada,(1999).
- [94] Bernstein, M. A., King, F. K. and Zhou, X. J.; Handbook of MRI Pulse Sequences, Elsevier, USA, (2004).
- [95] Laufs, H., Daunizeau, J., Carmichael, D. W. and Kleinschmidt, A., Recent advances in recording electrophysiological data simultaneously with magnetic resonance imaging, *NeuroImage* 40(2): 515-528 (2008).
- [96] Brown, M. A. and Semelka, R. C.; MRI Basic Principles and Applications, John Wiley & Sons, New Jersey, 3rd Ed.,(2003).
- [97] Hashemi, R. H., Bradley, W. G. and Lisanti, C. J.; MRI: The Basics. Lippincott Williams & Wilkins, Philadelphia, 2nd Ed. (2004).
- [98] P. Mansfield Multi-planar image formation using NMR spin echoes; J. Phys. C: Solid State Phys., Vol. 10, (1977).
- [99] Ray Hashman Hashemi, William G. Bradley Jr, and Christopher J. Lisanti, MRI: The Basics; 3rd edition published by Wolters Kluwer.
- [100] Dixon W. T., Simple proton spectroscopic imaging; Radiology, 153: 189 (1984).
- [101] McGinley, J., Srivastava, V. and DeMeester, G.; Passive shimming technique for MRI magnets, Magnetic Resonance Imaging 15(3), 9-10 (1997).
- [102] Elster A. Questions and answers in magnetic resonance imaging. St Louis: Mosby-Year Book, (1994).
- [103] Jacobs M. A., Ibrahim T. S., Ouwerkerk R.; AAPM/RSNA physics tutorials for residents: MR imaging: brief overview and emerging applications. Radiographics; 27:1213-29 (2007).

- [104] Stuart Currie, Nigel Hoggard, Ian J Craven, Marios Hadjivassiliou, Iain D Wilkinson; Understanding MRI: basic MR physics for physicians; Postgrad Med J. 89: 209-223 (2013;).
- [105] Ridgway JP., Cardiovascular magnetic resonance physics for clinicians: part I. J Cardiovasc Magn Reson, 12:71 (2010).
- [106] Hahn E. L., Spin Echoes; Phys Rev; 80:580-94 (1950).
- [107] Gutberlet, I., Simultaneous EEG and fMRI: recording, analysis, and application, Chapter 2.1: Recording EEG Signals Inside the MRI. Oxford University Press (2010).
- [108] M. Andersen, K. H. Madsen, and L. G. Hanson; Prospective motion correction for MRI using EEG-equipment, Proc. Intl. Soc. Mag. Reson. Med. 24, 4254 (2016).
- [109] R. R. Jensen, C. Benjaminsen, A. E. Hansen, R. Larsen, and O. V. Olesen; Markerless motion correction in MRI, Proc. Intl. Soc. Mag. Reson. Med. 23, 0587 (2015).
- [110] Allen, P. J., Josephs, O. and Turner, R., A Method for removing Imaging Artifact from Continuous EEG Recorded during Functional MRI. NeuroImage 12(2): 230-239 (2000).
- [111] Sandor G., and Erdman A.; Advanced Mechanism Design: Analysis and Synthesis; Prentice-Hall, New Jersey (1984).
- [112] Griffiths, D. J., Introduction to Electrodynamics Fourth Edition; Mishawaka, USA, Prentice Hall, (1981).
- [113] Jackson, J. D., Classical Electrodynamics Third Edition; New York, John Willey & Sons, Inc., (1998)
- [114] Bowtell, R. and Bowley, R. M.; Analytic Calculations of the E-Fields Induced by Time-Varying Magnetic Fields Generated by Cylindrical Gradient Coils, Magnetic Resonance in Medicine 44: 782-790 (2000).

- [115] Bencsik M., Bowtell R., Bowley R. M.; Electrical fields induced in a spherical volume conductor by temporally varying magnetic field gradients; *Phys. Med. Biol.* 47(4), 557-576 (2002).
- [116] Glover, P. M., Cavin, I., Qian, W., Bowtell, R. and Gowland, P. A.; Magnetic-field-induced vertigo: a theoretical and experimental investigation, *Bioelectromagnetics* 28: 349-361 (2007).
- [117] Bencsik, M., Bowtell, R. and Bowley, R. M.; Electric fields in the human body by time-varying magnetic field gradients: numerical calculations and correlation analysis, *Phys. Med. Biol.* 52: 1-17 (2007).
- [118] Syamal K. Sen and Gholam Ali Shaykhian; *Journal Neural, Parallel & Scientific Computations archive* Volume 19, Issue 1-2, 211-228 (2011).
- [119] M. E. H. Chowdhury, Karen J. Mullinger and Richard Bowtell, Simultaneous EEG-fMRI: evaluating the effect of the cabling configuration on the gradient artefact; *Phys. Med. Biol.* 60 N241-N250 (2015).
- [120] M. B. Vestergaard, J. Schulz, R. Turner and L. G. Hanson. Motion tracking from gradient induced signals in electrode recordings, *Proc. of the ESMRMB 28th Annual Meeting*, 368, (2011).
- [121] E. H. Bhuiyan, M. E. H. Chowdhury, P. M. Glover, and R. Bowtell. *Proc. Intl. Soc. Mag. Reson. Med.* 23, 1017 (2015).
- [122] J. Schulz, T. Siegert, E. Reimer, C. Labadie, J. Maclaren, M. Herbst, and M. Zaitsev R. Turner, "An embedded optical tracking system for motion-corrected magnetic resonance imaging at 7T," *Mag. Res. Materials in Phy., Bio. and Med.*, 25:443-453 (2012).
- [123] Maclaren J., Herbst M., Speck O., Zaitsev M. Prospective motion correction in brain imaging: a review, *Magn. Res. Med.* 69, 621-636 (2013).

- [124] Ooi MB, Krueger S., Thomas W. J, Swaminathan S. V., Brown T. R., Prospective real-time correction for arbitrary head motion using active markers *Magn. Reson. Med.* 62, 943-954 (2009).
- [125] James L. Crowley, Frank Wallner, Bernt Schiele; Position estimation using principal components of range data; *Robotics and Autonomous Systems* 23 267-276 (1998)
- [126] Baudouin Denis de Senneville, Abdallah El Hamidi, and Chrit Moonen; A Direct PCA-Based Approach for Real-Time Description of Physiological Organ Deformations; *IEEE Transactions on Medical Imaging*, Vol. 34, No. 4 (2015)
- [127] Glyn S. Spencer, EEG-fMRI: Novel Methods for Gradient Artefact Correction, PhD Thesis, SPMIC, University of Nottingham, UK (2014).
- [128] K. Mullinger, W. Yan, and R. Bowtell, "Reducing the gradient artefact in simultaneous eeg-fmri by adjusting the subject's axial position," *NeuroImage*, vol. 54, no. 3, pp. 1942-1950 (2011).
- [129] H. Mandelkow, P. Halder, P. Boesiger, and D. Brandeis, "Synchronization facilitates removal of MRI artefacts from concurrent EEG recordings and increases usable bandwidth," *NeuroImage*, vol. 32, no. 3, pp. 1120-1126 (2006).

Appendix A

Mathematical Calculations in Details

A.1 Rotation about the x-axis

After rotational transformation the change in induced voltage due to the application of the x-, y- and z-gradients can be calculated in the following way:

$$\begin{aligned}\delta V_x &= -V_{xat} + V = \dot{\mathbf{B}}_{xat} \cdot \mathbf{S}_{xat} - \dot{\mathbf{B}} \cdot \mathbf{S} = [g_x(z'c - y's + z_0) - \frac{1}{2}g_z x]S_x \\ &\quad + [g_y(z'c - y's + z_0) - \frac{1}{2}g_z(y'c + z's + y_0)](S_y c + S_z s) \\ &\quad + [g_x x + g_y(y'c + z's + y_0) + g_z(z'c - y's z_0 + z_0)](S_z c - S_y s) \\ &\quad - [g_x(z' + z_0) - \frac{1}{2}g_z x]S_x - [g_y(z' + z_0) - \frac{1}{2}g_z(y' + y_0)]S_y \\ &\quad - [g_x x + g_y(y' + y_0) + g_z(z' + z_0)]S_z \\ &= g_x[z'S_x c - y'S_x s + xS_z c - xS_y s - z'S_x - xS_z] \\ &\quad + g_y[(z'c - y's)(S_y c + S_z s) + (y'c + z's)(S_z c - S_y s) - z'S_y - y'S_z] \\ &\quad + g_y[(z_0 S_y + y_0 S_z)(c - 1) + (z_0 S_z - y_0 S_y)s] \\ &\quad - \frac{1}{2}g_z[xS_x + (y'c + z's)(S_y c + S_z s) - xS_x - y'S_y - 2(z'c - y's)(S_z c - S_y s) + z'S_z] \\ &\quad + g_z[(z_0 S_z + \frac{1}{2}y_0 S_y)(c - 1) - (\frac{1}{2}y_0 S_z + z_0 S_y)s].\end{aligned}\tag{A.1}$$

To make mathematical manipulation easier, let us consider the contribution of x-gradient as A_x which is given as follows:

$$\begin{aligned} A_x &= g_x[z'S_xc - y'S_xs + xS_zc - xS_ys - z'S_x - xS_z] \\ &= g_x[(z'S_x + xS_z)(c - 1) - (y'S_x + xS_y)s]. \end{aligned} \quad (\text{A.2})$$

By replacing $c = \cos \phi_x$ and $s = \sin \phi_x$, one obtains

$$\begin{aligned} A_x &= g_x[(z'S_x + xS_z)(\cos \phi - 1) - (y'S_x + xS_y) \sin \phi_x] \\ &= -g_x[2(z'S_x + xS_z) \sin^2 \frac{\phi_x}{2} + (xS_y + y'S_x) \sin \phi_x]. \end{aligned} \quad (\text{A.3})$$

Similarly, the contribution of y-gradient can be denoted by A_y and be given as follows:

$$\begin{aligned} A_y &= g_y[(z'c - y's)(S_yc + S_zs) + (y'c + z's)(S_zc - S_ys) - z'S_y - y'S_z] \\ &\quad + g_y[(z_0S_y + y_0S_z)(c - 1) + (z_0S_z - y_0S_y)s] \\ &= g_y[z'S_yc^2 - y'S_zs^2 - y'S_ysc + z'S_zsc + y'S_zc^2 - z'S_ys^2 + z'S_zsc - y'S_ysc - z'S_y - y'S_z] \\ &\quad + g_y[(z_0S_y + y_0S_z)(c - 1) + (z_0S_z - y_0S_y)s] \\ &= g_y[z'S_y(c^2 - 1) + y'S_z(c^2 - 1) - y'S_zs^2 - z'S_ys^2 + 2(z'S_z - y'S_y)sc] \\ &\quad + g_y[(z_0S_y + y_0S_z)(c - 1) + (z_0S_z - y_0S_y)s]. \end{aligned} \quad (\text{A.4})$$

By replacing $c = \cos \phi_x$ and $s = \sin \phi_x$, one obtains

$$\begin{aligned} A_y &= g_y[-z'S_y \sin^2 \phi_x - y'S_z \sin^2 \phi_x - y'S_z \sin^2 \phi_x - z'S_y \sin^2 \phi_x + (z'S_z - y'S_y) \sin(2\phi_x)] \\ &\quad + g_y[(z_0S_y + y_0S_z)(\cos \phi_x - 1) + (z_0S_z - y_0S_y) \sin \phi_x] \\ &= -g_y[2(z'S_y + y'S_z) \sin^2 \phi_x + (y'S_y - z'S_z) \sin(2\phi_x)] \\ &\quad + g_y[-2(z_0S_y + y_0S_z) \sin^2 \frac{\phi_x}{2} + (z_0S_z - y_0S_y) \sin \phi_x]. \end{aligned} \quad (\text{A.5})$$

Now the contribution of z-gradient, A_z are as follows:

$$\begin{aligned}
A_z &= -\frac{1}{2}g_z[xS_x + (y'c + z's)(S_y c + S_z s) - xS_x - y'S_y - 2(z'c - y's)(S_z c - S_y s) + 2z'S_z] \\
&\quad + g_z[(z_0 S_z + \frac{1}{2}y_0 S_y)(c - 1) - (\frac{1}{2}y_0 S_z + z_0 S_y)s] \\
&= -\frac{1}{2}g_z[y'S_y(c^2 - 1) + z'S_z s^2 + (y'S_z + z'S_y)sc - 2z'S_z(c^2 - 1) - 2y'S_y s^2 + 2(y'S_z + z'S_y)sc] \\
&\quad + g_z[(z_0 S_z + \frac{1}{2}y_0 S_y)(c - 1) - (\frac{1}{2}y_0 S_z + z_0 S_y)s].
\end{aligned} \tag{A.6}$$

By replacing $c = \cos \phi_x$ and $s = \sin \phi_x$, one obtains

$$\begin{aligned}
A_z &= -\frac{1}{2}g_z[-y'S_y \sin^2 \phi_x + z'S_z \sin^2 \phi_x + \frac{1}{2}(y'S_z + z'S_y) \sin(2\phi_x) \\
&\quad + 2z'S_z \sin^2 \phi_x - 2y'S_y \sin^2 \phi_x + (y'S_z + z'S_y) \sin(2\phi_x)] \\
&\quad + g_z[(z_0 S_z + \frac{1}{2}y_0 S_y)(\cos \phi_x - 1) - (\frac{1}{2}y_0 S_z + z_0 S_y) \sin \phi_x] \\
&= \frac{3}{2}g_z[y'S_y \sin^2 \phi_x - z'S_z \sin^2 \phi_x - \frac{1}{2}(y'S_z + z'S_y) \sin(2\phi_x)] \\
&\quad + g_z[-2(z_0 S_z + \frac{1}{2}y_0 S_y) \sin^2 \frac{\phi_x}{2} - (\frac{1}{2}y_0 S_z + z_0 S_y) \sin \phi_x] \\
&= \frac{3}{2}g_z[(y'S_y - z'S_z) \sin^2 \phi_x - \frac{1}{2}(y'S_z + z'S_y) \sin(2\phi_x)] \\
&\quad + g_z[-2(z_0 S_z + \frac{1}{2}y_0 S_y) \sin^2 \frac{\phi_x}{2} - (\frac{1}{2}y_0 S_z + z_0 S_y) \sin \phi_x].
\end{aligned} \tag{A.7}$$

By applying x-, y- and z-gradients, the change in induced voltage due to rotation about the x-axis is-

$$\begin{aligned}
\delta V_x &= A_x + A_y + A_z \\
&= -g_x[2(z'S_x + xS_z) \sin^2 \frac{\phi_x}{2} + (xS_y + y'S_x) \sin \phi_x] \\
&\quad - g_y[2(z'S_y + y'S_z) \sin^2 \phi_x + (y'S_y - z'S_z) \sin(2\phi_x)] \\
&\quad + g_y[-2(z_0 S_y + y_0 S_z) \sin^2 \frac{\phi_x}{2} + (z_0 S_z - y_0 S_y) \sin \phi_x] \\
&\quad + \frac{3}{2}g_z[(y'S_y - z'S_z) \sin^2 \phi_x - \frac{1}{2}(z'S_y + y'S_z) \sin(2\phi_x)] \\
&\quad + g_z[-2(z_0 S_z + \frac{1}{2}y_0 S_y) \sin^2 \frac{\phi_x}{2} - (\frac{1}{2}y_0 S_z + z_0 S_y) \sin \phi_x]
\end{aligned} \tag{A.8}$$

The change in induced voltage in the individual coil due to a rotation by an angle, ϕ_x , about the x-axis can be obtained by the given equations:

$$\begin{aligned}
\delta V_1 &= -2g_x(xS_z)\sin^2\frac{\phi_x}{2} - 2g_y(y'S_z)\sin^2\phi_x + g_y(z'S_z)\sin(2\phi_x) - \frac{3}{2}g_z(z'S_z)\sin^2\phi_x - \frac{3}{4}g_z(y'S_z)\sin(2\phi_x) \\
&\quad - 2g_y(y_0S_z)\sin^2\frac{\phi_x}{2} + g_y(z_0S_z)\sin\phi_x - 2g_z(z_0S_z)\sin^2\frac{\phi_x}{2} - \frac{1}{2}g_z(y_0S_z)\sin\phi_x \\
\delta V_2 &= -2g_x(z'S_x)\sin^2\frac{\phi_x}{2} + g_x(y'S_x)\sin\phi_x, \quad \delta V_3 = -2g_x(z'S_x)\sin^2\frac{\phi_x}{2} + g_x(y'S_x)\sin\phi_x \\
\delta V_4 &= -g_x(xS_y)\sin\phi_x - 2g_y(z'S_y)\sin^2\phi_x + g_y(y'S_y)\sin(2\phi_x) + 2g_y(z_0S_y)\sin^2\frac{\phi_x}{2} - g_y(y_0S_y)\sin\phi_x \\
&\quad + \frac{3}{2}g_z(y'S_y)\sin^2\phi_x - \frac{3}{4}(z'S_y)\sin(2\phi_x) - 2g_z(y_0S_y)\sin^2\frac{\phi_x}{2} - g_z(z_0S_y)\sin\phi_x \\
\delta V_5 &= -g_x(xS_y)\sin\phi_x - 2g_y(z'S_y)\sin^2\phi_x + g_y(y'S_y)\sin(2\phi_x) + 2g_y(z_0S_y)\sin^2\frac{\phi_x}{2} - g_y(y_0S_y)\sin\phi_x \\
&\quad + \frac{3}{2}g_z(y'S_y)\sin^2\phi_x - \frac{3}{4}(z'S_y)\sin(2\phi_x) - 2g_z(y_0S_y)\sin^2\frac{\phi_x}{2} - g_z(z_0S_y)\sin\phi_x.
\end{aligned} \tag{A.9}$$

For a very small rotation angle ϕ_x , i.e. $\phi_x \leq 5^\circ$ one can write $\sin\phi_x = \phi_x$ (in radians) and as a result the general equation for the change induced voltage in the coils due to rotation about the x-axis by an angle, $\delta\phi_x$ reduces to

$$\begin{aligned}
\delta V_1 &= -\frac{1}{2}g_x(xS_z)\delta\phi_x^2 - 2g_y(y'S_z)\delta\phi_x^2 + 2g_y(z'S_z)\delta\phi_x - \frac{3}{2}g_z(z'S_z)\delta\phi_x^2 - \frac{3}{2}g_z(y'S_z)\delta\phi_x \\
&\quad - \frac{1}{2}g_y(y_0S_z)\delta\phi_x^2 + g_y(z_0S_z)\delta\phi_x - \frac{1}{2}g_z(z_0S_z)\delta\phi_x^2 - \frac{1}{2}g_z(y_0S_z)\delta\phi_x \\
\delta V_2 &= -\frac{1}{2}g_x(z'S_x)\delta\phi_x^2 + g_x(y'S_x)\delta\phi_x, \quad \delta V_3 = -\frac{1}{2}g_x(z'S_x)\delta\phi_x^2 + g_x(y'S_x)\delta\phi_x \\
\delta V_4 &= -g_x(xS_y)\delta\phi_x - 2g_y(z'S_y)\delta\phi_x^2 + 2g_y(y'S_y)\delta\phi_x + \frac{1}{2}g_y(z_0S_y)\delta\phi_x^2 - g_y(y_0S_y)\delta\phi_x \\
&\quad + \frac{3}{2}g_z(y'S_y)\delta\phi_x^2 - \frac{3}{2}(z'S_y)\delta\phi_x - \frac{1}{2}g_z(y_0S_y)\delta\phi_x^2 - g_z(z_0S_y)\delta\phi_x \\
\delta V_5 &= -g_x(xS_y)\delta\phi_x - 2g_y(z'S_y)\delta\phi_x^2 + 2g_y(y'S_y)\delta\phi_x + \frac{1}{2}g_y(z_0S_y)\delta\phi_x^2 - g_y(y_0S_y)\delta\phi_x \\
&\quad + \frac{3}{2}g_z(y'S_y)\delta\phi_x^2 - \frac{3}{2}(z'S_y)\delta\phi_x - \frac{1}{2}g_z(y_0S_y)\delta\phi_x^2 - g_z(z_0S_y)\delta\phi_x.
\end{aligned} \tag{A.10}$$

The linear terms dominate in the expressions for the change in induced voltages in the coils. So one can exclude the non-linear terms

$$\begin{aligned}
\delta V_1 &= 2g_y(z'S_z)\delta\phi_x - \frac{3}{2}g_z(y'S_z)\delta\phi_x + g_y(z_0S_z)\delta\phi_x - \frac{1}{2}g_z(y_0S_z)\delta\phi_x \\
\delta V_2 &= g_x(y'S_x)\delta\phi_x, \quad \delta V_3 = g_x(y'S_x)\delta\phi_x \\
\delta V_4 &= -g_x(xS_y)\delta\phi_x + 2g_y(y'S_y)\delta\phi_x - g_y(y_0S_y)\delta\phi_x - \frac{3}{2}(z'S_y)\delta\phi_x - g_z(z_0S_y)\delta\phi_x \\
\delta V_5 &= -g_x(xS_y)\delta\phi_x + 2g_y(y'S_y)\delta\phi_x - g_y(y_0S_y)\delta\phi_x - \frac{3}{2}(z'S_y)\delta\phi_x - g_z(z_0S_y)\delta\phi_x.
\end{aligned} \tag{A.11}$$

Let us again consider Fig. 3.2, a system of five coils in which the centres lie on the x, y and z-axis and the axes of the coils are the x-, y- and z-axes, one can calculate the change

in induced voltage in each coil due to the rotation about x-axis by taking only the terms xS_x, yS_y, zS_z . The change in induced voltage in the five coils are given by a general equation:

$$\begin{aligned}\delta V_x = & -g_y(y'S_y - z'S_z) \sin(2\phi_x) + \frac{3}{2}g_z(y'S_y - z'S_z) \sin^2 \phi_x \\ & -g_y(z_0S_z - y_0S_y) \sin \phi_x - 2g_z(z_0S_z + \frac{1}{2}y_0S_y) \sin^2 \frac{\phi_x}{2}.\end{aligned}\quad (\text{A.12})$$

The change in induced voltage in the individual coil due to a rotation by an angle, ϕ_x , about the x-axis can be obtained by the given equations:

$$\begin{aligned}\delta V_1 = & g_y(z'S_z) \sin(2\phi_x) - \frac{3}{2}g_z(z'S_z) \sin^2 \phi_x - g_yz_0S_z \sin \phi_x - 2g_zz_0S_z \sin^2 \frac{\phi_x}{2} \\ \delta V_2 = & 0 \quad \delta V_3 = 0 \\ \delta V_4 = & -g_y(y'S_y) \sin(2\phi_x) - \frac{3}{2}g_z(y'S_y) \sin^2 \phi_x - g_yy_0S_y \sin \phi_x - g_zy_0S_y \sin^2 \frac{\phi_x}{2} \\ \delta V_5 = & -g_y(y'S_y) \sin(2\phi_x) - \frac{3}{2}g_z(y'S_y) \sin^2 \phi_x - g_yy_0S_y \sin \phi_x - g_zy_0S_y \sin^2 \frac{\phi_x}{2}.\end{aligned}\quad (\text{A.13})$$

For a very small rotation angle ϕ_x , i.e. $\phi_x \leq 5^\circ$ one can write $\sin \phi_x = \phi_x$ and as a result the general equation for the change induced voltage due to the rotation about the x-axis reduces to

$$\begin{aligned}\delta V_x = & -2g_y(y'S_y - z'S_z)\delta\phi_x + \frac{3}{2}g_z(y'S_y - z'S_z)\delta\phi_x^2 \\ & -g_y(z_0S_z - y_0S_y)\delta\phi_x + g_z[-2(z_0S_z + \frac{1}{2}y_0S_y)\delta\phi_x^2].\end{aligned}\quad (\text{A.14})$$

In case of a very small rotation angle the change in induced voltage in the five coils are then given as follows:

$$\begin{aligned}\delta V_1 = & 2g_y(z'S_z)\delta\phi_x - \frac{3}{2}g_z(z'S_z)\delta\phi_x^2 - g_y(z_0S_z)\delta\phi_x - 2g_z(z_0S_z)\delta\phi_x^2 \\ \delta V_2 = & 0 \quad \delta V_3 = 0 \\ \delta V_4 = & -2g_y(y'S_y)\delta\phi_x - \frac{3}{2}g_z(y'S_y)\delta\phi_x^2 - g_y(y_0S_y)\delta\phi_x - g_z(y_0S_y)\delta\phi_x^2 \\ \delta V_5 = & -2g_y(y'S_y)\delta\phi_x - \frac{3}{2}g_z(y'S_y)\delta\phi_x^2 - g_y(y_0S_y)\delta\phi_x - g_z(y_0S_y)\delta\phi_x^2.\end{aligned}\quad (\text{A.15})$$

A.1.1 Rotation about the y-axis

After the rotation about the y-axis the induced voltages are denoted by V_{yat} can be obtain by the following equation:

$$\begin{aligned}
-V_{yat} &= \mathbf{B}_{yat} \cdot \mathbf{S}_{yat} = [g_x(x's + z'c + z_0) - \frac{1}{2}g_z(x'c - z's + x_0)](S_xc - S_zs) \\
&\quad + [g_y(x' + z'c + z_0) - \frac{1}{2}g_zy]S_y \\
&\quad + [g_x(x'c - z's + x_0) + g_yy + g_z(x's + z'c + z_0)](S_xs + S_zc) \\
&= [g_x(x's + z'c)(S_xc - S_zs) + g_x(x'c - z's)(S_xs + S_zc)] \\
&\quad + g_xz_0(S_xc - S_zs) + g_xx_0(S_xs - S_zc) \\
&\quad + [g_y(x's + z'c)S_y + g_yy(S_xs + S_zc) \\
&\quad - \frac{1}{2}g_z[(x'c - z's)(S_xc - S_zs) + yS_y + g_z(x's + z'c)(S_xs + S_zc)] \\
&\quad + g_zz_0(S_xs + S_zc) - \frac{1}{2}g_zx_0(S_xc - S_zs)].
\end{aligned} \tag{A.16}$$

The change in induced voltage due to rotation about the y-axis is then given as follows:

$$\begin{aligned}
\delta V_y &= -V_{yat} + V = \dot{\mathbf{B}}_{yat} \cdot \mathbf{S}_{yat} - \dot{\mathbf{B}} \cdot \mathbf{S} \\
&= g_x(x'S_xsc + z'S_xc^2 - x'S_zs^2 - z'S_zsc \\
&\quad + x'S_xcs - z'S_xs^2 + x'S_zc^2 - z'S_zcs - z'S_x - x'S_z) \\
&\quad + g_x(z_0S_xc - z_0S_zs + x_0S_xs - x_0S_zc - z_0S_x - x_0S_z) \\
&\quad + g_y(x'S_ys + z'S_yc + yS_xs + yS_zc - z'S_y - yS_z) \\
&\quad + \frac{1}{2}g_z(-x'S_xc^2 + z'S_xsc + x'S_zsc - z'S_zs^2 \\
&\quad - yS_y + 2x'S_xs^2 + 2z'S_xsc + 2x'S_zsc \\
&\quad + 2z'S_zc^2 + x'S_x + yS_y - z'S_z) \\
&\quad + \frac{1}{2}g_z(2z_0S_xs + 2z_0S_zc - x_0S_xc + x_0S_zs) - 2g_zz_0S_z - x_0S_x).
\end{aligned} \tag{A.17}$$

Again for simple mathematical manipulation, let us consider the contribution of x-gradient:

$$\begin{aligned}
B_x &= g_x(x'S_xsc + z'S_xc^2 - x'S_zs^2 - z'S_zsc \\
&+ x'S_xcs - z'S_xs^2 + x'S_zc^2 - z'S_zcs - z'S_x - x'S_z) \\
&+ g_xz_0(S_xc - S_zs) + g_xx_0(S_xs + S_zc) - g_xz_0S_x - g_xx_0S_z \\
&= g_x[2(x'S_x - z'S_zS_z)sc + (z'S_x + z'S_z)(c^2 - 1) \\
&\quad - (x'S_z + z'S_x)s^2] \\
&\quad + g_x[(z_0S_x + x_0S_z)(c - 1) + (x_0S_x - z_0S_z)s].
\end{aligned} \tag{A.18}$$

By replacing $c = \cos \phi_y$ and $s = \sin \phi_y$, one obtains

$$\begin{aligned}
B_x &= g_x[2(x'S_x - z'S_zS_z) \sin \phi_y \cos \phi_y + (z'S_x + z'S_z)(\cos^2 \phi_y - 1) \\
&\quad - (x'S_z + z'S_x) \sin^2 \phi_y] \\
&+ g_x[(z_0S_x + x_0S_z)(\cos \phi_y - 1) + (x_0S_x - z_0S_z) \sin \phi_y] \\
&= g_x[(xS_x - zS_z) \sin(2\phi_y) - 2(zS_x + xS_z) \sin^2 \phi_y] \\
&- g_x[-2(z_0S_x + x_0S_z) \sin^2 \frac{\phi_y}{2} + (x_0S_x - z_0S_z) \sin \phi_y].
\end{aligned} \tag{A.19}$$

Similarly, the contribution of y-gradient:

$$\begin{aligned}
B_y &= +g_y(x'S_ys + z'S_yc + yS_xs + yS_zc - z'S_y - yS_z) \\
&= g_y[x'S_ys + yS_xs + z'S_y(c - 1) + yS_z(c - 1)] \\
&= g_y[(x'S_y + yS_x) \sin \phi_y - 2(z'S_y + yS_z) \sin^2 \frac{\phi_y}{2}].
\end{aligned} \tag{A.20}$$

Now, the contribution of z-gradient:

$$\begin{aligned}
B_z &= +\frac{1}{2}g_z(-x'S_xc^2 + z'S_xsc + x'S_zsc - z'S_zs^2 \\
&\quad - yS_y + 2x'S_xs^2 + 2z'S_xsc + 2x'S_zsc \\
&\quad + 2z'S_zc^2 + x'S_x + yS_y - z'S_z) \\
&+ g_zz_0(S_xs + S_zc) - \frac{1}{2}(S_xc - S_zs) - g_zz_0S_z + \frac{1}{2}g_zx_0S_x.
\end{aligned} \tag{A.21}$$

By replacing $c = \cos \phi_y$ and $s = \sin \phi_y$, one obtain

$$\begin{aligned}
B_z &= \frac{1}{2}g_z[-x'S_x(c^2 - 1) + 3z'S_xsc \\
&\quad + 3x'S_zsc + 2z'S_z(c^2 - 1) - z'S_zs^2 + 2x'S_xs^2] \\
&\quad + g_z[(z_0S_z - \frac{1}{2}x_0S_x)(c - 1) + (z_0S_x + \frac{1}{2}x_0S_x)s] \\
&= \frac{3}{2}g_z[x'S_x \sin^2 \phi_y + \frac{1}{2}(z'S_x + x'S_z) \sin(2\phi_y) - z'S_z \sin^2 \phi_y] \\
&\quad + g_z(z_0S_x + \frac{1}{2}x_0S_x) \sin \phi_y - 2(z_0S_z - \frac{1}{2}x_0S_x) \sin^2 \frac{\phi_y}{2} \\
&= \frac{3}{2}g_z[(x'S_x - z'S_z) \sin^2 \phi_y + \frac{1}{2}(z'S_x + x'S_z) \sin(2\phi_y)] \\
&\quad + g_z[(z_0S_x + \frac{1}{2}x_0S_z) \sin \phi_y - 2(z_0S_z - \frac{1}{2}x_0S_x) \sin^2 \frac{\phi_y}{2}].
\end{aligned} \tag{A.22}$$

Now the change in induced voltage due to rotation about the y-axis because of the application of the x-, y- and z-gradients becomes:

$$\begin{aligned}
\delta V_y &= B_x + B_y + B_z \\
&= g_x[(x'S_x - z'S_z) \sin(2\phi_y) - 2(z'S_x + x'S_z) \sin^2 \phi_y] \\
&\quad - g_x[-2(z_0S_x + x_0S_z) \sin^2 \frac{\phi_y}{2} + (x_0S_x - z_0S_z) \sin \phi_y] \\
&\quad + g_y[(x'S_y + yS_x) \sin \phi_y - 2(z'S_y + yS_z) \sin^2 \frac{\phi_y}{2}] \\
&\quad + \frac{3}{2}g_z[\frac{1}{2}(z'S_x + xS_z) \sin(2\phi_y) + (xS_x - z'S_z) \sin^2 \phi_y] \\
&\quad + g_z[(z_0S_x + \frac{1}{2}x_0S_z) \sin \phi_y - 2(z_0S_z - \frac{1}{2}x_0S_x) \sin^2 \frac{\phi_y}{2}].
\end{aligned} \tag{A.23}$$

The change in induced voltage in the individual coil due to a rotation by an angle, ϕ_y , about the y-axis can be obtained by the given equations:

$$\begin{aligned}
\delta V_1 &= -g_x(z'S_z) \sin(2\phi_y) - 2g_x(x'S_z) \sin^2 \phi_y + 2g_x(x_0S_z) \sin^2 \frac{\phi_y}{2} + g_x(z_0S_z) \sin \phi_y - 2g_y(yS_z) \sin^2 \frac{\phi_y}{2} \\
&\quad + \frac{3}{2}g_z(xS_z) \sin(2\phi_y) - \frac{3}{2}g_z(z'S_z) \sin^2 \phi_y + \frac{1}{2}g_z(x_0S_z) \sin \phi_y - 2g_z(z_0S_z) \sin^2 \frac{\phi_y}{2} \\
\delta V_2 &= g_x(x'S_x) \sin(2\phi_y) - 2g_x(z'S_x) \sin^2 \phi_y - 2g_x(z_0S_x) \sin^2 \frac{\phi_y}{2} + g_x(x_0S_x) \sin \phi_y + g_y(yS_x) \sin \phi_y \\
&\quad + \frac{3}{4}g_z(z'S_x) \sin(2\phi_y) + \frac{3}{2}(xS_x) \sin^2 \phi_y + g_z(z_0S_x) \sin \phi_y + g_z(x_0S_x) \sin^2 \frac{\phi_y}{2} \\
\delta V_3 &= g_x(x'S_x) \sin(2\phi_y) - 2g_x(z'S_x) \sin^2 \phi_y - 2g_x(z_0S_x) \sin^2 \frac{\phi_y}{2} + g_x(x_0S_x) \sin \phi_y + g_y(yS_x) \sin \phi_y \\
&\quad + \frac{3}{4}g_z(z'S_x) \sin(2\phi_y) + \frac{3}{2}(xS_x) \sin^2 \phi_y + g_z(z_0S_x) \sin \phi_y + g_z(x_0S_x) \sin^2 \frac{\phi_y}{2} \\
\delta V_4 &= g_y(x'S_y) \sin \phi_y - 2g_y(z'S_y) \sin^2 \frac{\phi_y}{2}, \quad \delta V_5 = g_y(x'S_y) \sin \phi_y - 2g_y(z'S_y) \sin^2 \frac{\phi_y}{2}.
\end{aligned} \tag{A.24}$$

For a very small rotation angle ϕ_y , i.e. $\phi_y \leq 5^\circ$ one can write $\sin \phi_y = \phi_y$ (in radians); as a result the general equation for the change induced voltage in the coils due to the rotation about y-axis by angle $\delta\phi_y$ reduces to

$$\begin{aligned}
\delta V_1 &= -2g_x(z'S_z)\delta\phi_y - 2g_x(x'S_z)\delta\phi_y^2 + \frac{1}{2}g_x(x_0S_z)\delta\phi_y^2 + g_x(z_0S_z)\delta\phi_y - \frac{1}{2}g_y(yS_z)\delta\phi_y^2 \\
&\quad + \frac{3}{4}g_z(xS_z)\delta\phi_y - \frac{3}{2}g_z(z'S_z)\delta\phi_y^2 + \frac{1}{2}g_z(x_0S_z)\delta\phi_y - \frac{1}{2}g_z(z_0S_z)\delta\phi_y^2 \\
\delta V_2 &= 2g_x(x'S_x)\delta\phi_y - 2g_x(z'S_x)\delta\phi_y^2 - \frac{1}{2}g_x(z_0S_x)\delta\phi_y^2 + g_x(x_0S_x)\delta\phi_y + g_y(yS_x)\delta\phi_y \\
&\quad + \frac{3}{2}g_z(z'S_x)\delta\phi_y + \frac{3}{2}(xS_x)\delta\phi_y^2 + g_z(z_0S_x)\delta\phi_y + \frac{1}{4}g_z(x_0S_x)\delta\phi_y^2 \\
\delta V_3 &= 2g_x(x'S_x)\delta\phi_y - 2g_x(z'S_x)\delta\phi_y^2 - \frac{1}{2}g_x(z_0S_x)\delta\phi_y^2 + g_x(x_0S_x)\delta\phi_y + g_y(yS_x)\delta\phi_y \\
&\quad + \frac{3}{2}g_z(z'S_x)\delta\phi_y + \frac{3}{2}(xS_x)\delta\phi_y^2 + g_z(z_0S_x)\delta\phi_y + \frac{1}{4}g_z(x_0S_x)\delta\phi_y^2 \\
\delta V_4 &= g_y(x'S_y)\delta\phi_y - \frac{1}{2}g_y(z'S_y)\delta\phi_y^2, \quad \delta V_5 = g_y(x'S_y)\delta\phi_y - \frac{1}{2}g_y(z'S_y)\delta\phi_y^2.
\end{aligned} \tag{A.25}$$

The linear terms dominate in the expressions for the change in induced voltages in the coils. So one can exclude the non-linear terms,

$$\begin{aligned}
\delta V_1 &= -2g_x(z'S_z)\delta\phi_y + g_x(z_0S_z)\delta\phi_y + \frac{3}{4}g_z(xS_z)\delta\phi_y + \frac{1}{2}g_z(x_0S_z)\delta\phi_y \\
\delta V_2 &= 2g_x(x'S_x)\delta\phi_y + g_x(x_0S_x)\delta\phi_y + g_y(yS_x)\delta\phi_y + \frac{3}{2}g_z(z'S_x)\delta\phi_y + g_z(z_0S_x)\delta\phi_y \\
\delta V_3 &= 2g_x(x'S_x)\delta\phi_y + g_x(x_0S_x)\delta\phi_y + g_y(yS_x)\delta\phi_y + \frac{3}{2}g_z(z'S_x)\delta\phi_y + g_z(z_0S_x)\delta\phi_y \\
\delta V_4 &= g_y(x'S_y)\delta\phi_y, \quad \delta V_5 = g_y(x'S_y)\delta\phi_y.
\end{aligned} \tag{A.26}$$

For the same position and orientation of the five coils shown in Fig. 3.2, one can calculate the change in induced voltage in each coil due to the rotation about the y-axis by angle, ϕ ,

by taking only the terms xS_x , yS_y , and zS_z . The change in induced voltage in the five coils are given by the general equation:

$$\begin{aligned}\delta V_y = & g_x(x'S_x - z'S_z) \sin(2\phi_y) + \frac{3}{2}g_z(x'S_x - z'S_z) \sin^2 \phi_y \\ & - g_x(x_0S_x - z_0S_z) \sin \phi_y - 2g_z(z_0S_z - \frac{1}{2}x_0S_x) \sin^2 \frac{\phi_y}{2}.\end{aligned}\quad (\text{A.27})$$

The change in induced voltage in the individual coil due to rotation about y-axis can then be obtained by the given equations:

$$\begin{aligned}\delta V_1 = & g_x(z'S_z) \sin(2\phi_y) - \frac{3}{2}g_z(z'S_z) \sin^2 \phi_y + g_x(z_0S_z) \sin \phi_y - 2g_z(z_0S_z) \sin^2 \frac{\phi_y}{2} \\ \delta V_2 = & g_x(x'S_x) \sin(2\phi_y) + \frac{3}{2}g_z(x'S_x) \sin^2 \phi_y - g_x(x_0S_x) \sin \phi_y + g_z(x_0S_x) \sin^2 \frac{\phi_y}{2} \\ \delta V_3 = & g_x(x'S_x) \sin(2\phi_y) + \frac{3}{2}g_z(x'S_x) \sin^2 \phi_y - g_x(x_0S_x) \sin \phi_y + g_z(x_0S_x) \sin^2 \frac{\phi_y}{2} \\ \delta V_4 = & 0 \quad \delta V_5 = 0.\end{aligned}$$

For a very small rotation angle ϕ_y , i.e. $\phi_y \leq 5^\circ$ one can write $\sin \phi_y = \phi_y$; as a result the general equation for the change induced voltage due to the rotation about y-axis by angle $\delta\phi_y$ reduces to

$$\begin{aligned}\delta V_y = & g_x(x'S_x - z'S_z)\delta\phi_y + \frac{3}{2}g_z(x'S_x - z'S_z)\delta\phi_y^2 \\ & - g_x(x_0S_x - z_0S_z)\delta\phi_y - 2g_z(z_0S_z - \frac{1}{2}x_0S_x)\delta\phi_y^2.\end{aligned}\quad (\text{A.28})$$

In the case rotation about the y-axis and a very small rotation angle, the change in the induced voltage in the five coils are consequently given as follows:

$$\begin{aligned}\delta V_1 = & 2g_x(z'S_z)\delta\phi_y - \frac{3}{2}g_z(z'S_z)\delta\phi_y^2 + g_x(z_0S_z)\delta\phi_y - 2g_z(z_0S_z)\delta\phi_y^2 \\ \delta V_2 = & 2g_x(x'S_x)\delta\phi_y + \frac{3}{2}g_z(x'S_x)\delta\phi_y^2 - g_x(x_0S_x)\delta\phi_y + g_z(x_0S_x)\delta\phi_y^2 \\ \delta V_3 = & 2g_x(x'S_x)\delta\phi_y + \frac{3}{2}g_z(x'S_x)\delta\phi_y^2 - g_x(x_0S_x)\delta\phi_y + g_z(x_0S_x)\delta\phi_y^2 \\ \delta V_4 = & 0 \quad \delta V_5 = 0.\end{aligned}\quad (\text{A.29})$$

A.1.2 Rotation about the z-axis

The voltage induced in the coil after the rotation about the z-axis can be denoted by V_{zat} are given by the following equation:

$$\begin{aligned}
-V_{zat} &= \mathbf{B}_{zat} \cdot \mathbf{S}_{zat} = [g_x z - \frac{1}{2} g_z (x'c + y's + x_0)](S_x c + S_y s) \\
&\quad + [g_y z - \frac{1}{2} g_z (-x's + y'c + y_0)](S_y c - S_x s) \\
&\quad + [g_x (x'c + y's + x_0) + g_y (-x's + y'c + y_0) + g_z z] S_z \\
&= g_x [z(S_x c + S_y s) + S_z (x'c + y's)] + g_x x_0 S_z \\
&\quad + g_y [z(S_y c - S_x s) + S_z (-x's + y'c)] + g_y y_0 S_z \\
&\quad + \frac{1}{2} g_z [-(x'c + y's)(S_x c + S_y s) - (-x's + y'c)(S_y c - S_x s) + 2z S_z] \\
&\quad - \frac{1}{2} g_z [x_0(S_x c + S_y s) + y_0(S_y c - S_x s)],
\end{aligned} \tag{A.30}$$

where ϕ is the rotational angle. The change in induced voltage in the coil due to rotation about the z-axis can be represented by the equation-

$$\begin{aligned}
\delta V_z &= -V_{zat} + V = \dot{\mathbf{B}}_{zat} \cdot \mathbf{S}_{zat} - \dot{\mathbf{B}} \cdot \mathbf{S} \\
&= g_x [z(S_x c + S_y s) + S_z (x'c + y's) - z S_x - x' S_z] \\
&\quad + g_y [z(S_y c - S_x s) + S_z (-x's + y'c) - z S_y - y' S_z] \\
&\quad + \frac{1}{2} g_z [-(x'c + y's)(S_x c + S_y s) - (-x's + y'c)(S_y c - S_x s) \\
&\quad \quad (S_y c - S_x s) + 2z S_z + x' S_x + y' S_y - 2z S_z] \\
&\quad - \frac{1}{2} g_z [x_0(S_x c + S_y s) - x_0 S_x + y_0(S_y c - S_x s) - y_0 S_y].
\end{aligned} \tag{A.31}$$

To make calculation as simple as possible, let us consider the contribution of x-gradient:

$$\begin{aligned}
C_x &= g_x [z(S_x c + S_y s) + S_z (x'c + y's) - z S_x - x' S_z] \\
&= g_x (z S_x c + z S_y s + x' S_z c + y' S_z s - z S_x - x' S_z) \\
&= g_x [z S_x (c - 1) + x' S_z (c - 1) + (z S_y + y' S_z) s].
\end{aligned} \tag{A.32}$$

By replacing $c = \cos \phi$ and $s = \sin \phi$, one obtain

$$\begin{aligned}
C_x &= g_x[zS_x(\cos \phi_z - 1) + x'S_z(\cos \phi_z - 1) + (zS_y + y'S_z) \sin \phi_z] \\
&= g_x[-2zS_x \sin^2 \frac{\phi_z}{2} - 2xS_z \sin^2 \frac{\phi_z}{2} + (zS_y + y'S_z) \sin \phi_z] \\
&= g_x[-2(zS_x + x'S_z) \sin^2 \frac{\phi_z}{2} + (zS_y + y'S_z) \sin \phi_z].
\end{aligned} \tag{A.33}$$

Similarly, the contribution of y-gradient:

$$\begin{aligned}
C_y &= g_y[z(S_y c - S_x s) + S_z(-x' s + y' c) - zS_y - y'S_z] \\
&= g_y[zS_y c - zS_y s - x'S_z s + y'S_z c - zS_y - y'S_z] \\
&= g_y[zS_y(c - 1) + y'S_z(c - 1) - (zS_x + x'S_z)s].
\end{aligned} \tag{A.34}$$

By replacing $c = \cos \phi_z$ and $s = \sin \phi_z$, one obtain

$$\begin{aligned}
C_y &= g_y[zS_y(\cos \phi_z - 1) - yS_z(\cos \phi_z - 1) - (zS_x + xS_z) \sin \phi_z] \\
&= g_y[-2(zS_y + yS_z) \sin^2 \frac{\phi_z}{2} - (zS_x + xS_z) \sin \phi_z].
\end{aligned} \tag{A.35}$$

Now the contribution of z-gradient:

$$\begin{aligned}
C_z &= \frac{1}{2}g_z[-(x'c + y's)(S_x c + S_y s) \\
&\quad -(-x's + y'c)(S_y c - S_x s) + x'S_x + y'S_y] \\
&\quad - \frac{1}{2}g_z[x_0(S_x c + S_y s) - x_0 S_x + y_0(S_y c - S_x s) - y_0 S_y] \\
&= \frac{1}{2}g_z[-x'S_x c^2 - y'S_x c s - x'S_y s c - y'S_y s^2 + x'S_y s c - y'S_y c^2 - x'S_x s^2 \\
&\quad + y'S_x s c + x'S_x + y'S_y] \\
&\quad - \frac{1}{2}g_z[(x_0 S_x + y_0 S_y)(c - 1) + (x_0 S_y - y_0 S_x)s] \\
&= \frac{1}{2}g_z[-x'S_x(c^2 + s^2) - y'S_y(c^2 + s^2) + x'S_x + y'S_y] \\
&\quad - \frac{1}{2}g_z[(x_0 S_x + y_0 S_y)(c - 1) + (x_0 S_y - y_0 S_x)s].
\end{aligned} \tag{A.36}$$

By replacing $c = \cos \phi_z$ and $s = \sin \phi_z$, one obtain

$$\begin{aligned}
C_z &= \frac{1}{2}g_z[-x'S_x(\cos^2 \phi_z + \sin^2 \phi_z) - y'S_y(\cos^2 \phi_z + \sin^2 \phi_z) + x'S_x + y'S_y] \\
&\quad - \frac{1}{2}g_z[(x_0S_x + y_0S_y)(\cos \phi_z - 1) + (x_0S_y - y_0S_x) \sin \phi_z] \\
&= \frac{1}{2}g_z[-x'S_x(\cos^2 \phi_z + \sin^2 \phi_z) - y'S_y(\cos^2 \phi_z + \sin^2 \phi_z) + x'S_x + y'S_y] \quad (\text{A.37}) \\
&\quad + g_z(x_0S_x + y_0S_y) \sin^2 \phi_z - \frac{1}{2}g_z(x_0S_y - y_0S_x) \sin \phi_z \\
C_z &= +g_z(x_0S_x + y_0S_y) \sin^2 \phi_z - \frac{1}{2}g_z(x_0S_y - y_0S_x) \sin \phi_z.
\end{aligned}$$

The change in induced voltage due to the rotation about the z-axis is as follows:

$$\begin{aligned}
\delta V_z &= C_x + C_y + C_z \\
&= -g_x[-(z'S_y + y'S_z) \sin \phi_z + 2(z'S_x + xS_z) \sin^2 \frac{\phi_z}{2}] \\
&\quad - g_y[(z'S_x + x'S_z) \sin \phi_z + 2(z'S_y + y'S_z) \sin^2 \frac{\phi_z}{2}] \quad (\text{A.38}) \\
&\quad + g_z(x_0S_x + y_0S_y) \sin^2 \phi_z - \frac{1}{2}g_z(x_0S_y - y_0S_x) \sin \phi_z.
\end{aligned}$$

The change in induced voltage in the individual coil due to a rotation by an angle, ϕ_z , about the z-axis can be obtained by the given equations:

$$\begin{aligned}
\delta V_1 &= +g_x(y'S_z) \sin \phi_z + 2g_x(xS_z) \sin^2 \frac{\phi_z}{2} - g_y(x'S_z) \sin \phi_z + 2g_y(y'S_z) \sin^2 \frac{\phi_z}{2} \\
\delta V_2 &= -2g_x(z'S_x) \sin^2 \frac{\phi_z}{2} - g_y(z'S_x) \sin \phi_z + g_z(x_0S_x) \sin^2 \phi_z + \frac{1}{2}g_z(y_0S_x) \sin \phi_z \\
\delta V_3 &= -2g_x(z'S_x) \sin^2 \frac{\phi_z}{2} - g_y(z'S_x) \sin \phi_z + g_z(x_0S_x) \sin^2 \phi_z + \frac{1}{2}g_z(y_0S_x) \sin \phi_z \quad (\text{A.39}) \\
\delta V_4 &= g_x(z'S_y) \sin \phi_z - 2g_y(z'S_y) \sin^2 \frac{\phi_z}{2} + g_z(y_0S_y) \sin^2 \phi_z - \frac{1}{2}g_z(x_0S_y) \sin \phi_z \\
\delta V_5 &= g_x(z'S_y) \sin \phi_z - 2g_y(z'S_y) \sin^2 \frac{\phi_z}{2} + g_z(y_0S_y) \sin^2 \phi_z - \frac{1}{2}g_z(x_0S_y) \sin \phi_z.
\end{aligned}$$

For a very small rotation angle ϕ_z , i.e. $\phi_z \leq 5^\circ$ one can write $\sin \phi_z = \phi_z$; as a result the general equation for the change induced voltage in the coils due to rotation about the z-axis

by angle $\delta\phi_z$ reduces to

$$\begin{aligned}
\delta V_1 &= +g_x(y'S_z)\delta\phi_z + \frac{1}{2}g_x(xS_z)\delta\phi_z^2 - g_y(x'S_z)\delta\phi_z + \frac{1}{2}g_y(y'S_z)\delta\phi_z^2 \\
\delta V_2 &= -\frac{1}{2}g_x(z'S_x)\delta\phi_z^2 - g_y(z'S_x)\delta\phi_z + g_z(x_0S_x)\delta\phi_z^2 + \frac{1}{2}g_z(y_0S_x)\delta\phi_z \\
\delta V_3 &= -\frac{1}{2}g_x(z'S_x)\delta\phi_z^2 - g_y(z'S_x)\delta\phi_z + g_z(x_0S_x)\delta\phi_z^2 + \frac{1}{2}g_z(y_0S_x)\delta\phi_z \\
\delta V_4 &= g_x(z'S_y)\delta\phi_z - \frac{1}{2}g_y(z'S_y)\delta\phi_z^2 + g_z(y_0S_y)\delta\phi_z^2 - \frac{1}{2}g_z(x_0S_y)\delta\phi_z \\
\delta V_5 &= g_x(z'S_y)\delta\phi_z - \frac{1}{2}g_y(z'S_y)\delta\phi_z^2 + g_z(y_0S_y)\delta\phi_z^2 - \frac{1}{2}g_z(x_0S_y)\delta\phi_z.
\end{aligned} \tag{A.40}$$

The linear terms dominate in the expressions for the change in induced voltages in the coils. So one can exclude the non-linear terms,

$$\begin{aligned}
\delta V_1 &= +g_x(y'S_z)\delta\phi_z - g_y(x'S_z)\delta\phi_z \\
\delta V_2 &= -g_y(z'S_x)\delta\phi_z + \frac{1}{2}g_z(y_0S_x)\delta\phi_z, \quad \delta V_3 = -g_y(z'S_x)\delta\phi_z + \frac{1}{2}g_z(y_0S_x)\delta\phi_z \\
\delta V_4 &= g_x(z'S_y)\delta\phi_z - \frac{1}{2}g_z(x_0S_y)\delta\phi_z, \quad \delta V_5 = g_x(z'S_y)\delta\phi_z - \frac{1}{2}g_z(x_0S_y)\delta\phi_z.
\end{aligned} \tag{A.41}$$

In this case, in Fig. 3.2, the system of five coils one take only zS_z , xS_x and yS_y , but there is no such term in the above expression. So there is no change in induced voltage in any the coils of our system due to rotation about z-axis. However, the change in induced voltage in the coils for our defined system (deviated from the axes) due to the rotation about z-axis are as follows:

$$\delta V_z = +g_z(x_0S_x + y_0S_y) \sin^2 \phi_z - \frac{1}{2}g_z(x_0S_y - y_0S_x) \sin \phi_z \tag{A.42}$$

The change in induced voltage in the individual coil due to rotation about the z-axis can then be obtained by the given equations:

$$\begin{aligned}
\delta V_1 &= 0 \\
\delta V_2 &= +g_z(x_0S_x) \sin^2 \phi_z \quad \delta V_3 = +g_z(x_0S_x) \sin^2 \phi_z \\
\delta V_4 &= +g_z(y_0S_y) \sin^2 \phi_z \quad \delta V_5 = +g_z(y_0S_y) \sin^2 \phi_z.
\end{aligned}$$

For a very small rotation angle ϕ_z , i.e. $\phi_z \leq 5^\circ$ one can write $\sin \phi_z = \phi_z$ (in radians); as a result the general equation for the change induced voltage due to the rotation about z-axis by angle $\delta\phi_z$ reduces to

$$\delta V_z = +g_z(x_0 S_x + y_0 S_y)\delta\phi_z^2 - \frac{1}{2}g_z(x_0 S_y - y_0 S_x)\delta\phi_z. \quad (\text{A.43})$$

In the case rotation about the z-axis and a very small rotation angle, the change in the induced voltage in the five coils are consequently given as follows:

$$\begin{aligned} \delta V_1 &= 0 \\ \delta V_2 &= +g_z(x_0 S_x)\delta\phi_z^2 & \delta V_3 &= +g_z(x_0 S_x)\delta\phi_z^2 \\ \delta V_4 &= +g_z(y_0 S_y)\delta\phi_z^2 & \delta V_5 &= +g_z(y_0 S_y)\delta\phi_z^2. \end{aligned}$$

Research Works

Conference Papers:

1. E. H. Bhuiyan, G. S. Spencer, P. M. Glover, and R. Bowtell; Tracking head movement inside an MR scanner using voltages induced in coils by time-varying gradients; Proc. Intl. Soc. Mag. Reson. Med. 25, 0965 (2017).
2. E. H. Bhuiyan, M. E. H. Chowdhury, P. M. Glover, and R. Bowtell; Movement Monitoring for MRI via Measurement of Changes in the Gradient Induced EMF in Coil Arrays, Proc. Intl. Soc. Mag. Reson. Med. 23, 1017 (2015).

Conferences and Contributed works:

1. E. H. Bhuiyan, G. S. Spencer, P. M. Glover, and R. Bowtell; Measuring position inside an MR scanner using voltages induced in coils by time-varying gradients; ISMRM British Chapter, The university of Leeds, 7-9 September (2016).
2. E. H. Bhuiyan, G. Presenza-Pitman, and M. Azzouz; Study the Optical Conductivity of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Material, National conference on Physics for development, Bangladesh University of Engineering & Technology, Dhaka, Bangladesh, (SSIT-II 02), 10-11 February (2011).
3. E. H. Bhuiyan and M. Azzouz, "Study the optical response of high-Tc superconducting materials within the rotating antiferromagnetism theory". Presented on Graduate

Student's Symposium, Laurentian University, Sudbury, ON, Canada, February 24-25 (2010).

Journals:

1. E. H. Bhuiyan and M. Azzouz; The Pseudogap and the Unusual Excitations in the Optical Conductivity of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Material; Physica C (2017) (Submitted).
2. E. H. Bhuiyan, M. M. Rahman and A. Z. Ziauddin Ahmed; Unusual Excitation in the Optical Conductivity of the $\text{YBa}_2\text{Cu}_3\text{O}_7$ Material, Bangladesh Journal of Physics, Vol. 19; (2016).
3. E. H. Bhuiyan, G. Presenza-Pitman and M. Azzouz, Study of the Optical Conductivity of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Material, Physica C 473, 6170 (2012) doi:10.1016/j.physc.2011.11.003.
4. E. H. Bhuiyan and M. Azzouz; Quantum Criticality in the High-Tc Cuprate Materials, Bangladesh Journal of Physics, 9 & 10, 117-127, (2010-2011)
5. E. H. Bhuiyan, A. Z. Ziauddin Ahmed and G. M. Bhuiyan and M. Shahjahan, Atomic transport properties of $\text{Ag}_x\text{Sn}_{1-x}$ liquid binary alloys; Physica B 403, 1695 (2008) doi:10.1016/j.physb.2007.09.090.

Book:

1. Optical Conductivity by Enamul Hoque Bhuiyan, LAP Lambert Academic Publishing, October, 2012, ISBN 978-3-659-26232-6, paperback, 136 Pages.

Thesis:

1. The pseudogap signature in the optical conductivity of the high-Tc cuprate materials by Bhuiyan, Enamul Hoque, Thesis (M. Sc.)-Laurentian University (Canada), 2010;

Publication Number: AAIMR65838; ISBN: 9780494658383; Source: Masters Abstracts
International, Volume: 49-02, page; 113 p.