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THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Detailed Sizing Optimisation of Industrial-Scale Composite Aeronautical Structures

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Declaration

I confirm that this thesis presented for the degree of Doctor of Philosophy, has

- i) been composed entirely by myself
- ii) been solely the result of my own work except stated otherwise
- iii) not been submitted for any other degree or professional qualification

Signed: Georgios Ntourmas

Date: Saturday 4th March, 2023

Dedication

Αφιερωμένη σε εσάς που μ' αγαπάτε.

Abstract

Carbon fibre reinforced polymers have been increasingly used in the aerospace industry over the last decades, because they offer reduced weight and enhanced mechanical characteristics compared to classic metallic structures. However, their application is also linked with several challenges concerning their design and optimisation process. One of the most widely studied topics is the detailed sizing optimisation of variable stiffness composite structures. This task involves determining the optimal thickness and stiffness distribution across the structure and translating that into a manufacturable stacking sequence design. In this study, uni-directional carbon fibre laminae are assumed to be used during manufacturing and therefore the stiffness variation across the span of a composite component is a result of multiple interconnected constant stiffness zones called patches. The task of the detailed design of a composite structure involves the computation of discrete stacking sequences satisfying both mechanical constraints, such as stability and strength, but also design and manufacturing guidelines commonly employed in the aerospace industry. When dealing with industrial-scale structures this optimisation is commonly split up in one gradient and one non-gradient based stage, in order to tackle the challenges of the task. These challenges are associated with mixed continuous and discrete design variables and constraints and the high computational cost of the mechanical constraints. A two stage-optimisation process has also been developed in the framework of this thesis.

In the first, gradient-based optimisation stage of the process, generic stacks are employed to model the thickness and stiffness distribution of the structure. A generic stack is comprised of a collection of layers whose orientations are fixed *a priori*, but thicknesses can vary independently enabling exploration of the design space. The result of this first stage is a continuous thickness and stiffness distribution. All mechanical constraints must be considered in the first stage and therefore the converged optimisation result satisfies these constraints. Before moving to the second stage of the optimisation, the total thickness of each patch is rounded up to the nearest number of manufacturable plies, which can be easily derived since

the thickness of the pre-impregnated uni-directional tape that will be used during manufacturing is known. In the second stage of the optimisation, the thickness distribution of the structure remains constant and the task is to retrieve stacking sequences which fulfil a set of predefined composite design and manufacturing rules, with blending or continuity between neighbouring patches being one of the most crucial ones. The objective function of this optimisation is to minimise the absolute difference between the stiffness of the continuous design computed in the first stage and the stiffness of the discrete candidate design in the second stage. The problem of retrieving layered designs that meet all of the prescribed composite rules is formulated as two Mixed Integer Linear Programming instances which mainly differ in how the blending is treated. Using either of the two formulations, mathematical programming algorithms can be employed to solve the problem to global optimality.

For the two-stage process to be successful, all constraints from both stages should be fulfilled. The main challenge is retrieving stacking sequences which satisfy the mechanical constraints of the problem, since these are not explicitly considered in the second stage of the optimisation but rather depend on the quality of the achieved stiffness match. To that extent, introducing as many composite design and manufacturing rules as possible in the first stage of the optimisation is of crucial importance in order to bridge the information gap between the continuous and discrete solution. This can be challenging due to the discrete nature of these constraints and the need to implement them in a gradient-based framework. Fortunately, the design space of the generic stacks allows multiple composite rules to be accurately formulated, which is not the case for alternative stiffness parametrisation techniques.

Overall, when applied to an academic demonstrator and compared to relevant studies, the developed approach is able to retrieve a design with a lower structural mass, while also satisfying all mechanical constraints. What is more, this can be achieved in only one iteration of the two-stage process. This result is a combination of having accurately formulated blending constraints in the first stage of the optimisation and of using a problem formulation in the second stage which does not limit the design freedom and is solved using a deterministic optimisation algorithm able to provide a globally optimum solution. The two-stage process has also been applied to the design of the composite wing skins of a large-scale unmanned aircraft. Both the first and second stage of the optimisation can efficiently handle the computational complexity of large case studies. For the second stage of the optimisation, a decomposition technique has been developed in order to derive good quality solutions in a significantly reduced time frame. In the case of the large-scale aircraft, minor constraint violations occur when evaluating the discrete design. These can be

conveniently handled by a repetition of the continuous optimisation, while keeping the discretised stacking sequences fixed. The optimiser can easily satisfy all remaining violations by a slight modification of the stiffness properties of other sub-components, such as the stringers, whose stiffness characteristics can be easily manipulated in the continuous domain. Overall, the weight penalty introduced between the continuous and discrete result which satisfies all constraints is approximately 2 Kg.

Finally, the work of the thesis has been extended towards the field of design for manufacturing. Automatic Fibre Placement manufacturing processes have become the aerospace industry standard for the production of large-scale composite components. The aspect of the layup time required to manufacture a composite component is introduced as an objective function in the first stage of the detailed sizing process. The methodology developed is able to identify how the material is going to be laid on the tool. The method is applied to the skin of the aforementioned aircraft wing and a trade-off between the structural weight and layup time is observed. Results demonstrate that the bi-objective optimisation is a promising tool for reducing the structural mass, while keeping the layup time to acceptable levels by benefiting from a more detailed structural modelling.

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List of Acronyms

AFP	Automated Fibre Placement
ATL	Automated Tape Laying
BCP	Bi-value Coding Parametrisation
BFS	Breadth First Search
CFRP	Carbon Fibre Reinforced Plastics
CLT	Classical Lamination Theory
DFS	Depth First Search
DMO	Discrete Material Optimisation
DMTO	Discrete Material and Thickness Optimisation
FE	Finite Element
FEM	Finite Element Model
GFEM	Global Finite Element Model
LP	Linear Programming
MALE	Medium Altitude Long Endurance
MDO	Multidisciplinary Design Optimisation
MILP	Mixed Integer Linear Programming
MIP	Mixed Integer Programming
MLW	Maximum Landing Weight
MTOW	Maximum Take-Off Weight
MZWFW	Maximum Zero Wing Fuel Weight
NLP	Nonlinear Programming
OptiMACS	Optimisation of Multifunctional Aerospace Composite Structures
RF	Reserve Factor
RRF	Reverse Reserve Factor
SF	Safety Factor
SFP	Shape Functions with Penalisation
SST	Stacking Sequence Table

Introduction

1.1 Composite materials in the aviation industry

Humans have always been fascinated by the idea of flying. Kites were first invented in Asia more than two thousand years ago, while the first sustained, controlled and powered flight in a heavier-than-air machine is attributed to the Wright brothers in 1903. The materials used for the construction of airplanes have since then transitioned from wood, canvas and wires to aluminium alloys which provided more strength and durability. Over the last decades, composite materials have been increasingly used in aircraft structures since they offer reduced weight and enhanced mechanical characteristics.

A composite material is comprised of at least two different materials which are combined to form a third one. Usually, the properties of each constituent material forming the composite one are dissimilar, leading to the unique properties of the latter. The earliest usage of composite materials likely occurred in the context of building construction with the utilisation of straw to reinforce mud bricks [9]. Composite materials, such as concrete, have been widely employed in the construction industry for centuries. In the aviation industry, glass fibre reinforced plastics were first used in the radomes of the aircraft. Nowadays, advanced composite materials such as Carbon Fibre Reinforced Polymers (CFRP) are actively used in the aerospace industry.

During the last decades, composite materials comprise a continuously increasing percentage of the aircraft mass, as seen in Fig. 1.1. Their application in the military aircraft sector preceded that of the civil aircraft, with the F-15 being among the first to employ boron reinforced epoxy on the skins of the empennage. Amongst the civil aircraft, the Airbus A310 was the first airliner to employ a CFRP material on the primary aircraft structure of the vertical tail plane. The usage of composite materials has increased since the A310, currently comprising half of the structural weight of the Airbus A350 and the Boeing 787.

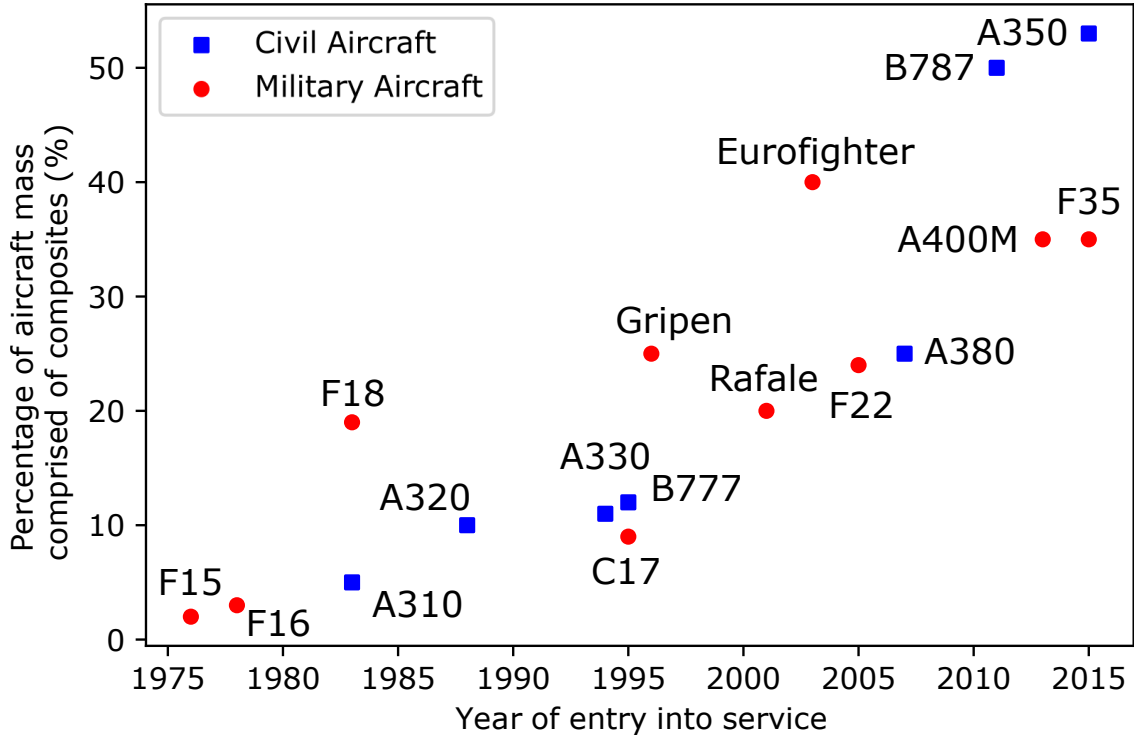


Figure 1.1: Percentage of aircraft mass comprised of composite materials over the course of the last decades, in the civil and military aircraft sector.

The reason for the drastic increase in the usage of composite materials has been the many benefits they offer over the traditionally used metallic materials. Amongst their qualities are the increased strength and stiffness, in combination with a lower weight and increased corrosion resistance. On the contrary, one of their drawbacks is the high cost of the raw materials and the manufacturing processes. This is also the reason why the usage of composite materials, such as CFRP, has been limited to few industries including aviation, high performance automotive and professional sports equipment.

The weight reduction which can be achieved by composite structures has been the main driving force behind their ever increasing adaptation in the aviation industry. According to Soutis [10], a 20% and 40% reduction in weight can be

achieved for primary and secondary aircraft structures respectively, when using CFRP instead of light metal alloys. Indeed, the usage of CFRP has the potential to reduce aviation carbon dioxide emissions by 15% [11], which is a necessary step towards the European Union’s goals of net-zero greenhouse gas emissions by 2050 [12]. The aforementioned reduction takes into consideration the increased emissions produced by CFRP manufacturing processes and assumes no recycling of thermoset composites.

A CFRP composite material is comprised of two different materials: the matrix, which in this case is a polymer and the fibres, made from graphite, which is a crystalline form of carbon. Fibres, which provide the strength and stiffness of the composite, are usually coated in a polymer epoxy matrix which binds the fibres together. This practice is called preimpregnation and the resulting preimpregnated fibres are called “prepregs”. The composition of prepregs is usually around 60% fibres and 40% epoxy. Prepregs used in the aviation industry mainly consist of unidirectional fibres available in the form of a tape. Alternatively, fibres may also be interwoven and preimpregnated and used in the form of a sheet.

For the scope of this work, the elements used to create the structure are unidirectional prepreg tapes of material. A unidirectional tape of materials is anisotropic, meaning that the mechanical characteristics vary between its longitudinal and transverse direction. The stiffness is of course significantly larger across the direction of the fibres when compared to the transverse direction. Multiple tapes of material can be placed in parallel to cover the entire span of a component, forming a single layer or lamina. The thickness of the prepreg tape used during manufacturing and therefore the thickness of a single layer, can be chosen based on the available products of the material supplier and usually ranges between 0.1 and 0.5 mm. Multiple layers are then placed or stacked on top of each other forming a laminate. Within each of these layers the unidirectional tapes can be placed at different orientations compared to a reference direction, as seen in Fig. 1.2a, in order to achieve the desired mechanical characteristics of the laminate. Therefore, by determining the optimal number of layers and the fibre orientation for each layer, the laminate can be tailored to meet the desired thickness and stiffness properties.

The main scope of this thesis is the determination of these optimal thickness and stiffness characteristics, or in other words the stacking sequence optimisation of a laminate. This task becomes increasingly challenging for industrial-scale aeronautical components because these are formed by multiple inter-connected laminates as seen in Fig. 1.2b. In order to derive the optimal thickness and stiffness distribution

across the span of the structure, each component is modelled using multiple constant stiffness zones which will be referred to as patches.

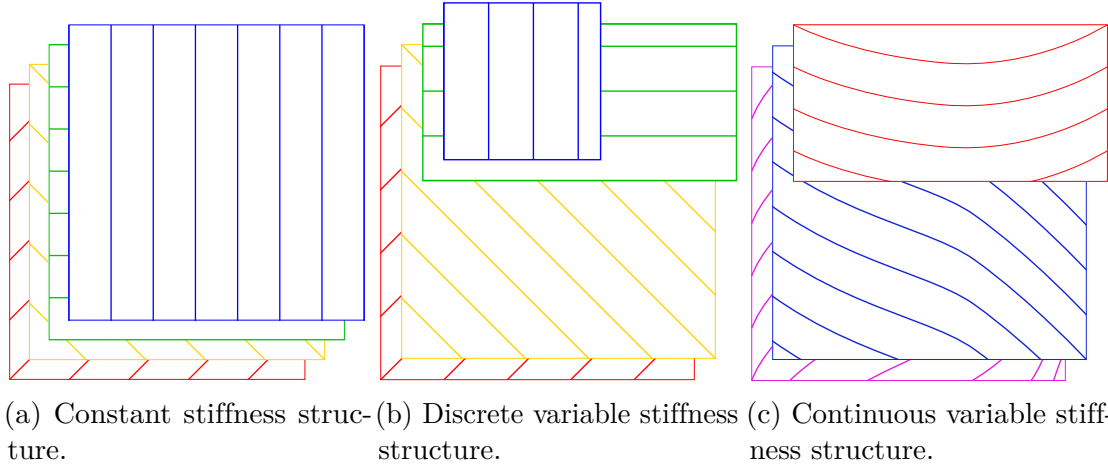


Figure 1.2: Illustration of constant and variable stiffness laminated structures. Stiffness variations can be achieved by introducing multiple patches, fibre steered laminates or a combination of the two. Figure adapted from [1].

On the side of manufacturing, various processes exist to fabricate CFRP components. These range from hand layup for interwoven fabrics, to automated tape placement and filament winding. In the aviation industry, large scale primary structures, such as the fuselage and wing skins, are manufactured using Automated Tape Laying (ATL) processes and more recently Automated Fibre Placement (AFP) processes. As seen in Fig. 1.3, the prepreg material is heated before being compacted by a roller on the surface of the mould of the component. The width of the tape loaded on the head of the ATL machine varies from 75 to 300 mm. The width chosen depends on the curvature of the manufactured part, with narrower tapes used for larger curvatures.

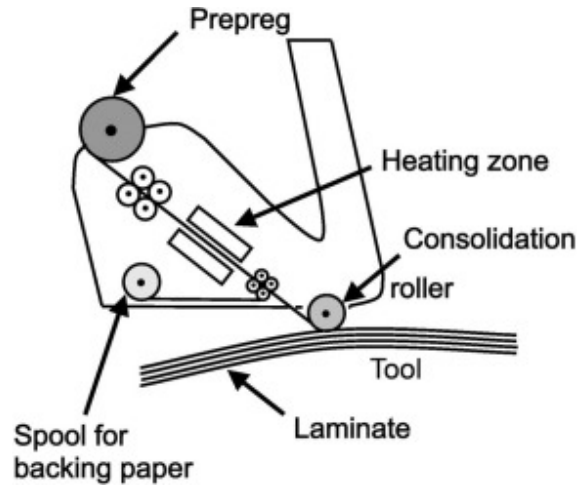


Figure 1.3: Representation of the head of the ATL machine [2].

The main difference between an ATL and an AFP manufacturing process is that, instead of a tape, the AFP head is fed a tow of material which is comprised of multiple strips i.e. narrower individual tapes. This allows for steering of the fibres across a laminate, which subsequently enables additional tailoring of the stiffness characteristics. An example of that can be seen in Fig. 1.2c. In this work, only straight fibre laminates are examined and the stiffness distribution across the structure is achieved by dropping or adding individual plies across the patches. Nonetheless, AFP processes are used in the industry even for constructing straight fibre laminates because they enable the manufacturing of structures with greater geometric complexities, while the material waste is smaller compared to that of ATL.

1.2 Context of the thesis

The work presented in this thesis has been conducted in the framework of the Optimisation of Multi-functional Aerospace Composite Structures (OptiMACS) European project. The aim of the project is to produce scientific and technological breakthroughs towards a Multidisciplinary Design Optimisation (MDO) methodology of enhanced efficiency and robustness for aerospace composite structures. The research contributions are to be incorporated within the existing in-house MDO platform employed in Airbus Defence and Space called *Lagrange* [13]. *Lagrange* has been actively developed over the last 4 decades and incorporates its own linear Finite Element (FE) solver.

Lagrange supports different kind of analyses including linear statics, linear dynamics, linear stability, and steady or unsteady aeroelastics. Different design parameters such as cross sections, shell element thicknesses, shape variables and in the case of steady aeroelastic analyses, trim variables, i.e. angles of attack of the aircraft and control surfaces, can be employed during an optimisation. A wide range of multidisciplinary requirements can be considered such as strength, stability, maximum displacements, natural frequencies, flutter speeds and basic manufacturing constraints for composites. The objective of an optimisation study can be the minimisation of the structural weight, the maximisation of the lift to drag ratio or other goals related to the mission performance of the aircraft.

The work of the early stage researchers in the OptiMACS project covers the introduction of: extended invariant failure criteria [14], predictive capabilities for process-induced shape distortions [15], new global-local strategies for non-regular structural areas [16] and interfaces for automated airframe evaluations in early

design stages. The work performed in this thesis aims to develop a robust optimisation framework for the stacking sequence optimisation of industrial-sized laminated structures.

1.3 Research questions and contributions

In summary, the research questions dealt with in this thesis are as follows:

1. Can a computationally efficient optimisation framework be setup for the detailed sizing of laminated composite structures when working with industrial scaled problems?
2. Can a detailed design fulfilling all requirements be achieved without significantly penalising the structural weight of the preliminary continuous design in an attempt to do so?
3. Can manual interactions between the two optimisation stages and reiterations of the design process be eliminated?
4. Can manufacturing requirements not normally considered during a detailed sizing optimisation, such as the minimisation of the layup time of the laminated structure, be included in the optimisation process?

The main contributions of this thesis are summarised as follows:

1. A maximum number of discrete composite design guidelines are introduced in the framework of a gradient-based optimisation stage which is part of a two stage optimisation process.
2. Two novel formulations of the stacking sequence optimisation are presented as part of the second stage of the optimisation process.
3. The developed process outperforms relevant works on an academic demonstrator.
4. The process can efficiently handle large-scale industrial problems, while minimising the reiterations of the design process and the weight penalisation inflicted upon the structure.
5. The layup time for an AFP manufacturing process is introduced as an objective function in the first stage of the optimisation, allowing trade-off analyses to be performed.

1.4 Publications

Below is the list of articles published or currently under review in peer reviewed journals and the conferences linked with the work performed in this thesis.

1.4.1 Articles published in international peer reviewed journals

- [1] **G. Ntourmas**, F. Glock, F. Daoud, G. Schuhmacher, D. Chronopoulos, E. Özcan, Generic stacks and application of composite rules for the detailed sizing of laminated structures, *Composite Structures* 276 (2021) 114487.
- [2] **G. Ntourmas**, F. Glock, F. Daoud, G. Schuhmacher, D. Chronopoulos, E. Özcan, Mixed Integer Linear Programming formulations of the stacking sequence and blending optimisation of composite structures, *Composite Structures* 264 (2021) 113660.
- [3] **G. Ntourmas**, F. Glock, S. Deinert, F. Daoud, G. Schuhmacher, D. Chronopoulos, E. Özcan, J. Ninić, Stacking sequence optimisation of an aircraft wing skin, *Structural and Multidisciplinary Optimization* 66 31 (2023).
- [4] G. Corrado, **G. Ntourmas**, M. Sferza, N. Traiforos, A. Arteiro, L. Brown, D. Chronopoulos, F. Daoud, F. Glock, J. Ninić, E. Özcan, J. Reinoso, G. Schuhmacher, T. Turner, Recent progress, challenges and outlook for multidisciplinary structural optimization of aircraft and aerial vehicles, *Progress in Aerospace Sciences* 135 (2022) 100861.

1.4.2 Articles under review

- [1] **G. Ntourmas**, F. Glock, F. Daoud, G. Schuhmacher, D. Chronopoulos, E. Özcan, J. Ninić, Layup time for an Automated Fibre Placement process in the framework of a detailed sizing optimisation, *Composites Part B: Engineering*.

1.4.3 International conferences

- [1] **G. Ntourmas**, E. Özcan, D. Chronopoulos, F. Glock, F. Daoud, Towards a streamlined stacking sequence optimisation methodology for blended composite aircraft structures, 8th European Conference for Aeronautics and Aerospace Sciences (EUCASS), 2019.
- [2] **G. Ntourmas**, F. Glock, F. Daoud, G. Schuhmacher, D. Chronopoulos, E. Özcan, J. Ninić Bi-objective manufacturing time and weight stacking sequence optimisation for an Automated Fibre Placement Process, *UKACM 2022 Conference*, 2022.

1.5 Structure of the Thesis

The thesis is structured as follows: In Chapter 2, a basic theoretical background is established for composite structures and optimisation problems. A review of the literature relevant to tackling the detailed sizing of composite structures is also presented. In Chapter 3, the first stage of the optimisation employing a gradient-based algorithm is outlined. Generic stacks which are used to model the thickness and stiffness distribution of the structure are introduced, together with the formulation of mechanical and composite constraints in the optimisation problem. In Chapter 4, two mathematical formulations of the blended stacking sequence retrieval are introduced as part of the second stage of the optimisation process and a decomposition technique of the problem is illustrated. The second stage of the optimisation is compared against relevant research works and results of the entire two-stage process are presented for an academic demonstrator. In Chapter 5, the efficiency and capabilities of the two-stage optimisation process are demonstrated on an industrial-size problem. In Chapter 6, the layup time required to manufacture a component using an AFP process is formulated as an objective function in the first stage of the optimisation. Finally, in Chapter 7, the findings of the work are summarised and potential future work is considered.

Background

In this chapter, the background for the work performed in the thesis is presented. More specifically, in section 2.1, an introduction to the mechanics of composites structures is presented along with commonly employed design and manufacturing guidelines for composite structures. In section 2.2, an introduction to the fundamentals of optimisation problems along with various classifications and algorithms are presented. In section 2.3, the challenges of the stacking sequence optimisation problem are explained and the many different approaches used to tackle it are discussed. Finally, in section 2.4 the methodology developed is outlined and the main contributions are summarised.

2.1 Composite structures

The design of composite structures entails the identification of its optimal characteristics. This includes the recovery of the optimal number of layers and their ideal stacking sequence i.e. unidirectional fibre orientation and order permutation of every lamina within each of the multiple interconnected laminates. In this section, the theoretical background used to link these laminate characteristics with the responses of the structure is presented. Common guidelines usually applied in the aerospace industry when designing laminated structures are also described.

2.1.1 Mechanics of composite laminates

In this section, the stress and strain relation is first introduced for a single lamina and subsequently Classical Lamination Theory (CLT) used to model the load and deformation behaviour of a laminate is explained.

2.1.1.1 Behaviour of a single lamina

Composite materials are anisotropic in nature and the relation between the stresses and strains can be presented using generalised Hooke's law [9]

$$\sigma_i = C_{ij}\varepsilon_j \quad i, j \in [1, 6] \quad (2.1)$$

where σ_i are the stresses illustrated on the three dimensional element of Fig. 2.1, C_{ij} is the stiffness matrix and ε_j are the strain components. The stiffness matrix C_{ij} of an anisotropic material is comprised of 36 constants. In fibre reinforced materials three mutually orthogonal planes of material symmetry exist and therefore they are characterised as orthotropic. For orthotropic materials, the stress strain relations are described by only 9 elastic coefficients in the stiffness matrix as

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix}. \quad (2.2)$$

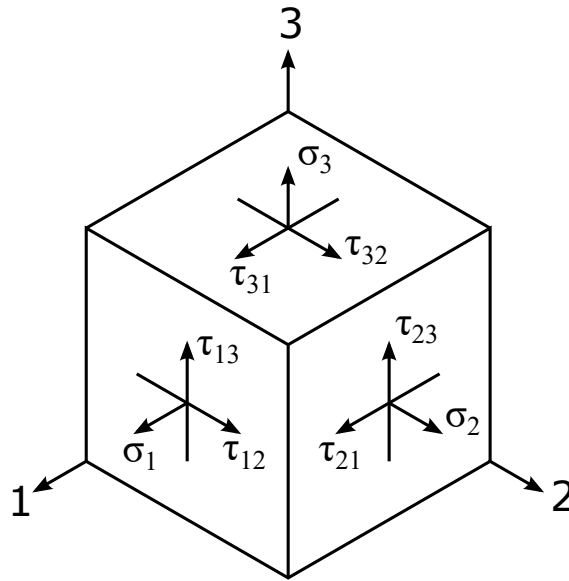


Figure 2.1: Stresses applied on a three dimensional element.

Assuming that a unidirectional lamina is only subjected to in-plane stresses, which is common for thin-walled structures like the ones used in aircraft, Eq. 2.2 can be further simplified to

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}. \quad (2.3)$$

where Q_{ij} are the plane stress reduced stiffnesses, which can be derived by using the plane stress conditions i.e. $\sigma_3 = 0$, $\gamma_{23} = \gamma_{13} = 0$ to simplify Eq. 2.2 and are given by:

$$Q_{ij} = C_{ij} - \frac{C_{i3}C_{j3}}{C_{33}} \quad i, j \in \{1, 2, 6\}. \quad (2.4)$$

The reduced stiffnesses can also be expressed in terms of the longitudinal modulus E_1 , the transverse modulus E_2 , the shear modulus G_{12} and the Poisson's ratios ν_{12} and ν_{21} of the material.

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{66} = G_{12} \quad (2.5)$$

The stress and strain relation for a plane stress condition shown in Eq. 2.3 is defined for the principal material coordinates (1, 2, 3) of the lamina. In general, these do not coincide with the global coordinate system (x, y, z) of a given problem. Therefore it is necessary to transform the stress-strain relations from one coordinate system to the other. The in-plane stress in the global coordinate system can be calculated as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [\mathbf{T}]^{-1} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \quad (2.6)$$

and the in plane strain as:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{\gamma_{xy}}{2} \end{Bmatrix} = [\mathbf{T}]^{-1} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{\gamma_{12}}{2} \end{Bmatrix} \quad (2.7)$$

where the transformation matrix $[\mathbf{T}]$ is defined as:

$$\mathbf{T} = \begin{bmatrix} \cos^2 \theta & \sin^2 \theta & 2 \cos \theta \sin \theta \\ \sin^2 \theta & \cos^2 \theta & -2 \cos \theta \sin \theta \\ -\cos \theta \sin \theta & \cos \theta \sin \theta & \cos^2 \theta - \sin^2 \theta \end{bmatrix} \quad (2.8)$$

with θ being the angle between the x-axis and the 1-axis. Consequently, the transformed stiffness matrix $[\bar{\mathbf{Q}}]$ is expressed as:

$$[\bar{\mathbf{Q}}] = [\mathbf{T}]^{-1} [\mathbf{Q}] [\mathbf{T}]^{-T}. \quad (2.9)$$

The terms $\bar{Q}_{16}$ and $\bar{Q}_{26}$ of the transformed stiffness matrix are non-zero for rotations $\theta \neq 0^\circ, 90^\circ$ and therefore the lamina appears to be anisotropic in the global coordinate system. However, because the lamina can still be expressed as an orthotropic one in the principal coordinate system, it is called generally orthotropic. The terms of the transformed stiffness matrix can be expressed so that parts of their formulation are invariant to rotations around the z-axis, based on the work of Tsai and Pagano [17] as:

$$\begin{aligned}
 \bar{Q}_{11} &= U_1 + U_2 \cos 2\theta + U_3 \cos 4\theta \\
 \bar{Q}_{12} &= U_4 - U_3 \cos 4\theta \\
 \bar{Q}_{22} &= U_1 - U_2 \cos 2\theta + U_3 \cos 4\theta \\
 \bar{Q}_{16} &= \frac{1}{2}U_2 \sin 2\theta + U_3 \sin 4\theta \\
 \bar{Q}_{26} &= \frac{1}{2}U_2 \sin 2\theta - U_3 \sin 4\theta \\
 \bar{Q}_{66} &= U_5 - U_3 \cos 4\theta
 \end{aligned} \tag{2.10}$$

where the terms U_i are the material constants defined as:

$$\begin{aligned}
 U_1 &= \frac{3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}}{8} \\
 U_2 &= \frac{Q_{11} - Q_{22}}{2} \\
 U_3 &= \frac{Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}}{8} \\
 U_4 &= \frac{Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}}{8} \\
 U_5 &= \frac{Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}}{8}
 \end{aligned} \tag{2.11}$$

2.1.1.2 Behaviour of a laminate

A laminate is formed by placing multiple laminae one on top of the other each at a specified angle θ . This allows the engineer to achieve adequate stiffness across multiple directions based on the requirements of a structure. In the framework of CLT, several hypotheses are being made in order to model the load and deformation behaviour of the laminate. First of all, the laminate is presumed to consist of perfectly bonded layers which are non-shear deformable. Additionally, the thickness of the laminate is assumed to be significantly smaller compared to its width and length. Finally, the Kirchhoff–Love hypothesis for shells is made which assumes that sections normal to the mid-surface of the laminate remain normal after bending. The mid-surface of a laminate with N layers is shown in Fig. 2.2 and serves as the reference plane for measuring the exact position of each layer.

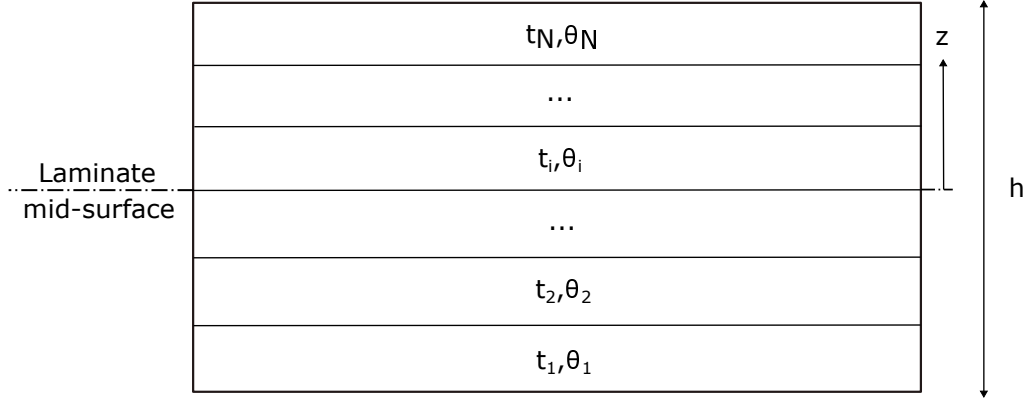


Figure 2.2: Illustration of a laminate consisting of N layers, each with a thickness t_i , fibre orientation θ_i and distance from the mid-surface z_i .

The stress strain relation for a plane stress state for the k^{th} layer of the laminate is given by:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x^o + z\kappa_x \\ \varepsilon_y^o + z\kappa_y \\ \gamma_{xy}^o + z\kappa_{xy} \end{Bmatrix} \quad (2.12)$$

where $\varepsilon_x^o, \varepsilon_y^o, \varepsilon_{xy}^o$ are the mid-surface strains and $\kappa_x, \kappa_y, \kappa_{xy}$ are the mid-surface curvatures. The strain varies linearly across the thickness of the stack but since the reduced stiffness matrix is different depending on the fibre orientation, the stress distribution across the stack is piecewise linear. In order to describe the behaviour of the plate in a more practical way, the stresses can be integrated over the thickness of the laminate to obtain the force $\mathbf{N}$ and moment $\mathbf{M}$ resultants

$$\begin{aligned} N_x &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x dz, \quad N_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y dz, \quad N_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} dz \\ M_x &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x z dz, \quad M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y z dz, \quad M_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} z dz. \end{aligned} \quad (2.13)$$

If the stress and strain relation of Eq. 2.12 is substituted to the above definitions, then the force and moment resultants take the form of

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (2.14)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (2.15)$$

where

$$\begin{aligned}
 A_{ij} &= \sum_{k=1}^N \left(\bar{Q}_{ij} \right)_k (z_k - z_{k-1}) \\
 B_{ij} &= \frac{1}{2} \sum_{k=1}^N \left(\bar{Q}_{ij} \right)_k (z_k^2 - z_{k-1}^2) \\
 D_{ij} &= \frac{1}{3} \sum_{k=1}^N \left(\bar{Q}_{ij} \right)_k (z_k^3 - z_{k-1}^3).
 \end{aligned} \tag{2.16}$$

In the equation above, $\mathbf{A}$ is the extensional stiffness matrix, $\mathbf{B}$ is the coupling stiffness matrix and $\mathbf{D}$ is the bending stiffness matrix. The so called ABD stiffness matrix contains a lot of information about the characteristics of a laminate. The terms A_{16} and A_{26} couple the shear and extension of the laminate, while the terms D_{16} and D_{26} couple bending and twisting. Finally, matrix $\mathbf{B}$ is responsible for the coupling between bending and extension.

2.1.2 Composite design guidelines

Some of the coupling behaviours, like bend and twist, are useful for multiple applications such as aeroelastic tailoring of aircraft wings [18, 19], wind turbine blades [20] and of race car spoilers [21]. However, in the aerospace industry, composite components are usually designed to fulfil several composite guidelines [22]. These guidelines are utilised for manufacturing laminates which are less prone to high stress concentration and unwanted mechanical coupling effects or to avoid problems during manufacturing [23]. Most of these rules have also been presented and used in previous works [24, 25, 26]. It is worth noting that while some of these design rules are omitted in some academic studies by using more sophisticated analyses [27, 28, 29] and manufacturing methods [30, 31, 32], the industry is following most of these rules [33]. Using well known and understood principles ensures a robust process and thus mitigates the risks associated with the certification of the aircraft. Besides that, it enables the usage of simplified analysis methods such as analytical buckling formulas which are only accurate for special classes of laminates. If these were not to be used, then there would be the need to resort to a Finite Element (FE) buckling analysis which assumes a finer mesh and is usually too computationally expensive to be used in optimisation studies. Below, the composite rules used in the context of this work are summarised.

The composite rules are grouped into two categories, i.e. design and manufacturing rules. Design rules regulate the stacking sequence of a single uniform laminate.

1. *Symmetry*. Symmetric laminates about the mid-plane are commonly used to avoid bending-extension coupling.

2. *Balance.* Balanced laminates consist of equal $+\theta$ and $-\theta$ orientations ($\theta \neq 0^\circ, 90^\circ$), in order to eliminate shear-extension coupling.
3. *Minimum percentage.* A minimum percentage of all fibre orientations used in a laminate might be enforced to minimise matrix degradation and favour a fibre-dominated failure mode instead.
4. *Damage tolerance.* External plies should never be in the direction of the main load path. Commonly, a 45° or -45° layer is placed in the outermost part of the laminate.
5. *Contiguity.* The maximum grouping of equally oriented layers is limited, in order to minimise interlaminar stresses and ensure a homogeneous stress distribution.
6. *Grouping.* Grouping $+\theta$ and $-\theta$ layers might be applied to reduce bending-twist coupling.
7. *Disorientation.* The difference in fibre orientation between two adjacent layers might be limited to 45° to minimise inter-laminar shear effects.

It should be noted that although grouping and disorientation design rules may both be advisable for a structure, their requirements are contradictory and hence they cannot be applied simultaneously.

The manufacturing rules consider the transitioning between laminates placed in neighbouring patches.

1. *Continuity/blending.* Continuity, also commonly referred to as blending, ensures manufacturability and structural integrity of the laminated composite. Various definitions of blending are available in the literature. Adams et al. [34] used two simplified blending definitions, i.e. outward (Fig. 2.3a) and inward (Fig. 2.3b) blending, in which only outermost or innermost plies, respectively, are allowed to be dropped. A ply drop indicates that a ply stops and does not continue on to the next patch of the structure. Campen et al. [35] introduced generalised (Fig. 2.3c and Fig. 2.3d) and relaxed generalised (Fig. 2.3e) blending. These two definitions allow for significantly greater design freedom, since they do not limit the ply drop position to an outer or inner ply. Generalised blending requires all plies in a thinner patch to continue in the adjacent thicker panels, while in relaxed generalised blending two neighbouring panels are considered to be blended if no dropped edges are in physical contact. In the current work, generalised blending is used instead of the relaxed generalised blending, as the latter leads to high stress concentrations.
2. *Maximum dropping.* The maximum number of plies dropped in transitions between neighbouring laminates is limited, to ensure smooth load distribution

throughout the structure.

3. *External covering ply.* At least one of the outer plies in a laminate is not allowed to be dropped in order to ensure structural integrity.
4. *Internal covering ply.* Not more than a certain number of consecutive plies are allowed to be dropped simultaneously so that potential delamination initiation zones are avoided.

Other manufacturing rules concerning the exact transitions within the taper area are not considered, since this level of detail is not in the scope of the current work. It should be mentioned that the taper area refers to the zone formed by the plies of two neighbouring patches merging into each other.

The full potential of the generalised blending definition is exploited in the current work (Fig. 2.3d), in a sense that the set of plies used in the thinnest patch must not necessarily extend to the rest of the patches in the structure. This is not the case for other implementations of generalised blending, such as the Stacking Sequence Tables (SSTs) [24], the Global Shared Layer Blending [36] or the modified Shepard's method [37], in which plies present in the thinnest patch must be continued across all other patches and not only the neighbouring ones (Fig. 2.3c). However, it should be noted that recent studies using the Ply Drop Sequence concept [38], multiple SSTs [39] and a general resolution scheme [40] have managed to eliminate some or all of the aforementioned drawbacks, offering increased design freedom.

2.2 Optimisation

The process of optimisation involves maximising or minimising the value of a function [41]. Optimisation problems are found in many fields such as engineering, computer science and operational research. In the aviation industry, multiple individual disciplines as for example aerodynamics, structures, manufacturing and flight trajectories are subject to optimisation. Traditionally, when complex numerical simulations were involved in the design process, it was carried out by manually adjusting design parameters until a sufficient result was reached. This process relied on a team of experienced engineers who could use their intuition and knowledge to improve the design goal or objective function. This strenuous and insufficient procedure is nowadays replaced with numerical optimisation techniques. The general

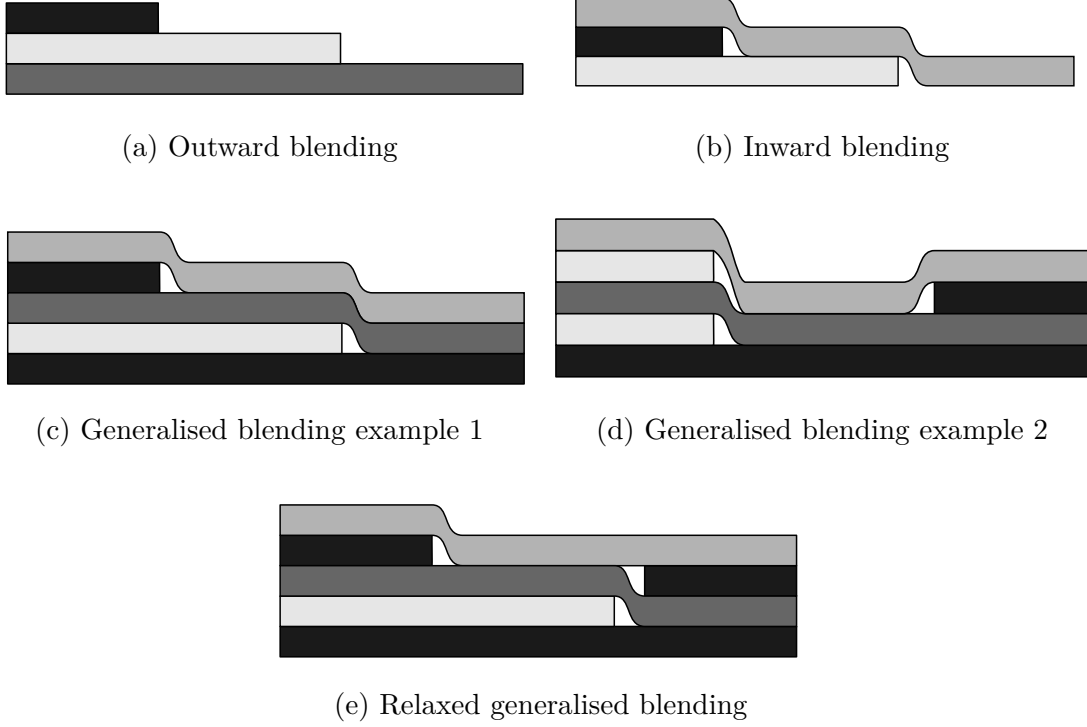


Figure 2.3: Blending examples between 3 patches.

formulation of a constrained optimisation problem can be defined as:

$$\begin{aligned}
 &\text{minimise} && f(t_n) \\
 &\text{w.r.t.} && t_n && n \in \{1, \dots, N_t\} \\
 &\text{subject to} && g_k(t_n) \leq 0 && k \in \{1, \dots, N_g\} \\
 &&& h_l(t_n) = 0 && l \in \{1, \dots, N_h\}
 \end{aligned} \tag{2.17}$$

where $f(t_n)$ is the objective function to be improved, with the improvement being a minimisation in this case. The values of the design variables t_n vary during the design problem in an attempt to retrieve the best possible solution. Even though an optimisation problem may be unconstrained, usually a number of inequality $g_k(t_n)$ and/or equality constraints $h_l(t_n)$ must be satisfied in order to achieve a feasible, or in other words realisable design solution.

Optimisation problems can be categorised in various different ways. The ones most relevant to the work performed in this thesis are listed below:

1. If either one of the objective functions or any constraints in the optimisation are nonlinear, then the problem is called a *nonlinear programming* (NLP) problem. On the contrary, if both the objective functions and the mathematical functions of the constraints are linear with respect to the design variables in the optimisation problem, then it is called a *linear programming* (LP) problem. Additionally, the

optimisation instance is called a *geometric programming* problem if the objective function and constraints are posynomial with respect to the design variables, meaning that the functions are expressed as a sum of power terms. Finally, the optimisation problem is classified as a *quadratic programming* problem if the objective function is quadratic and the constraints are linear functions.

2. Based on the allowable values for the design variables, the optimisation problem can be characterised as an *integer programming* or a *continuous programming* problem. In the former case, the permissible design variable values are limited to discrete numbers, while in the latter any real value is allowed. Of course in both cases, upper and lower limits may be imposed to the design variables. Optimisation instances which feature a combination of integer and continuous design variables are called *mixed integer programming* (MIP) problems.

Different optimisation problems are usually better suited for different optimisation algorithms. An optimisation algorithm defines the process based on which the design space is explored and the objective function is improved. In general, optimisation algorithms can be classified in two main categories, *deterministic* and *stochastic*.

Stochastic optimisation algorithms modify the values of the design variables in a random or semi-random manner and usually, no gradient information of the objective functions is used, which is the reason for them also being classified as zero order algorithms. The list of algorithms belonging to this class is extensive. The primary categories of stochastic algorithms are heuristics [42], such as evolutionary algorithms [43], meta-heuristics [44] like harmony search [45] or particle swarm optimisation [46, 47] and hyper-heuristics [48]. Although the terms of heuristics and meta-heuristics are often used interchangeably and the boundaries of the categories are not always clear, meta-heuristics are usually better suited for optimising more classes of problems, with heuristics being tailored more towards specific applications. The goal of hyper-heuristics is to automate the selection or the generation of simpler heuristics, in order to efficiently handle an optimisation problem. In the context of heuristics, efficiently handling a problem relies on the fine balance between the exploration and exploitation of the algorithm. Exploration ensures that multiple design regions are explored to avoid local minima, while exploitation aims to extensively search a local region for the local minimum.

Stochastic algorithms work very well with both discrete and continuous problems. Since they do not require any gradient information, they are also easy to apply to complex problems. A disadvantage of heuristics is that as the number of design variables increases, the number of iterations in order to achieve an adequate design

space exploration also increases drastically. Considering that each iteration is associated with the evaluation of the functions of the objective and the constraints, the computational time required for complex functions can quickly become infeasible. Therefore, although heuristic algorithms are ideal for discrete problems, they are not suitable for problems with many design variables and problems with computationally expensive and abundant constraint evaluations.

Deterministic optimisation algorithms use information based on the objective function to perform a directed search of the design space. They can be further sub-divided to zero, first and second order algorithms depending on the gradient order information used in the search. Zero order methods only use the information provided by the objective function, while first and second order methods utilise information of the objective function and of the first and second derivatives respectively. It should be noted that the sensitivities of the constraint functions must also be computed, besides those of the objective function. Deterministic optimisation methods converge to the same solution if the initial design point and relevant settings remain unchanged. This is not the case for stochastic algorithms due to the random decisions involved. In general, first and second order deterministic methods are suited for non-linear problems. They also converge to a solution in considerably fewer iterations compared to zero order methods. Their disadvantage is that they are suited for continuously formulated objective functions and constraints, because of the need for gradient computations. They are also prone to converging to a local instead of global optimum, if the design space is non-convex.

An example of zero order deterministic methods are interior point algorithms [49], simplex algorithm [50] and branch and bound algorithms [51, 52]. The two former algorithms are suited for continuous problems, while branch and bound is applicable to discrete optimisation problems. Branch and bound operates by dividing the feasible design space into smaller subsets, computing bounds on the objective value for each one of them and using this information to discard other subsets from consideration in the optimisation problem [53]. The bounds for the objective function are obtained with a continuous relaxation of the optimisation problem. The process is able to prove global optimality of a problem. Proving optimality or even obtaining feasible solutions becomes exponentially harder as the problem size increases. This disadvantage is mitigated to some extent by modern sophisticated branch and bound commercial solvers [54, 55].

The simplest example of a first order gradient-based optimisation method is steepest descent [56], in which the search direction is defined by the gradient of the objective

function. Concerning the computation of the gradients, the two main choices are, performing it analytically or numerically. Numerical strategies, like finite differences, are straightforward to implement but require $N_t + 1$ or $2N_t + 1$ evaluations of the objective function, depending on whether a forward or central differentiation scheme is employed, with the latter being more accurate. Complex step derivatives overcome a big drawback of finite differences linked to the high dependency of the gradient accuracy on the step size. However, the computational expense of both methods can become unbearable, depending on the application, since the number of objective functions and constraint evaluations is proportional to the number of design variables. More specifically the partial derivatives that need to be computed are $\frac{\partial f}{\partial t_n}$ for the objective function and $\frac{\partial g_k}{\partial t_n}, \frac{\partial h_l}{\partial t_n}$ for the inequality and equality constraints. This drawback of high computational effort is alleviated by analytical sensitivity methods, which are however significantly more demanding in their implementation due to the need for manual derivation of all the derivatives. Either the direct or the adjoint method can be applied depending on the problem type. The computational cost of the methods is equivalent to an evaluation of the function. However, the computational expense in the direct method is proportional to the number of design variables, while the expense in the adjoint method is linked with the number of objective and constraint functions [57]. Finally, the second derivative of the objective function, also called the Hessian matrix, needs to be computed for second order Newton methods. The convergence of such methods is superior to first order methods, but since the computation of the Hessian can be complex and time consuming, quasi-Newton methods can be employed.

The design problem studied in this work belongs to the class of structural optimisation problems. These can be classified in three general categories [58] as seen in Fig. 2.4. Sizing optimisation, an example of which can be seen in the first row of the figure, involves finding the optimal cross sectional areas of truss elements or the optimal thickness of plates. When performing shape optimisation, the topology of the structure is restricted but the material distribution can vary while respecting certain geometric constraints. Topology optimisation aims to find the optimal material layout and is not limited to predefined configurations.

In this work, the optimisation problem studied is that of the stacking sequence of laminated CFRP structures. The goal is to find the optimal thickness and stiffness distribution across a component. The design parameters that need to be identified are the number of layers in each patch in the structure and the optimal fibre orientation for each layer and patch. The final design must respect several design and manufacturing rules mentioned in Section 2.1.2 but also mechanical,

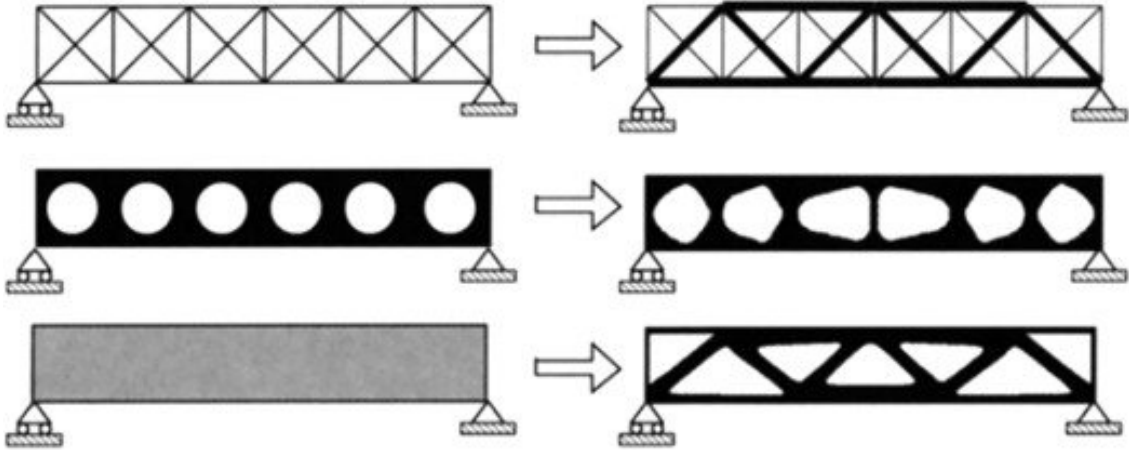


Figure 2.4: Classification of structural optimisation problems in sizing, shape and topology optimisation as seen from top to bottom. The initial states of the structure are depicted on the left side of the figure and the optimised states on the right. Figure reproduced from [3].

structural constraints such as stability and strength of the structure.

2.3 Detailed sizing optimisation of composite structures

There are two main challenges associated with the detailed sizing optimisation of large-scale composite structures. The first one is the discrete nature of many of the problem's design variables related to the stacking of the composite, as well as the discrete nature of the manufacturing constraints. This challenge alone is not a decisive problem, since heuristic algorithms are well suited to deal with such problems. The particular difficulties arise in combination with the second challenge, which is the size and complexity of real-world aircraft structures. This renders the sole use of heuristic algorithms prohibitive, with respect to the computational expenses, due to the great number of required iterations. Such computationally expensive applications are a much better fit for gradient-based optimisation algorithms that require significantly less iterations to converge. As stated in the previous section though, the disadvantage of gradient-based approaches is that they are mainly suitable for problems with continuous design spaces.

The problem of stacking sequence optimisation has been studied thoroughly by many researchers over the last decades. A bibliographic review of the different approaches on stacking sequence optimisation has been performed by Ghiasi et al. [59, 60]. In this section, a short overview of single stage optimisation approaches is first presented and the emphasis is then placed on two-stage optimisation approaches

which are better suited for large-scale applications. The different methods to model the thickness and stiffness of the laminate in the first stage of the optimisation are discussed, followed by the algorithms used to retrieve discrete laminates with optimally matching stiffness distributions.

2.3.1 Single stage approaches

Genetic algorithms have been extensively used to tackle the problem of stacking sequence optimisation, because they are especially suited to handle problems with discrete design variables and allow for a rather simple and straightforward implementation. One of the first implementations of a genetic algorithm optimised the fibre placement orientations of fixed thickness plies composing a laminate for which the allowable buckling capacity was maximised [61]. An implementation of a binary search tree to avoid re-evaluating identical laminate designs, utilising a local improvement scheme to allow for faster convergence, has also been performed [62]. Local improvement genetic algorithms were also used [63] in combination with a regression model to alleviate the computational time required by the Finite Element Model (FEM) analysis. The computational time is reduced by more than fifty percent, but the methodology is limited to simple problems. Different variations of classic Genetic Algorithms have also been considered. The works of both Todoroki and Haftka [64] and Liu et al. [65] involved the usage of repair strategies to implement some basic design rules during the optimisation procedure. Finally, new genetic operators were developed in order to take into consideration the changing number of design variables during the optimisation of the damping properties of hybrid laminates [66]. Genetic algorithms have also been used to optimise laminated structures subjected to blending constraints between different patches [4, 67]. Adams et al. [34] introduced the concept of a guide-based design where plies are dropped from a thick laminate which serves as a guide for the design of the composite by means of a genetic algorithm driven optimisation. In the work of Seresta et al. [68], a guide-based approach is used and the optimisation is performed by a genetic algorithm. In order to keep the computational costs low, only one individual per population is evaluated using a FE analysis. This one analysis in conjunction with locally adding or dropping only one ply, aims to provide compliance with the mechanical constraints of the structure.

In order to increase the design freedom provided by guide-based approaches and introduce a more complete set of design and manufacturing guidelines, Irisarri et al. [24] introduced the concept of a Stacking Sequence Table (SST). The optimisation is performed using an evolutionary algorithm, while the ply drop design guidelines are enforced by appropriate evolutionary operators. The study claims to have achieved

the best results out of all other methods up to then, when applied on a standard academic benchmark. However, the generalisation of the methodology to large structures is challenging, as mechanical constraints are incorporated in the objective function of the optimisation and therefore evaluated in each optimisation cycle.

Besides genetic algorithms, many other meta-heuristics have also been used to tackle the stacking sequence optimisation problem. Amongst them are Tabu search [69], ant colony optimisation [70], particle swarm optimisation [71], simulated annealing [72] and hybrid particle swarm and simulated annealing algorithms [73]. A one shot optimisation approach using only meta-heuristics is bound to fail for industry applications since the number of optimisation cycles grows significantly for an increased number of design variables, while the computational cost of each optimisation cycle is also considerably high.

As already mentioned, gradient-based algorithms provide significantly better convergence over zero order methods. Their disadvantage is that they are suited for continuously formulated problems whereas the optimisation problem at hand is highly discrete. Nonetheless, single stage approaches dealing with the discrete characteristics of the problem in a gradient-based optimisation framework have been developed. In order to achieve discrete solutions, penalty functions can be used to drive the value of continuous design variables to discrete allowables. The first research work performed towards that direction was by Stegmann and Lund [74] and Lund and Stegmann [75]. The methodology was labelled as Discrete Material Optimisation (DMO) and originates from the field of topology optimisation. Each ply in the laminate is modelled as a summation of different material properties. In the specific problem of stacking sequence optimisation, the CFRP material is predefined, therefore, the difference in material properties only stems from the orientation at which the ply is placed. In the specific case of four available standard orientations (-45° , 45° , 0° , 90°), four design variables model the specific properties of the ply. In order for the methodology to make physical sense, the four design variables need to be binary and in addition to that, only one of them can have the value of 1. Therefore, the optimisation algorithm needs to be able to push the values of the design variables to their binary limits through the usage of appropriate penalty functions. The main limitations of DMO are that the optimisation takes place for a fixed thickness of the laminate and that the number of design variables in the optimisation is increased e.g. four times the number of layers for four available ply orientations. In the work of Hvejsel and Lund [76], two interpolation and penalisation schemes are examined, concerning the convergence of the optimisation problem. In later work [77, 78], the DMO method is extended to Discrete Material and Thickness Optimisation (DMTO)

and different algorithmic schemes of Sequential Linear Programming and Sequential Quadratic Programming are tested on different plate sizing cases. The DMO and DMTO methodologies have also been extended to include strength and strain criteria [79]. Overall, the applicability of the methodologies is probably limited by the size of the problem and can only be used as an initial sizing step, since further processing is needed in order to derive structures that comply with design and manufacturing rules, especially blending.

In an attempt to reduce the design variables needed by DMO, Bruyneel [80] developed Shape Functions with Penalisation (SFP) inspired from shape functions in FE. SFP only works for an even number of candidate ply orientations and the number of design variables needed is the number of candidate ply orientations divided by two. According to the author, the methodology seems to be less likely than DMO to converge to a solution which represents a combination of the available ply orientations i.e. a non-discrete design. SFP was later extended in order to account for some of the composite design rules [81]. A Bi-value Coding Parametrisation (BCP) in which the number of design variables has a logarithmic dependency on the available ply orientations has also been developed [82]. However, for the case of four available ply orientations, the number of design variables would equal two, for either SFP and BCP. The most recent approach presented is normal distribution fibre optimisation which uses the generic form of the normal distribution function in order to select a discrete orientation angle amongst the available ones. In this method, only one design variable is needed per ply no matter how many available candidates exist [83]. In an attempt to bridge the gap between the two optimisation steps previously described, a bi-level approach was developed, during which the optimal stacking sequences and ply drop-offs are sought after each thickness optimisation loop has finished [84]. By using the backtracking method in each iteration the structure completely satisfies the blending rules.

Overall, single stage approaches using zero order optimisation algorithms are not suitable for large scale applications, because a large number of design variables leads to the need for many optimisation iterations, with each iteration requiring the solution of the FEM and therefore being computationally expensive for large applications. Single stage approaches employing gradient-based algorithms are more computationally efficient. However, even though laminated structures with discrete thicknesses and fibre orientations can be designed using them, these structures still fail to satisfy some of the discrete composite guidelines commonly required by aircraft manufacturers.

2.3.2 Two-stage approaches

In order to bypass the shortcomings of individual optimisation algorithms, many researchers have split up the stacking sequence problem in two stages. During the first stage, a gradient-based optimisation algorithm is used in order to derive the optimal continuous thickness and stiffness characteristics of the structure. At this stage, all mechanical constraints linked with the responses of the structure, commonly computed by a FE analysis, such as stability, strength and displacements need to be considered. The thickness of the structure is then usually rounded up to a thickness corresponding to an integer number of uni-directional layers. This rounding operation can be performed since the discrete thickness of the uni-directional CFRP tape that will be used during manufacturing is usually known, especially at later, more mature design phases of an aircraft.

The aim of the second stage of the optimisation is to derive a discrete, manufacturable design that fulfils all of the desired design and manufacturing rules. To that extent, a zero order optimisation algorithm is employed in the second stage. Usually, mechanical constraints are not considered because their inclusion at this stage introduces all of the drawbacks previously mentioned. The main challenge with two-stage approaches is attaining a structural design which also fulfils all of the mechanical constraints imposed in the previous stage. In order to achieve satisfaction of these constraints, the objective function of the second stage involves the minimisation of the absolute difference between the continuous and discrete stiffness characteristics of the structure. As will be discussed in the coming sections, this is not easy to achieve and failure to do so leads to the need of performing the two-stage process again. This results in both unnecessary computational effort but most importantly impairs the benefits emerging from the optimisation e.g. weight reduction.

In the next sections, the different approaches used to model the stiffness properties in the first gradient-based optimisation stage are summarised. Shortcomings arising in the scope of the entire two-stage process and ways to overcome these are also presented. Next, the main developments and approaches for the second stage of the optimisation, where the stiffness matching is performed are presented.

2.3.2.1 Stiffness modelling

This section summarises the various stiffness representations used in the first stage of bi-level optimisation approaches. Developments towards an overall efficient two-stage process are also synopsised.

The first and most straight forward way to represent the thickness and stiffness

characteristics of a laminated structure is by using the actual ply properties such as its thickness and fibre orientation. The work of Zhou et al. [85] included an initial free design stage, during which the total thickness of each pre-defined fibre orientation zone is optimised. The methodology is part of the commercial software OptiStruct provided by Altair and an application to detailed sizing of aerospace components of Bombardier is also demonstrated. Liu et al. [86] and Farzan Nasab et al. [87] used the total thickness of each standard fibre orientation as a design variable and approximated the bending stiffness matrix using the smeared stiffness method. Although this drastically reduces the number of design variables, it also over-simplifies the problem as no stacking sequence information is available in the gradient-based optimisation stage. The approaches of DMO [74, 75] and DMTO [78, 88] mentioned previously employ design variables related to the thickness and material of the laminate, where the material selection variable can be used as a substitute for the fibre orientation of a layer. Even though these methods have been presented as single stage approaches, they still fall under the category of direct stiffness modelling and are applied in a gradient-based framework.

The two disadvantages of modelling the properties of the structure using layer thicknesses and orientations are that a large number of design variables need to be employed and that the design space created by such parametrisations is highly non-convex. This is unfavourable, especially when dealing with gradient-based algorithms which are prone to getting stuck in local minima.

A widely used and accepted alternative to direct stiffness modelling techniques are lamination parameters. Lamination parameters which were first introduced by Tsai and Pagano [17] allow the stiffness properties of a composite laminate to be compactly described. More specifically, lamination parameters are non-dimensional through the thickness integrated trigonometric functions. In the most general case, 12 lamination parameters and the total thickness of the laminate can completely define the ABD matrix and therefore the stiffness properties of the laminate. If the laminate is also assumed to be symmetric, balanced and orthotropic, then only 4 lamination parameters in addition to the total thickness can appropriately model its stiffness. The lamination parameters are defined as:

$$\xi_{[1,2,3,4]}^A = \frac{1}{h} \int_{-\frac{z_i}{2}}^{\frac{z_i}{2}} [\cos(2\theta), \sin(2\theta), \cos(4\theta), \sin(4\theta)] dz \quad (2.18)$$

$$\xi_{[1,2,3,4]}^B = \frac{4}{h^2} \int_{-\frac{z_i}{2}}^{\frac{z_i}{2}} [\cos(2\theta), \sin(2\theta), \cos(4\theta), \sin(4\theta)] z dz \quad (2.19)$$

$$\xi_{[1,2,3,4]}^D = \frac{12}{h^3} \int_{-\frac{z_i}{2}}^{\frac{z_i}{2}} [\cos(2\theta), \sin(2\theta), \cos(4\theta), \sin(4\theta)] z^2 dz \quad (2.20)$$

where $-\frac{z_i}{2}$ and $\frac{z_i}{2}$ stand for the distance of the bottom and top surface of the i^{th} ply with respect to the mid-surface of the laminate as depicted in Fig. 2.2. The ABD stiffness matrix can then be expressed as:

$$\begin{Bmatrix} A_{11} \\ A_{22} \\ A_{12} \\ A_{66} \\ A_{16} \\ A_{26} \end{Bmatrix} = h \begin{bmatrix} 1 & \xi_1^A & \xi_3^A & 0 & 0 \\ 1 & -\xi_1^A & \xi_3^A & 0 & 0 \\ 0 & 0 & -\xi_3^A & 1 & 0 \\ 0 & 0 & -\xi_3^A & 0 & 1 \\ 0 & \xi_2^A/2 & \xi_4^A & 0 & 0 \\ 0 & \xi_2^A/2 & -\xi_4^A & 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix} \quad (2.21)$$

$$\begin{Bmatrix} B_{11} \\ B_{22} \\ B_{12} \\ B_{66} \\ B_{16} \\ B_{26} \end{Bmatrix} = \frac{h^2}{4} \begin{bmatrix} 0 & \xi_1^B & \xi_3^B & 0 & 0 \\ 0 & -\xi_1^B & \xi_3^B & 0 & 0 \\ 0 & 0 & -\xi_3^B & 0 & 0 \\ 0 & 0 & -\xi_3^B & 0 & 0 \\ 0 & \xi_2^B/2 & \xi_4^B & 0 & 0 \\ 0 & \xi_2^B/2 & -\xi_4^B & 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix} \quad (2.22)$$

$$\begin{Bmatrix} D_{11} \\ D_{22} \\ D_{12} \\ D_{66} \\ D_{16} \\ D_{26} \end{Bmatrix} = \frac{h^3}{12} \begin{bmatrix} 1 & \xi_1^D & \xi_3^D & 0 & 0 \\ 1 & -\xi_1^D & \xi_3^D & 0 & 0 \\ 0 & 0 & -\xi_3^D & 1 & 0 \\ 0 & 0 & -\xi_3^D & 0 & 1 \\ 0 & \xi_2^D/2 & \xi_4^D & 0 & 0 \\ 0 & \xi_2^D/2 & -\xi_4^D & 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix} \quad (2.23)$$

with h being the thickness and U_i the material constants defined in Eq. 2.11. Besides offering a framework with reduced design variables to model the stiffness characteristics, the lamination parameter design space is convex [89] and the critical buckling factor is a concave function over the bending lamination parameters [90]. A review on composite design using lamination parameters has been performed in the work of Albazzan et al. [91]. Their first application in the scope of a sizing optimisation was in the work of Miki and Sugiyama [92]. Since then multiple researchers have adopted them in two-stage approaches [93, 94, 95, 7].

Because the value of lamination parameters does not give direct information on the exact stacking sequence of the laminate, the problem that an optimal set of computed lamination parameters does not correspond to a feasible stacking sequence design which complies with the design rules might rise. Therefore, work has been conducted in order to precisely formulate the feasible region of lamination parameters for different types of laminates and sets of design rules [96, 95, 97, 98]. The feasible lamination parameters' space for the Tsai-Wu failure criterion has also been

derived [99]. Besides accurately describing the feasible design region of lamination parameters, using as many composite design rules as possible in the first stage of the optimisation is crucial for retrieving a discrete stacking sequence in the second stage of the optimisation. Macquart et al. [6] implemented blending constraints in a gradient-based optimisation using lamination parameters and demonstrated the importance of these constraints on an academic benchmark problem.

An alternative to lamination parameters are polar parameters which were introduced by Vannucci and Verchery [27]. The polar method is a formulation which uses true tensor invariants to represent a general planar tensor. Polar parameters have also been used in two-stage optimisation processes [100, 101, 102]. The polar method has also been extended to the first-order shear deformation theory [103] and third-order shear deformation theory [104]. Finally, blending constraints have been formulated in the polar parameter design space [40, 105].

2.3.2.2 Stiffness matching

This section summarises the approaches developed to retrieve discrete stacking sequence designs fulfilling a set of composite design and manufacturing rules. In general, the approaches employed do not significantly differ compared to single stage gradient-free algorithms discussed in section 2.3.1. Similar techniques can be applied to satisfy the composite design guidelines and instead of minimising the structural mass, the objective is altered to minimise the absolute difference between the continuous and discrete characteristics. Consideration of mechanical constraints is also omitted to make the process computationally efficient and the satisfaction of mechanical constraints is left to be examined after the discrete solution has been retrieved and depends on the quality of the stiffness matching which was performed.

Genetic algorithms were employed in one of the earliest two-stage approaches, in order to match the optimal lamination parameters derived by the first stage of the optimisation [106]. The same concept was also followed in the work of Herencia et al. [107]. Permutation genetic algorithms performing a shuffle amongst the plies of different zones, in order to match the optimum lamination parameters passed from a top level continuous optimisation, have also been used [108, 86, 109, 110, 111]. The works also include consideration of blending between neighbouring panels, which is in general performed successfully for laminates consisting of many plies. A comparative study between three meta-heuristic algorithms i.e. particle swarm optimisation, ant colony optimisation and a genetic algorithm on the lamination parameter matching problem, has also been carried out by Bloomfield et al. [112]. Ant colony optimisation was found to be the most reliable algorithm out of all

3, while the particle swarm optimisation and genetic algorithm performed better for some specific problem instances. IJsselmuiden et al. [113] presented a two step approach for the blended optimisation of laminated structures. During the first stage, a continuous optimisation using lamination parameters is performed and at the second stage a guide-based genetic algorithm is used. Instead of using meta-heuristics to solve the lamination parameter matching problem, Balabanov et al. [114] used an analytical solution based on the Groebner basis but the applicability of the approach appears to be limited.

In the work Peeters et al. [115], SSTs which were first introduced in a single-stage optimisation process [24] have been used. A two step approach is also used in Meddaikar et al. [37]. First, an optimisation using lamination parameters is performed and then the detailed design is retrieved with the use of SSTs. The novelty in this work concerns structural approximations which are used in order to ensure that the detailed design derived from the SST optimisation fulfils the mechanical constraints. A modified Shepard's method is used to perform the load approximations which are successively enhanced and updated by new design points. In the same work, a new methodology of matching the optimal design derived by the first stage optimisation is presented. Lamination parameters matching, which is mentioned as stiffness matching, is compared to an eigenstiffness matching which essentially requires each element of the stiffness matrix being greater or equal than that of the continuous optimum. The latter ensures mechanical constraint satisfaction but introduces weight penalisation. Another approach for the solution of the blending optimisation is known as the global shared-layer blending method, which involves a labelled graph and a SST to characterise the connectivity between different patches [36]. A stacking sequence optimisation using a permutation search algorithm has also been presented [116, 117]. Finally, a complete sizing optimisation framework has been demonstrated using the 18-panel horseshoe problem in later works by the authors [118, 119].

Silva Caprio et al. [120] presented an aeroelastic tailoring framework applied to the design of an industry scale aircraft wing. The optimisation approach is split up in two steps with the first one being a continuous optimisation using lamination parameters. The physical responses calculated during that step are recorded and used in order to set up a surrogate model that is used during the following discrete optimisation step. More specifically, the discrete optimisation is performed using an Evolutionary Algorithm and a developed method called the slice and swap method, which is used to determine the blended stacking sequence of the wing structure.

A deterministic beam search algorithm is developed in the work of Fedon et al. [121] and multiple repair strategies are used to derive laminates that fulfil the composite guidelines [26]. In another work, two nested backtracking algorithms were used to produce blended composite designs [122]. Assuming that the total number of layers per zone and the number of layers per orientation in each zone are known, the inner loop algorithm first creates stacking sequences which are admissible with the design rules by means of an enumeration tree. Then the outer loop algorithm generates stacking sequences for neighbouring panels so that blending in the structure can be achieved. The extendability of the approach might be limited for thicker structures containing hundreds of zones. The work was later extended to a one step optimisation approach where the thickness, stacking sequence and blending of the structure are optimised simultaneously [84]. While the authors claim that the approach is efficient, only simple buckling formulas are employed for the mechanical constraint satisfaction. Bruyneel et al. [123] followed a bi-level approach in which the total thickness and ply share percentage are first optimised using a gradient-based method and then a backtracking algorithm is utilised on enumeration trees of admissible stacking sequences.

A process for determining a blending stacking sequence of composite structures has been patented [124]. The blending problem is formulated using integer variables that represent the connectivity of different panels. The stacking sequence retrieval is performed by solving linear systems of equations in order to determine the angle of the plies that better match pre-computed optimal lamination parameters. The blending problem has also been formulated in the form of integer programming, during which the connectivity between sub-laminates is taken into account while also optimising the structure for ease of manufacturability [125]. In a later work [33], more insight is provided in the sub-laminate family generation and the overall design feasibility. Finally, the stacking sequence optimisation problem has been formulated as a mixed-integer problem by Allaire and Delgado [126].

2.4 Discussion

The detailed sizing optimisation process which was employed by Airbus Defence and Space at the beginning of this work is shown in Fig. 2.5a. The thickness and stiffness properties of the structure are modelled using generic stacks which are comprised of a collection of layers whose orientations are fixed *a priori*, but thicknesses can vary independently. Regardless of the number of generic layers used, all layers with the same fibre orientation were grouped together essentially only allowing for an optimisation of the total thickness and ply share in each patch. This leads to an

optimisation of only the flexural stiffness of the laminate. The continuous thickness distribution must then be rounded up to an integer number of layers. The retrieval of discrete stacking sequences which comply with the prescribed composite design and manufacturing rules was performed manually using engineering judgement and by deriving feasible stacking sequences by a “master” stack. Further manual interventions with respect to adding more layers locally may have been necessary to satisfy all design guidelines when incompatibilities appeared. When the discrete design was re-evaluated to examine the satisfaction of the mechanical constraints, very significant violations were usually observed, leading to multiple repetitions of the continuous optimisation and the manual stacking sequence retrieval cycle. This process, besides being time and labour intensive, also diminished the original optimisation result leading to a large weight increase and performance loss.

In this work, a two-stage optimisation process is developed as seen in Fig. 2.5b. While many of the two-stage approaches in the literature can probably be applied to large-scale structures, a big remaining challenge is retrieving a discretised stacking sequence which fulfils mechanical constraints such as strength and buckling, without the need to perform the two stage process again, which leads to over-dimensioning of the structure. In this work, in the first gradient-based step of the optimisation process, a direct stiffness modelling method using the thicknesses of generic stacks is employed. The number, stacking sequence and orientation of the layers in these generic stacks still remains fixed during the first stage of the optimisation, however design variables are now assigned to multiple layers across the generic stack, allowing for an optimisation of the bending stiffness of the structure. In the second stage, the stacking sequence retrieval satisfying all composite guidelines is performed with the objective of achieving a good quality stiffness match between the continuous and discretised design.

The developed two-stage process is able to retrieve designs satisfying both mechanical and composite constraints and is validated on both academic and industrial demonstrators. The benefit of using a direct stiffness modelling of the stiffness properties is that more composite guidelines can be expressed as constraints in the first stage of the optimisation which helps bridge the information gap between the two stages. Drawbacks associated with a more complex modelling process and a non-convex design space have not been detrimental to the process. In the second stage, the stacking sequence optimisation problem has been formulated as two different Mixed Integer Linear Programming (MILP) instances which can be used by many state of the art commercial solvers. Besides minimising simplifying assumptions found in other studies, the solvers used to optimise the specific class of

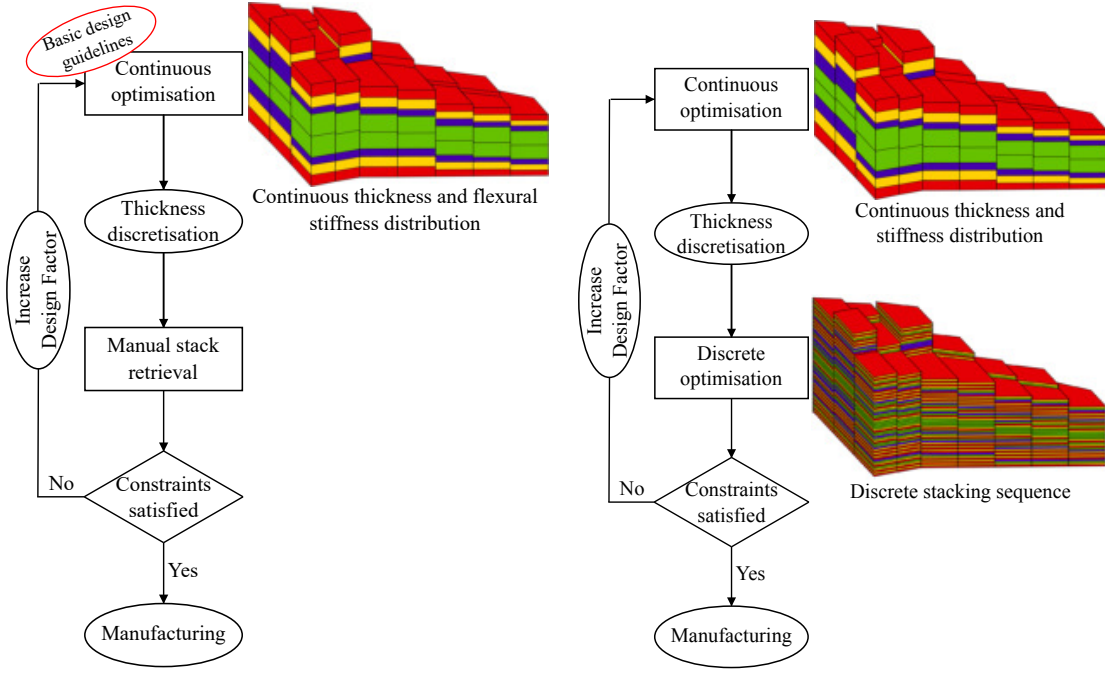


Figure 2.5: Comparison between the detailed sizing optimisation process before and after the completion of this PhD.

problems are also deterministic and optimality can be proven, further contributing to a good stiffness match which is essential for the success of the process. Finally, additional requirements concerning design for manufacturing have been included in the first stage of the optimisation.

Generic stack parametrisation

The majority of the two-stage approaches found in the literature use either lamination or polar parameters to describe the stiffness properties of the structure. Even though these allow for a compact parametrisation of the stiffness, with few design variables and a convex design space, they still present two drawbacks. First of all, composite design guidelines whose formulation applies on a discrete ply level, are hard to formulate or cannot be formulated accurately [6, 105]. Secondly, first ply failure criteria depend explicitly on the ply orientation and in the case of bending they also depend on the position of the ply within the stack. Therefore, it is only possible to use conservative envelopes for ply failure [99, 128] or homogenised invariant laminate failure criteria [129]. This conservatism negates some of the possible benefits and it does not allow to distinguish between different failure mechanisms such as matrix cracking or fibre breakage. The latter precludes the usage of progressive failure methods which can exploit the, often substantial remaining laminate strength, after initial matrix failure.

This chapter is based on the paper, *Generic stacks and application of composite rules for the detailed sizing of laminated structures* by G. Ntourmas, F. Glock, F. Daoud, G. Schuhmacher, D. Chronopoulos and E. Özcan which appeared in the Composite Structures journal, 276 (2021) 114487. Note: symbols may have been changed to maintain consistency throughout this thesis.

Generic stacks used in this work are based on a direct stiffness modelling of the structure. A generic stack is comprised of a collection of layers whose orientations are fixed *a priori*, but thicknesses can vary independently enabling exploration of the design space. Similar parametrisations were amongst the first to be used and are still employed. Mateus et al. [130] used both layer thickness and orientation to optimise the buckling strength of a composite laminate. An optimisation with a fixed stacking sequence and only the thicknesses of the individual layers used as design variables was performed by Schmit Jnr. and Farshi [131] and two manufacturing constraints were included in the optimisation by Costin and Wang [132]. Most recently, such approaches have been mainly applied to the design of wind turbine blades [133]. Discrete Material Optimisation [74, 75] and Discrete Material and Thickness Optimisation [78, 88] employ design variables related to the thickness or material of a laminate. Their application has been limited to a small number of design variables per structural zone to keep the complexity of the optimisation to a minimum.

This chapter focuses on the first stage of the optimisation process in which the stiffness properties of the structure are modelled using generic stacks. The motivation for using generic stacks is provided by the need to employ ply-based failure criteria and precisely accommodate discrete composite guidelines in the gradient-based optimisation, the importance of which has been identified in the literature [86]. In this chapter, the ability of generic stacks to adequately model the stiffness of a structure, while keeping the optimisation problem to manageable computational levels is demonstrated.

This chapter is structured as follows: In section 3.1, the formulation of the optimisation problem is presented and the parametrisation of the stiffness of the structure using generic stacks is explained. In section 3.2, the stability and strength constraints are described and in section 3.3 the formulation of the composite design and manufacturing rules in the gradient-based design space is illustrated. Finally, in section 3.4 results are shown on the influence of the generic stacks on the stiffness parametrisation and optimisation convergence, followed by a discussion in section 3.5.

3.1 Optimisation problem formulation and parametrisation

A generic stack is comprised of multiple generic layers. The exact orientation and stacking sequence of these plies is fixed during the optimisation whereas the individual thickness of each generic ply corresponds to a design variable which can take any real positive value. Several generic plies are used to form a generic stack as shown in Fig. 3.1. The number and stacking sequence of the generic stack needs to be chosen so that the resulting thickness and stiffness does not depend on the modelling decisions. Knowing the stacking sequence and thickness of each generic ply, the extensional stiffness matrix $\mathbf{A}$, the coupling stiffness matrix $\mathbf{B}$ and the bending stiffness matrix $\mathbf{D}$ can be calculated as:

The optimisation algorithm used in this gradient-based framework is NLPQLP [134, 135], a sequential quadratic programming algorithm which is available in *Lagrange* [13]. It is worth noting that the optimal stacking sequence attributes are not limited to solely being a function of the structural weight of the structure, but can also take into account performance related metrics such as the Breguet range. Similarly, constraints governing the optimal solution can also include structural eigenmodes or static aeroelastics amongst others. This work focuses on the demonstration of the methodology, therefore the optimisation cases deal with a mass minimisation subject to mechanical i.e. buckling and strength constraints and composite design and manufacturing constraints. The mathematical formulation of the problem is:

$$\begin{aligned} \min_{0 < t_{ij} \leq t_{ij}^u} \quad & \sum_{j=1}^J \sum_{i=1}^I t_{ij} S_j \rho \\ \text{subject to} \quad & g_k(t) \geq 0 \quad k \in \{1, \dots, K\} \end{aligned} \quad (3.1)$$

where t_{ij} is the thickness of the i^{th} layer in patch number j , S_j and ρ are the surface and material density of patch number j and $g_k(t)$ are the K constraints applied to the optimisation problem.

3.2 Mechanical constraints

For the examples demonstrated in this chapter and Chapter 4, the plates are assumed to be simply supported, biaxially compressed and specially orthotropic. Buckling constraints in the optimisation are then formulated as:

$$\frac{N_{x, cr}}{N_x} - 1 \geq 0. \quad (3.2)$$

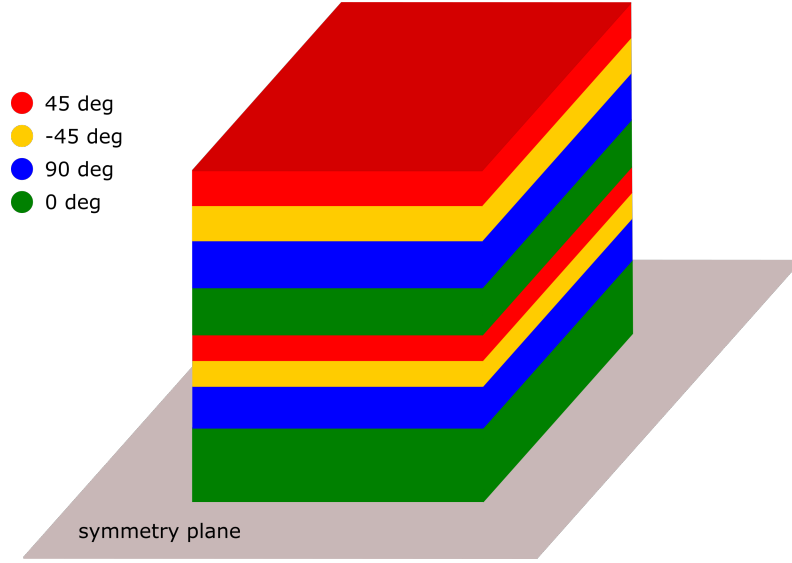


Figure 3.1: An indicative generic stack of 16 generic plies. Only half of the symmetric stack is shown.

The fraction in equation 3.2 will be referred to as a Reserve Factor (RF) and corresponds to the first eigenvalue of the biaxially compressed plate. In equations 3.2, 3.3, N_x is the applied load across the longitudinal direction, N_y the load across the transverse direction and $N_{x,cr}$ the critical buckling load in the longitudinal direction which is calculated as:

$$N_{x,cr} = \frac{\pi^2 \left(D_{11} \left(\frac{m}{a} \right)^4 + 2(D_{12} + 2D_{66}) \left(\frac{mn}{ab} \right)^2 + 2D_{22} \left(\frac{n}{b} \right)^4 \right)}{\left(\frac{m}{a} \right)^2 + \frac{N_y}{N_x} \left(\frac{n}{b} \right)^2} \quad (3.3)$$

where a and b are the lengths of the bi-axially loaded plate in the longitudinal and transverse direction respectively. The critical buckling load is the minimum one calculated for a set of m and n half wavelengths across the length and width of the plate respectively. The analytical formula of equation 3.3 is chosen in this work to maintain consistency when comparing with relevant studies in Chapter 4. Since the formula is based on Classical Lamination Theory the critical buckling load is less accurate for moderately thick and thick laminates.

Concerning strength constraints, the maximum strain criterion is applied in this work for fibre failure. These constraints are formulated as:

$$\left| \frac{\varepsilon_{1T}^u}{\varepsilon_1} \right| - 1 \geq 0 \quad or \quad \left| \frac{\varepsilon_{1C}^u}{\varepsilon_1} \right| - 1 \geq 0 \quad (3.4)$$

for extensional and compressive loads respectively where ε_{1T}^u is the ultimate strain of the material for tension, ε_{1C}^u the ultimate strain for compression and ε_1 is the strain along the fibre direction for a specific generic ply. Knowing the stiffness of a laminate and the force and moment resultants the strain ε and curvature κ

distributions can be calculated solving:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\kappa} \end{Bmatrix}. \quad (3.5)$$

Once the global strain distribution has been calculated, the strain per ply ε_1 can be computed by transforming the global strain into the local material directions which are known due to using the generic stack to describe the stiffness of the laminate. The reader is invited to further reading on the detailed calculation of strain [9].

3.3 Composite constraints

The list of composite guidelines commonly enforced in the aerospace industry in order to ensure a robust process and mitigate the risks associated with the certification of the aircraft were introduced in section 2.1.2. One of the most crucial factors determining whether the discrete stacking sequences in the second stage of the optimisation are going to be satisfying the mechanical constraints in the structure is whether discrete composite guidelines have also been introduced in the first stage of the optimisation. Therefore, in an attempt to bridge the gap between the two stages of the optimisation, as many composite design guidelines as possible must also be introduced in the framework of the generic stacks. The guidelines used in this work and their respective implementation are:

1. *Symmetry* can be achieved by linking the design variables on the two sides of the laminate formed by the mid-plane surface.
2. *Balance*, similarly to symmetry, may be achieved by linking the design variables corresponding to the $\pm\theta$ oriented layers.
3. *Damage tolerance* is accommodated by not placing external plies in the direction of the main load. Commonly, a 45° or -45° layer is placed in the outermost part of the laminate. This design rule can be facilitated in the optimisation by choosing a generic stack with a 45° or -45° generic layer on the outer part of the laminate in combination with increasing the minimum gauge of the relevant design variable to the thickness of the tape to be used during manufacturing.
4. A *minimum percentage* (p) of any fibre orientation θ_c used in a laminate in order to minimise matrix degradation and favour a fibre-dominated failure mode instead can be mathematically formulated as:

$$p \sum_{i=1}^I t_{ij} - \sum_{i'=1}^{I'} t_{i'j} \leq 0 \quad \forall j, i' \in \{I \cap \theta_{ij} = \theta_c\} \quad (3.6)$$

For laminates in which not only the four standard orientations ($45^\circ, -45^\circ, 90^\circ, 0^\circ$) are used in the design, it makes sense to reduce the limit of the minimum

percentage requirement, or to apply the constraint cumulatively for multiple fibre orientations [136], in order to avoid an infeasible design space while making sure that the laminate will still exhibit a fibre-dominated failure mode.

5. The *contiguity* design rule applied for a specific θ_c fibre orientation is formulated as:

$$\sum_{i'=1}^{I'} \left(\frac{t_{i'j}}{t_d} \right)^n \geq \frac{1}{2} + \frac{1}{2} \tanh \left(k \left(\frac{t_{i^*j} + t_{(i^*+1)j}}{t_d} \right) - M - c \right) \quad (3.7)$$

$$\forall j, i^* \in \{I \cap \theta_{ij} = \theta_c\}, i^* < i' < i^* + 1$$

The right hand-side of equation 3.7 is a Heaviside function which takes values in $[0, 1]$. The design variable driven part of the right hand-side of the equation captures the total thickness of two “consecutive” θ_c oriented plies i^* and $i^* + 1$ as shown in Fig. 3.2. It should be noted that when referring to the thickness of the generic plies, “consecutive” does not imply physical contact but rather the absence of any other θ_c oriented ply between the two of interest. Anyhow, modelling the laminate using two generic plies of the same fibre orientation which are in physical contact does not add any advantage but only increases the complexity of the optimisation. When the total thickness of these two “consecutive” θ_c oriented generic layers becomes larger than the maximum number of allowable consecutive layers of the same fibre orientation M , then the right hand side of the equation increases. Constant t_d denotes the thickness of the tape which will be used in manufacturing, k regulates the steepness of the Heaviside function and c is a tolerance value related with k which defines at which point exactly the sum of the two θ_c oriented plies starts creating a value for the right hand-side of the equation which is “significantly” larger than zero. The left hand-side of the equation captures the total thickness of all plies placed between the two “consecutive” θ_c oriented plies. When the right hand-side becomes larger than 0, then the left hand side must also increase to fulfil the constraint meaning that the thicknesses of the intermediate plies must also increase. Exponent n values larger than 1 can be used to enforce a single ply of a specific fibre orientation to be used instead of a collection of plies of various orientations whose thicknesses are smaller than one. A plot of the contiguity constraint value is shown in Fig. 3.3.

Manufacturing rules, affect the transitioning between laminates placed in neighbouring patches.

1. *Continuity/blending* ensures manufacturability and structural integrity of the laminated composite. In this work, the definition of generalised blending [35] is used. Generalised blending requires all plies in a thinner patch to continue in the

i^*	0 deg
i'_1	90 deg
i'_2	45 deg
i'_3	-45 deg
i^{*+1}	0 deg

Figure 3.2: Layer indexing relevant to the contiguity constraint for an indicative subset of generic layers.

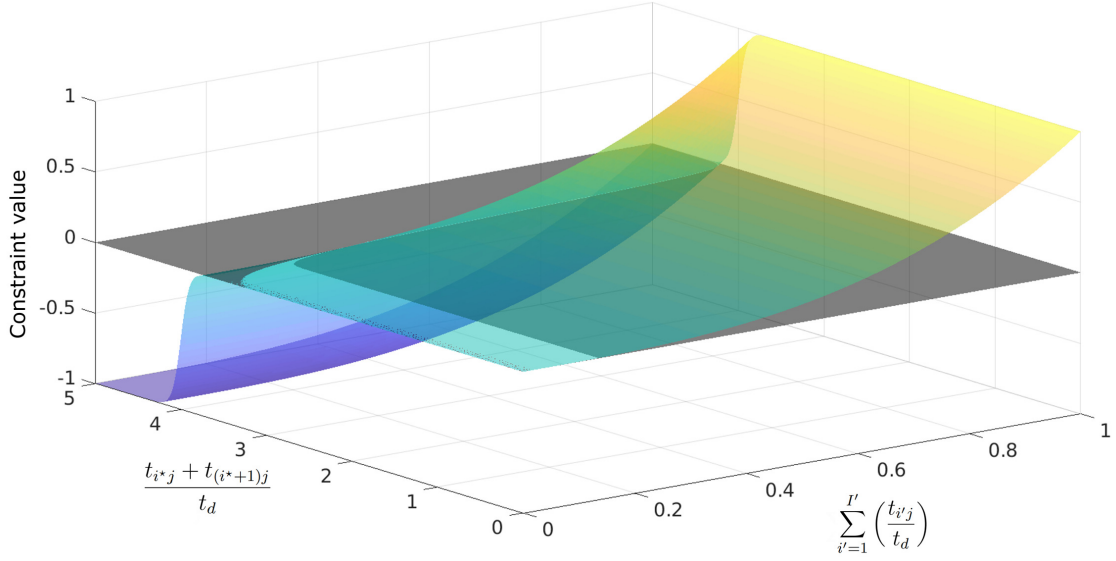


Figure 3.3: Plot of the contiguity constraint value as a function of the total thickness of two “consecutive” θ_c oriented plies and the total thickness of all plies placed between the two “consecutive” θ_c oriented plies. The plot has been generated using $M = 4$, $n = 2$, $k = 10$ and $c = 0.2$.

adjacent thicker panels. In this first stage of the optimisation this is formulated with respect to the continuous thickness of the generic layers. The mathematical expression for two neighbouring patches j_1 and j_2 is:

$$\begin{aligned}
 &\text{if } \sum_{i=1}^I t_{ij_1} \leq \sum_{i=1}^I t_{ij_2} \\
 &\quad \text{then } t_{ij_1} \leq t_{ij_2} \quad \forall i \in I \\
 &\text{else } \sum_{i=1}^I t_{ij_1} > \sum_{i=1}^I t_{ij_2} \\
 &\quad \text{then } t_{ij_1} > t_{ij_2} \quad \forall i \in I
 \end{aligned} \tag{3.8}$$

The above formulation can only be applied when patches j_1 and j_2 have the same generic stack. In the case of different number of generic plies or stacking sequence of these generic plies, the formulation must be altered to that of Eq.

5.4. The formulation of blending presented in Eq. 3.8 is non-differentiable for $\sum_{i=1}^I t_{ij_1} = \sum_{i=1}^I t_{ij_2}$. An alternative differentiable formulation is:

$$1 - \left(\frac{\sum_{i=1}^I t_{ij_2}}{\sum_{i=1}^I t_{ij_1}} \right)^{2 \left(\frac{t_{ij_1}}{t_{ij_2}} - 1 \right)} \geq 0 \quad \forall i \in I \quad (3.9)$$

2. According to the *maximum dropping* manufacturing rule, the maximum number of plies dropped in transitions between neighbouring laminates j_1 and j_2 is limited, to assist smooth load distribution throughout the structure. On a lamina level this is formulated as:

$$t_l - |t_{ij_1} - t_{ij_2}| \geq 0 \quad \forall i \in I \quad (3.10)$$

and on a laminate level

$$t_s - \left| \sum_{i=1}^I t_{ij_1} - \sum_{i=1}^I t_{ij_2} \right| \geq 0. \quad (3.11)$$

In equations 3.10 and 3.11 t_l and t_s are the absolute allowable differences in thickness for a ply and stack respectively.

3. According to the *external covering ply* guideline, at least one of the outer plies in a laminate is not allowed to be dropped to ensure structural integrity. This manufacturing rule can be enforced by setting the minimum gauge of the corresponding design variable to the thickness of the pre-impregnated unidirectional tape.

The complete optimisation problem is therefore formulated as:

Objective	$\min_{0 < t_{ij} \leq t_{ij}^u} \sum_{j=1}^J \sum_{i=1}^I t_{ij} S_j \rho$
Buckling	$\frac{N_x, cr}{N_x} - 1 \geq 0$
Strength	$\left \frac{\varepsilon_{1T}^u}{\varepsilon_1} \right - 1 \geq 0, \quad \left \frac{\varepsilon_{1C}^u}{\varepsilon_1} \right - 1 \geq 0$
Minimum percentage	$\sum_{i'=1}^{I'} t_{i'j} - p \sum_{i=1}^I t_{ij} \leq 0$
Contiguity	$\sum_{i'=1}^{I'} \left(\frac{t_{i'j}}{t_d} \right)^n - \frac{1}{2} - \frac{1}{2} \tanh \left(k \left(\frac{t_{i^*j} + t_{(i^*+1)j}}{t_d} \right) - M - c \right) \geq 0$
Continuity	$1 - \left(\frac{\sum_{i=1}^I t_{ij_2}}{\sum_{i=1}^I t_{ij_1}} \right)^{2 \left(\frac{t_{ij_1}}{t_{ij_2}} - 1 \right)} \geq 0$
Maximum dropping	$t_l - t_{ij_1} - t_{ij_2} \geq 0, \quad t_s - \left \sum_{i=1}^I t_{ij_1} - \sum_{i=1}^I t_{ij_2} \right \geq 0$

(3.12)

3.4 Results

In this section, the design space captured by different generic stacks is plotted in the lamination parameter design. A variety of initialisation schemes and generic stacks comprised of different generic layers are also applied to two simple demonstrator cases. The influence these have on the converged optimisation result is observed. The influence that the composite constraints have on the resulting generic stacks is also demonstrated.

3.4.1 Convergence and parametrisation

Parametrising the properties of the laminate employing generic stacks introduces a dependency of the stiffness on the modelling choices performed. For the flexural stiffness of a laminate, this dependency does not introduce complications in the case that the fibre orientations that will be used during manufacturing are also applied in at least one of the generic layers in the stack. However, the bending stiffness of the laminate does not only depend on the fibre orientations of the generic layers, but also on the number of these layers and their exact sequence across the stack. A visualisation of the stiffness design space that different generic stacks lead to is presented in Fig. 3.4. The stiffness is visualised as a function of the ξ_{1D} and ξ_{3D} lamination parameters to allow for a comparison with similar works from the literature [97]. The number of design variables that affect the thickness and stiffness of a generic stack depends on whether symmetry and balance constraints are used or not, since these are applied by linking the appropriate generic layer thicknesses as explained in section 3.1. It is assumed that each design variable can take any value contained in a vector of evenly spaced thicknesses. These are normalised from 0 to 1 since the absolute magnitude does not affect the dimensionless lamination parameters. In order to populate the design space, the design points are generated by calculating the Cartesian product between the design variables and the thickness vector. Initially, a $[45, -45, 90, 0]_s$ stack is used for which symmetry and balance are assumed, leading to a total of 3 design variables. The design space is very sparsely populated as seen in Fig. 3.4a, partly due to the fact that only 11 evenly spaced thicknesses are considered. When increasing the size of the thickness vector to include 31 evenly spaced thicknesses, the design space is further expanded (Fig. 3.4b). The value of ξ_{3D} increases as the number of 0° and 90° plies increases and decreases for a growing thickness of $\pm 45^\circ$ plies. However, plies placed further away from the mid-plane contribute more significantly to this behaviour and therefore the design space towards the upper bound of ξ_{3D} is less populated and cannot be captured with laminates that satisfy the minimum and maximum percentage

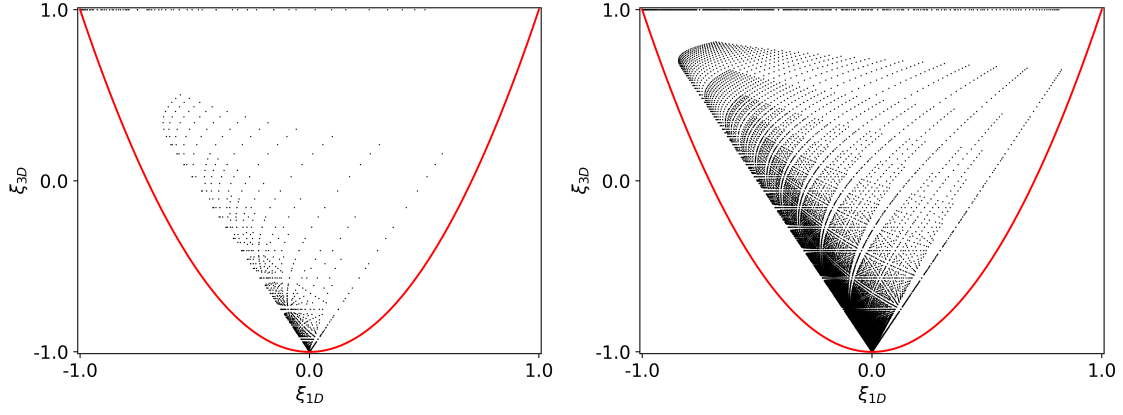
design rules. The opposite behaviour is observed in Fig. 3.4c when the order of the layers in the generic stack is flipped to $[0, 90, -45, 45]_s$. In Fig. 3.4d the design space is expanded to include $\pm 30^\circ \pm 60^\circ$ plies in the available design variables and as expected the design space expands towards the feasible lamination parameter design space which is plotted in red. Finally, when examining Figs. 3.4e and 3.4f the drastic increase in the available design space becomes apparent even while using a thickness vector containing only 11 thicknesses. The previously mentioned limitations of capturing stiffnesses towards the more extreme areas of the design space are lifted due to the ability of placing fibres across more locations in the stack. Plots with larger generic stacks are omitted because it becomes impossible to recognise individual design points.

As mentioned before, parameterising the properties of the laminate using generic stacks introduces local minima in the design space. What is more, the choice of number, stacking sequence and initial thickness of the generic plies can also influence the resulting thickness and stiffness of the structure.

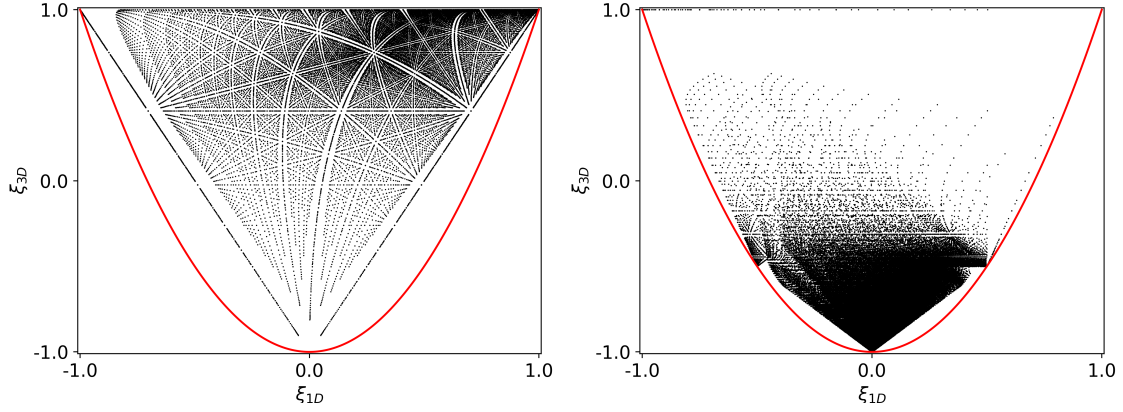
A simple problem consisting of only one panel is examined first. The panel is simply supported along its edges. A graphite-epoxy material (IM7/8552) is used. The modulus of elasticity across the fibre direction is $E_1 = 141GPa$, the modulus of elasticity in the transverse direction is $E_2 = 9.03GPa$, the shear modulus is $G_{12} = 4.27GPa$, the principal Poisson's ratio is $\nu_{12} = 0.32$ and the density is $\rho = 1.572 \times 10^{-6} Kg/mm^3$. The dimensions of the panel and the compressive loads applied are noted in Fig. 3.5a. Buckling and strength constraints are taken into consideration, initially examining only the first load case of Fig. 3.5a. The maximum strain criterion of equation 3.4 is used as a strength constraint. Instead of using the ultimate strain value for the material, a conservative value of the maximum allowable strain ($\varepsilon_{1T}^u = 5500\mu\varepsilon$ and $\varepsilon_{1C}^u = 3000\mu\varepsilon$) taking damage tolerance into consideration is used.

The stacking sequence of the generic stack is $[(45, -45, 90, 0)_4]_s$, combining to a total of 32 generic layers. Symmetry and balance composite constraints are enforced with design variable linking, leading to a total of 12 design variables. Three different schemes, as shown in Table 3.1, are used to examine the influence of the design variable initialisation on the result.

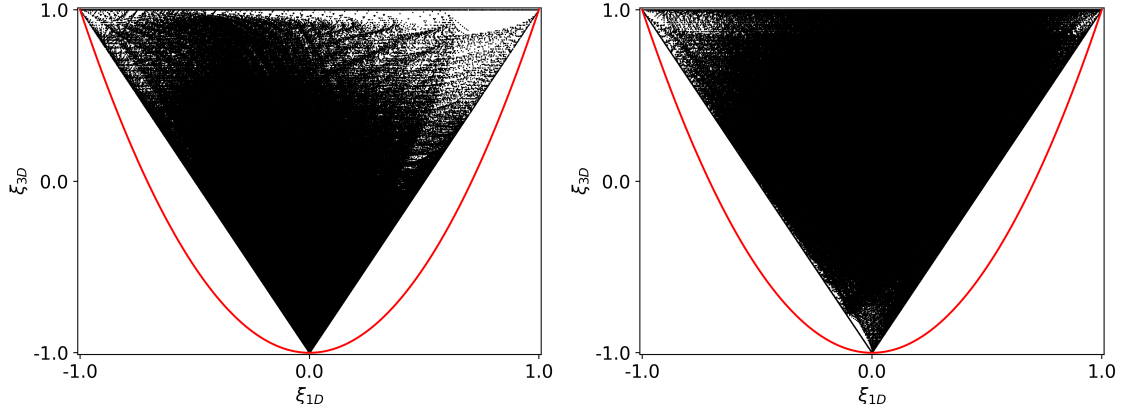
In order to visualise the results, the polar representation of the stiffness used by Dillinger et al. [93] and Bordogna et al. [137] is used. The thickness normalised elastic modulus of elasticity rotated 360 degrees around the reference axis of the



(a) $[45, -45, 90, 0]_s$ generic stack using 11 evenly spaced thicknesses between $[0, 1]$. (b) $[45, -45, 90, 0]_s$ generic stack using 31 evenly spaced thicknesses between $[0, 1]$.



(c) $[0, 90, -45, 45]_s$ generic stack using 31 evenly spaced thicknesses between $[0, 1]$. (d) $[\pm 45, \pm 30, \pm 60, 90, 0]_s$ generic stack, enforcing symmetry and balance using 11 evenly spaced thicknesses between $[0, 1]$.



(e) $[(45, -45, 90, 0)_2]_s$ generic stack, enforcing symmetry and balance using 11 evenly spaced thicknesses between $[0, 1]$. (f) $[(0, 90, -45, 45)_2]_s$ generic stack, enforcing symmetry and balance using 11 evenly spaced thicknesses between $[0, 1]$.

Figure 3.4: Stiffness plot of different generic stacks on the lamination parameter design space. The design points generated are an outcome of the Cartesian product between the design variables associated with the generic stack and a vector of normalised thicknesses ranging from $[0, 1]$. The number of design variables for each generic stack depends on the number of generic layers and whether symmetry and balance are enforced. The red curve in all the plots represents the feasible design space of the lamination parameters.

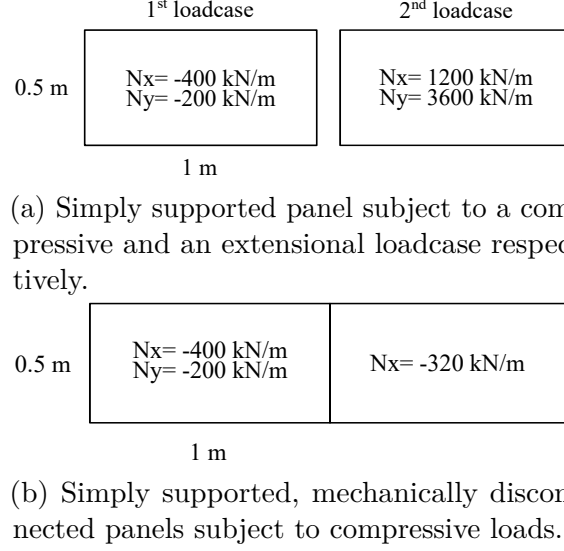


Figure 3.5: Geometry and loading conditions of the panel arrangements.

Scheme	Initial thickness(mm)			Final mass(Kg)	
	0°	90°	±45°	One load case	Two load cases
1	0.3	0.3	0.3	7.263	7.314
2	0.3	0.3	0.15	7.263	7.314
3	0.15	0.15	0.3	7.262	7.314

Table 3.1: Three different initialisation schemes applied to each fibre orientation in the generic plies.

laminate is calculated as:

$$\hat{E}_{11}(\theta) = \frac{1}{\hat{A}_{11}^{-1}(\theta)} \quad (3.13)$$

The thickness normalised inverse $\hat{A}^{-1}$ matrix as a function of any angle θ is given by:

$$\hat{A}^{-1}(\theta) = \mathbf{T}^T \hat{\mathbf{A}} \mathbf{T} \quad (3.14)$$

where

$$\hat{\mathbf{A}} = \frac{\mathbf{A}}{h} \quad (3.15)$$

and $\mathbf{T}$ is the transformation matrix defined in Eq. 2.8.

In equation 3.13, $\hat{A}_{11}^{-1}(\theta)$ is used to visualise the in-plane stiffness. Similarly, the out-of-plane stiffness can be visualised by using $\hat{D}_{11}^{-1}(\theta)$ with the thickness normalised bending stiffness matrix calculated as:

$$\hat{\mathbf{D}} = \frac{12\mathbf{D}}{h^3}. \quad (3.16)$$

The resulting in-plane stiffness shown in Fig. 3.6a is significantly different for each initialisation scheme. On the contrary, out-of-plane stiffnesses (Fig. 3.6b) are identical for all three schemes. Although the maximum mass difference is at the

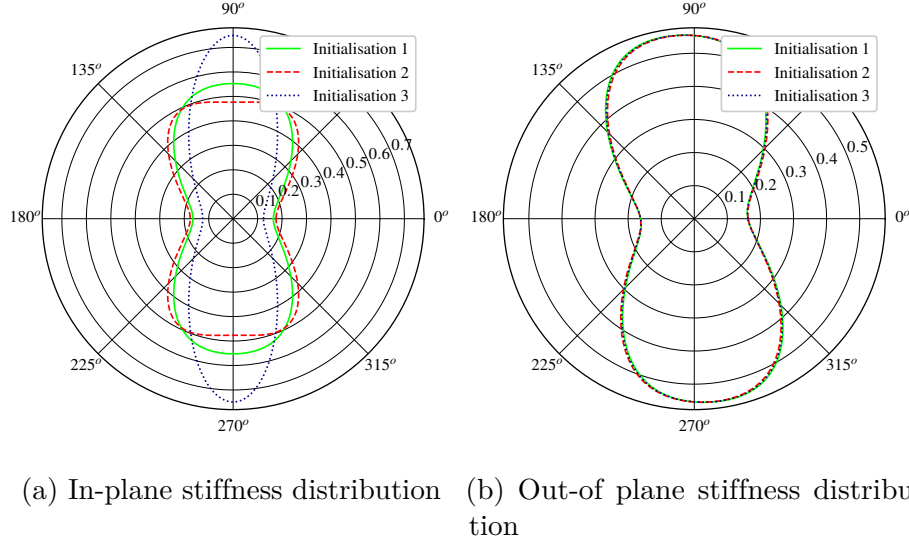


Figure 3.6: Comparison of the stiffness distributions for a panel consisting of 32 generic plies subject to compressive loads for different thickness initialisation schemes. Only buckling constraints are applied in this instance.

order of 0.01% (Table 3.1), the gradient-based optimisation has been trapped in different local minima. The different in-plane stiffnesses indicate different ply-shares between the four fibre orientations. However, since the design is only driven by buckling, the optimiser has computed identical out-of-plane stiffnesses by influencing the distribution of the ply-shares across the thickness of the laminate.

Although local minima due to different initial thicknesses do arise due to the non-convex design space of generic stacks, this drawback is mitigated for real-world problems. When designing aircraft structures, hundreds or thousands of load cases are taken into consideration, depending on the maturity of the design stage, leading to a huge criteria model. The criteria model, consisting of millions of constraints emerging from the design-driving requirements such as strength, damage tolerance, buckling, manufacturing, but also eigenmodes, flutter and aeroelastic effectiveness, is heavily restricting the design space and eliminates, from a practical point of view, the majority of the non-sensible local minima.

To highlight this effect, a second load case of extensive forces along the edges of the panel is also included in the study, as seen in Fig. 3.5b, to capture the sizing of the panel with respect to strength constraints. Examination of the stiffness distributions for the same three initialisation schemes shows identical in-plane stiffness and slightly different out-of-plane stiffness for the first scheme as shown in Figs. 3.7a and 3.7b respectively. The masses are equal for all three schemes.

The number of generic plies used also influences the computed design. This is

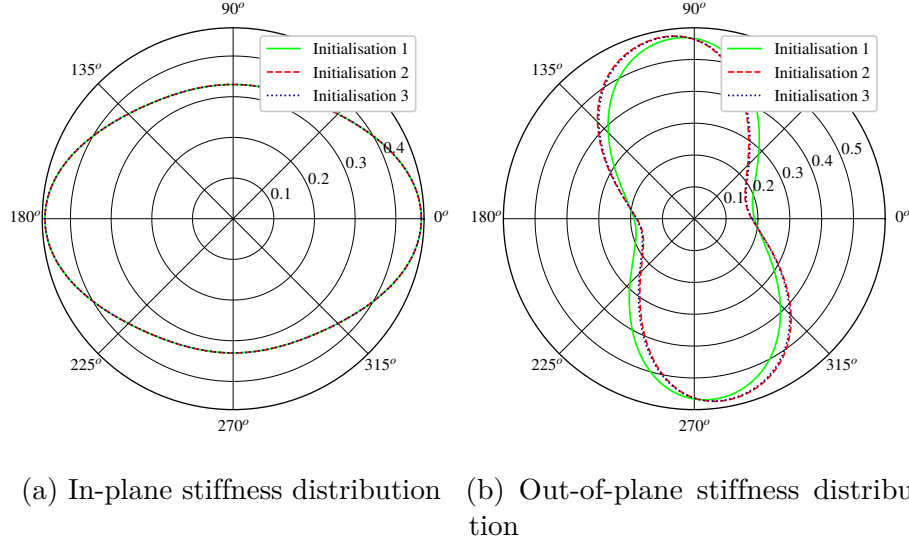


Figure 3.7: Comparison of the stiffness distribution for a panel consisting of 32 generic plies subject to compressive and extensional loads for different thickness initialisation schemes. In-plane stiffness distributions are now identical for all different initialisations due to sizing effects from both buckling and strength criteria.

demonstrated by optimising the same demo-problem subject to the two load cases and using a different number of the same repetitive generic blocks ($[(45, -45, 90, 0)]$) which form a laminate of 8, 16 and 32 generic layers respectively. Already when using 16 generic plies, the in-plane stiffness of the plate converges to the optimal solution (Fig. 3.8a) while the out-of-plane stiffness slightly differs between the case of 16 and 32 generic plies as seen in Fig. 3.8b. Experience gained from application of the two-stage process in many different instances indicates that a laminate which is expected to be approximately N layers thick, can be adequately modelled by a generic stack comprised of $N/4$ generic layers. This modelling guideline is not effective when applying the contiguity manufacturing constraint, where a larger number of generic plies needs to be introduced.

An additional factor that may influence the resulting membrane and bending stiffness of the laminate is the stacking sequence of the generic block repeated to form the generic stack. In Figs. 3.9a and 3.9b, the in-plane and out-of-plane stiffness distributions are plotted for the panel when using the $[(0, 90, 45, -45)]$ generic block instead of the one applied for the previous example ($[(45, -45, 90, 0)]$). For each of the blocks, 2 generic stacks are formed, one consisting of 8 generic plies and the other of 16. The resulting stiffnesses of the laminates formed by 8 generic plies are different, which is to be expected since the stiffnesses are also different for the case of 8 and 16 generic layers (Fig. 3.8). The influence of the repeated generic block on the stiffness ceases when forming a generic stack with 16 layers, which supports the

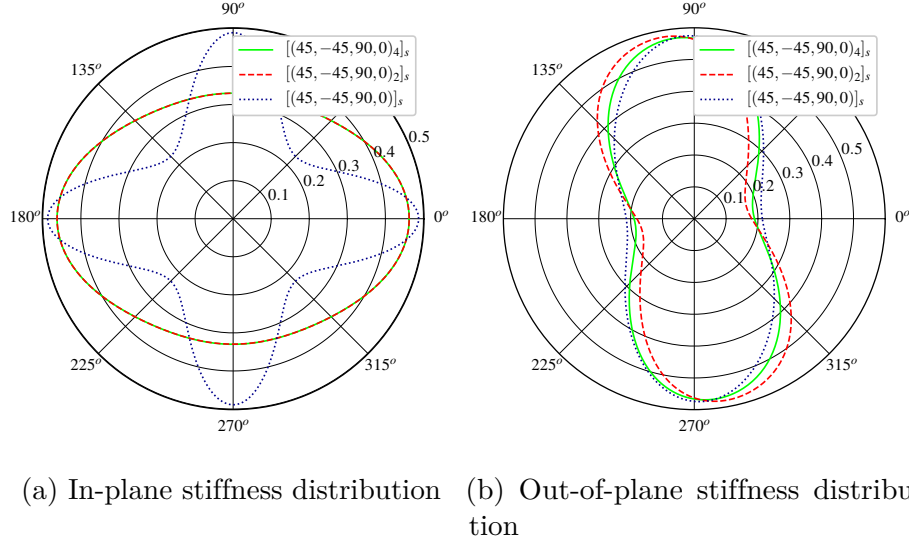


Figure 3.8: Comparison of the stiffness distributions for a panel consisting of 8, 16 and 32 generic plies subject to compressive and extensional loads. Both buckling and strength constraints are applied during the optimisation.

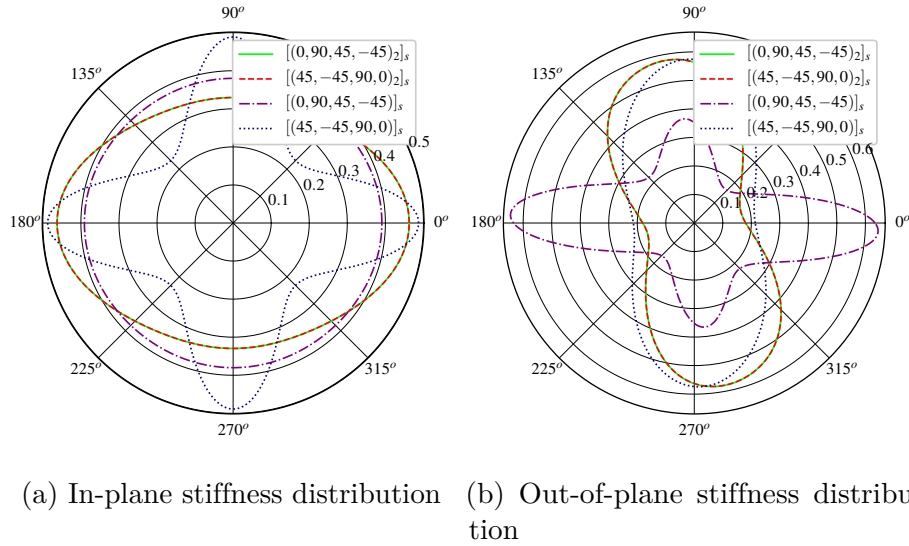
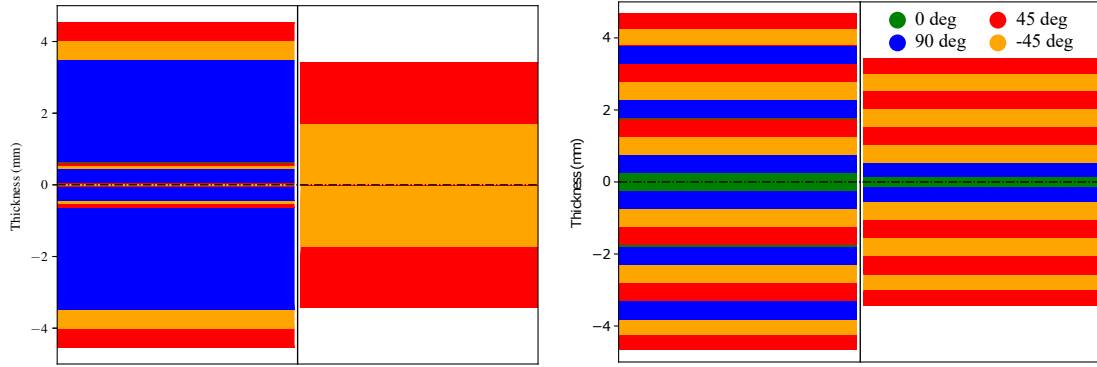


Figure 3.9: Comparison of the stiffness distributions for a panel comprised of 2 different generic blocks when forming a generic stack of 8 and 16 plies, subject to compressive and extensional loads. Both buckling and strength constraints are applied during the optimisation.

previous stiffness sensitivity results with respect to the number of generic plies.

3.4.2 Influence of the continuity and contiguity constraints

Finally, the influence of the continuity and contiguity constraints on the resulting thickness and stiffness of the structure is reported. A second patch is added to the demonstrator problem to enable the application of the continuity constraint.

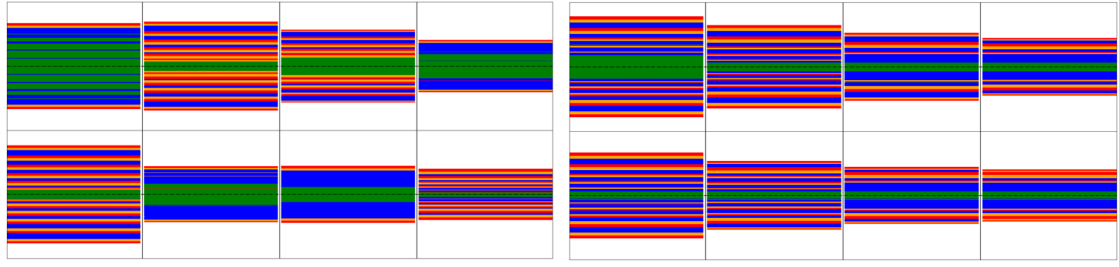


(a) Symmetry and balance design guidelines are enforced (b) Symmetry, balance, contiguity and blending composite guidelines are enforced

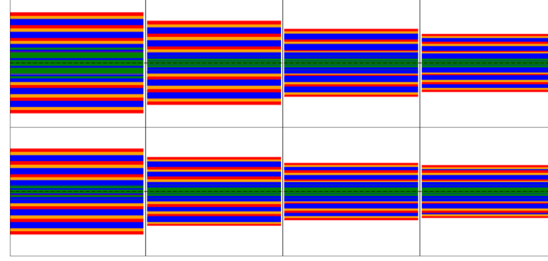
Figure 3.10: Resultant generic stack thickness distribution for the demonstrator problem of Fig. 3.5b.

The two patches are not mechanically connected so no load redistribution is taken into account. Only the continuity constraint couples the design variables of the two patches. A single buckling load case is considered, with the exact loads shown in Fig. 3.5b.

In Fig. 3.10a the resulting generic stack of an optimisation not using any composite design or manufacturing rules, besides symmetry and balance, is depicted. The optimum ply-share between the different fibre angles and their distribution across the stack is vastly different. If the contiguity and blending rules are also activated, then this leads to a significantly different structural solution whose stiffness distribution is homogenised (Fig. 3.10b). As for the application of the contiguity constraint, the assumed discrete ply thickness is 0.125mm and the maximum number of consecutive plies allowed equals 4. The results from another example of 8 mechanically disconnected panels arbitrarily loaded in order to force significantly different stiffness distributions amongst them is presented in Fig. 3.11. In Fig. 3.11a, the resulting thickness distribution of the generic stack is shown when only symmetry and balance design rules are applied. When the blending constraint is also activated, the stiffness distribution is smoothed across all patches (Fig. 3.11b). The inclusion of the contiguity constraint on top of the other constraints, leads to areas with multiple consecutive 0 degree fibres to be broken down into smaller blocks, by insertion of 90 degree layers as seen in Fig. 3.11c.



(a) Application of no manufacturing constraints besides symmetry and balance. (b) Application of symmetry, balance and blending constraints.



(c) Application of symmetry, balance, continuity and contiguity constraints.

Figure 3.11: Resultant generic stack thickness distribution for an arbitrary problem consisting of 8 rectangular patches arranged in a 4×2 grid.

3.5 Discussion

In this chapter, the modelling of the stiffness properties of the structure using generic stacks in the framework of a gradient-based optimisation is presented. The emphasis in this stage should be to formulate as many composite guidelines in the continuous optimisation as possible. The design space of generic stacks allows for more composite rules to be implemented, since it is a direct representation of the stack's sequence characteristics. Moreover, these formulations are more precise compared to the other modelling approaches, such as, lamination and polar parameters. Blending, in particular, can be exactly described in its continuous form. However, generic stacks do form a non-convex design space which is not welcome in any gradient-based optimisation. The influence of different starting points in the retrieved result is expected to be eliminated by the diversity of factors sizing real-world composite structures. The effect of the number of generic plies on the final stiffness distribution is also investigated, indicating that even for industrial scale structures which are expected to be thicker, the size of the continuous optimisation will still remain manageable.

Stacking sequence retrieval

The outcome of the first stage of the optimisation is a continuous thickness and stiffness distribution of the structure. The entire thickness of each patch is usually rounded up to the nearest integer or even number of plies which can be determined based on the thickness of the uni-direction CFRP tape that will be used during manufacturing. Therefore, the thickness of each patch remains constant during the second stage of the optimisation and the objective of it is deriving a discrete stacking sequence which best matches the target stiffnesses calculated in the first stage, while also satisfying all composite design and manufacturing rules. The computed stacking sequences must then be transferred into the FEM of the first stage so that an analysis can be performed to ensure that the mechanical constraints which were not considered during the second stage are still satisfied.

As mentioned in Chapter 2, the second stage of the optimisation is usually performed using a stochastic zero-order algorithm [108, 113, 123, 36]. In this work the composite manufacturing rules were formulated as two different Mixed Integer

This chapter is based on the paper, *Mixed Integer Linear Programming formulations of the stacking sequence and blending optimisation of composite structures* by G. Ntourmas, F. Glock, F. Daoud, G. Schuhmacher, D. Chronopoulos and E. Özcan which appeared in the Composite Structures journal, 264 (2021) 113660. Note: symbols may have been changed to maintain consistency throughout this thesis.

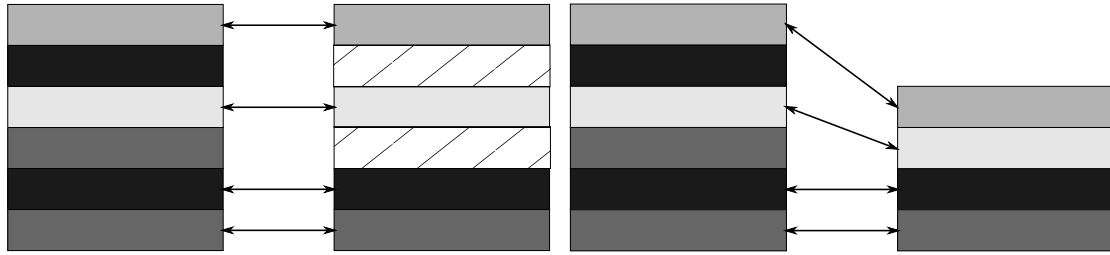
Linear Programming (MILP) instances. Although potential structural engineering applications of operational research have been identified [138], a complete MILP formulation of the stacking sequence optimisation has not yet been presented to the best of the author’s knowledge with the exception of some composite design rules which have been previously formulated in such a way [126]. Additionally, Kang and Blom [125] have worked towards that direction, having however treated the blending rules in the context of compliance with pre-computed laminates, which is not the case for the present study. Other studies focusing on tree structures, branch and bound algorithms and beam search algorithms [139, 140, 122, 7, 141, 121, 26], are algorithmic approaches built around the specific nature of the stacking sequence optimisation problem in question. In this work, two complete formulations accounting for all composite design and manufacturing rules which can be used in any optimisation solver able to handle a MILP problem formulation are presented.

This chapter is structured as follows: In section 4.1 the two MILP formulations namely explicit and implicit are introduced, followed by a decomposition strategy developed for the implicit formulation. In section 4.2 results on the convergence of the two formulations and the decomposition strategy are shown for various optimisation problem instances and then the developed second stage of the optimisation is compared against relevant algorithms from the literature. Moreover, results from the complete two-stage process are presented to highlight the impact of the blending constraint on the solution and also compare the developed two-stage approach with other relevant approaches found in the literature. Finally, in 4.3 the findings and conclusions made in this chapter are summarised.

4.1 Mixed Integer Linear Programming formulations

The input for the stacking sequence calculation is a simple geometric definition of the patches, together with the optimal number of the total layers and lamination parameters for each patch. The MILP formulation of this problem is discussed in this section. Two distinct formulations of the blended stacking sequence optimisation, namely explicit and implicit, have been developed. As far as the design rules are concerned, their expression does not fundamentally differ between the two formulations. The distinctive difference between the two is how blending and ply drops are modelled. Nevertheless, both the explicit and implicit formulation make use of the same principles of generalised blending described in section 2.1.2. Therefore, the resulting structure from each formulation is expected to be the same if the same input and constraints are used for each optimisation.

In brief, the explicit formulation considers all patches having the same number of layers, with that number being equal to the number of layers in the thickest patch of the structure. Since each patch has a different optimum number of layers, some of the patches will consist of plies that correspond to ply-voids as displayed in Fig. 4.1a. The implicit formulation demonstrated in Fig. 4.1b takes into account the fact that each patch has a different, known number of optimum layers, and defines the appropriate number of layers for each individual patch. The benefit of the explicit formulation is that the blending of the structure is rather straightforward, as continuity is ensured by placing plies of the same orientation one next to the other. The disadvantage of this formulation is that the locations of the ply drops are not known and therefore there are many possible “positions” one specific ply could have within the laminate. Therefore, since the order at which plies will appear is not known, the calculation of the lamination parameters for such a stack is cumbersome. The advantage of the implicit formulation is that the calculation of lamination parameters is straightforward as the position of each ply is clearly defined. The downside of the method is the fact that formulating the manufacturing rules and most importantly the continuity of the structure is more challenging.



(a) Schematic representation of the explicit formulation. (b) Schematic representation of the implicit formulation.

Figure 4.1: An illustrative example of the same blended stack for two neighbouring patches as represented by the explicit and implicit formulation. Each colour represents a different fibre orientation, ply-voids are illustrated using hatched plies and arrows indicate ply continuity.

4.1.1 Explicit formulation

Various design variables are introduced in order to formulate the blended stacking sequence problem. Each set of design variables, i.e. $x, y, b, c, d, z, u, g, f$ contains many individual members. The number of members for each set depends on the usage of the design variable and different indexes are used to distinguish between the individual members of each set of design variables. Index $i \in \{1, 2, \dots, I\}$ is used to denote the exact layer in a specific patch $j \in \{1, 2, \dots, J\}$. It is important to mention that the range of i is constant for all patches j , even if most of the patches are expected to have less layers than I which is the number of layers for the thickest

patch in the structure. Index $\theta \in \{1, 2, \dots, \Theta\}$ denotes the different available fibre orientations. Finally, $k \in \{1, 2, 3, 4\}$ distinguishes between the lamination parameter which is of interest. A summary of the indexes employed to differentiate between the members of each design variable set is given in Table 4.1.

Name	Description
i	Layer
j	Patch
θ	Fibre orientation
k	Lamination parameter

Table 4.1: Summary of indexes used to denote the various design variables in the explicit formulation.

The objective is to minimise the deviation from the optimal values of the continuous stiffness. The continuous thickness distributions of the generic stacks are translated into lamination parameters using Eqs. 2.18-2.20. The reason for using lamination parameters in the second stage of the optimisation is to maintain consistency between this work and other relevant academic studies so that results can be directly compared. It is worth mentioning that instead of minimising the absolute difference between the lamination parameters of the first and second stage of the optimisation, another possibility is to minimise the traces of the products of the flexural and bending stiffness matrices between adjacent laminates [142, 143]. The former objective function is preferred in this work because, especially for the case of the explicit formulation, the computation of the objective function is cumbersome due to fact that the positions of the layers across the laminate are not known. Therefore, the objective aiming to minimise the deviation between the resulting lamination parameters of the first and second stage is formulated as:

$$\min_{-1 \leq \xi_{kj}^{A,B,D} \leq 1} \sum_{j=1}^J w_k^{A,B,D} |\xi_{kj}^{A,B,D} - (\xi_{kj}^{A,B,D})_{\text{optimal}}| \quad \forall k \quad (4.1)$$

where $\xi_{kj}^{A,B,D} \in [-1, 1]$ are the lamination parameters calculated for a specific stack, whereas, $(\xi_{kj}^{A,B,D})_{\text{optimal}}$ are the optimal parameters delivered by the gradient-based optimisation. Weight coefficients $w_k^{A,B,D}$ may be used to emphasise specific parts of the objective function. In all numerical examples presented in this thesis, the value of the weight coefficients is set to $w_k^{A,B,D} = 1$. Two new sets of design variables need to be introduced to assist with the linear formulation of the objective function. The first one is $g_{kj}^{A,B,D} \in [0, 1]$, defined as $g_{kj}^{A,B,D} = |\xi_{kj}^{A,B,D} - (\xi_{kj}^{A,B,D})_{\text{optimal}}|$. The objective function is not linear in its current form, because of the absolute value, therefore, a simple linearisation needs to be performed. The additional set of design variables $f_{kj}^{A,B,D} \in \{0, 1\}$, is also introduced to enable the linearisation described by

the following set of equations

$$\begin{aligned}
 \xi_{kj}^{A,B,D} - (\xi_{kj}^{A,B,D})_{\text{optimal}} &\leq f_{kj}^{A,B,D} \quad \forall j, k \\
 (\xi_{kj}^{A,B,D})_{\text{optimal}} - \xi_{kj}^{A,B,D} &\leq 1 - f_{kj}^{A,B,D} \quad \forall j, k \\
 \xi_{kj}^{A,B,D} - (\xi_{kj}^{A,B,D})_{\text{optimal}} &\leq g_{kj}^{A,B,D} \quad \forall j, k \\
 (\xi_{kj}^{A,B,D})_{\text{optimal}} - \xi_{kj}^{A,B,D} &\leq g_{kj}^{A,B,D} \quad \forall j, k \\
 g_{kj}^{A,B,D} &\leq \xi_{kj}^{A,B,D} - (\xi_{kj}^{A,B,D})_{\text{optimal}} + 2(1 - f_{kj}^{A,B,D}) \quad \forall j, k \\
 g_{kj}^{A,B,D} &\leq (\xi_{kj}^{A,B,D})_{\text{optimal}} - \xi_{kj}^{A,B,D} + 2f_{kj}^{A,B,D} \quad \forall j, k
 \end{aligned} \tag{4.2}$$

Therefore, the objective function can now be formulated with respect to the $g_{kj}^{A,B,D}$ design variables as:

$$\min_{0 \leq g_{kj}^{A,B,D} \leq 1} \sum_{j=1}^J w_k^{A,B,D} g_{kj}^{A,B,D} \quad \forall k \tag{4.3}$$

To make the formulation of the problem possible, the binary design variables $x_{ij\theta} \in \{0, 1\}$ defining the orientation of a specific layer within a certain patch are introduced. More specifically, every layer consists of Θ design variables, each one representing whether the corresponding fibre orientation is used or not. Each layer may only have one orientation, or none, in the case the ply is dropped. Therefore, the following feasibility constraints need to be defined.

$$\sum_{\theta=1}^{\Theta} x_{ij\theta} \leq 1 \quad \forall i, j \tag{4.4}$$

Additionally, the number of layers in a patch must be equal to the optimum discretised thickness (n^j) which originates from the continuous optimisation and remains fixed throughout the discrete optimisation.

$$\sum_{i=1}^I \sum_{\theta=1}^{\Theta} x_{ij\theta} = n^j \quad \forall j \tag{4.5}$$

In many cases, laminates are designed to be symmetric. Whenever symmetry is required, the following constraints need to be incorporated in the optimisation model.

$$x_{ij\theta} = x_{(I-i+1)j\theta} \quad \forall i \in \{1, 2, \dots, I/2\}, j, \theta \tag{4.6}$$

Balanced laminates can be extracted from the optimisation by including the following constraints.

$$\sum_{i=1}^I x_{ij\theta_1} = \sum_{i=1}^I x_{ij\theta_2} \quad \forall i, j, \theta_1 = -\theta_2 \neq \{0, 90\} \tag{4.7}$$

The minimum percentage (p) design rule is implemented as:

$$\sum_{i=1}^I x_{ij\theta} \geq pn^j \quad \forall j, \theta \tag{4.8}$$

The damage tolerance rule is demonstrated below for the case that the outermost ply is equal to 45° . This layer also has to be continuous over all patches according to the external covering ply manufacturing rule. The following constraints fulfil both requirements.

$$x_{1j\theta_1} = 1 \quad \forall j, \theta_1 = 45 \quad (4.9)$$

The grouping design rule for the case of symmetric laminates is formulated as:

$$x_{ij\theta_1} \leq x_{(i+1)j\theta_2} \quad \forall i \in \{1, 2, \dots, I/2\}, j, \theta_1 = -\theta_2 \neq \{0, 90\} \quad (4.10)$$

For non symmetric laminates, the i range needs to be modified accordingly. The contiguity and disorientation constraints cannot be easily implemented in the explicit formulation and are hence only going to be presented for the implicit formulation in the next section.

In order to define the continuity manufacturing constraint between neighbouring patches, further design variables need to be introduced. A set of them is $y_{ij} \in \{0, 1\}$, defining the presence or absence of a specific ply within a patch. The continuity constraint is expressed as:

$$x_{ij_2\theta} = x_{ij_1\theta} y_{ij_2} \quad \forall i, j, \theta. \quad (4.11)$$

Because the constraint presented above is non-linear, it needs to be linearised in order to be handled by a MILP solver. The linearisation constraints for the product of two binary variables [144] are now introduced.

$$\begin{aligned} x_{ij_2\theta} &\leq x_{ij_1\theta} \quad \forall i, j, \theta \\ x_{ij_2\theta} &\leq y_{ij_2} \quad \forall i, j, \theta \\ x_{ij_2\theta} &\geq x_{ij_1\theta} + y_{ij_2} - 1 \quad \forall i, j, \theta \end{aligned} \quad (4.12)$$

In the above equation, patches j_1 and j_2 denote two neighbouring patches within the structure. An additional constraint which is not necessary for the proper formulation of the optimisation problem, but has been observed to assist the convergence of the optimisation, is used to define the number of ply drops per patch based on the known number of layers for each patch. The constraint likely aids in the reduction of the search tree.

$$\sum_{i=1}^I y_{ij} = n^j \quad \forall j \quad (4.13)$$

Before moving on to the definition of the rest of the constraints, it is worth formulating the lamination parameter values in a way that serves the specific approach to the problem. The formal definition of lamination parameters has already been presented

in Chapter 2 (equations 2.18-2.20).

$$\xi_{kj}^{A,B,D} = \sum_{i=1}^I \sum_{\theta=1}^{\Theta} s_{k\theta} a_{ij}^{A,B,D} x_{ij\theta} \quad \forall j, k \quad (4.14)$$

Coefficients $s_{k\theta}$ take into consideration the influence of the trigonometric terms, depending on which θ design variable is used out of $x_{ij\theta}$. Additionally, coefficients $a_{ij}^{A,B,D}$ depend on whether A, B or D lamination parameters are considered and on the position of the ply within the laminate. Due to the fact that ply drops are included in the problem formulation as design variables, the distance of each ply from the mid-surface of the laminate is not known *a priori*. Another set of design variables $b_{ij} \in \{0, 1, 2, \dots, M_1\}$ is used for that purpose, where the upper bound is $M_1 = i + 1$. The definition of b_{ij} is as follows:

$$b_{ij} = \sum_{m=1}^i y_{mj} \quad \forall i, j \quad (4.15)$$

In summary, the aforementioned coefficients $a_{ij}^{A,B,D}$ depend on the b_{ij} design variables and are defined as:

$$a_{ij}^A = \frac{1}{n^j} \quad \forall i, j \quad (4.16)$$

$$a_{ij}^B = -\frac{4}{(n^j)^2} b_{ij} + \frac{2n^j + 2}{(n^j)^2} \quad \forall i, j \quad (4.17)$$

$$a_{ij}^D = \frac{12}{(n^j)^3} (b_{ij})^2 - \frac{12n^j + 12}{(n^j)^3} b_{ij} + \frac{3(n^j)^3 + 6n^j + 4}{(n^j)^3} \quad \forall i, j \quad (4.18)$$

The term $(b_{ij})^2$ in equation 4.18 is not linear and therefore needs to be linearised. Due to the fact that the linearisation of a product of two bounded integer variables is cumbersome and involves piecewise linearisations, a trick is employed to calculate $c_{ij} \in \{0, 1, 2, \dots, (M_1)^2\}$ as:

$$(b_{ij})^2 = c_{ij} = \left(\sum_{m=1}^i 2d_{mj} \right) - b_{ij} \quad \forall i, j \quad (4.19)$$

In the above equation, a new set of design variables $d_{ij} \in \{0, 1, 2, \dots, M_1\}$ was introduced. These are defined as $d_{ij} = b_{ij}y_{ij}$ and once again need to be linearised. The linearisation of the product of two variables, one of which is binary and the other an integer with an upper bound of M_1 and a lower bound of 0, is defined by the following four inequalities [145].

$$\begin{aligned} d_{ij} &\leq M_1 y_{ij} \quad \forall i, j \\ d_{ij} &\leq b_{ij} \quad \forall i, j \\ d_{ij} &\geq b_{ij} - (1 - y_{ij})M_1 \quad \forall i, j \\ d_{ij} &\geq 0 \quad \forall i, j \end{aligned} \quad (4.20)$$

Finally, due to the fact that the product of $a_{ij}^{B,D} x_{ij\theta}$ appears in equation 4.14, two new sets of design variables $u_{ij\theta} \in \{0, 1, 2, \dots, M_1\}$, $z_{ij\theta} \in \{0, 1, 2, \dots, (M_1)^2\}$ need to be introduced and are defined as $u_{ij\theta} = b_{ij} x_{ij\theta}$ and $z_{ij\theta} = c_{ij} x_{ij\theta}$. The linearisation of these design variables follows the principles of equation 4.20.

The lamination parameter values can eventually be expressed as:

$$\xi_{kj}^B = \sum_{i=1}^I \sum_{\theta=1}^{\Theta} s_{k\theta} \left(-\frac{4}{(n^j)^2} u_{ij\theta} + \frac{2n^j + 2}{(n^j)^2} x_{ij\theta} \right) \quad \forall k, j \quad (4.21)$$

$$\xi_{kj}^D = \sum_{i=1}^I \sum_{\theta=1}^{\Theta} s_{k\theta} \left(\frac{12}{(n^j)^3} z_{ij\theta} - \frac{12n^j + 12}{(n^j)^3} u_{ij\theta} + \frac{3(n^j)^3 + 6n^j + 4}{(n^j)^3} x_{ij\theta} \right) \quad \forall k, j \quad (4.22)$$

The internal covering ply manufacturing constraint is formulated using the b_{ij} design variables as:

$$b_{(i+K)j_1} - b_{(i+K)j_2} - (b_{ij_1} - b_{ij_2}) \leq K - 1 \quad (4.23)$$

where j_1 and j_2 denote two neighbouring patches and K the number of maximum consecutive ply drops. The maximum dropping manufacturing rule is explicitly satisfied due to the fixed number of layers provided as an input to the optimisation and therefore does not require further constraints.

The entire optimisation problem for the explicit formulation is summarised as:

Objective	$\min_{-1 \leq \xi_{kj}^{A,B,D} \leq 1} \sum_{j=1}^J w_k^{A,B,D} \xi_{kj}^{A,B,D} - (\xi_{kj}^{A,B,D})_{\text{optimal}} \quad \forall k$
Feasibility	$\sum_{\theta=1}^{\Theta} x_{ij\theta} \leq 1 \quad \forall i, j$
Thickness match	$\sum_{i=1}^I \sum_{\theta=1}^{\Theta} x_{ij\theta} = n^j \quad \forall j$
Symmetry	$x_{ij\theta} = x_{(I-i+1)j\theta} \quad \forall i \in \{1, 2, \dots, I/2\}, j, \theta$
Balance	$\sum_{i=1}^I x_{ij\theta_1} = \sum_{i=1}^I x_{ij\theta_2} \quad \forall i, j, \theta_1 = -\theta_2 \neq \{0, 90\}$
Minimum percentage	$\sum_{i=1}^I x_{ij\theta} \geq pn^j \quad \forall j, \theta$
Damage tolerance	$x_{1j\theta_1} = 1 \quad \forall j, \theta_1 = 45$
Grouping	$x_{ij\theta_1} \leq x_{(i+1)j\theta_2} \quad \forall i \in \{1, 2, \dots, I/2\}, j, \theta_1 = -\theta_2 \neq \{0, 90\}$
Continuity	$x_{ij_2\theta} = x_{ij_1\theta} y_{ij_2} \quad \forall i, j, \theta$

	$b_{ij} = \sum_{m=1}^i y_{mj} \quad \forall i, j$
Auxiliary	$d_{ij} = b_{ij} y_{ij} \quad \forall i, j$
design	$c_{ij} = \left(\sum_{m=1}^i 2d_{mj} \right) - b_{ij} \quad \forall i, j$
variables	$u_{ij\theta} = b_{ij} x_{ij\theta} \quad \forall i, j$
	$z_{ij\theta} = c_{ij} x_{ij\theta} \quad \forall i, j$
Internal covering ply	$b_{(i+K)j_1} - b_{(i+K)j_2} - (b_{ij_1} - b_{ij_2}) \leq K - 1$
	$\xi_{kj}^A = \sum_{i=1}^I \sum_{\theta=1}^{\Theta} s_{k\theta} \frac{1}{n^j} x_{ij\theta} \quad \forall j, k$
Lamination	$\xi_{kj}^B = \sum_{i=1}^I \sum_{\theta=1}^4 s_{k\theta} \left(-\frac{4}{(n^j)^2} u_{ij\theta} + \frac{2n^j + 2}{(n^j)^2} x_{ij\theta} \right) \quad \forall k, j$
parameters	
computation	$\xi_{kj}^D = \sum_{i=1}^I \sum_{\theta=1}^4 s_{k\theta} \left(\frac{12}{(n^j)^3} z_{ij\theta} - \frac{12n^j + 12}{(n^j)^3} u_{ij\theta} + \frac{3(n^j)^3 + 6n^j + 4}{(n^j)^3} x_{ij\theta} \right) \quad \forall k, j$

Linearisation constraints applied to the problem are omitted to avoid confusion.

4.1.2 Implicit formulation

Similarly to the explicit formulation, various design variables need to be introduced. Each set of design variables, i.e. x, r, t, g, f , contains many individual members. Once again, index $i \in \{1, 2, \dots, I_j\}$ is used to denote the exact layer in a specific patch $j \in \{1, 2, \dots, J\}$. The range of i is not constant for all patches j , but rather takes a maximum value of I_j according to the patch of interest. Index $h \in \{1, 2, \dots, H\}$ accounts for all interfaces between neighbouring patches and index $p \in \{1, 2, \dots, P_h\}$ takes into account the different blending combinations between neighbouring patches. A more detailed explanation of these two indexes will be presented in the following paragraphs. Finally, indices θ and k follow the definition of the explicit formulation and denote fibre orientations and lamination parameter which is of interest respectively.

The formulation of the objective function is exactly the same as the one presented in equation 4.1 and the same goes for the set of design variables $g_{kj}^D \in [0, 1]$ and $f_{kj}^D \in \{0, 1\}$. The definition of the $x_{ij\theta} \in \{0, 1\}$ set of design variables follows the same logic with the exception that in the implicit formulation, each patch is assigned with as many I_j layers as needed. All design rules (equations 4.6-4.10) are formulated

similarly with the only thing changing being the number of maximum layers per patch I_j . An additional minor alternation applies for equation 4.4 in which the inequality sign needs to be replaced by the equality one, since the design variables $x_{ij\theta}$ in this approach cannot correspond to a ply drop.

Additionally, the two design rules which have not been formulated for the explicit approach can now be implemented. The contiguity constraint limits the number of consecutive layers having the same fibre orientation to a maximum of N . In a mathematical formulation this is expressed as:

$$x_{ij\theta} + x_{(i+1)j\theta} + \dots + x_{(i+N)j\theta} \leq N \quad \forall i \in \{1, 2, \dots, I_j - N\}, j, \theta \quad (4.24)$$

The disorientation constraint is expressed as:

$$x_{ij\theta_1} + x_{(i+1)j\theta_2} \leq 1 \quad \forall i, j, |\theta_1 - \theta_2| > 45 \quad (4.25)$$

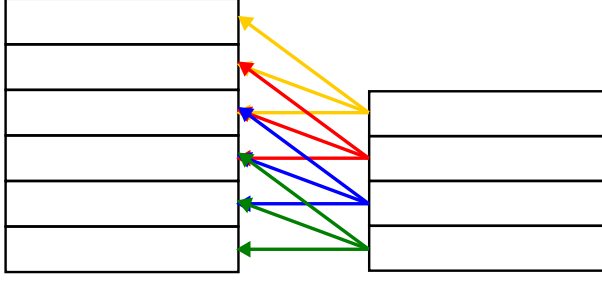
The calculation of lamination parameter values is now trivial. Equation 4.14 is once again used except this time, coefficients $a_{ij}^{B,D}$ can be easily pre-calculated due to the fact that ply drops have already been taken into account by using an exact number (I_j) of $x_{ij\theta}$ design variables for each patch.

The part of this second approach that becomes cumbersome is formulating the continuity between neighbouring patches. A new set of binary design variables $t_{ihp} \in \{0, 1\}$ is introduced and is used to regulate and keep track of whether a specific ply within a patch will be blended with one of the p possible neighbouring plies. The number of possibilities p per ply depends on the interface h examined and can easily be determined during the setup of the optimisation problem. A representation of the different plies that could blend between two patches is given in Fig. 4.2a. More specifically, for any of the interfaces h , the maximum number of possibilities per ply (P_h) can be calculated as:

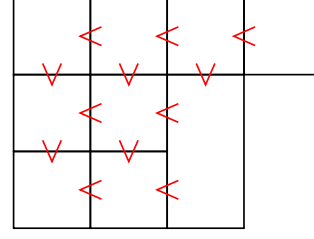
$$P_h = |n_{j_1} - n_{j_2}| + 1 \quad (4.26)$$

In the above equation, patches j_1 and j_2 denote the two patches which constitute an interface h , with j_1 being the thickest of the two, in case there is a difference in thickness between them. The number of interfaces H is easily calculated prior to setting up the optimisation problem based on the arrangement of the patches. In Fig. 4.2b, an example of the interfaces between a specific patch geometry is given.

Finally, a new set of design variables $r_{ih\theta p} \in \{0, 1\}$ is defined as $r_{ih\theta p} = x_{(i+p-1)j_1\theta} t_{ihp}$. Because the definition of $r_{ih\theta p}$ is non-linear, a standard linearisation needs to be performed. This linearisation of the product of two binary variables is similar to the one presented in equation 4.12.



(a) Knowing the number of ply drops between two neighbouring patches, the different blending possibilities can be determined for each ply independently.



(b) The interfaces between patches are marked with red ticks for the given patch geometry.

Figure 4.2: Illustrative examples showing the usage of indexes p and h .

The continuity manufacturing rule is then defined as:

$$x_{ij_2\theta} = \sum_{p=1}^{P_h} r_{ih\theta p} \quad (4.27)$$

This constraint alone is not enough to ensure manufacturable composite patches. Three further sets of constraints need to be defined to ensure the blending between individual layers is achieved in a meaningful manner. First of all, a left hand-side ply must blend with only one of the right hand-side plies. References to left and right hand-side plies follow the example illustrated in Fig. 4.2a.

$$\sum_{a=1}^A t_{(i-a+1-c)h(a+c)} \leq 1 \quad \forall h, i \in [1, I_{j_2} + P_h - 1]$$

$$\text{with } A = \begin{cases} i & \text{if } i \leq P_h - 1 \\ P_h & \text{if } P_h \leq i \leq I_{j_2} \\ I_{j_2} + P_h - i & \text{if } i \geq I_{j_2} \end{cases} \quad \text{and } c = \begin{cases} 0 & \text{if } i \leq I_{j_2} \\ i - I_{j_2} & \text{if } i > I_{j_2} \end{cases} \quad (4.28)$$

Secondly, a right hand-side ply must blend with exactly one of the left hand-side plies.

$$\sum_{p=1}^{P_h} t_{ihp} = 1 \quad \forall i, h \quad (4.29)$$

Thirdly, in order to avoid unmanufacturable designs that lead to plies intersecting (e.g. right hand-side 1 ply matched with left hand-side 3 and left hand-side 2 matched with right hand-side 2) the following constraints are defined.

$$\sum_{p=1}^{P_h} t_{(i-1)hp} (P_h - p + 1) \leq \sum_{p=1}^{P_h} t_{ihp} (P_h - p + 1) \quad \forall i \geq 2, h \quad (4.30)$$

Finally, the internal covering ply rule is formulated as:

$$\sum_{i=i^*}^{i^*+K} \sum_{a=1}^A t_{(i-a+1-c)h(a+c)} \geq 1 \quad \forall h, i^* \in [1, I_{j_2} + P_h - 1 - K]$$

$$\text{with } A = \begin{cases} i & \text{if } i \leq P_h - 1 \\ P_h & \text{if } P_h \leq i \leq I_{j_2} \\ I_{j_2} + P_h - i & \text{if } i \geq I_{j_2} \end{cases} \quad \text{and } c = \begin{cases} 0 & \text{if } i \leq I_{j_2} \\ i - I_{j_2} & \text{if } i > I_{j_2} \end{cases} \quad (4.31)$$

The presented set of equations is implemented using the Python interface of Gurobi.

A summary of the entire optimisation problem for the implicit formulation is:

Objective	$\min_{-1 \leq \xi_{kj}^{A,B,D} \leq 1} \sum_{j=1}^J w_k^{A,B,D} \xi_{kj}^{A,B,D} - (\xi_{kj}^{A,B,D})_{\text{optimal}} \quad \forall k$
Feasibility	$\sum_{\theta=1}^{\Theta} x_{ij\theta} = 1 \quad \forall i, j$
Symmetry	$x_{ij\theta} = x_{(I_j-i+1)j\theta} \quad \forall i \in \{1, 2, \dots, I_j/2\}, j, \theta$
Balance	$\sum_{i=1}^{I_j} x_{ij\theta_1} = \sum_{i=1}^{I_j} x_{ij\theta_2} \quad \forall i, j, \theta_1 = -\theta_2 \neq \{0, 90\}$
Minimum percentage	$\sum_{i=1}^{I_j} x_{ij\theta} \geq pn^j \quad \forall j, \theta$
Damage tolerance	$x_{1j\theta_1} = 1 \quad \forall j, \theta_1 = 45$
Grouping	$x_{ij\theta_1} \leq x_{(i+1)j\theta_2} \quad \forall i \in \{1, 2, \dots, \frac{I_j}{2}\}, j, \theta_1 = -\theta_2 \neq \{0, 90\}$
Contiguity	$x_{ij\theta} + x_{(i+1)j\theta} + \dots + x_{(i+N)j\theta} \leq N \quad \forall i \in \{1, 2, \dots, I_j - N\}, j, \theta$
Disorientation	$x_{ij\theta_1} + x_{(i+1)j\theta_2} \leq 1 \quad \forall i, j, \theta_1 - \theta_2 > 45$
Auxiliary	
design	$r_{ih\theta p} = x_{(i+p-1)j_1\theta} t_{ihp} \quad \forall i, h$
variables	

$$x_{ij_2\theta} = \sum_{p=1}^{P_h} r_{ih\theta p} \quad \forall i, h$$

$$\sum_{a=1}^A t_{(i-a+1-c)h(a+c)} \leq 1 \quad \forall h, i \in [1, I_{j_2} + P_h - 1]$$

$$\text{with } A = \begin{cases} i & \text{if } i \leq P_h - 1 \\ P_h & \text{if } P_h \leq i \leq I_{j_2} \\ I_{j_2} + P_h - i & \text{if } i \geq I_{j_2} \end{cases} \quad c = \begin{cases} 0 & \text{if } i \leq I_{j_2} \\ i - I_{j_2} & \text{if } i > I_{j_2} \end{cases}$$

$$\sum_{p=1}^{P_h} t_{ihp} = 1 \quad \forall i, h$$

$$\sum_{p=1}^{P_h} t_{(i-1)hp} (P_h - p + 1) \leq \sum_{p=1}^{P_h} t_{ihp} (P_h - p + 1) \quad \forall i \geq 2, h$$

$$\sum_{i=i^*}^{i^*+K} \sum_{a=1}^A t_{(i-a+1-c)h(a+c)} \geq 1 \quad \forall h, i^* \in [1, I_{j_2} + P_h - 1 - K]$$

Internal covering ply

$$\text{with } A = \begin{cases} i & \text{if } i \leq P_h - 1 \\ P_h & \text{if } P_h \leq i \leq I_{j_2} \\ I_{j_2} + P_h - i & \text{if } i \geq I_{j_2} \end{cases} \quad c = \begin{cases} 0 & \text{if } i \leq I_{j_2} \\ i - I_{j_2} & \text{if } i > I_{j_2} \end{cases}$$

Once again, linearisation constraints applied to the problem are omitted to avoid confusion.

4.1.3 Problem decomposition

Both formulations presented in the previous sections solve the stacking and blending optimisation problem considering all patches and constraints simultaneously. In this section, a decomposition strategy is presented for the implicit formulation of the optimisation problem. The reason for choosing implicit instead of explicit is because the strategy can be applied with greater freedom to any patch in the structure. This is due to the lack of ply-voids in the implicit formulation.

The implicit MILP formulation of the stacking sequence optimisation problem considers all design variables and constraints simultaneously for the entire arrangement of patches in the structure. However, this complete optimisation problem can become too computationally expensive for industrial-scale structures consisting of hundreds of layers. What is more, Gurobi and similar software based on branch and bound algorithms can significantly benefit from good initial feasible solutions by reducing the size of the search tree. Therefore, a technique for decomposing the stacking sequence optimisation problem has been developed, to speed up the overall convergence of the optimisation and to retrieve acceptable solutions in a fraction of the original computational time.

The flowchart of the decomposition strategy is presented in Fig. 4.3. A decomposition of the problem requires the definition of at least one path traversing all patches in the structure. The connectivity between the patches can be presented using an undirected graph where vertices represent patches and edges indicate the shared boundaries between neighbouring patches. All patches belonging to the same structural component are connected with at least one other patch. As for the paths to be examined during the decomposition of the problem, these can either be manually defined by the user or automatically generated using one of the algorithms shown in the “Define paths” module of Fig. 4.3. The *MAX* and *MIN* algorithm generates a path by starting from the thickest or thinnest patch in the structure

respectively and sequentially adding the next thickest or thinnest patch which shares an edge with one of the patches already existing in the path. *BFS* stands for a Breadth First Search algorithm during which, starting from any chosen root patch, all neighbouring patches are added to the path until all patches have been included. On the other hand, *DFS* is the Depth First Search algorithm which explores all patches in a branch before expanding to other branches.

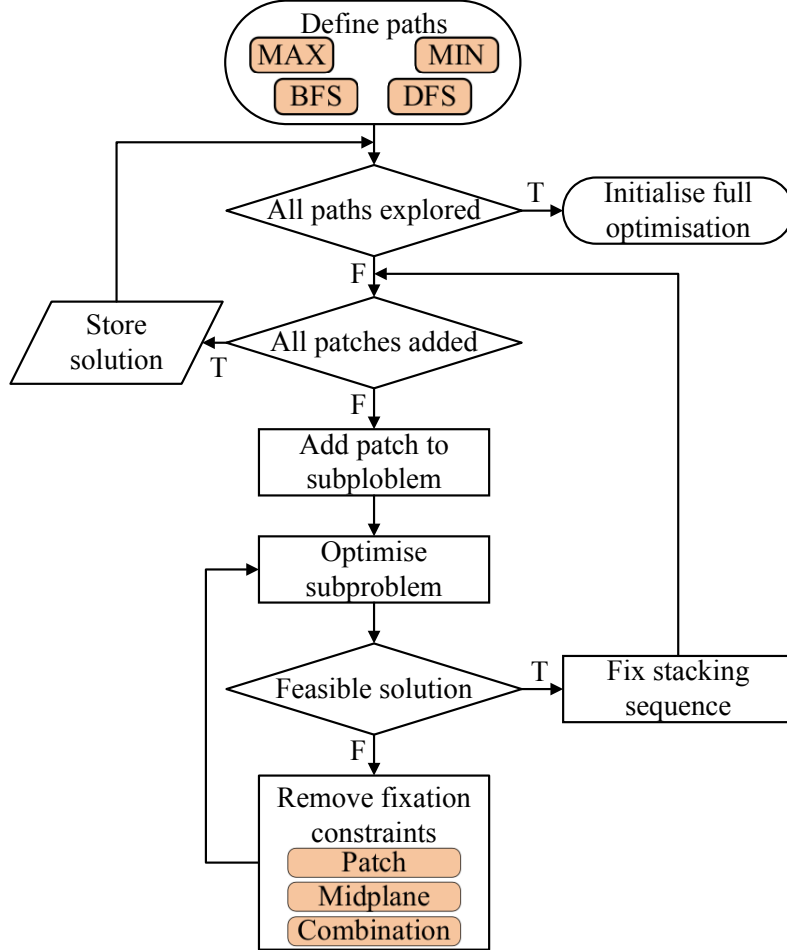


Figure 4.3: Flowchart of the optimisation using decomposition developed for the implicit formulation.

Once a path or set of paths have been defined, each of them is solved using the decomposed approach. Initially, only the first patch in the path is optimised individually, without any blending requirements with neighbouring patches. Afterwards, depending on the order of the patches in the path, one patch is added at a time to the optimisation sub-problem until all patches in the path have been added. Each time a new patch is added to the sub-problem, the stacking sequence of the patch or the patches optimised in the previous sub-problem is fixed. Therefore, the purpose of each sub-problem after the first one is to find the optimum solution that satisfies all requirements and blends with all the established neighbouring stacking sequences.

However, in many cases, these optimisation sub-problems might be infeasible given the fixed stacking sequence of the neighbouring patches. In such cases, constraints restricting the stacking sequence of these neighbouring patches must be lifted to allow for more design freedom. This is performed using one of the methods shown in the “Remove fixation constraints” module of Fig. 4.3 which are also visually demonstrated in Fig. 4.4. The sub-problem can be altered by backtracking to the previously added patch in the path and removing the fixation constraints for it. The fixation constraints are the constraints restricting the fibre orientation of each layer to a specific value and are applied by equality constraints on all $x_{ij\theta}$ design variables of a specific patch. This is repeated until a feasible solution can be obtained as shown in Fig. 4.4a. In the specific example of Fig. 4.4a, where only 3 neighbouring patches exist, the rightmost one being the one added last to the sub-problem, the first step of the fixation constraint removal process would be to remove the constraints for the second patch. If the optimisation problem is still infeasible, then a second step can be performed to remove the stacking sequence fixation constraints applied to the first patch. These potential fixation constraint removal steps are illustrated by the usage of different colour schemes in Fig. 4.4. Alternatively, instead of removing the fixation constraints for an entire patch, the constraints can be relaxed for all patches and for a certain number of layers around the midplane of the laminate as demonstrated in Fig. 4.4b. This number of layers increases until eventually a feasible solution is obtained. Finally, a combination of the two previously mentioned techniques depicted in Fig. 4.4c can be applied. More specifically, an increasing number of layers are removed for the patch added last in the path. In case the sub-problem remains infeasible, the process of removing an increasing number of layers around the midplane is repeated after backtracking to the previously added patch in the path. The goal of all of these techniques for removing the fixation constraints is to keep the size of the decomposition small and not turn the decomposed problem into one which is of a similar complexity as the original complete optimisation problem. Once all the paths have been explored, the solution with the best objective function can be used as an initial solution for the complete optimisation.

4.2 Results

In this section, results of the second stage of the optimisation are first presented with respect to the convergence between the two formulations and the impact of the decomposition. Next, a comparison between the developed second stage of the process and two alternative algorithms from the literature is presented. What

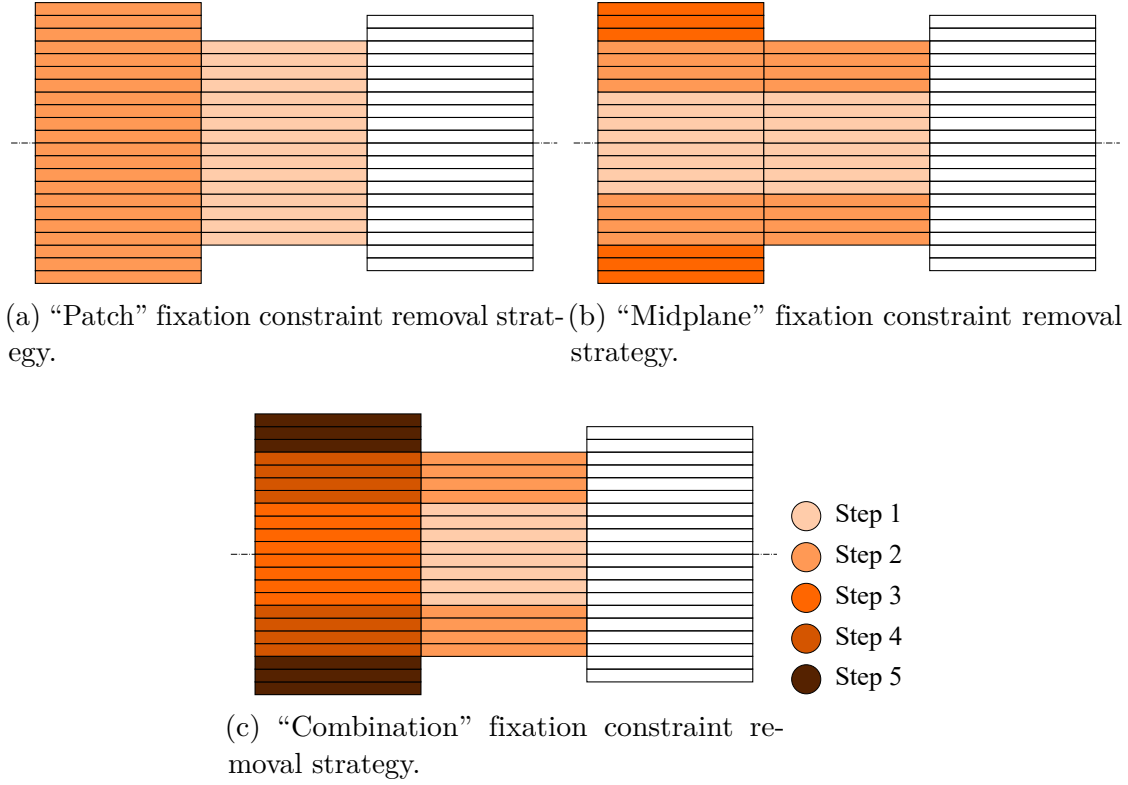


Figure 4.4: Visual representation of the fixation constraints removal strategies during the decomposition of the second stage of the optimisation for an example of 3 neighbouring patches. In all examples the rightmost patch is the one added last to the sub-problem. The different colour schemes of each step indicate the layers for which the stacking sequence fixation constraints would be removed. The number of steps required depends on whether the optimisation problem still remains infeasible or not after fixation constraints have been removed from the optimisation.

is more, results for the complete two-stage process are shown for a widely used academic benchmark. The academic benchmark used for the majority of results shown in this section consists of 18-panels arranged in a horseshoe configuration introduced by Soremekun et al. [4]. The panel arrangement, force resultants and dimensions are given in Fig. 4.5. N_x and N_y force resultants are given in lbf/in ($\times 175.1$ to convert into N/m). The graphite-epoxy material (IM7/8552) whose properties have already been described in section 3.4 is used. The thickness of the ply used is 0.191mm to maintain consistency with other studies.

4.2.1 Second stage optimisation

Initially, the explicit and implicit formulations are studied using some illustrative cases and then the horseshoe problem is used to assess the proposed discrete optimisation approach. Moreover, the MILP formulations are also compared against two approaches from the literature.

18 in.		20 in.			12 in.	
24 in.	1 $N_x = -700$ $N_y = -400$	2 $N_x = -375$ $N_y = -360$	3 $N_x = -270$ $N_y = -325$	4 $N_x = -250$ $N_y = -200$		5 $N_x = -210$ $N_y = -100$
			6 $N_x = -305$ $N_y = -360$	7 $N_x = -290$ $N_y = -195$		8 $N_x = -600$ $N_y = -480$
9 $N_x = -1100$ $N_y = -600$	10 $N_x = -900$ $N_y = -400$					
11 $N_x = -375$ $N_y = -525$	12 $N_x = -400$ $N_y = -320$	13 $N_x = -330$ $N_y = -330$	14 $N_x = -190$ $N_y = -205$	15 $N_x = -300$ $N_y = -610$		
		16 $N_x = -815$ $N_y = -1000$	17 $N_x = -320$ $N_y = -180$	18 $N_x = -300$ $N_y = -410$		

Figure 4.5: 18 panel horseshoe blending problem [4].

4.2.1.1 Explicit and implicit formulations

Both the explicit and implicit formulation are able to produce fully blended composite designs satisfying all specified design and manufacturing rules. In this section, the results obtained after applying the approaches to some methodology demonstration problems are presented. More specifically, blended stacking sequences that fulfil a prescribed set of design and manufacturing rules are manually defined and the accordant lamination parameters, which are used as target values during the optimisation, are calculated for each individual patch. Therefore the optimal objective function value for this set of problem instances is known *a priori* and is equal to zero. All design and manufacturing rules described in section 2.1.2 are used, with the exception of contiguity, and disorientation which is omitted because grouping is used instead. The fibre orientations for the demonstration cases are limited to $0^\circ, 90^\circ, \pm 45^\circ$. The demonstration problem instances are comprised of a different number of patches (2 or 4), maximum number of layers in the thickest patch (40 or 80) and number of ply drops between adjacent patches (4 or 8). In all demonstration cases, patches are placed sequentially one after the other.

In Table 4.2, the final MILP problem size in terms of the number of design variables, constraints and percentage of non-zero elements in the MILP optimisation matrix is

Description			Explicit formulation			Implicit formulation		
Patches	Layers	Ply drops	Constr.	Design var.	Non-zeros (%)	Constr.	Design var.	Non-zeros (%)
2	40	4	868	368	1.3	911	393	1.1
2	40	8	868	368	1.3	1146	492	0.9
2	80	8	1852	772	0.6	2745	1163	0.3
4	80	8	5071	1990	0.2	6823	2789	0.1

Table 4.2: Comparison of the MILP size for selected demo problems between the explicit and implicit formulation.

presented for a subset of the cases. The final MILP size does not directly correspond to the size one would theoretically calculate given the problem definition, because Gurobi, like other commercial solvers, applies pre-solve algorithms that might reduce the size of the problem. It can be observed that the size grows significantly for large instances, as expected. Moreover, using the implicit formulation results in slightly larger problem instances. Finally, it is worth noting that an increased number of ply drops increases the problem size of the implicit formulation, while the size of the explicit formulation stays the same. This is due to the fact that the design variables and therefore the constraints of the implicit formulation depend on the number of blending possibilities between neighbouring patches, which is analogous to the number of ply drops.

In Fig. 4.6, the runtimes of the two approaches for all the different demonstration problem instances are presented. The individual runtimes are depicted using half circles of logarithmically scaled radiuses. More specifically, the maximum radius in Fig. 4.6 represents a runtime of 442s while the smallest one 0.8s. All runs have been performed on a PC with an Intel Core i5-8250U @1.60GHz (4 cores, 8 threads) CPU. The algorithm is stopped if an objective value less than $10^{-2} \times (\text{number of patches})$ is found.

Results presented in Fig. 4.6, highlight the expected increase in run-time as the number of layers and patches increase. The implicit approach is better than the explicit approach for small sized problems, however, as the problem size increases the explicit formulation is either comparable or better than the implicit one in terms of time, as expected. Finally, the runtime of the implicit formulation strongly depends on the number of ply drops between adjacent patches, with more ply drops resulting in higher runtimes, while the explicit formulation does not necessarily depend on the number of ply drops. This can be explained by the fact that more design variables and constraints need to be introduced in the implicit formulation to account for

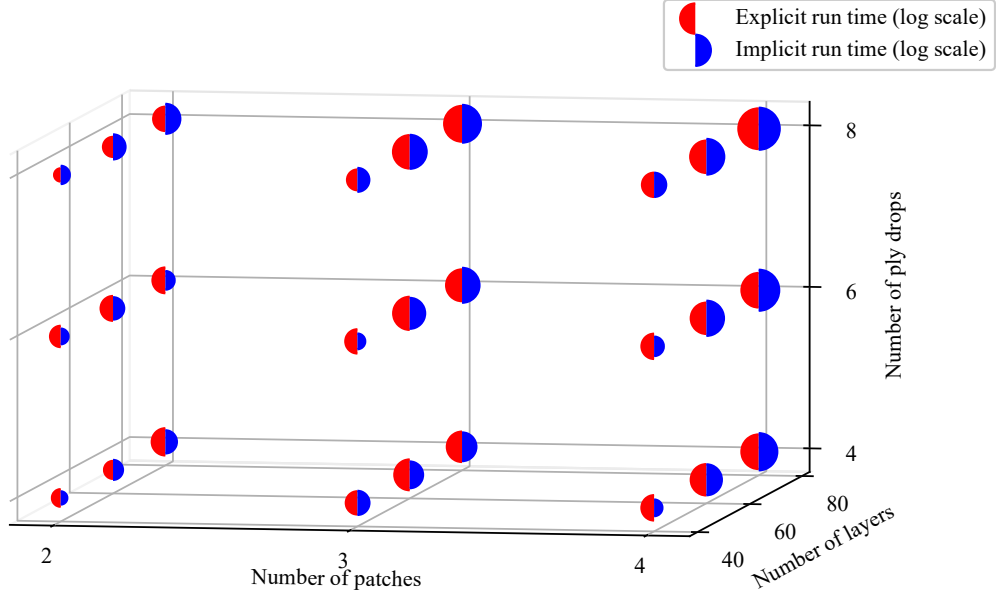


Figure 4.6: Illustration of run-times for the implicit and explicit formulations for the initial set of problem instances as the number of patches, layers and ply drops change.

more ply drops.

In Fig. 4.7 the convergence plots of the two different MILP formulations are given for the horseshoe problem [4]. The “best known objective” in Fig. 4.7 corresponds to the smallest objective function value of a solution satisfying the constraints of the optimisation, while “objective lower bound” refers to the minimum discovered bound for the value of the objective function which is known due to the the branch and bound based algorithm being employed. Optimality of the solution is proven when the value of the lower bound of the objective is equal to that of the best known objective. The optimisation is stopped after 600s because proving optimality becomes time consuming. As observed earlier, the implicit formulation outperforms the explicit one, since the maximum number of layers is rather small for this problem, as shown in Table 4.3. This is not necessarily the case for larger problems for which the explicit approach can converge to better solutions in less computational time. The disadvantage of the explicit formulation though is the lack of providing a good theoretical bound for the objective (Fig. 4.7), which in turn results in bigger computational times until proving convergence.

Patch	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Layers	34	28	22	20	16	22	20	26	38	36	30	28	22	20	26	32	20	22

Table 4.3: Number of layers for the patches of the 18-panel horseshoe problem.

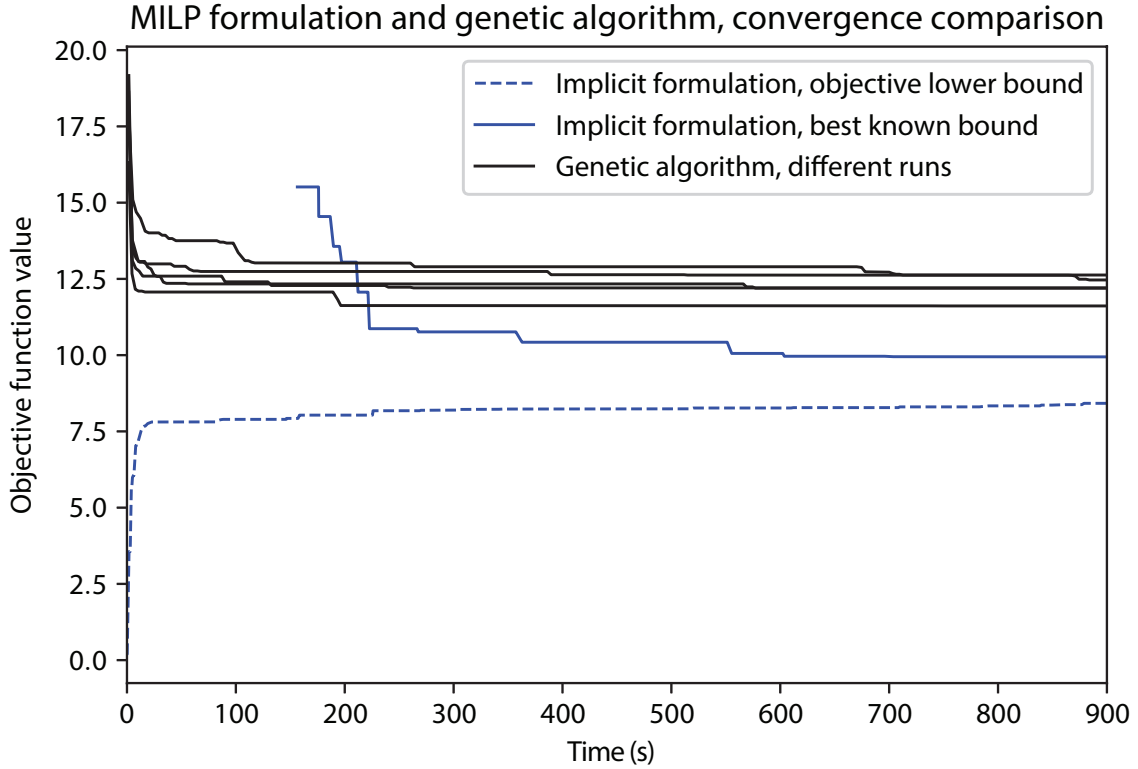


Figure 4.7: Convergence comparison between the explicit and implicit formulation for the 18 panel horseshoe problem.

4.2.1.2 Problem decomposition

Next, a comparison between the convergence of the decomposition strategy and the non-decomposed implicit formulation is performed for the 18-panel horseshoe problem. Three different paths, representing patch sequences, are defined to test the decomposition strategy as shown in Table 4.4. The first path begins from panel 5 in the top right of the horseshoe pattern and moves counterclockwise to patch number 18 in the bottom right following a specific pattern. The second path is built up starting from the thinnest patch in the structure and by gradually adding the thinnest possible neighbour each time. The third path is constructed by starting from the thickest patch in the structure and by choosing the thickest neighbour to add each time.

Path	Patch sequence
1	[5,8,7,4,3,6,2,1,10,9,11,12,13,14,15,16,17,18]
2	[5,4,3,7,8,6,2,1,10,9,12,11,13,14,17,18,15,16]
3	[9,10,1,2,3,6,4,7,8,5,11,12,16,13,14,17,15,18]

Table 4.4: Paths containing the sequence of patches used for the optimisation using decomposition of the 18 panel horseshoe problem.

Fig. 4.8 illustrates the progress plot for the objective value using the approaches

with and without decomposition, where the three triangles mark the objective value of each local minimum discovered by using the three different paths. The one used to initialise the optimisation without the decomposition is that of the best found solution which corresponds to the second path. For the specific manufacturing rules used, the average run-time of each individual run for the different paths is approximately 10 seconds, including the overhead of setting up each of the many sub-problems.

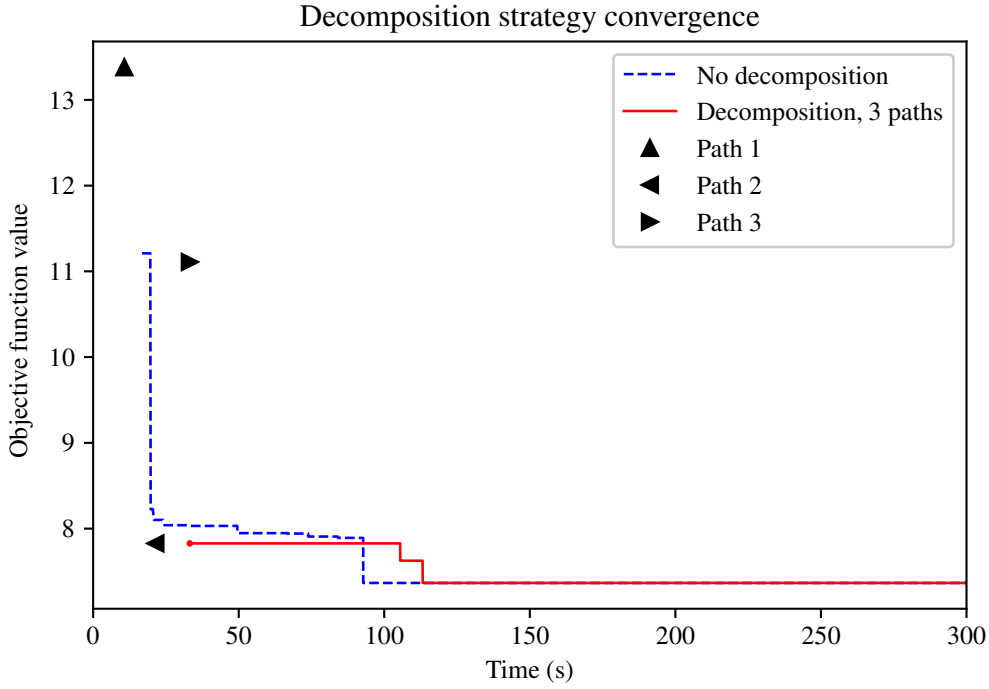


Figure 4.8: Convergence comparison between the implicit formulation and the decomposition strategy for the 18 panel horseshoe problem.

The fact that the path which resulted to the best solution of the optimisation with the decomposition is the one starting from the thinnest patch and moving to the thinnest neighbour is not incidental. This can be attributed to the fact that in thinner patches, the placement of a layer influences the stiffness more compared to a thicker panel. Therefore, allowing the thinner patches to be optimised first can prevent the decomposition from getting stuck at a local minimum favouring some of the thickest patches. Finally, as for the actual convergence, no significant speed up is observed for the examined case. The optimisation with no decomposition actually converges to what appears to be a global minimum faster than the optimisation approach using decomposition. However, the benefit of using decomposition becomes more evident as the size of a problem grows which is the case for realistic aircraft structures.

To demonstrate this effect, the loads applied to the original formulation of the horseshoe panel problem are multiplied by a factor of 8. The material and dimensions of the panels remain the same. After performing the first optimisation stage again, this increase in loads leads to a doubling of the panels' thicknesses. Fig. 4.9 depicts the improvement of the value of the objective function with respect to time, for the modified horseshoe problem when applying the optimisation with and without the decomposition technique. For this bigger optimisation problem, applying the optimisation with the decomposition yields a much better solution in a considerably shorter time frame. A second observation is that each of the 3 different paths now requires an average of approximately 30 seconds to optimise compared to 10 in the original problem definition. This is expected, since the number of layers have doubled and the size of the optimisation problem has increased accordingly. As the number of layers increases, the design space of the laminate becomes wider. A wider design space also means that the current thinnest patch in the structure does not influence the quality of the decomposition that much. Indeed, in this modified problem, the path which results to the lowest objective function is the third one which starts from the thickest patch in the structure. What is more, the deviation between the objective function values of the three paths is now considerably smaller compared to the original. Finally, because the layers increase, the potential to match the same target stiffness increases which leads to a lower global optimum for this modified problem.

Overall, both the explicit and implicit formulations provide stacking sequences that are fully compliant with respect to the specified design and manufacturing rules. Based on the observations and experience gained up to now, the explicit formulation can outperform the implicit one for large industrial problems. However, the explicit formulation still lacks the implementation of the disorientation and contiguity constraint as well as the decomposition technique.

4.2.1.3 MILP vs Genetic algorithm

Finally, the open source genetic algorithm developed by Vicente [5] is used to optimise the horseshoe problem described in the previous section. The genetic algorithm uses the SST encoding. All design and manufacturing rules are used, except for the grouping rule, because the disorientation design rule is activated instead. The settings used for the runs of the genetic algorithm are summarised in Table 4.5. These settings have been extracted from the ones used in the work of Vicente [5] and it should be noted that a different set of settings has the potential to provide a better convergence behaviour. The convergence history of various genetic algorithm

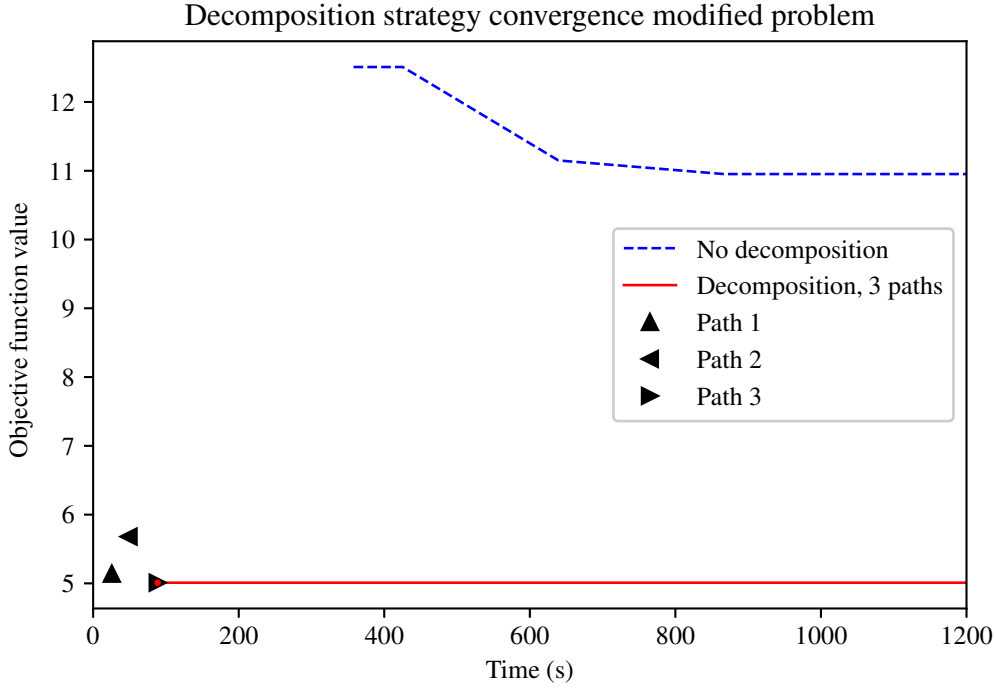


Figure 4.9: Convergence comparison between the implicit formulation and the decomposition strategy for the modified 18 panel horseshoe problem in which the applied loads are multiplied by a factor of 8.

Parameter	Value
Population size	200
Maximum number of generations	1000
Elitism percentage	4%
Probability of mutation	15%
Probability of crossover	75%

Table 4.5: Settings used in the genetic algorithm of Vicente [5].

runs and of the implicit formulation are presented in Fig. 4.10. All algorithm runs have been performed on the same 1.6GHz CPU using only one core. The implicit formulation converges to a much better solution. This can be attributed to two reasons. First of all, the blending formulation in this work allows for more design freedom, as explained in section 2.1.2, which leads to a smaller feasible objective function. Secondly, the mathematical programming algorithm used to solve the problem formulation, as presented in the current work, can consistently lead to the optimal value of the objective function, while the genetic algorithm gets trapped in different local minima, based on the contextual random number generator seed. However, the genetic algorithm is able to provide decent feasible solutions in less time.

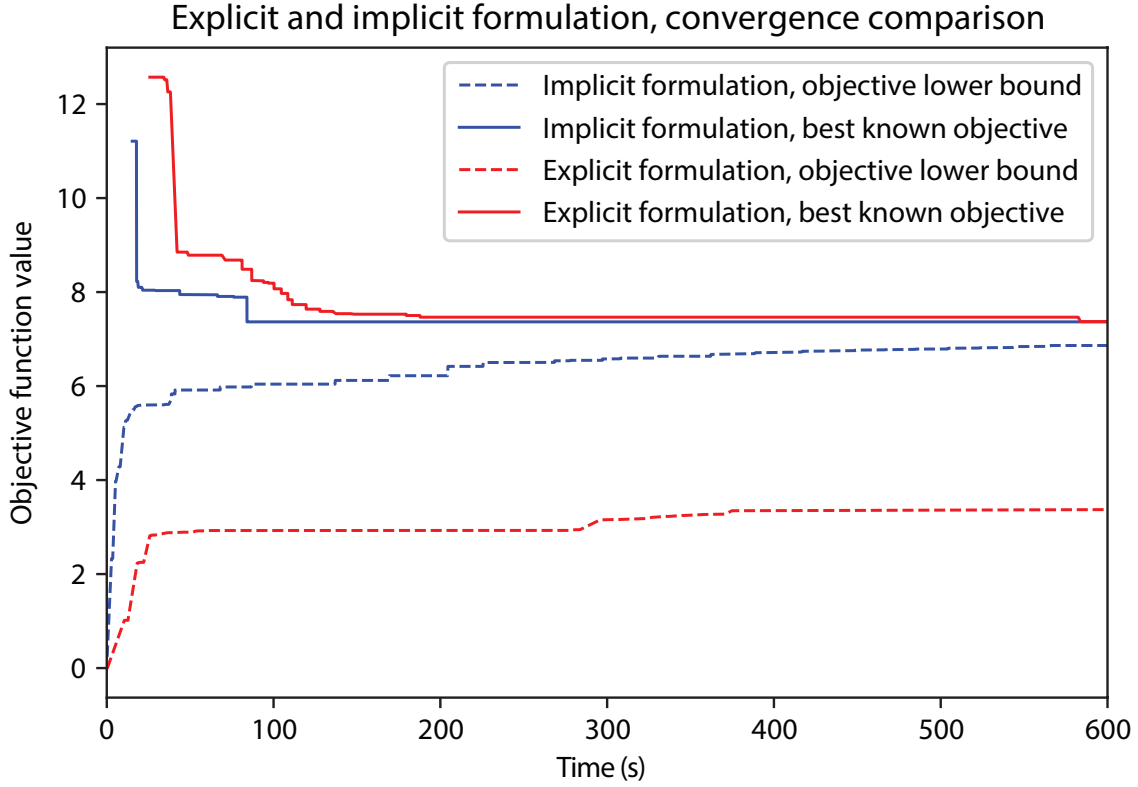


Figure 4.10: Convergence comparison between the implicit formulation and a guide-based genetic algorithm [5] for the 18 panel horseshoe problem.

4.2.1.4 MILP vs Branch and Bound

Liu et al. [7] introduced a logic-based method which is a combination of the branch and bound and global layerwise techniques, developed to retrieve stacking sequences that optimally match some given lamination parameters for single patches. Table 4.6 compares the results obtained from the presented methodology against the published results for various problems introduced by Liu et al. [7]. Each example uses a different set of design rules when matching the lamination parameters. In Liu et al. [7] two runtimes are provided for each instance; one is the time when the optimum solution ($t.$ to opt.) is found and the other one is the time (Total $t.$) until the search of the tree is completed and hence optimality is proven. The time shown for the implicit MILP formulation of this work is the time spent until optimality is proven. Overall, using a mathematical programming algorithm to achieve the same task is shown to outperform the logic-based method. This is due to the fact that a search of the entire tree is not necessary to prove optimality and because more sophisticated techniques such as presolving, cutting planes and heuristics are used to assist the optimisation convergence. It should also be noted that for example 1 in Table 4.6, an objective function with a lower value was calculated using Gurobi. The provided runtimes for the current work have been produced using 1 thread on a

Example	Results from [7]			MILP Implicit formulation	
	$t.$ to opt.(s)	Total $t.$ (s)	Obj.	$t.$ (s)	Obj.
1	4.77	3600+	0.0806	3.83	0.0794
2	0.52	7.10	0.1729	0.13	0.1729
10	3.76	3600+	0.0892	1.69	0.0892
11	0.23	5.95	0.0984	0.16	0.0984
13	0.19	0.90	0.3828	0.14	0.3828
14	0.97	1.59	0.3776	0.18	0.3776
15	6.40	9.06	0.1120	0.10	0.1120

Table 4.6: Single patch optimisation results, comparison against the work of Liu et al. [7].

1.6GHz computer compared to a 4GHz computer used in the work of Liu et al. [7].

4.2.2 Academic demonstrator

In this section, results for the complete two-stage process are presented. Firstly, the importance of the continuity constraint is demonstrated using the 18-panel horseshoe problem. Eventually, the developed two-stage process is compared against relevant academic studies performed on the same benchmark example.

4.2.2.1 Importance of continuity constraint

The significance of having the blending constraint in the gradient-based optimisation is demonstrated on the 18-panel horseshoe problem. The generic stack used to model the patches in the horseshoe benchmark is $[(45, -45, 90, 0)_3]_s$. The composite constraints applied are symmetry, balance, damage tolerance, a minimum and maximum allowed percentage of 10 and 60 % respectively and the external covering ply rule. If the blending constraint is also applied in the gradient-based optimisation, then the discrete optimisation stage is directly able to retrieve a design which fulfils all buckling constraints. On the contrary, if blending is not enforced in the continuous optimisation stage, the discrete optimiser is not able to retrieve a discrete stacking sequence which satisfies buckling constraints. In such a case, the design factor needs to be increased. This procedure is followed iteratively until the discrete design meets the buckling or any other mechanical constraints. The outcome of such an iterative effort is demonstrated in Table 4.7. Consequently, more computational time and manual effort need to be sacrificed. Worst of all, this iterative process leads to a significant weight penalisation, which for this case is 5%. This additional weight is due to overdimensioning several of the patches in the structure as can be seen from the maximum buckling RFs. This goes to show the importance of having

as many manufacturing constraints as possible in the gradient-based optimisation stage but also formulating these constraints as accurately as possible.

	Continuous results			
	No blending			With Blending
Design factor	1	1.2	1.25	1
Mass(Kg)	27.72	29.45	29.85	28.10
	Discrete results			
Min. Buck. RF	0.83	0.964	1.03	1.01
Max. Buck. RF	1.30	1.60	1.59	1.30
Mass (Kg)	28.93	30.52	30.79	29.31

Table 4.7: Structural mass and buckling RF comparison for the 18-panel horseshoe problem between a two-stage optimisation in which blending is not included in the first stage and one in which it is activated.

4.2.2.2 Comparison against relevant two stage processes

In this section, results for the 18-panel horseshoe problem are compared against results achieved by other two-stage optimisation approaches available in the literature. Published results on the benchmark stemming from the application of non-deterministic one-stage approaches [24, 38] are not included because both mechanical and composite constraints can be considered simultaneously, however their applicability to large-scale problems is limited. For the current comparison, the generic stack used is $[(45, -45, 15, -15, 30, -30, 60, -60, 75, -75, 90, 0)_3]_s$, to match the set of fibre angles used by IJsselmuiden et al. [113] and Macquart et al. [6]. In the first stage of the optimisation, besides blending, symmetry is also enforced to ensure no bending-extension coupling exists since this effect is not captured by the analytical buckling formulation of equation 3.3 used in the results presented here. Likewise, in the discrete optimisation only blending and symmetry are enforced and the allowed fibre orientations are the same as the ones used in the generic stack, which coincide with the ones used in the work of IJsselmuiden et al. [113] and Macquart et al. [6]. Picchi Scardaoni et al. [40] have allowed for any integer fibre orientation to be used in the discrete design. As far as composite design and manufacturing rules are concerned, all three studies [113, 6, 40] have used blending while additionally IJsselmuiden et al. [113] have enforced symmetry and balance and Picchi Scardaoni et al. [40] have applied a manufacturing rule prohibiting small changes in thickness of adjacent panels.

In the work of IJsselmuiden et al. [113], blending constraints were not used in the lamination parameter design space of the gradient-based optimisation. Therefore, the discrete stacking sequence derived initially, significantly violates the buckling RFs.

	Mass (Kg)	Min. buckling RF
IJsselmuiden et al. [113]	29.46	1.025
Macquart et al. [6]	28.61	0.979
Picchi Scardaoni et al. [40]	27.28	1.058
Current work	28.15	1.000

Table 4.8: Structural mass and buckling RF comparison for the 18-panel horseshoe problem including blending constraints in the continuous optimisation.

An iterative repair strategy of locally increasing thicknesses needs to be performed for several loops until arriving to the end result of Table 4.8.

In the work of Picchi Scardaoni et al. [40], blending constraints are derived for the polar parameter design space. The design derived by the ant colony optimisation employed for the second optimisation stage is lighter than the one achieved in this work. However, finite element buckling is employed to calculate the buckling RFs of the horseshoe problem instead of the analytical formula of equation 3.3 employed in this and the other two works [113, 6]. Evaluation of the buckling RFs of the discrete stacks published by the researchers [40] using the analytical expression of equation 3.3, results in a minimum RF of 0.65. Therefore, due to the different buckling modelling, the resultant masses cannot be fairly compared. Finally, buckling compliant discrete stacks are derived by employing a design factor of 1.1 leading to the minimum RF of 1.058 for the discrete stacks.

The work of Macquart et al. [6] is the one which can be most fairly compared to the results of the proposed methodology. Blending constraints are derived for the lamination parameter design space of the first gradient-based optimisation stage. The achieved mass of the current work is 1.6% smaller than the one achieved in the work of Macquart et al. [6]. The actual difference should actually be larger, since the reserve factor in the other work [6] is less than 1. Finally, another significant advantage of the present work is that these results are achieved after one pass of the two-stage process. In the lamination parameter space there is the need for a second loop using a design factor of 1.05. At least a third loop which was not performed in the study would eventually be required to satisfy the buckling constraints which would further increase the structural weight.

The accuracy of the blending formulation in the generic stacks is further highlighted in Fig. 4.11. The average difference between the continuous stiffness and the discrete stiffness is noticeably smaller for the current two-stage approach. This smaller distance originates from the accuracy of the blending formulation and leads to manufacturable stacking sequences in one pass of the optimisation process which

therefore reduces the mass penalty between the continuous and discrete result. The discrete stacking sequences retrieved in this work, as well as the corresponding buckling RFs are provided in Table 4.9.

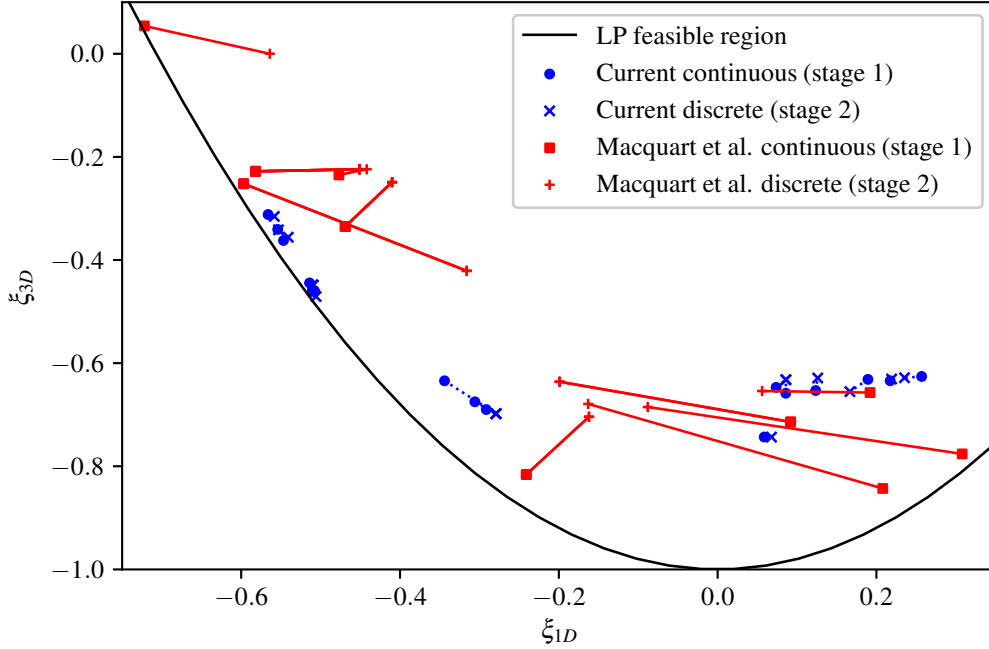


Figure 4.11: Continuous and discrete results of the current work and of Macquart et al. [6] depicted in lamination parameter (ξ_{1D}, ξ_{3D}) space.

It should be noted that the discrete optimisation stage in this work uses mathematical programming while the other study used a genetic algorithm [127]. Differences not related to the algorithmic side of the two second-stage approaches, but rather to the design freedom of each blending formulation, do exist, with the discrete optimisation of the current work having more flexibility. A quantification of the percentage each of the continuous and discrete optimisation contribute to a lower structural mass has not been performed. However, results of section 4.2.2.1 confirm that the ability of getting buckling constraint satisfaction in one pass of the optimisation process significantly drives mass down and therefore part of the lower structural mass can be attributed to using generic stacks.

4.3 Discussion

Both of the MILP formulations can be used in conjunction with mathematical programming algorithms and are able to retrieve laminated composite structures which fulfil a set of prescribed design and manufacturing rules. The objective function

Patch	No. of layers	Stacking sequence	Buckling RF
1	34	[-30, 30, 45, 30, -45, 30, -45, 60, 60, 45, -60, 60, -60, -75, -45, 90, 45] _s	1.160
2	28	[-30, 45, 30, -45, 30, 60, 60, -60, 60, -60, -75, -45, 90, 45] _s	1.012
3	22	[45, -45, 60, 60, -60, 60, -60, -75, -45, 90, 45] _s	1.222
4	18	[60, 60, -60, 60, -60, -75, -45, 90, 45] _s	1.040
5	16	[60, 60, -60, 60, -60, -75, -45, 45] _s	1.207
6	22	[45, -45, 60, 60, -60, 60, -60, -75, -45, 90, 45] _s	1.099
7	18	[60, 60, -60, 60, -60, -75, -45, 90, 45] _s	1.007
8	24	[60, 60, -60, 60, -60, -75, 75, -60, 90, -45, 90, 45] _s	1.032
9	38	[-30, 30, 30, 45, 30, -45, 30, -45, 60, -30, 60, 45, -60, 60, -60, -75, -45, 90, 45] _s	1.056
10	36	[-30, 30, 30, 45, 30, -45, 30, -45, 60, 60, 45, -60, 60, -60, -75, -45, 90, 45] _s	1.143
11	30	[-30, 45, 30, -45, 30, 60, -30, 60, -60, 60, -60, -75, -45, 90, 45] _s	1.083
12	28	[-30, 45, 30, -45, 30, 60, 60, -60, 60, -60, -75, -45, 90, 45] _s	1.007
13	22	[45, -45, 60, 60, -60, 60, -60, -75, -45, 90, 45] _s	1.150
14	18	[60, 60, -60, 60, -60, -75, -45, 90, 45] _s	1.103
15	24	[60, 60, -60, 60, -60, -75, 75, 90, -45, 90, -60, 45] _s	1.000
16	32	[45, -30, -45, 45, 30, -45, 30, 60, 60, -60, 60, -60, -75, -45, 90, 45] _s	1.056
17	18	[60, 60, -60, 60, -60, -75, -45, 90, 45] _s	1.021
18	22	[60, 60, -60, 60, -60, -75, 90, -45, 90, -60, 45] _s	1.065

Table 4.9: Optimal discrete stacks and the corresponding buckling RFs.

of this optimisation involves the minimisation of the absolute difference between the stiffness of the computed design and a set of target stiffness characteristics provided by the first stage of the optimisation.

The derived formulations, namely explicit and implicit, mainly differ in the way blending is treated. The explicit formulation offers a much more direct approach to the representation of blending, which however increases the complexity of defining the objective as a linear function of the available design variables. The derivation of blending constraints for the implicit formulation is more complex, however other composite rules and the objective function can be easily derived. Finally, a decomposition technique has been developed for the implicit formulation to assist with the discovery of good local optima in a much shorter time frame.

The results presented show that both formulations are able to tackle a widely used benchmark in a reasonable time frame. The explicit formulation is able to

handle problems of larger size in a shorter time. However, the utilisation of the decomposition technique on the implicit formulation is very promising for applications in industrial composite parts of much larger scales. Compared to an open-source genetic algorithm, the current formulations lead to better discrete stacks in a given time frame. This can be partly attributed to the fact that the current formulations offer more design freedom than most available blending representations and partly to mathematical programming algorithms, offering a more robust convergence towards the global optimum compared to heuristics. Comparison of the current MILP formulation against a custom-made Branch and Bound algorithm focusing on the stacking sequence optimisation of single patches, shows that optimality of the solution can be proven faster for all examined cases.

Finally, the importance of having an accurate blending formulation in the continuous optimisation is demonstrated by comparing the results from both including and excluding the constraint from the gradient-based optimisation. In the case that the constraint is included, the computational effort is reduced to the minimum as there is no need to perform extra two-stage optimisation loops and additionally significant weight savings are achieved. The presented two-stage approach is also applied on a widely used benchmark and is compared against other similar studies using lamination and polar parameters. The current methodology leads to a lower structural weight compared to previous works having used the same hypothesis for the benchmark. Moreover, the proposed two-stage methodology results in composite structures satisfying both discrete composite requirements but also buckling constraints after a single pass of the two optimisation stages.

Industrial demonstrator

In Chapter 3 and Chapter 4 two stages of the optimisation approach developed in this study were presented. In Chapter 4 results from the application of the complete two-stage process on a widely used academic benchmark were presented. While such two-stage approaches have been applied multiple times to academic case studies [113, 6, 40], there have only been few realistic applications to larger sized problems. Silva Caprio et al. [120] applied aeroelastic tailoring to a regional jet following a two-stage process. Satisfaction of the mechanical constraints is achieved due to a surrogate model employed in the second stage of the optimisation. Bordogna et al. [137] also performed aeroelastic tailoring on the wing of a regional wing using lamination parameters. Mechanical constraint violations are observed for the discretised design even when applying blending constraints in the first stage of the optimisation. Recently, Scardaoni et al. [146] applied a two-stage process using polar parameters to the sizing of a box-wing. Buckling constraint violations occurred in some of the sub-components studied.

This chapter is based on the paper, *Stacking sequence optimisation of an aircraft wing skin* by G. Ntourmas, F. Glock, S. Deinert, F. Daoud, G. Schuhmacher, D. Chronopoulos, E. Özcan and J. Ninić which appeared in the *Structural and Multidisciplinary Optimization* journal 66 31 (2023). Note: symbols may have been changed to maintain consistency throughout this thesis.

The current chapter focuses on the application of the two-stage optimisation process to the detailed sizing of the wing skin of an aircraft. The process is capable of handling large-scale optimisation problems without the computational runtime becoming prohibitive. Emphasis is placed on the retrieval of manufacturable stacking sequences that fulfil all mechanical constraints. It is shown that, the influence on the mechanical constraint satisfaction differs depending on the composite design rules applied in the second stage of the optimisation and the thickness discretisation performed between the two stages. Overall, the weight penalty introduced between the continuous and finalised discrete design which satisfies all constraints is shown to be negligible.

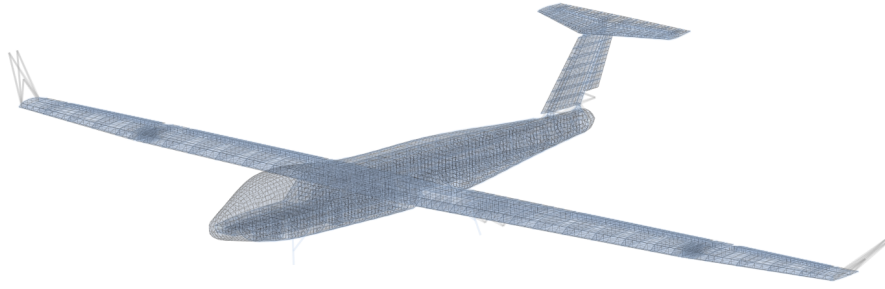
This chapter is structured as follows: In section 5.1 the details of the industrial-sized aircraft model used as a case study are introduced. More specifically, the elements and materials employed to model the structure are summarised, followed by a description of the considered load cases and the design variables and constraints in the optimisation problem. Afterwards, in section 5.2 results on the size and convergence of the two stages of the optimisation are presented, followed by a study on the influence of different design rules during the stacking sequence retrieval stage on the mechanical constraint satisfaction of the discretised structure. Finally, findings of the industrial demonstrator study are discussed in section 5.3.

5.1 Model setup

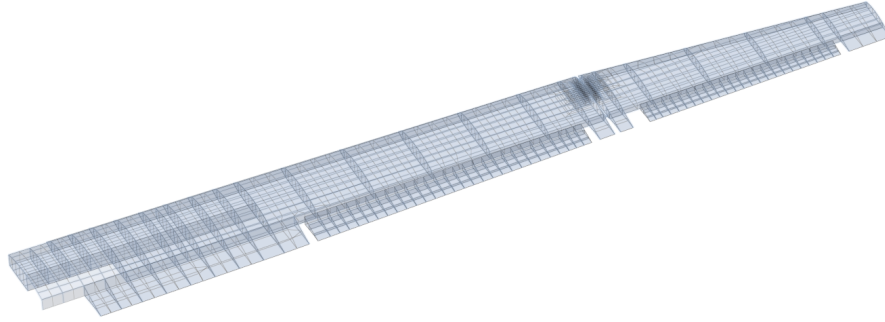
The two-stage optimisation process has been applied to the wing covers of a modified OptiMALE model [147], an industrial-scale demonstrator shown in Fig. 5.1. More specifically, OptiMALE is a Medium Altitude Long Endurance (MALE) aircraft which operates as an unmanned aerial vehicle. The wing span and length of the aircraft are approximately 30 and 15 meters respectively. In this study the focus is placed on the detailed design of the fibre-reinforced composite skins around the wing box of the aircraft. These wing skins are split up into 4 different sub-components, 2 for the lower and upper side of the wing and 2 for the inner and outer parts of the wing, with the latter designed to be detachable for storage and transportation purposes.

5.1.1 Elements and materials

The aircraft is modelled using a coarse Global Finite Element Model (GFEM) consisting of 1D and 2D structural elements. The skins of the wing are modelled using isoparametric membrane bending triangular and quadrilateral 2D plate elements.



(a) GFEM model of the entire aircraft.



(b) Close up view of the wing.

Figure 5.1: Structural model of the Medium Altitude Long Endurance OptiMALE aircraft.

More specifically, the elements used are *Lagrange* native elements CQUAD4 and CTRIA3, with mathematical formulation similar to that of the homonymous Nastran MSC elements. A composite material is assigned to these skin regions of the wing box. The spar webs and ribs of the wing are also modelled using the same plate elements however, the spar webs are modelled with a composite material while the ribs are metallic.

The remaining components in the wing box assembly are modelled using 1D elements. Particularly, rib caps and spar caps are modelled with CROD tension-compression elements and stringers are modelled using CBAR Timoshenko Beam elements of a “T” cross section. The difference in modelling stems from the fact that stringers must also be able to bear out-of-plane loading, which, in the case of spar and rib caps, is transmitted to and carried by the spar and rib webs accordingly. To that extent, the modelling of the reinforcements around a cutout area, which is designated for the main landing gear of the aircraft as shown in Fig. 5.2, is altered. More specifically, the reinforcements along the wingspan of the lower side of the wing are modelled using a CBAR “T” cross section element and the reinforcements along the chord of the upper side of the wing are modelled with CBAR “I” cross section elements. All the aforementioned 1D elements use an isotropic material which represents the elastic modulus of a composite material.

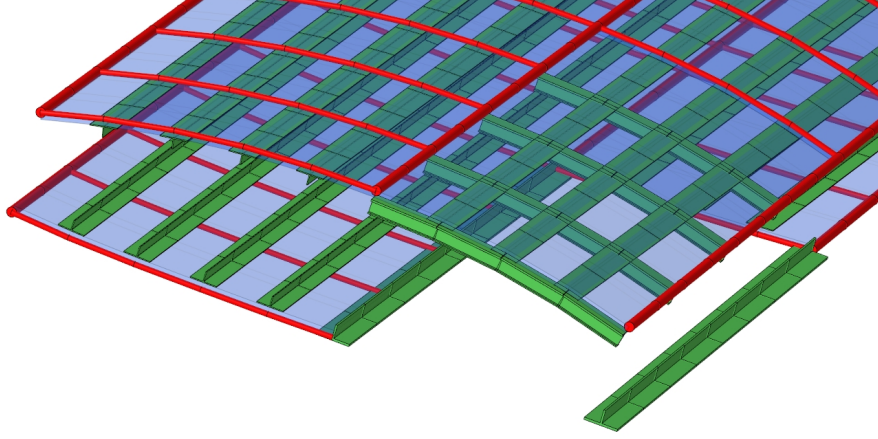


Figure 5.2: Modelling of the spar and rib caps around the main landing gear cutout area of OptiMALE. CROD and CBAR elements are depicted with a red and green colour, respectively.

5.1.2 Load cases

The model is subjected to 19 static load cases summarised in Table 5.1. These load cases cover critical instances of maximum and minimum vertical load factors originating from pull-up and push-over manoeuvres respectively, as well as maximum roll accelerations and gust cases at the given mass configurations. Additionally cruise load cases are considered at different envelope points. More specifically, n_z denotes the vertical load factor exerted on the aircraft and $\dot{p}$ the roll acceleration. The mass configurations considered are Maximum Take Off Weight (MTOW), Maximum Landing Weight (MLW) and Maximum Zero Wing Fuel Weight (MZFWF).

The aforementioned load cases were generated through an aeroelastic analysis of the flight states using a Doublet Lattice Method (DLM) coupled to the structural GFEM model. For this, the aerodynamic loads coming from the DLM model were combined with the inertia loads originating from accelerations on the structural FEM model. The resulting elastic deformations of the aircraft which alter the aerodynamic boundary conditions were then used to iteratively update the aerodynamic loads in DLM until convergence was reached. Each load case was trimmed to reach load equilibrium in the flight degrees of freedom using angles of attack and control surface deflections. The resulting loads on the model were frozen and applied as static loads during the first stage of the optimisation shown in this study.

5.1.3 Patch definition and design variables

Design variables need to be assigned to all structural components which are being sized. In practise, the level of detail during the design process of an aircraft increases

ID	n_z	Mach	Altitude (m)	$\dot{p}$ (deg/s ²)	Mass config.	Description
1	1	0.4	1000	0	MTOW	Cruise
2	1	0.4	8000	0	MTOW	Cruise
3	3	0.4	1000	0	MTOW	Pull-up
4	3	0.4	8000	0	MTOW	Pull-up
5	-1	0.4	1000	0	MTOW	Push-over
6	-1	0.4	8000	0	MTOW	Push-over
7	1	0.4	1000	150	MTOW	Roll
8	1	0.4	8000	150	MTOW	Roll
9	1	0.4	1000	0	MZWFW	Cruise
10	1	0.4	8000	0	MZWFW	Cruise
11	3	0.4	1000	0	MZWFW	Pull-up
12	3	0.4	8000	0	MZWFW	Pull-up
13	-1	0.4	1000	0	MZWFW	Push-over
14	-1	0.4	8000	0	MZWFW	Push-over
15	1	0.4	1000	150	MZWFW	Roll
16	1	0.4	8000	150	MZWFW	Roll
17	1	0.6	8000	0	MZWFW	Cruise
18	3.5	0.6	8000	0	MZWFW	Pull-up
19	1	0.12	1000	0	MLW	Cruise

Table 5.1: Flight load cases considered for the detailed sizing optimisation of OptiMALE.

as the project matures. In this study a sensible, adequately detailed discretisation of the design space has been chosen for all components being sized based on experience gained from past projects. The minimum size of any structural element is of course limited by the coarseness of the GFEM model. In practise, multiple elements are grouped together during the definition of the design variables. To maintain consistency and to simplify the setup of the optimisation problem, the boundaries of the design variables for each component are defined using the components adjacent to it.

More specifically, for the skins of the aircraft, which are the focus of the study, the patches that eventually define the zones according to which the variable stiffness laminates will be formed, are defined using the ribs, spars and stringers of the wing of the aircraft. The patch layouts for the 4 skin components of the wing are shown in Fig. 5.3. A smaller patch size has been chosen for the area near the wing root of the inner upper and lower wing skins as steeper thickness gradients are usually expected in these areas.

The thickness and stiffness properties of each patch are modelled using a generic stack. The number of generic layers used must be adequate for the expected

thickness of the laminate. As already mentioned in section 3.4, as a general rule of thumb, the stiffness properties of a laminate with N number of plies can be properly modelled using a generic stack with at least $N/4$ generic plies. In this example, the maximum expected thickness for laminates towards the root of the wing is equal to approximately 120 layers and therefore 32 generic layers have been used. The generic stack used is $[(45, -45, 90, 0)_4]_s$. This generic stack is applied to all thicker regions of the skins of the wing which are known due to previously performed preliminary optimisations. More specifically, it is used for patches 1 through 52 and 1 through 20 in sub-components I and III respectively. All other patches across each sub-component are modelled using a generic stack with 8 plies $[45, -45, 90, 0]_s$. The reason for this choice, is that if the number of generic plies used is significantly bigger than the actual physical plies corresponding to the thickness of a laminate, then the permitted design freedom is greater than what can be achieved during the discrete optimisation. This leads to unattainable stiffness distributions which in turn result to significant mechanical constraint violations of the discrete design.

1	2	3	4	5	6	7	11	14	17	26	37	45	47	49	51
8	9	10	12	13	15	16	18	19	20	39	43	46	48	50	52
21	22	23	24	25	27	31	34	36	41	53	54	55	56	57	58
28	29	30	32	33	35	38	40	42	44						

(a) Sub-component *I*: Upper wing inner skin.

1	3	7		
2	5	8	10	12
4	6	9	11	13

(b) Sub-component *II*: Upper wing outer skin.

1	2	3	4	5	6	7	11	14	17	23	30	38	42	45	48
8	9	10	12	13	15	16	18	19	20	29	36	40	43	46	49
				21	22	24	26	28	33	35	39	41	44	47	50
				25	27	31	32	34	37						

(c) Sub-component *III*: Lower wing inner skin.

1	3	7		
2	5	8	10	12
4	6	9	11	13

(d) Sub-component *IV*: Lower wing outer skin.

Figure 5.3: Patch arrangement for all the skins of OptiMALE's wing.

Besides the aforementioned skins of the wing, all other structural components of

the wing box are also sized during the optimisation. This includes the spar caps, stringers, rib webs, rib caps and the connecting structural elements between the detachable inner and outer part of the wing. These wing components are also discretised into design variables using their interfaces with neighbouring structural components. The exact definition of those design variables is not presented, since the focus of the study is on the design of the skin components of the wing. Due to local effects, the sizing of the spar webs and ribs has been performed outside of *Lagrange* and was kept constant during the optimisation.

5.1.4 Constraints

In this subsection, the formulations of the mechanical constraints applied to the first stage of the optimisation are presented. Additionally, the details concerning the application of the composite design guidelines in the continuous optimisation are summarised. The information regarding the application of the composite design guidelines in the second stage of the optimisation is presented on a case-to-case basis in Section 5.2.

5.1.4.1 Mechanical

Buckling and strength constraints are applied to the model. Concerning the stability constraints on the skins of the wing, each buckling field is assumed to be simply supported, subjected to in-plane loads and specially orthotropic. The mathematical formulation of these constraints in the optimisation problem is:

$$\frac{1}{2} \left(\frac{N_x}{N_{x,cr}} + \sqrt{\left(\frac{N_x}{N_{x,cr}} \right)^2 + 4 \left(\frac{N_{xy}}{N_{xy,cr}} \right)^2} \right) \leq 1. \quad (5.1)$$

The left hand-side in equation 5.1 indicates the combined Reverse Reserve Factor (RRF) of a buckling field for a combination of the critical normal loads and shear loads. In equation 5.1, N_x indicates the load applied on the longitudinal direction, N_{xy} the shear load applied on the plate, $N_{x,cr}$ and $N_{xy,cr}$ the critical normal and shear buckling loads respectively and N_y denotes the load applied on the transverse direction. The critical buckling load $N_{x,cr}$ across the longitudinal direction is given by equation 3.3. The critical shear load can be computed under the assumption of a long plate and is expressed as:

$$N_{xy,cr} = \begin{cases} \frac{4}{b^2} \sqrt[4]{D_{11} D_{22}^3} \left(8.12 + \frac{5.05}{\delta} \right) & \text{for } \delta \geq 1 \\ \frac{4}{b^2} \sqrt{D_{22} (D_{12} + 2D_{66})} (11.7 + 0.532\delta + 0.938\delta^2) & \text{for } \delta < 1 \end{cases} \quad (5.2)$$

where

$$\delta = \frac{\sqrt{D_{11} D_{22}}}{D_{12} + 2D_{66}}. \quad (5.3)$$

Alternatively, Finite Element buckling can also be applied to thicker laminates. In this study, the analytical buckling formulation is chosen to maintain simplicity.

To ensure that the stringer provides enough stability to the skin, a column buckling constraint is used. The well-established Johnson-Euler column buckling method is applied to an assembly of the stringer and its adjacent skins [148]. Local skin buckling under compression is taken into account by using the effective skin width via the Von-Kármán method [149]. The critical buckling load of this assembly is computed in an iterative manner until convergence of the effective skin width is reached. Concerning strength constraints, the maximum strain criterion described in section 3.2 is also applied here.

5.1.4.2 Composite

Manufacturing constraints are applied to both the continuous and discrete stage of the optimisation. In the first stage of the optimisation, symmetry and balance are enforced by linking the appropriate design variables in each patch. The stacking sequence of the generic stacks used in combination with gauges applied to the outermost generic plies, also facilitates the enforcement of the damage tolerance and external covering ply composite rules. A minimum of 10% and maximum of 60% is also applied to all laminates of the wing skin. Finally, between all neighbouring laminates, a maximum dropping of 30 plies is applied and continuity constraints are enforced. The blending constraints of Eq. 3.8 can only be applied between laminates that share the same generic stack. As mentioned in subsection 5.1.3, two different generic stacks are applied for sub-components I and III to appropriately model the stiffness of thinner and thicker patches. The blending constraints between neighbouring patches modelled with a different generic stack are applied to the sum of each fibre orientation instead of each generic ply. More specifically, the formulation of these constraints is:

$$\begin{aligned}
& \text{if } \sum_{i=1}^I t_{ij_1} \leq \sum_{i=1}^I t_{ij_2} \\
& \text{then } \sum_{i'=1}^I t_{i'j_1} \leq \sum_{i'=1}^I t_{i'j_2} \quad \forall i' \in \{I \cap \theta_{i'j} = \theta_c\}, \theta_c \\
& \text{else } \sum_{i=1}^I t_{ij_1} > \sum_{i=1}^I t_{ij_2} \\
& \text{then } \sum_{i'=1}^I t_{i'j_1} > \sum_{i'=1}^I t_{i'j_2} \quad \forall i' \in \{I \cap \theta_{i'j} = \theta_c\}, \theta_c
\end{aligned} \tag{5.4}$$

where θ_c are the fibre orientations used in the generic stack.

The contiguity constraint which limits the maximum number of equally oriented plies stacked consecutively is not applied in this example. The reason for this decision is that in order to provide meaningful results, it requires a laminate expected to be N layers thick, to be modelled with approximately $3N/4$ generic layers. When compared to the $N/4$ generic layers guideline mentioned in subsection 5.1.3, which is adequate to model the stiffness of the laminates, the application of the contiguity constraint would increase the number of design variables in the optimisation problem. However, most importantly, it would also lead to more neighbouring patches with a different number of generic plies, which would require the application of the blending constraints in the form of 5.4. This formulation of blending is less accurate than that of 3.8, since all plies of a specific orientation in a laminate are aggregated. Blending constraints have a greater influence on the stiffness output of the laminates than the contiguity constraint and therefore their exact application for as many neighbouring laminates as possible was preferred.

As for the composite rules applied to the discrete problem, these will be explained in more detail in the following section on a case-to-case basis.

5.1.5 Thickness discretisation under consideration of strength constraints

The optimal, continuous number of layers resulting from the first stage of the optimisation must be converted to a discrete number of layers which remains constant during the second optimisation stage. Since the nominal cured ply thickness of the pre-impregnated unidirectional tape that will be used during manufacturing is known, the total thickness of each patch can be rounded up to either the nearest integer number of layers or the nearest even number of layers.

During the examination of the large-scale demonstrator studied in this chapter, issues were observed with the satisfaction of strength constraints of the discretised design, especially when buckling and strength criteria simultaneously drove the design of a patch. One of the solutions to this problem is rounding-up all or some of the total individual fibre orientations e.g. $[0^\circ, 90^\circ, \pm 45^\circ]$ in the structure. More specifically, if the total continuous number of individual fibre orientations is denoted by $\nu_{j\theta}$ and the total discrete number of plies per fibre orientation and patch in the structure is denoted by $n_{j\theta}$, where index $\theta \in \{1, 2, \dots, \Theta\}$ denotes the different available fibre orientations and $j \in \{1, 2, \dots, J\}$ the patches in the structure, then

the individual thicknesses can be rounded up based on the following condition:

$$\begin{aligned}
 &\text{if } \nu_{j\theta} - \lfloor \nu_{j\theta} \rfloor \leq l \text{ then} \\
 &\quad n_{j\theta} \geq \lfloor \nu_{j\theta} \rfloor \\
 &\quad \text{else} \\
 &\quad n_{j\theta} \geq \nu_{j\theta}.
 \end{aligned} \tag{5.5}$$

In Eq. 5.5, $l \in [0, 1)$ is a value corresponding to the fractional part of $\nu_{j\theta}$ above which the individual number of layers for an orientation $n_{j\theta}$ are rounded up.

To maximise the benefits of this discretisation scheme, the discrete $n_{j\theta}$ values for the fibre orientations θ of interest must also be enforced as constraints in the upcoming discrete optimisation. However, if the number of layers for some fibre orientations is to remain fixed in the discrete optimisation, special considerations must be taken in order to ensure that the number of layers used complies with the design and manufacturing rules which will be enforced afterwards. More specifically, if strictly balanced laminates are to be derived then:

$$n_{j\theta_1} = n_{j\theta_2} \quad \forall j, \theta_1 = -\theta_2 \neq \{0, 90\}. \tag{5.6}$$

If slight imbalances of n_{imb} layers are allowed then:

$$\begin{aligned}
 &n_{j\theta_2} - n_{j\theta_1} \leq 0 \\
 &n_{j\theta_1} - n_{j\theta_2} \leq n_{imb} \quad \forall j, \theta_1 = -\theta_2 \neq \{0, 90\}.
 \end{aligned} \tag{5.7}$$

The minimum p_1 and maximum percentage p_2 design rules are formulated as:

$$\begin{aligned}
 n_{j\theta} &\geq p_1 \sum_{\theta=1}^{\Theta} n_{j\theta} \quad \forall j, \theta \\
 n_{j\theta} &\leq p_2 \sum_{\theta=1}^{\Theta} n_{j\theta} \quad \forall j, \theta.
 \end{aligned} \tag{5.8}$$

Finally, blending between any two neighbouring patches j_1 and j_2 is expressed as:

$$(n_{j_1\theta_1} - n_{j_2\theta_1})(n_{j_1\theta_2} - n_{j_2\theta_2}) \geq 0 \quad \forall j_1, j_2, \theta_1 \neq \theta_2. \tag{5.9}$$

The constraints in equations 5.5-5.9 are satisfied by multiple laminates. Therefore, the thickness discretisation task must be converted in an optimisation task of its own, albeit much simpler to solve than the continuous and discrete optimisation stages. Essentially, a third optimisation stage is added to the process. However, the optimisation process is still referred to as a two-stage process to avoid confusion in cases where the thickness discretisation task is simply performed by a rounding operation of the thickness of the laminate. The goal of this thickness discretisation

optimisation is to retrieve a solution which satisfies the aforementioned requirements while adding the minimum number of extra plies, hence the objective function is formulated as:

$$\min \sum_{j=1}^J \sum_{\theta=1}^{\Theta} n_{j\theta}. \quad (5.10)$$

The optimisation described above is an integer quadratically constrained programming problem which is solved using the commercial mathematical programming tool Gurobi [54].

5.2 Results

In this section, results of the two stage methodology applied to OptiMALE are presented. Initially, the continuous thickness and stiffness distribution of the wing skins are presented as well as the convergence of the two stages of the optimisation. The emphasis is then placed on the retrieval of discrete stacking sequences and on how the intermediate thickness discretisation stage and the choice of composite rules influences the satisfaction of the mechanical constraints.

5.2.1 Size and convergence

In total, the first stage of the optimisation problem is comprised of approximately 600000 constraints and 3000 design variables. *Lagrange* employs an active set strategy to filter out inactive constraints during each optimisation iteration. The continuous optimisation converges after 142 iterations with a Karush–Kuhn–Tucker condition of 10^{-5} . The total computational time is approximately 40 hours on a typical workstation employing 8 threads.

The continuous thickness and flexural stiffness distribution resulting from the first stage of the optimisation are shown in Fig. 5.4 for the four wing skin sub-components. As expected, the thickness of the wing decreases towards the tip. The flexural stiffness for each one of the patches is plotted in polar coordinates with a white colour. In these polar plots, the distance from the pole indicates the normalised modulus of elasticity, and the azimuth denotes the angle of rotation with respect to the reference axis of each laminate. More details on the visualisation of the membrane stiffness can be found in section 3.4. It can be seen that thin areas such as the outer parts of the upper and lower side of the wing which are predominantly sized by buckling have a higher flexural stiffness across the 45 degree axis. On the contrary, thicker areas towards the wing root have increased flexural stiffness across the 0 degree axis since strength constraints are also sizing these patches as will be discussed in

the following sections. The structural mass of the aircraft, following the continuous optimisation, is 11789 Kg.

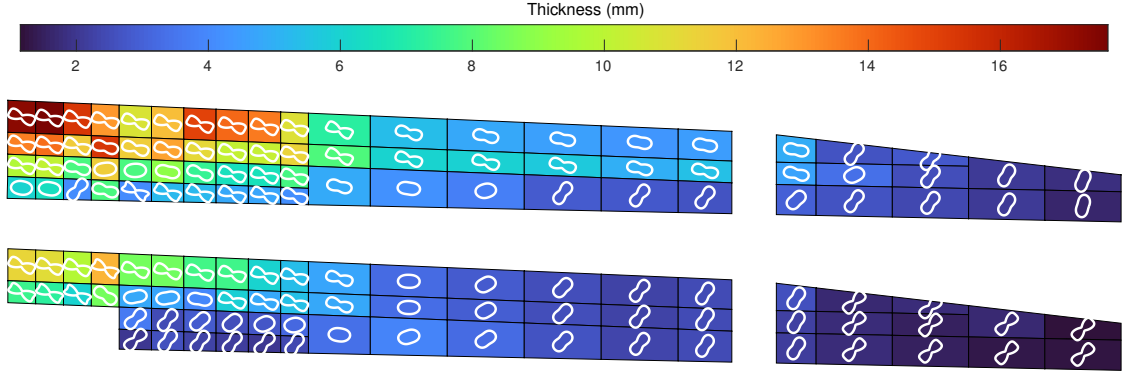


Figure 5.4: Thickness and flexural stiffness distribution across the wing skins of OptiMALE.

Fig. 5.5 illustrates the progress plot for the objective value of sub-component III, using the implicit MILP formulation and the process shown in Fig. 4.3 to retrieve a good feasible solution. Normally, one or two different path strategies are used to retrieve an initial discrete stacking sequence. In this case, 6 different paths are defined to demonstrate the impact of the chosen path to the objective function. The number indicated for the *DFS* and *BFS* strategies in Fig. 5.5 signifies the root patch used to initiate the path build. The average run-time of each individual run for the different paths is less than 5 minutes on a typical workstation employing 8 threads, including the overhead of setting up each of the many sub-problems for every path. One first observation is that paths generated using a thinner patch as the root i.e. *MIN*, *BFS-11* and *DFS-11* lead to a smaller objective function and therefore better stiffness match across all patches collectively. This can be attributed to the fact that in thinner patches, the placement of a layer influences the stiffness more compared to a thicker panel. Therefore, allowing the thinner patches to be optimised first can in many cases prevent the decomposition from getting stuck at a local minimum favouring some of the thickest patches. Secondly, the *DFS* strategy appears to also lead to a smaller objective function when compared to the *BFS* one. The reason behind this is that during the former, stiffness requirements of patches further across the structure are taken into consideration earlier on the decomposition, when there is still adequate design freedom to accommodate them.

After all paths have been explored, the solution of the one with the lowest objective function is used to initialise the complete optimisation problem. Once again, the “best known objective” in Fig. 5.5 corresponds to the smallest objective function value of a solution satisfying all constraints of the complete optimisation problem. On the other hand “objective lower bound” refers to the minimum discovered bound

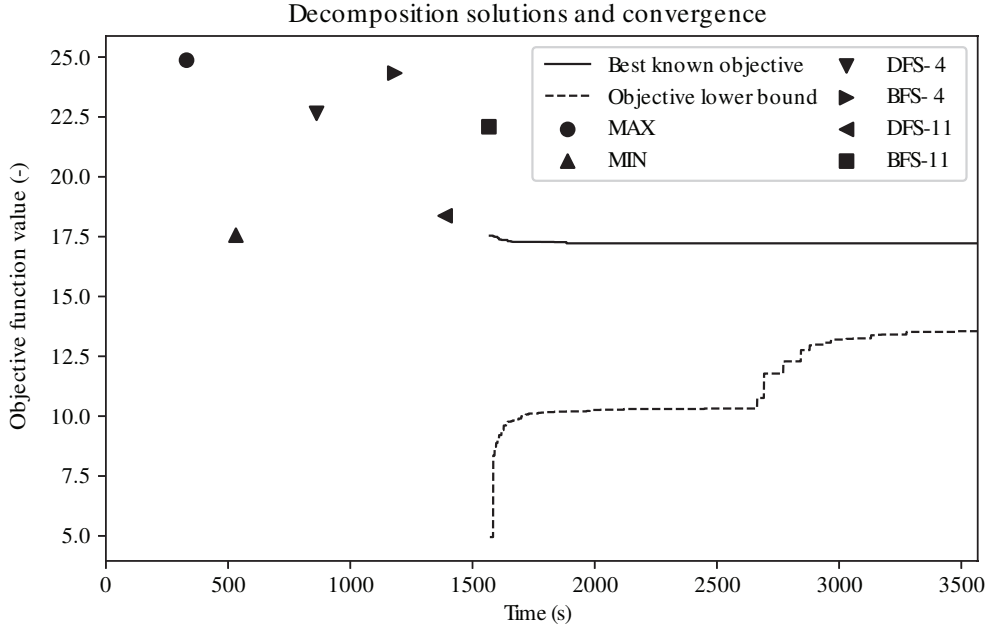


Figure 5.5: Comparison of different path generation strategies and convergence of the discrete stacking sequence optimisation problem for sub-component #3. The number indicated for the *DFS* and *BFS* strategies signifies the root patch used to initiate the path build.

for the value of the objective function. Optimality of the solution is proven when the value of the lower bound of the objective is equal to that of the best known objective. In this specific example, the runtime allotted to the optimisation of the complete MILP formulation is 2000 seconds. In the first half of it, a heuristic algorithm available in Gurobi Optimization [54] is employed to improve the best solution further, without solving the continuous relaxation of the MILP problem. In the latter half of the optimisation, the branch and bound algorithm of the commercial solver is used to prove optimality. The lower bound of the objective is substantially increased throughout this process, due to the continuous relaxation of the MILP problem being solved. Actually proving optimality for large-scale problems becomes time consuming and therefore the optimisation is stopped earlier. The quality of the solutions obtained through the decomposition and subsequent complete optimisation process are satisfactory, as shown by the results in the following section.

5.2.2 Retrieval of discrete stacking sequences

Before performing the second stage of the optimisation in order to retrieve discrete stacking sequences for all sub-components of the wing skin, the total continuous thickness of each patch is rounded up to the nearest even number of discrete plies assuming a tape thickness of $0.125mm$. Initially, the design and manufacturing rules

		Number of fields	
		Buckling	Strength
Skins	<i>I</i>	153	120
	<i>II</i>	36	26
	<i>III</i>	137	108
	<i>IV</i>	36	26
Stringers	<i>I</i>	371	319
	<i>II</i>	131	119
	<i>III</i>	339	295
	<i>IV</i>	131	119

Table 5.2: Number of fields to which strength and buckling constraints are applied, across the skins and stringers of the 4 sub-components of the wing of OptiMALE.

applied are the strict rules commonly employed in the industry, which have also been used in the first stage of the optimisation. More specifically, all composite design and manufacturing guidelines of Section 2.1.2 are applied, with the exception of the *disorientation* guideline which is not used because it contradicts the requirements of the *grouping* guideline which is used instead. The minimum and maximum percentages of layers allowed for each orientation in every patch are 10% and 60% respectively. No more than 4 plies of the same orientation are allowed to be grouped together and the number of consecutive plies that may be dropped simultaneously is capped to 4. The standard fibre orientations $[0^\circ, 90^\circ, 45^\circ, -45^\circ]$ commonly used in the industry are applied during the stacking sequence retrieval.

After obtaining the discrete stacking sequences satisfying these design guidelines, the continuous laminates of the wing skins are replaced by the discrete ones and an analysis of the model is performed. The minimum reserve factors and the number of fields in which violations appear for the buckling and strength constraints of the wing skins and the stringers are shown in Table 5.3. In the case of buckling constraints, each patch is usually comprised of 1-3 buckling fields depending on its size, while for strength constraints each element in the GFEM acts as a separate constraint field. The number of fields to which buckling and strength constraints are applied for the wings and stringers is shown in Table 5.2. The actual number of constraints in the optimisation and therefore the analysis model, is the number of fields to which constraints are applied multiplied with 19, the number of load cases applied to the model. Therefore, the actual constraint violations may be greater than the number of fields violated, shown in Table 5.3, if the constraint applied to one field is violated for multiple load cases.

As can be seen in Table 5.3 and Fig. 5.6, significant violations appear for the majority of the wing skin sub-components in both buckling and strength. The

		1-Strict	
		Buckling	Strength
Skins	<i>I</i>	-	0.952/42
	<i>II</i>	0.848/2	0.956/1
	<i>III</i>	0.888/3	0.964/8
	<i>IV</i>	0.586/14	-
Stringers	<i>I</i>	0.962/25	-
	<i>II</i>	0.933/2	-
	<i>III</i>	0.973/5	-
	<i>IV</i>	0.985/2	-
Weight (Kg)		11802	

Table 5.3: Minimum Reserve Factor and number of constraint violations for strength and buckling constraints across the skins and stringers of the 4 sub-components of the wing of OptiMALE. Strict design rules are used for the retrieval of the discrete stacking sequences.

minimum RF equal to 0.586 appears on patch number 5 of the lower, outer part of the wing skin. As mentioned in Section 5.1.3, sub-component IV is modelled using the following generic stack $[45, -45, 90, 0]_s$. The continuous number of layers for this generic stack and for patch number 5 is $[2.790, 2.790, 0.698, 0.699]_s$. The resulting discrete stack is $[45, -45, 45, -45, 0, 90, 90]_s$. First of all, the fact that the continuous thickness is 13.95 layers, which is rounded up to 14 layers for the stack retrieval, does not leave a generous safety margin for the inevitable stiffness change which occurs between the two stages. Most importantly, since symmetry, balance and minimum percentage are being strictly enforced, the stiffness match is sacrificed. The equivalent generic stack with a discrete thickness distribution of the discrete stack for patch 5 is $[2, 2, 2, 1]_s$. Sacrificing the number of 45 and -45 layers, which had it not happened would probably lead to a smaller constraint violation, is necessary in order to satisfy the minimum percentage guideline for 0 and 90 degrees. The adverse effects of the strict design rules on the stiffness matching between continuous and discrete are more evident in thinner patches.

Buckling constraint violations also occur for the stringers in all of these sub-components, even though the stringer design has not been discretised but rather remains fixed to the optimal solution found in the first, continuous stage of the optimisation. The reason behind the violations in the stringers is mainly internal load redistribution across the structure due to the discretisation of the wing skins. Essentially, some areas in the wing attract more load due to the increase in thickness and change in stiffness, which also leads to higher loads for the stringers which have not been sized for such loads.

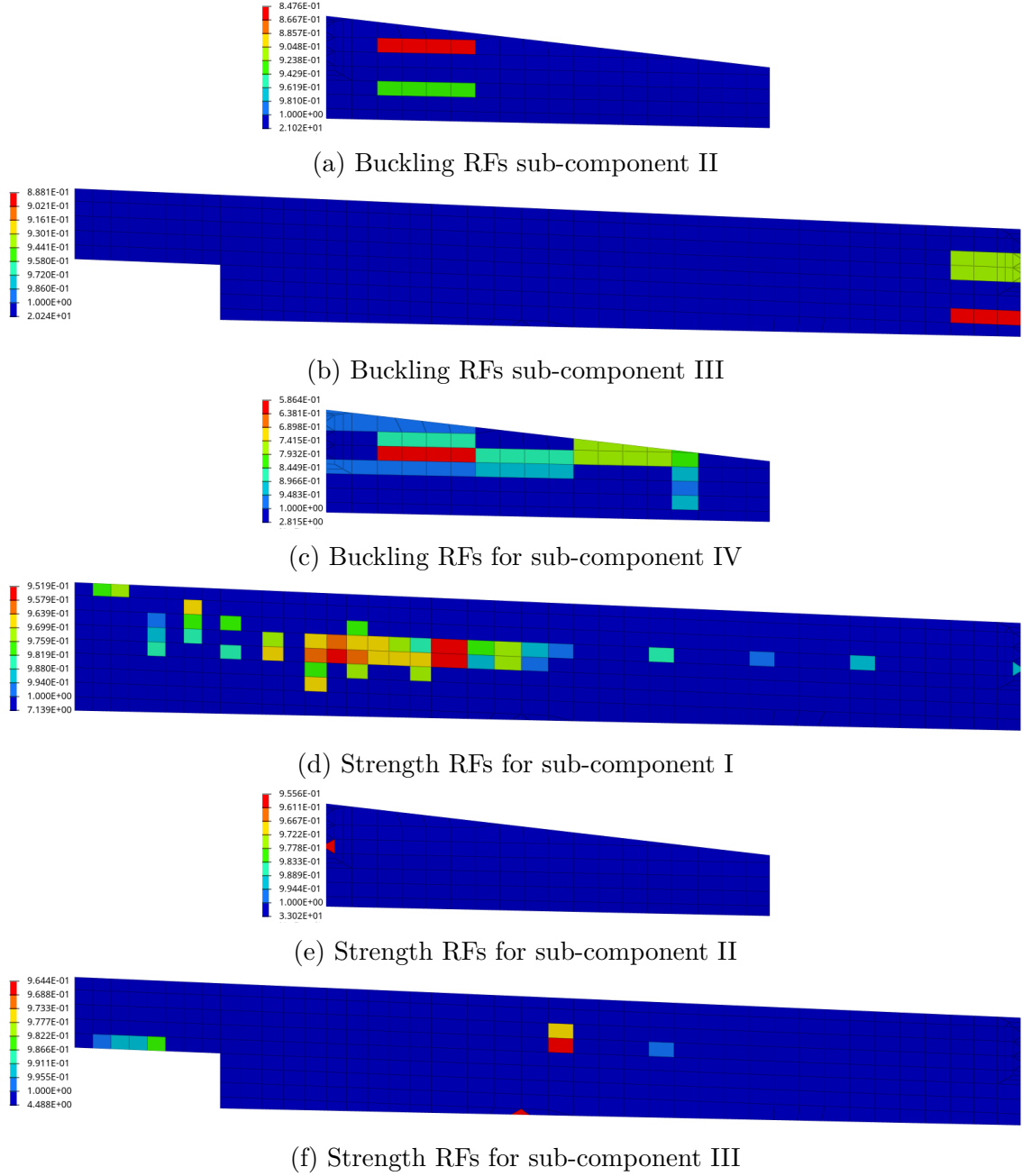
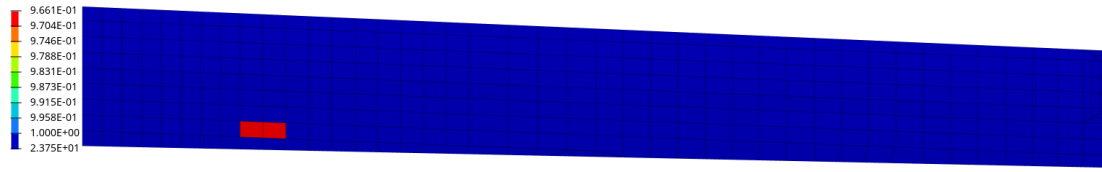


Figure 5.6: Minimum buckling and strength Reserve Factors for the discrete stacking sequences derived after strictly applying the design rules.

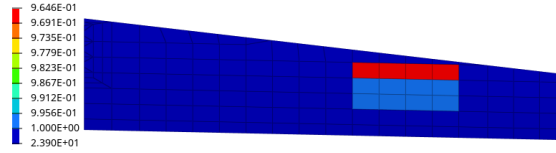
5.2.3 Considerations for buckling and strength constraints

As seen previously, the strict design rules lead to a sparsely populated design space which makes stiffness matching difficult. The strong influence of the design rules can be seen if the strict design rules applied in the previous section are relaxed. In particular, asymmetries are allowed up to 3 plies away from the midplane of each patch. Additionally, grouping of 45 and -45 layers is no longer strictly enforced and imbalances of up to one layer are permitted. Besides the relaxation of the design rules, the intermediate thickness discretisation step between the two stages of the optimisation is also modified, to allow rounding up the total patch thickness to the nearest integer number of plies, instead of rounding up to the nearest even number of plies. The analysis results for the new discrete stacking sequences are shown in Table 5.4 and Fig. 5.7, under the column named “2-Relaxed”. An average improvement of 6% is observed for the minimum values of the RFs and the number of constraint fields violated are reduced by 24%. The improvements are noticed for both buckling and strength constraints across the skins and stringers of the wing. It is worth noting that when examining Figs. 5.4 and 5.7 simultaneously, it becomes clear that areas in sub-component I bearing a larger flexural stiffness across the 0 degree axis, are also the areas that are sized by strength constraints. This can be seen from the fact that they are prone to constraint violations for small disturbances introduced by the discretisation of the design. On the other hand, areas in sub-component III in which constraint violations occur for buckling have a high flexural stiffness along the 45 degree axis.

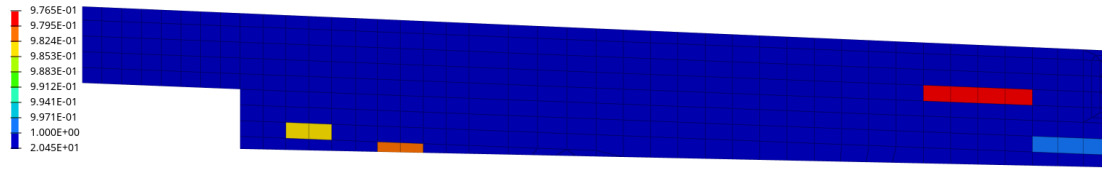
The remaining buckling violations on the wing skin mainly exist because of patches for which the difference between the total discrete and continuous thickness is very small. This difference is the thickness introduced during the intermediate rounding up step and acts as a safety net for the stiffness mismatches which inevitably occur whenever transitioning from a continuous to a discrete design. In order to highlight this effect, a Safety Factor (SF) is introduced during the thickness discretisation stage. Basically, the total continuous thickness of each patch is rounded up to the nearest integer number of plies for all patches, except the ones where this thickness difference is less than 0.2 layers. In those patches, the discrete thickness is further increased by an extra layer. The analysis results of this third alternative thickness discretisation, with the design rules remaining relaxed, are demonstrated in the column named “3-Relaxed with SF” of Table 5.4. The buckling RFs and the number of constraint violations are further improved in this case, since the discrete stiffness is increased for a sub-set of the patches. A negative side effect of the SF application in the thickness discretisations is that a lower minimum RF is observed for the



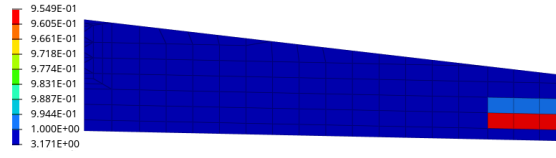
(a) Buckling RFs sub-component I



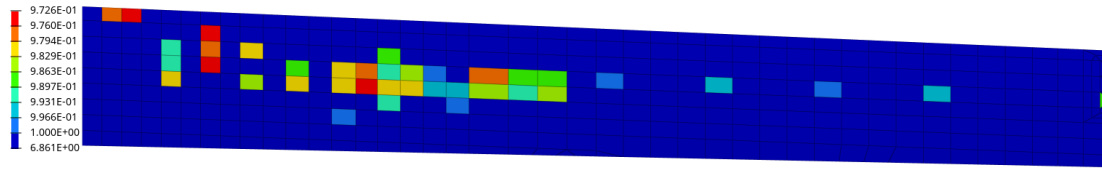
(b) Buckling RFs sub-component II



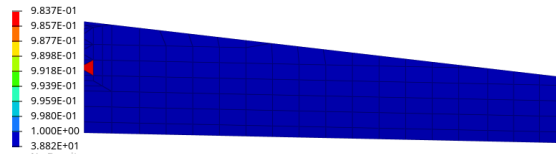
(c) Buckling RFs sub-component III



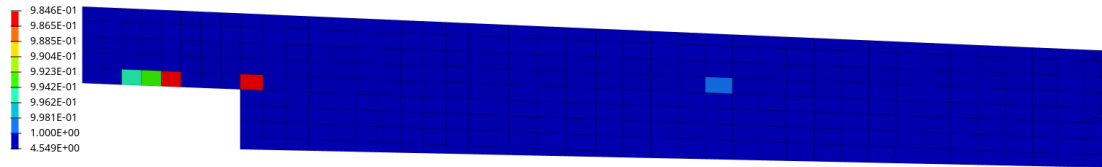
(d) Buckling RFs sub-component IV



(e) Strength RFs for sub-component I



(f) Strength RFs for sub-component II



(g) Strength RFs for sub-component III

Figure 5.7: Minimum buckling and strength Reserve Factors for the discrete stacking sequences derived after relaxing the design rules.

		2-Relaxed		3-Relaxed with SF		4-Strength	
		Buckling	Strength	Buckling	Strength	Buckling	Strength
Skins	<i>I</i>	0.966/1	0.973/38	-	0.961/26	-	0.967/10
	<i>II</i>	0.965/3	0.984/1	-	0.987/1	-	-
	<i>III</i>	0.977/4	0.985/5	0.991/2	0.961/4	0.948/5	0.977/4
	<i>IV</i>	0.955/2	-	-	-	-	-
Stringers	<i>I</i>	0.970/17	-	0.939/7	-	0.952/19	-
	<i>II</i>	0.991/1	-	0.989/1	-	0.959/3	-
	<i>III</i>	0.982/5	-	0.976/3	-	0.973/4	-
	<i>IV</i>	0.991/2	-	-	-	-	-
Weight (Kg)		11796		11802		11807	

Table 5.4: Minimum Reserve Factor and number of constraint violations for strength and buckling constraints across the skins and stringers of the 4 sub-components of the wing of OptiMALE. The discrete stacking sequences have been derived using relaxed design rules, relaxed design rules in combination with a thickness safety factor and the thickness discretisation considering strength constraints presented in Section 5.1.5.

stringers in sub-component one even though the number of fields in which the buckling constraints are violated is reduced. The introduction of this lower RF can be attributed to load redistribution in the structure, since the marginally thicker patches attract more load compared to what the stringers were designed for during the continuous optimisation.

Finally, in order to improve the strength constraint violations on the wing skins, the intermediate thickness discretisation method presented in Section 5.1.5 is applied. The value of parameter l in Eq. 5.5 is set to 0. After the integer quadratically constrained programming problem is solved, the number of layers for each individual fibre orientation are derived and are then enforced by equality constraints in the second stage of the optimisation. However, for thicker patches, these equality constraints can lead to infeasible optimisation problems due to the simultaneous application of the contiguity constraint. This constraint, unlike blending, balance, and minimum percentage constraints, cannot be considered in the alternative thickness discretisation stage due to lack of detailed stacking sequence information. In order to overcome the obstacle of infeasible designs, the constraints enforcing the number of layers are only strictly applied to 0 degree fibres and are allowed to be violated for ± 45 and 90 orientations. The constraint violations for the stacks derived using this method are shown in the column named “4-Strength” of Table 5.4. Although the minimum RF for the strength constraints on the wing skin is slightly increased and the number of fields violated is reduced, the number of buckling fields violated for the skins and stringers is increased. This is once again due to load redistribution

being amplified by the introduction of thicker patches, which can be observed by the increase in structural weight which is the biggest across all derived discrete stacks.

5.2.4 Feasible design

As seen in Tables 5.3 and 5.4, none of the four derived discrete stacks fulfils all mechanical constraints. In order to achieve a discrete manufacturable design which fulfils all mechanical constraints, the first stage of the optimisation must be performed again using a design factor. Unfortunately, a design factor leads to an unnecessary overdimensioning of the entire structure. In the case of OptiMALE, the discrete stacks #2-4 derived by employing a different thickness discretisation and design rule variations, all lead to smaller violations of the mechanical constraints when compared to the first discrete stack. All things considered, the best discrete stack is the second one using the relaxed design rules. The fact that it is the lighter solution out of all other candidates also leads to the best stiffness match and therefore to the least impact of load redistribution, as seen by the buckling constraint violations on the stringers.

In order to avoid introducing the design factor and because the constraint violations for the second discrete stack are rather insignificant, the following alternative procedure is followed. The continuous optimisation is performed again, only this time, the design variables corresponding to the skins of the wing are removed and the design is fixed to the derived discrete stacks. All mechanical constraints applied to the skins, as well as all other design variables and their respective constraints, remain intact. This modified optimisation converges in very few iterations to a design that satisfies all mechanical constraints in which the skin is discretised. Moreover, the structural weight of this final design is 11791 Kg which is lighter than that of the starting point of this optimisation at 11796 Kg. This means that the optimisation not only drives the design to a state at which no constraints are violated, but also manages to reduce the weight of other structural components because the discretised, thicker skins now attract part of the load previously carried by other components. Compared to the initial 11789 Kg weight of the continuous structure, only 2 Kg of mass were added for a design in which the wing skins have been discretised, while also satisfying all mechanical and composite constraints across the structure. This is virtually no change in structural weight considering the weight prediction precision of such a coarse GFEM structural model and in relation to the overall structural weight of the OptiMALE aircraft.

This study only examined the discretisation of the skins of the wing and no other composite parts of the structure. While it is true that discretising other components,

such as the stringers, might once again lead to load redistribution and hence, constraint violations, the aforementioned strategy to fulfil all mechanical constraints can be realistically applied even in these cases. The reason for this is that other sub-components exist, whose stiffness is not only a function of the discrete stacking sequence but is also influenced by continuous design variables. For example, in the case of a stringer with a “T” cross section, the stiffness also depends on the lengths of the flange and the web which can vary in the continuous domain.

5.3 Discussion

This chapter demonstrates the detailed sizing of a composite aircraft wing skin using the developed two-stage optimisation approach. Both stages of the presented approach can efficiently handle industrial-size optimisation problems. Concerning the capability of the methodology to retrieve manufacturable stacking sequences which fulfil all mechanical constraints, it is shown that the enforcement of strict design rules leads to large mechanical constraint violations especially for thin laminates. These violations are significantly reduced when the design rules imposed to the second stage of the optimisation are slightly relaxed.

Different thickness discretisation strategies are applied between the continuous and discrete optimisation stages, in an attempt to eliminate all mechanical constraint violations across all sub-components of the wing. As expected, a thickness safety factor introduced for patches in which the continuous thickness is marginally different than that of the discrete, reduces the number of buckling field violations in the wing skin. Alternatively, a thickness discretisation strategy considering strength constraint requirements slightly improves the satisfaction of strength constraints. However, both of these thickness discretisations lead to a higher structural mass, which also sets the ground for a worse stiffness match during the stacking sequence retrieval, leading to a load redistribution and constraint violations across other sub-components. In general, discretising the total thickness of each patch up to the nearest integer number of plies and applying slightly relaxed design rules, such as allowing asymmetries close to the midplane, provides the overall best results.

The minor constraint violations can be easily handled by a repetition of the continuous optimisation, using the discretised stacking sequences. The optimiser can quickly satisfy all remaining violations by a slight modification of the stiffness properties of other sub-components, whose stiffness characteristics can be easily manipulated in the continuous domain. Overall, the weight penalty introduced between the continuous and discrete result which satisfies all constraints is approximately 2 Kg.

Layup time objective function

This chapter investigates the benefits of introducing manufacturing cost requirements into the detailed design framework developed in this work. The manufacturing cost of a component depends on multiple factors including the type of manufacturing process employed, the material used, the geometric complexity of the parts and the assembly and labour costs. Zhao et al. [150] and Chen et al. [151] developed a method to analyse production cost which can take different manufacturing processes and materials into consideration. In the aerospace industry, ATL and AFP have been established as the methods used to manufacture large-scale CFRP components, such as wing and fuselage skins, despite their high original acquisition cost which is offset by the increased productivity rates that can be achieved compared to manual layup.

Therefore, optimising the ATL or AFP manufacturing process to get a maximum productivity is of high importance. The broad spectrum of research work performed on automated prepreg layup has been summarised by Lukaszewicz et al. [152] and

This chapter is based on the paper, *Layup time for an Automated Fibre Placement process in the framework of a detailed sizing optimisation* by G. Ntourmas, F. Glock, F. Daoud, G. Schuhmacher, D. Chronopoulos, E. Özcan and J. Ninić which is under review at the Composites Part B: Engineering journal. Note: symbols may have been changed to maintain consistency throughout this thesis.

Brasington et al. [153]. In this chapter, the emphasis is placed on the minimisation of the time spent during the machine layup. It should be mentioned that the time spent laying material is only a portion of the entire manufacturing time during the AFP process. A study from Electroimpact [154] found that only 27% of the total manufacturing time was spent during layup, which matches the 24% reported by Boeing [155]. On the contrary, the time required for inspection and repairs was measured to be higher at 32% and 63% respectively. Minimising the delays linked with inspection and repair has been more extensively researched. More specifically, the repair time can be minimised by optimising the path plan of the machine [156]. On the other hand, the inspection of the manufactured component is mainly performed manually in the aerospace industry. However, the inclusion of more non-destructive testing in the near future should significantly decrease the inspection time, making the layup time even more relevant for the overall reduction of the total manufacturing time.

The increase in productivity by a reduction of the layup time has been identified as an area of future research [152], which is however constrained by secondary operations, machine downtime and the design of the part. Design for manufacturing has also been listed as a research opportunity in a recent review of AFP [153]. The main challenge with design for manufacturing is the data exchange and communication between different disciplines involved in the process such as design, process planning, manufacturing and inspection [157].

Only few research works have combined structural and manufacturing considerations in an optimisation framework. Phillips and Guo [158] and Phillips [159] introduced the manufacturing time and cost in a Multidisciplinary Optimisation framework using a Process Based Method which is however better suited for preliminary design stages and not a more detailed sizing stage. In the work of Hagnell and Åkermo [160], the manufacturing time is estimated using geometry complexity metrics which fail to consider manufacturing time reductions which could occur with exploration of alternative thickness and stiffness distributions in the structure. The work of Ückert et al. [161] combines structural and layup time requirements in a late design stage with finalised stacking sequences. Time savings are achieved by avoiding incomplete ply courses and adapting ply contours. However, all design modifications are performed manually and not in an automated optimisation framework. Finally, Irisarri et al. [162] explore the manufacturing complexity of a composite component in a detailed sizing optimisation process however, the procedure is tailored to a specific manufacturing method called the Quilted Stratum Process.

In this chapter, the layup time is introduced as an objective function in the first of the two stages of the detailed sizing optimisation process. The rest of the optimisation problem remains the same, in a way that the structural weight minimisation objective function and the mechanical and composite constraints are still considered. The novelty of the work lies in the calculation and optimisation of the layup time required to manufacture the component, which is generalised to account for multiple interconnected patches of any shape. This is performed by treating the laminated structure as a series of shared courses of material, whose shape depends on the geometry of the structure and the fibre orientations that are to be laid during manufacturing. Each of these courses is further split up into sub-courses depending on the patch discretisation of the structure and the thickness distribution across the patches which varies between each optimisation iteration. The bi-objective optimisation has been applied to sub-component *I* of OptiMALE which is the upper inner skin of the wing as introduced in Chapter 5. The bi-objective optimisation study leads to a trade-off between a slight increase in structural weight for a reduction of the time required to lay down the material. Furthermore, it has been demonstrated that smaller patch sizes, which are usually avoided during the design of the aircraft because they are expected to lead to increased manufacturing complexity, should actually be preferred during the design. The reason for this is that the bi-objective optimisation can take advantage of the increased design freedom to reduce the weight while keeping the layup time similar to that of a design with larger patch sizes.

This chapter is structured as follows: In section 6.1, the formulation of the layup time as an objective function in the optimisation is formulated. Results of a bi-objective mass and layup time optimisation are presented in section 6.2 and the findings of this work are summarised in section 6.3.

6.1 Methodology

In this section, the methodology derived to model the AFP process using the amount of information available inside the gradient-based optimisation is demonstrated. The varying stiffness of the structure across its span is modelled by using multiple constant stiffness zones or patches. Each of these patches is modelled using a generic stack and therefore a continuous stacking sequence is known, which however is not compliant with discrete design and manufacturing criteria. Previous works [163, 8] have expressed the layup time required to manufacture one single rectangular ply of CFRP material using an ATL process. The layup time depends on the dimensions of the ply, the width and number of pre-impregnated uni-directional CFRP material strips or tows that the machine head can lay in parallel, the orientation at which

the material is laid and several parameters related to the speed, acceleration and delays of the machine. However, when considering large-scale aerospace components, examining the layup time of each single ply in a patch is not reasonable, since all of the patches share plies which will be manufactured in one go and not individually. Therefore, it is sensible to study the layup time for the component of interest as a collection of material courses as demonstrated in Fig. 6.1. The AFP machine manufactures each course by combining multiple tows to be laid simultaneously.

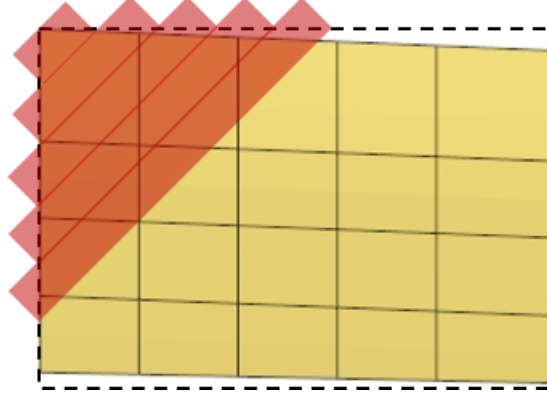


Figure 6.1: Illustration of the geometry of 45 degree courses computed for part of a component.

A different set of material courses spanning the component has to be computed for each different fibre orientation that is expected to be manufactured. The layup time required to manufacture one single course [8] is expressed as:

$$T_{course} = l_{course}/v_0 + v_0/2(1/a_{acc} + 1/a_{dec}) + T_{delays/course} \quad (6.1)$$

where l_{course} represents the length of the course, v_0 the maximum layup speed achieved by the machine, a_{acc} and a_{dec} the acceleration and deceleration of the machine head respectively and $T_{delays/course}$ additional delays associated with each course.

At this point, the assumptions made in order to compute the layup time of AFP manufacturing process, in the framework of the detailed sizing optimisation, have to be listed. An exact computation of the layup time can be preformed using dedicated software provided by the manufacturers of AFP machines or by CAD software able to perform path planning processes. Path planning is a widely researched topic [156] which in itself can be an area with many challenges regarding the structural defects arising, especially when working with curved fibre paths.

A first assumption made during the sizing optimisation performed in this work is that straight fibre paths are employed to manufacture the component. Although optimisation of fibre steered CFRP is an actively studied topic [60], the standard in

the aerospace industry, both in terms of design and also manufacturing, has been constant stiffness laminates. Besides treating only straight fibre paths of specific orientations, these orientations are also considered to be invariant, regardless of geometric complexities, such as doubly curved surfaces, which inevitably alter the true fibre orientation. Additionally, the methodology that will be further developed in this work is limited to relatively flat or small curvature paths. For more complex geometries or cylindrical components the v_0 maximum layup speed is not achievable and would have to be decreased across the length of the course.

Furthermore, the starting point for the first course of a given fibre orientation is chosen at a random corner position of the component. Even though the starting point may influence the quality of the manufactured part, especially when dealing with fibre steering [164], the effect should have less impact for constant stiffness laminates particularly due to the staggering of plies. Staggering refers to the shifting of the courses in the ply with respect to the courses in an identical ply laid previously, in order to spread out and minimise the structural impact of course overlaps and ply drops [165, 166]. The number of courses considered for each fibre orientation depends on the geometry of the component, the fibre orientation, the width of the course which will be used during manufacturing w_{course} and the overlap between courses placed adjacent $d_{overlap}$.

Finally, the order of the generic plies is not accounted for during the computation of the layup times. Instead, the continuous thicknesses of all generic layers with the same orientation within a patch are summed up to a total thickness per orientation and patch. This also means that any delays associated with re-positioning the machine from one location of the part to another one are not accounted for, since the exact stacking sequence during the first stage of the optimisation is unknown.

The mid-line of each course traverses through a set of patches. The order of the patches in this set and their corresponding lengths can be pre-calculated before the optimisation and remain constant throughout. For any course, a set of patches $\mathbf{J}$ is calculated with $\mathbf{J} = \{j_1, \dots, j_n\}$. The course will only be manufactured if all patches $j \in \{j_1, \dots, j_n\}$ share a common thickness. As mentioned previously, the thicknesses of all generic layers within a patch are summed up for each fibre orientation available in the set of manufactured fibre orientations Φ . Therefore, the cumulative thickness of each fibre orientation for each patch can be expressed as:

$$s_{j\theta} = \sum_{i=1}^I t_{ij} \quad \forall i, j \quad \theta_{ij} = \varphi \in \Phi \quad (6.2)$$

where θ_{ij} is the fibre orientation of a specific generic ply.

In the case that all patches belonging to a course do not share the same thickness, then multiple combinations of sub-courses arise. These combinations depend on the exact thickness distribution amongst the patches, which changes throughout the course of the optimisation. A visual example of this logic applied to a specific thickness distribution across 4 patches is given in Fig. 6.2. The way such a material distribution would be manufactured is that the minimum shared thickness of 0.8 mm would be laid across all patches, followed by 1.2 and 0.7 mm of thickness spanning across the first two and the last patch respectively. Finally, the remaining material of 0.6 mm thickness would be placed.

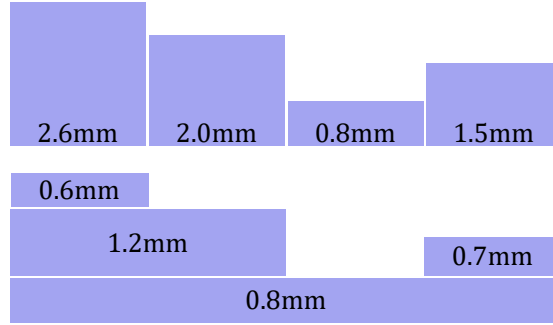


Figure 6.2: Schematic representation of the layup strategy used to manufacture the thickness profile of 4 patches. The indicated thicknesses refer to the same fibre orientation.

For a course spanning the ordered set of patches $\mathbf{J}$, where $n(\mathbf{J})$ is the number of patches in this set, the total number of different sub-course combinations that might arise is $N = \sum_{l=1}^{n(\mathbf{J})} l$, where index l indicates a group of sub-courses which span through $n(\mathbf{J}) + 1 - l$ patches, marked with red rectangular boxes in Fig. 6.3. Each group of sub-courses contains $k = l$ sub-courses. For example, the first group contains one sub-course which includes all patches that the mid-line of the course traverses through. The last group of sub-courses contains $k = n(\mathbf{J})$ potential sub-courses with each of them being comprised only one individual patch. As expected, only a few of all the different sub-course combinations can exist for any given thickness distribution. This depends on whether the minimum thickness that these patches share is 0 or not. For a given ordered set of patches, the minimum thickness for any fibre orientation θ is calculated as:

$$m_{lk\theta} = \min(s_{jl\theta}^*) \quad \forall l \in [1, n(\mathbf{J})], k \in [1, l], j \in \mathbf{J} \quad (6.3)$$

In the equation above, $s_{jl\theta}^*$ are the modified thicknesses for the different sub-courses and are computed as:

$$s_{jl\theta}^* = \begin{cases} s_{j\theta} & \forall j \in \mathbf{J}, l = k = 1 \\ s_{j\theta} - \sum_{\lambda=1}^{l-1} \sum_{k=1}^{\lambda} c_{j\lambda k} m_{\lambda k \theta} & \forall l \in [2, n(\mathbf{J})], k \in [1, l], j \in \mathbf{J}. \end{cases} \quad (6.4)$$

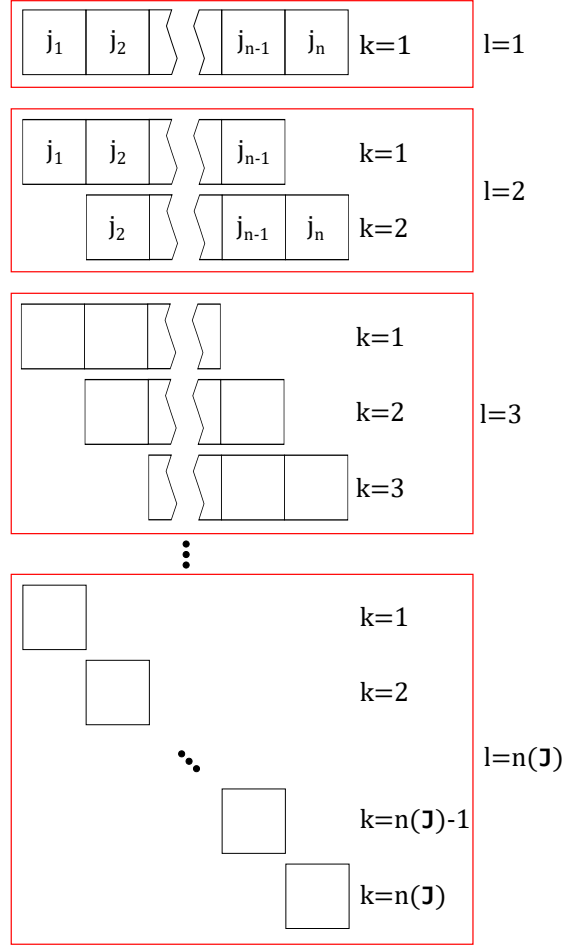


Figure 6.3: Overview of all potential sub-courses that might rise for any random thickness distribution, for a material course spanning across n patches. Each black square represents a patch.

For the first group of sub-courses ($l = k = 1$), the modified thickness $s_{jl\theta}^*$ is equal to the cumulative thickness for each fibre orientation ($s_{j\theta}$) as computed during the optimisation. For all subsequent groups, the modified thickness has to be adjusted to account for material that has already been accounted for in previous groups and sub-courses. In Eq. 6.4 $c_{j\lambda k}$ is a coefficient calculated as:

$$c_{j\lambda k} = \begin{cases} 1 & \forall \quad j \in [k, k + n(\mathbf{J}) - \lambda] \\ 0 & \forall \quad j \notin [k, k + n(\mathbf{J}) - \lambda] \end{cases} \quad (6.5)$$

The layup time for all the sub-courses within a course can then be calculated by slightly modifying Eq. 6.1 to:

$$T_{course} = \sum_{l=1}^{n(\mathbf{J})} \sum_{k=1}^l m_{lk\theta} / t_{tow} \left(l_{lk} / v_0 + v_0 / 2 (1/a_{acc} + 1/a_{dec}) + T_{delays/sub-course} \right). \quad (6.6)$$

In the above equation, t_{tow} is the thickness of the tow that will be used during manufacturing and therefore the ratio $m_{lk\theta} / t_{tow}$ denotes a continuous number of plies.

The length of each sub-course l_{lk} can easily be calculated since the corresponding length of each patch l_j in the course is known:

$$l_{lk} = \sum_{j=k}^{k+n(\mathbf{J})-l} l_j. \quad (6.7)$$

For patches along the outer edges of the component, an 100% boundary coverage is assumed for the calculation of the corresponding lengths l_j .

The delays $T_{delays/sub-course}$ considered for each sub-course manufactured, are limited to the time required to cut the ply and move the head of the machine downwards and upwards. The total layup time T for the entire component is calculated by summing all the lay-up times for each course and fibre direction

$$T = \sum_{\theta} \sum_{courses} T_{course}. \quad (6.8)$$

Therefore, the optimisation problem defined in Eq. 3.1 now becomes

$$\begin{aligned} &\text{minimise} && w_1 M(t) + w_2 T(t) \\ &\text{subject to} && g_k(t) \geq 0 \quad k \in \{1, \dots, K\} \end{aligned} \quad (6.9)$$

where w_1 and w_2 are weight factors applied to the two objective functions of structural mass $M(t)$ and layup time $T(t)$. Both objectives are a function of the thicknesses t of the generic layers. The layup time objective function is implemented in the gradient-based MDO platform of *Lagrange*. The derivatives for all constraints and objective functions are all computed analytically. The formulation of the total layup time for a component as shown in Eq. 6.8 involves multiple evaluations of the minimum of a given set of thicknesses as seen in Eq. 6.3. Assuming three thicknesses t_1 , t_2 and t_3 , the minimum of them can be mathematically formulated both exactly as:

$$\begin{aligned} \min(t_1, t_2, t_3) = & \frac{t_1}{2} \left(\frac{|t_3 - t_1| + |t_1 - t_2|}{|t_1 - t_2| + |t_2 - t_3| + |t_3 - t_1|} \right) + \\ & \frac{t_2}{2} \left(\frac{|t_1 - t_2| + |t_2 - t_3|}{|t_1 - t_2| + |t_2 - t_3| + |t_3 - t_1|} \right) + \\ & \frac{t_3}{2} \left(\frac{|t_2 - t_3| + |t_3 - t_1|}{|t_1 - t_2| + |t_2 - t_3| + |t_3 - t_1|} \right) - \\ & \frac{|t_1 - t_2| + |t_2 - t_3| + |t_3 - t_1|}{4} \end{aligned} \quad (6.10)$$

and inexactly as:

$$\min(t_1, t_2, t_3) = \lim_{k \rightarrow \infty} \sqrt[k]{t_1^k + t_2^k + t_3^k}. \quad (6.11)$$

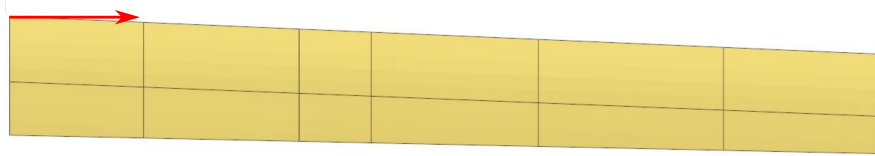
The issue with the exact calculation is that the absolute values introduce a lot of discontinuities in the derivative. Additionally, the computation becomes increasingly complicated as the number of thicknesses for which the minimum has to be computed for increases. On the other hand, the accuracy of the inexact formulation becomes problematic for thicknesses with a very small relative difference and exponents $k < -100$ resulted to numerical issues in the computation of the derivative value. Therefore, all minimum values are calculated using the respective function of the programming language and the computation of the derivative is performed numerically using finite differences. This does not hinder the computational performance of the optimisation process since the layup time is only a function of the thicknesses and is not linked with the response of the GFEM model as is the case for the structural constraints in the optimisation.

It should be noted that the methodology presented above is not limited to a modelling of the structure using generic stacks. Lamination parameters can also be employed in specific use cases. If for example the fibre orientations used are $\{0, 90, 45, -45\}$, then the number of layers for each of these orientations can be easily calculated during the optimisation using a system of linear equations, when knowing the total thickness of the laminate and the lamination parameters corresponding to the flexural stiffness matrix $\mathbf{A}$. Finally, it should be mentioned that the presented mathematical formulation of the layup time as an objective function can also be used as a constraint in the optimisation problem with very minor modifications.

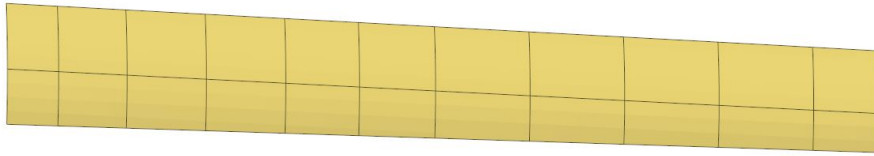
6.2 Results

The developed methodology is applied to the wing skin of OptiMALE which was discussed in more detail in Chapter 5. The bi-objective optimisation is only applied to one component in the structure, more specifically, the upper, inner part of the wing skin i.e. sub-component I. Only this part of the structure is sized during the optimisation while the other sub-structures in the wing i.e. remaining wing skins, spars, stringers and ribs, are sized based on the previously performed, single objective weight minimisation. This is done in order to eliminate all influences to the structural mass of the wing skin which would be stemming from load redistribution across the other sub-components on the wing and only focus on the effect the layup time has on the sizing and therefore mass of the skin. Besides strength and buckling constraints, manufacturing constraints are also applied to the skin of the wing. More specifically, blending, maximum ply drop and minimum percentage constraints are applied between neighbouring laminates. This leads to a total of more than 200000 constraints.

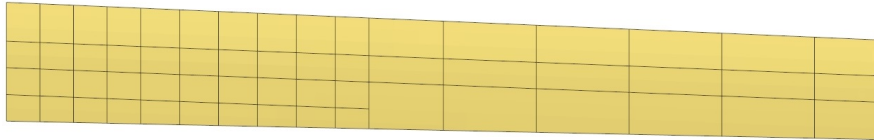
Regarding the discretisation of the structure into patches, three schemes of different coarseness are examined in the bi-objective optimisation. The 12, 22 and 58 patches are shown in Figs. 6.4a, 6.4b and 6.4c respectively, for the inner part of the suction side of the wing, which has rough dimensions of 10 and 1.3 meters in the span and chord direction respectively. Concerning the number of generic layers used to model the stiffness properties of the structure, a maximum of 32 generic layers resulting in 12 design variables, due to symmetry and balance requirements, have been used for the thickest regions of the wing covers. For the thinnest, outer parts of the wing, 8 generic layers leading to 3 design variables for each patch have been chosen to model the properties of the structure. The exact areas employing 32 and 8 generic layers have been discussed in more detail in Chapter 5.



(a) Patch discretisation #1



(b) Patch discretisation #2



(c) Patch discretisation #3

Figure 6.4: Three patch discretisations of different coarseness have been applied to model the inner part of the suction side of the skin of the wing. The orientation of the zero degree fibres is indicated by the red arrow in patch discretisation #1.

The bi-objective sizing optimisation is performed for all three patch discretisations shown in Fig. 6.4. By altering the ratio between the weight factors used for each objective function in Eq. 6.9, a trade-off between the structural mass and the layup time is observed. This trade-off is visualised for the three patch discretisations in the Pareto front of Fig. 6.5. A Pareto front includes all non-dominated optimisation solutions, meaning that none of the objectives can be further reduced without increasing the other one. The AFP machine parameters that have been applied to the case study are summarised in Table 6.1. The weight factors used for each objective function when deriving the design points of patch discretisation #1 are

summarised in Table 6.2. As the weight factor of the layup time increases, the algorithm converges to design points with a lower layup time. It should be noted that the optimisation algorithm handles the entire mass of the aircraft in tonnes and the layup time in seconds. The reason behind choosing the specific order of magnitude for the weight factor of the structural mass is because it also influences the aggressiveness of the convergence hence reducing the chances of the optimiser getting trapped in a local minimum.

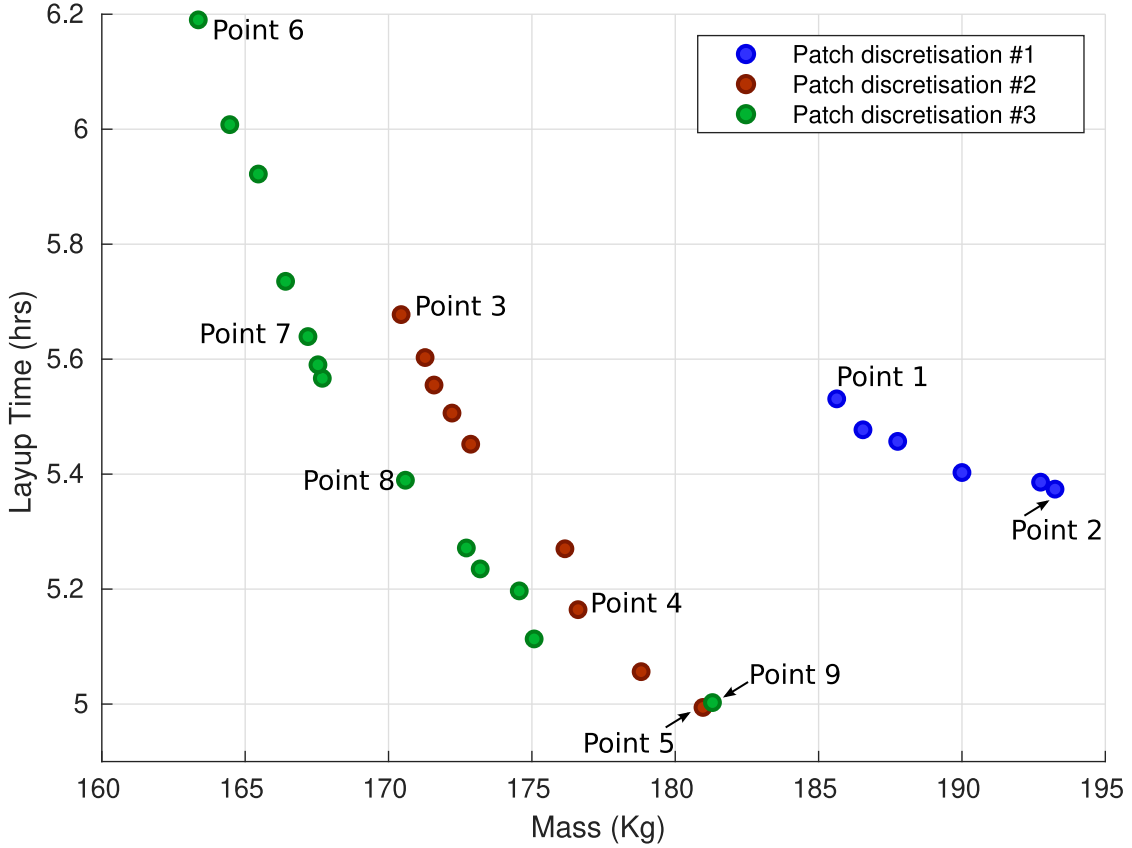


Figure 6.5: Pareto front distribution of the solution of the bi-objective optimisation applied to the three different patch discretisations.

Parameter	Value
v_0	1.4 m/s
a_{acc}	0.76 m/s^2
a_{dec}	0.76 m/s^2
$T_{delays/sub-course}$	3 s
t_{tow}	0.184 mm
w_{course}	150 mm
$d_{overlap}$	5 mm

Table 6.1: Performance data for a typical AFP machine. Data partly extracted from Haffner [8] and approximated from Airbus proprietary documents.

As expected, the minimum structural mass corresponds to the highest layup time, which also implies the highest manufacturing complexity. On the contrary, as the

w_1	100	100	100	100	100	100
w_1	10^{-6}	10^{-4}	2×10^{-4}	8×10^{-4}	10^{-3}	5×10^{-3}

Table 6.2: Weight factors used in the two objective functions to derive the design points of patch discretisation #1. The design points corresponding to the weight factors are in order of descending layup time.

layup time decreases, the structural mass of the wing increases. This trade-off can be explained when looking in more detail at the thickness distribution across the patches. In Fig. 6.6a the total thickness of each patch is shown for design *Points 1* and *2* for patch discretisation #1. The thickness is plotted in blue and red for each design point respectively. First of all, it can be seen that the reduced structural mass of design *Point 1* does not correspond to a decreased thickness in all patches. Instead, the patches towards the leading edge are thicker in design *Point 1* when compared to *Point 2* while the opposite is true for the patches closer to the trailing edge. Therefore, the design points resulting from the bi-objective optimisation in this case differ quite significantly in terms of thickness distribution.

The resulting thickness and stiffness properties of any design depend on multiple contradicting requirements coming both from the constraints but also the objective function of the optimisation. In Fig. 6.6b, the effect that the layup time objective function has on the thickness distribution of 45 degree plies can be seen. In this case, 0 degree fibres would be manufactured along the span of the wing. Therefore, when manufactured, 45 degree courses of material can traverse across patches neighbouring in the span direction, patches neighbouring in the chord direction, a combination of the two and finally patches placed diagonally and therefore not directly neighbouring. In design *Point 2*, which is plotted in red, it can be seen that the need to reduce the layup time has resulted in a smoothing of the thickness in patch groups “a”, “b”, “c” and “d” as seen in Fig. 6.6b. The thickness difference between patch “c” and the patch group “d” has also been eliminated. Therefore, one of the ways the optimiser is reducing the layup time is by minimising the number of ply cuts which are linked with the delays mentioned in Eq. 6.1. Of course, the minimisation of the layup time inside the optimisation framework depends on multiple other contradicting factors. For example, if minimising the ply drops between neighbouring patches for certain fibre orientations is advantageous for the reduction of the layup time, then this alters the optimal thickness and stiffness distribution of a minimal structural mass design. Any increase in mass is also unfavourable for the layup time of the laminated structure since more material must be deposited by the AFP machine. Finally, it is worth mentioning that different thickness profiles across the wing can also influence the complexity of support structures such as the stringers. This impact is beyond

the scope of the study but should be taken into consideration when examining the manufacturing time of an entire wing.

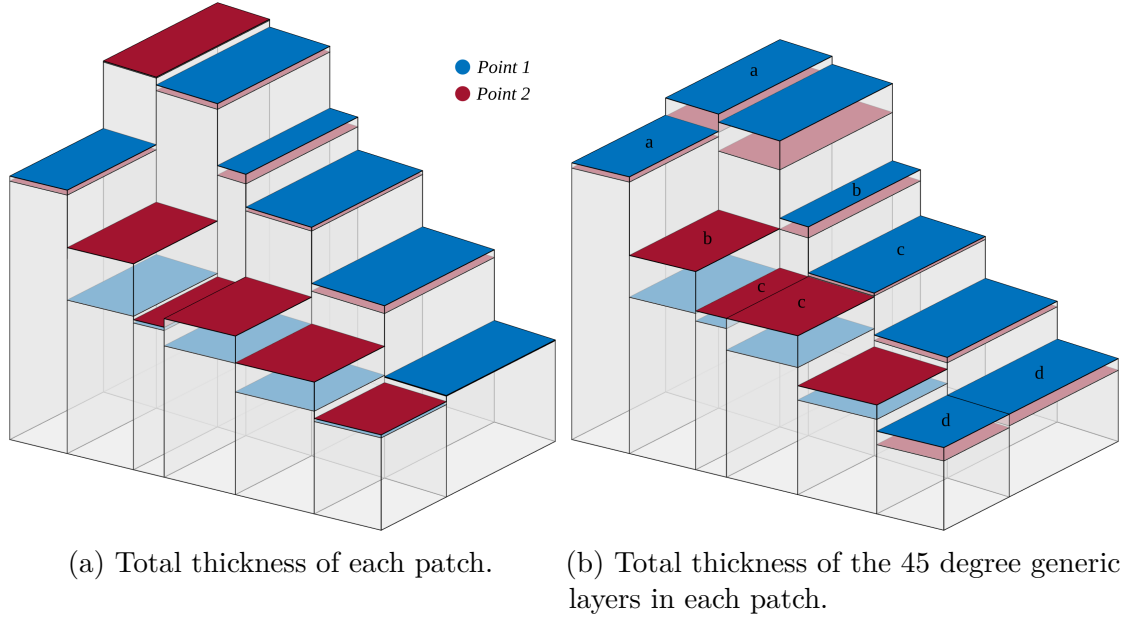


Figure 6.6: Thickness distribution across patch discretisation #1 for design *Points 1* and *2* of Fig. 6.5. Blue and red indicate the thickness distribution of design *Points 1* and *2* respectively. The orientation of the patches is the same as the one shown in Fig. 6.4, meaning that the top left patch is on the leading edge of the wing root. The thicknesses between Figs 6.6a and 6.6b are not in scale.

Further observations can be made when comparing the Pareto fronts for the different patch discretisations. First of all, the finer patch discretisation obviously leads to a design with a lower mass, as seen by the difference between *Points 1*, *3* and *6*. The design of *Point 6* is also linked with an even higher layup time. As the structural design freedom provided to the optimiser is increased by including more patches, the thickness and stiffness can be tailored to locally meet strength and stability requirements, without overdimensioning larger areas of the skin. This leads to more thickness differences between neighbouring patches which in turn implies the usage of more sub-courses of material. As mentioned earlier, since each sub-course is linked with its own time delays related to cutting the course and moving the head of the machine up and down, the overall layup time increases.

What is also worth mentioning is that the Pareto fronts for patch discretisations #2 and #3 appear to roughly coincide towards the design area of a minimum layup time as seen by *Point 5* and *Point 9*. This is reasonable due to the fact that these designs are driven by a smoothing out of thickness differences across patches to allow for the manufacturing of longer courses of material. The design freedom to perform such changes is achievable by both discretisations #2 and #3. In the case

of patch discretisation #1, the design corresponding to the minimum layup time corresponds to a higher structural mass compared to the equivalent designs of the other two discretisations. This occurs because the coarseness of discretisation #1 drastically limits the design freedom causing overdimensioning of the structure. This highlights the point that the layup time is not only driven by the number of ply cuts but also on the volume of material being laid.

On the matter of choosing a suitable design point from the Pareto front, one question that rises is whether a design with a higher mass and lower layup time will be preferred by the relevant aircraft design team. For example switching from *Point 3* to *Point 4* in the Pareto front of Fig. 6.5 increases the mass of the wing skin by approximately 6Kg (3.6%), while reducing the layup time by around 30 minutes (-9%). This decision may be justified for aircraft projects with a high production rate such as single aisle aircraft families.

Even if a design team is not willing to increase the structural mass in order to reduce the layup time, the bi-objective optimisation can still be a powerful tool. The discretisation of the structure into patches is performed manually, with structural sub-components such as ribs, spars and stringers acting as the boundaries when defining these constant stiffness zones. A coarser patch discretisation may be chosen over a finer one, because the latter is associated with a higher manufacturing complexity. This is true, since, if the only goal of the optimisation is to reduce the weight of the structure, then the layup time will increase as seen by the difference in design *Points 3* and *6*.

However, when introducing the layup time as an objective, the designer can still benefit from the increased design freedom by choosing a different point from the Pareto front. For example, if design *Point 7* is chosen then the mass is reduced by more than 3Kg while the layup time stays roughly the same when compared to that of design *Point 3* of the coarser patch discretisation #2. Alternatively, a reduction of approximately 30 minutes in layup time can be achieved for a similar structural mass if design *Point 8* is used instead of *Point 3*. Therefore, the bi-objective optimisation can assist with the reduction of weight while keeping the layup time to acceptable levels, by making use of an increased design space which would not be achievable by a single objective optimisation.

On a final note, the influence that the second stage of the optimisation has on the computed layup time is demonstrated for design *Points 3, 4* and *5*. The total continuous thickness of each patch resulting from the first stage of the optimisation is rounded up to the nearest integer number of plies and this thickness distribution

Design point	First optimisation stage		Second optimisation stage	
	Mass(Kg)	Layup time(hrs)	Mass(Kg)	Layup time(hrs)
3	170.4	5.68	171.8 (+0.8%)	5.75 (+1.2%)
4	176.6	5.16	177.9 (+0.7%)	5.22 (+1.2%)
5	181.0	4.99	182.5 (+0.8%)	5.07 (+1.6%)

Table 6.3: Mass and layup time comparison between the first and second stage of the optimisation for three selected design points. The layup time of the designs resulting from the second stage of the optimisation has been computed using the first stage of the optimisation and therefore follows the same assumptions.

remains fixed during the second stage of the optimisation. Since the objective function of the second stage is to minimise the absolute difference between the continuous and discrete stiffness characteristics, the ply shares, which correspond to the flexural stiffness of the patch are also well maintained. As seen in Table 6.3, the layup time is only slightly increased for each design point. This difference is mainly due to the total thickness being rounded up, as seen by the increase in the total weight, which inevitably increases the layup time. The assumption that the thickness of each fibre orientation is treated as a single layer, which is performed during the calculation of the layup time, neglects any impact that the actual stacking sequence has on the layup time of the component.

6.3 Discussion

In this chapter, the CFRP layup time of an AFP manufacturing process is included as an objective function in the detailed design optimisation process. The calculation of the layup time objective is introduced in the first stage of the optimisation. Even though the information available on the stacking sequence of the component is less exact at this point, there is enough design freedom to achieve meaningful trade-offs between the two objectives. The developed methodology is applied to the upper wing skin of OptiMALE. Different weight factors are applied to each objective function in order to acquire a Pareto front of different design points. The design with the lowest structural mass is also linked with the highest layup time and vice versa. More importantly, it is shown that the bi-objective optimisation can be used to bypass coarser patch discretisations, usually applied during the design process to avoid designs with a high manufacturing complexity. A greater design freedom is linked with more manufacturing complexities and a larger layup time. This is true for an optimisation in which the minimisation of the structural mass is the sole requirement. However, the second objective function can be employed to keep the layup time to an acceptable limit while benefiting from reduced structural weight due to the

increased design freedom. Alternatively, the layup time can also be implemented as a constraint in the optimisation framework to achieve the same result. Finally, instead of reducing the structural mass, it can be kept to satisfactory margins while the layup time is reduced by means of the bi-objective optimisation.

Conclusions

In this final chapter, the findings of the work are discussed and proposals for future work are presented.

7.1 Summary and conclusions

This work deals with the detailed sizing optimisation of large-scale composite structures. The optimisation process developed in this work consists of two stages. In the first stage of the optimisation, a gradient-based algorithm is used in order to derive a continuous thickness and stiffness distribution of the laminated structure. The gradient-based algorithm enables a fast convergence of the optimisation which is essential due to the high computational cost linked with each iteration. This stems from the need to evaluate all mechanical constraints such as stability and strength which depend on the reactions of the GFEM of the structure. The parametrisation of the thickness and stiffness of the structure is performed using generic stacks, formed by a set of layers whose exact fibre orientation and stacking sequence remains fixed during the optimisation while the individual thickness of each generic ply forming the stack can vary continuously.

Even though this parametrisation introduces a non-convex design space, it is shown that as the number of constraints increases, the local minima retrieved by the gradient-based algorithm do not meaningfully differ for different design variable

initialisation schemes and stacking sequence orders of the generic stacks. That is true as long as the generic stack used to model the stiffness of a patch has enough generic layers. The number depends on the approximate physical thickness of the patch being modelled. The number of generic layers can also depend on the composite constraints applied in the first stage of the optimisation, with the contiguity constraint requiring a larger generic stack. This has the drawback of increasing the number of design variables but most importantly introduces multiple neighbouring patches being modelled with a different generic stack. This in turn necessitates the application of a non-exact blending constraint between these neighbouring patches. Since the usage of exact blending constraints in the first stage of the optimisation is far more important than that of the contiguity constraints, minimising the number of different generic stacks in the component being optimised is deemed to be a more sensible design choice.

In the second stage of the optimisation, the number of layers across all patches is kept constant. The thickness value used for each patch is derived by rounding up its total thickness to the nearest integer number of layers or to the nearest even number of layers. This calculation can be performed since the thickness of the uni-directional tape that will be used during manufacturing is already decided at later design stages. The objective of the second stage of the optimisation is to retrieve a discrete stacking sequence whose stiffness characteristics optimally match the continuous stiffness attributes calculated in the gradient-based optimisation. The solution must also satisfy all of the composite design and manufacturing rules. The satisfaction of mechanical constraints which are not considered in the second stage is subject to the quality of the stiffness match performed. This does not only depend on the ability of the optimisation algorithm employed in the second stage to retrieve good quality solutions and the design freedom granted during the second stage, but also on how realistic the continuous stiffness distribution is, compared to the feasible discrete design space.

A deterministic branch and bound-based commercial algorithm is used in the second stage of the optimisation. The optimisation problem has been formulated in two different ways as a MILP instance. The two formulations, namely implicit and explicit, mainly differ in how blending between patches is modelled. Both formulations lead to the same discrete stacking sequences assuming that the same constraints are applied to the optimisation problem. The implicit formulation has eventually been used more extensively because it is better suited to accommodate a decomposition technique that helps speed up the solution of the optimisation for problems of larger scale. When compared against an open-source genetic algorithm,

the current formulations are able to achieve a better stiffness match in a given time frame. This can be partly attributed to the fact that the current formulations offer more design freedom than most available blending representations and partly to the branch and bound-based algorithm, offering a more robust convergence towards the global optimum compared to heuristics. The current MILP formulation outperforms a custom-made Branch and Bound algorithm focusing on the stacking sequence optimisation of single patches for all examined cases.

The overall two stage process is also compared against relevant academic studies on the basis of a commonly used benchmark study. The process developed in this work leads to a lower structural weight compared to previous works having used the same hypotheses for the benchmark. What is more, the proposed two-stage methodology leads to composite structures which satisfy both discrete composite requirements but also buckling constraints after a single pass of the two stages. This can be partially attributed to the fact that blending constraints can be exactly formulated on the generic stack design space and to the fact that the algorithm employed in the second stage of the optimisation can converge to the globally optimum solution.

The optimisation process has also been applied to OptiMALE, a large-scale unmanned aircraft demonstrator study. The size of the optimisation problem has been manageable for both the first and second stage using conventional computational means. In the second stage of the optimisation, the usage of the decomposition strategy becomes necessary for problems of larger scale. Unlike the academic study, mechanical constraint violations were observed when evaluating the discretised stacking sequences on the wing skins of OptiMALE. The root of these violations is load redistribution occurring across the structure due to the change of stiffness induced by the discretisation process. Higher constraint violations are observed when strictly enforcing the design and manufacturing rules during the stacking sequence retrieval. This is especially the case for thinner laminates for which the feasible discrete design space is significantly more condensed. The constraint violations are reduced when relaxing the design rules such as allowing for slight imbalances and asymmetries. Two different thickness discretisation schemes were also tested in an attempt to eliminate all constraint violations, each one focusing on stability and strength constraints respectively. Even though these schemes appear to slightly reduce the respective constraint violations, they lead to thicker patches and therefore a heavier structural design. Thicker patches introduce the drawback of driving the achievable discrete design space further away from the continuous solution of the first stage. Therefore, overall, rounding the thickness of each patch up towards the nearest integer number of plies seems to be the best choice in terms

of performing the best stiffness match and attaining the lowest constraint violations. In order to satisfy all mechanical constraints, the entire two-stage process may be performed again using increased design factors in the first stage of the optimisation. This would unavoidably penalise the structural weight since all patches would be over-dimensioned. An alternative to this strategy was applied in this work because the remaining constraint violations were rather insignificant, in the order of 0.96. The alternative way to satisfy all constraints involves fixing the design variables of the discretised wing skin and performing the first stage of the optimisation again using the remaining continuous design variables. The optimiser can quickly satisfy all remaining violations by a slight modification of the stiffness properties of other sub-components, whose stiffness characteristics can be easily manipulated in the continuous domain. For example, the stiffness of the stringers of the wing is not only a function of the discrete stacking sequence but is also influenced by continuous design variables. Overall, the weight penalty introduced between the continuous and discrete result which satisfies all constraints is approximately 2 Kg. Overall, the developed approach is very effective for highly constrained industrial optimisation problems. However, it is also clear that it does not generalise well to problems with no restrictions in terms of achieving an optimal stiffness e.g. fibre steered and non-conventional laminates.

An additional objective function besides the structural weight was also implemented in the first stage of the optimisation. This objective function captures the layup time required to manufacture a composite part using an AFP manufacturing process. By altering the weight factors applied to each objective function a Pareto front of different design solutions can be achieved, with the design exhibiting the lowest structural mass also linked with the highest layup time and vice versa. The most important outcome of the study, is that the bi-objective optimisation can be used to bypass limiting modelling practises, such as using coarser patch discretisations during the design process to avoid designs with a high manufacturing complexity. While it is true that in the case of a single objective mass minimisation study, a finer patch discretisation would lead to a more time consuming design, this can be bypassed when employing both objective functions. More specifically, designers can benefit from the increased design freedom offered by the finer patch discretisation, but attain a design requiring the same layup time and having a lower structural mass.

7.2 Future work

The work performed has resulted in an optimisation framework that can reliably derive manufacturable stacking sequences for large-scale aerospace components. A potential area of future work concerns the automation of the selection and generation process of the generic stacks across the patches in the structure. As discussed earlier, the number of generic layers depends on the approximate thickness of the patch and on the composite constraints applied during the first stage of the optimisation. This has not been an issue up to now because the generic stacks used are rather uniform across the structure. This stems from the fact that at the current state, avoiding interfaces between patches where a different generic stack is used is advisable, because otherwise, a non-exact formulation of the blending constraint in the continuous design space must be applied. However, other constraints such as the contiguity design rule, necessitate the usage of more customised generic stacks and therefore a framework to automate the selection and generation of these generic stacks in the GFEM and optimisation model. Before working towards this direction, devising a more accurate blending constraint between patches with different generic stacks would be necessary.

Another area in which improvements could be made are the two MILP formulations of the stacking sequence optimisation problem. Any alternative formulation reducing the number of design variables or constraints has the potential to further speed up the convergence of the optimisation. Alternatively, other solvers able to handle Mixed Integer Non Linear Problems can be employed, as this would significantly reduce the size of the optimisation, since all linearisations constraints and the respective design variables would not be required. As for the minor mechanical constraint violations observed after the second stage of the optimisation for the OptiMALE model, these are likely to occur for most industrial sized problems. Unfortunately load redistribution is an effect that cannot be captured in the second stage of the optimisation. The only thing that can be done is to perform the best possible stiffness match and to bridge the information gap between the continuous and discrete design by any possible means.

Finally, future work on the area of the layup time can focus on the influence of the assumptions used in the formulation of the layup time objective function. This can be achieved by retrieving discrete stacking sequences for distinct design points of the Pareto front and then creating a detailed AFP machine plan for the manufacturing of these designs. The layup times estimated during the continuous optimisation process can then be compared against the more accurate computations

for the discrete designs for the different design points. Alternatively, the layup time can also be implemented as a constraint in the optimisation framework to avoid experimentation with weight factors in the objective functions and rather specify a maximum allowable limit for the layup time of a component.

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